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Preface

This book comprises the forty-second volume of the Annual Transactions of the American Institute of Electrical Engineers. It consists almost entirely of papers and discussions presented at the four national meetings of the Institute during 1923 and its makeup and size follow the style first adopted with the 1922 volume. All of the material contained in this volume has appeared in the monthly Journals of the A. I. E. E., except that certain of the longest papers were printed in abridged form in the Journal, whereas they are given complete in this volume. The amount of material presented before the Institute during this year has been so large that it was found impossible to reproduce all of it in this volume, owing to the necessary limitations of size and expense. It has therefore been a matter of grave consideration for the Publication Committee to select from the year's production of papers those which were considered of greatest value for reference purposes and to eliminate such papers as might be considered of more transitory interest. The Journals of the A. I. E. E. for the year corresponding to these Transactions also contain numerous contributions of merit which have been omitted from the Transactions merely for lack of space. In order to make these contributions and the eliminated Institute papers available to the membership, they have been included in the index of the Transactions, referring to the issues of the Journal in which they were published. The arrangement of the articles in this volume is generally chronological and each group or groups of papers is followed directly by the discussion thereon. The contents of this volume are indexed both by title and author's name and are cross-indexed wherever possible. In addition to the index of authors, the names of all those taking part in the discussions are also included. The volume concludes with the annual report of the Board of Directors for the fiscal year ending April 30, 1923, together with the officers, committeemen, and Section and Branch officers for the corresponding year.

Preface

This book comprises the [illegible] Volume of the annual Transactions of the American Institute of Electrical Engineers. [illegible] of papers and discussions presented at the [illegible] of the Institute during [illegible] and its [illegible] it is not essential that [illegible] monthly [illegible] A. I. E. E. [illegible] the longest papers were [illegible] when they are given complete in this volume. [illegible] presented before the Institute during [illegible] impossible to reproduce all of the [illegible] to the [illegible] The Journal of the A. I. E. E. [illegible] and Transactions, referring to the pages of [illegible] The arrangement of the [illegible] authors [illegible] index [illegible] the authors [illegible] included [illegible]

The Apparent Dielectric Strength of Cables

BY ROBERT J. WISEMAN

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After reading with considerable interest, the two papers presented at the annual convention of the Institute, dealing with the dielectric strength of cables[1], the writer analyzed the data presented in each paper from an entirely different viewpoint and also the data given in each paper according to the views of the other paper.

The results obtained were quite interesting and are given below, so that the members of the Institute may be in a position to criticize the new viewpoint. Perhaps from all the views which have been given, we may be able finally to arrive at a complete understanding of what dielectric strength as applied to cables actually is.

We are all familiar with the formula which expresses mathematically the dielectric stress at any point in the insulation surrounding a round conductor when a voltage is applied between the conductor and sheath, namely:

$$K = \frac{0.434\,V}{x \log_{10} R/r}$$

where K is the dielectric stress at a distance x from the center of the conductor; V is the voltage between conductor and sheath; R the radius of the dielectric; and r the conductor radius.

Likewise we are all familiar with the fact that the maximum stress occurs at the surface of the conductor and the minimum stress at the surface of the dielectric. Again, as the diameter of the conductor is varied and the diameter of dielectric held constant, the computed value of K reaches a minimum value when the conductor diameter is $D/2.72$. This has been very clearly described in the Middleton, Dawes and Davis paper.

For a long while we have considered the maximum stress theory as described by Russell[2] as the proper one to use, but when we compute the maximum value of K from actual experimental data we observe quite a large variation in K maximum, when it should be constant. Osborne[3] presented a needle-point theory to explain the departure of practise from theory.

In 1914, Messrs. Middleton and Dawes[4] presented a paper before the Institute, in which they substituted an empirical formula for the logarithmic formula when expressing the breakdown strength of cables when the ratio R/r is greater than 2.72. Their present paper is a further analysis of the problem from the same viewpoint.

There seems to be a contradiction in their paper. According to the views expressed when developing their empirical law, they assume a fixed value of maximum stress and analyze their data with this in view. Later in the paper, they discuss the variation of the maximum stress with the ratio R/r. The empirical law should be corrected to bring in the change in maximum stress with the ratio R/r.

Simons takes the data which Fernie[5] published in 1921 and tends to disapprove his theory, showing that taking Fernie's data, the average stress theory is just as good.

Fundamentally, there seems to be no foundation for the average stress theory, as it implies that at the moment of breakdown the stress is a function only of the thickness of dielectric. From the nature of the shape of the electrostatic field in a single-conductor cable, experimental data, as well as the theoretical formula, we know this is not so.

The minimum stress theory is an interesting endeavor to find a formula by means of which the breakdown strength of cables may be computed when designing cables. It takes into account the shape of the electrostatic field—which is good. However, taking into consideration our ideas as to what the dielectric strength[6] of a dielectric is and knowing that we have a higher stress in the body of the dielectric at the moment

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[1] "Potential Gradient in Cables," by W. I. Middleton, C. L. Dawes and E. W. Davis.

"On the Minimum Stress Theory of Cable Breakdowns," by D. M. Simons.

[2] "Dielectric Strength of Insulating Materials and the Grading of Cables," by A. Russell, *Journal* I. E. E., Vol. 40, page 6, 1907.

[3] "Potential Stresses in Dielectrics," by H. S. Osborne, Trans. A. I. E. E., Vol. XXIX, page 1553, 1910.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

[4] "Voltage Testing of Cables," by W. I. Middleton & C. L. Dawes, Trans. A. I. E. E., Vol. 33, page 1185, 1914.

[5] "Insulating Materials," by Fernie, *Beama*, page 244, 1921.

[6] Osborne quotes very nicely from Maxwell on this point:

"If the electromotive intensity at any point in a dielectric is gradually increased, a limit is at length reached at which there is a sudden electrical discharge through the dielectric. The electromotive intensity when this takes place is a measure of what we may call the electric strength of the dielectric."

Osborne then writes:

"This critical value of the electric intensity in a dielectric is itself called, in common parlance, the electric strength of the dielectric.

Granting the existence of this definite physical constant, the electric strength of a dielectric that is, a definite value of electric intensity which cannot be continuously or repeatedly exceeded without disrupting the dielectric, the voltage required to break down a given design of insulation depends evidently upon two factors:

1. The electric strength of the dielectric.
2. The distribution of the electric intensity, or as one may now call it, the electric stress, in the dielectric."

of breakdown than the stress at the surface of the dielectric, it seems hard to support the minimum stress theory.

In all the theories which have been advanced so far, it has been assumed that the dielectric strength of the dielectric in a cable is a constant and independent of the shape of the electrodes after taking into account the shape of the electrostatic field for computing the dielectric strength from the theoretical law. Why is it not fair to assume that the dielectric strength as computed is not what we commonly called the "true dielectric strength" of the dielectric but really the "apparent dielectric strength?" That is, we mean the true dielectric strength must be corrected for the shape and size of the electrodes and that this correction factor will give us a means of obtaining a constant which we can call the true dielectric strength of the dielectric.

There are grounds for making the above assumption. We know that if a dielectric is tested for breakdown strength between parallel plates, it is necessary to state the size of the electrodes and the thickness of the dielectric or the results are not of great value. Mr. F. M. Farmer[7] in a very valuable paper shows this to be true. The writer believes from tests he has conducted that the dielectric strength of a dielectric when tested between parallel plates is a constant, provided a correction factor is used. Here again it is a question of true or apparent dielectric strength. This correction factor is very nearly the correction factor for the increase in capacity between two parallel plates due to end effect.

Knowing that the dielectric strength for parallel plates is a function of the area and thickness of dielectric, suppose we assume that the dielectric strength of a cable is a function of the *radii* or surface of the conductor and sheath.

The writer, in 1914 and 1915, conducted a research[8] on the present problem. The same methods were used as described in the Middleton, Dawes and Davis paper, that is, samples of cables were studied with constant conductor diameter and variable sheath diameter and variable conductor diameter with a constant sheath diameter. Instead of using the commercial kind of dielectric, ceresine wax was used, as it is more homogeneous than rubber, paper or varnished cambric, could be obtained free of impurities and absorbs practically no moisture.

It was possible to explain the results obtained by means of ionization. Although a great many investigators do not believe in free ions in solids, if we assume they do exist and then apply the laws of ionization, it is remarkable how well computed results agree with experimental data. If we take the law of corona for air about a wire and apply it to a solid insulation (with the proper constants), we can express the dielectric stress at the surface of a conductor in a cable in a form which will give good agreement between computed and experimental values. This corona law is:

$$K = K_0 \left(1 + \frac{a}{\sqrt{r}}\right)$$

where r is the radius of the conductor, a is a constant, K_0 is the true dielectric strength of the dielectric, and K is the apparent dielectric strength of the dielectric. K is the value which should be used when computing the breakdown voltage of a cable having a conductor radius r.

At the time of the research, the writer had only his own data to study and, consequently, was only able to apply the ionization theory to his data. Since the publication of the recent papers, a further study has been made along the lines described above and the empirical law

$$K = K_0 \left(1 + \frac{a}{\sqrt{r}}\right) \sqrt{R}$$

has been found to express the value of K in the cable formula

$$V = K\, r \ln R/r$$

This formula for maximum stress states that the dielectric strength of a dielectric in a cable is a function of the radii of the conductor and insulation. If we hold the radius of the conductor r constant and increase the radius of insulation R, K will increase on account of the term $\sqrt{R}$. If we hold the radius of insulation constant and decrease the conductor radius, K will increase. We know from data presented that this is so.

If we should plot $K/\sqrt{R}$ as ordinates against $1/\sqrt{r}$ as abscissas, the value of $K/\sqrt{R}$ for various values of $1/\sqrt{r}$ should fall along a straight line.

Let us now compare the four different viewpoints, namely, the minimum stress, the average stress, the increase in conductor diameter theory when $R/r > 2.72$ and the ionization theory.

Table I gives the value of dielectric strength computed for each viewpoint for the data given in Table II of Fernie's paper. The minimum stress seems to apply very well, considering experimental data. The average stress, although giving a good average deviation from an average stress value, does not seem to apply, as the average stress is decreasing with a decrease in the ratio R/r, whereas it should be independent of ratio R/r. For the ionization theory, the equation for K maximum was found by plotting $K/\sqrt{R}$ against $1/\sqrt{r}$ to be

$$K \text{ maximum} = 131.6 \left(\frac{1.935}{\sqrt{r}} - 1\right) \sqrt{R} \text{ kv. per cm.}$$

Taking this formula, the value of K maximum was calculated with the results given in column 7. A comparison of columns 6 and 7 gives an idea of the

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[7] "Dielectric Strength of Thin Insulating Materials," by F. M. Farmer, TRANS. A. I. E. E., Vol. 32, page 2097, 1913.

[8] "A Study of the Dielectric Strength of Cables," by R. J. Wiseman, Thesis, Electrical Engineering Department, Massachusetts Institute of Technology, June 1915.

TABLE I

DATA TAKEN FROM FERNIE, TABLE II.

R mm.	r mm.	t mm.	R/r	Break down Voltage kv./cm.	K max. Actual kv./cm.	K max. Calc. kv./cm.	Per cent Diff.	K min. kv./cm.	K aver. kv./cm.	$K_1 = \frac{5.44\ V}{D}$
11.32	2.43	8.89	4.65	160	428	410	− 4.2	92.0	180	384
17.97	5.27	12.70	3.41	217	335	295	− 11.9	98.3	171	328
12.59	3.69	8.89	3.40	130	288	338	+ 17.3	84.3	146	260
14.16	5.27	8.89	2.69	134	257	262	+ 1.9	95.6	151	
16.26	7.37	8.89	2.21	140	240	212	− 11.7	108.7	158	
11.62	5.27	6.35	2.21	91.5	219	237	+ 8.2	99.5	144	
17.06	8.17	8.89	2.09	114	189	198	+ 4.8	90.6	128	

Average Diff. 8.6 per cent (Col. 8)

Average Dev. { 5.9 per cent (Col. 9)
8.7 per cent (Col. 10)

$$K \text{ maximum} = 131.6\left(\frac{1.935}{\sqrt{r}} - 1\right)\sqrt{R} \text{ kv. per cm.}$$

TABLE II

DATA TAKEN FROM FERNIE, TABLE III.

R mm.	r mm.	t mm.	R/r	Break down Voltage kv./cm.	K max. Actual kv./cm.	K max. Calc. kv./cm.	Per cent Diff.	K min. kv./cm.	K aver. kv./cm.	$K_1 = \frac{5.44\ V}{D}$
7	2	5	3.50	54.3	216	214	− 0.9	61.8	109	211
9	3	6	3.00	69.0	209	191	− 8.6	69.7	115	208
11	4	7	2.75	65.3	161	176	+ 9.3	58.6	93	162
13	5	8	2.60	83.9	175	168	− 4.0	67.5	105	
16	6	10	2.67	94.2	160	165	+ 3.1	60.1	94	

Average Diff. 5.2 per cent (Col. 8)

Average Dev. { 6.4 per cent (Col. 9)
7.6 per cent (Col. 10)

$$K \text{ maximum} = 40.0\left(\frac{3.29}{\sqrt{r}} - 1\right)\sqrt{R} \text{ kv. per cm.}$$

agreement between K maximum actual and K maximum calculated, assuming the corona law to hold. The average difference between K maximum actual and K maximum calculated is 8.6 per cent, which compares favorably with the average deviations of minimum stress and average stress. The Middleton theory gives less values for K_1 than K maximum, as computed from the formula. The stress is not constant, decreasing with a decrease in ratio R/r.

Table II contains an analysis of the various views for the data given in Table III of Fernie's paper. Both the minimum and average stress theories apply well here. The Middleton theory again gives decreasing values of stress with a decrease in ratio R/r. Attention is directed to the very close agreement of K maximum actual and K computed from $K_1 = 5.44\ V/D$. The equation for K maximum for ionization theory is

$$K \text{ maximum} = 40.0\left(\frac{3.29}{\sqrt{r}} - 1\right)\sqrt{R} \text{ kv. per cm.}$$

Column 7 gives the calculated value of K maximum, using this equation. The average difference between Columns 6 and 7 is 5.2 per cent, which is slightly better than the average deviations for minimum stress and average stress.

Table III is a reproduction of Table I in the Middleton, Dawes and Davis paper. As in the case for the two previous tables, this table gives a comparison of the values of K for the different views. The minimum stress and average stress theories do not hold. The value of $K_1 = \dfrac{5.44\ V}{D}$ increases as the ratio R/r decreases. The samples tested had a constant diameter of insulation D. Therefore, we should expect a curve similar in shape to curve $A\ B\ C$ in Fig. 2 of their paper. Fig. 3 of their paper shows the points to be widely scattered and the curve lies between the theoretical curve as computed by the logarithmic law, and a curve similar to $A\ B\ C$ of Fig. 2. Although allowance must be made for deviations of points from true values, still it seems as if we should get better agreement than they present.

Turning now to the ionization theory, Column 7 gives the calculated value of stress after finding the equation

$$K \text{ maximum} = 710\left(1 + \frac{0.1832}{\sqrt{r}}\right)\sqrt{R} \text{ volts per mil.}$$

Fig. 1 is a plot of K maximum/$\sqrt{R}$ against $1/\sqrt{r}$ to show how the points fall along a straight line. This is given in order to compare with Fig. 11 of their paper which shows K maximum plotted against D/d as a straight line. Column 8 gives the per cent difference between columns 6 and 7. It shows a variation plus and minus and an average difference of 4.2 per cent, which is a good agreement for experimental data.

Table IV is the same as Table II of the Middleton, Dawes and Davis paper. The minimum stress theory does not hold. There is a constantly increasing value of average stress with decrease in ratio R/r but not of great amount. The average value is 243 volts per mil with a deviation from the average of 6.6 per cent. The authors in their paper state their empirical law, $K_1 = 5.44\ V/D$ does not hold. In this case K_1 decreases with a decrease in R/r. Column 7 gives the calculated value of maximum stress, using the formula

$$K \text{ maximum} = 149.2\left(\frac{2.02}{\sqrt{r}} - 1\right)\sqrt{R} \text{ volts per mil},$$

which was obtained from the straight line plotted in Fig. 1 for K maximum$/\sqrt{R}$ against $1/\sqrt{r}$. Column 8 gives the per cent difference between columns 6 and 7. The average difference is 4.3 per cent, which is a good agreement and better than the average deviation for the average stress values.

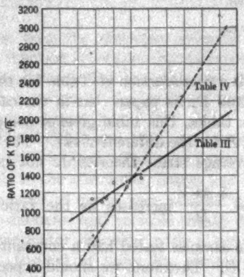

FIG. 1—RELATION BETWEEN RATIO OF CONDUCTOR STRESS TO SQUARE ROOT OF INSULATION RADIUS AND RECIPROCAL OF SQUARE ROOT OF CONDUCTOR RADIUS FOR CABLES IN TABLES III AND IV.

Table V is also taken from the Middleton, Dawes and Davis paper, Table III. The minimum stress and average stress theories do not hold. Instead of using $K_1 = 6.27\ V/D$ to compute the stress according to the author's views, the formula $K_1 = 5.44\ V/D$ was used. The factor 1.15 has already been used in the development of the formula. This will not detract from the nature of the study as it only gives different values of K_1 from those presented in their paper. K_1 given in the second last column shows reasonably consistent values with an average deviation from the average of 3.9 per cent. The last column gives the values of K maximum$/\sqrt{R}$. Here also we get reasonably consistent values with an average deviation from the average of 3.2 per cent or a shade better than 3.9 per cent It is a question as to which view to choose here. Column 7 gives the calculated value of stress after taking the average value of $K/\sqrt{R} = 3100$ and multiplying it by $\sqrt{R}$ for each point. In view of the small variation in $K/\sqrt{R}$ for each point from the average value, we should expect as good agreement in the calculated value of K maximum.

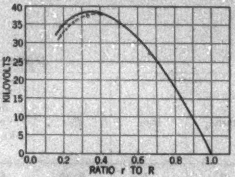

FIG. 2—RELATION BETWEEN RUPTURING VOLTAGE AND THE RATIO FOR DATA IN TABLE VI

The average difference between actual and calculated values of K maximum is 3.1 per cent, which is very close agreement.

The data in Table VI are taken from the thesis of the writer. Constant diameter over insulation and variable conductor diameter were used. The dielectric strength has been computed for the various viewpoints. Fig. 2 is a plot of V against r/R, taking as a value of K the average of the values of K for ratios $R/r < 2.72$ as was done by Middleton, Dawes and Davis for the data in their Table I. The actual curve for values of $R/r > 2.72$ or $r/R < 0.368$ is slightly higher than the theoretical and more nearly like the theoretical than should be expected according to the Middleton theory. Attention is drawn to the small variation in K maximum over the whole range of values of R/r from 1.198 to 5.35. It would be allowable to take an average of all values and the average deviation is only 2.9 per cent from the average. This is better than the results in any other table. The writer believes this slight change in dielectric strength is due to the greater homogeneity of the dielectric than for dielectrics used by others. It would be fair to state from the values given here that the dielectric strength is a constant and the theoretical law is correct. However, the fact that the dielectric strength decreases with a decrease in ratio R/r causes us to modify the above statement. While studying the data given in the table, the writer found the law $K = K_0\,(1 + a/\sqrt{r})$ to apply. It was found recently that the term "$\sqrt{R}$" would further improve the empirical equation for maximum value of dielectric

TABLE III
DATA TAKEN FROM MIDDLETON PAPER, TABLE I.
$D = 0.375$ in.

Size Cond.	r mils.	t mils.	R/r	Break down Voltage kv.	K max. Actual V/mil.	K max. Calc. V/mil.	Per cent Diff.	K min. V/mil.	K aver. V/mil.	$K_1 = \frac{5.44\ V}{D}$ V/mil.
24 sol.	10.0	178	18.65	27.7	942	868	− 7.9	50	156	402
20	16.0	172	11.70	29.5	748	751	+ 0.3	64	171	428
14	32.5	155	5.77	33.9	592	619	+ 4.6	103	219	492
	45.0	142.5	4.17	35.3	549	572	+ 4.2	132	248	512
8	64.0	123.5	2.93	39.1	568	529	− 6.7	194	317	568
6	81.0	106.5	2.31	33.5	493	505	+ 2.4	213	315	
5	93.0	94.5	2.02	31.8	487	492	+ 1.1	242	336	
2	129.0	58.5	1.45	23.8	494	463	− 6.3	340	407	
3 str.	130.0	57.5	1.44	28.0	587			407	487	

Average Diff. 4.2 per cent (Col. 8)

$$K \text{ maximum} = 710 \left((1 + \frac{0.1832}{\sqrt{r}} \right) \sqrt{R} \text{ volts per mil.}$$

(r and R in inches)

TABLE IV
DATA TAKEN FROM MIDDLETON PAPER, TABLE II.
$D = 0.375$ in.

Size Cond.	r mils.	t mils.	R/r	Break down Voltage kv.	K max. Actual V/mil.	K max. Calc. V/mil.	Per cent Diff.	K min. V/mil.	K aver. V/mil.	$K_1 = \frac{5.44\ V}{D}$ V/mil.
24 sol.	10.0	178	18.65	39.9	1355	1240	− 8.4	72.6	224	580
20	16.0	172	11.70	38.3	971	968	− 0.3	82.9	223	556
14	32.5	155	5.77	34.9	612	660	+ 7.7	107.0	225	508
	45.0	142.5	4.17	36.8	571	552	− 3.3	137.0	258	535
8	64.0	123.5	2.93	31.6	460	452	− 1.8	157.0	256	459
6	81.0	106.5	2.31	26.9	395	395	+ 0.0	171.0	253	
5	93.0	94.5	2.02	24.0	368	364	− 1.0	183.0	254	
3	114.5	73.0	1.637	16.8	297	322	+ 8.3	181.0	230	
2	129.0	58.5	1.453	15.7	326	299	− 8.3	224.0	268	

Average Diff. 4.3 per cent (Col. 8)
Average K aver. V/mil. 243. (Col. 10)
Average Dev. 6.6 per cent (Col. 10)

$$K \text{ maximum} = 149.2 \left(\frac{2.02}{\sqrt{r}} - 1 \right) \sqrt{R} \text{ volts per mil}$$

(r and R in inches.)

TABLE V
DATA TAKEN FROM MIDDLETON PAPER, TABLE III
$d = 20.1$ mils.

D ins.	R ins.	t mils.	R/r	Break down Voltage kv.	K max. Actual V/mil.	K max. Calc. V/mil.	Per cent Diff.	K min. V/mil.	K aver. V/mil.	$K_1 = \frac{5.44\ V}{D}$	$K/\sqrt{R}$
0.0938	0.0469	26.8	4.69	9.93	643	670	+ 4.2	137	370	583	2980
0.125	0.0625	42.4	6.22	13.6	744	775	+ 4.0	119.5	321	591	2970
0.1563	0.0782	58.1	7.77	17.8	870	867	− 0.3	112	306	620	3110
0.1875	0.0938	73.7	9.33	23.0	1026	950	− 7.4	110	312	668	3350
0.250	0.1250	104.9	12.45	27.3	1083	1094	+ 0.8	87	260	595	3060
0.3125	0.1563	136.2	15.55	34.2	1244	1224	− 1.6	80	251	596	3150

Average Diff. 3.1 per cent (Col. 8) Average: 609 3100
Maximum Dev. + 9.7 per cent (Col. 11)
+ 8.1 per cent (Col. 12)
Minimum Dev. − 4.3 per cent (Col. 11)
− 4.2 per cent (Col. 12)
Average Dev. 3.9 per cent (Col. 11)
3.2 per cent (Col. 12)

strength. Column 6 gives the calculated value of maximum stress from

$$K \text{ maximum} = 281\left(1 + \frac{0.0546}{\sqrt{r}}\right)\sqrt{R} \text{ kv. per cm.}$$

Very close agreement is obtained with the actual value of maximum stress as given in column 5, and the average difference between actual and calculated values is only 0.9 per cent. The minimum stress and average stress theories do not hold.

The writer also conducted tests with a constant diameter of conducter ($2r$) and variable diameter ($2R$) over insulation. The results are not given here because the values of maximum dielectric strength decreased with increase in ratio R/r and none of the previously expressed views were applicable.

TABLE VI.
$D = 0.952$ cm.

r cm.	t cm.	R/r	Break down Voltage kv.	K max. Actual kv./cm.	K max. Calc. kv./cm.	Per cent Diff.	K min. kv./cm.	K aver. kv./cm.	$K_1 = \frac{5.44\ V}{D}$ kv./cm.
0.0891	0.387	5.35	34.3	230	229	−0.4	21.4	88.6	196
0.1024	0.374	4.65	35.8	228	227	0.4	24.5	96.0	205
0.1194	0.357	3.99	37.0	224	224	0.0	28.1	104	211
0.1588	0.317	3.00	37.3	214	220	+2.8	35.7	118	213
0.179	0.297	2.66	38.5	220	219	−0.5	43.3	130	
0.238	0.239	2.00	35.9	218	215	−1.4	54.5	150	
0.318	0.1585	1.50	27.3	212	212	0.0	71.5	172	
0.398	0.0785	1.198	15.0	208	211	+1.4	87.0	191	

Average: 219

Average Diff.	0.9 per cent	(Col. 7)
Max. Dev.	+5.0 per cent	(Col. 5)
Min. Dev.	−5.0 per cent	(Col. 5)
Average Dev.	2.9 per cent	(Col. 5)

$$K \text{ maximum} = 281\left(1 + \frac{0.0546}{\sqrt{r}}\right)\sqrt{R} \text{ kv. per cm.}$$

Considering the data which have been presented, it does not seem possible to state that the minimum stress theory and the average stress theory will explain the difference between experimental and computed values of voltage breakdown of a dielectric when used in a cable.

The empirical law, which states that the voltage breakdown is a function of the diameter over the insulation when the ratio of diameter over insulation to a conductor diameter is greater than 2.72 and that the dielectric within a diameter the value of which is $D/2.72$ does not add to the insulating properties of the conductor, seems to have some foundation. Further investigation along this line is desirable and in conjunction with the investigation the writer suggests a study be made of the variation of $K_1 = \dfrac{5.44\,V}{D}$ with the geometry of the cable, for example, with d, D or D/d. This is necessary, as in its present form the formula cannot be used to determine beforehand the breakdown voltage of a cable, unless previous data had been obtained for the cable.

The ionization theory has been applied satisfactorily to all the data available. Two assumptions were made in developing the theory; first, ionization takes place in a solid dielectric and the dielectric should follow the laws of ionization; and, second, the dielectric strength of a dielectric is a function of the size and shape of the electrodes, in the case of cables it being found a function of the square root of the radius of the dielectric.

The law $K_1 = 5.44\ V/D$ is limited in its application to cables having ratios of R/r greater than 2.72.

The law $V = K_0\left(1 + \dfrac{a}{\sqrt{r}}\right)\sqrt{R}\ r \log R/r$ is applicable to all ratios as shown very clearly in the tables. It may therefore be considered as a more general formula than $K_1 = 5.44\ V/D$. The term $r \log R/r$ of the mathematically developed law is retained which is the term involving the shape of the electrostatic field.

What has been considered the dielectric strength of the dielectric is not the "true dielectric strength" as was supposed, but an "apparent dielectric strength" which is a function of the radii of the conductor and insulation, as was originally assumed.

This new viewpoint is given with the hope that others may consider it in connection with any data they may have and perhaps in time, from the various viewpoints, a correct understanding of what we mean by the "dielectric strength of a cable" will be found.

Discussion

John B. Whitehead: If I understand Mr. Wiseman aright, he has found an agreement between the results of observation on the dielectric strength of cables and the application of certain empirical laws, based on the behavior of gases. As regards ionization, his suggestion is that it may be possible to explain the laws of cable failures in terms of ionization of the dielectric.

It seems to me there are two reasons why we must go slowly here. The first is that the results of observation on the behavior of dielectrics in cables are not consistent. We are not yet able to bring the various people working in this field together, and as we hear succeeding papers, we find differences in points of view, and differences in results.

The second reason, why we must go slowly, is because there is no evidence of a free mobility of ions in solid dielectrics, as recognized by the author. In the case of gases we find consistent laws, and various experimenters agree; we have a satisfactory explanation in ionization by collision, of the breakdown of gases. But even though we have these empirical laws, and an elaborate theory explaining the phenomena, we still are not in a position to say just what is going on in a gas when it breaks down.

As you know, we have a definite law for stating the critical gradient at which air breaks down, in terms of the shape of the dielectric, but that is a far cry from saying what is going on in the molecule of gas, when it lets go. The law connecting the diameter of the conductor with the breakdown strength of air, while simple on paper, nevertheless does not tell us what is going on in the molecule.

R. W. Atkinson: The author has shown what seems an interesting mathematical expression for the data of the various groups analyzed. However, I do not believe that there is justification for the physical intrepretation which is made nor even for the giving of the equation for K on page 166 as expressing an empirical law covering the data analyzed.

There is reason to suppose that a formula similar to that applying for corona in air may apply in many cases to solid or semi-solid insulators. As a whole however the data quoted by the author tends as much to disprove as to prove this theory. There is very little foundation in the data presented for the introduction of the new term in the formula.

E. W. Davis: We quite agree with Mr. Wiseman that if the voltage rupturing of dielectrics on cables is carefully analyzed from all angles, we may finally arrive at a complete understanding of what voltage breakdown actually is. We would like, however, to emphasize the fact that because of the erratic nature of breakdown tests, many more actual tests must be made before such a complete understanding can be arrived at. By analyzing the few sets of data available today, we hardly produce conclusive deductions. An example of this is found in the data published by Mr. Fernie. Further tests made by him quite upset the theory that he at first advocated.

The so-called Middleton formula ($K = 5.44\ V/D$) was only proposed by its authors as applying to cables whose ratio R/r is greater than 2.72. For cables whose ratio is less than 2.72, the simple theoretical formula ($K = 0.434\ V/r \log_{10} R/r$) has been found to apply.

Comparing the values of K (or maximum stress) as computed by Mr. Wiseman's formula with the values computed by the Middleton formula, as shown in Tables I to IV, we find that by the latter formula, the values generally show fewer deviations from the averages and that the averages for the cables whose ratios R/p are greater than 2.72, check more closely with the actual stresses for cables whose ratio are less than 2.72. This is shown in the following table.

Table	Average K for $R/r > 2.72$ Wiseman	Average K for $R/r > 2.72$ Middleton	Max. Deviations from Average Wiseman	Max. Deviations from Average Middleton	Actual K for $R/r < 2.72$
I	347	324	21%	18.8%	226
II	197	197	10.7	15.9	168
III	668	480	30.0	18.3	512
IV	774	527	60.2	12.9	346
V	920	609			
VI	224	206	2.22	4.85	215

It should be noted that for Tables III, IV and V, the results obtained by the Middleton formula are much more consistent than those obtained by the formula proposed by Mr. Wiseman, while in Tables I, II and VI there is little choice between the two formulas.

G. B. Shanklin: Mr. Wiseman's paper is the third vital paper on dielectric strength presented before the Institute in the past few months. Our knowledge of dielectric strength is thereby vastly increased.

The first by Mr. Middleton, presented at Niagara Falls Convention, indicated that the dielectric strength of cables should be expressed by empirical formulas and not by a maximum, minimum or average stress theory. In the discussion which followed Mr. Middleton's paper an illuminating ray of light was thrown on the problem by the words of Dr. Steinmetz, Mr. Peaslee and Mr. Peek. Their discussions predicted and guided the trend of the investigation in a truly remarkable way.

Afterwards, Dr. Wagner presented a paper at Chicago on the "Electrical Breakdown of Solid Dielectrics," ascribing breakdown to a true accumulative conduction, concentrated in small, weakened spots of the insulation thickness. His work is in such good agreement with the beliefs of Dr. Steinmetz, Mr. Peaslee and others, and is, in itself, so convincing that it is sure to have a decided bearing on future work. Dr. Wagner dealt only with parallel dielectric fields and did not apply his theory to the more complicated fields met with in cable cross sections.

The present paper by Mr. Wiseman is an elaboration of the empirical corona formula suggested by Mr. Peek and shows without any question of doubt, that the law of breakdown in cables must be expressed empirically. It appears to me, however, as misleading to attempt to explain this as due to some form of ionization, such as corona in air. It is hard to see how there could be ionization of this nature in solid material. The mean free path of electrons is entirely too small. That an almost similar "energy distance" formula will express the law of breakdown in cables does not necessarily mean that the phenomena are similar. Rather, it means that the shape of the dielectric field is involved in both cases. The shape of the dielectric field determines the distribution of voltage gradient, so perhaps it is more accurate to say that the voltage gradient is involved in both cases.

In the discussion previously referred to, both Dr. Steinmetz and Mr. Peaslee indirectly show how Mr. Wiseman's empirical results can be explained by Dr. Wagner's conduction breakdown theory. The density of the conduction current, which is determined by the voltage gradient, must reach a critical value some distance out from the conductor before actual breakdown occurs. In the case of corona in air, the voltage gradient determines the velocity of the ions and therefore the required critical distance for ionization by collision. The two cases are similar only in the above respects, but it so happens that this is sufficient to give them the same form of mathematical expression.

W. N. Eddy: Although Mr. Wiseman's detailed analysis of the available breakdown data on cables is very interesting and valuable, it leaves the impression that in studying the breakdown of cable insulation he considered only the voltage impressed on the insulation, the thickness of the insulation and the dimensions of the electrodes. No reference to either the temperature of the cable or the time of application of the voltage could be found in the paper. Such a point of view, if true, gives a misleading idea of the problem and while a formula developed from such data may represent the test data satisfactorily it will not give an adequate idea of the behavior of the cable in service at constant voltage and varying temperatures. In this connection it is interesting to note the statement made by Dr. Steinmetz in a recent discussion.*

"An extensive investigation of this problem of the mechanism of breakdown of solid insulation has been carried out during the last few years in my laboratory by Mr. Hayden and his assistants in which we derived the conclusion, or rather were forced to the conclusion, that there exists no such thing as a definite breakdown voltage or breakdown gradient of solid insulation."

It seems to be well established that the breakdown of insulations is caused by local overheating. This in turn may be caused by unhomogeneity of the insulation, by overloading and therefore overheating of the cable at normal voltage, by the application for a short time of voltage greatly in excess of normal or by the application for a longer time of voltage slightly in excess of normal. For instance, a cable in service is under practically constant voltage throughout its life. After more or less time the various temperature cycles finally cause a point in the insulation to develop that begins to take a little more current than the other points, this increase of current results in more local heating which in turn means more current and so on until the insulation carbonizes and breakdown results.

For such reasons it would seem reasonable that any investigation on the dielectric strength of cables should devote as much or more attention to the effect of temperature and time than to that of the voltage alone.

Wm. A. Del Mar: The various attempts of Jona, Middleton, Atkinson, Peek and Wiseman to find some constant value of the electric stress which will determine the breakdown voltage of cables have been only partially successful because it is not yet possible to make a number of samples of cable of identical dimensions which will breakdown at the same voltage. Indeed, apparently identical flat samples of impregnated paper show a variation of at least 10 per cent above and below the average in breakdown voltage.

It therefore seems hopeless to expect electrical uniformity of material in samples of different dimensions in an attempt to find a law connecting breakdown voltage and dimensions.

Whereas leakage of current depends upon average conditions, breakdown voltage depends upon maxima and minima. It is probable that a solid or semi-solid dielectric would never breakdown under stress if it were absolutely homogeneous. Increasing the potential gradient would simply increase the leakage and nothing would happen unless the heat generated were confined. Failure results from some local facility afforded to ionic flow. In the case of impregnated paper this may be a spot where the paper has low baffling effect to ionic motion, or a portion of the oil rich in free ions or a vapor pocket where vapor ionizes and sets up local surges.

The only way to overcome the difficulties due to the inherent heterogeneity of dielectrics, is to test a very large number of samples. The deductions of Middleton, Atkinson, Peek and Wiseman, have been drawn from too limited a number of tests. Such tests are very expensive and seriously tax laboratory facilities in the factories. It seems as if cooperative research would be necessary to reach a solution of this problem.

In the meanwhile some statistical data are obtainable from factory records extending over long periods. Thus during six consecutive months of 1922, the records of one company show that the breakdown voltage of sector cables runs about 100 kv. for each centimeter of insulation thickness between conductors. Occasionally this figure would run up to 125 or down to 75, but the general run was unmistakably represented by 100 kv. per cm.

During the same period, two sizes of high-voltage single-conductor cables were made and numerous samples tested. Their dielectric strength as derived from Peek's formula was found to be about 100 kv. per cm. with occasional samples running above and below to about the extent mentioned for sector cables.

The dielectric strength of impregnated paper should be of this order because that of petrolatum is about 30 kv. per cm. by Peek's formula, and the multiplying effect of the paper is found experimentally to be of the order of 3.

Mr. Wiseman's experiments appear to have been made on ceresine and not on impregnated paper. These two materials exhibit totally dissimilar characteristics under stress and I am inclined to give very little weight to any formula derived from tests on the former, when applied to the latter. Possibly the free ions in ceresine are not determining factors, but C. F. Hanson and I, in an Institute paper last year, gave experimental evidences of the existence of free ions in petrolatum and since we have obtained additional experimental evidence of ionic stimulation in impregnated paper under electric stress and have developed a practical measure of the baffling effect of paper to ionic motion. Whether true or false, this theory has been of great practical use in cable making, as it has enabled the dielectric strength to be varied over a range from 50 to 150 kv. per cm., at will. All of this is inconsistent with the idea that impregnated paper obeys a law based upon tests on a solid dielectric, such as ceresine.

The formula developed by Mr. Wiseman may, of course, prove to be of great value in the case of solid insulation.

R. J. Wiseman: The writer agrees with Dr. Whitehead that we should go slowly in applying the empirical laws for ionization of gases to solids. We know very little about ionization in solid dielectrics. I started with the assumption there may be something in it and was surprised to obtain what I consider sufficient agreement to warrant further investigation. It is a problem for the physicist and I hope someone else will take up the study and give it a thorough analysis, either to prove or disprove what has been advanced. It must not be laid aside just because there are no visible or physical effects present on account of ionization. The writer does not believe such effects need to be a necessary adjunct to ionization. We can have ionization in gases without any visible or physical effects, why is it not possible in solids?

It would have been a great help to us if Mr. Atkinson had shown why he believes the data quoted in the paper tends as much to disprove as to prove the theory advanced. The data given in five of the six tables were obtained from papers presented at the Niagara Falls Convention. Considering the different sources of data and the fact that the new empirical formula gave better agreement than other formulas seems to tend more to prove the theory rather than to disprove it.

The formula presented states that the dielectric strength is a function of the radii of conductor and insulation as well as their ratio. Consequently we cannot average up the values of K in the theoretical formula for dielectric strength if the radius of conductor varies as has been the case in the tables given in the paper. Bearing this in mind, the table Mr. Davis presents has no weight. Take Table IV of the paper which is a copy of Table II of the Middleton, Dawes and Davis paper, we have a range of K by the theoretical formula from 1355 volts per mil to 326 volts per mil, K decreasing as the radius of the conductor increases. It is obvious we have no right to average such a wide range in K.

Mr. Eddy's remarks pertaining to no reference to either temperature of the cable or the time of application of the voltage bring out a serious offense against what we should do when presenting data and the writer apologizes for not including this information in the paper. The tests given in Table VI from the writer's previous work were made at room temperature and an increase of voltage of about 1000 volts per second. We all admit that very long time constant voltage tests when the voltage is near the rupturing voltage and varying temperatures give different results than short time voltage tests and constant temperature.

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*A. I. E. E. Journal, December, 1922.

The writer is unable to appreciate Mr. Del Mar's remark that "it is probable that a solid or semi-solid dielectric would never breakdown under stress if it were absolutely homogeneous." The writer agrees with Mr. Del Mar that it is necessary to conduct many tests before we can feel we have solved the problem in question. However it is necessary to make a start at drawing conclusions after a reasonable number of tests have been made. The data presented by Messrs. Middleton, Dawes and Davis are the results of many tests at each point and the data given by the writer in Table VI are the result of at least 10 tests at each point. The average deviation for each point was low and therefore the average value could be accepted as representing as well as experimental work was possible, the phenomenon taking place for the conditions under test.

Considering the agreement which the new formula gives between computed and actual values of K for each of the tables given in the paper and the fact that all but five of the tables were taken from papers by authors and on different dielectrics gives the formula more weight and that there may be something more worth while in it than Mr. Del Mar seems to believe. Very true, ceresine and impregnated paper exhibit totally dissimilar characteristics and this is brought out in the expression $K = K_0 (1 + a/\sqrt{r}\,\sqrt{R})$. The dissimilarity would be shown in the value for a, K and whether a positive or negative sign would occur. The baffling effect referred to by Mr. Del Mar is due to the higher amount of free ions and the greater freedom of mobility of ions in petrolatum than in paper. Change the freedom of mobility in paper and you will change the characteristics of impregnated paper.

I am very glad Mr. Del Mar reports evidence of free ions in petrolatum. It helps to substantiate what the writer has believed for many years. The writer views the problem as to what is a dielectric, as follows, all substances, gases, liquids, and solids have free ions, different substances having different amounts and each substance having a different mobility, both of which determine the dielectric strength of the substance. The number of free ions in gases is large and freedom of motion great, therefore we get low dielectric strength. Liquids may be classed as the next higher order of dielectric, having less free ions and less mobility than gases. Solids present the case of where free ions are few and freedom of motion greatly limited, resulting in high dielectric strength. The above refers to homogeneous materials. Where we have a non-homogeneous substance such as rubber each of the pure substances in the rubber has its own influence on the resulting material. For a substance like impregnated paper in a cable we have the combination of liquid, and paper impregnated with a liquid. The liquid alone has a property of its own but the impregnated paper is a combination of liquid and the solid paper. The resulting material may have any number of characteristics as the liquid and paper alone and in combination vary. It may even happen that for the same liquid and paper, two cables may exhibit different properties on account of some slight change in the treatment or manufacture which would change the ionic characteristics of each dielectric.

The writer appreciates greatly the remarks of Mr. Shanklin in referring to the views of Dr. Steinmetz, Dr. Wagner and Messrs. Peaslee and Peek. The writer's paper was written in August, 1922 prior to the publication of Dr. Wagner's paper and the discussion of the papers presented at the Niagara Falls Convention.

Without being too bold, since reading Dr. Wagner's paper and the remarks of Mr. Peaslee the question has arisen in the mind of the writer, are we all referring to the same thing? Is Dr. Wagner's conduction breakdown theory and Mr. Peaslee's ideas the same as the views expressed by the writer but expressed differently. They view the subject from a changing conduction angle whereas the writer looks upon it as an ionization phenomenon. Corona such as we call the visible glow in air when the ionization becomes great enough to make the air conducting is not necessary before we say ionization takes place. Mr. Bergen Davis[1] in a paper presented before the Institute in 1914 showed that ionization and corona are practically identical. Some of his ideas present a good foundation for a study of the phenomenon of ionization. Ionization in gases takes place as soon as the voltage stress is great enough to cause the free ions to move and create other ions by impact. It seems reasonable to suppose that the same will be true for the free ions in a solid. Furthermore it is not necessary that there be any physical effects resulting from the formation of ions in the dielectric. If we agree to these statements, then it is not so difficult to explain some of the phenomena of the electrical breakdown of dielectrics. Taking Mr. Peaslee's discussion of the papers presented at the Niagara Convention. Is the current he refers to partly the current due to ionization and not entirely a conduction current obtained from resistance? As the voltage increases and therefore the voltage stress, any free ions present will be set in motion, causing other ions to be formed by collision with neutral molecules. These ions are moving toward the positive electrode to be discharged. If the positive electrode is near the negative electrode, the ions will be discharged, and a further increase in voltage stress is necessary in order to produce a sufficient number of ions to cause the dielectric to become conducting by breakdown. If the positive electrode is not close to the negative electrode the ions do not have time to reach the positive electrode before a large number have been produced in the region of the negative electrode. This may result in the dielectric around the negative electrode exhibiting the properties of a conductor and the electrode has been virtually extended into the substance of the dielectric. This will result in a greater increase in the number of ions without further increase in voltage and finally, puncture will occur. The voltage necessary to cause breakdown will not be proportional to the voltage required for small distances between the electrodes. In the case of alternating voltages, there is the additional consideration that the time phase is likely to change before the ions have been able to discharge, so that they will simply accumulate around the electrodes and the dielectric will become conducting at the electrodes with the results previously stated. Is the current Mr. Peaslee refers to at different parts of the insulation the ionization current described above. The writer believes it is and that his curve of current density can be explained by ionization of the dielectric as the voltage is increased.

It may be of interest in connection with Mr. Shanklin's reference to "energy distance" to report that calculations were made for the data given in the paper with the following results.

The value of the voltage stress was computed at a radius equivalent to that for the energy distance as found by means of the ionization formula for each table. The voltage stress at the energy distance should be constant. The following average deviation was found for each table.

Table	I	II	III	IV	VI
Average dev. per cent.........	10.4	6.0	0.1	7.6	0.3

For Table I, II and IV, the formula for K had a negative sign, whereas for Table III and VI, the formula for K had a positive sign. Although a constant value of K was not obtained for the stress at the energy distance for each table, in two cases it was practically obtained and for the other three, fair agreement obtained.

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[1] Theory of Corona, by Bergen Davis. PROCEEDINGS, A. I. E. E., Vol. 33, April, 1914.

Experimental Determination of Short-Circuit Currents in Electric Power Networks

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Review of the Subject.*—No means has yet been discovered whereby the abnormal rise of current occurring during short circuits is avoided. Protection against its destructive effects remains, therefore, a subject of major importance. Among the problems requiring short-circuit current determinations, the following are the chief ones:*

(1) Selection of oil circuit breakers of the required interrupting capacity.

(2) Determination of the size of current-limiting reactors.

(3) Determination of relay settings in relay systems depending on selective action from over-current and directional relays.

(4) Calculation of mechanical stresses in the structural elements of apparatus subject to short-circuit electromagnetic forces.

Each of these problems requires the knowledge of the magnitude of short-circuit currents; relay problems frequently require, in addition, the relative phases of currents and voltages at different points of the system during short circuits; in item (4) above the wave-form of the short-circuit current sometimes has to be considered. The available information on the latter subjects, i. e. on phase relations and on wave form during short circuits is relatively meager, probably because it has been required in special cases only. Nevertheless, the demands for these data are increasing—on account of both the tendency towards increased sensitiveness of protective devices and the rapid increase in the magnitude of the short-circuit currents to be handled—and it will be worth while, therefore, if this added information is obtained. This paper is confined to the problems of the determination of the magnitude of short-circuit currents.

The magnitude of short-circuit currents depends on a multiplicity of factors which have been enumerated and dealt with in other publications.[1] *When the impedances of all the circuit elements affected by the short-circuit are known together with the current-time decrement characteristics of all machinery capable of supplying current to the short circuit,*[2] *the problem of short-circuit-current determination resolves itself into one of current division in a given network of electrical conductors under given electromotive forces.*

In the following paper, the comparative merits of three methods—calculation, a-c. test, and d-c. test—of determining short-circuit currents in networks are briefly discussed. Two d-c. experimental methods applicable to the "short-circuit calculating table" are analyzed in detail. The accuracy of its results, by both methods, is obtained for a variety of circuit conditions. The proper field of use of the short-circuit calculating table, and the best method of its application are determined.

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[1] See Bibliography No. 1, 2.

[2] See Bibliography No. 1, 2, 3.

INTRODUCTION

SHORT-CIRCUIT currents in networks may be determined by calculation from the circuit constants and the connection diagram, or by tests with an experimental circuit equivalent to the system under consideration. The calculations for complicated networks by the elementary procedure using Kirchhoff's law without employing simplifications or mathematical shortcuts are extremely lengthy and not practical. Although the complications are reduced by the use of circuit transformations and by mathematical methods,[3] test methods in many cases give quicker results of sufficient accuracy for routine short-circuit-current determinations.

The most obvious test method is that in which all the essential elements in the system under consideration are reproduced in a miniature system in true proportion. This kind of test calls for a miniature a-c. system. The chief disadvantages of this type of a-c. test system are due to the necessity of both resistor and reactor units and to the limitations of the low-range a-c. meters: the miniature apparatus has to be either of relatively large current rating to accommodate the available a-c. instruments and thus becomes quite expensive, or delicate low-range instruments are required. These conditions have brought about the d-c. miniature line in the form of the short-circuit calculating table. Its use is based on an approximation whereby the complete a-c. system under short-circuit conditions is reduced to an approximately equivalent d-c. network. The magnitudes of the currents in the various circuit branches of this equivalent d-c. system under short-circuit are determined by test, and converted by the use of the proper factor of proportionality into the corresponding a-c. currents of the original system.

One method of employing this short-circuit calculating table is to make the ohmic resistance values in the equivalent d-c. system proportional to the reactances of the various circuit elements, *i. e.* of generators, reactors, transformers, lines, etc. This method may be called the *reactance method.*

The second method of applying the short-circuit calculating table is the *impedance method,* in which the impedances in all the system elements are assumed to be in phase and set up as resistances on the calculating table. This method is not in general use.

The details of construction of this apparatus and the customary method of its application have been fully described in several publications.[4]

OBJECT

It is the object of this paper to determine the merits and limitations of the short-circuit calculating table,

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[3] See Bibliography No. 4, 5, 6.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

[4] Descriptions of several forms of short-circuit calculating table are found in the following articles: Bibliography No. 7, 10, 11, 12 and 13. For reference to the reactance method of short-circuit-current determination see Bibliography No. 8, 9 and 13.

by a consideration of its proper field of use, its accuracy and the methods of its application.

CHARACTERISTICS OF SHORT-CIRCUIT CALCULATING TABLE

The use of the short-circuit calculating table depends on the following features:

(1) Substitution of single-phase circuit for polyphase circuit.

(2) Neglect of capacity current.

(3) Neglect of phase differences arising from the inequality of the impedance angles of the various circuit elements represented.

(1) *Substitution of Single-Phase for Polyphase Circuit.* When three-phase short-circuits in a three-phase circuit are considered, the reactance values represented on the calculating table are the values per phase; the conversion of the three-phase circuit to a single-phase circuit is thus quite simple. This simple procedure is not generally applicable to problems of line-to-ground short-circuits on grounded-neutral systems, or to single-phase, line-to-line short-circuits on polyphase circuits because the short-circuit currents are not equally divided among all the phases. In such cases it is necessary first to determine the paths taken by the components of short-circuit current. Then the equivalent reactance (or impedance) of the current paths may, in some cases, be found experimentally by the calculating table.[5] In other cases, complete solution by calculation or by a-c. miniature test is necessary. An a-c. miniature polyphase testing equipment having resistance, reactance and capacity units is described by Gray, Bibliography No. 15. The circuit constants of this system are substantially fixed, since it is intended for application to one power system only.

A miniature 3-phase a-c. generating and transmission system with adjustable resistance, reactance and capacity units has been constructed and operated in the General Engineering Laboratory of the General Electric Company for the solution of network problems. It is hoped that the author will be permitted to present the details of this miniature system in a later paper.

To summarize: the chief application of the short-circuit calculating table is for the solution of balanced short circuits, such as three-phase short circuits in a three-phase circuit; other short circuits, *i. e.* those with unequal currents in the various phases, require either a solution combining theoretical analysis with calculating-table tests, or complete solution by calculation, or solution by a-c. miniature test.

In this connection attention should be called to the fact that three-phase short-circuit currents are generally used as the basis for selecting circuit breakers for three-phase systems, because three-phase short circuits impose, as a rule a heavier duty on the circuit breakers than other short circuits.[6]

(2) *Consideration of Capacity Currents.* Capacity charging currents require consideration in short-circuit-current determinations in the following cases:

(a) for short circuits far remote from generating station on long lines.

(b) for line-to-ground short circuits on grounded-neutral systems when the short-circuit current is limited such as by neutral resistors or reactors, to a value comparable to the capacity charging current.

The following example will serve to illustrate item (a) applied to a three-phase short circuit on a 200-kv. line 200 miles long having aerial conductors of 350,000 circular-mil copper at 20 ft. spacing. For a distant-end short circuit, omitting the capacity current, the computed short-circuit current at the generator is about 6 per cent higher than the correct short-circuit current including consideration of distributed line capacity. For short circuits, occurring at points less than 200 miles remote from the generator,—excepting line-to-ground short circuits—the capacity current may, therefore, be neglected.[7]

For line-to-ground short circuits, the capacity current[8] is readily calculated from the physical data for the circuits involved.

To summarize: the neglect of capacity charging currents introduces errors smaller than 10 per cent for three-phase short circuits occurring less than 200 miles remote from the generator. For line-to-ground short circuits the capacity current is sometimes a considerable part of the total short-circuit current, and is determined by calculation or by a-c. miniature test.

(3) *Consideration of Phase Differences.* When a system having circuit elements of dissimilar impedance angles is represented on the short-circuit calculating table errors are encountered, because scalar quantities only can be used on the table. The error due to this approximation will be considered first with the aid of several examples taken from typical circuits. In addition to these examples, calculated data are presented. The latter data cover a wide range of circuit constants, including not only those of the more common types of circuits but also those of special and extreme cases. In all the cases considered, the accuracy of the reactance method and of the impedance method for short-circuit calculations was determined—either by test or by calculation.

In the following examples, three types of systems may be considered:

Systems of aerial conductors

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[5] See W. W. Lewis, Bibliography No. 14.

[6] See discussion by E. G. Merrick, A. I. E. E. TRANS. 1918, pp. 153-154.

[7] Short-circuit-current data on long aerial lines, up to 3000 miles in length, are given by Lewis, Bibliography No. 17.

[8] See Gray, Bibliography No. 15; and Lewis, Bibliography No. 16, p. 638.

Underground cable systems

Systems having both aerial conductors and underground cables.

Sytems of Aerial Conductors. The error most readily determined is that due to the impedance angles of generators and of distribution lines. When all the principal lines affected by a short circuit have equal impedance angles, these lines may for this purpose be consolidated into one equivalent impedance of the same angle. When, furthermore, in such circuits all sources of short-circuit current may be replaced by one equivalent generator, all short-circuit currents will have the same error. Circuits of this type will be considered first. The resistance of generators, when of the order of 1/20 of their reactance, will be neglected.

The examples given apply to three-phase systems, and the resistance and reactance data are the values per conductor. Three-phase short-circuits are considered.

Example 1

Series circuit consisting of generating station, transformer bank and aerial conductors, all in series; reference kv-a. = 11,000.[9]

	Resist.[9] per cent	React.[9] per cent	Imped. per cent
Generators, total capacity 11,000 kv-a.; 6600 volts, 60 cycles, 3-phase	..	28.0	28.0
Transformer bank 6.6/44 kv	0.9	4.8	4.9
Lines 44 kv., of 116,000 cir. mil copper and of 179,000 cir. mil aluminum; total resistance and reactance of lines to point of short circuit 124 miles from generating station, all lines having practically the same impedance angle	16.5	31.0	$35.1\underline{/62}$

Total impedance of all circuit elements = $17.4 + j\,63.8 = 66.1$ per cent.

Scalar sum of all impedances = 68.0 per cent.

Hence the error of the short-circuit current by the reactance method is indicated by the ratio $u_x = \frac{66.1}{63.8} = 1.04$, meaning that the result is 4 per cent high.

The corresponding error of the impedance method, obtained from the impedance accuracy factor $u_z = \frac{66.1}{68.0} = 0.97$, is 3 per cent low.

NOTE: The accuracy factor u_x is the ratio of short-circuit current by reactance method to the correct short-circuit current; u_z is the corresponding ratio applied to the impedance method.

It is seen that the error of the short-circuit current by the reactance method is not excessive and is on the safe side. The chief source of error is the resistance of the lines. It should be noted that the impedance angle of the lines is 62 deg., while that of generators and transformers is substantially 90 deg.

Example 2

A three-phase generating station with transformers supplies a 110-kv. 60-cycle aerial line. Reference kv-a. = 60,000.

	Resist. per cent	React. per cent	Imped. per cent
Generators, 60,000 kv-a. aggregate rating, total combined reactance		12.0	12.0
Transformer unit, 13.2/110 kv., 20,000 kv-a. rating	1.5	19.4	19.4
Line, 110 kv., 250,000 cir. mils copper short-circuited 88 miles from station	10.0	35.9	$37.2\underline{/74.4}$

Total impedance of all circuit elements up to point of short circuit = $11.5 + j\,67.3 = 68.3$ per cent.

Scalar sum of all impedances = 68.6 per cent.

For the short-circuit current by reactance method,

$$u_x = 68.3/67.3 = 1.015$$

which means that the short-circuit current is 1.5 per cent high. Likewise, for the impedance method

$$u_z = 68.3/68.6 = 0.996$$

which means an error of 0.4 per cent low. The errors of the short-circuit current are seen to be negligible.

Example 3

One 60-cycle, 3-phase generating station with transformer bank; circuits affected by short-circuit: two series-connected aerial lines fed from the generating station; reference kv-a. = 25,000.

	Resist. per cent	React. per cent	Imped. per cent
Generators with transformers 13.2/88 kv. aggregate rating 43,750 kv-a., impedance components of entire station	0.5	13.3	13.3
Line 88 kv. No. 1/0 B & S copper, 29 miles	4.8	8.1	$9.5\underline{/59.4}$
Line 88 kv. No. 1/0 B & S aluminum, 51 miles	13.9	13.3	$19.3\underline{/43.8}$

Both lines in series, point of short-circuit 80 miles from generating station.

Total impedance of all circuit elements up to point of short-circuit = $19.2 + j\,34.7 = 39.7\underline{/61.0}$.

Scalar sum of all impedances = 42.1 per cent.

For the short-circuit current, by reactance method

$$u_x = 39.7/34.7 = 1.14$$

---

[9] It is often convenient to express resistance and reactance in terms of the per cent voltage drop through them; *i. e.* the percentage reactance drop—frequently called briefly the "per cent reactance"— is the voltage drop, expressed as a percentage of normal voltage, across the reactance when traversed by a current due to a specified balanced kv-a. load on the circuit. If x is the reactance in ohms, kv-a. the reference kv-a., and kv. the line voltage in kv. in a 3-phase circuit, the "per cent reactance" is per cent $x = \frac{x\,(\text{kv-a.})}{10\,(\text{kv.})^2}$ (see Bibliography No. 9).

which means that the short-circuit current is 14 per cent high. Likewise for the impedance method

$$u_s = 39.7/42.1 = 0.94$$

which means that the error is 6 per cent low. The somewhat greater errors in this case are caused by the following factors: The impedance of the distribution lines between the generating station and the point of short-circuit has a fairly low angle (49 deg.) and a relatively large magnitude (more than twice that of the generating station impedance); in other words, the circuit elements of low impedance angle constitute the larger part of the total system impedance. This example illustrates the accuracy of the short-circuit current by the calculating table for a fairly remote short-circuit on a system having distribution lines of an impedance angle in the vicinity of 45 deg.

Example 4

Occasionally the distribution lines of a system have dissimilar impedance angles. This condition may arise from the use of unequal wire sizes, or of dissimilar metals, or of reactors. In the example chosen, a generating station with step-up transformers feeds two aerial lines, of unequal wire sizes, connected in parallel and terminating in a common bus. The data are as follows: Reference kv-a. 15,000, frequency 60 cycles.

	Resist. per cent	React. per cent	Imped. per cent
Generators with step-up transformers total generating capacity 26,000 kv-a.	0.60	9.20	9.21
Line No. 1, 38 miles long, No. 2 B & S copper, 60 kv.	13.70	13.10	$18.95\underline{/44}$
Line No. 2, 38 miles long, No. 4/0 B & S copper, 60 kv., connected in multiple with line 1	4.25	11.80	$12.55\underline{/70}$

Short-circuit at terminal of lines.

By calculation from these data, by the customary method, the errors of the total short-circuit current and of the branch currents in lines No. 1 and No. 2 are as follows:

	Reactance Method	Impedance Method
Total short-circuit current	7 per cent high	1 per cent low
Branch current 1	24 per cent high	4 per cent low
Branch current 2	9 per cent low	4 per cent low

It is seen that the total current by the reactance method is not badly in error and is on the safe side, while the branch current No. 1, is 24 per cent high. It will be observed that the reactance method gives a high result for the branch of low impedance angle, (44 deg.), and a low result for the branch of higher impedance angle, (70 deg.), while the impedance method gives equal errors (only 4 per cent low) in the two parallel branch lines.

The results of the preceding four examples may be summarized as follows: In examples 1, 2 and 3, systems having distribution circuits of the same impedance angle were considered. The errors of the reactance method were on the safe side in all cases (*i. e.* high results); in the first two cases the error was below 5 per cent, but in the third case the error was 14 per cent because the distribution lines had impedance angles of the order of 45 deg. and impedance magnitudes amounting to as much as two thirds of the total system impedance. In the fourth example, one of occasional occurrence, parallel circuits of widely dissimilar impedance angles gave by the reactance method a permissible error for the total short-circuit current, but an error of slightly over 20 per cent for one of the branch currents. The impedance method gave low values in this case, but none more than 4 per cent in error. The conditions illustrated by examples 3 and 4 are considered in detail in the more general analysis presented later.

Systems of Underground Cables. Cables generally have lower impedance angles than aerial lines. When short circuits occur near a generating station, the impedance limiting the short-circuit current is largely due to the generators and station reactors and therefore has a large angle. The total short-circuit current, as determined by the calculating table, will then not be badly in error. The minor components of short-circuit current in the longer cable lines may however have greater errors. Similarly for remote short circuits the impedance angle of the cables may materially lower the accuracy of the total short-circuit current.

The case of a near short circuit on a cable system having No. 4/0 B & S 3-phase, 3-conductor, 15,000-volt cables, at 60 cycles, will be illustrated by the following example:

Example 5

Reference kv-a. = 40,000

	Resist. per cent	React. per cent	Imped. per cent
Generating station	0.50	16.5	16.5
Combined data for cable circuits between generating station and point of short circuit; this impedance is equivalent to that of two parallel No. 4/0 B & S 15,000 volt 3-phase cables, each one mile long	3.06	1.55	$3.43\underline{/27 \text{ deg.}}$

The total circuit impedance to the point of short circuit is

$$3.56 + j\,18.05 = 18.4 \text{ per cent}$$

The scalar sum of the circuit impedances is 19.9 per cent. Hence, we have by the reactance method

$$u_z = 18.4/18.05 = 1.02$$

or an error of 2 per cent (high). Likewise, by the impedance method,

$$u_s = 18.4/19.9 = 0.92$$

representing an error of 8 per cent (low).

Example 6

In this example, the system of example 5 will be considered. but the short circuit will be located remote from the generating station, such that the combined impedance of the cable circuits between generating station and point of short circuit is equivalent to 2.5 miles of No. 4/0 B & S copper 3-conductor cable. Using 40,000 kv-a. as the reference kv-a. the data are,

	Resist. per cent	React. per cent	Imped. per cent
Generating station..............	0.50	16.5	16.5
Combined data for cable circuits between generating station and point of short circuit..........	15.3	7.75	17.15 /27 deg.

The total impedance to the point of short circuit is

$$15.8 + j\,24.3 = 29.0 \text{ per cent}$$

The scalar sum of the circuit impedances is 33.7 per cent.

Hence, by the reactance method,

$$u_x = 29.0/24.3 = 1.19$$

representing an error of 19 per cent high. Likewise by the impedance method:

$$u_s = 29.0/33.7 = 0.86$$

representing an error of 14 per cent low.

Example 7

In this example, the network shown in Fig. 1 will be considered. The data given in this diagram and in Table I are miniature test data representing an 11,000-volt, 60-cycle underground 3-conductor cable system, having current-limiting reactors (giving a 3 per cent voltage drop in the cables when carrying full-load cable current) in each of the lines *A*, *B*, *C*, *D*, *E*. The cables are of No. 4/0 B & S and 250,000 cir. mil copper. Those in lines *F*, *G*, *H*, and *I* have no reactors, and therefore have low impedance angles, of the order of 26 to 31 deg. The results by a-c. test, by reactance and by impedance methods, are given in Table I, for a 3-phase short circuit at *b*. The total short-circuit current by reactance method is 5 per cent high, that by impedance method is 4 per cent low. These errors are small because the impedance angle of the cable network from generator bus to the point of short-circuit is 56 deg. The three principal components, *B*, *C* and *F* of current fed into the short circuit have errors, by the reactance method, from 7 per cent low to 16 per cent high, the negative error occurring in *B*, *i. e.* in the path of highest impedance angle (67 deg.). The current in line *F* (error 16 per cent high) is partly dependent on reactances of lines having low impedance angles (as low as 26 deg.). Errors of over 20 per cent high, by reactance method, occur in three lines more remote from the point of short circuit. However, these lines have the lowest currents, each carrying less than 15 per cent of the total short-circuit current, and two of them are over four miles long.

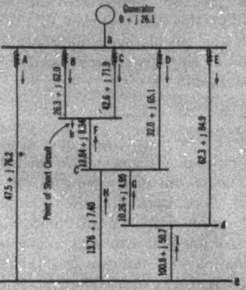


FIG. 1—CIRCUIT DIAGRAM FOR EXAMPLE 7

Short-circuit current determination. Arrows indicate direction of short-circuit current. Note: Figures opposite lines and generator represent resistance and reactance values, expressed as complex numbers; line-to-neutral values in ohms per phase. Reactance of reactors included with line impedances for lines *A*, *B*, *C*, *D*, *E*.

The short-circuit currents by the impedance method have a maximum error of 6 per cent low. In this

TABLE I

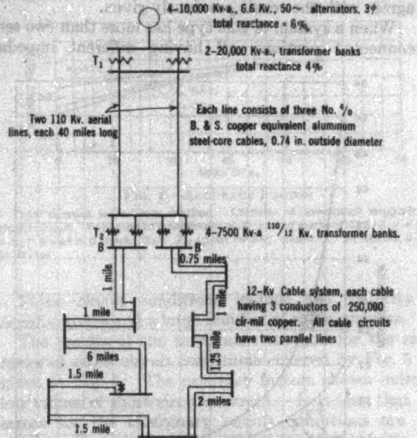
Circuit constants and miniature test data for short-circuit current determination in cable network of Fig. 1. The data given represent in true proportion the conditions of the actual network. The ohmic values given are 56 times the actual system values.

	Length miles	Miniature circuit data			Miniature a-c. test short-cir't. current amperes	Reactance Method		Impedance Method	
		Resist. ohms.	React. ohms	Impedance ohms		Short-circuit current	accuracy factor	Short-circuit current	Accuracy factor
gen.		——	26.1	26.1 /90	9.70	10.13	1.05	9.27	0.96
A	3.1	47.5	76.2	89.8 /58.1	1.30	1.550	1.10	1.218	0.94
B	2.1	26.3	62.0	67.3 /67.0	2.96	2.765	0.93	2.875	0.97
C	3.3	42.6	71.9	83.5 /59.3	2.37	2.385	1.01	2.320	0.98
D	2.4	32.0	65.1	72.5 /63.8	1.88	1.99	1.06	1.761	0.94
E	4.1	62.3	84.9	105.3 /53.7	1.17	1.438	1.23	1.098	0.94
F	1.1	13.84	8.34	16.20 /31.1	4.28	4.98	1.16	4.075	0.95
G	0.7	10.26	4.95	11.4 /25.7	1.21	1.507	1.24	1.145	0.95
H	1.0	13.76	7.40	15.6 /28.3	1.25	1.480	1.18	1.172	0.94
I	5.9	100.8	50.7	112.8 /26.7	0.05	0.069	1.4	0.047	0.94

NOTE: The reactance and impedance values for lines *A*, *B*, *C*, *D* and *E* include the reactances of cables and of the reactors in series with the cables.

example cables with reactors in series were considered jointly as a single circuit element. By this procedure the magnitude of the errors of the impedance method is reduced slightly. The results of the reactance method are, of course, not affected by this combination.

The results of the examples on cable circuits may be summarized as follows: In cable systems having cables of the same impedance angle (without current-limiting reactors), a cable impedance angle of the order of 30 deg. does not cause serious errors by the reactance method in the near short-circuit of example 5, but the error becomes nearly 20 per cent when, for a more remote short-circuit, in the same system, the cable circuit impedance is larger than the station impedance. Reactors inserted in some of the cables (example 7) cause series-parallel combinations of lines of dissimilar impedance angles giving, by the reactance method, a wider range of errors, exceeding 20 per cent in some of the longer cable lines carrying minor components of short-circuit current. The impedance method under these conditions (example 7) gives better accuracy, the errors being 6 per cent low or better. The use of reactors in all cables will, of course, tend to equalize the impedance angles of the cable circuit elements and improve the accuracy of the reactance method. More general data on the errors due to parallel-connected circuit elements of dissimilar impedance angles are given in a later paragraph.



4–10,000 Kv-a., 6.6 Kv., 50~ alternators. 3φ total reactance = 6%
T_1
2–20,000 Kv-a., transformer banks total reactance 4%
Two 110 Kv. aerial lines, each 40 miles long
Each line consists of three No. 4/0 B. & S. copper equivalent aluminum steel-core cables, 0.74 in. outside diameter
T_2
B
B
4–7500 Kv-a 110/12 Kv. transformer banks.
0.75 miles
1 mile
1 mile
1 mile
12–Kv Cable system, each cable having 3 conductors of 250,000 cir-mil copper. All cable circuits have two parallel lines
1.25 mile
6 miles
1.5 mile
2 miles
1.5 mile

FIG. 2

Reference kv-a. for all impedance data = 20,000

Systems having both Aerial Conductors and Underground Cables: In some transmission systems energy is transmitted over aerial circuits for a number of miles and then distributed underground in cables. Such a system is that shown in Fig. 2, for example 8.

Example 8

For a reference basis of 20,000 kv-a., the system data are as follows, from Fig. 2.

	Resist. per cent	React. per cent	Imped. per cent
Generating station		6.0	6.0 $\underline{/90}$ deg.
Transformers at T_1, total equivalent	0.4	4.0	4.02 $\underline{/84}$ deg.
110 kv. lines, total equivalent	0.9	1.9	2.10 $\underline{/65}$ deg.
Transformers at T_2, total equivalent	0.6	5.3	5.33 $\underline{/84}$ deg.
12 kv. cable system from busses *B B* to point of short circuit *x*, total equivalent	6.3	3.1	7.02 $\underline{/26}$ deg.

By vector addition, the total system impedance is

$$8.2 + j\,20.3 = 21.9\underline{/68} \text{ deg.}$$

The scalar sum of all impedances is 24.5 per cent. Then the error of the reactance method is determined by

$$u_x = 21.9/20.3 = 1.08$$

which means an error of 8 per cent (high). Likewise the error of the impedance method is found from

$$u_z = 21.9/24.5 = 0.89$$

giving an error of 11 per cent (low). Although the cable portion of this system has an impedance angle as low as 26 deg., the total system impedance angle is 68 deg., since the circuit elements of high impedance angle prevail; accordingly, the error of the reactance method is not excessive. In this type of circuit the elements of dissimilar impedance angle occur in series connection only. A series-parallel combination of circuit elements of dissimilar impedance angles, due to cables and aerial lines, is illustrated in the next example.

Example 9

In the system under consideration, shown in Fig. 3, a three-phase short-circuit is located at *a*. The principal current path to the short-circuit is over line *A*. All the other lines may be considered as being consolidated into a single circuit *K* representing *B, C, D, E, F, G, H, J*, in combination. The impedance of *K* then is

$$z_k = 8.03 + j\,1.65 = 8.19\underline{/11.6} \text{ ohms}$$

The combination of this impedance, together with that of *A*, namely

$$z_A = 1.11 + j\,2.48 = 2.72\underline{/65.9} \text{ ohms}$$

is equivalent to the distribution network of Fig. 3, for a three-phase short-circuit at *a*. Circuits *A* and *K* are in multiple. The accuracy of the short-circuit currents in generator, branch *A* and branch *K* are as follows:

	Per cent error react. method	Per cent error imped. method
Total short-circuit current, in generator	95 per cent high	6 per cent high
Short-circuit current in line *A*	5 per cent low	2 per cent low
Short-circuit current in *K* representing *B, C, D, E, F, G, H, J* in combination	330 per cent high	2 per cent low

This circuit condition, while somewhat unusual, serves to illustrate the magnitude of the errors that may arise from the use of the reactance method in systems having parallel connected circuit elements of widely dissimilar impedance angles. The circuit construction of Fig. 3 has the disadvantage of giving large interchange currents in normal system operation and is, therefore, not common.

ANALYSIS OF CIRCUITS

From the preceding examples it is seen that, for the determination of the accuracy of the short-circuit calculating table, electric power systems may be divided into two classes:

(a) those systems in which the circuit elements of dissimilar impedance angles occur in series connection only; *i. e.* all parallel connected circuit elements are of the same impedance angle; systems of this class are illustrated by examples 1, 2, 3, 5, 6, and 8.

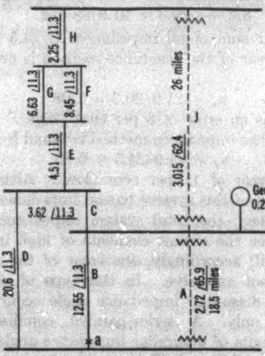


FIG. 3

Full vertical lines indicate 10,000-volt 3-conductor cables. Dotted lines indicate 50,000-volt aerial 3-phase circuits. All impedances are expressed in ohms with their impedance angles. Impedances of circuits *A* and *J* include transformer impedances.

(b) those systems in which the circuit elements of dissimilar impedance angles occur in multiple as well as in series. Such circuits are represented by examples 4, 7 and 9.

The following analysis of circuits is made on the basis of this classification.

Systems in which the circuit elements of dissimilar impedance angles occur in series connection only. The simplest circuit of this type is that of Fig. 4, in which z_G represents the impedance of the generating station and z_1 represents the impedance of the distribution network to the point of short-circuit, all distribution lines having the same impedance angle. The accuracy of the reactance and impedance methods was determined by calculation of the accuracy factors, as explained in example 1. These factors for the reactance method are plotted in Fig. 5 for a wide variety of circuit constants covering a range of variation of z_G from 0.2 z_1 to 2.0 z_1 and a wide range of variation of impedance angle of z_1 from 90 deg. (pure reactance) to 0 deg. (pure resistance). The application of these curves will be illustrated by the following case employing the system of example 6:

z_G = impedance of generating station = $0.5 + j\,16.5 = 16.5\underline{/88}$ deg.

z_1 = impedance of distribution system to point of short-circuit = $15.3 + j\,7.75 = 17.15\underline{/27}$ deg.

With practically no error, the angle of z_G may be taken as 90 deg. The ratio of $z_G/z_1 = 0.96$. In sheet 5, for an abscissa of 27 deg. the ordinate for $z_G = 0.96\,z_1$ is, by approximate interpolation, 1.19 (*i. e.* slightly greater than the ordinate for the curve $z_G = z_1$). The error by reactance method is, therefore, 19 per cent high and agrees with the result previously given.

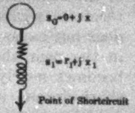


FIG. 4—SERIES CIRCUIT DIAGRAM

When a system of this type has more than two series-connected portions, all having different impedance angles, the data of Fig. 5 may be applied as long as all parallel-connected circuit elements have the same impedance angle. Example 8 is of this type. This system of five major portions may be reduced to two series-connected impedances, namely:

z_G = generating station and $T_1 = 0.4 + j\,10.0 = 10.0$

z_1 = series combination of 110 kv. lines, T_2 and 12 kv. cables = $7.8 + j\,10.3 = 12.9\underline{/53}$ deg.

It follows that $z_G = (10.0/12.9) \times z_1 = 0.78\,z_1$

Correspondingly, the accuracy factor of the short-

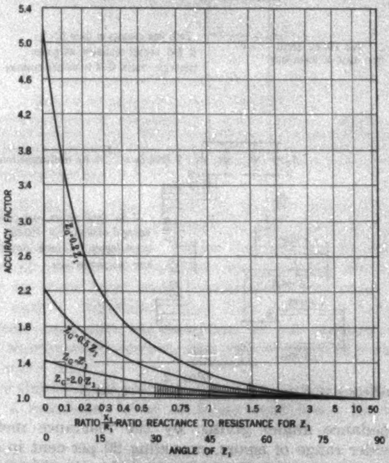


FIG. 5—ACCURACY FACTOR

Total current, reactance method. Circuit: A generator supplying a single circuit. Variation of magnitude of Z_G and of angle of Z_1; $/Z_G/ = 0.2, 0.5, 1.0$ and 2.0 times $/Z_1/$; Z_G angle, 90 deg. constant; Z_1 angles, 0 deg. to 90 deg.

circuit current by reactance method is, from Fig. 5, for an abscissa of 53 deg. approximately 1.08, a value identical with that found by previous calculations.

The shaded parts of Fig. 5 indicate circuit conditions for which errors safely below 20 per cent are obtained by the reactance method in this type of system. Typical circuit conditions resulting in errors within this limit are the following:

Generating station reactance not lower than distribution system impedance and angle of the latter not lower than 30 deg.

Generating station reactance not lower than one half the distribution system impedance and angle of the latter not less than 45 deg.

Generating station reactance not lower than one fifth the distribution system impedance and angle of the latter not less than 55 deg.

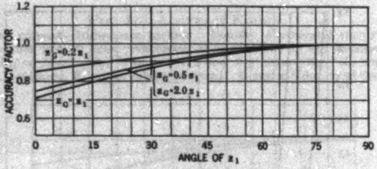


Fig. 6—Accuracy Factor

Total current, impedance method. Circuit: A generator supplying a single circuit. Variation of magnitude of Z_C and of angle of Z_1; $/Z_G/ = 0.2$, 0.5, 1.0 and 2.0 times $/Z_1/$; Z_G angle, 90 deg. constant; Z_1 angles, 0 deg. to 90 deg.

These circuit conditions include the majority of aerial systems and a large number of cable systems.

The results of the impedance method for the same range of series circuit conditions covered by Fig. 5 are given in Fig. 6. The accuracy factors shown indicate low values of short-circuit current. It is seen that the errors for the customary circuit conditions are not consistently smaller than those obtained with the reactance method.

It follows from these considerations that the reactance method is preferable to the impedance method under the circuit conditions stated above (as covered by the shaded portions of Fig. 5), the former method giving values of short-circuit current slightly high, and not necessitating impedance determinations.

Application of Results for Series Type of Circuits. The determination of the error for systems within this class requires the knowledge of the impedance angle of the distribution system (62 deg. in the case of example 1) and of the ratio of generating-station reactance to distribution-system impedance. The latter ratio is approximately equal to the product of the sine of the distribution system impedance angle times the ratio of generating station reactance to distribution system reactance, the latter being obtained from the test data by the calculating table when completely connected up for short-circuit tests.

Systems in which parallel-connected circuit elements have impedance angles less than 15 deg. apart may be considered as belonging to the class of systems under consideration. It should be pointed out here that a reactor placed between a distribution line and the bus from which it is fed must be considered as a separate circuit element for a short circuit occurring in the line near its junction to the reactor. The reactor will then be in multiple with other impedances, generally of lower impedance angle, if there are other lines feeding short-circuit current from the bus towards the fault. In such cases, the results derived above for series circuits do not apply.

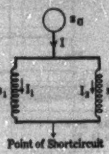


Fig. 7—Parallel Circuit Diagram

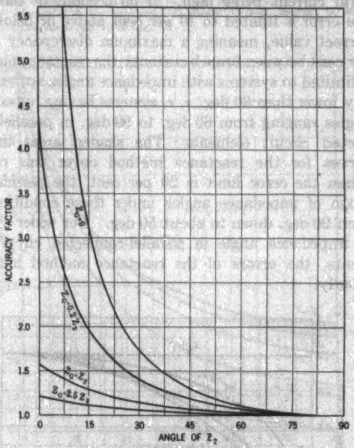


Fig. 8a—Accuracy Factor

Total Current—Reactance Method. Circuit: A generator supplying two parallel branch lines. Variation of magnitude of Z_G and of angle of Z_2: $/Z_1/ = /Z_2/$; $/Z_G/ = 0$, 0.2 and 2.5 times $/Z_2/$; Z_G angle, 90 deg. constant; Z_1 angle, 90 deg. constant; Z_2 angles, 0 deg. to 90 deg.

Systems having Multiple-Connected Impedances of Dissimilar Angles. The simple circuit chosen for the consideration of multiple circuits is that of Fig. 7, where z_G represents a generating station (of substantially pure reactance) supplying two parallel impedances z_1 and z_2. It frequently occurs that part of a network under short circuit consists of a number of impedances of substantially the same angle θ_1, the remaining portion having impedances of another angle, θ_2. Networks of this type may be reduced to the circuit of Fig. 7. The accuracy factors were determined by the customary calculations and plotted for the total current I and for each of the branch currents I_1 and I_2. The circuit conditions covered by the curves so obtained are tabulated in Table II. These circuit

TABLE II

Circuit Conditions Covered by Accuracy Data in Figs. 8, 9 and 10. The Data Apply to Circuits of the Type Shown in Fig. 7.

Fig. No.	Method of short-circuit-current determination	Constants of Generator and of Branch Circuits (Fig. 7)					
		z_g		z_1		z_2	
		Magnitude	Angle	Magnitude	Angle	Magnitude	Angle
8A, B, C........ 8D, E..........	React. method Imped. method	from 0 to 2.5 times z_1	90 deg.	$= z_2$	90 deg.	$= z_1$	from 0 deg. to 90 deg.
9A, B, C........ 9 D, E..........	React. method Imped. method	$= z_1$	90 deg.	$= z_2$	from 15 deg. to 90 deg.	$= z_1$	from 0 deg. to 90 deg.
10 A. B, C...... 10 D, E........	React. method Imped. method	$= z_1$	90 deg.	$= 0.2\ z_2$	from 15 deg. to 90 deg.	$= 5\ z_1$	from 0 deg. to 90 deg.

conditions include variations of magnitude of generator and branch-circuit impedances as well as variations of impedance angle for each of the branch circuits.

Reactance Method. From an examination of the curves in Figs. 8A, B, C, 9A, B, C and 10A, B, C it is seen that in a number of cases one of the branch currents is low, the current in the other branch and the total current being high. If on account of this fact the error is limited to 10 per cent above or below the correct value, meaning a maximum discrepancy of 20 per cent between branch currents, the reactance method is limited to systems with impedance angles not materially lower than 60 deg., *i. e.* systems having impedance angles ranging from 60 deg. to 90 deg. in parallel-connected circuit elements. The shaded areas in the curves for the reactance method cover this range. When the error limit is 20 per cent, the permissible range of impedance angles under these conditions is from 90 deg. down to about 50 deg. For wider ranges of impedance angle in parallel-connected circuit elements, the errors of the reactance method increase rapidly.

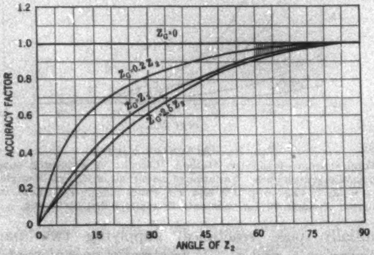


FIG. 8B—ACCURACY FACTOR

Branch current I_1—Reactance method. Circuit: A generator supplying two parallel branch lines. Variation of magnitude of Z_G and of angle of Z_2; $/Z_1/ = /Z_2/$; $/Z_G/ = 0$, 0.2, 1.0 and 2.5 times $/Z_2/$; Z_G angle, 90 deg. constant; Z_1 angle, 90 deg. constant; Z_2 angles, 0 deg. to 90 deg.

Impedance Method. The curves in Figs. 8D, E, 9D, E and 10D, E indicate that some types of systems for which the reactance method gives excessive errors may be solved by the impedance method with better accuracy. The prevailing tendency of the impedance method is to give low values of short-circuit current. High values occur occasionally but these are only slightly large for impedance angles as low as 30 deg. For an error limit of 15 per cent, indicated by the shading in Figs. 8D, E, 9D, E and 10D, E, the impedance method is applicable to systems with parallel-connected circuit elements of widely dissimilar impedance angles including angles as low as 30 deg. The following example will illustrate a case of this kind in a circuit similar to Fig. 7. Let the percentage values of impedance be as follows:

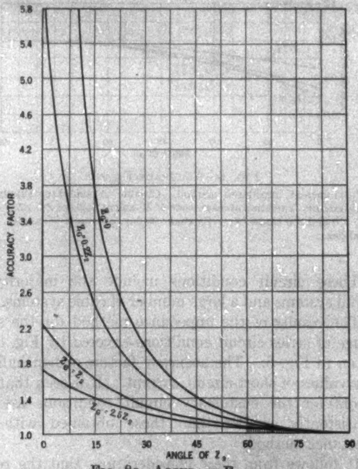


FIG. 8C—ACCURACY FACTOR

Branch current I_2—Reactance method. Circuit: A generator supplying two parallel branch lines. Variation of magnitude of Z_G and of angle of Z_2; $/Z_1/ = /Z_2;/$ $/Z_G/ = 0$, 0.2, 1.0 and 2.5 times $/Z_2/$; Z_G angle, 90 deg. constant; Z_1 angle, 90 deg. constant; Z_2 angles, 0 deg. to 90 deg.

$$z_G = 10\ \underline{/90}\ \text{deg.}$$
$$z_1 = 10\ \underline{/90}\ \text{deg.}$$
$$z_2 = 10\ \underline{/40}\ \text{deg.}$$

where z_1 represents a feeder reactor (the short circuit being just beyond the reactor in a cable), and z_2 the impedance of the other cable paths feeding into the short circuit from the generator. By the impedance method, the accuracy factor for the total short-circuit current is 1.01 from Fig. 9D. This represents an error of 1 per cent high. For each of the two branch currents, the error is 8 per cent low, obtained from Fig. 9E from the accuracy factor 0.92 measured as the ordinate to the curve for an angle of z_1 equal to 90 deg. for an abscissa of 40 deg.

This discussion leads to the conclusion that for systems not covered by the reactance method but having impedance angles not materially below 30 deg. the impedance method gives errors well below 20 per cent and commonly below 10 per cent. The short-circuit current values are generally low. When parallel-connected circuit elements have dissimilar impedance angles ranging from 90 deg. to angles materially below 30 deg., excessive errors may occur.

Additional Factors. While the results given above were derived from an analysis of the two simple types of circuit shown in Figs. 4 and 7, they do not apply

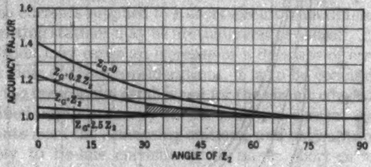


FIG. 8D—ACCURACY FACTOR

Total current—Impedance method. Circuit: A generator supplying two parallel branch lines. Variation of magnitude of Z_G and of angle of Z_2; $/Z_1/ = /Z_2/$; $/Z_G/ = 0$, 0.2, 1.0 and 2.5 times $/Z_2/$; Z_G angle, 90 deg. constant; Z_1 angle, 90 deg. constant; Z_2 angles, 0 deg. to 90 deg.

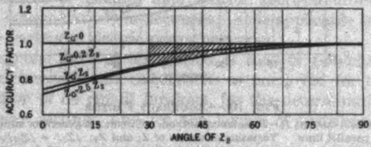


FIG. 8E—ACCURACY FACTOR

Branch currents—Impedance Method. Circuit: A generator supplying two parallel branch lines. Variation of magnitude of Z_G and of angle of Z_2; $/Z_1/ = /Z_2/$; $/Z_G/ = 0$, 0.2, 1.0 and 2.5 times $/Z_2/$; Z_G angle, 90 deg. constant; Z_1 angle, 90 deg. constant; Z_2 angles, 0 deg. to 90 deg.

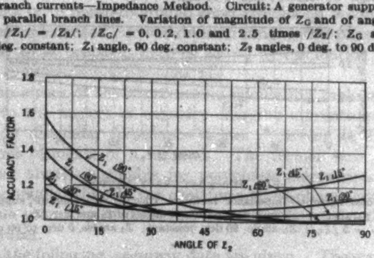


FIG. 9A—ACCURACY FACTOR

Total current—Reactance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_2/ = /Z_G/$; Z_G angle, 90 deg. constant; Z_1 angles, 15 deg, 30 deg., 60 deg., 90 deg.; Z_2 angles, 0 deg. to 90 deg.

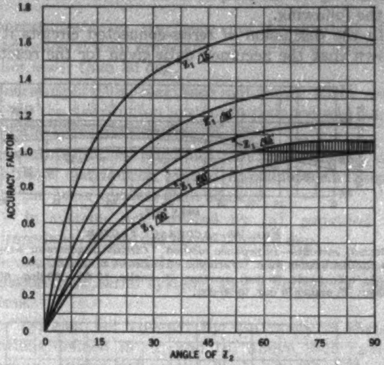


FIG. 9B—ACCURACY FACTOR

Branch current I_1—Reactance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_2/ = /Z_G/$; Z_G angle, 90 deg. constant; Z_1 angles, 15 deg., 30 deg., 60 deg., 90 deg.; Z_2 angles, 0 deg. to 90 deg.

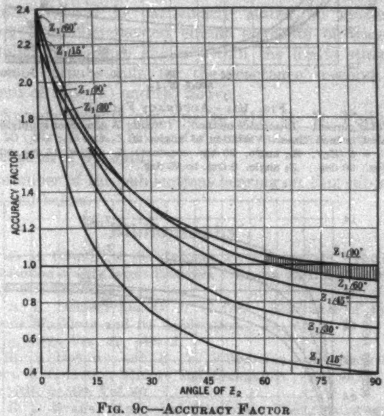


FIG. 9C—ACCURACY FACTOR

Branch current I_2—Reactance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_2/ = /Z_G/$; Z_G angle, 90 deg. constant; Z_1 angles, 90 deg. constant; Z_1 angles, 15 deg., 30 deg., 60 deg., and 90 deg.; Z_2 angles, 0 deg. to 90 deg.

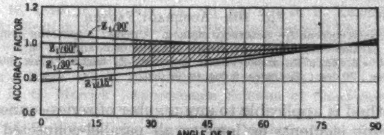


FIG. 9D—ACCURACY FACTOR

Total current—Impedance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_2/ = /Z_G/$; Z_G angle, 90 deg. constant; Z_1 angles, 15 deg., 30 deg., 60 deg., 90 deg.; Z_2 angles, 0 deg. to 90 deg.

exclusively to the simpler circuits, as will be indicated in the following.

In the discussion of series-connected circuits it was shown that the accuracy data given for the reactance method is applicable to any number of series-connected circuit elements as long as all parallel-connected circuit elements have substantially the same impedance angle.

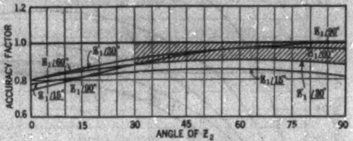


Fig. 9e—Accuracy Factor

Branch currents—Impedance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_2/ = /Z_G/$; Z_G angle, 90 deg. constant, Z_1 angles, 15 deg., 30 deg., 60 deg., 90 deg.; Z_2 angles, 0 deg. to 90 deg.

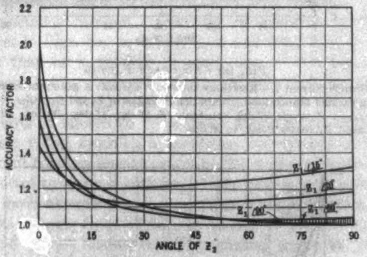


Fig. 10a—Accuracy Factor

Total current—Reactance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_G/$; $/Z_2/ = 5\,/Z_G/$; Z_G angle, 90 deg. constant; Z_1 angles, 15 deg. 30 deg., 60 deg., 90 deg.; Z_2 angle, 0 deg. to 90 deg.

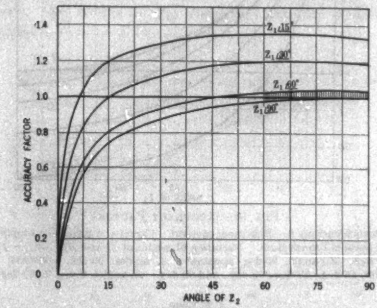


Fig. 10b—Accuracy Factor

Branch current I_1—Reactance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_G/$ $/Z_2/ = 5\,/Z_G/$; Z_G angle, 90 deg. constant, Z_1 angles, 15 deg., 30 deg., 60 deg., 90 deg.; Z_2 angles, 0 deg. to 90 deg.

Likewise, for parallel circuits of three or more multiple impedances ranging from pure reactance to pure resistance, the maximum errors for the impedance method cannot exceed those determined for two multiple impedances in accordance with the preceding analysis. This is due to the fact that the sum of the magnitudes of any number of inductive admittances can never be more than 1.414 times their vector sum.

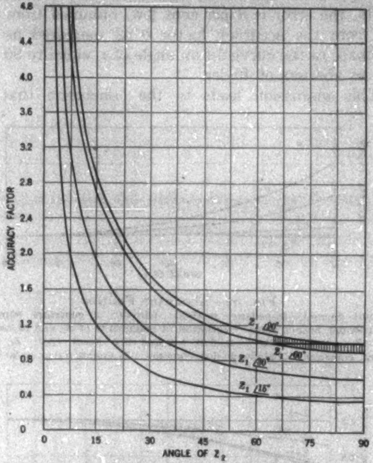


Fig. 10c—Accuracy Factor

Branch current I_2—Reactance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_G/$; $/Z_2/ = 5\,/Z_G/$; Z_G angle, 90 deg. constant; Z_1 angles, 15 deg., 30 deg., 60 deg., 90 deg.; Z_2 angles, 0 deg. to 90 deg.

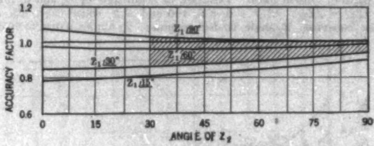


Fig. 10d—Accuracy Factor

Total current—Impedance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_G/$; $/Z_2/ = 5\,/Z_G/$; Z_G angle, 90 deg. constant; Z_2 angles, 0 deg. to 90 deg.

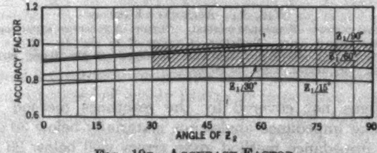


Fig. 10e—Accuracy Factor

Branch currents—Impedance method. Circuit: A generator supplying two parallel branch lines. Variation of angles of Z_1 and Z_2; $/Z_1/ = /Z_G/$; $/Z_2/ = 5\,/Z_G/$; Z_G angle, 90 deg. constant; Z_1 angles, 15 deg. 30 deg., 60 deg., 90 deg.; Z_2 angle, 0 deg. to 90 deg.

Furthermore, the likelihood of obtaining maximum errors diminishes as the number of parallel connected impedances in Fig. 7 increases beyond two.

The error of currents in low-impedance tie lines interconnecting substations in systems having elements of widely dissimilar impedance angles is likely to be greater than that in the feeders supplying these stations on account of distorted voltage drops (such lines are *F*, *G*, and *H* in Fig. 1).

The currents supplied to a short-circuit from different generating stations may be mutually out of phase. This condition may cause an increase or a decrease of error.

In view of these considerations, it is to be concluded that the error limits established for the circuit constants specified apply primarily to the principal current components from the generating stations, and only approximately to tie-line currents or to minor short-circuit-current components.

CONCLUSIONS

The short-circuit calculating table is a simple practical means of making approximate short-circuit-current determinations in networks. It is applicable to a large variety of electric-power networks.

Its chief field of application is that of polyphase short-circuit-current calculations as employed for the selection of oil circuit breakers, for the determination of relay settings, and for the layout of distribution systems.

It is not generally applicable to phase-to-phase short-circuits or to phase-to-ground short-circuits on account of the unbalanced current division among the phases. These latter problems may be solved by calculation or by a-c. test.

The errors of the short-circuit calculating table are due to the neglect of capacity charging currents and due to dissimilar impedance angles occurring in the various elements of a network. The former error is almost always negligible for polyphase short-circuits, being smaller than 10 per cent for a three-phase short-circuit occurring in an aerial line 200 miles from the generator.

In the following, accuracy data are given. The error limits established for the circuit conditions specified apply primarily to the principal current components from the generating stations, and only approximately to tie-line currents.

In a great many systems the resistance component of all the impedances in the network may be entirely neglected for short-circuit-current determinations. This method, the reactance method, generally gives erroneously high current values. The errors are safely below 20 per cent, and commonly under 10 per cent,

(a) in systems in which circuit elements of dissimilar impedance angles occur in series connection only, (*i. e.* parallel-connected circuit elements have impedance angles less than 15 deg. apart), provided the circuit constants are within a definite range of the following character:

Generating-station reactance not lower than distribution system impedance, and angle of the latter not lower than 30 deg.

Generating-station reactance not lower than one-half the distribution system impedance and angle of the latter not less than 45 deg.

Generating-station reactance not lower than one-fifth of distribution system impedance and angle of the latter not less than 55 deg.

(b) in systems not covered by (a), provided all impedance angles are larger than 55 deg.

The systems covered by (a) and (b) above include the majority of aerial systems and a variety of cable systems.

For systems not covered by the range of circuit conditions defined in (a) and (b) above, but having impedance angles not materially below 30 deg., the impedance method is preferable and gives errors well below 20 per cent and commonly below 10 per cent. The short-circuit current values are generally low. Cable systems with current-limiting reactors in some of the lines come within this class. While the impedance method is particularly suited to systems having parallel-connected circuit elements of widely dissimilar impedance angles within the limits stated, it is not a substitute for the reactance method in systems of type (a) and (b) above.

Acknowledgments are due to Mr. G. B. Phillips and to Mr. A. R. Miller for their assistance in the preparation of the data.

A number of the publications bearing on this subject are listed below.

Bibliography

1. E. M. Hewlett, J. N. Mahoney, G. A. Burnham. Rating and Selection of Oil Circuit Breakers. TRANS. A. I. E. E. 1918, p. 123, Vol. 37.

2. R. E. Doherty and O. E. Shirley. Reactance of Synchronous Machines and its Application. TRANS. A. I. E. E. 1918, p. 1209, Vol. 37, part 2.

3. R. E. Doherty and E. T. Williamson. Short-circuit Current of Induction Motors and Generators. TRANS. A. I. E. E. 1921, p. 509, Vol. 40.

4. R. D. Evans. Analytical Solutions. *Elec. Jour.*, August 1919, p. 345.

5. Charles Fortescue. Development of Analytical Solutions. *Elec. Jour.*, Aug. 1919, p. 350.

6. A. E. Kennelly. The Equivalence of Triangles and Three-Pointed Stars in Conducting Networks. *Elec. Wld. & Eng.*, Sept. 16, 1899, p. 413, Vol. 34, No. 12; also A. E. Kennelly, The Application of Hyperbolic Functions to Electrical Engineering Problems. *University of London Press*, 1912, p. 222.

7. A Device for Calculating Currents in Complete Networks of Lines. *G. E. Rev.*, October 1916, p. 901.

8. E. G. Merrick. Approximate Solution of Short-circuit Problems. *G. E. Rev.*, June 1916, p. 470.

9. H. R. Wilson. An Approximate Method of Calculating Short-circuit Current in an A-C. System. *G. E. Rev.*, June 1916, p. 475.

10. W. W. Lewis. Calculation of Short-circuit Currents in Alternating-Current Systems. *G. E. Rev.*, Feb. 1919, p. 140.

11. W. R. Woodward., Calculating Short-circuit Currents in Networks. *Elec. Jour.* Aug. 1919, p. 344.

12. W. W. Lewis. A New Short-circuit Calculating Table. *G. E. Rev.*, Aug. 1920, p. 669.

13. E. A. Hester, O. C. Traver, R. N. Conwell, L. N. Crichton. Transmission-Line Relays, II. A. I. E. E. Jour., Nov. 1922, p. 839.

14. W. W. Lewis. Short-circuit Currents on Grounded-Neutral Systems. *G. E. Rev.*, June 1917, p. 524.

15. G. H. Gray. Design, Construction and Tests of an Artificial Power-Transmission Line for the Telluride Power Co. of Provo, Utah. Trans. A. I. E. E. 1917, p. 789, Vol. 36.

16. W. W. Lewis. Present Status of High-Voltage Transmission of Power. Part I. *Gen. Elec. Rev.*, Oct. 1922, p. 628.

17. W. W. Lewis. Present Status of High-Voltage Transmission of Power. Part II. *G. E. Rev.*, Nov. 1922, p. 674.

Discussion

R. E. Doherty: I wish to discuss two points bearing upon this paper: One regarding accuracy; the other, regarding the extension of the use of miniature models to determine, in addition to the short-circuit current distribution under steady state, as covered by the present paper, the most desirable feeder connections for normal load transfer; and further, the behavior of stations and generating units under transient state.

Although the impedances of lines, cables, reactance coils, etc., can be calculated with a fair degree of accuracy, the calculated value of *transient reactance*[1] of the generators is not so accurate. It can, of course, be determined with fair accuracy by actual short-circuit tests; but in calculations an error as great as 15 or 20 per cent may be expected in some cases. The principal reason for relatively large errors in the calculated values is the effect of abnormal saturation of local flux paths under short circuit. The phenomenon of saturation is extremely complicated and different types of machine construction involve different degrees of saturation. Hence it becomes very difficult to calculate the result. Therefore, in the calculated, or rather estimated, values of *transient reactance*, such errors as I have mentioned must be expected.

This, of course, is no reason, as may be suggested, why the accuracy of the "calculating table" itself should not be better than 20 per cent. The former source of error is necessary, unless tested results are available; but that is no reason why another *known* error of 20 per cent should be superposed, perhaps in the same direction. I therefore agree with the author that by the proper use of the various curves given in the paper, together with a study of the examples, excessive error from the table itself can be avoided. However, the possible error in generator reactance should be kept in mind, especially in those cases where the external reactance is relatively low.

Another source of error is the set of decrement curves usually applied in this connection. I refer to the Hewlett, Mahoney and Burnham curves. The calculating table gives the "initial" or the "sustained" current, as the case may be. In selecting oil circuit breakers, the time element enters; and if such curves are applied to determine the effect of the time element, that is, the decrease in current due to the transient, the possible error in such curves should be kept in mind. As everyone knows, who has read the Hewlett, Mahoney and Burnham paper, these curves represent the first step in the direction of giving some basis for selection of switches, and combining into a single set of curves, as they do in the interest of simplicity, numerical values covering such wide limits, they must necessarily contain sources of considerable error. These errors, however, are, I believe, usually on the safe side, thus giving current values too high. The point is, that having obtained the "initial" value, perhaps by the calculating table, there is possible further error in applying the decrement curves.

Hence, from the foregoing discussion, error may arise from three sources, namely: The generator reactance, the calculating table, as the author brings out, and the application of the Hewlett, Mahoney and Burnham curves. It has, therefore, seemed advisable to offer the suggestion that, in the use of the calculating table, these additional sources of error be kept in mind.

What appears to be a promising extension of the application of miniature models is the representation of certain isolated sections of a network, including receiving circuit impedances as well as the feeder impedances, for the purpose of determining the most desirable connections of the feeders. This is usually a compromise between the connection which is the most favorable for copper on the one hand and simplicity, relay settings, etc., on the other. Such an extension would obviously involve the case of both resistance and reactance units and to this extent would be more complicated than the direct-current table. This scheme was proposed by G. M. Armbrust of the Commonwealth Edison Company of Chicago, who is investigating the feasibility of such an outfit.

Referring to the possibility of reproducing the transient phenomena in generating stations under short-circuit conditions, it may be of interest to mention that a study and detail design has been made of a miniature generating system, reproducing in small generators the same relation of momentum, reactance, armature reaction, etc., as exists in the actual large units. It thus appears feasible to represent with a small number of machines a relatively large variety of combinations of units. It may be of interest also that these miniature machines are not a geometrical reproduction of the larger machine on a small scale, since such a machine, even if practically feasible to build, cannot be made to represent the actual machines.

G. M. Armbrust: Direct-current calculating tables have been used extensively in large systems to determine short-circuit currents for use in determining relay settings, design of systems, etc. A high degree of accuracy has been obtained in systems having considerable reactance. There has been some doubt, however, as to how extensively the method could be used on cable systems owing to errors introduced by varying impedance angles of the component parts of the system.

Mr. Schurig's paper clears up much of this doubt, and considerably extends the use of the calculating table by outlining its limitations and defining the probable degree of error under a classification of conditions representing most distribution problems.

The Commonwealth Edison Company of Chicago operates some 900 miles of transmission cable, the system being practically entirely underground. Using Mr. Schurig's method described in his paper, I have checked a number of typical cases on this system and obtained results of a reasonable degree of accuracy, agreeing with his conclusions.

The layout of a system such as this, a combination of network and radial distribution, is determined by economy, limitations due to possible short-circuit currents, and feasibility of relay protection. Theoretically the best transmission economy is obtained by an extensively inter-connected network, but the necessity for limiting short-circuit current to the ability of existing equipment to handle it, makes necessary considerable sectionalizing of the system. Determination of the economical design with this limitation requires the solution of many such cases having a wide variation of characteristics—also the changing standards of equipment used, such as larger-sized cables having greater impedance angles, use of reactors of various values, complicate the problem and make it a continuous one.

These considerations make desirable the use of an alternating-current calculating table representing both resistance and reactance, though not to replace the direct-current table which, because of its simplicity of operation offers a quick and sufficiently accurate solution for most problems. Another valuable use of an alternating-current representation of this system could be obtained by designing the scheme to include units representing loads and their power factors for the purpose of determining the most economical arrangement of the transmission system for load distribution within the limitations of operation, short-circuit currents, etc.

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[1] The generator reactance which is active under sudden short circuit.

O. R. Schurig: Both Mr. Doherty and Mr. Armbrust have referred in their discussions to the problem of determining experimentally, by miniature equivalent circuit, the division of normal current flow in a network. The following discussion is devoted to the question of determining whether this problem may be solved satisfactorily by a d-c. miniature system such as the "short-circuit calculating table." There are three reasons why a d-c. device is not to be recommended for general application to load-current division problems; viz:

(1) For normal current flow in the lines of a network a higher degree of accuracy is generally required than for short-circuit currents.

(2) Capacity currents frequently are a material portion of the total current flowing.

(3) Tie-line current flowing between generating stations or between substation busses may be greatly in error, when determined by a d-c. method.

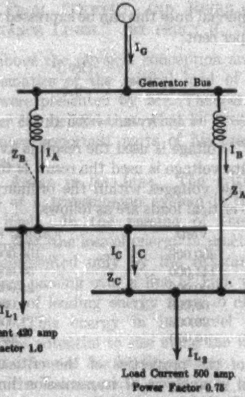

Fig 1—Diagram of Circuit Illustrating Determination of Normal Current Flow (due to loads) in a Distribution Network

Circuit at Z_A represents three cables in multiple with a current limiting reactor in each cable at the generator end.

Circuit at Z_B represents two cables in multiple with a current limiting reactor in each cable at the generator end.

The first point may be considered with reference to the accuracy data presented in the paper: If, for example, the power factors of the loads in a system range from 0.70 to 0.95, the corresponding range of impedance angles is from 46 deg. to 18 deg., and the load reactances in these two cases are 71 per cent and 31 per cent, respectively, of the load impedance magnitudes. Since the load currents depend on impedances, rather than reactances, it will be seen at once that the reactance method is entirely unsuitable under load conditions similar to those mentioned. Thus the impedance method, only, comes into consideration. Its accuracy for load current division will be considered below.

Capacity currents cannot be represented together with load currents in a d-c. miniature system, because of the wide phase difference between them. It follows, then, that for systems having large capacity charging currents—such as high-voltage aerial systems with long lines—the d-c. method applied to the determination of normal-current division cannot give representative results.

Interchange currents flowing over tie lines between generator busses are frequently displaced in phase from the generator currents feeding these busses. Since a d-c. representation of these currents cannot take account of their respective phase relations, considerable errors in these interchange currents may be encountered if the d-c. method is applied.

Similarly, the currents flowing in tie lines between substation busses are likely to be greatly misrepresented by the d-c. method, when the power factors of the loads on these busses differ materially. This will be shown by the following example. Let the circuit be that of Fig. 1. The load currents I_{L1} and I_{L2} are given and will be considered fixed in magnitude and phase. The problem is to determine currents I_A, I_B, I_C, and I_G by the d-c. method employing impedance magnitudes. Let

I_{L1} = 420 amperes at power factor 1.0

I_{L2} = 500 amperes, lagging 41 deg. behind I_{L2}, (*i. e.* at power factor 0.75)

$Z_A = 0.307 + j\,0.599 = 0.673 \;\underline{/\,63}$ deg. ohms.

$Z_B = 0.276 + j\,0.485 = 0.558 \;\underline{/\,60}$ deg. ohms.

$Z_C = 0.246 + j\,0.148 = 0.287 \;\underline{/\,31}$ deg. ohms.

Note: These line impedances are for 12-kv., 3-conductor, cables at 60 cycles per sec. The values of Z_A and Z_B include current limiting reactors.

The results are as follows, for the above fixed values of I_{L1} and I_{L2}.

Current symbols referring to Fig. 1	Correct currents determined by complete a-c. solution. Amp.	Currents by d-c. method employing impedance magnitudes. Amp.	Error of approximate currents
I_A	406	417	3% too high
I_B	459	503	10% too high
I_C	135	—3	98% too low
I_G	863	920	7% too high

It is seen that the currents over circuits *A* and *B* and in the generator are not over 10 per cent in error, while the tie-line current I_C is entirely misrepresented by the d-c. method. The large error is due to the phase displacement between load current I_{L1} and tie-line current I_C, the latter current being substantially "wattless." This example, therefore, illustrates how the d-c. method may grossly mislead in the determination of tie-line current for systems having loads of dissimilar power factors.

It should be pointed out here that in short-circuit current determinations with the calculating table the likelihood of excessive errors in tie-line currents is far more remote since there is only one load—*i. e.* the short-circuit—and since parallel lines of widely dissimilar impedance angles are generally avoided on account of the interchange currents they produce in normal operation.[2]

In a strictly radial system of distribution, *i. e.* with no tie lines between substations, the last-mentioned errors will not occur in a d-c. representation of the system. On the other hand, it may be argued that in the purely radial type of distribution system calculation of load current division is less complicated and the need for a miniature system less urgent than for systems having a complex distribution network.

To summarize: For the determination of normal current division in a-c. distribution networks, the d-c. miniature test method is not generally applicable on account of errors due to dissimilar load impedance angles (*i. e.* dissimilar load power factors) and to the neglect of capacity currents. Under these conditions, the solution of this problem by miniature test calls for an a-c. miniature system having reactors and resistors, and, in some cases, condensers. These conclusions corroborate those presented by Mr. Armbrust and by Mr. Doherty.

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[2] This particular cause of interchange currents in normal operation is largely eliminated by designing parallel distribution circuits to have equal impedance angles, *i. e.*, to have substantially equal ratios of reactance to resistance. Equality of line impedance angles does not, however, do away with tie-line currents caused by the nature of the *loads* on a system.

Qualitative Analysis of Transmission Lines

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Review of Subject.*—The process proposed in this paper is called "Qualitative Analysis of Transmission Lines" because of its similarity to chemical qualitative analysis.*

Based upon a previous presentation by Mr. Percy H. Thomas the "critical load" for transmission lines is introduced with a simple formula for determining it and various rules are given for the variation of power factor voltage, etc., of a line by relating the actual load to the critical load.

The most important parts of Mr. Thomas's presentation are abstracted and presented showing the fundamental physical and mathematical basis for this method of analysis.

The detail development of the formula for the critical load is then given and the effect of the various line constants upon it are briefly discussed.

Working from the critical load a vector interpretation of the action of a long high-voltage transmission line is given. From this vector interpretation it is possible to predict certain essential operating characteristics of transmission lines.

Based on the previous generalities practical examples of transmission lines operating at critical load are then given showing how it is possible by the qualitative analysis to predict desired conditions of operation. Two of these examples show the operation of a 3100 mile copper transmission line of 220 kv. nominal potential. Another shows a 3875-mile aluminum transmission line.

In the conclusion an appeal is made for further work along these lines and it is pointed out particularly that this is not an approximate method but an exact method for "Qualitative Analysis of Transmission Lines."

INTRODUCTION

THIS paper is entitled "Qualitative Analysis of Transmission Lines" because of the similarity of the process set forth to *qualitative* analysis in chemistry. Transmission line calculations according to the well-known formulas can be considered as in the class of *quantitative* chemical analysis; results are obtained not only giving the character of the quantities but also their amounts.

The value of *qualitative* analysis of transmission lines lies first in the results obtainable by the simple rules set forth. But it has a second and greater value to one trying to arrive at a conception of what is really happening in a transmission line; it gives a basis for a mental picture and a true conception of the evolutions in the line and the effects upon them of changes in any of the constants.

CRITICAL LOAD

The qualitative analysis proposed is based upon the fact that for each transmission line there is a *critical load* which at unity power factor behaves practically like direct current on the line.

Analyses are made by relating the load conditions for a given line to this critical load.

The critical load was first discovered and presented to the Institute by Mr. Percy H. Thomas in 1909[1]. Unfortunately little practical use has been made of Mr. Thomas's discovery. The physical concept behind the critical load formula can not be better given than in Mr. Thomas's own words which will be quoted later.

Starting from Mr. Thomas's conclusions, the formula for the critical load of any transmission line has been derived and is as follows:

$$\text{kv-a.} = \frac{10^3\,(\text{kv.})^2}{\sqrt{L/C}} \qquad (1)$$

For commercial lines this can be expressed with accuracy within 5 per cent:

$$\left.\begin{aligned} \text{kv-a.} &= \frac{(\text{kv.})^2}{0.4} \\ \text{or:} \quad \text{kv-a.} &= 2.5\,(\text{kv.})^2 \end{aligned}\right\} \qquad (2)$$

If the "Y" voltage is used the result is kv-a. per phase. If the delta voltage is used the result is total kv-a.

For a few voltages within the ordinary commercial range the critical loads are as follows:

Volts	Critical Kv-a.
200,000	100,000
141,000	50,000
100,000	25,000
60,000	9,000
20,000	1,000
10,000	250

Some of the properties of the critical load which are useful in analysis of transmission lines of present commercial lengths are as follows:

(1) The critical load when placed on a transmission line at unity power factor will be transmitted at unity power factor. The current will be constant throughout the line. The voltage drop will be the same as for direct current ($I\,R$). The power loss will be equal to the direct-current power loss ($I^2 R$).

(2) If the kv-a. load is greater than the critical load, the power factor at the sending end of the line will be always more lagging than the power factor at the receiving end of the line.

(3) If the kv-a. load is less than the critical load, the power factor at the sending end of the line will always be more leading than the power factor at the receiving end of the line.

(4) If the load is the critical kv-a. but at a leading power factor, the power factor at the sending end of the line will be still leading but nearer unity.

(5) If the load is the critical kv-a. but at a lagging power factor, the power factor at the sending end will still be lagging but nearer unity.

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[1] "Output and Regulation in Long Distance Lines"; Percy H. Thomas, TRANSACTIONS, A. I. E. E. 1909.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

If the line is longer than any line now in commercial service either leading or lagging critical loads should finally reach the stable condition at a very slight leading power factor, usually better than 0.995.

For a line of unusually large diameter at a small spacing or unusually small diameter at a large spacing the value of $\sqrt{L/C}$ should be taken from Fig. 1 and used in formula (1). The accuracy of formula (2) is however within 5 per cent plus or minus for present commercial transmission systems.

The values of the critical kv-a. with various voltages may easily be read from the slide rule by setting it as shown in Fig. 2. Set "4" on scale B opposite "1" on scale A. The rider is set to the voltage on scale C and the critical load read on Scale A. The decimal point can best be determined by remembering that the critical load for 100 kv. is 25,000 kv-a.

QUOTATIONS FROM "OUTPUT AND REGULATION IN LONG DISTANCE LINES," BY PERCY H. THOMAS

As stated above the physical conception and fundamental mathematics of the critical load of a transmission line were presented by Mr. Thomas in 1909. Nothing better can be said than what he wrote at that time. The more important parts of his presentation are as follows:

"*Electrical Action in the Line.* The critical electrical feature of a transmission system is the action of the line itself. In the passing of energy over a line there is, first, the loss of energy in practical overhead work represented only by the resistance losses. But with power passing over a line at high voltage, a certain amount of leading energy flows to charge the line capacity. This energy in its travel causes a tendency for the potential to rise along the line in the direction of the flow of the leading energy. Similarly there flows to the line a quantity of lagging energy, by virtue of the inductance of the line and the flow of current therein. This lagging energy tends to cause a drop of potential in the line in the direction of its flow. The resultant effect is the difference of the leading and lagging energy-flow, superimposed, of course, upon the true power transmitted and the resistance-drop. *****

"When the lagging energy predominates over the leading energy, the tendency is for a drop in potential along the line in the direction of the lagging flow—the well known inductive drop, which is combined with the ohmic drop.

"When the leading energy predominates there is a tendency for a rise of potential in the direction of the leading energy-flow, which must be combined with the ohmic drop as before. The rise of potential effect of the leading energy will be opposed to the ohmic drop when the transmission of power and the flow of leading energy are in the same direction. * * * * *

"Now consider the case when the leading and the lagging energies are equal and when they are opposite in phase; that is, when the load power factor = 1. In this case there will evidently be no appearance in the line of demand on the generator for either lagging or leading energy, and no tendency either for a rise or a drop of voltage therefrom, and the ohmic drop is the only loss of voltage. This condition will be actually realized in practise if the leading and lagging energies of the line are equal and with a load of unity power factor. If, however, the load current has a lag or a lead, the leading and lagging energies, while they may be equal, will not be opposite in phase and will not neutralize each other. The rather remarkable conclusion follows —that in this case of complete neutralization the loss of voltage is only the ohmic drop and is independent of frequency and is of the same value as with the direct current. Thus if this condition can be realized, and there enters no secondary effect, there will be no further economy in direct-current transmission as far as energy loss or voltage line drop is concerned, the same effective line voltage being used. But the diffculty is that this condition of equality of the leading and the lagging energy in the line is usually not met with in practical transmission of relatively short length and often cannot be economically obtained.

"The unfortunate condition exists that the leading energy, being the condenser energy, is practically constant, whatever the load on the line, since it depends only on the voltage of the line. On the other hand, the lagging energy depends only on the actual amount of power or current being transmitted, since it varies as the square of the line current. Thus with half load or no load, a line that was balanced for full load would have a large excess leading energy flow.

* * * * * *

"It should be carefully borne in mind that the leading and lagging energy and energy-loss so far discussed are wholly confined to the line and have no necessary relation to the leading or lagging current in the load.

"The factors by which the relative values of the leading and the lagging energy of the line are controlled are: the line voltage, the actual load or current, and the ratio of the line inductance and capacity per unit length. The energy taken by the capacity of the line varies as the square of the line voltage, so that this factor is of extreme importance. The lagging energy of the load varies as the square of the current, as has already been mentioned. Neutralization of the leading and lagging line energies can be obtianed only when the power factor of the load current is unity, for only then the leading and lagging energies are opposite in phase.

"The current of the line, of course, decreases directly with the increase of line voltage chosen, if the power transmitted be kept constant.

"The energy flowing to the capacity of the line per unit length is porportional to pCV^2, where pC is the capacity susceptance and V the voltage. The lagging energy will be proportional to pLI^2, where pL is the inductive ohms and I the current. If these are equal then $V/I = \sqrt{L/C}$, which is the condition of balanced out-of-phase energy. It is clear that this can be reached for any line and voltage by properly choosing the current, and for any current by properly choosing the voltage. It is also clear that the condition is independent of frequency.

"Equation (**34**) of the companion paper gives the line voltage equation when $R = O$. Here if the current or voltage is chosen to fulfill the above condition.... the voltage of the line is constant as it should be. The resistance is chosen as zero, as otherwise the change of voltage along the line will destroy the exact balance of the leading and the lagging energy.

"Thus as the net result, the ratio of the leading to the lagging energy varies as the fourth power of the voltage when the power transmitted remains the same. The capacity of the line can be increased by adding condensers in parallel across the line at regular and sufficiently frequent intervals; the inductance can similarly be increased by the addition of choke coils in the same manner. But these expedients have never found much favor. As the power and voltage are often fixed by the other considerations, some other method of altering this ratio than change of power or voltage would be desirable. * * * * * * * *

"It should be said that in this matter of adjustment of the leading and lagging line energies, a change of frequency is of no avail, since both elements, capacity and inductance, are affected in the same manner. Furthermore, while a certain voltage may be most favorable in some particular case for balancing the line effects and reducing the line-drop to the ohmic equivalent, it does not necessarily follow that this voltage will give the best transmission condition, for the advantage of a higher voltage may easily more than offset the favorable adjustment of the leading and lagging energy of the line. Furthermore, the most favorable line conditions will not always be the most favorable for other parts of the system and any complete design will be a compromise, as usual. * * * *

"In the main the electrical action of the line has now been briefly outlined. The most notable features are:

"1. That by properly balancing the leading or capacity energy and the lagging or inductive energy in the *line itself*, the full-load transmission will be as though the line had only ohmic resistance.

"2. That a reasonable constancy of voltage conditions on the line *for all loads* may be obtained by supplying the leading energy in the condition when the load is off or light, partly or wholly from the receiving end. * * * * * * * *

SUMMARY

"The more important definite conclusions arrived at may be summarized as follows:

"1. To obtain the most economical transmission of power over a long-distance line, it is necessary to have balanced the leading and lagging energies of the line itself, and at the same time to have unity power factor in which case the transmission is similar to direct-current both as to voltage drop and energy loss.

"2. The power factor at the load end may be made unity or otherwise controlled by synchronous machines, generators or motors, located at the receiving end of the line.

"3. To obtain satisfactory regulation for high-power long-distance transmission, the leading current taken by the line at light loads should be fed mainly from the receiving end. By this means the voltage rise from the receiving end; that is, the drop from the power house to the receiving end, can be kept nearly constant for all loads. Adjustment of the line-drop can be accomplished at any load by varying the amounts of the leading energy or charging current supplied to the respective ends of the line.

"4. The voltage regulation and line losses may be controlled for any commercial frequency and for any percentage load condition—as far as its electrical action is concerned—using a conductor of sufficient section, by making the leading and lagging line-energies equal at full load, and by feeding suitable proportions of the charging current at other loads from the respective ends of the line, the greater portion being from the receiving end.

"5. The adjustment of the leading and lagging energies taken by the line itself may be accomplished by the choice of voltage and load, or by adjusting the ratio of the capacity and inductance of the line. . . .

"6. Calculations of these long high-voltage, high-power lines may be made approximately only by formulas founded on the assumption of localized capacity. The error in the voltage may be several per cent. The approximate formulas are especially unreliable when the leading and lagging energy in the line are equal.

* * * *

"9. While it is possible by a cut-and-try method for any given case to arrive at the best conditions of capacity, resistance, inductance, and the power-factor at the load end of the line, yet, the best results will be obtained most directly by planning the line in accordance with the principles here explained.

"While comparatively few high-tension power-transmitting systems now installed are of sufficient magnitude to require as careful a treatment as that outlined here, the development of this art cannot proceed much further without some such method. It is the hope of the writer that here, either directly or, perhaps more likely, by suggestion, he has somewhat forwarded this development."

EVOLUTION OF FORMULA FOR CRITICAL LOAD

Mr. Thomas, as quoted above, gives the fundamental relation for the critical load: $V/I = \sqrt{L/C}$. This can be converted into the more convenient form given in the earlier part of this paper as follows:

Let kv-a. = total kv-a.

$(\text{kv-a.})_p$ = kv-a. per phase

E = delta voltage in kv.

$e = \dfrac{V}{1000} = Y$ voltage in kv.

I = current per phase

Then kv-a. $= 3\,(\text{kv-a.})_p$

$e\,I = (\text{kv-a.})_p$

$$I = \frac{(\text{kv-a.})_p}{e}$$

Substituting:

$$V/I = \frac{1000\,e}{I} = \frac{1000\,e}{\dfrac{(\text{kv-a.})_p}{e}} = \frac{1000\,e^2}{(\text{kv-a.})_p} = \sqrt{L/C}$$

$$\therefore\ (\text{kv-a.})_p = \frac{1000\,e^2}{\sqrt{L/C}} \qquad (3)$$

$$\text{and kv-a.} = \frac{1000\,E^2}{\sqrt{L/C}} \qquad (4)$$

These last two equations, (3) and (4), are of exactly the same form. The difference in notation shows that if the Y voltage is used the result is the critical kv-a. per phase, while if the delta voltage is used the result is the total kv-a.

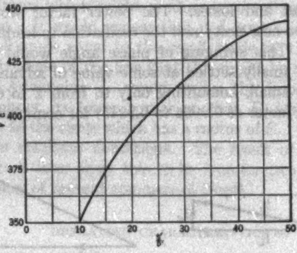


FIG. 1—CONDUCTORS IN ONE PLANE

S' = Distance between conductors in feet; D' = Conductor diameter in inches. For conductors in triangular arrangement: divide $\dfrac{S'}{D'}$ by 1.26 and use result. $\sqrt{\dfrac{L}{C}}$ for all sizes and spacings of aerial high-tension lines.

By applying practical values of conductor size and spacing to Fig. 1 it is found that $\sqrt{L/C} = 400$ is a fair average value. Substituting this value in equation (4) gives:

$$\text{kv-a.} = \frac{1000\,E^2}{400}$$

$$\text{kv-a.} = \frac{E^2}{0.4} \qquad (5)$$

in which E is expressed in kilovolts.

DISCUSSION OF CRITICAL LOAD FORMULA

Equation (4) for the critical load may be rewritten:

$$\text{kv-a.} = 1000\,E^2\,\sqrt{C/L} \qquad (6)$$

In this form it shows plainly that an increase in the capacitance of the transmission line will increase the critical load; a decrease in capacitance will decrease the critical load. In contrast to this, an increase in the reactance of a line will decrease the critical load and a decrease in the reactance will increase the critical load. Since the capacitance and reactance of transmission lines fundamentally bear a reciprocal relation it is seen that any change in size or arrangement of conductors will affect the reactance and inductance inversely. This will change the value of the quantity under the root sign by the square of the factor of change. But since this is under the root sign it will only affect the kv-a. directly as the change in either one of the quantities. For instance if it were possible to increase the capacitance of a circuit 50 per cent and therefore decrease the inductance a similar amount the critical load would only be increased 50 per cent.

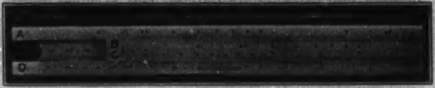


FIG. 2

For very high-voltage lines the full loads will correspond quite closely to the critical loads. However light loads will depart very far from the critical loads and those experienced with such lines know that light load conditions often present greater problems than full load. Looking at equation (6) to determine what is necessary for light loads, it is seen that the critical load may be reduced by decreasing the capacitance of the line or by increasing the inductance. To decrease the capacitance of the line is a difficult thing, but remembering the derivation of the equation it is easy to see that the same result may be produced by reducing or neutralizing the charging current. This can be done by connecting reactance coils across the circuit at various points or by the use of synchronous phase-modifiers operated under-excited so as to draw a lagging current from the line. This second has the advantage that under full-load conditions the phase-modifiers could be over-excited to draw a leading current and so increase the critical load. These devices need not be uniformly distributed but can be lumped at different points as determined for each line. As a matter of fact any line so long as to require modifying devices at other than the receiving end would probably have several distributing substations at which could be located the necessary modifying devices for proper operation at light-load, and increasing the critical load for full-load conditions. The decrease of the critical load for light load operation by increasing the inductance presents practical difficulties so that it will probably not be used for power transmission. This is the "loading coil" method used on telephone circuits where it is highly practical and the methods of decreasing the equivalent capacitance are impractical.

VECTOR INTERPRETATION OF CRITICAL LOAD TRANSMISSION

The best conception of what is happening in a transmission line carrying its critical load at unity power factor can be obtained by following Mr. Thomas's suggestion of assuming $R = O$. Mr. Thomas points out that under these conditions the effective values of current and voltage will be constant throughout the length of the line. In addition to this it should be noted that the vector position will change throughout the line. Starting from the receiving end with the current and voltage in phase and traveling toward the sending end the vectors will advance in a counter-clockwise direction (Fig. 3). At the end of a quarter wave-length both current and voltage vectors will be 90 degrees ahead of the vectors at the receiving end. At one-half wave-length the vectors will have advanced 180 degrees and will be in exact opposition to those at the receiving end. Continuing, at the end of one wave-length the vectors will be in phase with those at the receiving end but one revolution in advance. This can be continued indefinitely. Since electricity travels with the speed of light the wave-length for 60 cycles is 3100 miles and a quarter wave-length is 775 miles. Since the voltage and current are in phase at each point, full power is transmitted throughout. Resistance is then the only impediment to long distance transmission of alternating current electrical energy.

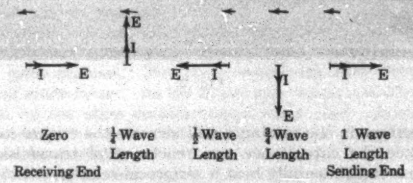


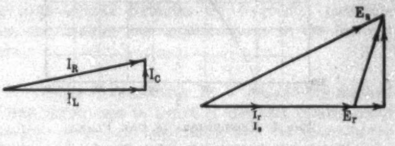
FIG. 3

It is therefore necessary to investigate the effect of resistance on the transmission of the critical load. Mr. Thomas has pointed out that the voltage drop with the critical load at unity power factor will be $I\,R$. That is, the increase in voltage from the receiving end toward the sending end is directly proportional to the resistance involved. The critical load being proportional to the *square* of the voltage, it will increase at a rate dependent on the square of the resistance (R^2). The actual load is increasing by the line losses which are $I^2\,R$ involving only the first power of R. This means the actual load is not keeping pace with the critical load and there must therefore be a change in phase relation of the current and voltage.

Going back to Mr. Thomas's fundamental conception, this disproportionate rise in voltage causes a flow of leading energy too great to be balanced by the lagging energy of the load current. This then must exhibit itself by a leading component of the current.

Fig. 4 shows the condition. I_L is the load current. I_c is the capacity current due to the increase of voltage above that corresponding to the actual load. I_R is the resultant current causing the power loss and voltage drop in the line. This resultant current enters as the square and since it is larger than I_L in some measure offsets the R^2 effecting increase in voltage, but the effect is very slight.

The leading current I_R has another effect and that is to decrease the voltage drop according to the familiar effect of a leading current flowing through inductance. These two actions will continue until the losses are at too rapid a rate for the slowly increasing potential. The action will then reverse and the current swing back in phase until the voltage is again sufficient to maintain the load and losses. The power factor would then again be near unity and the oscillation to leading would recur. This swinging of phase angle would continue until it finally settled at some value of advance which is very small, amounting only to from 2 to 6 degrees (power factor of 0.9994 to 0.9945) for commercial lines.

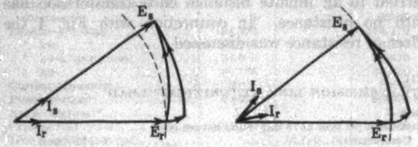


FIGS. 4 AND 5

It should be borne in mind that the relative conditions just described refer to the relation at different points along the line and not to varying conditions at one point. With the load constant the conditions at any point will be constant. But the phase relations will oscillate as different points along the line are considered.

The ideas presented in the previous paragraphs and some other conceptions can probably better be grasped by comparison with the familiar vector diagram of a current flowing through a lumped resistance and reactance without appreciable capacitance. In Fig. 5 E_r and I_r indicate the voltage and current at the receiving end. E_s and I_s indicate the voltage and current at the sending end. It is to be noted that I_r and I_s are equal and in phase. E_r and E_s are connected by the impedance triangle of the reactance. The power factor is unity at the receiving end and lagging at the sending end.

Fig. 6 shows a similar diagram for a transmission line having distributed inductance and capacity with the critical load at unity power factor. The effect of the distributed inductance and capacity is to curve the sides of the impedance triangle and reduce the difference between E_r and E_s to an amount equal to IR. From the discussion in the first part of this section it is evident that I_s would lead E_s by an angle too small to be shown on the diagram.



FIGS. 6 AND 7

Proceeding from Fig. 6 it is interesting to consider the conditions which might produce zero voltage drop in the line. It would appear that if I_r were advanced by a small angle equal to the small angle of the impedance triangle of the line that this might be accomplished. Advancing I_r would swing the impedance triangle about its lower corner so that E_s would be thrown in to the circle which has a radius of E_r. Fig. 7 shows the resultant conditions. The result of calculations given later show this to be true for two transmission lines of present commercial length.

PRACTICAL EXAMPLES OF TRANSMISSION LINES OPERATING AT CRITICAL LOAD

Many of the generalities discussed in the preceding sections of this paper can be much better appreciated by a few practical calculations. The results of some of these will therefore here be given.

All of these calculations have been made by the use of the rigid hyperbolic formula and are of a high order of accuracy. A true sine wave has been assumed. The conductors have been considered as placed in one plane with sufficient transpositions to make them symmetrical.

Critical Load for Practical Lines:—In order to show the relation of the critical load to various lines now in service Table I has been prepared for a few lines of 110,000 volts and above. This table shows the critical load to be quite within the range of operation of most of these lines and its use would help in analyzing and understanding their behaviors.

TABLE I

$\sqrt{L/C}$ AND "CRITICAL KV-A." FOR VARIOUS HIGH-TENSION LINES

Company	Nominal kv.	$\sqrt{L/C}$	Receiver Volts kv.	Critical kv-a.
Southern California Edison	220	387	200	103,000
Pacific Gas and Electric	220	384	200	104,000
Great Western	165	391	150	57,500
Au Sable	140	420	125	37,000
Utah Power & Light	130	396	118	35,000
Tennessee Power	120	404	110	30,000
Connecticut Riv.	120	402	110	30,000
Pacific Gas & Electric	110	388	100	26,000
Au Sable	110	400	100	25,000
Hydro-Electric Commission Ontario	110	375	100	27,000
Georgia Railway & Power	110	381	100	26,000

Short Copper Transmission Line at Critical Load. Table II gives first the general specifications for a copper transmission line with very large conductors

TABLE II

OPERATING CHARACTERISTICS OF COPPER TRANSMISSION LINE AT "CRITICAL" LOAD

General Data

Length: 193.75 miles (¼ of ¼ wave-length)		Resistance per mile at 20 deg. cent., 98 per cent Conductivity	0.0558 Ohms
Nominal Line Voltage	220,000 Volts	Reactance per mile	0.791 Ohms
Frequency	60 Cycles	Susceptance " "	5.38 Micro-mhos.
Conductor, copper, stranded	1,000,000 Cir. Mils.	$\sqrt{L/C} = 383.3$	
" diameter	1.152 Inches		
Spacing—flat	20 Feet		

There will be no corona at nominal voltage. In the calculations it has been assumed that there is no leakage loss.

Item	Predicted Flat Voltage		Power Factor 0.707 Lagging		Power Factor 0.707 Leading	
	Receiving End	Sending End	Receiving End	Sending End	Receiving End	Sending End
Volts (delta)	200,000	200,000	200,000	248,800	200,000	145,500
" (Y)	115,500	115,500	115,500	143,700	115,500	84,000
Load kv-a. (total)	104,400	107,100	104,400	91,000	104,400	93,210
" kw. "	104,200	106,900	73,800	75,750	73,800	77,400
" kv-a. (per phase)	34,800	35,700	34,800	30,330	34,800	31,070
" kw. " "	34,733	35,633	24,600	25,250	24,600	25,800
Current (Amperes)	301.2	309.0	301.2	211.0	301.2	370.2
Power Factor*	0.9975 *d*	0.9982 *d*	0.707 *g*	0.833 *g*	0.707 *d*	0.83 *d*
Angle (degrees)*	4.04 *d*	3.42 *d*	45.00 *g*	33.63 *g*	45.00 *d*	33.94 *d*
Critical Load (kv-a.)	104,400	104,400	104,400	161,700	104,400	55,300
Line Loss (kw.) (total)	2,700		1,950		3,600	
" " (kw.) (per phase)	900		650		1,200	
Line Drop (volts) (delta)	0		48,800		− 54,500†	
" " " (Y)	0		28,200		− 31,500†	
Per Cent Line Loss	2.59		2.64		4.9	
" " " Drop	0		24.4		− 27.3†	

**d* = leading; *g* = lagging. †rise

and almost 200 miles in length. The results of the calculations for various conditions are then given. The voltage, current, load, etc. are shown at the receiving and sending ends of the line and the losses, etc. in the line. The first condition given is that for "flat" voltage predicted as described in connection with Figs. 6 and 7. This shows the accuracy with which flat voltage can be predicted for shorter lengths of line. It is to be noted that the critical load is constant throughout on account of the constant voltage. Due to the line losses the load kv-a. and kw. are both increased at the sending end above the receiving end. It is then to be expected that the current would tend to loose some of its original angle of lead on the voltage. This is shown by the decrease in angle from 4.04 degrees to 3.43 degrees.

It is interesting to note the very high power factor throughout the line which shows the necessity of dealing with the angle rather than with the power factor in calculating at critical load.

TABLE III

OPERATING CHARACTERISTICS OF ALUMINUM TRANSMISSION LINE AT "CRITICAL" LOAD

General Data

Length: 193.75 miles (¼ of ¼ wave-length)
Nominal Line Voltage.................................. 220,000 Volts
Frequency.................................. 60 Cycles
Conductor, aluminum, stranded.................................. 500,000 Cir. Mils.
" Diameter.................................. 0.81 Inches
Spacing—Flat.................................. 20 Feet
Resistance per mile at 75 deg. Fah., (62 per cent Conductivity).................................. 0.179 Ohms
Reactance " " .... 0.833 Ohms
Susceptance " " .... 5.09 Micro-mhos.
$\sqrt{L/C} = 405$

There will be no corona at nominal voltage. In the calculations it has been assumed that there is no leakage loss.

Item	Predicted Flat Voltage		Load Power Factor 0.707 Lagging		Load Power Factor 0.707 Leading	
	Receiving End	Sending End	Receiving End	Sending End	Receiving End	Sending End
Volts (delta)	200,000	201,500	200,000	255,000	200,000	157,000
" (Y)	115,500	116,300	115,500	147,300	115,500	90,700
Load kv-a. (total)	98,800	107,100	98,800	88,800	98,800	95,490
" kw. "	96,600	105,600	69,840	75,900	69,840	80,700
" kv-a. (per phase)	32,933	35,700	32,933	29,600	32,933	31,830
" kw. " "	32,200	35,200	23,280	25,300	23,280	26,900
Current (Amperes)	285.2	307.0	285.2	201.0	285.2	351.0
Power Factor*	0.9777 *d*	0.9862 *d*	0.707 *g*	0.855 *g*	0.707 *d*	0.845 *d*
Angle (degrees)*	12.13 *d*	9.58 *d*	45.00 *g*	31.25 *g*	45.00 *d*	32.35 *d*
Critical Load (kv-a.)	98,800	100,200	98,800	160,600	98,800	60,900
Line Loss (kw.) (total)	9,000		6,060		10,860	
" " (kw.) (per phase)	3,000		2,020		3,620	
Line Drop (volts) (delta)	1,500		55,000		− 43,000†	
" " " (Y)	800		31,800		− 24,800†	
Per Cent Line Loss	9.3		8.7		15.5	
" " " Drop	0.7		27.5		− 21.5†	

**d* = leading; *g* = lagging. †rise

In order to demonstrate practically some of the rules given at the beginning of the paper there are also shown in Table II calculations for lagging and leading power factors. These show very plainly the tendency of the critical load when at lagging or leading power factor to increase toward unity power factor. The voltage, current and other conditions are well demonstrated in the table and can best be understood by a direct study of the table.

Short Aluminum Transmission Line at Critical Load: As was shown in the general section of this paper resistance really is the controlling factor with critical load transmission. In order to show the effect of resistance Table III has been prepared in every way similar to Table II except that a comparatively small aluminum conductor has been used instead of the large copper conductor.

Calculation was made for predicted flat voltage as before and this is practically checked by the result. The line power loss is 9.3 per cent. The large angles involved with the increased resistance are in great contrast to those involved with the copper line.

The results for lagging and leading power factor also show the effect of the increased resistance on many of the factors.

Characteristics of Very Long Transmission Line: In the general section of this paper it was shown in connection with Fig. 3 how the critical load could be carried to an infinite distance on a transmission line with no resistance. In connection with Fig. 4 the effect of resistance was discussed.

In order to show these points by a practical example, a transmission line 3100 miles long equal to one wave length at 60 cycles has been calculated and the results are shown in Table IV. The conditions at the receiving end have been chosen to give constant power factor and uniform relation between the voltage and current throughout the line. The conditions at each quarter wave-length have been calculated and show clearly the uniform conditions throughout the line.

The changes in the different items are very interesting and can be followed through the table.

The total length of this line, 3100 miles, is sufficient to span this country from the Atlantic to the Pacific.

The table shows that such a line could be operated with practically unity-power factor throughout. With 200,000 volts at the receiving end it would require only 256,300 volts at the generating end. This would

TABLE IV

OPERATING CHARACTERISTICS OF VERY LONG COPPER TRANSMISSION LINE AT UNIFORM POWER FACTOR

General Data

Nominal line voltage.................... 220,000 Volts
Frequency.................... 60 Cycles
Conductor, copper, stranded.................... 1,000,000 Cir. Mils.
" diameter.................... 1.152 Inches
Spacing—flat.................... 20 Feet

Resistance per mile at 20 deg. cent., 98 per cent
Conductivity.................... 0.0558 Ohms
Reactance per mile.................... 0.791 Ohms
Susceptance " ".................... 5.38 Micro-mhos.
$\sqrt{L/C} = 383.3$

There will be no corona at nominal voltage. In the calculations it has been assumed that there is no leakage loss.

Item	Point 1 Receiving End	Section 1-2	Point 2	Section 2-3	Point 3	Section 3-4	Point 4	Section 4-5	Point 5 Sending End
Section Length (miles)..........		775		775		775		775	
Total Length (miles)..........		775		1,550		2,325		3,100	
" " (wave lengths)....		¼		½		¾		1	
Volts (delta)..........	200,000		212,700		226,400		240,700		256,300
" (*Y*)..........	115,500		122,790		130,730		139,080		148,030
Angle with Potential at Point 1* (degrees)..........	0		92.09 *d*		184.17 *d*		276.26 *d*		368.34 *d*
Load kv-a. (total)..........	104,400		118,110		133,740		151,290		171,600
" kw. "	104,340		118,050		133,680		151,200		171,510
" kv-a. (per phase)..........	34,800		39,370		44,580		50,430		57,200
" kw. " "	34,780		39,350		44,560		50,400		57,170
Current (amperes)..........	301.2		320.6		341.0		362.9		386.2
Power Factor*..........	0.9994 *d*		0.9994 *d*		0.9994 *d*		0.9994 *d*		0.9994 *d*
Angle (degrees)*..........	2.02 *d*		2.02 *d*		2.03 *d*		2.02 *d*		2.03 *d*
Critical load (kv-a.)..........	104,400		118,000		133,900		151,050		171,500
Line Loss (kw.) (total)..........		13,710		15,630		17,520		20,310	
" " " (per phase).....		4,570		5,210		5,840		6,770	
Line Drop (volts) (delta)..........		12,630		13,750		14,460		15,500	
" " " (*Y*)..........		7,290		7,940		8,350		8,950	
Per Cent Line Loss (section)....		13.14		13.24		13.12		13.43	
" " " " (total).....		13.14		28.13		44.9		64.4	
" " " Drop (section)....		6.31		6.47		6.39		6.43	
" " " " (total).....		6.31		13.2		20.4		28.1	

**d* = leading; *g* = lagging.

TABLE V

OPERATING CHARACTERISTICS OF VERY LONG COPPER TRANSMISSION LINE

General Data

Nominal Line Voltage.................... 220,000 Volts
Frequency.................... 60 Cycles
Conductor, copper, stranded.................... 1,000,000 Cir. Mils.
" diameter.................... 1.152 Inches
Spacing—flat.................... 20 Feet

Resistance per mile at 20 deg. cent., 98 per cent
Conductivity.................... 0.0558 Ohms
Reactance per mile.................... 0.791 Ohms
Susceptance " ".................... 5.38 Micro-mhos
$\sqrt{L/C} = 383.3$

There will be no corona at nominal voltage. In the calculations it has been assumed that there is no leakage loss.

Item	Point 1 Receiving End	Section 1-2	Point 2	Section 2-3	Point 3	Section 3-4	Point 4	Section 4-5	Point 5 Sending End
Section Length (miles)..........		775		775		775		775	
Total Length (miles)..........		775		1,550		2,325		3,100	
" " (wave lengths)....		¼		½		¾		1	
Volts (delta)..........	200,000		212,700		226,300		241,100		256,200
" (*Y*)..........	115,500		122,790		130,660		139,200		147,920
Angle with Potential at Point 1 (degrees)*..........	0		94.00 *d*		184.38 *d*		277.97 *d*		368.75 *d*
Load kv-a. (total)..........	104,400		118,000		133,920		151,500		171,720
" kw. "	104,200		118,000		133,650		151,500		171,300
" kv-a. (per phase)..........	34,800		39,330		44,640		50,500		57,240
" kw. " "	34,730		39,330		44,550		50,500		57,100
Current (amperes)..........	301.2		320.2		341.4		362.6		386.8
Power Factor*..........	0.9975 *d*		1.0000 *d*		0.9980 *d*		1.0000 *d*		0.9985 *d*
Angle (degrees)*..........	4.04 *d*		0.23 *d*		3.62 *d*		0.63 *d*		3.22 *d*
Critical Load (kv-a.)..........	104,400		118,000		133,800		151,700		171,300
Line Loss (kw.) (total)..........		13,800		15,660		17,850		19,800	
" " (kw.) (per phase).....		4,600		5,220		5,950		6,600	
" Drop (volts) (delta)..........		12,700		13,600		14,800		15,100	
" " " (*Y*)..........		7,290		7,870		8,540		8,720	
Per Cent. Line Loss (section)....		13.25		13.30		13.35		13.06	
" " " " (total).....		13.25		28.3		45.4		64.4	
" " " Drop (section) ..		6.31		6.41		6.53		6.26	
" " " " (total).....		6.31		13.13		20.5		28.1	

**d* = leading; *g* = lagging.

then indicate that by designing a line properly in relation to its critical load it is now possible to transmit at any commercial frequency many times as far it would be commercially practical to do so.

It must be noted particularly that the results in this table are for full load only and are not set forth to show the operation of a line with variable load. With such a line as this it would be necessary from a financial point of view to keep the line operating at 100 per cent load factor so that light load conditions need not be considered. It would be necessary to consider how to get such a line into service, but this could be accomplished in a number of ways if the line were otherwise commercially feasible.

Table V shows a line in every way similar to that in Table IV except that the load power factor is slightly more leading, being the same as Table II for flat voltage. Table V shows that with this longer line there is practically no difference in the voltage drop. The effect of the flat voltage on the first 200 miles of the line being overcome by the greater drop in the remainder of the 775 miles. Table V does show the variation of the phase angle from side to side of the stable angle of 2.02 degrees used in Table IV.

To visualize the results of Tables IV and V better they have been plotted in Fig. 8. Near the top the vector position at five points on the line is given showing plainly the complete revolution through the length. Yet, at each point the current and voltage are practically in phase. The voltage, current and load rise almost uniformly from the sending end to the receiving end. The power factor curves show the uniform power factor predicted and calculated in Table IV, and the oscillating power factor of Table V gradually approaching the uniform value of Table IV. The very large scale used for the power factor should be appreciated for the variation is really very slight.

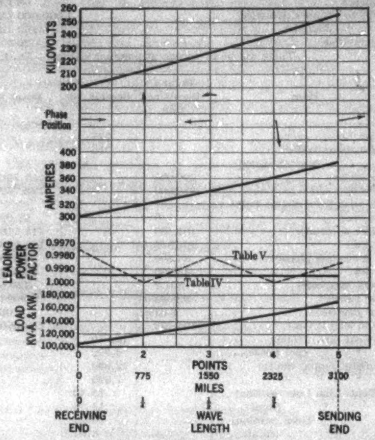


FIG. 8—OPERATING CHARACTERISTICS OF VERY LONG COPPER TRANSMISSION LINE

TABLE VI

OPERATING CHARACTERISTICS OF VERY LONG ALUMINUM TRANSMISSION LINE

General Data

Nominal Line Voltage	220,000 Volts	Resistance per mile at 75 deg. Fah., 62 per cent Conductivity	0.179 Ohms
Frequency	60 Cycles	Reactance per mile	0.833 Ohms
Conductor, aluminum, stranded	500,000 Cir. Mils.	Susceptance " "	5 09 Micro-mhos.
" diameter	0.81 Inch	$\sqrt{L/C} = 405$	
Spacing—flat	20 Feet		

There will be no corona at nominal voltage. In the calculations it has been assumed that there is no leakage loss.

Item	Point 1 Receiving End	Section 1-2	Point 2	Section 2-3	Point 3	Section 3-4	Point 4	Section 4-5	Point 5	Section 5-6	Point 6 Sending End
Section Length (miles)		775		775		775		775		775	
Total Length (miles)		775		1,550		2,325		3,100		3,875	
" " (wave lengths)		¼		½		¾		1		1¼	
Volts (delta)	120,000		143,400		170,100		200,000		239,100		279,500
" (Y)	69,300		82,800		98,200		115,500		138,100		161,470
Angle with Potential at Point 1* (degrees)	0		76.78 *d*		176.26 *d*		262.17 *d*		359.62 *d*		448.23 *d*
Load kv-a. (total)	35,940		50,040		70,260		98,800		138,300		193,800
" kw. "	35,340		47,610		70,200		96,540		138,300		191,100
" kv-a. (per phase)	11,980		16,680		23,420		32,930		46,100		64,600
" kw. " "	11,780		15,870		23,400		32,180		46,100		63,700
Current (amperes)	172.9		201.5		238.5		285.2		334.0		400.0
Power Factor*	0.983 *g*		0.951 *d*		0.9991 *g*		0.9773 *d*		0.9995 *d*		0.9870 *d*
Angle between Current & Potential (degrees)*	10.67 *g*		18.05 *d*		2.46 *g*		12.13 *d*		1.77 *d*		9.09 *d*
Critical Load (kv-a.)	35,500		50,800		71,500		98,800		141,200		193,000
Line Loss (total) (kw.)		12,290		22,590		26,400		41,700		52,800	
" " (per phase) (kw.)		4,090		7,530		8,800		13,900		17,600	
Line Drop (volts) (delta)		23,400		26,700		29,900		39,100		40,400	
" " " (Y)		13,500		15,400		17,300		22,600		23,370	
Per Cent. Line Loss (section)		34.7		47.4		37.6		43.2		38.2	
" " " " (total)		34.7		98.7		173.0		201.0		441.0	
" " Line Drop (section)		19.5		18.6		17.6		19.6		16.9	
" " " " (total)		19.5		41.7		66.6		99.3		133.0	

**d* = leading; *g* = lagging.

Characteristics of Very Long Aluminum Line: The line drop and losses were so low for the copper line that an extra long aluminum line has been calculated to reach more quickly the conditions that might obtain with a copper line 10,000 or 15,000 miles long. Table VI shows the operating characteristics of this very long aluminum transmission line. This is in every way similar to Table V except that the length has been extended to a total of 3875, being 1¼ wave-lengths. The conditions were not chosen for constant power-factor but so that "Point 4" with 200,000 volts would correspond to "Point 1" of the copper line in Table V, and the aluminum line in Table III for "predicted flat voltage." This again shows that the flat voltage condition cannot be extended for a great length of line due to the power losses; the effort to do this just causes a swinging of the power factor. The decrease of swing of the power factor as the sending end is approached is evident; the angle for stable operation is 6.06 degrees.

CONCLUSION

The formula for the critical load and various rules for its use were set forth at the beginning of the paper and therefore will not be repeated here.

It is undoubtedly possible that the form of expression of the rules can be improved and considerably extended. It is therefore hoped that this paper will not only lead many to a better conception of what is going on in a high-tension transmission line, but will also lead them to develop and present their thoughts so that others may have the benefit of a further practical understanding of the nature of transmission of electrical energy.

Included in the paper are examples of very long transmission lines to show how their action may be predicted, rather than as examples of economical lines for any particular condition.

It must be remembered that while approximate results can be obtained in some cases that the formulas are *not* approximate formulas but for the purpose of obtaining quickly in advance of accurate calculations a "Qualitative Analysis of Transmission Lines."

Discussion

R. D. Evans: In discussing Mr. Goodwin's paper, we wish to comment on three points, as follows: 1. The conditions for the most efficient transmission of power; 2. Stable operating conditions for long transmission lines; 3. The desirability of the circle diagram for calculating performance of transmission lines.

The first statement given under the caption of "Summary," suggests that the transmission of the critical load will be most efficient at unity power factor at the receiver. We wish to point out, however, that the most efficient transmission of power for a given transmission line and receiver voltage occurs at a slightly lagging power factor. This condition can best be explained by means of the circle diagram.

In Fig. 1, is shown a circle diagram for a transmission line, 125 miles in length. In this diagram, the receiver load is plotted in kw. along the *X* axis and in reactive kv-a. along the *Y* axis. The circle with solid line, with radius C_R is a graph of the different receiver load conditions which satisfy certain definite voltage conditions at the supply and receiver ends of the transmission line. The dotted circle represents a similar graph for a different supply voltage condition. The transmission losses for a definite transmission line and with constant receiver voltage may be represented by a series of concentric circles, only segments of which are plotted. It will be noted from the loss circles, that the line *l m* represents the receiver load and supply voltage conditions for minimum transmission loss or maximum transmission efficiency for the particular amount of power to be transmitted. The critical load for the transmission line, is indicated graphically as a load at the point *x*. It will be observed, that lower transmission line losses will occur if the critical load is operated at a slightly lagging power factor and at a slightly higher generator voltage. As indicated by the condition for the point *y*, most transmission line loads are of lagging power factor and synchronous condensers are required to improve the power factor and thus, regulate the system voltage. Hence, operation of a transmission line at the critical load, as indicated by the point *x* of Fig. 1, will occasion greater transmission losses, higher condenser losses and require greater condenser capacity than would occur under the more favorable operating condition indicated by the point *y*.

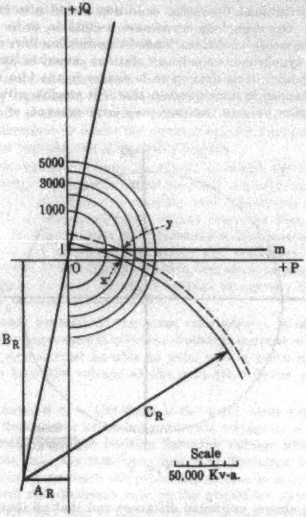


Fig. 1

The second point we wish to discuss is in reference to stable operating conditions of transmission lines. In Mr. Goodwin's paper, there is included under the heading of "Practical Examples of Transmission Lines Operating at Critical Load," a list of existing lines with their critical loads; also some data as to very long transmission lines. By inference, it might be supposed that it is practicable to transmit over 100,000 kw. at 60 cycles over a distance of 775 miles. The circle diagram for this very long transmission line, given in Table No. 4 of Mr. Goodwin's paper, will appear as indicated in Fig. 2. The critical load occurs at the point *x* and it will be noted that the transmission system will not deliver any appreciable load in excess of the critical load. If this line supplied synchronous machinery at the receiver, the operation would be unstable and difficulty would be experienced in maintaining synchronism. Furthermore, with the actual, transformers would be required which would further limit the amount of power that might be transmitted with satisfactory operating conditions. The circle diagram also shows that, in order to maintain the same voltage at no load as at full load, the reactive kv-a. required would be very nearly equal to the maximum kv-a. that might be transmitted over the transmission line. With a 60-cycle line of 775 miles in length, there would be a very real problem in limiting transmission line voltages, in case the load were dropped for any reason and the problem of making the generators non-self-exciting would be extremely difficult. Mr. Goodwin has pointed out that there would be difficulties incident to the operation of these very long transmission lines at light loads, but the speaker wishes to emphasize that the full-load operating condition would also be unsatisfactory. On such long transmission lines in order to insure stable operating conditions, loaded transmission lines with intermediate synchronous condenser stations should be employed.

The speaker feels that there is danger in the idea which has been advanced in some quarters that is it possible with unloaded transmission lines to transmit very large amounts of power per circuit to almost unlimited distances and that all that is necessary is to build a 220-kv. line and close the switches and the system will operate satisfactorily. The problem, however, is not as simple as that although it may be solved readily enough. There are factors such as unstable operating conditions that must be given consideration because of the large amount of power which may be involved and which may require loading of the transmission lines.

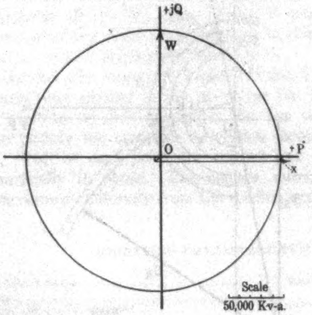


FIG. 2

In connection with studies of transmission line performance, it seems desirable to emphasize the advantages resulting from the use of the circle diagram method of solution. This method was rather fully discussed in articles by H. K. Sels and the speaker, published in the *Electric Journal*, during 1921-1922. The advantages of this method arise from the graphical presentation of the different voltage, load, and transmission loss conditions. This method readily shows the conditions for maximum efficiency on the transmission system for different conditions of supply and receiver voltages. The diagram also gives directly, information as to the probable stable operation of the transmission system. It is to be emphasized that a mathematical solution of a transmission line may not be a stable operating condition.

Summarizing, the maximum efficiency of a transmission line does not occur for the critical load at unity power factor, but at a slightly lagging power factor. Second, critical load conditions on very long transmission lines will have unstable operating characteristics, which may be avoided in a practical way, only by the use of loaded transmission lines with intermediate condenser stations. Third, the circle diagram method of calculating transmission lines has many desirable characteristics, including a graphical presentation of the various conditions with regard to variation in generator and receiver voltages, maximum transmission efficiency, and stable operating characteristics.

V. Karapetoff: I should like to discuss the first page of Mr. Goodwin's paper, particularly the statements regarding the properties of the critical load. Mr. Goodwin refers to Mr. Percy H. Thomas's paper of 1909, and so I looked up that paper for a confirmation or a proof, of the statement made here, but I found none. I looked up Mr. Thomas' companion paper, his mathematical discussion of the same date, but there also I could find no proof, and so I tried to check these statements mathematically. They do not seem to check, and I feel that Mr. Goodwin owes us an appendix to the paper, proving the statements made on the first page.

The general equations of a transmission line are quoted on p. 135 of my paper on the "Heavisidion," and I shall only refer to them without repeating them here. Mr. Goodwin's critical load is one for which $E_2 = Z_s I_2$, so that the second term in my eqs. (1) and (2) is equal to zero. Thus, the critical load is characterized not only by the voltage and the current, but also by a definite phase angle which depends on the line constants.

In the light of this theory, I should like to quote some of Mr. Goodwin's statements: "The critical load when placed on a transmission line at unity power factor will be transmitted at unity power factor." While I realize that the phase angle of the surge impedance may be small, nevertheless, I feel that in a statement of this kind, it should be explained that this only is so provided that the resistance and the leakage are neglected. I am afraid that the above statement may later be used and quoted as a reference, and the fact that it is only approximate may not be noted.

The second part of the statement reads: "The current will be constant throughout the line." This is not quite true, because both the current and the voltage have a decremental factor, αS. Again, neglecting the resistance and the leakage, the foregoing statement is true, but this limitation must be definitely stated.

The second statement is as follows: "If the kv-a. load is greater than the critical load, the power factor at the sending end of the line will be *always* more lagging than the power factor at the receiving end of the line." We cannot compare the critical load with any other load unless we take into account the power factor. Such a general statement does not seem to be true without further qualifications. Moreover, the power factor varies from leading to lagging and back, as you proceed along the line. If you start with a receiver condition, you can assume the generator end to be anywhere you please, one mile, seven hundred miles, or fifteen hundred miles, from the receiver end. There is no definite generating end in that sense, and as you proceed from the receiver end, you will meet line conditions of both leading and lagging power factor; thus, the above statement, as you may see, depends on where you take the generating end.

For all these reasons, I feel that Mr. Goodwin owes us much more explanation and proof than he has given us.

C. P. Steinmetz: A transmission line has four constants, two representing the energy dissipation, and two the energy storage, as depending on current and on voltage respectively.

1. The *resistance* r, calculated in the usual manner from material and size of wire, represents the *energy dissipation* depending on the line *current:* $r i^2$.

2. The *shunting conductance* g, representing the *energy dissipation* depending on the line *voltage*, such as by leakage through insulation, corona, dielectric hysteresis, etc. in transmission lines. This usually is so small as to be neglected.

3. The *inductance* L, representing the *energy storage* depending on the *current*, in the *magnetic field* surrounding the line, and giving rise to the *inductance voltage* or counter e. m. f. of self inductance.

4. The *capacity* C, representing the *energy storage* depending on the *voltage*, in the *electrostatic field* surrounding the line, and giving rise to the *capacity current* or charging current of the line.

The inductance, per unit length of line conductor, is given by:

$$L_0 = 2 \log L/R \times 10^{-9}\ h \qquad (1)$$

or, in common logarithms:

$$L_0 = 4.6 \log L/R \times 10^{-9}\ h \qquad (2)$$

where R = radius of line conductor, and L = distance from return conductor.

The line capacity, per unit length of line conductor, is given by:

$$C_0 = \frac{1}{c^2 L_0} \qquad (3)$$

where:

$c = 3 \times 10^{-10}$ cm. is the velocity of light

Instead of the inductance L and the capacity C of the line the surge frequency f_0 and the surge resistance r_0 may be used as line constants, and in some respect give a better physical insight into the behavior of the transmission line.

Any change of circuit conditions, such as current or voltage of a transmission line, means a change of the energy stored by inductance and by capacity, and thus a readjustment of the stored energy. This occurs usually by an oscillation, that is, electro-magnetic energy of current in the inductance changes to electro-static energy of voltage in capacity and back. It thus follows, that the rate of change, that is, the frequency of the oscillation, and the ratio of the voltage to current must be determined by the inductance and the capacity. The ratio of voltage to current is of the nature of a resistance or impedance, and therefore called the *surge resistance*, or *surge impedance*, or *natural impedance* of the line, and denoted by r_0 or z_0, while the frequency is called the *surge frequency* or *natural frequency* f_0 of the line.

It is:[2]

$$f_0 = \frac{1}{4\sqrt{L\,C}} \qquad (4)$$

$$r_0 = \sqrt{L/C} \qquad (5)$$

Substituting (1) and (3) into (4) and (5), gives the *surge frequency:*

$$f_0 = \frac{l}{4\,c} \qquad (6)$$

where l = length of line, and c = velocity of light or, the *wave length:*

$$\begin{aligned} l_0 &= f_0\, c \\ &= 4\, l \end{aligned} \qquad (7)$$

that is, the wave length is 4 times the length of the line.

and

$$\left.\begin{aligned} r_0 &= 2\, c \log L/R \times 10^{-9} \\ &= 4.6\, c \log L/R \times 10^{-9} \end{aligned}\right\} \qquad (8)$$

or, substituting for c:

$$r_0 = 138 \log L/R \qquad (9)$$

It is interesting to note that:

The surge frequency of a transmission line does not depend on size and distance of conductors or line construction, but depends only on the length of the line.

The surge resistance of a transmission line does not depend on the length of the line, but only on its construction, and is proportional to the logarithm of the ratio of the radius of the conductor, into the distance of the return conductor.

f_0 and r_0 therefore segregate the energy storage line characteristics into two constants of which the former depends only on the length, the latter only on the construction of the line.

All the infinite variety of transients which may occur in a circuit, such as traveling waves, stationary waves, discharges, etc., therefore must always occur at a definite frequency, the surge frequency or its multiples, and in every component wave or impulse, the ratio of voltage to current must always be the same, the surge resistance r_0.

The natural frequency of the line is familiar, but the meaning of the surge resistance r_0 is relatively little understood, though this constant probably is more important than the frequency, since it determines or limits the current which a transient voltage wave or impulse can give, and inversely.

As r_0 is the ratio of voltage to current, at which the readjustment of energy storage by inductance and capacity occurs, it follows that at this ratio r_0, capacity and inductance balance, thus no shift or change of phase occurs along the line, and no reflection. In other words, if a transmission line is closed by a resistance r_0, any transient passes into this resistance without reflection, while at any higher or lower resistance or impedance, at open circuit or short circuit, a voltage or current reflection must occur.

In lightning protection, the surge resistance r_0 is of fundamental importance, since it gives the discharge current, which the protective device must be able to pass, with a given lightning voltage, to keep the voltage at the protective device down to safe value.

For instance, if r_0 = 180 ohms is the total surge resistance, if the line flashes over at an instantaneous voltage e_0 = 200 kv. and this therefore is the limiting lightning voltage which may occur on the line, the maximum lightning discharge which as traveling wave may reach the protective device, is e_0/r_0 = 1100 amperes, and the discharge rate of the protective device thus must be sufficiently high so that when discharging 1100 amperes, the voltage across the impedance of the arrester does not back up to a value endangering the apparatus.

Inversely, with a given and known discharge rate and discharge voltage of a lightning arrester, and the known maximum instantaneous value of voltage, to which the protective apparatus can be safely exposed, we can calculate the maximum discharge current and therefrom the lightning potentials, up to which protection may be expected.

For instance, with a lightning arrester containing a series resistance of 200 ohms, and a maximum permissible (momentary) voltage of 25,000 on the protected transformers, 125 amperes would be the maximum discharge, which the arrester could take, and at a surge resistance of 360 ohms, this would give a lightning voltage of 45,000; therefore, if the voltage induced by lightning rises higher than this (which it often does), the arrester will probably fail to protect.

This ratio r_0 between current and voltage however applies only to a single component wave or impulse, such as a traveling wave running along the line, or the discharge of the bound charge induced by a thundercloud on a distribution circuit. If reflec-

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[1] This does not include the magnetic field inside of the conductor.

[2] Due to the effect of the resistance (and shunted conductance) f_0 and r_0 are slightly changed, so that the above strictly are only approximations.

tion occurs, the main wave and the reflected wave superimpose, and in the resultant wave the voltage is the sum and the current the difference of the component waves (or inversely), and for the resultant waves, this simple relation between voltage and current does not hold any more. That is, in the study of circuit protection, the effect of reflection (and refraction) and the voltage rise possible thereby, must be separately studied.

An interesting application of the conception of the surge impedance r_0 is given in the paper.

The standard equations of the transmission line consist of two components; each of the current components equals the corresponding voltage component divided by r_0, and in each of the components the voltage current ratio thus is constant and the phase relation between current and voltage constant, that is, the power factor constant. However, in the voltage the two components are added, in the current subtracted (or inversely), so that in the resultant wave phase relation, power factor and voltage current ratio along the line change. The first component decreases, the second one increases with increasing distance from the power supply. The latter therefore is the reflected wave. If then in the load at the receiving end the current is chosen so that r_0 is the voltage current ratio, then the second component wave vanishes, there being no reflection at the end of the line, as discussed above. This is the "critical load" referred to in Mr. Goodwin's paper.

Thus, the "critical load" of Mr. Goodwin's paper is given by: Volts divided by amperes equals surge resistance, where the surge resistance is given by equation (9).

A table of surge resistances for different sizes of transmission conductors are given in the following, for different distances between the conductor and the return conductor, and for different resistances between the conductor and the ground. The latter is the surge resistance which comes into consideration in discharges to ground, such as lightning. These latter values are approximate only, as they depend on the conductivity of the ground. The assumption has been made that the neutral plane lies 15 feet below the ground surface.

As seen, with very wide variations of conductor sizes and distance of the return conductor, the surge resistance varies relatively little, within the range of commercial transmission line porportions, so that for approximate estimate of discharge rates, etc., we may assume 360 ohms as the surge impedance of a line conductor against another line conductor, and 500 ohms as the surge impedance of a line conductor against ground.

TABLE OF SURGE IMPEDANCE

$z_0 = 138.15\ L/R$ ohms per conductor

	Size of Wire No.	R	Distance of Return Conductor, L, = Distance above Ground[1]							
			0.30 1′	0.60 2′	1.20 4′	2.40 8′	4.80 16′	4.50 15′	9 30′	18 m. 60′
	3	0.115	278	320	362	403	446	524	549	579
	1	0.145	264	306	348	389	430	510	535	565
	00	0.183	251	292	333	376	416	496	521	552
Cable	0000	0.250	232	273	315	357	397	477	502	533
	350000	0.325	212	256	297	339	381	462	486	517
	500000	0.390	206	247	289	330	372	451	476	505

~ 360 ohms. ~ 500 ohms

[1] Assuming neutral plane 15 ft. below ground.

C. L. Fortescue: Mr. Goodwin's paper suggests to my mind old thoughts expressed in a new way. In this world we seldom get something for nothing, and this applies with particular truth to power transmission. In fact, a transmission line, like all electro-magnetic apparatus, requires for successful operation reactive power. As a matter of fact, reactive power on a transmission line is something like taxes—you cannot get away from it.

An engineer may elect to supply this reactive power entirely from the generator end, but his object is to transmit power from a locality where it is cheap to one where it is dear and to do this as expeditiously as possible.

To deliver real power requires a prime mover in addition to a generator, but reactiv epower does not require a prime mover, but it may be done by that wonderful machine called the synchronous condenser, which we may conveniently call a wattless power generator.

To transmit this wattless power is an expensive process, and we have to pay for it not only in the wattless power which we transmit to the point where it is required, but we have to produce more wattless power in order to transmit that amount. So that it is a very expensive process to transmit that power over a whole transmission line to the point where it is required. However, we can elect to transmit our wattless power or supply it at any point in the line where we please.

Suppose we have a transmission line of a certain length; our first idea is to transmit our power and supply also the wattless power required from the generator. To supply the real power, of course, we must have a prime mover, but the wattless power does not require a prime mover. We can use a synchronous condenser.

We may divide the line into sections and supply the wattless power to these sections by means of a synchronous condenser. They have to transmit the wattless power for the section in which they are placed a short distance only. This arrangement is absolutely necessary, so that we can supply the necessary wattless current at the least possible cost, and in so doing we do not gum up our power transmission, by transmitting this wattless current over the line. Therefore, the supplying of this wattless energy at the point where it is needed, not only gives us a better operating system, and a more stable transmission line, but gives us more economical power transmission for the same amount of wattless generating apparatus; in other words, with the proper use of synchronous condensers, we can transmit more power over a given distance.

Mr. Goodwin has treated the only case where the wattless power is applied at each point just as it is needed, through the medium of the distributed capacitance of the line that is the critical load condition, but for a practical operating line, we have to supply the necessary wattless power under all conditions of load, and the synchronous condenser enables us to do that because it can supply either condensive energy or inductive energy, according to the excitation.

As an actual fact, the total amount of condenser capacity required for a line so supplied with its wattless energy is much less for a given amount of power transmitted than one supplied at one end only. The economical spacing of the condenser depends on the cost of condensers, on the cost of substations, maintenance, etc. The idea that power can be transmitted without supplying the necessary magnetic energy which seems to be rather prevalent among some engineers does not hold.

There is another point which I wish to emphasize which Mr. Evans has brought out, and that is that the mathematical solution of the transmission line is not necessarily a practical solution. The peculiarities of the apparatus used for regulating the line, have to be considered as for instance, not only the synchronous condenser, but also the voltage regulators. Under certain conditions of load the system becomes unstable and is no longer operable.

F. G. Baum: I am glad to see engineers take up the matter of the constant potential transmission line. This type of transmission must finally come to be our standard for large power transmission because we must have (1) a safe and stable transmission, (2) a reversible transmission, and these can be had only with (3) a constant potential transmission system. But with a constant potential transmission the calculations become extremely simple because the charging current per unit length of line is constant and therefore the charging current can be expressed as a simple linear equation in terms of the length of the line. This makes the solution very simple. See diagram and explanation, Fig. 3.

If we are to have all points of the line at one voltage, then the resultant line pressure for all loads fall on the arc of a circle. (See Figs. 2 and 3).* Fixing this condition then makes the calculations merely the solution graphically or analytically of the simple trigonometric functions of the circle. The calculations for a 150-mile line section have been carried out on the following page on a small sheet and yet the calculations are as accurate as the constants of the line are known and as accurate as the measurements could be made on the completed line. By placing synchronous condensers say every 150 miles we get a line that is practically independent of the generators for voltage regulation and the charging current and no load voltage conditions are under control. I don't see any other way of operating a long transmission system, and we must have such a large capacity transmission system to solve the power problems of the country today.

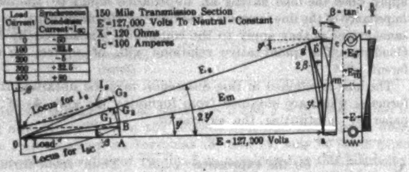

FIG. 3—CONSTANT POTENTIAL TRANSMISSION SYSTEM

CASE 1: ($R = O$, I in Phase with E).

For E = Constant, the resultant line pressure must be tangent to circular arc $a\ g$ drawn with radius $o\ a = E$ with o as center. This means that the current must be kept in phase with E at all points of the line. From similar circular sectors we have (since $a\ g = I\ X$, practically):

$$I_c/I = \frac{I\ X}{E} \text{ or } I_c = \frac{I^2\ X}{E} \qquad (1)$$

(or capacity current varies as the square of I). Draw $a\ b = I\ X$ at Rt. $\angle$s to $o\ a$ at a. Bisect $a\ b$ at m, making $a\ m = m\ b = \frac{I\ X}{2}$. Draw arc $b\ g$ with $m\ b$ as a radius. Now arc $b\ g$, the voltage compensation, is $\left(\frac{I\ X}{2}\right)$ 26. But $2E = I\ X$ or $= \frac{I\ X}{2\ E}$.

Therefore: Arc $b\ g = \frac{I^2\ X^2}{2\ E}$ or combined with (1).

$$b\ g = \frac{I^2\ X}{E}\ (X/2) = \frac{I_c\ X}{2} \qquad (2)$$

From (1) we also get

$$I = \sqrt{\frac{I_c\ E}{X}} = \sqrt{\frac{E^2\ C\ \omega}{X}} = E\sqrt{\frac{L}{C}}$$

(Where C = Capacity and L = Inductance). Hence it is seen that I is proportional to voltage but independent of the length of the line. Also from (1) $I = \sqrt{\frac{C\ \omega}{X}} = E/400$ (Practically), or kv-a. per phase (E in

$$\text{Kilovolts}) = \frac{1000\ E^2}{400} = 2.5\ E^2 \qquad (3)$$

For E = 127,000 this is kv-a. 3 phase = 121,000. This may be called natural capacity of the transmission system. In equation (1) I Load current = 325 ampres. or about 80 per cent of full load.

CASE 2: (R = 15 ohms, I in phase with E).

Draw $a\ c$ making angle $\beta = \tan^{-1}\frac{R}{X}$ with $a\ b$. Project $b\ c$ at Rt. $\angle$s to $a\ c$. Project c to b' at Rt. $\angle$s to $a\ b'$. ($b\ c = I\ R$). To compensate for this drop in voltage we must have concentrated capacity at load end so that line $a\ c$ will be rotated to $a\ b$, then the distributed capacity as per (2) will rotate point b to g. Now $b'\ c = b\ c$ (practically) and $b\ c$ must be compensated for by $I_{sc}\ X$ where I_{sc} is the leading current or synchronous condenser current at the load end of the line. Therefore

$$I\ R = I_{sc}\ X \qquad (4)$$

To compensate for both $I\ R$ drop and reactance drop we must have, combining equations (2) and (4):

$$I_{sc}\ X + \frac{I_c\ X}{2} = I\ R + \frac{I^2\ X^2}{2\ E} \qquad (5)$$

$$I_{sc} = I\ R/X + 1/2\left(\frac{I^2\ X}{E} - I_c\right) \qquad (6)$$

For power factors below unity we must of course balance the lagging kv-a. by leading kv-a. of the synchronous condenser. For angle of lag θ draw line making angle θ with $a\ c$ and project c at Rt. $\angle$s to this line. The distance c to the line gives the kv-a. The diagram shows visually the results and effect of varying the different factors.

In the current diagram correction for $I\ R$ drop results in generator current changing from $O\ G$ to $O\ G_3$. The length $G_2\ G_3$ is practically equal to $A\ B$, so that we have the same current and voltage relations (except that current has increased to make up line losses) as we had at the beginning of the line. Therefore we may add any number of such line sections to form a transmission line of any length. Condensers placed every 150 miles will thus control the potential for all load conditions and give (1) a stable line, (2) a constant potential line, (3) a reversible line. Only by such a line can the big power problems of the U. S. A. be solved.

For the ordinary transmission system we purchase generators for say 0.8 power factor and 125 per cent normal voltage. This means extra capacity of 125 per cent × 125 per cent or 156 per cent of the actual available capacity. This same extra capacity is required in transformers, and as the transformation is repeated several times (we have two to four or more transformations) the burden of extra plant capacity becomes very great. Also the transmission and distributing line capacities must be increased for the low power factors = 100/power factor or for 0.8 power factor the increase is 25 per cent. A tremendous burden is therefore being carried by the power systems due to the low power factors of the systems, and it is inevitable that the burden must be relieved. There are two means of relief, (1) install more synchronous motors instead of induction motors, (2) install synchronous condensers to correct the power factors; the burden of the cost of the synchronous condensers would fall on the class of customers having low power factors. But if we can reduce the total burden of the power installation as shown above by the power factor correction, the cost of the system per unit delivered will be less than the cost when carrying the burden, then it is good business to modernize our method of power transmission. See the paper on voltage regulation, etc. referred previously.

The solution of the Pit River power and transmission problem and the reasons therefore, as described in the *Electrical World* for January 27th, 1923, may be of interest.

J. R. Dunbar: Mr. Goodwin has presented some very interesting qualitative facts regarding the operation of transmission lines under certain conditions of load, but his formulas are not based on the hyperbolic theory. In this discussion, the formulas based on the hyperbolic theory are derived, and it is shown that those in the paper are approximations for the rigid formulas. To engineers familiar with the hyperbolic theory, this discussion may help to clarify some obscure points in the paper.

In the paper, a load called the "critical load" is used, and it is defined as a non-inductive load having an equivalent resistance equal to $\sqrt{L/C}$. In the derivation of this expression, certain assumptions were made which are strictly true only for an infinitesimal line, or for a line with resistance and leakance both absent. For short aerial transmission lines, where the resistance is small compared to the inductance and the leakance may be assumed zero, the assumptions are approximately true, and the formulas are sufficiently accurate to form the basis of engineering computations.

In the transmission line shown in Table IV of Mr. Goodwin's paper, it is stated that the conditions at the receiving end have been chosen to give constant power factor throughout the line. In order to do this, a load equal in kv-a. to the "critical load" and having an indeterminate proportion of reactive kv-a. has been taken. No data are given in the paper to indicate how the amount of reactive kv-a. required has been determined, so it was probably determined by a trial and error process. In this way Mr. Goodwin has determined approximately the load for uniform power factor along the line. The exact formulas for this load will now be developed using the hyperbolic theory.

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*TRANS. A. I. E. E., 1921. Vol. XL, p. 1017.

Let:

E_A, I_A, P_A = the voltage, current and volt-amperes, respectively, at the point A, the sending end of the line, expressed in vector form. Similarly for any point on the line, P, and for the receiving end, B.

z_0 = the "surge impedance" of the line.

$\alpha = \alpha + j\,\alpha^2$ = the "hyperbolic angle" per mile of line.

r, l, g, c = the resistance, inductance, leakance, and capacitance, respectively, per mile of line.

x = the distance from A to the point P.

The formulas for z_0 and α are as follows:

$$z_0 = \sqrt{\frac{r + j\,l\,\omega}{g + j\,c\,\omega}}$$

$$\alpha = \sqrt{(r + j\,l\,\omega)\,(g + j\,c\,\omega)}$$

If we consider a load having an equivalent impedance equal to the surge impedance of the line, connected at the receiving end, we have what Dr. Kennelly has named the virtually infinite line.[3] That is to say, the line behaves as if it were infinite in extent. Dr. Kennelly has considered this line, and the formulas for this case are as follows:

$$E_P = E_A\,e^{-\alpha x} = E_A\,e^{-\alpha' x - j\alpha_2 x}$$

where e is the base of natural logarithms.

$$I_P = I_A\,e^{-\alpha x} = I_A\,e^{-\alpha' x - j\alpha_2 x}$$

$$P_P = P_A\,e^{-2\alpha_1 x}$$

From these formulas, it is seen that the magnitudes of the voltage and of the current fall off from A toward B according to a simple exponential law. Also the angle between the voltages at any two points varies directly as the distance between them. The same holds true with respect to the currents. Furthermore, the volt-amperes fall off at double the rate of the current and the voltage, but still in accordance with an exponential law, and the power factor is constant over the whole line. It is apparently this load that Mr. Goodwin was searching for and he has hit upon a close approximation for ordinary commercial transmission lines. In fact, when $r = 0$ and $g = 0$, z_0 becomes equal to $\sqrt{L/C}$.

The formulas for the virtually infinite line are very readily adaptable for use with ordinary logarithmic tables. As already pointed out, the change in angle is directly proportional to the change in distance. The length corresponding to an angle of 360 deg. is the wave length of the line and is equal to $\frac{2\,\pi}{\alpha_2}$. The angle corresponding to any other length is found by direct proportion from this value. For the transmission line given in Table IV, the wave length is 3043.9 miles (4898.5 km.). The wave length of a line containing resistance depends not only on the frequency but also on the relative values of resistance, inductance and capacitance.

In order to determine the magnitude of the voltage at any point, the formula is $E_P = E_A\,e^{-\alpha_1 x}$. This may be transformed as follows:

$$\log_e E_P = \log_e E_A - \alpha_1 x$$

$$\log_{10} E_P = \log_{10} E_A - 0.43429\,\alpha_1 x.$$

In the last formula, all the quantities may be determined from ordinary logarithmic tables, so that the calculation of the voltage distribution on any transmission line for one particular load by means of the rigid hyperbolic theory is a very simple matter, and is well within the capabilities of any electrical engineer. The current at any point is directly proportional to the voltage, and the volt-amperes is equal to the product of the current and the voltage. The power factor is uniform.

In order to illustrate the application of these formulas, the transmission line in Table IV has been calculated for a receiving end voltage of 115,500 and for a load having an impedance equal to the surge impedance. The computation is shown in Table A. In Table B, the results are tabulated in such a manner that they may be compared with the results given in Table IV. It should be noted that the points used in Tables IV and B are not quarter wave length points.

A comparison of the results in Tables IV and B show some discrepancies, but the differences are not beyond reason. It is therefore not unreasonable to believe that Mr. Goodwin's general conclusions respecting the qualitative operation of transmission lines, loaded with the "critical load" as used in the paper, will apply when the load having an impedance equal to the surge impedance of the line is used as the basis of the argument, to the same extent as they apply to the conditions considered by Mr. Goodwin. The quantitative relations will, of course, be different.

The load mentioned in this discussion may be obtained from formulas which are obtained from formulas (3) and (4) of the paper by substituting the expression

$\sqrt{\frac{r + j\,l\,\omega}{g + j\,c\,\omega}}$ for the expression $\sqrt{L/C}$. Thus, from formula (3) we obtain

$$(\text{kv-a.})_p = \frac{1000\,e^2}{\sqrt{\frac{r + j\,l\,\omega}{g + j\,c\,\omega}}}$$

and from (4)

$$\text{kv-a.} = \frac{1000\,E^2}{\sqrt{\frac{r + j\,l\,\omega}{g + j\,c\,\omega}}}$$

TABLE A

OPERATING CHARACTERISTICS OF VERY LONG TRANSMISSION LINE AT UNIFORM POWER FACTOR

r = 0.0558 ohms

$\omega\,l$ = 0.791 ohms

$\omega\,c$ = 5.38 × 10^{-6} mhos

g = 0

Length = 3100 miles

E_B = 115,500 volts to neutral.

$z = r + j\,\omega\,l = 0.0558 + j\,0.791 = 0.79297\underline{/85.°965}$

$y = j\,5.38 \times 10^{-6} = 5.38 \times 10^{-6}\,\underline{/90°}$

$z_0 = \sqrt{z/y} = 383.92\overline{\backslash 2.°0175}$

$\alpha = \sqrt{z\,y} = .0020655\underline{/87.°983}$

$= .0000721104 + j\,.0020642$

Wave length $= \frac{2\,\pi}{.0020642} = 3043.9$ miles

Power factor = 99.9375 per cent leading

$\alpha_1\,x$ for 1 wave length (3043.9 miles) = 0.21949

Distance from A	Wave Lengths	0.43429 $\alpha_1\,x$	$\log_{10} E_P$	E_P	Angle with E_A	I_p	Kv-a.	Kw.
0	0	0	5.15967	144,430	0°	376.2	54,340	54,300
380.5	0.125	0.01192	5.14775	140,520	45°	366.0	51,440	51,400
761.0	0.25	0.02383	5.13583	136,720	90°	356.1	48,690	48,660
775	0.2546	0.02427	5.13539	136,580	91°.66	355.8	48,590	48,560
1141.5	0.375	0.03575	5.12392	133,020	135°	346.5	46,090	46,060
1521.9	0.5	0.04766	5.11199	129,420	180°	337.1	43,630	43,600
1550	0.5092	0.04854	5.11112	129,160	183°.32	336.4	43,450	43,420
1902.4	0.625	0.05958	5.10009	125,920	225°	328.0	41,300	41,270
2282.9	0.75	0.07149	5.08817	122,510	270°	319.1	39,090	39,070
2325	0.7638	0.07281	5.08685	122,140	274°.08	318.1	38,860	38,830
2663.4	0.875	0.08341	5.07616	199,170	315°	310.4	36,990	36,970
3043.9	1	0.09533	5.06434	115,970	360°	302.1	35,030	35,010
3100	1.0184	0.09708	5.06258	115,500	366°.64	300.9	34,750	34,730

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[3] "Artificial Electric Lines; Their Theory, Mode of Constuction, and Uses"—A. E. Kennelly, A. M., Sc. D., pages 43 to 46.

TABLE B

Item	Point 1 Receiving End	Section 1-2	Point 2	Section 2-3	Point 3	Section 3-4	Point 4	Section 4-5	Point 5 Sending End
Section Length (miles)		775		775		775		775	
Total Length (miles)		775		1.550		2.325		3.100	
" " (wave lengths)		0.2546		0.5092		0.7638		1.0184	
Volts (delta)	200,000		211,500		223,700		236,600		250,200
Volts (*Y*)	115,500		122,140		129,160		136,580		144,430
Angle with potential at Point 1 (degrees leading)	0		91°.66		183°.32		274°.98		366°.64
Load kv-a. (total)	104,250		116,580		130,350		145,770		163,020
" kw. (total)	104,190		116,490		130,260		145,680		162,900
" kv-a. (per phase)	34,750		38,860		43,450		48,590		54,340
" kw. (per phase)	34,730		38,830		43,420		48,560		54,300
Current (amperes)	300.9		318.1		336.4		355.8		376.2
Power factor (leading)	0.9994		0.9994		0.9994		0.9994		0.9994
Angle (degrees leading)	2.0175		2.0175		2.0175		2.0175		2.0175
Line loss (kv. total)		12,300		13,770		15,420		17,220	
" " (kw. per phase)		4,100		4,590		5,140		5,740	
Line drop (volts delta)		11,500		12,200		12,900		13,600	
" " (volts star)		6,640		7,020		7,420		7,850	
Per cent line loss (section)		11.8		11.8		11.8		11.8	
" " " " (total)		11.8		25.0		39.9		56.3	
" " " drop (section)		5.76		5.76		5.76		5.76	
" " " " (total)		5.76		11.85		18.3		25.1	

In the particular case when $r = 0$ and $g = 0$, these reduce to (3) and (4). Thus in the present overhead transmission lines, the load as given by formulas (3) and (4) may be taken as an approximate value of the load as considered in this discussion. For cable systems, the approximation does not hold.

C. F. Wagner: On the first page of his paper, the author attributes certain properties to transmission lines operating at their critical load. The fifth property states, "If the load is the critical kv-a. but at a lagging power factor, the power factor at the sending end will still be lagging but nearer unity." Since there is no discontinuity at the critical load, one could increase the kv-a. load by such a small amount that the above condition still exists, *viz.*, that the power factor at the sending end will still be lagging but nearer unity. However, as soon as the load is increased ever so slightly above the critical load, the second property becomes applicable *viz.*, "If the kv-a. load is greater than the critical load, the power factor at the sending end of the line will be always more lagging than the power factor at the receiving end of the line." In this case, the two properties are contradictory. Attention is therefore called to the desirability of adding the limitations to each respective property as affected by length of line, power factor, per cent of critical load, resistance of line, etc.

Percy H. Thomas: No one who has not attempted to decide on the best conditions for which to lay out an important transmission line would appreciate what a great help an understanding of the principle discussed by Mr. Goodwin, namely, the inherent relation between voltage and best load, will be in arriving at the desired result. While it is seldom that a line will work out practically to exactly meet the best condition layed down by his equation kv-a. $= E^2/0.4$, deviations can be made in the way of slightly lower power factor or higher power factor, higher load or lower load, as may be called for by the particular case in hand and these variations can be intelligently made in such a way as to proceed directly to the results sought.

It is very surprising that resistance, even if numerically but a small fraction of the reactance of the line, still plays a dominating part in the performance of a line operating at approximately the most efficient load. Mr. Goodwin has shown the fundamental nature of these relations by extending his calculations to lines thousands of miles long and find that the results were very much the same order as with shorter lines.

From the point of view of full-load line losses the possibility of operating under substantially d-c. conditions is most favorable but there are certain other characteristics of such a line which are not altogether favorable and call for careful consideration. For example, while no great harm results from under-loading such line, provided the voltages are maintained substantially constant at both ends, it is still utterly incapable of taking any immaterial overload. Even with the far end of the line dead-short-circuited, the current flow will not, in a long line increase greatly over the full-load current appropriate to the load given by Mr. Goodwin's formula. This characteristic is most marked in those lines having a resistance low in comparison with the reactance.

One of the practical questions to be considered in applying this principle for securing the most economical transmission is the fact that a power factor approaching unity is highly desirable. In most transmission systems securing such a high-load power factor will be a more or less expensive process, since synchronous condensers of large capacity will usually have to be provided. This condition is, however, already well recognized.

The curve shown in Fig. 8 of the paper under heading "Leading Power Factor," showing the variation of phase angle plus and minus along the transmission, is an interesting and curious freak of these lines.

Taking it all in all, the design and layout of what we may call superpower lines turns out to be quite a different thing from the usual transmission line problem as known for the last twenty years and requires quite a different handling especially with regard to the terminal connections and terminal apparatus and the various provisions for automatic regulation. I believe this subject should still receive further study in the near future.

H. Goodwin, Jr.: It is the pleasure of the author to take most of the discussion as amplifying the paper and confirming the good of and need for "Qualitative Analysis." The confirmation of the *need for* Qualitative Analysis is of two kinds: Direct expression of such opinion; and, incomplete or misstatements by those contributing to the discussion. The latter will be dealt with in detail.

It is a particular pleasure to draw attention to the discussion by Dr. C. P. Steinmetz which develops many points in a most interesting and helpful manner.

In general reply to some criticism in the discussion, the author frankly acknowledges the sketchiness of some sections of the paper. He felt, however, that to develop all the interesting phases of the subject would make the paper long and burdensome—almost require a book; also that this would entail further delay in making this most helpful method generally available to the profession.

Mr. Evans first takes exception to the first statement under the caption "Summary" which is in regard to the most economical transmission of power over a long distance line.

First it should be noted that this is *quoted* from Mr. Thomas's paper and is not a direct statement by the author. It has only been inserted to complete better the picture which Mr. Thomas paints. "Qualitative Analysis" deals with the qualitative operating characteristics of transmission lines and carefully avoids quantitative statements. The author hopes to present something in a future paper on the subject of efficiency of transmission which will develop the subject fully, logically and in a new light. But since Mr. Evans has gone to such length on this point it may be well here to discuss the matter further that others be not misled. Notice first that Mr. Thomas's statement is in regard to "a long-distance line," while Mr. Evans' example is concerned only with a short line 125 miles long.

Measuring from Mr. Evans's Fig. 1, it would appear that his claim is that the transmission would be more economical with the current at the receiving end lagging behind the voltage at an angle of 10 deg., which corresponds to a power factor of 0.9848. This means an increase in current at the receiver end of 1.6 per cent and a total increase in losses of about 3 per cent. His statement that the generator voltage would be higher under his conditions is correct, according to the rules of "Qualitative Analysis." It would therefore appear that Mr. Thomas's statement is more accurate than Mr. Evans's deduction from his diagram.

The circle diagram for long transmission lines has often been called into question and various efforts have been made to correct its deficiencies. It is based on assumptions. In an exposition of the circle diagram in the *Electric Journal* for December, 1921 by Mr. R. D. Evans and another author, there is given on page 533 a derivation of the "loss circle diagram." One of the bases is the statement: "the losses *neglected*—are *practically* constant" (italics by present author). In general, the derivation is so incomplete and involved as to make checking most difficult. But in opening this article on the circle diagram, the authors state: "The primary object of an approximate graphical solution is not one of accuracy—the rigid mathematical solution may be applied to the particular case with any further degree of accuracy that may be desired." It is but natural to suggest that Mr. Evans apply a rigid mathematical solution to his assumed load and line and prove thereby that the lagging power factor condition is more efficient than the unity power factor condition suggested by Mr. Thomas from the simple, fundamental, physical conception. When he has thus demonstrated to his own satisfaction that Mr. Thomas is correct, he might try a few degrees leading (about half the small angle of the impedance triangle of his assumed short line). Mr. Evans has not given sufficient data on his assumed line to allow others to do this.

Mr. Evans then calls upon his circle diagram as given in his Fig. 2 to prove the "instability" of some of the long lines given as examples by the author to show some of the characteristics of long lines operating at fixed critical loads. The author's calculations were made by the rigid mathematical formulas on which Mr. Evans' circle diagram is supposed to be founded. Is it but reasonable to suggest that Mr. Evans again follow his own advice and check the conditions which he proposes and the conditions which the author has suggested by the rigid mathematical formula. Fortunately Mr. J. R. Dunbar has in later discussion shown the results of repeated calculations on one of the lines calculated by the author which for the present purposes check with very fair accuracy, particularly in regard to the question of power factor.

But this aside. Acknowledge that starting with certain receiver conditions it is possible to figure the conditions at a supply point 100 miles or several hundred miles distant; again make this point and load the receiver conditions for another length of line; repeat this as often as desired. What is there then to question in the result if each step has been made accurately? How can a "diagram," based on very short lines and modified by approximation for longer lines, be called upon to over-throw the result with one stroke of the compasses?

Enough has probably already been said to counter Mr. Evans' third point of "the desirability of the circle diagram," etc. His discussion rather proves the desirability and necessity for the use of "Quantitative Analysis" based on simple, physical conceptions by all transmission engineers, even though they may be expert with various methods of "Qualitative Analysis."

Just one more point—a warning is necessary in this connection: The greatest care must be used in the application of any formulas, which have any assumptions whatsoever, to lines operating at the critical load near unity power factor.

Mr. Karapetoff called upon the author for proof of the statements on the first page and suggested the possibility of proving them by making many calculations on his "Heavisidion." If the section of the paper headed "Vector interpretation of critical load transmission" is studied sympathetically and thoroughly the bases of the rules given on the first page will readily be understood. As for the matter of checking by actual calculation, the author has checked these rules by numerous calculations and by comparing with the results of accurate calculations by others, and complete agreement has always been found.

As for the derivation of the rules from the hyperbolic formula, Mr. J. R. Dunbar has very kindly contributed in discussion such a derivation. Mr. Dunbar notes that the rules in the paper are approximations for the rigid formula. This is readily agreed. The rules are given in a simple form and are stated as applicable to "transmission lines of *present commercial lengths*."

The last of Mr. Karapetoff's discussion is fully covered by noting again that the rules are given to apply only to "transmission lines of present commercial length."

In regard to Dr. Steinmetz's discussion, the author has little to say except to thank him for his confirmation of the author's work and his fuller explanation of many of the interesting points. However, in his last paragraph he states "we may assume 360 ohms as the surge impedance of a line conductor against another line conductor." In view of the values given in Table I of the paper for various high-voltage transmission lines, it would appear that while 400 may be a very little above the average there are no lines which have a value so low as 360. Therefore, the factor 0.4 in the approximate formula for the critical load would seem to be entirely satisfactory.

Mr. Fortescue's discussion, as does a section of Mr. Evans's, dealing with distributing synchronous condensers along the line really goes by the whole point of the paper and shows again the necessity for "Qualitative Analysis" and understanding the reversal of conditions which take place when the load on a line passes the critical load.

In Mr. Fortescue's seventh paragraph he says "but for a practical operating line," etc. The author maintains that the information which he has set forth in this paper is much more practical and useful than any discussion on the use of synchronous condensers distributed at different points on a long transmission line. We have with us now hundreds of transmission lines showing the characteristics expounded in this paper, but we have not as yet one transmission line with synchronous condensers scattered along it at various points for the delivery of reactive power.

The author fully appreciates the application that may be made of synchronous condensers in this manner and has considered them in certain definite applications but he maintains that their use will be much better appreciated and their application made more logically if the principles of "Qualitative Analysis" are thoroughly understood and applied before detailed figures are started on the use of synchronous condensers.

In Mr. Fortescue's last paragraph, he amplifies some of Mr. Evans' criticisms and says finally "under certain conditions of load the system becomes unstable and is no longer operable." If there is any question about instability and impossibility of operation of very long lines, it will not be at the critical load. The instability and difficulties of operation will come at other loads and the one stable condition which the line will naturally and easily maintain itself is the condition of critical load.

Mr. Baum's discussion is very interesting, as is the work which he has done on constant potential transmission. However, the author would like to suggest to any one following Mr. Baum's work that he thoroughly familiarize himself first with "Qualitative Analysis" which will greatly aid him in understanding the exact qualitative figures given by Mr. Baum.

Mr. Dunbar's discussion is greatly appreciated. The angle of lead in Table 4 which he questions is half the small angle of the impedance triangle. It was obtained by trial and error, if you will, but it took but one trial to determine it originally, it being apparent from the physical consideration set forth in the paper that this should be the correct angle. Many calculations made since the original assumptions have checked its accuracy.

Attention of others who have discussed this paper is drawn particularly to Mr. Dunbar's sixth paragraph in which he particularly confirms the points disputed by Mr. Evans.

The author has no contention with Mr. Dunbar on the wave length of the lines given. 3100 miles is generally considered by practical engineers as a wave length at 60-cycles. According to Mr. Dunbar's figures, this is long by less than 2 per cent. The wave lengths for the aluminum line and the copper line are different, and the only way to put them on an easily comparable basis is to assume the distances the same. Appreciation of the fact that 3100 miles is not the exact wave length is shown by the angles given and by the vectors in Fig. 8 of the paper.

Mr. C. F. Wagner questions the rules given on the first page of the paper under conditions when "the load is increased ever so slightly." Mr. Wagner's attention is drawn to the sentence introducing the rules in which these rules are set forth for commercial transmission lines. The increases which he speaks of are far beyond the accuracy of any commercial instruments and beyond the accuracy of commercial engineering. He can see that the author appreciates them by following through the latter part of the paper and studying the calculations for extra long lines. It is hoped that Mr. Dunbar's discussion may also assist him. None of this, however, changes the general application of the rules given.

In closing, the author again wishes to draw attention from this discussion to the great need for the general application of the practical and useful principles involved in "Qualitative Analysis of Transmission Lines."

The "Heavisidion"

A Computing Kinematic Device for Long Transmission Lines[1]

BY VLADIMIR KARAPETOFF
Fellow, A. I. E. E.
Professor of Electrical Engineering, Cornell University

Subject of the Paper.—*A kinematic computing device is described which can be set to represent vectorially the voltage and the current at any point of a long transmission line with uniformly distributed properties and with a given load. The device has been named after Oliver Heaviside who was among the first to establish and to solve the fundamental differential equations of such a line. The parts of the device are made to assume at will different positions corresponding to different points on the line. The constants of a line to be represented by the device are adjustable at will, and a complete set of performance curves of a given transmission line can be obtained. Conversely, by a few simple trials, the best constants of a line and the necessary kv-a. of a synchronous condenser may be found, to give the required performance characteristics. The current, the voltage, and the power factor (or the phase angle) can be read off directly on the device for any desired point of the line, including the generator and the load end.*

Results of computations are shown for two transmission lines, 1000 and 300 miles long, values having been obtained in the usual tedious way by means of hyperbolic functions of a complex variable, and also read off directly on the Heavisidion. The agreement is as good as could be desired, showing that the device is reliable and that a considerable saving in time is possible with it. The device consists mainly of steel and celluloid bars, proportional dividers, parallel double tongs, and other simple kinematic linkages. Two sharp-edged wheels are used, similar to those used in planimeters. The parts are so combined as to satisfy the familiar exponential vectorial expressions for the sinusoidal voltage and the current, the independent variable being the distance from one of the ends of the line.

A. INTRODUCTION

The Meaning of the Word Heavisidion. The device was named after the noted English scientist and engineer, Oliver Heaviside, Honorary Member of the A. I. E. E., who was among the first to establish and to solve the fundamental differential equations of an electric line with distributed resistance, inductance, capacitance, and leakance.

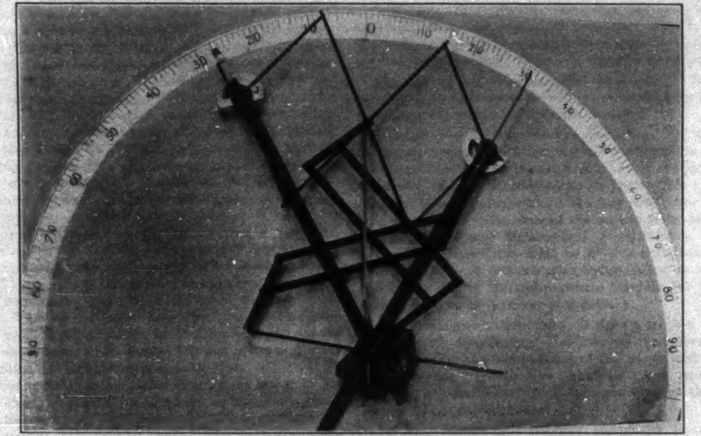

FIG. 1—THE FIRST EXPERIMENTAL HEAVISIDION

What the Heavisidion is. A combination of movable bars and linkages (Figs. 1 and 2) which can be set to represent, to a certain scale, the current and the voltage at any point of a line as vectors, in their proper relative phase position. The line constants are taken into account by properly setting the adjustable portions of the device. The parts of the device are kinematically so constrained that when it is set correctly for one point of the line, say the receiving end, it gives correct values for any other point. Each degree on the large circular scale corresponds to a certain number of miles along the line. By moving the two pointers by the same number of degrees in the opposite direc-

[1] The investigation upon which this paper is based was supported by a grant from the Heckscher Foundation for the Advancement of Research, established by August Heckscher at Cornell University.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

tions, the device is reset for another desired point on the line.

The Purposes of the Device. (1) To enable a designer to predetermine quickly and accurately the voltage and current characteristics of a transmission line, at any desired load and terminal voltage, taking the distributed capacitance and leakage correctly into account; without hyperbolic functions, without complex quantities, and without short-cut approximate methods. (2) To enable a designer or an operating engineer to determine quickly the effect of a proposed change in the line constants, in the load, in the voltage, in the length of the line, etc., upon the resulting distribution of current and voltage along the

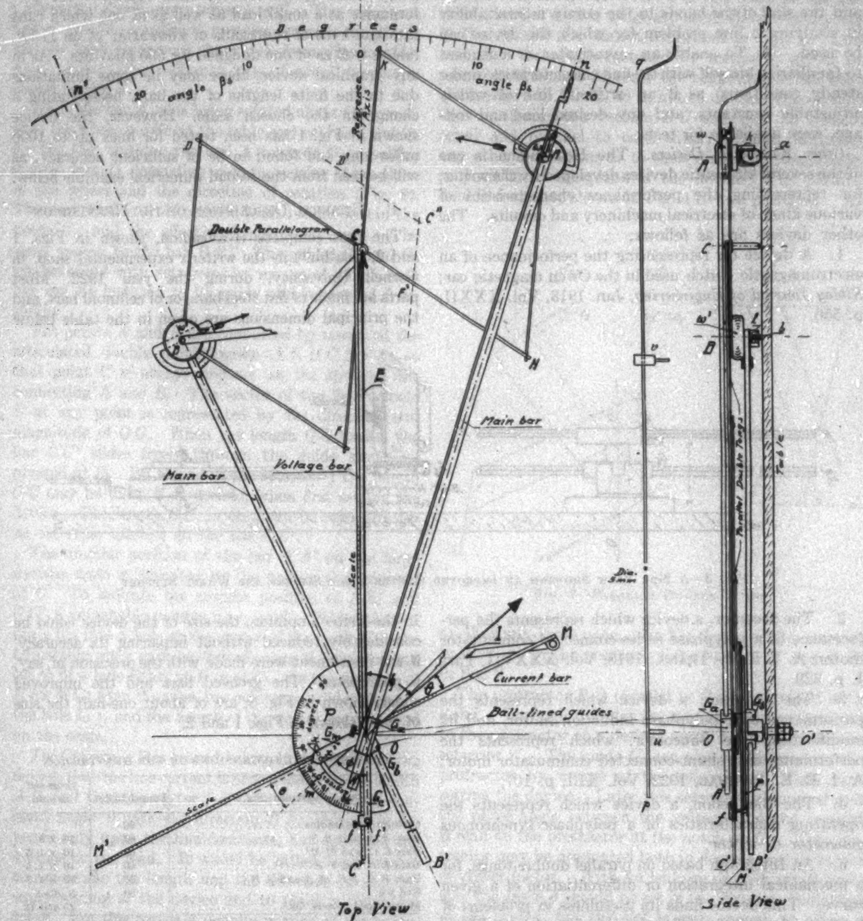
Degrees
20
10
0
10
20
angle β_0
angle β_0
n'
n
K
q
Reference Axis
D
D'
C''
C
Double Parallelogram
F'
E
f
H
Main bar
Main bar
Voltage bar
Scale
I
M
Current bar
θ
Ball-lined guides
P
G
G_a
G_b
O
O'
u
e
m
scale
M'
f
f'
C'
B'
w
a
A
w'
l
b
B
v
Parallel Double Tongs
Table
Dia. 5mm
p
Top View
Side View

FIG. 2—THE TOP VIEW AND THE SIDE VIEW OF THE HEAVISIDION WITH THE PARALLEL DOUBLE-TONGS LEFT OUT (SEE FIGS. 1 AND 4.)

line. (3) To make it possible to select the proper kv-a. of a synchronous condenser at a desired point of the line, so as to obtain the desired line voltage characteristics. (4) For some purposes to take the place of the involved analytical theory and to permit to visualize the actual electrical relationships along the line. (5) To add the judgment of the eye and the skill of the hands to the purely mental ability in studying a line problem for which the device can be used. (6) To enable an investigator or a student to familiarize himself with the line characteristics (under steady conditions) as if an artificial line of widely adjustable constants, and any desired load and voltage, were available for tests.

Other Kinematic Devices. The Heavisidion is one of the several kinematic devices developed by the writer, for representing the performance characteristics of various kinds of electrical machinery and circuits. The other devices are as follows:

1. A device for representing the performance of an electromagnetic clutch used in the Owen magnetic car; *Sibley Journal of Engineering*, Jan. 1918, Vol. XXXII, p. 550.

2. The Secomor, a device which represents the performance of a polyphase series-connected commutator motor; A. I. E. E. TRANS., 1918, Vol. XXXVII, Part I, p. 329.

3. The Indumor, a device which represents the performance of a polyphase induction motor, and its modification, the Shucomor, which represents the performance of a shunt-connected commutator motor; A. I. E. E. JOURNAL, 1922, Vol. XLI, p. 107.

4. The Blondelion, a device which represents the operating characteristics of a polyphase synchronous generator or motor.

5. An Integraph based on parallel double-tongs, for a mechanical integration or differentiation of a given curve. This device finds its usefulness in problems of hunting of machinery, fly-wheel design, ship stability, etc. *Optical Soc. of Amer. and Review of Scientific Instruments, Journal*, 1922, Vol. VI, p. 978.

6. The C. P. S'er (named after Dr. C. P. Steinmetz), a device for the automatic addition of impedances and admittances (description in preparation).

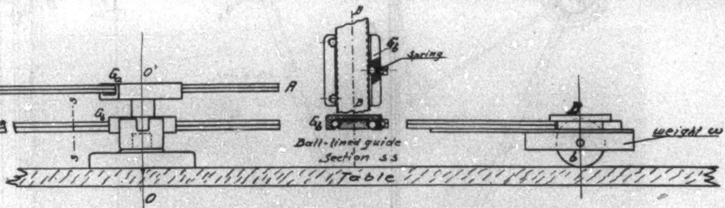

FIG. 3—A SIDE VIEW SHOWING AN IMPROVED CENTER PIECE SLIDERS AND WHEEL SUPPORT

7. A device for the study of transient phenomena in lines (in preparation).

Limits of Current and Voltage. Being a graphical device, the Heavisidion requires certain scales to be chosen for each particular problem. A convenient scale has to be selected for volts and another for amperes. The device can therefore represent the performance at a small load as well as at one which runs into hundreds of thousands of kilowatts; of an 11-kv. line as well as of one designed for 550 kilovolts. As in any graphical device, there may be some limitations due to the finite lengths of the bars, necessitating a change in the chosen scale. However, the device shown in Fig. 1 has been tested for lines up to 1000 miles long and found to be of sufficient accuracy, as will be seen from the second numerical example below.

B. GENERAL DESCRIPTION OF THE HEAVISIDION

The first complete Heavisidion, shown in Figs. 1 and 2, was built in the writer's experimental shop, in Cornell University, during the year 1922. Most parts are made of flat steel bars, or of celluloid bars, and the principal dimensions are given in the table below In the writer's opinion, the size of the device could be considerably reduced without impairing its accuracy, if the instrument were made with the precision of, say, a planimeter. The grooved bars and the improved slider, shown in Fig. 3, are of about one-half the size of those shown in Figs. 1 and 2.

THE PRINCIPAL DIMENSIONS OF THE HEAVISIDION

Part or distance (Fig. 2)	Length in cm.	Cross-sec. in mm.
Circular scale, radius	80	..
Bars $A\,A'$ and $B\,B'$	80	25 × 3
Bars $C\,C'$ and $M\,M'$	65	10 × 1.5
Bars $D\,H$ and $F\,K$	60	10 × 1.5
Bars $B\,D$, $B\,F$, $K\,A$ and $A\,H$	30	10 × 1.5
Rod $p\,q$	80	5 (dia.)
Sharp-edged wheels, dia	..	25 and 12

The Voltage. In the side view (Figs. 2 and 3) $O\,O'$ is a vertical axis about which the guide sleeves G_a and G_b can revolve. In the top view the steel bars $A\,A'$ and $B\,B'$ are shown passing through these guides. Each guide is lined on the inside with steel balls to reduce the friction to the movement of the bars (Fig. 3). A fork k (Fig. 2) is pivoted about a vertical axis at A and serves as a bearing for the sharp-edged wheel a which can roll on the horizontal table which supports the device. By means of the protractor shown at A, the vertical plane of the wheel a may be set at any desired angle to the longitudinal center line of $A\,A'$. A similar fork l guides the sharp-edged wheel b at B. The weights w and w' provide a sufficient pressure between the wheels and the table to prevent the wheels from slipping sidewise. A somewhat different type of wheel support is shown in Fig. 3.

When the bar $A\,A'$ is being rotated about $O\,O'$ as an axis, the wheel a rolls on the table and acts as a rudder for the bar, pushing the end A towards O or pulling it away from O, depending upon the setting of the plane of the wheel and the direction of rotation (Fig. 8). Thus, as the bar $A\,A'$ rotates, it also slides within the guide G_a, and the effective length $O\,A$ varies. The same applies to the bar $B\,B'$. The guides G_a and G_b are provided with setscrews by means of which each bar can be fastened to its guide and prevented from sliding.

The points A and B are connected by means of the articulated double parallelogram $A\,K\,H\,C\,D\,F\,B$, so that point C is always midway on the straight line connecting A and B. The vector of the line voltage E at any point is represented by the direction and magnitude of $O\,C$. When the length $O\,C$ varies, the bar $C\,C'$ slides freely through the guide sleeve G_c pivoted at O. By means of the setscrew f the length $O\,C$ may be fixed, if so desired, when first setting the device. The length $O\,C$, in cm., can be read directly on the scale marked on the bar $C\,C'$.

The angular position of the bar $A\,A'$ on the large circular scale is indicated by the pointer $O\,n$, pivoted at O. To indicate the angular position of $B\,B'$ and $C\,C'$, a detachable pointer $p\,q$ is used. The stationary pin u fits into a hole at the center of the guide G_a; the adjustable pin v may be shifted along the pointer and set to fit a hole either at B or at C. As the distance of B (or of C) from O varies, the pointer $p\,q$ slides through the hole in v, and the length of the vector may be read on the scale.

The Current. It is proved below, in the theory of the device, that the line current is proportional to the length $A\,B$, and that the vector I of the current forms a constant angle θ with the direction $B\,A$; this angle depends only upon the line constants, but not upon the voltage or the load. It would be rather inconvenient to determine the length and the direction of $A\,B$ for every setting of the device and to make a correction for θ. For this reason a definite portion of the vector $B\,A$ is transferred parallel to itself to the bar $M\,M'$, as the vector $O\,M$. This bar slides in the guide G_m, also pivoted at O. The guide G_m is provided with the setscrew x, by means of which the length $O\,M$ may be fixed in the original setting of the device. The length $O\,M$, in cm., can be read directly on a scale marked on the bar $M\,M'$. The angular position of $O\,M$ may be read on the protractor P, or more accurately on the large outer scale, by applying the pointer $p\,q$.

In the particular Heavisidion shown in Figs. 1 and 2 it has been found convenient to make $O\,M$ equal and parallel to $B\,C$ or $C\,A$. For this purpose the author's parallel double tongs are used[2] shown separately in Fig. 4, and also shown connected in Fig. 1. This kinematic linkage is fastened by means of pivots at points B, C, O, and M (only the beginnings of the bars being shown in Fig. 2, so as not to obscure the sketch) and constrains the vectors $B\,C$ and $O\,M$ to remain equal and parallel to each other, without imposing any other limitations as to their direction, magnitude, or position in the plane of the device. The parallel double tongs are shown in Fig. 4 in two positions. At (a) they are shown open, as in Fig. 1, and at (b) crossed, as they are in the position of the bars $A\,A'$ and $B\,B'$ in Fig. 2.

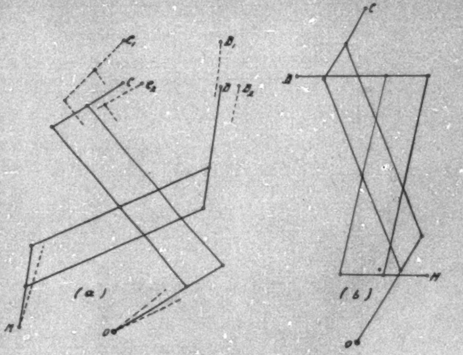

Fig. 4—Parallel Double Tongs

The protractor P is pivoted at O and provided with a slot. Through this slot the guide G_m can be fastened to the protractor at a desired angle θ, by means of the setscrew x'. The guide G_c can also be fastened to the protractor by means of the setscrew f', when first setting the device. Later this setscrew is loosened and the phase angle between the voltage and the current is read on the protractor at the notch on G_c. The protractor may be graduated both in degrees and in values of power factor. With no leakage or with small leakage, the angle θ should be set in the direction indicated in the sketch, but the slot is long enough to allow a setting of G_m on the other side of the zero division.

With the parallel double tongs attached at C, the current scale is fixed. Should it be desired to make this scale independent of the voltage scale, an adjustable link $D' C'' F'$ may be added to the double parallelogram and the connection to the parallel double tongs made at C'', instead of C. It is also possible to turn the vector $O M$ by the angle θ so as to represent the true phase position of the current. This can be done, for example, by using the generalized proportional dividers described in the paper on the Indumor (*loc. cit.*, p. 111).

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[2] *American Machinist*, 1921, Vol. 55, p. 1050.

C. AUXILIARY CHARTS OF LINE CONSTANTS

In order to use the Heavisidion for a line of given constants (resistance, reactance, capacitance and leakance, all per unit length), it is necessary to determine the following auxiliary constants (see also the list of symbols at the end of the paper):

τ, the so-called *imperfection angle* of the line; the sharp-edged wheels a and b are set at this angle (Fig. 8).

θ, the angle between $O M$ and the true direction of current vector I, Figs. 2 and 7; it is proposed to call this angle the *differential angle* of the line.

z_s, the *surge impedance* of the line, which determines the scale of the line current;

β, the *wave length constant* of the line.

The angle β is the angle by which each of the bars, $A A'$ and $B B'$, must be turned (in the opposite directions) in order to obtain a setting for a point of the line one km. or one mile distant from the given setting. In other words, β is measured in degrees (or radians) per unit length. If β is measured in degrees, then (360 deg./β deg) gives the number of miles for which each of the bars is turned by 360 deg. The bars are then again in their original positions and the values of current and voltage repeat themselves. The length 360 deg./β deg. is therefore called the wave length of the line; it depends upon the line constants and the

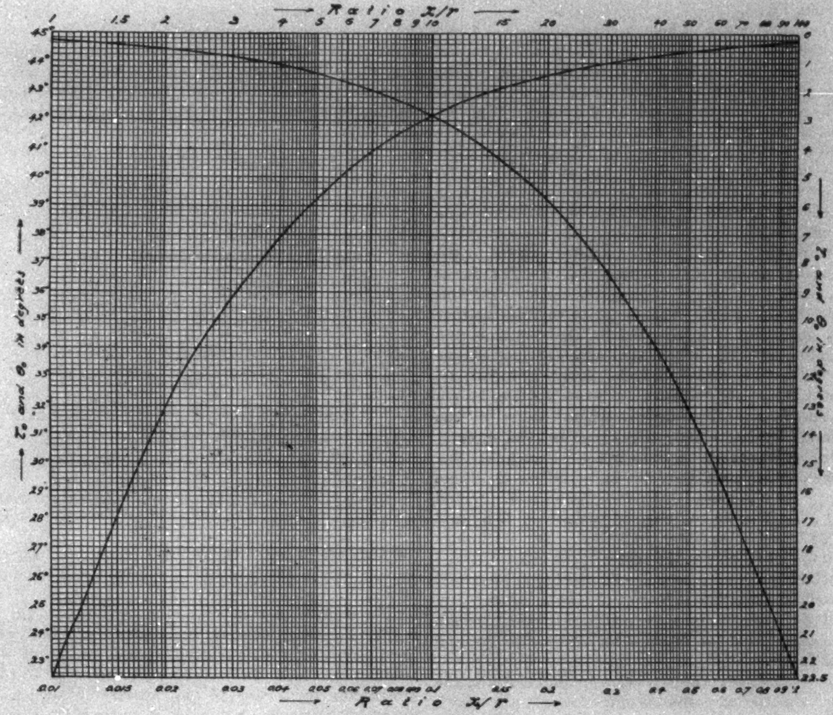

Fig. 5

FIG. 5—A CHART OF VALUES OF IMPERFECTION ANGLE τ_0 AND OF DIFFERENTIAL ANGLE θ_0 FOR LINES OF ZERO LEAKAGE ($g = 0$); IN THIS CASE $\theta_0 = \tau_0$. *Examples*: $x/r = 3$, $\tau_0 = 10$ DEG. 13 MIN.; $x/r = 0.4$, $\tau_0 = 34$ DEG. 06 MIN.

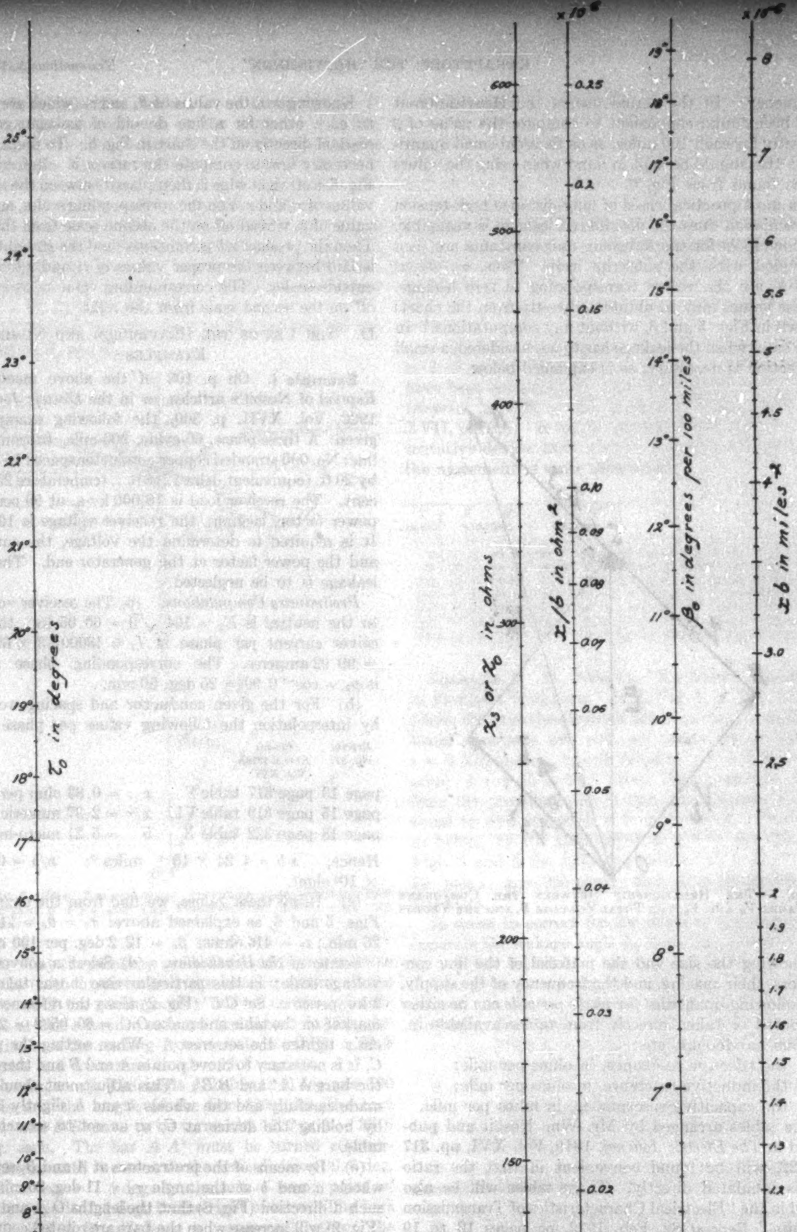

FIG. 6—A CHART OF VALUES OF SURGE IMPEDANCE z_0 AND OF THE WAVE LENGTH CONSTANT β_0, FOR LINES OF ZERO LEAKANCE ($g = 0$).

frequency. In the actual use of the Heavisidion it has been found convenient to compute the value of β directly for each 100 miles, so as to avoid small quantities; this should be kept in mind when using the values of β_0 found from Fig. 6.

In most practical cases of long-distance high-tension transmission lines the distributed leakage is negligible. The symbols for the foregoing four constants are then provided with the subscript zero. Thus, τ_0, θ_0, z_0 and β_0 are the values corresponding to zero leakage. These values may be obtained directly from the charts shown in Figs. 5 and 6, without any computations. In rare cases when the leakage has to be considered, a small correction is necessary, as is explained below.

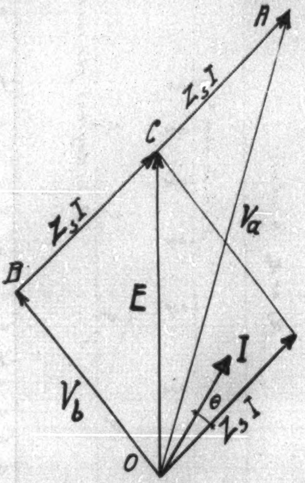

FIG. 7—THE RELATIONSHIP BETWEEN THE COMPONENT VOLTAGES V_a AND V_b, THE TOTAL VOLTAGE E, AND THE VECTOR $Z_s\,I$.

Knowing the size and the material of the line conductors, their spacing, and the frequency of the supply, the following quantities *per phase per mile* can be either computed or taken directly from tables available in various handbooks, etc.:

r the effective resistance, in ohms per mile;
x the inductive reactance, in ohms per mile;
b the capacitive susceptance, in mhos per mile.

The tables arranged by Mr. Wm. Nesbit and published in *The Electric Journal*, 1919, Vol. XVI, pp. 317 to 322, will be found convenient in that the ratio x/r is tabulated directly. These tables will be also found in the "Electrical Characteristics of Transmission Circuits," *Reprint* 82, Feb. 1922, on pages 12 to 19 (published by the Westinghouse Electric & Manufacturing Co.).

Knowing x/r, the values of θ_0 and τ_0 (which are equal to each other for a line devoid of leakage) can be read off directly on the chart in Fig. 5. To find z_0, it is necessary first to compute the ratio x/b. Referring to Fig. 6, a straight edge is then placed between the known values of τ_0 and x/b on the corresponding scales, and the value of z_0 is read off on the second scale from the left. Then the product $x\,b$ is computed and the straight edge is laid between the proper values of τ_0 and $x\,b$ on the outside scales. The corresponding value of β_0 is read off on the second scale from the right.

D. THE USE OF THE HEAVISIDION AND NUMERICAL EXAMPLES

Example 1. On p. 103 of the above mentioned *Reprint* of Nesbit's articles, or in the *Electric Journal*, 1920, Vol. XVII, p. 300, the following example is given: A three-phase, 60-cycle, 300-mile, transmission line; No. 000 stranded copper conductor spaced 10 by 10 by 20 ft. (equivalent delta 12.6 ft.); temperature 25 deg. cent. The receiver load is 18,000 kv-a. at 90 per cent power factor, lagging; the receiver voltage is 104 kv. It is required to determine the voltage, the current, and the power factor at the generator end. The line leakage is to be neglected.

Preliminary Computations. (a) The receiver voltage to the neutral is $E_2 = 104\sqrt{3} = 60.05$ kv; the receiver current per phase is $I_2 = 18000/(3 \times 60.05) = 99.92$ amperes. The corresponding phase angle is $\phi_2 = \cos^{-1} 0.90 = 25$ deg. 50 min.

(b) For the given conductor and spacing we find by interpolation the following values per phase:

Reprint No. 82.	*Electric Journal* 1919, Vol. XVI.			
page 13	page 317	table V	x	$= 0.83$ ohm per mile
page 15	page 319	table VII	x/r	$= 2.37$ numeric
page 18	page 322	table X	b	$= 5.21$ micro-mhos.

Hence, $x\,b = 4.24 \times 10^{-6}$ miles^{-2}; $x/b = 0.159 \times 10^{6}$ ohm^2.

(c) Using these values, we find from the charts in Figs. 5 and 6, as explained above: $\tau_0 = \theta_0 = 11$ deg. 26 min.; $z_0 = 416$ ohms; $\beta_0 = 12.2$ deg. per 100 miles.

Setting of the Heavisidion. (d) Select a convenient voltage scale; in this particular case it was taken as 3 kv. per cm. Set $C\,C'$ (Fig. 2) along the reference line marked on the table and make $O\,C = 60.05/3 = 20.02$ cm.; tighten the setscrew f. When setting the point C, it is necessary to move points A and B and therefore the bars $A\,A'$ and $B\,B'$. This adjustment should be made carefully and the wheels a and b slightly lifted by holding the device at C, so as not to scratch the table.

(e) By means of the protractors at A and B, set the wheels a and b at the angle $\tau_0 = 11$ deg. 26 min. in such a direction (Fig. 8) that the lengths $O\,A$ and $O\,B$ (Fig. 2) will increase when the bars are rotated counterclockwise.

(f) Set the slide G_m at the angle $\theta_0 = 11$ deg. 26

min., in the direction marked + on the protractor P, and set the protractor P at the angle ϕ_2 = 25 deg. 50 min. to the voltage bar $C\,C'$. This means that the actual angle between the bars $C\,C'$ and $M\,M'$ is 25 deg. 50 min. + 11 deg. 26 min. = 37 deg. 16 min., although the reading on P is only 25 deg. 50 min.

(g) The current scale is determined by the voltage scale and by the previously found value of z_0. Namely, with 3000 volts per cm., the current scale is, 1 cm. = 3000/416 = 7.2 amperes. Therefore the length $O\,M$ = 99.92/7.2 = 13.9 cm.

(h) Tighten the setscrew x on G_m, and also the setscrews on G_α and G_β.

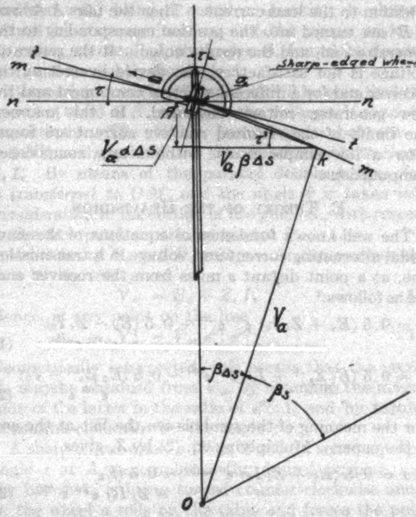

FIG. 8—THE RELATIONSHIP BETWEEN THE PROPAGATION CONSTANT $\alpha + j\,\beta$ AND THE IMPERFECTION ANGLE τ OF THE LINE.

(i) Lay the pointer $p\,q$ over $O\,B$, and turn the whole device (without disturbing the relative setting of the bars) until the readings at n and n' on the opposite sides of the reference axis are the same. In this particular case the reading from zero to n will be +29.5 degrees and from zero to n' − 29.5 degrees.

Readings. (j) The angle β_0 being 12.2 deg. per 100 miles and the length of the line 300 miles, the bars $A\,A'$ and $B\,B'$ have to be turned by 12.2 × 3 = 36.6 deg. each. The bar $A\,A'$ must be turned counterclockwise until it comes to the scale division 36.6 deg. − 29.5 deg. = 7.1 deg. to the left of zero; point n' must come to 7.1 deg. to the right of zero. The bars $A\,A'$ and $B\,B'$ will have to cross each other; the double parallelogram and the parallel double tongs are mounted in different planes so as not to interfere with such motion. It is easier first to move one bar by a certain amount, leave it there, then move the other bar, etc.[3] In the final position of the bars, the lengths $O\,C$ and $O\,M$ are measured to the same scale as before, and we find that the generator voltage E_1 = 70.4 kv., the generator current I_1 = 96 amperes, and the phase angle ϕ_1 = 21 deg. 30 min., with the current leading.

(k) If it is required to find the current and the voltage at several points along the line, say every 50 miles, the bars $A\,A'$ and $B\,B'$ are moved by an angle of 12.2/2 = 6.1 deg. at a time, and readings are taken as before.

In the following table the "primed" values are those read off on the Heavisidion, while the unprimed values have been computed by means of hyperbolic functions (*Reprint*, p. 105, or *The Electric Journal*, 1920, Vol. XVII, p. 309). It will be seen that even with a purely tentative device, built without any claim at precision, the agreement is quite satisfactory.

L miles	E kilovolts	E'	I amperes	I'	ϕ	ϕ'	Current
0	60.05	60.05	99.92	99.92	25 deg. 50 min.	25 deg. 50 min.	lags
50	63.17	63.50	93.66	93.30	19 deg. 53 min.	19 deg. 40 min.	"
100	65.77	66.00	88.94	88.0	12 deg. 34 min.	12 deg. 30 min.	"
150	67.81	68.20	86.37	86.5	4 deg. 14 min.	4 deg. 00 min.	"
200	69.31	69.80	86.47	87.5	4 deg. 42 min.	4 deg. 10 min.	leads
250	70.24	70.00	89.33	90.0	13 deg. 18 min.	12 deg. 30 min.	"
300	70.65	70.40	94.75	96.0	20 deg. 53 min.	21 deg. 30 min.	"

Example 2. In Pernot's "Electrical Phenomena in Parallel Conductors," 1918, Vol. I, p. 101, a three-phase, 60-cycle transmission line is quoted for which the linear constants *per mile per phase* are as follows: r = 0.275 ohm; x = 0.769 ohm; $b = 5.51 \times 10^{-6}$ mho; $g = 0.15 \times 10^{-6}$ mho. This example differs from the preceding one in that the leakance g is not equal to zero, although it is quite small. Proceeding as before, on the assumption of $g = O$, we find from Figs. 5 and 6 the following values: $\tau_0 = \theta_0$ = 9 deg. 50 min.; z_0 = 385 ohms, and β_0 = 23.95 deg. per 100 miles.

In order to correct for the effect of the leakage, we compute the leakage angle θ_g, from the relationship tan $\theta_g = g/b$ = 0.15/5.51. See eq. (18) below.

This gives θ_g = 1 deg. 34 min.; with this value we correct the above obtained values as follows. See *Correction for Small Leakage* at the end of the paper:

$\tau = \tau_0 + 0.5\,\theta_g$ = 9 deg. 50 min. + 0.5 × 1 deg. 34 min. = 10 deg. 37 min.;

$\theta = \tau_0 - 0.5\,\theta_g$ = 9 deg. 50 min. − 0.5 × 1 deg. 34 min. = 9 deg. 03 min.;

$\beta = \beta_0\,(\cos\tau/\mathrm{Cos}\,\tau_0)$ = 23.95 (0.9829/0.9853) = 23.9 deg. per 100 miles.

It will thus be seen that with a small angle θ_g the correction for β is negligible. The same is true for the surge impedance z_s; with small leakage it may be taken as equal to z_0 without leakage.

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[3] An additional kinematic connection could be added between the bars $A\,A'$ and $B\,B'$ to constrain them to move simultaneously by the same angle in the opposite directions, should a special problem warrant such a complication.

Knowing the above constants, the Heavisidion was set for a given receiver load and voltage, and by moving the bars in steps of 200 miles the "primed" values were obtained given in the table below.

L miles	E kilovolts	E'	I amperes	I'	ϕ	ϕ'	Current
0	50.0	50.0	25.0	25.0	25 deg. 00 min.	25 deg. 00 min.	lags
200	48.8	49.3	49.8	50.7	58 deg. 33 min.	58 deg. 15 min	leads
400	40.7	40.05	94.7	95.0	65 deg. 09 min.	65 deg. 00 min.	leads
600	28.9	29.0	128.5	128.5	50 deg. 01 min.	50 deg. 00 min.	"
800	24.3	24.6	143.7	146.0	10 deg. 24 min.	9 deg. 00 min.	"
1000	34.7	36.0	140.0	142.5	19 deg. 21 min.	18 deg. 30 min.	lags

The unprimed values were computed by Mr. Pernot using exponential expressions (*ibid.*, p. 107). Here again the agreement between the computed and measured values is quite satisfactory, especially considering the great length of the line.

Procedure with Extra-Long Lines. The circular scale of the Heavisidion as actually built (Fig. 1) extends only up to about 100 deg. on each side of the center zero. Therefore, for an extra-long line, or at a high frequency, the bars $A\,A'$ and $B\,B'$ (Fig. 2) may be at the limits of the scale before the other end of the line has been reached. In such a case the Heavisidion is used as *its own optical image*.

Referring again to Fig. 1, let the pointers of the main bars be somewhere on the lower (non-existing) semicircle of the scale. Imagine a plane vertical mirror passing through both 90 deg. points and the center line $O\,O'$ of the device, the reflecting plane being turned towards the fictitious semicircle. The optical image of such an impossible setting lies in the upper semicircle, and therefore can be realized on the actual device, being in its real range. While the bar $A\,A'$ moves counter-clockwise, its image moves clockwise. The reverse is true for the bar $B\,B'$.

Therefore, when the bars get near or past the 90 deg. points, proceed as follows: Tighten the setscrews of guides G_a and G_b, and note the amount by which the bars have passed the 90 deg. points, for example, $+\rho_1$ and $-\rho_2$ degrees, respectively. Turn the wheels a and b by the angle τ on the other side of zero so that the lengths $O\,A$ and $O\,B$ would increase if rotated clockwise. Move the bars to the positions corresponding to $-\rho_1$ and $+\rho_2$ degrees with respect to the 90 deg. points. Loosen the setscrews and continue moving each bar in the direction opposite to its previous motion. By this means the range of the device can be increased indefinitely.

Branch Loads. Let the Heavisidion be set for the receiver end and then the bars $A\,A'$ and $B\,B'$ rotated towards the generator end, as explained above. Let there be a branched concentrated load somewhere on the line. Having come to this point, the voltage bar $C\,C'$ is fastened so that it cannot move, while the current bar $M\,M'$ and the bars $A\,A'$ and $B\,B'$ are reset to take into consideration the added branch current. Then the motion of the device is resumed as heretofore, to the next load, where the new current is added vectorially, etc.

Synchronous Condenser. Usually a very long transmission line has to have a synchronous condenser at the receiver end, in order to keep voltage fluctuations within reasonable limits. The required maximum kv-a. of such a condenser can be determined with the Heavisidion by a few simple trials. The device is set for the desired receiver voltage and for a reasonable reactive current to be supplied by the condenser, in addition to the load current. Then the bars $A\,A'$ and $B\,B'$ are turned into the position corresponding to the generator end, and the results noted. If the generator voltage is not satisfactory, the device is reset at the receiver end for a different reactive component and the new generator voltage measured. In this manner, the limits of the required reactive current are found after a few simple trials, without any complicated computations.

E. THEORY OF THE HEAVISIDION

The well-known fundamental equations of the sinusoidal alternating current and voltage in a transmission line, at a point distant s miles from the receiver end, are as follows:[4]

$$E = 0.5\,(E_2 + Z_s\,I_2)\,\epsilon^{\alpha s}\,\epsilon^{j\beta s} + 0.5\,(E_2 - Z_s\,I_2)\,\epsilon^{-\alpha s}\,\epsilon^{-j\beta s} \qquad (1)$$

$$I = 0.5\,(E_2\,Z_s^{-1} + I_2)\,\epsilon^{\alpha s}\,\epsilon^{j\beta s} - 0.5\,(E_2\,Z_s^{-1} - I_2)\,\epsilon^{-\alpha s}\,\epsilon^{-j\beta s} \qquad (2)$$

For the meaning of the symbols see the list at the end of the paper. Multiplying eq. (2) by Z_s gives

$$Z_s\,I = 0.5\,(E_2 + Z_s\,I_2)\,\epsilon^{\alpha s}\,\epsilon^{j\beta s} - 0.5\,(E_2 - Z_s\,I_2)\,\epsilon^{-\alpha s}\,\epsilon^{-j\beta s} \qquad (3)$$

Combination of E and $Z_s\,I$ out of V_a and V_b. Let us introduce, for the sake of abbreviation, component voltages V_a and V_b, where

$$V_a = (E_2 + Z_s\,I_2)\,\epsilon^{\alpha s}\,\epsilon^{j\beta s} \qquad (4)$$

$$V_b = (E_2 - Z_s\,I_2)\,\epsilon^{-\alpha s}\,\epsilon^{-j\beta s} \qquad (5)$$

For a given point on the line, V_a and V_b are vectors measured in effective volts. Eqs. (1) and (3) become

$$E = 0.5\,(V_a + V_b) \qquad (6)$$

$$Z_s\,I = 0.5\,(V_a - V_b) \qquad (7)$$

These relations are shown graphically in Fig. 7.

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[4] See, for example, C. P. Steinmetz, "Transient Phenomena," Section III, Transients in Space; F. E. Pernot, "Electrical Phenomena in Parallel Conductors," pp. 96-98. In the following equations the customary dots under E and I, to denote vectors, are omitted.

V_a is a geometrical sum and V_b a geometrical difference of E and $Z_s I$; that is,

$$E + Z_s I = V_a \qquad (8)$$

$$E - Z_s I = V_b \qquad (9)$$

Adding these equations and dividing by 2, eq. (6) is obtained; subtracting and dividing by 2, gives eq. (7).

Thus, the voltage E at a point on the line can be formed out of the vectors V_a and V_b for the same point. Similarly, the vector $Z_s I$ can be formed in a different manner out of the same vectors. But for a given line Z_s is a known constant, see eqs. (26) to (29) below. Therefore, knowing $Z_s I$, the current vector I can be obtained. Z_s, being a complex quantity, is characterized by its magnitude z_s and the phase angle θ. Therefore, to obtain I, the vector $Z_s I$ must be divided by z_s and turned by the angle θ.

Comparing now Figs. 7 and 2, it will be seen that the bars $A\,A'$ and $B\,B'$ represent the voltages V_a and V_b respectively. The double parallelogram between A and B fixes the point C and thus determines both E and $Z_s I$. By means of the parallel double tongs, $Z_s I$ is transferred to $O\,M$, and the angle θ is taken into consideration in the setting of the guide G_m with respect to the zero mark on the protractor P.

Variation of V_a and V_b with s. At the receiver end ($s = 0$) eq. (4) gives

$$V_{ar} = E_2 + Z_s I_2 \qquad (10)$$

Hence, at any point on the line

$$V_a = V_{ar}\, \epsilon^{\alpha s}\, \epsilon^{j\beta s} \qquad (11)$$

Geometrically interpreted, this means that the vector V_a may be obtained from V_{ar} by changing the magnitude of the latter in the ratio of $\epsilon^{\alpha s}$: 1, and by turning it counter-clockwise by the angle $\beta\, s$.[5]

A sharp-edged wheel a (Figs. 2 and 8), set at a proper angle τ at A, accomplishes the desired purpose. As the bar $A\,A'$ is being turned counter-clockwise about O, the wheel a rolls on the table and forces the point A away from O. The wheel tracks a logarithmic spiral and consequently causes the length $O\,A$ to increase in a geometric progression, when the angle $\beta\, s$ increases in an arithmetical progression.[6]

In order to express the angle τ through α and β (Fig. 8), let $m\,m$ be a small portion of the curve traced by the wheel a on the table. Let $t\,t$ be a tangent to the curve at A, and let $A\,k$ be a small arc of the curve which practically coincides with the chord $A\,k$. Let $O\,k$ represent the value of V_a at k; then $O\,A$ is the increased value of V_a due to an increase $\beta\,\Delta\,s$ in the angle $\beta\,s$. Here $\Delta\,s$ is a small element of length along the transmission line. The angle which the tangent to the curve makes with $n\,n$ (perpendicular to $O\,A$) is denoted by τ. This angle has been called the imperfection angle of the line.[7] For a line devoid of resistance and leakage, $\tau = 0$ and the end of the vector V_a describes a circle.

The line $c\,k$ is drawn at right angles to $O\,A$, that is, parallel to $n\,n$. The angle at k is not quite equal to τ, but approaches this value as the point k indefinitely approaches A. Let the angle at k be called τ'. When the angle $\beta\,\Delta\,s$ is very small, $k\,c$ practically coincides with the arc $k\,c$ of radius V_a, drawn from O as a center. Therefore, in the limit, replacing $\Delta\,s$ by $d\,s$, we have

$$k\,c = V_a\,\beta\,d\,s \qquad (12)$$

The increment in the length of the vector V_a, that is, the algebraic difference between $O\,A$ and $O\,k$, is approximately equal to $c\,A$, since, with a small angle $\beta\,\Delta\,s$, $O\,k$ is nearly equal to $O\,c$. But the increase in the length of V_a, apart from the change in the direction, is equal to

$$\Delta\,V_a = V_a\,\epsilon^{\alpha\Delta s} - V_a \qquad (13)$$

Expanding into a Maclaurin series, we get

$$\Delta\,V_a = c\,A = V_a\,(\epsilon^{\Delta\alpha s} - 1) = V_a\,[\alpha\,\Delta\,s + 0.5\,\alpha^2\,(\Delta\,s)^2 + \text{etc.}] \qquad (14)$$

When ultimately $\Delta\,s$ is reduced to an infinitesimal $d\,s$, all the terms of the expansion, except the first, can be neglected, and we get

$$c\,A = V_a\,\alpha\,d\,s \qquad (15)$$

Therefore, in the limit, when $\tau = \tau'$,

$$\tan\tau = \tan\tau' = c\,A/k\,c = \alpha/\beta \qquad (16)$$

The foregoing discussion, being based on eq. (4), applies to vector V_a. A similar result can be obtained for vector V_b on the basis of eq. (5), except that the exponents are negative; consequently the rotation is clockwise, and the vector decreases instead of increasing in magnitude. Thus, in order to obtain the values of V_a and V_b for different points on the line, the bars $A\,A'$ and $B\,B'$ (Fig. 2) must be moved in the opposite directions, each by the angle β per unit length of the line. The wheels a and b then automatically change the lengths of these vectors in the proper ratio. If the lengths and the positions of V_a and V_b have been properly selected at some one point of the line, they remain correct at any other point, thus justifying the construction of the device.

F. THEORY OF THE AUXILIARY CHARTS

The practical use of the Heavisidion is very much simplified by means of the charts shown in Figs. 5 and 6. The use of the charts is explained under C above, so that it remains only to review the theoretical relationships of the various constants of a transmission line, in order to make clear the underlying theory of these charts. For the elementary theory of electric lines the reader is referred to numerous works on the subject, among others to those mentioned above.

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[5] For the theory of the "turning operator," $\epsilon^{j\beta s} = \text{Cos}\,\beta + j\,\text{Sin}\,\beta$, see for example V. Karapetoff's "Electric Circuit," Arts. 33 and 34; also his paper on superimposed imaginary e. m. fs., in the A. I. E. E. JOURNAL, 1922, Vol. 41, p. 11.

[6] Pernot, *ibid.*, p. 114.

[7] See V. Karapetoff, Trigonometric Expressions for the Phenomena Occurring in Long Transmission Lines; *Electrical World*, 1915, Vol. XLVI, pp. 857 and 914.

The Imperfection Angle Expressed Through the Line Constants. The factors α and β can be expressed through the linear constants of the line; therefore the angle τ, eq. (**16**), can also be expressed through these constants. Referring to Fig. 9, we introduce the *resistance angle*, θ_r, defined by the expression

$$\tan \theta_r = r/x \tag{17}$$

and the *leakage angle*, θ_g, defined by

$$\tan \theta_g = g/b \tag{18}$$

For a line devoid of resistance and leakage $\theta_r = \theta_g = 0$. For the series impedance Z and the shunted admittance Y of the line, per unit length, we can therefore write

$$Z = z\, \epsilon^{j(0.5\,\pi - \theta_r)} \tag{19}$$

$$Y = y\, \epsilon^{j(0.5\,\pi - \theta_g)} \tag{20}$$

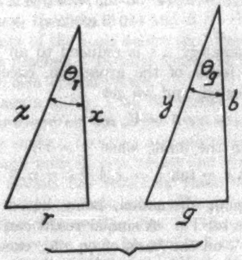

FIG. 9—THE RESISTANCE ANGLE AND THE LEAKAGE ANGLE OF THE LINE

For the complex propagation constant we have

$$\alpha + j\beta = \sqrt{Z\,Y} = \sqrt{z\,y}\, \epsilon^{0.5\,\pi - 0.5\,j(\theta_r + \theta_g)} \tag{21}$$

Hence $\alpha = \sqrt{z\,y}\,\mathrm{Sin}\, 0.5\,(\theta_r + \theta_g)$ (**22a**)

$$\beta = \sqrt{z\,y}\,\mathrm{Cos}\, 0.5\,(\theta_r + \theta_g) \tag{22b}$$

In the expression for β it is sometimes convenient to use x and b in place of z and y. From Fig. 9 we have

$$z = x/\cos\theta_r\,;\; y = b/\mathrm{Cos}\,\theta_g \tag{23}$$

The foregoing expression for β then becomes

$$\beta = \sqrt{\frac{x\,b}{\mathrm{Cos}\,\theta_r\,\mathrm{Cos}\,\theta_g}} \times \mathrm{Cos}\, 0.5\,(\theta_r + \theta_g) \tag{24}$$

Substituting the values of α and β from eqs. (**22**) in eq. (**16**), gives

$$\tan\tau = \tan 0.5\,(\theta_r + \theta_g)$$

from which, since τ is less than 90 deg.,

$$\tau = 0.5\,(\theta_r + \theta_g) \tag{25}$$

Thus, knowing the angles θ_r and θ_g from eqs. (**17**) and (**18**), the imperfection angle τ, at which the wheels a and b must be set, can be computed from eq. (**25**), without first determining the values of α and β.

Surge Impedance and its Angle Expressed through the Line Constants. By definition, the surge impedance of a line

$$Z_s = z_s\,\epsilon^{-j\theta} = \sqrt{Z/Y} \tag{26}$$

where Z and Y are expressed by eqs. (**19**) and (**20**) respectively. Z_s is a complex quantity, and is characterized by its magnitude z_s and the phase angle θ. It is convenient to use the minus sign before θ, because in ordinary power transmission lines this operator turns the current vector clockwise; compare the directions of $Z_s I$ and I in Figs. 2 and 7. Substituting the values of Z and Y in eq. (**26**), we get

$$z_s\,\epsilon^{-j\theta} = \sqrt{z/y}\,\epsilon^{-0.5\,j(\theta_r - \theta_g)} \tag{27}$$

from which

$$z_s = \sqrt{z/y} \tag{28}$$

and

$$\theta = 0.5\,(\theta_r - \theta_g) \tag{29}$$

It is proposed to call θ the *differential angle*, or the *distortion angle* of the line. It characterizes the difference between the imperfection angle θ_r due to the line resistance and the imperfection angle θ_g due to the leakage. When $\theta_r = \theta_g$, $\theta = 0$, and from eqs. (**17**) and (**18**) we get the familiar condition used in telephone engineering for a distortionless line: $r/L = g/C$.

It is sometimes convenient to express z_s in terms of x and b. Using eqs. (**23**) in eq. (**28**), we obtain

$$z_s = \sqrt{\frac{x\,\mathrm{Cos}\,\theta_g}{b\,\mathrm{Cos}\,\theta_r}} \tag{30}$$

Case of Zero Leakage. In most practical cases of power transmission lines it is permissible to put $g = 0$, unless the performance is desired in stormy weather, with poor insulation, excessive corona loss, or other abnormal conditions. When $g = 0$, θ_g is also equal to zero, and the foregoing expressions are somewhat simplified. The symbols which refer to this specific case are provided with the subscript zero. Eq. (**25**) gives

$$\tau_0 = 0.5\,\theta_r \tag{31}$$

and from eq. (**29**) $\theta_0 = 0.5\,\theta_r$ (**32**)

Consequently

$$\tau_0 = \theta_0 = 0.5\,\theta_r = 0.5\tan^{-1}(r/x) \tag{33}$$

The chart in Fig. 5 is constructed to represent this relationship, and saves the time of looking up trigonometric tables. From eq. (**24**)

$$\beta_0 = \sqrt{(x\,b/\text{Cos}\,2\,\tau_0} \times \text{Cos}\,\tau_0 \qquad (\mathbf{34})$$

and from eq. (**30**)

$$z_0 = \sqrt{\frac{(x/b)}{\text{Cos}\,2\,\tau_0}} \qquad (\mathbf{35})$$

The alinement chart in Fig. 6 gives values of β_0 and z_0 for known values of τ_0, $(x\,b)$, and (x/b), in accordance with eq. (**34**) and (**35**), and makes the actual use of these equations unnecessary. The scales are logarithmic so that a product is replaced by a sum. It would lead too far to explain here in detail the laying out of an alinement chart; it suffices to state that the chart in Fig. 6 takes the place of eqs. (**34**) and (**35**), and thus saves considerable time in obtaining the values of β_0 and z_0.[8]

Corrections for Small Leakage. Even under extreme conditions of transmission line operation, the uniformly distributed leakage is comparatively small, and the angle θ_g is not over 2 or 3 degrees. Therefore, the above described charts can still be used, and then small theoretical corrections applied to the values read off. For this purpose the angle θ_g must be computed from eq. (**18**). Comparing eqs. (**25**) and (**31**) we have

$$\tau = \tau_0 + 0.5\,\theta_g \qquad (\mathbf{36})$$

Thus, to the value of τ_0 obtained from Fig. 5 for the case of no leakage, a small angle, $0.5\,\theta_g$, must be added to obtain the actual value of τ with the leakage. Similarly, comparing eqs. (**29**) and (**32**) we get

$$\theta = \theta_0 - 0.5\,\theta_g \qquad (\mathbf{37})$$

We thus see that when leakage is present, the angles τ and θ are no more equal to each other.

In eq. (**30**) Cos θ_g can still be assumed to be equal to unity, so that with a small leakage $z_s = z_0$, and the expression (**35**) needs no correction. It must be noted that in this expression the angle τ_0 is used, and not τ. In other words, the value of z_0 obtained from Fig. 6 may be used with small leakage as well as without leakage.

In eq. (**24**) Cos θ_g may also be put equal to unity, thus giving

$$\beta = \sqrt{(x\,b)/\text{Cos}\,2\,\tau_0} \times \text{Cos}\,\tau \qquad (\mathbf{38})$$

Comparing this expression with eq. (**34**), we get

$$\beta = \beta_0\,(\text{Cos}\,\tau/\text{Cos}\,\tau_0) \qquad (\mathbf{39})$$

In other words, β_0 can be taken from Fig. 6 and the value so obtained corrected in the ratio of Cos τ to Cos τ_0.

* * * *

The writer's assistant, Mr. O. K. Marti, actually built the device shown in Fig. 1 and performed all the measurements described above. To him credit is also due for several mechanical details and for the drawings and charts used in this paper. The author wishes to express to him his sincere appreciation of the valuable assistance rendered. Mr. C. H. Dagnall, an instructor in Electrical Engineering in Cornell University, made a preliminary investigation of the possibilities of the proposed principle and performed some valuable computations and measurements. The author is under obligation to him for his active interest in this problem and for the use of some data from his thesis. Professor A. E. Wells, of Mechanic Arts, Mr. D. B. Green, Foreman of Machine Shop, and Mr. G. A. Culligan, Mechanician, all of the staff of the College of Engineering, gave generously of their time and skill in the making of the parts of the device, and it is only through their hearty cooperation that the device was completed within a comparatively short time.

LIST OF SYMBOLS

b	capacitive susceptance, in mhos per mile
E	voltage vector at the point under consideration, in effective volts
E_2	same at the receiver end
f	frequency, in cycles per second
g	leakage conductance (leakance), in mhos per mile
I	current vector at the point under consideration, in effective amperes
I_2	same at the receiver end
$j =$	$\sqrt{-1}$
o	subscript meaning "no leakage"
r	effective resistance, in ohms per mile
s	distance from the receiver end to the point under consideration, in miles
V_a, V_b	component voltage vectors defined by eqs. (**4**) and (**5**), in effective volts
V_{ar}, V_{br}	same at the receiver end
x	inductive reactance, in ohms per mile
$Y = y\,\epsilon^{j\,(0.5\,\pi - \theta_g)}$	admittance of the line, in mhos per mile
$Z = z\,\epsilon^{j\,(0.5\,\pi - \theta_r)}$	impedance of the line, in ohms per mile
$Z_s = z_s\,\epsilon^{-j\theta} = \sqrt{Z/Y}$,	surge impedance of the line, in ohms
$Z_0 = z_0\,\epsilon^{-j\theta_0}$	same without leakage
α	attenuation constant of the line, a numeric per mile
β	wave-length constant of the line, in radians per mile; in numerical applications and in Fig. 6, β is in degrees per 100 miles
β_0	same without leakage
ϵ	base of natural logarithms
θ	phase angle of the impedance Z_s, with the minus sign, or the differential angle of the line, in degrees or radians
θ_0	same without leakage
θ_g	leakage angle, in degrees or radians
θ_r	resistance angle, in degrees or radians
τ	imperfection angle of the line, in degrees or radians, defined by eq. (**25**).
τ_0	same without leakage

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[8] For the theory of alinement charts see M. d'Ocagne, Calcul Graphique et Nomographie; T. B. Peddle, The Construction of Graphical Charts.

A New Method for the Routine Testing of Alternating-Current High-Voltage Paper-Insulated Cable

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Review of the Subject.—*The usual methods of testing alternating-current high-voltage paper-insulated cables are based upon the ability of the insulating material to withstand excessive potentials in order to determine its condition. Accordingly, these methods are not suitable for routine tests imposed for the purpose of determining the degree of deterioration existing in any particular cable.*

In an effort to develop a routine test, which will serve to detect an impending fault, use has been made of the kenotron as the source of high-potential direct-current. By means of it a large volume of data has been secured concerning the input-current, for a cable, as a function of time after complete electrification at a constant high potential. These data when plotted, as curves, show by their shape the condition of the cable insulation.

Curves showing a sharp decrease in the magnitude of input-current during the first minute and a gradual but persistent rate of decay for the succeeding six or seven minutes indicate that the insulation is in an acceptable condition from the operating point of view.

Curves showing little, if any, decrease or a persistent tendency to increase during the time interval of the test indicate that the insulation has deteriorated to such a point that the cable may be expected to fail at an early date if retained in service. The degree of deterioration is indicated first: by the time which has elapsed since complete electrification before the increase occurrs, and second: by the sharpness of the upward trend of the curve in any instance.

The value of this method of testing has been demonstrated by its actually having detected a considerable number of impending cable faults before they became a menace in operation.

Additional refinements in the methods of measuring the input-current for different classes of cables may be desirable. Further investigations are being carried on to determine this necessity. Investigations of certain theoretical features are also being carried on. These include, among other things, oscillographic studies of the input-current and voltage under test conditions.

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Introductory—The Problem

WITH the expansion of underground transmission alternating-current networks, as regards both extent and operating voltages, the problem of determining the condition of the feeders has become more and more complex. The size and cost of alternating-current equipment for testing cables in modern underground systems is a matter of no small importance. With the high-transmission voltages and increased feeder lengths, these two factors approach limits which are almost prohibitive.

Also, the usual alternating-current test schemes have only determined the ability of a specific cable to withstand a particular test voltage; usually much in excess of that in normal operation. In our opinion a more desirable test method must include not less than the following features:

1. Should, if possible, be capable of detecting abnormal conditions in a cable at a sufficiently early date to permit of their location and elimination prior to the time at which they would constitute a menace to the system.
2. It should be effective at an impressed potential sufficiently low to insure that the insulating material will not be over-stressed.
3. Simplicity.

Due to the desire to develop a test, independent of the actual destruction of the material, to determine its acceptability, no consideration has been given the question of relative effects of test voltages impressed on cables by the kenotron or by the usual alternating-current tests; *i. e.* the ratio of kenotron to alternating-current potential for equal electrical stresses on the cable has not been considered in this investigation. Also, no attention has been given the problem of determining the geographical location of any particular fault.

Accordingly we have, for over four years, been investigating the possibility of developing such a test method. Because of certain rather outstanding conditions existing in the alternating-current feeders for the Edison system, and the necessity of safeguarding against interruptions due to cable failures, that portion of the transmission network was utilized in the study.

At the time this study was inaugurated The Edison District of The Philadelphia Electric Company was supplied with energy principally by means of 6,000-volt, a-c., 60-cycle, two-phase, three-conductor underground transmission feeders. These feeders constituted direct ties between the generating and substations. They had been installed for a number of years and were manufactured prior to the production of low dielectric loss cables. The cables referred to are practically all 250,000 cir. mil cross-section, insulated with 5/32-in. by 5/32-in. paper impregnated with rosin oil compound. Their lengths vary between 12,000 and 18,000 feet.

The alternating-current supply system for the Edison District has been considerably augmented, beginning the latter part of 1920, by the addition of high-efficiency 13,200-volt, three-phase paper-insulated lead-covered transmission cables.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

During the season of 1920 there were a number of unfortunate experiences with the operation of this portion of the transmission systems. This condition was brought about by several circumstances, among which the following may be noted:

1. The Edison load at that time was closely approaching the rated capacity of the substations and on that account if trouble developed, at any point, it was almost certain to cause a rather general interruption in the Edison District by tripping out other substations. This condition was especially true if a transmission feeder failed in service, since it probably would cause other feeders, working in parallel with it, to trip out.

2. The heavy loading of the alternating-current feeders, supplying the Edison system, tended to develop, through heating, inherent weaknesses in their insulation. Consequently, it became imperative that the utmost care be used in operating this portion of the underground system.

Several years ago The Philadelphia Electric Company secured a 60-kv., 0.1-ampere single-tube kenotron set in order to determine what, if any, possibility there was of applying a high-voltage direct-current test to underground transmission feeders to ascertain the general operating condition of such cables. The first thought was to consider the kenotron as a high-voltage megger, and from the observed values of current and impressed voltage to calculate the insulation resistance. It was immediately found that the current varied over an extremely wide range, from a large magnitude, at the instant of electrifying the cable, down to a value of 1/3 to 1/4 as great after several minutes of electrification. From this it was seen that a wide range of calculated insulation resistance values could be readily obtained and no decision could be made as to which of these, with respect to elapsed time, should be considered as representing the condition of the cable.

In addition to the apparent wide deviation in insulation resistance just noted it was observed that the current for a given cable when electrified to the same potential, but on different days, might vary over an extremely wide range; possibly of the order of 200 to 300 per cent variation. This latter situation is, undoubtedly, associated with the temperature of the insulating material at the time of test. However, as no feasible method was available for determining the temperature of a cable throughout its length, the combination of circumstances giving rise to wide variations of current seemed to preclude the use of the kenotron.

In view of the operating difficulties experienced in 1920, added consideration was given to the possible application of the kenotron for the purpose originally in mind; *i. e.* determination of the condition of cable insulating material. From what had been done earlier it was apparent that another method of analysis must be developed. Accordingly, consideration was given to plotting the tabular data on rectangular coordinate cross-section paper. It was hoped that the resulting curves might indicate the general character of the cable insulation. Here again, however, the wide variation between tests on successive days seemed to complicate the problem.

The next step was to analyze some of the tabular data[1] with the hopes that the equation expressing the relation of the constants in the curve might be determined. It was very soon found that these constants apparently obeyed no definite law.

Another method of analysis employed was to plot the data on semi-logarithmic paper. The results confirmed the earlier deductions that the current-time curves were not represented by a simple mathematical function.

The very high initial value of current, together with its more or less uniform rate of decay, apparently indicated characteristics similar to those of an electrical condenser. Since it required a long period of time—of the order of many minutes—before the current reached a value which was reasonably constant it was also evident that the condenser characteristics exhibited were not those usually considered, but must represent a charge absorbed within the dielectric. Finally, the approximately constant value of current observed after an extended period probably represents the true leakage current through the insulation.

From the foregoing it was concluded that the shape, rather than the magnitude, of the current-time curves for any particular cable might be indicative of the condition of its insulation.

It was then determined that a large volume of data should be obtained with the hope that, possibly, its analysis might produce something more indicative than had been found in the limited material previously available. During the accumulation of this added information some rather interesting developments were noted in the results of the tests and it was deemed advisable to follow them further.

METHODS OF TESTING

Due to the many variables complicating this problem, an early effort was made to minimize their influences by conducting the kenotron tests in as nearly a uniform manner as possible.

In order that the data for different tests might be comparable, it was deemed advisable to energize the cables to the same potential in each case. Because of the age of the feeders in the 6000-volt system, an aversion to subjecting them to abnormal potential stresses, together with lack of knowledge as to what effect the kenotron tests might have upon cable insulation, it was decided to use 9000-volts, which approximates the peak value of the wave for these two-phase circuits, on 6000-volt feeders when testing them by kenotron.

As it had been decided that a study should be made to determine what, if anything, could be learned from the shape of the current-time curves, it was felt advisable to electrify the cables under test at a standard uniform rate. It was found that the kenotron voltage could be raised very readily at a rate of 1500-volts per second, that is, a time interval of six seconds would be required in which to bring it up to test value. Therefore, this rate of electrification was adopted as standard. Operators making tests were accordingly instructed to close the high-voltage test switch at 54 seconds and to raise the voltage, as uniformly as possible, at a rate such that the test voltage, 9000 volts, would be reached at 60 seconds exactly. See Fig. 1.

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[1] Steinmetz, Engineering Mathematics.

In order to eliminate, as far as practicable, any unexpected effects due to adjustment of the kenotron voltage the attention of the operator was called to the necessity for care in approaching the final test value. This was done to minimize the possibility of imposing upon the cable a final voltage exceeding that desired; thereby necessitating reduction.

Having fixed the rate at which electrification should be applied, the time intervals at which readings were to be taken were arbitrarily set as follows:

Initial reading upon completion of electrification and subsequent readings at 15, 30 and 45 seconds and at 1, 2, 3, 4, 5, 6, 7 and 8 minutes after completion of electrification; a complete time interval of 480 seconds after full test voltage had been reached.

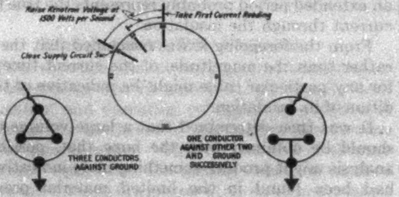

Raise Kenotron Voltage at 1500 Volts per Second
Take First Current Reading
Close Supply Circuit Sw
THREE CONDUCTORS AGAINST GROUND
ONE CONDUCTOR AGAINST OTHER TWO AND GROUND SUCCESSIVELY

FIG. 1 —METHOD OF TIMING AND CONNECTIONS USED IN DETERMINATION OF CABLE CURRENT TIME CURVES

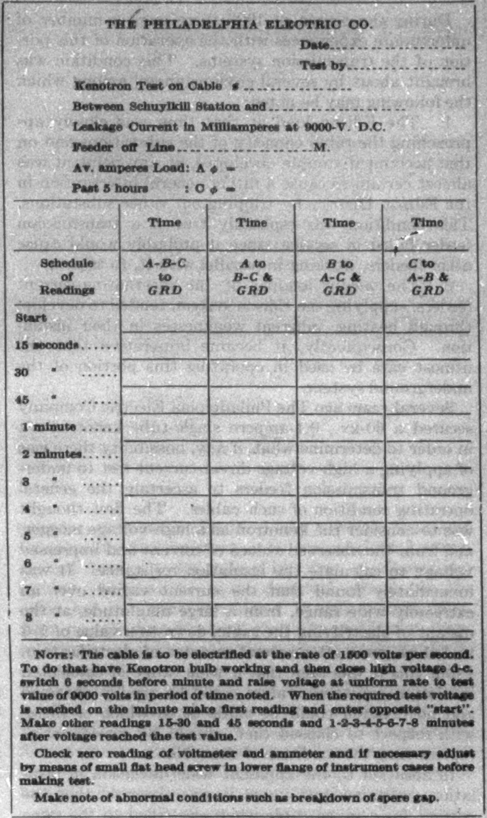
The sequence of tests selected, see Fig. 1, was as follows:

1. Three conductors together against sheath and ground.
2. Phase *A* against *B-C* and sheath to ground.
3. Phase *B* against *A-C* and sheath to ground.
4. Phase *C* against *A-B* and sheath to ground.

Therefore, omitting the time required in changing connections and other unavoidable delays, not less than 32 minutes has been required to test a single feeder.

The foregoing instructions were prepared in concise form for the guidance of those in the Station Operating Department who were responsible for routine electrical testing, and mimeographed forms for tabulating the data were furnished. See Fig. 2. This placed the operation as nearly as possible on a routine basis, which was desirable for the purpose of learning how practical this scheme of testing would prove.

Upon receipt of the tabular material, for the several tests, from the Station Operating Department, the observed values of current were reduced to units of current per mile in order to facilitate direct comparison of results observed on different feeders. Much of the material has been plotted in curves similar to those which appear here.



THE PHILADELPHIA ELECTRIC CO.

Date..................

Test by..................

Kenotron Test on Cable # ....

Between Schuylkill Station and ....

Leakage Current in Milliamperes at 9000-V. D.C.

Feeder off Line ....M.

Av. amperes Load: A ϕ =

Past 5 hours : C ϕ =

	Time	Time	Time	Time
Schedule of Readings	*A-B-C* to *GRD*	*A* to *B-C* & *GRD*	*B* to *A-C* & *GRD*	*C* to *A-B* & *GRD*
Start				
15 seconds......				
30 "				
45 "				
1 minute				
2 minutes......				
3 "				
4 "				
5 "				
6 "				
7 "				
8 "				

NOTE: The cable is to be electrified at the rate of 1500 volts per second. To do that have Kenotron bulb working and then close high voltage d-c. switch 6 seconds before minute and raise voltage at uniform rate to test value of 9000 volts in period of time noted. When the required test voltage is reached on the minute make first reading and enter opposite "start". Make other readings 15-30 and 45 seconds and 1-2-3-4-5-6-7-8 minutes after voltage reached the test value.

Check zero reading of voltmeter and ammeter and if necessary adjust by means of small flat head screw in lower flange of instrument cases before making test.

Make note of abnormal conditions such as breakdown of spere gap.

FIG. 2

DATA AND OBSERVATIONS

Referring to Figs. 3 and 4 which show a group of current-time curves from successive tests on feeder 112 for a period of several weeks, some interesting things may be observed.

These data are plotted to semi-logarithmic coordinates with the logarithm of the current, in milliamperes per mile at 9000 volts, as the ordinate and the time, in seconds, plotted uniformly as the abscissa. It is observed that for the tests of February 25, 1921, (Fig. 3), there is a very sharp decrease in current, from its initial value, during the first 30 seconds. For the remainder of the eight minutes, the current decreased rather slowly and at a fairly uniform rate. It is also interesting to note the variation in the characteristics of the insulation for each of the three conductors of the cable, as brought out by the curves representing tests on each of the several conductors when measured against the other two connected to the sheath and to ground.

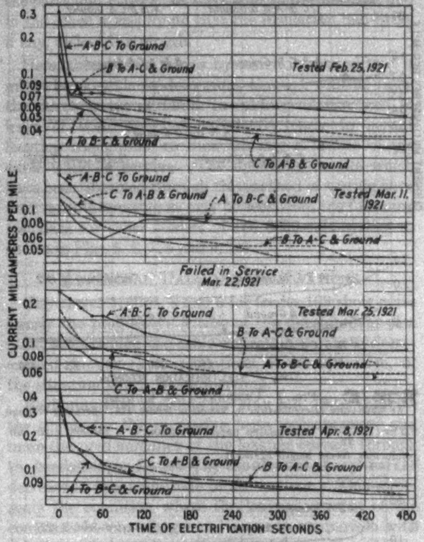


FIG. 3—CURRENT-TIME CURVES

At 9000-volts electrification by kenotron feeder 112; operating voltage . kv.; $\frac{5 \times 5 \text{ in.}}{32}$ impregnated insulation, 3 × 250,000 = cir. mil. conductors; feeder length 17,487 ft.

The tests of March 11th (Fig. 3) show an entirely different situation both as to absolute value of the initial current and outline of the curves. Here again, special attention should be paid to the curves for each of the several conductors. It is noticed that the plotted data for phase *B*, when tested against the other two conductors and the sheath to ground, and phase *C*, tested in a similar fashion, have a rather uniform downward slope. On the other hand, the curve representing phase *A* when tested against *B-C* and ground, shows a decreasing characteristic during the first 60 seconds after which time the current abruptly increased. At the end of two minutes, after electrification of the cable, the value of current on phase *A* is only about 25 per cent less than its initial value and it continues for the balance of the eight-minute period at a magnitude not differing materially from this. Sufficient data had not been obtained, up to the time this material was plotted, to give a clue as to what this sharp upward trend, or inversion, of the curve indicated. Subsequent events seemed to bring out this matter very clearly, for on March 22nd, or eleven days after the test under discussion, the cable failed in service.

Tests made March 25th and April 8th, (Fig. 3), after the cable had been repaired and restored to service, showed characteristics similar to those on February 25th; *i. e.* a gradual decay of the current from the instant of electrification.

The plotted data for tests of April 22nd, (Fig. 4), again showed that an abnormal condition existed. During the first 60 seconds the current for all three phases, tied together and tested against the sheath and ground, decreased quite sharply, but after that time the current arose very abruptly and at the end of 8 minutes was approximately 30 times its initial value. With this indication the kenotron test was discontinued and the cable was connected to the regular high-voltage a-c. test set and it was found that the insulation on phase *A* broke down at a low impressed a-c. potential.

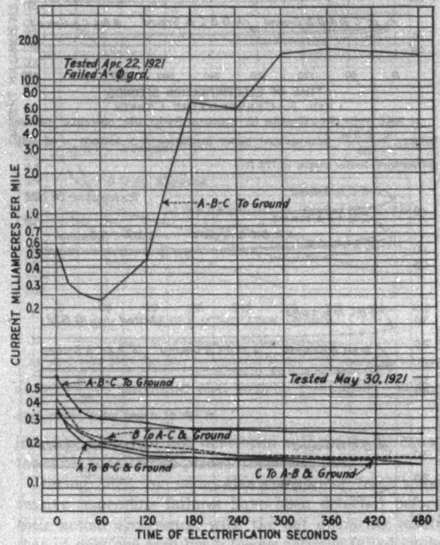


FIG. 4—CURRENT-TIME CURVES

At 9000-volts electrification by kenotron feeder 112; operating voltage 6.0 kv.; $\frac{5 \times 5 \text{ in.}}{32}$ impregnatedins ulation; 3 × 250,000 = cir. mil. conductors; feeder length 17,487 ft.

Tests made again on May 30th, (Fig. 4), produced a series of curves apparently free from abnormal characteristics.

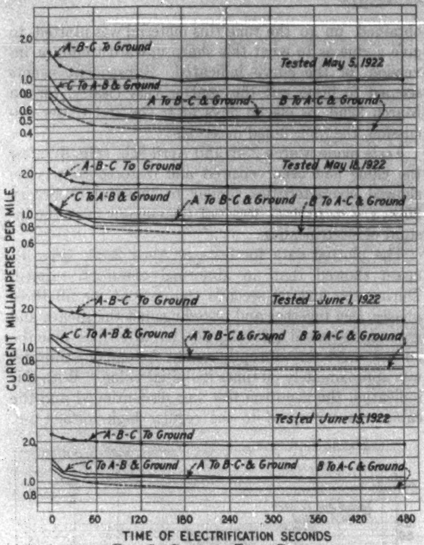


FIG. 5—CURRENT-TIME CURVES

At 9000 volts electrification by kenotron feeder 109; operating voltage 6.0 kv.; $\frac{5 \times 5 \text{ in.}}{32}$ impregnated paper insulation; 3 × 250,000 = cir. mil. conductors; feeder length 19,725 ft.

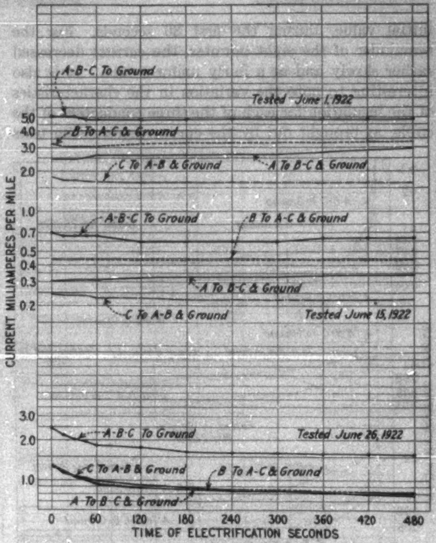


FIG. 7—CURRENT-TIME CURVES

At 9000-volts electrification by kenotron feeder 116; operating voltage 6.0 kv.; $\frac{5 \times 5 \text{ in.}}{32}$ impregnated paper insulation; 3 × 250,000 = cir. mil. conductors; feeder length 12,075 ft.

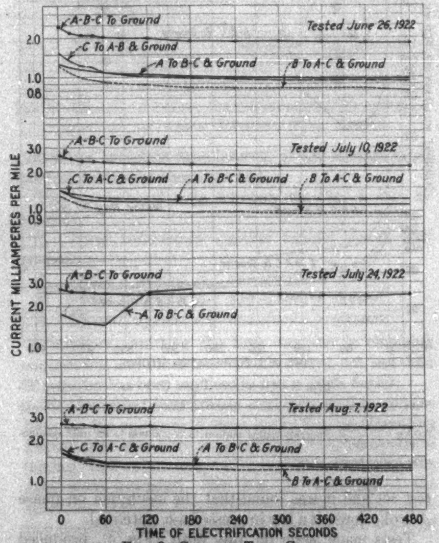


FIG. 6—CURRENT-TIME CURVES

At 9000-volts electrification by kenotron feeder 109; operating voltage 6.0 kv.; $\frac{5 \times 5 \text{ in.}}{32}$ impregnated paper insulation; 3 × 250,000 = cir. mil. conductors; feeder length 19,725 ft.

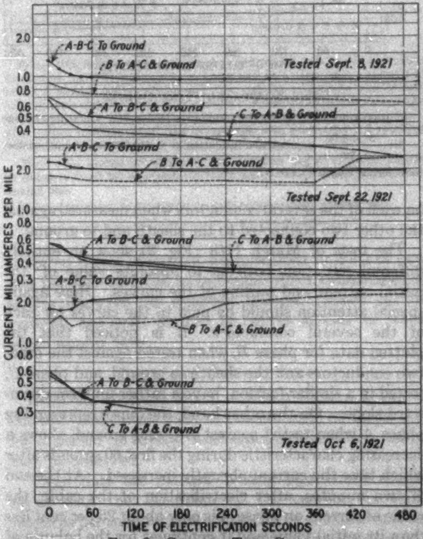


FIG. 8—CURRENT-TIME CURVES

At 9000-volts electrification by kenotron feeder 122; operating voltage 6.0 kv.; $\frac{5 \times 5 \text{ in.}}{32}$ impregnated paper insulation; 3 × 250,000 = cir. mil. conductors; feeder length 13,234 ft.

Recognition of Apparent Solution

Referring again particularly to the tests of March 11th, (Fig. 3), and April 22nd, (Fig. 4), it was felt that the sharp inversion of the curves must be indicative of an unsatisfactory condition of the insulation, since the inversion was followed by an actual breakdown of the particular cable a few days later in the one case, and in the other it was found that the insulation had little, if any, dielectric strength.

Here, then, was a means by which it seemed that it might be possible to determine, through routine tests, the development of faults in cable insulation and to learn of their presence sufficiently early to permit locating them prior to their causing a failure of the cable while in service. The operating results which have been obtained through this method of testing have been most interesting as may be seen from the following test data for other feeders of the 6000-volt system as shown in Figs. 5, 6, 7, 8 and 9.

Additional Data and Observations

The curves of Figs. 5 and 6 for feeder 109 show quite clearly the progressive nature of the development of a fault, even though the cable was retained in active service and there were no indications, in its operation, of the situation which was impending.

Beginning with the tests for May 5th, 1922, (Fig. 5), the curves for the successive tests indicate more and more the predominance of the leakage current through the insulation, as disclosed by the absence of any marked decrease in the magnitude of the current. This represents the condition where the leakage current of the condenser becomes larger and larger in comparison with the charging currents. Finally, with the test of July 24th, (Fig. 6), the cable was found to be defective. The flatness of the curves, from the test following repairs to the cable, appeared to indicate that the feeder was not yet in really good condition.

The curves of Fig. 7 for feeder 116 show that two impending failures were detected before their occurrences in service. The tests of June 1st and 15th, (Fig. 7), brought to light conditions in the paper insulation such that complete breakdown occurred at 7500-volts on the a-c. test set. The tests of June 26th showed the cable to be in much better condition, following the elimination of the faults located earlier, as indicated by the decrease of current with elapsed time when compared with the slight decrease, or even marked rise, detected in the tests of June 1st and 15th.

The curves of Figs. 8 and 9 for feeder 122 show quite clearly the progressive deterioration of cable insulating materials, as well as the wide differences that may develop in the characteristics of the insulation of the different conductors in a cable. Curve B in the tests of September 22nd and October 6th, (Fig. 8), shows quite clearly, both by the magnitude and form, that the condition of the insulation of that conductor is radically different from that of the o her two.

During the year 1922 twelve 6000-volt feeders have been subjected to a routine test by the kenotron at regular intervals of about two weeks. Whenever the test data indicated that the current had a persistent tendency to increase, the cable has been subjected to a high-voltage a-c. test after the completion of the kenotron test. In some twenty instances it has been found that the insulation failed at from 2000 to 7500 volts under a-c. potential. In a few instances the tabular data have indicated a rise in the current value but the incipient fault had not developed sufficiently to cause the cable to fail when placed under a-c. test.

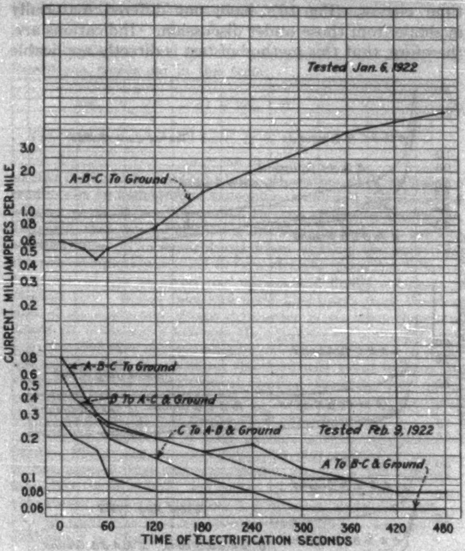


Fig. 9—Current-Time Curves

At 9000-volts electrification by kenotron feeder 122; operating voltage 6.0 kv.; $\frac{5 \times 5 \text{ in.}}{32}$ impregnated paper insulation; 3 × 250,000 = cir. mil. conductors; feeder length 13,234 ft.

During 1922, by using this method of test, some twenty impending failures have been detected before their actual occurrence, with the net operating result that in only one case has a feeder actually failed while in service. This one instance was undoubtedly due to the fact that it was necessary to discontinue the routine kenotron tests temporarily. The failure occurred during the time the tests were not being made.

Examination of a considerable number of faults which have been predetermined by the kenotron has not disclosed any evidence of over-stressing of the insulating material. The several faults which have been detected to date have had legitimate mechanical explanations, as for example, cracked lead, lead worn through where the cable has come in contact with the edge of the duct, wrinkled insulating paper or non-uniformity in the construction of splices.

In addition to the investigation which has been carried out on the 6000-volt feeder system some work has also been done in the application of this proposed test method to 13,200-volt, three-phase cables. While the volume of data, which has been obtained to date from these latter tests, is not as extensive as for the tests on the 6000-volt feeders, the resulting current-time curves, (Fig. 10), have not differed materially in shape from those under discussion. Indications are, therefore, that this method of test is directly applicable to cables operating at nominal pressures other than 6000-volts.

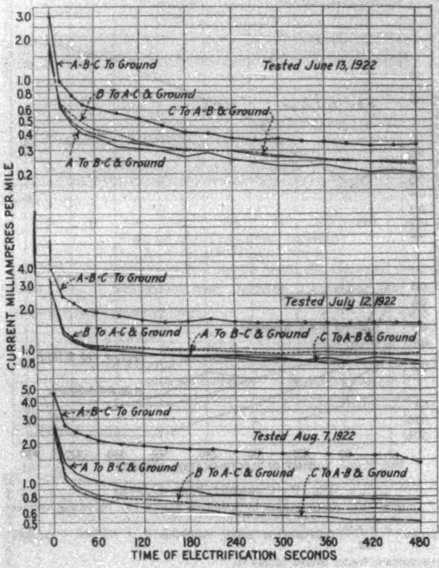


FIG. 10—CURRENT-TIME CURVES

At 15,000-volts electrification by kenotron feeder 345; oprating voltage 13.2 kv.; $\frac{7 \times 7 \text{ in.}}{32}$ impregnated paper insulation; 3 × 250,000 = cir. mil. conductors; feeder length 12,924 ft.

However, due to the fact that the total current from the kenotron into the cable consists of three components, viz. capacity current, absorption current, and leakage current, and since these vary in magnitude according to the length and condition of insulation of the cable a wide variation in the amount of current to be measured when testing different classes of cables may be expected to occur. Accordingly, refinements in the method and apparatus used for reading the current values may be desirable and even necessary for some cables. Also, since the shape of the current-time curves is indicative of the condition of the insulation, as has been shown, it is evident that any refinements possible in obtaining accurate test values will be of aid in detecting an impending fault at an earlier stage of its development than is now possible.

CONCLUSION

The material which has been presented leads to the conclusion that the method of testing here outlined provides a means by which it is feasible to determine the operating condition of paper insulation on high-voltage alternating-current transmission cables. It will also be readily seen that this scheme of testing conforms with the desired characteristics of a routine test method which were previously outlined.

Since the major part of this investigation has been carried out on cables which were known to have been manufactured prior to the production of the present day low-dielectric-loss, high-efficiency cable it is believed that a large amount of supplemental information should be obtained.

In the following there are listed a number of points which should be investigated further. They readily arrange themselves in two divisions, viz. practical and theoretical.

PRACTICAL PROBLEMS

1. Determine characteristic current-time curves for high-efficiency cable.
2. Make mechanical study of faults detected by kenotron.
3. Determine feasibility of extending use of kenotron to actual development of fault, without recourse to alternating-current test sets.
4. Ascertain necessity for more sensitive indication of impending failure than indicated by inversion of current-time curves.
5. Determine what, if any, necessity exists for a source of energy for the operation of the kenotron, independent of the station bus supply.

THEORETICAL PROBLEMS

1. Ascertain what, if any, modifications of shape or magnitude, of the current-time curves results from changes in the rate of electrification.
2. What, if any, modification of the current-time curves results from electrifying cables to different ultimate potentials?
3. Determine what modification of the current-time curves results because of different operating temperatures of the cables.
4. Make comprehensive study of wave shape of the current and voltage as supplied by the kenotron under test conditions.
5. What relation, if any, exists between milliamperes per unit of length and condition of cable insulation?

6. Determine significance and effect of flashover of the protective sphere gaps, provided on kenotron sets, upon this method of testing.

It is believed the results of investigations now in progress, along some of the lines indicated, will prove of great value.

The assistance that has been received, in the preparation of this paper, from Dr. Steinmetz, Mr. J. L. R. Hayden and Mr. E. E. F. Creighton, of the General Electric Company, and Mr. H. P. Liversidge, Assistant Chief Engineer of The Philadelphia Electric Company, is gratefully acknowledged. We also wish to express our appreciation of the efforts of those in the Station Operating Department and Laboratory Division of The Philadelphia Electric Company who assisted in obtaining the information desired.

Discussion

J. L. R. Hayden: The great value of Messrs. Phelps and Tanzer's paper is its method of indicating the gradual deterioration of a cable insulation. This makes it possible to determine the approach of a failure sufficiently, so that the cable can be broken down out of service by high-potential test, and repaired before it gives trouble in service.

There is probably a slow deterioration of cable insulation while in operation. It is a serious matter to have a cable fail in service. Therefore cables are sometimes given an a-c. high-potential test at regular intervals, so that, if deteriorated, they break down in test rather than in service. But the high test voltage, especially when applied for some time, rapidly deteriorates the cable insulations, so that such periodic high-potential testing may materially reduce the life of the cable, without guarding against a break down in service. The cable may fail under normal voltage, shortly after it was tested, if the deterioration occurring under the high-voltage test has almost reached the failure point, and then is completed to the failure point by the succeeding normal voltage.

The method of testing described by Messrs. Phelps and Tanzer offers the advantage of not deteriorating the cable by exposure to abnormal voltage.

The evidence is continuously becoming more conclusive, that electric failure of solid insulation, especially compound insulation as in a cable, is not a puncture by electric overstress by the voltage, but is of the nature of a gradual deterioration by the combined action of voltage, time and temperature. Therefore a judgment on the progress of the deterioration should be made possible by methods of test showing the internal condition of the insulation. This is the case with d-c. tests, but not with a-c. tests. The alternating current taken by the cable under test is the charge on the metal terminals of the cable as condenser, and does not enter or pass through the insulation thus showing nothing about it. The voltage merely shows why the insulation fails, and then destroys its evidence.

The only feature which has a bearing on the insulation, in the a-c. test, is the power factor, which indicates the losses in the insulation. The power factor therefore is justly considered as an important feature, and low power factor specified. While low power factor is valuable and desirable as meaning low losses and therefore lower temperature, it may be questioned whether for instance, a cable with a somewhat higher power factor, but a power factor which does not vary much with voltage, time, temperature, etc., may not give a better life than a cable in which the power factor greatly varies with voltage, time and temperature.

In the d-c. test, the final value of the current is that which is conducted through the total resistance of the insulation, and depends on the latter. Therefore it has a slow transient current, which is the current that passes a few seconds after the initial rush of condenser charging current has ceased. This continues to decrease gradually for several minutes. It is a current which passes into the insulation, therefore passes through some of the resistance, and forms internal electric charges within the insulation. It depends on the specific capacities and the resistivities of the different parts of the insulation, and its study should therefore afford information on what goes on inside of the insulation.

D. W. Roper: According to the textbooks, the equation of the curve shown by these authors is as follows:

$$i = \delta_o + \delta_1 \epsilon^{-ct} \qquad (1)$$

That equation, with the half transposition, and taking the logarithms, comes out in this form:

$$\log_\epsilon \frac{1}{i - \delta_o} = c\,t + \log_\epsilon 1/\delta_1 \qquad (2)$$

This equation, you will note, is in the form:

$$y = mx + b,$$

so that it should be a straight line. However, when we take the readings on an underground transmission line and plot them and analyze them, it was discovered by one of my assistants, Mr. Halperin, that instead of getting one straight line, you get two straight lines. The curve found when plotting the points for the second equation seemed to be a straight steep line followed by a curved line in about one minute and finally a less-steep straight line. If you take and work backwards from the first part of the curve and extend it in order to see what the curve would be, if the curve followed the equation for a straight line, then you find that the current goes down to a minimum in about one minute. Or, if you follow the third part curve back to the vertical axis, the initial current reading would be only about one-half of what you actually get; so that apparently the equation as given in the text books needs some revision.

We have endeavored to carry out one of these suggestions of Messrs. Phelps and Tanzer in applying this method of analysis to the modern high-efficiency cables, having a low dielectric loss; but unfortunately, we do not have in these cables, up to the present time, any dielectric loss failures; and it was failures of that particular kind which were discovered in advance and eliminated in Philadelphia.

We have taken quite a few readings and made a number of analyses of tests on these high-efficiency cables, and also on some of the rosin oil cables, which showed very much the same shape of curve; but up to the present time we have not been able to secure practical results from the analysis, but it is interesting to us, at least, to know the peculiar shape of the curve we get upon analysis.

Messrs. Phelps and Tanzer in their paper mentioned that the cables which they tested showed a maximum ratio of 405 to 1, between the initial reading, and the final reading. On some of our 33-kv. cables, we found that the pressure was as high as 150, the current ranging from fifteen milliamperes for the initial reading to 1/10 milli-ampere for the final reading, also, in some of the tests we noticed some interesting differences between the leakage through the cambric insulation and through the impregnated paper insulation.

We have made a number of tests on a 15-mile 33-kv. line, with 19/64 paper around each conductor, and 7/64 belt; and for the test lead between the positive side and the transmission line, we had about 400 ft. of cambric insulated conductor, with about twice the thickness of insulation between the copper and the lead. In making observations on this combination of 400 ft. of cambric cable and the 15-mile line, we find the losses in the paper cable are approximately three-fifths of the total leakage, while there was about 40 per cent of the leakage in the 400 ft. of cambric insulation in the test lead.

Charles P. Steinmetz: Messrs. Phelps and Tanzer's paper shows that from the study of the slow cable transient the deterioration of the insulation can be estimated and its failure thereby anticipated. This is rational, as this slow transient depends on the internal conditions of the insulation. It is a true electrical transient, and of special interest by its long duration, often many minutes, while the external transient of charging the condenser terminals over an inductive supply circuit, usually vanishes within a fraction of a second.

This slow cable transient (the so-called soaking in of the charge, etc.) does not exist if the insulation is perfectly homogeneous; but if the insulation consists of n dielectrics of different specific capacities and resistivities, a slow transient of $(n - 1)$ terms appears.

To illustrate the nature of this transient, let us consider a cable as condenser, in which the dielectric consists of layers of two materials of different specific capacities and different resistivities (paper and rosin oil for instance).

The voltage impressed upon the condenser produces an electrostatic flux between the condenser terminals. A line of electrostatic flux must either return into itself, or terminate in an electrostatic charge. At the first moment of the impressed voltage there can be no internal charges in the dielectric, since there was no time to conduct them through the high resistance of the dielectric. Thus in the first moment, the electrostatic flux density is the same in both dielectrics, and the voltage is distributed between the two dielectrics by their specific capacity, that is, the voltage gradients in the two dielectrics are inversely proportional to the specific capacities.

Due to the finite (though extremely low) conductivity of the dielectric, a current is conducted, or leaks, through the dielectric, and the current density of this current in the two dielectrics is proportional to their respective conductivities, and to their voltage gradients, which latter in the initial moment of the phenomenon depend on the specific capacities. Unless then the resistivities of the two dielectrics happen to be exactly proportional to their specific capacities, the current densities in the two component dielectrics must be different. This means that at every boundary between the two dielectrics a change of current density occurs, and more current flows towards the boundary from the one side, than leaves it on the other side, and consequently, an electrostatic charge builds up at the boundary between the two dielectrics, due to the difference between the two current densities on the two sides of the boundary. This electrostatic charge lowers the voltage gradient and thus the current density in the dielectric of higher current density, and raises it in the dielectric of lower current density, until the current densities in both component dielectrics have become equal and any further building up of internal charge ceases.

The final condition thus is that of uniform current density in both component dielectrics, so that the voltage gradients are proportional to the respective resistivities of the two dielectrics.

Thus in a condenser with a compound dielectric, like a cable, after initial electrification by the rapid external charging transient, an internal voltage adjustment occurs, from the initial voltage distribution corresponding to the specific capacities, to the final voltage distribution corresponding to the respective resistivities of the two dielectrics, with a change from uniform electrostatic flux density and non-uniform current density, to uniform current density and ununiform flux densities terminating at internal electrostatic charges at all the boundaries between the two dielectrics. As the internal static charges have to be conducted through the high resistance of the dielectric, this occurs by a very slow transient, of a duration usually many thousand times greater than the external charging transient.

In the final condition, the stored energy of the condenser is greater than in its initial condition, and its capacity therefore higher.

It can be shown that of the energy of this slow internal transient, half is converted into heat in the resistance of the dielectric, the other half stored as electrostatic energy of the internal charges which after discharge partly appears at the terminals as "residual charge."

Numerous other interesting conclusions follow, for instance that the $i^2 r$ loss in the cable dielectric with alternating current may be many times greater than with direct current, and this additional $i^2 r$ loss is independent of the frequency. The difference between the effective resistance of a cable, measured with alternating current, and the d-c. resistance, is not all dielectric hysteresis, but partly at least true resistance, etc.

As seen, this slow transient gives some information on the individual component dielectrics of such a compound structure as a cable, and in view of its importance in estimating cable deterioration, it may be of interest to give in a later paper the equations of it and thus its relation to the resistivities, capacities, etc. of the different component dielectrics.

G. B. Shanklin: Messrs. Phelps and Tanzer have developed a method of test based on sound principles with which it is possible to discover cable faults during their incipient stage, thereby filling a long felt and urgent need in underground transmission. They claim only that this method of test has worked out successfully on comparatively low-voltage lines (6000 volts) of the old rosin-oil type cable and offer no predictions as to its future usefulness on lines of other and higher voltages.

Considering the principles upon which their method is based, there is no reason why it should not be just as successful on longer, higher-voltage lines of the new type cable. It is a question of obtaining data to prove this point and such work, I understand, is now under way.

The leakage current through a fault during its incipient stage is extremely small, even smaller than the leakage through the rest of the total cable length. Its detection is, therefore, very difficult and the success Messrs. Phelps and Tanzer have had can be attributed to certain factors in their test method:

1st. The use of d-c. voltage, thereby eliminating charging current which would be so high in comparison with leakage current as to completely obscure the results.

2nd. The use of *high* d-c. voltage, which has a tendency to search out and penetrate weak spots, forcing more current through in proportion to leakage current over rest of line. High d-c. voltage also brings the total current within readable values, eliminating the possibility of errors introduced by stray currents, etc. In view of this the curves should be taken at the highest voltage compatible with safety from damage to the sound portion of the line.

3rd. The opportunity of comparing the current vs. time curves of the individual conductors. Single-conductor cable does not offer this opportunity, but perhaps a comparison of the curves taken on cables of the same circuit in adjacent ducts will do just as well.

The above factors all tend to furnish a method of test that is almost unbelievably sensitive, searching out a weak spot perhaps several inches in length in a line miles in length. A careful study of the conductivity of various types of cable faults during their progressive stages of formation is necessary before an exact estimate of the efficacy of this method of test can be made. Such a study will involve serious difficulties. The study would, obviously, have to be made in the laboratory on short lengths, for it is impossible to locate faults on long lines during their initial stages of development, without first reducing them by burning, thereby destroying the desired evidence. In laboratory study the faults would have to be artificially formed and might not represent the same type as those formed in service. When the laboratory and field experiences are combined we may, in time, hope to see this method of test reach its full development and furnish a reliable means of preventing short circuit on all types of transmission and distribution lines.

W. N. Eddy: In testing insulation with direct current, the question naturally arises as to what d-c. voltage should be used.

It is known that a gap in air will break down at a d-c. voltage equal to the a-c. maximum or 1.41 times the a-c. effective. It is known that the d-c. puncture voltage for most solid insulations is higher than the a-c. maximum, that is, the ratio between the d-c. and the a-c. maximum is often greater than 1.0. The ratio is known to decrease with increasing temperature and also to vary with the type, dimensions and arrangement of the insulation and even with the method of application of the voltage.

At present the ratio to be expected on any given type of insulation cannot be foretold without making actual d-c. breakdown tests on samples of the insulation.

A detailed investigation of the causes and reasons for the variation of the d-c. to a-c. ratio that is being carried on, has made such progress that it will be possible to publish some exceedingly interesting and valuable results on this subject in the immediate future.

S. J. Rosch: The cable user seems to have neglected one very important factor in cable work, and that is, the importance of insulation resistance.

On reading over some of the earlier text books and papers on the subject of dielectrics, I find that insulation resistance had been considered of no moment when referring to impregnated paper insulation. Several of the papers recently published however, have shown that the insulation resistance of cables which have been impregnated in a given compound, has a definite relation to their dielectric loss.

I bring this point up, to point out another line of investigation to Messrs. Phelps and Tanzer, in conjunction with the work they are now doing on cables. While presenting his paper, Mr. Phelps stated that after he had made his voltage tests on the cable, he could find no evidence that the latter had been overstressed. The question in my mind is, how can he tell when a cable has been overstressed? As known to most of you, insulation resistance varies inversely with temperature. If we were to take a certain cable whose insulation resistance happened to be about 300 megohms per mile, subject it to a certain test voltage, and if upon the completion of this test voltage, we should find that the insulation resistance had decreased to about 200 megohms per mile, there would be two ways to account for this decrease. (1) The temperature of the cable had been increased. (2) The insulation of the cable had been overstressed.

Analyzing these two possible explanations, we would have to discard the first, because it would be practically impossible for the temperature of the cable to change ten or even five degrees within a period of five minutes, without the external application of heat, so that the only other explanation in this case, would be that the insulation had been overstressed.

In my opinion therefore, the tests which are now being conducted by Messrs. Phelps and Tanzer, are incomplete if they do not measure the insulation resistance of their cables before and after the application of the voltage tests. As to the feasibility of making these tests, I want to say that insulation resistance is not necessarily a laboratory measurement. The writer has personally measured insulation resistance in the field, by setting up a galvanometer on a tripod, and has obtained very good results that way.

H. L. Wallau: It has just occurred to me that this method of testing laminated insulation in cables might possibly be carried out and applied to laminated insulation in high-voltage generator armatures and transformers.

If we could determine the proper time to rewind an armature before it failed in service, it would be of great value to us.

E. D. Tanzer: Mr. Roper, in presenting some additional information on tests, similar to those which we have made in Philadelphia, calls attention to the large difference between the initial input current and its final value and cites a particular instance in which the ratio was approximately 15 to 0.1 or 150. I find this to be generally true that the condition of the cable insulation determines more or less the ratio between the initial input and the final value of the current. We have felt that on different types of cables, especially on the new types of cables having inherently better insulation, that perhaps more refined methods of reading these current values would be necessary in order to obtain sufficiently accurate data.

We do not believe, however, that this will nullify the theory that we have proposed, that is, that the inversion of the current-time curve is a measure of the condition of the cable insulation. Dr. Steinmetz has very clearly and interestingly presented the conditions taking place in the insulation of a cable, from the time of initial excitation to the time at which the input current becomes nearly constant. As it was pointed out, there is evidently an accumulation of charge in the cable insulation, and this is responsible for the slow transient that occurs. I am very much interested in Dr. Steinmetz's method of analysis and hope that he may find it convenient to present this in greater detail at some future time.

Mr. Rosch, speaks of a point which we have not fully covered in the paper, *i.e.*, whether or not the method of test imposes any undue stress upon the cable. He has presented to us a point which we can welcome and take up in the work which is now being undertaken in Philadelphia along this particular line, that is, to find out if possible, if there is any difference in the insulation resistance before and after our method of test on a particular cable has been made.

I would like to call attention, however, to the fact that this particular method of testing we have proposed is essentially a routine field test and not a laboratory test. Consequently, we are not able to hold any one of the fifty-seven variables exactly constant as might be done in the laboratory, in as much as it is a routine proposition applicable to field results only.

I am rather interested in Mr. Wallau's point in regard to the application of this method as a possible means of testing armature windings and things of that sort. I personally believe, as does Mr. Phelps, that this method of test does have a possibly wider application than we have made of it. Up to the present time our experience has been confined to cables of one particular class, that is, those having insulation of impregnated paper. I understand, however, that some further corroborative evidence in regard to rubber insulated cables has been secured elsewhere at this time, although the data era not available to be released just not. So far as I can see there would be nothing which would prevent the application of this particular method of testing to other insulations.

The only thing to which I would again call attention is the probability of refinement in the measurements of the current values which might be necessary as the insulation to be tested becomes better and better. As the insulation resistance is steadily increased, then along with the increased value of the impressed voltage necessary, the current to be measured would be less and less, and accordingly more refined methods of measuring this current would undoubtedly be necessary, or at least very desirable.

We believe that in the near future that we will be able to predict an impending fault, at a longer interval of time before it becomes a menace to the system than we are now able to do. This development will be in the nature of refinements in the method of conducting the test and will not mitigate against the method of test at all.

Public Address Systems

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THIS paper aims to present the problems encountered in the development of electrical systems for amplifying the voices of public speakers and music; and to describe the equipment as brought to a commercial state and now in use in the United States and various other countries.

The two main requirements of a successful public address or loud speaking system are, first, that it shall reproduce the sounds, such as speech or music, faithfully; and second, that this faithful reproduction shall be loud enough and sufficiently well distributed, for all of the audience to hear it comfortably. Most of the development work has been directed toward obtaining these two results under the various conditions which surround the operation of these systems.

The faithful and natural reproduction of sound depends upon many factors, of which the following are some of the more important: The acoustics of the space in which the sound originates, the characteristic of the loud speaking system itself and the acoustics of the space in which the sound is reproduced. Where the sound is picked up and reproduced in the same space, as is the case when the speaker is using one of these systems to address a large audience locally, there is a reaction between the horns and the transmitter or pick-up mechanism which is controlled by the acoustic conditions under which the system is operated.

ACOUSTICS OF THE SPACE

In connection with the acoustics of the space in which the sound originates, or in which it is reproduced, four factors stand out. These concern the effects of reverberation, of echo, of resonance, and of diffraction. In the specific cases where the sound is reproduced in the same space or room in which it originates, another effect is encountered, which has generally been termed "singing," and is evidenced if sufficiently great by the emission of a continuous note from the equipment.

Reverberation is caused by reflection and is evidenced by the persistence of the sound after its source has ceased emitting. When the reverberation in the space in which the initial sound is being picked up is sufficient to cause one sound to hang over and become mingled with succeeding sounds, in other words, so that the sound from one syllable interferes with that of the succeeding syllable, it is practically impossible to improve the acoustic conditions solely by the use of the public address system. In such a case, the first procedure is to place material which absorbs sound in the space. The purpose of this absorbing material is to lower the time required for the sound to die away after the source ceases to emit. The amount which any given material lowers the time of reverberation depends not only upon the amount of material introduced, but also upon its disposition within the space.

The term "echo" applies to a similar phenomenon, but is generally used where there is sufficient time lag between the reflected sound and that originally emitted, so that two distinct impressions reach the ear.[1]

The troubles encountered from echoes usually occur only in large buildings or large open spaces surrounded by buildings, trees, or other obstacles and are generally associated with interferences with the reproduced sound rather than with the original sound. There are cases, however, particularly in auditoriums, where some of the walls or ceiling are large curved surfaces, in which case localized echoes may result. The speaker's voice or extraneous sounds from the audience may be reflected from one or more of these surfaces to focus spots where the volume of sound is consequently abnormally great. It is important, therefore, that the transmitter which is picking up the sound shall not be located at one of these spots. These points of localized echo are particularly troublesome also when they occur in the space occupied by the audience. Under these conditions not only is the sound intensity too great, but the character of the sound is altered and very often badly confused. The avoidance of such difficulties is a matter of test and the proper arrangement of the reproducing mechanism, as will be seen later in some detail.

The effect of resonance seldom occurs in connection with the amplified and reproduced sound, inasmuch as the spaces dealt with are large and their natural frequencies are too low to be troublesome. Resonance usually becomes of importance in connection with mounting the pick-up apparatus or transmitter. It generally results from attempts to conceal the transmitter by placing it in some form of small enclosure. The best form of housing from an acoustic standpoint consists of a screen cover which protects the instrument from being struck or injured but in no way affects the sound reaching it.

Resonance produces a distortion which it has been customary to consider as of two varieties. First, there is an unequal amplification of sounds of various frequencies and second, there is the introduction of transients. These transients occur whenever the sound changes but are most easily recognized audibly by their continuation after the source has ceased emitting. They also have frequency characteristics which depend not only on the sound which started them but also upon the character of the resonant portion of the system.

The troubles introduced by diffraction are seldom of

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[1] Collected Papers on Acoustics, Wallace Clement Sabine, *Harvard University Press*, 1922.

very great importance except where the sound is reflected from regularly spaced reflectors or passed through regularly spaced openings. Quite serious diffraction troubles have been encountered when operating a loud speaker in a large field, surrounded by an open work board fence, the trouble being evidenced by very distinct areas, particularly at the outskirts of the audience where the sounds were badly distorted.

The difficulties encountered as a result of "singing" form one of the most troublesome problems connected with the actual operation of these systems. When a portion of the sound emitted by the projectors reaches the transmitter with sufficient intensity, that its reproduction is as great as the originally emitted sound from the projectors, and with such phase relation that it tends to aid the original sound, the system will emit a continuous note. Moreover, when the portion of the sound from the projectors which reaches the transmitter is not sufficient to cause a continuous note, it may be sufficient to cause considerable distortion of the speech or music. In excessively reverberant halls these conditions are often fulfilled when the actual amplification is so small that the people at the distant points are scarcely able to hear the speaker. In all cases in our experience the difficulty has been sufficiently overcome by properly placing the transmitter with respect to the projectors. The situation is very much helped by the presence of the audience, which adds considerably to the acoustic damping of the room.

It will be seen, therefore, that the acoustic conditions of the space in which loud speakers are used are of considerable importance.

CHARACTERISTICS OF THE SYSTEM

The first requirement of the system itself is that it shall reproduce speech or music faithfully. A system is said to do this, or in other words, its quality is called perfect, when the reproduced sound contains all of the frequencies, but no others, contained in the original sound striking the "pick-up" mechanism, and when these frequencies have the same relative intensities that they had in the original.

An imperfect or distorting system is one which fails to fulfill this requirement. There are two main types of distortion which had to be considered; the first being the unequal amplification of the system for the various frequencies constituting the sound and the second being the introduction of frequencies not present in the original sound. For simplicity of discussion, this last class will be divided in three parts, namely: the effect of transients, the effect of asymmetric distortion and the effect of disturbing noises.

The effect of transients has already been mentioned in connection with acoustics and they, of course, produce the same type of distortion whether they occur in the acoustic or the electrical system. Transients occur whenever the sound changes either in pitch or intensity, and are introduced at the beginning and ending of each speech sound. This modification of the characteristics of the speech sounds acts to lower the intelligibility. It probably causes more trouble in speech transmission than the fact that the sound continues after the source ceases.

Asymmetric distortion affects one half of the wave differently from its other half. This causes the introduction of frequencies which, in some cases, produce very serious disrurbances in the transmission of music and speech. The most noticeable troubles are from the formation of sum and difference tones.[2] Such tones are likely to give rise to dissonances with the other sounds occurring in the music. In the case of speech asymmetric distortion manifests itself by a lower intelligibility.

The effect of foreign noises sometimes encountered is twofold. First, they influence the ability of the listener to hear the characteristic speech sounds and hence tend to lower the intelligibility. Secondly, the constant attempt of the hearer to sort out the speech sounds or music through the disturbing noises tires him appreciably. In order that this strain shall be inappreciable, it is desirable that the sound delivered by the system shall be at a power level at least 10,000 times that of the noise.

The second general requirement which is placed on a successful system is that it shall deliver its faithful reproduction loud enough for all the audience to hear it comfortably and enough above noise for good intelligibility. In this connection there have arisen one or two interesting points bearing on the psychology of hearing. One of the most striking of these is concerned with the coordination between hearing and seeing. Although the projectors are usually mounted twenty or more feet above the speaker's head, and in some exceptional cases, slightly to one side of him, the majority of the audience is conscious of only one source of sound, and that appears to be the speaker himself.

This phenomenon is so marked that in several cases the question has been raised in the minds of the listeners as to whether the system was functioning. They could only be convinced that it was by having it shut down for a few seconds when their inability to hear made them realize how successfully the system could operate.

Another of these psychological phenomena deals with the apparent distortion of the voice when its intensity at the ears of the listener is too great or too small. If the speaker is talking in a normal conversational tone, his voice contains a larger percentage of low frequencies than is the case when he is raising his voice to a considerable volume. If the loud speaker so amplifies this voice that it reaches the audience with such volume that their instinct tells them that the speaker should be shouting, the system appears to make his voice sound quite heavy and somewhat unnatural. It has been found necessary, therefore, to so regulate the amount of amplification that the people at the furthest portion of the hall can hear comfortably and the volume of sound should not be permitted to become any louder than necessary to meet this condition. On the other hand, if the volume is insufficiently loud, certain of the weaker speech sounds are entirely lost, and it becomes difficult to understand.[3]

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[2] **Origin of Combination Tones in Microphone-Telephone Circuits. E. Waetzmann, *Annalen der Physik*, Vol. 42, 1913.**

SOLUTION OF THE PROBLEM

With these considerations in mind it may be interesting to take a brief survey of the whole problem and the method by which the solution was reached. Two general methods of attack were considered. The first was to attempt to make each unit of the system faithfully reproduce its input, while the second was to make any distortions of one part of the system, cancel those of another portion, so that the complete system would operate satisfactorily. In either case, it was desirable to keep each unit free from asymmetric distortion, as this type of distortion cannot be easily compensated for.

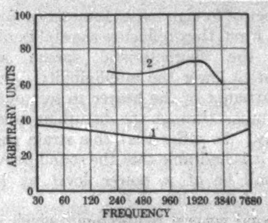

FIG. 1—1-CONDENSER TRANSMITTER WITH ASSOCIATED AMPLIFIER
2-CARBON TRANSMITTER WITHOUT AMPLIFIER

While it would probably have been simpler to follow the second line of attack, the greatly increased flexibility of a system in which each part is correct in itself was of sufficient value to cause the attempt to be made that way. When it is realized that these systems, to be commercially successful, must be capable of operating for various sized audiences, ranging possibly from one thousand to several hundred thousand, that they must be used in connection with long distance telephone lines, as well as with either radio broadcasting or receiving stations, the desire for flexibility can be understood.[4]

As a result of attempting the development in the manner already described, there has resulted a system which involves four functional units; a "pick-up" mechanism or transmitter unit, a preliminary amplifier unit, commonly called the speech input equipment, a second amplifier unit commonly called the power amplifier, and a receiver-projector unit for transforming the amplified currents back into sound, and properly distributing it throughout the space to be covered.

It may be interesting at this place to determine how successfully these various units and the system as a whole fulfill the requirements of equal sensitivity to all frequencies within the important speech or music range. Fig. 1 shows the relative sensitiveness of the transmitter as a function of frequency. The ordinates are proportional to the logarithm of the power delivered for constant sound pressure at the diaphragm and the abscissa to the logarithm of the frequencies employed. The lower of the two figures refers to the condenser transmitter with its associated input amplifier.[5] The upper refers to the push-pull carbon-type transmitter. These transmitters will be described in detail later.

Fig. 2 shows a similar curve for the complete amplifier system, comprising a three-stage speech input amplifier, and a power stage capable of delivering approximately 40 watts of speech frequency electrical power without distortion. In connection with these amplifiers a sharp distinction should be made between their gain rating, or amplification, and their overload or power rating. Gain measures the power amplification which can be obtained provided the input is small enough so that the equipment at the output end is not overloaded. Overload or power rating refers to the maximum power which can be supplied by the amplifier without causing distortion of the currents being amplified. Although the power rating of power equipment is usually determined by the heat which can be dissipated, a marked distortion of wave form takes place when the iron in any of the apparatus is worked beyond the straight line portion of the magnetization curve. In the case of amplifiers, the maximum power obtainable is limited by the power output at which distortion occurs rather than by the heat which can be dissipated.

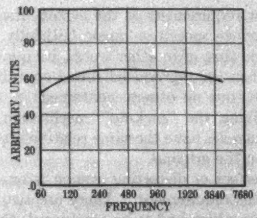

FIG. 2—PUBLIC ADDRESS SYSTEM AMPLIFIERS

Fig. 3 shows a chart for the characteristics of the complete system, including the carbon transmitter, the speech input and power amplifiers and the receiver projector unit.

In connection with the requirements for equal amplification of all frequencies, it is interesting to note that a system, which does not fail to reproduce equally all frequencies in the speech range by more than a ratio of 10 to 1 in reproduced power, is indistinguishable, from a perfect system or from the speaker, himself. It seems probable that this effect is, in some way, connected with variations in the frequency sensitiveness curve of normal ears. Normal ears show a sensitiveness variation with frequency as great as 10 to 1 and the frequencies of maximum sensitiveness vary materially from one individual to another.[6]

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[3] Physical Examination of Hearing R. L. Wegel, *Proceedings of the National Academy of Sciences*, Volume 8, Number 7, July 1922.

[4] Use of Public Address Systems with Telephone Lines, W. H. Martin and A. B. Clark. Presented before A. I. E. E. Feb. 14, 1923.

[5] The Sensitivity and Precision of the Electrostatic Transmitter for Measurement of Sound Intensities. E. C. Wente, *Physical Review, N. S.* Vol. 19, No. 5, May 1922.

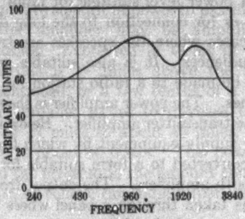

Fig. 3—Complete Public Address System with Carbon Transmitter

It has been found that in order to transmit speech with entire satisfaction for loud speaker purposes, that is, sufficiently well so that the audience is not aware of the contribution of the mechanical equipment, it is necessary for the system to operate with essentially uniform amplification over a range of frequencies from 200 to 4000 cycles. While there are, in speech, frequencies slightly outside of this range, the loss in naturalness and intelligibility by the system's failure to reproduce them, is slight.[7]

While no such frequency range is required for intelligibility only, it has been found that systems not covering substantially this frequency range, sound unnatural. When the lower frequencies are missing, the reproduction sounds "tinny." When the higher frequencies are missing it sounds heavy and muffled. The requirements for thoroughly natural reproduction of music are probably more severe, particularly in the low-frequency region, than are the similar requirements for speech, but, at the present time, complete data are not available to indicate the contribution of these frequencies to naturalness.

In connection with the flexibility of the system, it is interesting to note that the speech input equipment has been designed to raise the power delivered by the transmitter to such an extent that it is sufficient for long distance telephone transmission or for the operation of a radio transmitting set. The power amplifier is designed to receive power at approximately this level and deliver it to the projector units sufficiently amplified to operate them satisfactorily.

Future Development

In viewing the loud speaker field from the point of view of future development there are two lines of attack along which work is being done, and which give promise of success. These are the improvements in frequency characteristics and increase in the range of loudness which the system can accommodate satisfactorily.

The improvement to be expected from a more uniform frequency characteristic is mainly an increase in naturalness, especially where music is being reproduced. A slight increase in intelligibility may be hoped for, although this factor is of little importance, as the present system is satisfactory in this respect.

The other improvement mentioned, namely, the volume range, is probably the more difficult, but is necessary before music can be reproduced in a perfect manner. Rough experimental data indicate that the loudness in an orchestra selection may vary from one part of the selection to another by a ratio as great as 50,000 to 1. While the present equipment does not operate with entire satisfaction over this range of loudness, it has been found relatively easy to obtain good results by manual adjustment of the amplification during the rendering of the selection. If the gain is varied in small enough steps, the change is not noticeable to the listeners.

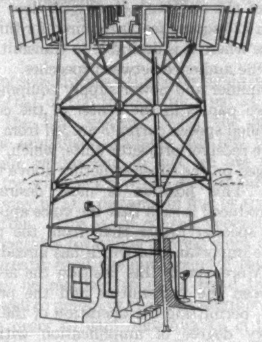

Fig. 4

An increase in the loudness range would render the manual adjustment unnecessary and would make the reproduction a faithful duplicate of the music as actually played.

Technical Description of the System

The foregoing discussion having described the requirements which must be met in order that the public address system shall successfully transmit speech and music, the system in its commercial form will now be described. In order to make clear the arrangement of the equipment, a typical installation is shown in Fig. 4, this being an installation where the audience and speaker are in the open air, and where no connection is made with the long distance lines. It might be well to state here that with the equipment shown an audience of 700,000 can be adequately covered.

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[6] Frequency Sensitiveness of Normal Ears, by H. Fletcher and R. L. Wegel, *Physical Review*, July 1922.

[7] The Nature of Speech and its Interpretation. Harvey Fletcher, *Journal of the Franklin Institute*, Vol. 193, No. 6, June 1922.

Some of the sound leaving the speaker's mouth is picked up by the transmitter, on a reading-desk type of pedestal, which is normally mounted at the front of the platform.

Fig. 5

The feeble currents from this transmitter are led by carefully shielded leads to the amplifiers in the control room, which is usually located directly beneath or to one side of the speaker's stand. A floor space of not more than 125 square feet is required for this room, even in the case where it is desirable to transmit phonograph music to the audience between speeches.

The amplifier and power supply equipment is shown on the two panels in the center of the control room. The amplified speech currents are led from these amplifiers to the receiver projector units, which, in this case, are arranged on the super-structure above the speaker's platform. This position is most desirable, as the illusion produced is such that the voice appears to come from the speaker rather than from the projectors, a factor, the importance of which has already been mentioned. Moreover in this position the acoustic coupling between the transmitter and the projector is a minimum, permitting the operation of the system at a satisfactory degree of amplification with an ample margin below the point where singing troubles are encountered.

A public address equipment, similar to that just described, but with a somewhat lower power output, has been developed for use at [illegible] smaller open air meetings, and in all but the largest indoor auditoriums. Fig. No. 5 shows one of these equipments, mounted on an automobile truck, which has been employed at a number of points in the eastern part of the United States. This smaller system has characteristics equally as good as the larger system in regard to faithful reproduction of speech and music, with a power output in the order of one-tenth as great. An audience of 50,000 can be adequately covered at an outdoor meeting with this system.

Fig. No. 6 is a schematic arrangement of the equipment at an installation of the type shown in Fig. 4. At the extreme left are the transmitters where the sound waves are picked up. The output from these transmitters is taken to a switching panel where means are provided for cutting in the various transmitters. From this panel the transmitter currents are taken to the transmitter amplifier, which is capable of amplifying them to a power level suitable for input to the power amplifier, or for connection to the long distance lines, in those cases where the speeches are transmitted to distant audiences. It is also suitable where connection is to be made to a radio station for broadcasting the speeches. The power amplifier is shown just to the right of the transmitter amplifier. Below it is indicated the power supply equipment by which the commercial power is converted to a form suitable for the vacuum tubes in both amplifiers. The output from the power amplifier is taken through a panel where switches and a multi-step auto-transformer are provided for the regulation of several projector circuits. Just above this panel is an indicating instrument, known as the volume indicator, provided in order that the operator may know what volume output is being delivered to the projectors.

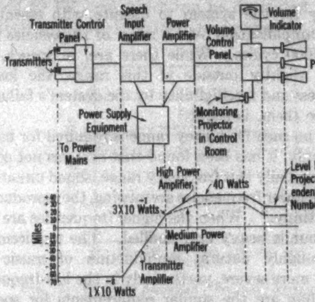

Fig. 6

The projectors, at the extreme right of the figure, consist of the motor or receiver unit transforming the speech currents into sound waves, and a horn to provide the proper distribution of the sound.

It is interesting to note the power amplification which is obtained in the larger of the two systems from the transmitter to the projectors. Referring to Fig. No. 6, a chart will be seen which indicates the power levels through the system drawn to a scale based on miles of standard telephone cable, our usual reference unit. The output of the high quality transmitter is of the order of 65 miles below zero level, this latter being the output from a standard telephone set connected to a common battery central office by a line of zero resistance. Expressed in watts the output of this transmitter under average conditions of use with the public address system is of the order of 10^{-8} watts. Incidentally, this is of the same order as the speech power picked up by the transmitter, or in other words, the transmitter does not amplify the speech power received by it as is the case with the transmitter used for regular telephone service.

This very minute amount of power in passing through the transmitter amplifier may be amplified about 120,000,000 times. Expressed in terms of telephone power levels, this is 17 miles above the zero level previously mentioned, or a few tenths of a watt.

The power amplifier serves to increase this power from a level of 17 miles to about 40 miles, the latter corresponding to about 40 watts. This power is then distributed to the projectors, the amount consumed by each projector, of course, depending upon the number connected.

An idea of what this amount of power at speech frequencies means may be given by the statement that it is sufficient to operate at a volume level slightly above that considered commercial all of the 12,000,000 telephone receivers in use in the Bell system if these were directly connected to the amplifier.

In describing the various pieces of equipment which together make up the system, we will follow the order in which the power is carried through the system from the transmitter to the receiver-projector units where the amplified sound waves are propagated.

TRANSMITTERS

In the early work on the public address system, an air-damped, stretched diaphragm condenser transmitter was employed, having a thin steel diaphragm about 2¼ inches in diameter, constituting one plate of the condenser. The other plate was a rigid disk, the dielectric being an air film 1/1000 of an inch in thickness. Due to the stretching of the diaphragm and the stiffness of the air film, the diaphragm of this transmitter had a natural period of approximately 8000 cycles per second which is well above the important frequencies in the voice range. This high natural period, in conjunction with the damping due to the thin film of air, resulted in a transmitter of very high quality of reproduction. However, its extremely small capacity (in the order of 400 micro-microfarads) made it necessary to use leads of very low capacity between the transmitter and the first amplifier, and due to the high impedance of the transmitter and its associated input circuit to voice frequency currents, these leads were very susceptible to electrostatic and electromagnetic induction. It was necessary to limit them to a length of 25 feet, and to provide complete shielding. Moreover the output of this transmitter was less than one five-thousandth of that of the transmitter now used, and for the early installations of the system, it was necessary to provide a preliminary amplifier beneath or to one side of the speaker's stand in order to keep the transmitter leads short and to provide sufficient power to properly operate the main amplifiers. Work was therefore undertaken to provide a transmitter, having quality practically as good as the condenser transmitter, volume output sufficient to operate the main amplifiers, and not requiring the elaborate precautions as to shielding the leads.

The high quality transmitter which was the result of this development work is of the granular carbon type with two variable resistance elements, one on each side of the diaphragm and is commonly known as a push-pull transmitter. It has nearly the same high quality reproduction characteristics as the condenser transmitter, due to the use of the same stretched diaphragm and air damping structure. It introduces no appreciable distortion over the range of frequencies required for good reproduction of speech, but it must be understood, that this quality was obtained only at the sacrifice of sensitiveness, the latter being in the order of 1/1000th that of the transmitter used at telephone stations in the Bell system. With the multi-stage vacuum tube amplifiers available this low volume efficiency is not serious and in fact we are using this transmitter for what is known as distant talking, *i. e.*, the speaker may be at a distance of five or six feet from the transmitter. This is, of course, necessary in any transmitter suitable for public address work as it is not possible to greatly limit the movement of the speakers, nor can they be required to use a hand transmitter. It might be well to point out that this sacrifice in volume efficiency in order to gain high quality is possible at the transmitting end of the system, but not at the opposite end where the electrical power is transformed into sound waves and propagated, as the device at this point must be capable of handling large amounts of power with minimum distortion.

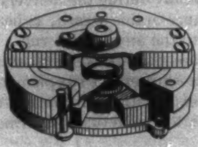
FIG. 7

Referring to Fig. No. 7 which is a cut-away view of this push-pull high quality carbon transmitter, the granular carbon chambers will be seen. The electrical path through each of these variable resistance elements is from the rear carbon electrode through the carbon granules to a gold-plated area on the diaphragm itself. The resistance of this path is about 100 ohms and as the two buttons are in series for the telephone currents, the transmitter is designed to work into an impedance of 200 ohms. The double button construction almost completely eliminates the distortion caused by the non-linear nature of the pressure-resistance characteristics of granular carbon.

As this instrument has a practically flat frequency characteristic no collecting horn or mouth piece is used with it as resonance is introduced by such chambers, with accompanying distortion. To insure the insulation of the transmitter from building vibrations, a simple spring suspension has been provided. To protect the transmitter from injury, two types of transmitter mountings have been used, both arranged for the suspension of the transmitter in a screen-enclosed space—the first adapted to take a single transmitter for indoor use only, while the second for outdoor use, mounts two transmitters within a double screen enclosure to prevent any noise effects due to winds. This second type is arranged to attach to a simple pedestal-type of reading desk, which it has been found desirable to provide as there is a slight tendency for the speaker to remain fairly close to the desk. In this connection it is interesting to note that we have found a small rug, so placed as to cover the area which the speaker should occupy during the delivery of his speech, is of great assistance in this regard, as he unconsciously confines himself to the area of this rug. Both of these measures to insure the speaker remaining in proper relation to the transmitter, are supplemented, wherever possible, by an explanation of the system to all the speakers previous to the actual performance.

TRANSMITTER SWITCHING PANEL

Resuming the path of the speech currents through the system, the output from the transmitters is taken to a panel designed to enable the operator to switch quickly from one transmitter to another, as with some public functions, the speeches are made at different points during the ceremonies. This switch is arranged to short-circuit the output of the power amplifier when passing from one transmitter to another, to prevent clicks in the projectors. In certain cases, the equipment is arranged to permit two or more transmitters to be connected to the amplifiers at one time, as is desirable when solo singers and an orchestra are to be picked up in a theatre, with proper adjustment of their respective volumes.

The amplifier equipment has been built in four units which may be grouped as necessary under the various conditions encountered in commercial installations. The proper amplifiers are determined, first, by the source of the voice frequency current to be amplified, that is whether a distant talking or a close talking transmitter is to be used, or whether the speeches are brought in over a telephone line, and secondly, the size of the space in which the amplified sounds are to be delivered to the audience. It was found that four units would provide for all the conditions occurring in practise, two of these being speech input or transmitter amplifiers with different gains and two being power amplifiers of different power ratings. These units and other equipment used with the system, are made up in panels, of uniform width, in order that the proper equipment for any installation may be assembled on two vertical angle iron racks arranged to be fastened to the control room floor.

SPEECH INPUT AMPLIFIER—FIRST TYPE

The first of the speech input amplifiers is shown schematically in the upper part of Fig. 8. It is a three-stage amplifier. Two potentiometers provide adjustment of the gain over a large range, and switching arrangements allow the output to be connected directly to the input of the power amplifier, when the program is to be transmitted to a local audience; or to be connected, through a transformer of proper impedance, to the long distance lines when the program is to be transmitted to a distant audience, or to a radio-broadcasting station. The filaments of the tubes are supplied from a 12-volt storage battery, while the plate circuits obtain direct current at 350 volts from the power supply equipment mentioned later. Arrangements are also provided for using 130 volts instead of 350 volts under certain conditions. The proper grid potentials are obtained by utilizing the drop over a resistance in the filament circuits of the first two tubes, and for the third tube small dry cells furnish the grid potential. The maximum gain with this amplifier is 85 miles, which expressed as a power ratio is 1.2 x 10^8. The maximum power output is approximately 3/10 of a watt. The front and rear views of this amplifier, mounted on the supporting rack, are shown in Fig. 13, where the gain regulating potentiometer, the rheostats for controlling the filament and transmitter currents, the three tube mountings

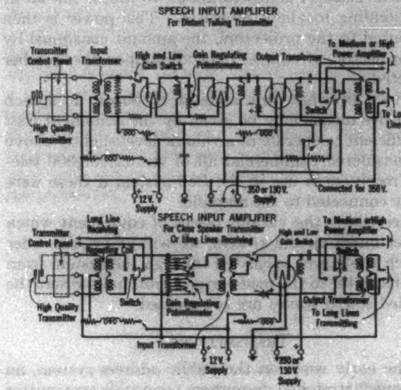
SPEECH INPUT AMPLIFIER
For Distant Talking Transmitter
Transmitter Control Panel
Input Transformer
High and Low Gain Switch
Gain Regulating Potentiometer
Output Transformer
To Medium or High Power Amplifier
Switch
To Long Lines
High Quality Transmitter
12 V. Supply
350 or 130 V. Supply
Connected for 350 V.
SPEECH INPUT AMPLIFIER
For Close Speaker Transmitter Or Long Lines Receiving
Long Line Receiving
Transmitter Control Panel
Repeating Coil
High and Low Gain Switch
To Medium or High Power Amplifier
Switch
Switch
High Quality Transmitter
Gain Regulating Potentiometer
Output Transformer
To Long Lines Transmitting
Input Transformer
12 V. Supply
350 or 130 V. Supply

FIG. 8

with protective gratings and the jacks which permit the connection of instruments for determining the current flow in the filament plate and transmitter circuits, will be noted. Great care was taken in the design of this amplifier to obtain as nearly as possible equal amplification of all frequencies in the important voice range. The transformers, and the retardation coils in the plate circuits were chosen with this consideration in mind.

POWER AMPLIFIERS

For practically all of the larger installations the maximum power possible with the system is required and the output from the transmitter-amplifier is taken directly to the high power amplifier. Referring to Fig. 9 it will be seen that this is a four-tube amplifier so connected that but one stage of amplification is obtained.

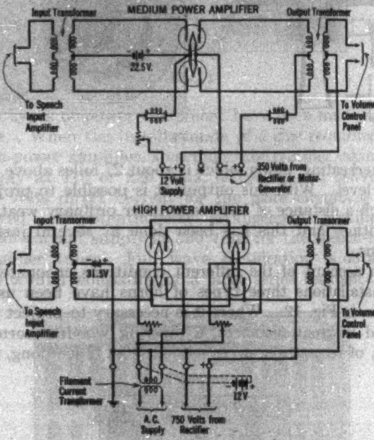

FIG. 9

Usually alternating current at 12 to 14 volts is used for heating the filaments of these tubes, the latter being connected in what we know as a push-pull arrangement. It will be seen that each side of the push-pull arrangement consists of two power tubes in multiple. It is interesting to note that this push-pull arrangement of the tubes will deliver somewhat more power for equal quality than the same number of tubes connected in the ordinary multiple arrangement, as the tubes may be worked beyond the straight part of their characteristic. The grid potential is chosen to permit the largest variation of current without distortion and is obtained from a group of small flashlight batteries.

The output transformer at the right of the figure is designed to match accurately the impedance of the tubes to that of the number of receiver-projector units which has been found to give the greatest flexibility under the varying conditions of commercial operation. This amplifier, speaking in telephone terms is worked at a gain of 23 miles, a power amplification ratio of about 200. Its maximum power output is about 40 watts. The plate circuits of the tubes are supplied at a d-c. potential of 750 volts. As has been pointed out previously, this amplifier gives a practically uniform gain for all the important frequencies in the voice range. This high-power amplifier is shown mounted on the supporting rack, in Fig. 13. The apparatus on the rear of the panel is protected with a sheet metal cover and integral with this cover is a disconnecting switch, which, when the cover is removed, cuts off the high potential from all the exposed parts on the set.

For indoor installations where the audience is small the power output given by the high-power amplifier is not required and a medium-power amplifier has been developed for this use. It is arranged to connect directly to the transmitter amplifier and the output is taken to the projectors through the volume control panel. It is worked at a gain of 17 miles or a power amplification ratio of about 33. The maximum output is about 4 watts, or about one-tenth of the power obtainable from the high-power amplifier.

The schematic of this amplifier, is shown in Fig. 9. The input coil is the same as is used in the high-power amplifier. The push-pull connection of the tubes is also used in this amplifier, although but two power tubes are used. The filaments of these tubes are supplied from a 12-volt storage battery while the plate circuits are supplied at 350 volts direct current from a motor generator set which will be described later.

SPEECH INPUT AMPLIFIER—SECOND TYPE

A speaker using the system may read his speech from his home or office and in such cases it is unnecessary to use the push-pull carbon transmitter in the distant-talking manner. For use when this transmitter is spoken into from a distance of a few inches, a second form of speech input amplifier has been made available having a gain of the proper value to supply either of the power amplifiers, or a long distance line if desired. This gain is relatively small as the output of the transmitter when used for close talking is about 10,000 times that when it is used for distant talking.

Fig. 8 shows the schematic of this amplifier which is a single-stage one, employing one tube and having the same over-load characteristic as the first form of speech-input amplifier. A two-way switch permits the connection of the transmitter or an incoming long distance line to the amplifier. To the right of this switch is a potentiometer for regulation of the gain. To the right of the tube is a second two-way switch for connecting the output either to the power amplifier or through an output transformer to an outgoing long distance line. The power supply for the tubes and transmitters, is the same as was described under the first form of speech input amplifier.

The switching means provided on this amplifier allow it to be used in a number of ways. Announcements from a close talking transmitter may be made from the projectors through a power amplifier or may be sent out on the telephone lines to a distant public address system installation or a radio-broadcasting station. In addition to these uses, incoming speech over the long distance lines may be put out on the projectors through the power amplifier or may be sent out on the long distance lines to a distant installation.

VOLUME CONTROL

As discussed heretofore, it is necessary to give the operator control of the volume put out by each projector or group of projectors. The equipment provided for this purpose is mounted on a panel uniform with the others and consists essentially of an auto-transformer connected across the output of the power amplifier with 11 taps multipled to the contacts of eight dial switches, the arrangement being shown schematically in Fig. 10. Seven of the dials control projector circuits on each of which one or more projectors may be grouped, the eighth dial being reserved for controlling the volume of the operator's monitoring projector in the control room. A key is associated with each dial for opening the circuit and a master key is provided for cutting off all of the projectors simultaneously.

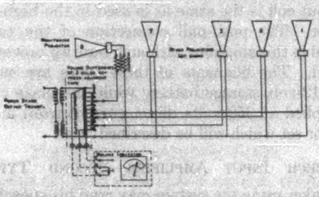

FIG. 10

The device shown as "Volume Indicator" in this figure consists of a vacuum tube detector bridged across the output terminals of the power amplifier. The rectified current is taken to a sensitive direct-current meter of the moving coil type, the degree of deflection of this meter measuring the output from the power amplifier when connected at a proper place in the circuit. The deflections of the meter therefore serve as a basis for determining the adjustment required on the transmitter-amplifier to give the required output when switching from one transmitter to another or for different speakers.

RECEIVER—PROJECTORS

From the control panel the power is taken to the projectors, each of these consisting of a loud-speaking receiver mechanism to transform the speech-currents into sound waves, and a horn to distribute the sound. The receiver is so designed that it will carry several watts with small distortion. It is shown in Fig. 11 where it will be seen that a light spring-supported iron armature is mounted between the poles of a permanent magnet and passes through the center of the coils carrying the voice currents. A light connecting link ties one end of this armature to the diaphragm which is of impregnated cloth, corrugated to permit vibrations of large amplitude. A stamped metal cover protects the parts from mechanical injury, and a cast iron case, in which the whole assembly mounts, is provided for protection against moisture.

One of these receivers equipped with the largest projector provided, will carry without serious distortion or overheating, power which is about 27 miles above the zero level. With this output, it is possible to project speech a distance of 1000 feet under ordinary weather conditions and this has been done at several installations.

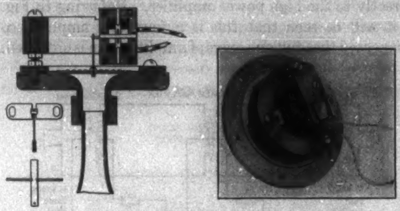

FIG. 11

On account of the different conditions encountered in installations three types of horns have been used, shown in Fig. 12. Where it is necessary to project the sound to great distances, a tapering wooden horn is used, of rectangular cross section, 10-1/2 feet long, the walls being stiffened to prevent lateral vibration. For most installations these large horns are not required, and two types of fibre horns are used. One of these is straight in the body, with a flaring open end, while the other used in the control room, is bent.

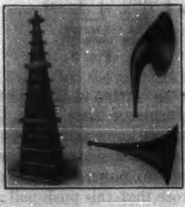

FIG. 12

The grouping of projector units on the volume control switches differs with the type of installation. In outdoor performances, the necessity of correcting the volume in certain directions due to varying winds makes it advisable to group adjacent projectors on a single switch. This is not the case with indoor performances, as no wind effects are possible. Instead, symmetrically placed projectors which will always require equal volume are grouped on a single switch.

POWER SUPPLY EQUIPMENT

In order to convert the commercial electric power supply to forms suitable for supplying the filament and plate current for the vacuum tubes in the amplifiers, two types of power equipment have been made available. When the installation is of a size requiring the high-power amplifier, a vacuum tube rectifier taking its supply at 110 or 220 volts, 60 cycles, and delivering 750 volts direct current for the plate circuits is employed. A potentiometer arrangement provides a direct-current supply at 350 volts for the speech-input amplifier tubes. Full wave rectification is obtained and a filter consisting of a large series reactance coil and bridged condensers is used to render the direct-current output suitable for this use. Included in the power equipment is a step-down transformer for supplying the filament of the power amplifier. For the larger installations, employing the rectifier, the total power drawn from the commercial supply is 1500 watts.

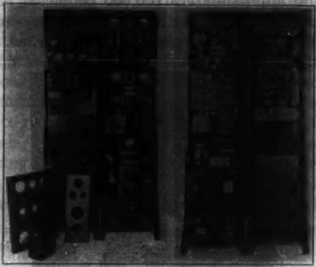

Fig. 13

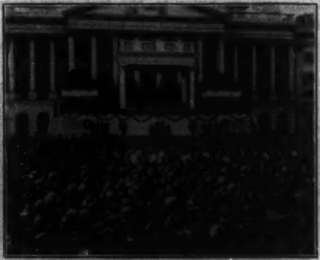

Fig. 14A

For installations of a size not requiring the use of the high-power amplifier, a compact motor generator set is provided consisting of a 350-volt d-c. generator driven by a suitable motor, the total power drawn from the supply mains being about 600 watts. A low-voltage generator for supplying direct current at 12 volts for the operation of the amplifier tubes is incorporated in this motor generator set. A filter is necessary and a reactance coil and a 12-volt storage battery is floated across its output. This supplies the transmitters and the tube filaments. The necessary indicating meters are provided on a meter panel for observing the voltages and currents of all the items of equipment which do not have individual meters associated with them.

Fig. 14B

OBSERVING SYSTEM

In addition to the monitoring projector provided in the control room for the guidance of the operator, it has been found necessary in all but the simplest installations to provide observing stations at various points in the audience. The observers stationed at these points are equipped with portable telephone sets, by which they may immediately communicate with the operator, who is provided with a telephone set consisting of a head receiver and a breast transmitter. The value of these observation stations for regulation of output volume during a program will be apparent.

Fig. 15

In the case of an open air performance a variable wind may make it necessary to increase the volume of certain projectors and decrease the volume of others in order to cover the audience uniformly. Without the observers, the control operator would be unable to take care of these changes.

Considerable preparation is required where the equipment is being used for the first time, in order that the performance of the public address system installation will be of the highest order. Where the acoustic conditions are unfavorable it is necessary to make tests with various arrangements of the projectors, in order to determine the most satisfactory one. It has been found advisable to carry out the entire program previous to the performance in order that the operating force may become familiar with the sequence.

Fig. 16A

CONCLUSION

The usefulness of such systems is very well illustrated by a few of the results which have actually been obtained. Fig. 14 shows a crowd of approximately 125,000 people, every one of whom was able to hear clearly and distinctly all of the words spoken in President Harding's inaugural address in March 1921. This crowd was relatively small, compared with the crowd which could be accommodated by one of the larger type systems. Some insight into the number of people which could be accommodated can be gained from the fact that such a system will cover comfortably a complete circle whose diameter is 2000 feet when the projector units are placed at the center.

Fig. 16B

One of the largest and most successful uses to which this equipment has been put took place on Armistice Day in 1921 when 20,000 people in San Francisco, 35,000 people at Madison Square Garden, New York City, and approximately 100,000 people at Arlington Cemetery near Washington joined in the impressive ceremonies which took place at the burial of the Unknown Soldier. Figs. 15, 16 and 17 are views at the three cities, during the ceremonies.

Fig. 17B

Some of the other uses which have been made of the public address system are the Republican and Democratic Conventions prior to the last presidential election, after-dinner speaking in large ballrooms and in halls where speakers have to address large audiences.

Fig. 17A

There is one more application of this type of equipment which is gaining rapidly in its use. This last is the application of the speech input equipment to radio broadcasting. The broadcasting of the opera, Aida, from the Kingsbridge Armory, and of the Philharmonic Concerts from the Great Hall of the College of the City of New York are two of the successful uses where music and speech were concered, while broadcasting of the results of the football games from the various distant cities indicate possibilities for the dissemination of interesting information[8].

The social and economic possibilities of the system are scarcely realized by the public as a whole at the present time, when the method resorted to for reaching large numbers of people is usually the printed word. While this method is effective, it leaves much to be desired in that the personal touch between the man with ideas and the people to receive them is entirely lost. The difficulty for any but those possessing the strongest voices to reach an appreciable number of people at one time has led to a decline in oratory as a means of conveying public messages to large numbers, for it is not always the man with the best ideas or the best ability of presenting them, who is blessed with a powerful voice. A system such as the one which has just been described enables the speaker, even though his voice be relatively weak, to address at one time and in one gathering, several hundred thousand persons, and if the system be used in connection with long distance telephone lines or radio broadcasting, the number which may be reached is increased almost indefinitely. The value of such a situation can hardly be overrated in times of nationa emergency or stress, when it is necessary for those in responsible positions in the Government to get their message to the people directly.

The development of the apparatus just described has been the result of the efforts of such a large number fo investigators working cooperatively that no attempt hasbeen made to acknowledge thein dividual contributions.

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[8] Use of Public Address Systems with Telephone Lines. W. H. Martin and A. B. Clark.

Discussion

For discussion of this paper see p. 83.

Use of Public Address System with Telephone Lines

BY W. H. MARTIN and A. B. CLARK
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The combination of the public address system and the telephone line makes it possible for a speaker to address simultaneously audiences located at a number of places. This paper discusses various applications of this combination, states the requirements for the lines, shows the circuit arrangements used and describes some of the important operating features.

A description is given of the system used on Armistice Day, 1921, when large audiences at Arlington, New York and San Francisco joined in the ceremonies attending the Burial of the Unknown Soldier at the National Cemetery at Arlington, Virginia.

The application of the public address system apparatus and methods to radio broadcasting is also briefly discussed.

THE public address system which is described in a paper by I. W. Green and J. P. Maxfield, was developed and first used for the purpose of extending the range of the voice of a speaker addressing an audience. With the aid of this system enormous crowds extending from the speaker's stand to points a thousand feet and more distant have in reality become an audience and have easily understood the speaker whose unaided voice covered only that portion of the crowd within a hundred feet or so from him.

When this system, consisting of a high quality telephone transmitter, distortionless multi-stage vacuum tube amplifiers, powerful loud speaking receivers and projectors, had so shown its capabilities, in reproducing speech sounds, a logical extension of its application was to use it with telephone lines. By connecting the transmitting and receiving elements of the public address system through a suitable telephone line a system is provided whereby a speaker can address an audience at a distant point. Also with a complete public address system at the point where the speaker is located, connected by lines to receiving elements of the public address system located at one or more distant points, the speaker is enabled to address a large local audience and to be heard simultaneously by audiences at one or more remote points. This last arrangement was first used on Armistice Day, 1921, when audiences at Arlington, New York and San Francisco joined together in the ceremonies attending the burial of the Unknown Soldier at the National Cemetery at Arlington, Virginia.

By means of the public address system, the meeting of this Institute at New York, at which this paper is presented is attended and participated in by Institute members at a meeting in Chicago. This is the first occasion on which complete public address systems installed at meetings in two cities have been connected together by telephone lines so that speakers at each

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

meeting address the local and distant audiences simultaneously.

With the transmitting element of the public address system working into the radio transmitter of a broadcasting station and with the receiving elements of the system connected to the output of the radio receiving sets, a system is provided whereby a number of people can be reached by each radio receiver.

The combination of these wire and radio communication channels with the elements of the public address system is, therefore, without limit in the number of persons who may be reached simultaneously by one speaker. Such combinations may prove extremely serviceable for occasions of nation-wide interest and importance.

The public address system apparatus has been used not only for transmitting speech sounds but also for music, both vocal and instrumental. The paper[1] describing the public address system has pointed out that the requirements for such a system are that for a wide frequency range it be practically distortionless, that is, transmit and reproduce with equal efficiency all frequencies in that range. This requirement must apply likewise to lines which are used with the loud speaker system. It has been found that a circuit which transmits without material distortion the frequency range from about 400 to 2000 cycles, can be used with the public address system to reproduce speech sounds which are fairly understandable under favorable conditions, although sounding unnatural. In general it is important to extend this range at both ends in order to improve the intelligibility of the sounds and increase the naturalness. For vocal and for some types of instrumental music the melody can be reproduced with the above frequency range, but these tones also are lacking in naturalness. Since some of the musical instruments are used to produce tones three and even four octaves below middle C, it is evident that the proper reproduction of music requires a further extension of the lower limit of the transmitted band than does speech. While the fundamentals of the higher musical tones lie in general in the range mentioned above, it is the harmonics in musical tones which distinguish those produced by different instruments and which give what musicians term "brilliance." The true reproduction of many musical selections requires the distortionless transmission of a frequency band of from about 16 cycles to above 5000 cycles. Many musical selections, however, employ only a part of this range and accordingly can be satisfactorily reproduced by systems not transmitting the whole range. Also, even with slight distortion obtained with somewhat narrower ranges, reproductions may be given which are agreeable to many popular audiences.

LINE REQUIREMENTS

In general the same line requirements which make for satisfactory transmission of speech over commercial telephone circuits also make for satisfactory transmission when telephone circuits are associated with loud speakers. There is this difference however. The loud speakers tend to make the line distortion much more noticeable and serious. Speech transmitted over a particular telephone line is, in general, more difficult to understand when listening to loud speakers than when listening to telephone receivers.

In commercial telephone service the main requirement is intelligibility while, with the loud speaker, the naturalness of the reproduced speech sounds is very important. People are accustomed to hearing transmission through the air with very little distortion and naturally expect the same results with loud speakers.

The above constitute the reasons why, for transmitting voice currents over telephone lines with loud speakers, it is necessary to pay unusual attention to the electrical characteristics of the lines. Evidently when music is to be transmitted, particularly music of a fairly high grade, it is necessary to place even more severe electrical requirements on the lines.

An analysis of what constitutes the electrical requirements of a telephone line which make for good transmission, particularly when loud speakers are employed, will now be given.

In the first place, as explained above, it is essential that a sufficiently broad frequency range be transmitted. As explained in another paper[2] it is not sufficient that a telephone circuit transmit sustained alternating currents within a given frequency range. It must also transmit short pulses of alternating currents within the proper frequency range without introducing oscillations of its own or "transient effects." This requires that loaded circuits for loud speaker use have a high cut-off frequency and hence have the frequencies of the predominant natural oscillations high. It has been found that when the cut-off frequency of loaded circuits is about 5000 cycles, good results are secured with loud speakers.

The two types of telephone circuit which best meet the requirements of transmitting a broad band of frequencies, both when sustained and when applied in short pulses, are non-loaded open-wire lines and extra-light loaded cable circuits. These are suitable for transmission over very long distances. For transmission over short distances, say from one point in a city to another point in the same city, non-loaded cable circuits equipped with distortion networks or attenuation equalizers for equalizing the attenuation, give good results.

A good idea of the range of frequencies which can be transmitted over high grade telephone circuits can be secured from Fig. 1, which shows the transmission efficiency at different frequencies for the New York-

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[1] Green and Maxfield, "Public Address System."

[2] Clark, Telephone Transmission over Long Cable Circuits, see p. 86.

San Francisco circuit. This circuit is a non-loaded No. 8 B. W. G. open wire line equipped with twelve telephone repeaters and is 3400 miles long. Its frequency characteristic meets very well the requirements for easy understanding of voice transmission although it causes some loss of naturalness.

The frequency range which can be transmitted with approximately constant efficiency is limited at the lower end by the fact that composite sets are employed in order to make it possible to superpose direct current telegraph circuits. The elimination of these composite sets would make it possible to improve the transmission of low frequencies and thus improve the operation of the circuit in connection with loud speakers. The resulting improvement, however, would not be of importance for commercial telephone service and would render it more difficult to avoid noise on circuits exposed to induction from paralleling power or telegraph circuits.

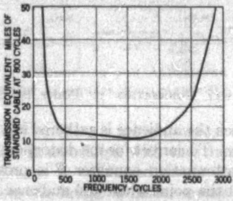

Fig. 1—Transmission Characteristic of Transcontinental Circuit—New York to San Francisco

At high frequencies the range is limited because these same wires are equipped with apparatus to permit super-position of multiplex carrier telegraph circuits above the voice range. This limitation also is not important for commercial telephone service although it is of importance for loud speaker use. To raise the upper limit of the voice transmission range would require giving up some of these facilities.

Fig. 2 will give an idea of how the distortion introduced by a length of non-loaded cable can be corrected by employing distortion networks or attenuation equalizers. This figure shows the transmission frequency characteristic of about 10 miles of non-loaded No. 19 A. W. G. cable. Curve *A*, in the figure, shows the characteristic when uncorrected, while Curve *B* shows the characteristic for the circuit when equipped with an attenuation equalizer.

After choosing the proper types of telephone circuits for use in connection with loud speakers, there remains to be considered a number of other important matters.

The maintaining of the telephone power within proper limits at different points in the circuit is very important. The power must not be allowed to become too weak, otherwise the extraneous power induced from paralleling circuits would tend to obliterate the telephone transmission. On the other hand, the telephone power must not be amplified to such an extent that the telephone repeaters will be overloaded or severe cross-talk be induced into paralleling circuits.

To keep the telephone power throughout the circuit between the above limits, requires careful study and adjustment. For handling regular telephone connections, the circuits are laid out and equipped with repeaters at proper points so that each circuit will be able to handle the varying volumes applied at the terminals when different subscribers are connected without getting into serious difficulties. When loud speakers are employed it is necessary to maintain the volume at the terminals of the toll lines at least within these limits and it is preferable to do somewhat better than this.

With the public address system, the high quality transmitter which picks up the sound at the sending end is usually associated with an amplifier whose adjustment is varied, depending on the output of voice currents from the transmitter. In order to obtain the proper adjustment of this amplifier, it is necessary to have some means for quickly indicating the volume of transmission. For this purpose, there has been developed a device which is called a "volume indicator." This consists of an amplifier detector working into a direct-current meter. With this volume indicator connected across the output of the transmitter amplifier, the volume of transmission delivered to the line is indicated by the deflections on the meter. By adjusting the amplifier, therefore, to keep the deflections of this meter reasonably constant at some deflection determined by previous calibration, it is practicable to keep the telephone power within the required limits. Obviously, this same device may also be employed to keep the telephone power constant at any other point in the system.

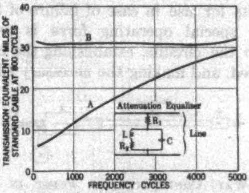

Fig. 2—Transmission Characteristic of No. 19 Gage Non-Loaded Cable Circuit
A—Without Attenuation Equalizer
B—With Attenuation Equalizer

While the necessity for keeping the power applied to the toll lines within proper limits cannot be overemphasized, it should also be noted that this is not sufficient. It is also essential that all parts of the toll circuit, including the repeaters, be maintained at prescribed efficiency so that the power levels at all intermediate points in the circuit will also be kept within proper limits. Long telephone lines are designed with special emphasis on this matter of constant efficiency so that, in general, no special precautions are required when using these circuits in connection with loud speakers.

In another paper,[3] the "echo" effects which may occur on long telephone circuits are explained. When setting up two-way circuits for loud speaker use, it is necessary to pay particular attention to effects of this sort. Furthermore, there is another source tending to produce echoes in circuits arranged for two-way use with loud speakers. This is the tendency for the sound delivered from the loud speaker projectors to enter the sensitive transmitter and be returned to the distant end of the circuit as an echo. Owing to the relatively slow velocity of transmission of sound through air the lag in such an echo may be great enough to be serious, although the line is a short one with high transmission velocity. It is, therefore, evident that this coupling through the air between the loud speaker projector and the transmitter must be kept small. If a very sensitive transmitter arranged so that a speaker may stand several feet away from it is employed, this problem becomes even more difficult.

There is one thing more that remains to be considered: the necessity for special operation. When a large number of people are assembled at some point to hear an address delivered at a distant point, it is evident that delay in establishing the connections cannot be tolerated. It is, therefore, necessary to establish such connections ahead of time and it is usually also necessary to set up spare circuits for use in case of failure of the regular circuits. A special operating force is required for checking up the circuits, establishing the connections when required, and making the necessary adjustments.

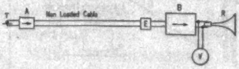

Fig. 3—One-Way Connection to Point in Same City

Rehearsals are necessary on important occasions to insure proper functioning of the circuits and proper coordination of the handling of the circuits with the programs at different points.

Typical Circuit Combinations of Public Address System and Lines

Following are a number of typical combinations of the public address system and telephone lines. The combinations by means of which one-way service may be rendered, are given first, following which certain combinations for giving two-way service are discussed.

By one-way service is meant service in which no provision is made for anyone in the distant audience to talk to the place where the speaker is located. Two-way service provides for speakers at either of two or more points addressing all of the other points. This is similar to the two-way service rendered by regular telephone circuits.

Fig. 3 shows the circuit arrangement which would be used when a speaker at one point in a city, for example, at his office, is to address an audience at another point in the same city. A high quality close talking transmitter *T*, together with a fixed gain single-stage amplifier *A*, are provided at the point where the speaker is located. This combination is designed to deliver to the line the same amount of power as a commercial type substation set. Connecting this point with the point at which the audience is gathered is a non-loaded cable circuit. To correct for the distortion in this cable circuit, an attenuation equalizer *E* is provided. The apparatus at the point where the audience is located is the equipment of the public address system without the transmitter and its associated amplifier. In Fig. 3, *B* is the amplifier for delivering sufficient power to the group of loud speaker projectors indicated by *R*. A volume indicator *V* associated with the amplifier *B* is used in maintaining constant the volume of sound delivered from the projectors.

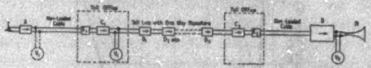

Fig. 4—One-way Connection to Point in Distant City

Fig. 4 shows the circuit combination required when a connection is to be established to a distant city where the loud speaking receivers are located. In the city where the speaker is located, connection is made to the toll office by means of a non-loaded cable circuit equipped with an equalizer similar to Fig. 3. A volume indicator V_1 is associated with the amplifier C_1 at the toll office to enable proper adjustment of amplifier C_1 to be made so that the power delivered to the toll line will be within the proper limits. As explained above if the volume at the toll office is allowed to become too great, the telephone repeaters on the toll line will be over-loaded and serious distortion will result, while if the volume is allowed to become too weak, extraneous noise and crosstalk will tend to obliterate the direct transmission. If a distant talking transmitter is used for the speaker, a multi-stage adjustable amplifier is associated with it. In this case the volume indicator is located at the output of this amplifier as shown by the dotted lines in Fig. 4. When the volume indicator is employed at this point it is necessary to take into account the loss introduced by the non-loaded cable and the equalizer E_1, together with the gain of the repeater C_1, in order to deliver volume within proper limits to the toll line. The toll line, shown equipped with repeaters D_1, D_2, etc., extends to the toll office in the distant city.

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[3] Clark. *loc. cit.*

At this point the amplifier C_2 is located, together with another equalizer E_2, for correcting the distortion in the local non-loaded cable circuit. The apparatus at the point where the audience is located is similar to that shown in Fig. 3.

Fig. 5 shows the circuit combination employed when a local address is to be given, while at the same time the same address is delivered to one or more distant points. In order to allow the local audience to hear the address by means of the loud speakers, the power amplifier *B* supplying energy to these is bridged across the output of the amplifier *A* associated with the transmitter *T*. A volume indicator *V*, connected across the circuit at the point where the bridge is made, makes it possible to maintain constant volume both for the local loud speakers and for the transmission applied to the toll lines by suitable adjustment of amplifier *A*. At the toll office means are indicated for connection to two distant points *X* and *Y*. Owing to the fact that amplifiers C_1 and C_2 are one-way devices, no interactions can occur between lines *X* and *Y* or between these lines and the local loud speaking system. The arrangements for reaching the distant points *X* and *Y* are similar to the one illustrated in Fig. 4.

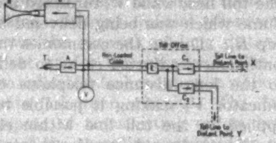

Fig. 5—One-way Connection for Addressing Local and Distant Audiences Simultaneously

All of the circuit arrangements which have so far been described are arranged simply so that a speaker may address one or more local or distant points. When it is desired that the speaker and the audience at the sending end be able to hear, also, a speaker at the distant point, more complicated arrangements are required.

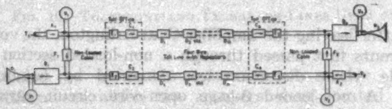

Fig. 6—Two-way Four-Wire Connection for Addressing Local and Distant Audiences

Fig. 6 shows a circuit arranged for such two-way service, the line being operated on the four-wire principle, *i. e.*, two separate transmission paths are provided, one for transmission in each direction. The circuits connecting transmitter T_1 with the projector group R_2 and transmitter T_2 with the projector group R_1 are similar to the circuit in Fig. 4. By-pass connections F_1 and F_2 are added at the two ends which allow part of the output of each transmitter to pass into the local loud speakers. These by-pass connections are so arranged that transmission can pass only in the proper direction. Two volume indicators are provided at each end. Referring to the left-hand terminal, volume indicator V_1 is provided to insure that power is supplied to the toll line within the proper limits of volume, as explained above. V_3 is provided to facilitate adjustment of the by-pass circuit F_1 and of amplifier B_1 so as to deliver proper volume from R_1 both for the local talking and for the reception of the addresses from the distant end of the circuit. The volume indicators V_2 and V_4 at the right-hand end of the circuit have functions similar to those of V_1 and V_3 respectively.

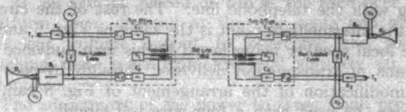

Fig. 7—Two-way Two-wire Connection for Addressing Local and Distant Audiences

Fig. 7 is similar to Fig. 6 with the exception that the toll line is of the two-wire type. At each end of the toll line, which may, or may not, contain two-way repeaters, transformers and networks N_1 and N_2 are placed for converting the two-wire circuit into a four-wire circuit. The equalized cable circuits at the two ends thus form two sides of short four-wire circuits. The conditions of balance between the networks and the toll lines prevent more than a very small amount of the direct transmission from each local transmitter from entering the local loud speaking receiver circuit at the points where the local circuits connect to the toll line. Practically all of the transmission from transmitter T_1 to projector group R_1 and from transmitter T_2 to projector group R_2 is delivered through the adjustable by-pass circuits F_1 and F_2, respectively.

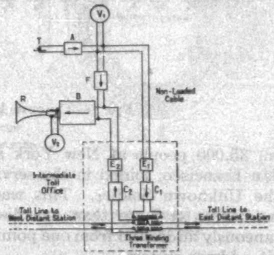
V1
T
A
F
Non-Loaded Cable
R
B
V2
E2
E1
Intermediate Toll Office
C2
C1
Toll Line to West Distant Station
Toll Line to East Distant Station
Three Winding Transformer

Fig. 8—Arrangement for Connecting Third Point to Circuit of Fig. 7

For connections requiring to and fro conversations between three or more points, all of which may be equipped with loud speakers, intermediate points may be connected to a two-wire telephone circuit by employing the arrangement shown in Fig. 8. A three-winding transformer is inserted in the toll line which is so constructed that the impedance which it introduces into the circuit is small enough to avoid a serious irregularity. Talking currents are put out on to the toll line through this transformer. The received transmission is obtained from a high impedance bridge across the midpoints of two of the windings of the three-winding transformer. Amplifiers C_1 and C_2 introduce sufficient gain to overcome the losses due to the inefficient coupling with the telephone line. The rest of the circuit at the intermediate point is the same as Figs. 6 and 7, the local speaker being heard by his own audience by means of transmission delivered through by-pass F. A modification of the arrangement of Fig. 8 can, of course, be used with a four-wire toll circuit.

ARRANGEMENTS FOR ARMISTICE DAY, 1921

Fig. 9 shows the circuit which was employed on Armistice Day, 1921, when audiences of 100,000 people at Arlington, 35,000 people at New York and 20,000 people at San Francisco, joined in the services at the burial of the Unknown Soldier. This was the first time that audiences at more than one distant point were simultaneously addressed from one point by means of the public address system.

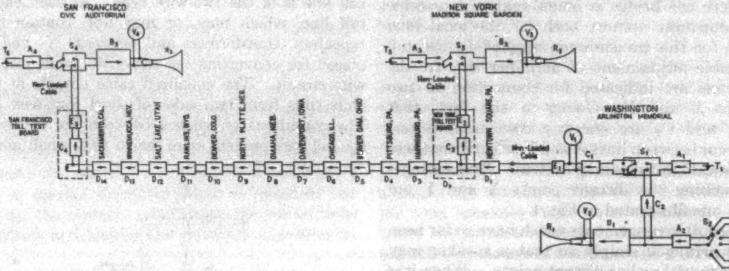
SAN FRANCISCO
CIVIC AUDITORIUM
T_6
A_4
S_4
B_3
V_4
R_3
Non-Loaded Cable
SAN FRANCISCO TOLL TEST BOARD
E_3
C_4
SACRAMENTO, CAL.
WINNEMUCCA, NEV.
SALT LAKE, UTAH
RAWLINS, WYO.
DENVER, COLO.
NORTH PLATTE, NEB.
OMAHA, NEB.
DAVENPORT, IOWA
CHICAGO, ILL.
BEAVER DAM, OHIO
PITTSBURG, PA.
HARRISBURG, PA.
NEW YORK TOLL TEST BOARD
NEWTOWN SQUARE, PA.
D_{14}
D_{13}
D_{12}
D_{11}
D_{10}
D_9
D_8
D_7
D_6
D_5
D_4
D_3
D_2
D_1
NEW YORK
MADISON SQUARE GARDEN
T_5
A_3
S_3
B_2
V_3
R_2
Non-Loaded Cable
E_2
C_3
WASHINGTON
ARLINGTON MEMORIAL
Non-Loaded Cable
V_1
E_1
C_1
S_1
A_1
T_1
C_2
V_2
R_1
B_1
A_2
S_2
T_2
T_3
T_4

FIG. 9—CIRCUIT USED FOR CEREMONIES ON ARMISTICE DAY, 1921

At Arlington three different transmitters T_2, T_3 and T_4 were used for the different parts of the ceremonies. T_2 was used for the musical selections, T_3 for the speeches made in the amphitheatre, and T_4 for the speeches at the grave of the Unknown Soldier. Another transmitter T_1 was provided for the use of an announcer who kept the audiences at New York and San Francisco advised of the proceedings. The speech currents leaving these transmitters were brought up to moderate volume by means of amplifiers A_2 and A_1, the former taking care in turn of the three different transmitters employed during the ceremonies.

The voice currents from the transmitters which were employed for the ceremonies, after passing through amplifier A_2, separated into two branches, one branch going to the local amplifier B_1, which supplied the local loud speakers R_1, the other going to the telephone circuit through amplifier C_2, switch S_1 and amplifier C_1. The switch S_1 was provided for connecting either the announcing transmitter T_1 or one of the transmitters for picking up the ceremonies to the end of the toll line. V_1 and V_2 are volume indicators, V_1 being employed to indicate that the proper power was being put into the toll line, while V_2 furnished an indication of the volume which was being delivered by the projector group R_1. During the ceremonies the amplifier C_2 was continuously adjusted so as to deliver proper volume to the long distance telephone circuit, the volume indicator V_1 making it possible to keep the volume applied to the toll line within close limits. At the same time independent adjustments were made of the amplifier B_1 to take care of the varying conditions introduced by the different talking conditions as well as the varying conditions introduced by shifting of the crowds listening to the ceremonies.

After leaving the amplifier C_1 at Arlington, the voice currents first passed through a non-loaded section of cable whose distortion was corrected by equalizer E_1. A non-loaded 8-gage open-wire circuit carried the voice currents to New York City. At this point, the circuit again branched, one branch delivering a part of the voice currents to the apparatus at Madison Square Garden, the other branch going to San Francisco over one of the non-loaded No. 8 gage transcontinental circuits. The arrangements employed at Madison Square Garden and at the Civic Auditorium in San Francisco were similar, switches being provided at each point to connect to the projector groups the circuit from Arlington or from the local transmitter.

The difficulties involved in transmitting voice currents for the first time to loud speaker installations at distant points, as well as the great importance of the occasion, made it necessary to take elaborate precautions in order to insure the success of the undertaking. The long distance telephone circuits were carefully inspected ahead of time and all of the amplifiers and other apparatus employed were subjected to numerous careful tests. For checking the complete circuit, alternating currents of different frequency were applied at Arlington and measured simultaneously at New York and San Francisco. The curve on Fig. 1 was obtained from the results of one of the measurements made on this occasion.

To guard against possibility of failure of the circuits, emergency circuits were provided, these emergency circuits taking different routes wherever possible. Fig. 10 shows the network of long distance circuits which was set up for this occasion. The solid lines in this figure indicate telephone circuits while the broken lines indicate telegraph circuits. The latter were for the purpose of transmitting orders between different units of the operating organization at different points.

At Arlington the nature of the ceremonies and the place in which they were held presented many difficulties from the acoustic standpoint. The main addresses were made in an open amphitheatre surrounded by a double colonnade of marble. The platform on which the speakers were located was partially covered by a marble arch. The floor of the amphitheatre is of cement on which are arranged marble benches. Temporary seats also were placed on top of the colonnade. During the ceremonies large crowds surrounded the amphitheatre on all sides. The arrangement of the amphitheatre and the surroundings is shown by Fig. 11.

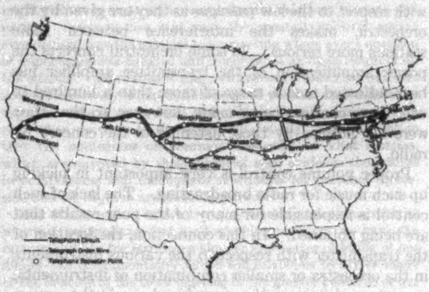

Fig. 10—Telephone and Telegraph Lines Used on Armistice Day, 1921

In order that the crowds outside of the amphitheatre might hear the speakers, loud speaking receivers and their associated projectors were placed on top of the colonnade. They were arranged in four groups as shown on Fig. 11, the projectors referred to, being numbered from 1 to 21 inclusive. Those in the east group were on top of the structure forming the main entrance to the amphitheatre. The projectors were carefully directed to cover uniformly the area around the amphitheatre and were supplied with sufficient power so that the speaker could be heard for at least a thousand feet from the outside of the amphitheatre. It was found, however, that while these projectors are highly directive, some of the sound from them could be heard inside the amphitheatre. This sound leakage at the western side was particularly serious because of the fact that it reached the rear seats inside of the amphitheatre sufficiently far enough ahead of the corresponding sounds directly from the speaker to be noticeable. To overcome this, the small projectors 29, 31, 32 and 34, placed on top of the arch over the platform, were directed at the rear seats and given sufficient volume output to overcome the sound reaching these seats from the loud speakers on the colonnade.

The adjustment of the power to these small projectors required great care because if given too great volume, bad reflections would be set up in the amphitheatre. On the other hand if this volume were not great enough, the outside projectors would cause serious interference. The small projectors 27 and 28 were used to overcome the sound leakage effects on the top of the west side of the colonnade. The projectors 35 and 36 covered the top of the colonnade on the east side.

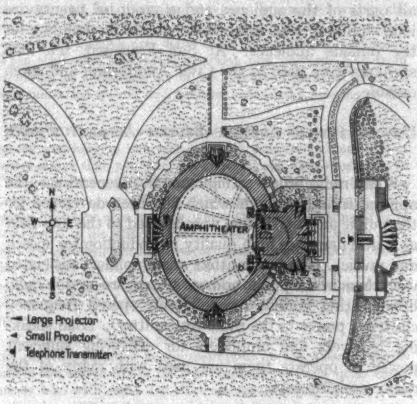

Fig. 11—Arrangement of Projectors at Arlington Amphitheatre

Fig. 11 shows also the location of the three transmitters used during the ceremonies, *a*, on the platform for the speakers, *b*, in front of one of the boxes in which were placed the singers and behind which was located the band, and *c*, at the grave. When the transmitter at the grave was tested it was found that serious interference was obtained between the speaker's voice and the sound from the projectors 16 to 19 inclusive. For the ceremonies at the grave, therefore, these loud speakers were disconnected and those numbered 22 to 26 used instead. Also in order to properly cover the inside of the amphitheatre during the ceremonies at the grave, the small projectors 30 and 33 were used. These were located on the arch over the platform and were directed at the front seats in the amphitheatre.

The projectors were divided up into a number of small groups and so connected that the volume of sound delivered by each group could be varied without affecting the other groups. This was necessary in arriving at the power to be delivered by each projector to give uniform distribution and to avoid interference between different groups.

By means of these arrangements all parts of the ceremonies were carried to all parts of the audience at the National Cemetery and were also delivered by means of the lines to the audiences in the distant cities.

At New York, a group of fifteen loud speakers was used in Madison Square Garden to satisfactorily reach all parts of the audience and a group of twenty-one loud speakers was suspended outside the building for the outside audience. At San Francisco, ten loud speakers were used in the Civic Auditorium and seven outside.

USE OF PUBLIC ADDRESS SYSTEM APPARATUS WITH RADIO

When radio broadcasting came into general use, the apparatus and methods which had been developed for the public address system were applied to this new field as it also demands high quality reproduction for speech and music. The transmitters and amplifiers associated with them in the public address system are used in radio broadcasting studios for delivering speech frequency electrical power to the radio transmitter. Loud speaking receivers and amplifiers for delivering sufficient power to operate them are used with many of the radio receiving sets.

The methods which have been employed to connect public address system transmitters with toll lines are being used for the broadcasting by radio of speeches and music given at points remote from the radio station. In such cases the transmitter and its associated amplifier are operated and controlled in the same way as described above for toll lines. In some cases the radio station is in the same city as the place where the speech or music is given and in other cases the two have been in different cities. In the first case the output of the transmitter amplifier is carried to the radio station over non-loaded cable circuits which are equalized by means of distortion correction networks to have uniform efficiency over a wide frequency range, in some cases up to 5000 cycles. Where the two points are in different cities, the non-loaded cable circuit goes to the toll office and there is connected to the toll lines which are operated in the same manner as described above for loud speaker use.

For some of the higher grade music, such as that given by symphony orchestras, the less efficient, but slightly higher quality condenser type transmitter has been used instead of the double button carbon type. This requires the use of ana dditional two stage amplifier in front of the regular three stage transmitter amplifier.

The output of the transmitter amplifiers is controlled with the aid of a volume indicator bridged across the output terminals of the amplifier. For best results, particularly in reproducing music, it is necessary to adjust the gain of these amplifiers to compensate partially for the large range in the volume of the music. If the amplifiers are set high enough in gain to send through the low passages of the music with sufficient volume so that it will override the static and the interference from other sending stations, the loud parts of the music will seriously overload the radio transmitter system, unless it is of very large capacity, and will in general overload the receiving sets. Furthermore, putting out these loud parts at the same relative level with respect to the low passages as they are given by the orchestra, makes the interference between radio stations more serious. In some orchestral concerts the power amplification of the transmitter amplifier has been adjusted over a range of more than a hundred to one, these changes being made, however, so that they were not noticed by those listening to the concert by radio.

Proper volume control is very important in picking up such music for radio broadcasting. The lack of such control is responsible for many of the poor results that are being obtained. In this connection, the location of the transmitter with respect to the various instruments in the orchestra or smaller combination of instruments, so as to maintain in the reproduced music the proper balance between the several parts is, of course, of great importance.

An interesting illustration of the combination of the public address system, telephone lines and radio broadcasting was used in connection with reporting a football game played in Chicago in the fall of 1922. By means of high quality transmitters and amplifiers located at the football field, announcements of the plays and the applause of the spectators were delivered to a cable circuit extending to the toll office in Chicago. This circuit was connected there to a toll line to New York where it delivered the telephonic currents to a radio broadcasting transmitter. In Park Row, in New York City, was located a truck on which was mounted a radio receiving set arranged to operate a public address system. By this means the reports of the plays of the football game in Chicago were delivered to a large crowd in the streets of New York.

Discussion

DISCUSSION ON "PUBLIC ADDRESS SYSTEMS"

(GREEN AND MAXFIELD) and

"USE OF PUBLIC ADDRESS SYSTEM WITH TELEPHONE LINES"

(MARTIN AND CLARK)

New York, N. Y., February 14, 1923

Frank B. Jewett: To you, members of the American Institute of Electrical Engineers, and to you, our guests assembled together in New York and Chicago, I wish in opening this first evening meeting of our Midwinter Convention especially to call your attention to the unique significance of the occasion which brings us together and of the epoch in engineering history which it marks, not only in electrical development but in the conduct of human affairs as well.

For the first time in the history of the world, groups of men and women, separated by hundreds of miles, are gathered together in a common meeting under a single presiding officer to listen to papers presented in cities separated by half the span of a continent, and to take part in the discussion of these papers with an ease characteristic of discussions in small and intimate gatherings. At the same time unnumbered thousands in their homes are auditors of our deliberations through radio broadcasting.

Who can picture the limits of the effect and influence which will flow from the developments of electrical science and electrical engineering which have made this night possible. In truth, we are participants in an historical event and our children, nay even many of us, may see the agency we here use employed with mighty effect in controlling our collective relations in state and nation. Someone has said that the greatest political engine ever devised was the colonial town meeting where every question of importance was debated and discussed in an open forum of all the citizens. Be that as it may, we are well aware that our best government today is found in those small political units where the town meeting in some form is feasible and still persists. It is only when we come to the larger units of city, state and nation that the limitations on common discussion of vital matters, which are imposed by sheer physical size, evidence themselves in cumbersome and inadequate substitutes for personal discussion and oft-times in unsatisfactory results.

May it not be that in this two-way working telephone with its sensitive transmitters and loud speaking receivers we have the instrumentality for insuring a simpler and better ordering of our affairs—an instrumentality which will enable us to derive many of the benefits of the town meeting in the greater concerns of our national life. The mechanism which we are here using is one adapted to permit many speakers in many distant audiences to be heard by all who care to listen and take part in a common discussion. In voicing the opinion that this mechanism is destined profoundly to affect our political and economic machinery, I intend to convey no thought or picture of a pure democracy but only of a representative form of government in which all questions that would be helped by oral discussion can be so discussed without restraint from the physical limitations of the human voice or of distance. They might be discussions between widely separated groups on some matter of common concern or between a designated representative and his constituents, or in any of the thousand and one ways in which human beings better their conditions by oral discussion.

In the hands of the members of this Institute the telephone has been developed from the first crude concept of Alexander Graham Bell's great contribution to knowledge and great gift to mankind, into a machine of incalculable influence in human affairs. The successful realization of a dream, a hope and an inspiration dating back to the pioneer days of Bell himself, namely, the production of a loud speaking telephone which would reproduce the human voice faithfully but in stentorian tones and in many places simultaneously has now been accomplished. In the solemn exercises attendant upon the burial of the Unknown Soldier at Arlington on Armistice Day a year ago, we had the curtains of the future drawn partially aside and saw vast audiences in New York and San Francisco co-mingled, as it were, with those who listened on the banks of the Potomac, but those exercises and all subsequent exercises and demonstrations have been in the nature of one-way transmission; that is, they have been arrangements in which the speakers were at one place and the audience at one or more other places, or they have been arrangements where a single audience listened to speakers in different places. Tonight, for the first time, we are witnessing and making use of the next step in the development by employing an arrangement which enables many speakers and many audiences in many places to meet as one body.

From the arrangement at Arlington to that of tonight may seem a logical next step. So it is for those who have the imagination to picture the future, but so too was the concept of a loud speaking telephone a simple logical next step to anyone who had witnessed the operation of an ordinary telephone. In both cases the next step that was easily logical in the imagination has been technically difficult in reality. It is neither my purpose nor my place as the presiding officer to trench upon the papers and discussion intended to describe and exemplify this new instrumentality. Before proceeding to the more formal part of our program, however, I wish merely to call your attention to the fact that we lack only direct distant vision to make this joint meeting one in very fact save only for the element of the personal proximity of those in attendance, and who among us is hardy enough to believe that this last limitation may not be removed within the lifetime of many of those now here. Fundamental and applied research are opening new doors daily in bewildering succession. When they open that last door which now prevents man from exercising at a distance all of his powers of personal intimate communication we will have another noteworthy meeting of this Institute.

Tonight, however, we must be content with full powers of speech and hearing but with partially defective vision. So it comes about that you, Mr. Schuchardt, our Vice President, in Chicago, and you, Mr. Rhodes, my Vice-chairman, are each in a unique position, a position in which one of you is acting in place of the President at a meeting where the President himself is in attendance in his official capacity, and the other is acting similarly for the President in his capacity as presiding officer. Some day this may not be necessary, but for the moment we are really acting as the eyes of this great joint meeting and are transferring as best we can the functions of distant vision by means of human speech.

R. F. Schuchardt: Mr. President, members and guests of the Institute gathered in the metropolis on the Atlantic shores and in this active city of the great central west, and those many others who with radio detectors are participating in this truly national meeting:

This is indeed a wonderful experience for all of us—an epoch-making event in history, as you, President Jewett, have so well said. Through the agency of devices which are already the familiar tools of telephone engineers, though still marvels to most of us, we are living for a few hours this evening in the identical mental atmosphere though vast stretches of our country separate us physically. In contemplating the means which make all this possible we recognize the debt that the world owes to the men whose achievements this represents—the patient, enthusiastic, talented engineers of the American Telephone & Telegraph Company and of that splendid Research Bureau of the Western Electric Company where Col. Frank B. Jewett has played such a leading role.

Never in history has there been such advance in the things that progress civilization, chief among which is transportation—transportation of material things, as done by the ocean greyhounds that bring Europe within a few days of our eastern coast and by the railroads that cross the continent in even less time, and now also by the still swifter airships; transportation of energy from hydro or steam plants over great electric transmission lines to distant markets; and that transportation of thought, the broadcasting of intelligence, that we enjoy this evening. All of these things annihilate distance and bring people closer together, into more intimate kinship. Yet we have the paradoxical situation that in the past decade, in the midst of this advance, there occurred the greatest outburst of barbarism known to the history of civilization.

The Phoenix that rises from the ashes must be largely the work of the engineer, and what can be more potent here than this contribution of the telephone engineer. President Jewett has eloquently pictured its possibilities in relation to our national affairs and no doubt his picture will materialize. May we not hope that the ease with which peoples on opposite sides of boundary lines in Europe can thus be brought together, ear to ear, if not face to face, will serve powerfully to show them that their fundamental interests and their aspirations are common and in reality they are all kin. The engineers of those countries must and will lead the way.

This wonderful work of the American telephone engineer, so forcefully brought home to us all by this evening's experience, will serve as an inspiration to engineers in other fields. It emphasizes splendidly the great value of engineering research, it stimulates the imagination, and will encourage others to renewed efforts. The engineer appreciates his obligation for worthwhile accomplishment for the common weal, and he rejoices in all opportunities in which to express this appreciation.

In conclusion, Mr. President, may I tell you how deeply the Great Lakes District feels the honor and the special favor of being permitted to join in this memorable meeting so uniquely, and we thank you and your associates for making this opportunity possible.

E. B. Craft: The papers to which we have listened this evening establish another milestone in the art of electrical communication.

The system which has been described and demonstrated for us would seem to almost complete the picture of the possibilities of the transmission and reception of the spoken word.

During the past ten years we have seen the range of the wire communication systems extended so that it is now possible for an individual to converse directly with any other *individual* within the confines of the continent.

The developments in radio broadcasting during the same period have made it possible for an individual to speak directly and simultaneously to *unlimited numbers* of individuals located at a similar number of different points.

Now this latest development which we have christened the public address system, adds another and tremendously important possibility which is that an individual may converse directly with *groups* of individuals of practically unlimited size. Furthermore, they can be assembled in direct view of the speaker or through the agency of wires or radio these large groups may be located anywhere within the confines of our land, on the islands adjacent or on the ships at sea.

Truly we have in these agencies all that would seem essential to Man's dream of universal intercommunication.

It may be of interest to consider briefly some of the possible uses to which this new instrument may be put.

It is a noteworthy fact that many of our largest and most beautiful places of assembly are, in spite of their costliness and artistic perfection, sadly deficient in their acoustic qualities. Through the use of the system which we are now considering, the limitations which seem to be inherent in the design and construction of many of these great structures, are removed, their usefulness as places of public assembly fully attained.

In outdoor spaces Man's voice is a rather puny thing, but now science has brought to his aid a perfectly controlled volume of energy by which his voice may be made to reach audiences of thousands of persons, many of whom would, under former circumstances, be merely indifferent onlookers.

The growth of our urban centers has reached such a point that for an individual to speak directly and at one time to any considerable portion of its inhabitants has become a physical impossibility. These physical limitations have been removed and the public speaker will now be able to much more effectively play his part in developing and molding public opinion.

In rural communities it is seldom, if ever, that any considerable number of its inhabitants have the opportunity to hear the voices of our National figures. On great public occasions whether they be State or National, it will now be possible for these people to gather in their local centers and have the voices of public men and the music of great bands or orchestras brought directly and naturally to their ears, whether the event itself be a hundred or a thousand miles away.

One could go on indefinitely and enumerate specific examples of the valuable applications of this new development, but it seems sufficient to point out some of these broader applications and to express the confidence that as new uses arise, the technical means will be available to accomplish the desired results.

B. E. Sunny: The American Institute of Electrical Engineers is tonight collecting an extra dividend on its investment, *in time and effort*, in helping for so many years in the development of the electrical industry.

It is holding a joint meeting in New York and Chicago by means of the long distance telephone and the loud speaker. The occasion is a scientific triumph, in the achievement of which, the Institute will cheerfully be given a generous share.

For forty years it has been the international agency for assembling and coordinating the human elements required by a new industry of unlimited significance in its possibilities for public service.

It has been the common gathering place,—the home, in fact, of the fraternity, where theories and problems have been fully and frankly discussed. It has been the clearing house for electrical ideas, plans and methods. In its meetings the rivalries growing out of keen and continuous competition have been forgotten, and a spirit of unity has always prevailed.

The success in the various branches,—electric lighting, transportation, telephone, etc., has necessarily come by slow degrees, but at a very much earlier date than could have been the case, were it not for the opportunities and assistance given by the Institute.

The awarding of the Edison Medal is one of the inspiring and stimulating processes of the Institute, by which 14,000 or more men are held together, in an earnest and enthusiastic campaign for the advancement of the profession.

The Edison Medal is an impressive tribute to the men who, in a large degree, represent the remarkable strides that have been made in modern electrical application.

In the list are men who differed in their theories and practises with respect to manufacture and operation, but these differences have long since been disposed of by the law of the survival of the fittest, which seems to have unusual applicability to electrical ideas.

The public service is no respector of persons and, as the final judge, accepts or rejects our efforts, without emotion.

The contributions to the art by each of these men have been tremendously important and valuable, and they are highly deserving of the distinguished recognition they have received. In their genius, industry, integrity, leadership and achievements, they are our glory, and we are indebted to the Institute for having placed them in our Hall of Fame.

We members of the Institute who have felt that the many important discoveries and inventions in electricity should give us permanently a place in the center of the stage, in the strong light of public approval, are gratified at the announcement by our brothers in the medical profession that they have found the cause of influenza, and discovered a remedy for diabetes. We extend to them our hearty congratulations, and if they will forego dislodging us entirely and be satisfied with a division of the spotlight, we shall be glad to welcome and share with them.

The marvelous thing about history of science and medicine is that so comparatively little was accomplished up to fifty years ago and so much has been accomplished since then. If we today lifted out of the office, shop, home and from the streets, the electrical devices that have been put into them in recent years, what a demoralization would result.

In the practise of medicine and surgery, where the same type of patient, determined and skillful men have been diligently laboring, the average length of life has been increased some 25 per cent in fifty years.

The electric lamp, X-Ray and other electric appliances have greatly helped to this result. In other lines gratifying progress has been made.

Unlike our frequent experiences in civil and governmental matters, in science we hold onto our advances and continually go forward. There is never danger that we will give up electric light and go back to the candle, or give up the trolley and return to the horse car. We hold fast to the serums as they come along, for prevention and cure. But in our ordinary affairs and governmental matters, we ofttimes go forward and slip back, the succeeding generations repeating the old mistakes. Most of the things that are wrong in the economic situation in the nation duplicate the experiences immediately after the civil war, while the situation with respect to the merchant marine is substantially identical to that which existed 142 years ago.

Our hope must be that as the years go on, one after another of these problems can be worked out on a *scientific* basis, where the gain can be held, and we will not slip back and have to begin all over again.

We can therefore congratulate ourselves that in the scientific advances that we are making we are securing permanent values, well worth all of the expenditures that may be made in their seeking.

John J. Carty: Of all the agencies employed in the electrical communications of the world, the telephone is the most wonderful. More than any other the telephone art is a product of American institutions and reflects the genius of our people. The story of its wonderful development is the story of our own country. It is a story of American enterprise and American progress. This story forms part of the proud record of the American Institute of Electrical Engineers.

It was one of our members, Alexander Graham Bell, who revealed to mankind the method of electrically transmitting the tones of the human voice to distant places. He was the first to provide the apparatus to do this marvel; he was the first to speak through the electric speaking telephone; and his voice was the first to be heard in the telephone receiver. It was another of our members, Thomas A. Watson, who made for Bell the first telephone, and who ran the first wire, and who heard the first words which were ever transmitted by electricity.

It is a cause for special gratification tonight that in the office of President, which Bell himself once occupied, we have such a distinguished successor. In those laboratories in which Bell was the original worker, President Jewett and his associates who are also our fellow members, have transmuted into realities those grand visions of science which inspired the founders of our Institute.

Nothing could appeal more to the genius of our Institute than

out to the uttermost limits of the earth with the sublime hope that some day it will be utilized in bringing together the people of all the nations of the earth into one common brotherhood.

Strange to say, it is from the discoveries of the new school of biologists that we realize how fundamental electrical communication systems are in the tremendous evolution of the human race which is now being manifested in the organization of society, and how vital to the welfare of mankind is the daily work of the members of this Institute.

Speaking always of communication in its broadest meaning, but emphasizing the importance of speech, Wilfred Trotter, one of our great authorities in biology, says: "The capacity for free intercommunication between individuals of the species has meant so much in the evolution of man, and will certainly come in the future to mean so incalculably more, that it cannot be regarded as anything less than a master element in the shaping of his destiny." And again, in speaking of human society as a gregarious unit, he says: "The ultimate and singular source of inexhaustible moral power in a gregarious unit is the perfection of communion amongst its individual members." As long as intercommunication was limited, he tells us, the full possibilities of evolution were concealed. But at length appeared man a creature endowed with speech, in whom this capacity for intercommunication could develop indefinitely. Then Trotter goes on to say: "At once a power of a new magnitude was manifest. Puny as were his individuals, man's capacity for communication soon made him master of the world . . . . . In his very flesh and bones is the impulse towards closer and closer union in larger and larger fellowships. Today he is fighting his way towards that goal, fighting for the perfect unit which nature has so long foreshadowed, in which there shall be a complete communion of its members, unobstructed by egoism or hatred, by harshness or arrogance or the wolfish lust for blood. That perfect unit will be a new creature, recognizable as a single entity; to its million-minded power and knowledge no barrier will be insurmountable, no gulf impassible, no task too great."

Here we have portrayed the foward march of humanity toiling ever onward to attain its goal. The realization that their wonderful art is destined to play such an important part in the final attainment, opens up a never-ending source of power and inspiration for electrical engineers everywhere. It adds new dignity to their calling.

We are living in the golden age of communications which has achieved the extension of the spoken word throughout both space and time. But this golden age has not yet ended; it has only just begun. Already, the human voice has been carried with the speed of light across the Atlantic Ocean, and across our continent, and far out into the Pacific. But still greater things are to come. We may hope for the time foretold by the poet,

> "Wherein each earth-encircling day shall be
> A Pentecost of Speech, and men shall hear,
> Each in his dearest tongue, his neighbor's voice
> Tho' separate by half the globe."

I believe that when mankind is prepared to receive the message, a great voice will be heard throughout the world which will proclaim "Peace on earth, good will toward men."

the telephone, for not only have our members built upon it an electrical system of communication of transcendant magnitude and usefulness, but they have made it into a powerful agency for the advancement of civilization, eliminating barriers to speech, binding our people together into one nation, and now reaching

Telephone Transmission Over Long Cable Circuits

BY ALVA B. CLARK
Member, A. I. E. E.
American Telephone & Telegraph Co.

Review of the Subject.—*The application of telephone repeaters has made it possible to use small gage cable circuits to handle long distance telephone service over distances up to and exceeding 1000 miles. A general picture of the long toll cable system which is being projected for use in the northeastern section of the United States was presented recently by Mr. Pilliod before this Institute.**

Many of the circuits in these toll cables are so long electrically that a number of effects, which are comparatively unimportant in ordinary telephone circuits, become of large and sometimes controlling importance. For example, the time required for voice energy to traverse the circuits becomes very appreciable so that reflections of the energy may produce "echo" effects very similar to echoes of sound. The behavior of the circuits under transient impulses, even when two-way operation is not involved so that "echoes" are not experienced, is very important. In order to keep within proper limits of variation of efficiency with frequency over the telephone range special corrective measures are necessary. Owing to the small sizes of the conductors, the attenuations in the longer circuits are very large. Special methods are, therefore, required to maintain the necessary stability of the transmission, including automatic means for adjustment of the repeater gains to compensate for changes in the resistance of the conductors caused by temperature changes.

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Important Tests and Adjustments. (390 w.)
Automatic Transmission Regulators. (390 w.)

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*JOURNAL of the A. I. E. E. Vol. XLI, 1922, August, p. 585.

THIS paper aims to present an idea of what is involved in the transmission of voice currents over long toll cable circuits. Because of the breadth of the subject covered, no attempt has been made to make the discussions of the various items complete, or to include many of the results of the experimental and theoretical work which contributed to a solution of the problems and which has involved the cooperative efforts of a large number of engineers and investigators. This paper should be considered merely as an introduction to the subject. It is hoped that subsequent papers will be presented dealing with these matters in more detail.

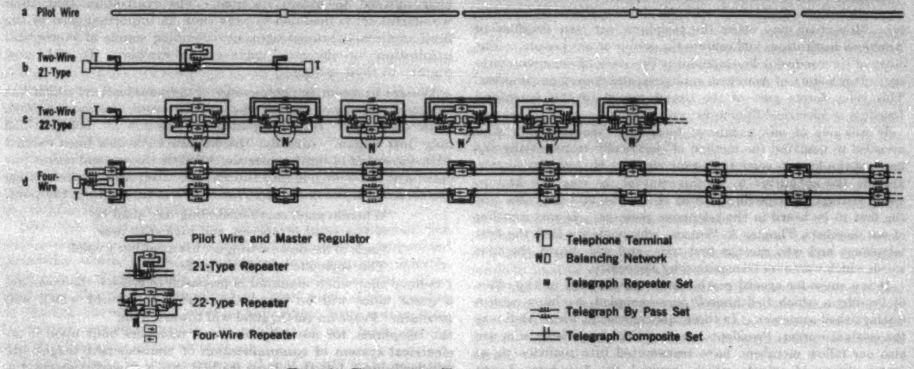

FIG. 1—DIFFERENT TYPES OF CABLE CIRCUITS

For the benefit of those who are not intimately in touch with telephone transmission work, the different types of circuits used in toll cables are first briefly reviewed. The important characteristics of the loading systems are then presented. Following this, various important effects encountered in long cable circuits are discussed and their reactions on the design of cable systems indicated.

In view of the discussion on telephone repeaters given in the Gherardi-Jewett paper,[1] which was presented before this Institute on October 1, 1919, it will be assumed that the reader of the present paper is familiar with the general features possessed by the various types of such devices and, accordingly, no descriptions of them are given, their over-all performance only being of interest in the present connection.

DIFFERENT TYPES OF CIRCUITS

The different types of circuits used in toll cables are illustrated in diagrammatic form in Figure 1. Circuit "b" is a two-wire telephone circuit employing a 21-type telephone repeater. This type of circuit is employed

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[1] TRANSACTIONS of A. I. E. E., Vol. XXXVIII, Part II, p. 1287.

only for handling connections on which but one telephone repeater is involved. Circuit "*c*" is a typical two-wire circuit on which the familiar 22-type telephone repeaters are operated. Circuit "*d*" is of the four-wire type which employs two transmission paths, one for each direction. The function of the pilot wire circuits, "*a*", will be taken up later.

With the exception of circuit "*b*", which possesses the limitation that it cannot advantageously be connected to another circuit containing telephone repeaters, the circuits shown in the figure may be connected, when required, to circuits of the same or other types, such as open-wire circuits, to build up various telephone connections. In general, circuits such as "*c*," employing 22-type repeaters, are used for handling connections of moderate lengths, while circuits such as "*d*" of the four-wire type, are employed for the longer connections where the transmission requirements are more severe.

In addition to employing the cable conductors for furnishing telephone service, these may also be arranged to furnish d-c. telegraph service. Apparatus for compositing the circuits so as to permit this superposition of the d-c. telegraph is indicated on the drawing. In general, the method of compositing the small gage cable circuits is the same as that employed for compositing open-wire lines. The telegraph circuits in cable, however, operate with a metallic instead of a grounded return and employ much weaker currents than those common on open wires. Telegraph currents employed in the cables are comparable in magnitude with the voice currents.

The two-wire circuits in toll cables employ conductors of No. 19 or No. 16 A. W. G. while for the four-wire circuits, No. 19 wire conductors are usually employed. (No. 19 wire weighs 20½ pounds per wire mile, or 5.8 kilograms per kilometer. No. 16 wire weighs twice as much).

Loading Characteristics

Two weights of loading are usually employed. These are commonly known as "medium heavy loading" and "extra light loading" and in this paper they will be referred to for brevity as "m. h. l." and "x. l. l." respectively. The medium heavy loading employs coils having an inductance of about 0.175 henry in the side circuits, spaced 6000 feet apart, (approximately 1.8 kilometers); the extra light loading employs coils having an inductance of about 0.044 henry for the side circuits with the same spacing. The capacity per loading section for the side circuits is approximately 0.074 microfarad.

The medium heavy loaded side circuits have a characteristic impedance of about 1600 ohms, and a cut-off frequency of about 2800 cycles. The extra light loaded side circuits have an impedance of about 800 ohms and a cut-off frequency of about 5600 cycles.

Fig. 2 shows the attenuation-frequency characteristics of No. 19 and No. 16 side circuits with the two types of loading. It will be observed that the m. h. l. circuits have lower attenuation for frequencies below about 2500 cycles, as should be expected from the fact that the inductance per mile introduced by the loading coils is greater. However, the attenuation is more nearly equal at different frequencies in the case of the x. l. l. circuits, this being particularly true at the higher voice frequencies.

Another important characteristic of loaded circuits when repeaters are involved is their velocity of propagation. Since the inductance per mile of x. l. l. circuits is only ¼ of that for m. h. l. circuits, the velocity of propagation is twice as great for the x. l. l. circuits as indicated by the well-known approximate formula

$$V = (L\,C)^{-\frac{1}{2}}$$

Where V is the velocity in unit lengths per second, L is the inductance in henries per unit length and C is the capacity in farads per unit length, the unit of length for expressing velocity, inductance and capacity being the same.

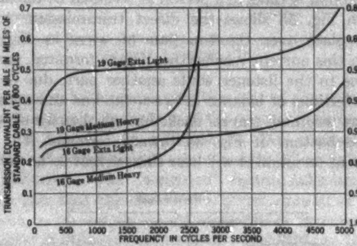

Fig. 2—Attenuation-Frequency Characteristics of Loaded Cable Side Circuits

The x. l. l. type of loading is best for the longer circuits, because of the more nearly equal attenuation of currents of different frequencies, its higher velocity of propagation which permits more efficient operation of telephone repeaters, and also its comparative freedom from transient effects, as will be explained in more detail later. For the shorter circuits where these effects are not so important, the m. h. l. type is satisfactory electrically and is therefore employed, since fewer repeaters are required owing to the lower attenuation.

"Echoes"

As is well known, whenever points of discontinuity or unbalance occur in a telephone circuit, reflections of electrical energy take place. If the circuit is long so that the time for transmission is appreciable and if also the losses are not so great as to cause the reflected energy to become inappreciably small before it reaches the ear of a listener, echo effects will be experienced. While, in general, reflections take place in any telephone circuit actual echoes are never appreciable unless telephone repeaters are employed. In the case of circuits with repeaters, the electrical length is usually great enough so that an appreciable length of time is required for the voice currents to travel to some discontinuity and back again. Furthermore, the repeater gains keep the reflected voice currents large.

It should be understood that the echo effects which are experienced in long repeatered circuits are due to the same unbalances, which, on shorter circuits, bring in trouble due to "singing," or distortion of the voice waves due to "near-singing." On electrically long circuits, due to the comparatively great time lags involved, the echo effects become of controlling importance. Consequently, it is, in general, necessary on such circuits to work the repeaters at gains well below those at which "singing" or distortion due to "near-singing" is experienced.

The echo effects which occur in four-wire circuits will first be discussed, since the effects are simpler in this case than they are in the case of a two-wire circuit.

Fig. 3a shows a four-wire circuit in diagrammatic form, while Fig. 3b shows the echoes which are caused by the unbalances at the terminals. When someone at terminal *A* talks to a person at terminal *B*, the heavy line in Fig. 3b shows the direct transmission, which takes place over the top pair of wires in Fig. 3a. When this current reaches the distant terminal, part of it goes to the listener while another part, due to the imperfections of balance between the line and network at that terminal, travels back through the pair of wires at the bottom of Fig. 3a toward terminal *A*. The talker at terminal *A* will hear this current as an echo if the four-wire circuit is long enough so that the time lag is appreciable. This first echo heard by the talker divides at terminal *A* in the same way as did the direct transmission at terminal *B*, part of it taking the upper path of Fig. 3a back toward the listener. The listener will, therefore, first receive the direct transmission and then a little later an echo. This process is repeated producing successive echoes which are received at both terminals *A* and *B* as indicated.

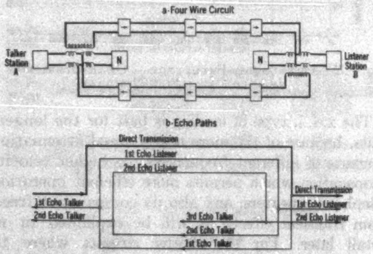
a-Four Wire Circuit
Talker Station A
N
N
Listener Station B
b-Echo Paths
Direct Transmission
1st Echo Listener
2nd Echo Listener
1st Echo Talker
2nd Echo Talker
3rd Echo Talker
2nd Echo Talker
1st Echo Talker
Direct Transmission
1st Echo Listener
2nd Echo Listener

FIG. 3—ECHO PATHS IN FOUR-WIRE CIRCUIT

A four-wire circuit 1000 miles (1600 kilometers) long has been set up in which the balances at the two ends were deliberately made poor so as to exaggerate the effects. More than a dozen successive echoes could be heard before they became inaudible. Since for each echo the voice energy traveled 2000 miles (3200 kilometers) this energy must have travelled the distance around the world before becoming inaudible.

In order that a circuit will be satisfactory for regular telephone use, the echoes must be kept small as compared to the direct transmission. Evidently if the first echoes are small as compared to the direct transmission, the later echoes will be much smaller in magnitude. For example, if the power in the first echo, heard by the listener, is 1/10 as great as the directly transmitted power, the second echo will have only 1/100 as much power, the third echo 1/1000, etc.

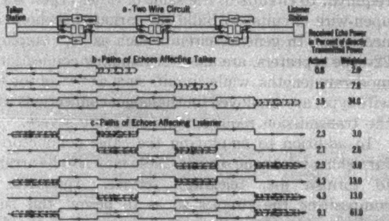
Talker Station
a-Two Wire Circuit
Listener Station
Received Echo Power in Per cent of directly Transmitted Power
Actual
Weighted
b-Paths of Echoes Affecting Talker
0.8
2.9
1.6
7.8
3.5
34.0
c-Paths of Echoes Affecting Listener
2.3
3.0
2.1
2.6
2.3
3.0
4.3
13.0
4.3
13.0
9.1
61.0

FIG. 4—ECHO PATHS IN TWO-WIRE REPEATERED CIRCUIT

The velocity of an x. l. l. circuit is approximately 20,000 miles (32,000 kilometers) per second, while the velocity with m. h. l. is only 10,000 miles (16,000 kilometers) per second. It is thus seen that the time required for voice energy to travel from one end to the other of an x. l. l. circuit 1000 miles (1600 kilometers) long is 0.05 second. An echo traveling from one end of the circuit to the other and back again would, therefore, arrive 0.1 second behind the impulse which started the echo. With m. h. l. circuits these times are of course doubled.

Fig. 4 illustrates the condition existing in a two-wire circuit. For simplicity, the first echoes only are shown, the later echoes being less important owing to their comparative weakness as explained above. In such a circuit reflections occur not only at the terminals, but at a number of intermediate points in the circuit, the condition of balance between the networks associated with the telephone repeaters and the corresponding lines being necessarily imperfect. This imperfection of balance is due in part to lack of perfect balance of the apparatus closely associated with the repeater, and in part to the small irregularities which exist in the make-up of any practical loaded line. A further cause is the reflection at the adjacent repeaters, due to the difference between the repeater impedance and the line impedance.

It will be noted that three sets of echoes are shown which affect the "talker." In addition to these which involve one or more repeaters, a comparatively small amount of power is reflected back to the "talker" from the various irregularities between the "talker" station and the nearest repeater. These reflections have not been indicated since their effects are of negligible importance. Six sets of echoes affect the "listener." Both for the echoes affecting the "talker" and the "listener," the dotted lines indicate reflections from a number of different points where irregularities exist as explained above.

In circuits containing a larger number of repeaters the numbers of sets of echoes affecting the talker and listener are, or course, greater. The number of sets of first echoes affecting the talker is equal to the number of repeaters. The number affecting the listener is equal to

$$\frac{N\ (N + 1)}{2}$$

where N is the number of repeaters.

It is, of course, obvious, that for either four-wire or two-wire circuits, if the circulating energies are large, they will have an adverse effect on the ability of two people to carry on a conversation over a telephone circuit. Not only will the transmission received by the listener be adversely affected, but the talker will be considerably distracted, particularly when the time of the transmission over the circuit is so long that he hears a distinct echo of his words.

Experiments have shown that the effects of the echoes both on the listener and talker become more serious as their time lag is increased. This means that as telephone circuits are made longer it is necessary either to improve balances or to design the telephone circuits so that the velocity of propagation will be higher. This necessity for making the velocity of propagation high on long circuits was one of the principal reasons which led to the selection of extra light loading for the longer circuits.

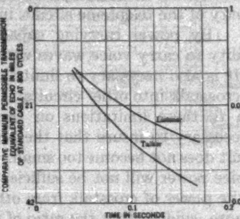

FIG. 5—EFFECT OF ECHOES ON TALKER AND LISTENER

Fig. 5 shows very approximately how the effects of the echoes vary with the length of time by which they are delayed. One curve is given for the effect on the "talker," another for the effect on the "listener." Both curves indicate, for various time lags, the comparative magnitudes of echoes which are small enough to be inappreciable when ordinary telephone conversations are carried on. The curve applying to the "listener" is referred to the direct power which he receives, while the curve for the "talker" is referred to the power which he puts into the circuit.

In Fig. 4 showing the condition existing in a two-wire circuit, the comparative magnitudes of the power in each echo are indicated, a typical condition of the lines being assumed. For the listener the echo power is expressed as a percentage of the directly transmitted power which he receives. In the case of the talker, it is expressed as a percentage of the power which he puts into the circuit. In addition to the comparative amounts of power in each echo, "weighted" magnitudes are indicated. The "weighted" figures take account of the fact that the effects of a given amount of echo become more serious as the time lag is increased as indicated by the curves in Fig. 5. Referring to Fig. 4, it will be noted that the "weighted" magnitudes of the power in the echoes are largest for the long paths. In general, this condition exists in the case of the majority of long two-wire repeatered circuits in cable.

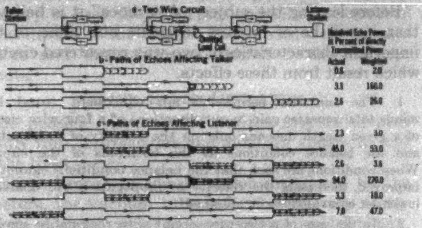

FIG. 6—ECHO PATHS IN TWO-WIRE REPEATERED CIRCUIT WITH OMITTED LOADING COIL

In order to compare the behavior of a four-wire circuit with a two-wire circuit, consider again Figs. 3 and 4. It will be observed that in Fig. 4, showing the two-wire circuit, there is one echo received by the talker which travels from one end of the circuit to the other. Referring to Fig. 3 showing a four-wire circuit, it will be seen that this echo corresponds to the one labelled "1st echo, talker." Similarly for the echoes affecting the listener, the echo whose path is longest in the two-wire circuit corresponds to a similar echo in the four-wire circuit. Since many additional echo paths are present in the two-wire circuit, it is evident that, other things being equal, the over-all transmission result obtainable from the two-wire circuit cannot be made as good as that obtainable from the four-wire circuit.

In a two-wire circuit it is, of course, obvious that any defect in the lines which will cause a large irregularity will result in a considerable impairment of the circuit. Fig. 6 shows the effect of omitting a loading coil at an intermediate point in a circuit, the conditions in this circuit being assumed to be the same as those in Fig. 4

with the exception of the omitted loading coil. The omitted loading coil introduces a large impedance irregularity which causes certain of the echoes to be made much greater in comparative magnitude as indicated. In order to reduce the echoes in the circuit with the omitted loading coil sufficiently to make the circuit satisfactory for telephone use, it is necessary to reduce the repeater gains. In this particular case it is necessary to lower the total gain about four miles, which increases the over-all transmission equivalent of the circuit from about 10 miles for the normal condition to about 14 miles for the condition with the omitted loading coil.

Before leaving the subject of "echoes" it is believed that it will be of interest to point out some of the important characteristics of two-way repeatered circuits which result from these effects.

1. The minimum permissible net equivalent (total loss minus total repeater gain in one direction) of a four-wire circuit of a given length depends only on the velocity of propagation and the balance conditions at the terminals of the circuit. When conditions are such that the balance conditions cannot be improved, increasing the velocity of propagation will enable a lower net equivalent to be obtained.

2. In the case of a two-wire circuit with reasonably smooth lines, the exact location of the repeaters and the gains at which individual repeaters are worked have little effect on the over-all result so far as echo effects are concerned. This follows from the fact that the echo paths from end to end of such a circuit are usually of more importance than the shorter echo paths. Evidently, moving the individual repeaters about or altering their gains has no effect on the longest paths, provided the total gain in each direction is kept constant.

3. In the case of a two-wire circuit of a given length, the velocity of propagation and smoothness of the lines are of most importance in limiting the possible net equivalent, the line attenuation being of secondary importance.

For example, in the case of the transcontinental (New York-San Francisco) open-wire line, the original circuit was loaded. (Although this paper deals particularly with repeaters on cable circuits this example was selected because it illustrates this point so well). The velocity of propagation was such that voice currents required about 0.07 second to travel from one end of the circuit to the other. The total line equivalent was equal to about 56 miles of standard cable. By applying repeaters to this circuit it was possible to obtain a working net equivalent of about 21 miles.

The unloading of the circuit increased the velocity so that the time of transmission was reduced to 0.02 second, about 0.3 of the time required when the circuit was loaded. The attenuation was increased so that the total line equivalent without repeaters was equal to about 120 miles of standard cable, a little more than twice the equivalent of the loaded circuit. By applying repeaters of an improved type to this circuit so as to keep the quality good in spite of the increased attenuation and correspondingly increased gain required, it was possible to obtain a working equivalent of only 12 miles of standard cable as compared to the original figure of 21 miles. This means that with the same amount of speech power applied at one end, the power received over the non-loaded circuit is seven times as large as that formerly received over the loaded circuit.[2]

The example of the transcontinental line, above, may well bring up the question as to why it is that cable circuits are loaded. This is done for two reasons: In the first place, it is in general cheaper to load cables than it is to make up the increased attenuation by means of more repeaters. In the second place the loading lessens the amount of distortion introduced by the cable circuits. In the case of open wire circuits, their series inductance is sufficient to keep the distortion small.

ATTENUATIONS AND CORRESPONDING AMPLIFICATIONS—POWER LEVELS

Owing to the fact that the weight of loading applied to the longest cable circuits is very light, the attenuation of such circuits is very great. A four-wire x. l. l. 19 gage circuit 1000 miles long has the enormous line equivalent of 500 miles of standard cable. The total power amplification applied to this circuit by the repeaters exceeds 10^{47}. This amount of amplification is more than enough to talk half way around the world at the equator using non-loaded No. 8 Birmingham wire gage open wire commonly employed for handling very long distance business (No. 8 B. w. g. copper weighs 435 pounds per wire mile, or 120 kilograms per kilometer).

In order to obtain an idea of how enormous this amplification is, assume that no repeaters were employed and an attempt were made to apply enough power at one end of the circuit to enable the normal amount of speech power to be received at the distant end. The power applied at the sending end would then have to be about 50 quadrillion times as great as the total power which it is estimated is radiated by the sun.

While the total amount of power amplification is very great, the amount of amplification put in at any one point is, of course, limited. The maximum amount of power at a repeater point is limited partly by the capacity of the vacuum tubes and partly by the power carrying capacity of the telephone circuit, including the loading coils. (By power carrying capacity is here meant the ability to carry voice waves without serious distortion.) It is also necessary to limit this power to avoid serious crosstalk into other circuits.

In addition to these limitations on the maximum power, it is necessary to insure that the power at any point in a circuit does not become too small. Otherwise the normal voice power will not be sufficiently large as compared to the power of crosstalk from other circuits. It is, furthermore, evident that the ratio of power from extraneous sources, such as parallelling telegraph circuits and power supply circuits, to the voice power should be as small as practicable in order to keep the circuits free from noise.

Fig. 7 will give an idea of how the telephone power attenuates and is amplified in a long circuit. The circuit shown is similar to those which it is proposed to employ between New York and Chicago, *i. e.,* it is a four-wire x. l. l. No. 19 wire circuit largely in aerial cable equipped with automatic means for compensating for the changes in attenuation caused by the effects of varying temperatures on the resistance of the conductors. (These automatic devices are described in a later section of this paper.) For simplicity, the power levels for transmission in one direction only are shown. The solid lines show the power levels when the temperature is a maximum so that the attenuations are greatest while the dotted lines show the levels when the temperature is a minimum and the losses are, therefore, also a minimum. The shaded areas between the lines represent the changes which take place during the course of a year.

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[2] A material improvement in the telephone quality was also effected by the unloading of the circuit.

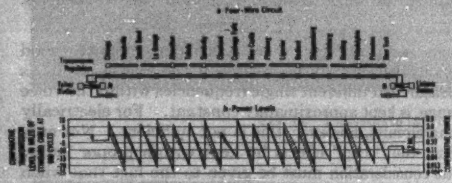

FIG. 7—POWER LEVELS IN NEW YORK-CHICAGO EXTRA LIGHT LOADED FOUR-WIRE CIRCUIT

When the requirement is introduced that transmission must take place in both directions it is found that at the points in the circuits going in one direction where the power is a maximum, the power going in the opposite direction in other circuits is a minimum. This represents a very bad condition for crosstalk from one four-wire circuit into another. In order to overcome this the conductors carrying strong voice power are kept electrically separated or shielded from those carrying weak power as indicated schematically in Fig. 8. The conductors which carry strong voice power are shown heavy, while those carrying weak power are shown light. In the cable proper the separation is effected by grouping the conductors in two bunches, one for transmission in one direction, the other for transmission in the opposite direction, taking care that these two bunches of conductors are separated electrically as far as possible. In the loading coil pots the coils employed on the circuits for transmission in the two directions are similarly kept separated. In the offices the separation is effected by arranging the repeaters and other apparatus as shown in the figure. It will be observed that no special separation is shown between the repeaters transmitting in the two directions, since to keep the conductors carrying weak power separated from those carrying strong power, it is merely necessary to keep the apparatus and cabling connected to the inputs of the repeaters separated from the apparatus and the cabling connected to the repeater outputs.

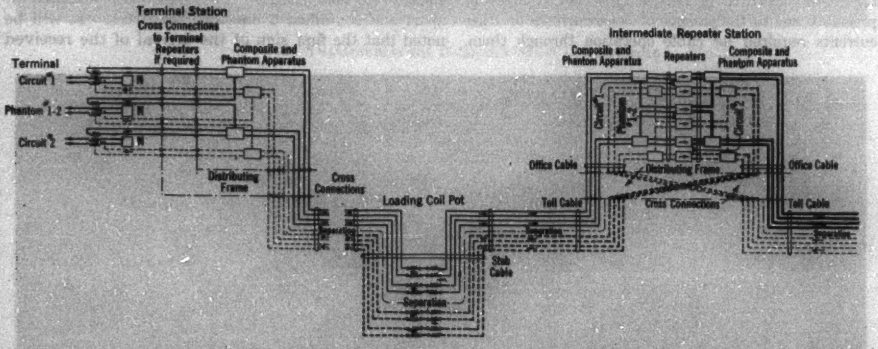
Terminal Station
Cross Connections to Terminal Repeaters If required
Composite and Phantom Apparatus
Terminal
Circuit 1
Phantom 1-2
Circuit 2
N
N
N
Distributing Frame
Cross Connections
Separation
Loading Coil Pot
Separation
Stub Cable
Separation
Intermediate Repeater Station
Composite and Phantom Apparatus
Repeaters
Composite and Phantom Apparatus
Circuit 1
Phantom 1-2
Circuit 2
Office Cable
Distributing Frame
Office Cable
Toll Cable
Cross Connections
Toll Cable
Separation

FIG. 8—FOUR-WIRE SYSTEM—SEGREGATION METHOD TO REDUCE CROSS-TALK

STEADY STATE DISTORTION

The possible sources of distortion may be divided broadly into (1) repeaters and auxiliary apparatus and (2) the lines.

With reference to the distortion introduced by the repeaters, the vacuum tube is fortunately very nearly perfect, at least in so far as concerns practical telephony. At one time, for purposes of test, a circuit was set up containing 32 vacuum tubes in tandem. On this circuit the distortion was so small that when listening to ordinary conversation it was difficult to detect any difference in the quality of transmission before and after traversing the 32 vacuum tubes.

It is beyond the limits of this paper to enter into the problems of design which were encountered in the development of the repeater circuits. For the present purpose of considering the over-all performance of repeatered circuits in cable no serious error will be made if it is assumed that the complete repeater circuits meet the requirements for an ideal repeater as set up in the Gherardi-Jewett paper.

Considering next the lines, it is necessary to make the loading very regular so that balance difficulties will not cause an undue amount of trouble on two-wire circuits. Regularity of the loading is also essential in order to avoid irregular transmission of different frequencies. In order to secure this regularity of loading, it is necessary that the spacing between loading points be made very uniform and that the cable be manufactured so that the electrostatic capacity of its circuits be held within close limits. The loading coils themselves must be closely alike in their electrical properties and furthermore, the coils must be stable, *i. e.* these electrical properties must not change appreciably due to the passage of voice currents or other currents required for cable operation through them.

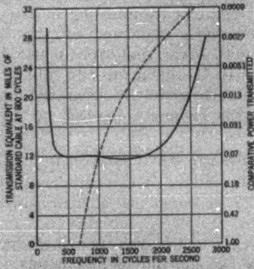

Fig. 9—Transmission-Frequency Characteristic of Long Extra Light Loaded Four-Wire Circuit

Fig. 10—Transients in No. 13 Medium Heavy Loaded Cable

Next, it is necessary to design the repeaters and associated apparatus used on the longer circuits, particularly the four-wire circuits, so as to put in different amounts of gain at different frequencies, thereby making the over-all transmission at different frequencies approximately constant in spite of the fact that the loss introduced by the cable circuits at different frequencies is not constant. Fig. 9 shows the over-all or net transmission equivalent plotted against frequency for an x. l. l. four-wire circuit 1080 miles long (1750 kilometers) which was set up for purposes of test. The heavy line in this figure shows the over-all result which was actually obtained with repeaters and associated apparatus designed to equalize the transmission, while the dotted line shows what the characteristic would have been had the repeaters introduced exactly the same amount of gain at all frequencies.

TRANSIENTS

In comparatively short telephone circuits, good quality will usually be assured if the transmission, as measured at different single frequencies within the voice range, is kept approximately constant. For electrically long circuits, however, this is not sufficient. Not only must the "echo" effects be kept within proper limits, but consideration must be given to the fact that when electrical impulses are applied to such circuits, peculiar transient phenomena are experienced. These transient phenomena occur in equal degree in two-way circuits and in circuits arranged to transmit in one direction only, that is, they are not related to "echo" effects.

In order to give an idea of the nature of some of the transient effects, some oscillograms are shown in Figs. 10, 11, 12 and 13. Fig. 10 shows an 1800-cycle current before and after traversing a cable circuit of an earlier type 1050 miles (1700 kilometers) long. This particular circuit was No. 13 A. w. g. weighing 82 pounds per wire mile (23 kilograms per kilometer) loaded with inductance coils of 0.2 henry spaced 1.4 miles (2.25 kilometers) apart and contained 6 one-way repeaters. It will be noted that the first sign of the arrival of the received current occurs about 0.1 second after the wave is put on at the sending end. This time checks with the formula for velocity given above. The time required after arrival of the first impulse (point "*a*") until the wave builds up to a practically steady-state condition at point "*b*" is about 0.055 second. The steady condition is interrupted at point "*c*" by the arrival of the break transient, the time interval between points "*b*"

FIG. 11—TRANSIENTS IN NO. 13 MEDIUM HEAVY LOADED CABLE

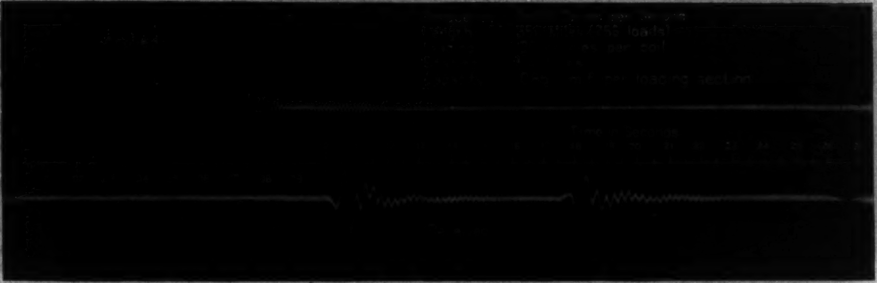

FIG. 12—TRANSIENTS IN NO. 13 MEDIUM HEAVY LOADED CABLE

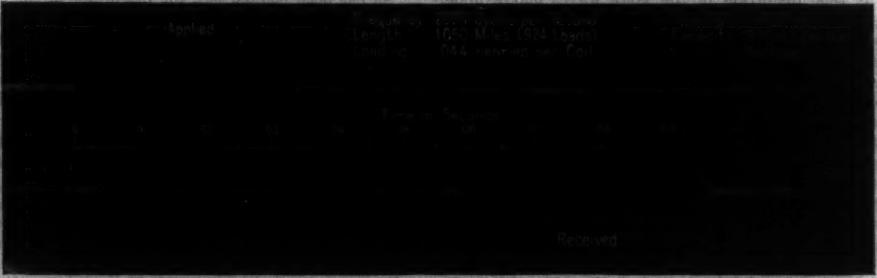

FIG. 13—TRANSIENTS IN NO. 19 EXTRA LIGHT LOADED CABLE

and "*c*," representing the period when the wave is in the steady-state, being only 0.01 second. The wave required about 0.055 second to die out—interval between points "*c*" and "*d*."

It is interesting to note the behavior of the current during the building-up and dying-out intervals. During the building-up process the frequency of the received current increases from a very low value at point "*a*" until at point "*b*" it becomes the same as that of the source. The magnitude of the received current also increases until at point "*b*" it reaches a value corresponding to the steady-state transmission equivalent of the line. The interval "*a-b*" is determined solely by the structure of the line and has nothing to do with the time during which the current is supplied at the sending end.

The dying-out process can be considered to be caused by the application at the time of break of a second current equal in value to the current originally applied but opposite in phase, so that the sum of the two currents will be zero. Hence, it is to be expected that the received current will disappear by adding to the steady-state a transient similar to the building up transient in the interval "*a-b*." That this is true is indicated by the behavior during the interval "*c-d*." At first the low-frequency current of the break transient produces a displacement of the axis of the steady current. As the frequency approaches a steady value a beating effect becomes noticeable which grows smaller until complete opposition of phase obtains and the received current disappears.

Fig. 10 clearly indicates that a pulse of voice current having a frequency in the neighborhood of 1800 cycles, even though received in proper volume if steadily applied, would be badly distorted.

When carrying on a conversation over such a circuit as this, distortion of the voice waves makes understanding difficult while peculiar metallic ringing sounds are very noticeable.

Next consider a circuit of the same character with half the length. The effect of a circuit of this length on an 1800-cycle wave is shown in the oscillogram of Fig. 11. It will be observed that the propagation time has been cut in half while the lengths of time for the received wave to build up and die out have also each been cut in two. This checks with theoretical work, indicating that the severity of this type of transient effect is directly proportional to the length of the circuit. This fact that the transient effect is porportional to the length of the circuit furnishes the reason why a short circuit may give tolerably good results, while a long circuit gives poor results.

Fig. 12 is of interest as indicating what takes place when we apply a current at the sending end of the circuit whose frequency is so high that no appreciable amount of the steady current will pass through the circuit. In this case only transient oscillations appear at the receiving end of the circuit. This particular circuit was of the same type as the above, although it was only 350 miles long (570 kilometers).

A large number of oscillograms of this sort have been taken in connection with the study of these transient effects. From these and theoretical considerations[3] it has been proved that the effects in a given circuit are much worse at high frequencies than at low frequencies, the severity of the effects, within certain limits, being a function of the ratio of the frequency being transmitted to the frequency of cut-off of the loaded circuit. The gage of the circuit has practically no effect.

Since in order to give good quality it is necessary to transmit fairly well all frequencies up to at least 2000 cycles, it is obvious that on long circuits in order to keep the transient effects small, the frequency of cut-off must be kept high. In order to do this, it is necessary either to make the loading coils of very low inductance or to space them very close together. This is another one of the reasons why extra light loading was adopted for the long cable circuits. (It will be remembered that the inductance of the side circuit loading coils is only 0.044 henry and the spacing 6000 feet).

The effect of lighter loading on the transient behavior of telephone currents, is shown in Fig. 13, which shows a 2000-cycle wave transmitted over an x. l. l. circuit about 1050 miles (1700 kilometers) long. This circuit contained 23 one-way repeaters. It will be observed that both the building-up and dying-out transient periods are very much reduced, which means that all pulses of telephone currents up to at least 2000 cycles will pass through such a circuit with very little distortion.

STABILITY

As has been pointed out, the magnitude of the line transmission loss in a repeatered circuit is of comparatively small importance in determining its possible transmission equivalent, whether the circuit be worked on a four-wire or two-wire basis. However, it is of extreme importance to be sure that the repeater gains are kept adjusted so as to compensate exactly for a large part of the transmission loss in the circuit, so that the difference between the total loss in the circuit and the total gain, which represents the net equivalent of the circuit, will be kept constant.

On certain of the long circuits this difference is very small as compared to the quantities which are subtracted. For example, in the case of a 1000-mile four-wire circuit using x.l.l. 19 gage conductors, the total line transmission loss is about 500 miles. Not counting the gain required to make up for losses in apparatus and office cabling, the total gain is about 488 miles, the difference, 12 miles, representing the net equivalent.

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[3] John R. Carson—"Theory of the Transient Oscillations of Electrical Networks and Transmission Systems." TRANSACTIONS of A. I. E. E. Vol. XXXVIII, page 407.

Evidently only a very small percentage change in either the transmission losses or the gains will have a large effect on the net equivalent. This represents about the most severe condition. Some examples of less severe conditions are:

2-Wire No. 19 m. h. l. circuit 200 miles long (320 kilometers). Line equivalent 58 miles. Repeater gain exclusive of gain required to make up for loss in apparatus and office cabling 46 miles. Net equivalent 12 miles.

4-Wire No. 19 m. h. l. circuit 500 miles long (800 kilometers). Line equivalent 145 miles. Repeater gain exclusive of gain required to make up for loss in apparatus and office cabling 133 miles. Net equivalent 12 miles.

In order to maintain the necessary constancy of the over-all or net transmission equivalent of long repeatered circuits in cable, it is necessary first of all to maintain the gains of the individual repeaters within close limits. In addition, periodic transmission measurements are required over the complete circuits, supplemented by suitable adjustment of certain of the individual repeaters whenever the over-all equivalent falls outside of the prescribed limits. Also, on the very long small gage circuits, the changes in attenuation, due to the resistance changes caused by temperature variations, become so large that it is practically essential to provide automatic means for overcoming these effects.

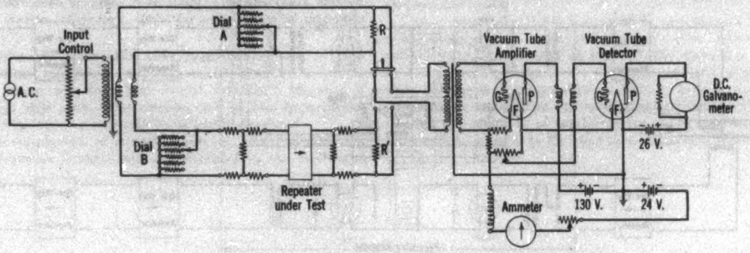

Fig. 14—Device for Measuring Telephone Repeater Gains

The methods employed in maintaining the gains of the individual repeaters and of the over-all transmission equivalents within proper limits will first be described, after which the automatic transmission regulators will be discussed.

Important Tests and Adjustments

In order to hold the repeater gains constant, close inspection limits are placed on the vacuum tubes during the course of manufacture to insure great uniformity of the product, as well as consistency of performance. In operating the repeaters, considerable care is taken to maintain constancy of the operating currents and voltages. The operating limits of currents and potentials together with the corresponding gain variations for one of the types of tube in common use are given in the following table:

Variable Quantity	Prescribed Limits	Gain Variation
Plate Potential..........	130 ± 5 volts	± 0.2 mile
Grid Potential...........	9 ± 1 volt	± 0.3 mile
Filament Current.......	1.25 ± 0.05 ampere	very small for new tube—one mile for tube just before replacement.

In addition to maintaining the tube currents and voltages within the required limits, the gains of the individual repeaters are checked periodically. Suitable adjustments are made when the repeater gains fall outside of the prescribed limits. When the filament emission of a tube becomes so low that the above specified variation in the filament current results in more than one mile gain variation, the tube is replaced.

A gain measuring device as indicated schematically in Fig. 14 is employed for this purpose. The measurement of gain is effected by comparison of the voltages across two resistances, one of which forms part of a circuit which includes the repeater, the other being simply a reference circuit. An amplifier-detector combination amplifies the voltages across these resistances and then rectifies them so as to obtain an indication on a d-c. galvanometer. Equality of voltages across the two resistances, which are designated as *R* and *R′* in the figure, is thus indicated by equal deflections of the galvanometer. When this condition is secured, the repeater gain is read directly from the dials *A* and *B*. By means of this device, it is readily possible to measure the gain of a repeater within a few tenths of a mile. Owing to the fact that the measuring circuits are comprised entirely of resistances, the readings of the set are independent of frequency, so that gains can be measured at all important telephone frequencies.

As pointed out above, transmission measurements over the complete circuits including the telephone repeaters are required at periodic intervals in order to insure that proper transmission standards are being maintained. By means of such measurements, the variations in the over-all equivalent of the circuits due to the cumulative effect of small gain variations, slight variations which remain after the automatic transmission regulators have compensated for the major variations in the conductors and variations from other causes including the effect of different conditions of humidity on the wiring in the offices, are determined and compensated for. These measurements are made by applying a known electromotive force through a known resistance to one end of the circuit and receiving the current at the distant end with a suitable calibrated arrangement employing an indicating meter. Since this type of measurement is similar in principle to the method employed for measuring the gains of the individual repeaters, it will not be described.

Automatic Transmission Regulators

Since the resistance of long cable circuits employing small gage conductors is comparatively large, it is, of course, evident that changes in this resistance caused by temperature changes to which the cable circuits are subject will have a large effect on transmission. For example, in the case of an x. l. l. No. 19, 1000 mile circuit (1600 kilometers) in aerial cable, the total attenuation changes more than 110 transmission miles during the course of a year. This corresponds to a variation in the received power of more than 10^{10} or ten billion times.

It is, of course, essential to provide special means to counteract these effects. Furthermore, since the temperature changes which occur in an aerial cable are very rapid, it is practically essential to make these means automatic. In the case of x. l. l. No. 19 circuits whose variation is greatest, it is necessary to locate the automatic regulators, in general, at every third or fourth repeater station in order to keep the transmission levels within proper limits. In Fig. 1a, a typical method of locating the regulating devices along a cable is indicated. In this sketch each square indicates a master automatic transmission controlling device while the loops extending in either direction from the squares indicate the cable circuits which control the functioning of these devices.

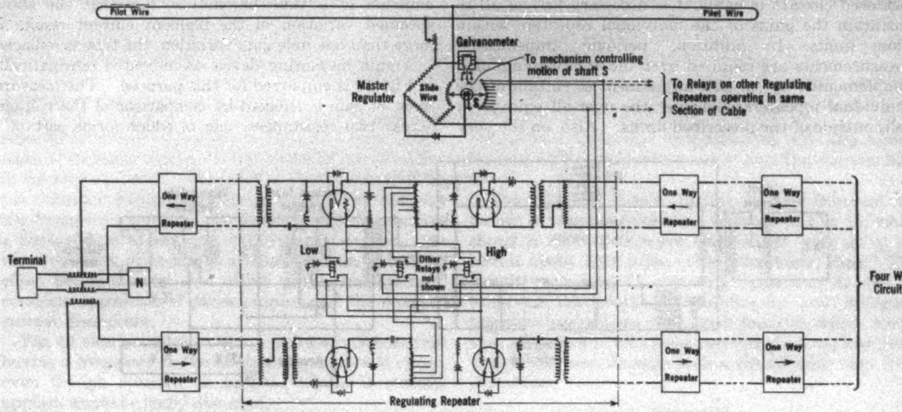
Pilot Wire
Pilot Wire
Galvanometer
To mechanism controlling motion of shaft S
Master Regulator
Slide Wire
S
To Relays on other Regulating Repeaters operating in same Section of Cable
One Way Repeater
One Way Repeater
One Way Repeater
Terminal
N
Low
Other Relays not shown
High
Four Wire Circuit
One Way Repeater
One Way Repeater
One Way Repeater
Regulating Repeater

Fig. 15—Pilot Wire Automatic Transmission Regulator

An automatic transmission regulator is shown schematically in Fig. 15. The device comprises a Wheatstone bridge arrangement. In one arm of the bridge, pilot wire pairs, extending in either direction in the cable, are included as indicated in the figure. The Wheatstone bridge has associated with it certain apparatus which will not be described here in detail, which functions in such a manner as to keep the bridge automatically balanced at all times. In the process of maintaining balance of the bridge, angular motion is conveyed to a shaft which is proportional to the resistance variations which the cable circuits undergo. The movement of the shaft causes different contacts to be made and thus controls relays which in turn control the gains of the telephone repeaters, one way of doing this being indicated in the figure. The repeater gains are thus caused to be raised and lowered automatically, and thereby overcome the differences in attenuation caused by the temperature changes in the cable conductors.

Discussion

James J. Pilliod: Metallic conductors interconnecting cities and territories must be used in ever increasing quantities to meet the requirements for long distance telephone service. For many conditions, open wires carried on pole lines provide the best method and the one with which we are all generally familiar. The other available method and the one with which this paper specifically deals, is to bunch the wires together within the limits of a small lead sheath and this, as might well be supposed, introduces new problems and some of the most important of these are problems of transmission. The two methods mentioned are complementary and both have wide fields of application.

For some sections of the country the cable method of providing for the service offers advantages which are of almost controlling importance and it is to some of these advantages that I will briefly refer in the following discussion.

Continuity of service is of prime importance under all conditions, but to insure this in a substantial degree in the case of heavy open wire lines under extreme weather conditions requires very expensive construction. The cable method is, therefore, an important step in the direction of providing greater insurance to continuous service. The successful development of the telephone repeater has made it possible to operate long telephone circuits by the use of wires of a smaller gauge than good engineering practise requires from the standpoint of mechanical strength in the case of open wire lines. To use these smaller conductors, means, therefore, that the wires must be placed in cables.

These cables are practically free from damage due to sleet and wind storms, and while an occasional interruption to service is likely to occur, just as a leak might develop in the roof of this building, still such interruptions are of less consequence than if the structure as a whole were badly damaged.

From the standpoint of conservation, as many as 300 long distance telephone circuits can be provided in one cable and several cables can be carried on one pole line. To provide for as many circuits as can be placed in one cable with open wire construction would require ten pole lines, using poles of considerably greater size. As is well known, timber is becoming scarcer each year and the number of available through routes not already occupied with pole lines is extremely limited. It is easy to imagine the difficulty that would be experienced in trying to find routes for ten to twenty new pole line routes at this time between New York and Boston or New York and Washington, but this, and even more, would be required if cable could not be used for telephone circuits connecting these points. The same applies to the section generally between New York and Chicago and between other points. Another item is the matter of copper. For the 300 circuits on the basis of open wire lines, we would require at least 70,000 lb. of copper per mile, while for three hundred circuits in cable, 13,000 lb. is sufficient. Therefore, for each mile of cable we leave 57,000 lb. available for other purposes.

I believe that these few examples make it sufficiently clear that the development work, an important part of which is described in this paper, which has been done to make it possible to operate long distance telephone circuits in cables is of great importance from the standpoint of conservation and economics and from the standpoint of reliability of service which the people of this country not only expect, but require and demand. It will, however, be evident from a reading of the paper that to obtain these things it is necessary to be liberal in the expenditure of technical ability.

I have been very much interested in the diagram showing power levels at the various points along a New York-Chicago cable circuit as illustrated in Fig. 7. The discussion of this, as well as the requirements for considering velocity of propagation, echoes, and transmission regulation to compensate for variations in attenuation due to temperature changes illustrates forcibly the necessity of careful design and a consideration of circuits from one end to the other as a unit if satisfactory results are to be obtained.

Theory of Electric Wave Filters Built up of Coupled Circuit Elements

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Review of the Subject.—*This paper is essentially a treatment of certain types of coupled circuit networks by methods similar to those used in the discussion of the properties of long lines with distributed constants. This method of treating those coupled circuit chains to which it can be applied far surpasses other methods of treatment in several important ways. First, the number of elements in the chain can be made as large as desired without complicating the problem in any way whatsoever. Second, the very method of treatment leads directly to the rational design of selective systems of a type which the older methods of treatment did not even show to exist. Third, this method of attacking the problem is much better adapted to a transient state treatment (following the methods of J. R. Carson and T. C. Fry) than the older methods of handling the coupled circuit problem.*

In the second section of this paper there is developed the general theory of the properties of identical circuits coupled so as to form a chain. The equations giving the current and voltage of any circuit in the chain are identical in form with those giving the current and voltage at any point of a long line with distributed constants. The propagation constant however, instead of being an algebraic function of the circuit constants and the impressed frequency as in the case for the line with distributed constants, is a transcendental function of the circuit constants and the frequency of the current being transmitted. The nature of this transcendental function is such that sharp changes occur in the characteristic curve which portrays the attenuation constant as a function of the frequency.

The third section shows how, by proper termination and design, these sharp changes in the attenuation frequency characteristic can be employed to build up filter networks.

The fourth section gives an application of the general theory by presenting a detailed treatment of simple series circuits magnetically coupled so as to form a chain. In this section a number of curves is presented. These curves give a visual picture of the general theory and bring out points useful in the designing of selective networks which must meet preassigned requirements.

The fifth section discusses the problem of building up filters using sections of many different types. General methods of attacking this problem are given and design formulas for three different types of filter sections which may be used together in building up a filter system, are derived. Curves are given which illustrate the methods of building up desirable characteristics. The design of selective systems is put upon a rational basis.

I. INTRODUCTION AND PURPOSE

The increase in the use of radio communication and the introduction of carrier current telephony has led to a very great interest in the problem of separating currents having frequencies which lie in a given band from currents of all other frequencies. Systems of circuits used to perform this separation have been called electric wave filters, or simply wave filters. Some of the properties of coupled circuits which make them valuable as wave filters have been known and employed for some time. It is the purpose of this paper to point out some new properties of certain types of systems built up of coupled circuits, and to show how systems having preassigned characteristics may be built up in a rational way by an application of the theory presented.

In this paper chains built up of coupled circuits will be treated in a manner similar to that employed in arriving at the properties of long lines with distributed constants. In the telephone art it is common practise to give the transmission properties of long lines, or of any piece of apparatus by stating the loss in miles of standard cable or more rationally in attenuation units, incurred by inserting the line or the piece of apparatus between two points of known impedances. This method is very well adapted to a discussion of the properties of wave filters, and will be employed in this paper.

Wave filters built up of series and shunt impedances have been discussed by G. A. Campbell[1] and K. W. Wagner.[2] The general theory presented in the following sections is somewhat broader than that presented by Campbell and Wagner, as it includes as a special case the filter built up of series and shunt elements as the shunt element may be considered as a coupling element.

In this paper illustrations of the general theory will be given by discussing particular types of filters which are not built up of series and shunt elements. Illustrations from this class of filters are used because it is the application of long line theory to such coupled circuit chains which forms the main contribution of

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*During the summer of 1921 the writer was employed as an assistant in filter design by the Western Electric Company. During this period he became very familiar with the series shunt type of filter element and with the methods used by the engineers of this company in handling filter design problems. This experience with the Western Electric Co. is reflected in this paper in such things as the loss method of treatment, the use of sections of different types to build up given characteristics, and the methods of obtaining the design formulas. The material for this paper was inspired by the previous work on coupled chains and on selective systems in general carried out by Professors Bennett, Crothers and the author at the University of Wisconsin. The general use of different equations in handling artificial line problems was acquired by the author from the published papers of Campbell and Carson. The conception of circuital and transfer impedance as a broad basis for filter theory and the type of filter section to which the treatment leads directly are thought to be original.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

[1] Campbell, U. S. Pat. Specs. No. 1,227,113; 1,227,114 of 1917.

[2] Wagner, *Archiv fur Elektrotechnik*, vol 8, page 145, (1919).

this paper to the literature on wave filters. The series shunt type of structure has already been described by Campbell and Wagner, and has been used for some time in the plant of the Bell Telephone System.

II. Current Propagation Through a Chain Built up of Identical Circuits Coupled Together

(1) *General Equations and the Propagation Constant.* Consider the chain of circuits shown in Fig. 1. Each circuit is identical with every other circuit. The impedance of each circuit when removed from the chain and isolated is represented by Z_c. That is with the circuit taken out of the group and E volts impressed in it the current flowing would be E/Z_c. Z_c will be called the circuitual impedance. In general it will be of the form $a + j\,b$. The coupling is represented by the square box. The transfer or mutual impedance is represented by Z_m. That is the voltage in the $(q+1)$th circuit due to the current in the qth circuit is $I_q Z_m$. In general Z_m may also be of the form $a + j\,b$. The source of power, represented by a generator works through an impedance Z_0 into a terminating circuit of half the normal circuitual impedance. The chain ends at the other end in a half circuit closed by a load impedance Z_n.

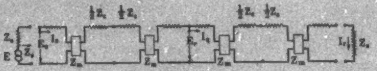


FIG. 1

Now applying Kirchoff's law to the qth circuit we obtain:

$$- Z_m I_{q-1} - Z_m I_{q+1} - Z_c I_q = o \qquad (1)$$

The voltage across the mid-point of the qth circuit is:

$$E_q = - Z_m I_{q-1} - Z_c/2\, I_q \qquad (2)$$

Equation (1) is a difference equation relating the currents in three adjacent circuits. Its solution is:

$$I_q = A\,\epsilon^{q\tau} + B\,\epsilon^{-q\tau} \qquad (3)$$

Substituting (3) in (1) there results:

$$Z_c/Z_m\,(A\,\epsilon^{q\tau} + B\,\epsilon^{-q\tau}) + A\,\epsilon^{-\tau}\,\epsilon^{q\tau} + B\,\epsilon^{\tau}\,\epsilon^{-q\tau} + A\,\epsilon^{\tau}\,\epsilon^{q\tau} + B\,\epsilon^{-\tau}\,\epsilon^{-q\tau} = 0 \qquad (4)$$

Collecting terms:

$$Z_c/Z_m\,(A\,\epsilon^{q\tau} + B\,\epsilon^{-q\tau}) + A\,\epsilon^{q\tau}\,(\epsilon^{\tau} + \epsilon^{-\tau}) + B\,\epsilon^{-q\tau}\,(\epsilon^{\tau} + \epsilon^{-\tau}) = 0 \qquad (5)$$

Factoring:

$$(A\,\epsilon^{q\tau} + B\,\epsilon^{-q\tau})\,(Z_c/Z_m + \epsilon^{\tau} + \epsilon^{-\tau}) = 0 \qquad (6)$$

If the first term were identically zero, no current would exist anywhere in the chain. The first term is therefore different from zero and we have:

$$Z_c/Z_m + (\epsilon^{\tau} + \epsilon^{-\tau}) = 0 \qquad (7)$$

or

$$\cosh\tau = - \frac{Z_c}{2\,Z_m} \qquad (8)$$

In the long line theory τ has been called the propagation constant. This name will be retained here. Equation (8) is the fundamental equation relating the propagation constant to the circuit constants.

The constants A and B of equation (3) are arbitrary and may be determined in the terms of any two currents, any two voltages, or in terms of a current and a voltage at some mid-circuit position.

(3) *The Voltage Equation.* Upon substituting for $I\,q - 1$ and $I\,q$ in equation (2) we obtain:

$$E_q = - Z_m\,(A\,\epsilon^{q+1\,\tau} + B\,\epsilon^{-(q+1)\,\tau}) - Z_c/2\,(A\,\epsilon^{q\tau} + B\,\epsilon^{-q\tau}) \qquad (9)$$

From equation (5) we obtain the relation,

$$Z_c/2\,(A\,\epsilon^{q\tau} + B\,\epsilon^{-q\tau}) = - Z_m/2\,(\epsilon^{\tau} + \epsilon^{-\tau})\,(A\,\epsilon^{q\tau} + B\,\epsilon^{-q\tau}) \qquad (10)$$

Substituting (10) in (9) there results

$$E_q = (- Z_m \sinh\tau)\,[A\,\epsilon^{q\tau} - B\,\epsilon^{-q\tau}] \qquad (11)$$

From (8)

$$\cosh\tau = - \frac{Z_c}{2\,Z_m}\,; \qquad \sinh\tau = \sqrt{\frac{Z_c^2}{4\,Z_m^2} - 1}$$

$$E_q = - \sqrt{Z_c^2/4 - Z_m^2}\,[A\,\epsilon^{q\tau} - B\,\epsilon^{-q\tau}] \qquad (12)$$

The term $\sqrt{Z_c^2/4 - Z_m^2}$ plays the same role here as the characteristic impedance, plays in long line theory. Therefore, this quantity will be called the characteristic impedance. It will be represented by the symbol Z_s. That is:

$$Z_s = \sqrt{Z_c^2/4 - Z_m^2} \qquad (13)$$

$$E_q = - Z_s\,[A\,\epsilon^{q\tau} - B\,\epsilon^{-q\tau}] \qquad (14)$$

(4) *Determination of the Constants A and B. In Terms of the Current and Voltage of the First Circuit.* When $q = o$ let $E_q = E_0$ and $I_q = I_0$.
Then from (14) we obtain:

$$E_0 = - Z_s\,(A - B) \qquad (15)$$

$$I_0 = A + B \qquad (16)$$

$$A = 1/2\,(I_0 - E_0/Z_s) \qquad (17)$$

$$B = 1/2\,(I_0 + E_0/Z_s) \qquad (18)$$

The current and voltage equation then become

$$I_q = 1/2\,(I_0 - E_0/Z_s)\,\epsilon^{q\tau} + 1/2\,(I_0 + E_0/Z_s)\,\epsilon^{-q\tau} \qquad (19)$$

$$E_q = 1/2\,(E_0 - Z_s\,I_0)\,\epsilon^{q\tau} + 1/2\,(E_0 + Z_s\,I_0)\,\epsilon^{-q\tau} \qquad (20)$$

Represent the real and imaginary parts of τ by h and k respectively so that

$$\tau = h + j\,k \qquad (21)$$

Equations (19) and (20) may now be written in two alternate forms, one involving the trigonometric functions and the other involving the hyperbolic functions.

The trigonometric forms are:

$$I_q = 1/2\,(I_0 + E_0/Z_s)\,\epsilon^{-hq}\,(\cos k\,q - j \sin k\,q) + 1/2\,(I_0 - E_0/Z_s)\,\epsilon^{hq}\,(\cos k\,q + j \sin k\,q) \qquad (22)$$

$$E_q = 1/2\,(E_0 + Z_s I_0)\,\epsilon^{-hq}\,(\cos k\,q - j \sin k\,q) + 1/2\,(E_0 - Z_s I_0)\,\epsilon^{hq}\,(\cos k\,q + j \sin k\,q) \qquad (23)$$

The hyperbolic forms are:

$$I_q = I_0 \cosh q\,\tau - E_0/Z_s \sinh q\,\tau \qquad (24)$$

$$E_q = E_0 \cosh q\,\tau - Z_s I_0 \sinh q\,\tau \qquad (25)$$

(5) *The Current and Voltage of Any Circuit in Terms of the Initial Voltage and the Terminating Impedance.* For some purposes it is useful to have the equations for current and voltage in terms of the sending end voltage and the terminating or load impedance. We next proceed to write these equations in terms of these constants.

The current in the end or *n*th section is:

$$I_n = I_0 \cosh n\,\tau - (E_0/Z_s) \sinh n\,\tau \qquad (26)$$

The voltage in the *n*th section or across the terminating impedance is:

$$E_n = E_0 \cosh n\,\tau - Z_s I_0 \sinh n\,\tau \qquad (27)$$

Also

$$Z_n I_n = E_n \qquad (28)$$

From (**26**), (**27**) and (**28**) we obtain:

$$I_0 = E_0 \frac{\cosh n\,\tau + Z_n/Z_s \sinh n\,\tau}{Z_n \cosh n\,\tau + Z_s \sinh n\,\tau} \qquad (29)$$

In (**29**) the reciprocal of the quantity multiplying E_0 is the impedance of the string of circuits looking into the generator end of the line. We represent this impedance by Z_L. That is:

$$Z_L = \frac{Z_n \cosh n\,\tau + Z_s \sinh n\,\tau}{\cosh n\,\tau + (Z_n/Z_s) \sinh n\,\tau} \qquad (30)$$

And

$$I_0 = E/Z_L \qquad (31)$$

Equations (**22**) and (**23**) may now be written:

$$I_q = 1/2\,E_0\,(1/Z_L + 1/Z_s)\,\epsilon^{-hq}\,(\cos k\,q - j \sin k\,q) + 1/2\,E_0\,(1/Z_L - 1/Z_s)\,\epsilon^{hq}\,(\cos k\,q + j \sin k\,q) \qquad (32)$$

$$E_q = 1/2\,E_0\,(1 + Z_s/Z_L)\,\epsilon^{-hq}\,(\cos k\,q - j \sin k\,q) + 1/2\,E_0\,(1 - Z_s/Z_L)\,\epsilon^{hq}\,(\cos k\,q + j \sin k\,q) \qquad (33)$$

Equations (**24**) and (**25**) may now be written

$$I_q = E_0\,(1/Z_L \cosh q\,\tau - 1/Z_s \sinh q\,\tau) \qquad (34)$$

$$E_q = E_0\,(\cosh q\,\tau - (Z_s/Z_L) \sinh q\,\tau) \qquad (35)$$

(6) *Comparison of the Equation with the Equation for the Smooth Line.* The equations which have been written down for current and voltage in the coupled circuit chain are identical in form with the equations for the current and voltage at any point of a long line with uniformly distributed constants. Thus equations (**22**), (**23**), (**24**) and (**25**) apply identically to the line with uniformly distributed constants. However the expressions for Z_s and τ are different for the smooth line than for the coupled circuit chain. The most striking and fundamental difference occurs in the propagation constants for the two cases. For the smooth line the propagation constant is an algebraic function of the line constants, while for the coupled circuit chain the propagation constant is a transcendental function of the circuit constants. It will be shown later that this difference portrays the difference in the characteristics of the two types of systems. In the equation for the coupled circuit chain, q is a positive integer. For the smooth line q need not be an integer.

(7) *Conditions under which the Reflected Wave Disappears.* In equations (**22**) and (**23**) the first terms decrease as the circuit under consideration recedes from the generator end of the chain. In the case of the smooth line this term has been called the main wave of current, and voltage respectively. The second term of each of these equations increases in magnitude as the circuit under consideration recedes from the generator end. In long line theory these terms have been called the reflected waves. This terminology is both useful and convenient and will be adopted in the discussion of the coupled circuit chain. In the case of the smooth line the reflected wave can be made to disappear in two ways, first by making the line very long, second by terminating in an impedance equal to the characteristic impedance of the line. We proceed to show that if these same conditions are fulfilled, the reflected waves disappear from the equation for current and voltage in any circuit of the coupled circuit chain.

In equation (**30**), which relate Z_L to the circuit constants, divide both numerator and denominator by $\cosh n\,\tau$. This gives

$$Z_L = \frac{Z_n + Z_s \tanh n\,\tau}{1 + (Z_n/Z_s) \tanh n\,\tau} \qquad (36)$$

$$\lim_{n \doteq \infty} Z_L = Z_s \qquad (36)$$

So that for a very great number of circuits in the chain $E_0/Z_s = I_0$; $I_0 Z_s = E_0$ and the terms representing the reflected wave disappears from the equations for current and voltage.

Now consider the chain to be terminated by an impedance equal to the characteristic impedance of the chain. That is, let $Z_n = Z_s$.

Then it can readily be seen from equation (**30**) or (**36**) that $Z_L = Z_s$, and the reflected wave is again absent.

Thus, when the chain is again made up of many circuits, or when it is terminated in an impedance equal to its characteristic impedance, the impedance looking into the chain is equal to the characteristic impedance and the terms representing the reflected wave are absent from the equations for current and voltage. The current and voltage in the qth circuit are then given by the following simple equation.

$$I_q = I_0\, \epsilon^{-q\tau} \qquad (37)$$

$$E_q = E_0\, \epsilon^{-q\tau} \qquad (38)$$

$$I_q = I_0\, \epsilon^{-hq} (\cos k\, q - j \sin k\, q) \qquad (39)$$

$$E_q = E_0\, \epsilon^{-hq} (\cos k\, q - j \sin k\, q) \qquad (40)$$

(8) *The Attenuation Constant and the Wave Length Constant.* In the equation for current and voltage in the qth circuit of the chain of coupled circuits, the propagation constant τ is in general a complex number. We have represented τ by $h + j\,k$. Now consider the general equation for current or voltage. If we confine our attention to the main wave alone or to the reflected wave alone, then equations (22) and (23) show that h affects the amplitude of the wave without affecting its phase position. For this reason it is called the attenuation constant. k, on the other hand, affects only the phase position and in a number of circuits such that $k\,q = 2\,\pi$ the phase of the current in either the main or reflected wave shifts through 360 deg., or when we pass over a number of circuits q_w given by

$$q_w = \frac{2\,\pi}{k} \qquad (41)$$

We have gone the equivalent of one wave length on a smooth line. For this reason k is called the wave length constant.

The propagation constant τ has already been evaluated in terms of the circuit constants. For many purposes it is convenient to evaluate h and k separately in terms of these constants.

From equation (8) we have

$$\cosh \tau = \cosh (h + j\,k) = -\frac{Z_c}{2\,Z_m} \qquad (8)$$

Let $Z_c/Z_m = u + j\,v = \gamma\,\underline{/\phi}$ (42)

Expanding (8)

$$\cosh (h) \cos (k) + j \sinh (h) \sin (k) = -\frac{u + j\,v}{2} \qquad (43)$$

Since h and k are assumed real, $\sinh h$, $\cosh h$, $\sin k$, and $\cos k$ are all real. Equating real and imaginary parts there results:

$$\left.\begin{aligned} \cosh (h) \cos (k) &= -u/2 \\ \sinh (h) \sin (k) &= -v/2 \end{aligned}\right\} \qquad (44)$$

Squaring the equations (44)

$$\left.\begin{aligned} \cosh^2 h \cos^2 k &= u^2/4 \\ \sinh^2 \sin^2 k &= v^2/4 \end{aligned}\right\} \qquad (45)$$

$$(\cosh^2 h - 1)(1 - \cos^2 k) = v^2/4 \qquad (46)$$

$$\cos^2 k = 1 - \frac{v^2}{4\,(\cosh^2 h - 1)} \qquad (47)$$

$$(\cosh^2 h) \left[1 - \frac{v^2}{4\,(\cosh^2 h - 1)} \right] = u^2/4 \qquad (48)$$

$$\cosh^4 h - (\cosh^2 h) \left[1 + \frac{u^2 + v^2}{4} \right] = -u^2/4 \qquad (49)$$

$$\cosh (h) = \left[\frac{1}{2} \left\{ \left(\frac{u^2 + v^2}{4} + 1 \right) + \sqrt{\left(\frac{u^2 + v^2}{4} + 1 \right)^2 - u^2} \right\} \right]^{\frac{1}{2}} \qquad (50)$$

$$\cosh (h) = [\,1/2\,\{\,(\gamma^2/4 + 1) + \sqrt{(\gamma^2/4 + 1)^2 - \gamma^2 \cos^2 \phi}\,\}\,]^{\frac{1}{2}} \qquad (51)$$

In like manner we obtain the relations

$$\cos k = \frac{-u}{\sqrt{u^2}} \left[\frac{1}{2} \left\{ \left(\frac{u^2 + v^2}{4} + 1 \right) - \sqrt{\left(\frac{u^2 + v^2}{4} + 1 \right) - u^2} \right\} \right]^{\frac{1}{2}} \qquad (52)$$

$$\cos k = \frac{-u}{\sqrt{u^2}} \left[\frac{1}{2} \left\{ \left(\frac{\gamma^2}{4} + 1 \right) - \sqrt{\left(\frac{\gamma^2}{4} + 1 \right)^2 - \gamma^2 \cos^2 \phi} \right\} \right]^{\frac{1}{2}} \qquad (53)$$

Equations (52) and (53) do not determine the sign (or the quadrant) of k. If the sign of k is determined so as to satisfy the relation:

$$\sin k = \frac{-v}{2 \sinh (h)} \qquad (53a)$$

then equations (50), (51), (52) and (53) always give real values to h and k which satisfy equations (41). The term $-u/\sqrt{u^2}$ is introduced into equations (52) and (53) so as to make $\cos k$ change sign with u as it must in order that equations (41) may be satisfied.

III. SELECTIVE PROPERTIES OF THE COUPLED CIRCUIT CHAIN

(9) *General Attenuation Characteristics.* Fig. 2 represents a chain of coupled circuits terminated by an impedance equal to Z_s so as to eliminate the reflected wave. The current in the qth circuit then is given by equation,

$$I_q = I_0\, \epsilon^{-q\tau} \qquad (37)$$

or

$$I_q = I_0\, \epsilon^{-qh} (\cos k\, q - j \sin k\, q) \qquad (38)$$

Where

$$\cosh \tau = -\frac{Z_c}{2\,Z_m} = -1/2\,(u + j\,v) \qquad (4)$$

Let us now define an ideal filter as one built up of coils and condensers in which no energy dissipation takes place. Let the circuit under consideration be an ideal filter. Then Z_c and Z_m are both pure imaginary quantities and Z_c/Z_m is a real number, that is, v is equal to zero. For the ideal case then:

$$\cosh \tau = -\frac{Z_c}{2\,Z_m} = -u/2$$

where u is a real number. For any particular structure Z_c/Z_m is a function of the frequency of the applied voltage and therefore also is a function of the frequency of the current being transmitted.

If $Z_c/Z_m < -2$ then $\cosh \tau > 1$ and τ is a positive real number; that is h has some positive value and $k = o$. Thus currents having frequencies that make $Z_c/Z_m < -2$ decrease in magnitude but retain the same phase position as the circuit under consideration recedes from the generator end of the chain.

If $Z_c/Z_m > 2$ then $\cosh \tau$ is a negative number less than -1. Under these conditions $h \neq o\ K = \pi$. Thus currents having frequencies that make $Z_c/Z_m > 2$ decrease in magnitude as the circuit under consideration recedes from the generator end of the string. There is also a phase shift of 180 deg. in passing from one circuit to the next adjacent circuit. If $-2 < Z_c/Z_m < 2$ then $\cosh \tau$ is a real number which in absolute value is less than unity. Under these conditions h is zero and k is, in general, different from zero. Thus currents having frequencies that give values of Z_c/Z_m that lie between -2 and $+2$ are transmitted without diminution through the chain. The frequencies that give to Z_c/Z_m the value plus or minus 2 for the idealized filter will be called the cut-off frequencies. The frequencies that give to Z_c/Z_m values between -2 and $+2$ will be called the transmitted band of frequencies. Frequencies that cause Z_c/Z_m to take the values outside the range -2 to $+2$, are said to be in region of attenuation or in the attenuated region.

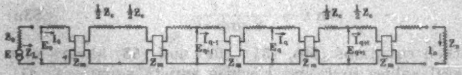


Fig. 2

It is to be noted that the attenuation characteristics described above are independent of any power loss in the filter network as the assumption was made that the coils and condensers used in building up the network were free from power losses. The characteristics come about solely through complex interaction of one circuit on the other. When particular types of structure are considered it will be shown that resistance causes attenuation in the transmitted region and lowers the attenuation in the attenuated region. We can, however, approach very near to the characteristic of the ideal filter by making the losses in coils and condensers as low as feasible.

(10) *General Wave Length Constant or Phase Shift Characteristics.* The general phase shift characteristics are most easily obtained from the general relation of equation (**8**).

$$\cosh \tau = \cosh (h + j\,k) = \frac{Z_c}{-2\,Z_m} = -\frac{u + j\,v}{2}$$

In the ideal filter we have shown that v is always zero, so for the ideal structure we may write $\cosh (h + j\,k) = -u/2$. When $u < -2$, τ is real and $k = o$. When $u > -2$, but less than $+2$, τ is a pure imaginary and as u varies from -2 to $+2$, k varies from o to π. That is within the transmitted band k varies from o at one cut-off frequency to π at the other cut-off frequency. As the frequency approaches one of the cut-off points from the attenuated region u is continually less than -2 and k is continually zero, as the other cut-off frequency is approached from the attenuated region u is continually greater than $+2$ and k continually equals π.

Sections (**9**) and (**10**) taken together give some very interesting properties of the ideal coupled circuit structures. In the attenuated regions, the phase shift or wave length constant remains at a constant value o or π, while the attenuation constant continually, takes on different values. For all frequencies within the transmitted band, the attenuation constant remains constant while the phase shift constant varies from zero to π.

(11) *General Impedance Characteristics.* Consider again the circuit of Fig. 2. Let the problem be that of transferring maximum power at frequencies within the transmitted band or within a portion of the transmitted band from the generator to a utilization device terminating the filter. Let the generator impedance be represented by Z_0. For the theory of section (**9**) to apply the utilization device must have the impedance Z_s or must be corrected to Z_s by means of a suitable transformer. It will be shown later that Z_s may be a pure resistance.

Let the terminating impedance Z_n be a pure resistance R_n. If R_n were directly across the generator terminals it would receive maximum power when $Z_0 = R_n$, or was corrected to R_n by means of a transformer. If the termination is such that $R_n = Z_s$ then the theory of section (**9**) applies and the resistance R_n receives the same power as though it were across the generator terminals. Therefore, for maximum power delivery from the generator to the terminating device, the system should be so designed that $Z_0 = R_n = Z_s$.

If Z_n and Z_0 are to be fixed resistances then in general we can satisfy the above relation for only one particular frequency within the transmitted band as Z_s, although it may be made a pure resistance within the transmitted band, varies with the frequency. The detailed manner in which it varies with the frequency will have to be taken up for each type of structure considered. In general, however, it may be stated that the impedance curve is fairly flat for quite a range of frequencies within the transmitted band, but falls to zero at the cut-off points. In the consideration of special types of structure it will be shown that this change in characteristic impedance has the effect of introducing a loss in the transmitted band. Over the greater share of the transmitted band this loss may be made small but at the cut-off points it may have an appreciable value.

(12) *Filters with Dissipation in Coils and Condensers.* In the discussion thus far contained in Part III it has been assumed that the coils and condensers used

in building the filter did not dissipate energy. This assumption insured that the ratio of the circuital to the transfer impedance should be a real quantity. This in turn leads to the sharp breaks in the characteristics of the attenuation and phase shift constants described in sections (**9**) and (**10**). In any real filter both coils and condensers dissipate energy. For any real filter then the impedance ratio Z_c/Z_m is in general a complex number and the ideal filter theory does not rigorously apply. We can, however, approximate the ideal filter by using coils and condensers in which the ratio of resistance to reactance is small. We would expect the widest divergence between the characteristic of the ideal filter and the real filter at those points where the ideal filter characteristic takes a sudden break, namely at the cut-off points. This expectation will be verified when the actual characteristics of some particular types of structures are given.

IV. APPLICATION OF THE GENERAL THEORY TO A FILTER BUILT UP OF SIMPLE SERIES CIRCUITS MAGNETICALLY COUPLED TOGETHER

(13) *Conventions and General Method of Handling the Problem.* Before starting the discussion of particular types of filters, it will be of advantage to give a general survey of the method to be employed in treating the selective properties of these systems. The method has been employed by telephone engineers for some time in the discussion of transmission through all types of networks.

In section (**11**) it was shown that maximum power was delivered to the filter when the filter was so designed that its characteristic impedance was equal to the generator impedance which was taken to be a pure resistance, or was connected to the generator impedance by means of a suitable transformer. In the ideal filter within the transmitted band this power is transmitted undiminished to a utilization resistance if this resistance has a value equal to the characteristic impedance, Z_0, of the filter. Under these conditions we would say that the loss through the filter, or attenuation of the filter is zero because the power delivered to the utilization device is the same as though this device were connected directly across the generator terminals.

Now since our utilization devices can always be reduced to equivalent resistance it is obvious that instead of discussing power ratios, we may discuss current ratios. That is in order to obtain the loss occasioned by the filter it is sufficient to obtain the ratio of the current delivered to the utilization device when placed across the generator terminals to the current delivered to the device with the filter interposed between the generator and the utilization device. Let the current with the filter interposed be I_0 and let the current delivered with the utilization resistance across the generator be I_0^1. Then the ratio I_0''/I_0 gives the effect of interposing the filter. Instead of using this ratio it is more convenient to use the natural logarithm of the ratio. Then in the system to be used the loss will be one attenuation unit when the natural logarithm of the ratio of the current with the utilization device across the generator terminals to the current in the utilization device with the filter interposed between it and the generator is unity. The convenience and application of this unit will become apparent as the discussion proceeds. As an example of its usefulness however, consider the following.

A filter chain is terminated so that the reflected wave is absent. The impedances are such that the generator delivers maximum power to the filter. The current in any section of the filter then is:

$$I_q = I_0\,\epsilon^{-q\gamma}$$

$$|I_q| = |I_0|\,\epsilon^{-hq}$$

But I_0 is also the current which would flow if the terminating impedance were across the generator terminals, and we have,

$$|I_q'/I_q| = \epsilon^{qh}$$

$$\log_e |I_q'/I_q| = q\,h$$

Or the attenuation per section is h attenuation units. For q sections it is $h\,q$ attenuation units.

(14) *Chain of Simple Series Circuits Magnetically Coupled. Ideal Case.* (*a*) *Cut-off Points.* The general theory thus far presented is very general and applies to all chain systems built up of identical circuits associated in such a way that the impedances of the system can be broken up into circuital impedances and transfer impedances. The illustrations which follow, however, give rise to filter systems which in general cannot be broken up into series and shunt elements.

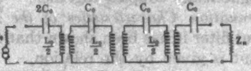


FIG. 3

As a first application of the theory consider the system shown by Fig. 3. This system obviously fulfills the conditions of the general theory. For the purpose of synthetically building up filter systems it is convenient to think of Fig. 3 as made of elements, as shown in Fig. 4. Each of these elements will be called a filter section. Fig. 4 represents a two section filter.

We will first assume that Fig. 4 represents an ideal filter; that is, a filter in which the coils and condensers do not dissipate energy. The circuital impedance is,

$$Z_c = j\left(\omega L_0 - \frac{1}{\omega C_0}\right) \qquad (54)$$

The mutual or transfer impedance is:

$$Z_m = j\,\omega M$$

$$\text{Let} \quad M = \frac{K_0 L_0}{2}; \quad |K_0| < 1$$

K_0 may be positive or negative.
Then,

$$Z_m = \frac{j\,\omega\,L_0\,K_0}{2} \tag{55}$$

$$Z_c/Z_m = \frac{2\left(\omega\,L_0 - \dfrac{1}{\omega\,C_0}\right)}{\omega\,L_0\,K_0} \tag{56}$$

The angular velocities corresponding to the cut-off frequencies are such as to make,

$$Z_c/Z_m = \frac{2\left(\omega\,L_0 - \dfrac{1}{\omega\,C_0}\right)}{\omega\,L_0\,K_0} = \pm 2 \tag{31}$$

$$\omega\,L_0 - \frac{1}{\omega\,C_0} = \pm K_0\,\omega\,L_0$$

The cut-off angular velocities are,

$$\omega_A = \sqrt{\frac{1}{L_0\,C_0\,(1-K_0)}} = \omega_{r0}\sqrt{\frac{1}{1-K_0}} \tag{57}$$

$$\omega_B = \left(\sqrt{\frac{1}{1+K_0}}\right)\omega_{r0} \tag{58}$$

Where ω_{r0} is defined by the equation

$$\omega_{r0} = \frac{1}{\sqrt{L_0\,C_0}} \tag{59}$$

It is interesting to note that the cut-off angular velocities are given by the same equation that give the natural angular velocities of two simple series circuits magnetically coupled together.

(*b*) *Attenuation Characteristics of the Ideal Filter.* For the ideal filter it has been shown that the propagation constant is given by the relation,

$$\cosh \tau = -\frac{Z_c}{2\,Z_m} = -u/2 - j\,o \tag{8}$$

For the structure under consideration this gives

$$\cosh\,(\tau = h + j\,k) = (1/K_0)\,[\,(\omega_{r0}/\omega)^2 - 1] \tag{60}$$

At this point we are chiefly interested in the general shape of the attenuation curve; that is, in the general shape of the curve which has values of h for ordinates and angular velocity or frequency for abscissa. Within the transmitted band, that is for values of ω lying between ω_B and ω_A, the cut-off angular velocities, h is zero.

When $\omega = o_1$ $\cosh\,(h + j\,k) = \infty$ and $h = \infty$. When $\omega = \infty$, $\cosh\,(h + j\,k) = -1/K_0$. For the ideal filter equation (50) gives $\cosh h = \pm \dfrac{u}{2}$; $h = \cosh^{-1}(|u/2|)$

$$\frac{d\,h}{d\,\omega} = \frac{d\,h}{d\,u}\,\frac{d\,u}{d\,\omega}$$

$$= \frac{\pm 2\,\omega_{r0}^2}{\omega\,K_0\,\sqrt{(1/K_0)^2\,(\omega_{r0}^2 - \omega^2)^2 - \omega^4}}$$

When $\omega = o$, $\left|\dfrac{d\,h}{d\,\omega}\right| = \infty$

When $\omega = \infty$ then $\dfrac{d\,h}{d\,\omega} = o$

When $\omega = \omega_A = \omega_{r0}\sqrt{\dfrac{1}{1-K_0}}$; $\dfrac{d\,h}{d\,\omega} = \infty$

When $\omega = \omega_B = \omega_{r0}\sqrt{\dfrac{1}{1+K_0}}$; $\left|\dfrac{d\,h}{d\,\omega}\right| = \infty$

The general attenuation characteristic then has a verticle tangent at $\omega = o$ and becomes great without limit as $\omega \doteq o$. As ω is increased from zero the slope remains finite and different from zero until $\omega = \omega_B$; here the curve has another verticle tangent and h takes the value zero and remains zero until $\omega = \omega_A$.

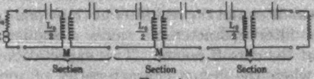


FIG. 4

At $\omega = \omega_A$ the curve has another verticle tangent and as the frequency becomes great without limit the value of h approaches the value $\cosh^{-1}(1/K_0)$. The general attenuation characteristic is, therefore, as shown by the sketch of Fig. 5.

When ω_A and ω_B are both finite the attenuation constant of the ideal filter for angular velocities lying between ω_A and ω_B is zero and the structure is called a band pass filter.

If $K_0 = 1$, $\omega_A = \infty$ and the attenuation constant is zero for all angular velocities above ω_B and the filter is called a high pass filter.

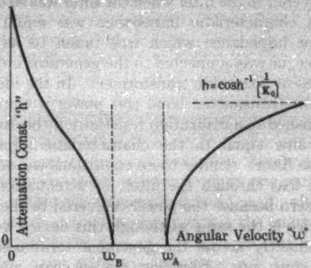


FIG. 5

(*c*) *Phase Shift Characteristics of the Ideal Filter.* The manner in which the phase shift or wave length constant varies with the frequency can most readily be obtained by using the equation:

$$\cosh h \cos k = -u/2 \tag{45a}$$

$$\sinh h \sin k = -v/2 = o \tag{45b}$$

Case I K_0 positive

From equation (56) we obtain:

$$Z_c/Z_m = u = 2/K_0\,[1 - (\omega_{r0}/\omega)^2\,]$$

For values of ω less than the lower cut-off angular velocity u is negative and it has been shown that h is different from zero. Sinh h is thus different from zero and to satisfy (**45b**), sin k must equal zero. The value of k must therefore be o or π. In equation (**45a**) cosh h is positive, u is negative, and therefore cos k must be positive. Therefore, for frequencies below the lower cut-off frequency, k must equal zero.

For values of ω greater than the upper cut-off angular velocity, sinh h is different from zero so that sin k must equal zero and k must equal o or $\pm \pi$. Now u is positive so also is cosh h so that cos k must be negative. $k = \pm \pi$.

When $\omega = \omega_{r0}$, $u = o$ so that $\cos k = o$, $k = \pm \pi/2$. Sinh h is also zero so equation (**45b**) becomes:

$$\sin k \frac{-v = o}{2 \sinh h = o} \tag{61}$$

If resistance were allowed in the circuits:

$$Z_c/Z_m = u + j\,v = 1/K_0 \left[\frac{a \pm j\,b}{j\,s} \right]$$
$$= 1/K_0 \left[\pm b/s - j\,a/s \right] \tag{61a}$$

(**61a**) shows that when K_0 is positive v is negative and that k must lie in the first or second quadrant. We then may assign values to k as follows. Below and including ω_B, $k = o$. At ω_{r0} $k = \pi/2$. Including and above ω_A, $k = \pi$.

Inside the transmitted band of an ideal filter k is given by

$$k = \cos^{-1} 1/K_0 \left[(\omega_{r0}/\omega)^2 - 1 \right] \tag{61b}$$

Taking the derivative with respect to ω, we obtain:

$$\left| \frac{d\,k}{d\,\omega} \right| = \frac{2\,\omega_{r0}^2}{\omega \sqrt{K_0^2\,\omega^4 - [\omega_{r0}^2 - \omega^2]^2}} \tag{61d}$$

When $\omega = \omega_B$ then $\left| \frac{d\,k}{d\,\omega} \right| = \infty$

When $\omega = \omega_A$ then $\left| \frac{d\,k}{d\,\omega} \right| = \infty$

The wave length constant-angular velocity curve thus has a verticle tangent at each cut-off point. The general shape of the curve for K_0 positive is given by the full line curve of Fig. 6.

Case II. K_0 negative.

The derivatives for this case are as given above for case I. For angular velocities below ω_B, sinh h is different from zero so $\sin k = o$, $k = o$ or $\pm \pi$. u is positive. So also is cosh h and therefore cos k must be negative. $k = \pm \pi$.

For values of ω greater than ω_A sinh h is different from zero so sin k must equal o. $k = o, \pm \pi$ or $\pm 2\,\pi$. u is negative, so cos k must be positive. $k = o$ or $\pm 2\,\pi$.

When $\omega = \omega_{r0}$, u is zero and cos k is zero. $k = \pm \pi/2$. Sin k is again indeterminate. However from (**61a**) when K_0 is negative, v is positive and we see that k must lie in the second or third quadrant. Then k must equal $- \pi/2$. We then may assign values to k as follows: For ω equal to and less than ω_B, $k = -\pi$. For $\omega = \omega_{r0}$, $k = -\pi/2$. For ω equal to or greater than ω_A, $k = o$.

The general shape of the wave length constant curve for K_0 negative is as given by the dotted curve of Fig. 6.

(*d*) *Characteristic Impedance of the Ideal Filter and Design Formula.* The characteristic impedance is given by the relation:

$$Z_s = \sqrt{Z_c^2/4 - Z_m^2} = \sqrt{(Z_c/2 - Z_m)(Z_c/2 + Z_m)} \tag{13}$$

For the idealized structure under consideration:

$$Z_c/2 - Z_m = 1/2\,j \left[\omega\,L_0\,(1 - K_0) - \frac{1}{\omega\,C_0} \right]$$

$$Z_c/2 + Z_m = 1/2\,j \left[\omega\,L_0\,(1 + K_0) - \frac{1}{\omega\,C_0} \right]$$

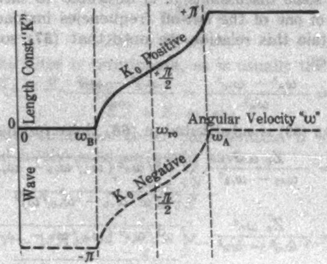


Fig. 6

$$Z_s^2 = -1/4 \frac{1}{\omega^2\,C_0^2} [\omega^2\,L_0\,C_0\,(1 - K_0)$$
$$- 1]\,[\omega^2\,L_0\,C_0\,(1 + K_0) - 1] \tag{62}$$

$$Z_s^2 = -1/4 \frac{1}{\omega^2\,C_0^2} [\,(\omega/\omega_{r0})^2\,\{\,(\omega/\omega_{r0})^2\,(1$$
$$- K_0)^2 - 2\,\} + 1] \tag{63}$$

At this point it is well to pause and consider the manner in which the constants of the filter are to be fixed. ω_{r0} is fixed by the location of the band to be transmitted. K_0 is determined by the width of the band of frequencies to be transmitted. Within the transmitted band the filter should have an impedance which matches the generator and power receiving device. That is Z_s at some point within the transmitted band is fixed by the impedance of the devices between which the filter is to be placed. We choose to fix the impedance for the angular velocity ω_{r0} and we wish to determine L_0 and C_0 in such a way that at ω_{r0}, Z_s shall equal Z_0. Upon applying this condition to equation (**63**), there results

$$Z_0^2 = -1/4 \frac{1}{\omega_{r0}^2\,C_0^2} [-K_0^2] = 1/4 \frac{K_0^2}{\omega_{r0}^2\,C_0^2} \tag{64}$$

$$Z_0 = \frac{K_0}{2\,\omega\,C_0} \tag{65}$$

It is to be noted that Z_0 is a pure resistance. Since in general, as stated before, Z_0 is fixed by the conditions of the problem so (**65**) really determines C_0 in such a way that the filter shall have the impedance Z_0. That is, C_0 is given by the design formula:

$$C_0 = \frac{|K_0|}{2\,\omega_{r0}\,Z_0} \tag{66}$$

The inductance is determined by the relation:

$$\omega_{r0}^2 = \frac{1}{L_0\,C_0}$$

$$L_0 = \frac{2\,Z_0}{|K_0|\,\omega_{r0}} \tag{67}$$

Upon substituting (**66**) in (**63**) there results

$$Z_s = Z_0 \sqrt{(\omega/\omega_{r0})^2 - 1/K_0^2\,(\omega/\omega_{r0} - \omega_{r0}/\omega)^2} \tag{68}$$

For some discussions it is desirable to have Z_s in terms of one of the cut-off frequencies instead of K_0. To obtain this relation we note that (**57**) solved for K_0 gives,

$$K_0 = \frac{\omega_A^2 - \omega_{r0}^2}{\omega_A^2} = \frac{\omega_{r0}^2 - \omega_B^2}{\omega_B^2} \tag{69}$$

When (**69**) is substituted in (**68**) there results:

$$Z_s = \frac{Z_0\,\omega_A^2}{\omega_{A2} - \omega_{r0}^2}\sqrt{\omega^2/\omega_A^2\,(\omega_A/\omega_{r0} - \omega_{r0}/\omega_A)^2 - (\omega/\omega_{r0} - \omega_{r0}/\omega)^2} \tag{70}$$

$$= \frac{Z_0\,\omega_B^2}{\omega_{r0}^2 - \omega_B^2}\sqrt{\omega^2/\omega_B^2\,(\omega_{r0}/\omega_B - \omega_B\,\omega_{r0})^2 - (\omega/\omega_{r0} - \omega_{r0}/\omega)^2} \tag{71}$$

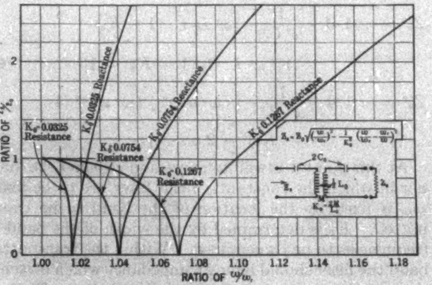
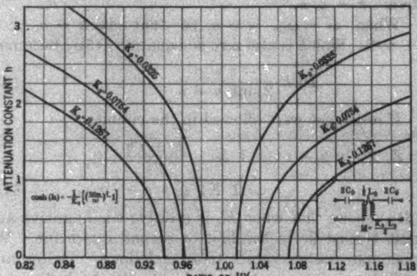


FIG. 7—CHARACTERISTIC IMPEDANCE FOR COUPLED CIRCUIT WAVE FILTER

Equation (**70**) and (**71**) show that in the ideal filter Z_s is real or is a pure resistance through the transmitted band. At the cut-off points Z_s falls to zero and outside the transmitted band it is a pure reactance. The curves of Fig. 7 give a more accurate picture of the manner in which the characteristic impedance varies with the frequency in the ideal filter. Only the part of the curve for frequencies above the resonant frequency is plotted since in each case the other half is nearly a reflection in a line through the resonant frequency of the part plotted.

(*e*) *Actual Curves for the Attenuation and Phase Shift Constants.* We are now in a position to actually plot up some curves showing the manner in which the attenuation and phase shift constants of the simple magnetically coupled filter vary with the frequency of the current being transmitted. In order to make these curves more general, for abscissa we will use the ratio ω/ω_{r0} and the coupling coefficient or band width constant will be used as a parameter.



$$\cosh(h) = -\frac{1}{K_0}\left[\left(\frac{\omega_{r0}}{\omega}\right)^2 - 1\right]$$

FIG. 8—ATTENUATION CONSTANT CURVES FOR IDEAL FILTER

We have the relation:

$$Z_c/Z_m = \frac{2\left(\omega\,L_0 - \dfrac{1}{\omega\,C_0}\right)}{\omega\,L_0\,K_0} \tag{56}$$

This equation can be put in the form:

$$Z_c/Z_m = 2/K_0\,[1 - (\omega_{r0}/\omega)^2] = u + j\,v \tag{72}$$

$$\cosh\,[\tau = h + j\,k] = 1/K_0\,[\,(\omega_{r0}/\omega)^2 - 1] \tag{73}$$

Both h and k are readily obtained from (**73**) for the following reasons. For angular velocities below ω_B, k is zero so (**73**) gives h directly. Within the transmitted band h is zero so (**73**) gives k directly. For angular velocities above ω_A, $k = \pi$ so h is again readily obtained. Within the transmitted band it is more convenient to use the relation:

$$\cos k = 1/K_0\,[\,(\omega_{r0}/\omega)^2 - 1] \tag{74}$$

for obtaining values of k.

Actual curves for h and k are given by Figs. (**8**) and (**9**).

(*f*) *Line Impedance Characteristics.* The line impedance is given by the equation:

$$Z_L = \frac{Z_n \cosh\tau\,n + Z_s \sinh\tau\,n}{\cosh n\,\tau + Z_n/Z_s \sinh n\,\tau} \tag{30}$$

In order to make the curves independent of the terminating impedance Z_n, the ratio of Z_L/Z_n will be used as ordinates. Dividing both sides of (30) by Z_n we obtain:

$$Z_L/Z_n = \frac{\cosh n\,\tau + Z_s/Z_n \sinh n\,\tau}{\cosh n\,\tau + Z_n/Z_s \sinh n\,\tau} \tag{75}$$

The computation for Z_L/Z_n is straight forward until it is attempted to find the value of this ratio at a cut-off point. At the cut-off points $\sinh n\,\tau$ is zero, so also is Z_s. We have therefore in the denominator of (75) an indeterminate form ϕ/o to evaluate. We proceed to obtain the equation which gives the value of Z_L/Z_n at the cut-off points.

At a cut-off point $\tau = o$ or $j\,\pi$. For either value of τ $\sinh n\,\tau = o$. Now within the transmitted band of an ideal filter $\tau = j\,k$, and we have the relations:

$$\cosh n\,\tau = \cosh j\,n\,k = \cos n\,k$$

So that at a cut-off point (75) becomes:

$$(Z_L/Z_n)_{\omega_A \text{ or } \omega_B} = \frac{\cos n\,k}{\cos n\,k + Z_n \dfrac{\sinh n\,\tau}{Z_s}} \tag{76}$$

Now we have the relation:

$$\cosh \tau = \frac{-Z_c}{2\,Z_m}$$

So

$$\sinh \tau = \sqrt{\frac{Z_c^2}{4\,Z_m^2} - 1}$$

also

$$Z_s = \pm Z_m \sqrt{\frac{Z_c^2}{4\,Z_m^2} - 1} \tag{77}$$

$$Z_s = \pm Z_m \sinh \tau \tag{78}$$

The sign to be used with (78) must be such as to cause the real part of Z_s to be a real positive number. Now inside the transmitted band of an ideal filter $\tau = j\,k$, so we have:

$$\sinh \tau = \sinh j\,k = j \sin k$$

Also inside the transmitted band of an ideal filter Z_m is a pure imaginary or:

$$Z_m = \pm j|Z_m|$$

$$Z_s = \pm|Z_m|\sin k$$

Now from (53a) the sign of k is opposite to the sign of v so within the transmitted band of an ideal filter we have:

$$Z_s = \frac{-v}{\sqrt{v^2}}\,|Z_m|\sin k \tag{79}$$

Also $\sinh n\,\tau = \sinh j\,n\,k = j \sin n\,k$
So that

$$\frac{\sinh n\,\tau}{Z_s} = \frac{j \sin n\,k}{\dfrac{-v}{\sqrt{v^2}}\,|Z_m|\sin k}$$

$$\lim_{\kappa = o \text{ or } \pi} \frac{\sinh n\,\tau}{Z_s} = \frac{-v}{\sqrt{v^2}}\,\frac{j}{|Z_m|}\,\frac{n \cos n\,k}{\cos k} \tag{80}$$

$$k = o \quad \text{or } \pi$$

And equation (76) becomes

$$(Z_L/Z_n)_{\omega_A \text{ or } \omega_B} = \frac{1}{1 - j\,n\,(Z_n/|Z_m|)\,(v/\sqrt{v^2}) \cos k}\,; \qquad (k = o \quad \text{or } \pi) \tag{81}$$

Now equation (81) contains the term $v/\sqrt{v^2}$. Now this term was introduced solely to fix a sign and thus avoid the necessity of carrying a double sign. However in an ideal filter v is zero. We must therefore use the sign which v would give if some small resistance were in the filter coils. This sign is readily determined from physical considerations or from equations like (99) which are derived hereafter.

Equation (81) is general and applies to all types of coupled circuit wave filters. For the type of structure now under consideration it may be put in a form more convenient for use as follows:

$$|Z_m| = 1/2\,\omega\,L_0|K_0|$$

$$L_0 = \frac{2\,Z_0}{|K_0|\,\omega_{r0}}$$

$$|Z_m| = Z_0\,\omega/\omega_{r0}$$

If the filter is terminated, as is usually the case, so that $Z_0 = Z_n$ then:

$$|Z_m| = Z_n\,\omega/\omega_{r0} \tag{82}$$

In the structure under consideration if loss were allowed in the coils we would have:

$$Z_c/Z_m = u + j\,v = \frac{a \pm j\,b}{j\,K_0\,S}$$

$$u + j\,v = 1/K_0\left[\frac{j\,a \pm b}{-s}\right] = 1/K_0\,[\pm b/s - j\,a/s]$$

Or v has a sign opposite to K_0. This result could also be obtained from equation (99). We then may write:

$$v/\sqrt{v^2} = \frac{-K_0}{\sqrt{K_0^2}} \tag{83}$$

Upon substituting (82) and (83) in (81) there results:

$$(Z_L/Z_n)_{\omega_A \omega_B} = \frac{1}{1 + j\,n\,(\omega/\omega_{r0})\,(K_0/\sqrt{K_0^2}) \cos k} \tag{84}$$

This is the equation giving the values of Z_L/Z_n at the cut-off points of the simple filter under discussion.

In applying equation (75) to obtain values of Z_L/Z_n within the transmitted band it is well to note that for certain frequencies the value of this ratio can be obtained with very little calculation if curves for the ratio Z_s/Z_n and of k have already been plotted. These frequencies are those which make $\cosh n\,\tau$ equal to zero and those which make $\sinh n\,\tau$ equal to zero. When $\cosh n\,\tau = 0$ then:

$$Z_L/Z_n = (Z_s/Z_n)^2 \tag{85}$$

When $\sinh n\,\tau = 0$ then:

$$Z_L/Z_n = 1 \tag{86}$$

Cosh $n\,\tau$ is zero for those frequencies which cause $n\,\tau$ to equal $j\,p\,\pi/2$ where p is an odd integer. Sinh

$n\,\tau$ is zero for those frequencies which cause $n\,\tau$ to equal $j\,q\,\pi$ where q is a positive integer. These frequencies are readily found from curves like those of Fig. 9.

A line impedance curve for $K_0 = 0.0754$ and $n = 3$ is shown by Fig. 10, only the part of the curve for frequencies above the resonant frequency is shown as the other part has the same general shape. This curve is flatter for a greater portion of the transmitted band than the curve giving the characteristic impedance for $K_0 = 0.0754$. A flat impedance curve is desirable in those cases where the filter is used in circuits in which impedance irregularities are undesirable.

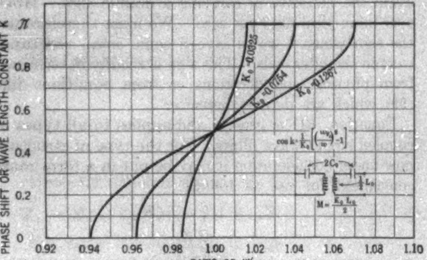


Fig. 9—Phase Shift or Wave Length Constant for Ideal Filter

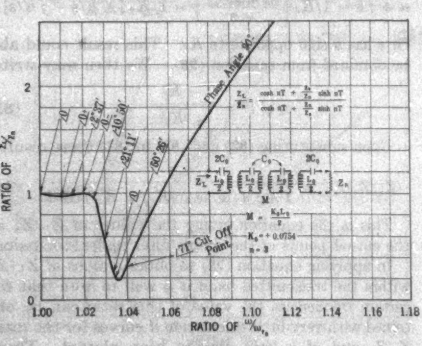


Fig. 10—Ideal Filter Line Impedance

(*g*) *Total Loss Characteristics of the Ideal Filter.* From equation (**34**) we have for the current in the qth section of a coupled circuit wave filter the relation:

$$I_q = E_0\,[1/Z_L \cosh q\,\tau - 1/Z_s \sinh q\,\tau] \qquad (\mathbf{34})$$

This equation gives the current in terms of the voltage of the first section. We wish $I\,q$ in terms of the impressed voltage. The relation between E_0 and the impressed voltage is

$$E_0 = \frac{E\,Z_L}{Z_0 + Z_L} \qquad (\mathbf{87})$$

$$I_q = \frac{E}{Z_0 + Z_L}\,[\cosh q\,\tau - Z_L/Z_s \sinh q\,\tau] \qquad (\mathbf{88})$$

If the terminating impedance were across the generator terminals, the current delivered to it would be:

$$I_n' = \frac{E}{Z_0 + Z_n} \qquad (\mathbf{89})$$

The ratio of the current delivered to the terminating impedance to the current which would be delivered to it if it were across the generator terminals is given by the equation:

$$I_n/I_n' = \frac{1 + Z_n/Z_0}{1 + Z_L/Z_0}\,[\cosh n\,\tau - Z_L/Z_s \sinh n\,\tau] \qquad (\mathbf{90})$$

If, as is generally the case, $Z_n = Z_0$ then (**90**) becomes:

$$I_n/I_n = \frac{2}{1 + Z_L/Z_0}\,[\cosh n\,\tau - Z_L/Z_s \sinh n\,\tau] \qquad (\mathbf{91})$$

In accordance with the plan announced at the beginning of this section the effect of interposing the filter between the generator and terminating impedance will be specified by giving the loss in attenuation units which results from the introduction of the filter. This loss has been defined by the relation:

$$A = \log_e I_n'/I_n \qquad (\mathbf{92})$$

The calculation of the current ratio I_n/I_n' is straight forward until a cut-off point is reached. At a cut-off point $\sinh n\,\tau$ is zero, so also is Z_s. This gives rise to the indeterminate form:

$$\frac{\sinh n\,\tau}{Z_s} = -\,o/o$$

This indeterminate form was encountered before and its value is given by equation (**80**). Upon substituting (**80**) in (**91**) and letting $\omega = \omega_A$ or ω_B we have:

$$(I_n/I_n')_{\omega = \omega_A \text{ or } \omega_B} = \frac{2 \cos n\,k}{1 + Z_L/Z_n}\,[1 + (v/\sqrt{v^2})\,j\,n\,(Z_L/|Z_m|) \cos k] \qquad (\mathbf{93})$$

$$k = o \quad \text{or } \pi$$

Equation (**93**) is a general equation applying to all types of ideal coupled circuit wave filters. For the simple filter under consideration (**93**) can be simplified by the use of equations (**82**) and (**83**). Then for the filter under consideration we may write:

$$(I_n/I_n')_{\omega = \omega_A \text{ or } \omega_B} = \frac{2 \cos n\,k}{1 + Z_L/Z_n}\,[1 - j\,n\,(Z_L/Z_n)\,(\omega_{r0}/\omega)\,(K_0/\sqrt{K_0^2}) \cos k\,] \qquad (\mathbf{94})$$

$$k = o \quad \text{or } \pi$$

A total loss characteristic for frequencies lying above the resonant frequency is given by the curve of Fig. 11. This curve is for a three section filter for which $K_0 = +0.0754$. The circuit constants are such that at the resonant frequency $Z_0 = Z_s = Z_R$. This is of course the same filter for which curves of h, k, Z_s and Z_L have already been given.

A comparison of the curve of Fig. 11 with three times the ordinates of the attenuation constant curve for the same filter as given by Fig. 8 shows that, with the exception of frequencies in the vicinity of the cut-off frequency and for very large frequencies, the loss figured from the attenuation constant alone gives a very close approximation to the total loss occasioned by the introduction of the filter. This is a very important fact as it allows preliminary design of ideal filters to be based upon the attenuation constant alone.

The variation of the total loss curve from the total attenuation curve for frequencies much larger than the resonant frequency is evident when we remember that the attenuation constant curve approaches $\cosh^{-1}|1/K_0|$ as $f \doteq \infty$ whereas the total loss curve approaches ∞ as $f \doteq \infty$ due to the fact that the line impedance becomes infinite when the frequency becomes infinite.

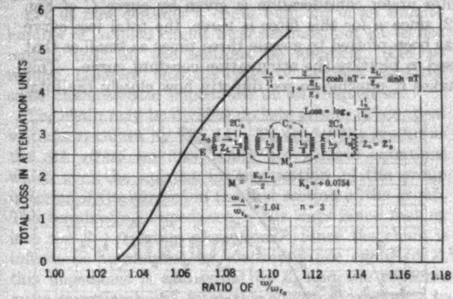


FIG. 11—TOTAL LOSS CURVE FOR IDEAL FILTER

The marked deviation of the total loss curve from the total attenuation curve for frequencies in the vicinity of the cut-off frequency is due to the fact that in this vicinity the characteristic impedance differs widely from the terminating impedance. This gives rise to a large reflected wave term in the current equation. The total loss through the filter is equal to the number of sections times the attenuation constant only when the reflected wave term is zero or very small compared to the main wave term.

(15) *Chain of Simple Series Circuits Magnetically Coupled. Dissipative Case.* (*a*) *General Statement and Scope of this Section.* Real filters differ from ideal filters in that the former always have some power losses in coils and condensers whereas the latter have not. The characteristics of the real filter can be made to approach those of the ideal filter by making these losses as low as possible. The losses in condensers can be made negligible but it is more difficult to construct coils with negligible power losses. It is the purpose of this section to give briefly the characteristics of the filter built up of simple series circuits magnetically coupled together, in which the coils are assumed to have energy losses.

All of the general equations of part II apply to the dissipative filter and the greater share of the special forms of these equations developed in section (**10**) also apply as well to the dissipative filter as to the filter without dissipation.

(*b*) *Cut-off Points, Attenuation and Phase Shift Characteristics.* As was pointed out in section (**12**) a filter which has losses in its coils has no sharp cut-off points, but currents of some frequency are very freely transmitted while others are transmitted very poorly. Since the dissipative filter has no definite cut-off points. the cut-off points of the idealized filter will be taken as the cut-off points. It is well to note that in this case the loss may be considerable at the cut-off frequencies.

The total propagation constant is given by equation (**8**). This equation may be used for finding both the phase shift constant and the attenuation constant by making use of a table of complex hyperbolic functions. In most cases it is more convenient to use equations (**50**) and (**52**) for determining these constants. The first step is to obtain an expression for the impedance ratio Z_c/Z_m.

For most coils used in high-frequency work the effective resistance varies with the frequency. For small frequency ranges this variation may be assumed to be a straight line function and we may

Let

$$\frac{R}{\omega L} = d \qquad \textbf{(96)}$$

Then for the structure under consideration,

$$Z_c = (d + j)\,\omega L_0 - \frac{j}{\omega C_0} \qquad \textbf{(97)}$$

$$Z_m = j\,\omega L_0 K_0/2 \qquad \textbf{(98)}$$

$$Z_c/Z_m = u + j\,v = 2/K_0\,[1 - (\omega_{r0}/\omega)^2 - j\,d] \qquad \textbf{(99)}$$

In order to obtain a picture of the characteristics of the dissipative filter and to be able to compare its characteristics with those of the ideal structure curves will be given for the following conditions:

$$K_0 = +0.0754 \qquad d = 0.002,$$

Curve A of Fig. 12 gives the attenuation constant characteristic for frequencies above the resonant frequencies. A comparison of this curve with the corresponding curve of Fig. 8 shows that, with the exception of frequencies in the vicinity of the cut-off frequency, the attenuation constant for this dissipative filter is nearly the same as for the ideal filter. The dissipative filter, however, has an attenuation constant different from zero in the transmitted band. The value of the attenuation constant within the transmitted band and the deviation of the attenuation constant curve for the dissipative filter from that for the ideal filter depends upon the value of *d*.

Curve *B* of Fig. 12 gives the phase shift constant for frequencies above the resonant frequency. This curve is practically the same as the one given by Fig. 9 for the corresponding ideal filter.

(*c*) *Characteristic Impedance and Line Impedance.* The characteristic impedance is given by the relation

$$Z_s = \sqrt{Z_c^2/4 - Z_m^2} \qquad \textbf{(13)}$$

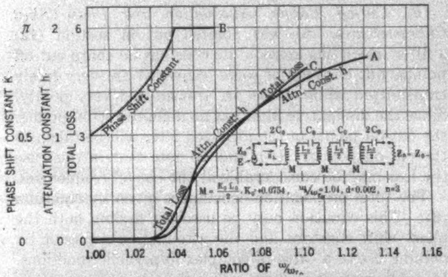


FIG. 12—PHASE SHIFT CONSTANT, ATTENUATION CONSTANT AND TOTAL LOSS CHARACTERISTICS FOR A DISSIPATIVE FILTER

If we use the design formulas as given by equations (**66**) and (**67**) we can write:

$$Z_m = 1/2\,j\,\omega\,L_0\,K_0 = \pm\,j\,Z_0\,\omega/\omega_{r0} \qquad \textbf{(100)}$$

$$\begin{aligned} Z_c &= (j + d)\,\omega\,L_0 - \frac{j}{\omega\,C_0} \\ &= \frac{2\,Z_0}{|K_0|}\,[d\,\omega/\omega_{r0} + j\,(\omega/\omega_{r0} - \omega_{r0}/\omega)\,] \end{aligned} \qquad \textbf{(101)}$$

The characteristic impedances for frequencies above the resonant frequency are given by curve *A* of Fig. 13. A comparison of this curve with the corresponding curve for the ideal filter, given by Fig. 7 shows that, for all frequencies except those in the vicinity of the cut-off frequency, the curves are nearly identical. The characteristic impedance of the ideal filter falls to zero at the cut-off point whereas in the real filter it has a value other than zero at the cut-off point. In the ideal filter the characteristic impedance is a pure resistance within the transmitted band and a pure reactance without the transmitted band. In the real filter the characteristic impedance has a very small phase angle over the greater share of the transmitted band. In the vicinity of the cut-off point this phase angle increases rapidly to 45 deg. at the cut-off point. Beyond the cut-off point the phase angle increases rapidly to 90 deg. The deviation from the ideal case of course depends upon the value of *d*. The line impedance is given for the real filter as well as for the ideal filter by equation (**75**). The line impedance curve for a three section dissipative filter in which $K_0 = +\,0.0754$, $d = 0.002$, is given by curve *B* of Fig. 13. This curve differs but little from the corresponding curve for the ideal filter given by Fig. 10. The two curves of Fig. 13 give a good comparison of the characteristic impedance and the line impedance for a three section dissipative filter. The line impedance is flatter over a greater portion of the transmitted band than the characteristic impedance. For frequencies outside the transmitted band and slightly removed from the cut-off frequency the two curves coincide to within a fractional part of a per cent.

(*d*) *Total Loss Characteristic.* The total loss characteristic will again be computed by the use of equations (**91**) and (**92**). All of the terms entering into these equations have already been given by curves or can readily be found from the data given by the curves. The calculation of the current ratio by means of (**91**) is straightforward everywhere because for the real filter no indeterminate forms appear.

The total loss characteristic for the three section filter under consideration is given by curve *C* of Fig. 12, since *n* for this filter is three, the total loss curve has been plotted to a scale three times as small as that used in plotting the attenuation constant curve. This makes it possible to see directly how closely the total loss is given by three times the attenuation constant. A comparison of curves *A* and *C* show that the loss computed on the basis of the attenuation constant alone is a very close approximation to the total loss. A comparison of curve *C*, Fig. 12, with the curve of Fig. 11 will show that the total loss curve for the ideal filter gives a close idea of the total loss in the real structure.

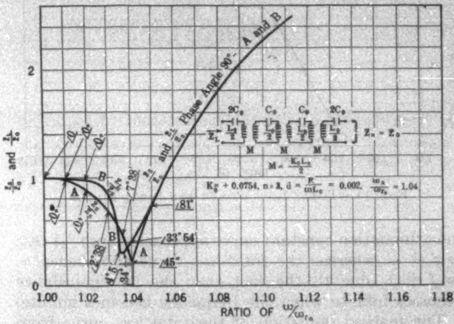


FIG. 13—CHARACTERISTIC IMPEDANCE AND LINE IMPEDANCE FOR A DISSIPATIVE FILTER

This section and the preceding one have shown that a very good idea of the selective properties of a filter may be obtained from a consideration of the attenuation constant curves for the corresponding ideal structure. Further, the loss curve of a filter network designed in accordance with the formulas given is obtained very closely by multiplying the ordinates of the attenuation constant curve by the number of sections in the filter. These two facts aid very much in the design of filters which must meet preassigned requirements.

V. FILTERS BUILT UP OF SECTIONS HAVING DIFFERENT ATTENUATION CHARACTERISTICS

(16) *General Considerations of the Problem of Building a Filter of Different Types of Sections.* If we have a chain built up by joining together an infinite number of identical coupled circuit filter sections then it has been shown that the transmission loss between any two sections is equal to the attenuation constant times the number of sections which separate the two under consideration. This follows because the reflected wave term drops out of the current equation and the current is given by a simple exponential expression. In filters with a finite number of sections we approach this condition by terminating the filter in such a way that over the greater portion of the transmitted band the structure acts as though it were in its infinite chain.

Let us now consider a chain built up of an infinite number of coupled circuit filter sections, but let us not require that all the sections be identical. Let us require, however, that all types of sections have the same characteristic impedance at all frequencies.

The chain may be divided up into groups of identical sections as shown by Fig. 14. To any of the groups as the Sth group we may apply our general equations which give the current in any section of the group in terms of the current in the first section of the group. The terminating impedance is the impedance looking into the $S + 1$ group of sections. Since the chain is infinite or terminated by an impedance equal to Z_s this impedance is also equal to Z_s and the reflected wave term is absent from the equations applying to the Sth group. The transmission loss through the group is therefore $h_s\, n$. Also the loss through group $S - 1$ is $n'\, h_{s-1}$ and so on. The total transmission loss between points A and B is $n'\, h_{s-1} + h_s\, n + n''\, h_{s+1}$. From this it follows that the total transmission loss between any two sections is equal to the sum of the attenuation constants for all the sections separating the two under consideration.

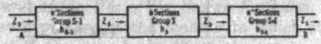


FIG. 14

That which has been said concerning the infinite chain also applies to the structure terminated by an impedance equal to the common characteristic impedance of the elements.

It follows from the above discussion that the problem of building up a filter of different types of sections is solved when we can so design the sections that the different types have the same characteristic impedance. In the sections which follow we will base the design of two types of filter sections upon the design of the section discussed in part IV in such a way that all three types have the same characteristic impedance and therefore can be used together in building up filters.

In sections which follow, attenuation characteristics alone will be employed in giving examples of built up filters. This is justifiable because it has been shown that the loss computed from the attenuation constant alone gives a good idea of the total loss characteristics of a structure.

(17) *The Filter Element Derived from Two Simple Series Circuits with Electrostatic and Electromagnetic Coupling Between Them.* The filter element to be considered here is shown by Fig. 15. The mutual inductance between the circuits is given by:

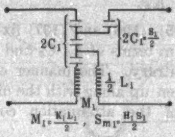


FIG. 15

$$M_1 = \frac{L_1 K_1}{2}$$

The mutual elastance is given by the relation

$$S_m = \frac{S_1 H_1}{2}.$$

In accordance with the principles laid down in section (**16**) we proceed to determine design formulas for this type of section so that it will have the same characteristic impedance at all frequencies as the simple section discussed in part IV. In the discussion which is to follow, all constants referring to the simple type of section or basic section will bear the subscript *o* while those referring to the section of Fig. 15 will bear the subscript 1.

The condition which we are to satisfy is stated by the equation:

$$Z_{s0} = \sqrt{Z_{c0}^2/4 - Z_{m0}^2} = Z_{s1} = \sqrt{Z_{c1}^2/4 - Z_{m1}^2} \tag{102}$$

Upon putting the values of Z_c and Z_m in (**102**) and squaring both sides there results:

$$\left.\begin{aligned} &\omega^2 L_0^2 - \frac{2 L_0}{C_0} + \frac{1}{\omega^2 C_0^2} - \omega^2 K_0^2 L_0^2 \\ &\quad = \omega^2 L_1^2 - \frac{2 L_1}{C_1} + \frac{1}{\omega^2 C_1^2} \\ &\quad - \omega^2 L_1^2 K_1^2 + \frac{2 K_1 H_1 L_1}{C_1} \\ &\qquad - \frac{H_1^2}{\omega^2 C_1^2} \end{aligned}\right\} \tag{103}$$

Since (**103**) is to hold for all angular velocities, we may equate coefficients of like powers of ω.

From the constant terms there results:

$$L_0/C_0 = L_1/C_1\,(1 - K_1 H_1) \qquad \textbf{(104)}$$

From the terms in ω^2 there results:

$$L_1^2 = L_0^2\,\frac{1 - K_0^2}{1 - K_1^2} \qquad \textbf{(105)}$$

From the terms in $1/\omega^2$ there results:

$$C_1^2 = C_0^2\,(1 - H_1^2) \qquad \textbf{(106)}$$

Upon substituting (**105**) and (**106**) in (**104**) there results:

$$\frac{(1 - K_1^2)\,(1 - H_1^2)}{1 - K_0^2} = (1 - K_1 H_1)^2 \qquad \textbf{(107)}$$

Equations (**105**), (**106**) and (**107**) fix L_1 and C_1 and either H_1 or K_1 in terms of L_0, C_0 and K_0. The ratio of H_1/K_1 is arbitrary. The manner of choosing this ratio will be taken up along with the discussion of the general shape of the attenuation constant characteristics.

The attenuation constant is a function of the impedance ratio $Z_c/Z_m = u + j\,v$, so we proceed to determine this ratio for the filter, section under discussion:

As before let $d = \dfrac{R}{\omega L}$

Then

$$Z_c = (d + j)\,\omega L_1 - \frac{j}{\omega C_1}$$

$$Z_m = j/2\left(\omega K_1 L_1 - \frac{H_1}{\omega C_1}\right)$$

$$Z_c/Z_m = u + j\,v = \frac{2\,[\,(\omega/\omega_{r1})^2 - 1 - j\,d\,(\omega/\omega_{r1})^2\,]}{(\omega/\omega_{r1})^2 K_1 - H_1} \qquad \textbf{(108)}$$

Where

$$\omega_{r1} = \frac{1}{\sqrt{L_1 C_1}}$$

For the ideal filter in which $d = o$ this becomes

$$u = \frac{2\,[\,(\omega/\omega_{r0})^2 - 1]}{(\omega/\omega_{r1})^2 K_1 - H_1} \qquad \textbf{(109)}$$

The attenuation constant for the ideal filter is determined by the relation:

$$\cosh\,(h) = |u/2|$$

There are four cases depending upon the relative values of K_1 and H_1

Case I $H_1 = o$

This is the case of the filter discussed in detail in part IV.

Case II $K_1 = o$

This is the case of the filter section with electrostatic coupling only. The general shape of its attenuation constant characteristic will now be determined for the ideal filter.

When $\omega = o, \quad u = -\,2/H_1 \quad h = \cosh^{-1}\left(\dfrac{1}{H_1}\right)$

When $\omega = \infty, \quad u = \infty$

$h = \cosh^{-1}\infty = \infty$

The attenuation constant characteristic for this section is then as shown by Fig. 16.

Case III

$H_1 > K_1$

At $\omega = o$; $\cosh h = 1/H_1$

At $\omega = \infty$; $\cosh h = 1/K_1$

When $\omega/\omega_{r1} = \sqrt{H_1/K_1}$ $\cosh h = \infty$ and the attenuation constant is infinite. That is the ratio of H_1/K may be thought of as a design parameter which fixes the frequency at which infinite attenuation occurs.

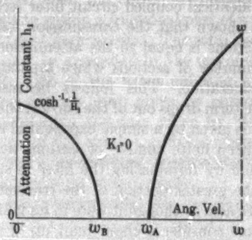


FIG. 16

The general shape of the attenuation characteristic for a section coming under Case III is shown by Fig. 17.

An actual attenuation constant curve for a dissipative filter coming under Case III is given by curve *A* of Fig. 20.

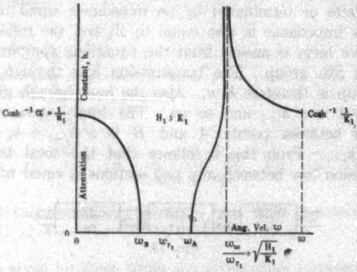


FIG. 17

Case IV. $H_1 < K_1$

Here again at $\omega = o \cosh h = 1/H_1$ at $\omega = \infty$ $\cosh h = 1/K_1$ at $\omega/\omega_{r1} = \sqrt{H_1/K_1}$; $\cosh h = \infty$.

The general shape of the attenuation constant for a filter section coming under case IV is shown by Fig. 18. The design of all four cases of the filter section shown by Fig. 15 has been based upon the design of Case I in such a way that at all frequencies the filters have the same characteristic impedance. Now at a cut-off point the characteristic impedance for the ideal structure fall to zero and changes from a pure resistance to a pure reactance and this is the only place at which this occurs so the different sections having the same characteristic impedance will also have the same cut-off points. For the ideal filter section, shown by Fig. 15 and designed in accordance with the formulas given, the characteristic impedance and the cut-off points can be computed from the equations given in part IV.

The cut-off points can also be found by determining the value of ω in equation (**109**) which will cause u to equal ± 2. These cut-off angular velocities are found to be

$$\left.\begin{aligned} \omega_{A \text{ or } B} &= \omega_{r1}\sqrt{\frac{1 + H_1}{1 + K_1}} \\ \omega_{A \text{ or } B} &= \omega_{r1}\sqrt{\frac{1 - H_1}{1 - K_1}} \\ \text{where} \qquad \omega_{r1} &= \frac{1}{\sqrt{L_1 C_1}} \end{aligned}\right\} \qquad (\mathbf{110})$$

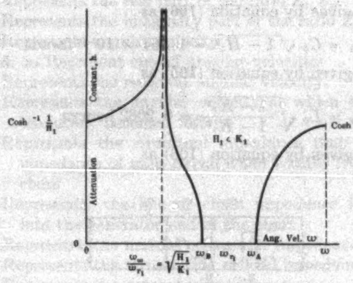


Fig. 18

It can be shown that the ω_A and ω_B determined from (**110**) are the same as those determined from (**57**) and (**58**) by substituting in (**110**) value for $L_1 C_1$, H_1 and K_1 from equation (**105**), (**106**) and (**107**).

(18) *Examples Built-of up Attenuation Characteristics and of the Use of the Design Formulas.* In order to give a picture of the manner in which attenuation characteristics may be built up, the curves of Figs. 19, 20 and 21 were plotted. Curve A of Fig. 19 is the attenuation constant characteristic for a section in which $H_1 = o$, $K = K_0 = 0.0325$, $d = 0.001$. Curve B of Fig. 19 is the attenuation characteristic for a two section filter in which the first section is the one described above and section 2 has the following constants $H_1 = K_0 = 0.0325$, $K_1 = o_1$ $d = 0.001$. This combination gives a symmetrical band pass filter; that is, the attenuation for $\omega = A$ is the same as the attenuation for $\omega = 1/A$. This combination has infinite attenuation at $\omega/\omega_{r0} = o$ and $\omega/\omega_{r0} = \infty$. Curve A of Fig. 20 is the attenuation curve for a section in which $K_0 = 0.0325$, $K_1 = 0.2859$, $H_1 = 0.3153$, $d = 0.001$. Curve B is an attenuation curve for a two section filter. One section has the constants given above. For the other section the values of H_1 and K_1 have been interchanged. This combination of sections gives a symmetrical band pass filter in which the points of infinite attenuation have been brought in from ∞ and o to

$$\omega/\omega_{r1} = \sqrt{\frac{0.3153}{0.2859}} \quad \text{and} \quad \omega/\omega_{r1} = \sqrt{\frac{0.2859}{0.3153}}$$

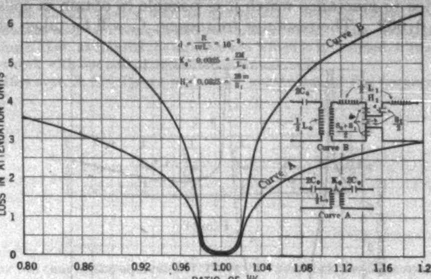


Fig. 19—Attenuation Curves for Band Pass Wave Filter

The curve of Fig. 21 is the attenuation characteristic of a three section filter obtained by adding a section for which $K_0 = 0.0325$, $K_1 = o$, $H_2 = 0.0325$, $d = 0.001$ to the filter just described above.

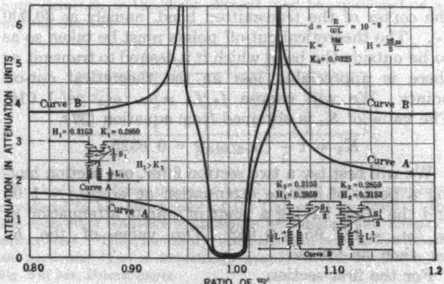


Fig. 20—Attenuation Curves for Coupled Circuit Wave Filters

As an illustration of the use of the design formulas let us consider the following problem:

It is desired to introduce between a generator whose impedance is 600 ohms, and a power receiving device of 600 ohms, resistance, a filter having the following characteristics:

The filter shall pass frequencies from 29,700 ~ to

30,300 ~ with as small a loss as possible. The variation in loss over this transmitted band not to exceed 0.2 of an attenuation unit. The loss at 31,000 ~ and above and at 29,000 ~ and below shall exceed 5 attenuation units. Required, the constants of the filter.

We will assume that the value of the ratio of resistance to reactance which it is possible to obtain in coils is 0.002. The resonant frequency, f_{r0} will be taken at the center of the transmitted band, namely as 30,000 ~. The theoretical cut-off points must be taken so as to be outside the band which it is desired to transmit as there is appreciable loss at the theoretical cut-off points. We will choose $f_A/f_{r0} = \omega_A/\omega_{r0}$ as 1.016. The value of K_1 is obtained from equation (**57**).

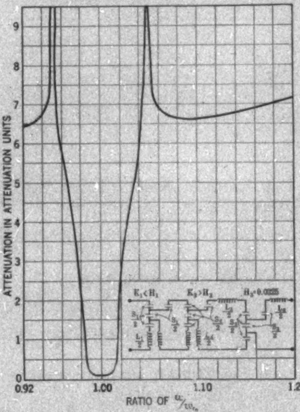


Fig. 21—Attenuation Curve for Coupled Circuit Wave Filter

$$K_0 = 1 - (\omega_{r0}/\omega_A)^2 = 0.0325$$

We will first try a two section filter, one section having a point of infinite attenuation at $\omega_\infty/\omega_{r1} = 1.05$ and the other having a point of infinite attenuation at $\omega_\infty/\omega_{r1} = 1/1.05 = 0.952$. The value of the K_s are then as follows:

For the first section,

$$\sqrt{H_1/K_1} = 1.05; \quad H_1/K_1 = 1.105$$

Substituting this in equation (**107**) we obtain

$$K_1 = 0.2859$$
$$H_1 = 0.3153$$

For the second section we obtain in like manner,

$$K_2 = 0.3153$$
$$H_2 = 0.2859$$

The losses in this two section filter based upon the attenuation constants alone is given by curve *B* of Fig. 20. This filter does not quite meet the requirements. To this filter we will add a third section for which $\omega_\infty/\omega_{r1} = \infty$. For this section then $K_3 = o$. Equation (**107**) gives H_3 as follows:

$$H_3 = K_0 = 0.0325$$

The loss through this three section filter based upon the attenuation constants alone is shown by the curve of Fig. 21. This filter meets the requirements. Its constants are found as follows:

C_0 is given by equation (**66**) as

$$C_0 = \frac{|K_0|}{2\,\omega_{r0}\,Z_0}, \quad \text{where} \quad Z_0 = 600$$

$$C_0 = 1.437 \times 10^{-10} \text{ farads}$$

L_0 is given by equation (**67**) as

$$L_0 = \frac{2\,Z_0}{|K|\,\omega_{r0}} = 0.1958 \text{ henrys}$$

L_1 is given by equation (**105**) as

$$L_1 = L_0 \sqrt{\frac{1 - K_0^2}{1 - K_1^2}} = 0.204 \text{ henrys}$$

C_1 is given by equation (**106**) as

$$C_1 = C_0 \sqrt{1 - H_1^2} = 1.364 \times 10^{-10} \text{ farads}$$

L_2 is given by equation (**105**) as

$$L_2 = L_0 \sqrt{\frac{1 - K_0^2}{1 - K_1^2}} = 0.2062 \text{ henrys}$$

C_2 is given by equation (**106**) as

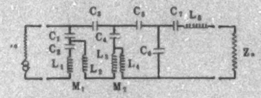


Fig. 22

$Z_0 = Z_n = 600$ ohms
$C_1 = 8.652 \times 10^{-10}$ farads $\quad L_1 = 0.102$ henrys
$C_2 = 3.974 \times 10^{-10}$ " $\quad L_2 = 0.102$ "
$C_3 = 1.958 \times 10^{-10}$ " $\quad L_3 = 0.1031$ "
$C_4 = 3.974 \times 10^{-10}$ " $\quad L_4 = 0.201$ "
$C_5 = 1.678 \times 10^{-10}$ " $\quad L_5 = 0.0979$ "
$C_6 = 8.843 \times 10^{-10}$ " $\quad M_1 = 0.02916$ "
$C_7 = 2.971 \times 10^{-10}$ " $\quad M_2 = 0.03251$ "
$K_0 = 0.0325, \quad K_1 = 0.2859, \quad K_2 = 0.3153$
$H_1 = 0.3153, \quad H_2 = 0.2859, \quad H_3 = 0.325$
$\omega_a/\omega_{r0} = 1.016, \quad \omega_{\infty 1}/\omega_{r0} = 1.05, \quad \omega_{\infty 2}/\omega_{r0} = 0.952$
$\omega_{r0} = (30,000)\,2\pi, \quad d = 0.001$
Attenuation characteristics given by Fig. 21

$$C_2 = C_0 \sqrt{1 - H_2^2} = 1.377 \times 10^{-10} \text{ farads}$$

L_3 is given by relation (**105**) as

$$L_3 = L_0 \sqrt{1 - K_0^2} = 0.1958 \text{ henrys}$$

C_3 is given by (**106**) as

$$C_3 = C_0 \sqrt{1 - H_3^2} = 1.437 \times 10^{-10} \text{ farads}$$

The filter goes together as shown by Fig. 21. After combining elements, the constants of the filter are shown by Fig. 22.

COMPILATION OF SYMBOLS

γ	Defined by $Z_c/Z_m = \gamma \underline{/\phi}$
C	Stands for capacitance
d	Stands for the ratio of resistance to reactance of coils $= R/\omega L$
E	Stands for voltage
ϵ	Represents the naperian base of logarithms
f	Represents frequency
h	Represents the attenuation constant
H	Represents the coefficient of electrostatic coupling
I	Represents current
j	Represents the rotative operator $\sqrt{-1}$
k	Represents the wave length or phase shift constant
K	Represents the coefficient of electromagnetic coupling
L	Represents self inductance
M	Represents mutual inductance
R	Represents resistance
S	Represents elastance
S_m	Represents mutual elastance
τ	Represents the propagation constant
u	Represents the real part of the ratio Z_c/Z_m
v	Represents the imaginary part of the ratio Z_c/Z_m
ω	Represents angular velocity
ω_A & ω_B	Represent cut-off angular velocities
ω_{r0}	Represents the resonant angular velocity
ω_∞	Represents the angular velocity at which the attenuation becomes infinite
Z_c	Represents the circuitual impedance that is the impedance of each circuit when isolated from the chain
Z_L	Represents the line or chain impedance looking into the generator end of the chain
Z_m	Represents the mutual or transfer impedance
Z_n	Represents the terminating or load impedance
Z_0	Represents the generator impedance
Z_s	Represents the characteristic impedance of the chain $= \sqrt{Z_c^2/4 - Z_m^2}$

Discussion

Kenneth S. Johnson: There are certain statements made in the Peters paper to which I wish to take exception and which, unless pointed out, I believe are likely to be misleading.

The coupling or mutual impedance, Z_m, is represented in the paper symbolically as a network with *two* pairs of terminals. Since, however, it introduces the same impedance into both adjacent sections and is completely defined by a single impedance, Z_m, it is actually only a network with *one* pair of terminals. Consequently with no loss of generality whatsoever Fig. 1 of the paper can be represented by the following sketch.

But since any network with only one pair of accessible terminals is always completely represented by a single impedance element the above sketch and hence Fig. 1 of the paper may be shown with complete generality by Fig. 2. Comparing this with one of Campbell's figures[1] it is evident that the two figures are exactly the same. It should be noted, however, that there is a difference in the method of designating the impedances. Although the Z_m of the Peters paper is exactly equal to the Z_2 of the Campbell paper, Campbell defines his Z_1 so that it includes only the series impedance component while Peters' Z_c comprises the series impedance plus twice the shunt impedance. That is

$$Z_c\,(\text{Peters}) = Z_1 + 2\,Z_2\,(\text{Campbell})$$

Evidently the two figures are identical. Either of them properly represents any periodic recurrent structure. We should understand, however, that Campbell's Z_1 and Z_2 or Peters' Z_c and Z_m (assuming one assigns to his work the broadest application) are symbolic and may not be actually realizable impedance elements as indicated. This is a commonplace to those familiar with electric circuit theory and its symbolism. Any of these impedances may be a complicated network in itself. The essential thing is that, as regards the structure in which it is inserted, it is a network with *one* pair of accessible terminals only.

However, if Mr. Peters prefers the symbolism of the "square box" to that of the equivalent "*T*" or "*π*" network the filter may be represented as shown in Fig. 3.

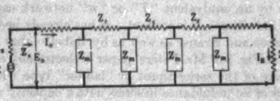


FIG. 1

Each box represents a network with *two* pairs of terminals and is completely defined by three impedance parameters. The theory and formulas of the filter may then be developed in terms of these parameters[2] without explicit reference to the equivalent "*T*" network as is done by Zobel.[3] This procedure, of course, gives no gain whatsoever in generality. Mr. Peters, evidently not appreciating the equivalence of the circuits noted above, makes the following statements in his paper.

"The conception of circuital and transfer impedance as a broad basis for filter theory and the type of section to which the treatment leads directly are thought to be original."

"The general theory presented in the following sections is somewhat broader than that presented by Campbell and Wagner as it introduces, as a special case, the filter built up of series and shunt elements, as the shunt element may be considered as a coupling element."

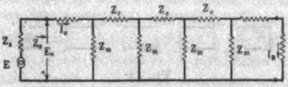


FIG. 2

Evidently the forms discussed by Mr. Peters may be properly considered to be special cases of the Campbell treatment. In this connection reference to an article by Luschen and Krause[4] is of interest. They, incidently, also discuss in more or less detail the particular type of inductively coupled structure shown in Fig. 3 of Mr. Peters' paper.

In the next to the last paragraph on page 449 he states that "If Z_n and Z_0 are to be fixed resistance then in general we can satisfy the above relation for only *one* particular frequency within the transmitted band . . .". This statement is not, in general, true as may be seen by reference to Fig. 5 on page 18 of the paper by Mr. Zobel which has already been referred to. In fact it is possible in certain types of band pass filters to satisfy the above relation for as many as *four* particular frequencies within the transmitting band.

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[1] See Fig. 4 in G. A. Campbell's paper on the "Physical Theory of the Electric Wave Filter," *Bell Technical Journal*, Vol. 1, No. 2, (Nov. 1922).

[2] See "Cisoidal Oscillations" by G. A. Campbell. TRANS. A. I. E. E., 1911, Vol. XXX, Part II.

[3] "Theory and Design of Uniform and Composite Electric Wave Filters," *The Bell System Technical Journal*, January 1923, Page 32.

[4] See Vol. 1, Third Issue (Nov. 1921), Wissenschaftliche Veroffentlichungen aus dem Siemens-Konzern.

A few sentences later we read that "In general . . . . the impedance curve is fairly flat for quite a range of frequencies within the transmitted band but falls to *zero* at the cut-off points." The characteristic impedance of a large number of filters does become zero at the cut-off points but for practically an equally large number of filters it becomes of infinite value at one or more of the cut-off points.

With regard to the specific forms of wave filters discussed by Mr. Peters the type shown in Fig. 3 of the paper appears to have been investigated several times previously.[5] Since two separate inductances coupled together by a mutual inductance are always replaceable by an equivalent "*T*" or "*π*" network each arm of which is an inductance, it is evident—as has already been pointed out by Luschen and Krause as well as by Zobel[6]—that the structure shown in Fig. 3 of Mr. Peters' paper is electrically equivalent to a structure of the series-shunt or "ladder" type whose series arms consist of an inductance in series with a capacity and whose shunt arms consist merely of an inductance. In a like manner the structure shown in Fig. 15 of Mr. Peters' paper is reducibel by the same procedure to a structure of the ladder type whose series arms comprise an inductance and condenser in series and whose shunt arms are likewise comprised of an inductance in series with a condenser. Complete design formulas for these latter structures, which have been used commercially and which are equivalent to the ones which Mr. Peters has shown in Fig. 3 and 15, are given on page 42 (V_1 and $VIII_1$) of Mr. Zobel's paper to which we have already referred.

On page 456, Mr. Peters states that " . . . . . with the exception of frequencies in the vicinity of the cut-off frequency and for very large frequencies the loss figured from the attenuation constant alone gives a very close approximation to the total loss occasioned by the introduction of the filter." It should be pointed out that this statement is not true for very low frequencies where the reflection effects in band pass wave filters are equally as large as they are for very high frequencies and consequently greatly increase the loss caused by the introduction of the filter.

The statement is also made at the top of the second column on page 456 "That the losses in condensers can be made negligible."

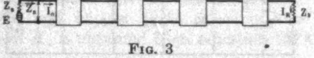
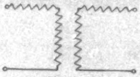


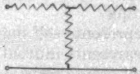
FIG. 3

This is never true with reference to paper condensers and is by no means true even with mica condensers, at relatively high frequencies. For example, the losses in the best mica condensers will approximate the losses assumed by Mr. Peters to exist in the coils *i. e.* the ratio of their resistance to their effective reactance is of the order of magnitude of 0.001. It should be pointed out however in this connection, that the value of 0.001 for "*d*," which is the value Mr. Peters has assumed in computing the curves shown in Figs. 19, 20 and 21, is very much smaller than has ever been encountered commercially so far as we know. To our knowledge coils with a value of "*d*" of 0.005 are about as efficient as have ever been commercially constructed. The effect of this greater amount of dissipation, is, of course, to increase the loss proportionally in the center of the transmitted band and to make the attenuation curves much more rounded near their cut-off frequencies.

L. J. Peters: I greatly appreciate Mr. Johnson's discussion of my paper, first, because he has called attention to some points of which I was aware but stated rather loosely, and second, because he has called attention to some things of which I was not aware. However, in regard to the broadness and suggestiveness of the series—shunt impedance treatment as given by Campbell and others and the circuitual—transfer impedance treatment as given by myself, I would like to state clearly in what way I regard the latter treatment as being the broader. I agree with Mr. Johnson that either treatment can be used to design or to investigate all of the types of filter sections which we have in mind. I further agree that one treatment can be thrown over into the other by a suitable transformation of the equations. My contention is that the circuitual—transfer impedance terminology and view point is broader in its suggestiveness, at least to the average engineer, than the series-shunt impedance terminology. In order to state more clearly what I mean, I will designate as Class 1 those filters actually constructed as shown by Fig. 4, and as Class 2 those filters actually constructed as shown by Fig. 5. Then my contention is that the circuitual-transfer impedance terminology at once suggests filter sections of Class 1 and Class 2 whereas the series-shunt impedance terminology suggests to the average engineer only filters of Class 2. I infer that the series-shunt viewpoint did not immediately suggest filters of Class 1 even to those very familiar with the electrical equivalence of various kinds of networks, as Dr. Campbell's original patent specifications, to which I have referred in my paper, do show diagrams for the possible variations of Class 2 filters but show no diagrams even indirectly suggesting a Class 1 type of filter. Also the published works with which I was familiar at the time my paper was written did not suggest to me at least that filter sections of Class 1 had been considered.



Class 1
FIG. 4



Class 2
FIG. 5

In this connection I would like to point out that there is a vast difference between having enough general theoretical knowledge to cover a given phenomenon and the actual consciousness that the phenomenon exists. As an illustration of what I mean, consider the thermionic vacuum tube. Nearly every engineer and physicist who came into contact with the two element tube had enough electrodynamic theory to know that the interposition of the third element would convert the tube into a device having untold possibilities. Yet the third element was not put in for years after the discovery of the two element device, and even after it was inserted, the possibilities of the device developed slowly over a number of years. That is, a realization of the possibilities of the device came about only by stating the general electrodynamic theory in such a way as to specifically cover the applications. I claim that the circuitual-transfer treatment is broader than the series-shunt treatment because it does state filter theory in such a way as to specifically cover filters of both Class 1 and Class 2, whereas the series-shunt treatment does not.

Now besides the broader suggestiveness of the circuitual-transfer impedance viewpoint over that of the series-shunt impedance viewpoint, there is also a correspondingly greater broadness in its immediate applicability. Even though the sets of equations arrived at by the two treatments are equivalent in that one set may be transformed into the other, the set of equations which I have given apply directly to both Class 1 and Class 2 filters, whereas before the series-shunt equations can be used to treat Class 1 filters, either the equations must be transformed into the form given in my paper or else the Class 1 network must be replaced by an electrically equivalent Class 2 network. That is, in order to use the series-shunt treatment to handle Class 1 networks, one must have in addition to the filter theory an insight into the possibilities of the replacement of one type of net-work by an electrically equivalent one. Under these conditions I believe that it would be customary to regard the circuital-transfer impedance treatment as being more general than the series-shunt impedance treatment.

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[5] See the article by Luschen and Krause previously referred to. See also K. W. Wagner's article on "Multiplex Telephony and Telegraphy with High Frequency Alternating Currents," *Elektrotechnische Zeitschrift*, August 14, 1911, Note 1 on page 395.

[6] See paragraph 1 on page 30 of the paper already mentioned.

Lest it be overlooked by some, I would like to call attention to the dates of the wave filter papers by Campbell and Zoebel referred to in Mr. Johnson's discussion. I do this, not to show that the filters of Class 1 were just recently considered by Mr. Johnson and his co-workers as Mr. Johnson has just sent me a memoranda from his files which shows that he considered this class of filter as early as 1919, but to call attention to the fact that these papers were not available when my paper was written.

The Automatic Train Control Problem

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Circumstances which make train control a pressing problem at the present time are mentioned. Certain desiderata are briefly discussed; the automatic control should act as a check on the proper exercise of manual control, not as a substitute; should conform to established safe signaling practises; should be designed for the severe conditions of railway operation; should not conflict with existing signals or otherwise introduce new hazards; should so far as possible conform to existing clearance lines; should not impede traffic; may be desired to exercise signal as well as control functions; may ultimately be desired for interlocking plants as well as main line tracks. Methods of transmitting and indication of track conditions to the train are described, (1) by intermittent mechanical and electric contact; (2) by intermittent induction through the use of permanent or electromagnets; (3) by continuous induction from the rails. Two general types of controlling action on the train are described, with some of their variations; (1) unconditional stop effective at a stop signal, with overlapping signal control, and (2) conditional stop effective at a caution signal. Relations between the type of controlling action and traffic capacity are discussed.

THE problem of mechanically enforcing observation and obedience of railway signals has attracted inventors and engineers for many years; but until very recent years railway officials have been inclined to view automatic train control as something rather Utopian; highly desirable no doubt, but too remote to command serious attention when the resources of the railroads were already taxed to provide other and more pressing improvements in the interest of safety; for example, extensions of existing signal systems and elimination of grade crossings. Within the last few years, however, a large number of disastrous wrecks attributed to disregard of signal indications have brought home the need for some effective check on the functioning of human, and therefore fallible engineers. The Interstate Commerce Commission has commented on the need with increasing emphasis, and finally has ordered automatic control installed on a large mileage of important lines. The train control problem thereby becomes so urgent that it is believed a brief review of the characteristics desired in automatic train control devices and of the broad features of some systems now available may have some general interest.

The roadside signal systems already installed on the principal railroads are adequate in so far as regards the detection and indication by visible signals of the presence of trains or other danger conditions along the track beyond the signal. The present demand is for an automatic device to bridge the gap between signals properly displayed, and trains properly controlled in accordance with the signal indications. The kernel of all modern roadside signal systems is the continuous track circuit consisting of the track rails, extending through the length of the block, a source of low-volt d-c. or a-c. power at one end and a relay at the other end. So long as the track is unoccupied and the rail circuit intact, the relay is held closed by current passing through the rails from end to end of the block, and establishes local circuits to cause the clear, or safe signal indications to be displayed. But if the rail circuit is interrupted, or short-circuited by a train in the block, the relay is deprived of power and opens, and the signal reverts to its restrictive indication. Similarly, the kernel of the train control problem is the transmission to the train of an indication of conditions in one or more blocks ahead. Given an indication on the train capable of controlling a relay, almost any desired cycle of automatic operation can be established to control the train. The principal problem beyond the relay is to determine what operations are desired.

Before discussing any specific solution of the train control problem, it will be well to examine briefly some of the requirements or desiderata which constitute the conditions of the problem.

DESIRED CHARACTERISTICS

1. The primary function of automatic train control is, of course, to stop the train before it enters upon unsafe territory. But it is not necessary or desirable that it should interfere with manual control by the engineer, so long as that conforms to the requirements of safety. To substitute even a very perfect mechanical safeguard for human vigilance would be, to say the least, a questionable gain. Perhaps the mechanical

device would respond to the foreseen conditions for which it was designed with more than human certainty; but it could not act intelligently in response to unforeseen conditions. The real gain in safety is to be made by reinforcing the vigilance of the engineer, not by supplanting it. With an automatic control, acting as a second line of defense behind the engineer we have two independent and successive safeguards. Either one may fail, and doubtless will on rare occasions; but the chance that both will fail simultaneously is exceedingly remote.

The automatic apparatus, therefore, should not relieve the engineer of complete responsibility for the safe operation of the train. It should act as a continuous check on the proper exercise of that control, intervening to stop the trains automatically only after the engineer has had ample opportunity to act, and has failed. There is evidence to show that automatic control thus applied, tends to increase, and not relax, the vigilance of the engineer, because the automatic application of the brakes acts as an immediate tell tale if he ever passes a danger signal. It may be desirable as an aid to discipline to have the apparatus record each automatic brake application or preserve some evidence of it in the way of broken seals or the like.

2. The existing fixed signal systems have been developed to a very high degree of reliability in the course of a good many years. Prudence would dictate that the principles and practises which experience has shown to be safe for fixed signals should be followed so far as they are applicable in the installation of automatic control. The most fundamental of these practises is the so-called "closed circuit" rule, which requires that every "clear" or "proceed" indication be made dependent on the integrity of energized closed circuits, so that any interruption of the circuits or of the power supply will result in a danger indication. In practise, the rule is construed more broadly than this literal interpretation. Not only circuits, but every element of the system is so arranged that it is predisposed to the position or condition corresponding to a danger indication, and will return to that condition automatically upon failure of any part. In the construction of each element every precaution is taken to reduce to a minimum the chance that friction, insufficient clearance, residual magnetism, sticking of contacts or any other cause will bring about a failure to return to the danger indication, when de-energized. It is chiefly due to the rigid application of this rule that such failures as occur in signal systems are almost invariably failures on the safe side—false indications of danger, rather than false indications of safety.

3. Obviously the apparatus should be designed to stand the severe conditions of shock, dirt, weather, etc. that are incident to railway operation, and should not be put out of action by any weather conditions that permit trains to run.

4. The train control devices should not increase the hazards of operation in any respect. They should be free from insecure or dangerous parts, and in particular they should not impair the protection given by existing signal equipment; for regardless of the cab signaling features, that may be incorporated with the automatic control apparatus, railroad officials would be naturally and properly reluctant to abandon the known security they obtain from the existing signals before it has been conclusively established by experience that the new apparatus is at least as reliable as the old. Even if the old apparatus were to be abandoned, it would be highly desirable that the new and the old should cooperate without interference during the transition stage.

5. If possible, it is desirable to avoid conflict with established clearance lines. Where third rail has been installed for traction purposes, it has been necessary to reserve a zone exclusively for the third rail and the contact shoes, excluding from the reserved zone all other parts of equipment and roadway structures. The same thing can be accomplished for train control apparatus if it must be; but it is by no means so simple a matter as it sounds. Bridges, tunnels, station platforms and innumerable roadway structures have been built on the basis of existing clearance lines. Any change in the lines now involves many complications and much expense. The same is true of rolling stock. At least one electrified road has been obliged to gage every foreign freight car accepted for the third rail division, to check its clearance. The most available location for contact devices if used will probably be about the same as that adopted for traction third rails, when not used or reserved for the latter purpose. Third-rail lines will have to find some different location.

6. The automatic control should place no avoidable restriction on the traffic capacity of the road. An effective argument in favor of automatic roadside signals has been the fact that they "keep the trains moving" and increase the traffic capacity. It will be difficult to reconcile the railroads to automatic control if it is found to impede traffic in any noticeable degree. This is an interesting phase of the problem which will be discussed at more length later.

7. It may be desirable in some cases to enforce automatically, local speed restrictions, at curves for example, or heavy down grades, or a uniform speed restriction at all points.

8. The accidents which it is desired to prevent are due in some cases to negligence on the part of the engineers, but in other cases they are due to actual inability to distinguish and interpret the roadside signals correctly, because they are obscured by weather conditions or by steam from other trains or the like. In such cases a clear indication of the safety or danger of the track ahead, given within the engine cab, would enable the engineer himself to avoid danger Such an

indication can be combined with the automatic control, in some cases at least, with comparatively little increase of complication or expense.

9. All of the foregoing has referred primarily to "block signaling" territory, *i. e.*, to main lines between terminals when the sole problem is to space trains at safe intervals; and the recent order of the Interstate Commerce Commission seems to be limited to this territory. But there is another very large portion of the total signaling of railways involved in interlocking plants where switches, derails and the like, as well as signals are controlled by an operator. There is no inherent reason to prevent at least a partial application of automatic control, or cab signals, or both in the interlocking as well as the block signal territory. But the need is less urgent, because speeds are low and signals are observed at shorter range. And the problems involved are more complex, because extensive net works of inter-related tracks are involved, and varied indications of routes and movements are required. Comparatively little thought has thus far been given to automatic train control in interlocking territory.

METHODS OF TRANSMITTING AN INDICATION TO THE TRAIN

The simplest system of automatic control and the one that has been used on the largest scale consists of an air valve hung on the car truck near the rail level, and stop arms operated in conjunction with the fixed signals. Whenever a stop signal is displayed, the stop arm is raised into the path of the air valve handle. If the train overruns the stop signal, the valve handle strikes the stop arm, and is knocked open, producing an emergency application of the brakes. This system has given excellent service on the subway and elevated lines where it has been employed but it is not commonly considered adaptable to steam roads in general, because it would be exposed to severe weather conditions mixed equipment, high speed impact, parts dragging on trains, and rougher usage generally than it has been subjected to in existing installations.

A refinement of the mechanical trip idea which has been used and reported to be successful on a number of limited stretches of steam railroads consists essentially of a fixed third rail ramp at each signal location, and a shoe carried by the train. In passing over the ramp the shoe is lifted and operates a brake valve, unless it receives current at the same time from the ramp. But if current is received it acts through a magnetic device to lock out the brake application valve. The ramp is energized by the roadside signal circuits when the signal indicates "proceed" but deenergized when the signal is set for "stop" or "caution". Various control schemes are on the market which have these broad features in common, but differ considerably in construction. In at least one case, provision has been made for giving several distinctive indications by varying the polarity and characteristics of the current supplied to the ramp.

The most fundamental weakness of this type of control, in common with all intermittent indication systems, is that failure to produce any indication necessarily acts as a safety, and not a danger signal. If the absence of an indication were made a danger signal, the trains would receive continuous danger signals from one ramp to the next whether the track ahead was clear or occupied. The train must be permitted to proceed in the absence of any indication between ramps; and if, owing to misadjustment or any other cause, the ramp fails to function, the train can continue past it, regardless of conditions in the block ahead. Nothing happens on the train to indicate that it has passed out of the block to which the last preceding ramp admitted it. The electric circuits in detail can be made conformable to the "closed circuit" rule previously discussed; but the system as a whole cannot operate on the closed circuit principle in its broadest sense, which requires every proceed signal to be given by affirmative action, and every failure of affirmative action to produce a stop signal. Intermittent track circuits in roadside signal practise are a somewhat parallel case. No ingenuity in their arrangement can make them indicate the presence of a train that has somehow got by them and stopped on the dead section between track circuits. In modern systems they have been superseded by continuous track circuits from end to end of the block.

The ramp type of control requires the establishment of special clearance lines, reserving some portion of the roadway cross section exclusively for the ramps and shoes. It seems hardly possible that it can be wholly free from trouble due to snow, sleet, and obstructions in the path of the shoe. On the other hand it is simple, and has behind it the record of more actual use than any other system, except the plain mechanical trip first mentioned. It can undoubtedly be made to give a valuable measure of protection if not the almost absolute protection (within their own scope) that is demanded of roadside signals.

An ingenious departure from the ramp type of control is a stationary flexible metallic brush beside the track, designed to make contact with a fixed shoe on the train. The brush is connected to the rail through a relay contact which is held open to indicate safety. The brake valve is held closed by a locally energized circuit on the locomotive, and released by short circuiting of this circuit in passing over the brush, unless the rail connection to the latter is held open in the relay. Unfortunately this scheme flagrantly violates the "closed circuit" principle in its most essential link, the shunt circuit through the shoe and brush to the rail.

In the permanent magnet induction type of control, large permanent magnets laid between the rails and below the rail level take the place of ramps. An iron core on the locomotive surrounded by a coil takes the place of the contact shoe. Stationary coils are arranged in the region of the magnet poles. When the track is clear they are energized from the roadside signal system and serve substantially to neutralize the field of the permanent magnet. The magnet then has no effect on the train. But when the track is occupied, the stationary coils are dead, and as the train passes over the magnet it induces a momentary current impulse in the coil on the locomotive, which is located to pass over the magnet with a safe clearance. This impulse (acting through a relay) is utilized to cause a brake application.

Aside from the inherent limitations of all intermittent systems three questions suggest themselves, the permanence of the magnets, the degree of accuracy in neutralizing the magnet field, and the effect of passage over the magnet at low speed. The first question can be dismissed when we consider the permanence of magnets in voltmeters and ammeters—not merely approximate constancy such as is required in the track magnets but almost invariable strength over periods of many years. The answer to the second question is not so clear. The balance between the neutralizing coils and the magnet could never be exact; and if the supply voltage varies as much as is often the case in signal systems, it would be at times extremely rough. The locomotive coil, then, must distinguish not between a magnetic field and no field, but between a field of the normal strength and a reduced field of, for example, one third normal strength. If the apparatus is adjusted not to respond to the reduced field, it would seem that its factor of safety in responding to the normal field would be rather small, judged by signal standards; the more so when it is recalled that the apparatus must take care of a wide variation of speed as well, and some slight variation of air gaps.

Regarding the action of the magnet at low speeds, there will presumably be some minimum, such as four or five miles per hour required to produce an indication. So far as direct danger from collision goes, five miles an hour is equivalent to a stop. But there is a possibility that a train may enter a block at low speed for other reasons than a signal stop. Precautions should be taken to insure that the engineer will not then construe the failure to receive a stop indication as a sign that the track is clear. If he should so construe it, he could accelerate to a dangerous speed within the block.

The permanent magnet system has the marked advantage that it requires no modification of existing roadway and equipment clearance lines; and the further marked advantage that it has no contact parts and therefore no question of insuring contact in unfavorable weather conditions.

A very simple and ingenious modification of the permanent magnet scheme has a laminated iron core in place of the permanent magnet, with a corresponding core and coil on the locomotive. A winding on the track core is short-circuited through a signal relay when the track is clear, and open when the track is occupied. The locomotive coil is continuously energized by a battery. When the track coil is short circuited it serves to choke back any magnetic impulse in its core (like a short-circuited transformer winding). Under this condition passage over the track element produces no considerable change in the locomotive circuit. But when the track coil is open, the magnetic surge in the track element as the energized locomotive element passes over it induces in the latter an inductive kick which results in a brake application. The advantages and disadvantages of this scheme are identical in many respects with the advantages and disadvantages of the permanent magnet arrangement. It would seem to have some advantage over the latter in respect to simplicity and perhaps a material advantage in the fact that its neutralizing circuit is entirely independent of variations in the voltage of the roadside signal system.

An entirely different system of control is the so-called "continuous induction" system. In this case the indicating energy for the locomotive is supplied directly to the rails of the track circuit in the form of alternating current, and collected inductively on the locomotive either by a coil located ahead of the leading wheels and surrounding a portion of the alternating field produced by the rails themselves, or by a small transformer surrounding the leading axle which acts as a single turn primary for the transformer. Detecting apparatus energized by the current induced in the locomotive circuit controls the application of the brakes or the indications of cab signals. When the track ahead is clear, the detecting apparatus is continuously energized and maintains the proceed indication on the locomotive. But when the track is occupied, or when the indicating current supply fails for any other reason, the locomotive apparatus reverts to its deenergized condition and a brake application results. The detecting circuit and apparatus on the train correspond exactly to the track circuit relay in roadside signal systems.

A train in a block automatically maintains a stop indication behind it in the same block by short circuiting the track circuit. But in order to stop a following train before it enters the block, it is necessary to carry the stop indication back to the next section in the rear. Several circuit arrangements which accomplish this are available, but none which utilizes the existing track circuits where roadside signals are installed without some additions. If desired, it is possible to give several distinctive indications on the train by varying the characteristics of current supplied to the track, or by superposing currents of different characteristics. The electrical efficiency of power transmission from the point of supply to the train is exceedingly low. The total energy input is the power consumed in a short-circuited rail circuit which may be several thousand feet long, while the energy collected on the train corresponds at most to the loss in a few feet of the rail circuit. In order to control the train without excessive power consumption in the track circuit a high degree of sensitiveness is necessary in the receiving apparatus on the train. The required sensitiveness is secured without sacrifice of reliability by the use of a vacuum tube amplifier.

The chief defects of the continuous induction system are the necessity of modifying existing track circuits (but without detriment to the roadside signals which they control) and the requirement of exceptionally sensitive receiving apparatus on the train. The outstanding advantage is a degree of safety superior to anything attainable with any intermittent system, and fully equivalent to the continuous track circuit as used for the control of roadside signals. The system is, in fact, merely an adaptation of the standard continuous track circuit to the conditions introduced by placing the relay on the train instead of beside the track. And it is superior to the stationary continuous track circuit in that its indications are immediately effective on the train when a dangerous condition arises, while the indications of a stationary signal even when perfectly observed and acted on are not available until the train comes within view of the next signal. An incidental advantage is adaptability to the exercise of any desired control or signaling functions on the train by the use of distinctive indications from the track. In particular it is perhaps the only one of the systems enumerated which is capable of giving adequate block signal protection on the train without the use of roadside signals. As an accessory to roadside signals it is unlikely that it can ever be the cheapest system. If it is selected it will be on grounds of superior protection. But as a substitute for roadside signaling, it would not be surprising if it should prove the cheapest as well.

A very large number of other train control schemes of various degrees of merit has been proposed; but the bulk of the systems now being actively advocated falls within one or another of the three classes outlined above; *i.e.*, intermittent control by mechanical and electric contact, intermittent control by electromagnetic induction, or continuous control by induction from the track circuit.

CONTROLLED FUNCTIONS

Thus far we have considered chiefly the means by which a controlling indication is transmitted to the train; but with any of the systems mentioned there are varied possibilities as regards the kind of control to be exercised after the indication has been received. Discussion of these possibilities will be facilitated if we consider for a moment the conditions that exist in an ordinary block signal system with roadside signals.

In Fig. 1 is indicated a stretch of track protected by semaphores 1, 2, 3, 4, 5. The track from 2 to 1 being clear, signal arm 2 is vertical indicating "proceed". Just ahead of the signal 3 is a standing train. This arm is therefore horizontal, indicating "Stop and proceed under control prepared to stop short of an obstruction in the block". Obviously it is useless to display a stop signal unless the signal can be observed in time to stop the train short of the danger which it announces; *i. e.*, in the case illustrated, at the signal itself. Hence the next signal in the rear, (No. 4), is set at 45 degrees indicating "Approach next signal prepared to stop." The first clear, or "proceed" signal in the rear is No. 5, two full blocks behind the obstruction, and this is the closest spacing on which trains can follow one another while running under full-speed signals. The length of the blocks may be two or three miles or more; but when signals are located for the maximum safe traffic it is made about equal to the braking distance for the fastest trains. Trains operating under full-speed signals are then separated by twice the safe braking distance or more.

If an intermediate signal is placed between each two signals of Fig. 1 with overlapping control, so that the caution indication from signal No. 3 extends back to signal No. 4 as before, we get the arrangement shown in Fig. 2, with the clear signal advanced from No. 5 to No. 4A and the minimum full-speed spacing reduced to 1½ times the maximum braking distance. By further subdivision of blocks with overlapping control the spacing could theoretically be still further reduced, with the actual distance inherently necessary for safety as a limit; but in practise the number of signals required would soon become prohibitive. If each caution signal is further made to indicate the speed corresponding to the braking distance between it and the stop signal ahead, slower moving trains can close up to a shorter spacing corresponding in each case to the safe braking distance for the speed at which the train is moving.

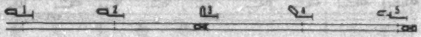

FIG. 1

If automatic control is applied to the signal layout shown in Fig. 1, it must take effect at the caution signal No. 4. It would do no good to apply the brakes at the stop signal No. 3, directly behind the standing train. But before reaching the signal No. 4 the engineer has had no intimation that he is required to stop; nor do the present operating rules require him to apply brakes at the caution signal. The present practise without automatic control is to apply the brakes only as required to stop the train at the signal No. 3. A train running at moderate speed may traverse half the length of the caution block or more before a brake application is necessary. An unconditional automatic brake application at the signal No. 4 would thus take the control out of the engineer's hands without previous warning, while he is still operating it in accordance with safe practise. This violates the principle that the engineer must retain full responsibility so long as he conforms to the signal indications. Two alternatives, then, are open. Either the signal No. 4 must be made a second stop signal with a caution indication at the preceding signal No. 5, as shown in Fig. 3, or the automatic action at the signal No. 4 must be made dependent on the action taken by the engineer at the same point or beyond.

In the former case it is evident that the minimum headway for full-speed signal indications is increased from two blocks to three, and the potential traffic capacity of the track is correspondingly reduced. But it should be noted that the traffic capacity of this particular section may still exceed the limit established for the whole operating division by conditions other than headway on the main line. It may easily happen in some cases that the traffic restriction here referred to would not in practise reduce the number of trains which can be operated over the whole division. In this connection it is interesting to note that extremely heavy traffic was operated for years in the New York subway system under exactly the conditions described; and when the growth of traffic demanded increased capacity, the need was met by improved air brakes which reduced the braking distance, and by special signaling in the station approaches (which were the actual limitation on track capacity) to permit closing in of trains at reduced speed.

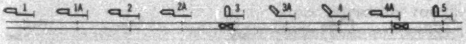

Fig. 2

The second alternative mentioned was to cause a conditional instead of an unconditional brake application at the caution signal. Several methods of accomplishing this conditional control are proposed.

The simplest of these is a cut-out under the engineer's control or the joint control of the engineer and fireman. If the caution signal is acknowledged while passing it, by operating the cut-out, the train proceeds under manual control without interference. In other words instead of actually enforcing train operation in conformity to the signal indications, acknowledgment of caution signals is enforced, on the theory that an engineer sufficiently attentive to acknowledge the signals can be depended on to take the necessary action when required. But acknowledgment of a signal does not absolutely assure that it has been correctly interpreted. It would be possible for the engineer to operate his cut-out at every signal, regardless of its indication.

Another proposal involves a modification of the present operating rules. A definite speed limit is established for the caution block, and the engineer on passing a caution signal is required to reduce his speed immediately to the prescribed limits. If he makes a sufficient brake application for this purpose promptly at the caution signal, the automatic control apparatus does not interfere; but if he fails, the automatic control does it for him. In one system a second indication for full stop is provided at a sufficient distance from the stop signal to bring the train to rest from the speed prescribed for the caution block. This plan restricts train movement in so far as it enforces a speed reduction earlier than safety and the present operating rules require. In the case of long blocks this may become a substantial restriction. The scheme appears to be safe in principle.

Still another method is to hold the automatic brake application in suspense from the time the caution indication is received to the time the brakes must be applied to stop at the stop signal. The actual time of application varies with the speed of the train, just as it does in manual control. In other words, the point of brake application varies in accordance with the curve of braking distances for various speeds. For convenience we can designate this as the "braking curve" type of speed control. Several methods of determining the point of brake application have been worked out. In one of these the caution indication is repeated at diminishing intervals as the stop signal is approached, the distance between each two successive indications corresponding to a certain time interval when the train is moving at the greatest speed permissible at that location. Two indications received in less than the established time interval show that the speed exceeds the limit set in the spacing of the indication points, and cause an automatic brake application. This principle has been applied for establishing speed limits at certain points in the New York subway system. The successive indications in that case took the form of roadside signals located at short intervals, with an automatic stop arm associated with each signal. The time intervals were measured by the relays which controlled the signals.

Another method involves a cam driven from an axle of the train. As the train proceeds through the caution block the cam establishes diminishing speed limits, and a speed-indicating element causes the brake application whenever the actual speed exceeds the limit established by the cam.

Still another method comprises a flywheel retarded periodically from a speed proportional to that of the train. The angular movement of the flywheel during retardation is a measure of the distance required to arrest the train; and when the distance remaining to the stop signal has been reduced to the indicated braking distance, the brakes are automatically applied.

All speed controls of the "braking curve" type require the automatic caution indication to be effective at a uniform distance from all stop signals. This type in common with the type based on limited speed in the caution block restricts train movement in one respect. In manual operation it often happens that an engineer after passing a caution signal finds the succeeding signal at caution instead of stop when it comes in view. The train which caused the original stop and caution signals has passed on to the second block ahead. Under these circumstances he accelerates again, and prepares to stop at the second signal. But in the case of any automatic cycle initiated by an intermittent indication from the track, the cycle would continue automatically to completion. The train would stop, and then proceed in accordance with the signal received at the next indicating point.

In connection with Fig. 2 it was shown that the headway of trains under manual control can be substantially reduced by subdividing the signal spacing determined by the braking distance of the fastest trains. The same thing applies with equal force to automatic control; and in fact involves less added complication than in the case of roadway signals; for the semaphores themselves, with their supporting masts or bridges are a major item in the latter case. Multiplication of signals would soon become prohibitive. In the case of automatic control on the other hand the increased equipment consists chiefly in the added circuits, with their associated relays, etc. Furthermore the number of speed indications which can be intelligibly given by semaphores is limited, while any reasonable number of indications can be provided for in the locomotive cab. Fig. 4 illustrates a case in which the maximum braking distance is subdivided into three indicating sections. In each section the speed is limited to a value which permits of stopping the train between the exit of that section and the entrance to the occupied block. At the entrance to each section an unconditional stop is assumed for any train running at a higher speed than the limit effective in that section. A warning (corresponding to a caution signal) is therefore necessary at the entrance to the preceding section. The warning signals are represented in Fig. 4 by circles inclosing the speed to be indicated. They may be roadside signals or signals displayed in the cab. The example illustrated gives a minimum headway of $1\frac{2}{3}$ blocks for operation under full-speed signals, as compared with two blocks for manual control in Fig. 1, and three blocks for unconditional stop with one block overlap in Fig. 3, with shorter headway for lower speeds. Theoretically, the headway could be further reduced by one-third block by combining the subdivided blocks with any of the plans already discussed for qualified control initiated at the caution signal.

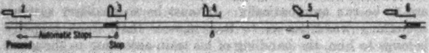
2
3
4
5
6
Automatic Stops
Proceed
Stop

Fig. 3

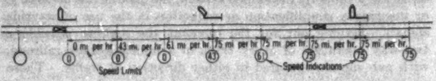
0 mi. per hr.
43 mi. per hr.
61 mi. per hr.
75 mi. per hr.
75 mi. per hr.
75 mi. per hr.
75 mi. per hr.
0
0
0
43
61
75
75
75
Speed Limits
Speed Indications

Fig. 4

To illustrate the relation of the various control cycles here discussed to headway and traffic capacity, the headway attainable under certain assumed conditions has been tabulated in Fig. 5. Headway is expressed in seconds rather than feet in order to show more directly the bearing on the potential traffic capacity of the track, in trains past a given point per hour. The conditions assumed are:

Time counted from the head of one train to the head of the following train.

Train length, 1000 ft.

Maximum speed 75 mi. per hr.

Train speeds 40, 60 and 75 mi. per hr.

Retardation 1. 5 mi. per hr. per second.

No allowance for time consumed in operation of semaphore signals.

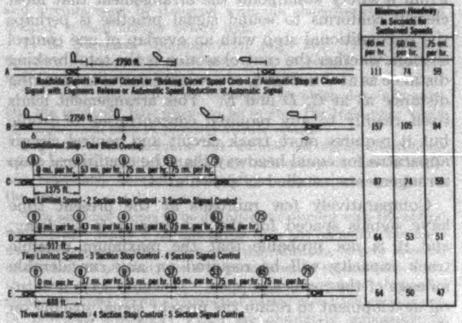
Minimum Headway in Seconds for Sustained Speeds
40 mi. per hr.
60 mi. per hr.
75 mi. per hr.
A
2750 ft.
Roadside Signals - Manual Control or "Braking Curve" Speed Control or Automatic Stop at Caution Signal with Engineers Release or Automatic Speed Reduction at Automatic Signal
111
74
59
B
2750 ft.
Unconditional Stop - One Block Overlap
157
105
84
C
0
0
0
53
75
0 mi. per hr.
53 mi. per hr.
75 mi. per hr.
75 mi. per hr.
1375 ft.
One Limited Speed - 2 Section Stop Control - 3 Section Signal Control
87
74
59
D
0
0
0
43
61
75
0 mi. per hr.
43 mi. per hr.
61 mi. per hr.
75 mi. per hr.
75 mi. per hr.
917 ft.
Two Limited Speeds - 3 Section Stop Control - 4 Section Signal Control
64
53
51
E
0
0
0
37
53
65
75
0 mi. per hr.
37 mi. per hr.
53 mi. per hr.
65 mi. per hr.
75 mi. per hr.
75 mi. per hr.
688 ft.
Three Limited Speeds - 4 Section Stop Control - 5 Section Signal Control
64
50
47

Fig. 5

The following train is assumed to approach each signal just as it assumes the permissive indication for the speed at which the train is running. The permissive indication for manual operation is considered to be the "proceed" signal at all speeds; for although it is permissible to pass the caution signal at speed, it is not permissible to maintain the speed throughout the block, unless the following signal is visible before the time comes to apply the brakes; and the range of visibility varies with location and weather. In most cases of manual operation at speeds below the maximum, a smaller headway than the table shows can be maintained by running under caution signals. The same is true of "condition" control by continuous induction without reservation as to visibility.

Line *A* refers to manual operation with signals spaced at the maximum braking distance. The same headway is required for any of the "conditional stop" arrangements with the same signal spacing and the stop indication at the caution signal. *B, C, D,* and *E* are for one section overlap with an unconditional stop effective at the rear stop signal and the necessary caution signals back of the stop. They show respectively one, two three and four automatic stop sections in the length of the maximum braking distance. With the subdivided blocks, progressive speed limits are provided. Line *C* with the braking distance subdivided in two sections gives the same headway at the higher speeds as manual control without overlap. The smaller subdivisions permit reduced headway. Lines *B, C, D* and *E* respectively require two, three four and five distinctive control and signal indications.

CONCLUSIONS

The substance of the matter seems to be that practical means are available for any kind of control likely to be required in automatic territory, though only a few of the arrangements described have been worked out in actual railroad service on any considerable scale. From a safety standpoint the arrangement that most clearly conforms to sound signal practise is perhaps the unconditional stop with an overlap of one control section, whether the control section is the total braking distance as at *B* in Fig. 5, or a subdivision of the braking distance as at *C, D* and *E*. This arrangement lends itself readily to any required concentration of traffic but it requires more track circuit and control circuit apparatus for equal headway than the conditional stop arrangements installed as shown at *A*.

Comparatively few railroads at the present time have signals spaced for the minimum safe headway, and it is not probable that the maximum possible track capacity will be required on any considerable mileage of the roads equipped. It would seem a natural development to retain the present signal spacing on the major portions of the roads, either with unconditional stops and one block overlap, or with conditional stops effective at the caution signal; and to provide the needed capacity at points of special congestion by relocation of signals when they are not now spaced at the minimum permissible distance, or by subdivision of the blocks where signals are already so spaced. Where extensive changes of existing signals are required, it would be worth while to consider whether the same result can be more economically obtained with cab signals.

With automatic control in force, it seems likely that the future trend of signal development will be toward cab signals rather than roadside signals; first because they are inherently capable of giving greater protection, and secondly because they involve less apparatus over and above that required in any case for the automatic control. If this is true, the railroads may find it desirable in making their present automatic control installations to give some consideration to possible future requirements for cab signals.

Discussion

Frank J. Sprague: I wish to refer to the tests which have been going on on the New York Central Railroad since the 20th day of February, a year ago, on so-called automatic train control. What is the object sought in train control? Simply to marry together the traffic conditions which exist on a railroad, no matter how indicated or governed, whether by manual signals or by automatic signals, with the braking system of a train in such manner as to insure the complete protection of that train movement, the reinforcement of any wayside indications, the amplification of those indications in the cab of the engine, or their replacement, without encroaching upon the prerogatives of the engineer when properly exercised, and without limiting the capacity of the railroad operation.

That means not an "automatic stop" in its simple form, because it has little value on a modern railroad system. It has to be placed either too far back, so as not to interfere with traffic, or too far forward, so that it will not itself protect traffic. Nor must there be an automatic system, in the sense that trains are to be run automatically. It must be an auxiliary system, something which leaves to the engineer that which properly belongs to him, the handling of his train according to the great varieties of conditions that exist in operation, all the while protecting him, but in no case unnecessarily interfering with him.

Therefore the system should be called an "auxiliary" and not an automatic, system of train control, one where operation is automatically initiated, but under the control, so far as proper, of the engineer himself.

There have been several systems mentioned. They may be divided into two broad classes. First, intermittent; and, in this class, we have the rail-contact ramp devices, also the inductive method of initiating impulses on the locomotive, either by induced electromotive forces in the control circuits, or by direct magnetic action; and then there is on the other hand the so-called continuous system, illustrated by wireless or by the picking up of induction from the rails themselves, as well as other methods unnecessary to mention at the present moment.

Train control has had a hard road to hoe, as those who are interested in it know. There have been over a thousand proposals before the United States Government, and one of the leading signal engineers of the country said there had been over four thousand proposals before the railroads.

Opponents of train control have largely ignored the possibilities of a properly developed system, the making less necessary, or insuring a simplification or reduction in the use of wayside signals,—leaving out for the moment the making possible a two-way operation of every track of a multiple-track railroad, with cab signals operated in either direction of traffic. Those are perhaps the two larger results to be obtained, not merely the humanitarian one.

The United States Government, however, has looked more particularly to the matter of safety, and the Interstate Commerce Commission recently has ordered 49 railroads in this country, to each equip, within two years, a complete engine division in certain specified localities. Their order covers only a very small proportion of the mileage of the large railroads, but this work has got to go ahead now and in the course of two years there will be a goodly number of systems in operation.

Some will bring, no doubt, disappointing results to the railroads, and with the possibility of inducing dangers which they were intended to eliminate. Some of these cannot be avoided, but we hope, as to others, the results will be satisfactory. Up to the present the few systems in use in this country are of the so-called rail-impact type.

This has one fundamental and serious objection, which I will specify and illustrate by reference to three exhibits laid before the Government at Washington. It is based upon the idea that at every block over which a train passes there must be operated a shoe whose object it is to apply the brakes, but such application is to be prevented when the block is clear, by the supply of electric current to that particular ramp rail.

You would be surprised to see and know perhaps how few times automatic operation should be called into play. I will illustrate very briefly by facts taken from the official records:

On three railroads, which I will designate as A, B and C, we have the ratio of needed brake applications to the number of blocks passed over as follows: A, 1 in 414; B, 1 in 1491; and C, 1 in 37. If you have an apparatus which is operated every time you go past a block, it is operated a hundred or more times, as frequently as called upon by adverse traffic conditions.

The great difference between that kind of a system and the induction system, no matter of which type, is that the apparatus operates only when it is required, and not the many times not necessary. In other words, it is required probably on an average of perhaps once in one hundred blocks, based upon the average density of train service in the United States.

The Joint Committee of the American Railroad Association and the Interstate Commerce Commission about a year and a half ago started to have certain sundry tests made in this country, and among others there was one initiated by my own company, carried on on the New York Central Railroad on the No. 2 track, the same track used by the 20th Century. It started a year ago and for the past two months it has been under closed sealed tests. Some idea of the intensity of that test may be indicated when I state that the number of brake tests have been from thirty to five hundred as many as occurred on the roads above mentioned, and the brake applications made during the two months just closed is what would have occurred, in average density of operation, on a train five times encircling the globe at the Equator.

Mr. Blake's paper omits reference to what has been done over a very considerable period, and those facts should ultimately be placed upon the record.

Azel Ames: It is to be regretted that Mr. Blake uses the word "engineer" throughout to denote the locomotive operator rather than the official designation of "engineman". It is suggested that the terms "main line" and "terminal" territory would be more precise than the terms "block signaling" and "interlocking" territory used in paragraph 9 for the reason that nowadays automatic blocking is carried through all main line interlockings and automatic train control would naturally be carried through them when installed on any given section of main line. In terminals where slower speeds are called for, automatic train control would of course be less necessary and vastly complicated.

The author points out the superiority from the safety standpoint of "continuous" as compared with "intermittent" control and also the greater facility which "continuous" control offers for the installation of cab signals. On the other side it may be said that with existing types of continuous control considerable inconvenience is likely to be encountered especially with work or construction trains due to the impracticability of pushing cars, snow plows, or other types of equipment ahead of the engine. It is also well to remember that while the principal protective feature of "continuous" control lies in causing a stop application in the event of a switch being opened or a train pulling out of a siding after the approaching train has entered the block, we cannot be sure that these untoward events will always happen when the approaching train is full braking distance, or even half that distance from the point of obstruction.

In noting that all speed controls of the "braking curve type" require the automatic control indication to be effective at a uniform distance from all stop signals, the author is undoubtedly assuming level track conditions, as the distance would vary with the grade of the approach to the stop signal.

Mr. Blake's paper refers principally to the signaling and train spacing features of the automatic train control problem and to the various means employed to produce an impulse or effect in the engine cab as the result of certain conditions upon the roadway. It would have been very interesting if he had touched upon that much neglected phase of the problem, namely, that of braking. For years discussions of automatic train control have seemed to assume that if a magnetically controlled valve somewhere in the braking system could be opened to permit a reduction of air pressure when it was desired to stop a train, the problem had been solved. Our present highly efficient automatic brake systems have been designed entirely for manual operation and many of their characteristics might be quite different if they had been designed for automatic control. Perhaps the most serious phases of the automatic train control problem after all are the great variety in the braking power of trains, especially freight trains, the large time elements involved in application and release in long trains, the necessity for release to accomplish a second application on long down grades and the enormous cost of changes in the existing equipment.

Most of these are matters for mechanical rather than electrical solution, but it is very desirable that the electrical engineer studying automatic train control should realize that it is the brake that must stop the train and that his electrical apparatus and circuits must be designed with the characteristics of the braking system always before him.

T. S. Stevens: Quoting from page 27: "The primary function of automatic train control is, of course, to stop the train before it enters upon unsafe territory."

This has been true with reference to older developments, but the tendency now seems to be to stop the train only when the engineman fails to properly reduce speed.

Page 32: "Furthermore the number of speed indications which can be intelligibly given by semaphore is limited while any reasonable number of indications can be provided for in the cab."

This sentence seems to be definitely at discord with the actual conditions. While I do not agree with the signaling methods used on a considerable mileage of the railroads, largely because of the lack of necessity for a large number of speed indications, the facts are that on very many railroads, where three-arm signals are used, quite a number of varied speed indications can be provided intelligibly which are absolutely impossible to provide in the locomotive cab. So far, if the stop indication is included, the manufacturers have been able to provide four speed indications: high, medium, slow and stop. As a point of fact the majority combine the slow and stop indications so that only three are provided.

M. E. Pipkin: Mr. Blake's paper gives the impression that the author considers continuity of indications on the engine a highly desirable feature in train control, if not an absolute essential. In this connection, it is interesting to compare the situation with that which obtains in the present roadside signal systems. In the roadside systems and in all of the automatic systems described by the author the track is divided into definite blocks and the control required is with definite relation to these blocks. Specifically the requirement is that the train stop at a definite point, namely at the entrance to an occupied block. Furthermore, the indication that a stop is required becomes effective in all cases at a definite location namely, the beginning of one or another block section. In the case of roadside signals, the necessary warning is given at the logical point, namely the point where it becomes effective. In other words the fixed roadside signal is essentially an intermittent indication confined to a definite location for each block, precisely as is true in the case of the intermittent control systems described. In both cases the control of the indication including track circuits and control circuits from the opposing train back to the actual indicating point is properly continuous, but the indication itself is intermittent.

E. J. Blake: Mr. Sprague's discussion emphasizes effectively two points mentioned in the paper; the necessity for avoiding anything that will supersede the engineman when he functions right, and the insufficiency of the plain automatic stop when traffic capacity has to be maintained. I agree with him that we can expect, and should provide for, simplification of roadside signalling and increased facility of train operation, as well as increased safety with the adoption of automatic train control. That has been the history of roadside signalling. It was introduced as a safety measure, but has become recognized equally as a measure for facilitating train movement. The particular case cited by Mr. Sprague is not altogether peculiar to automatic control; two way operation of multiple tracks has been provided for in a number of cases by the use of roadside signals.

Mr. Sprague does not make wholly clear just what is meant by the "ratio of needed brake applications to the number of blocks plassed over." If he means that the engineman disregarded stop indications so that automatic control should have come into play in one block out of every 414, or 1491, or 37, the last figure seems surprisingly small. In any case we can agree that actual operations of the automatic control necessitated by failure of the engineman ought to be exceedingly infrequent. But it does not seem especially important what the control apparatus does the ninety nine times it is not required (assuming that it does not interfere with the operation of the train). The essential thing is certainty of operation in the hundredth case when it is needed. We have a somewhat similar case in roadside signalling. The signals ordinarily operate at the passage of every train, irrespective of the presence of a following train for which the signal indication is required. It is to be hoped that the results of the New York Central tests to which Mr. Sprague refers will be published in due time.

Major Ames's comments on the terminology of the paper are well founded. "Engineman" "main line" and "terminal" territory are more precise than the terms used in the paper. I cannot agree with him that the principal advantage of continuous control from a safety standpoint is response to danger arising after the train has entered the block. It has this advantage, but as Major Ames points out, nothing can protect a train against a danger that arises too near it to permit of stopping. The fundamental claim of the continuous systems to superior safety as I see it is the fact that they can be arranged to respond to *every* cessation of energy as a danger indication, just as is true of roadside signal apparatus. Every intermittent system necessarily responds to some *action* at the indicating point so that absence or failure of the action is, in effect, a false proceed indication. Protection for equipment pushed ahead of the locomotive is a real difficulty; but in the case of work trains at least it would seem reasonable to operate with the automatic control cut out, just as they are frequently operated under "work orders" without regard to fixed signals or the direction of traffic.

The reference to uniform location of control indications for "braking curve" control should have been qualified as suggested with regard to grades.

Major Ames points out that proper air brake control after suitable traffic indications have been received is a serious problem in itself. This is true in some degree for all control cycles, and particularly, for the cycles based on slowing down to a prescribed speed at caution signals; for in this case the automatic device must check the sufficiency of the brake application even when it is initiated by the engineman. But it should be borne in mind that the automatic control is intended to stop the train only as a last resort in the infrequent cases where the engineman fails to do so. Under the circumstances, rough and even dangerous handling of the brakes might be tolerated in preference to continuing to run unchecked into the obstruction.

The first passage quoted by Mr. Stevens, when read with the sentence which follows it seems to say substantially what Mr. Stevens says in his comment. But the second passage (page 32) may be unintentionally misleading. It is true that the number of simple and desirable aspects available with semaphore signals is limited because the semaphore (and its light indications at night) must be read at long range and under unfavorable conditions; and it is true that no such limitation exists for cab signals. As an extreme illustration the engineman is able to distinguish clearly seven hundred and twenty distinct minutes in twelve hours on the face of his watch. But it is not true that unlimited indications can be transmitted from the roadway to the train without very great complications. Practical developments to date have been confined to the simplest available circuits giving three or at most four indications on the train as stated by Mr. Stevens. It is likely that these indications will suffice for a great majority of future installations; but it is far from impossible to give a much larger number if the need should justify the cost.

Mr. Pipkin cites an interesting parallel between train operation under roadside signals and under automatic control, and concludes that intermittent indications on the train are a logical result of the subdivision of the track into blocks at definite points. The analogy is not wholly valid because the roadside signals are commonly visible through a considerable part, and often through the whole of the block. In the latter case the indications received on the trains are strictly continuous, notwithstanding the intermittent location of the signals themselves. Furthermore the point at which action is required to bring the train to a stop at the entrance to a block is not definite but varies with the speed of the train. An engineman approaching a stop signal often sees the signal change to caution before he is obliged to set the brakes; or sees the signal change before the train has come to rest, releases his brakes and proceeds. In either case he is taking advantage of a continuous indication, extending back to the visible range of the signal. It is the universal custom to locate roadside signals with a view to getting the longest practicable range of visibility—that is to give continuous indications to the engineman as nearly as possible; and even where the range is shortest the indication is by no means comparable with an instantaneous impulse during passage by a definite point.

Discussion of Application and Economics of Automatic Railways Substations

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Review of the Subject.*—The automatic substation is defined, and its history is traced from the first station placed in service in 1914 to the present time. The number of automatically controlled railway stations up to the beginning of 1922 is given as 142 equipments totaling 86,000 kw. Outside the city of Cleveland, two-unit stations have been installed in but four instances, although a number of these stations are now under construction. With manually operated stations, especially in interurban work, the location of stations is determined by local conditions, availability of labor, etc. rather than by load conditions. But with the advent of the automatic substation it is possible to locate stations more nearly at the proper points to take care of the system loads giving improved voltage conditions with a minimum of feeder copper. Accessory advantages, in addition to the labor saving item, include a reduction in running light losses, a reduction in distribution losses, lower cost of substation and, in some cases, an actual retirement of existing feeder copper. An example is cited of a double track line 80 miles long with figures from a report which shows a net annual return on the investment, including 6 per cent interest, of 34.9 per cent. The labor saving is based on two twelve-hour shifts, and it is further calculated that with three eight-hour shifts the annual saving would be increased by an appreciable figure. The Des Moines City Railway was the first system to install a group of automatic substations for city service, while the Cleveland Railway Company is given credit for installing the first two-unit automatic substation for metropolitan railway service. Reasons are advanced for using two-unit stations in Cleveland in contrast to the general practise of employing single-unit stations in interurban work. Emphasis is laid on the necessity for absolutely uninterrupted service in a large city system, even under such emergencies as either an interruption of the a-c. power supply, or short circuit due to breaking of the trolley wire. A further comparison between the operation of manual and automatically controlled substations is taken from one of the surveys made in Cleveland. The tabulation indicates a net annual saving of $13,900 per year for the automatic over the existing stations.*

As to the reliability of automatic control equipments, conclusion is reached that an operator is very unlikely to anticipate trouble, whereas with automatic control the protective equipment operates so successfully that various interruptions are a rare occurrence. There is also the possibility of an operator making an error in judgment, thereby causing trouble which might not otherwise occur. A further consideration which should not be overlooked is the possibility of loss of operators due to strikes or other causes. An outline of the equipment used in the Cleveland stations is given with special reference to outgoing feeders and the proposed method of eliminating current-limiting resistance. Description is also given of the remote control equipment now being manufactured, which will enable the several automatic stations to be supervised from a central point by a dispatcher.

PERHAPS for the benefit of some of those who have not had the opportunity to follow this development a definition of an automatic substation will not be out of order. Messrs. Allen and Taylor define the automatic substation as a substation in which the functions of starting and connecting machines to the line whenever there is a demand for power and finally shutting down, after the demand for power has been satisfied, are all performed in their proper sequence without the assistance of an operator, either in it or at adjacent stations.

Further, it might be added that to make this scheme possible, there must necessarily be arrangements made for taking care of the equipment in such a substation, and protecting service under all of the operating conditions which railway power equipment is subjected to.

The first successful installation of an automatic railway substation, as far as I am aware at least, is the installation of a 300 kw. 600 volts, 25 cycles, 3-phase, synchronous converter on the Elgin & Belvedere Electric Railway, an interurban road. The equipment in this substation was rearranged for automatic control and placed in service December, 1914.

The idea of automatic control for this station, as I understand, originated with an engineer in the employ of one of the large electrical manufacturing companies, who upon presenting the idea to his company was informed that it was impractical. He, however, thinking otherwise, broached the subject to Mr. B. J. Arnold, who, I believe, was interested in the Elgin & Belvedere Railway Company. In any event, after securing a miscellaneous assortment of relays, contactors, and such other equipment that was applicable to this control system, and was at that time more or less standard in steel mill control work, the station referred to was put into successful operation.

Needless to say, after the practicability of the idea had been demonstrated, the manufacturing companies adopted it. It is, therefore, noteworthy that the history of the automatic substation does not extend over an appreciable length of time. But, in this period there has been a wide adaption of the scheme throughout this country.

A summary of automatically controlled synchronous converters and motor-generator sets in railway service up to this time a year ago, discloses the fact that there are some 80,750 kw. of converters and some 5200 kw. of motor-generator sets in railway service automatically controlled. The capacity is distributed over 129 installations and involves a total of 142 equipments. The majority of these being stations containing a unit of 300 to 500 kw., although there are some 1000-kw. converters involved. Of the 129 installations there are but four cases, except Cleveland, where two units were used in one station. There are, of course, numerous

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recent installations not included in this list which have been placed in operation since the tabulation was made. Today, I understand, there are about half again as many orders for equipments of this type upon the books of the two manufacturing concerns building this equipment.

There is as far as this discussion is concerned, three general types of electric railways. The first of these groups is the interurban system, with which as far as we are concerned may be included small suburban systems. In this type of service there is found by far the greatest number of installations of the automatic substation. The idea originated on an interurban system, as was noted, and it is in this class of service that the greatest savings are possible when compared with the manually operated plant. Further, the more infrequent the service the greater will be the proportional saving beyond the item of labor saved. A brief discussion of some of the principal items is as follows:

With manually operated plants in interurban work, it is too often the case that the location of the plants instead of being decided by the load conditions, is of necessity influenced by having to place the station either in or near a center of inhabitants wherein the station operator and his family may live, for without at least some of the conveniences of modern civilization, the railway companies are presented with the inability to secure competent help or are compelled to pay exorbitant wages to hold a sufficient number of competent men to operate the stations.

The necessity of having two or in some cases three shifts of skilled attendants for each station also influences the number and size of such stations, that is, the number of stations provided are held at a minimum, which in turn is reflected in the amount of feed copper necessary to connect the station with the load. Therefore, it is readily seen that if the human element could be eliminated from the operation of a substation, that the station could be located with respect to load only. It is desirable to have as many stations feeding a line as are economically possible, for by this method the feed copper may be held to a minimum, the operating potential of the line may be maintained, and a greater guarantee of service is gained by reason of the numerous points of supply. The approach to this ideal has been made possible by the advent of the automatic substation development. There are other items of savings referred to previously made possible by the use of this type of station on interurban lines. Principal among these items, is the elimination of light load running losses. These losses on an average interurban line may amount to considerably over a period of twenty-four hours where the headway between cars is infrequent. With automatic equipment the station will shut down after the car has run beyond the territory served by the station and the station will again start as the next car comes into the territory. On single-track lines with cars spaced an hour apart, and possibly meeting somewhere in the vicinity of the station, there are instances where the substation is required to run for only fifteen or twenty minutes out of an hour. This frequent starting and stopping of the equipment is not done if the station is manually operated, consequently with the automatic substation, the average load carried on the equipment may approach more nearly the point of highest operating efficiency. Distribution losses are reduced on account of the possibility of utilizing more stations and locating them where required without having to consider increased labor in operating costs. The cost of the substation building may be materially reduced, for it is unnecessary to provide the usual clearances around equipment, likewise, the apparatus may be arranged with regard to a minimum size of building, rather than to ease of operating as is necessary in arranging for manual control, also facilities for the comforts of the operator are eliminated.

The advent of the automatic substation for interurban work has resulted in several instances in a complete rearrangement of the power facilities of some of these lines, while on others, automatically equipped stations have been installed between existing manual stations, eliminating the necessity of installing additional feeder copper to care for increase in load, in several instances allowing copper to be removed, at the same time increasing the average trolley voltage and at no increased cost of operation. An instance where the power facilities of an installation could be completely rearranged at a considerable saving in operating cost is illustrated by the following summary of a report made on an eastern property. This property operates a double track line approximately eighty miles long.[1] The schedule is essentially one car each way every forty minutes. Local cars are operated at schedule speed of 23 mi. per hr., and limited trains at 30 mi. per hr. The power is supplied to the trolley at 600 volts d-c. from eight manually operated substations spaced approximately ten miles apart. There is a .0000 trolley wire over each track and in addition a 500,000 cm. feeder running the entire length of the line.

It was proposed to retain five of the present substation locations. Three of the present substations are to be moved to new locations and there will be four additional substations made up of equipment now used as spare apparatus in other stations. This will make a total of seven new buildings and eleven automatic control equipments to be supplied, together with high-tension switching for each station.

The balance sheet for this proposed installation of automatic control on this interurban is as follows:

Investment Required	
Seven new buildings..........................	$20,000
Auto. control, etc. high-tension switching..	125,975
Installing of switching equip. and reinstallation of rotaries and transformers....	16,300
Total..........................	$162,275

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[1] From report by Mr. C. A. Butcher, Westinghouse Electric & Mfg. Company.

Rearrangement of feeder system		$6,030	
Removal of 66.58 miles of 500,000 cm. cable at $50		3,330	
Extension of transmission system		4,350	
Gross Investment		$175,985	
Credits			
85-1/10 miles of 500,000 cm. cable at 18c. per pound	$123,000		
Salvage, other materials	6,000		
Total	$129,000		
Net Investment		$46,985	
Annual Savings			
Operators' wages eliminated		$22,330	
Power saved		2,860	
Other misc. substation savings		200	
Total		$25,390	
Annual Charges			
Inspectors' wages	$4,725		
Fixed charges, not including interest on investment	4,230		
Total	$8,955		
Net Annual Saving		$16,435	
Return on investment including 6 per cent interest			34.9 per cent
Return on investment not including interest			28.9 per cent

Total investment would therefore be retired in approximately 3½ years.

The labor saving shown in the table is based on two twelve-hour shifts of one man per each station. At the time this report was made an eight-hour day was in demand. This would increase the item of labor by a third and under this condition the net annual saving would be increased to $23,878. By this new arrangement of power and distributing system the trolley voltage at the car would be increased by twenty volts. This would result in an increase in running speed and therefore a possible decrease in platform expense, that is, train crews.

You may now realize that the automatic substation has adapted itself ideally to the interurban field where the savings made possible by its application are most apparent. Until about 1918 the automatic equipment was limited almost entirely to the interurban field. This was due, I believe, to the greater cost of the control equipment compared with manual equipment of the same capacity which was thought apparently could not be justified unless savings made possible by its application were without question large, also the scheme was looked upon by quite a few as more or less of a makeshift for use only on the small suburbans.

To Des Moines, Iowa, a city of 126,000 inhabitants belongs the credit of advancing the application of this type of station one step from the purely interurban service. The Des Moines City Railway operates 91.27 miles of single track and 150 cars of all types including interurban, had in operation in 1921, eight or nine automatic substations of 500 kw. capacity each, in city and interurban work. These installations, as I recall, represent simply the use of the interurban type of station to move city cars, and no new departures were made by reason of the different service.

The next step in progression was made in Cleveland. The Cleveland Railway Company has in operation today, three automatic installations which represent the first case on record of the adaptation of the automatically controlled substations to metropolitan service. These installations also enjoy the distinction of being the first two unit automatic substations of large capacities having been constructed. Each of these stations contains two 1500-kw., 60-cycle, converters. I might add that to date they are the only installations of their kind in existance.

While the application of automatic substation to the interurban system is now on more or less of a standard basis, variations being slight, and principally in kw. capacity, the question of applications on metropolitan systems does not lend itself to standardization nearly so readily. This fact must be realized when the great variety of conditions, principally brought about by operating practises, are considered on various large city systems.

When the problem of layout or rearranging of power generation and distribution facilities of a large system is considered, it will be found that there are numerous items which enter into and effect immensely the ultimate results, that do not appear in planning for the interurban system, and further the weight of these various problems regarding the whole, change, dependent upon what particular city one may be working in.

The various problems are closely co-related. Unfortunately the time available will not allow a full discussion of all the various phases, but, briefly, the results obtained in a survey depend primarily upon the importance of the station or stations to the balance of the system, that is, whether the station is located on the outskirts of the city, in which event it may be considered of relatively less importance than one located perhaps in a congested area serving possibly several important lines of transportation. The area or zone to be supplied by a given station, that is the current density, which also fixes the kw. capacity of the station, is in general determined by land value, building cost and restriction, reserve capacity, a-c. and d-c. feeders, investment necessary in building equipment and feeders, together with annual charges and losses. The paramount necessity, however, is that the system must be conducive of a high degree of reliability of service.

Still another factor referred to previously as "operating practise" has a direct bearing upon the question. It is this phase that tends to make it almost impossible to standardize or make comparisons of proper power facilities for metropolitan systems. By "operating practise" I refer particularly to the quality or type of service rendered by the operating company to the car riding public. An illustration will perhaps make this more clear. The principal source of trouble to the power department of a railway company comes from the falling of a trolley wire, bad order car equipments, and overloads due to traffic delays, etc. When a trolley wire breaks and grounds, it is of course disconnected from the source of power at the station by automatic breakers, that is, if manual stations are used. On a great number of roads it is the practise to hold these grounded feeders or sections open until the wire has been picked up either by the car crews or the line emergency crews. The power plant is then notified and power is restored. By this procedure the amount of trolley wire annealed or destroyed is held at a minimum as is also the matter of maintenance on station equipment. The great disadvantage of this method, however, lies in the fact that important lines are tied up for indefinite lengths of time, schedules are disarranged which on heavy lines may require the remainder of the day to rectify, and most important of all, thousands of patrons of the line are delayed, in some instances seriously.

The progressive management is really trying to produce the best possible service, free from interruption, realizing that to have cars standing in the street without power, does not, to say the least, improve public relationship.

For example, the management of the railway in Cleveland is most insistent in its demands that power be supplied under all conditions. When a short circuit occurs upon a feeder section, instead of leaving the section out or dead, until the trouble has been cleared, every attempt is made to maintain the power supply upon the faulty section, for, with power the cars continue to move. To make this possible, it can readily be appreciated that the layout of the generating and distributing system will be considerably more extensive, more complicated and expensive, than on a property where power service is discontinued when trouble arises on the line.

The question of the size (kw. capacity) and the number of units to be installed within the automatic substation in metropolitan service is one which also is receiving much discussion at present. There is no question but that to increase the number of points of supply to a system will result in a better average operating voltage at the car, and with a reduction in the amount of feeder copper necessary, also by reducing the potential gradient of the rails do much to eliminate electrolysis. It would, therefore, appear that the ideal layout of automatic substations should consist of a series of small single-unit installations located at relatively short distances from each other and in a very simple, small inexpensive building. With low land values and building cost, and the absence of building restrictions, which are encountered generally in small cities, sparingly settled areas or suburban districts of larger cities, there seems to be no doubt but that a number of single-unit installations answer the requirements admirably. On the other hand, where the cost of land and buildings is high and there are numerous restrictions to be complied with as is the case in Cleveland the multiple unit station works to the greatest economy, taking every item into consideration. On the metropolitan system where continuity of service is the one ruling factor, it is necessary (as was previously noted) to protect or make possible this continuous service by installing reserve capacity in equipment and distribution system. To exemplify this, take but one of the phases necessary for insurance to continuity of service, the a-c. power supply.

In small cities where underground construction of high-tension lines is not necessary, the cost of connecting the a-c. power supply to the numerous small single-unit stations is not much of a factor. On the other hand, in large cities, where underground construction is necessary, the cost of supplying all of the small stations with a-c. lines becomes an expensive matter. In this connection, the installation of duplicate a-c. feeders to a station of importance containing more than one unit is warranted, insuring as it does freedom from station "shut downs" arising from a-c. line failures. In the case of less important stations having but a single unit, this feature of safety to service is sacrificed, for the duplication of supply cable being in the case hardly permissible. Where the multiple-unit station proves the most efficient and economical type to install, the factor of service insurance is further increased by reason of the scheme of automatic sequence which is a part of this type of station.

On the average interurban system the load taken as a whole upon the system tends toward a flat characteristic, that is, each of the stations upon the system is subjected to numerous peaks during the period of operation of the line. These peaks occurring when there is a train within the territory supplied by any particular station and as noted previously as the train moves out of the territory, the station shuts down. On the large city system, the load is of an entirely different nature. There is a continuous load on the station which varies more or less gradually from the peak condition, morning and evening, to the base running schedule period. In the former case, that of the interurban, the power equipment is being worked to the highest possible over-all efficiency. In city service, however, the frequent starting and stopping of equipment is impossible, for with all stations upon the system arranged to feed into what results, by reason of emergency tie feeders, a general network of feeders, and with very low ohmic resistance (0.03 approximately) between stations, each and every machine in service will attempt to assume its portion of the system load. This tends to keep equipment in service when there is really no necessity for same. This would result detrimentally to the all-day station conversion efficiencies. There is, of course, a limit that can be established above which an increased average trolley potential will not reduce platform or other expenses materially. Beyond this limit it is obviously unnecessary to have converter equipment operating at the resulting low-load factor.

The two-unit station has an advantage in this connection, in that the two units will operate to capacity during the peaks, while the remainder of the day one unit will shut down, leaving the remaining unit in operation.

There were numerous reasons for the adoption of automatic equipment in Cleveland. Primarily,the company was presented with the necessity of supplying additional power to portions of the city lying within the district wherein, by reason of recent developments and natural traffic growth, the power supply was in some cases inadequate. It was found impractical to attempt to increase the operating potential in most of these territories by additional feeders from existing plants, for not only were feeders necessary, but additional conversion equipment would have to be installed at the various plants to absorb the increased load, together with losses. The alternative was, of course, additional power facilities, located within the territory in which the load originated. The results taken from one of the several surveys made to determine the most economical procedure with respect to a given territory, indicates the following:

Power supplied from existing plants locations		
Proportional part of building, together with new equipment necessary to be installed for the load.		$ 89,398
Total feeder copper (existing plus additional copper		274,251
Total		325,251
Annual charges		53,636
*Power supplied for manually operated substations**		
Building, land, equipment and feeders		$221,263
Annual charges		44,240
Operation	$ 8,290.	
Feeder losses	11,644.	
Power supplied for automatically operated substations		
Buildings, land, equipment and feeders		$242,129
Annual charges		39,735
Inspection	$ 400.	
Feeder losses	11,644.	

Basis upon which annual charges are calculated

Buildings	8¾ per cent	(Insurance, taxes, depreciation, interest).
Land	6 per cent	(Taxes, interest).
Equipment	10¼ per cent	(Insurance, taxes, depreciation, interest).
Maintenance		(Buildings, equipment).
Operation		or Inspector.
Feeder copper	9 per cent	(Taxes, interest).
Feeder losses		$11,644.

*Used for comparison purposes only.

This indicates a net annual saving of $13,900 comparing the automatic substation with the scheme of supplying the territory from existing plants.

In comparing the automatic with the manually operated plant of like capacity an annual saving of $4500 is indicated in favor of the automatic substation, notwithstanding the fact that $20,866 increase investment is made. Incidentally, a comparison of the cost of feeders in the first and second plants would emphasize the relation which this item bears to the economical location of power facilities with regard to load, particularly when working with low voltages. It might be of interest to note that if the present power facilities of the company were all on an automatic basis today, calculation indicates that a saving in operating expense could be made amounting to approximately $175,000 annually.[3]

With these possible savings in mind, it remained but to ascertain the reliability of automatic equipment. From operating records available it appeared that this equipment, where properly installed and maintained, was more reliable than the manually operated. From experience this is borne out.

This is easily seen when it is considered that the chances of an operator being able to anticipate trouble are remote. Abnormal conditions develop so quickly and are often so obscure that before he has the opportunity to realize what is occurring the damage is often done, or if not, his principal duty then will be to limit the extent of injury and to confine it to the particular piece of apparatus or circuit in which it originated. While he is so engaged service is, in all probability, at a standstill. With automatic control the equipment is protected as the trouble occurs, and is prevented from spreading. This is accomplished by the various protective devices peculiar to this type of control. Interruptions to service are, therefore, held at a minimum.

No matter how well trained the operators may be, there are certain errors of judgment that must necessarily enter into the operation. In large centers of population, where the tendency normally is for high labor turnover, there will always be a larger percentage of new operators on the system than in some of the less active cities. This element of inexperienced operators will result in operating mishaps, which are avoided by use of automatic control. In connection with the item of labor, there is the intangible but undoubtedly great value obtained in adopting automatic control in the assurance gained by the knowledge that labor difficulties cannot interfere with the success or continuity of service of a system so equipped.

As has been previously mentioned, the installation of the automatic substation in Cleveland, represents the first attempt to utilize this type of equipment in metropolitan service. These stations then represent pioneer work, there being no precedent. Utilizing the experience and knowledge of operating conditions in Cleveland, continuous studies were made of the adaptability of this type of control, as it was being planned, with the result that changes and additions were made in the layout prior to its manufacture, and in fact, while in the process of manufacture. These were necessary to bring the equipment to a point where it would meet with all of the operating conditions and where absolute dependence could be placed upon it under all conditions. The results have been exceedingly gratifying, for aside from the d-c. outgoing feeder control scheme, there have been comparatively few changes found necessary considering the immensity of the problem. Some of the changes necessitating the redesign, addition or rearrangement of relays, while the majority were accomplished by changing circuit connections.

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[3] Comparison made with present system, including steam plant

Referring to the control of outgoing d-c. feeders, the solution to this phase has been the most troublesome of the whole development. There was installed and in fact still exists in these stations, a type of feeder control, known as the current limiting resistance method of control, which has met with success in interurban and small city work. At the time of installation it was the only scheme available. With this scheme, in the event of a short circuit or overload occurring on the feeder, the current flow and voltage to the feeder is reduced by inserting the resistance. If this abnormal condition persists, the section is finally cut off from the source of power entirely. This method of operation on the property where a premium is placed upon continuity of service is not acceptable. Recently a new method of feeder control has been devised. This scheme eliminates the current limiting resistance entirely. By the use of a current transformer on the d-c. feeder, an arrangement is secured whereby the feeder will be disconnected only in the event of a short circuit occurring on the feeder. This is a step towards the continuity of service we wish to obtain, however, it still does not fill all the requirements, for, in our desire for continuous service, it is necessary that wherever possible, power must be supplied to faulty feeders for the movement of cars and to clear the feeder of fault. To accomplish this and other phases of operation which are desired, it has been necessary to devise a method of remote control. By utilizing an auxiliary or transfer bus in each station with feeders arranged for automatic transfer on short circuits only, a most flexible system is obtained by the addition of this control. Provisions are had for load shifting in case of severe load congestion, failures of conversion equipment, a-c. feeders, etc. This is accomplished by transferring load to adjacent plants over tie feeders, these same feeders furnishing the means of "burning off" grounds on faulty sections, thus replacing the present current limiting resistance.

A brief outline of the remote control equipment which is at present being manufactured, will be of interest, A dispatcher's office will be established at a centralized point from which there will extend two pairs of telephone conductors to each automatic substation. At the dispatcher's office, arranged in a semi-circular desk, each substation will be represented by a small panel containing a series of telephone lamps behind the customary lenses. On the table of this desk is to be arranged a series of buttons, controlling relays by which combinations may be set up sending impulses to any particular substation where, it will cause to operate certain relays which in turn perform required changes in the operating conditions of the stations. As these changes take place at the station, the dispatcher is advised by the signal lamps mounted on his desk. These lamps enabling him to ascertain at any time the positions of the controlling relays or contactors at any station. The functioning of these devices by reason of automatic operation will also set up signals at the dispatcher's board. To enable the dispatcher to know the load conditions of the stations, that he may act intelligently in cases of emergency, arrangement is had to duplicate in the dispatching center, continuous loading of each station upon a graphic chart. It might be well to state that this system of remote control does not supersede the automatic substation control in any sense. All the various benefits derived from full automatic control still remain. This system is so arranged that in the event of the remote control becoming inoperative from any reason whatsoever, it is automatically disconnected from the substation control leaving the station still under full automatic control.

The present plan of the railway company contemplates the use of automatically controlled stations for the entire property eventually. It is proposed where practical to break up the existing manually operated stations into units of one and two machines each. With this in mind, the remote control has been designed with facilities for increasing the number of station positions to thirty.

Regarding the future of automatic control: While I have limited myself to the railway applications only, the development has not been limited to this field. Engineers in other fields of electric generation and distribution have not failed to grasp the idea and apply it. There are several installations in 220-volt d-c. Edison service, likewise, there are several industrial applications. The idea has also been utilized in one or two instances in hydro-generating plants. In this particular field automatic control should, it seems to me, have as wide an application as in the railway field, for by the use of this control, thousands of small water power sights are made possible of development and may produce power economically whereas before, this was impossible.

Each and every application of automatic equipment, no matter in what field, will present a variation of control features by reason of new or different operating conditions met with upon each new application, but with the engineering talent and ingenuity displayed by the men heading the control divisions of the principal companies engaged in this particular field, together with the inclination of the engineer in charge of the property to cooperate and study conditions and requirements, the limitations of automatic control are not in sight.

Systems of Single-Phase Regeneration for Use with Series Type Commutator Motors

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Review of the Subject. *— The alternating-current series motor may be adapted to regeneration in a number of important respects more readily than a similar direct-current series motor on account of the following features: (1) The a-c. motor always employs a cross-field winding which prevents armature distortion. (2) A relatively low impressed commutator voltage is used reducing the danger from flashing. (3) With the a-c. system a transformer can be used to obtain desired variation in motor voltage. The advantages of this feature are outlined at length with reference to its effect on regeneration.*

The several schemes of regenerative connections are divided into four classes as follows: (1) Series excitation; (2) self-excitation, or cross excitation; (3) shunt excitation; (4) separate excitation. The first two connections have not been commercially used. A number of fundamental assumptions as to the theory of the single-phase motor are given as a basis for vector diagrams, which are used to analyze the conditions for both motoring and regeneration for the four types of connections. A detailed presentation of the conditions which obtain with these connections is then given by the author.

When using the first connection for regeneration it is necessary with series excitation to reverse the main field with reference to the motoring conditions. With the alternating-current series motor a sudden decrease in the line voltage will not increase the regenerated current as in the case of direct current, but will tend to decrease it. In a like manner an increased speed with any given impressed voltage will also decrease the regenerated current. Objectionable characteristics are: first, a strong tendency to pick up as a direct-current generator with the secondary transformer winding acting as a short circuit; second, a like tendency for low-frequency currents to be set up; third, undesirable speed torque characteristics. These features are cited as the reasons for not seriously considering this type of motor and connection for railway application with regeneration in this country.

The second system employs an extra set of brushes located midway between the normal brushes on the commutator. This system is handicapped by the fact that armatures and also the commutators of the motors must have increased capacity because of the necessity for carrying exciting as well as load currents during regeneration. This is particularly objectionable on account of the space limitations existing in connection with railway application. Another disadvantage is the necessity for extra brushes around the commutator. This method of connection, however, has the advantage of giving a shunt speed torque characteristic and permitting power factor compensation during regeneration.

The third connection using the so-called shunt excitation has the advantage of simplicity and reliability. The chief disadvantage lies in the fact that power factor correction cannot be made during regeneration. It is handicapped to some extent, due to the fact that the same continuous torque between armature and field winding cannot be obtained as was possible with the series connection. In a majority of commercial applications, however, this connection will provide sufficient tractive effort. Only two units are required to obtain regeneration: first, a substantial reactor and, second, a change-over switch for controlling the same.

The fourth system utilizes separate exciters either of the (1) constant speed or (2) variable speed type. Under the constant speed systems, either phase converters or motor generator sets may be used. With the variable speed, either separate axle generators or one of the main motors is used as an exciter. This system, although in general requiring more apparatus for regeneration (hence costing and weighing more) than some of the other systems, is also more flexible. The desired speed torque characteristics can be obtained and, with a constant speed exciter, power factor compensation is available as well as power for driving auxiliary apparatus.

The conclusion is reached that regeneration can be properly and successfully obtained with one or more of the several systems outlined. In the several appendices, explanation is made of the vector diagrams by means of which the author analyzes the conditions for each system of connection.

It has long been known that under certain conditions a motor driving an electric locomotive or car could be converted into a generator and that as such it could be used to exert a braking effect upon the vehicle either for "holding" a train while descending a grade, or for reducing the speed of the train when making a stop.

Numerous articles have appeared in the technical press during the past few years covering various systems of control for use in connection with three-phase, split-phase and direct-current systems of regeneration.

However, practically no mention has been made of systems of regeneration for use with single-phase series type commutator motors.

Probably the reader is better acquainted with the direct-current system at the present time than with any other type. Hence, in the following general discussion the writer will use the d-c. system for comparative purposes.

The Westinghouse Electric & Manufacturing Company tested a regenerative single-phase locomotive on its test track for several months at East Pittsburgh as long ago as 1905, and a few years later a similar system was applied to the Midi locomotive in France.

At that time there was practically no demand whatsoever for regenerative equipments in the commercial field, and the system referred to above was used primarily in connection with a number of important gear tests which were being made at that time. The regenerative feature was utilized to give a relatively cheap and flexible load, two similar locomotives being used during the test, one regenerating while the other motored.

The a-c. series motor lends itself to regeneration in a number of important respects more readily than the corresponding direct-current series type motor. This type of a-c. motor is always provided with a cross-field winding preventing armature distortion, and as it always has a relatively low impressed commutator

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voltage, the danger of flashing is practically eliminated.

The a-c. system also has the advantage that the existence of a transformer on the vehicle makes the variation of motor voltage very simple and economical. This, in turn, means that weakened fields at high speeds are not a necessity and that regeneration at practically all speeds down to a standstill can be accomplished with practically any desired field strength. This permits the obtaining of any desired torque with any desired speed, with the armature and field currents adjusted to give the minimum root mean square currents in either member. Thus, in general, it is possible to effect regeneration with a lower root mean square current in the armature and over a greater range in speed than is possible in the case of the direct-current system with its limited and less flexible voltage range; also, due to the presence of the transformer and due to the fact that connections can be completed inductively, as well as conductively, the a-c. series motor lends itself more economically to a larger number of regenerative connections than appears possible with the d-c. system.

The various schemes of regenerative connections available for use with this type of motor can be divided into four distinct classes, or combinations of these classes, depending upon the type of field excitation employed. The four classes can be designated as follows:

1. Series Excitation
2. Self-excitation, or cross excitation.
3. Shunt Excitation.
4. Separate Excitation.

The third and fourth classes have been commercially applied and detailed description of a few of the various forms of connections possible in these two classes will be given in the following pages.

In so far as is known by the writer, no commercial applications of the first and second classes have been made as yet, and only a general discussion of a single form of each of these two classes will be given.

For the sake of clarity it is necessary to give in a brief manner several fundamental assumptions which will act as a basis for the vector diagrams which follow. In all cases it is assumed that the electrical windings are such that:

1. A motoring torque will be exerted when the field flux and armature current of a given machine have a component 180 deg. out of phase with each other.

2. A braking or regenerating torque will be exerted when the field flux and armature current have a component in phase with each other.

3. Power will be taken from the line when the armature current has a component in phase with line voltage.

4. Power will be returned to the line when the armature current has a component 180 deg. out of phase with the line voltage.

5. A lagging armature current will be taken from, or returned to, the line when the resultant wattless current required for the armature circuit is lagging.

6. A leading armature current will be taken from or returned to the line when the resultant wattless current required for the armature circuit is leading.

The following well known fundamental laws will be used frequently and are stated in order that the reader may more easily follow the writer's various presentations.

(1) The sum of the voltages in any closed circuit must be equal to zero, that is, the sum of all the counter voltages must be equal and opposite to the impressed voltage.

(2) When a voltage is applied to a reactance, the current produced will lag 90 deg. behind the impressed voltage. The reactive drop, or counter voltage, produced by this current must be 180 deg. away from the impressed voltage in order to produce a balanced condition, and hence the reactive drop must lag 90 deg. behind the current that produces it.

(3) Likewise, when a voltage is applied to a resistance, the current produced will be in phase with the impressed voltage. The $R I$ drop, or counter volts, produced by this current must be 180 deg. away from the impressed voltage in order to produce balance, and hence, it is 180 deg. away from the current.

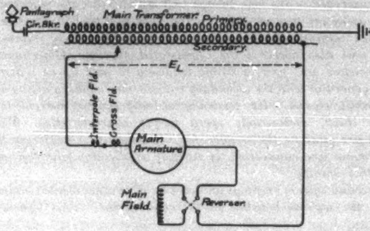

FIG. 1—SERIES MOTOR

(4) The counter electromotive force set up, due to a rotating conductor cutting a given flux, will be in phase with the flux cut.

Also, it might be noted that all vector diagrams are arranged so that any leading vector is always drawn counter-clockwise with respect to any lagging vector.

MOTORING WITH SERIES EXCITATION[1]

In line with the above the operation of the series motor as a motor assumes the form shown in Fig. 1 which is a schematic of a typical main circuit connection.

The transformer voltage applied across the motor terminals sets up a circulating current in the motor circuit. The motor circuit is inductive and therefore the current lags the impressed voltage. The circulating current thus set up produces a counter voltage equal and opposite to the impressed voltage. At standstill this counter voltage is composed entirely of the impedance drop around the armature circuit. The impedance drop consists of a resistance and reactance drop. The reactance drop is at right angles to the resistance drop. The resistance drop is 180 deg. away from the armature current and in phase with the field current.

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[1] See Appendix A for detailed Vector Analysis.

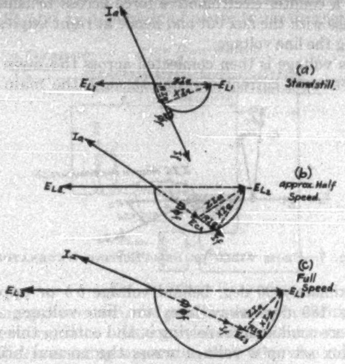

Fig. 2—Series Motor

As the speed of the motor increases a counter electromotive force is set up across the armature which adds directly to the resistance drop in the circuit. Hence at any speed except zero the resultant counter voltage is composed of the counter electromotive force as well as the impedance drop in the armature circuit. As this counter electromotive force grows in value with any given constant impressed voltage the impedance drop and hence the armature current must decrease in value to maintain the proper resultant value of countervoltage. Hence, with a fixed terminal voltage the armature current and consequently the propelling torque, decreases as the speed increases.

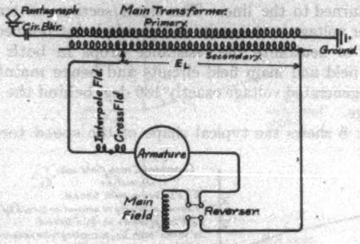

Fig. 3—Series Generator

Fig. 5 shows the typical shape of the speed torque characteristic obtained with the series connection.

Regeneration with Series Excitation[2]

In order to regenerate after motoring with the series connection the main field must be reversed. Thus Fig. 3 shows a typical main circuit schematic for this connection. The operation regenerating is similar to the operation motoring except that the field current, and consequently the field flux and resultant torque produced, has been reversed. Thus at standstill the counter voltages set up are exactly the same as in the motoring connections at standstill. However, as the speed increases from standstill the counter electromotive force set up is in the opposite direction and hence opposes the resistance drop but remains at right angles to the reactance drop. Thus at some definite speed where the counter electromotive force is just equal and opposite to the $R\,I$ drop the armature current will lag the line voltage by exactly 90 deg. since the only counter voltage available is the reactance drop around the armature circuit.

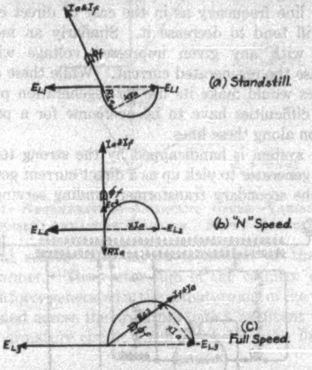

Fig. 4—Series Generator

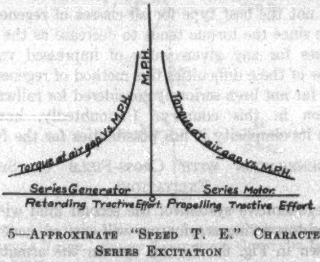

Fig. 5—Approximate "Speed T. E." Characteristic. Series Excitation

At this point the generator is supplying its own losses and hence is drawing no power from the line. As the speed increases above this value the counter electromotive force overbalances the resistance drop and the resultant counter voltage is equal to the vector sum of the reactance drop and the difference between the resistance drop and counter electromotive force and the resultant vector relations are such that power is returned to the line. This can be readily seen if the difference between the counter electromotive force and the resistance drop is considered as a negative resistance drop. The presence of a positive resistance drop indicates the use of power and hence the presence of a negative resistance drop must represent the generation of power.

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[2] See appendix B for detailed Vector Analysis.

Thus it is seen that the alternating-current series motor can be made to operate as a series generator merely by reversing the fields. A suddenly decreased line voltage will not increase the regenerated current of the line frequency as in the case of direct current, but will tend to decrease it. Similarly an increased speed with any given impressed voltage will also decrease the regenerated current. While these characteristics would make its use for regeneration possible, other difficulties have to be overcome for a practical solution along these lines.

The system is handicapped by the strong tendency of the generator to pick up as a direct-current generator with the secondary transformer winding serving practically as a short circuit. Even if direct currents are made impossible, as in the case of a repulsion motor, there is a strong tendency for low-frequency currents to be set up, causing trouble. Also the speed torque characteristics obtained with this connection (See Fig. 5) are not the best type for all classes of regenerative service since the torque tends to decrease as the speed increases for any given value of impressed voltage. In view of these difficulties this method of regeneration has so far not been seriously considered for railway application in this country. Undoubtedly, however, due to its simplicity it has possibilities for the future.

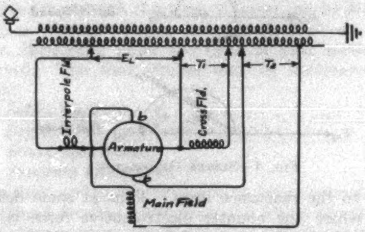

FIG. 6—CROSS FIELD OF SELF EXCITING GENERATOR

REGENERATION WITH CROSS-FIELD OR SELF-EXCITATION[3]

A regenerative system of the second kind with the main motors acting as armature (self-excited) generators is shown in Fig. 6. In this system the armature is supplied with an extra set of brushes *b b* which are located around the commutator midway between the normal or motoring set. The cross field is connected across a section of the main transformer. The resultant current and flux set up in the cross field circuit is approximately 90 deg. behind the line voltage since the field circuit is highly inductive. The armature conductors revolving in and cutting the cross-field flux set up by a counter electromotive force across brushes *b b* in phase with the flux cut and hence at right angles and lagging the line voltage.

This voltage is then connected across the main field and sets up a current and flux through the main field

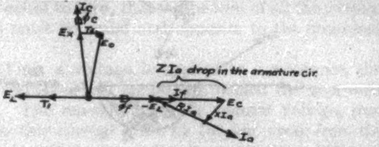

FIG. 7—CROSS FIELD OF SELF ECITING GENERATOR

approximately 90 deg. behind voltage *b b* or approximately 180 deg. away from the line voltage. The armature conductors revolving in and cutting this main field flux set up a voltage across the normal brushes approximately 180 deg. away from the line voltage. Hence, when these terminals are connected to the transformer the resultant voltage difference will set up a circulating current in the circuit. If the line voltage impressed is greater than generated voltage a motoring torque will be produced and power will be drawn from the line. If the generated voltage exceeds the line voltage a braking torque will be exerted and power will be returned to the line. The small section of transformer voltage in the excitation circuit is introduced in order to neutralize the resistance drops in both the cross field and main field circuits and hence maintain the regenerated voltage exactly 180 deg. behind the line voltage.

Fig. 8 shows the typical shape of the speed torque characteristic obtained with this connection.

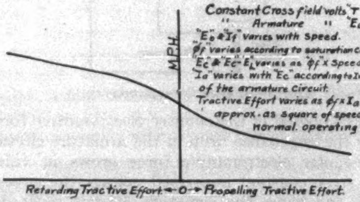

FIG. 8—APPROXIMATE SPEED-TRACTIVE EFFORT CHARACTERISTIC WITH CROSS-FIELD EXCITATION SYSTEM

This system is handicapped due to the fact that the armatures, and also the commutators of the main motors, have to be increased in capacity because they carry the exciting currents in addition to the load currents. This is quite a serious handicap in view of the space limitations existing for the main motors in connection with railway work. Another disadvantage of the system is the necessity of additional brushes around the commutator.

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[3] See appendix C for Vector Analysis.

However, such a system is advantageous in so far as it gives a shunt or flat speed-torque characteristic (see Fig. 8) and as it permits of power factor compensation during regeneration, and possibly during motoring, within certain limits. Hence, it eliminates the necessity of auxiliary rotating apparatus that otherwise might be required to obtain this power factor compensation feature and therefore probably has certain possibilities for the future.

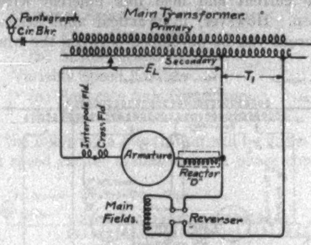
Pantagraph
Cir. Bkr.
Main Transformer
Primary
Secondary
E_L
T_1
Interpole Fld.
Cross Fld.
Armature
Reactor "D"
Main Fields.
Reverser

FIG. 9—SHUNT GENERATOR

REGENERATION WITH SHUNT EXCITATION[4]

The simplest form for a shunt type regenerative system is shown in Fig. 9. The main fields are connected directly across a section of the main transformer. The resulting field current and flux set up in the circuit lags approximately 90 deg. behind the line voltage. The armature conductors revolving in and cutting the field flux set up a counter electromotive force across the armature circuit in phase with the flux cut and hence approximately 90 deg. behind the line voltage.

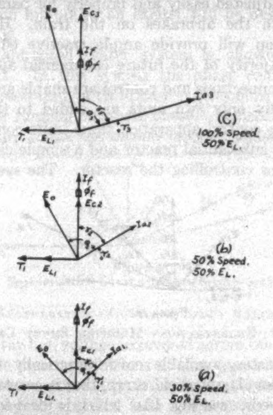
E_o
E_{c3}
I_f
ϕ_f
I_{a3}
T_3
T_1
E_{L1}
(C)
100% Speed.
50% E_L.
I_f
ϕ_f
E_o
E_{c2}
I_{a2}
T_2
T_1
E_{L1}
(b)
50% Speed.
50% E_L.
I_f
ϕ_f
E_o
E_{c1}
I_{a1}
T_1
E_{L1}
(a)
30% Speed.
50% E_L.

FIG. 10—VARIABLE SPEED—REGENERATING. CONSTANT IMPRESSED ARMATURE VOLTS. SHUNT EXCITATION. CONSTANT IMPRESSED FIELD VOLTS

The armature is thus connected in series with an external reactor and across a section of the main transformer. The vector sum of the counter electromotive force generated in the armature and of the voltage impressed across the armature give a resultant voltage in the armature circuit which lags behind the line voltage by some angle depending upon the relative values of the impressed and generated voltages: This resultant voltage sets up a circulating current in the armature circuit which lags by approximately 90 deg. since

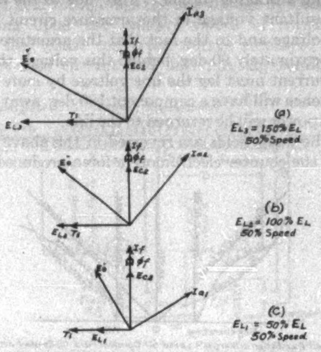
E_o
I_f
ϕ_f
E_{c3}
I_{a3}
T_1
E_{L3}
(a)
E_{L3} = 150% E_L
50% Speed
E_o
I_f
ϕ_f
E_{c2}
I_{a2}
E_{L2} T_1
(b)
E_{L2} = 100% E_L
50% Speed
I_f
ϕ_f
E_o
E_{c1}
I_{a1}
T_1
E_{L1}
(C)
E_{L1} = 50% E_L
50% Speed

FIG. 11—REGENERATION—CONSTANT SPEED—VARIABLE IMPRESSED ARMATURE VOLTAGE. SHUNT EXCITATION

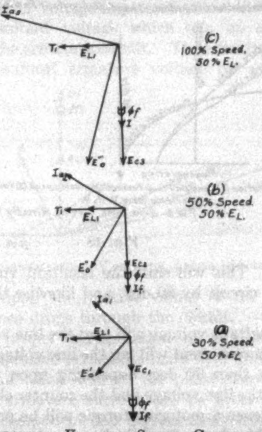
I_{a3}
T_1
E_{L1}
(C)
100% Speed.
50% E_L.
ϕ_f
I
E_o
E_{c3}
I_{a2}
T_1
E_{L1}
(b)
50% Speed
50% E_L.
E_o
E_{c2}
ϕ_f
I_f
I_{a1}
T_1
E_{L1}
(a)
30% Speed.
50% E_L
E_o
E_{c1}
ϕ_f
I_f

FIG. 12—MOTORING—VARIABLE SPEED—CONSTANT IMPRESSED ARMATURE VOLTS. SHUNT EXCITATION, CONSTANT IMPRESSED FIELD VOLTS.

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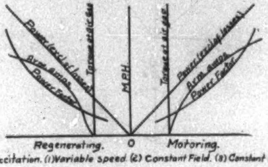
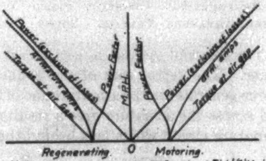
[4] See appendix D for Vector Analysis.

the armature is highly inductive due to the external reactor. Hence, since the field current and flux are also lagging the line voltage the armature current has a component in phase, the field flux, and is therefore exerting a braking torque. Also, due to the fact that the resultant voltage in the armature circuit lags the line voltage and to the fact that the armature current is approximately 90 deg. behind this voltage, the armature current must lag the line voltage by more than 90 deg. hence will have a component 180 deg. away from it. Thus, power will be returned to the line.

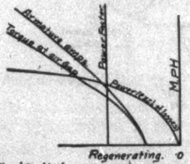
If the main fields are reversed in the above connections, the counter electromotive force produced will be reversed. This will shift the resultant voltage in the armature circuit by 90 deg. and likewise the armature current.



(a) Shunt Excitation. (1) Variable speed (2) Constant Field. (3) Constant arm. Volts.



(b) Shunt Excitation. (1) Constant Speed. (2) Constant Field (3) Variable arm. Volts.



(C) Shunt Excitation (1) Constant Power Factor (2) Variable speed. (3) Variable Field. (4) Variable arm. Volts

Fld. & Arm Volts Varying directly with speed.

FIG. 13

The resultant voltage will lead the line voltage while the armature current will lag the line voltage, by something less than 90 deg. depending upon the relative values of the line voltage and the counter electromotive force. Hence a motoring torque will be produced and power will be drawn from the line.

Fig. 13 (A, B, and C) gives the typical shape of the different speed torque characteristics for both motoring and regenerating obtainable with this connection.

The chief handicap of this system lies in the fact that it does not permit of power factor correction during the regenerating period and must always draw a resultant wattless component of current from the line for excitation. It is also handicapped to some extent due to the fact that the same continuous torque between the armature and field windings, possible with the series connection, cannot be obtained in the shunt connection due to the phase difference between the armature current and field flux inherent with this connection. However, in practical applications sufficient torque can be developed between the windings such that when added to the losses the resultant retarding effort available will be more than sufficient for holding any train while descending a given grade that can be hauled up the same grade with the series connection. Also, when the obtainable torque is not sufficient for any given application the speed-torque characteristics can be adjusted easily and properly for parallel operation with the airbrakes on the train. Hence, this connection will provide ample tractive efforts for a large majority of the future commercial applications.

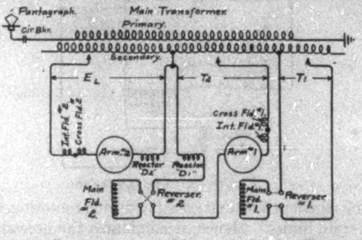


FIG. 14—REGENERATION—MODIFIED SHUNT EXCITATION

The connections and control are simple and reliable. Practically only two units are added to the normal motoring control apparatus, these units consisting of a simple substantial reactor and a simple change-over switch for controlling the reactor. The speed torque characteristics available cover practically the entire range from (1) a vertical curve, that is constant torque at all speeds, (see Fig. 13A) which is ideal for straight deceleration purposes, to (2) a shunt or relatively flat curve, that is increasing torque with increasing speed, (see Fig. 13C) which is the desirable characteristic for use in holding a train at some given desired speed when descending a grade.

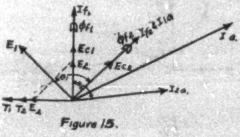


FIG. 15—REGENERATION—MODIFIED SHUNT EXCITATION

Hence, this form lends itself readily to practically any class of service and will undoubtedly be in great demand for future commercial applications.

A modified form of the shunt connection is shown in Fig. 14 and is described in appendix D.

SEPARATE EXCITATIONS[5]

The regenerative systems utilizing separate exciters during the regenerating period can be divided into two general classes as follows:

1. Constant Speed Exciter Systems
    (a) Phase Converters
    (b) Motor Generator Sets.
2. Variable Speed Exciters
    (a) Separate Axle Generators.
    (b) Main Motor as an Exciter.

In the following a description will be given of a constant speed exciter system using a phase converter as an exciter. A variable speed exciter system using one of the main motors as an exciter will be described in appendix E.

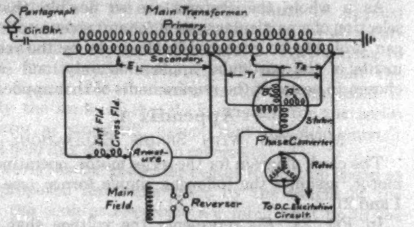

Pantagraph
Cir. Bkr.
Main Transformer
Primary
Secondary
E_L
T_1
T_2
"B"
"A"
Stator
Phase Converter
Int. Fld.
Cross Fld.
Armature.
Rotor
Main Field.
Reverser
To D.C. Excitation Circuit.

FIG. 16—REGENERATION—CONSTANT SPEED EXCITER SYSTEM

A constant speed exciter system is shown in Figs. 16 and 17. In this system the phase converter or exciter is essentially a two-phase synchronous or induction motor running on single phase. Hence, a voltage is generated across the second or open phase at right angles to the line voltage impressed across the motoring phase. In most applications of this type a two phase synchronous motor will probably be employed in order that it may be operated as a synchronous condenser during the motoring period for power factor correction. Also, with the use of the direct-current excitation in the rotor the main motor field excitation can be supplied in part if not entirely without drawing it all in the form of wattless lagging current from the line.

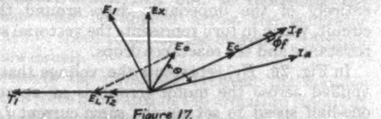

E_1
E_x
E_o
E_c
I_f
ϕf
I_a
Θ
T_1
E_L T_2
Figure 17.
Regeneration - Constant Speed Exciter System.

FIG. 17—REGENERATION—CONSTANT SPEED EXCITER SYSTEM

The right angle voltage obtained from the generating phase of the phase converter is connected in series with the main field of the traction motor and across a section of the main transformer. The vector sum of these two voltages gives a resultant voltage which sets up a field current and field flux through the field circuit. The resultant field voltage lags the line voltage by some angle depending upon the relative values of transformer voltage and phase converter voltage employed. The resultant field current and field flux lag behind this voltage by 90 deg. since the field circuit is highly inductive, and therefore lags behind the line voltage by more than 90 deg.

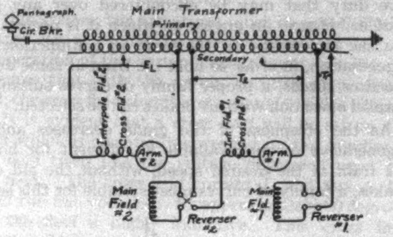

Pantagraph.
Cir. Bkr.
Main Transformer
Primary
Secondary
E_L
T_2
T_1
Interpole Fld. #2
Cross Fld. #2
Arm. #2
Int. Fld. #1.
Cross Fld. #1
Arm. #1
Main Field #2
Reverser #2
Main Fld. #1
Reverser #1

FIG. 18—REGENERATION—VARIABLE SPEED EXCITER SYSTEM

The main armature conductors revolving in and cutting this flux set up a counter electromotive force across the armature in phase with the flux cut. The vector sum of this counter electromotive force and of the transformer voltage impressed across the armature gives a resultant voltage which sets up circulating current in the armature circuit. The armature current lags the resultant armature voltage by some angle depending upon the relative values of the resistance and reactance drops through the circuit.

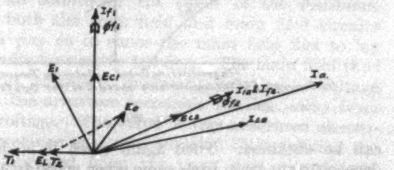

I_{f1}
ϕf_1
E_1
E_{c1}
E_o
I_a.
I_{1a} & I_{f2}.
ϕf_2
E_{c2}
I_{2a}
T_1
E_L T_2

FIG. 19—REGENERATION—VARIABLE SPEED EXCITER SYSTEM

By the proper manipulation of the transformer voltages impressed across the main field and armature circuits (and when desired, the phase converter) the circulating armature current can be given a leading or lagging component 180 degrees away from the line voltage, and at the same time exert a braking torque, or it can be given a leading or lagging component in phase with the line voltage, and at the same time exert a motoring torque. Thus all requirements for regeneration can be fulfilled.

Figure 20A shows three different typical approximate forms of speed torque characteristics, not drawn to scale, obtainable with this system of regeneration. It is possible to obtain a family of curves for practically any desired form of characteristics, by the proper manipulation and choice of the various applied voltages. In fact, with the proper control facilities, it is possible to change from any one family to one or more different families of curves, during the regenerating period. Thus, it is possible to choose the proper form of curve most suitable for each of the various forms of regenerative duty that may be encountered over any given profile. Hence, on grades on which it is necessary to use the air brakes on the train in conjunction with regeneration, in order to hold the train at some desired speed or speeds, a proper family of curves suitable for parallel operation with air brakes can be selected.

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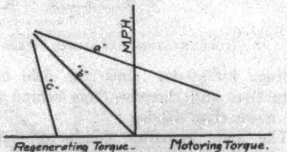
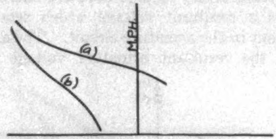
[5] See appendix E for Vector Analysis.

As the steepness of the grade decreases and the regenerative torque available is sufficient for holding the train at the desired speed, without the aid of air brakes, a family of curves most suitable for this service can be obtained. Then again when it is desired to decelerate the train from some given speed to standstill, the family of curves most suitable for such service can be chosen. Thus this system, although in general requiring more regenerative apparatus, (and hence costing and weighing more), than some of the previously described systems, is also more flexible and hence will also do more than these systems. The control and apparatus required for regeneration are relatively simple and reliable.



(a) Separate Excitation.—Constant Speed Exciter System.



(b) Separate Excitation.- Variable Speed Exciter System.

FIG. 20

The proper speed torque characteristics can be obtained for practically any application and if desired power factor correction during the regenerating period can be easily obtained.

In the constant speed exciter systems the value of torque obtainable is limited by the current capacity of the motor windings only and hence, if desired will give the same continuous torque at the air gap as is obtainable in the series motor connection. Also in this system the constant speed exciter can be utilized for power factor compensation as well as driving auxiliary apparatus during the motoring periods.

Hence, this system, in one or more of its possible forms, will also undoubtedly be used to a large extent in future commercial applications and especially where extremely heavy torque regenerative duty is required.

CONCLUSION

In general, it can be safely stated that probably any future regenerative application that may be desired, can be properly and successfully taken care of with one or more of the various single-phase systems available at the present time.

As a whole the systems are so flexible that the selection of a particular system for any given application can probably be governed in the main by the requirements of the particular application only, and can be chosen to best suit the requirements of that application.

Appendix A

MOTORING WITH SERIES EXCITATION:

The clock diagram for the series motor, operating as a motor, assumes the following simple forms, (see Figs. 1 and 2).

In Fig. 2A, E_{L1} represents the voltage that must be applied across the motor terminals at standstill to set up a current I_a through the motor circuit. The motor circuit is inductive and, therefore, the current lags the impressed volts. The motor is drawing power from the line (equal to the losses in the motor circuit only at standstill), and hence has a component in phase with the line voltage, and since it is exerting a propelling torque, it is 180 degrees away from the field current I_f and field flux ϕ_f.

The counter-voltage at standstill is composed entirely of the impedance drop around the motor circuit, which in turn represents the vectorial sum of the resistance and the reactance drops.

In Fig. 2B, E_{L2} represents the voltage that must be applied across the motor terminals at approximately one-half speed to set up the same current I_a. E_{L2} is greater than E_{L1} due to the fact that the impressed voltage must overcome the back electromotive force of the motor E_{C2}, as well as the impedance drop around the circuit. As E_C grows in value due to speed with any given impressed voltage, the impedance drop and hence the armature current must decrease in value to give the correct vector sum of counter-voltages

In Fig. 2C, E_{L3} represents the voltage that must be applied across the motor terminals at full speed to set up current I_a. E_{L3} is greater than E_{L2}, due to the fact that E_{C3} is greater than E_{C2}.

The power component of current I_a in Fig. 2B and Fig. 2C is greater than the corresponding component in Fig. 2A, due to the fact that the motor is delivering power, hence must draw power from the line.

It should be noted that in both Fig. 2B and 2C the counter electromotive force E_c and the $R\,I$ drop are added directly and their resultant added vectorially to the $X\,I$ drop. When the series connection is utilized for regenerative purposes, it so happens that E_c and the $R\,I$ drop are subtracted from each other and the net resultant thus obtained is then added vectorially to the $X\,I$ drop.

Appendix B

REGENERATION WITH SERIES EXCITATION

In order to regenerate with the series connection after motoring, the main fields must first be reversed. The clock diagram for this connection then assumes the simple form shown in Figs. 3 and 4.

E_{L1} represents the voltage that must be applied across the motor terminals at standstill to set up a current I_a through the generator circuit. By referring to Fig. 2A for the series motor, it will be seen that the clock diagram for the series motor at standstill is exactly the same as the clock diagram for the series generator at standstill, except that the field current I_f and field flux ϕ_f have been reversed.

However, as the generator begins to pick up speed as it is propelled down the grade by the weight of the train, the counter electromotive force built up in the armature begins to counteract the resistance drop in the generator circuit until at some speed N, the counter electromotive force (see Fig. 4B), is just equal and opposite to the $R\,I$ drop. At this point the armature current has swung around until it lags the line voltage by exactly 90 deg. This means that the generator is drawing no power from the line whatsoever, and hence at this speed it is supplying its own losses.

As the speed of the train still further increases, the counter electromotive force set up in the armature circuit increases; hence, E_{L3}, Fig. 4C represents the voltage that must be applied across the generator terminals at full speed to set up the current I_a through the armature circuit.

E_{c3} represents the difference between the counter electromotive force generated in the armature circuit at this speed and the resistance drop through the circuit, due to I_a. The vector sum of E_{c3} and $X\,I_a$ is equal and opposite to E_{L3} and, hence a balanced condition exists. The current I_a has a component 180 degrees away from the line voltage, and hence it is delivering power to the trolley. Also, the current I_a is in phase with the field flux ϕ_f, and hence the generator is exerting a braking torque.

It should be noted that as the speed, and consequently the counter electromotive force in the armature circuit increases, the line voltage must also increase in order to maintain the same value of I_a otherwise as the counter electromotive force increases, due to increased speed, the $X\,I$ drop and consequently the current I_a would have to decrease in order to maintain the proper counter-voltage. Hence, with a fixed terminal voltage, the armature current and consequently the retarding torque, decreases as the speed increases.

Thus, Fig. 5 shows the typical shape of the speed-torque characteristics obtained with the series connection.

Appendix C

REGENERATION WITH CROSS-FIELD OR SELF-EXCITATION

In this system, the armatures must be supplied with an extra set of exciter brushes as shown in Fig. 6. The operation of this system is as follows: (see Fig. 7).

Assume that with the motors rotating at a certain speed, the cross field circuit is connected to a line voltage T_1. It is evident that a cross field will be set up, the size of which is essentially governed by the number of cross field turns and the voltage impressed upon the cross field. The cross field current I_c and flux ϕ_c lag behind the line voltage E_L by approximately 90 degrees, since the field is highly inductive. This cross field will induce by rotation a voltage E_x between the exciter brushes $b\,b$, which is proportional to and in phase with the cross field, and hence also lags behind the line voltage by approximately 90 degrees.

This voltage is then connected in series with a small section of the main transformer voltage T_2, and the resultant voltage E_0, is connected across the main fields. Since the main field circuit is highly inductive, the field current I_f and flux ϕ_f will lag behind the resultant excitation voltage E_0 by approximately 90 deg., and hence will be approximately 180 deg. away from the line voltage T_1. The small section of transformer voltage T_2 in the excitation circuit is introduced in order to counteract the effect of the resistance drops in both the cross field and main field circuits in such a way as to cause the main field flux to lag the line volts by exactly 180 deg. The main field thus set up will, therefore, induce by rotation a voltage E_c across the armature terminals 180 deg. away from the line voltage. The value of this generated electromotive force can easily be adjusted by regulating the voltage impressed upon the cross field circuit, and hence can be made sufficiently in excess of the impressed armature voltage E_L so that a leading regenerated current I_a will be returned to the line. Also, since I_a has a component in phase with ϕ_f, a braking torque will be exerted. Hence, all requirements for regeneration are fulfilled by this arrangement.

Appendix D

REGENERATING WITH SHUNT EXCITATION

The simplest form of connections for a regenerative system of the third kind is shown in Figs. 9 and 10. In this system the main field is connected directly across a section of the transformer voltage T_1 and the armature is connected in series with a current limiting and phase-shifting reactor D, and across a section of the transformer voltage E_L.

The transformer voltage T_1 sets up a current I_f and flux ϕ_f, through the field circuit. I_f and ϕ_f lag behind the line voltage T_1 by approximately 90 deg. The armature conductors revolving in and cutting ϕ_f set up a counter electromotive force E_C across the armature circuit in phase with ϕ_f, and hence approximately 90 deg. behind the line voltage T_1 or E_L. The vector sum of E_C and E_L, or E_0 represents the resultant voltage in the armature circuit which sets up a circulating current I_a through this circuit. The current I_a lags the resultant voltage E_0 by some angle θ, depending upon the relative values of resistance and reactance in the armature circuit, and hence depending in the main upon the value of the external reactor D.

I_a has a component of current in phase with ϕ_f and 180 deg. away from the line voltage E_L. Hence, with this connection a braking torque is exerted and power is returned to the line.

In Figs. 10, 11, 12 and 13 the resistance losses in the various circuits have been neglected for the sake of clarity.

Figs. 10 (A, B and C), show the operation of the generator at three different speeds for a constant field excitation and a constant armature impressed voltage.

From these three diagrams, it is seen that the torque, being equal to the product of the wattless component of I_a and the field flux ϕ_f, remains constant as the speed changes. Likewise, the power returned to the line, being proportional to the product of the watt component of I_a and line voltage E_L, varies directly with the speed, which is as it should be, since a constant torque is being developed at the various speeds.

Fig. 11 (A, B and C), shows the operation of the generator at a constant speed and field strength for three different values of impressed armature voltage.

From these three diagrams it is seen that the torque is directly proportional to the impressed armature voltage, since the wattless component of I_a varies directly with the impressed armature voltage. Likewise, the power returned to the line varies directly with the impressed armature voltage since the watt component of armature current I_a remains constant at the different voltages.

If, in the above connections, (see Fig. 9), the main fields are reversed, a motoring torque will be produced in place of a braking torque. The torque and power characteristics motoring are of the same type as those obtained in the regenerating connection, that is, for any given impressed armature and field voltage, the torque will be constant while the power taken from the line will vary as the speed changes.

Fig. 12 (A, B and C) shows the operation of the shunt motor under these conditions. Likewise, for a constant field strength at a constant speed, the torque produced and the power taken from the line will vary directly with the value of impressed armature voltage.

Fig. 13 (A and B) gives the approximate typical shape of the various characteristic curves, not drawn to scale, for the shunt connection both motoring and regenerating for: (1) constant applied armature and field voltage and variable speed, and (2) constant speed and field strength and variable armature volts.

Fig. 13C gives the typical appearance of the same various characteristic curves, not drawn to scale, for constant power factor and variable speed and automatically controlled field and armature voltage.

Another form of the shunt connection is shown in Figs. 14 and 15. This modified form is only feasible when there are two or more main traction motors on the locomotive or car for use during the regenerating period. This system, as it could be applied to a two-motor proposition, is shown in Fig. 14.

In this system, the main field of motor No. 1 is connected directly across a section of the transformer winding T_1, and the armature is connected in series with the main field of motor No. 2, a reactor D_1, and across a section of the transformer winding T_2. The armature of motor No. 2 is then connected in series with a second reactor D_2 and across a section of the transformer winding E_L.

The transformer voltage T_1 sets up a current I_{f_1} and flux ϕ_{f_1}, through the field circuit of No. 1 motor. I_{f_1} and ϕ_{f_1} lag behind the line voltage T_1 by approximately 90 deg. Number one armature conductors revolving in and cutting ϕ_{f_1}, set up a counter electromotive force E_{C_1} across the armature circuit in phase with ϕ_{f_1}, and hence approximately 90 deg. behind the line voltage T_1 or T_2. The vector sum of E_{C_1} and T_2 or E_1 represents the resultant voltage in No. 1 armature circuit which sets up a circulating current I_{1_a} through this circuit.

The current I_{1_a} lags voltage E_1 by some angle θ_1, depending upon the value of external reactor D_1 and sets up a flux ϕ_{f_2} in the main field of motor No. 2, ϕ_{f_a} being in phase with I_{1_a}.

Then the armature conductors of motor No. 2 revolving in and cutting flux ϕ_{f2}, set up a counter electromotive force E_{C_2}, across the armature in phase with ϕ_{f_2}. The vector sum of E_{C_2} and E_L or E_2 represents the resultant voltage in No. 2 armature circuit which sets up a circulating current I_{2_a}. The current I_{2_a} lags the voltage E_2 by some angle θ_2 depending upon the value of reactor D_2.

The vector sum of I_{1_a} and I_{2_a} gives a total resultant armature current I_a, which has a component 180 degrees away from the line voltage, and hence is being returned to the line. Armature current I_{1_a} has a component in phase with ϕ_{f_1} and current I_{2_a} has a component in phase with ϕ_{f_2}. Hence, both the motors are exerting a braking torque and are returning power to the line.

This system can be extended in the same manner for three or more motors, each additional step giving an improvement in the resultant power factor and torque.

The main advantage of these modified forms lies in the fact that the power factor and torque are improved and that the resultant kv-a. required in the external reactors are decreased by an amount equal to the value of main field reactance utilized in the various armature circuits.

The chief disadvantage lies in the added control complications entailed due to multiplication of the armature circuits.

Appendix E

REGENERATION—SEPARATE EXCITATION

The clock diagram for the regeneration system using a phase converter as an exciter is shown in Figs. 16 and 17.

Referring to Fig. 16, it is seen that a voltage T_1 is applied across the motoring phase A of the phase converter. This induces a voltage E_x across the open or generating phase B. This generating phase voltage E_x is connected in series with a section of the transformer voltage T_2 and the resultant voltage E_1 thus obtained is connected across the main field of the main traction motor. This voltage E_1 sets up a field current I_f and field flux ϕ_f through the field circuit. I_f and ϕ_f lag the voltage E_1 by approximately 90 deg. since the main field is highly inductive.

The armature conductors revolving in and cutting ϕ_f set up a counter electromotive force E_{C} across the armature circuit in phase with the flux cut. The vector sum of E_{C} and E_{L} gives a resultant voltage E_{O} which sets up a circulating current I_a in the armature circuit. I_a lags E_{O} by some angle θ depending entirely upon the relative values of the resistance and reactance drops in the armature circuit. I_a has a component of current 180 deg. away from line voltage and hence power is being returned to the line. Also I_a has a component in phase with ϕ_{C} and hence a braking torque is being exerted. Thus all requirements for regeneration are being fulfilled.

A variable speed exciter system is shown in Figs. 18 and 19. This system is only feasible when there are two or more main motors available for regenerative duty. In this system the main field of No. 1 motor is connected across some section of transformer voltage T_1 and the armature of the same motor is connected in series with the main field of motor No. 2 and across a section of the main transformer T_2. The armature of motor No. 2 is then connected across a section of transformer voltage E_{L}.

The voltage T_1 sets up a current I_{f_1}, and flux ϕ_{f_1} through the field circuit. The armature conductors of No. 1 motor revolving in and cutting ϕ_{f_1}, set up a counter electromotive force E_{C_1}, across the armature. The vector sum of E_{C_1} and T_2 or E_1 sets up a circulating current I_{1_a} or I_{f_2} and flux ϕ_{f_2} through the field circuit of No. 2 motor. I_a or I_{f_2} and ϕ_{f_2} lag behind E_1 by approximately 90 deg. since the field circuit is highly inductive. The armature conductors of motor No. 2 revolving in and cutting ϕ_{f_2} sets up a counter electromotive force E_{C_2} across No. 2 armature. The vector sum of E_{C_2} and E_{L} or E_{O} sets up a circulating current I_{2_a} in this circuit. I_{2_a} lags E_{O} by some angle θ depending entirely upon the relative values of the resistance and reactance drops in No. 2 armature circuit. The vector sum of I_{1_a} and I_{2_a} or I_a has a component 180 deg. away from line voltage and hence power is being returned to the line. Also I_{1_a} has a component in phase with θ_{f_1} and I_{2_a} has a component in phase with ϕ_{f_2} and hence both motors are exerting a retarding torque. Thus all requirements for regeneration are again fulfilled.

It is interesting to note that the main motor acting as an exciter is not only furnishing the excitation for the other motor but it is also regenerating, that is, it is exerting a braking torque and is returning power to the line. This feature is radically different from the similar type of direct current system. In the direct current system where one of the main motors is used as a generator the motor acting as an exciter returns no power to the line whatsoever and the braking torque exerted is due to the losses in the excitation circuit only. The above feature is due to the fact that the excitation circuit connections, that is, No. 1 motor connections, are essentially the same as those used for straight shunt regeneration, the main field of No. 2 motor serving as the reactor.

Fig. 20B shows the approximate forms of the speed torque characteristics, not drawn to scale, that may be obtained with this type of system.

The "Blondelion"

A Kinematic Device which Indicates the Performance of a Polyphase Synchronous Generator or Motor[1]

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Review of the Subject.—*There are many engineering problems which can be solved step by step, and there are others which contain two or more inter-dependent unknown quantities and therefore cannot be solved step by step. For example, the diameter of a short transmission shaft, mainly subjected to torsion, can be computed directly for a desired shearing stress; then the pulley can be designed, the belt for the pulley, etc. This is a step-by-step solution.*

On the other hand, let it be required to determine the diameter D and the length L of a cylindrical tank, to hold a given quantity of liquid and to possess a given cooling surface. This problem either requires two simultaneous equations for D and L, or else has to be solved by trials.

The problem of determining theoretically the internal voltage drop and voltage regulation of a given synchronous generator at a given load is one of the second kind, that is, several conditions must be satisfied simultaneously. The problem of predetermining the armature current and the power factor of a given synchronous motor at a given excitation and load is of a similar nature.

The purpose of the kinematic device described in the paper is to enable one to solve these two problems on synchronous machines mechanically, almost without any mathematics and without constructing vector diagrams. The device can be used even by a person who does not know its theory, just as thousands of computers who use the slide rule could not explain the theory of its scales.

A knowledge of the performance characteristics of a synchronous machine is of considerable practical importance, both to its designer and to the user; it has therefore been deemed advisable to simplify and to standardize the procedure so as to make the process as nearly automatic as possible. This should give more time to progressive engineers to think about other problems connected with synchronous machinery, problems which are only partly solved, such as the temperature rise, insulation, transient phenomena, etc.

Two factors in particular complicate the performance of synchronous machines (especially with salient poles), namely the armature reaction and the variable saturation of the magnetic circuit. The effect of the armature reaction is to weaken (or to strengthen) the field flux and also to shift it sidewise. The method of predetermination of voltage regulation given in the A. I. E. E. Standards is admittedly an approximate one, and in designing the Blondelion a more accurate method of taking these two components of the armature reaction into account has been used. This method is known as Blondel's theory of two armature reactions. The curves in Figs. 10 to 13 show a close check between the experimental curves of certain machines and the points read off on the kinematic device. The Blondelion can be readily simplified for use in accordance with the A. I. E. E. method. The variable saturation of the magnetic circuit is taken care of in the device automatically, by means of a proper linkage.

To illustrate the principle upon which the Blondelion is built, let us take again the above-mentioned tank problem and see how a mechanical device could be made for its solution. Let various values of tank diameter D be marked on a certain scale and let the values of tank length L be marked on another scale. Assume that each scale has an index which can slide along it and that these two indexes are kinematically so interconnected that a pointer indicates directly the corresponding volume of the tank on a third scale. The arrangement is somewhat indefinite in that the same volume can be obtained with an infinite number of combinations of values of D and L.

Let now the same indexes be also interconnected by another kinematic linkage, such that another pointer, on a fourth scale, gives directly the area of the cylinder. Here again, an infinite number of combinations of D and L will give the same area. In order, however, to make the third pointer indicate a desired volume, and the fourth pointer simultaneously indicate a desired area, the D and L indexes have to be set at perfectly definite points on their respective scales. These points are readily found by shifting the indexes back and forth. The problem is thus solved, and to use the device one does not even have to know either the cylinder formulas or the nature of the kinematic linkages.

In a synchronous machine there are certain four conditions (explained in the paper), which must be satisfied simultaneously. Therefore, the Blondelion has four independent linkages each of which may assume almost any desired shape. When, however, these linkages are interconnected and certain points and lengths are fixed, the remaining links assume perfectly definite positions, and the desired unknown lengths (which stand for electrical quantities) can be read off directly. In other words, the four linkages represent a system of four simultaneous equations with four unknown quantities, just as the above described tank linkage represents two equations with two unknown quantities. In both cases the solution is entirely automatic, and with properly designed linkages cannot be wrong if the particular setting is right.

INTRODUCTION

The Meaning of the Word Blondelion. The device was named for the noted French scientist and engineer, Professor André Blondel, and is based on his theory of two armature reactions in a synchronous machine.

What the Blondelion is. A combination of movable and adjustable bars, linkages, and straight and curved guides (Fig. 1) which can be set to represent, to a certain scale, a vector diagram of voltages, currents, m. m. fs., and fluxes in a synchronous generator or motor of any desired constants. An important feature of the device is a saturation curve of adjustable shape. The parts of the device are kinematically so constrained that when it is set correctly for one load it remains so at any other load, irrespective of a changing excitation, saturation, or terminal voltage of the machine.

The Purposes of the Device. (1) To enable a designer to select the best electrical constants and a no-load

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[1] The investigation upon which this paper is based was supported by a grant from the Heckscher Foundation for the Advancement of Research, established by August Heckscher at Cornell University.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y. February 14-17, 1923.

saturation curve, and to "test" a synchronous generator or motor before it has been actually built. (2) In some applications to take the place of an involved analytical theory; it is often difficult to investigate analytically or numerically the effect of separate factors upon the resultant performance characteristics, especially with considerable saturation. (3) For some purposes to take the place of various proposed circle diagrams which, with a saturated magnetic circuit, are of more than doubtful value and accuracy. (4) To add the judgment of the eye and the skill of the hands to the purely mental ability in selecting the constants of a machine for a desired performance, or in judging the characteristics for assumed constants. (5) To enable an investigator or a student to familiarize himself with a machine as if he had one available for tests. This is of particular importance with large or special machines, for which no facilities may be available for testing under full-load conditions.

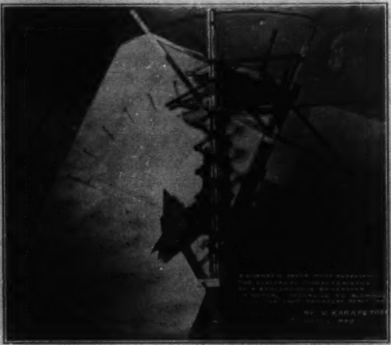

FIG. 1—A GENERAL VIEW OF THE BLONDELION

The Quantities which Can be Represented on the Blondelion. Current, terminal voltage, internal voltage (or the stator flux), input, output, power factor, internal phase angle, synchronizing torque, and the field current. These quantities may be obtained at a constant applied voltage, at a constant input (or output), or at a constant field excitation, as desired; or else, these factors may be varied in any desired manner.

The Factors Which May be Taken Into Account and Varied Separately at Will in the Blondelion. Per cent armature resistance and reactance; the direct and the transverse armature reaction; the degree of saturation in the magnetic circuit; the field ampere-turns; the copper and core losses; friction and windage.

Other Kinematic Devices. The Blondelion is one of the several kinematic devices developed by the writer, for representing the performance characteristics of various kinds of electrical machinery and lines. The other devices are as follows:

1. A device for imitating the performance of an electromagnetic clutch used in the Owen magnetic car; *Sibley Journal of Engineering,* Jan. 1918, Vol. XXXII, p. 55.

2. The Secomor, a device which imitates the performance of a polyphase series-connected commutator motor; A. I. E. E. TRANS., 1918, Part 1, Vol. XXXVII, p. 329.

3. The Indumor, a device which imitates the performance of a polyphase induction motor; also its modification, the Secomor, which represents the performance of a shunt-connected polyphase commutator motor; A. I. E. E. JOURNAL, 1922, Vol. XLI, p. 107.

4. The Heavisidion, a device which represents the operating characteristics of an elecrtical transmission line with distributed capacitance and leakage A. I. E. E. JOURNAL, 1923, Vol. XLII, p. 127.

5. The C. P. S'er (named for Dr. C. P. Steinmetz), a device for the automatic addition of impedances in series and admittances in parallel. (in preparation).

6. An Intergraph based on the author's parallel double-tongs, for a mechanical integration or differention of a given curve. This device finds its usefulness in problems of hunting of electrical machinery, flywheel design, ship stability, etc. *Optical Society of America and Review of Scientific Instruments, Journal* 1922, Vol. VI, p. 978.[1]

Limits of Rating and Output. Like any other graphical device, the Blondelion requires certain scales to be chosen for each particular problem. A convenient scale has to be selected for volts, another for armature amperes, still another for the field current, etc. The device can therefore represent the performance of a small machine as well as of one whose output runs into tens of thousands of kilowatts; of a 110-volt machine as well as of one wound for 22,000 volts. As in any graphical device, there are some limitations due to a finite length of the bars and to an interference of the links in certain positions. With one setting the device can be made to give an accurate performance for all values of practical interest, say between one-quarter full-load and twice the full-load, and between 60 and 100 per cent power factor. For points beyond these limits some bars may have to be replaced or a different scale selected.

B. GENERAL DESCRIPTION OF THE BLONDELION

The first complete Blondelion, shown in Fig. 1, was built in the writer's experimental shop, in Cornell University, during the winter of 1921-22. The same device is represented by a single-line diagram in Fig. 2; the construction details are shown in Figs. 6 to 9. Most parts are made of flat steel bars, or of celluloid bars, not over two centimeters wide and a few millimeters thick. Some bars are of constant lengths, others are of adjustable useful lengths, holes being drilled every few millimeters. Most bars are connected to each other by means of pivot joints. Some points are constrained to move along straight or curved guides.

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[1] For a description of the parallel double tongs see the *American Machinist,* 1921, Vol. 55, p. 1050.

The device is assembled on a table provided with a groove *XX* (Fig. 2.) A stationary rail *H H* supports the free end of the main bar *O T* which can turn about *O* as a center. The bar *O T* is provided with a longitudinal slot which serves as a guide for the slide *F F*, roller *S*, linkage *O Q G*, *T*-square *t t*, and the linkage *G K C*. By means of the *T* square *t t*, the bar *p p'* is made to remain at right angles to the bar *O T*. The proportional dividers *A N C D* are guided by *p p'* at *D*.

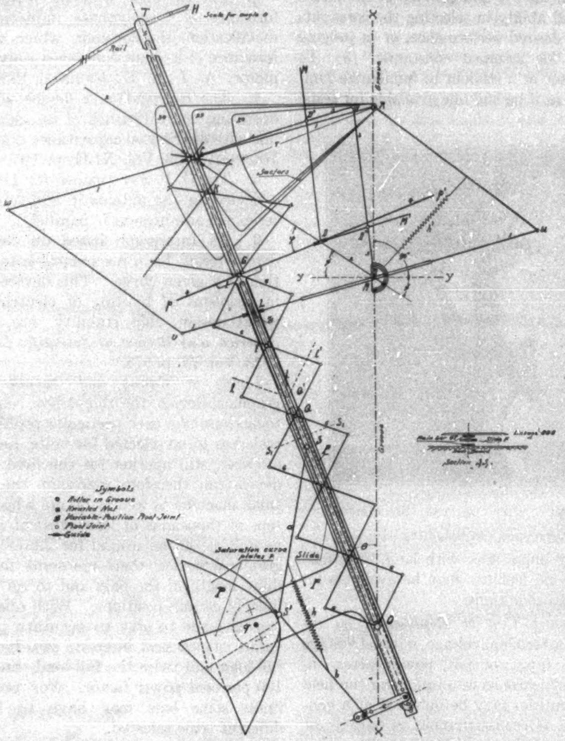
Scale for angle θ
Rail
Groove
Sectors
Symbols
Roller in Groove
Knurled Nut
Variable-Position Pivot Joint
Pivot Joint
Guide
Saturation curve plates P
Slide
Section A-A
Main bar OT
Slide F
Linkage OQG
X
Y
O
Q
S
F
P
q
h
b
a

FIG. 2—A SINGLE-LINE DIAGRAM OF THE BLONDELION

Different bars are placed in different horizontal planes to enable them to cross each other without interference. In Fig. 2 the bars nearest to the table are marked 1, those immediately above them are marked 2, etc. The particular sequence selected is not essential since the device is intended to represent a vector diagram in a plane.

A detailed description of the functions of the different parts of the Blondelion is given below, in connection with its theory. It suffices to state here that the device is set for chosen design constants and for a desired shape of the no-load saturation curve. The setting consists (a) in the selection of a suitable scale for volts, (b) in the selection and adjustment of a proper curve plate (or plates) to correspond to the desired saturation curve and (c) in the adjustment of the lengths of a few bars. The Blondelion then represents vectorially the following four simultaneous relations which together characterize the machine:

(a) The voltage drop in the armature winding.

(b) The component induced e. m. fs.

(c) The main magnetic circuit, modified by the armature reaction.

(d) The relationship between the flux, the net induced e. m. f., and the resultant m. m. f.

After having been properly set and connected to represent the above relations, the aggregate of the linkages becomes a kinematic system of a limited number of degrees of freedom. Thus for example, if the field current and the power factor of the load are kept constant, the parts of the device can be moved only in such a way as to represent variations of the terminal voltage with the armature current; the device then possesses only *one degree of freedom*. If the terminal voltage is kept constant, it is possible to vary the energy component of the current independently, and its reactive component by varying the field excitation; the device then is a system with *two degrees of freedom*; etc.

Having set the device for the desired operating conditions of the machine, certain of its parts can be shifted in the grooves or along the guides, thus causing all the other members to assume new positions. At any position, readings can be taken of the voltage, current, power factor (or phase angle), field excitation, output, deviation of the field structure from its no-load position, etc. In other words, at least as many characteristic quantities can be read directly as with an actual load or brake test, with an additional advantage of having the currents and the voltages directly and automatically combined into a vector diagram.

Lengths are measured on the centimeter scales marked along the bars and the grooves, and the phase angles are read by means of protractors, or with a sighting goniometer (see the author's paper on the Indumor, mentioned above). Or else, angles may be computed from the measured projections of the vectors.

C. The Performance Diagram of a Polyphase Synchronous Generator

The vector diagram in Fig. 3 represents the terminal voltage, the current, and the induced electromotive forces in a polyphase synchronous generator with salient poles, on a partly inductive load. The relations are shown from the point of view of Blondel's theory of the direct and transverse armature reactions. The notation is that used in the author's "Magnetic Circuit," Chap. 8, to which the reader is referred for explanations and proofs. One phase only is represented in the diagram; e is the terminal voltage; i is the armature current; $i\,r$ is the ohmic drop and $i\,x$ is the reactive drop in the armature, per phase. E is the resultant induced voltage considered as a geometric sum of the voltages E_n and E_t induced by the real and fictitious poles of the machine respectively. The current i is shown lagging behind the terminal voltage e by an angle ϕ. The ohmic drop $i\,r$ is in phase and the inductive drop $i\,x$ is in quadrature with the current. The induced voltage E is also the resultant of e, $i\,r$, and $i\,x$, and is a measure of the total stator flux which approximately determines the core loss.

In accordance with the theory explained on p. 151 of the "Magnetic Circuit," the armature current i is resolved into two components, $i_d = i$ Sin ψ and $i_t =$ Cos ψ. The component i_d is such that the m. m. f. wave due to it (in all the three phases) glides along the air gap in a direct juxtaposition to the field m. m. f., weakening or strengthening the latter. This part of the armature m. m. f. is known as the *direct armature reaction*; hence the subscript d. The components i_t in the three phases form an m. m. f. also gliding along the air gap, but with its maxima and minima midway between the main poles. This is known as the *transverse armature reaction*, and the corresponding symbols are provided with the subscript t.

The direct armature reaction is arithmetically subtracted from (or added to) the field excitation, and the resultant m. m. f. produces the useful flux which induces in the armature winding the e. m. f. E_n; the subscript n stands for the word "net." The transverse armature reaction gives rise to a flux which induces in the armature winding the e. m. f. E_t. Since the two revolving fluxes are in *space* quadratures, the corresponding e. m. fs. are in *time* quadrature. When the machine is acting as a generator and the current is lagging, E_t lags behind E_n. The transverse m. m. f. and its flux being proportional to the component i Cos ψ of the current, it is convenient to extend $B\,D$ to its intersection with $O\,G$ at C. The angle at D being equal to ψ, same as at O, the triangle $C\,D\,G$ is similar to the current triangle. Therefore, $C\,D$ is proportional to the total current i on the same scale on which $D\,G$ is proportional to its component i_t. For the value of the coefficient of proportionality see the "Magnetic Circuit," p. 156, eq. (84), in which $E_t' = C\,D$.

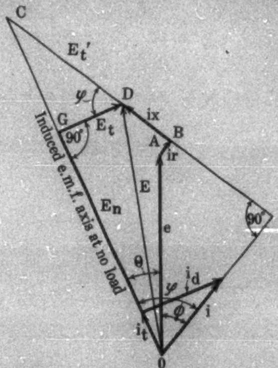

Fig. 3—The Vector Diagram of a Synchronous Generator

The diagram in Fig. 4, on which the Blondelion is based, is identical with that in Fig. 3, except that the ohmic drop $i\,r$ is neglected, since in usual 25- and 60-cycle alternators of medium and large size its value is small as compared to $i\,x$. This simplifies the construction of the device, although $i\,r$ could be taken into account, as in the Indumor. The direction of the terminal voltage, e, is taken along the X-axis. The current vector is omitted, since $B\,D$, $D\,C$, and $B\,C$ are proportional to the current, and any of the three may be taken as a measure thereof. In interpreting the diagram, it must be remembered that $B\,D$, considered as the armature current, is turned ahead by 90 degrees, while as the reactive drop it is in its true phase position. The same remark applies to $D\,C$.

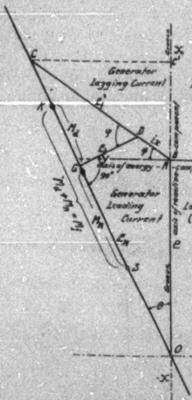
X
C
C'
Generator Lagging Current
Motor Leading Current (overexcited)
E'
K
M_d
φ
D
ix
φ
B
Y
G
axis of energy-R-component.
90°
Generator Leading Current
Motor Lagging Current (underexcited)
$M_d + M_n = M_f$
M_n
E_n
e
S
θ
O
-X

Fig. 4—An Alternator Diagram as Represented by the Blondelion

The direct armature reaction, being proportional to i Sin ψ, is also proportional to $G\,C$, since $C\,D$ is proportional to the current, and the angle at D is equal to ψ. However, the coefficient of proportionality of the direct armature reaction (Magnetic Circuit, p. 153, eq. 79), is different from that of the transverse reaction; moreover, the direct reaction, M_d, is expressed in ampere-turns, while the effect of the transverse reaction is taken into account by means of the e. m. f. E_t. For these reasons the direct armature reaction is represented not by $G\,C$, but by $G\,K = M_d$. In the Blondelion, the ratio of $G\,C$ to $G\,K$ may be adjusted at will and then kept constant for any setting corresponding to a given machine.

Let furthermore $S\,G = M_n$ represent the net field excitation necessary for inducing the net e. m. f. $O\,G = E_n$. Then the actual number of ampere-turns, M_f, on the field poles must be equal to the sum of M_n and M_d, since the amount M_d is counterbalanced by the direct armature reaction. The values of M_n, M_f and E_n are taken from the no-load saturation curve, but this curve should be corrected for a greater magnetic leakage at full load; this is actually done by careful designers.

We thus see that the various linkages of the Blondelion must be such that with any setting for a given machine, the following four vectorial relationships be fulfilled:

(a) The geometric sum of the terminal voltage e and the reactive drop $i\,x$ must be equal to the total induced e. m. f. $O\,D = E$.

(b) The sum of the induced e. m. fs. E_n and E_t must be equal to the same induced voltage $O\,D = E$. Besides, E_n and E_t must remain in quadrature with each other, and both must have the values imposed by the field excitation and the armature reaction.

(c) The length $D\,C$ must remain proportional to $B\,D$ and be in line with it, both $B\,D$ and $D\,C$ being proportional to the armature current. $G\,K$ must remain proportional to $G\,C$.

(d) The field excitation M_f, corrected for the effect of the direct armature reaction M_d, must be equal to the net excitation M_n which corresponds to E_n on the no-load saturation curve of the machine. The proper relationship between M_n and E_n must be preserved at all loads, and the effect of a variable saturation correctly taken into account.

D. THE KINEMATIC DETAILS OF THE BLONDELION

Having established the necessary relationships which must exist between the vector diagram in Fig. 4 and the linkages in Fig. 2, it remains to show how the various points of the device are constrained to move along certain paths in order that the above-mentioned four conditions remain fulfilled with any desired setting.

I. THE SATURATION DEVICE

The saturation device (lower part of Figs. 1 and 2) is that component mechanism of the Blondelion which permits to vary simultaneously the induced voltage E_n and the corresponding field current, or the m. m. f. M_f, in accordance with a chosen no-load saturation curve of the machine (Fig. 5). In order to simplify the construction and the use of the device, it is convenient to replot to a standard scale a given no-load saturation curve, to be used in the Blondelion. In the particular instrument shown in Figs. 1 and 2, the induced voltage E_n is represented by the length $O\,G$ of five rhombi,

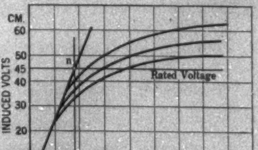

Fig. 5—No-Load Saturation Curves of Different Machines Replotted to a Standard Scale

while the corresponding excitation, on the straight part of the saturation curve, is represented by that of the two rhombi, $Q\,G$. Therefore, the no-load saturation curve must be plotted to such a scale that on its straight part, when the field excitation is represented by say a cm., the corresponding voltage is represented by (5/2) a cm. Moreover, with the chosen dimensions of the device, it is convenient to select such a voltage scale that, for the rated voltage, $O\,G$ is equal to about 45 cm.

In Fig. 5 the point n corresponds to a rated voltage of 45 cm. and to a field m. m. f. of (5/2) 45 = 18 cm. With these assumptions, the extended straight parts of all no-load saturation curves pass through the same point n, and the only difference lies in the upper curved parts of these curves. An actual study of many saturation curves showed that the differences in the upper parts, when re-plotted to a standard scale, are comparatively small, so that a setting within wide limits is possible with only three curve plates.

The saturation device (Fig. 2) is mounted on the slotted bar $O\,T$, pivoted at O. A rhombic articulated lattice $O\,G$ may be expanded and contracted along the bar $O\,T$, and the length $O\,G$ represents the net induced e. m. f. E_n (Figs. 3 and 4). When $O\,G = 45$ cm., $G\,Q = (2/5) \times 45 = 18$ cm. Therefore, with this particular setting, $G\,Q$ represents the excitation corresponding to point n in Fig. 5. In other words, as long as the machine operates on the straight part of the saturation curve, the above described linkage, contracted or expanded, gives the correct relationship between the induced voltage and the field current.

In order to take care properly and automatically of the upper curved part of the no-load saturation curve, the linkage $a\,b\,c$ is added, with rollers S and S'. The roller S is guided by the bar $O\,T$, the roller S' runs along the curved plates P. A tension spring h, between a and b, causes the roller S' to press against the plates P. These plates are supported by the slide $F\,F$ which is fastened to the rhombic linkage at Q and can move along the bar $O\,T$ in ball bearings. In order to represent synchronous machines with low, medium, and high saturation, a set of three curved plates is provided. Parts of these plates are combined by trials to form a desired saturation curve. Any plate can be turned into a desired position and fastened with the set screw q. Furthermore, the point of attachment and the length of the link c are adjustable. Actual experience shows that by properly setting the plates, by using the right portions of their edges, and by correctly adjusting the link c, any reasonable saturation curve can be duplicated with sufficient accuracy.

The excitation corresponding to the voltage $O\,G$ on the upper curved part of the no-load saturation curve is no more represented by the length $G\,Q$, but by $G\,S$. The arrangement is such that when $O\,G$ is short, points Q and S coincide and move together, giving the straight part of the saturation curve. As $O\,G$ increases in length, the slide $F\,F$, by virtue of its connection at Q, is forced upward. Hence, the curved plates move upwards, allowing the roller S' to move to the left and to change the position of S. The shape of the curves is such that the distance $S\,G$ varies as the exciting current, or as the ampere-turns M_n, when $O\,G$ varies as the induced voltage E_n.

In some applications it may be desirable to make the field excitation scale independent of the voltage scale, and not to use always the same point n (Fig. 5). This can be done by adding a separate adjustable linkage $l\,Q'\,l'$ (Fig. 2) which can be attached to the main rhombic linkage at any desired place, by means of tapped holes and screws, as in Fig. 6. The plate $F\,F$ is then attached to Q', instead of Q, and the roller S must coincide with Q' on the straight portion of the no-load saturation curve.

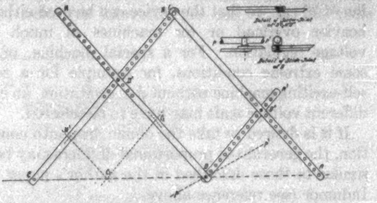

Fig. 6—Proportional Dividers for the Reactance Drop and for the Transverse Armature Reaction

II. The Proportional Dividers for the Reactance Drop

In Figs. 2, 3, and 4 both $B\,D$ and $D\,C$ are proportional to the armature current, and therefore, for a given machine, their ratio must be kept constant for all values of the current, that is, for all possible settings of the device. The proportional dividers used for this purpose are shown in Fig. 6, the corresponding points being denoted by the same letters as in Fig. 2. The dividers consist of four members; the bars $A\,N$, $C\,N$, and $D\,M$ are of the same length; the bar $D\,M$ is of about half their length. An extra linkage, $C_2\,C_1\,N'$ may be added if so desired, to have the current represented by a separate vector $A\,C_1$. However, the distances $A\,D$, $A\,C$ and $D\,C$ are all proportional to the current, so that it is only a matter of the scale.

By opening or closing the dividers, the distance $A\,C$ may be varied at will, and the distances $A\,D$ and $D\,C$ always remain proportional to it. To enable the ratio of $A\,D$ to $D\,C$ to be set in accordance with the constants of the machine, holes are drilled and tapped in all the bars, as shown in the detail sketch. Countersunk machine screws are used for fastening the bars together at the desired points; these screws do not prevent free rotation of the bars relatively to each other. The point D is provided with a pivoted guide shown in the detail sketch; the cylindrical bar $p\,p'$ (Figs. 2 and 7) can move freely through this guide. The purpose of the bar $p\,p'$ is to complete the vector polygon $O\,B\,D\,G$ (Fig. 4).

The bars $A\,N$ and $C\,N$ must be long enough to allow the dividers to open for the highest desired value of the armature current. The shorter bar and the holes along the bars must allow of a setting for the highest percentage of reactance drop and of transverse armature reaction that may be encountered under the extreme practical conditions. To illustrate, let the length of the vector $O\,A$ of the rated terminal voltage be 45 cm., and let, for a machine of normal characteristics, the sum of the reactive drop $i\,x$ and of the voltage E_t' (Figs. 3 and 4), at the rated armature current, be say 50 per cent of the terminal voltage. Then, if performance characteristics are desired up to twice the full-load current, the length $A\,C$, with the dividers fully opened, should be not less than 45 cm. The lengths of the bars in the above described dividers allow an opening of 70 cm., so that the device can be used either for a heavier overload or for machines of much poorer voltage regulation. For a special machine, or under some extreme conditions, for example for a reactive self-exciting machine without d-c. excitation, an entirely different voltage scale may have to be selected.

If it is desired to take the ohmic drop into consideration, the generalized proportional dividers may be used, similar to those described in the author's paper on the Indumor (see reference above).

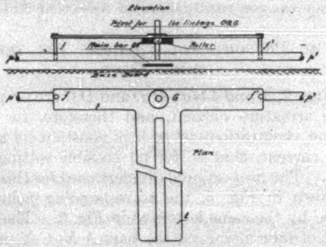

FIG. 7—*T*-SQUARE FOR THE DIRECT AND THE TRANSVERSE ARMATURE REACTION

III. THE PROPORTIONAL DIVIDERS FOR DIRECT ARMATURE REACTION

In the description of Fig. 4 it has been explained that the direct armature reaction, $M_d = G\,K$, is proportional to the length $G\,C$. In order to realize this relationship in the Blondelion, rhombic proportional dividers are used, with centers at C, K, and G (Fig. 2). These dividers are independent of the rhombic linkage $O\,G$. In their general construction the dividers $G\,K\,C$ are similar to those shown in Fig. 6. The sides of one of the rhombi are provided with holes (Fig. 1) for adjusting the ratio of $G\,C$ to $G\,K$ for a given machine. The total field excitation, M_f, is proportional to the length between the points S and K, and is measured on a centimeter scale placed between these points. When the performance is desired at a constant field current, a link of constant length may be placed between these points.

IV. THE SLOTTED T-SQUARE FOR KEEPING THE DIRECT AND THE TRANSVERSE REACTION AT RIGHT ANGLES TO EACH OTHER

It has been explained in the description of Figs. 3 and 4 that with any setting of the device the vectors $O\,C$ and $G\,D$ must remain perpendicular to each other. This is obtained by means of the T-square $t\,t$ (Figs. 2 and 7) fastened to the round rod $p\,p'$. The T-square is made of sheet brass. Its shorter arm is pivoted at G and its longer arm is guided in the direction $O\,G$ by the roller pin t' of the rhombic linkage $O\,G$. The T-square and the rod $p\,p'$ are fastened together by means of pins f and f' and the pivot G (Fig. 7). At D (Fig. 2) the rod $p\,p'$ passes through a guide, shown in the detail sketch in Fig. 6. The T-square is shown in Fig. 1 with the right-hand side of its shorter arm cut off. The T-square and the bar $p\,p'$ are in different planes above and below the main bar $O\,T$, so as to allow the slide at D to move over the whole length of the bar $p\,p'$ without interference with the pivot G or with the pins f and f'. The main bar $O\,T$ can therefore cross all the other bars.

V. THE ARMATURE CURRENT, THE TERMINAL VOLTAGE, AND THE PHASE ANGLE

As is explained above, the magnitude of the armature current is proportional to the opening of the proportional dividers $A\,C$ (Figs. 2 and 6). The length $A\,C$ can be measured on a centimeter scale (Fig. 8) pivoted at A and constrained to slide in the sleeve g at C. The A end of the scale is made into an indicating pointer by means of which the phase angle (or the power factor) of the current can be read off directly on the protractor. Since in the Blondelion the current vector is turned by 90 deg. with respect to its true position, the terminal voltage and the current are in phase, when the bar $A\,C$ is perpendicular to the axis $X\,X$. For this reason 0 deg. and 100 per cent power factor are marked in the center of the protractor.

The terminal voltage $e = O\,A$ (Fig. 4) is measured on the centimeter scale $k\,k$ (Fig. 8) which forms one of the two guides for the center A, with its horizontal roller r'. The center line of the groove $X\,X$ represents the direction of e. The groove is cut in the drafting board or the table on which the whole device is mounted. The distance is measured from O to one of the edges of the protractor and a correction is made for its radius.

When it is desired to investigate the performance of a machine at a constant armature current, the set-screw j is tightened, thus fixing the length $A\,C$. To test the performance of the machine at a constant terminal voltage, the set-screw j' is tightened. To find the characteristics at a constant power factor the set-screw j'' is fastened.

VI. The Load Device

The electrical output of a generator, per phase, is equal to the terminal voltage $O\,A$ (Fig. 9) multiplied by the component $C\,C'$ of the current $A\,C$. It must be remembered that in the Blondelion the current vector is turned by 90 deg. with respect to its true phase position, so that $C\,C'$ is the energy component of the current, in phase with the voltage $O\,A$. For any setting of the device, the distance from point C to the center $X\,X$ of the groove can be readily measured and thus the output computed. The same applies to the electrical input of a synchronous motor.

In some uses of the Blondelion it may be desirable to read the electric power directly, on a suitable scale, and for this purpose the arrangement shown in Fig. 9 has been devised. It consists of the articulated parallelogram $u\,v\,w\,R$ and of a celluloid sector $R\,R'$, of radius r. The parallelogram is constrained as follows: At R there is a vertical pin with a horizontal roller which can move along the groove $X\,X$. The construction is similar to that of the ball bearing guide at A, shown in the detail sketch in Fig. 8. The bar $u\,v$ has a slot which forces the center line of $u\,v$ always to pass through point A (Fig. 8). The bar $w\,R$ can slide through a guide at C. It is proved below that with any setting of the device the power is proportional to the length $O\,L$ on the main bar $O\,T$.

If it is desired to investigate the performance of a generator at a constant output (or that of a motor at a constant input) the double slide at L is fastened to the bar $O\,T$ by means of the set screw z. This causes the length $O\,L$ to remain constant, but does not otherwise hamper the motion of the parallelogram, since the two slides at L can turn with respect to each other. The construction is similar to the double slide at point C, shown in the detail sketch in Fig. 8. The sector $R\,R'$ is forced to remain in contact with the side of the bar $O\,T$ by means of the tension spring h' shown between the bars $u\,v$ and $w\,R$. With changes in the setting, the sector rolls on $O\,C$. The power scale depends upon the radius r of the sector.

The theory of this linkage is as follows: The power, being equal to $O\,A \times C\,C'$, is proportional to the area of the triangle $O\,A\,C$. But the same area is equal to $O\,C \times A\,A'$, where $A\,A'$ is the altitude of the triangle with respect to the base $O\,C$. Since $u\,v$ is always parallel to $R\,w$, the triangles $O\,A\,L$ and $O\,R\,C$ are similar to each other, so that $O\,C/r = O\,L/A\,A'$. Hence,

$$\text{Power} = k_e\,k_i\,O\,A \times C\,C' = k_e\,k_i\,O\,C \times A\,A'$$
$$= k_e\,k_i\,.\;r\,.\,O\,L.$$

In this expression k_e is the voltage scale coefficient, that is, volts (or kilovolts) per cm.; k_i is the scale coefficient of the armature current, that is, amperes per cm. Both coefficients are constant, and so is the radius r, with a given sector. Hence, the power is proportional to the length $O\,L$.

As an example, let the Blondelion be set for a three-phase, Y-connected alternator, the current scale being $k_i = 50$ amp./cm., and the e. m. f. scale $k_e = 100$ volt/cm., the voltage being measured per phase of Y. Let the radius of the sector be $r = 25$ cm. To find the power scale, we compute the length $O\,L$ corresponding to an output of say 1000 kw. per phase. We have

$$1000 \times 1000 = 100 \times 50 \times 25 \times O\,L,$$

from which $O\,L = 8$ cm. Hence, the power scale is $1000/8 = 125$ kw. per cm. The scale for the total power for the three phases is 375 kw. per cm.

It is convenient to have several sectors of different radii (Fig. 2) since each sector gives the best results only within a certain range of settings.

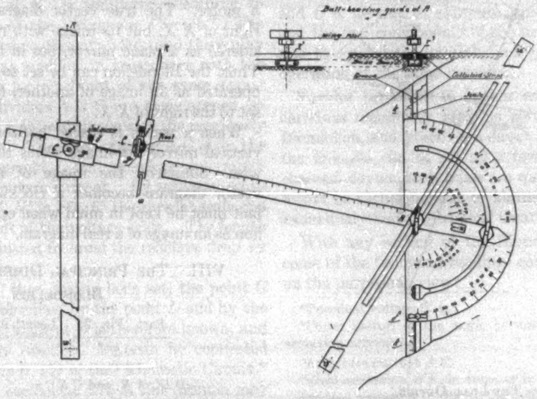

Fig. 8—The Protractor and the Scale Bar $A\,C$

VII. THE GENERATOR AND THE MOTOR RANGE

So far the parts and the setting of the Blondelion have been described in a position corresponding to the vector diagram in Fig. 4, that is, with the machine working as a generator, with a lagging current. This range of operation corresponds to the position of the current vector *A C* in the upper left-hand quadrant comprised between the axes *X X* and *Y Y*. When the vector *A C* (Fig. 4) is directed along the *Y Y* axis to the left, the generator load is non-inductive. When *A C* is directed along the *X X* axis upward, the current is lagging by 90 deg. behind the terminal voltage *e*. This is because the true position of the current vector *i* is that indicated in Fig. 3, lagging by 90 deg. with respect to *A C*.

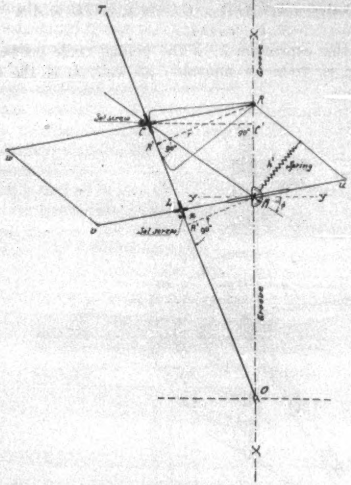

FIG. 9—THE LOAD DEVICE

When the vector *A C* lies in the lower left-hand quadrant, it has an energy component to the left and a leading reactive component. Therefore this range corresponds to the operation as a generator with the current leading the terminal voltage. All the linkages of the Blondelion are so made as to enable a setting with either a lagging or a leading generator current.

In both right-hand quadrants the energy component of the current is reversed, so that when *A C* lies to the right of *X X*, the machine is operating as a synchronous motor. When the reactive component of the vector *A C* is directed upward, the machine furnishes a lagging current to the line, or as is more customary to express it, the motor takes a leading current from the line. It is known from the theory of the synchronous motor that for such an operating condition the machine must be overexcited. Conversely, the operation in the lower right-hand quadrant corresponds to a lagging component taken from the line, and to an underexcited motor.

It would be rather difficult so to design all the linkages of the Blondelion that they could be turned by 360 degrees about point *A*, without interfering with each other. Therefore, the device is assembled to cover the operation either mainly to the left or mainly to the right of the axis *X X*, according to whether the generator range or of the motor range is of most interest. In either case the transition from one range to the other can be readily observed.

A complete study of both the generator range and the motor range is also possible with one setting, by using the principle of images or reflection. Let the Blondelion be set for the generator range, and let it be desired to investigate the performance of the same machine as a motor. The true vector diagram then lies to the right of *X X*, but its image with respect to *X X*, considered as a plane mirror, lies in the generator range. Thus, the Blondelion can be set to the left of *X X* and operated as an image of another, fictitious Blondelion, set to the right of *X X*.

When a watch is placed horizontally in front of a vertical mirror, its image seems to run counter-clockwise. Similarly, the image of a counter-clockwise vector diagram becomes a clockwise diagram. This fact must be kept in mind when operating the Blondelion as an image of a real diagram.

VIII. THE PRINCIPAL DIMENSIONS OF THE BLONDELION

Part (Fig. 2)	Length in Centimeters
O T	100
A C	70
D D', *A N*, and *C N*	35
p p'	70
a	25
b	20
c	10
Radius of arc *H-H*	100
w R and *v u*	85
v w and *u R*	40
Radii of load sectors	19, 24, 29
Corresponding distances to the center line of *O T*	20, 25, 30

E. THE USE OF THE BLONDELION

The reader is now supposed to be familiar with the purpose and the construction of the separate parts shown assembled in Figs. 1 and 2. It remains to indicate how the device may be set for a given numerical problem. Let us assume the simplest case of an alternator for which the following data are either known from tests or have been estimated from the dimensions of the machine:

The no-load saturation curve;

The true armature reactance (*not* the synchronous reactance);

The direct armature reaction;

The transverse armature reaction.

Let it be required to set the Blondelion for this machine, for some given values of the terminal voltage e, armature current i, and phase angle ϕ.

First of all, suitable scales are selected for volts and for field amperes (or ampere-turns per pole). With these scales, the saturation device is set by trials to represent the given no-load e. m. f. curve as closely as possible. For details see the description of this device given above. Then the center A is fastened in the groove $X\ X$ at the desired point, to represent the terminal voltage e. The direction $A\ C$ is set with the protractor at A, to correspond to the desired value of the power factor or angle ϕ. The length $A\ D = i\ x$ is computed for the desired value of the armature current.

Next, the value of E_t' is computed according to eq. (**84**) on p. 156 of the "Magnetic Circuit." The value of the empirical coefficient (0.30) used there may have to be somewhat modified in accordance with the best data available for machines of the same type.[3] The proportional dividers $A\ N\ C$ are then opened so that $A\ C = E_t' + i\ x$, in accordance with the previously chosen voltage scale. The same opening $A\ C$ gives the desired armature current i, and therefore determines the scale of the armature current. By using an additional linkage ($C_2\ C_1\ N'$ in Fig. 6) the current scale can be made independent of the voltage scale. The length $A\ D$ can be now adjusted to equal the reactive drop $i\ x$ in the machine.

The figure $O\ A\ C$ thus having been set, the point G is automatically determined by the point D and by the direction $p\ p'$. The angle at D is therefore known, and the direct armature reaction, M_d, can be computed from formula (**79**) on p. 153 of the "Magnetic Circuit." The semi-empirical coefficient 0.75 in this formula may need some modification to suit the particular type of the machine investigated (see reference to Arnold above). The linkage $C\ K\ G$ is then set in such a way that $K\ G$ is equal to M_d to the same scale to which the field excitation is measured between G and Q.

The whole device is now set for that particular load, and after the desired readings have been taken, the parts can be shifted to any other desired performance point, and the new lengths and angles measured. In particular the following quantities may be kept *constant*:

(a) The field excitation, by placing a link of constant length between points S and K.

(b) The terminal voltage, by fastening the protractor A to one of the guides of the groove $X\ X$.

(c) The armature current, by fastening the bar $A\ C$ to its guide at C.

(d) The power factor, by fastening the bar $A\ C$ to the protractor at A.

(e) The power output, by fastening the guide at L (through which $u\ v$ passes) to the bar $O\ T$.

Even with one of these constraints, the device has more than one degree of freedom, so that it is possible to impose two constraints simultaneously. For example, both the field current and the load power-factor may be kept constant, and the relationship found between the armature current and the terminal voltage.

Sometimes the load characteristics of a machine are given, and from these it is required to estimate its armature reaction and reactance. The values of $A\ D$ and $K\ G$ are then adjusted by trials until the vector relationships obtained between the armature current, the terminal voltage, and the field excitation satisfy the available curves as well as possible (see the numerical example below). The final settings of $A\ D$, $A\ C$, and $G\ K$ permit to compute the armature reactance, and the two components of the armature reaction, and also to extend the available curves to other ranges of operation of the same machine.

Various problems in design and operation of synchronous machinery may be solved by means of the Blondelion, and wherever a direct setting is not possible, the linkages can be set by a few trials. With a mechanical device, such trials are quite simple, while they would be rather tedious or almost impossible with the usual analytical or graphical treatment.

With any setting of the Blondelion (Fig. 2), all or some of the following readings can be taken, depending on the purpose in view:

*Terminal voltage $O\ A$.

*Power factor, or the angle between the voltage e and the armature current i.

*Armature current $A\ C$.

*Field excitation, $S\ K$, in amps. or in ampere-turns.

Induced voltage $O\ G$, due to the main poles.

Induced voltage $G\ D$, due to the transverse armature reaction.

Total induced voltage $O\ D$, which is also a measure for the armature flux.

Reactance drop $A\ D$.

Direct armature reaction, $G\ K$, in field amps. or in ampere-turns.

Angle of deviation θ of the poles with reference to the terminal voltage.

Electrical input or output $O\ L$.

*Projection of i on the X and Y axes.

Any other angles between the vectors that may be of interest.

For obtaining the usual performance characteristic of the machine, only the quantities marked with an asterisk need be measured. From the quantities enumerated above the following results can be computed:

Voltage regulation curves.

Field excitation curves at a constant voltage.

Load saturation curves.

V-curves.

Input or output in kw.

Separate losses.

Efficiency.

Synchronizing force.

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[3] For values of this coefficient, k_q, and also of the coefficient of direct reaction, k_o, mentioned below, see E. Arnold, *Wechselstromtechnik*, Vol. IV, 1913, p. 34.

In an article on "Hunting and Parallel Operation of Synchronous Machines," *Sibley Journal of Engineering*, March 1920, the author has outlined a method of computation of the synchronizing force, taking into account the no-load saturation curve of the machine. The treatment is based on Blondel's theory of two armature reactions, and in the first approximation the no-load saturation curve is replaced by two straight lines at an angle. Even then the expression for the power transmitted electromagnetically between the field pole structure and the armature, as a function of the angle θ, is quite involved. The derivative of this power with respect to θ by definition is the synchronizing power, and its expression is also rather complicated. In the Blondelion the increase in the power $O\,L$, for a certain small variation of the angle θ, can be read directly, and the synchronizing force, $\Delta\, O\,L/\Delta\,\theta$, easily computed. Thus, the device should prove useful in flywheel computations and in problems on hunting of synchronous machinery.

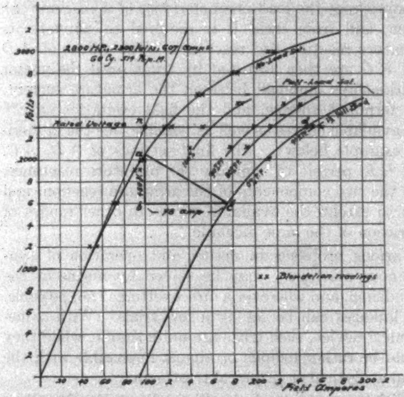

FIG. 10—NO-LOAD AND LOAD SATURATION CURVES OF A WESTINGHOUSE SYNCHRONOUS MACHINE OPERATING AS A GENERATOR

F. NUMERICAL EXAMPLES AND COMPARISON WITH EXPERIMENTAL DATA

Complete performance characteristics of two actual machines have been obtained by means of the Blondelion and the results compared with the available experimental data on the same machines. In Figs. 10 to 13, the curves represent the actual test results, while the crosses are the readings taken on the Blondelion. The no-load saturation curves (Figs. 10 and 12) were read on the Blondelion with both an increasing and a decreasing voltage, and a slight discrepancy between the two is due to some lost motion in the device. Load curves were checked for each machine working both as a generator and as a motor, and the agreement in all cases has been found satisfactory.

The test curves on the two machines and the other data quoted below were kindly furnished to the author by the engineers of the Westinghouse Electric and Manufacturing Co., and of the Allis-Chalmers Co., to whom he wishes to express his sincere gratitude. The principal data are as follows:

Make	Westinghouse	Allis-Chalmers
Rating as synchronous motor, H. P.	2800	500
No. of phases	3	3
Frequency in cycles	60	60
Rated voltage	2300	2200
Amperes per phase	607	105
Rated at a power factor of	0.90	1.00
Speed, rev. per min.	514	450
No. of poles	14	16
Stator connection	*Y Y*	*Y*
Slots per pole per phase	3	3
Coil lies in slots	1 and 8	1 and 8
Conductors per slot	6	6
Armature I^2R in kw	9.9 (at 75 ° C)	4.1
Field turns per pole	68.5	83.5

First of all, the saturation device was set to represent the no-load saturation curve of the Westinghouse machine shown in Fig. 10. The point *n*, defined in Fig. 5, corresponds in this case to a field current of 100 amperes and to a terminal voltage of 2300. The saturation device was set accordingly for the lengths 100 amperes = 18 cm. and 2300 volts = 45 cm., on the straight part of the saturation curve. The curved plates were then adjusted to give the upper part of the no-load characteristics. It will be noted that the "delta" or terminal voltage was used, and not the *Y*-voltage; all other voltage vectors in the Blondelion were set in terms of this voltage.

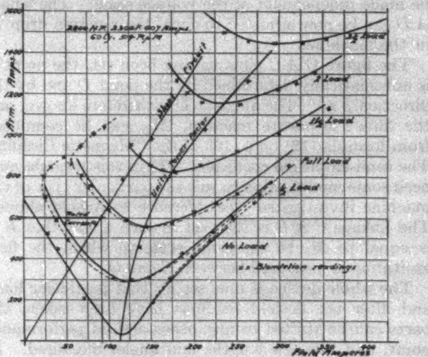

FIG. 11—THE SHORT-CIRCUIT CURVE AND THE *V*-CURVES OF A WESTINGHOUSE SYNCHRONOUS MOTOR

The next step was to estimate the direct armature reaction and the reactive drop, from the load saturation curve at zero power factor. The no-load saturation curve of the machine was traced on a piece of thin paper and shifted by trials diagonally, parallel to itself, until it fitted as well as possible the upper part of the load saturation curve at zero power factor. The line *a c* (Fig. 10) gives the distance and the direction of the shift. The horizontal distance, $b\,c = 78$ field amperes, represents the direct armature reaction; the vertical distance, $a\,b = 450$ volts, gives the reactive drop.[4]

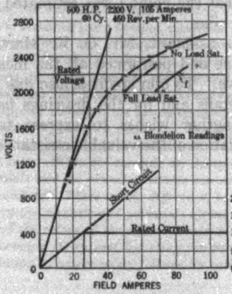

Fig. 12—Performance Characteristics of a Allis-Chalmers Synchronous Machine Operating as a Generator

These values hold well within the operating range of the machine, but are a little large for the short-circuit point, on account of the well-known increase in the reactance and leakage with the saturation.

The above obtained value of the direct reaction can be checked by means of eq. (**79**) on p. 153 of the "Magnetic Circuit." Namely, when $\phi = 90$ deg., the angle ψ is so nearly equal to 90 deg. that sin ψ is almost equal to unity. We therefore have,

$$M_d = 0.75 \times 0.96 \times 0.94 \times 3 \times 9 \times 0.5 \times 607 = 5550 \text{ ampere turns.}$$

In this expression 0.5 is introduced, because the two halves of the armature winding are connected in parallel. In terms of the field winding this m. m. f. corresponds to

$$5550/68.5 = 81 \text{ amperes}$$

which checks well with the value 78 amperes derived above from the experimental curves. The value of the reactive drop, 450 volts, was also checked theoretically from the dimensions of the machine, and a satisfactory agreement found between the two.

The scale for the field current being 100 amperes = 18 cm., the distance *G K* at the rated armature current must be equal to $18 \times (78/100) = 14$ cm.

The next step was to estimate the transverse armature reaction. Using expression (**84**) on p. 156 of the "Magnetic Circuit," with a different numerical coefficient, we get

$$E_t' = 0.43 \times 0.96 \times 0.94 \times 3 \times 9 \times 0.5 \times 607 \times 2300/(100 \times 68.5) = 1070 \text{ volts.}$$

In this expression the coefficient 0.43 was used in place of 0.30, in accordance with the later data than those upon which the values given in the "Magnetic Circuit" were based.[5]

Thus, at the rated current, the proportional dividers must have an opening

$$A\,C = (1070 + 450) \times 45/2300 = 30 \text{ cm.}$$

The current scale is $607/30 = 20.2$ ampere per cm.

The point *D* was determined so as to give the ratio

$$D\,C : A\,D = 1070 : 450$$

and the setting was done for the nearest ratio possible with a limited number of holes.

After this, the device was set with $O\,A = 2300$ volts $= 45$ cm., the power factor $\text{Cos}\,\phi = O$, $A\,C = 30$ cm., and $G\,K = 14$ cm. This gave the point *d* (Fig. 10). Then, keeping the power factor constant, the point *A* (Fig. 2) was shifted up and down along *X X* and the remaining points on the curve checked.

In a similar manner the points on the other load curves were determined, at the corresponding values of power factor. Finally the center *A* was shifted as near as possible to *O*, and a few points checked on the short-circuit curve (Fig. 11). In this case the length *A C* was varied, to get different values of the armature current.

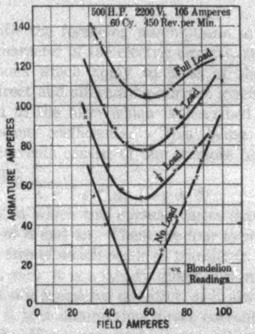

Fig. 13—*V*-Curves of an Allis-Chalmers Synchronous Motor

The *V*-curves, shown in Fig. 11, were checked as follows: The machine being rated as a motor at 607 amperes and 90 per cent power factor, the bar *A C* was set at $0.9 \times 607 = 546.3$ amperes at right angles to the groove *X X*. By means of the set screw *j′* (Fig. 8) the point *A* was fastened to keep the terminal voltage constant and equal to 2300 volts. The saturation device and the load device then automatically assumed their proper positions. The set-screw z at L (Figs. 2 and 9) was then tightened to keep the power input into the machine constant. The field current was read.

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[4] See the author's "Experimental Electrical Engineering," Third Edition, 1923, Vol. I, pp. 564 to 566.

[5] E. Arnold, *loc. cit.* For the ratio of the pole arc to pole pitch of 0.70 he gives $k_q = 0.446$.

After this, the bar $A\ C$ was turned stepwise, by a few degrees at a time, and the new field and armature currents were read at each step. This was done for a lagging and a leading current and the points so obtained were marked by crosses on the full-load V-curve shown in Fig. 11. The length $O\ L$ was then reduced to one half, and similar points were obtained, shown on the half-load V-curve. Subsequently the V-curves at 1½, 2 and 2½ times full load were checked, by setting $O\ L$ equal to that many times its length at the rated load. The no-load V-curve was checked by making $O\ L = O$. The curve which connects the lowest points of the V-curves, that is, the unity power factor curve, was checked by setting the screw j' so as to keep the power factor at 100 per cent, and varying the field current and the armature current. The load device was not used in this last test.

The curves of the Allis-Chalmers machine, shown in Figs. 12 and 13, were checked in a similar manner, and it would be a needless repetition to quote here the numerical data. It must be stated, however, that no actual load tests were made on this machine running as a generator. The maker of the machine stated to the author that the curves in Fig. 12 at 90 and at 100 per cent power factor were computed by the A. I. E. E. method and that the point f was estimated from tests on similar machines.

The writer's assistant, Mr. O. K. Marti, actually built the device and performed all the measurements described above. To him credit is also due for several mechanical details and for the drawings used in this paper. The author wishes to express to him his sincere appreciation of the valuable assistance rendered.

Transient Conditions in Electric Machinery

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Review of the Subject:— *The vector method is just as useful in solving problems involving transient conditions in electric circuits as it has proved to be when the currents and potentials are steady sinusoids. As far as the writer is aware, the vector method for determining transients in rotating electric machines was first used by L. Dreyfus. Previously the method had been applied to fixed combinations of resistances, inductances and capacitances by Kennelly and others.*

By making certain assumptions that are, however, quite reasonable in many cases, the transient currents in nearly all of the common types of electric machinery are damped sinusoids. Fortunately the damping is exponential and is thus readily accounted for. It is interesting to trace the development of the method. In the solution of all problems in direct currents the potentials, currents, and circuit constants are real numbers. In the corresponding problem in which the applied potentials are steady sinusoids, these quantities are all represented by complex numbers. In all other respects the working out of the solution is identical with that followed in the direct-current case. When the currents are damped sinusoids, they and the potentials and the circuit constants can still be represented by complex numbers. There is this difference, however; the vectors which represent the currents and potentials shrink exponentially as they rotate and the values of the circuit constants depend not only upon the frequency of the current, but also upon its rate of shrinking. Again the solution of any problem follows the same procedure that would the corresponding one in which the currents are steady sinusoids. In both the steady and damped sinusoidal cases the circuit constants depend upon the angular velocity of the vectors which represent the currents. In the former, the angular velocity is purely imaginary while in the latter it is complex, the real part being the rate at which the current vector shrinks and the imaginary portion, its angular velocity. In electric machinery in which rotating magnetic fields are produced, these fields shrink exponentially as they rotate when the currents are damped sinusoids. If these rotating magnetic fields are represented by vectors, the vectors will have a complex angular velocity just as do the currents. The e. m. f. which is produced by a steady sinusoidal variation of flux lags the flux by 90 degrees, whereas if the flux variation is a damped sinusoid, the angle of lag is less than 90 degrees, depending upon the damping. The mathematical relation, however, is the same, viz., the e. m. f. is proportional to the negative of the product of the flux and its angular velocity. It is then readily appreciated that the form of the solution for the transient state is the same as that which is used for the steady state. Before the method can be expected to give as accurate results as are obtained when predicting the steady operation, considerable experimental data must be obtained in order to determine the best methods of measuring the necessary constants, for these may be somewhat different during the transient period than during steady operation.

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THE problem of calculating the transient currents in alternating-current apparatus has rightly been considered a difficult one. Fortunately, the difficulty lies not so much in the problem itself as in the inadequacy of the mathematical treatment that has been employed. If a vector method of analysis is used, very great difficulties are removed. This was first appreciated by the author early in 1921 while studying the paper "Short-circuit Current of Induction Motors and Generators" by Doherty and Williamson. Later it was found that the vector method had already been developed by L. Dreyfus.[1] Notwithstanding this previous publication, the subject was considered of sufficient importance to warrant the presentation of this paper.

As is the case with many other complex problems, the method of attack is presented most clearly when certain ideal conditions are assumed. These conditions are that the resistances and inductances are constant and, in the case of rotating machines, that the speed is constant and that the space distribution of air-gap flux is sinusoidal. After one is familiar with the method in its simplest form, it may be possible to introduce refinements which will account for iron losses, variable permeability and harmonics.

The form of the transient current in any system of mutually inductive circuits is determined by the differential equations of the fall in electric pressure through them. With the foregoing assumptions in regard to the ideal character of the electric circuits, the differential equations which apply to most of the common types of direct and alternating-current machinery may be reduced to the form which is described as "linear with constant coefficients." This being true, the transient currents consist of one or more terms of the general form:

$$i = I\,\epsilon^{-\alpha t} \sin(\omega t + \theta)$$

The way in which this current varies depends upon the values of α and ω. There are four types of variations, only two of which occur as transients. If α and ω are both zero, it is a steady direct current of value, $I \sin\theta$. If α is a positive quantity, as it always is, and ω is zero, it is a diminishing direct current of value, $I\,\epsilon^{-\alpha t} \sin\theta$. If α is zero and ω is a positive quantity, it is a steady sinusoidal current of value, $I \sin(\omega t + \theta)$. And finally, if neither α nor ω are zero, it is a diminishing sinusoidal current. In each of these cases the current, whether steady or transient, may be represented by a vector which in general has a length of $I\,\epsilon^{-\alpha t}$ and which makes an angle of $(\omega t + \theta)$ with the horizontal axis. The projection of this vector on the vertical axis is the instantaneous value of the current. The four cases are illustrated in Fig. 1, *a*, *b*, *c* and *d*.

In each of the diagrams I marks the initial position of the vector. In the first case (*a*) the vector is fixed both in magnitude and in angular position. In the second case (*b*) the magnitude of the vector diminishes exponentially although its angular position does not change. In the third case (*c*) the vector has a constant magnitude but rotates with an angular velocity of ω. In the fourth and general case (*d*) the magnitude of the vector diminishes exponentially while it rotates at a constant angular velocity of ω. Thus the general form of transient current may be represented by a vector which rotates at a constant angular velocity and whose terminus travels along a logarithmic spiral.

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[1] See Bibliography.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

The velocity of the terminus of the vector has two components, one along the vector and one perpendicular to it. It is convenient to measure these component velocities in multiples of the instantaneous length of the vector. The term angular velocity is thus appropriate. This is a broader meaning of angular velocity than is ordinarily assigned to it, and for this reason, it is spoken of as the generalized angular velocity. Differentiation shows that the generalized angular velocity of this vector is, $m = (-\alpha + j\,\omega)$. See Fig. 2. Notice that the usual significance is attached to the symbol j. It indicates that the vector $j\,\omega$, lags the vector, $-\alpha$, by 90 degrees.

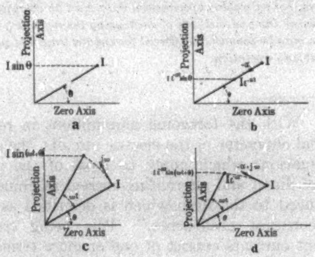

FIG. 1—THE END OF EACH VECTOR WHICH IS DRAWN FROM THE ORIGIN IS INDICATED BY A LARGE PERIOD; THE END OF SMALL VECTORS DRAWN FROM THESE PERIODS IS INDICATED BY ARROWHEADS.

The form of the current variation is determined by the generalized angular velocity; the magnitude is determined by the coefficient I, and the phase, by the angle, θ.

It is interesting to note that the current is passing through a maximum value at the moment that the current vector makes an angle arc tan α/ω with the projection axis. This occurs when the vector is in such a position that the angular velocity m, drawn from its terminus, is perpendicular to the projection axis. See Fig. 3. A steady sinusoidal current is maximum at the moment that the vector representing it lies in the projection axis.

The behavior of damped sinusoidal currents in electric circuits, having resistance inductance and capacitance, has been the subject of considerable investigation, and is familiar, especially to those who have been interested in radio communication. The field of application of this theory, however, is much broader than this, and may easily be extended to include practically all electric machinery, both alternating and direct-current.

Currents and pressures that have the same generalized angular velocity can, of course, be combined by the customary principles of vector addition. The resulting vector diagrams will then be similar to those which apply to the steady state, save that the electromotive forces due to induction, both self and mutual, are not in time quadrature with the current, as we shall presently see. It is further understood that, as they rotate, all of the vectors shrink exponentially.

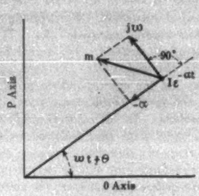

FIG. 2

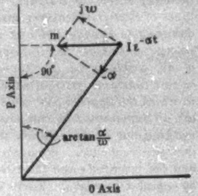

FIG. 3

Inasmuch as those who are interested in the operation of electric machinery have had little contact with the theory of damped sinusoidal currents, it may be well to show how these currents and the electric pressures they produce can be represented by vectors. In all of the following analyses any capacitance that the circuit may possess will be disregarded. This, of course, places a definite limitation upon the scope of the analysis and prevents the discussion of certain transient conditions, particularly those of relatively high frequency, which are very important in transmission circuits.

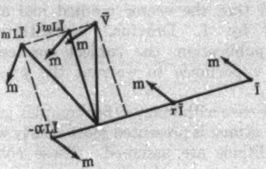

FIG. 4

When a current of the general form flows through a non-reactive resistance the *fall* in potential is $r\,I\,\epsilon^{-\alpha t}$ $\sin(\omega\,t + \theta)$. This may be represented vectorially as in Fig. 4. The *fall* in potential through an inductance is:

$$L\frac{d\,i}{d\,t} = -\,\alpha\,L\,I\,\epsilon^{-\alpha t}\sin(\omega\,t + \theta) + \omega\,L\,I\,\epsilon^{-\alpha t}\cos(\omega\,t + \theta)$$

The first of these components may be represented by a vector in opposition to the current vector, and the second by a vector which leads the current by 90 degrees.

See Fig. 4. In this figure and hereafter a line drawn above a letter indicates that it represents a current or pressure of the general form.

Thus: $\bar{V} = r\,\bar{I} + (-\,\alpha + j\,\omega)\,L\,\bar{I}$

or $\bar{V} = (r + m\,L)\,\bar{I}$

where m represents the generalized angular velocity of the current vector; the generalized impedance is $(r + m\,L)$.

In Fig. 4 notice that there is a *rise* in potential through the circuit, numerically equal to $\alpha\,L\,I$, which is in time phase with the current. This indicates that electric energy is being generated within the circuit, due to the diminution of the magnetic field. The power so generated, $\alpha\,L\,I^2$, partially supplies the copper loss, $I^2\,r$, in the circuit. The vector representing the fall in pressure due to self inductance leads the vector representing the current by arc tan α/ω more than 90 deg. It is in this respect that the vector diagrams representing the transient state differ from those that represent the steady state.

If the electric circuit is linked with a magnetic circuit in which eddy current and hysteresis losses may occur, it is customary, though strictly speaking it is incorrect, to represent both the current and the magnetic flux by vectors when they have a steady sinusoidal variation.

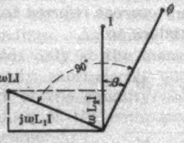

Fig. 5

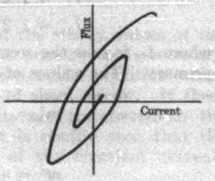

Fig. 6

In this case the flux is represented as lagging the current which produces it by a constant angle β. See Fig. 5. This is equivalent to assuming that the hysteresis loop is an ellipse and that the eddy currents are all in time quadrature with the flux in the core. The fall in reactive pressure $j\,\omega\,L\,I$, leads the flux, ϕ, by 90 deg., but the current by an angle which is somewhat less. This being the case, it is necessary to represent the inductance by a complex number, i. e. $L = L_1 - j\,L_2$; where $L_2/L_1 = \tan\beta$. Thus $j\,\omega\,L\,I = j\,\omega\,(L_1 - j\,L_2)\,I$

$$= \omega\,L_2\,I + j\,\omega\,L_1\,I$$

Similarly if the current is of the general form,—and we make the same assumption in regard to the constancy of the phase angle between the flux and the current,—the fall in pressure due to inductance is $(-\,\alpha + j\,\omega)\,(L_1 - j\,L_2)\,\bar{I}$, which may still be written $m\,L\,\bar{I}$, understanding now that both m and L are complex numbers. On expansion, this becomes $[\,(-\,\alpha\,L_1 + \omega\,L_2) + j\,(\omega\,L_1 + \alpha\,L_2)\,]\,\bar{I}$. If $\alpha\,L_1$ equals $\omega\,L_2$, i. e. if $\alpha/\omega = \tan\beta$, the fall in pressure due to inductance is in exact quadrature with the current and the energy given up by the magnetic field due to its collapse is just sufficient to supply the core losses. The power absorbed in the coil is then equal to the copper loss. If, with the same frequency, the rate of collapse of the magnetic field is greater than this, the coil absorbs less power than the copper loss.

If there is another electric circuit linking the same magnetic core, the fall in electric pressure through it produced by the action of the current in the first circuit is $j\,\omega\,M\,\bar{I}_1$. Notice that this is a vector which leads the current vector by more than 90 deg. When there are core losses in the mutual magnetic circuit the mutual inductance, M_1, might likewise be represented by a complex number of the form, $M_1 - j\,M_2$.

Only considerable research can determine the value of this suggestion that the self and mutual inductance be represented by complex numbers when the currents are decaying sinusoids. It should be noted that there is a constant lag angle of flux with respect to magnetizing current only if the hysteresis "loop" is a diminishing ellipse such as shown in Fig. 6. This is, of course, not true and whether it is justifiable can only be determined by experiment.

In any inductive circuit the current cannot immediately respond to an abrupt change in the magnitude or form of the applied electromotive forces. There is always a transient condition of current flow, the duration of which is determined by the inductance and resistance of the circuit. Moreover, the form of this transient flow is determined solely by these constants and is wholly independent of the character of the applied electromotive forces. The magnitude and phase of the transient, however, are determined by the initial and final values of the current in the circuit, and these are fixed by the applied electromotive forces.

Two Circuits Having Mutual Inductance but no Motion with Respect to Each Other

The simplest case we shall consider is that of a single-phase transformer which is simultaneously short-circuited on both primary and secondary sides.

With the primary winding short-circuited the fall in pressure through it is zero.

$$\bar{I}_1\,r_1 + \bar{I}_1\,m\,L_1 + \bar{I}_2\,m\,M = 0 \qquad (1)$$

r_1 and L_1 represent the resistance and self inductance, not leakage inductance, of the primary. M is the mutual inductance between primary and secondary. Likewise:

$$\bar{I}_2\,r_2 + \bar{I}_2\,m\,L_2 + \bar{I}_1\,m\,M = 0 \qquad (2)$$

The primary and secondary currents have the same form and thus the same generalized angular velocity. Eliminating $\bar{I}_1$ and $\bar{I}_2$ from equations (1) and (2) gives:

$$(r_1 + m\,L_1)\,(r_2 + m\,L_2) = m^2\,M^2$$

Expand and divide each term by $L_1\,L_2$ giving,

$$\sigma\,m^2 + (k_1 + k_2)\,m + k_1\,k_2 = 0 \qquad (3)$$

where $\sigma = 1 - \dfrac{M^2}{L_1\,L_2}$ $k_1 = r_1/L_1$ and $k_2 = r_2/L_2$

By some writers σ is called the leakage coefficient, whereas others define the leakage coefficient as $\frac{L_1 L_2}{M^2} - 1$. In the latter case $\left(\frac{L_1 L_2}{M^2} - 1\right) = \sigma' = \frac{\sigma}{1-\sigma}$; or $\sigma = \frac{\sigma'}{1+\sigma'}$. In a transformer in which the resistance of the windings may be neglected this leakage coefficient, σ, is the ratio of the current on open circuit to that on short circuit with the same applied potential. k_1 and k_2 are the reciprocals of the time constants of the two windings on open circuit.

The solution for the generalized angular velocity is

$$m = -\frac{k_1 + k_2}{2\,\sigma} \pm \frac{\sqrt{(k_1 + k_2)^2 - 4\,\sigma\, k_1\, k_2}}{2\,\sigma} \qquad (4)$$

Since σ is less than unity, the values of m are both real, and the transient current consists of two direct-current components one of which diminishes much faster than the other. These currents may be represented by vectors as in Fig. 1B.

In this case, in which both windings are short-circuited, there will be no current in either winding after the transient has disappeared. Thus the only current after short-circuit occurs is the transient. Let the vectors I_{10} and I_{20} represent the primary and secondary currents before short circuit occurs. Also let $\bar{I}_1'$ and $\bar{I}_1''$ represent the components of the transient current in the primary and similarly for the secondary. Then at the moment of short circuit, the sum of the instantaneous values of the two transient components must equal the instantaneous value of the initial current in both primary and secondary. The transient components might thus be represented by an indefinitely great number of vectors since it is only necessary that their projection on the given axis have specified values. It is more convenient, however, if they are chosen so that their vector sum is equal to the vector representing the initial current before short circuit. Thus we will write:

$$I_{10} = \bar{I}_1' + \bar{I}_1'' \qquad (5)$$

and

$$I_{20} = \bar{I}_2' + \bar{I}_2'' \qquad (6)$$

Remember that the vectors $\bar{I}_1'$, etc., are diminishing exponentially and that this equality holds only at the moment of short circuit. Further, notice that the vectors I_{10} and I_{20} are drawn to represent the *maximum* values of the currents before short circuit. Each of these transient components must satisfy equations (**1**) and (**2**). Substitute in equation (**1**) and write in the following form:

$$\bar{I}_1' \frac{(r_1 + m' L_1)}{m' M} = -\,\bar{I}_2'$$

also

$$\bar{I}_1'' \frac{(r_1 + m'' L_1)}{m'' M} = -\,\bar{I}_2''$$

where m' and m'' are the two values of m as indicated by equation (**4**). If these equations are added and $(I_{10} - \bar{I}_1')$ is substituted for $\bar{I}_1''$ the solution for $\bar{I}_1'$ is:

$$I_1' = \frac{m'\, m''}{k_1\,(m' - m'')}\,\{\,(k_1/m'' + 1)\,I_{10} + \sqrt{1-\sigma}\,\sqrt{L_2/L_1}\,I_{20}\,\} \qquad (7)$$

also

$$I_1'' = \frac{m'\, m''}{k_1\,(m'' - m')}\,\{\,(k_1/m' + 1)\,I_{10} + \sqrt{1-\sigma}\,\sqrt{L_2/L_1}\,I_{20}\,\} \qquad (8)$$

The product of the two roots divided by their difference is

$$\frac{m'\, m''}{m' - m''} = \frac{k_1\, k_2}{\sqrt{(k_1 + k_2)^2 - 4\,\sigma\, k_1\, k_2}}$$

In a transformer the ratio of the self-inductances of the primary and secondary is practically equal to the square of the ratio of the turns. Thus as is commonly said, $\sqrt{L_2/L_1}\, I_{20}$ is the secondary current referred to the primary. In order to reduce these expressions to a more understandable form, we will assume that k_1 equals k_2. This is very nearly true. If this is done the two components of the primary current become:

$$\bar{I}_1' = 1/2\,(I_{10} + I_{20}) \qquad (9)$$

$$\bar{I}_1'' = 1/2\,(I_{10} - I_{20}) \qquad (10)$$

where I_{20} is now the secondary current referred to the primary. The values of m reduce to:

$$m' = -\frac{r}{L + M}$$

$$m'' = -\frac{r}{L - M}$$

where r and L are the resistance and self-inductance of the primary winding, and M is the mutual inductance on the assumption that both windings have the same number of turns. In this case the difference of the self and mutual inductances is the leakage inductance of one winding. The two components of the transient secondary current are similar to those of the primary. Thus at the moment of short circuit each of the currents in the two windings breaks up into two components, one of which is one-half of the no-load current; $(I_{10} + I_{20})$ is the no-load current. This component decays at a rate determined by the resistance and the sum of the self and mutual inductances of the two windings. It is due to the energy stored in the magnetic core of the transformer. Its rate of decay is much slower than that of the other component. The latter's rate of decay is determined by the resistance and leakage inductance of the windings, and this component of the currents is due to the energy stored in the leakage field. The vector diagram and oscillograph record are given in Fig. 7. In this figure the vectors I_1 and I_2, representing the primary and secondary currents, are rotating at an angular velocity of ω. At the moment of short circuit each of these vectors abruptly stops and breaks up into two component vectors which diminish at different rates.

The more usual transient condition would be one in which the short circuit occurred on the secondary side alone. In this case the current immediately after short circuit consists of the transient current together with the steady value of the short-circuit current. The conditions that must be satisfied are now:

$$I_{10} = \bar{I}_1' + \bar{I}_1'' + I_{sc1} \qquad (11)$$

and

$$I_{20} = \bar{I}_2' + \bar{I}_2'' + I_{sc2} \qquad (12)$$

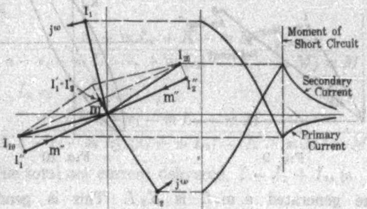

Fig. 7

where I_{sc1} and I_{sc2} represent the steady values of the short-circuit currents in the primary and secondary windings. As before this is a vector equation which holds only at the moment of short circuit. If these steady short-circuit currents are transferred to the left of the equality sign it is readily seen that the values of the components of the transient currents become:

$$I_1' = 1/2\,(I_{10} + I_{20}) - 1/2\,(I_{sc1} + I_{sc2}) \qquad (13)$$

$$I_1'' = 1/2\,(I_{10} - I_{20}) - 1/2\,(I_{sc1} - I_{sc2}) \qquad (14)$$

As before the secondary components are similar to the primary components. It is now necessary to determine the phase relation between the steady short-circuit current and the current in the transformer before the short circuit occurs.

In order to keep the diagrams as simple as possible, we will make the customary assumption that the primary and secondary currents are equal and in phase opposition. In that case the first component of the transient current is zero, and the second component is equal to the current before short circuit minus the steady short-circuit current. [See equations (**13**) and (**14**)] with $I_{10} = -I_{20}$ and $I_{sc1} = -I_{sc2}$. The error thus introduced is usually very slight. The vector diagram at the moment of short circuit is given in Fig. 8.

Before short circuit occurs only the four vectors representing the primary and secondary potentials and current appear. They are rotating at a uniform angular velocity, ω. At the instant after short circuit all of these except the primary potential disappear and in their place spring the steady short-circuit currents. I_{sc1} and I_{sc2} and the transient currents I_1'' and I_2''. The former continue to rotate at an angular velocity of ω without change in magnitude, while the latter remain fixed in angular position but diminish exponentially at a rate, α, depending upon the resistance and leakage inductance of the windings. The arrows at the ends of the vectors indicate the direction of their motion. Observe that, at the moment of short circuit, the current before short circuit is the vector sum of the steady short-circuit current and the transient current. Since the steady short-circuit current is from 10 to 25 times the full-load current, it makes little difference whether the short circuit comes at a time of full load or no load. It may be a little more severe at no load. On account of the excessive value of the steady short-circuit current, the vectors representing *it* and the *transient current* are essentially in phase opposition at the moment of short circuit. Half a cycle after short circuit occurs the vector I_{sc1} will thus have overtaken the stationary vector I_1'' and their vector sum will be a maximum. It at once follows that short circuit is most severe when it occurs at a moment that the steady short-circuit current would be a maximum.[2] Since the steady short-circuit current and applied potential are nearly in quadrature, for example, the power factor of a certain 1000-kv-a., 60-cycle transformer is 0.15 at short circuit,—the worst moment for a short circuit to occur is at about the time that the applied potential is passing through zero. In the transformer just cited the ratio of the resistance to the leakage inductance is 56 and thus in the time of one half cycle the transient current will have fallen to $\epsilon^{-\frac{56}{120}}$ or 63 per cent of its initial value. Thus the first current rush would be to a value which is 63 per cent greater than the maximum value of the steady short-circuit current which is itself more than 20 times the maximum value of the full-load current. The

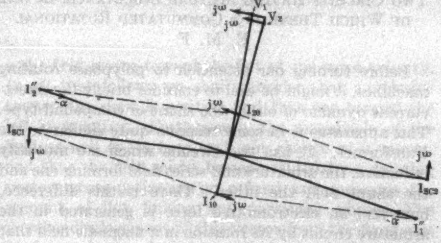

Fig. 8

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[2] To determine the exact moment at which the short-circuit current would be a maximum involves the solution of a mixed trigonometric and exponential equation, which is decidedly not worth while in the present problem.

forces between turns vary as the square of the current, and thus the first shock on this transformer may be 265 per cent of its ultimate maximum value.

When these forces have their maximum value they occur one-one-hundred-and-twentieth (1/120) of a second after short circuit in a sixty-cycle transformer and one-fiftieth (1/50) of a second after short-circuit in a twenty-five-cycle transformer. Nevertheless with transformers having the same short-circuit characteristics (volts, amperes, and watts on short-circuit test), the maximum forces that the windings must sustain are the same for all frequencies.

If the transformer is but a portion of the electric circuit in which the transient takes place the characteristic equations (**1**) and (**2**) must be written:

$$\bar{I}_1 r_1 + \bar{I}_1 m L_1 + \bar{I}_2 m M = \bar{V}_1 \qquad \textbf{(15)}$$

$$\bar{I}_2 r_2 + \bar{I}_2 m L_2 + \bar{I}_1 m M = \bar{V}_2 \qquad \textbf{(16)}$$

$\bar{V}_1$ and $\bar{V}_2$ are the transient potentials which appear at the terminals of the transformer. If the transformer has a relatively small exciting current little error will be made if it is treated like a simple inductive reactance. In this case the relation between the primary and secondary pressures is much simpler.

$$\bar{V}_1 - \bar{I}_1 (R + m L) = a \bar{V}_2 \qquad \textbf{(17)}$$

R and L are the equivalent resistance and leakage inductance of the two windings and a is the ratio of transformation at no load.

TWO CIRCUITS HAVING MUTUAL INDUCTANCE, IN ONE OF WHICH THERE IS A COMMUTATED ROTATIONAL E. M. F.

Before turning our attention to polyphase rotating machines, it might be well to consider briefly the direct-current dynamo of either the shunt or compound type. This apparatus is in some respects quite similar to the transformer. It has two circuits which are mutually inductive, the armature and series field forming one and the shunt field the other. There is this difference, however: an electromotive force is generated in the armature circuit by its rotation in a magnetic field that is due to the joint action of the current in both circuits. With constant speed this generated e. m. f., is proportional to the magnetic field. The relation between the magnetic field and the currents in the armature and shunt field circuits which produce it is similar to the well-known hysteresis loop, for which there is no simple mathematical expression. In some problems in which the exciting current is a steady alternating one, it has been found convenient, however, to assume that this hysteresis loop is an ellipse. If this assumption should prove sufficiently accurate in the present case also, it will simplify the problem greatly, for as has been already suggested, we may then represent the magnetic field and the rotational e. m. f. which it produces, together with the net exciting ampere-turns by vectors even when these quantities are damped sinusoids, *i. e.*, of the general form. This vector relation is shown in Fig. 9, which is drawn for the steady state.

The flux, ϕ, and the rotational e. m. f. E which is proportional to it, lag the exciting current I by a constant angle β. Thus we may write

$$E = (K_2 - j K_2)\, I \qquad \textbf{(18)}$$

$$\text{or } E = K\, I$$

where K is a complex number, $K_1 - j K_2$, having the dimensions of resistance. The vector equation shows that when the instantaneous value of the current is zero the generated e. m. f. is $K_2 I$. This is generally referred to as the residual voltage. At the moment that the current has its greatest value the rotational e. m. f. is $K_1 I$. If the exciting current is going through a steady cyclic variation, it is a simple matter to determine these two components of the e. m. f. See Fig. 10. When the current is a damped sinusoid, however, the vector representing it is shorter at the moment that it is perpendicular to the time axis than it was earlier in the cycle when the current was passing through its maximum value. See Fig. 11. In this case the best method of determining the constants K_1 and K_2, in order to assure the greatest accuracy in the calculations can only be determined by experiment.

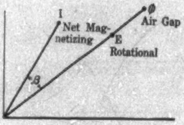

Fig. 9

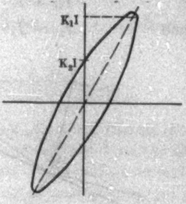

Fig. 10

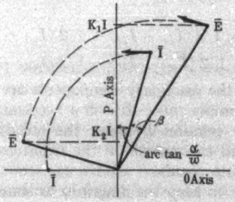

Fig. 11

Let $\bar{I}_1$ and $\bar{I}_2$ be the currents in the armature and shunt field circuits and similarly, r_1 and L_1, the resistance and self-induction of the one, and r_2 and L_2, the same constants of the other. Also let M be the mutual inductance between the two circuits. The e. m. f., $\bar{E}$, generated in the armature circuit by rotation is:

$$\bar{E} = K\, (N_1/N_2\, \bar{I}_1 + \bar{I}_2);$$

where N_1 and N_2 are the effective turns in each circuit acting on the main magnetic axis. Let $N_1/N_2 = a$. Also let $\overline{V}$ be the rise in terminal pressure across both the armature and shunt field circuits.

Then we have:

$$\overline{V} = \bar{E} - \bar{I}_1 r_1 - \bar{I}_1 m L_1 - \bar{I}_2 m M \qquad \text{(armature circuit)}$$

$$= \bar{I}_1 (K a - r_1 - m L_1) + \bar{I}_2 (K - m M) \qquad (19)$$

also $\overline{V} = - \bar{I}_2 r_2 - \bar{I}_2 m L_2 - \bar{I}_1 m M$ (shunt field circuit)

$$= - \bar{I}_2 (r_2 + m L_2) - \bar{I}_1 m M \qquad (20)$$

If these equations, (19) and (20), are solved for $\bar{I}_1$ and $\bar{I}_2$ we have:

$$\bar{I}_1 = \frac{r_2 + m L_2 + K - m M}{(K a - r_1 - m L_1)(r_2 + m L_2) - (K - m M) m M} \overline{V}$$

and

$$\bar{I}_2 = - \frac{K a - r_1 - m L_1 + m M}{(K a - r_1 - m L_1)(r_2 + m L_2) - (K - m M) m M} \overline{V}$$

The total net current delivered, $\bar{I} = \bar{I}_1 + \bar{I}_2$, is

$$\bar{I} = \frac{r_1 + r_2 + K(1 - a) + m(L_1 + L_2 - 2M)}{(K a - r_1 - m L_1)(r_2 + m L_2) - (K - m M) m M} \overline{V}$$

The ratio of the fall in terminal potential, $-\overline{V}$, to the total current is the generalized impedance, Z, of the dynamo. Thus we have:

$$Z = \frac{(K - m M) m M - (K a - r_1 - m L_1)(r_2 + m L_2)}{r_1 + r_2 + K(1 - a) + m(L_1 + L_2 - 2M)}$$

For further discussion of this problem the reader is referred to the paper by Doherty on "Exciter Instability" presented at the Pacific Coast Convention, August, 1922.

TWO SYSTEMS OF BALANCED POLYPHASE CIRCUITS HAVING A CONSTANT RELATIVE ANGULAR VELOCITY

The author believes that the simplest method of presenting the analysis of the transient condition in polyphase induction and synchronous machines is that which considers the rotating magnetic fields in the air gap and the reactions which they produce. When these polyphase machines are operating under balanced conditions, either steady or transient, the currents in the several phases of the polyphase winding may be represented by vectors of equal length equally spaced with respect to each other. For the steady condition the vectors are of constant length and rotate at a constant angular velocity. This representation has been used for many years. The *only* difference between the steady and the transient condition is that the vectors which represent the latter state diminish exponentially as they rotate. If this conception of the transient state is borne in mind the following analysis is readily followed by anyone who is familiar with the steady operation of these machines. In all that follows the relative speed of the two windings is assumed to remain constant throughout the time considered. Since the maximum current occurs within about a half-cycle after the transient begins the angular velocity of the moving part has little opportunity to change before the first shock is over.

The parallel lines in Fig. 12 represent the uniform air gap of a three-phase induction motor. The phase belts of the stator conductors are indicated at the letters a, b, c. The pole pitch is from $+a$ to $-a$. If this winding is properly designed, there will be very small harmonics in the space distribution of the air-gap flux density when the alternating currents are balanced. We will neglect whatever harmonics may exist and assume that this distribution may be represented by a sinusoidal curve such as β. The line $X - X$ is the axis of this curve. If the current in phase b lags that in phase a by 120 deg. this magnetic field will move toward the right at a constant angular velocity of ω radians per second; where ω equals the frequency of the alternating current multiplied by 2π. On the other hand, if the current in phase b leads that in phase a the field moves toward the left at the same velocity. In all that follows we shall consider that a velocity toward the right is positive and that one toward the left is negative. Furthermore, the magnitude of the field as well as its form, will not change as it moves, and thus the field might be represented by a rotating vector whose projection on a fixed axis indicates the amount of flux linking any particular phase. The angular displacement of this vector from the fixed axis is always the angle between the axis of the moving field and the axis of the phase considered. See Fig. 13. One of the most important points in the theory of rotating fields that are produced in this manner in a uniform air gap is that the flux through any phase is a maximum at the moment that the current in that phase is also a maximum. For example, when the current in phase a is maximum the field is in the position β_a. One sixth of a cycle later, the current in $-c$ will be a maximum and in the same time interval, the flux distribution will have moved one sixth of two full pole pitches toward the right and be a maximum through this phase, $-c$. In a similar manner, it will have moved into the position β_b by the time that the current in phase b is a maximum. This can be represented vectorially as in Fig. 13. The current and flux are

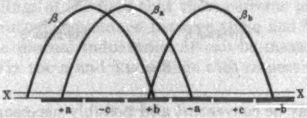

FIG. 12

represented by the rotating vectors I and φ. The instantaneous value of the current in any phase or the flux through that phase is the projection of the corresponding vector on the axis of that phase. In Fig. 13 the vectors are drawn for the time that the current in phase $-c$ has its maximum value. Notice that for both the phases a and b the currents in them and the fluxes through them are each one half of their maximum values. There is this difference, however, for while the values are decreasing for phase a they are increasing for phase b, in which they will be a maximum 1/6 of a cycle later. Ordinarily, we are content to know the current and flux in any one phase, a, for example, and then the projection axes for the other phases are not drawn.

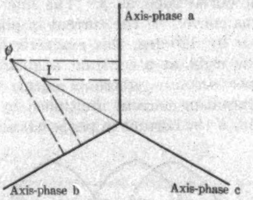

Fig. 13

It may be convenient and possibly necessary in some problems to consider magnetic fields that rotate to the left, that is, with a negative angular velocity. In this case the vector, φ, representing the flux through any phase would likewise have a negative angular velocity and would thus move in the clockwise direction. If the trigonometric expressions for the instantaneous currents in the phases are written it will be noticed that giving a negative value to ω is equivalent to reversing the phase order of the currents. Such a reversal of phase order reverses the direction of rotation of the magnetic field produced by the current. If the current vector in Fig. 13 is rotated in the negative or clockwise direction it will be noticed that the current in phase b will lead that in phase a by one-third of a cycle, that is the phase order of the currents is reversed. We must be most careful, however, to remember that, if it is necessary to combine or operate on these vectors, whether they have positive or negative angular velocities the operator j *always* indicates a phase rotation of one quadrant in the positive or counter clockwise direction. If this is not done we may become hopelessly confused. If the air gap is not uniform, as is the case with salient pole machines, the curve of flux density will alter both its shape and magnitude as it rotates, but that is a complication which it is well for the present to disregard, leaving its consideration until some future time.

In general if the currents in the several phases of a symmetrical n-phase winding have values equal to the projections of a "current" vector on each of a system of n equally spaced axes, the magnetic flux linking any winding is equal to the projection of a "flux" vector, which is drawn in phase with the "current" vector, on the axis corresponding to that winding. This assumes that the windings are so designed that there are essentially no harmonics in the space distribution of the magnetic field and that each winding will, if acting alone, produce exactly the same flux per ampere.

If the currents are of the general form both the "current" and "flux" vectors shrink exponentially as they rotate. The space distribution of the magnetic field is still sinusoidal at all times but its magnitude diminishes as it moves through the air gap. Compare Fig. 13 with Fig. 14. In the latter figure let us assume that the currents took on the general form at the moment that the flux distribution was at the point indicated by the curve β, and that their generalized angular velocity is $(-\alpha + j\,\omega)$. The equation of the maximum density for any later position of the field such as β_1 is: $B_m = B_o \epsilon^{-\alpha t}$; where t is the time required for the field to move from the position β to β_1, *i. e.*, through an angular distance X.

The flux through any phase as well as the current in it may still be represented by a vector although this vector is now of the general type—that is, it shrinks exponentially as it rotates. A current or flux variation of the general form reaches its maximum value in any phase at the moment that the vector which represents it is still arc tan α/ω from the projection axis of this phase. Because the field is shrinking as it rotates the flux linking phase a is greater when the field is in the position β_1 than when it is in the position β_2 at which moment the axis of the field is coincident with the axis of the phase. See Fig. 15. Thus the flux through any phase is a maximum at the moment that the current in the phase is a maximum, but the flux distribution is no longer directly opposite the phase when this occurs, as was the case when the currents were steady sinusoids.

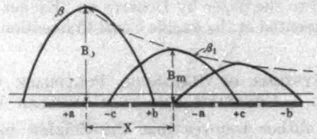

Fig. 14

It is then evident that even in the transient state the flux through any phase and the current in that phase may still be represented by coincident vectors.

The electromotive force generated by this rotating field is the negative of the time rate of change of the flux linkages. Thus, in general, when this time rate of flux change in any phase is zero the e. m. f. is zero and the flux linking the phase is a maximum. In Fig.

16 the flux in phase a is a maximum when the vector representing it is in the position shown. At this moment the e. m. f. is zero and the vector, $\bar{E}$, representing it must, therefore, be perpendicular to the projection axis. An additional proof of this is interesting even if unnecessary. Consider Fig. 15. When the field is in the position β_1 the e. m. f. produced in the phase a has two components, one due to the motion of the field and the other due to its shrinking. The motional e. m. f. is proportional to the velocity of the field and to the sine of the angle, $\omega\, t$, between the field and the phase winding. If at the point $+\,a$, the direction of the flux density is upward and the motion of the field is to the right the e. m. f. generated in the belt, $+\,a$, is away from the observer. On the other hand if the field within a circuit changes an e. m. f. is set up which is proportional to the rate of change and is in such a direction as to oppose the change. The e. m. f. thus produced is proportional to $\alpha \cos \omega\, t$, and is toward the observer in the belt of conductors $+\,a$. If the angular displacement, $\omega\, t$, is so chosen that the resulting e. m. f. is zero, $\omega \sin \omega\, t$ equals $\alpha \cos \omega\, t$. That is to say, the angular displacement of the flux distribution is arc tan α/ω. Observe that the fall in potential, $-\,\bar{E}$, leads the flux, $\bar{\phi}$, by an angle, $\pi/2$ + arc tan α/ω, just as does the fall in potential due to self induction. Thus, in a polyphase winding we may combine the fall in pressure due to the so-called leakage flux in one phase with the fall in pressure in this phase due to the mutual air-gap flux which is produced by all phases. This is commonly done in the analysis of synchronous machines and the resulting loss in pressure is the so-called synchronous inductance, or reactance drop. If L represents the synchronous inductance, a transient current, $\bar{I}$, having a generalized angular velocity m, will produce a fall in potential of $m\, L\bar{I}$. This fall in potential is represented by a vector which is $\pi/2$ + arc tan α/ω ahead of the current vector.

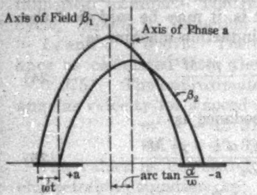

Fig. 15

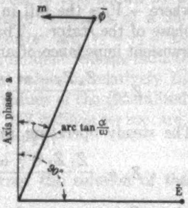

Fig. 16

This conception of the action of a rotating field of the transient type may be summarized as follows: Consider two groups of symmetrical polyphase windings, a and b, the first of which is stationary and the second, moving in the positive direction at an angular velocity of p radians per sec. The projection axis for the stationary winding is stationary, while the projection axis for the moving winding is moving at the same angular velocity, *viz.* p, as indicated in Fig. 17. If the angular velocity of the currents in the stationary, or a, winding, is ω radians per sec., the relative angular velocity of the magnetic field due to these currents with respect to the b winding is $(\omega - p)$ radians per sec. in the positive direction. The flux linking any phase of the b winding is a maximum arc tan $\dfrac{\alpha}{\omega - p}$ before the flux vector lies in the projection axis of this phase. If the field is moving with respect to the b winding in the positive direction the flux will be a maximum when the flux vector is at $\bar{\phi}_a'$. See Fig. 17. If the field is moving relatively in the negative direction, *i. e.* if p is greater than ω, the flux will be a maximum when the flux vector is at $\bar{\phi}_a''$. At the moment that the flux is a maximum through any phase, the generated e. m. f. in this phase is zero and the vector representing it will be at E_b' for a positive relative angular velocity and at E_b'' for a negative velocity. This generated e. m. f. is equal to the product of the current in one phase of the a windings, the relative generalized angular velocity with respect to the b windings of the field that these currents produce, and the mutual inductance between the a and b windings. The mutual inductance M can be measured as follows: Fix the a and b windings with respect to each other and send balanced polyphase currents through either, the a for windings for example. The mutual inductance M is the e. m. f. generated in one phase of the other, or b winding, divided by the product of the current in one phase of the a winding and angular velocity of this current. By the time that the flux has advanced $\pi/2$ – arc tan $\dfrac{\alpha}{\omega - p}$ with respect to the b windings, the generated e. m. f. will have reached its maximum value, but the current will usually not reach its maximum value until somewhat later depending

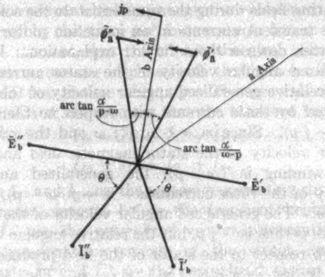

Fig. 17

upon the constants of the *b* windings and their terminal pressure. In any case, however, the current will be represented by a vector equal to $\frac{\bar{E}_b - \bar{V}_b}{r_b + m\,L_b}$; where $\bar{V}_b$ is the terminal pressure, r_b the resistance and L_b the synchronous inductance of one of the *b* phases. By the time the current in the winding has reached its maximum value the field will have advanced a further relative angle of θ. Thus whether the motion of the *a* field is positive or negative, its action on the *b* winding will develop a field which will always lag the *a* field in its motion with respect to the *b* winding. Consider an induction machine in which the magnetic field due to the stator, *a* windings, rotates in the positive direction at ω radians per sec. If the rotor turns in the same direction at less than this synchronous speed the relative motion of the stator field with respect to the rotor is positive. The rotor component of magnetic field actually and apparently lags the stator field. If the rotor turns in the same direction at a speed greater than synchronous, the relative motion of the stator field with respect to the rotor is negative and the rotor component of field apparently leads when observed from the fixed stator, though it actually lags, as would be observed from any point on the rotor itself. It is interesting that these well-known facts should apply to the transient state as well as to the steady operating condition.

Having established these general principles in regard to rotating fields during the transient state the solution for the transient currents in an induction motor may be written down without further explanation. If the generalized angular velocity of the stator currents is *m* the relative generalized angular velocity of the field produced by these currents with respect to the rotor is $(m - j\,p)$. Since $m = -\alpha + j\,\omega$ and the relative angular velocity of the stator magnetic field and the rotor winding is $(\omega - p)$, the generalized angular velocity of the rotor currents is $-\alpha + j\,(\omega - p)$, *i. e.*, $m - j\,p$. The generalized angular velocity of the rotor currents is thus $m - j\,p$, but the relative angular velocity with respect to the stator of the field produced by them is $[m - j\,p - (-j\,p)]$, *i. e.*, *m*. The second $(-j\,p)$ is the relative angular velocity of the stator with respect to the rotor. Thus we have:

$$\bar{E}_2 = -(m - j\,p)\,M\,\bar{I}_1 \qquad (19)$$

$$\bar{E}_1 = -m\,M\,\bar{I}_2 \qquad (20)$$

$\bar{E}_2$ and $\bar{E}_1$ are respectively the generated e. m. f. due to the stator currents in the rotor and that due to the rotor currents in the stator. The rotor current, $\bar{I}_2$, is:

$$\bar{I}_2 = \bar{E}_2/Z_2 \qquad (21)$$

where Z_2 is the generalized self impedance of the rotor calculated for a generalized angular velocity of $m - j\,p$. The stator current $\bar{I}_1$, is:

$$\bar{I}_1 = \frac{\bar{E}_1 - \bar{V}_1}{Z_1} \qquad (22)$$

where Z_1 is similarly the generalized self impedance of the stator calculated for a generalized angular velocity of *m*, and $\bar{V}_1$ is the transient terminal pressure of the stator. Combining these four simple relations gives the characteristic equation for the stator current of an inductor motor during the transient period. That is:

$$\bar{I}_1 = -\frac{Z_2}{Z_1\,Z_2 - m\,(m - j\,p)\,M^2}\,\bar{V}_1 \qquad (23)$$

$$= \frac{-\bar{V}_1}{Z_T}$$

where $-V_1$ is the fall in terminal pressure across one phase of the stator. That is, it may be said that the transient impedance of an induction machine is:

$$Z_T = \frac{Z_1\,Z_2 - m\,(m - j\,p)\,M^2}{Z_2} \qquad (24)$$

The steady operating impedance is

$$Z_0 = \frac{Z_1\,Z_2 - j\,\omega\,(j\,\omega - j\,p)\,M^2}{Z_2} \qquad (25)$$

where $Z_1 = r_1 + j\,\omega\,L_1$ and $Z_2 = r_2 + j\,(\omega - p)\,L_2$. If the transient occurs when the stator winding is symmetrically short-circuited or if it occurs when the stator is connected to a circuit whose resistance and inductance are many times smaller than the similar constants of the motor, the transient terminal pressure of the stator is essentially zero. It is thus evident that the transient impedance is also zero. Equating the transient impedance Z_T to zero gives the characteristic equation for an induction motor and determines the generalized angular velocity of the stator current.

Since $Z_1 = r_1 + m\,L_1$ and $Z_2 = r_2 + (m - j\,p)\,L_2$ equating the transient impedance to zero gives $(r_1 + m\,L_1)\,[r_2 + (m - j\,p)\,L_2] - m\,(m - j\,p)\,M^2 = 0$ Expanding and collecting terms and dividing by $L_1\,L_2$ gives the following quadratic in *m*:

$$\sigma\,m^2 + (k_1 + k_2 - j\,p\,\sigma)\,m + k_1\,k_2 - j\,p\,k_1 \qquad (26)$$

where $k_1 = r_1/L_1$, $k_2 = r_2/L_2$ and $\sigma = 1 - \frac{M^2}{L_1\,L_2}$

Notice that this characteristic equation for the induction motor is identical with that for a transformer with the exception of two terms, $-j\,p\,\sigma\,m$, and $-j\,p\,k_1$. These terms, however, completely alter the form of the transient. The solution for the generalized angular velocity, *m*, is:

$$m = \frac{-(k_1 + k_2)}{2\,\sigma} + \frac{j\,p}{2} \pm \frac{\sqrt{(k_1 + k_2 - j\,p\,\sigma)^2 - 4\,\sigma\,(k_1\,k_2 - j\,p\,k_1)}}{2\,\sigma} \qquad (27)$$

If $k_1 = k_2$, the quantity under the radical is real and will be negative unless the equivalent resistance is somewhat greater than the equivalent reactance as measured with the rotor blocked, a most unusual condition except possibly in very small motors. Furthermore the decrement $\frac{k_1 + k_2}{2\,\sigma}$ is very nearly equal to the resistance of the stator winding divided by the equivalent inductance, L_0, as determined from locked saturation test. Thus under ordinary conditions the two values of the generalized angular velocity are approximately:

$$m' = -\,r_1/L_0 + j\,p \tag{28}$$

$$m'' = -\,r_1/L_0 + j\,O \tag{29}$$

These are the values given by Doherty and Williamson. The magnitude of the error is best shown by a numerical example. The relative constants of a 25-cycle, 2600-h. p. induction motor are $r_1 = r_2 = L_1 = L_2$ and $\sigma = 0.075$. The equivalent leakage inductance as calculated from these values is relatively $L_0 = 0.0751$. The approximate values of the generalized angular velocity for a speed of 157 radians per sec. are:

$$m' = -\,13.31 + j\,157$$

$$m'' = -\,13.31 + j\,O$$

The values as calculated from the solution of the characteristic equation are:

$$m' = -\,13.33 + j\,155.8$$

$$m'' = -\,13.33 + j\,1.2$$

The error in the approximate values in this case at least is very small.

The form of the generalized angular velocity shows that the transient current during short circuit consists of two components, one having a frequency slightly below that corresponding to the speed of the motor, and the other a very low frequency. Notice that the sum of these frequencies always equals that corresponding to the speed of the motor. If the ratios of resistance to synchronous self inductance are not the same for the stator and rotor windings the components of the transient current will decay at different rates, otherwise their rates of diminution are ordinarily equal. During this transient short circuit the angular velocity of the rotor is greater than the angular velocity of the stator currents so that the machine is operating as an induction generator. In this case, the *sum* of the angular velocities of the corresponding stator and rotor component currents is equal to the angular velocity of the rotor. In calculating the performance of an induction machine under steady operating conditions it is customary to use an effective resistance of the stator winding and a true, or ohmic, resistance of the rotor winding. Similarly, it may prove more accurate to use an effective resistance of stator and a true resistance of rotor when calculating m', and, vice versa, a true resistance of stator and an effective resistance of rotor when calculating m''. If, for example, the ratio of effective to true resistance for both stator and rotor is 1.5, the rate of decay of the transient currents is about 25 per cent greater,[3] than would be calculated by using the true resistances. Again, if the ratios of true resistance to synchronous self inductance are the same for both stator and rotor, the rates of decay of the two transient components would be slightly different if effective resistances are substituted for true resistances as suggested.

Having determined the form of the transient it remains only to determine its magnitude. The fundamental principle is that the current in any inductive circuit cannot abruptly change its value. Thus, if the currents in the stator and rotor windings immediately before the transient occurs are represented by the vectors I_{10} and I_{20} in Fig. 18 the vector sum of the currents in these windings immediately after the transient begins must be respectively I_{10} and I_{20}. The case is somewhat different from the single-phase transformer inasmuch as there are now two or more stator or primary circuits and a corresponding number of rotor or secondary circuits in which the instantaneous values of the currents immediately before and after the transient begins must be the same. It has already been pointed out that for any instantaneous values of balanced polyphase currents in symmetrical windings both the currents and the magnetic field which they produce may be represented by *vectors* in phase with each other. If the currents in the stator and rotor windings do not abruptly change at the moment that the transient begins, it is evident that the component magnetic fields they produce in the air gap do not change in magnitude or in position. That is, the vectors which represent the component fields produced by the stator and rotor currents do not change at the moment that the transient begins. Since the current vectors are in phase with the flux vectors they also do not change abruptly. The general relation that holds at the moment that the transient begins is:

$$I_{10} = \bar{I}_1' + \bar{I}_1'' + I_{1s} \tag{30}$$

$$I_{20} = \bar{I}_2' + \bar{I}_2'' + I_{2s} \tag{31}$$

where I_{10} and I_{20} are the vectors representing the stator and rotor currents before the transient begins; $\bar{I}_1'$ and $\bar{I}_1''$ are the generalized vectors representing the two transient components of the stator currents; similarly $\bar{I}_2'$ and $\bar{I}_2''$ are the generalized vectors representing the transient components of the rotor current; I_{1s} and I_{2s} are the vectors which represent the steady currents in the stator and rotor which will exist alone after the transient has disappeared. As it is more convenient if the rotor currents are determined on the basis of a one to one ratio of transformation this will be done throughout. In this case $L_1 = L_2$, at least approximately. The steady operating conditions before and after the transient occurs are sufficient to determine the steady current vectors I_{10}, I_{1s}, etc. The following relations must hold for the transient currents:

$$\bar{I}_1' + \bar{I}_1'' = I_{10} - I_{1s}$$

$$\bar{I}_2' + \bar{I}_2'' = I_{20} - I_{2s}$$

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[3] A. I. E. E. Transactions, page 525.

If an induction motor or generator is short-circuited on all phases I_{1s} and I_{2s} are both zero since there will ultimately be no current in the machine. If an induction motor or generator is connected to a line when it carries no current, I_{10} and I_{20} are both zero. We shall work out in detail only the first of these cases since nothing is gained in generality by doing more.

Each of the transient components must satisfy the vector relation:

$$(r_1 + m\, L_1)\, \bar{I}_1 + m\, M\, \bar{I}_2 = O$$

and thus we may write:

$$(r_1 + m'\, L_1)\, \bar{I}_1{}' + m'\, M\, \bar{I}_2{}' = O \qquad (32)$$

$$(r_1 + m''\, L_1)\, \bar{I}_1{}'' + m''\, M\, \bar{I}_2{}'' = O \qquad (33)$$

also at the moment of short circuit

$$\bar{I}_1{}' + \bar{I}_1{}'' = I_{10} \qquad (34)$$

and $$\bar{I}_2{}' + \bar{I}_2{}'' = I_{20} \qquad (35)$$

Solving these four simultaneous equations for the components of the transient currents in the stator winding gives:

$$I_1{}' = \frac{m'\, m''}{m' - m''}\, [M/r_1\, I_{20} + (1/k_1 + 1/m'')\, I_{10}]$$

and $$\bar{I}_1{}'' = \frac{m'\, m''}{m'' - m'}\, [M/r_1\, I_{20} + (1/k_1 + 1/m')\, I_{10}]$$

These expressions are readily reduced to understandable forms if suitable approximations are made. If $k_1 = k_2 = k$ the values of m' and m'' are approximately

$$m' = -\, k/\sigma + j\, p \qquad (36)$$

$$m'' = -\, k/\sigma + j\, O \qquad (37)$$

Also $$M = \sqrt{1 - \sigma}\, L$$

$$= (1 - \sigma/2)\, L \qquad \text{approximately}$$

where L is the synchronous self inductance of either the stator or rotor winding. The approximate values of the transient components become:[4]

$$I_1{}' = I_{10}/2 - 1/\sigma\, I_n$$

$$I_1{}'' = I_{10}/2 + 1/\sigma\, I_n$$

The error in these values is well within 10 per cent ordinarily. Notice that the transient currents depend almost entirely upon the no-load current, I_n, and the leakage coefficient, σ. It makes little difference whether short-circuit occurs at no-load or full load. The first of these components rotates at approximately synchronous speed while the second is practically stationary. Each shrinks exponentially at the same rate. The first maximum value of the transient current in any phase is greatest if it occurs at such a time that the transient components lie as nearly as possible in the projection axis for that phase at the moment that the transient begins. For, in that case, about one-half a cycle later the first component will have overtaken the second and the resulting projection on the axis of the phase will have its greatest value. During this time, however, each of the components will have shrunk to $\epsilon^{-\frac{k}{\sigma}\frac{\pi}{p}}$ of their initial values. The first maximum current rush may be to an approximate value of

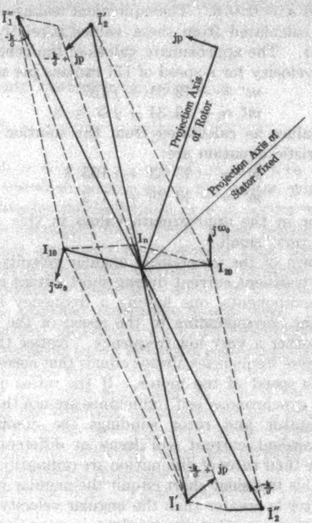

FIG. 18

$$\frac{2\, I_n}{\sigma}\, \epsilon^{-\frac{k}{\sigma}\frac{\pi}{p}} \qquad (38)$$

where I_n is the maximum value of the exciting current before the transient begins. This approximate solution checks with that given by Doherty and Williamson although the latter is in a slightly different form. The vector diagram is given in Fig. 18, which is drawn at the moment that the transient begins. The projection axes of one phase of the stator winding and of one phase of the rotor winding are shown. The former is fixed in position while the latter rotates at a speed

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[4] More exact solutions for the component currents, based on the approximate values of m are:

$$\bar{I}_1{}' = \left(1 + j\,\frac{k}{\sigma\, p}\right)\left(I_{10}/2 - \frac{\sqrt{1-\sigma}}{\sigma}\, I_n\right)$$

and $$\bar{I}_1{}'' = \left(1 + j\,\frac{k}{\sigma\, p}\right)\left(\frac{1 - j\,\frac{k}{\sigma\, p}}{1 + j\,\frac{k}{\sigma\, p}}\, I_{10}/2 + \frac{\sqrt{1-\sigma}}{\sigma}\, I_n\right)$$

equal to that of the rotor, *viz.*, p radians per sec. When s is the slip, p equals $(1 - s)\ \omega_0$. Before the transient begins only the vectors representing the stator and rotor currents appear. They are rotating at a uniform angular velocity, ω_0. At the moment of short circuit both of these vectors vanish and in their places spring the vectors which represent the transient currents. The arrows at the ends of these vectors represent their angular velocities.

SYMMETRICAL POLYPHASE AND SINGLE-PHASE SYSTEMS MOVING WITH RESPECT TO EACH OTHER AT A CONSTANT ANGULAR VELOCITY

The analysis of this problem is somewhat more difficult than that of the preceding. When a polyphase winding is acted upon by a rotating magnetic field it reacts and produces another magnetic field which always rotates at the same speed and in the same direction as does the first. This is not true of a single-phase winding, however, and it is herein that the difficulty lies. When a single-phase winding is excited by a rotating magnetic field, there is produced in it an alternating current of a frequency corresponding to their relative angular velocities. An alternating current in a single-phase winding produces a magnetic effect that is equivalent to two magnetic fields which rotate in opposite directions with respect to the winding at an angular velocity equal to that of the current in the winding. This well-known method of analysis is ascribed to Ferraris. The magnitude of each of these fields is one-half of the *maximum* value of the actual alternating field produced by the single-phase current.

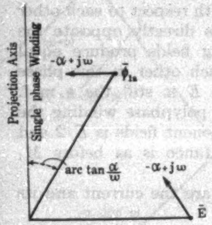

Projection Axis
Single phase Winding
-α + jω
$\bar{\phi}_{1a}$
arc tan $\frac{\alpha}{\omega}$
-α + jω
Ē

FIG. 19

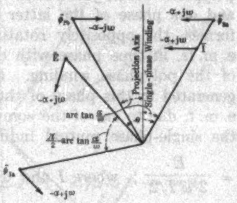

$\bar{\phi}_{2b}$
-α - jω
-α + jω
$\bar{\phi}_{2a}$
-α + jω
Ī
Ē
-α + jω
Projection Axis
Single-phase Winding
arc tan $\frac{\alpha}{\omega}$
θ
$\frac{\pi}{2}$ - arc tan $\frac{\alpha}{\omega}$
$\bar{\phi}_{1a}$
-α + jω

FIG. 20

We have already established the relation that exists between a rotating field and the field which may be produced by its action on a polyphase winding. It now becomes necessary to establish a corresponding relation when the rotating field acts on a single-phase winding. In Fig. 19 let $\bar{\phi}_{1a}$ represent a positively rotating magnetic field which has a complex angular velocity $(-\alpha + j\ \omega)$. The flux linking the single-phase winding will be a maximum at the moment when $\bar{\phi}_{1a}$ is in the position shown. The generated e. m. f., E, in the winding will be zero at this instant. By the time that $\bar{\phi}_{1a}$ has advanced $(\pi/2 - \text{arc tan}\ \alpha/\omega)$ this generated e. m. f. will be a maximum. The current however will not reach its maximum value until a time still later that is determined by the generalized self impedance of the single-phase winding. At the time that the current is a maximum the vector representing it is arc tan α/ω from the axis of the winding as shown in Fig. 20. At this moment one of the oppositely rotating fields $\bar{\phi}_{2a}$ is coincident with the current and the other, $\bar{\phi}_{2b}$, makes the same angle with but is on the other side of the axis of the winding. Of course, the axis of the field due to a single-phase current always coincides with the axis of the winding.

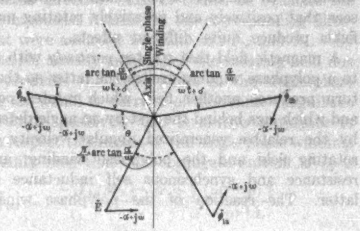

Single-phase Winding
Axis
arc tan $\frac{\alpha}{\omega}$
arc tan $\frac{\alpha}{\omega}$
ωt + δ
ωt + δ
$\bar{\phi}_{2b}$
Ī
$\bar{\phi}_{2a}$
-α + jω
-α + jω
-α - jω
θ
$\frac{\pi}{2}$ - arc tan $\frac{\alpha}{\omega}$
-α + jω
Ē
-α + jω
$\bar{\phi}_{1a}$

FIG. 21

One of these oppositely rotating fields travels at the same speed and in the same direction as does the initial field, $\bar{\phi}_{1a}$, while the other travels at the same speed but in the opposite direction. The condition at a time angle $\omega\ t + \delta$ after the current in the single-phase winding is a maximum is shown in Fig. 21. The oppositely rotating fields are represented by the vectors

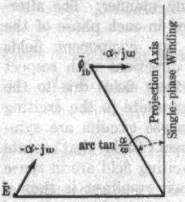

-α - jω
$\bar{\phi}_{1b}$
Projection Axis
Single-phase Winding
arc tan $\frac{\alpha}{\omega}$
-α - jω
Ē

FIG. 22

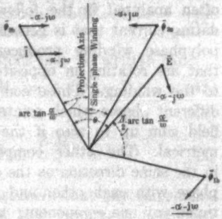

-α - jω
$\bar{\phi}_{2b}$
Ī
-α - jω
-α + jω
$\bar{\phi}_{2a}$
Projection Axis
Single-phase Winding
Ē
-α - jω
arc tan $\frac{\alpha}{\omega}$
θ
$\frac{\pi}{2}$ - arc tan $\frac{\alpha}{\omega}$
$\bar{\phi}_{1b}$
-α - jω

FIG. 23

$\bar{\phi}_{2a}$ and $\bar{\phi}_{2b}$: the subscripts a and b indicate the direction of rotation. The constant phase relation between $\bar{\phi}_{1a}$ and $\bar{\phi}_{2a}$ will be determined presently.

If the first magnetic field had been rotating negatively with a complex angular velocity of $(-\alpha - j\ \omega)$ the flux linking the single-phase winding would have been maximum at the moment shown in Fig. 22. By the time the current had reached its maximum value this field would have reached the position indicated in

Fig. 23. The oppositely rotating components of the single-phase field are now shown at $\bar{\phi}_{2a}$ are $\bar{\phi}_{2b}$. At a time angle $\omega t + \delta$ later the vectors shown in Fig. 23 will have moved to the positions shown in Fig. 24. It is important to observe that the oppositely rotating components of the single-phase field are exactly the same in Fig. 21 and in Fig. 24. That is, as far as the reaction produced in the single-phase winding is concerned, there is no difference between positively and negatively rotating fields provided they are always conjugate with respect to the axis of the single-phase winding. In the case of polyphase windings we have seen that positively and negatively rotating magnetic fields produce quite different effects.

A magnetic field that rotates *positively* with respect to a polyphase winding excites the latter so that it in turn produces another field which rotates *positively* and which lags behind the first by an angle determined by the relative generalized angular velocity of the rotating field and the polyphase winding, and the resistance and synchronous self inductance of the latter. The reaction of the polyphase winding is often analyzed in the following manner: The alternating current that is generated in each phase of the polyphase winding produces two component fields that are rotating in opposite directions with respect to the winding. Those component fields due to the different phases which rotate opposite to the exciting field sum up to zero if the phase circuits are symmetrical. The other component fields that rotate in the same direction as the exciting field are in space phase with each other and their resultant is thus n times any one component; where n is the number of phases of the polyphase winding. That is, we may consider that the current in each phase of a polyphase winding consists of two equal components, hereafter designated by the subscripts a and b, each equal to *one-half* of the *maximum phase current.* These two component currents produce equal but oppositely rotating magnetic fields. If the winding were single-phase each of these magnetic fields would exist, but with a symmetrical polyphase winding only the fields exist which rotate in the same direction as does the exciting field. Those that rotate in the opposite direction neutralize one another, so that their resultant is zero.

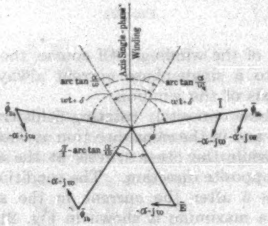
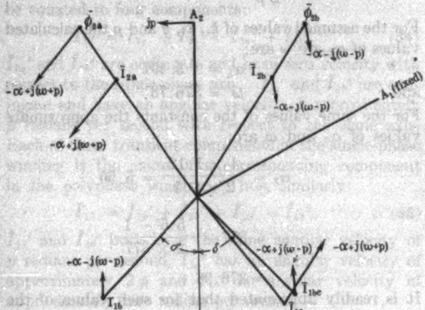


FIG. 24

Let M_1 be the e. m. f. generated in one phase of the polyphase winding by *one ampere* of *either* the a or b component of the single-phase current when varying at the rate of one ampere per second. Similarly let M_n be the e. m. f. generated in the single-phase winding by one ampere of the a component of the polyphase current, provided that is the one which produces a magnetic field, when varying at the rate of one ampere per second. Thus defined the n-phase mutual inductance is n times the single-phase mutual inductance. If the single-phase winding is supplied with an alternating current having an angular velocity of ω and the winding is rotated at this same velocity, ω, with respect to the polyphase winding, one of the oppositely rotating fields due to the single-phase current will be stationary with respect to the polyphase winding and thus will generate no e. m. f. in it. The other field will rotate at a relative angular velocity of $2\,\omega$ and generate an e. m. f., E, in each phase of the polyphase winding. If the single-phase current is I, the single-phase mutual inductance is $M_1 = \dfrac{E}{2\,\omega\, I/2}$. If the single- and polyphase windings are fixed with respect to each other and one phase of the latter is directly opposite the former, the oppositely rotating fields produce equal e. m. fs. in time phase with each other in this phase of the polyphase winding. If E is still the e. m. f. generated in this phase of the polyphase winding the e. m. f. due to *one* of the component fields is $E/2$ and the single-phase mutual inductance is as before $M_1 = \dfrac{E}{2\,\omega\, I/2}$; where I and ω are the current and its angular velocity in the single-phase winding. If the two windings are fixed with respect to each other and balanced currents of I amperes per phase are supplied to the polyphase winding the polyphase mutual inductance is the e. m. f. generated in the single-phase winding divided by $\dfrac{\omega\, I}{2}$; where ω is the angular velocity of the polyphase currents and I is the current per phase. Thus measured, the n-phase mutual inductance will be n times the single-phase inductance.

Let us assume that we have a three-phase winding fixed in position and a single-phase winding which is rotating in the positive direction at an angular velocity of p radians per second. If the single-phase winding carries a current having an angular velocity of ω radians per second there will be produced in the air-gap two rotating fields, one traveling in the positive direction at an angular velocity of $(\omega + p)$ and the other in the negative direction at a velocity of $(\omega - p)$ radians per second. If p is greater than ω both of these fields travel in the positive direction. The following relations can be worked out on the assumption that ω is either greater or less than p. In accordance with the established custom when analyzing polyphase induction motors and generators, we will choose the first of these assumptions, *viz.*, that ω is greater than p.

The alternating current in the single-phase winding having an angular velocity, ω, produces two oppositely rotating fields each having this same angular velocity. The single-phase winding, having a positive angular velocity of p radians per second, gives to each of these fields an added angular velocity, so that one rotates positively with an angular velocity of $\omega + p$ and the other negatively with angular velocity of $\omega - p$ radians per second. Each of these fields produces by its action on the polyphase winding a rotating field which has the same angular velocity. This is illustrated in Fig. 25. In this figure A_1 represents the fixed axis of one phase of the polyphase winding, and A_2 represents the axis of the single-phase winding; the latter having a positive angular velocity of p radians per second. The oppositely rotating fields due to the single-phase current, which has a complex angular velocity of $- \alpha + j\,\omega$ radians per second, are represented by the vectors $\bar{\phi}_{2a}$ and $\bar{\phi}_{2b}$. The former has an angular velocity of $- \alpha + j\,(\omega + p)$ and the latter a velocity of $- \alpha - j\,(\omega - p)$ radians per second. Since these component fields are proportional to the current in the single-phase winding, this current can be divided into similar components as indicated in the figure, $\bar{I}_{2a}$ and $\bar{I}_{2b}$. Observe that each of these component currents is one-half of the maximum value of the single-phase current, and that at any moment the actual current in the single-phase winding is their vector sum. The first of these rotating fields, $\bar{\phi}_{2a}$, produces a current in the short-circuited polyphase winding $\bar{I}_{1a}$ where



$\bar{\phi}_{2a}$
jp
A_2
$\bar{\phi}_{2b}$
$-\alpha - j(\omega - p)$
$\bar{I}_{2a}$
$\bar{I}_{2b}$
$-\alpha + j(\omega + p)$
$-\alpha - j(\omega - p)$
A_1 (fixed)
$-\alpha + j(\omega + p)$
σ
δ
$-\alpha + j(\omega - p)$
$-\alpha + j(\omega + p)$
$-\alpha - j(\omega - p)$
$\bar{I}_{1bc}$
I_{1b}
$\bar{I}_{1a}$

Fig. 25

$$I_{1a} = \frac{- \bar{I}_{2a} M_1 (m + j\,p)}{r_1 + (m + j\,p) L_1}$$

$$= \frac{- \bar{I}_{2a} M_1 (m + j\,p)}{Z_{1\,(m+jp)}} \qquad (39)$$

M_1 is the single-phase mutual inductance, $m = - \alpha + j\,\omega$ is the generalized angular velocity of the single-phase current, r_1 and L_1 are respectively the resistance and synchronous self inductance of the polyphase winding. The other rotating field $\bar{\phi}_{2b}$ produces a current in the short-circuited polyphase winding, $\bar{I}_{1b}$, which has a *negative* angular velocity of $\omega - p$ radians per second. In this case the current is:

$$\bar{I}_{1b} = \frac{- \bar{I}_{2b} M_1 [- \alpha - j\,(\omega - p)]}{r_1 + [- \alpha - j\,(\omega - p)] L_1} \qquad (40)$$

Each of these polyphase currents produces two oppositely rotating component currents in the single-phase winding. The sum of the components which rotate in the same direction must of course be equal to the initial components rotating in the corresponding direction that were assumed to exist, *viz.*, $\bar{I}_{2a}$ and $\bar{I}_{2b}$. By the principle already established the rotating field due to $\bar{I}_{1b}$, produces the same effect in the single-phase winding as would a current of the same magnitude which rotates positively and is always conjugate to $\bar{I}_{1b}$ with respect to the axis of the single-phase winding. It is readily seen that this conjugate current, $\bar{I}_{1bc}$, is

$$\bar{I}_{1bc} = \frac{- \bar{I}_{2a} M_1 [- \alpha + j\,(\omega - p)]}{r_1 + [- \alpha + j\,(\omega - p)] L_1}$$

$$= \frac{- \bar{I}_{2a} M_1 (m - j\,p)}{Z_{1\,(m-jp)}} \qquad (41)$$

The stator current $\bar{I}_{1a}$ and the conjugate current $\bar{I}_{1bc}$, which replaces $\bar{I}_{1b}$, both rotate positively and their resultant effect on the single-phase winding is thus the same as the effect of their vector sum. The positively rotating component of the single-phase current is thus given by:

$$2\,\bar{I}_{2a} = \frac{- (\bar{I}_{1a} + \bar{I}_{1bc})\, 3\, M_1 (- \alpha + j\,\omega)}{r_2 + (- \alpha + j\,\omega) L_2}$$

$$\bar{I}_{2a} = \frac{- (\bar{I}_{1a} + \bar{I}_{1bc})\, 3/2\, M_1\, m}{Z_{2\,m}} \qquad (42)$$

These characteristic equations may be written thus:

$$\bar{I}_{1a} Z_{1\,(m+jp)} + \bar{I}_{2a} M_1 (m + j\,p) = 0 \qquad (43)$$

$$\bar{I}_{1bc} Z_{1\,(m-jp)} + \bar{I}_{2a} M_1 (m - j\,p) = 0 \qquad (44)$$

$$\bar{I}_{2a} Z_{2\,(m)} + (\bar{I}_{1a} + \bar{I}_{1bc}) \frac{3\,M_1}{2}\, m = 0 \qquad (45)$$

In these equations it is assumed that any impedance outside the armature and field circuits is negligible, otherwise the zeros at the right of the equality signs would be replaced by the proper *fall* in *terminal pressure.*

If $\bar{I}_{1a}$ and $\bar{I}_{1bc}$ as determined by the first two equations are substituted in the third equation, the latter becomes, on dividing by I_{2a}:

$$Z_{2\,(m)} - 3/2\, M_1\, m \left(\frac{M_1 (m + j\,p)}{Z_{1\,(m+jp)}} + \frac{M_1 (m - j\,p)}{Z_{1\,(m-jp)}} \right) = 0 \qquad (46)$$

If the values of the impedances are substituted, this reduces to the following cubic in m:

$$m^3 + \frac{(k_1(1+\sigma) + k_2)\, m^2}{\sigma} + \frac{(k_1^2 + 2\, k_1\, k_2 + \sigma\, p^2)\, m}{\sigma} + \frac{(k_1^2 + p^2)\, k_2}{\sigma} = O \tag{47}$$

where $k_1 = r_1/L_1$, $k_2 = r_2/L_2$ and $\sigma = 1 - \dfrac{3\, M_1^2}{2\, L_1\, L_2}$

r_1 and r_2 are the resistances of one phase of the polyphase winding and of the single-phase winding; L_1 and L_2 are the synchronous self-inductance of the polyphase winding, and the self inductance of the single-phase winding. At least one root of a cubic equation must be real and since the coefficients are all positive this root must be negative. This indicates that one of the components of the single-phase current is a diminishing direct current. The other roots of the cubic equation are conjugate imaginaries. This indicates two alternating currents, having positive and negative angular velocities but the same rate of diminution. In the polyphase winding, however, the three components of the current will all have different angular velocities. The first component of the single-phase current will produce a current having an angular velocity of p radians per second and the same rate of diminution that it itself has. The second component of the single-phase current, having a positive angular velocity will produce a current which has an angular velocity of approximately $2\,p$ radians per second. The third component of the single-phase current having a negative angular velocity, produces a current in the polyphase winding of very small angular velocity. The sum of the angular velocities of these two latter components of the polyphase current is exactly $2\,p$, and each diminishes at the same rate as does the alternating current in the single-phase winding.

A literal cubic equation is practically impossible to solve but if numerical values are given to k_1, k_2 and σ, the real root can be found to any degree of approximation by Horner's method. The equation may then be reduced to a quadratic, the roots of which can be readily found. Approximate values of the roots may be found as follows: The largest term in any of the coefficients is p^2. With a 60-cycle machine this would be 142,122. The ratios of resistance to self-inductance k_1 and k_2, are of the order ten, while σ is of the order one-tenth. For these values of k_1, k_2 and σ the real root of the characteristic equation is approximately

$$m = -\, k_2/\sigma \tag{48}$$

The degree of approximation of course depends upon the values of the constants. If $k_1 = k_2 = 1.0$, $\sigma = 0.1$ and $p = 377$;

$$m = -\,10.06$$

whereas the approximate value would be

$$m = 10.0$$

In this case the error is about one-half of one per cent; which is better than the analysis warrants.

Since the sum of the roots must be the coefficient of m^2 in the characteristic equation with its sign changed and since the remaining roots are conjugate imaginaries of the form $-\,\alpha_2 \pm j\,\omega$ we have

$$-\,k_2/\sigma - 2\,\alpha_2 = -\,\frac{k_1(1+\sigma) + k_2}{\sigma} \tag{49}$$

That is:

$$\alpha_2 = -\,\frac{k_1(1+\sigma)}{2\,\sigma} \tag{50}$$

Furthermore since the product of the roots is equal to the last term in the characteristic equation we have:

$$\alpha_1(\alpha_2^2 + \omega^2) = (k_1^2 + p^2)\, k_2/\sigma$$

From this, the approximate value of ω is seen to be

$$\omega = \sqrt{p^2 - (\alpha_2^2 - k_1^2)}$$

$$= p - \frac{\alpha_2^2 - k_1^2}{2\,p} \quad \text{approximately} \tag{51}$$

For the assumed values of k_1, k_2, σ and p the calculated values of α_2 are ω are:

$$\alpha_2 = 5.497$$

and $\omega = 376.73$

For the same values of the constants the approximate values of α_2 and ω are:

$$\alpha_2 = \frac{1 \times 1.12}{2 \times 0.1} = 5.50$$

$$\omega = 377 - \frac{5.5^2 - 1}{2 \times 377}$$

$$= 376.96$$

It is readily appreciated that for such values of the constants as were chosen the roots are approximately

$$m' = -\,k_2/\sigma \tag{52}$$

$$m'' = -\,\frac{k_1(1+\sigma)}{2\,\sigma} \pm j\,p \tag{53}$$

It may be desirable, however, to extend the analysis so that it will apply to the case in which it is necessary to consider the constants of circuits outside of the machine itself. In that event the approximate values of the roots may not be sufficiently accurate. Again if the constants are considerably larger, as for example:

Let $k_1 = 15$, $k_2 = 20$, $\sigma = 0.15$ and $p = 377$ the actual values of m are:

$$m' = -\,143.2 + j\,o$$

$$m'' = -\,52.57 \pm j\,360.3$$

The approximate values of m are:

$$m' = -\,133.3$$

$$m'' = -\,57.5 \pm j\,373$$

In this case the error in the approximate calculation is quite considerable.

The general principle that determines the magnitude of the various component currents is that the resulting distribution of magnetic field in all parts of the machine cannot abruptly change, and what is even more important the component fields due to the different currents cannot abruptly change. That is, in mathematical language, the magnetic field at every point is a continuous function of the time, and since the magnetic fields are assumed to be proportional to the currents, the latter are also continuous functions of the time. If it were possible to construct mutually inductive circuits which had a leakage coefficient of zero, the currents in these circuits might abruptly change, *i. e.*, they might be discontinuous functions.

In order to illustrate the method of determining the component currents we will calculate their approximate values when the alternator is suddenly short-circuited. The general method of treating the single-phase current is to represent it by pairs of oppositely rotating vectors. In the present problem there are two such pairs one having no velocity and the other an angular velocity of p radians per second. That is, the transient current $\bar{I}_{2T}$ in the single-phase winding may be equated to four components:

$$\bar{I}_{2T} = \bar{I}_{2a}' + \bar{I}_{2b}' + \bar{I}_{2a}'' + \bar{I}_{2b}'' \qquad \textbf{(54)}$$

$\bar{I}_{2a}'$ and $\bar{I}_{2b}'$ are conjugate and have zero velocity with respect to the single-phase axis. $\bar{I}_{2a}''$ and $\bar{I}_{2b}''$ are conjugate and have an angular velocity of approximately p radians per second with respect to the same axis. Each of these transient components in the single-phase winding is the cause of a corresponding component in the polyphase winding. Thus, similarly:

$$\bar{I}_{1T} = \bar{I}_{1a}' + \bar{I}_{1b}' + \bar{I}_{1a}'' + \bar{I}_{1b}'' \qquad \textbf{(55)}$$

$\bar{I}_{1a}'$ and $\bar{I}_{1b}'$ both have the same angular velocity of p radians per second, $\bar{I}_{1a}''$ has an angular velocity of approximately $2\,p$ and I_{1b}'' an angular velocity of approximately zero radian per second.

The fundamental relation at the moment of short circuit is that:

$$I_{10} = \bar{I}_{1T} + I_{1s} \qquad \textbf{(56)}$$

and

$$I_{20} = \bar{I}_{2T} + I_{2s} \qquad \textbf{(57)}$$

where I_{10} is the current in the polyphase winding, before short circuit and I_{1s} is the steady current after the transient $\bar{I}_{1T}$ occasioned by the short circuit has disappeared. I_{20} and I_{2s} are similar values for the single-phase winding.

If the field excitation is not changed at or after short circuit $I_{20} = I_{2s}$ and thus the transient current in the single-phase winding must be zero at the moment of short circuit. Since the components of the single-phase current are conjugate in pairs it follows that at the moment of short circuit:

$$I_{2a}' = -\,I_{2a}'' \qquad \textbf{(58)}$$

and

$$I_{2b}' = -\,I_{2b}'' \qquad \textbf{(59)}$$

The corresponding pairs of single- and polyphase component currents must satisfy the two equations (**43**) and (**44**), the *a* components satisfying (**43**) and the *b* components, (**44**). Thus:

$$I_{1a}'\,[r_1 + (-\,k_2/\sigma + j\,p)\,L_1] + I_{2a}'\,M_1\,(-\,k_2/\sigma + j\,p) = O \qquad \textbf{(60)}$$

$$I_{1b}'\,[r_1 + (-\,k_2/\sigma + j\,p)\,L_1] + I_{2b}'\,M_1\,(-\,k_2/\sigma + j\,p) = O \qquad \textbf{(61)}$$

$$I_{1a}''\left[r_1 + \left(-\,k_1\,\frac{(1+\sigma)}{2\,\sigma} + j\,2\,p\right)L_1\right] + I_{2a}''\,M_1\left(-\,k_1\,\frac{(1+\sigma)}{2\,\sigma} + j\,2\,p\right) = O \qquad \textbf{(62)}$$

$$I_{1b}''\left(r_1 + (-\,k_1\,\frac{(1+\sigma)}{2\,\sigma}\;L_1\right) + I_{2b}''\,M_1\left(-\,k_1\,\frac{(1+\sigma)}{2\,\sigma}\right) = O \qquad \textbf{(63)}$$

If each of these equations is divided by L_1, it will be seen that since k_1 is much smaller than k_1/σ, k_2/σ or p, the following relations between the polyphase and single-phase components are approximately true:

$$\bar{I}_{1a}' = -\,M_1/L_1\,\bar{I}_{2a}'$$
$$\bar{I}_{1b}' = -\,M_1/L_1\,\bar{I}_{2b}'$$
$$\bar{I}_{1a}'' = -\,M_1/L_1\,\bar{I}_{2a}''$$

but

$$\bar{I}_{1b}'' = -\,M_1/L_1\,\frac{(1+\sigma)}{(1-\sigma)}\;\bar{I}_{2b}''$$

If the conditions set forth in equations (**55**), (**58**) and (**59**) are applied to these four equations we have:

$$\bar{I}_{1T} = M_1/L_1\,\frac{2\,\sigma}{1-\sigma}\,I_{2b}' \qquad \textbf{(64)}$$

If it is deemed necessary a more accurate value of I_{2b}' can be calculated by substituting in equations (**60**) to (**63**) inclusive the actual values of the generalized angular velocity as determined by equation (**47**) and by recognizing that I_{2a}' and I_{2b}' are conjugate with respect to the single-phase axis.

Since the current in the polyphase winding before short circuit is much smaller than the steady short-circuit current:

$\bar{I}_{1T} = -\,I_{1s}$ approximately, at the moment of short circuit. The steady short-circuit current is calculated by dividing the e. m. f. generated in the polyphase winding on open circuit by the synchronous impedance. The open circuit e. m. f. is

$$E = -\,j\,p\,M_1\,I_{20} \qquad \textbf{(65)}$$

Since the resistance is much less than the synchronous reactance the steady short-circuit current is approximately.

$$I_{1s} = -\,\frac{j\,p\,M_1\,I_{20}}{j\,p\,L_1} = -\,M_1/L_1\,I_{20} \qquad \textbf{(66)}$$

From this it follows that:

$$I_{2b}' = I_{20}\,\frac{(1-\sigma)}{2\,\sigma} \qquad \textbf{(67)}$$

The direct-current component of the single-phase current has an initial value of $I_{2a}' + I_{2b}'$ which is $I_{20} \dfrac{(1-\sigma)}{\sigma}$. The alternating component of the single-phase current is equal and opposite to this initially, but one-half cycle later the vectors representing all of the component currents are in phase. During this time, however, the direct and alternating components have shrunk different amounts. The first maximum value of the single-phase current will be, approximately:

$$I_{20} \frac{(1-\sigma)}{\sigma} \left\{ \epsilon^{-\frac{k_2}{\sigma} \frac{\pi}{p}} + \epsilon^{-\frac{k_1 (1+\sigma)}{2\sigma} \frac{\pi}{p}} \right\} \tag{68}$$

For $k_1 = k_2 = 1$, $\sigma = 0.1$, this is $17\, I_{20}$.

The component of the polyphase current which has an angular velocity of p radians per second is $\bar{I}_{1a}' + \bar{I}_{1b}'$ which initially equals $\dfrac{1-\sigma}{\sigma} I_{1s}$. The component which has an angular velocity of approximately $2\,p$ is $\bar{I}_{1a}''$ which initially equals $-\dfrac{1-\sigma}{2\,\sigma} I_{1s}$. The component which has a very small angular velocity is I_{1b}'' which initially equals $-\dfrac{1+\sigma}{2\,\sigma} I_{1s}$. The first of these components shrinks at a rate of k_2/σ while each of the others shrinks at a rate of $\dfrac{k_1\,(1+\sigma)}{2\,\sigma}$. Notice that initially the first component is opposite in phase to each of the others. Thus one-half cycle after the transient begins, the three components will be in phase. If the transient begins at such a time that the steady short-circuit current in one phase would naturally be passing through its maximum value, this phase will carry the maximum possible transient current one-half cycle later. This maximum transient current is approximately:

$$I_{1s} \left[\frac{1-\sigma}{\sigma} \epsilon^{-\frac{k_2}{\sigma} \frac{\pi}{p}} + \frac{1}{\sigma} \epsilon^{-k_1 \frac{(1+\sigma)}{2\sigma} \frac{\pi}{p}} \right] \tag{69}$$

for $k_1 = k_2 = 1$, $= 0.1$ this is $18\, I_{1s}$, *i. e.* eighteen times the maximum value of the steady short circuit current.

TWO SINGLE-PHASE WINDINGS HAVING A CONSTANT RELATIVE ANGULAR VELOCITY

A current of angular velocity ω in one single-phase winding produces in the other winding currents having angular velocities of $\omega + p$ and $\omega - p$; where p is the relative angular velocity of the two windings. Each of these currents reacts on the first winding and produces additional currents having angular velocities of $\omega + 2\,p$ and $\omega - 2\,p$. These currents in turn produce others in the second winding having angular velocities of $\omega + 3\,p$ and $\omega - 3\,p$, and so on ad infinitum. It is then evident that the solution of this problem cannot be reached by the method here developed unless all but a few of the component currents are neglected. The solution will not be attempted at this time.

Carson[5] has suggested a functional form of solution of this problem and Dreyfus[6] also gives a solution based on certain assumptions.

In conclusion the writer hardly needs to say that it has been his sole purpose to show how certain low-frequency transient conditions in electric machinery may be analyzed by a vector method. Before accurate results can be obtained by this method considerable experimental work is probably necessary in order to determine the most satisfactory methods of measuring the required constants. An interesting development would be the solution of the transient conditions in certain unsymmetrical arrangements of polyphase machines similar to those so ably discussed by Mr. Fortescue.[7]

Transient conditions may be the result of a variety of causes, one of which is short circuit. The vector method, however, is equally applicable in any case in which the original premises are reasonably true. The actual working out of a numerical solution is rather complex and this has been purposely avoided in order that the general principles might not be obscured a mass of calculation. Furthermore, it seemed best at this time not to attempt to generalize the theory and present it in such form that it would apply to combinations of electric machines that might even be unsymmetrical.

Appendix

For those who are interested in comparing the vector method of solution with the differential equations which apply to these problems the following development is given.

Consider a symmetrically wound three-phase induction motor having Y-connected stator and rotor phases as shown diagrammatically in Fig. 26. The stator and rotor currents are represented by the letters x and y respectively and their assumed positive directions are indicated by the arrows. Fig. 27 shows a development of the air gap. The progression of the phases, 1, 2 and 3 is toward the right for both stator and rotor, and the latter moves at a constant angular velocity of p radians per second toward the right.

Let r_1, r_2, L'_1 and L_2' represent the resistances and self-inductances of one phase of stator and rotor respectively.

Let M_s represent the mutual inductance between any

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5. Bibliography No. 7.
6. Bibliography No. 4.
7. A. I. E. E. Vol. XXXVII, p. 1027.

two phases of the stator. Due to symmetry these three mutual inductances are equal.

Let M_y represent the mutual inductances between any two phases of the rotor. These mutual inductances are likewise equal.

Let $2/3\,M$ represent the maximum value of the mutual inductance between one phase of the stator and one phase of the rotor. At a time when the angular displacement of these phases of stator and rotor is $p\,t$ the momentary value of this mutual inductance is $2/3\,M \cos p\,t$, if the space distribution of the air-gap flux density is sinusoidal. This assumption of a sinusoidal distribution of flux density is not much in error if the motor is well designed. It is also necessary to assume that r_1, r_2, L_1', L_2', M_x, M_y and $2/3\,M$ are constant. This is commonly done in engineering practise.

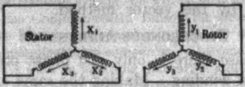

FIG. 26

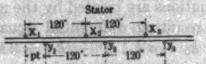

FIG. 27

With the stator phases short-circuited as indicated in Fig. 26 the following differential equations may be written,—considering first phases 1 and 2 and then phases 2 and 3.

$$\Big\{ r_1 x_1 + L_1' \frac{d\,x_1}{d\,t} - M_x \frac{d\,x_2}{d\,t} - M_x \frac{d\,x_3}{d\,t}$$

$$+ \frac{d}{d\,t} [\, y_1 \frac{2}{3} M \cos p\,t + y_2 \frac{2}{3} M \cos (p\,t + 120\ )$$

$$+ y_3 \frac{2}{3} M \cos (p\,t + 240°) \,] \Big\}$$

$$- \Big\{ r_1 x_2 + L_1' \frac{d\,x_2}{d\,t} - M_x \frac{d\,x_3}{d\,t} - M_x \frac{d\,x_1}{d\,t}$$

$$+ \frac{d}{d\,t} (y_2 \frac{2}{3} M \cos p\,t + y_3 \frac{2}{3} M \cos (p\,t + 120°)$$

$$+ y_1 \frac{2}{3} M \cos (p\,t + 240°) \Big\} = 0 \qquad (1)$$

There is a negative sign before M_x for the reason that positive currents in phases 2 and 3, for example, produce a flux through phase 1 which is opposite in direction to the flux that phase 1 produces through itself.

The equation which applies to phases 2 and 3 is:

$$\Big\{ r_1 x_2 + L_1' \frac{d\,x_2}{d\,t} - M_x \frac{d\,x_3}{d} - M_x \frac{d\,x_1}{d\,t}$$

$$+ \frac{d}{d\,t} [y_2 \frac{2}{3} M \cos p\,t + y_3 \frac{2}{3} M \cos (p\,t + 120°)$$

$$+ y_1 \frac{2}{3} M \cos (p\,t + 240°) \,] \Big\}$$

$$- \Big\{ r_1 x_3 + L_1 \frac{d\,x_3}{d\,t} - M_x \frac{d\,x_1}{d\,t} - M_x \frac{d\,x_2}{d\,t}$$

$$+ \frac{d}{d\,t} [y_3 \frac{2}{3} M \cos p\,t + y_1 \frac{2}{3} M \cos (p\,t + 120°)$$

$$+ y_2 \frac{2}{3} M \cos (p\,t + 240°) \,] \Big\} = 0 \quad (2)$$

Since the sums of the stator and rotor currents are each equal to zero, we may substitute:

$$x_3 = - x_1 - x_2 \quad \text{and} \quad y_3 = - y_1 - y_2$$

Also replace $L_1' + M_x$ by L_1. Multiply the second equation by 2 and add it to the first; again, subtract the second equation from the first. The results of these operations are respectively:

$$r_1 x_1 + L_1 \frac{d\,x_1}{d\,t} + \frac{2}{\sqrt{3}} M \frac{d}{d\,t} [y_1 \cos (p\,t + 30°)$$

$$+ y_2 \cos (p\,t + 90°) \,] = 0 \qquad (3)$$

$$r_1 x_2 + L_1 \frac{d\,x_2}{d\,t} + \frac{2}{\sqrt{3}} M \frac{d}{d\,t} [y_1 \cos (p\,t - 90°)$$

$$+ y_2 \cos (p\,t - 30\ ) \,] = 0 \qquad (4)$$

By a similar procedure differential equations may be written for phases 1 and 2 of the rotor. These equations will be found to be similar to (3) and (4) except that the signs before the phase angles will be reversed. Thus we will find that:

$$r_2 y_1 + L_2 \frac{d\,y_1}{d\,t} + \frac{2}{\sqrt{3}} M \frac{d}{d\,t} [x_1 \cos (p\,t - 30°)$$

$$+ x_2 \cos (p\,t - 90°) \,] = 0 \qquad (5)$$

$$r_2 y_2 + L_2 \frac{d\,y_2}{d\,t} + \frac{2}{\sqrt{3}} M \frac{d}{d\,t} [x_1 \cos (p\,t + 90°)$$

$$+ x_2 \cos (p\,t + 30°) \,] = 0 \qquad (6)$$

The following treatment was suggested to the author by Dr. F. L. Hitchcock as being the simplest method of reducing these equations to one containing a single variable.

Let $\quad r_1 x_1 + L_1 \dfrac{d\,x_1}{d\,t} = Z_1 x_1$

(Z_1 is an impedance operator)

Similarly $\quad r_1 x_2 + L_1 \dfrac{d\,x_2}{d\,t} = Z_1 x_2$

Also $\quad r_2 y_1 + L_2 \dfrac{d\,y_1}{d\,t} = Z_2 y_1$

And $\quad r_2 y_2 + L_2 \dfrac{d\,y_2}{d\,t} = Z_2 y_2$

If D and D^{-1} represent the direct and inverse differential operators, we shall have on integrating equations (3) and (4):

$$D^{-1} Z_1 x_1 + 2/\sqrt{3}\, M\, [y_1 \cos (p\,t + 30°)$$

$$+ y_2 \cos (p\,t + 90°] = 0 \qquad (7)$$

$$D^{-1} Z_1 x_2 + 2/\sqrt{3}\, M\, [y_1 \cos (p\,t - 90°)$$

$$+ y_2 \cos (p\,t - 30°] = 0 \qquad (8)$$

Solve these two equations for y_1 and y_2

$$y_1 = \frac{2}{\sqrt{3}\,M}\{\cos(p\,t + 90^\circ)\,D^{-1}Z_1\,x_2 - \cos(p\,t - 30^\circ)\,D^{-1}Z_1\,x_1\} \qquad (9)$$

$$y_2 = \frac{2}{\sqrt{3}\,M}\{\cos(p\,t - 90^\circ)\,D^{-1}Z_1\,x_1 - \cos(p\,t + 30^\circ)\,D^{-1}Z_1\,x_2\} \qquad (10)$$

Differentiation of equation (**9**) gives:

$$\frac{d\,y_1}{d\,t} = \frac{2}{\sqrt{3}\,M}\{\cos(p\,t + 90^\circ)\,Z_1\,x_2 - p\sin(p\,t + 90^\circ)\,D^{-1}Z_1\,x_2 - \cos(p\,t - 30^\circ)\,Z_1\,x_1 + p\sin(p\,t - 30^\circ)\,D^{-1}Z_1\,x_1\}$$

The expression for y_2 may be differentiated in a like manner.

If the values of y_1, y_2, $\frac{d\,y_1}{d\,t}$ and $\frac{d\,y_2}{d\,t}$ are now substituted in equations (**5**) and (**6**), the resulting equations may be written as follows:

In order to simplify the writing of these equations let:

$$\begin{array}{ll} \cos(p\,t - 30^\circ) = a & \sin(p\,t - 30^\circ) = a_1 \\ \cos(p\,t + 30^\circ) = b & \sin(p\,t + 30^\circ) = b_1 \\ \cos(p\,t - 90^\circ) = c & \sin(p\,t - 90^\circ) = c_1 \\ \cos(p\,t + 90^\circ) = d & \sin(p\,t + 90^\circ) = d_1 \end{array}$$

Equation (**5**) becomes:

$$\{(-r_2\,a + p\,L_2\,a_1)\,D^{-1}Z_1 - L_2\,a\,Z_1 + M^2(a\,D - p\,a_1)\}\,x_1 + \{(r_2\,d - p\,L_2\,d_1)\,D^{-1}Z_1 + L_2\,d + M^2(c\,D - p\,c_1)\}\,x_2 = 0 \qquad (11)$$

Equation (**6**) becomes:

$$\{(r_2\,c - L_2\,p\,c_1)\,D^{-1}Z_1 + L_2\,c\,Z_1 \quad (-r_2\,b + L_2\,p\,b_1)\,D^{-1}Z_1 - L_2 b\,Z_1 + M^2(d\,D - p\,d_1)\}\,x_1 + \; + M^2(b\,D - p\,b_1)\}\,x_2 = 0 \qquad (12)$$

In the process of eliminating x_1 and x_2 from equations (**11**) and (**12**) all of the harmonic terms in the coefficients vanish, giving as a final result:

$$(r_2^2 + L_2^2\,p^2)(D^{-1}Z_1)^2 + L_2^2\,Z_1^2 + (D^2 + p^2)\,M^4 + 2\,r_2\,L_2\,Z_1\,D^{-1}Z_1 - 2\,(r_2\,D + p^2\,L_2)\,M^2\,D^{-1}Z_1 - 2\,M^2\,L_2\,Z_1\,D = 0 \qquad (13)$$

Differentiate this equation twice, collect the terms in descending powers of D and after dividing each term by $L_1^2\,L_2^2$, let $1 - \frac{M^2}{L_1\,L_2} = \sigma$, $r_1/L_1 = k_1$ and $r_2/L_2 = k_2$. This gives:

$$\sigma^2\,D^4 + 2\,\sigma\,(k_1 + k_2)\,D^3 + [\,(k_1 + k_2)^2 + 2\,\sigma\,k_1\,k_2 + p^2\,\sigma^2]\,D^2 + [2\,k_1\,k_2\,(k_1 + k_2) + 2\,p^2\,k_1\,\sigma]\,D + k_1^2\,(k_2^2 + p^2) = 0 \qquad (14)$$

If equation (**26**) in the body of the paper is written for the conjugate roots, *i. e.*, for a negative value of the rotor angular velocity it would be:

$$\sigma\,m^2 + (k_1 + k_2 + j\,p\,\sigma)\,m + k_1\,k_2 + j\,p\,k_1 = 0 \qquad (\mathbf{26_1})$$

The product of equations (**26**) and (**26**$_1$) will contain all four roots, and will be found to be identical with the differential equation (**14**).

This proves that the two methods are identical in regard to the results they produce, and that, of the two, the vector method is much less laborious. Furthermore, it is much simpler to determine the constants of integration by the vector method.

SYNCHRONOUS ALTERNATOR

In this case there is only one phase on the rotor, *viz.* the field winding. Making the same assumptions in regard to the resistances and inductances as were made before, the following differential equations may be written:

$$r_1\,x_1 + L_1\frac{d\,x_1}{d\,t} + M\frac{d}{d\,t}(y\cos p\,t) = 0$$

$$r_1\,x_2 + L_1\frac{d\,x_2}{d\,t} + M\frac{d}{d\,t}[y\cos(p\,t - 120)\,] = 0$$

$$r_2\,y + L_2\frac{d\,y}{d\,t} + M\frac{d}{d\,t}[x_1\cos(p\,t - 30^\circ) + x_2\cos(p\,t - 90^\circ)\,] = 0$$

If these equations are treated by the method already outlined the resulting differential equation is:

$$\sigma\,D^3 + (k_1 + k_2 + \sigma\,k_1)\,D^2 + (k_1^2 + 2\,k_1\,k_2 + \sigma\,p^2)\,D + k_2\,(k_1^2 + p^2) = 0$$

In this case $\sigma = 1 - 3/2\,\frac{M^2}{L_1\,L_2}$

Again the equation obtained by the vector method and the differential equation are identical.

Bibliography

Following is a short bibliography. Some of the papers listed give a bibliography of their own.

1. Freie Magnetische Energie Zwischen Verketteten Mehrphasensystemen. L. Dreyfus E. U. M. 29 S. 891, 1911.
2. Ausgleichvorgange in der Symmetrischen Mehrphasenmaschine. L. Dreyfus E. U. M. 30 S 25, 121, 139, 1912.
3. Calculation of Sudden Short-Circuit Phenomena of Alternators, N. S. Diamant A. I. E. E. p. 2237, Vol. XXXIV, 1915.
4. Ausgleichvorgange Beim Plotzlichen Kurzschluss von Synchron generatoren. L. Dreyfus, *Archiv fur Electrotechnik*, 5 S 103, 1916.
5. Reactance of Synchronous Machines and its Application. R. E. Doherty and O. E. Shirley, A. I. E. E., p. 1209, Vol. XXXVII, 1918.
6. Short-circuit Current of Induction Motors and Generators. R. E. Doherty and E. T. Williamson, A. I. E. E., p. 509, Vol. XL, 1921.
7. Theory and Calculation of Variable Electrical Circuits. J. R. Carson, *Physical Review*, Vol. XVII, p. 116, 1921.
8. Ausgleichsvorgange beim Parallelschalten von Synchronmaschienen. J. Biermanns, *Archiv. fur Electrotechnik*, 10 S185, 1921.
9. Exciter Instability, R. E. Doherty, A. I. E. E., 1922.
10. Sur les phénomènes électro-magnétiques résultant de la mise court-circuit bursque d'un alternateur. P. Boucherot. *International Electrical Congress*, 1911, Vol. II, p. 49.

Discussion

D. W. Roper: Professor Lyon talks about transients in electrical machinery and develops equations for their analysis. It would be interesting if he would apply the same type of analysis to the transients which we have in our large transmission and distribution systems as well as in the machines.

The Commonwealth Edison Company is shortly going to develop and propagate some transients artificially by means of tests, and if Professor Lyon can develop equations which will give him sone clue to the transients which occur in a transmission or distribution system, we will be glad to offer him the facilities of the Commonwealth Edison for checking his conclusions. We hope by cooperating in this way with the scientists and the manufacturing companies, to become more familiar with transients such as lightning discharges and the transients which follow short circuits on the system. We would like to have those transients named and classified.

R. F. Franklin: Professor Lyon demonstrates how the transient current of electrical apparatus may be calculated by the use of the *Generalized Angular Velocity Method.* I believe that the solution of a number of transient electrical problems could be simplified by this method.

The author makes the interesting suggestion that core loss might be taken into account by representing it by a component $-j L_2$ of the inductance; thus the inductance would be, $L = (L_1 - j L_2)$. In the calculation of transformers, induction motors, etc., the core loss is usually represented by an effective resistance. The chief merit in the proposed scheme of using an inductance $-j L_2$ instead of an effective resistance lies in the fact that the watts consumed in this inductance is, like the core loss, a function of the frequency. While of course, the core loss does not vary directly with the frequency as is assumed in this scheme, nevertheless, it is a closer approximation than the use of a resistance which is independent of the frequency. As stated by the author, the core loss can usually be neglected in the calculation of short-circuit currents.

In calculating the short-circuit current of electrical apparatus the question of the best method of determining the inductances and resistances arises. The most accurate method is to determine them from the currents measured under actual short-circuit conditions, the inductances being determined by the magnitudes of the currents and the resistances by the rate of decay of the currents. However, this method is not always possible. The inaccuracy in the determination of these constants by any other method lies in the difficulty of reproducing the flux densities and distribution that actually exist under short-circuit conditions.

In a polyphase short circuit the flux distribution in an induction motor is as follows: normal flux (*i. e.* flux corresponding to the impressed voltage) in the stator core behind the teeth; approximately normal flux in the rotor core behind the teeth; the sum of these, *i. e.*, approximately two times normal flux, in the leakage paths at one instant during the cycle, the instantaneous value varying from practically zero to nearly two times normal flux in the leakage paths. This condition of flux distribution can be approximated in the following manner.

With the rotor and stator in the position of maximum flux enclosure and the rotor circuit open, a voltage is impressed on the stator. The voltage that appears at the rotor rings is that induced by the mutual flux. If now a voltage equal in magnitude and in phase with the voltage appearing across the rings is impressed on the rings zero current will flow in the rotor, just as when two alternators of equal voltage and in phase are connected together. The stator current will obviously be the same as before, viz. merely, the exciting current. If now the rotor and stator voltages are maintained constant and the rotor turned through one cycle then the currents in the rotor and stator at different points of the cycle will be those that would exist at the same points of the cycle under sudden short-circuit condition, provided there is no transient. The reason for this is that the flux distribution is the same, assuming no transient, as would exist during the cycle under the short-circuit condition.

The ordinary static impedance test affords another, but less accurate means of determining the motor constants. In this case the flux distribution is as follows: full flux (*i. e.* corresponding to the impressed voltage) in the stator core behind the teeth; zero flux[1] in the rotor core behind the teeth; and full flux in the leakage paths. In other words the stator flux returns entirely through the leakage paths. For reasons already discussed the effect of saturation in this case causes error.

For synchronous machines which involve a single-phase secondary or field winding, or in induction machines with one single-phase member the leakage impedance can be approximately determined by a static impedance test by short-circuiting the single-phase winding, that is the field winding, and impressing a voltage on a primary phase which is lined up in space position with the field winding. This method approximates in accuracy that of the static impedance test of the induction motor discussed above.

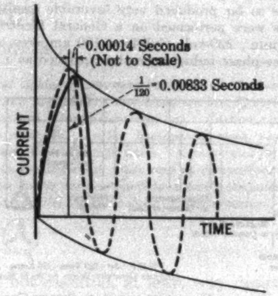

Fig. 1

An interesting point of investigation is the determination of the effect of the large short-circuit torque of a rotating machine on the magnitude of the short-circuit current. The maximum short-circuit current occurs one-half cycle from the instant of short circuit, as stated by the author. This is based on the assumption that during the short circuit the rotor maintains constant speed and does not drop back in space position. Actually, the large short-circuit torque causes it to drop back and thus the maximum short-circuit current occurs at a later time. Thus, in Fig. 1, if the rotor had maintained constant speed and space position the short-circuit current would be represented by the dotted curve with the maximum at one-half cycle or 1/120 second for 60-cycle current. However due to the slowing up or falling back in space position of the rotor the short-circuit current will actually follow the full line, with the maximum value at a later time. Therefore, due to the transient the peak for the actual condition is less than that for the constant speed rotor. However, consideration of the following calculation will show that the difference in magnitudes is not appreciable.

If the power supply of an average induction motor is cut off, then normal torque applied to the shaft will bring it to rest from full speed in from one to two seconds. Taking the lower limit of one second, the deceleration under constant torque is 100 per cent speed change per second, and the total speed change in 1/120 second is 1/120 × 100 per cent = 0.833 per cent. This calculation assumes constant normal torque. Actually, the maximum value of the short-circuit torque is, say, six times normal. Assuming sine wave of torque the average value is $2/\pi \times 6 = 3.82$ times normal torque. Due to this increased torque the total speed change in 1/120 second is 3.82 × 0.833 per cent = 3.18 per cent. Thus in 1/120 second the rotor speed would have decreased 100 per cent to 96.82 per cent. The average speed during this interval would be 98.41 per cent, and in order to move the rotor one-half pole pitch at this reduced speed it would require 1/120 × 100/98.41 = 0.00847 second instead of 1/120 or 0.00833 second. Thus a difference of only 0.00014 second. Obviously, the difference in magnitude of the peaks, due to the transient, in this small interval of time is not appreciable.

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[1] Or more strictly magnetic interlinkages, and further assuming zero resistance.

J. O'R. Coleman: A method of attack so useful and widespread in its application as that presented by Professor Lyon in his paper on "Transient Conditions in Electrical Machinery" is of vital interest from both a theoretical and practical viewpoint. Whenever a new method is introduced, doubt arises in the minds of many as to its application to practical cases. A limited amount of experimental work has been done along these lines, and has so far produced very favorable results.

These tests were performed on a General Electric 75-h. p., 900-rev. per min., 220-volt (full-load 180 amperes, no-load 60 amperes) three-phase induction motor operated as a generator.

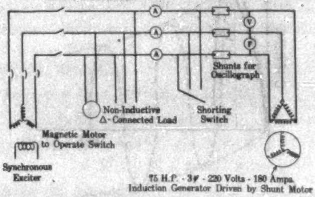
A
A
A
V
F
Shunts for Oscillograph
Non-Inductive Δ-Connected Load
Shorting Switch
Magnetic Motor to Operate Switch
Synchronous Exciter
75 H.P. - 3φ - 220 Volts - 180 Amps.
Induction Generator Driven by Shunt Motor

FIG. 2—DIAGRAM OF CONNECTIONS FOR SHORT-CIRCUIT TEST ON AN INDUCTION GENERATOR

This machine was driven by a shunt motor of like size. The excitation for the induction generator was supplied by a 30-kw. synchronous generator. Although the circuit breakers would not remove this machine until late in the transient, the impedance of the path through the oscillograph shunts and the induction generator was so large compared with that of the short-circuiting switch that the disturbance of the synchronous exciter would not alter appreciably the record of the oscillograph. The diagrammatic arrangement of the units is shown in Fig. 2.

The vector diagram for a sudden short circuit occurring at no-load is shown in Fig. 3. The vectors I_{1B}, I_{1C} and I_{1A} represent the current in the three phases just before the short circuit occurs. The short begins at the moment just before the current in phase B reaches its positive maximum. At this moment I_{1B}, for example, breaks up into two components I_{1B}' and I_{1B}'', whose vector sum is equal to the initial vector. After short circuit the terminus of the vector I_{1B}' travels along the logarithmic spiral shown at a velocity of 360 circular radians per second and is damped at the rate of 96 hyperbolic radians per second. The terminus of the vector I_{1B}'' travels in a like manner along a logarithmic spiral as shown at a velocity of 17 circular radians per second and is damped at the rate of 60 hyperbolic radians per second. The value of the transient current at any instant is the projection of the vector sum of the two vectors on the time axis. The first maximum occurs about 0.008 of a second after the short circuit. The positions of the vectors at this instant and the projection of their vector sum is shown in the diagram. The figure at the right of the diagram shows the resultant current obtained in this manner plotted in rectangular coordinates.

An oscillogram of the transient current during short circuit taken when operating at no load is shown in Fig. 4. It will be noticed that the short circuit occurred in phases A and B nearly 0.001 of a second before that in phase C. Due to a very decided change in the slope of the transient curves of phases A and B when short circuit occurred on phase C, the latter point was taken as the point of short circuit in the calculations.

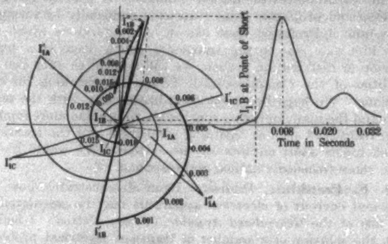

FIG. 3—VECTOR DIAGRAMS FOR THE SUDDEN SHORT CIRCUIT OF A 3-PHASE INDUCTION MOTOR

$R_{1ohm} = 0.0172 \quad R_{2ohm} = 0.0455 \quad L_{1c} = 0.00569$

$L_2 = 0.00996 \quad S = 0.0483$

Taking into consideration the effective resistance

$m_1' = 96 + j\,360 \quad m_1'' = 60.4 + j\,17.5$

$I_{1B} = 90.5 \sin 85°$

$i_{1B}' = I_{1B}' \epsilon^{-96t} \sin (360\,t + 258°)$

$i_{1B}'' = I_{1B}' \epsilon^{-60.4t} \sin (17.5\,t + 78.3°)$

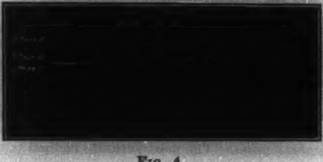

FIG. 4

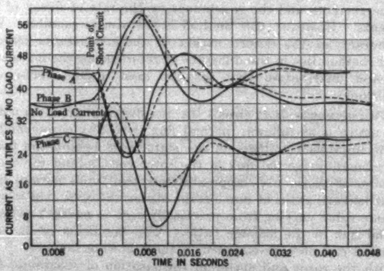

FIG. 5—CURVES OF SUDDEN SHORT-CIRCUIT OF A 75-H. P., 3-PHASE, 60-CYCLE, 900-REV. PER MIN., 220-VOLT INDUCTION MOTOR OPERATING AS A GENERATOR AT NO-LOAD.
------- CALCULATED CURVE
———— OSCILLOGRAM

Fig. 5 shows a reproduction of the actual oscillogram compared with the calculated values as obtained from the vector diagram described above. For phases A and B the agreement is very satisfactory for both magnitude and phase position. Phase C while agreeing as to phase position does not agree very well as to its magnitude. Due to the position of phase C at the time of short circuit, a very slight change in the point of short circuit taken for the calculations will make a large difference in the magnitude of the calculated curve. Although the difference in the time of short circuit is very small it is doubtless one of the reasons why the oscillogram and calculated curve do not agree so well for Phase C.[2]

W. V. Lyon: The author is much interested in Mr. Roper's statement. The cooperation of which he speaks when carried out with a teacher brings the important engineering problems of the day to the attention of those young men who will soon be responsible for our progress. From direct contact with the problem the teacher can speak authoritatively. Aside from the technical knowledge that the student absorbs the mental stimulus he receives of thus being brought into contact with a vital problem may be of inestimable importance.

The author wishes to express his appreciation for the suggestions that Mr. Franklin makes in regard to the measurement of the constants that should be used in determining the transient currents. Mr. Franklin's estimate of the probable error involved by assuming that the speed is constant during the time of the first current rush is also interesting. It bears out the feeling that the author had that the error would probably be very small.

Other students who have continued Mr. Coleman's investigations have been able, by reason of his experience, to check the observed and computed transient currents with surprising accuracy.

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[2] This discussion submitted in partial fulfillment of the requirements for the degree of Master of Science from M. I. T. carried out under the the direction of Professor Lyon, and with the assistance and cooperation of Mr. W. Ferguson.

1922 Developments in Autovalve Lightning Arresters

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The work done in the past year in developing commercial forms of lightning arresters based on the "autovalve" principle is reported. Fundamental data given by Dr. Slepian at the 1922 Midwinter Convention are briefly reviewed. The balance of the paper deals with new material and covers further research, development and test. Further research supports the original findings and gives specific information on basic relations. In the development work, required characteristics are first determined, and then structures are developed to give these characteristics. Commercial forms of the arresters are shown and the performance characteristics described, in fundamental units and in comparison with standard electrolytic arresters. Tests in the laboratory and observation under service conditions verify the calculated results and show the "autovalve" arresters designed for station service to parallel the electrolytic in performance. Test methods and results are described and service results reported.

AT the midwinter convention in February 1922 reference was made in discussion by Dr. Slepian, to a new principle of lightning arrester construction with which work was being carried on. At that time there had been a considerable amount of laboratory experience and a small amount of service experience with devices on the new principle and the results had been so uniformly promising that an intensive program of development was inaugurated.

Owing to the nature of the conditions which a lightning arrester is called upon to meet, it was felt that in this case as in others, the ordinary laboratory proof of the experimental and design work should be supplemented by extensive trials of experimental installations under close observation and under actual service conditions.

Since that time the work of one year has been done. Results of increasing promise and of continued interest have been secured. This work is here described.

The new principle, which is being referred to by the name "autovalve," consists in the imparting to an arrester of the spark gap type, the valve or counter e. m. f. characteristics which are generally recognized to be, if not essential, at least very desirable in lightning arresters for all classes of service. For a spark gap to possess valve characteristics, it is necessary that the breakdown voltage and the voltage across the gap when current is flowing be at least approximately equal. Current flow across an air gap may be in the form of an arc or of a glow discharge. The voltage of an arc is low, of the order of 50 volts or less, while that of a glow is of the order of several hundred volts. For practical reasons the glow is preferable because of the higher voltage. Studies have shown that up to 5 mils length, the glow discharge voltage is practically constant over a wide range of current density which is a further desirable characteristic of the glow discharge.

If the electrodes of the gap are made of material of considerable resistivity, the discharge is distributed over the face of the electrode, the current is prevented from concentrating in any local area, local heating is not possible and therefore an arc cannot form, and the discharge is maintained as a glow.

Assuming that a glow discharge is to be used, it is necessary that the breakdown voltage be in the vicinity of 350 volts, which is the voltage across the glow. Studies have shown that the breakdown voltage of very small gaps is as shown in Figure 1. This means

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Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

that for the required 350 volts, the gap length should be such as to give the minimum breakdown voltage for air at atmospheric pressure, or approximately 0.0003 in. in length. This seems at first sight to be impractical. It has, however, been found that use may be made of the well known fact that when conductors are separated by insulating spacers, breakdown tends to take place over the edge of the spacer at voltages lower than would be required for the gap. Trials have demonstrated that when the electrodes are separated by mica spacers of the thickness to give correct gap lengths, 3 to 5 mils, from the standpoint of voltage across the discharge, the breakdown voltage remains at the minimum value of 350.

Thus, a structure composed of a column of flat electrodes separated by mica spacers approximately 5 mils in thickness will operate as a valve, the critical voltage being equal to approximately 350 per gap, and the characteristic for voltages above this value being dependent on the resistance of the electrodes between plane surfaces. The discharge current capacity may be controlled by proper selection of disk area and thickness to give the characteristics required for any particular service. The voltage at which the discharge starts is 350 per gap within close limits and the speed of operation has been found to be very high.

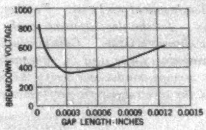

FIG. 1—BREAKDOWN VOLTAGE OF SMALL GAPS IN AIR

In the work of development of lightning arresters made on this principle, there were three major problems; namely, (1) experimental work consisting of the determination of characteristic relations and of the development of a suitable material for the electrodes, (2) design, consisting of the determination of the proper proportions to give the necessary performance characteristics and of the development of structures, (3) testing which consists of the determination of durability and performance characteristics by comparison in the laboratory with established forms of lightning arresters and by observation under actual service conditions.

In the *fundamental experimental work*, the initial attack was research with small glow discharges to determine breakdown voltage, the relation between voltage and current density, the distribution of voltage across minute gaps, the variation of voltage distribution with current density and various similar points.

From the data secured in these studies, it is desired to select a set of conditions which will give equal values for the breakdown voltage and for the voltage across the discharge while current is flowing and under which conditions the voltage across the discharge will be approximately constant over a considerable range of current density.

For gaps up to 5 mils in length, the discharge consists almost entirely of the cathode dark space and cathode glow, both of which are practically constant in voltage over a large range of current density. When larger separations are used, a portion of the positive column is included, introducing a variation of voltage with current change. From this standpoint, the limit of gap length is about 0.005 in. which gives a voltage across the discharge which remains practically constant at 350 volts over any anticipated range of conditions.

Two means are available to secure the desired breakdown volts. If plane electrodes of ordinary manufacture are placed in contact, there will be actual contact at several points and over the rest of the surface there will be separations up to say 2 or 3 mils. For low voltages, the current will be that due to conductivity at the contacts. At voltages above 350 per gap however, the gaps break down at the points where the separation is correct for the minimum breakdown voltage and the discharge is then distributed over the face of the electrode by reason of the resistivity. The alternative and preferred means depends on the fact referred to above that, when conductors are separated by an insulating spacer, breakdown tends to take place over the edge of the spacer at voltages lower than required for the gap. Trials demonstrated that when the electrodes are separated by mica spacers of the thickness required to give correct separation from the standpoint of voltage across the discharge (that is 0.003 in. to 0.005 in.) the breakdown voltage remains at the minimum value of 350.

The unit construction therefore becomes thin flat plates of resistance material separated by gaps of not more than 0.005 in. by mica spacers. For convenience in manufacture, the disks are made circular in form and the spacers in the shape of circular washers.

With these points fixed, *development work* was begun using disks of resistance material with various resistivities to determine the limit of resistivity required and to verify the findings of the research work by performance with structures of the size required for lightning arresters.

A considerable problem was encountered in the development of a material for the disks which would have the right characteristics and be feasible in manufacture. Ultimately however, a composition material was devised which is sufficiently durable, being mechanically more or less similar to porcelain, and which is electrically right, having a conductivity controlled by the methods of manufacture. With this material it has been found possible to manufacture disks of the desired resistivity and which, in sizes required for the structures, are remarkably true in dimensions. The nature of the material is such as to establish confidence in the permanence of characteristics.

The resistivity of the disk material required to give proper distribution of discharge over the face of the disk in order to insure that the discharge is maintained as a glow, depends on the type of material used for the electrodes, upon the gap length, and upon differences in gap length over the area of any particular gap. Experimental trials in the laboratory, in which observation was made by oscillographic record as to whether or not power current follows the static discharge, demonstrate that the material used in the structure for service tests and in commercial production is such that under the conditions of manufacture and application, no power current flows. This means that the discharge is distributed sufficiently so that a counter voltage is definitely maintained. A considerable factor of safety is provided in order to allow for variations in service conditions and manufacture.

On the basis of a critical or counter voltage of 350, instantaneous value, per gap and assuming that a reasonable factor of safety is desirable, the basis is established that the number of disks in series in a structure is such as to give one gap per 200 r. m. s. line volts maximum. This gives a factor of safety of approximately 25 per cent.

With the factors thus far determined, a valve type arrester is provided which begins to operate at 25 per cent over voltage and which stops the current flow when the voltage is decreased to this value, and it only remains to determine the resistance required in the contemplated arresters to give the necessary freedom of discharge and to select a disk area which will give this value. On this particular factor, there are years of service experience with the electrolytic arresters as a guide. If the same freedom of discharge can be secured with the proposed arrester as is given by the electrolytic, satisfactory results will be secured. As a check however, the study shown in the curves of Figs. 2 and 3 was made. In the figures, various characteristics of ordinary transmission lines are plotted against line volts. The curve in Fig. 3 of surge impedance is intended to represent average practise. The curve Fig. 2 of surge voltage induced in the line is made directly proportional to an assumed height of line above ground, starting with the initial assumption of 500,000 volts for a line 20 feet above ground. The curve of permissible surge voltage represents the peak value of double line voltage. This is of course an assumption. From these data, a curve, Fig. 3, is given showing the results secured with electrolytic arresters of the present standard design. From all this information a disk area of 61 square cm. was chosen, giving resistance and performance characteristics as shown in Figs. 2 and 3, in the curves for station type autovalve arresters. The data from which the curves are plotted are tabulated in Table 1.

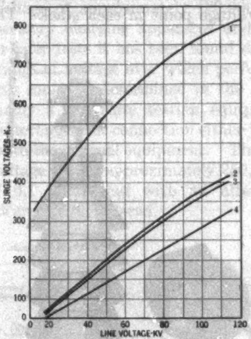

Fig. 2—Voltage Relations
1 = Induced Surge Voltage
2 = Resultant Voltage Electrolytic Arrsester
3 = Resultant Voltage Autovalve Arrester
4 = Peak Value of Max. Line Volts

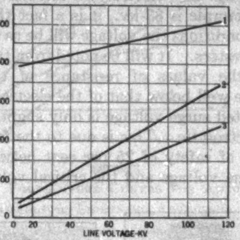

Fig. 3—Surge Impedance and Arrester Resistances
1 = Surge Impedance
2 = Equivalent Resistance Electrolytic
3 = Equivalent Resistance Autovalve

The arrester resistance values of Fig. 3 were determined by laboratory tests under surge conditions *i. e.* with steep wave front impulses applied by a condenser discharge at voltages corresponding to the peak value of double line voltage as described more in detail later.

The permitted surge voltages are calculated from the assumed values of induced surge voltage, surge impedance and arrester resistance as follows: The excess of impressed voltage over the critical or counter voltage of the arrester is reduced by the arrester current in the ratio of arrester resistance to the sum of arrester resistance and surge impedance. The permitted surge voltage is then the sum of the critical voltage and this reduced surge voltage. Thus, for a 22-kv. station type autovalve arrester, the assumed impressed surge voltage is 400 kv., the critical voltage 42 kv., making the excess 358 kv. The arrester resistance is 45 ohms, the assumed surge impedance 400 ohms, and the ratio of reduction $\frac{45}{400 + 45} = 0.101$. The permitted excess surge voltage is then $0.101 \times 358 = 36$ kv. and the total surge voltage $36 + 42 = 78$ kv.

It must be kept in mind, in considering the curves and data of Fig. 2 and 3, and Table 1 that they are based on assumptions in factors which are of vital importance, namely, in the values of surge voltage impressed on the lines. Assumptions are also made in the values of surge impedance, but in this case the values given are known to be fairly representative. The other values given and calculated are, with a fair degree of accuracy, correct. Thus, while the intention is not to state that the maximum voltage which may be applied to insulation, protected by an electrolytic arrester on a 22 kv. line, for example is 83-kv., it is a fact that under the assumed conditions the voltage will be as shown for the various arresters. It will be noted from these data that although the values of resistance of the autovalve arrester are well below that of the electrolytic arrester, the permitted surge voltage is only slightly lower than for the electrolytic. This is due to the fact that there is a factor of safety of 25 per cent between the peak value of line voltage and the critical voltage with the autovalve arrester while no such factor of safety exists with the electrolytic since the critical voltage is determined by the line voltage.

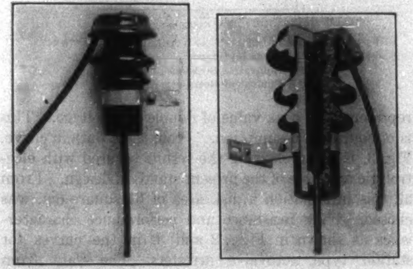

FIG. 4—7500-VOLT DISTRIBUTION TYPE AUTOVALVE ARRESTER

In addition to the factors which control performance required and performance delivered it is necessary in selecting or in designing lightning arresters to take account of economic factors. An arrester which gives perfect protection but which costs more to install and maintain than it is worth in saving in service and apparatus is not justified.

From this standpoint and since it is possible to make quite appreciable reductions in manufacturing cost by a reduction in disk area, it has been found desirable to develop a line of low-voltage arresters of reduced capacity for the protection of very small installations. In this line of arresters, a disk area of 15 sq. cm. is used giving the characteristics shown in curve Fig. 3 for the distribution type autovalve arrester.

In the average installation made with this size of arrester, the reduction in the "discharge current capacity" is off-set to some extent by the fact that transformers are installed on the same system fairly close together and the energy of any particular disturbance is taken care of by several arresters in parallel.

Figs. 4 to 6 show the forms in which arresters involving the autovalve principle have been made. Fig. 4 shows the distribution type, primarily intended for protection of distribution transformers, and therefore made for cross arm mounting, in unit form, entirely self contained in a porcelain case and with leads for connection to the line and ground wire. The station or large capacity arresters are built up of units such as are shown in Fig. 5. The assembly of these units into structures is shown in Fig. 6.

FIG. 5—STATION TYPE AUTOVALVE LIGHTNING ARRESTER UNIT

A change which has been made from the previous standard practise is in the use of three complete columns for a three-phase structure instead of the former standard, four phase legs in the "multiplex" connection. An interconnection between phase stacks is provided at the phase voltage point in each stack. This connection is based on the thought that the requirements from the standpoint of discharge current capacity are fixed by the most severe conditions, which are those imposed by disturbance of external or atmospheric origin. Such disturbances are imposed on the three lines alike and in flowing to ground through the arrester when the multiplex connection is used, the charges from the individual lines flow first through the corresponding

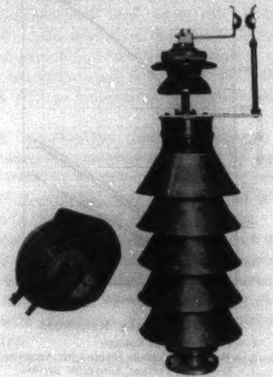

FIG. 6—PHASE ELEMENT OF 37-KV. STATION TYPE AUTOVALVE LIGHTNING ARRESTER

Rain sheds for gap and top units are removed to show structure.

phase legs and then together through the ground leg. The result is that the impedance of the path to ground from any wire is equivalent to four times the impedance of a single-phase leg. Since each phase leg is proportional to 58 per cent of line voltage, the total impedance is equivalent to that of a structure proportional to 2.32 times line voltage. With the connection used in the autovalve structures this impedance is reduced to that of an arrester for line voltage only. The performance is in this respect improved in the ratio of 2.32 to 1 while the material added is increased in the ratio of 3 to 2.32, is other words an addition of 29 per cent in the material used increases the performance by 132 per cent. In respect to the performance under the less severe conditions of surges of internal origin in which the voltage is applied between lines, this connection is the same as the former standard connection.

Tests for establishment of the various characteristics referred to and for proof of the performance and durability of the structures resulting have been made both in the laboratory and by observation of results in service.

For the early laboratory work, use was made of a testing equipment consisting of a condenser of 0.2 microfarads capacity, capable of being charged to a maximum voltage of approximately 130,000, together with the necessary charging apparatus and discharge circuit. For later tests it has been found desirable to extend the range of voltage over which performance of the structures could be determined in the laboratory and accordingly a duplicate testing equipment was provided and arranged for connection either in multiple or in series with the original equipment. The available maximum voltage is thus approximately 250,000 with a capacity of 0.1 microfarad with an alternative of 120,000 volts and 0.4 microfarad.

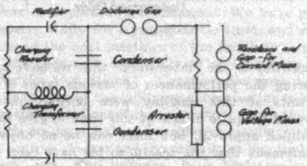

FIG. 7—CONNECTIONS FOR SURGE TESTS

Tests for durability consist in general of the connection to the arrester of a power voltage of the maximum value for which the arrester is intended and the simultaneous discharge through the arrester of a condenser such as to give the assumed maximum of double the peak value of line voltage across the arrester. Where it is possible, that is with the arresters for the lower voltages, or smaller capacities additional tests are made at surge conditions just under the flashover limit. It was intended to determine something in regard to the arrester life by a count of the number of discharges of double line voltage peak value required to cause some sort of failure, but practically indefinite tests do not cause any observable change in the arrester either from the standpoint of performance or appearance. As the surge conditions are increased in intensity, failure occurs by flashover at a voltage of the order of 15 to 20 times the voltage for which the arrester is designed. When surges just below the value necessary to cause flashover are repeatedly applied until failure occurs, the type of failure depends on the cycle of test. If the tests are close enough together to cause excessive heating, failures have been noted due to destruction of the valve characteristics and the consequent flow of power current. Parallel tests on commercial arresters of other types show the same general type of failure to occur under the same conditions, which are so much more severe than any we can expect, except in the case of a direct stroke that the test is interesting only as an establishment of limits. If the discharges are timed far enough apart to eliminate excessive heating, the life of the structure seems to be indefinite.

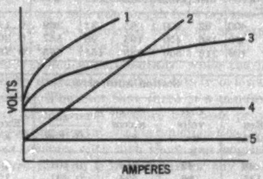

FIG 8—TYPICAL VOLT-AMPERE CURVES FOR AUTOVALVE AND ELECTROLYTIC LIGHTNING ARRESTERS

1 = Distribution Autovalve
2 = Electrolytic
3 = Station Autovalve
4 = Critical Voltage Autovalve
5 = Critical Voltage Electrolytic

As would be expected, the only failures that have been observed in service to date have been due to flashover and the flashover point of the commercial design has been increased considerably from the early samples with which this trouble was noted in the field. The total failures of arresters installed in service to date are something under 1 per cent.

The testing method to determine performance characteristics is to discharge through the arrester condenser charges of various characteristics. The performance may be shown by means of volt-ampere curves determined by such tests in which the surge conditions are varied over a considerable range. The current through the arrester is measured by a spark gap measurement of voltage across a resistance of known and permanent value which is connected in series with the arrester. The voltage is determined by a direct spark gap measurement across the arrester.

The fact that consistent results of the type reported can be secured in this test demonstrates the remarkable speed with which the autovalve arresters operate. The measurements of voltage across the arrester were made with a 125-millimeter sphere gap connected directly across the arrester. No resistance was used in series with the gap. In spite of the extreme speed of such a gap, the discharge started in the arrester, and discharge currents of several thousand amperes flowed through the arrester before the sphere gap broke down. When this type of test is made with an arrester which has a high dielectric spark lag, as compared to that of the sphere gap used for measurement of voltage, the indicated voltages will be determined in general by the factors which control initial breakdown of the arrester rather than by the impedance drop in the arrester. This does not mean that in such a case the measurements are untrue, but merely that they give no dependable indication as to the discharge impedance of the arrester.

TABLE I
CALCULATION OF ARRESTER PERFORMANCE

Line kv.	Ht. ft.	Surge kv.	Ins. str. kv. inst.	Z ohms	Electrolytic				Exc. kv.	Total kv.
					Equip res. ohms	Crit. volts kv.	Surge excess kv.	$\frac{R}{R+Z}$		
22	20	400	62	400	65	31	369	0.14	52	83
66	32½	650	186	450	195	93	557	0.302	168	261
110	40	800	312	500	325	155	645	0.393	253	409

Line kv.	Station Autovalve					Total kv.
	Res. ohms	Crit. volts kv.	Surge excess kv.	$\frac{R}{R+Z}$	Excess kv.	
22	45	42	358	0.101	36	78
66	135	126	524	0.231	121	247
110	225	210	590	0.31	183	393

The circuit used in these tests is shown in Fig. 7 and typical volt-ampere curves for both the distribution and station type arresters are shown in Fig. 8. For comparison a volt-ampere curve of a standard electrolytic arrester is also shown in Fig. 8. The data of Table I and the curves based thereon shown in Figs. 8 and 3 are calculated from resistance values determined from the volt-ampere curves of voltage corresponding to the assumed maximum permissible surge voltage.

The tests under service conditions give in general less specific information than laboratory tests, but they are valuable from the fact that they act as a sort of a court of last appeal. A total of 527 units has been installed on lines ranging up to 25,000 volts, both in the factory and on lines of various operating companies for periods up to 17 months. There failures have been observed with low voltage distribution units installed at the beginning of last year and the conditions were such as to point almost conclusively to flashover as the cause of failure. Several other failures were noted among the earliest arresters installed and these failures are known to have been caused by errors in design which were discovered after the arresters were installed. The major portion of these service installations were made with 2500-volt distribution arresters of an early design, very close to the present type but using a disk area of 8.5 sq. cm. These tests, while they do not furnish specific values for the characteristics such as are available from laboratory tests, give final proof of the fundamentals, and thus add sufficient support to the laboratory tests to assure the results shown in Figs. 2 and 3.

It is fitting that mention should be made that the work reported here has been contributed by a considerable group of people in various degrees, but each in important measure. These include Dr. Slepian, the originator of the device and Messrs. G. M. Little, I. R. Smith, L. R. Golladay, S. M. Pineles, and E. J. Haverstick as well as the writer.

Discussion

D. W. Roper: The designers of the arrester described in Mr. Atherton's paper, having the courage of their convictions that they had a real lightning arrester, were willing to submit it to the scrutiny of the "laboratory" that we have in Chicago, for comparing the performances of various types of arresters.

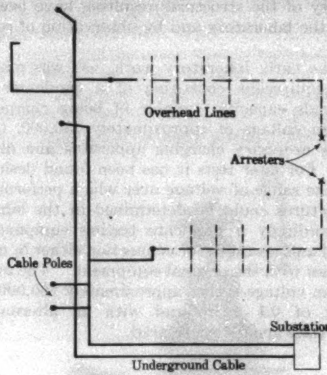

FIG. 1

The results of this scrutiny were entirely favorable. I think it would be quite unsafe at this time to give the arrester our unqualified approval, but there can be no objection to a modest statement that the results so far have been favorable. In fact we are placing more arresters of this design on our lines this year.

The test of this arrester was unsatisfactory in one respect during the past year. From a scientific viewpoint, the results were disappointing because we had so little lightning, the amount being only about 30 or 40 per cent of the average for the past ten years. However, there were one or two storms that gave us some interesting data. In one of them two arresters were affected at the same time. Fig. 1 is a diagram of the installations:

The sketch shows a substation feed system, distribution circuits, four-wire, three-phase, with the grounded neutral. The two arresters, indicated by the X's, were on different circuits, so that there was no actual or line connection between the two except around through the station. In one severe storm following a discharge apparently of great severity, trouble was observed at both these locations. One of the arresters was destroyed, but failed to clear itself from the line. It saved the transformer, however, and that is the purpose of an arrester. But it also gave indications of the highest voltage discharge that we have noted in our records up to the present time, so we will hardly call the failure of the arrester to clear itself, a point against it. Apparently that point will be taken care of in future designs.

There is one other practical detail. In one of the slides there was shown a form of contact which I believe very illuminating. This is a form of contact made by taking a piece of bronze and shaping it so as to give a contact and also making a spring connection between the metal electrode at the end, and the disk. A plate, on top, is screwed down on this contact but the lightning discharges are so great that this spring contact is not feasible in a lightning arrester; it will burn through with a heavy discharge. The various makers of arresters who are co-operating in our experiments have all been informed of this fact, so they now use a connection which gives the necessary pressure, and, with another soldered wire connection, provides the proper carrying capacity.

The remarkable feature of this paper, from my point of view, is the fact that one manufacturing company is applying the principle of the glow discharge to widely different types of apparatus.

Charles P. Steinmetz: Mr. Atherton's paper is very interesting in the information it gives on the autovalve lightning arrester, but still more interesting for showing the great progress made during the last year, as the result of the development of the kenotron high-voltage rectifier, in the production of apparatus and methods of testing lightning arresters under representative service conditions. I refer to the "lightning generator", as the apparatus shown in Fig. 7 of Mr. Atherton's paper has become to be called. It consists of a considerable number of large high-voltage condensers charged with high unidirectional voltage from a kenotron rectifier set, and giving a very high power impulse or oscillation of very short duration and of definite predeterminable shape, and so more nearly approaching the direct or secondary lightning discharge than any apparatus available for test. It therefore appears that we rapidly approach the condition when lightning arrester tests will not merely be comparisons of different types, but give exact numerical and reproducable results under specifications, which can be standardized, and with an exactness like that of high-potential tests of apparatus for instance. We have given the matter extensive study for a considerable time, and expect to be able at one of the next meetings of the Institute to present a paper on "Methods and Specifications of Lightning Arrester Tests for Service Conditions." Considerable work has to be done to make the use of the lightning generator for testing safe, to determine the limitations of the apparatus; study the methods of tests which are least liable to erroneous results; investigate the possible sources of error and the precaution required to guard against them, etc. For instance, it is little realized what serious effect the inductance of the leads has in retarding the impulse and limiting the frequency; the possibility of erroneous results due to local low-power high-frequency oscillations in isolated capacities such as that of the sphere gap has to be guarded against; the location of the ground is important in its effect on local oscillations caused by the capacity of the system against ground. Also specifications are required on the relation of the spheres to the length of the gap to insure instantaneous action, since even the sphere gap is instantaneous only within a limited range. For instance, the 125 mm. sphere gap in Mr. Atherton's paper asumes a time lag for gaps of less then 30 mm. Furthermore, the development of test gaps of definite and known time lag is desirable to determine the time lag of the protective devices and the apparatus to be protected by them, etc.

J. L. R. Hayden: I would like to ask Mr. Atherton about the first curve in his paper, Fig. 1,—whether the data given in this are actual results of tests made at atmospheric air pressure, or whether they are calculated from values observed at lower air pressure. We have been doing a lot of work on these small gaps, and find, it is quite some job to get consistent results down to gaps of one ten-thousandth of an inch. At the same time we found that the proportionality between gap length and air pressure holds for a limited range only, and the voltage of a very small gap can not be calculated from that of a gap of very low air pressure.

Joseph Slepian: Mr. Hayden asks about the curve on the first page of Mr. Atherton's paper giving the relation between the breakdown voltage and gap length for very small gaps, and he asks whether it is a calculated curve or an experimental.

It is not a calculated curve, the part to the right of the minimum has been verified experimentally, while the part to the left is obtained by extrapolation, from data from lower pressures than atmospheric. The fact that there is a minumum breakdown for air at atmospheric pressure has been proven experimentally. The experimental establishment of this fact presented great difficulties because with such short gap lengths one must be absolutely sure the gap is free of dust or any loose particles that may get into and bridge the gap. Because of the experimental difficulties there was originally much disagreement as to whether there was or was not a minimum breakdown voltage. The later experiments were done with the greatest care, and show that until you get actual contact the gap will not break down. When I say actual contact, I mean separation of the order of wave-lengths of light. Until separations of that order are reached the gap will not break down with less than 350 volts.[1]

In this later experimental work, small spheres were used. When the gap length was less than this distance of three-tenths of a millimeter, the spark instead of passing between the nearest points of the spheres, would strike from points separated by a greater distance—showing that the smaller distance between the electrodes was stronger dielectrically than the longer path from the sides. Because of this striking of the spark over the longer distances, the actual curve showing the increase of break down voltage with decrease of gap length could not be obtained at atmospheric pressures. Results are however readily obtained at lower gas pressures, and the results obtained at the lower gas pressures can be translated into higher pressures by using what is called Paschen's law for the breakdown of gaps in gases. Paschen's law states that for plane electrodes, the breakdown voltage is a function only of the amount of gas between the electrodes. Thus if you halve the pressure of gas and double the distance between the electrodes, the breakdown will be unaltered. That is, lowering the pressure has the effect of increasing the distance corresponding to any particular breakdown voltage. At a low pressure then, the distances become large and manageable so that we can actually experimentally make a complete curve showing the relation between the gap length and breakdown voltage. Curves obtained in this way show that as the separation decreases the voltage falls to the minimum voltage and then rises again. If a series of such curves is obtained for increasing gas pressures, it will be found that these curves are all similar, differing only in that the abscissas representing gap lengths crowd down toward the origin as the pressure increases. The presumption is overwhelming that at atmospheric pressure this continues to hold, except that the dimensions have shrunk down to where it is impossible to run an actual curve experimentally.

Summing up, I may say that experimentally there is a definite

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1. *Proc. Roy. Soc.* LXXI., p. 374, 1903; W. R. Carr.
Proc. Roy. Soc., LXXIII., p. 337, 1904; P. E. Shaw.
Phil. Mag. (6), 16, p. 456, 1908; J. E. Almy.
Phys. Rev., 31, p. 216, 1910; E. H. Williams.

proof that there is a minimum breakdown voltage at atmospheric pressures, and by extrapolation from curves taken at lower pressures, there is very definite proof that the voltage rises above this minimum breakdown voltage when the gap length gets sufficiently short.

K. B. McEachron: The application of the flow discharge to lightning arrester service as demonstrated in Mr. Atherton's paper is very interesting. We have been in the habit of associating the glow discharge with currents of very small value and it is surprising, indeed, that an arrester built on this principle should be able to handle thousands of amperes even for a very short interval of time.

As a matter of fundamentals, I would like to ask the author what determines the change from the glow discharge to the arc discharge with a given area of electrode. If it is a question of current and time, which in this case means energy, what is the relation between these quantities which marks the borderline between the glow and the arc discharge?

I am also interested in knowing the method used by the author when he found as a result of test that a current of several thousand amperes passed through the arrester before the sphere gap connected in parallel with the arrester broke down. In other words, how was the speed of the arrester measured?

A. L. Atherton: There are one or two points brought out on which comments are desirable. First, I want to acknowledge the assistance that was rendered us in the development work by the cooperation of Mr. Roper in Chicago. We were very fortunate to be able to place the trial arresters of his system where he has collected data for comparison and where his ideas are so definitely crystallized. One point which he has brought out is of special interest. During the season three failures occurred among the trial arresters installed in Chicago. A close examination was made of the arresters and every indication was that those failures were by flashover outside of the column of disks between the casing and the column. On calculating back from the line characteristics and the arrester characteristics, it developed that the surge voltage to cause such a failure must have been extremely high, as stated by Mr. Roper. In fact, it probably was a service condition that would result in the failure of any type of distribution arrester now available. However, we have felt it advisable to make changes to raise the flashover point since a ready means was at hand. Moreover, an increase has been made in disk area since the trial arresters were built so that the protection afforded has been increased to about double or a little over, giving voltages at the arrester something under half those which were encountered in the trial installations in Chicago. These two changes increase the factor of safety against flashover many times.

Dr. Steinmetz, in speaking of the test, brought out a point of great interest and one on which a great deal of work has yet to be done. The matter of proper timing of measuring gaps is hard to handle, and the results are rather obscure and hard to translate. Of course, in practically all of the tests that are made on lightning arresters in the laboratory, measurements are made with gaps of which the timing is under control.

The results reported in the paper were selected as rather startling because the gaps that were used in those tests were without any retarding means. They were as rapid as they could be under the test conditions. It is an interesting and valuable point that the results secured with such gaps checked reasonably with what would be expected from the characteristics of the material put into the arrester. That simply demonstrates that the arrester itself is a very high-speed device.

The general question, mentioned in one discussion, of the heat generated in the gaps and the effects to be anticipated from this heat is best evaluated by reference to the tests. Single-column distribution-type arresters have been subjected to repeated discharges of approximately 7000 amperes which is several times the maximum current anticipated in service, and with a duration of the order of that of the surges met in service. After many such surges, tests and visual examination showed the disks to be unchanged.

It is also interesting that this same sort of energy concentration is present in all high grade arresters, since the same energy has to be handled by all arresters with equal discharge resistance. In general, if we assume the manufacturer's descriptions to be correct, in the types of arresters in which puncture of solid material takes place in operation, the energy is concentrated more than it is in this arrester since the total current is stated to flow through a few small punctures. In the autovalve arrester the discharge is very definitely spread out over the area of the electrodes. This brings us to the first question raised by Mr. McEachron. The form of discharge between given electrodes is, as pointed out in the paper, determined by the electrode temperature. This naturally varies with power input and duration. In any given discharge the temperature gradually rises and, if the power and duration are sufficiently above service conditions, the electrode temperature will reach that required for an arc. At this time quantitative data as to limits are not fully available. It is established however that maximum anticipated service conditions, excepting direct strokes of lightning must be greatly exceeded before the glow discharges change to arcs.

As to the inference of speed of the arrester from the tests with parallel gap, it is a safe assumption that if the gap with no series resistance breaks down first the arrester will not break down, for the voltage across the gap is low after breakdown and the gap with leads is a small part of the total circuit. But the arrester did break down. Therefore its speed is of the order of that of the measuring gap.

Machine Switching Telephone System for Large Metropolitan Areas.

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Review of the Subject—*From the earliest forms of telephone switchboards to the modern types, the development of the switchboard has been marked by the increasing use of automatic methods to supplement the manual operation wherever this would result in better service to the public or more efficient operation.*

In addition to all that has been done in developing and introducing automatic operations with manual switchboards, it has been found desirable and practicable to go further in the direction of introducing automatic operation in the telephone plant and a machine switching system has been developed in which the bulk of the connections are established without the aid of an operator.

The complexity of a large metropolitan area and the exacting requirements which a machine switching system must meet are outlined briefly, and the system which has been developed to meet these requirements is described.

The application of the system to a typical large metropolitan area and the means provided for permitting its gradual introduction into the existing plant are discussed.

It is the purpose of this paper to outline briefly certain important developments in connection with machine switching telephone systems and to discuss the application of the results of these developments to the problem of providing telephone service in large metropolitan areas.

The telephone was invented in 1876. Amost immediately thereafter it was recognized that, for it to attain its greatest field of usefulness, switchboards and switching centers would have to be established for effecting interconnection between subscriber's lines.

Professor Bell's vision of the future was given in a statement to prospective investors. He said:

> "It is conceivable that cables of telephone wires could be laid underground, or suspended overhead, communicating by branch wires with private dwellings, country houses, shops, manufactories, etc., etc.—uniting them through the main cable with a central office where the wires could be connected as desired, establishing direct communication between any two places in the city. Such a plan as this, though impracticable at the present moment, will, I firmly believe, be the outcome of the introduction of the telephone to the public. Not only so, but I believe in the future wires will unite the head offices in different cities, and a man in one part of the country may communicate by word of mouth with another in a distant part.
>
> "Believing, as I do, that such a scheme will be the ultimate result of the telephone to the public, I will impress upon you all the advisability of keeping this end in view, that all present arrangements of the telephone may be eventually realized in this grand system."

Early Developments

The only apparatus available at that time for this purpose was that employed in telegraph, messenger, fire and burglar alarm services. Some of this apparatus, such as wire, insulators, batteries, annunciators, etc., was found to be useful in the new art; other apparatus had to be developed. The switchboards of that day employed this apparatus. They were small in size, and could accommodate only a limited number of lines.

It soon became evident that the requirements of the telephone exchange service demanded signaling and switching equipment different from that employed in any of the other branches of the electrical industry, and it became necessary to create an entirely new art, involving many branches of science, before commercial telephone service could be given on an adequate scale.

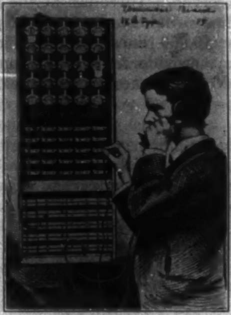

Fig. 1—Early Type Switchboard

The switchboards grew from small boards, capable of handling a few lines, as shown in Fig. 1, to the very complex arrangements providing signaling, switching, and transmission facilities for as many as ten thousand lines in a single board, of the type shown in Fig. 2.

As the subscribers increased in number it was found that beyond a certain point it was no longer practicable or economical to have all of the subscribers' lines brought to one center. It was therefore necessary to have several centers, the number depending upon many

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factors, the most important of which are the size and telephone needs of the community.

The consequence of all this is that in large metropolitan areas the number of centers is large, and the trunking system complex, as each center must be provided either directly or indirectly with trunks to every other center.

As an illustration, take the New York Metropolitan area, shown in Fig. 3, where the telephone plant is of the greatest intricacy because of the very large number of subscribers served. There are at the present time 158 central office switchboards, many of them having equipment for 10,000 lines. These offices and the associated plant provide for intercommunication between 1,400,000 telephones, and approximately two trillion possible connections. It is estimated that by the year 1940 there will be 300 central office switchboards within the New York Metropolitan area, serving some 3,300,000 telephones—or nearly two and a half times the present number.

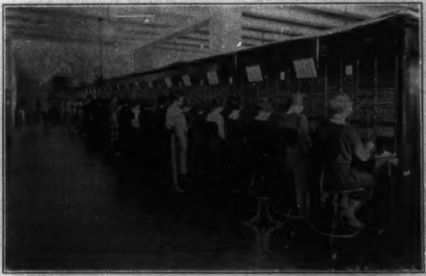


FIG. 2—MODERN TYPE COMMON BATTERY SWITCHBOARD

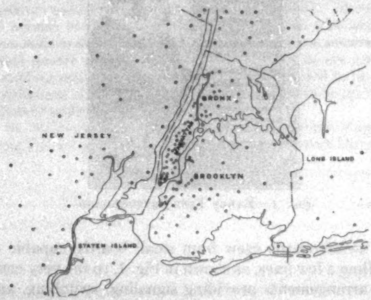


FIG. 3—MAP SHOWING LOCATION OF CENTRAL OFFICES IN NEW YORK METROPOLITAN AREA

MANUAL SWITCHBOARDS

The system most commonly employed today for connecting subscribers' lines together is the so-called "manual" system; that is, a system in which operators are employed to make the actual connections between subscribers' lines, although so many of the functions are performed automatically that, except in name, it is to a large degree automatic.

It is a long step from the early switchboards to the modern common battery multiple manual switchboards. The history of the development of switchboard equipment and apparatus shows that enormous progress has been made in this art in a comparatively few years. As the telephone subscribers have grown in number and as the amount and complexity of the traffic have increased, it has been only by the most intensive development that it has been possible to keep ahead of the demand for telephone service, and that telephone engineers have been able to get the speed, efficiency and accuracy that are obtained today in so-called manual operation. It is worthy of note in this connection that the attainment of these ends was made possible by the extensive introduction of automatic features.

A very brief description at this point of the type of manual switchboard more commonly employed will be helpful.

In this switchboard the subscriber's line terminates at the central office in so-called "jacks." Associated with each line is a lamp, individual to it, which automatically lights when the subscriber removes his receiver from the hook. This serves as a signal to the operator that a connection is desired.

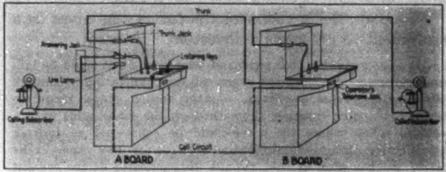


FIG. 4—DIAGRAM SHOWING MANUAL INTEROFFICE CONNECTION

The operator answers this call by inserting one end of a cord in the jack associated with the calling subscriber's line, operates a listening key which connects her telephone set to the subscriber's line, and asks for the number desired. When this is obtained the operator completes the connection by inserting the other end of the cord in the jack of the desired subscriber's line, and the subscriber's bell is rung. Suitable lamp signals are provided so that the operator may know when the called subscriber answers, when either subscriber desires further attention, or when either or both of them have finished talking and have hung up their receivers.

If the subscriber desired is connected to a distant office, the operator receiving the call would, instead of plugging directly into the subscriber's line, directly connect the subscriber's line to a trunk terminating in the desired office, where the connection would be completed by a second operator, known as the "*B*" operator, as shown in Fig. 4. Such communication between the two operators as is necessary to establish this connection takes place over a special pair of wires known as a "call circuit."

The method, of which the above is a bare outline, is that used in completing ordinary connections. Different arrangements and different operating methods have to be provided for handling short haul toll calls, long distance calls, calls from coin boxes, and calls of many other kinds.

In the simplest types of manual systems, the subscriber, in order to signal the central office, turns a crank thus operating a magneto generator. This throws a drop in front of an operator at the central office. In the switchboards developed to meet the needs of the larger areas, electric lamps are substituted for the drop, and relays automatically controlled by the subscriber bring them into play at the proper time. Electric lamps which serve as visual signals to the operator to indicate the status of the connection are also associated with the cords that the operator uses for connecting subscribers together. The operation of these lamps is automatic and is under the control of the switchhook at the subscriber's station.

Many other arrangements of an automatic character have been developed and are used as occasion requires—not merely because they are automatic in character but only when it has been established that they make for better service to the public or for efficiency and economy of operation, or both. Among these may be mentioned automatic ringing, automatic listening, and many forms of automatic signaling. Many of these arrangements are highly ingenious and contribute greatly to the efficiency and economy of operation. Thus, the trend of switchboard development has been more and more in the direction of automatic operation and automatic methods.

In addition to all that has been done in developing and introducing automatic operation with so-called manual switchboards, it has been felt for a long time that in large and complex telephone areas, such for example as New York City, the time would ultimately come when it would be desirable to go further in the direction of introducing automatic operation in the telephone system. This whole matter has been the subject of much thought on the part of engineers of the Bell System and, as a result, there has been developed and recently put into operation a system in which the work of establishing most of the local connections is done entirely by machinery.

The introduction of this system will eventually make a considerable reduction in the telephone company's requirements for operators which are becoming more difficult to fulfill year by year. Operators will be required, however, to handle toll and many special classes of local calls and for this reason, together with the constant growth of the business and the considerable period of time that will be required to introduce the new system completely, we can expect little or no reduction in the present operating forces for some time to come, and no operator will find herself out of employment on account of the introduction of the machine switching system.

MACHINE SWITCHING

It is the purpose of this paper to describe this system sufficiently in detail to give a general picture of it, but because of the limitation as to space no attempt will be made to go into the intricacies of circuits and apparatus, which doubtless would be of interest only to the telephone engineering specialists.

Among other requirements, the following must receive special consideration in the design of a machine switching system.

The functions to be performed by the telephone subscriber in getting a connection must be simple and easily understood.

It must work efficiently and with accuracy and speed, and of course must be capable of handling the various types of calls that the subscriber wishes to make.

The system must not require modifications in the existing rate structure, otherwise than desirable. If the rate structure calls for message register operation, coin boxes, etc., means must be provided for automatically operating the register and collecting the coins on such calls, and for preventing a charge on calls not answered, calls for free lines, busy lines, etc.

The system should employ, as nearly as practicable, the conventional numbering scheme.

It should work with the existing telephone network, so that its introduction does not require wholesale number changes and extensive rearrangement or the abandonment of existing switchboards or other plant. Its introduction must, of necessity, be on a gradual basis.

It must be sufficiently flexible in design to care for growth and such changing traffic conditions as occur from time to time.

In large telephone areas, such as the New York Metropolitan area, there is a great variety of calls to be handled and many different classes of service furnished the public, such as message rate, flat rate, official, coin box, non-attended pay station, attended pay station, special services such as information, etc. Not only individual lines but party lines, and private branch exchanges must be cared for, and provision must be made for thousands of toll messages which must be recorded, supervised and timed.

A call originating in a machine switching office in New York City may have as its destination any one of a great number of points. It may be for another subscriber in the same office or for one in another nearby machine switching or manual office; it may be for one of a large number of surburban toll points, or it may be to some point in a distant city.

The machine switching system, which is the subject of this paper, meets these requirements. After long-continued laboratory experiments, supplemented by field trials, power-driven apparatus of the panel type has been found to be the most suitable, and is now in successful operation in New York City and in other large cities in the country.

General Plan of Operation

At the expense of some repetition it seems desirable, in order to give as clear an understanding as possible as to the operation of the system, to first give a brief outline of how the call is handled and a description of the more important elements of the equipment, before going into a detailed description of the operation of the system.

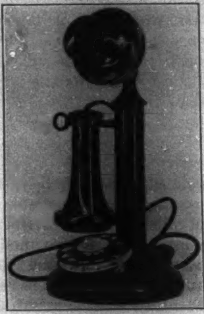


Fig. 5—Desk Stand Equipped with Dial

The subscriber's station is equipped with the usual form of telephone instrument and, in addition, with a calling device known as a "dial," mounted at the base of the desk stand, as shown in Fig. 5. This dial has ten finger holes bearing letters and figures, as shown in Fig. 6.

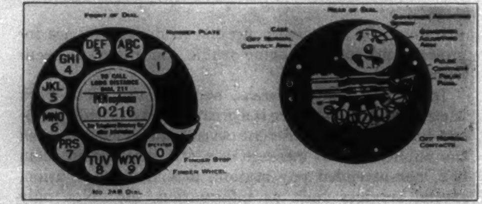
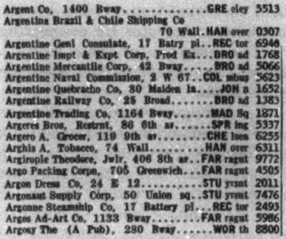


Fig. 6—Subscriber's Dial

In making a call the subscriber will, of course, first refer to the telephone directory. He will find in the directory a listing that is only slightly different from that to which he is accustomed. Typical samples of this new form of telephone listing for New York City are shown in Fig. 7. As will be noted, these conform to the present manual listings, except that the first three letters of the office name are set out prominently. This numbering system will be discussed later in this paper.

Having secured the desired telephone number from the directory, which we will assume is "ACAdemy 1234," the subscriber will first remove his receiver from the hook and will hear the so-called "dial tone," which indicates that the apparatus is ready to receive the call. He will then insert his finger in the hole over the letter *A*, rotate the dial until the finger comes in contact with the metal stop shown in the picture, then release the dial, which will automatically return to normal. He will repeat this operation for the letters *C* and *A*, and in turn for the four numerals, *1, 2, 3, 4*.

This operation of dialing on the part of the subscriber is exactly the same, whether the telephone number he desires is in a manual or in a machine switching office.



Argent Co, 1400 Bway..................	GRE eley	5513
Argentina Brazil & Chile Shipping Co 70 Wall.	HAN over	0307
Argentine Genl Consulate, 17 Batry pl..	REC tor	6946
Argentine Impt & Expt Corp, Prod Ex...	BRO ad	1768
Argentine Mercantile Corp, 42 Bway.....	BRO ad	5066
Argentine Naval Commission, 2 W 67..	COL mbus	5623
Argentine Quebracho Co, 80 Maiden la....	JOH n	1652
Argentine Railway Co, 25 Broad.......	BRO ad	1383
Argentine Trading Co, 1164 Bway.......	MAD Sq	1871
Argeres Bros, Restrnt, 86 6th av.......	SPR ing	5337
Argero A, Grocer, 119 9th av.........	CHE lsea	6255
Arghis A, Tobacco, 74 Wall...........	HAN over	6311
Argiropie Theodore, Jwlr, 406 8th av..	FAR ragut	9772
Argo Packing Corpn, 706 Greenwich...	FAR ragut	4505
Argon Dress Co, 24 E 12............	STU yvant	2011
Argonaut Supply Corp, 50 Union sq..	STU yvant	7476
Argonne Steamship Co, 17 Battery pl...	REC tor	2493
Argos Ad-Art Co, 1133 Bway.........	FAR ragut	5986
Argosy The (A Pub), 280 Bway.......	WOR th	8800

Fig. 7—Typical Examples of New Form of Listing Telephone Numbers

Similarly, the method employed by a subscriber who is connected to a manual office in getting a subscriber connected to a machine switching office, is the same as though the desired subscriber were connected to another manual office.

The progress of a call originating in a machine switching office is briefly as follows:

As will be seen from Fig. 8, the line of the calling subscriber, whom we will assume to be a subscriber in the Academy office, appears in a so-called "line finder" frame. When the subscriber's receiver is removed from the switchhook preparatory to dialing, the line is selected by a "line finder" and connected to an idle "sender" by means of a "sender-selector."

Upon completion of these operations which take but a fraction of a second, the dial tone is sent out to the calling subscriber as previously mentioned. When the subscriber dials, electrical impulses on a decimal basis are transmitted to the sender which receives and registers them, translating them in turn to the proper basis for the control of the selectors which are not operated on a decimal basis. The sender automatically causes the particular "district selector" which is permanently associated with the line finder originally used, to select a trunk to the office desired.

Assuming that the call is for a subscriber in the same office, Academy, the trunk chosen will terminate at an "incoming selector" frame and the sender above referred to will cause the call to be routed through the incoming selector to a final selector, and thence to the particular line desired. When the connection is thus completed, audible signals will be sent back to the calling subscriber to indicate that the station is being rung or that the line is busy.

If the call had been for a subscriber in another machine switching office, namely, Pennsylvania, the call would be routed from the district selector to the office desired, either directly or through an "office selector" in case the total number of trunks to all offices is too large to be placed on the district selector mulitple. These trunks terminate on incoming selectors at the Pennsylvania office which select the subscriber's line through final selectors, as described above.

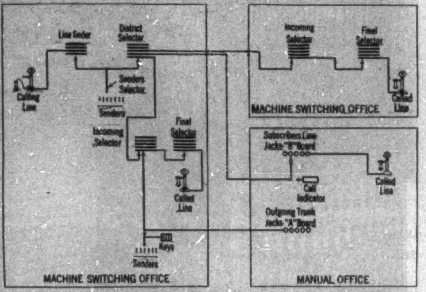


FIG. 8—DIAGRAM SHOWING CONNECTIONS FROM MACHINE SWITCHING TO MACHINE SWITCHING, MACHINE SWITCHING TO MANUAL AND MANUAL TO MACHINE SWITCHING.

If the call is for a subscriber connected to, say, the Worth Office, which is a manual office, the call would be routed from a district selector directly or through an office selector to the "B" board in the Worth Office, where the number desired appears in front of the operator at a "call indicator position" in the form of visible numbers on the keyshelf. The operator is advised of the trunk to which the call is connected by suitable signals, and the call is completed by plugging this trunk into the desired subscriber's line.

Calls originating in a manual office and intended for a machine switching office reach the machine switching office over trunks from the "A" operators in the manual office. At the machine switching end these trunks terminate in incoming selectors, which have access to the final selectors on which the subscriber's lines are located. The selectors are under the control of a special group of senders, and operators are provided with suitable keys for setting up in these senders the number of the desired subscriber. These operators at the machine switching office receive the information as to the desired number from "A" operators in the distant manual office, exactly as is done in the case of manual operation.

The introduction of machine switching equipment does not require radical changes in private branch exchanges. The private branch exchange is provided with dials, and calls to the central office are dialed by the private branch exchange attendant or by the extension user in the same way that the ordinary subscriber dials. No change in the private branch exchange is required for handling incoming calls. An idle trunk in the private branch exchange group is selected by the mechanism in the machine switching office, in much the same way as an individual subscriber's line is selected.

NUMBERING SYSTEM

One of the unique advantages of the plan developed for designating telephone numbers, to which reference has already been made, is that it does not necessitate the abandonment of the existing manual listings. It requires no change except that the first three letters of the office name are set out more prominently. Simple as this change in the form of listing appears, until it was developed by the Bell System no satisfactory method of designating telephone numbers for machine switching offices in large cities was known.

Many plans had been proposed, to all of which there were serious objections. Some of them required changing the whole system of manual designation, others the use of combinations difficult for the subscriber to use. In small cities a numbering plan employing only digits is sometimes practicable, but in such a large area as we are considering, such a plan would involve seven digits. The subscriber's number would take the form of say 786-3549. Such numbers would be difficult for operators to use and for the subscribers to carry in mind and would require that every subscriber's number in the entire area be changed before the first machine switching office could be cut into service. With the new system, the subscriber's number and office in general remains as before. It is necessary to change only a few conflicting office names in order to make them fit into the system.

DESCRIPTION OF THE EQUIPMENT

A detailed description of each unit employed in this system would be impracticable, in this connection, but a brief description of the more important ones will be of interest.

Sender. The use of the sender makes practicable the introduction of machine switching in large metropolitan areas where, of necessity, the service conditions are extremely complex. It is, in effect, the brains of the system, dealing with the subscriber and controlling the selection until the destination is reached, as an operator deals with the subscriber and controls the selection in a manual system. The number dialed conveys the same information to the sender in a machine switching system as the number spoken by the subscriber does to an operator in a manual system.

The sender is an arrangement of relays, sequence switches, and selectors, so worked out as to perform the following more important functions:

1. It receives a succession of electrical impulses from the subscriber's dial which are on a decimal basis, stores them and translates them to a non-decimal basis, corresponding to the particular group of lines and trunks that is involved in the path of the call.

2. It controls selecting mechanisms which build up the connection to the called party in such a manner that each mechanism is given the exact time required to perform its functions without any waste of time, independently of the rate received from the dial.

3. It makes the central office designations entirely independent of the arrangement of the trunk groups on the selector frames. This is a very important matter, inasmuch as it allows the selectors to be used to full efficiency. It provides the desired flexibility for growth and permits any desirable rearrangement of the trunks on the selector frames that the telephone company may find desirable at any time.

4. The sender is capable of distinguishing the class of office at which the connection terminates. That is, if the call is to terminate at a mechanical office, the sender will arrange to govern the selection accordingly. If the call is to terminate at a manual office, the sender recognizes this and arranges to send out impulses to the call indicator equipment in the manual office.

5. For the completion of certain calls, traffic conditions require the introduction of tandem centers as discussed later. The sender recognizes calls to be routed via tandem centers and arranges to handle these correctly. The tandem center may be manual or it may be mechanical, and the control must be determined accordingly.

6. Certain senders are arranged to serve lines supplied with coin boxes. These senders are arranged to make a test to determine whether a coin has been deposited and do not allow the connection to be cut through so that conversation can take place until the coin is deposited. If the subscriber does not deposit the coin, after a reasonable time has elapsed the sender connects an operator to the subscriber, and this operator notifies the subscriber of his omission. After the coin has been deposited, the sender allows the called subscriber to be rung and permits the conversation. In case the called subscriber is busy or does not answer, or if the call is to a free line, the sender returns the coin to the calling party. If the called party answers, the sender causes the coin to be collected.

The sender makes a test of the calling line after the subscriber has completed dialing, to insure the deposit of the coin, and recognizes whether a coin has actually been deposited or whether some abnormal condition exists, in which case the call will be routed to an operator who causes an investigation to be made.

7. In large areas, such as the New York Metropolitan area, there are distant points, connection to which requires toll charges. In such cases the subscriber is instructed to dial a special operator who will ascertain his wishes, complete the call, and make the proper charge. Should a subscriber attempt to dial outside of his own local service area, his call will automatically be routed to an operator.

Panel Type Selecting Mechanism. An important mechanism of a machine switching system is the selector and its associated multiple bank. It is a device by means of which trunks or lines are connected together as required. It performs the same function as the switchboard cord and plug which in a manual exchange can be plugged by the operator into any one of a number of jacks which are the terminals of trunks or lines.

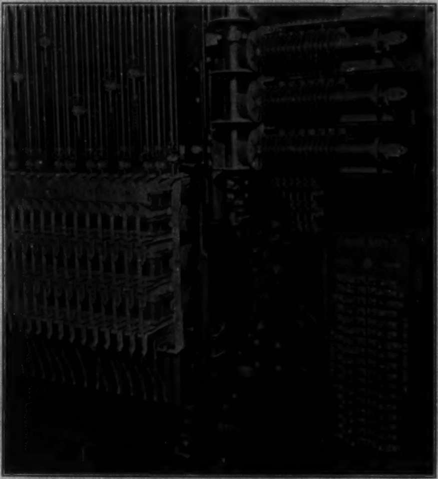


FIG. 9—VIEW OF THE SELECTOR

Fig. 9 shows the mechanical elements of the selectors. The movable member corresponds to the cord and plug of the manual system and the fixed terminals or multiple, to which the movable member can make connection, corresponds to the jacks of the manual system.

Fig. 10 shows the fixed terminals or multiple to which the selectors connect. This multiple consists of flat punchings about 3½ feet long and 1 inch wide overall. Each of these strips has lugs on each side with which the selectors can make contact. In this particular panel, three hundred of these strips are piled one above the other, separated by insulation, and securely bolted together, forming a panel about 15 inches high.

This panel provides a multiple consisting of "tip," "ring" and "sleeve" connection for one hundred lines appearing sixty times; that is, thirty on each side. The insulating material consists of special impregnated paper and is of such a nature that, after the panel is assembled and baked, it becomes inert and is not adversely affected by any conditions met with in a central office. It is this panel which has given the name to the system.

FIG. 9A—VIEW OF THE SELECTOR

As shown in Fig. 9, the selector consists of a metal tube supported in bearings allowing vertical motion and carrying five sets of brushes. Each one of the five sets of brushes is arranged to make connections to the tip, ring and sleeve terminals of the panel banks before which it normally stands, and the tip, ring and sleeve contact members of all five of these brushes are multiplied together. They are normally free from contact with the terminals, but any set may be tripped mechanically, so that that set will contact successively over terminals as the selector rises.

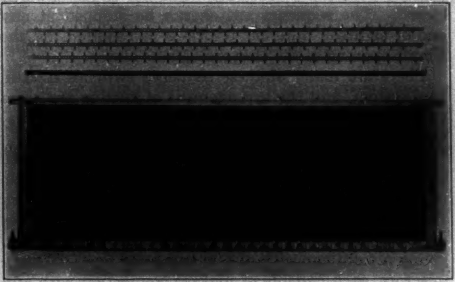


FIG. 10—SELECTOR MULTIPLE BANK

A friction clutch is provided at the base of each selector, so arranged that the selector can be raised or lowered by power supplied by a constantly rotating small motor, common to 60 selectors. A magnet is also provided for tripping, by means of a rotating rod, any one of the five sets of brushes into mechanical engagement with the terminals. In choosing a trunk or line, that one of the five sets of brushes which has access to the panel in which the desired trunk or line happens to be, is tripped so that it makes contact with the bank terminals before it. The selector then moves upward, under the proper control, until the tripped brush engages the desired line or trunk. The selector is then held in this position by a pawl associated with the clutch. When the connection is to be taken down, the pawl is withdrawn, and the selector is carried back by means of the power drive controlled by the clutch. When the selector reaches its normal position the tripped brush is reset.

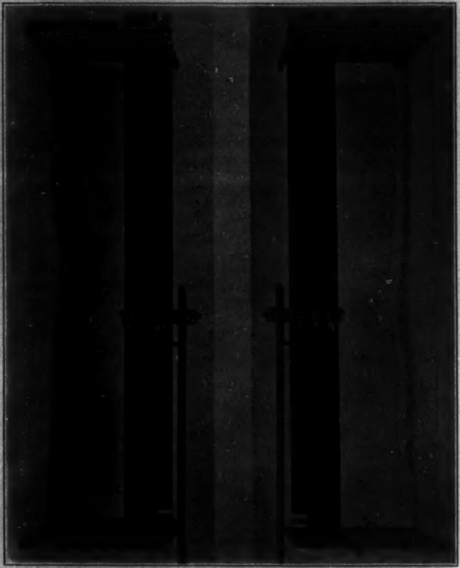


FIG. 11—COMMUTATOR FOR CONTROLLING VERTICAL MOVEMENT OF SELECTING MECHANISM

Selectors used for different purposes are arranged to move their brushes upward at different speeds. The speed most commonly employed moves the brushes over the terminals at the rate of 60 trunks per second. At the top of the frame, just above the fifth bank, are located commutators as shown in Fig. 11, one for each selector. The multiple wiring of the brushes on the selector leads to other brushes which move over strips on these commutators, and thereby completes the connection from the movable selector to the rest of the circuit, thus avoiding flexible wire connections with their attendant troubles. This commutator, also, performs the more important service of controlling the travel of the selector. Brushes moving over conducting segments separated by insulation produce impulses which, when sent back to the sender, indicate to it the exact position of the selecting mechanism.

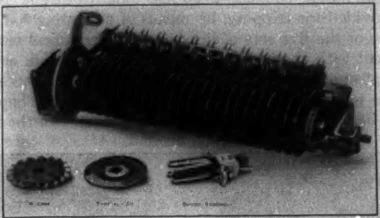


FIG. 12—SEQUENCE SWITCH ASSEMBLY

FIG. 13—SELECTOR FRAME COMPLETELY EQUIPPED

Sequence Switch. Another device of great importance is the "sequence switch," shown in detail in Fig. 12. It is operated through an electromagnetic switch from the same motor that drives the selectors.

The sequence switch may be described as a circuit controller or device whose function is to establish in a definite sequence such circuit conditions as are required in the operation of the system. It is made up of circular disks called cams mounted rigidly on a shaft. The plates of the cams are cut so that brushes come in contact with the plates only when the circuit is to be closed. The sequence switch can be stopped at any one of eighteen different positions as required, by the simple opening of the electromagnetic clutch.

FIG. 14—GROUP OF TYPICAL SELECTOR FRAMES

There are many of these sequence switches used in this system, and the arrangement of cutting the cams varies, depending upon the particular circuit combinations which it is desired to establish.

Selector Frames. Fig. 13 shows thirty selectors with all of the associated mechanism mounted upon one side of a frame ready for operation in an exchange. Both sides of the frame are alike. Five panels of 100 lines each are mounted in this frame, one above the other, giving a total capacity of 500 trunks or lines. Thirty selectors, each capable of making connection with any one of the 500 trunks or lines, are placed adjacent to each other on each side of these panels; the entire frame thereby having a capacity of sixty selectors, each of which has access to 500 trunks or lines.

Immediately to the right of the selectors are the sequence switches and, under protective covers, such relays as are used in connection with the selectors upon the frame shown.

Selecting apparatus of this general type, but differing in details of design, is used during the different stages of the call as line finders, district selectors, incoming selectors and final selectors, reference to which has been made before. Fig. 14 shows a section of a machine switching office with some of the typical frames.

The use of apparatus of the substantial construction just described is made possible only through the use of the sender which receives impulses from the subscriber at the rate they are dialed and receives impulses from the selecting mechanism at the rate it is traveling. This obviates the necessity for restrictions in the design of either the dialing circuit or the selecting circuit, such as would be necessary if they were tied together.

Power Supply Arrangements. Since most of the operations normally required in handling a call in a machine switching office are carried out mechanically, it is evident that a considerably larger amount of power is required than with the manual system. Selectors and sequence switches are propelled mechanically by rotating shafts driven continuously by small motors mounted on each frame.

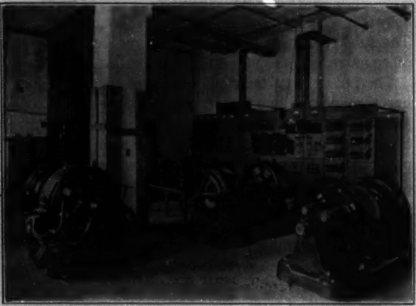


FIG. 15—POWER MACHINE AND CONTROL EQUIPMENT FOR TWO 10,000-LINE UNITS

The use of small motors on each frame gives a flexible and reliable source of power particularly since the motors now being used are of the special "duplex" type developed for the purpose. They consist of two motor elements in one frame, one element being normally driven from the commercial power service and the other being driven by the telephone reserve storage battery to which it is automatically connected by a relay inside the motor when the regular power fails. A power failure, therefore, causes no interruption to the drive. The selectors are arranged so that not more than half in any one group are driven by the same motor which insures continuous service in case of motor failure.

The main power requirement is for direct current at about 24 and 48 volts which is furnished from motor generator sets (Fig. 15) of special construction to reduce noise, converting the commerical alternating or direct power current into current which is regulated as to voltage and is free from variations which would cause noise in the telephone circuits.

Storage batteries (Fig. 16) floating across the current supply bus-bars insure regulation. In addition to stabilizing the voltage and reducing noise interference from the machines and between telephone circuits, the batteries perform the important function of keeping the exchange in operation during interruptions to the commercial power service. Small motor generators furnish current for ringing subscribers' bells and drive commutators supplying various tones and signals. Batteries or machines supply current for operating coin boxes and pulse machines provide impulses for the operation of certain of the machine switching apparatus.

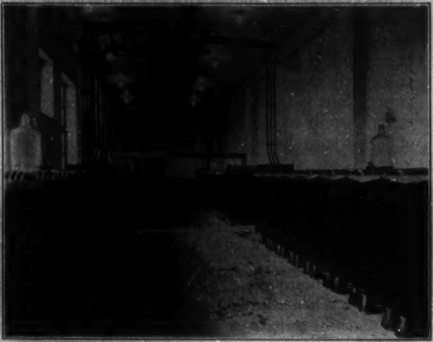


FIG. 16—BATTERY ROOM FOR TWO 10,000-LINE UNITS

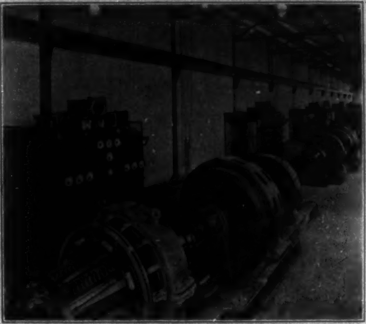


FIG. 17—165 H. P. GAS ENGINE GENERATING SET FOR EMERGENCY USE

Whenever practicable, two or more commercial power services from independent generating stations are secured, either of which will keep the office supplied. Where independent generating systems are not available a reserve gas engine supply (Fig. 17) is installed to take the place of the incoming power service, such engine also being equipped for emergency operation on gasoline.

All of the essential power machines and batteries are provided in duplicate and as indicated, are arranged to come into action automatically wherever this is necessary to insure continuity of service in the event of loss of power or trouble with any of the power equipment. Alarms are provided to detect variations in battery voltage, blowing of fuses, stopping of machines or any failure of service on all power busses which feed energy to the telephone or signaling circuits. The power plant is thus designed to give an uninterrupted energy supply at all times even when the usual sources of power may have been temporarily discontinued.

DETAILED PLAN OF OPERATION

The following will give in some detail the plan of operation for handling typical calls between various types of offices in a large metropolitan area such as New York City.

Calls Originating in Machine Switching Offices. Fig. 18 shows schematically the path of a call originating in a machine switching office. The pair of wires of a subscriber's line is attached to one of the sets of fixed terminals in a panel bank appearing before a group of selectors of the type which has been described. By putting fewer lines in these panels and increasing the number of selector brushes, we attain the speed necessary at this stage of the connection. These selectors are called "line finders," since their function is to find calling lines. The terminals correspond to the answering jacks and the selectors to the "A" operators' answering cords of the manual system.

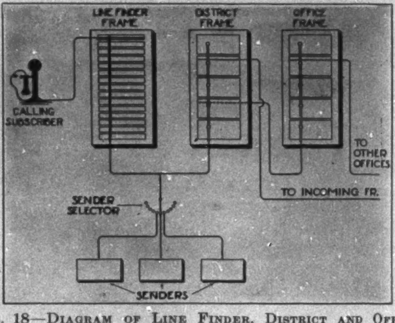


FIG. 18—DIAGRAM OF LINE FINDER, DISTRICT AND OFFICE FRAMES

When the subscriber removes his receiver, he closes the circuit of his line, causing a relay at the central office in series with his line, to operate. This relay causes an idle line finder, having access to his line, to trip the proper brush and then move upward to his line. At the same time a sender selector attached to that line finder is choosing, out of a common group, an idle sender. The sender selector is a small selector of a type in which the brushes are driven by a magnet over contacts arranged as shown in Fig. 19.

The sender having been attached in this manner to the calling line, a low humming sound, known as the dial tone, is heard by the subscriber, advising him that the mechanism is ready for him to dial. The entire sequence of events just described takes place in a fraction of a second, so that ordinarily the subscriber finds the dial tone when the receiver reaches his ear.

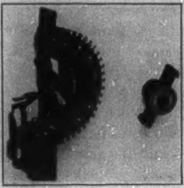


FIG. 19—SENDER SELECTOR

The subscriber now dials the required letters of the office name, and the numerals of the called number.

The pulses from the dial come over the subscriber's line through the line finder and sender selector to the sender which records and translates them to control the setting up of the connection. As soon as the connection has been established, the sender is released and is ready to be used for a new call, being kept in use only a few seconds for each call.

The first step in completing the connection is to choose an idle trunk in the proper direction. To the nearby offices there are groups of direct trunks, whereas the more distant offices are reached through tandem centers described later.

The line finder leads to the movable element of a panel selector known as a "district selector." This district selector has capacity for 450 working outgoing trunks, the other 50 trunks being used for control purposes. In a small city 450 trunks would be sufficient to reach all points, but in the case of the New York offices 450 outgoing trunks are not sufficient. Accordingly, only a few of the trunk groups outgoing from these offices leave directly from the district selectors. To obtain access to the remaining trunks there are, on every district selector frame, groups of trunks leading to so-called "office selectors." These office selectors are of the panel type and each has a capacity for 450 outgoing trunks.

The path of a call through a district and office selector will now be traced. The district selector starts upward under the control of the sender. As the district selector moves upward, it produces pulses by means of the brushes which slide over the commutator at the top of the selector. These pulses are transmitted back to the sender, and are there counted. When the sender has counted the number of pulses which indicates to it that the district selector has proceeded to the proper position, the sender opens the fundamental circuit to the selector and causes it to stop. This method of controlling the movement of the selector is termed the reverse control method.

The first selection made chooses the set of brushes to be tripped into engagement with the terminals. Assume, as shown in Fig. 18, the desired trunk appears on the second panel from the bottom. Therefore, the district selector is allowed to make two pulses and is then stopped by the sender. The brush-tripping device is thus set in position to trip the second brush, and the selector is started again by a signal from the sender, which operation completes the process of tripping the brush.

The selector now continues upward, making a pulse for every group of trunks which it passes over, until, having reached the desired group, as indicated by the number of pulses counted by the sender, it is again stopped by the sender at the beginning of this group. The selector is now started again, and this time under its own control, hunts for an idle trunk in the group. Busy trunks are grounded on the third or signaling terminals, whereas idle trunks are open. A testing relay, associated with the selector, keeps the selector moving upward until a trunk with an open third wire is found, whereupon the selector stops, makes connection with this trunk, and renders it busy to other selectors by grounding the signaling strip.

This trunk, as indicated in Fig. 18, leads to an office selector. The same process is repeated by the office selector, under control of the sender, to trip first the proper brush, then choose the proper group, and finally to choose an idle trunk in the group. The connection is now extended to an outgoing trunk. The sender still remains attached to the connection, since it must still control the further setting up of the connection.

The sizes of the working trunk groups on district and office selectors can vary from 5 to 90, depending upon the traffic to be handled.

Calls Between Machine Switching Offices. If the call is for a subscriber in a machine switching office it is completed as shown in Fig. 20. This figure shows a diagram of the apparatus used to connect an incoming full mechanical trunk to a subscriber's line, whether this line is in the originating machine switching office or in another which must be reached over interoffice trunks.

The incoming trunk to the machine switching office terminates on an "incoming selector," which is of the type already described. The machine switching office has a capacity for 10,000 numbers, but the incoming selector has capacity of only 500 trunks, so that the same arrangement is employed as on the district selectors; that is, the incoming selector chooses one of a number of other selectors, called "final selectors," which have access to the subscribers' lines. Since each group of final selectors has access to 500 subscribers, 20 groups of finals will be necessary to care for the full 10,000 numbers. On the incoming selector frames, therefore, appear 20 groups of trunks, each group leading to a different frame of final selectors.

The method of selection is the same as described for the district and office selectors; that is, first the incoming selector, under control of the sender in the originating office, trips the proper brush, chooses the proper group, and finally chooses an idle trunk leading to a final selector. The final selector then goes through the process of brush, group, and subscriber's terminal selection. The terminal selection is under the control of the sender which counts line by line in the group of ten, until the desired one is reached. If the called line is idle, it is rung, and the calling subscriber is advised of that fact by hearing the audible ringing signal. If the called line is busy it is not connected, but an intermittent buzz, recognized as the busy signal, is sent back to the calling subscriber. If the called number is that of a P. B. X. having several trunks, the final selector automatically hunts for an idle one. If the final selector, after testing all the P. B. X. trunks finds them all busy, it sends back the busy signal.

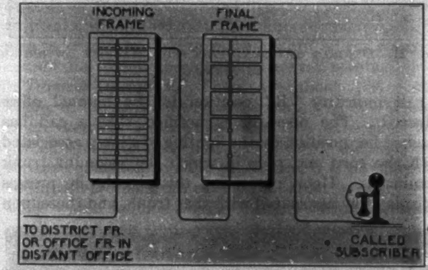


Fig. 20—Diagram of Incoming and Final Frames

As soon as the called line is reached, the sender is dropped from the circuit to be available for another connection. It is not held during the period of ringing, during the time that the busy signal is being given, if the line is busy, or during any part of the period of conversation.

It will be noted that the method of selection is not on a decimal basis. The first selection is to choose one of five brushes on the incoming selector as already explained; that is, we choose that particular fifth of the terminals in which the called line happens to be, and since 1/5 of 10,000 is 2000, we choose the 2000 group desired. The next selection is by groups of 500, which is again non-decimal. This "translation," as it is called, of the number from the decimal notation, as dialed by the subscriber, into the notation as needed by the selectors, is taken care of very simply in the senders.

Calls from Machine Switching to Manual Offices. Calls from machine switching to manual offices are handled at the manual office on call indicator "B" positions. Fig. 21 shows a diagram of the equipment used to connect such a call to a subscriber in the manual office.

The call progresses through the district and office selector in the same manner as described for the machine switching call, but the trunk which it takes up leads to a call indicator "B" position in the manual office selected. The operator is notified that a call has reached her position by the lighting of a lamp associated with the cord and plug in which the incoming trunk terminates. Upon perceiving this signal, she presses a display key associated with that trunk, and thereupon the called subscriber's number is displayed on a bank of numbered lamps located on this operator's keyboard. The operator picks up the plug, tests the called line and, if it is found idle, plugs in; or, if it is found busy, she plugs into a special jack which is arranged to send the intermittent busy tone back to the calling subscriber.

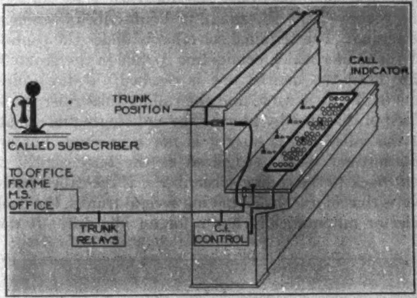


Fig. 21—Diagram of Connection from Machine Switching to Manual

Fig. 22—Incoming Trunk Position in a Manual Office Arranged for Call Indicator Operation

The called subscriber's number is displayed in the following manner. Associated with the operator's position, and with her call indicator, is a group of relays. When the display key is depressed, this group of relays is attached to the trunk. The sender which has meanwhile been waiting on the connection, is thereby given a signal, and sends the number called by means of code pulses which are received by the group of relays. These relays, in turn, light the set of lamps on the call indicator corresponding to the digits of the called number, as shown in Figs. 22 and 23. The code pulses employed for sending this called number are positive and negative, strong and weak, and are translated by the sender from the decimal dial pulses to this type of pulse to reduce the time required and to simplify the receiving apparatus.

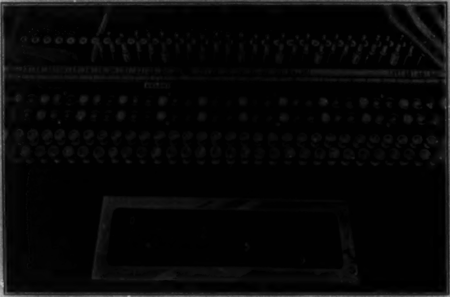


Fig. 23—Call Indicator at an Incoming Trunk Position in a Manual Office

Incoming Calls from Manual to Machine Switching Offices. Calls from manual offices are handled at the machine switching office on the cordless "B" positions. Fig. 24 shows a diagram of the equipment used to connect a call originating in a manual office destined for a subscriber in a machine switching office. Such a call is answered by the "A" operator in the manual office in the usual manner. She takes up the call circuit by depressing her call circuit key to the machine switching office desired, passes the called subscriber's number, and receives a trunk assignment in exactly the same manner as if the call were going to another manual office. The

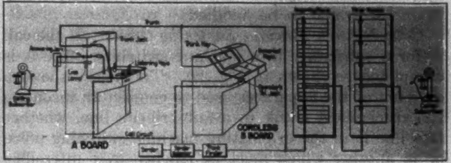


Fig. 24—Diagram of a Connection from a Manual to a Machine Switching Office

cordless "B" operator, upon assigning a trunk, presses the assignment key of that trunk, which temporarily attaches her keyboard to a sender and simultaneously to the incoming trunk which she has assigned. As shown in Fig. 24, the incoming trunk terminates on an incoming selector which has access to final selectors on which the called number appears, in the same manner as described before.

The operator now sets up on her numbered keys the number desired, and this information is transmitted immediately to the sender. These keys, which lock mechanically, are released after a fraction of a second by a magnet controlled by the sender and are ready for the next call. The "B" operator's sender now controls the incoming and final selectors in the same manner as the subscribers' senders, causing the incoming selector to choose an idle trunk to a final selector having access to the desired group of 500 numbers. The final selector reaches its destination in the manner previously described and, as soon as the line is found, the sender is released.

Fig. 25—Cordless "B" Positions in Machine Switching Office

Fig. 25 shows a line of cordless positions. The section at the left is the cable turning section, having nothing to do with the operation of the board.

Manual Positions Required in Machine Switching Offices. While regular calls between two subscribers will be completed in this system without the aid of operators, certain classes of calls, such as toll calls to suburban points and calls for discontinued or changed numbers, etc., will require the assistance of an operator. Special manual positions are therefore provided in the machine switching office for this service. These positions also care for cases where the subscriber may need the assistance of an operator for other reasons than the above, and are in addition to the cordless "B" positions previously described.

The operators are called "Special Service Operators." The subscriber signals them by dialing "Zero," which on the dial is also marked with the word "Operator." The connection then progresses in the same general manner, through the district and office selectors, as for any originating call. An idle trunk appearing on the office selector leading to an answering jack before the special service operator is chosen and the sender released. Should a subscriber in any local service area dial a subscriber in another area, the sender will automatically route the call to a special service operator.

The special service operator in large areas has before her a number of cord circuits having one end terminating in a cord and plug. She also has upon a keyboard a set of keys similar to those described for the cordless "B" position, except that there are additional strips of keys upon which she can write up an office code. The operator answers the subscriber by inserting one of the plugs in the answering jack and, having ascertained the desires of the subscriber, directs the connection to the proper destination by setting up on her keys the proper numerical code. Senders are furnished for these positions so that, as soon as the information from the keyboard has been registered on the sender, the keys are released and are ready for another call.

The other end of the special service operator's cord circuit terminates in a district selector which, either directly or through other selectors, has access not only to trunks which the subscriber himself might call, but also to trunks leading to more distant offices which he cannot dial directly because they are toll points.

Tandem Operation. There are about 158 central offices in the area shown on the map, Fig. 3. While it is an essential requirement that any subscriber connected to any of these offices be able to reach any subscriber connected to any other office, it is obvious that to furnish trunks from each office direct to every other office would require a great number of long trunks in small groups carrying a very light load most of the time.

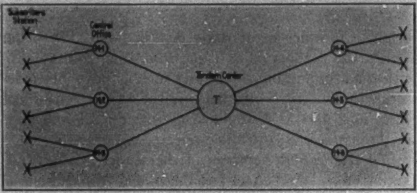


Fig. 26—Typical Tandem Trunking Plan

In order to eliminate the inefficiency that such an arrangement would entail, it has been the practise in manual operation to handle the traffic from one part of the area to another part of the area over main trunk routes. The collecting and distributing points on these trunk routes are known as "tandem centers," and the plan of operation is known as "tandem operation."

Fig. 26 shows an arrangement of offices in a typical tandem trunking plan. Offices marked *M* are local offices, either manual or machine switching. The office marked *T* is a tandem office. If a call is originated by a subscriber in office M-1 for a subscriber in offices

M-4, M-5, or M-6, to which no direct trunks are provided, the call is routed at office M-1 to trunks terminating at tandem office T. At this point they are connected to trunks leading to the proper office, where the connection is completed to the desired subscriber in the usual manner. Likewise, calls from offices M-2 and M-3 are completed over the same groups of trunks rom the tandem office T to offices M-4, M-5, or M-6.

The plan described above is typical of that followed in the New York Metropolitan area for many years, the completion of the call being controlled at the tandem office by operators.

The machine switching system is not only adapted to fit into the existing tandem plan, either when used in the local central office or at the tandem office, but also makes available possibilities for considerably extending the field of usefulness of the tandem system, due to certain advantages in handling calls at tandem points by the use of machinery.

The use of a sender at the machine switching office which is capable of routing a call in any way desired permits locating the selectors which have access to the interoffice trunks at any convenient point either at the originating office or at some distant point. In other words, the tandem office T shown on Fig. 26 may consist of a group of office selectors such as have been described previously. In this case the trunks from offices M-1, M-2 and M-3 would lead from district selectors in these offices to the office selectors at office T which would select, under control of the sender in the originating office, an idle trunk to office M-4, M-5, or M-6, as desired. At the terminating office the call would be completed through incoming and finals if it is a machine switching office, or call indicator "B" positions if it is a manual office, exactly as described previously.

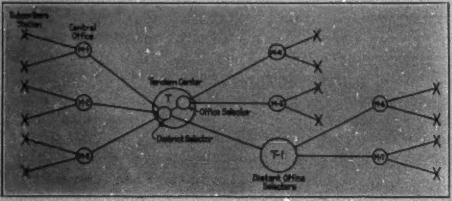


FIG. 27—TANDEM TRUNKING PLAN SHOWING DISTANT OFFICE SELECTOR

If the number of points to be reached through the tandem office is greater than the capacity of a group of office selectors, a group of district selectors may be provided at the tandem office which have access to groups of office selectors located at the same office or at some distant point, as described above.

Fig. 27 shows schematically a tandem plan using the above method. Tandem office T is provided with district selectors on which terminate trunks from local offices M-1, M-2 and M-3. These selectors have access to office selectors in the same office through which offices M-4 and M-5 are reached, and to office selectors located in the distant tandem office T-1 through which offices M-6 and M-7 are reached.

To handle calls at a machine switching tandem office originating from manual offices, operators are required at the tandem office. These operators handle calls in much the same manner as cordless "B" operators in a machine switching office, as already described. The operator receives the desired office name and number from the originating operator over a call circuit and sets it up on her keyboard, which is similar to the cordless "B" board, except that it has office keys in addition to the numerical keys. The number is received by a sender which then controls the operation of the selecting mechanism in the tandem office and other offices through which the call may pass, to the desired local office and subscriber's line.

Many different combinations of the above are possible and are employed when desired.

MAINTENANCE

As will have become apparent from the description already given there is, in the machine switching telephone central office, a large amount of apparatus which, in order to insure service of good quality, must be maintained in proper working condition. Consequently, the subject of maintenance has been very carefully kept in mind throughout the design of the system. For instance, all new pieces of apparatus used in this system have been subjected to the most rigid tests to insure that they will have a satisfactory life and that their margins of adjustment will be adequate.

When maintaining machine switching equipment, the main reliance is placed on preventive measures, so that incipient faults will be detected and corrected before they have got to the point of interfering with service. Ingenious automatic testing arrangements have been designed to aid in this preventive maintenance work. They subject the various circuits in the exchange to routine tests, and are arranged so that they will automatically test all of the circuits, one by one, under conditions more severe than they will ever be called upon to meet in service. In case some feature of any circuit has deteriorated from its normal standard of adjustment—which includes a wide margin—so that it will not meet this severe testing condition, the testing apparatus automatically stops and by supervisory lamps indicate the location of the trouble. An audible alarm is also sounded which notifies the maintenance man responsible that something requiring his attention has been found. The circuit in trouble may still be capable of giving service, but is below the standard set and may soon give service trouble if not corrected.

As applied to the sender, for example, the automatic routine test equipment picks up each sender in turn and puts it through its regular process of operation, under conditions more severe than are encountered in practise. If the sender under test meets the operating conditions without failure, the sender is dropped and the test equipment moves to the next sender. If any trouble develops an alarm is given, which summons the maintenance man who is able to determine by the condition of the apparatus the location of potential trouble.

The operation of the testing equipment may be varied by suitable keys, so that all the features of each sender may be tested once, or so that any one feature of the sender may be tested as many times as desired.

All the equipment in the office occurs in groups, and arrangements are made for readily taking out of service for readjustment any piece of apparatus which may have been found to have potential trouble—the other members of the group continuing to handle the calls.

APPLICATION

In the preceding pages there has been briefly described a switching system which meets the exacting and complex requirements of telephone service in the largest cities and in which, so far as is practicable, the various switching operations are performed automatically. Only such operators are required in connection with this system as are necessary for handling special classes of service and certain operations in connection with the interchange of calls between manual and machine switching central offices during the transition period.

Variations in the arrangements which have been described have been developed and are available for use whenever the conditions warrant. An illustration of this is the so-called key indicator, which permits the handling of calls from manual to machine switching offices without the aid of the cordless "B" operators. This is effected by providing the operators in the manual offices with special keys and equipment for controlling directly the selection of the subscriber's line in the machine switching office.

This machine switching system marks a very important advance in a development which began shortly after the telephone was invented, and which has been most vigorously prosecuted by the engineers of the Bell System from then to the present time. Throughout this entire development period the tendency has been to introduce automatic methods and apparatus whenever they gave a better result to the public, or whenever they were attended by an economy of any kind.

How this system works has been briefly explained. What arrangements are provided for handling regular machine switching calls, calls to and from existing manual offices, private branch exchanges, etc., has been described. How the introduction of this system into a telephone network is affected will now be discussed briefly.

Obviously, the problem of introducing machine switching equipment into such an extensive and complex structure as is the telephone plant of a big city, is a large one. It is impracticable to introduce it all at once. Its introduction must be effected gradually and this is accomplished by using it for growth and such replacements as are necessary, later extending its use as conditions warrant.

The fundamental engineering studies which have to be made and which must precede the manufacture and installation of the equipment for a machine switching office are, in all important respects, the same as those which must precede the manufacture and installation of the equipment in a new manual office. They involve a careful study of the telephone needs of the area, with a view to determining ultimately the quantities of the different kinds of arrangements necessary to give the service. This requires a study of the commercial requirements at the time when the equipment should be cut over and for several years thereafter. Data must be collected as to the probable rates of calling, the average duration of the calls and the amount of trunking to and from other offices.

With these data available, the size and arrangement of the trunk groups on the selector frames, the number, grouping and type of selectors and senders required, and the size of the power plant can be determined. From this the cabling arrangement can be worked out, and suitable floor plans prepared.

Manufacturing specifications can then be prepared in accordance with which the equipment of the office is manufactured and installed. Before the equipment is cut into service, the various arrangements are thoroughly tested individually, and when in proper condition the whole is checked up by making complete operation tests.

If time and space permitted, it would be of interest to discuss the methods of actually cutting the equipment into service, and the comprehensive program which is worked out for the training of the employees who are to handle the equipment and advising the public which is to use it. All these matters are of the utmost importance, and must be carried out systematically in order that there may be no reactions on the general service at the time of the cut-over.

Discussion

Frank B. Jewett: There is very little that I can add to what Mr. Morehouse has outlined to you in the paper except to point out one or two collateral features of this very substantial trend which we are experiencing toward the increased use of machinery to perform functions which ordinarily require human intelligence.

My principal contact with this machine switching problem has been on the engineering and manufacturing side, and the thing that has struck me most forcibly is the fact that we appear to be trying to the best of our ability to make a mechanical human being.

The register to which Mr. Morehouse refers embodies I believe, the nearest approach to human intelligence that has yet been invested in a mechanism. It has some truly remarkable characteristics which heretofore have been associated in our minds largely with the characteristics of human beings.

The attempt to go farther and farther in the direction of making inanimate material take the place of animate human beings and perform their functions not only today but tomorrow and the next day and to perform them economically, has imposed an increasing necessity not only for greater accuracy and care in design but also greater care in the selection of the materials which we employ. In turn these refinements are increasing the pressure on us to enquire deeper and deeper into the fundamental characteristics of materials and into the ultimate construction of matter, in order that we may put into the machine those things that will best serve the purpose for which they are designed and serve them for an indefinite time, and so far as possible be things which will give us a preliminary warning of failure.

If we have a human being in a chain of things which require a human being to be present, and that human being becomes defective, when we replace him by another we replace many functions. When we put the characteristics of a human being into machinery and the machinery becomes defective, it is not the simple process of substituting one biped for another and so replacing many functions, but we have to do it more or less piecemeal. The result is that so far as possible we must obviate the possibility of failures which, if they occur, will greatly complicate and deteriorate our service and at the same time give us no adequate opportunities for immediate replacement. The telephone service of today is a thing which will stand for no material degradation and will tolerate no material delays in replacing parts or defective arrangements.

There is another point which, in passing, may be of interest. In summarizing his paper, Mr. Morehouse has pointed out that the net result ultimately would be a reduced requirement for operators, not necessarily a smaller total than we now have but a smaller total than we might have required had we gone along on some other basis. This does not mean that the total amount of human labor involved is decreased to the same extent that we can reduce the number of operators, because to a certain extent what we are doing by this process of substituting machines for human beings in the making of telephone connections is to transfer human labor from one place to another. In other words, to a certain extent we are transferring the human labor of the operator to the human labor of the factory because the machine switching mechanism increases the total amount of manufacturing effort that is required.

The factor just referred to is something we must take into account when considering the economics of the machine switching problem. The net result of machine switching is likely to be a conservation of human labor in preparing for and in providing telephone service, but insofar as we transfer human labor to the factory to make machines which are more accurate and complicated than the machines of a simpler structure, we have increased the problems of our manufacturing department beyond what they now are.

Bancroft Gherardi: A natural question always arising in the case of a new development is—who did it? People unfamiliar with development work of this kind sometimes think that that question can be answered by naming one or two, or perhaps half a dozen people. The reverse is the fact in this case. This machine switching system which Mr. Morehouse and the other authors of the paper have so ably described, is literally the result of the combined work of thousands of people, of whom I will venture to guess more than 100 are in this room.

Like most successful developments in many respects this is an evolution rather than a revolution. It builds to a considerable extent on what preceded it.

Looking at the pictures of the machine switching board, we note that it does not look much like the manual common battery switchboard; and yet I would like to point out a few of the elements of the common battery switchboard that will be found equally important in the machine switching board.

It employs, of course, the metallic circuit and the twisted pair. It employs the standard, three-wire switchboard arrangement, by which there are two-wire talking wires and a third wire within the central office, used for various signaling purposes. It employs the common battery talking and signaling system; in fact, if you were to take the circuits of a machine switching office and separate from the signaling circuits the talking circuit itself with the battery supply arrangements, you would find it to be identical with the talking circuit of the standard common battery switchboard. You would also find that the line relay and the supervisory relay were present in the machine switching circuits in the same general way, but performing different detailed functions from those in the manual system. I could go on with many other points of this kind but I think I have given you enough to illustrate what I have in mind.

You may be interested in the extent of the application of this system. There have been two of the machine switching offices of the kind described, already placed in service in Omaha. One is in service in Kansas City, and a small one in Newark, also one in Paterson, New Jersey. Three have already been placed in service in New York, and many others are under way in New York and elsewhere.

There is one more point which is of considerable importance in connection with this development. Some of you know perhaps that the lawyers recognize it as a principle of cross-examination that you should never ask a question of a witness unless you already know what the answer will be, or you don't care.

Now, we have somewhat the same situation in experimental work that involves the public. We do not wish, when it is possible to avoid it, to do any experimental work involving the public, unless we already know it is going to come out all right, or we don't care. Now, we always care, so the alternative is that we must practically carry on our development work so that we know before we try to experiment, so far as it is humanly possible to know, that the result will come out as we hope and anticipate.

That has been a difficult matter in connection with this machine switching system, and yet the result has been accomplished. In one way or another, we have been able during the progress of the experimental work, when it got beyond the point that the answer could be obtained in the laboratories, to try out element by element of the system in service, in such a way as to get the answer we wanted, and at the same time, take no chances on the service that was being rendered in the meanwhile.

The result was that when the first complete office went in service in Omaha, we felt a moral certainty that all would go well and everything did go well. In fact, the newspaper and public comments on the following day were practically unanimously favorable.

Fred L. Baer: It has been my pleasure, for the past twenty years, to be connected intimately with the operation and promotion of automatic switching equipment of the step by step type. The function of telephone switching equipment of any kind is essentially that of connecting together subscribers in different parts of an area and then affording them facilities for easy conversation. As Mr. Gherardi has pointed out, in a machine switching system, after the connection has once been set up, the facilities for conversation are virtually the same as in manual practise, with which most of you are familiar.

In the early years our principal endeavor in designing and producing automatic switching equipment was to take care of the large percentage of normal calls such as local city calls. The number of toll calls formed only a small percentage of the total traffic and were handled in each case in the most expedient manner. Later, the general advancement of the art and the more exacting requirements of the telephone using public were met by a corresponding development in the art of automatic switching, so that it can now be safely said that telephone requirements of every nature can be satisfactorily met with machine switching equipment.

The problem that now confronts us is the one usually experienced in the development of any art, *i. e.* having accomplished certain desired results, to find ways and means of effecting economies so that these results may be obtained with the lowest possible expenditure.

Regardless of the question of cost there are some cases where automatic switching equipment is useful in meeting certain definite conditions, for instance, where climatic conditions make the maintenance of an adequate operating staff almost prohibitive, or where there is a polyglot population and it becomes almost impossible to maintain an operating staff sufficiently conversant with the various languages used locally to give satisfactory service.

One of the questions that Mr. Gherardi touched upon, that of the public attitude, was very interesting to me for the reason that I had always made it a point to learn the public reaction to the service, wherever possible. I believe we, in the telephone business, have been unduly apprehensive. While the telephone business is our principal activity, it is only an incidental thing to the public. If machine switching service meets the public requirements in a more satisfactory manner than has previously been done, the reaction is favorable. If it does not, the reaction will be unfavorable. Fortunately, however, I know of no case where this reaction has been unfavorable because of the fact that the service given by machine switching telephone systems is, from many angles, superior to service given manually.

F. J. Chesterman: We in Philadelphia have very similar problems in a smaller way, to the New York problems, and we are engaged at the present moment in preparing for the Sherwood cut-over.

I think we should each of us bear in mind that in introducing developments into a telephone system such as we have in New York and in Philadelphia, it must be done in such a way as not to interfere with the orderly operation of the system. In the plan which has been adopted, with the exception of the subscriber who is directly involved and who has a dial on his own telephone, there is no change in the method of making a telephone call. In other words, whether you are calling a manual or a machine switching subscriber, the procedure is identical in either case. This seems to be the point which has bothered the layman more than any other, and in talking with a public service Commissioner recently, he had the impression that two telephones were required at the subscribers' premises, one for calling machine switching subscribers, and one for calling manual subscribers. If this were true, each subscriber would have to know the type of service which every other subscriber had, and this, in a city such as Philadelphia or Pittsburgh, is, of course, impracticable; so that one of the prime requisites of a machine switching system, which necessarily must be installed a unit at a time, is that each machine switching telephone must be able to communicate, as in the past, with the whole telephone system.

There is another requirement of machine switching, which Mr. Morehouse dwelt upon, and that is, it must be able to handle all classes of calls; not only flat and message rate, but also coin boxes. It must be able to handle calls to toll points, calls for information, and a thousand different types of calls which are encountered in the operation of a complete telephone system. Not only must the machine switching system fulfill these requirements, but it must be such a system that its use will not entail material changes in the method of operating the existing system; I refer now to the manual offices with which the machine switching system must co-ordinate.

It would be obviously a very material problem to train the manual operators to handle connections from machine switching offices in a totally dissimilar manner from that in which they handle calls from other manual offices; and Mr. Morehouse pointed out very distinctly, I think, the pronounced similarity in the method of handling calls from the two types of offices.

In a similar manner, calls from a manual to a machine switching office are handled in practically the same way that calls from one manual office to another manual office are handled. By this means, the effect of machine switching operation is localized practically to the particular unit that we are installing, and does not interfere or change or modify in any material sense the operation of the telephone system as a whole.

I think therefore the machine switching system as described, fulfills all of the main essential requirements that we have been reviewing just now, but in addition to that, it is meeting all of the requirements for a complete telephone system.

E. B. Craft: I am simply going to touch upon some of the problems, that we have had to deal with in connection with this development, pertaining particularly to the mechanical and electrical design of the mechanism itself.

The necessity for large groups of trunks in Metropolitan areas has brought forth the necessity for large-sized selectors, and the panel mechanism which has been illustrated and described, embodies a number of features mechanically that are of general interest, not only to telephone specialists, but to any engineer who has to do with the design and construction of machinery.

Let us consider for a moment just what this mechanism has to do. We have in the selector rod that goes over the face of these panels, a piece of apparatus which is about seven feet long and weighs about a pound and a half. It must be so arranged that it can travel over the face of the terminals at the rate of possibly 60 terminals per second. As these terminals are 1-8 of an inch apart, you can gather some idea of the precision with which this rather substantial mass of material must be controlled. Of course it is not a new thing to select groups of terminals by machinery. The outstanding feature of this mechanism, however, is, that we are dealing with a much larger mass of material which must be controlled, and with much greater distance over which we must travel. This large size precludes operation by the usual forms of electro magnetic devices, and it has been necessary therefore to use continuously-applied power means. This is the reason for using the power drive.

Now, in the introduction of this power drive, we are getting into some new fields in that we must by all means insure continuity of operation. This power drive mechanism, both motors and friction roll drives which are associated with each other, must operate every hour in the day, every day in the week, and every week in the year. Such seemingly simple problems as lubrication have given a great deal of trouble. These mechanisms as I say, must operate continually, and there are real and fundamental problems of lubrication which have entered into the design of them. On the other hand, practically all parts of the mechanism, outside of the power drive itself, must operate without lubrication. Because of the lightness of the construction of some of the smaller fast-moving parts, the application of lubricating materials will oftentimes interfere with their operation, so that we have many problems where the design and the materials must be such that parts will operate for long continued periods of time without any lubrication whatever.

In this connection, it may be interesting to refer to the motor which is used to drive this selective mechanism. In order to insure this continuity of operation, there has been provided a so-called duplex motor for the operation of the lifting drives. This motor is so arranged that it normally operates directly from the usual public service supply, either alternating or direct current. In case of failure of this power supply, however, the motor is so arranged, that without any outside manipulation whatever, it automatically switches to the power leads from a storage battery located in the building, so that its continuous operation is insured. The throw-over from the main source of supply to this emergency source, is such that the motor will not stop and the mechanism is maintained in continuous operation, even though the primary source of power supply is removed.

The terminal bank which is illustrated in the paper and shown on the screen, involved some mechanical problems a little bit out of the ordinary. This rigid structure is about 39 inches long and 15 inches high, made up of 300 strips of metal, with an equal number of insulating strips between them. This whole group of parts must be so assembled that, in its finished state, these terminals are properly centered within dimensional limits of 85/10,000 of an inch. This is done to provide as wide limits as possible in other portions of the mechanism, particularly in the moving parts. Therefore, in the fixed portions, this great precision is adhered to, to reduce to a minimum other adjustments that may have to be made.

Furthermore, these parts must be insulated from each other to withstand unusually high potentials that may occasionally be applied and in order to insure this they are tested with a potential of 500 volts. The proper insulation resistance between the parts must be maintained at a proper point at all times.

Now, in a full-sized central office, such as we have been describing, there may be as many as 4000 of these selecting mechanisms, 4000 of these rods, and in the ultimate plan for the Metropolitan area, it is expected there may be as many as 300 offices, so that we may have over a million of these devices that may be involved in the giving of service in an area such as the Metropolitan district. Each one of these selecting mechanisms must be constructed with sufficient precision that it may be removed, and a new one introduced, or so that any part of the mechanism can be replaced, without recourse to machinery for making the replacement. This presents a problem of interchangeable manufacture which is in a class by itself.

Now, we go to the dial. The only intelligence that is transmitted from the substation in this machine switching system to control the central office mechanism is through the agency of electrical impulses. Therefore, it goes without saying, that these must be very accurately controlled.

The dial which has been described to you is arranged to transmit pulses at the rate of ten per second, with a variation of plus or minus one pulse. There must be a definite relation between the period of make and break of the electric circuit so that this little device located at the substation, which is used to transmit electrical impulses, has a period of make, held between the limits of twenty-five-thousandths of a second, and fifty-thousandths of a second, and a period of break between forty-five-thousandths of a second, and one hundred thousandths of a second. What we really have here at the substation, subject to all the rough usage that devices of this sort usually receive, is a real instrument of precision.

H. P. Charlesworth: There is one matter that comes to mind which is of considerable importance from the subscriber's standpoint and which may be of interest here.

Mr. Chesterman has mentioned that one of the fundamental requirements that a satisfactory machine switching system must meet is that any subscriber shall not be required to perform essentially different operations in reaching different switchboard systems from the same telephone. In other words, the machine switching subscriber must not have to differentiate between a manual telephone and a machine switching telephone and neither must the manual subscriber have to similarly differentiate on his part.

This naturally brings us immediately to the telephone numbering system which is referred to in the paper and perhaps just one word regarding it may be of considerable interest. The automatic system receives a call, of course, from the subscriber through the medium of the dial. The call must come in as a suitable number of electrical impulses. We would, therefore, naturally think of adopting for machine switching subscribers, telephone numbers consisting entirely of numerals, and this plan is in fact followed in small exchanges. However, in the case of New York, for example, this would mean seven numerals, three to control the selection of the desired office and four for the number itself. A letter would also have to be added if a manual party line station was being called.

Such numbers would be very difficult for both the subscriber and operators to use. Furthermore, it is obvious that it would be impracticable, in a large city, to change the form of listing of all the existing manual telephone numbers coincident with the introduction of the first machine switching office. On the other hand, it would also be undesirable to list, in the directory, machine switching subscribers entirely in numerals and manual subscribers in central office names and numerals. It will be seen, therefore, that the subscriber's numbering system to be employed in machine switching operation is an extremely important matter and in fact, for a time presented one of the most difficult problems in connection with large city operation. The solution when found, however, was extremely simple.

You will note from the photographs in the paper that by putting letters on the dial, the subscriber in dialing a letter would, in effect, be dialing a number. For example, in dialing PEN 4256, the subscriber is unconsciously dialing 736, 4256, or in other words, just what the machinery requires to complete the call. Obviously then, by simply capitalizing the first three letters of the office name and selecting satisfactory office names, which usually means the existing office names except where their first three letters conflict, it is very simple to arrange so that the subscriber can use the same form of listing for all calls. Thus the subscriber does not have to know whether the office called is a machine switching office or a manual office which, as previously stated, is the arrangement to be desired when introducing machine switching operation in large Metropolitan areas.

Wind Shielding Between Conductors of Telegraph and Telephone Lines

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An investigation of the action of wind on telegraph and telephone lines is described in the following paper. The results of the tests made during this investigation show that the relation between wind velocity and wind pressure on ice covered conductors conforms closely to the generally accepted formula $P = KV^2$. *They also indicate that the wires of such lines, when coated with ice, shield each other from the wind, both when carried on the same crossarm and on different arms.*

OBSERVATIONS of the behavior of heavy telegraph lines during severe ice and wind storms have shown that such lines frequently withstand more severe ice and wind conditions than would be expected on the basis of theoretical considerations. Such observations have naturally given cause to question the soundness of the methods ordinarily followed in determining the ability of pole lines to resist heavy wind storms, and have indicated that the pressure of the wind on lines carrying many wires is not as great as the calculated pressure on one wire, multiplied by the number of wires on a line. In other words, it has for some time been felt that a shielding effect between wires exists when the wires are covered with ice, and are carried in such numbers and with such close spacing as is common practise on telegraph and telephone lines. In order to obtain information on this subject, a series of tests was conducted during the years 1916 to 1920 inclusive, for the purpose of answering, so far as practicable, the following questions:

(a) Is the formula $P = 0.0025\,V^2$, which is commonly used for calculating wind pressures on cylindrical surfaces, substantially correct when applied to the design of aerial pole and wire lines?

(b) Does shielding exist between telegraph and telephone wires carried on the same and on different crossarms, and to what extent?

The results of these tests were felt to be of such interest to all those concerned with the construction of telegraph and telephone lines, that this paper has been prepared with a view to making the information obtained generally available.

The tests were conducted on the Jersey meadows in the vicinity of Elizabethport, N. J. This location was chosen because the flat and open nature of the land permitted a free sweep of wind, unobstructed by anything which might cause unusual eddy currents. The nearest buildings, hills or depressions, in the direction from which fully 90 per cent of the winds blew were at least five miles away, while the distance in most directions was even greater.

DESCRIPTION OF TESTING EQUIPMENT

In order to carry out the proposed investigation it was necessary to devise a structure which simulated a section of telegraph line and, at the same time, was adapted for accurate measurement of the wind loads imposed upon it. Since the heaviest loading ordinarily considered for determining transverse strength of a pole line is that due to wind acting at right angles to the line on conductors coated with ice ½ in. in radial thickness, it was obvious that actual wires could not satisfactorily be used in carrying out the test. It was finally decided to employ wooden rods, 10 ft. long and 1-1/8 in. in diameter, to serve as the wires of the dummy line. Each rod thus represented a 10-ft. length of wire 1/8 in. in diameter, loaded with ice ½ in. in thickness.

The problem of designing a structure to carry these wires, and at the same time permit accurate measurement of the wind pressure on them, was one that required considerable experimentation before arriving at a satisfactory solution. One of the first features decided upon, however, and one that was retained throughout all tests, was that the wires, subject to a definite restraining force, should be free to move under the influence of the wind, and that the amount of deflection should be used as a measure of the force on the wires. By devising an instrument to record these deflections, and by applying known horizontal forces to the wires and noting the corresponding indication on the recording instrument, it was possible to determine exactly how much pressure on any wire unit was necessary to produce a certain deflection on the recording instrument.

A structure was built in the winter of 1916-1917 which carried all wires in a swinging frame, the motion of this frame being transmitted by mechanical means to recording meters. This arrangement was generally effective, but it proved lacking in refinements and accuracy, so that after constant experimentation during a period of two years, it was replaced by the structure and recording apparatus hereinafter described.

The structure used in the final tests, as shown in Fig. 1, was erected to represent to scale a 10 ft. section of a fifty-wire pole line, pivoted so it could always swing directly into the wind. The entire wire structure was mounted in the framework K, pivoted on ball bearings at F and G, and provided with a vane H.

The wires were spaced 12 in. apart and supported at each end by light steel channels (B, Fig. 2), 10 wires being carried between each pair of channels to represent the load on one crossarm. (For convenience, each complete unit of ten wires and two channels will hereinafter be referred to as a "crossarm"). The crossarms were spaced two feet apart vertically. Each crossarm was mounted independently of the other arms and carried on four hangers (*C*, Fig. 2) equipped with ball bearings at both ends, as shown at *D* in the same figure.

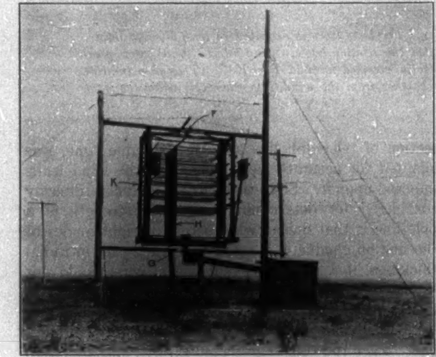

Fig. 1—Wind Test Structure

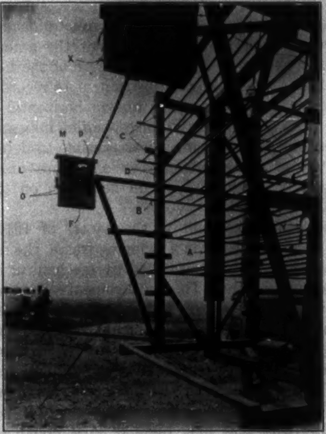

Fig. 2—Arrangement of Equipment on Wind Test Structure

The motion of the crossarms was restrained by springs, attached to two opposite hangers on each arm and to the framework, and adjusted so that all wires would move as nearly as possible in parallel planes.

In order to determine the relation between wind pressure on numbers of wires carried on crossarms and a single isolated wire, a 1-1/8 in. rod, representing one wire, was mounted about five feet in front of the structure and at an elevation approximately midway between the top and bottom crossarms. This single wire was supported in the same manner as the crossarms. The hangers may be seen at *L*, Fig. 2, the ball bearings at *M*, and the restraining springs at *O*, the arrangement of the latter also illustrating the method of attaching restraining springs to the crossarm hangers. The single wire hangers, springs, etc., were boxed in, in order to eliminate the effect of wind pressure on the hanger arms.

The instrument employed for recording the observations is shown in Fig. 3. It will be seen that this consisted of a specially designed recorder containing six independent recording mechanisms and pens, and six charts driven simultaneously by a single, long roller. In order to transmit the deflections of the single wire and the various crossarms to the recording meter, the Bristol electric, long distance, motion recording system was employed. This system operates on the balanced induction principle and consists of two pairs of solenoids, arranged to swing horizontally back and forth over the end of soft iron cores, and connected in parallel to an alternating current circuit. One pair of solenoids is used for transmitting and the other for receiving, any motion of the transmitting coils being followed by a corresponding movement of the receiving coils and of the recording pen.

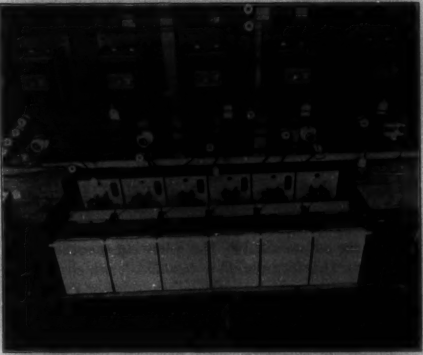

Fig. 3—Apparatus for Recording Crossarms and Wire Deflections

The complete measuring apparatus thus consisted of a transmitter mounted at each end of each crossarm and of the single wire, and of the corresponding recording mechanisms and charts, the recording instrument being arranged for making simultaneous records of the deflections of the single wire and of each of the five crossarms. The transmitters were connected to the hanger arms by means of light metal rods. One single wire transmitter may be seen at *P*, Fig. 2, while those for the crossarms were mounted in two vertical boxes, one of which appears at *A*, in the same figure. The reason for using a transmitter at each end of the single wire and of each arm was to enable the recording instrument to record the average deflection of the wire or arm. By using the transmitters in pairs and cutting out one coil in each transmitter, it was possible to record the average movement of the two ends of the unit and thus compensate for any irregularities which might be caused by slight inaccuracies in adjustment.

The recording apparatus was mounted in a hut placed alongside the structure. The electrical connections between the transmitters and recorder were carried through the hollow pivot at the base of the framework by means of collector rings.

During early experiments, it was found that the records contained certain irregularities which were apparently caused by differences in the periods of the various recorder mechanisms. This difficulty was overcome by damping the motion of the recording pens, and of the arms and single wire, by the use of oil dash pots. By varying the amount of oil in the dash pots, it was possible to adjust all six recording systems so as to have approximately the same period.

In addition to the structure and apparatus described above for measuring wind pressure on a single wire and on ten-wire crossarms, it was also necessary to provide an instrument for indicating wind velocity in order to carry out that part of the investigation directed toward determining the relation between pressure and velocity, as expressed by the Formula $P = K V^2$. Various devices, such as an anemometer, a U-tube, and a Venturi tube, were experimented with for this purpose and the latter instrument finally decided upon as giving the most accurate results. The Venturi tube, connected to a direct reading wind gage, is a type of apparatus used by the United States Government in making tests of wind velocity in connection with aeroplane work. The complete unit, consisting of a Venturi tube and direct reading wind gage, used in this investigation was calibrated in the wind tunnel at the Bristol Company's factory to read actual wind velocities in miles per hour, and was subsequently standardized against a U-tube which was calibrated in the wind tunnels of the Navy Department at Washington, D. C. The Venturi tube was mounted on the test structure immediately alongside the single wire, and may be seen at *X*, in Fig. 2, the associated gage being shown at *Y* in the same figure.

METHOD OF MAKING TESTS AND WORKING UP RESULTS

Preliminary to making any tests, it was of course necessary to calibrate carefully all instruments and equipment. The direct reading wind gage was calibrated as described above. The instruments for recording the deflections of the single wire and crossarms were calibrated by applying known horizontal forces to each of the units. A piece of flexible cord was fastened at each end of the wire or arm to be calibrated, and run over a pulley a few feet distant. Known weights were then suspended from the free end of each cord and the deflections on the corresponding charts of the recording instrument noted. A large number of readings were taken in this manner for each arm and for the single wire, care being taken to perform the work when there was no wind blowing. Similar calibrations were also made under impact loads, in order to make sure that inertia characteristics were equalized as nearly as possible.

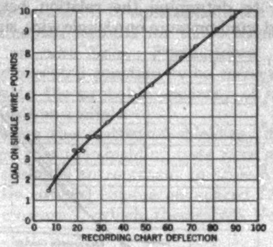

FIG. 4—CALIBRATION CURVE—SINGLE WIRE

In the early experiments, it was found that the best records could not be obtained at low wind velocities without sacrificing considerable accuracy and efficiency in the upper ranges. The recording instruments and wire units, therefore, were so adjusted that the greatest accuracy would be obtained with loads corresponding to actual velocities of approximately 30 miles per hour or over. Sample calibration curves are shown below, Fig. 4 showing the calibration for the single wire, and Fig. 5 for crossarm *C*, the middle arm on the experimental line. (For convenience, the several crossarms are designated by letters *A*, *B*, *C*, *D*, and *E*, reading from the top downward).

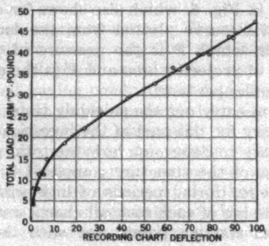

FIG. 5—CALIBRATION CURVE—ARM "C"

Having placed all equipment and apparatus in working condition and having completed all calibrations, it was only necessary to wait for favorable weather conditions, in order to obtain the desired records. The majority of test observations and readings were made during the months of February and March, since it was found that the steadiest and highest winds occurred at that time. All equipment was frequently calibrated during this period so as to prevent any possibility of changed conditions affecting the accuracy of the final results.

In order to determine the relation between wind velocity and wind pressure on a single wire, simultaneous observations were made, during periods of heavy wind, of the readings on the wind velocity gage and those on the chart recording the deflections of the single wire. Numerous tests were conducted in this manner and the velocities as obtained from a great number of readings, plotted against the corresponding pressures on the wire. The results thus obtained are shown by the dot and dash curve in Fig. 6, which also shows for comparison the theoretical curve plotted from the formula $P = 0.0025\,V^2$, in which P is the pressure in pounds per square foot of projected area and V is the actual wind velocity in miles per hour.

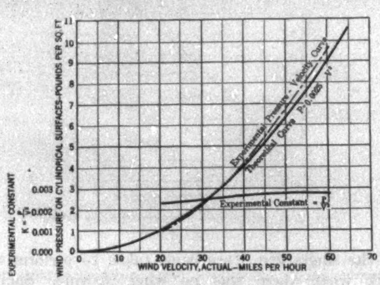

Fig. 6—Pressure Velocity Curve

In order to carry out the shielding tests, and obtain data necessary for this part of the investigation, it was only necessary to determine how the total pressure on all the wires of the structure compared with that on the single wire, during periods of high wind velocity. At the beginning of each test, all charts were carefully marked with starting points and with the name or letter of the units whose motion they were to register. The roller carrying all six charts was then set in motion and the deflections corresponding to simultaneous pressures on all units recorded.

After the completion of each test, the charts were laid out so that the deflections recorded at the same instant by the single wire and the various crossarms could be noted. Points were then chosen at suitable intervals on the single wire deflection curve and the deflections which crossarms recorded at the same instant tabulated opposite these single wire deflections. The following table shows a sample of some of the actual test results obtained in this manner, each value of crossarm deflections being the average of numerous readings.

SAMPLE OF TEST RESULTS
(50 WIRES ON LINE)

Deflection Single Wire	Average Deflection Arm C.
7.5	6.5
12.5	9.0
17.5	12.1
22.5	15.1
27.5	19.0
32.5	23.3
37.5	26.9
42.5	31.7
47.5	35.0
52.5	39.7
57.5	44.0
62.5	49.6
67.5	53.0
87.5	72.0
92.5	78.3

Tests were made in the above manner with all five crossarms in place on the structure. Tests were also made with 40, 30, 20, and 10 wires on the line, one arm of wires being removed just previous to each series of tests, the removal of arms being in the order *A*, *E*, *B*, and finally *D*.

In order to put the data obtained from the above tests in convenient form for study, curves were plotted from the tabulated deflections, the deflections of the crossarms being plotted as ordinates and those of the single wire as abscissas.

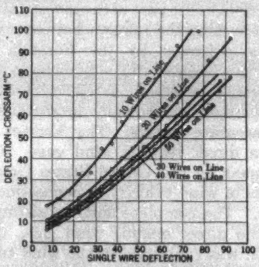

Fig. 7—Arm *C*—Observed Deflections

The lower curve in Fig. 7 is plotted from the data given in the foregoing table and shows the deflections of Arm *C* when there was a total of 50 wires on the line. Similar curves were also plotted for each of the other arms, and in addition, a set of curves for each wire load, 40, 30, 20, and 10 respectively. The complete set of curves for arm *C* in shown in Fig. 7.

By applying the proper calibration curves to the deflection curves above described, it was possible to transform deflections to loads and to obtain new sets of curves showing the direct relation between forces on the several crossarms and that on the single wire. The lower curves in Fig. 8 were obtained in this manner, and show the force of the wind on each crossarm during the test of 50 wires. The upper curve in this figure shows the total load on all 50 wires, and was obtained by adding together the corresponding ordinates for the five ten-wire crossarms. Curves similar to Fig. 8 were made up to show total loads on 40, 30, 20, and 10 wires.

In discussing wind test results, however, it is desirable to refer to pressure in terms of pounds per square foot, rather than to total pressures on any number of given wires and, therefore, these latter curves were again transformed to a new set based on unit pressures. Fig. 9 was obtained by dividing the total loads plotted in Fig. 8 and the other similar curves by the projected area in square feet of the wire or wires on which the pressures were observed. These curves show admirably the relation between the unit pressure on a single wire and that on a number of wires subjected to the same wind velocity. The curves in Fig. 10, which show the actual per cent of shielding at different wind velocities, were next made up by dividing the unit pressure on the line (shielded pressure in Fig. 9) by the unit pressure on the single wire (unshielded pressure in Fig. 9). In order to make the curves more convenient for general use, they were plotted against velocities instead of pressures, the velocity ordinates being determined from the test pressures by the use of the formula $P = 0.0025\ V^2$.

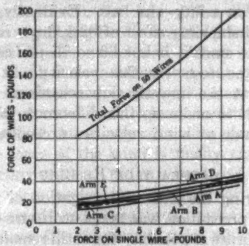

Fig. 8—Wind Load on Crossarms (50 Wires on Line)

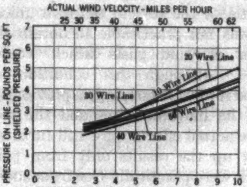

Fig. 9—Shielded and Unshielded Pressures

Discussion of Results

It will be seen that the experimental curve in Fig. 6, plotted from the results of the tests made to determine the relation between wind velocity and wind pressure on a single wire, differs very little from the theoretical curve plotted from the Formula $P = 0.0025\ V^2$. In fact, the experimental curve gives for the constant $K = P / V^2$, a value which averages close to the commonly used value of $K = 0.0025$, no point differing more than 8 per cent from this figure. The results of this test, therefore seem to substantiate the use of the commonly accepted formula for the determination of wind loads on wire lines. Furthermore, this test, assuming the accepted constant to be correct, gives an indication as to the probable accuracy of the results of the wind tests as a whole.

In considering the matter of shielding, it must be borne in mind that the shielding tests were made upon a structure designed to represent to scale a telegraph line having ice covered conductors spaced 12 in. apart on ten-pin crossarms, which themselves were spaced two feet apart, center to center. The results described below, therefore, may only be considered strictly applicable to telegraph or telephone lines loaded as indicated, and having conductor and crossarm characteristics similar to those represented by the test structure. It is, of course, reasonable to suppose that shielding would probably be greater with a closer spacing of conductors or crossarms, or an increased thickness of ice on the wires, and also that it would be smaller with a greater separation or less ice, but the tests were not carried far enough to warrant any definite statements in this respect.

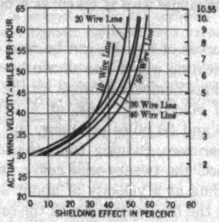

Fig. 10—Variation in Shielding with Wind Velocity

In analyzing the results of the tests, consideration should first be given to the behavior of arm *C*, the middle arm, with various numbers of additional wires on the line. By referring to Fig. 7, it will be noted that the deflection of arm *C*, corresponding to any given deflection of the single wire, decreases as the number of crossarms or wires on the line is increased. Thus the deflection of arm *C* corresponding to a deflection of 60 for the single wire, is 80, with 10 wires on the line, but only 46.5, when the number of wires is increased to 50.

These values of actual comparative deflections, considered entirely without reference to calibrations for wind pressures or velocities, prove unquestionably that the deflection, and consequently the pressure, on one arm decreases as the number of wires on the line increases. In other words, they establish definitely the fact that shielding exists between crossarms.

Next considering the curves shown in Fig. 9, it is possible to gain an idea of the shielding between wires carried on the same crossarm, and also of the amount of shielding under different wind conditions. For example, when the pressure on the single wire amounted to 8 pounds per square foot, the pressure recorded on a line of ten wires was only 4.6 pounds per square foot, demonstrating that there is a very marked shielding effect between wires carried on the same crossarm, as well as between wires carried on different crossarms. Furthermore, corresponding to the pressure of 4.6 pounds per square foot on a ten-wire line, the tests showed a unit pressure of only 3.4 pounds per square foot when there were 50 wires on the line. A similar relation will be found to exist when any other unit pressure on the single wire is taken as a basis of comparison.

From the curves of Fig. 10, can be read directly the per cent of shielding which was shown by the tests to exist with various numbers of wires on the line and at different wind velocities. Thus, at a wind velocity of between 25 and 30 miles per hour, the shielding is apparently near zero, judging from the slope of the curves. In fact, the twenty-wire curve shows a zero value of shielding at a velocity of about 30 miles per hour, although the fifty-wire curve at the same velocity indicates approximately 20 per cent of shielding. The small shielding found at these comparatively high velocities is probably due to the fact that, as previously stated, it was found impracticable to adjust the recording instruments to give accurate results at low wind velocities without sacrificing accuracy in the upper ranges. It will be noted that the shielding increases with the wind velocity, varying from about zero at velocities of from 25 to 30 miles per hour to as high as 59 per cent at velocities of about 62 miles per hour, with fifty wires on the line. Furthermore, it appears that the shielding increases most rapidly between 30 and 40 miles per hour, above which the rate of increase gradually becomes less. Few observations were made at wind velocities higher than 60 miles per hour, but the curves indicate that the shielding does not increase greatly above this velocity. At an actual velocity of 57 miles per hour, which corresponds to the pressure of 8 pounds per square foot that is ordinarily assumed as the condition of heavy loading, the shielding effect varies from a minimum of about 43 per cent on a ten-wire line to approximately 57 per cent on a fifty-wire line.

In designing telegraph and telephone lines for situations of hazard, it has often been the practise in calculating transverse loads to assume a wind pressure of 8 pounds per square foot of projected area on wires covered with ice ½ in. in radial thickness, and then to neglect one-third of the number of wires carried—that is, to assume a shielding effect of 33-1/3 per cent. In the present investigation, tests show, as indicated in Fig. 10, that under a wind velocity of 57 miles per hour, which corresponds to a pressure of 8 pounds per square foot, the shielding effect varies from a minimum of about 43 per cent on a ten-wire line to approximately 57 per cent on a fifty-wire line. Evidently, therefore, the assumed shielding effect of 33-1/3 per cent is very conservative, since it is but 60 to 75 per cent of that shown by the tests for this velocity. It is hardly probable that the investigation contained any errors large enough to make up this difference, especially in the tests made at wind velocities exceeding 40 miles per hour.

ACCURACY OF RESULTS

While during the entire investigation of wind pressures every precaution was taken to insure the results being as accurate as possible, there still remain a number of factors whose influence should not be overlooked.

In the first place, the structure used to support the crossarms was necessarily within such distance of the wires that it may have influenced the wind currents to some extent. Furthermore, there was probably a slight "end effect" at the ends of the wooden rods where they were supported by the thin steel channel crossarms. At the most, however, this latter effect could not have altered the results by more than one per cent since the length of the span was roughly 100 times greater than the diameter of the rods.

Secondly, it was not found possible to obtain exactly similar characteristics in the several recorders nor to make the inertia of the single wire exactly proportional to the inertia of the several ten-wire units. For this reason, it was not practicable to obtain simultaneous observations of wind gusts which would be truly comparative. Although this condition was largely compensated for by oil damping, as previously described, applied to the movable wires and the recording apparatus, the actual effects were still further eliminated by working up the results only from points on the charts which were practically free from sudden changes in wind velocity. It is believed that this procedure, together with the fact that large numbers of points were averaged, practically eliminated all errors due to inertia.

Another condition which possibly influenced the results was that the steel suspension members from which the crossarms hung were not shielded from the wind. On the basis of the relatively small area these hangers presented to the wind, the error from this cause could not have exceeded one per cent and, therefore, was neglected. The hangers supporting the single wire were shielded from the wind and hence were not a source of error.

Another difference between the experimental line and a real line is that in the latter, the swaying of wires and poles causes constant variation in the position of the wires with respect to each other, while in the experimental line, the sags of the wooden rods were more constant, although not entirely uniform. While this characteristic of a real line could not, of course, be duplicated, the wires of the experimental line, nevertheless, assumed positions quite similar to those of a real line, since the natural warping of the wooden rods caused many irregularities in the sags of the different wires.

The above paragraphs discuss the principal sources of error which are known to exist. There is still another respect in which the conditions in a real open wire line could not be simulated in the experimental structure. The inertia of wires heavily loaded with ice tends to absorb much of the energy of sudden gusts of winds, before the force of the wind is finally transmitted to the supporting poles. On the other hand, the swaying of heavy wires under the influence of sudden and variable gusts of wind probably at times causes momentary forces on the supporting poles which may even exceed the direct pressure of the wind at any one instant. These conditions are obviously too complex to be duplicated in a short span line such as used in the present investigation, but their absence from the experimental line could hardly have affected the relation between the wind pressures on the single wire and the multiple wire units, which relation was the primary thing that the investigation was to determine.

It will be seen from the foregoing discussion that such inaccuracies as may have entered into the results of the tests were comparatively small. In fact, the relation between wind pressure and wind velocity was found to be approximately the same as that which other investigators have determined, the maximum difference between the experimental constant K, and the commonly accepted value, being within 8 per cent and the average difference only 3.6 per cent. It therefore seems safe to assume, for practical purposes, that the wind pressures recorded in these tests and the results deduced from them were accurate within 10 per cent.

CONCLUSIONS

The final results of the wind pressure investigation described in this paper may be summarized as follows:

1. The formula $P = 0.0025\,V^2$, in which P is the pressure in pounds per square foot of projected area on cylindrical surfaces, and V is the wind velocity in miles per hour, may be used in calculating wind pressures on *unshielded* ice covered conductors.

2. A definite shielding effect exists between ice covered telegraph and telephone conductors carried on the same crossarm and closely spaced (about 12 in. center to center) as in modern telegraph and telephone practise.

3. A similar shielding effect also exists between such wires carried on different crossarms spaced two feet apart.

4. The amount of shielding on any number of wires increases with the wind velocity, and vice versa.

5. The shielding at any wind velocity varies with the number of wires, increasing as the number of wires increases, and vice versa.

6. On telegraph and telephone lines having conductors coated with ice ½ in. in radial thickness, the total shielding effect at wind velocities corresponding to a pressure of 8 pounds per square foot, even allowing a 10 per cent reduction from the test results in order to compensate for possible inaccuracies, will run from a minimum of 39 per cent on a ten-wire line to as high as 51 per cent on a fifty-wire line.

The author desires particularly to acknowledge his obligation to Mr. R. Leedom, who personally carried out most of the work of the investigation above described, and his appreciation of the valuable assistance rendered by Mr. C. P. Siedler in the preparation of this paper.

Discussion

F. L. Rhodes: Mr. Howe's valuable paper advances our knowledge of a subject which has occupied the attention of engineering investigators for more than three hundred years. Study of the resistance of plates to the motion of fluids against them, goes back to the time of Galileo. Sir Isaac Newton, about 1687, announced the general formula based on theoretical considerations, indicating that the pressure varies with the square of the velocity, that is, $P = K\,V^2$.

Since Newton's time a vast amount of experimental work has been done along two lines. (1) The verification of the second degree relation. (2) The evaluation of the constant. Some investigators have added to the second degree term a small constant and sometimes first and third power terms. Fig. 6 in in Mr. Howe's paper, indicating a slight tendency for the pressures at the higher velocities to exceed those computed from the second degree formula, is interesting in the light of this early work. It is now, however, generally agreed that the equation $P = K\,V^2$ is sufficiently precise for all practical purposes.

Much experimental work has been devoted to the evaluation of the constant in the Newtonian formula. Rouse and Smeaton in 1759, evolved a value for K of substantially 0.0050. The weight of Smeaton's reputation carried this coefficient into use far beyond conditions similar to those from which it was derived, it having been based chiefly upon experiments with windmills. Many subsequent investigators determined values for K, working with plates and solids having small surfaces borne by rotating arms which were whirled rapidly. The methods employed were subject to several sources of error.

Within the past half century more carefully conducted experiments have indicated that a constant of from 0.0040 to 0.0043 more nearly represents the actual relation than the Smeaton constant of 0.0050. These values apply to the pressure on plane surfaces normal to the direction of the wind. As early as 1798 Samuel Vince determined that the resistance offered by the convex front of a hemisphere was only about 0.4 of that offered by its flat base front. Subsequent experiments by Col. Duchemin and others showed that the resistance was much affected by the shape of the front surface. Duchemin made use of a coefficient of 0.6 to give the relation between the pressure on cylindrical surfaces as compared with the pressure on flat surfaces of the same diameter. At the present time a coefficient of about ⅝ is generally used, and applying this to the constants given above for plane surfaces gives results of from 0.0025 to 0.0027, which are closely confirmed by Mr. Howe's experimental results for a single wire.

In considering the precision of the Newtonian formula, it should not be forgotten that, strictly speaking, corrections should be made for temperature and barometric pressure. For example, the pressure decreases about 1 per cent for each 4½ deg. fahr. rise in temperature, and increases about 1 per cent for each 0.36 inch of barometric pressure.

Turning now to the question of shielding, Fidler, writing in 1887 on Bridge Construction, pointed out that when the wind blows upon a grating or lattice girder, the pressure on the grating or girder is somewhere between that of the entire area enclosed by its perimeter and that of the actual front surface of the bars or plates that compose it, and the pressure on a second girder in the rear of the first would usually be less than that on the first. He suggested the need of practical tests and stated that it is very probable that the amount of shelter varies somewhat with the shape and arrangement of the lattice bars, and very certain that it varies very greatly with the distance between the girders.

Quantitative results as to shielding have been conflicting. For example: Baker, in connection with the Forth Bridge Investigation between 1884 and 1890, found that if two similar disks were placed exactly in the rear of each other, but at distances apart of 1, 2, 3 and 4 diameters, the total pressure on the combination amounted respectively to 1.0; 1.4; 1.6 and 1.8 times the pressure on the front disk alone, and that the total pressure was but little if any increased by the insertion of intermediate disks. On the other hand, Thibault had found as much as 1.7 pressures at single diameter distances.

The allowance for shielding effect has apparently gradually crystallized into the use of one and one-half times the projected area of latticed structures.

It has long been known that some shielding effect occurs in the case of ice loaded wires on crossarms, and, as pointed out in Mr. Howe's paper, conservative assumptions have been made in the past to take account of this effect. It is gratifying to have available this data from actual measurements conducted on a full scale section representative of an actual wire line.

The author has called attention to a factor which is not ordinarily taken into account directly in engineering computations of wire lines, namely the inertia and the swaying of the ice loaded wires. Another factor is the swaying of the poles themselves, which sometimes, when the ground is not frozen, results in enlarging the pole hole transversely to the line, so that considerable motion of the poles and wires may result. There is also, as shown by Langley, the fact that, particularly in high winds, the air moves in a tumultuous mass, the velocity at a single fixed point sometimes jumping almost instantaneously from one extreme to the other. The variables involved in wire line engineering are many and their relations are complex. The mathematical treatment of the problems of design is of value in developing clearly the effects of changes in the various important factors involved thus acting as a guide to progress, and in smoothing out inconsistencies that would otherwise be liable to occur in the case of lines having widely different characteristics.

The final basis of the design must rest on the results of actual experience as reflected in the evaluation of the constants appearing in the mathematical relations. In this respect the author's paper contributes information of importance to all those who are interested in the design of pole lines carrying closely spaced wires.

E. C. Keenan: I am obliged to differ with Mr. Howe's conclusion that there is a shielding effect between wires, which will run from a minimum of 39 per cent on a 10-wire line to as high as 51 per cent on a 50-wire line.

On account of many failures of pole lines and wires along the railroads that I represent, which failures have seriously interfered with train operation, I have been carefully studying the question of proper strength of poles to safely carry our wires. The wind shielding effect of wires has been offered, upon several occasions, as a reason for lessening the initial strength in the pole line structure, but, after investigation and mature deliberation, we have rejected this assumption as unwarranted by such facts as are known and the experience we have had.

My principal reason for not accepting Mr. Howe's conclusion is based upon the fact that, in my judgment, the tests with the model conducted by Mr. Howe, did not represent conditions as they actually are found in the field. The model was designed to simulate a 10-foot section of an actual pole line, the wires being represented by wooden rods 1⅛ in. in diameter, so that these rods would represent a wire with a coating of ice ½ in. thick. The model, as constructed, maintains the wooden rods, or wires, on practically an even plane. While there may have been some slight irregularity, due to the warping of the rods, it seems to me that such irregularity cannot compare with the irregularity that exists in the wires as actually maintained.

In order to determine whether or not telegraph and telephone wires are maintained so that the wires of each crossarm are on practically a horizontal plane, I had measurements taken at sixteen places along the railroad on railroad wires and wires of other companies. These tests include leads carrying from three to seven arms of wires. I give below five examples. The figures give the distances in inches of each wire from an assumed horizontal datum line.

Case No. 1—*1st Crossarm*—0, 2½, 1¾, 4¼, 4½, 5, 10½, 6;
Case No. 2—*2d Crossarm*—6, 5½, 5, 3½, 6, 2, 1, 0;
Case No. 3—*3rd Crossarm*—½, 0, 2¼, 2¾, 5⅜, 7½, 6¾, 7½;
Case No. 4—*4th Crossarm*—3½, 2½, ½, 12½, 11½, 2½, 4½, 0, 1, 2½;
Case No. 5—*5th Crossarm*—5, 1½, 7½, 0, 1½, 0, 10, 6½, 9½, 12½.

At the other eleven places where tests were made, the results were similar to the first three cases, the last two cases being the worst found.

From these tests of the actual conditions in the field, it seems to me that very little shielding could result from wires that are, of necessity, maintained in such irregular positions.

While it may be said that the cases for which measurements are given, showing great irregularity in the sag of wires, do not represent average conditions, I desire to assure you that I have observed this condition on the ground, and I am satisfied that the sixteen tests represent the conditions that exist to a considerable extent.

The wires in Mr. Howe's model were non-flexible as compared with wires actually in service, and due to the shortness of the model span, the model wires, it seems, could not have been subject to such swaying motions as occur in actual practise in spans from 100 to 150 feet in length. Such swaying motions as occur in actual practise, undoubtedly cause the wires to deviate from the plane, thus exposing more surface to wind pressure.

In actual practise, in addition to the swaying effect, there is vibration effect, the wires doing what is commonly known in the field as "dancing," or vibrating up and down. Under these conditions the wires are very far from being on the same plane and, therefore, afford little, if any, shielding.

The supporting structure in the model does not simulate a pole support in that a pole in actual service sways considerably and, consequently, there is a general swaying effect of both poles and wires caused by gusts of wind.

The model used by Mr. Howe does not represent the actual condition of a pole line constructed on *curves* where the poles are inclined from the vertical position, and the crossarms, consequently, are not horizontal, so that the wires are not on a horizontal plane.

The above conditions seem to me to be most important, as they indicate wide variations as encountered in practise from those of the model.

In the second sentence of the first paragraph of Mr. Howe's paper, the statement is made that some observation indicates that the pressure of the wind on lines carrying many wires, is not as great as the calculated pressure on one wire multiplied by the number of wires on the line, from which the conclusion is drawn that the reason the lines did not fail was due to the shielding effect of one conductor upon another. This, it seems to me, is a supposition, and after thoroughly investigating, it might be found that the apparent greater strength of the line was due to the conditions that were not as severe as anticipated, rather than to the shielding effect.

In the last paragraph of the first column, attention is called by Mr. Howe to the fact that the tests were made in the Jersey meadows on account of the flat and open nature of the land, permitting a free sweep of the wind, unobstructed by anything that might cause unusually heavy currents, the nearest buildings, hills and depressions in the direction from which 90 per cent of the wind blows, being at least five miles away. In this connection I should like to call attention to the observations of Dr. Humphreys, Professor of Interior, Meteorological Physics, U. S. Weather Bureau, who says:

> "Near the surface of the earth the wind is always in a turmoil owing to friction and to obstacles of all kinds that interfere with the free flow of the lower layers of the atmosphere and thereby allow the next higher layers to plunge forward in irregular fits, swirls and gusts with all sorts of irregular velocities and in every direction. Indeed, the actual velocity of the wind near the surface of the earth often and abruptly varies from second to second by more than its full average value, and the greater the average velocity, the greater, in approximately the same ratio, are the irregularities or differences in the successive momentary velocities."

From the above, which is generally admitted by those who have studied the freakish and unusual action of the wind, it appears that actual conditions generally prevailing along telegraph and telephone rights of way, are more severe than in the flat, open country where the tests were made.

On page 32 of the report attention is called to the inaccuracies of the recording instruments. It would be interesting, if it were practicable to determine, the extent of these inaccuracies, because on page 25, second paragraph of the first column, the report leads me to think that it probably was considerable, as, apparently, no shielding was noticed at a velocity of 30 miles per hour, which is a rather high velocity; and then for the shielding to suddenly jump to 50 per cent at 50 miles per hour, leads me to believe that inaccuracies in the recording instruments which were noted, may have been considerably more than Mr. Howe thinks.

Attention is called to the fact that it has often been the practise in calculating transverse loads, to assume a shielding of 33⅓%. While this may have been done in certain instances, I do not believe that the practise has generally been followed by engineers, because there is not sufficient engineering basis for such assumption.

In the same column, second paragraph, "Accuracy of Results," attention is called to the effect of the structure itself upon the shielding. While the small steel channel supporting the wooden rods could have very little effect, it occurs to me that the frame itself, which is rather large and exposes quite a surface, would produce a very appreciable effect upon the wires. It would also seem that at high wind velocities if the frame happened to be rotated so as to be in the back of the supporting structure, considerable variation might be expected.

In conclusion I desire to say that, in my opinion, telegraph and telephone pole lines and wires should be constructed under the full, proper assumed loading of wind and ice for the territory in which they are located, with a factor of safety of, at least, two. Under these assumptions many variable factors must be taken into consideration, such as strengths of woods and wire, wind velocities, thickness of ice-coating, etc. The prevailing engineering, fundamental principles that have so far been followed after many years' experience, indicate that no lessening of the assumptions should be permitted unless one is absolutely sure of them, which is especially true when one considers the small factor of safety employed in our telegraph and telephone pole line and wire work.

K. L. Wilkinson: The design of wood pole lines has always been more or less unsatisfactory from the standpoint of accuracy of calculations and the designer has had to make certain assumptions which were based on experience with overhead line construction and not on any scientifically proven physical factors. The transverse load on the line has been taken as a steady and uniform wind pressure acting equally on all wires and the pole surfaces. No allowance was made directly for impact, inertia, shielding, etc., but a rough allowance was made for these factors by using factors of safety for wood poles considerably lower than those usually assumed for wooden bridges, building and the like. Experience has shown this procedure to be justified because for many years it has been observed that certain wood pole lines stand up under storm conditions which, according to the theoretical design formulas, should cause the lines to fail. This has been widely observed particularly in telephone and telegraph lines with their large numbers of closely spaced wires and has been taken into account in their design.

The question of wind shielding, as applied to pole lines, has in the past been the subject of occasional controversy between engineers, not because its existence was doubted but because it had not been demonstrated and measured. Mr. Howe's paper is therefore particularly valuable to those interested in overhead lines construction, first because it proves that wind shielding is an important factor and second because it indicates the quantitative limits of this factor.

Shielding of one body by another in streams of fluid pressure has been a subject of consideration for many years and has been taken into account in the design of steel bridges and similar latticed structures. Experiments have been made from time to time in wind tunnels and these have yielded many valuable indications on this subject. In recent years it has been necessary to carefully study wind pressures in connection with the design of aircraft and this has yielded information which has indicated how currents of air moving at high velocities are broken up by objects of different sizes and shapes. All of these experiments and tests have tended to show that parallel objects in a stream of air do produce a certain shielding effect on each other and that this shielding depends upon the velocity of the stream of air and upon the ratio of the size of the objects to the distance between them.

I think it can therefore be said, without detracting in any way from the value of Mr. Howe's paper that he has not presented a new subject. He has, however, admirably demonstrated in a practical and scientific way the factor of wind shielding in telegraph and telephone lines and has established beyond reasonable doubt that this factor should be considered in the design of such lines.

P. J. Howe: The presentation by Mr. Rhodes of such various data relative to prior investigations of wind pressures constitutes, in effect, a comprehensive introduction to the writer's own paper on this subject. The close conformity existing between the fundamental principles previously established and the facts arrived at through the recent experiments goes far toward indicating that the design of wire carrying structures cannot have a proper engineering basis until some allowance is made for the shielding effect which closely spaced wires exert on each other.

Similarly, Mr. Wilkinson's statement, obviously based on the experience of a great telephone system, that wood pole lines stand up under storm conditions which, according to the theoretical design formulas, should cause the lines to fail, shows that there is some factor, such as shielding or the equivalent, that serves to lessen the effect of wind pressures on pole lines carrying closely spaced wires.

In his discussion of the subject on the other hand, Mr. Keenan disputes the conclusion that there is a shielding effect between wires. Unfortunately, his remarks give few engineering facts to support the many personal opinions that are expressed and in consequence, his discussion serves principally to indicate that the presence of shielding between wires is not agreeable to the speaker. In order, however, that the Institute records may contain both sides of the various questions which have been raised, it seems desirable to make a brief answer to at least a few of the assertions made by Mr. Keenan.

In the first place, he states that the question of proper strength of poles has been carefully studied by him on account of the many failures of pole lines and wires that have occurred along the railroads which he represents and that in view of the facts obtained from such experiences, he has rejected the assumption regarding shielding. The truth of the situation is that pole lines and wires have failed many times on account of inadequate strength or maintenance, but no facts were submitted by Mr. Keenan or have ever been obtainable by the writer to show the failure from ice and wind storm of a line which theoretically ought not to have failed. The fact that lines have failed in the past is no reason whatever why shielding may or may not exist between wires.

Frequent reference is made by the speaker to the fact that the wooden rods which represented wires in the experimental structure were practically on an even plane, whereas actual wires on a pole line occupy irregular positions and vibrate up and down during high winds; also that wires on curves depart considerably from a horizontal level, due to the fact that the poles on curves are inclined from the vertical. He further asserts that wires which are very far from being on the same plane afford little if any shielding.

Whether the plane of the wires in the experimental structure was horizontal or not really makes very little difference. According to Mr. Keenan's own quotation from Dr. Humphreys of the United States Weather Bureau, the wind near the surface of the earth is in a turmoil, full of gusts, swirls and varying velocities, and hence must be considered as blowing from all directions, inclined as well as horizontal. The investigation made by the writer was made out of doors under actual out of door conditions and the results, therefore, apply to the conditions that exist during the extreme disturbances of heavy wind storms and gales. Furthermore, not only did the normal level of the test wires vary considerably, as will appear from Fig. 1 of the paper, but when subjected to the forces of 50 to 60 mile per hour gales, these wires or rods fairly danced and oscillated, much as do the wires in actual pole lines. The implication that wires must be in the same plane in order to afford shielding has little justification, in view of the findings of these experiments.

Another point made by Mr. Keenan was that, according to actual measurement, the wires on individual cross-arms vary all the way from 0 to as much as 12½ in. from a horizontal level. These figures undoubtedly refer to very special cases of lines which are either of small importance or in need of repair. To believe that they are at all representative of actual conditions on a great railway would be a sad reflection on those who are responsible for the maintenance of the railroads' communication system. As a matter of fact, anyone who has traveled the great railroad systems of this country knows that mile after mile of the wires on the paralleling telegraph lines flash by with seldom a noticeable deviation from the uniform level of the wires carried on the various crossarms.

In commenting on the quotation from Dr. Humphreys, Mr. Keenan reaches the conclusion that conditions in the open country where the tests were made are not as severe as those along telegraph and telephone rights of way. Just why this might be so is not evident. Certainly human experience associates the highest winds with localities where the wind has a free sweep, unimpeded and unobstructed. In any event, results of the tests are applicable to either situation since the actual wind pressures and velocities were measured and the conclusions based on these measurements.

In discussing the possible sources of error in the tests, Mr. Keenan thinks that because no shielding was noticed at velocities much below 30 miles per hour, the inaccuracies in the measuring instruments may have been more than the writer thinks. The exact accuracy or inaccuracy of the tests is, of course, indeterminable, but it must be remembered that shielding from one object to another results solely from air disturbances that extend back of the first object, and that the distance which such disturbances extend depends on the size and shape of the first object and the velocity of the wind. It is not to be expected that any shielding whatever will be experienced until the wind attains a high enough velocity for the disturbance to reach from one wire to its neighbor. This limiting velocity is shown by the curves of Fig. 10 to range between 25 and 30 miles per hour for wires of the size and spacing employed in the tests. It would undoubtedly be considerably lower, however, with larger wires or closer spacing, and higher with smaller wires or wider spacing.

Another of the writer's statements which is questioned, is "that it has often been the practise in calculating transverse loads, to assume a shielding of 33⅓ per cent," Mr. Keenan stating that there is not sufficient engineering basis for such an assumption. The statement questioned said "it has often been the practise in calculating transverse loads . . . to neglect ⅓ of the number of wires carried—that is, to assume a shielding effect of 33½ per cent." This neglecting of ⅓ of the wires is prescribed not only in the National Electrical Safety Code issued in 1920 by the Bureau if Standards, but also in mandatory regulations, mostly of more recent date, that have been drawn up and put into effect by the Public Utility Commissions of various states such as Ohio, North Dakota, Wisconsin, Kansas and Oklahoma. The engineering basis for the assumption that shielding exists seems to be thoroughly established, as pointed out in the discussions of both Mr. Rhodes and Mr. Wilkinson, by the long continued practise of structural engineers in allowing a considerable reduction in the theoretical area of latticed structures exposed to wind pressures. The ratio between the separation of members and their size is often much greater in latticed towers and structures than in the experimental line used in the wind tests.

One of the final points raised by Mr. Keenan is that the structure which supported the experimental pole line would produce a very appreciable effect upon the wires. This, of course, is a possibility which is difficult to appraise. The main point to keep in mind, however, is that this effect, whatever it may have been, doubtless affected all wires and, therefore, should have exerted only slight influence on the relative pressures on the different wires and the shielding of the wires, one to another.

Mr. Keenan's remaining criticisms are for the most part either immaterial or already answered in the Paper under discussion. His final remarks relative to proper strengths and factors of safety for telegraph and telephone pole line construction have no bearing whatever on the matter of shielding and need no answer.

The Wave Antenna

A New Type of Highly Directive Antenna

BY HAROLD H. BEVERAGE, CHESTER W. RICE, and EDWARD W. KELLOGG
of the Radio Corporation of America — Assoc. A. I. E. E. of the General Electric Co. — Assoc. A. I. E. E. of the General Electric Co.

Review of the Subject.—*A small bungalow in a grove of oak trees just outside of Riverhead, Long Island, with a line of poles along a country road, carrying two copper wires, and ending by a stream nine miles southwest of Riverhead,—this in brief describes the Atlantic coast "ear" of the Radio Corporation of America, where the wireless messages from England, France, Germany and Norway are received, disentangled, amplified, converted into current of telephonic frequency and automatically relayed over telephone circuits to the Broad St. Office in New York, where operators take the messages by ear, or automatic recorders mark the dots and dashes on tape.*

The present paper deals with the two copper wires on the line of poles, for they constitute the wave antenna which has not only marked a distinct advance in the reduction of interference and "static," but because of its aperiodic nature and effectiveness as an energy collector, has made possible the sumultaneous reception of a large number of messages by one antenna, and the automatic relaying of the messages over land wires.

The use of two wires is not an essential feature of the wave antenna but permits flexibility in the location of the receiving station. In its elementary form the wave antenna consists of a straight horizontal conductor. (See Fig. 10) of the order of a wave length long, parallel to the direction of propagation of the desired signal, with the receiving circuit located at the end farthest from the sending station and with the end nearest the sending station grounded through a resistance of the proper value to practically prevent reflections. Under these conditions the desired signal waves produce comparatively feeble currents at the end nearest the sending station and strong currents at the receiver end, while disturbances coming from the opposite direction cause feeble currents at the receiver end and strong currents at the end farthest from the receiver (nearest the transmitting station) (See Fig. 2). This comparative immunity of the receiving set to disturbances coming from a direction opposite to the desired signal is lost if reflections are permitted to occur at the end farthest from the receiver. The growth of current in the direction of travel of the space wave depends on the velocity of propagation of waves on the antenna in comparison with the velocity of the space waves, the received current being strongest if the two are equal. If the characteristic wave velocity on the antenna is less than that of the space waves (or less than the velocity of light) increasing the length of the antenna increases the received current up to a certain point, after which further increase in antenna length reduces the received current. The length for maximum signal depends on the velocity ratio and wave length. The slower the antenna or the shorter the wave length received the shorter the length for maximum signal. It is very frequently the case, however, that the best directive properties are obtained with an antenna longer than that which gives the strongest signal.

The effect of the space wave is to produce in the wire a signal frequency electromotive force which affects the different parts of the antenna progressively as the space wave passes over the line. On this basis the received current can be calculated in terms of the frequency and intensity of the induced electromotive force, the direction of the space waves and the length and electrical constants of the antenna. By assuming the direction to be changed while all other factors remain the same, and calculating the relative value of received current for various directions of signal wave, we can determine the directive properties of the antenna. The result is best shown by means of a polar directive curve. For each assumed direction for which the received current has been calculated, a radius is drawn, with length proportional to the received current. The curve drawn through the ends of these radii is known as the directive curve for the antenna. Directive curves are given (Figs. 35 to 41) which bring out the effects of antenna length, relative to the wave length, velocity of propagation, and line attenuation. The directive curves are for the most part drawn with the maximum radius taken as unit length, since this makes comparisons of directive curves easier. In general, it is found that moderate line losses are not appreciably detrimental to the directive properties of the antennas, while the fact that velocities obtainable with unloaded lines are materially below that of light, results in an actual improvement in directive properties in most cases. As a rule the longer the antenna the sharper its directive curve. While it is possible to obtain fair directive properties with antennas less than a half wave length long, this length is considered about the shortest that can be recommended.

By a process of balancing, it is possible to produce a "blind spot" or direction of zero reception, at any angle more than 90 deg. from the signal. One method of obtaining this result is by producing reflections of certain phase and intensity at the end opposite to the receiver. Reflections at the receiver end of the antenna, on the other hand, do not alter the directive properties of the antenna.

Experimental work thus far has given a qualitative check on the theory and calculations of the wave antenna, and it is hoped that further observations and measurements will shortly be made. Experimental data on wave front tilt, on which the action of the wave antenna depends, is especially meagre.

Data on wave velocity and line losses on an existing antenna can be obtained by means of a radio frequency oscillator and one or two hot-wire milliammeters. Measurements taken by the writers show much higher attenuation and lower velocities for ground return circuits than for metallic circuits. Ground resistance explains this effect. The mean depth of return currents at the longer radio wave lengths appear to be of the order of several hundred feet. The more wires in multiple in the antenna, the lower the velocity and the higher the rate of attenuation (See Fig. 66 and 68).

Reduction of atmospheric disturbances or "static", has probably received more attention from experimenters than any other one phase of radio reception. Of the various lines of attack none has been more fruitful than the employment of directive receiving systems. Every increase in directivity has resulted in an improvement in stray ratio. The wave antenna carries the principle farther than any previous type. Conditions on the eastern coast of North America are especially favorable for taking advantage of differences in direction, for the European stations are to the northeast while the predominating direction of static is from the southwest.

Various practical engineering problems in connection with the wave antenna, including its application to short wave reception, are discussed toward the close of the paper.

HISTORICAL

THE wave antenna is a long horizontal antenna and therefore belongs to the class of antennas usually referred to as ground antennas, of which it is an outgrowth.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

The first work with long horizontal antennas appears to have been done in the pioneer days of radio by Marconi[1], Braun[2], Secher[3]. A short historical sketch

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1. Marconi, English Patent No. 12039, 1896.
2. F. Braun, D. R. P. No. 115081, 1898.
3. E. Secher, *Phy. Ztschr.* 4, 320, 1903.

of ground antenna work is given by Zehnder[4] who was also an early worker in this field.

More recently Kiebitz[5] has studied the transmission and reception using certain forms of ground antennae. A discussion of the operation has been given by Burstyn[6]. Further experiments and comments on the operation of ground antennae are given by Kiebitz, Burstyn, Hansrath, Mosler[7].

In this country early work was done by Clark, Rogers and Taylor. This work has been recorded in the classical papers by Taylor[8].

Alexanderson's[9] barrage receiver made use of ground antennae and may be considered as the starting point of our work.

The capabilities of the wave antenna were discovered through work done by Beverage in studying the properties of long ground antennas, of the order of a half wave length or more long, in which he discovered that under certain circumstances they showed marked unidirectional properties. One of his antennas consisted of a No. 14 B & S rubber covered wire approximately six miles long laid on the scrub oak and sand of Long Island from Eastport to a point near Riverhead. This northeasterly direction was chosen in order to best receive the European stations. The antenna is pictured diagramatically in Figure 1.

With the receiving set connected between the antenna and ground at the Eastport end and with the Riverhead end grounded, Beverage observed strong signals from Europe, but when the conditions were reversed and the receiver was inserted at Riverhead the European signals were very weak and the static and stations to the southwest were strong. This marked unidirectional property of the antenna was found only when the end opposite to the receiver was grounded.

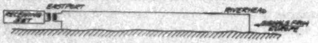

FIG. 1

Beverage next investigated the effect of antenna length by listening in series with the antenna at different points along the antenna. The receiver was mounted in a Ford truck and trips were made back and forth along the antenna, stopping to listen at approximately every mile. By this process the observations given in Fig. 2 were obtained.

In commenting on these observations Beverage's log reads - - - - - "It is evident that the antenna is too long for maximum reception at the S. W. end. It may be due to the fact that the velocity of the current in the wire is considerably slower than the velocity of light, and the currents from remote parts of the antenna lag in phase behind currents from parts of the antenna near the receiver. If so, condensers, distributed along the line, could be used to increase the velocity of currents flowing in the wire and make the velocity the same as the velocity of light." By inserting series condensers in the line Beverage found that he could shift the location of the points at which the signal maxima occurred and thus utilize longer antennas.

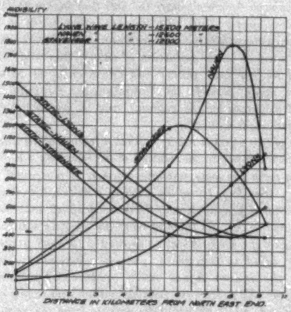

FIG. 2—GROWTH OF SIGNAL CURRENTS IN LONG HORIZONTAL ANTENNA

At about this point in the work Mr. Alexanderson called Rice's attention to Beverage's important discovery. Rice and Kellogg, before learning of the results of Beverage's work, had concluded on purely theoretical grounds that a long horizontal antenna, pointing in the direction of a sending station, would constitute a unidirectional receiver provided the receiver were placed at the end of the antenna remote from the sending station, while the end nearest the transmitting station was grounded through a non-inductive resistance, equal to the surge impedance of the antenna. If open or dead grounded at the end nearest the transmitter, reflections would occur, and the antenna should show bidirectional properties. Rice explained this theory of the operation to Mr. Alexanderson, suggesting that the necessary damping required to make the antenna unidirectional was due to the resistance of the ground connections combined with the high attenuation to be expected in an antenna of No. 14 B & S rubber covered wire lying on the sandy soil and bushes.

At Mr. Alexanderson's request, Rice joined Beverage at Eastport, and a study was made of the properties of the rubber covered lines by means of an oscillator. They also made listening tests which indicated that some resistance, in addition to the ground resistance, was best for unidirectional effects.

The use of the oscillator made it possible to determine the characteristics of the lines with considerable accuracy. Two methods were followed. The first consisted in measuring the input current to the line as the frequency is varied. This gives a series of maxima and minima as the line passes through successive modes of oscillation. If the far end is open, current maxima will occur with 1/4, 3/4, 5/4, etc. standing waves on the line, while if the far end is grounded the current maxima correspond to an even number of quarter wave-lengths. From this data the velocity on the line may be computed.

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[4] L. Zehnder, *Jahrb. d. drahtl Tele.*, Vol. 5, 1911, p. 594.

[5] F. Kiebitz, *Jahrb. d. drahtl Tele.*, Vol. 5, 1911, p. 349, Vol. 6, 1912 p. 1 and p. 554. Also *Electrician*, Vol. LXII, p. 972 and Vol. 68, 1912 p. 868, 936, 978, 1020.

[6] W. Burstyn, *Jahrb. d. drahtl Tele.*, Vol. 6, 1912, p. 10 and 333.

[7] *Jahrb d. drahtl Tele.*, Vol. 6, 1912, p. 359 and p. 570.

[8] A. Hoyt Taylor, I. R. E., Vol. 7, 1919, p. 337 and p. 559.

[9] E. F. W. Alexanderson, I. R. E., Vol. 7, 1919, p. 363.

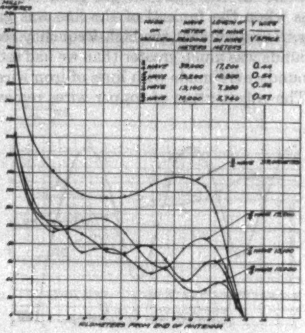

Fig. 3—Oscillator Tests of Rubber Covered Antenna Wire. Current Distribution with Far End Open

The second method of testing consisted in supplying the line with a constant current from the oscillator and measuring the current at intervals along the line with a hot-wire milliammeter. Figs. 3 and 4 show typical curves obtained in this manner. The high attenuation and low velocity characteristic of lines of this type were brought out, and the irregularities of the current distribution curves showed the effects of changes in line constants which would be expected in view of the fact that portions of the line lay on practically bare ground, and other parts were well above ground.

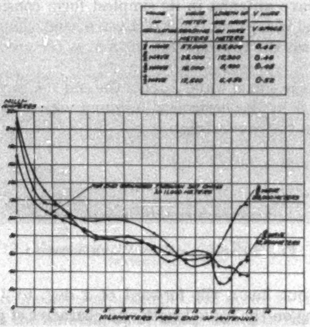

Fig. 4—Oscillation Tests of Rubber Covered Antenna Wire. Current with and without Damping Resistance

Further studies were made of the effects of series condensers. Fig. 5 compared with Figs. 3 and 4 shows the increase in velocity brought about by the insertion of series condensers. While it was shown that the effective velocity could be raised to the desired value, the limit of useful length was now set by the high attenuation.

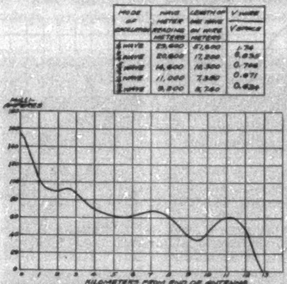

Fig. 5—Oscillator Tests of Rubber Covered Antenna Wire with Twenty-two 1/10 Microfarad Condensers in Series. Current Distribution with Far End Open

A midwinter blizzard put a stop to further tests on the rubber covered lines, but sufficient data had been gathered to warrant confidence in the theory of the wave antenna and indicate its value. Acknowledgment should be made of the assistance during this work given by Mr. P. S. Carter and Mr. R. D. Greenman.

The New Riverhead Antenna

Since the full realization of the possibilities of the wave antenna called for a line of high velocity and low attenuation, and free from points of reflection, the next logical step was to build a high grade line of the best possible type. Bare copper wire on poles was recommended and Mr. Alexanderson, who had enthusiastically followed the work at Eastport, had the line constructed by the Radio Corporation. Work was begun as soon as the frost was out of the ground and in June the line was ready for experimental use. Fig.

Fig. 6

6 shows a photograph of part of the line. A flexible type of line construction was adopted which would make it possible to try numerous experiments such as a series of loops or series of verticals connected through a transmission line, etc. Two cross arms were provided, one at a height of 18 feet and the other 30 feet above ground. The upper arm carried two No. 10 B & S copper wires and the lower arm four similar wires. All lines were broken every ten poles, and down leads were provided so that connections could be readily changed. The line ran seven miles approximately southwest from Riverhead along an unfrequented sand road, and was later extended to Terrell River, making a total length of nine miles. The line is as straight as it was feasible to build it, and its direction is substantially in line with the principal European long wave stations, the reception of which was a matter of primary interest.

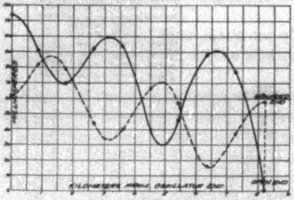

FIG. 7—OSCILLATION TESTS OF RIVERHEAD ANTENNA. FOUR WIRES IN MULTIPLE. λ = 9400

The first tests on the new line showed that the hoped for results had been realized. Instead of an optimum length of six or seven kilometers, the signals became stronger and stronger as the receiving set was moved toward the southwest, and the signal strength there was several times greater than had been obtainable with the rubber covered antennas. While the stray ratios observed on the rubber covered antennas had seemed excellent, the new antenna fully met our expectations of improvement.

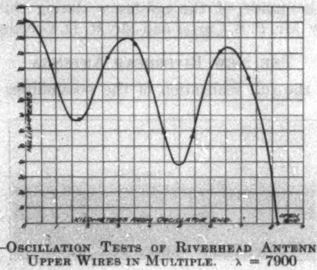

FIG. 8—OSCILLATION TESTS OF RIVERHEAD ANTENNA. TWO UPPER WIRES IN MULTIPLE. λ = 7900

Oscillator tests on the new antenna showed that the velocity was high and attenuation low compared with the rubber covered wires, and that it had no serious reflection points. Figs. 7, 8 and 9 show typical curves obtained with the new antenna. A comparison with similar curves, taken on the rubber covered ground wires, brings out the improved electrical properties.

The failure at the start to get good short circuit reflections when the far end of the line was grounded caused us to suspect that the high-frequency resistance of the grounds was much greater than we had estimated. These grounds were made with lines of iron wire laid in water. The substitution of copper wires removed this difficulty. The adjustment of resistance for unidirectional effects was now clean cut and in accord with the theory. A dead ground at the N. E. end gave as bad a stray ratio on the average as an open circuit.

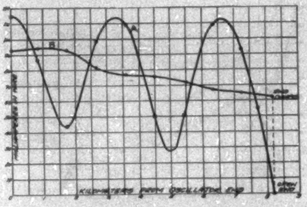

FIG. 9—OSCILLATOR TESTS ON RIVERHEAD ANTENNA. SINGLE UPPER WIRE, λ = 7500. *A* END OPEN. *B* END GROUNDED THROUGH 600 OHMS.

EXPLANATION OF THE ACTION OF THE WAVE ANTENNA

The wave antenna in its simplest form consists of a horizontal wire of the order of a wave length long pointing towards the transmitting station, as pictured in Fig. 10.

When the signal wave reaches the end "*A*" an e. m. f. is induced in the horizontal antenna wire, due to the fact that the wave front is not perpendicular to the ground, but has a tilt forward of 1 deg. to 10 deg., depending on the wave length and character of ground. Thus, at the end *A*, a little wave starts to run down the antenna towards the receiving station, and if it travels with the same velocity as the radio wave in space, the space wave follows right along with it, supplying energy to it as it goes and building it up, until at the end *B* it has reached a magnitude many times that which it had at *A*. This is illustrated in Fig. 11 which shows a single wave at successive time intervals. If the velocity of the wave on the wire is not equal to the space

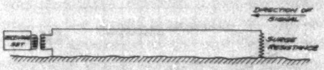

FIG. 10—SIMPLE FORM WAVE ANTENNA

wave (velocity of light) interference effects develop, the wave on the wire building up for a certain distance and then decreasing in amplitude. The velocities on actual antennas, however, are nearly enough equal to that of light so that for considerable lengths the wave on the wire builds up as it would on a light velocity line.

A signal coming from the opposite direction to that which we have been considering, will build up a wave on the wire in a similar way, from a small value at *B* to a large value at *A*. If now the line were open or grounded at *A*, the wave would be reflected back over the antenna to the receiver end *B*, and would be heard. On the other hand, if we damp the end *A* in such a manner as to prevent reflections, the antenna becomes unidirectional. A non-inductive resistance, having the value $R = \sqrt{L/C}$ ohms, where *L* and *C* are the inductance and capacity of the antenna per unit length, constitutes a practically perfect damper.

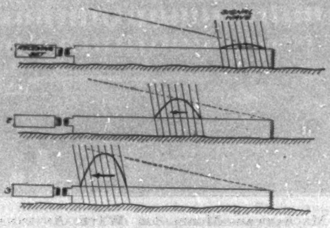

Fig. 11—Building Up of Wave on Wire as Space Wave Progresses

Fig. 12—Mechanical Model of Wave Antenna—Stationary View

Many mechanical analogies will occur to the reader, such as the building up of water waves in the direction of the wind. An interesting experiment is to run over thin ice. If you run at just the speed of wave propagation of the ice surface you can build up a larger wave. If you run too slow or too fast little effect is produced. To demonstrate the manner of building up of waves on a wave antenna, Rice and Kellogg built the mechanical model shown in Figs. 12 to 17. The upper line represents the space wave and the lower line the wave on the wire. Each line consists of a series of wooden sticks strung on a pair of small steel wires a half inch apart in the case of the lower line and an inch and a half apart in the upper line. The rockers of the upper line are loaded with lead. The inertia of the rocking stick is analogous to line inductance, while the elasticity of the connection between successive sticks is analogous to the capacity between the conductors of an electrical line. The velocity can be changed by varying the tension on the wires. The dashpots at the ends practically stop reflections. The upper line provides a means of imparting energy progressively to the different portions of the lower line, as a space wave imparts energy to the antenna line. A slight coupling between the two lines is supplied by light rubber bands stretched from short hooks on the upper sides of the lower sticks to corresponding hooks on the under side of the upper sticks. Waves are imparted to the upper line by moving one end by hand for an impulse or by a motor driven rocker for continuous waves.

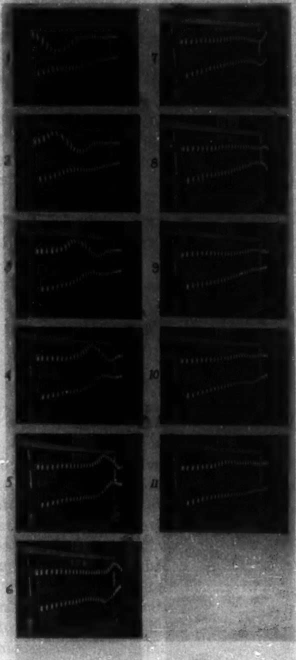

Fig. 13—Mechanical Model of Wave Antenna with End Damped

Fig. 12 is a stationary view of the machine.

Fig. 13 shows a series of views taken with a moving picture camera, when an impulse is imparted to the upper line. The growth of the wave on the lower line as it approaches the far end is readily seen. The wave on the lower line appears to pass off the end leaving the line practically stationary.

Fig. 14, compared with 13 shows the effect of removing the dashpot, thus permitting a free end reflection. We notice in these pictures a return wave which on reaching the near end, causes a movement of the first rocker. This illustrates the loss of unidirectional properties if the end is not damped.

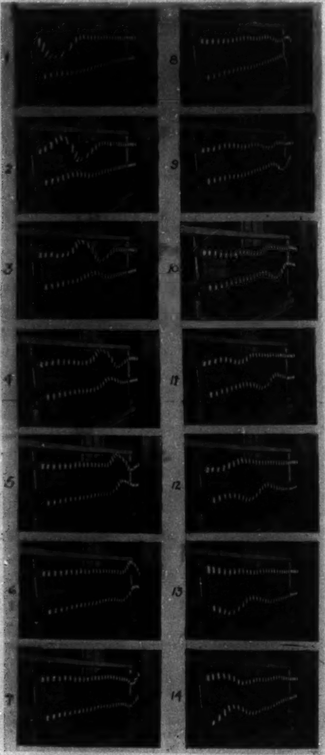

Fig. 14—Mechanical Model of Wave Antenna with Free-end Reflection

Time exposures, with continuous waves of constant amplitude supplied to the upper (space wave) line, bring out the amplitudes developed on various parts of the lower line, or the equivalent of current distribution in a wave antenna.

Fig. 15 shows the increase in amplitude on the lower line (antenna) as the waves progress from left to right. That the upper line carries practically pure traveling waves (*i. e.* has no return waves) is shown by the nearly uniform amplitude throughout its length.

Fig. 15—Mechanical Model of Wave Antenna—Full Velocity, Building Up

Fig. 16—Mechanical Model of Wave Antenna—Full Velocity, Reflection at End

Fig. 17—Mechanical Model of Wave Antenna—Slowed Down

Fig. 16 shows the effect of removing the dashpot at the right hand end of the lower line. Considerable movement at the extreme left end now appears, due to the reflected wave. The forward wave (left to right) built up on the line, is very small near the left end, so we find the amplitude there nearly uniform, as would be expected with waves traveling in one direction only (right to left). On the other hand toward the right we see very clear standing wave effects, for here the forward (left to right) wave has an amplitude more nearly equal to that of the return wave.

In Fig. 17 the right hand end is again damped, and the lower line has been slowed down by reducing the tension on the wires. This illustrates the building up and down of the waves on the antenna when its velocity is much below that of the wave in space.

REDUCTION TO PRACTICAL FORM

After the construction of the Riverhead antenna, considerable time was devoted to experiments of various kinds with the new antenna, tests and comparisons of different antenna arrangements, an oscillographic study of static, and tests of station apparatus. Messrs. Beverage, Rice, Kellogg, P. S. Carter, R. D. Greenman and E. P. Lawsing participated. A number of practical developments were evolved in the course of this work.

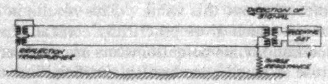

FIG. 18—ARRANGEMENT FOR LOCATING RECEIVING SET AT SAME END AS SURGE RESISTANCE

The "Reflection Transformer" Circuit. In order that the receiving set might be located at the same end of the line as the surge impedance, thereby facilitating adjustment, the arrangement shown in Fig. 18 was proposed by Kellogg. The two wires work in multiple as an antenna, but act as a balanced transmission line to bring the signal currents back from the southwest end. This scheme obviates the use of extra wires for the return transmission line and thus avoids the problem of preventing detrimental effects of nearby conductors. When the primary of the "reflection transformer" was opened the receiving set was quiet, showing that the transmission line although it was not transposed, was not introducing any undesirable electromotive forces. Thus a horizontal loop receives neither static or signal.

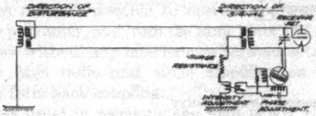

FIG. 19—FIRST BALANCING CIRCUIT EMPLOYED FOR OBTAINING ZERO RECEPTION FROM THE BACK END

Compensation for Back Wave. Beverage showed that while the resistance could be adjusted to give a minimum for static while listening to European signals, still better stray ratios were obtained by combining with the signal currents brought in over the transmission line, a small amount of the currents flowing to ground at the northeast end of the antenna. This he accomplished with a phase adjuster and intensity coupler of the type used by Mr. Alexanderson in the barrage receiver. The circuit arrangement is shown in Fig. 19.

The two long wave stations New Brunswick (13,600 meters) and Annapolis (16,900 meters) were of great assistance in making tests and adjustments. Either station could be entirely put out by using the phase and intensity adjustments of Fig. 19. It was found that when the adjustments were made for putting New Brunswick out, the stray ratio was best on the European stations whose wave lengths were near that of New Brunswick, and that when Annapolis was put out the adjustments were such as to give the best possible stray ratio for the longer wave European stations. In other words, the best stray ratio was obtained when the end conditions were adjusted to put out the image of the desired European station. If a light-velocity wave antenna is an exact number of half wave lengths long, the mathematical analysis shows that it is unidirectional provided the true surge impedance is connected between antenna and ground at the end nearest the transmitting station. This is illustrated in Fig. 29. Whether the true surge impedance is non-inductive or contains a capacity or inductive component will depend upon the characteristics of antenna and ground at the frequency under consideration. Kellogg pointed out that since the antenna was not an exact number of half wave lengths long, the back wave effect would prevent the antenna from being unidirectional even though the true surge impedance had been used, and it was for this reason that Beverage found that balances were required to obtain the best stray ratio. Another method proposed by Kellogg of supplying this necessary compensation was to insert a circuit consisting of inductance, resistance, and capacity in series in the neutral at the end nearest the transmitting station, as shown in Fig. 20. By adjusting the resistance and varying the capacity through the point where it tuned out the reactance of the coil, a wave of any desired intensity and phase could be reflected down the antenna to exactly compensate for the back wave effect and thus render the antenna unidirectional.

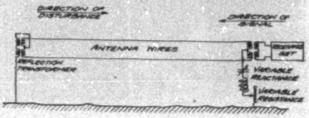

FIG. 20—METHOD OF BALANCING BACK END CURRENTS BY REFLECTIONS FROM DAMPED END

If only one station is to be received the circuit shown in Fig. 20 is as satisfactory as any that the writers have found. For reception of long wave stations an antenna output transformer was used having a step-down ratio of 200 to 10 turns, and having a complete iron magnetic circuit of about 3/4 square inch cross section, made of

0.0015 enamelled sheet iron of the kind developed for the Alexanderson alternators. The secondary was connected in series with the first tuned circuit of the receiving set.

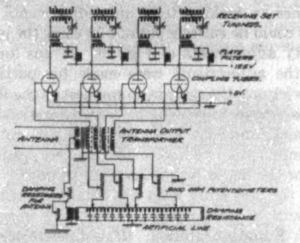
RECEIVING SET TUNINGS.
PLATE FILTERS
+125V
COUPLING TUBES.
+6V.
0
ANTENNA
ANTENNA OUTPUT TRANSFORMER
DAMPING RESISTANCE FOR ANTENNA
5000 OHM POTENTIOMETERS
DAMPING RESISTANCE
ARTIFICIAL LINE

Fig. 21—Multiplex Receiving System

Multiplex Reception. The simultaneous reception of a number of stations was one of the next objects of our work. If the surge impedance is set at the best value for a mean wave length, and no finer adjustment of the back-end compensation is attempted, then it is only necessary to provide more secondaries for the antenna output transformer, Fig. 20. In order that one set should not sap too much energy from another, transformers were designed with very slight reaction between the secondaries. Some data were taken of the best resistance and reactance in the ground circuit, as a function of wave length, and networks were figured out which would give the desired impedances at the wave lengths of the stations which it was most important to receive. This system of multiplexing, however, did not appeal to the writers as the most satisfactory solution of the problem. A system was wanted in which all the adjustments for each station to be received could be made without reacting on the adjustments for the others.

The arrangement shown in Fig. 21 was worked out, and proved entirely satisfactory. Each coupling tube feeds a receiving set, and the antenna output transformer and artificial line, with its sliding contacts and potentiometers, serve to impress the desired potentials on the grids of the tubes. Since the load is negligible there is no reaction between the different sets. Grounded shields between the secondaries of the antenna output transformer prevent electrostatic reactions. The desired component of the currents or potentials in the ground circuit can be obtained in any desired phase by moving the sliding contact along the artificial line, and in the needed intensity by adjusting the potentiometer. The artificial line has a characteristic impedance of about 400 ohms and reflections are prevented by a resistance of about this value. This results in a phase adjustment which gives practically constant intensity. Five thousand ohms potentiometers are used, and these constitute so small a load on the artificial line that the adjustment of any one does not appreciably alter the potential distribution on the line. The artificial line, damped as it is at the far end, acts as a practically pure resistance in the antenna surge impedance circuit. By adjustment of the series and shunt resistance boxes the antenna damping resistance may be set at the best average value, leaving only a small residual to be "cleaned up" by the artificial line adjustments.

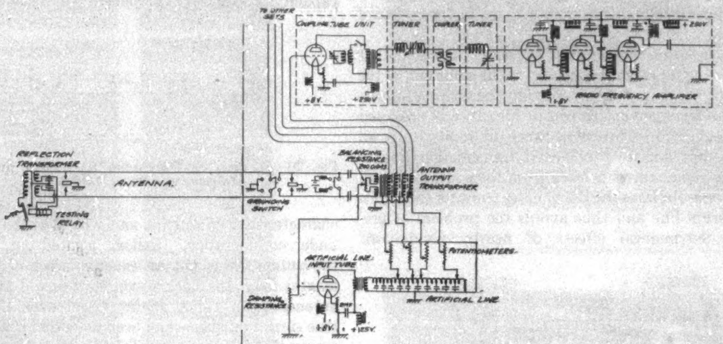
TO OTHER SETS
COUPLING TUBE UNIT
TUNER
COUPLER
TUNER
+250V
+6V.
+250V
+6V
RADIO FREQUENCY AMPLIFIER
REFLECTION TRANSFORMER
ANTENNA.
TESTING RELAY
GROUNDING SWITCH
BALANCING RESISTANCE 200 OHMS
ANTENNA OUTPUT TRANSFORMER
POTENTIOMETERS.
ARTIFICIAL LINE INPUT TUBE
DAMPING RESISTANCE
+6V.
+125V.
ARTIFICIAL LINE

Fig. 22—Wave Antenna Circuits. Present Atrangement

Fig. 22 is a more detailed diagram showing some changes which have been made since the original installation.

Shielded Sets. The aperiodic nature of the wave antenna and the success of the coupling tube multiplex system made it clear that there would be call for operating a number of receiving sets in the same building. The artificial line and antenna output transformer had been designed for operating four sets, but this was not necessarily the limit. In fact, later, when the new Riverhead station was laid out a total capacity of nine receiving sets was planned, six of which are now in daily operation. The amplifiers, detectors, and tuners previously used in the receiving sets of the Radio Corporation were unshielded, and the practise had been, where two sets were in use in the same station, to keep them well separated and operate from separate batteries. To meet the new situation, a new line of apparatus was developed. Each piece of apparatus was in a metal lined box, the metal lining being grounded, and connections between the boxes were made through shielded cable. All tuned inductances consisted of astatic pairs of coils, of compact form, thus reducing chances of magnetic coupling. The radio amplifier was shielded between stages as well as externally. The amplifiers and detector have individual plate and filament filters in the supply lines. A two stage filter was introduced in the circuit between the detector and the audio amplifier in order to prevent radio frequency currents and potentials from getting into the audio circuits where they might cause back coupling or interference between sets. Low resistance telephones were used to minimize electrostatic coupling. These precautions made it possible to operate the several sets in close proximity, and from the same plate and filament batteries, without any interference between sets, and to employ high radio and audio amplification without trouble from back coupling.

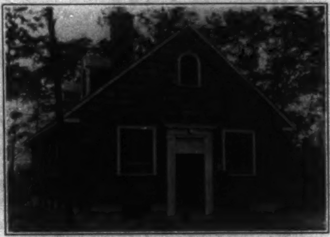

FIG. 23—RECEIVING STATION, RIVERHEAD, LONG ISLAND

It was usual in receiving the high power European stations to develop a high-frequency potential of about 7 volts at the plate of the last tube in the radio amplifier, with a useful current of about 0.2 milliamperes. By connecting the output of the audio amplifier to a good telephone line, satisfactory tone signals were received and copied in New York City. Beverage arranged a rectifier for the audio frequency currents and operated a telegraph sounder, obtaining very satisfactory signals in this way when static was moderate or light. During the spring of 1921, considerable commercial traffic was received directly in New York in this manner, using the private telegraph wire which connected the Riverhead experimental station with the Broad St. Office of the Radio Corporation. One demonstration which aroused considerable interest consisted in putting Carnarvon's signal on the telegraph line at Riverhead, and automatically repeating it at New York into the New Brunswick control line, so that the operators in the British station heard their own signal coming back on New Brunswick's wave.

When the success of the wave antenna had been demonstrated at Riverhead similar antennas were constructed at Chatham, Mass. and Belmar, N. J. where the Radio Corporation was already operating receiving stations. These new antennas gave satisfactory performance in commercial service until the long wave traffic was finally concentrated in the new Riverhead station which the Radio Corporation constructed during the summer of 1921. Fig. 23 shows the present Riverhead Station. Fig. 24 shows an output transformer, artificial line and potentiometers, Fig. 25 shows shelves with two receiving sets. The building does not provide space for operators, since all signals are transferred to telephone lines and copied directly by operators

FIG. 24—ANTENNA PANEL, RIVERHEAD RECEIVING STATION

FIG. 25—SHIELDED RECEIVING SETS, RIVERHEAD, LONG ISLAND

in the New York Office, or recorded automatically on tape.

THEORY[10]

In order to work out a formula by which the directive properties of a wave antenna can be calculated, we shall consider space waves of some specified frequency, since the directive properties of a given antenna depend on the wave length to be received.

Case 1. Signal Direction Parallel to Antenna, Zero Loss Antenna. As the space wave travels along over the antenna, it induces an electromotive force successively in the different portions of the line. We may represent the electromotive force at the end A of the antenna shown in Fig. 26 by

$$e_a = E_0 \sin \omega t \qquad (1)$$

volts per kilometer

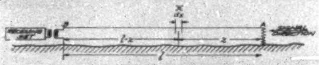

FIG. 26—MEASUREMENTS REFERRED TO IN DERIVATION OF EQUATIONS

Since it takes x/v seconds for the space wave traveling with a velocity v to reach the point X, the electromotive force e_x in the wire at the point X will be behind e_a in phase, or

$$e_x = E_0 \sin \omega\,(t - x/v) \qquad (2)$$

Let us confine our attention for the present to the effects of the electromotive force induced in a small section of the wire $d\,x$ kilometers in length, and situated x kilometers from the end A. Since e_x is expressed in volts per kilometer, the voltage induced in $d\,x$ kilometers of wire will be

$$e_x\, d\,x \text{ or } E_0\, d\,x \sin \omega\,(t - x/v)$$

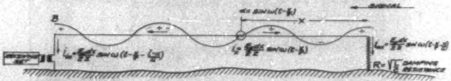

FIG. 27—CURRENTS RESULTING FROM VOLTAGE INDUCED IN SMALL SECTION OF ANTENNA

We may think of this little section of line as an alternator Fig. 27, supplying current to two transmission lines in series, one running to A and the other running to B. A line damped at the far end, so as to prevent reflections, shows an impedance $Z = \sqrt{L/C}$ [11] ohms at the input end, whatever the length of the line. We have here assumed such damping. If each line has an impedance Z ohms the alternator must work through an impedance $2\,Z$ and will produce a current at X,

$$d\,i_x = \frac{e_x\, d\,x}{2\,Z} = \frac{E_0\, d\,x}{2\,Z} \sin \omega\,(t - x/v) \qquad (3)$$

The alternator at X in forcing the current $d\,i_x$ through the line, gives rise to a train of waves moving toward B and another train of waves moving toward A. The resulting currents at the ends of the line $d\,i_{bx}$ and $d\,i_{ax}$ will be retarded in phase, as compared with $d\,i_x$. If u is the velocity of wave propagation along the wire, it will take $\frac{l - x}{u}$ seconds for the forward waves to reach B, and x/u seconds for the backward waves to reach A.

Then $$d\,i_{bx} = \frac{E_0\, d\,x}{2\,Z} \sin \omega \left(t - x/v - \frac{l - x}{u}\right) \qquad (4)$$

$$d\,i_{ax} = \frac{E_0\, d\,x}{2\,Z} \sin \omega\,(t - x/v - x/u) \qquad (5)$$

The total current at the end of the line is the sum of the currents produced by each of the sections $d\,x$ long in the line. We may perform the summation by integrating equation (4) or (5) between the limits $x = o$ and $x = l$, or by means of a vector diagram as shown in Fig. 28.

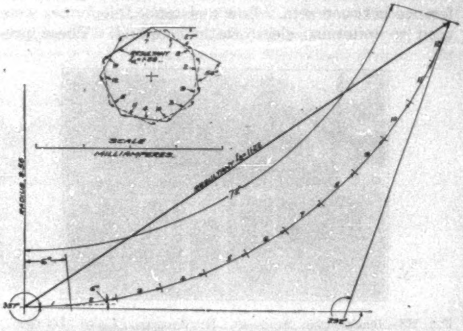

FIG. 28—DETERMINATION OF CURRENTS AT END OF WAVE ANTENNA BY VECTOR DIAGRAMS

The current at the end of the line $d\,i_{bx}$ or $d\,i_{ax}$, resulting from the induced electromotive force in a section $d\,x$ long, situated x kilometers from A, has a maximum cyclic value or vector length $\frac{E_0\, d\,x}{2\,Z}$ and a phase angle which varies with x, as shown in expression (4) and (5). Hence the summation will consist in adding a series of vectors, each $\frac{E_0\, d\,x}{2\,Z}$ long, and each at its proper phase angle. Taking the phase of e_a for reference, the angle of lag of $d\,i_{bx}$ as shown in (4) is

$$\psi_{bx} = \omega \left(\frac{x}{v} + \frac{l - x}{u}\right)$$

---

[10] The Mathematical work of this paper is due to Kellogg.

[11] This expression for Surge Impedance is strictly correct only for zero loss lines, but at radio frequencies it is a very close approximation for ordinary lines.

which may be written

$$\psi_{bx} = \frac{\omega}{u}\left\{ l - x\left(1 - \frac{u}{v}\right)\right\} \tag{6}$$

and from (**5**) the angle of lag of $d\, i_{ax}$ is

$$\psi_{ax} = \omega\left\{ x/v + x/u \right\} = \frac{\omega}{u}\, x\left(1 + \frac{u}{v}\right) \tag{7}$$

A further change in the form of these expressions will make for convenience.

The velocity u is equal to $\frac{1}{\sqrt{L\,C}}$[12] where L and C are the series inductance and shunt capacity per unit length of line. Then $\omega/u = \omega\sqrt{L\,C}$, which will be recognized as the line wave length constant, for which the symbol β is frequently employed.

If we let n stand for the velocity ratio u/v and substitute β for ω/u we get

$$\psi_{bx} = \beta\,\{l - x\,(1 - n)\,\} \tag{8}$$

$$\psi_{ax} = \beta\, x\,(1 + n) \tag{9}$$

For the purpose of numerical calculation it is convenient to express β in terms of λ and n as follows:

$$\beta = \omega/u = \frac{2\,\pi\,f}{n\,v} \text{ and since } \lambda = \frac{v}{f},$$

$$\beta = \frac{2\,\pi}{n\,\lambda} \text{ radians per kilometer} \tag{10}$$

Let us illustrate the vector diagram method of summing up the currents by a numerical example. Assume the antenna length l to be 12 kilometers, its wave velocity u equal to 0.8 of that of light, and its impedance Z equal to 500 ohms, and a signal having a wave length of 15 kilometers and an intensity $E_0 = 10$ millivolts per kilometer. In the present case $\beta = \frac{2\,\pi}{n\,\lambda} = \frac{2\,\pi}{(0.8)\,(15)} = 0.522$ radians per kilometer. Take $d\,x$ as 1 kilometer, giving x the values 0.5, 1.5, 2.5, 3.5— to 11.5 kilometers. The corresponding phase angles, figured from expressions (**8**) and (**9**) are

x. (Km.)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
$\psi\, b$ x radians	6.23	6.12	6.02	5.91	5.8	5.7	5.6	5.5	5.38	5.28	5.17	5.06
degrees	357	351	345	339	333	327	321	315	309	303	297	292
$\psi\, a$ x radians	0.47	1.41	2.35	3.29	4.23	5.16	6.10	7.04	8.0	8.93	9.87	10.80
degrees	27	81	135	189	243	297	352	405	459	513	567	622

Each vector will have a length $\frac{E_0\, d\, x}{2\, Z} = \frac{0.01 \times 1}{1000} = 0.00001$ amperes or 10 microamperes. Fig. 28 shows the vector diagrams for the currents at the two ends of the antenna, giving receiver end current $I_b = 112.2$ microamperes and back end current $I_a = 12.8$ microamperes.

These diagrams help us to see how we may write out an expression for the resultant currents. If we divide the line into a large number of very short sections the series of vectors form an arc of a circle instead of a polygon. The length of this arc is the length of one vector $\frac{E_0\, d\, x}{2\, Z}$ multiplied by the number of sections $\frac{l}{d\, x}$ which gives $\frac{E_0\, l}{2\, Z}$. If the vectors all lay on a straight line $\frac{E_0\, l}{2\, Z}$ would be the length of the total current vector. This condition is met at the end B when $u = v$. For any other line velocity, or for the current at the end A, the arc while of the same total length, will have a certain curvature, and the length of the chord subtending the arc, which is the resultant vector, will depend on the curvature. For a given length of arc the curvature is measured by the total angle, or the angle between the first and last vectors, corresponding to $x = 0$ and $x = l$ in (**8**) or (**9**). Thus in the case of the I_b vector addition we put $x = 0$ and $x = l$ in equation (**8**) to obtain the initial and final directions of the arc, which gives

$$\psi_{b0} = \beta\, l$$

and

$$\psi_{bl} = \beta\, l\, n$$

The difference or total angle through which the arc turns is

$$\psi_{b0} - \psi_{bl} = \beta\, l\,(1 - n)$$

The radius of curvature r is given by $r = \frac{\text{arc}}{\text{angle}}$ or

$$r = \frac{\frac{E_0\, l}{2\, Z}}{\beta\, l\,(1 - n)} = \frac{E_0\, l}{2\, Z\, \beta\, l\,(1 - n)}$$

The length of the chord which is the vector of the resultant current I_b is given by

$$\text{chord} = 2\, r \sin 1/2 \text{ (angle) or}$$

$$I_b = 2\,\frac{E_0\, l}{2\, Z\, \beta\, l\,(1 - n)} \sin \frac{1}{2}\, \beta\, l\,(1 - n)$$

Cancelling the $2\, l$ in the numerator and denominator gives

$$I_b = \frac{E_0}{Z\, \beta\,(1 - n)} \sin \frac{1}{2}\, \beta\, l\,(1 - n) \qquad \textbf{(11)}^{13}$$

By the same process using equation (**9**) we get

$$I_a = \frac{E_0}{Z\, \beta\,(1 + n)} \sin \frac{1}{2}\, \beta\, l\,(1 + n) \tag{12}$$

---

[12] This expression ignores line losses, but is a very close approximation in all cases with which we shall have to deal. The same is true of the expressions $Z = \sqrt{L/C}$ and $\beta = \omega\sqrt{LC}$

[13] A formula which correctly showed the end currents as functions of the antenna length and the velocity ratio, was first worked out during the early Eastport work by Mr. P. S. Carter.

These expressions give $I_b = 112$ microamperes and $I_a = 12.2$ microamperes for the problem worked graphically in Fig. 28. The difference between these values and those found from the graphical solution is due to the inaccuracy involved in the graphical method using so few sections.

It will be noticed that if the sign of n is changed in (**11**) we get (**12**). We may consider the sign of the signal velocity v, to be negative when the signal comes from the opposite direction. Since n is defined as u/v, this would give n a negative sign. Therefore, there is only one formula required, and to find the back end current I_a we consider the signal direction to be reversed.

We showed that if $u = v$ the arc is a straight line and $I_a = \frac{E_0\, l}{2\, Z}$. If we set $n = 1$ in (**11**), we get the indeterminate form O/O, and to evaluate the expression we make use of the relation that $\frac{\sin a}{a}$ approaches 1 as a approaches O, where the angle a is expressed in radians. Thus in (**11**)

$$\frac{\sin \frac{1}{2}\, \beta\, l\, (1 - n)}{\beta\, (1 - n)}$$

which may be written

$$\frac{\frac{1}{2} l \sin \frac{1}{2}\, \beta\, l\, (1 - n)}{\frac{1}{2}\, \beta\, l\, (1 - n)}$$

approaches $\frac{1}{2}\, l$ as $\frac{1}{2}\, \beta\, l\, (1 - n)$ approaches O, and expression (**11**) takes the value

$$I_b = E_0/Z \times \frac{1}{2}\, l \text{ or } \frac{E_0\, l}{2\, Z}$$

Hence when $u = v$

$$I_b = \frac{E_0\, l}{2\, Z} \qquad (\mathbf{11a})$$

Figure 29[14] shows the receiver end and back end currents calculated by equations (**11a**) and (**12**) for antennas of various lengths, assuming

$u = v = 3 \times 10^5$ kilometers per second
$\lambda = 12$ kilometers
$E_0 = 0.010$ volts per kilometers
$Z = 500$ ohms

These curves bring out the unidirectional properties of the wave antenna.

Case II. Signal at an Angle to Antenna, Zero Loss Antenna. So far we have considered only signal waves traveling in the direction of the antenna. To calculate the directive properties of the antenna, we must find the effect of a signal of the same wave length and intensity coming at an angle to the antenna. In the first place there will be a difference in the electromotive force E_0 induced in a unit length of wire. In Fig. 30A, we imagine ourselves looking down at two horizontal wires, each of unit length, situated in the midst of a signal wave whose magnetic lines are shown as dotted lines in the figure. Only the magnetic lines in a very thin layer immediately over the plane of the wires are shown. These magnetic lines have a horizontal movement indicated by the arrow, and a downward movement resulting from the forward tilt of the wave front. The horizontal movement causes no cutting of the conductor. Owing to the downward movement, the wires will be cut by all of the lines which cross them in the figure. The numbers of magnetic lines which cross wires No. 1 and No. 2 are in the ratio 1 to cos θ, and the induced electromotive force per unit length will be in the same ratio. Another point of view is illustrated in Fig. 30B, in which we imagine ourselves as looking at the ends of the magnetic lines, which now appear as dots. Here the same two wires are shown, and the actual direction of motion of the magnetic lines is indicated. Only

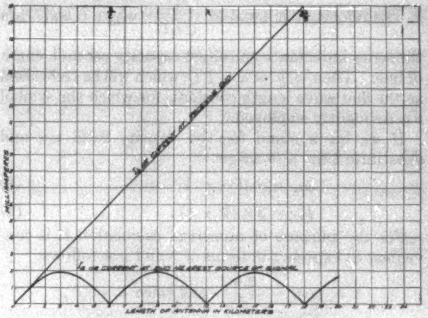

FIG. 29—RELATIVE CURRENTS AT THE TWO ENDS OF A WAVE ANTENNA

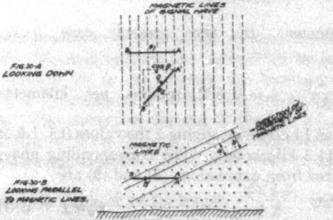
DIRECTION OF WAVE PROPAGATION
MAGNETIC LINES OF SIGNAL WAVE
FIG 30-A LOOKING DOWN
cos θ
MAGNETIC LINES
FIG 30-B LOOKING PARALLEL TO MAGNETIC LINES.

FIG. 30—EFFECT OF ANTENNA ANGLE ON INDUCED ELECTROMOTIVE FORCE PER UNIT LENGTH

those lines within the region a will cut wire No. 2 which appears in this projection as $A' - B'$, while all those in region b will cut No. 1, and the ratio of a to b is again $\cos\theta$. We shall therefore multiply by $\cos\theta$ to take account of the difference in induced electromotive force. That is to say if a signal coming in the direction of the antenna induces E_0 volts per kilometer, a signal of the same intensity coming from an angle θ to the antenna will induce $E_0 \cos\theta$ volts per kilometer.

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[14] This figure is practically a reproduction of a curve plotted by Mr. P. S. Carter, based on his calculations previously mentioned.

The angle which the signal direction makes with the antenna also effects the time required for the wave front to pass over the antenna, and therefore affects the relative phases of the electromotive forces induced in the different parts of the antenna. From the time the maximum electromotive force occurs at A to the time the same thing occurs at X the wave has only to travel a distance $x \cos\theta$ as indicated in Fig. 31, and this will require $\frac{x \cos\theta}{v}$ seconds, where v is the velocity of the signal wave. If the induced electromotive force at A is $e_a = (E_0 \cos\theta) \sin \omega t$ then at X it will be

$$e_x = (E_0 \cos\theta) \sin \omega \left(t - \frac{x \cos\theta}{v} \right) \qquad \textbf{(13)}$$

Comparing (**13**) with (**2**) we see that we now have the factor $\frac{v}{\cos\theta}$ where we had simply v, and where we had E_0 we now have $E_0 \cos\theta$. Replacing v by $\frac{v}{\cos\theta}$ mean changing u/v or n to $n \cos\theta$. Making these changes in (**11**) gives us, for the current due to a signal coming from a direction θ from the antenna.

$$I_b = \frac{E_0 \cos\theta}{Z\beta\,(1 - n\cos\theta)} \sin \frac{1}{2}\beta\, l\,(1 - n\cos\theta) \qquad \textbf{(14)}$$

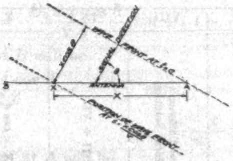

Fig. 31—Effect of Signal Angle on Time Required for Wave to Traverse Antenna

If the signal is traveling in the direction of the antenna, $\theta = 0$, and $\cos\theta = 1$, and (**14**) becomes the same as (**11**). If the signal is from the opposite direction $\theta = 180$ deg., $\cos\theta = -1$ and (**14**) becomes the same as (**12**).

Let us apply formula (**14**) to the calculation of the directive curve of the antenna whose currents for $\theta = 0$ and $\theta = 180$ deg. were determined in Fig. 28. The calculations are shown in Table I. Column IX, multiplied by E_0/Z would give the currents corresponding to the value assumed for E_0. To show directive properties, however, it is more satisfactory to give the current for any direction in terms of its ratio to the current which the same signal would produce if it came from the direction for which the antenna gives maximum reception. Thus column X gives the relative current strengths as found by dividing all the figures of column IX by the largest figure in the column which is 5.6 and corresponds to $\theta = 0$. Fig. 32 shows the directive curve of the antenna, obtained by plotting the relative currents as shown in column X radially at the corresponding angles.

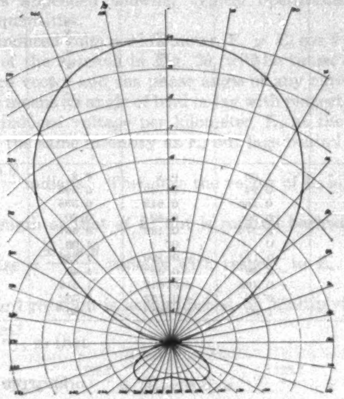

Fig. 32—Directive Curve of Wave Antenna. $\lambda = 15$ km. $l = 12$ km., $\alpha = .0$, $u = 0.8\,v$

Case III. Line Losses Considered. The factor *line attenuation* has so far been left out of account in order to simplify the problem. We assumed that the current at the end of the line, resulting from a wave which started at some point X, had the same strength as at X, although of different phase.

In the case of bare copper wire on poles, the errors due to ignoring line losses are not in general large enough to give misleading results. There are many instances however in which line losses are high or the antenna unusually long, where these effects cannot be neglected.

When there are no reflections to cause standing wave effects, the decrease in current strength as we go farther from the source is expressed by the relation $I_x = I_0\, \epsilon^{-\alpha x}$, in which I_0 is the current strength at the source, I_x the current at a distance x from the source, and α is the "attenuation constant" of the line. A value of 0.05 per kilometer for α, means that the current decreases about 5 per cent with every kilometer which the wave travels.

In deriving equations (**4**) and (**5**), we regarded the

TABLE I
DIRECTIVE CURVE OF WAVE ANTENNA ZERO ATTENUATION

$$I = \frac{E_s \cos \theta}{Z \dfrac{\omega}{u}\left(1 - \dfrac{u \cos \theta}{v}\right)} \sin \frac{1}{2} \frac{\omega}{u} l \left(1 - \frac{u \cos \theta}{v}\right)$$

$$= \frac{E_s \cos \theta}{Z\, 0.524\, (1 - 0.8 \cos \theta)} \sin 180 \text{ deg. } (1 - 0.8 \cos \theta)$$

$l = 12$ km.
$\lambda = 15$ km.
$u = 0.8$ v.
$\alpha = 0$

I θ deg. Signal Angle	II cos θ	III 0.8 cos θ	IV 1 − 0.8 cos θ	V 0.524 (1 − 0.8 cos θ)	VI 180 deg. (1 − 0.8 cos θ)	VII Sin 180 deg. (1 − 0.8 cos θ)	VIII VII ÷ V	IX VIII × cos θ	X Relative Current IX ÷ 5.6
0	1.00	0.8	0.2	0.105	36	0.588	5.6	5.6	1.00
20	0.94	0.752	0.248	0.13	44.8	0.704	5.4	5.08	0.91
40	0.766	0.612	0.388	0.203	70	0.94	4.62	3.54	0.63
60	0.50	0.40	0.60	0.314	108	0.951	3.02	1.51	0.27
80	0.174	0.139	0.861	0.450	155	0.423	0.94	0.164	0.03
90	0	0	1.00	0.524	180	0	0	0	0
100	− 0.174	− 0.139	1.139	0.596	205	− 0.423	− 0.71	0.124	0.022
120	− 0.50	− 0.40	1.40	0.734	252	− 0.951	− 1.29	0.645	0.115
140	− 0.766	− 0.612	1.612	0.845	290	− 0.94	− 1.11	0.85	0.152
160	− 0.94	− 0.752	1.752	0.918	315	− 0.707	− 0.77	0.724	0.129
180	− 1.00	− 0.80	1.80	0.945	324	− 0.588	− 0.622	0.622	0.111

TABLE II
DIRECTIVE CURVE OF WAVE ANTENNA

Vector $$I_s = \frac{E_s \cos \theta\, \epsilon^{-j \frac{2 \pi \cos \theta}{\lambda} l}}{2 Z \left[\alpha + j \dfrac{2 \pi}{n \lambda} (1 - n \cos \theta)\right]} \left\{ 1 - \epsilon^{-\left[\alpha + j \frac{2 \pi}{n \lambda} (1 - n \cos \theta)\right] l} \right\}$$

$\lambda = 12$ km.
$l = 12$ km.
$u = 0.8$ v.
$\alpha = 0.05$
$\epsilon^{-\alpha l} = .55$

Arranging Formula for Calculation of Magnitude and Substituting Numerical Values of Constants, we have

Magnitude $$I = \frac{E_s}{2 Z} \frac{\cos \theta}{\sqrt{(0.05)^2 + (0.654)^2 (1 - 0.8 \cos \theta)^2}} \left\{ \begin{array}{c} \text{Vector Difference of } 1.0 \text{ and } 0.55 \\ \text{at Angle } 450 \text{ deg. } (1 - .0.8 \cos \theta) \end{array} \right\}$$

I θ Signal Angle	II cos θ	III 0.8 cos θ	IV 1 − 0.8 cos θ	V 0.654 (1 − 0.8 cos θ)	VI $\sqrt{(0.05)^2 + (0.654)^2 (1 - 0.8 \cos\theta)^2}$	VII 450 deg. (1 − 0.8 cos θ)	VIII (Vector Difference) Graphical Solution	IX VIII ÷ VI	X IX × cos θ	XI Relative Current X ÷ 8.14
0	1.00	0.80	0.2	0.131	0.140	90	1.14	8.14	8.14	1.0
20	0.940	0.752	0.248	0.162	0.169	112	1.31	7.75	7.3	0.896
40	0.766	0.613	0.387	0.253	0.258	174	1.55	6.0	4.6	0.565
60	0.50	0.40	0.6	0.393	0.396	270	1.14	2.89	1.45	0.178
80	0.1736	0.139	0.861	0.564	0.566	388	0.58	1.02	0.177	0.022
100	− 0.1736	− 0.139	1.139	0.744	0.745	513	1.51	2.03	0.35	0.0431
120	− 0.50	− 0.40	1.40	0.916	0.916	630	1.14	1.25	0.62	0.0762
140	− 0.766	− 0.613	1.613	1.055	1.055	726	0.45	0.427	0.326	0.040
160	− 0.94	− 0.752	1.752	1.146	1.146	789	0.96	0.838	0.78	0.0958
180	− 1.00	− 0.80	1.80	1.18	1.18	810	1.14	0.97	0.97	0.119

point X in Fig. 26 as the source of waves on the antenna. We considered the current $d\,i_{ax}$ and $d\,i_{bx}$ at the ends of the line to have the same strength as the current $d\,i_x$ at x, since we were treating the case of a zero loss line. We now wish to consider a line on which there is attenuation, so the current $d\,i_{bx}$ at the end B, which is $(l-x)$ kilometers from the source, will be weaker than the current $d\,i_x$ at X, in the ratio $\epsilon^{-\alpha(l-x)}$, and the current $d\,i_{ax}$ at the end A, which is x kilometers from the source, will be weaker in the ratio $\epsilon^{-\alpha x}$. The phase relations as before are those corresponding the time differences $\dfrac{1-x}{u}$ seconds and $\dfrac{x}{u}$ seconds required for waves to travel from X to B and X to A respectively. The current at X is $d\,i_x = \dfrac{e_x\,d\,x}{2\,Z}$, or substituting the value of e_x given in (**13**)

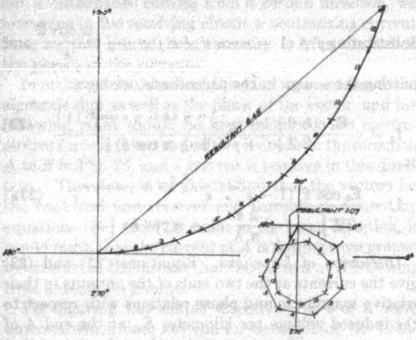

FIG. 33—DETERMINATION OF END CURRENTS FOR LINE WITH ATTENUATION

$$d\,i_x = \frac{E_0\,d\,x\cos\theta}{2\,Z}\sin\omega\left(t-\frac{x}{v}\right)$$

The current at B will be

$$d\,i_{bx} = \frac{E_0\,d\,x\cos\theta}{2\,Z}\,\epsilon^{-\alpha(l-x)}\sin\omega\left(t-\frac{x}{v}-\frac{l-x}{u}\right) \qquad (\mathbf{15})$$

and the current at A will be

$$d\,i_{ax} = \frac{E_0\,d\,x\cos\theta}{2\,Z}\,\epsilon^{-\alpha x}\sin\omega\,(t-x/v-x/u) \qquad (\mathbf{16})$$

The total end currents may be found by dividing the line into sections and drawing a series of vectors which represent the end currents resulting from the induced electromotive forces in the successive sections, as was done in Fig. 28. In the present case the vectors will differ progressively in length as well as in phase. Fig. 33 shows diagrams for finding the end currents for a signal direction parallel to the antenna. The conditions of the problem are the same as in the case of Fig. 28 except that for Fig. 33 an attenuation of 0.05 per kilometer is assumed. The corresponding vectors in the two figures have the same phase angles, but in Fig. 33 the vectors form a spiral instead of a circular arc.

The total end currents can be found by integrating equations (**15**) and (**16**) in their present form, but the work is simplified if we make use of vector notation.

In what follows, the symbols in bold faced type, such as **E** or **I** stand for vector quantities of the form $a+j\,b$, or its equivalent. For reference a summary is appended showing typical operations with vector quantities.

The induced volts per kilometer, $\mathbf{E}_a = \mathbf{E}_0\cos\theta$ at the end A of the antenna in Fig. 26, will be taken as the reference vector and the phase angle of any current or voltage means its angle of lead or lag with respect to $\mathbf{E}_a$.

The induced voltage per kilometer, $\mathbf{E}_x$ at the point X is of the same intensity as $\mathbf{E}_a$ but lags behind $\mathbf{E}_a$ by $\omega\dfrac{x\cos\theta}{v}$ radians. Therefore the vector of $\mathbf{E}_x$ is of the same length as that of $\mathbf{E}_a$ but is rotated backward, or clockwise $\omega\dfrac{x\cos\theta}{v}$ radians with respect to $\mathbf{E}_a$. This backward rotation is accomplished by multiplying by $\epsilon^{-j\omega\frac{x\cos\theta}{v}}$, so that

$$\mathbf{E}_x = \mathbf{E}_a\,\epsilon^{-j\omega\frac{x\cos\theta}{v}} = (\mathbf{E}_0\cos\theta)\,\epsilon^{-j\,\omega\frac{x\cos\theta}{v}} \qquad (\mathbf{17})$$

The voltage induced in $d\,x$ kilometers of wire at X is $\mathbf{E}_x\,d\,x$, and this, acting through an impedance of $2\,\mathbf{Z}$ ohms produces a current at X of

$$d\,\mathbf{i}_x = \frac{\mathbf{E}_x\,d\,x}{2\,\mathbf{Z}} = \frac{\mathbf{E}_0\,d\,x\cos\theta}{2\,\mathbf{Z}}\,\epsilon^{-j\,\omega\frac{x\cos\theta}{v}} \qquad (\mathbf{18})$$

The waves on the wire must travel $(l-x)$ kilometers from X to reach the end B of the antenna. Therefore compared with the current $d\,\mathbf{i}_x$ at X, the current $d\,\mathbf{i}_{bx}$ at B, is smaller in the ratio $\epsilon^{-\alpha(l-x)}$ and retarded in phase by the angle $\omega\dfrac{l-x}{u}$ radians. Hence

$$d\,\mathbf{i}_{bx} = d\,\mathbf{i}_x\,\epsilon^{-\alpha(l-x)}\,\epsilon^{-j\frac{\omega}{u}(l-x)}$$

Combining exponents and substituting β for ω/u we have

$$d\,\mathbf{i}_{bx} = d\,\mathbf{i}_x\,\epsilon^{-(\alpha+j\beta)(l-x)} \qquad (\mathbf{19})$$

Substituting in this the value of $d\,\mathbf{i}_x$ shown in (**18**) we have

$$d\,\mathbf{i}_{bx} = \left\{\frac{\mathbf{E}_0\,d\,x\cos\theta}{2\,\mathbf{Z}}\,\epsilon^{-j\,\omega\frac{x\cos\theta}{v}}\right\}\epsilon^{-(\alpha+j\beta)(l-x)}$$

Combining exponents and collecting the x terms gives

$$d\,\mathbf{i}_{bx} = \frac{\mathbf{E}_0 d\,x\cos\theta}{2\,\mathbf{Z}}\,\epsilon^{-\left\{(\alpha+j\beta)\,l+(\alpha+j\beta-j\,\omega\frac{\cos\theta}{v}x\right\}} \qquad (\mathbf{20})$$

The waves from X to A travel x kilometers on the wire, and are reduced in intensity in the ratio $\epsilon^{-\alpha x}$ and retarded in phase by $\omega\, x/u$ or $\beta\, x$ radians. Therefore

$$\begin{aligned} d\,\mathbf{i}_{ax} &= d\,\mathbf{i}_x\,\epsilon^{-\alpha x}\,\epsilon^{-j\beta x} \\ &= d\,\mathbf{i}_x\,\epsilon^{-(\alpha + j\beta)x} \\ &= \left\{\frac{\mathbf{E}_0\,d\,x\cos\theta}{2\,\mathbf{Z}}\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v}\right\}\epsilon^{-(\alpha + j\beta)x} \\ &= \frac{\mathbf{E}_0\,d\,x\cos\theta}{2\,\mathbf{Z}}\,\epsilon^{-(\alpha + j\beta + j\,\omega\,x\,(\cos\theta)/v} \end{aligned} \tag{21}$$

The total current $\mathbf{I}_b$ is the sum of all the currents $d\,\mathbf{i}_{bx}$, corresponding to all values of X, or in other words we integrate the expression (**20**) for $d\,\mathbf{i}_{bx}$ between the limits $x = o$ and $x = l$.

$$\mathbf{I}_b = \frac{\mathbf{E}_0\cos\theta}{2\,\mathbf{Z}}\,\epsilon^{-(\alpha + j\beta)\,l}\int_{x=0}^{x=l}\epsilon^{(\alpha + j\beta - j\,\omega\,x\,(\cos\theta)/v}\,d\,x$$

Performing the integration and substituting the limits gives

$$\mathbf{I}_b = \frac{\mathbf{E}_0\cos\theta\,\epsilon^{-(\alpha + j\beta)\,l}\left\{\epsilon^{(\alpha + j\beta - j\,\omega\,l\,(\cos\theta)/v} - 1\right\}}{2\,\mathbf{Z}\left(\alpha + j\beta - j\,\omega\,\dfrac{\cos\theta}{v}\right)}$$

This expression gives the value of $\mathbf{I}_b$, but it is more convenient for calculation if several changes are made in its form.

Putting $\epsilon^{-(\alpha+j\beta)\,l}$ inside the parenthesis gives

$$\mathbf{I}_b = \frac{\mathbf{E}_0\cos\theta\left\{\epsilon^{-j\,\omega\,l\,(\cos\theta)/v} - \epsilon^{-(\alpha + j\beta)\,l}\right\}}{2\,\mathbf{Z}\left(\alpha + j\beta - j\,\omega\,\dfrac{\cos\theta}{v}\right)}$$

Taking $\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}$ out of the parenthesis gives

$$\mathbf{I}_b = \frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}\left\{1 - \epsilon^{-(\alpha + j\beta - j\,\omega\,l\,(\cos\theta)/v}\right\}}{2\,\mathbf{Z}\left(\alpha + j\beta - j\,\omega\,\dfrac{\cos\theta}{v}\right)}$$

Since $\omega/v = \dfrac{\omega\,n}{u} = \beta\,n$, we may substitute

$j\,\beta\,(1 - n\cos\theta)$ for $j\,\beta - j\,\dfrac{\omega\cos\theta}{v}$, giving

$$\mathbf{I}_b = \frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l(\cos\theta)/v}\left\{1 - \epsilon^{-\{\alpha + j\beta\,(1 - n\cos\theta)\,l\}}\right\}}{2\,\mathbf{Z}\,[\alpha + j\beta\,(1 - n\cos\theta)\,]} \tag{22}$$

A more satisfactory form of the equation for purposes of calculation is obtained by separating the attenuation and phase angle factors in the expression $\epsilon^{-[\alpha + j\beta\,(1 - n\cos\theta)\,]\,l}$ and substituting $\dfrac{2\,\pi}{\lambda}$ for ω/v

and $\dfrac{2\,\pi}{n\,\lambda}$ for β

This gives

$$\mathbf{I}_b = \frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j(2\pi l/\lambda)\cos\theta}\left\{1 - \epsilon^{-\alpha\,l}\,\epsilon^{-j\frac{2\,\pi\,l}{n\,\lambda}\,(1 - n\cos\theta)}\right\}}{2\,\mathbf{Z}\left[\alpha + j\,\dfrac{2\,\pi}{n\lambda}\,(1 - n\cos\theta]\right)} \tag{22a}$$

This is the complete expression for the current at the end B farthest from the signal source.

The total current at A is found similarly by integrating expression (**21**) for $d\,\mathbf{i}_{ax}$ between the limits $x = o$, and $x = l$

$$\begin{aligned} \mathbf{I}_a &= \frac{\mathbf{E}_0\cos\theta}{2\,\mathbf{Z}}\int_{x=0}^{x=l}\epsilon^{-(\alpha + j\beta + j\,\omega\,x\,(\cos\theta)/v}\,d\,x \\ &= -\,\frac{\mathbf{E}_0\cos\theta\left\{\epsilon^{-[\alpha + j\beta + j\,\omega\,l\,(\cos\theta)/v]} - 1\right\}}{2\,Z\left[\alpha + j\beta + j\,\dfrac{\omega\cos\theta}{v}\right]} \end{aligned}$$

Substituting $j\,\beta\,(1 + n\cos\theta)$ for $j\,\beta + j\,\dfrac{\omega\cos\theta}{v}$ and

putting the $-$ sign in the parenthesis, we have

$$\mathbf{I}_a = \frac{\mathbf{E}_0\cos\theta\left\{1 - \epsilon^{-\{\alpha + j\beta\,(1 + n\cos\theta)\,]\,l\}}\right\}}{2\,\mathbf{Z}\,[\alpha + j\beta\,(1 + n\cos\theta)\,]} \tag{23}$$

or

$$\mathbf{I}_a = \frac{\mathbf{E}_0\cos\theta\left\{1 - \epsilon^{-\alpha\,l}\,\epsilon^{-j\frac{2\,\pi}{n\,\lambda}\,l\,(1 + n\cos\theta)}\right\}}{2\,\mathbf{Z}\left[\alpha + j\,\dfrac{2\,\pi}{n\,\lambda}\,(1 + n\cos\theta)\right]} \tag{23a}$$

Discussion of Equations. Equations (**22**) and (**23**) give the currents at the two ends of the antenna in their relative magnitude and phase relations with respect to the induced voltage per kilometer E_a, at the end A of the antenna. The assumptions throughout are that the antenna is straight and uniform and free from reflections at the ends, and that the signal wave causes no other electromotive forces in the circuit than that in the horizontal wire. The assumption that the induced electromotive force per unit length of conductor is the same in all parts of the antenna, and is independent of the amplitude of the wave on the wire, means that the reduction in intensity of the space wave by divergence or ground absorption has been neglected, and that no saturation effect has been considered. The absorption and divergence of the space wave may be estimated with fair approximation, and in most cases with antennas of moderate length, will be found to be negligible. No evidence of saturation has so far come to the writers' attention. In cases where the antenna has exceeded the length which gave maximum signal, the limit to the amplitude of the waves on the wire, has been set by the velocity difference or the line losses.

It will be noticed that (**22**) is the same as (**23**) except that the algebraic sign before the term $\cos\theta$ is reversed, and the factor $\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}$ appears in the numerator of (**22**). The latter term has no effect on magnitude but effects the phase. If we should calculate the current at the end nearest the signal source by using formula (**22**) and assuming the signal to come from the opposite direction, or in other words take θ as 180 deg. instead of 0 deg., we would get the same numerical value for the current as though we had used equation (**23**), but the phase angle would be different because in (**23**) the phase is expressed with reference to the voltage produced by the signal at the end nearest the source of signal, whereas in using (**22**) and reversing θ to find the "back end" current, we refer the phase to that of the signal voltage at the end farthest from the signal source.

We are concerned with the question of phase only when currents from two or more sources are to be combined. For example, in calculating the effects of reflections or of voltages induced in the end verticals, it would be necessary to know the phase relations of the currents which are combined. Or when in order to put out a disturbance coming from a certain direction, we introduce in the receiving circuit a neutralizing current from another source, it is necessary to take account of the phases of the currents.

In making calculations of this kind we must watch the algebraic sign as well as the phase of the vector, and the following point should be kept in mind. An electromotive force is taken as positive if it acts in the direction A to B in Fig. 26, and a current is positive in this direction. Therefore, if we should find that the vectors for the back end and receiver end currents as figured by equations (**23**) and (**22**) point in the same direction, it would mean that the current at A is flowing from ground to antenna at the instant that the current at B is flowing from antenna to ground.

For figuring the simple directive curve of a wave antenna which does not call for determining the phase of the end currents, we may work entirely with equation (**22**) omitting the factor $\epsilon^{-j\omega l(\cos\theta)/v}$. The expression in the bracket is a vector difference and may be evaluated graphically or by use of the familiar formula for the third side of a triangle when two sides and the angle between them are given. We have two vectors, one having a length 1, and the other having a length $\epsilon^{-\alpha l}$, and the angle between them is $\beta\,(1 - n\cos\theta)\,l$ radians or $\frac{360\,l}{n\,\lambda}\,(1 - n\cos\theta)$ degrees. The vector difference is $\sqrt{1 + (\epsilon^{-\alpha l})^2 - 2\,\epsilon^{-\alpha l}\cos\frac{360\,l}{n\,\lambda}(1 - n\cos\theta)}$

The expression $\alpha + j\;\beta\,(1 - n\cos\theta)$ in the denominator must be given in terms of its vector length $\sqrt{\alpha^2 + \beta^2\,(1 - n\cos\theta)^2}$, in order to calculate the current. Therefore, if the phase of the end current, relative to current or voltage in other parts of the antenna, is not sought, but only the magnitude of the end current, we may use the following formula, which is obtained by making the changes just mentioned in (**22**)

$$I_B = \frac{E_0\cos\theta\sqrt{1 + (\epsilon^{-\alpha l})^2 - 2\,\epsilon^{-\alpha l}\cos\frac{360\,l}{n\,\lambda}(1 - n\cos\theta)}}{2\,\angle\,\sqrt{\alpha^2 + \beta^2\,(1 - n\cos\theta)^2}} \qquad (\mathbf{24})$$

This equation becomes equivalent to (**14**) if the line has no attenuation. If we set $\alpha = 0$ in (**24**), and substitute $\beta\,l$ for $\frac{360\,l}{n\,\lambda}$ we get,

$$I_B = \frac{(E_0\cos\theta)\sqrt{2 - 2\cos\beta\,l\,(1 - n\cos\theta)}}{2\,Z\,\beta\,(1 - n\cos\theta)}$$

Making use of the trigonometric relation $1 - \cos a = 2\sin^2 1/2\,a$ we get under the radical in the numerator

$$2 - 2\cos\beta\,l\,(1 - n\cos\theta) = 4\sin^2\frac{1}{2}\beta\,l\,(1 - n\cos\theta)$$

and with this substitution the expression reduces to

$$I_B = \frac{(E_0\cos\theta)\sin\frac{1}{2}\beta\,l\,(1 - n\cos\theta)}{Z\,\beta\,(1 - n\cos\theta)}$$

which is the same as (**14**).

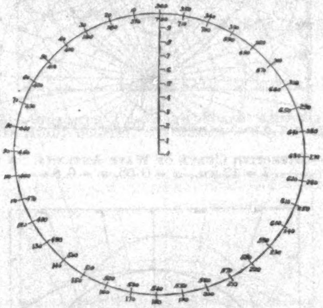

FIG. 34—CHART FOR EVALUATING

$1 - \epsilon^{-\left\{\alpha + j\frac{2\pi}{n\lambda}(1 - u\cos\theta)\right\}}$

Fig. 34 shows a chart for use in finding the value of the vector difference in the bracket of (**22**). The radius of the circle is taken as unity, ten centimeters being a convenient radius. Scale divisions from 0 to 1 are shown on one radius. From a point on this scale corresponding in value to $\epsilon^{-\alpha l}$, we measure the distance to a point on the circumference corresponding to the angle $\beta\,l\,(1 - n\cos\theta)$ radians or $\frac{360\,l}{n\,\lambda}\,(1 - n\cos\theta)$ degrees, and this measurement is the vector difference sought.

Let us apply formula (**22**) to the calculation of the directive curve of an antenna 12 kilometers long receiving a signal of 12 kilometers wave length, having an attenuation constant $\alpha = 0.05$ per kilometer, and a wave velocity $= 2.4 \times 10^5$ kilometers per second, or $n = 0.8$. The calculations are shown in Table II. Since the purpose of the directive curve is to show the relative receiver currents for different signal directions, the factor $\frac{E_0}{2\,Z}$ which is the same for all directions, is omitted (or in other words assumed to have a value of 1) and the relative currents shown in column XI are obtained by dividing all the calculated currents of column X by the largest one of those, which corresponds to $\theta = 0$. The corresponding directive curve is shown in Fig. 35.

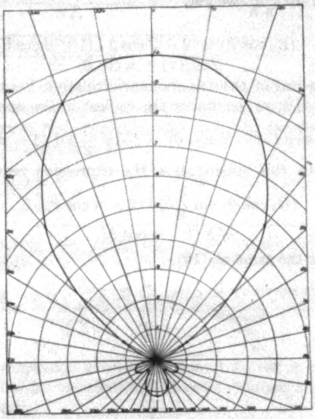

Fig. 35—Directive Curve of Wave Antenna. $\lambda = 12$ km., $l = 12$ km., $\alpha = 0.05$, $u = 0.8\,v$

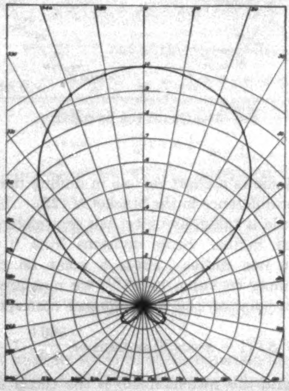

Fig. 36—Directive Current of Ideal Wave Antenna, One Wave Length Long. $\alpha = 0$, $n = 1$, $l = 12$, $\lambda = 12$

Figs. 32 to 38 bring out the effects of length, velocity and attenuation on the directive properties of a wave antenna. The value 0.05 per kilometer for α, and $0.8\,v$ for u, used in calculating Fig. 35, are mean values observed for long waves (7000 to 25,000 meters) on bare 0.102 inch (0.259 cm.) diameter copper wire supported on poles. Fig. 36 shows the directive curve for an antenna one wave length long on the assumption of zero attenuation and light velocity. By analogy with transmission line practise, we have referred to a wave

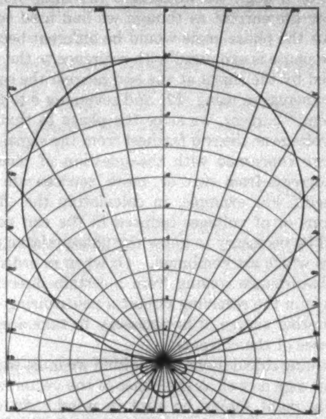

Fig. 37—Directive Curve of Wave Antenna One Half Wave Length Long. $\lambda = 12$ km., $l = 6$ km., $\alpha = 0.05$, $u = 0.8v$

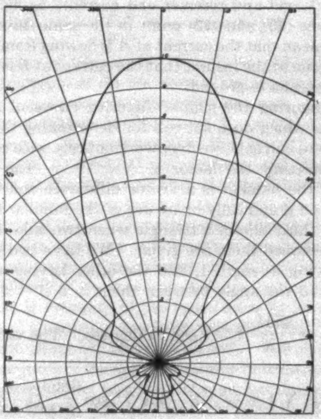

Fig. 38—Directive Curve of Wave Antenna Two Wave Lengths Long. $\lambda = 12$ km., $l = 24$ km., $\alpha = 0.05$, $u = 0.8\,v$

antenna as "ideal" if it has zero attenuation and light velocity. While Fig. 36 was calculated for a 12 kilometer wave, the directive curve is applicable to any antenna a wave length long, and having full velocity and zero attenuation. The effect of length is shown by a comparison of Figs. 37, 35 and 38, which show the directive curves for antennas of the same constants and 1/2, 1 and 2 wave lengths long respectively. Fig. 39 compared with Fig. 35, shows the effect of reducing the velocity from .8 v to .6 v on a one wave length antenna, and Fig. 40 shows the effect of raising the velocity to 2 v. Fig. 41 is calculated for the same conditions as Fig. 35, except that for Fig. 41, the line losses are twice as high.

Calculation of Phase Angle. Equation (**22**) is a vector equation which determines the magnitude of $\mathbf{I}_b$ and its phase relation with respect to the reference vector $\mathbf{E}_a$.

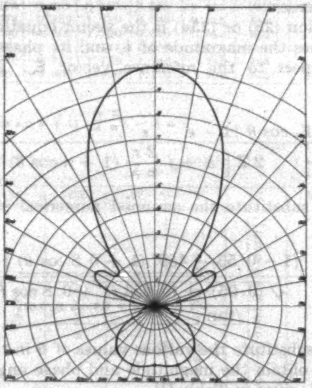

Fig. 39—Directive Curve of Low Velocity Wave Antenna. $\alpha = 0.05$, $n = 0.6$, $l = 12$, $\lambda = 12$

It should be recalled that $\mathbf{E}_a$ is defined as the voltage induced per kilometer of antenna at the end *"A"*. For convenience of discussion we will rewrite equation (**22a**) here

$$i_b = \frac{\mathbf{E}_0 \cos\theta}{2\,\mathbf{Z}}\;\frac{\epsilon^{-j\frac{2\pi l}{\lambda}\cos\theta}\left\{1 - \epsilon^{-\alpha l}\,\epsilon^{-j\frac{2\pi l}{n\lambda}(1 - n\cos\theta)}\right\}}{\alpha + j\,\dfrac{2\pi}{n\lambda}(1 - n\cos\theta)} \qquad \textbf{(22a)}$$

In order to find the phase angle of the entire expression, we must first determine the phase angles of the individual vectors of which it is a product.

The first term $\dfrac{\mathbf{E}_0 \cos\theta}{2\,\mathbf{Z}}$ is a purely numerical multiplier with zero phase angle, when we assume $\mathbf{Z} = \sqrt{L/C}$ and take $\mathbf{E}_0$ as having zero phase angle.

The term $\epsilon^{-j(2\pi l/\lambda)\cos\theta}$ is a unit vector having a phase angle $-\dfrac{2\pi l}{\lambda}\cos\theta$ radians or $-\dfrac{360\,l}{\lambda}\cos\theta$ degrees.

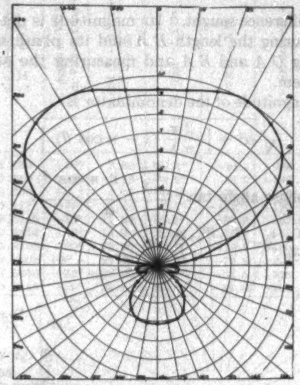

Fig. 40—Directive Curve of Wave Antenna with Excessive High Velocity. $\alpha = 0.05$, $n = 2.0$, $l = 12$, $\lambda = 12$

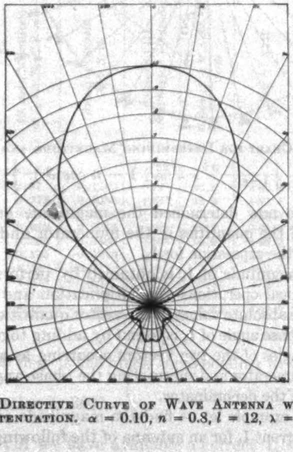

Fig. 41—Directive Curve of Wave Antenna with High Attenuation. $\alpha = 0.10$, $n = 0.8$, $l = 12$, $\lambda = 12$

The bracket is the difference of two vectors, the first a unit vector with zero phase angle, and the second a vector having the absolute value $\epsilon^{-\alpha l}$ and the phase angle $-\dfrac{2\pi}{n\lambda}\,l\,(1 - n\cos\theta)$ radians or $-\dfrac{360°\,l}{n\lambda}$ (1 − n

$\cos\theta^\circ$) degrees. The graphical method of finding the magnitude and phase of this vector difference is shown in Fig. 42. Here the unit vector of zero phase angle OA is drawn horizontally to the right. The vector OB is then drawn with the length $\epsilon^{-\alpha l}$ and phase angle $-\dfrac{360^\circ l}{n\lambda}(1-n\cos\theta^\circ)$. Then BA is the vector difference sought. Its magnitude is determined by measuring the length BA and its phase angle by extending OA and BA and measuring the angle between them.

The magnitude of the denominator is

$$\sqrt{\alpha^2+\left[\frac{2\pi}{n\lambda}(1-n\cos\theta)\right]^2}$$

and its phase angle $\tan^{-1}\dfrac{\frac{2\pi}{n\lambda}(1\text{-}n\cos\theta)}{\alpha}$.

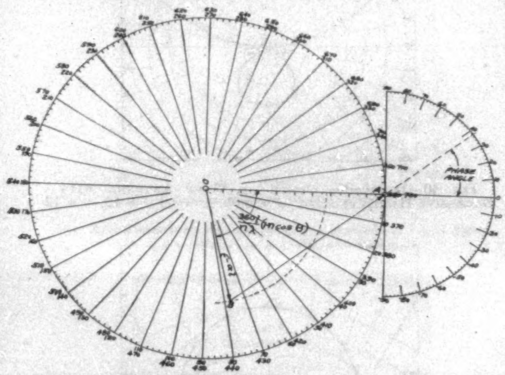

FIG. 42—CHART FOR DETERMINING MAGNITUDE AND PHASE OF $\left\{1-\epsilon^{-\alpha 2}\epsilon^{-\frac{j2\pi}{n\lambda}(1-n\cos\theta)}\right\}$

We have now determined the magnitude and phase angles of the quantities in the numerator and denominator of the expression for $\mathbf{I}_b$.

The magnitude of $\mathbf{I}_b$ is obtained by performing the arithmetical operations of multiplication and division using the absolute values of the vector quantities.

The phase angle of $\mathbf{I}_b$ is found by adding together the phase angles of the two vectors occurring as a product in the numerator and subtracting the phase angle of the vector in the denominator.

As an example let us find the magnitude and phase of the current $\mathbf{I}_b$ for an antenna of the following characteristics:

Wave length $\lambda = 12$ kilometers
Antenna length $l = 12$ kilometers
Attenuation constant $\alpha = 0.05$ per kilometer
Space wave velocity $v = 3\times10^5$ kilometer per second
Antenna wave velocity $u = 2.4\times10^5$ kilometers per second
Ratio $n = u/v = 0.8$ (numeric)
Ratio $\dfrac{\mathbf{E}_0}{2Z}$ assumed $=$ 1 ampere per kilometer
Angle at which signal strikes antenna $=\theta$ degrees

If we substitute the above values in equation (**22a**) using the form (absolute magnitude) $\underline{/\text{Phase angle}}$, to designate a vector quantity, we obtain

$$\mathbf{I}_b = \frac{\{1\underline{/-360^\circ\cos\theta}\}\{1\underline{/0^\circ}-0.55\underline{/-450^\circ(1-0.8\cos\theta)}\}\cos\theta}{\sqrt{(0.05)^2+[0.654(1-0.8\cos\theta)]^2}\,\underline{/\tan^{-1}(0.654\,(1-0.8\cos\theta)/0.05)}}$$

If we substitute various values of θ in this equation, we obtain the magnitude and phase of $\mathbf{I}_b$ as given in Table III. The calculation of magnitudes is taken from Table II, since the constants assumed are the same in the two cases.

Equation (**23**) or (**23a**) is the vector equation which determines the magnitude of $\mathbf{I}_a$ and its phase relation with respect to the reference vector $\mathbf{E}_a$. Equation (**23a**) is

$$\mathbf{I}_a = \frac{\mathbf{E}_0\cos\theta\,\{1-\epsilon^{-\alpha l}\epsilon^{-j\frac{2\pi}{n\lambda}l(1+n\cos\theta)}\}}{2\,\mathbf{Z}\left[\alpha+j\dfrac{2\pi}{n\lambda}(1+n\cos\theta)\right]} \qquad (\mathbf{23a})$$

If we substitute the assumed numerical values we obtain

$$\mathbf{I}_a = \frac{\{1-0.55\underline{/-450^\circ(1+0.8\cos\theta)}\}\cos\theta}{\sqrt{(0.05)^2+[0.654\,(1+0.8\cos\theta)]^2}\;\underline{/\tan^{-1}(0.6\text{[illegible]}\,(1+0.8\cos\theta)/0.05)}}$$

If we substitute the various values of θ in this equation, we obtain the magnitude and phase angle of the current $\mathbf{I}_a$ given in Table IV.

Compensation of Back End Currents. An example of a problem which requires the calculation of both the magnitude and phase of the end currents is the determination of the directive curve of an antenna in which the current due to waves coming from a particular "back end" direction is "balanced out" by means of some of the current from the other end of the antenna.

Let us imagine there is an intense source of disturbance directly behind the antenna whose directive curve is given in Fig. 35, and for which we have just calculated the magnitude and phase of $\mathbf{I}_b$ and $\mathbf{I}_a$, and that we wish to neutralize the effect of this disturbance by an adjustment of the damping circuit at A.

From equation (**22**) or Table III we have for $\theta =$ 180 deg., $\mathbf{I}_b = -0.968\,\underline{/+301\text{ deg.}}$

From equation (**23**) or Table IV we have for $\theta =$ 180 deg., $\mathbf{I}_a = -8.15\,\underline{/-40\text{ deg.}}$

A fraction of $\mathbf{I}_a$ is to be reflected in such phase that it will cancel $\mathbf{I}_b$ in the receiver.

The current in receiver at B is $\mathbf{I}_b = -0.968\,\underline{/+301}$ deg.

TABLE III

MAGNITUDE AND PHASE OF I_b

l = 12 Km. α = 0.05
λ = 12 Km. u = 0.8v.

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
θ Signal Angle	Cos θ	0.8 cos θ	1 − 0.8 cos θ	0.654 (1 − 0.8 cos θ)	$\sqrt{(0.5)^2 + \{0.654 (1 - 0.8 \cos \theta)\}^2}$	− 450 deg. (1 − 0.8 cos θ)	Vector Difference $\{1 - 0.55 \underline{/VII}\}$ from Diagram	VIII/VI	IX cos θ Magnitude of I_b for $\frac{E^0}{2Z} = 1$	Relative Current X ÷ 8.15	Angle of VIII from Vector Diagram	$\frac{0.654 (1 - 0.8 \cos \theta)}{0.05}$	Tan $^{-1}$ XIII	XII − XIV	− 360 deg. cos θ	XV + XVI Phase Angle of I_b
0	1.000	0.800	0.200	0.131	0.140	−90°	1.14	8.15	8.15	1.000	+29°	2.62	69.1°	−40.1	−360	−400.1°
20	0.940	0.752	0.248	0.1625	0.1698	−112	1.30	7.65	7.18	0.880	+24°	3.25	72.9°	−48.9	−338	−386.9°
40	0.766	0.613	0.387	0.253	0.258	−174.0	1.55	6.02	4.61	0.565	+2 °	5.07	78.8°	−76.8	−276	−352.8°
60	0.500	0.400	0.600	0.393	0.396	−270.0	1.14	2.88	1.44	0.177	−29°	7.85	82.7°	−111.7	−180	−291.7°
80	0.174	0.139	0.861	0.563	0.565	−387	0.57	1.01	0.175	0.0215	+26°	11.30	85.0°	−59.0	−62.6	−121.6°
100	−0.174	−0.139	1.139	0.744	0.745	−512	1.51	2.03	−0.353	−0.0433	+10°	14.9	86.2°	−76.2	+62.6	− 13.6°
120	−0.500	−0.400	1.400	0.916	0.916	−630	1.14	1.245	−0.623	−0.0765	−29°	18.30	86.9°	−115.9	+180	+ 64.1°
140	−0.766	−0.613	1.613	1.055	1.055	−726	0.46	0.434	−0.333	−0.0408	+8°	21.10	87.3°	−79.3	+276	+ 196.7°
160	−0.940	−0.752	1.752	1.146	1.146	−788	0.95	0.826	−0.778	−0.0955	+33°	22.90	87.5°	−54.5	+338	+ 283.5°
180	−1.00	−0.800	1.800	1.176	1.176	−810	1.14	0.970	−0.970	−0.1185	+29°	23.6	87.60°	−58.60	+360	+ 301.4°

TABLE IV

MAGNITUDE AND PHASE OF I_a

l = 12 km. α = 05
λ = 12 km. u = 0.8v

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV
θ Signal Angle	Cos θ	0.8 cos θ	1 + 0.8 cos θ	0.654 (1 + 0.8 cos θ)	$\sqrt{(0.05)^2 + \{0.654 (1 + 0.8 \cos\theta)\}^2}$	− 450 deg. (1 + 0.8 cos θ)	Vector Difference $\{1 - 0.55 \underline{/VII}\}$	VIII/VI	IX cos θ Magnitude of I_a for $E_0/2Z = 1$	Relative Current X ÷ 8.15	Angle of VIII from Vector Diagram	$\frac{0.654 (1 + 0.8 \cos \theta)}{0.05}$	Tan $^{-1}$ XIII	XII − XIV Phase Angle of I_a
0	1.000	0.800	1.800	1.176	1.176	−810°	1.14	0.97	0.97	0.1185	+29°	23.6	+87.6	−58.6°
20	0.940	0.752	1.752	1.146	1.146	−788°	0.95	0.826	0.778	0.0955	+33°	22.9	+87.5	−54.5°
40	0.766	0.613	1.613	1.055	1.055	−726°	0.46	0.434	0.333	0.0408	+8°	21.1	+87.3	−79.3°
60	0.500	0.400	1.400	0.916	0.916	−630°	1.14	1.245	0.623	0.0765	−29°	18.3	+86.9	−115.9°
80	0.174	0.139	1.139	0.744	0.745	−512°	1.51	2.03	0.353	0.0433	+10°	14.9	+86.2	−76.2°
100	−0.174	−0.139	0.861	0.563	0.565	−387°	0.57	1.01	−0.175	−0.0215	+26°	11.3	+85.0	−59.0°
120	−0.500	−0.400	0.600	0.393	0.396	−270°	1.14	2.88	−1.44	−0.177	−29°	7.85	+82.7	−111.7°
140	−0.766	−0.613	0.387	0.253	0.258	−174°	1.55	6.02	−4.61	−0.565	+2°	5.07	+78.8	−76.8°
160	−0.940	−0.752	0.248	0.1625	0.1698	−112°	1.30	7.65	−7.18	−0.880	+24°	3.25	+72.9	−48.9°
180	−1.000	−0.800	0.200	0.131	0.140	−90°	1.14	8.15	−8.15	−1.000	+29°	2.62	+69.1	−40.1°

The required compensating current at B is $\mathbf{I}_{cb} = -0.968 \underline{/+301} \text{ deg.} - 180 = -0.968 \underline{/+121} \text{ deg.}$

We have taken the compensating current as one half cycle or 180 deg. behind $\mathbf{I}_b$ in phase.

To allow for attenuation, and the time required for propagation from A to B, we must produce by the reflection at A, a current

$$\mathbf{I}_{ca} = \frac{-0.968}{\epsilon^{-\alpha l}} \underline{/+121° + 450°} = -1.76\underline{/+571°}$$

The 450 deg. is the angle corresponding to the time of propagation or $360° \frac{l}{n\lambda}$

Thus at A we have available a current $\mathbf{I}_a = -8.15 \underline{/-40}$ deg. and wish to produce for the purpose of compensation, a current $\mathbf{I}_{ca} = -1.76 \underline{/+571}$ deg.

This represents a phase advance of 611 deg.

We cannot reflect a wave before it arrives at the reflection point, which such a phase advance implies.

A current, however, which is two complete cycles behind the desired value; that is, to say a current $-1.76\ \underline{/+571\text{ deg.}} - 720$ deg. or $-1.76\ \underline{/-149\text{ deg.}}$ will give cancellation of all but the first two waves of the train, and this current which is 109 deg. behind $\mathbf{I}_a$ can be obtained by reflection. Experience has shown that in spite of the failure to neutralize the first two waves in a train, a very high degree of balance is obtainable, both on signals and static.

The terminal impedance required to give a specified reflection, may be determined from the vector relation

$$\mathbf{Z}_t = \mathbf{Z}\,\frac{\mathbf{I}_1 - \mathbf{I}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \mathbf{Z}\,\frac{1 - \mathbf{I}_2/\mathbf{I}_1}{1 + \mathbf{I}_2/\mathbf{I}_1} \qquad (\mathbf{25})^{15}$$

in which $\mathbf{Z}_t$ is the terminal impedance, $\mathbf{Z}$ the surge impedance of the line, $\mathbf{I}_1$ the current due to the oncoming wave and $\mathbf{I}_2$ the current due to the reflected wave. In the present case $\mathbf{I}_1$ is $\mathbf{I}_a$ and $\mathbf{I}_2$ is the desired $-1.76\ \underline{/-149\text{ deg.}}$

Then

$$\mathbf{I}_2/\mathbf{I}_1 = \frac{-1.76\underline{/-149^\circ}}{-8.15\underline{/-40^\circ}} = 0.216\underline{/-109^\circ}$$

$$= -0.070 - 0.204\,j$$

$$1 - \mathbf{I}_2/\mathbf{I}_1 = 1.070 + 0.204\,j$$

$$1 + \mathbf{I}_2/\mathbf{I}_1 = 0.930 - 0.204\,j$$

$$\frac{1 - \mathbf{I}_2/\mathbf{I}_1}{1 + \mathbf{I}_2/\mathbf{I}_1} = \frac{1.070 + 0.204\,j}{0.930 - 0.204\,j} = 1.055 + 0.450\,j$$

Substituting this value in (**25**) we have

$$\mathbf{Z}_t = Z\,(1.055 + 0.450\,j)$$

If the line surge impedance $\mathbf{Z}$ is 500 ohms, (with zero phase angle) the terminal impedance to give the desired neutralization would be 500 $(1.055 + 0.450\,j)$ or 527 ohms of non-inductive resistance and 225 ohms of inductive reactance.

Placing this impedance in the ground circuit at A will cause a similar reflection of all waves of the same length reaching A, whether due to a signal in line with the antenna or to a disturbance coming from a different direction. That is to say the reflected wave will in each case be 0.216 of the magnitude of the oncoming wave, and 109 deg. behind it in phase. When the reflected wave reaches the end B, its amplitude will be reduced to $0.55 \times 0.216 = 0.119$ of that of $\mathbf{I}_a$ and its phase will be further retarded by 450 deg. giving a total lag of 559 deg. behind I_a. Thus to find the directive curve of the antenna with the impedance described at A, we calculate I_a by equation (**22**), for each value of θ deg., determining the phase angle of $\mathbf{I}_a$ as well as its magnitude, multiply its magnitude by 0.119, retard its phase by 559 deg. and find the resultant when it is added vectorially to $\mathbf{I}_b$, calculated by formula (**23**) for the same value of θ deg.

Fig. 43 shows the relations between $\mathbf{I}_b$, $\mathbf{I}_a$ and the reflected current, for $\theta = 180$ deg.

In Table V we have given the calculations for the directive curve of the compensated antenna. The magnitudes and phase angles of $\mathbf{I}_a$ and $\mathbf{I}_b$ are taken from Tables III and IV. Instead, however, of rotating 0.119 I_a backward by 559 deg. and adding it vectorially to I_b, we have rotated it 180 deg. less, and subtracted it vertically from $\mathbf{I}_b$, which is an easier operation to perform and gives the same result. This backward rotation of 379 deg., is, for the purpose of finding the vector difference, equivalent to one of (379 deg. − 360 deg.) = 19 deg. The table shows the values of 0.119 I_a, rotated backward 19 deg., and the vector difference found by subtracting this from the corresponding $\mathbf{I}_b$. Thus to find the directive curve of any compensated antenna, we first choose the direction for which we wish to have zero reception. Then we compare $\mathbf{I}_a$ and $\mathbf{I}_b$ for this value of θ deg. and determine the factor by which $\mathbf{I}_a$ must be multiplied and the angle through which it must be rotated to make it coincide with $\mathbf{I}_b$. Then for each other value of θ deg. we multiply $\mathbf{I}_a$ by the same ratio and rotate it through the same angle, and subtract the vector so found, vectorially from $\mathbf{I}_b$.

Inspection of the table shows that in the example we have worked out, the effect of the reflection on the receiver current is negligible for values of θ less than 90 deg. Therefore, the front end of the directive curve is not appreciably altered or the reception of the desired signal affected by the reflection we have been considering.

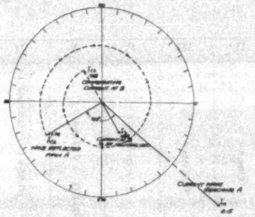

Fig. 43—Vector Relations in Compensation Problem

Fig. 44 shows the directive curve plotted from the calculations given above. This is for the same antenna and same conditions as Fig. 35, except that the "reflection balance" has been applied to give zero reception from the back.

In the case of an antenna producing relatively large back end currents, as, for example, a quarter wave length antenna, the front end of the directive curve would be considerably affected by a reflection designed to produce zero back end reception.

Effect of Reflection at Receiver End. The effect of reflection at the receiver end of the antenna may be shown as follows:

Let $\mathbf{I}_b$ = Wave built up on wire by signal at end B (Equation **22**)

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15. See page for derivation.

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TABLE V
CALCULATION OF COMPENSATED ANTENNA

$l = 12$ $\alpha = 0.05$
$\lambda = 12$ $u = 0.8v$

I	XVIII	XIX	XX	XXI	XXII	XXIII	XXIV	XXV	XXVI
θ Signal Angle	Magnitude of $\mathbf{I}_a$ Table IV Column X	Magnitude of Compensating Current $-\mathbf{I}_c$ XVIII × 0.119	Magnitude of $\mathbf{I}_b$ Table III Column X	Angle of $\mathbf{I}_a$ Table IV Coumn XV	Angle of $\mathbf{I}_c$ XXI − 18.5	Angle of $\mathbf{I}_b$ Table III Column XVII	Angle between $\mathbf{I}_b$ and $\mathbf{I}_c$ XXIII − XXII	Vector Difference $\mathbf{I}_b - \mathbf{I}_c$	XXV ÷ 8.05
0	0.97	0.115	8.15	−58.6°	−77.1	−400.1°	−323.0	8.05	1.00
20	0.778	0.093	7.18	−54.5°	−73.0	−386.9°	−313.9	7.12	0.883
40	0.333	0.040	4.61	− 79.3°	− 97.8	−352.8°	−255.0	4.62	0.573
60	0.623	0.074	1.44	−115.9°	−134.4	−291.7	−157.3	1.51	0.187
80	0.353	0.042	1.75	−76.2°	−94.7	−121.6°	−26.9	1.72	0.213
100	−0.175	−0.021	−0.353	−59.0°	−77.5	−13.6°	+ 63.9	0.343	0.043
120	−1.44	−0.172	−0.623	−111.7°	−130.2	+64.1	+194.3	0.79	0.098
140	−4.61	−0.55	−0.333	−76.8°	−95.3	+196.7°	+292.0	0.53	0.066
160	−7.18	−0.855	−0.778	−48.9°	−67.4	+283.5°	+350.9	0.16	0.020
180	−8.15	−0.970	−0.970	−40.1°	−58.6	+301.4°	+360.0	0	0

$\mathbf{I}_B$ = Total oncoming wave at *B* (Resultant of $\mathbf{I}_b$ and other waves traveling in same direction resulting from reflections at *A*.)

$\mathbf{I}_R$ = Receiver current or total current at *B*, = resultant of total oncoming wave $\mathbf{I}_B$ and reflected wave at *B*.

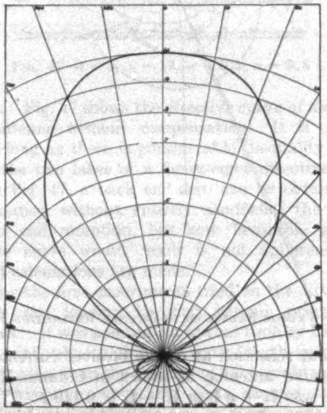

FIG. 44—DIRECTIVE CURVE OF WAVE ANTENNA. COMPENSATED BY REFLECTION FOR ZERO BACK END. $\lambda = 12$ KM., $l = 12$ KM., $\alpha = 0.05$, $u = 0.8\,v$

$\mathbf{I}_a$ = Current built up on wire at *A* by signal (Equation 23)

$\mathbf{I}_A$ = Total oncoming wave at *A* (wave moving in direction *B* to *A*) or resultant of $\mathbf{I}_a$ and other waves in same direction resulting from reflections at *B*.

b = reflection coefficient[16] at *B* or vector ratio of reflected to oncoming waves at *B*.

d $= \mathbf{b}\,\epsilon^{-(\alpha + j\beta)l}$

a = reflection coefficient at *A*

c $= \mathbf{a}\,\epsilon^{-(\alpha + j\beta)l}$

At *B* we have the oncoming wave $\mathbf{I}_B$ and reflected wave $\mathbf{b}\,\mathbf{I}_B$ giving the resultant current

$$\mathbf{I}_R = \mathbf{I}_B\,(1 + \mathbf{b}) \tag{26}$$

The wave $\mathbf{b}\,\mathbf{I}_B$ reflected from *B* becomes $\mathbf{b}\,\mathbf{I}_b\,\epsilon^{-(\alpha+j\beta)l}$ or $\mathbf{d}\,\mathbf{I}_B$ when it reaches *A*. It combines with $\mathbf{I}_a$ (the wave built up on the wire toward *A*) producing a total wave at *A*

$$\mathbf{I}_A = \mathbf{I}_a + \mathbf{d}\,\mathbf{I}_B \tag{27}$$

Likewis the wave $\mathbf{a}\,\mathbf{I}_A$ reflected from *A* becomes $\mathbf{a}\,\mathbf{I}_A\,\epsilon^{-(\alpha + j\beta)l} = \mathbf{c}\,\mathbf{I}_A$ when it reaches *B*. It combines with $\mathbf{I}_b$ (built up on the wire between *A* and *B*) giving a total wave reaching *B*

$$\mathbf{I}_B = \mathbf{I}_b + \mathbf{c}\,I_A \tag{28}$$

From (28)

$$\mathbf{I}_A = \frac{\mathbf{I}_B - \mathbf{I}_b}{\mathbf{c}}$$

Equating this to (27)

$$\frac{\mathbf{I}_B - \mathbf{I}_b}{\mathbf{c}} = \mathbf{I}_a + \mathbf{d}\,\mathbf{I}_B$$

Solving for $\mathbf{I}_B$

$$\mathbf{I}_B - \mathbf{I}_b = \mathbf{c}\,\mathbf{I}_a + \mathbf{c}\,\mathbf{d}\,\mathbf{I}_B$$

$$\mathbf{I}_B\,(1 - \mathbf{c}\,\mathbf{d}) = \mathbf{I}_b + \mathbf{c}\,\mathbf{I}_a$$

$$\mathbf{I}_B = \frac{\mathbf{I}_b + \mathbf{c}\,\mathbf{I}_a}{1 - \mathbf{c}\,\mathbf{d}} \tag{29}$$

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[16] The reflection coefficient is shown in equation (44) to be

$$\mathbf{I}_2/\mathbf{I}_1 = \frac{1 - \mathbf{Z}_t/\mathbf{Z}}{1 + \mathbf{Z}_t/\mathbf{Z}}$$ in which

$\mathbf{I}_1$ = current of oncoming wave
$\mathbf{I}_2$ = current of reflected wave
$\mathbf{Z}$ = surge-impedance of line
$\mathbf{Z}_t$ = impedance of terminal circuit

From (**26**) the total current in the receiver is

$$\mathbf{I}_R = \mathbf{I}_B (1 + \mathbf{b}) = \frac{1 + \mathbf{b}}{1 - \mathbf{c}\,\mathbf{d}} (\mathbf{I}_b + \mathbf{c}\, I_a) \qquad \textbf{(30)}$$

When there was no reflection at *B* we had a receiver current

$$\mathbf{I}_R = \mathbf{I}_b + \mathbf{c}\, I_a$$

We now have a receiver current in which $\mathbf{I}_b$ and $\mathbf{I}_a$ are combined in exactly the same manner, but their resultant is multiplied by the factor $\frac{1 + \mathbf{b}}{1 - \mathbf{c}\,\mathbf{d}}$, which being a function of line length and terminal conditions only, and not a function of $\mathbf{I}_a$, $\mathbf{I}_b$, or signal direction θ deg. does not alter the directive properties of the antenna.

The relation shown in (**30**) is strictly true only for steady state conditions. Let us illustrate what occurs when a train of waves first reaches the antenna. To take a simple case we may assume $\mathbf{I}_a = 0$, so that (**30**) becomes $\mathbf{I}_R = \mathbf{I}_b \frac{1 + \mathbf{b}}{1 - \mathbf{c}\,\mathbf{d}}$ as the ultimate current.

When the first wave $\mathbf{I}_b$ reaches *B* and is reflected the total current at *B* is

$$\mathbf{I}_b (1 + \mathbf{b})$$

The reflected wave reaches *A* with a value $\mathbf{d}\,\mathbf{I}_b$, is reflected back and reaches *B* with a value $\mathbf{c}\,\mathbf{d}\,\mathbf{I}_b$ giving a total wave $\mathbf{I}_b (1 + \mathbf{c}\,\mathbf{d})$ which by reflection at *B* produces a receiver current $(1 + \mathbf{b})\, \mathbf{I}_b (1 + \mathbf{c}\,\mathbf{d})$.

After a second double reflection the total oncoming wave at *B* is made up of the wave $\mathbf{I}_b$ reaching the receiver for the first time, a wave $\mathbf{c}\,\mathbf{d}\,\mathbf{I}_b$ which has been reflected down the line and back, and a wave $\mathbf{c}^2\, \mathbf{d}^2\, \mathbf{I}_b$ which has been down the line and back twice. The total oncoming wave $\mathbf{I}_b (1 + \mathbf{c}\,\mathbf{d} + \mathbf{c}^2\,\mathbf{d}^2)$ by reflection at *B* produces a receiver current

$$(1 + \mathbf{b})\, \mathbf{I}_b (1 + \mathbf{c}\,\mathbf{d} + \mathbf{c}^2\,\mathbf{d}^2)$$

After a large number of reflections the receiver current is

$$(1 + b)\, \mathbf{I}_b (1 + \mathbf{c}\,\mathbf{d} + \mathbf{c}^2\,\mathbf{d}^2 + \mathbf{c}^3\,\mathbf{d}^3 + \mathbf{c}^4\,\mathbf{d}^4 + \ldots)$$

$$= (1 + \mathbf{b})\, \mathbf{I}_b \frac{1}{1 - \mathbf{c}\,\mathbf{d}} = \mathbf{I}_b \frac{1 + \mathbf{b}}{1 - \mathbf{c}\,\mathbf{d}} \text{ as given by } \textbf{(30)}$$

If the reflection coefficient at either *A* or *B* is small (which is generally true) or if the line attenuation is high, $\mathbf{c}\,\mathbf{d}$ will be small and the steady state value of receiver current is approached very quickly.

Let us apply equation **30** to the case of the compensated 12 kilometer antenna whose directive curve is shown in Fig. 44. Here in order to give zero reception for $\theta = 180$ deg., we produced a reflection at *A* such as to give a current at *B* whose magnitude is 0.119 of that of $\mathbf{I}_a$ and whose phase is 559 deg. behind $\mathbf{I}_a$. Hence

$$\mathbf{c} = 0.119\, \epsilon^{j\,(-559°/57.3°)}$$

Suppose that instead of the surge impedance at the receiver end, we have a transformer having 400 ohms inductive reactance and 100 ohms effective resistance. Taking the surge impedance of the antenna as 500 ohms (non inductive) the reflection coefficient at *B* will be

$$\mathbf{b} = \frac{1 - \mathbf{Z}_t/\mathbf{Z}}{1 + \mathbf{Z}_t/\mathbf{Z}} = \frac{1 - (0.2 + 0.8\,j)}{1 + (0.2 + 8\,j)}$$

$$= \frac{0.8 - 0.8\,j}{1.2 + 0.8\,i} = 0.12 - 0.6\,j$$

$$= 0.612\, \epsilon^{j\,(-79°/57.3)}$$

Since

$$\epsilon^{-(\alpha + j\beta)\,l} = 0.55\, \epsilon^{j\,(-450°/57.3)}$$

$$\mathbf{d} = \mathbf{b}\, \epsilon^{-(\alpha + j\beta)\,l} = 0.336\, \epsilon^{j\,(-529°/57.3)}$$

and

$$\mathbf{c}\,\mathbf{d} = 0.04\, \epsilon^{j\,(-1088°/57.3)} = 0.04\,(\cos 1088° + j \sin 1088°) = 0.0396 - 0.0056\,j$$

$$\frac{1 + \mathbf{b}}{1 - \mathbf{c}\,\mathbf{d}} = \frac{1.12 - 0.6\,j}{0.9604 + 0.0056\,j} = 1.163 - 0.63\,j$$

$$= 1.323\, \epsilon^{j\,(-28.5°/57.3)}$$

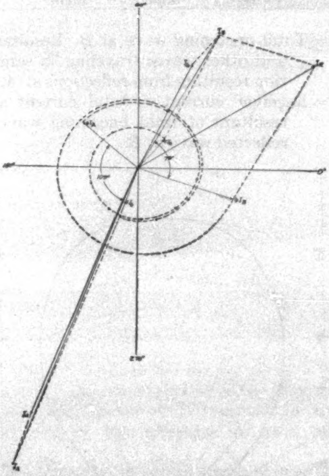

FIG. 45—VECTOR RELATIONS IN DOUBLE REFLECTION PROBLEM

Without reflection at *B*, the receiver current of the compensated antenna, as shown in Column XVII of Table V was $\mathbf{I}_b + \mathbf{c}\,\mathbf{I}_a$ or $\mathbf{I}_b + \mathbf{I}_a \times 0.119\, \epsilon^{j\,-559°/57.3)}$

With the reflections caused by the receiver circuit we have just been considering the current in the transformer primary will be

$$1.323\, \epsilon^{j\,(-28.5°/57.3)}\, (\mathbf{I}_b + \mathbf{I}_a \times 0.119\, \epsilon^{j\,(-559°/57.3)})$$

or a current of 32.3 per cent greater for each value of θ than that given in column XVII of Table V. The directive curve or *relative* current for different values of θ is the same as before. Fig. 45 shows the vector relations of the currents at the ends of the line, for the signal direction $\theta = 120$ deg. Both $\mathbf{I}_b$ and $\mathbf{I}_a$ as given by formulas (22) and (23) have negative signs, and the negative signs are retained throughout rather than reversing the vectors in the diagrams, which would somewhat obscure the angular relations. If the entire diagram is turned 180 deg. (looked at up side down) the vectors appear in their true positions.

Short Antennas. The question of the possibility of obtaining in less space directive properties approaching those of a full wave length antenna is of considerable interest. Fig. 47 shows the directive curve of a quarter wave antenna without compensation. It is obvious that so long as there is considerable inequality in the size of the two lobes of a figure-eight directive curve such as Fig. 47, a back end zero can be obtained by compensation without entirely sacrificing the signal, or front end reception, but how favorable a total directive curve would result is not apparent until detailed calculations are made.

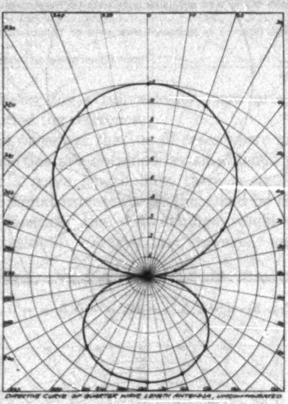

Fig. 47—$l = 3$, $\lambda = 12$, $\alpha = 0.5$, $n = 0.8$

Among the first experiments tried on the Riverhead antenna was loading to give various propagation velocities. It was pointed out at that time by Rice that reducing the velocity on a full wave length antenna to something less than the velocity of light would result in a sharper directive curve, but tests showed that the natural velocity of the line was so nearly equal to the best velocity that there was little to be gained by loading. The urgent problem at the time was to obtain the best possible reception of European signals, utilizing as much space as seemed conducive to this result. Tests had shown the half wave antenna, with back end compensation, to be definitely inferior to the full wave length antenna. The question of short antennas was therefore not investigated until some months later when the principal engineering problems connected with multiplex reception with the wave antenna had been worked out.

At the suggestion of Mr. R. H. Ranger, of the Radio Corporation, calculations were made by Kellogg of the directive properties of short antennas on which the velocity had been reduced by loading to the best value for the wave length and antenna length in question. Mr. Ranger reasoned that since reducing the velocity sharpened the directive curve of a full wave length antenna, it might be possible, by sufficiently reducing the velocity, to compensate for reduced length and perhaps obtain a good directive curve in very small space. The most favorable velocity was found to be that which gave zero (or minimum) reception at the back end, without compensation. The condition for this is that

$$v/u = \lambda/l - 1$$

in which l is the length of the antenna
v is the velocity of light
u is the antenna velocity
λ is the wave length

Figs. 48 and 49 show directive curves for quarter and eighth wave length antennas with velocities equal to one third and one seventh of that of light respectively in accordance with the above equation.

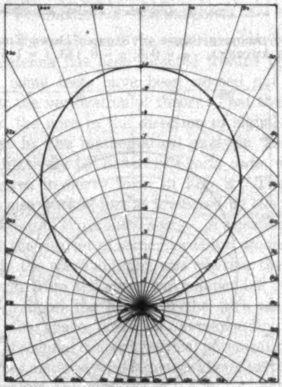

Fig. 48—Directive Curve of Slowed Down Quarter Wave Length Antenna $\alpha = 0.5$, $n = 0.333$, $l = 3$, $\lambda = 12$

By way of comparison, the directive curves Figs. 50 and 51 have been calculated for compensated antennas without loading. The back end areas of these curves are seen to be slightly greater.

Multiplex reception with different compensation for each wave length is possible with the unloaded, compensated antennas. On the other hand, with the slowed down antenna, the velocity is right only for one wave length. In order to receive other wave lengths, compensation would be employed in addition to the loading.

Fig. 52 shows the directive curve for an 18 kilometer wave length of the slowed down antenna, whose directive curve for $\lambda = 12$ kilometers is shown in Fig. 48. The dotted line *A* shows the directive properties without compensation and the solid line *B* is the directive curve with compensation. Fig. 53 shows the directive curve for the same antenna receiving an eight kilometer wave. We observe that the slowed down antenna can be multiplexed by compensation and receive longer waves than that for which its velocity is adjusted, but that for shorter waves it is unfavorable.

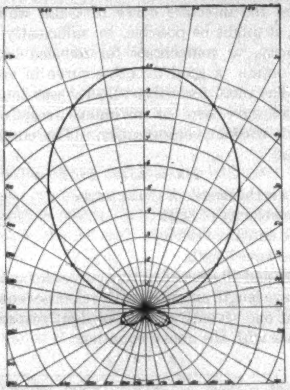

Fig. 49—Directive Curve of Slowed Down Eighth Wave Length Antenna. $\alpha = 0.5$, $n = 0.143$, $l = 1.5$, $\lambda = 12$

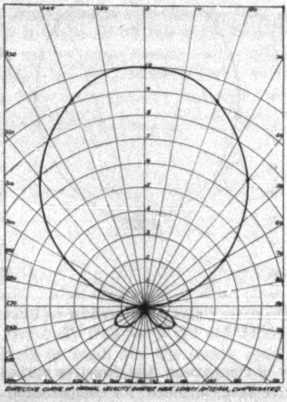

Fig. 50—Directive Curve of Normal Velocity Quarter Wave Length Antenna, Compensated. $l = 3$, $\lambda = 12$, $\alpha = 0.5$, $n = 0.8$

Compared with the simple unloaded, compensated antenna, the slowed down antenna has a better directive curve at one wave length, but a narrower range of satisfactory multiplex reception. The signal strength is of the same order of magnitude on both—considerably less than on an uncompensated full velocity antenna of the same length. The intensity factor or receiver current per unit value of the quantity $\frac{E_0}{2Z}$ in equation (**22**), is shown in Table VI for the short antennas whose directive curves are given in Figs. 47 to 53. The

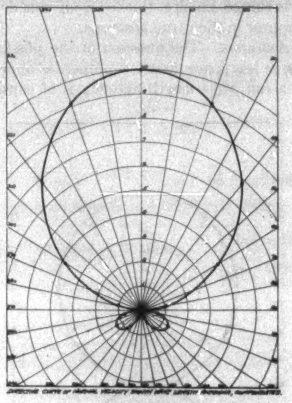

Fig. 51—$l = 1.5$, $\lambda = 12$, $\alpha = 0.5$, $n = 0.8$

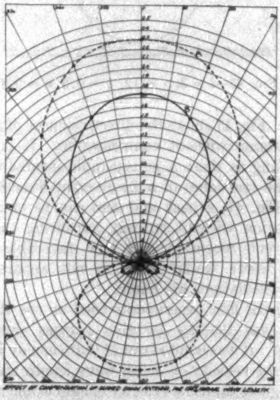

Fig. 52—*A* Uncompensated. *B* Compensated. $l = 3$ km., $\lambda = 18$, $\alpha = .05$, $n = 0.333$

TABLE VI
RELATIVE SIGNAL INTENSITIES ON SHORT ANTENNAS

	Antenna Length Kilometers l	Wave Length Kilometers λ	Velocity Ratio n	Attenuation Constant α	Directive Curve Figure No.	Intensity Factor
Ideal Antenna, one eighth wave length long, Uncompensated	1.5	12	1.0	0.0		1.5
Average Unloaded Antenna, $l = 1/8\ \lambda$, Uncompensated	1.5	12	0.8	0.05		1.44
Same, Compensated for zero reception at $\theta = 180$ deg.	1.5	12	0.8	0.05	51	0.33
Slowed-down Antenna $l = 1/8\ \lambda$, No compensation	1.5	12	0.143	0.05	49	0.44
Ideal, Quarter-wave-length Antenna, Uncompensated	3	12	1.0	0		3.0
Average Unloaded Antenna $l = 1/4\ \lambda$, Uncompensated	3	12	0.8	0.05	47	2.8
Same, Compensated to give zero reception for $\theta = 180$ deg.	3	12	0.8	0.05	50	1.92
Slowed-down, Quarter-wave Antenna, $l = 1/4\ \lambda$, No Compensation	3	12	0.333	0.05	48	1.78
Same, Compensated to give zero reception at $\theta = 180$ deg., for $\lambda = 18$	3	18	0.333	0.05	52	1.73
Same, Compensated to give zero reception at $\theta = 180$ deg., for $\lambda = 8$	3	8	0.333	0.05	53	0.44
Average Antenna, one wave length long	12	12	0.8	0.05	35	8.15

intensity factor for a full velocity, zero attenuation line is included for reference.

It will be observed that in all cases where unidirectional properties are obtained on an antenna a small fraction of a wave length long, there is considerable sacrifice of signal intensity. If this could be made up with amplification the disadvantage would not be serious. Residual voltages are, however, inevitable in radio receiving systems. For example, electromotive forces are induced in the end verticals, of which no account is taken in the calculations given here; foreign circuits at crossings or unavoidable short parallels induce electromotive forces in the antenna wires, transmission lines used to carry signal or compensating currents are not absolutely quiet, transformer or receiving circuit coils pick up some disturbances, and where high amplification is employed tube noises are a factor. With strong signals such as obtained with full length antennas, these effects may, by careful engineering, be kept negligibly small compared with the signal, but if the signal produced by the antenna is weak, or if it is a small remainder after comparatively strong currents are combined to give neutralization for a certain direction, the directive properties actually obtained are likely to be decidedly inferior to those indicated by calculations, based on assumed ideal conditions.

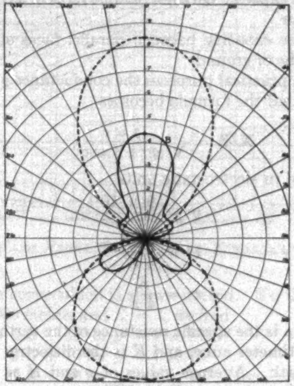

Fig. 53—*A* Uncompensated. *B* Compensated. $l = 3$, $\lambda = 8$, $\alpha = 0.5$, $n = 0.333$

Wherever possible, therefore, the writers have advocated building full wave length or at least, half wave length antennas for commercial reception.

Fundamental Experiments

While for the most part the experimental work on the wave antenna has been directed toward practical utilization, some tests have been carried out whose purpose was the verification of theory: That the waves built up on the wire in the direction of signal travel, as indicated by the mathematical analysis, had been shown qualitatively by Beverage's early data, some of which is given in curve form in Fig. 2. The series condensers also worked just as predicted by the theory.

Tests Bearing on Wave Tilt Theory. A question which was at first the subject of some discussion among those interested in the wave antenna was whether the collection of energy from the space wave depended on wave front tilt, or upon the height of the wire above the earth and the space potential corresponding to its position. If a signal causes a vertical potential gradient $+G$ volts per meter, a wire h meters above earth would tend to assume the potential $+Gh$. A half wave length away where the potential gradient due to the signal wave is $-G$; the wire would tend to assume the potential $-Gh$, and the potential difference along the wire would give rise to a current. According to such a picture the wave antenna would be equivalent to an infinite number of small static antennas whose charging current is carried over the line to the ends. If a formula for the end current is worked out on this basis it shows the same directive properties as the formula we have developed, except that the factor $\cos\theta$ by which E_0 is multiplied in (**22**) is omitted. Thus in Table II we should use the figures in column IX instead of column

X to obtain the directive curve. The difference is especially marked in the case of a half wave length antenna with signals coming at right angles to the antenna. If the half wave length wave antenna were acting like a series of static antennas the reception for $\theta = 90$ deg. would be about half of the full intensity or half that for $\theta = 0$. On the other hand if the wave antenna depends on wave front tilt the reception from 90 deg. to the antenna would be zero for all lengths. As a test of this the Belmar wave antenna was cut at a point about 5.8 kilometers from the station, giving a short antenna, with the 200 kw. New Brunswick sending station ($\lambda = 13{,}600$ meters) on the side. The reception of New Brunswick, only 50 miles away, was of the same order of intensity as European stations, thus indicating that the half wave antenna was substantially dead on the side.

When the new antenna at Riverhead was completed, it afforded an opportunity to test a long loop, as a receiving circuit. The two top wires which were 9 meters above ground were connected through from Riverhead to Terrell River or the full length of the antenna, and two of the lower wires which were about 5½ meters above ground, were similarly connected through. The upper and lower wires were connected together through a damping resistance at Riverhead (the north-east end), and the receiving set was connected between the upper and lower wires at Terrell River. European signals were extremely weak on this loop, as compared with intensity obtained when the receiving set was connected from either pair or wires to ground. In the latter case there appeared to be no choice between the upper and lower wires.

In other words there is no differential effect due to the difference in height of the wires. The effect of height was again tested at Schenectady using a wave length of 120 meters and a wire one wave length long at heights ranging from 0.8 meters to 2.9 meters. The received current was found to be practically independent of the height. This is what would be expected from an antenna which works by virtue of the forward tilt of the wave front. If the currents in the antenna were due to the space potential of the wire above ground the received current would be proportional to the height of the wire.

There is another conclusion which follows from the theory that the action of the wave antenna depends upon the tilt of the wave front. Low, wet ground, or salt marsh would be an unfavorable location so far as signal intensity is concerned. No experimental evidence on this point has come to the writers' attention. Before conclusions can be drawn from tests or comparisons it is necessary to make sure that conditions are the same in other respects. In one instance with which the writers are familiar, two antennas, in locations several miles apart were constructed parallel and as much alike as possible. One followed a small stream and the other was over comparatively dry ground. The two antennas gave substantially equal signal intensities. Presumably in this case the low resistance ground was too restricted and local to have any appreciable effect on the tilt of the twelve to fourteen kilometer waves on which the observations were made. For ground of a given resistance the wave front tilt increases as the wave length decreases, and for short waves (less than 1000 meters) there is a substantial tilt, sufficient for satisfactory operation even over wet ground.

Zenneck[17] has worked out equations for the forward tilt of the wave front as a function of wave length, and the specific resistance and dielectric constant of the soil. The tilt is expressed as the ratio of the horizontal to the vertical potential gradient of the space wave.

In its general form Zenneck's formula is

$$\mathbf{X}/\mathbf{Z} = \sqrt{\frac{g + j\,\omega\,c}{g' + j\,\omega\,c'}} \qquad (31)$$

in which

X is the horizontal potential gradient of the space wave

Z is the vertical potential gradient of the space wave

g is the leakage conductance between parallel faces of a centimeter cube of air

g' is the leakage conductance between parallel faces of a centimeter cube of ground

c is the capacity between parallel faces of a centimeter cube of air

c' is the capacity between parallel faces of a centimeter cube of ground

For all practical purposes the conductance g for air is zero, so that the formula becomes

$$\mathbf{X}/\mathbf{Z} = \sqrt{\frac{j\,\omega\,c}{g' + j\,\omega\,c'}} \qquad (32)$$

Expressed in electrostatic units

$$g' = \frac{9 \times 10^{11}}{\rho}$$

$$c = \frac{1}{4\,\pi}$$

$$c' = \frac{K}{4\,\pi}$$

in which ρ is the specific resistance of the earth in ohms for a centimeter cube, and K is the dielectric constant of the earth. Average values of ρ and K as given in Fleming[18] are shown in Table VII.

TABLE VII.

	ρ = Ohms per centimeter cube	k = Dielectric constant
Sea Water	100	80
Fresh Water	10,000 to 100,000	80
Moist Earth	1000 to 100,000	5 to 15
Dry Earth	1,000,000 and up	2 to 6
Wet Sand	100 to 10,000	9
Dry River Sand	very large	2-3
Wet Clay	100 to 10,000	
Dry Clay	1,000,000 and up	2-5

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[17] See "Principles of Electric Wave Telegraphy and Telephony" by J. A. Fleming, Third Edition, P. 801.

[18] Principles of Electric Wave Telegraphy and Telephony by J. A. Fleming, Third Edition P. 800.

Fig. 55 shows the values of **X/Z** for various values of wave length λ, specific resistance ρ and dielectric constant K. With long waves and low-resistance soils $\omega c'$ is negligible compared with g', in which case

$$\mathbf{X/Z} = \sqrt{\frac{j\,\omega\,c}{g'}} \qquad (\mathbf{33})$$

This is a function of wave length and specific resistance, and is shown in the sloping lines of Fig. 55. On the other hand if the waves are so short and the soil resistance so high that g' is negligible compared with $\omega c'$ we have

$$\mathbf{X/Z} = \sqrt{C/C'} = \sqrt{1/K} \qquad (\mathbf{34})$$

which is independent of wave length or soil conductivity and is shown for several values of K in the horizontal lines at the top of the figure. To find the value of **X/Z** for a certain wave length and a soil of a given resistivity and dielectric constant we use whichever curve (the sloping line of equation (**33**) or the horizontal line of equation (**34**) gives the lower value for **X/Z**. If near the intersection of the two straight lines we use the transition curve which is shown dotted. To illustrate, if λ = 1000 meters and K = 4, we find

$\mathbf{X/Z} = 1.3 \div 10$ for $\rho = 1 \times 10^5$
$2.5 \div 10$ for $\rho = 4 \times 10^5$
$3.9 \div 10$ for $\rho = 1 \times 10^6$ (on transition curve)
$4.9 \div 10$ for $\rho = 4 \times 10^6$ or greater. (On horizontal line for K = 4).

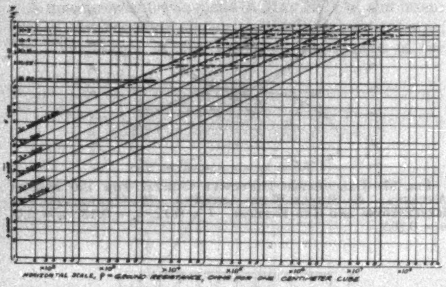

FIG. 55—WAVE FRONT TILT, X/Z BY ZENNECK'S FORMULA

It will be noted that in Zenneck's formula (**32**), the ratio **X/Z** is a vector quantity whose phase angle ranges from 0 deg. to 45 deg. By far the most common condition is that g' greatly exceeds $\omega c'$ (*i. e.* the earth carries current by conduction rather than by capacity) in which case the phase angle is nearly 45 deg. The phase difference means that the vertical and horizontal potential gradients do not become zero simultaneously, but the electric field is a rotating one. Under these circumstances if a straight conductor is held in various positions in a vertical plane parallel to the direction of wave propagation, there will be no direction of the conductor in which the electromotive force induced in it is zero. This may account for the doubtful results obtained in attempts to measure wave tilt by observing the angle of zero or minimum electromotive force in a straight conductor rotated in a vertical plane parallel to the signal direction. Wherever the tilt is considerable so that it might be readily measured, the minimum is correspondingly dull.

More satisfactory as a test of the theoretical conclusions, would be quantitative measurements of the relative magnitudes of the electromotive forces induced in horizontal and vertical conductors, for various wave lengths and ground conductivities. Determination of the phase relations would provide a further check.

An observation of relative signal intensities on a large loop and a wave antenna, indicated a wave tilt of the order of magnitude called for by Zenneck's formula, but little data of this kind have been taken. The large values of horizontal voltage gradient found in the measurements mentioned below by Beverage and Weinberger were at first considered greater than could be accounted for by wave tilt. Assuming probable values of ground resistance, the ratio of horizontal to vertical potential gradient according to Zenneck's formula is of the order of magnitude of one or two per cent, whereas the measured horizontal gradient was about 30 per cent of the vertical gradient calculated by Austin's formula. The space potential theory of action however is still less capable of accounting for the potentials observed. If we assume ground water to be 100 feet (30 meters) below the surface of the ground, and the earth above ground water level to have a specific resistance of 2×10^6 ohms per centimeter cube, which is about the value found by measurement, we find for a 15,000 meter wave length that the potential difference between ground water and surface would be less than that corresponding to a difference of elevation of two feet in the space above ground. Considering the ground as constant potential, and expressing the vertical potential gradient as $G\,\epsilon^{-j(2\pi x/\lambda)}$ (in which $\epsilon^{-j(2\pi x/\lambda)}$ expresses the change of phase with distance x measured in direction of propagation) the potential of the wire at a height h with respect to ground would be $h\,G\,\epsilon^{-j(2\pi x/\lambda)}$ and the potential gradient along the wire would be $\frac{d}{dx}\,h\,G\,\epsilon^{-j(2\pi x/\lambda)}$ $= -j\frac{2\pi}{\lambda}\,h\,G\,\epsilon^{-j(2\pi x/\lambda)}$. If the height of the wire is 10 meters and the wave length 15,000 meters the magnitude of the horizontal potential gradient would be $\frac{2\pi h}{\lambda}G = \frac{2\pi \times 10}{15{,}000}G = 0.0042\,G$ or 0.0042 of that of the vertical gradient, which is less than Zenneck's formula gives for the horizontal potential gradient due to wave tilt. Theoretical analyses agree moreover, that there is no electromotive force induced in a horizontal wire over a perfectly conducting earth, and therefore the space potential picture of operation is untenable.

Experimental Directive Curve. Fig. 56 shows a directive curve obtained experimentally. A transmitting set was operated, supplying about 5 kilowatts to a small vertical antenna, at 120 meters wave length. On a field about 600 meters from the transmitting station a system of wave antennas was erected consisting of twenty-four lines each 55 meters long and about one meter above ground, radiating from a central point like the spokes of a wheel. By joining two opposite spokes together at the center, a wave antenna was obtained 110 meters long. Using the next pair gave a similar antenna 15 deg. from the last. A ground of about 20 ohms resistance was provided at each end of each antenna. The current at one end of the antenna was measured with a thermocouple and galvanometer, the opposite end being damped by a resistance. Measurements were taken successively on the several antennas, while the radiation was kept as nearly constant as possible. From the series of readings thus obtained Fig. 56 was plotted.

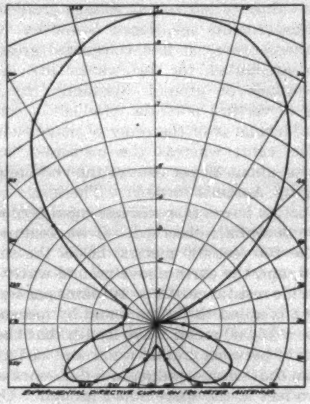

FIG. 56—$l = 110$, $\lambda = 120$

A number of factors was present to cause a difference between the shape of the experimental directive curve Fig. 56 and the curve *A* of Fig. 57 calculated by equation (22). The resistance used at the end opposite the ammeter was not the true surge impedance of the line, as determined by later measurements. The end verticals were high enough in comparison with the length of the antenna to cause currents of considerable relative magnitude, the ground was not perfectly level, and the divergence of the waves was appreciable on account of the nearness of the sending station. Additional observations had been planned, but the work was interrupted before another directive curve could be obtained. As it stands the directive curve shown in Fig. 56 serves as a qualitative check on the theory.

During the same series of short wave tests, readings were taken to show the building up of the current in an antenna. A wire sectionalized every ten meters was used in the antenna which pointed toward the transmitting station. The line was broken successively at the sectionalizing points and the current read with the thermocouple and galvanometer. The values of current are plotted in Fig. 58. Humps such as appear in the curve might be due in part to imperfect damping, but such humps are to be expected from the theory. The total current at any point *X* in the antenna Fig. 26 is the resultant of the forward wave built up on the part of the antenna between *A* and *X*, and the back wave from the part between *X* and *B*.

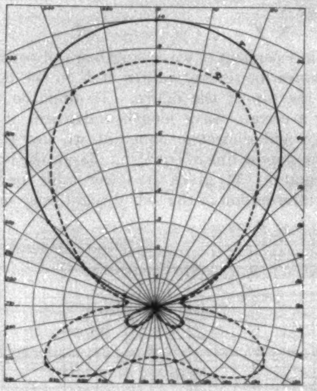

FIG. 57—CALCULATED DIRECTIVE CURVES FOR 120 METER ANTENNA. A—SIMPLE WAVE ANTENNA. B—CORRECTED FOR REFLECTIONS AND END EFFECTS. $l = 0.12$, $\lambda = 0.13$, $\alpha = 2.0$, $n = 0.865$.

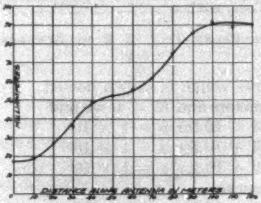

FIG. 58—OBSERVED CURRENT DISTRIBUTION IN 120 METER ANTENNA

The complete expression for the current at a point *X* on an antenna of length *l*, then becomes,

$$\mathbf{I}_x = \frac{E_0 \cos\theta\, \epsilon^{-j(2\pi x \cos\theta)/\lambda}}{2\,\mathbf{Z}} \times \left\{ \frac{1 - \epsilon^{-\alpha x}\, \epsilon^{-j\frac{2\pi x}{n\lambda}(1 - n\cos\theta)}}{\alpha + j\dfrac{2\pi}{n\lambda}(1 - n\cos\theta)} + \frac{1 - \epsilon^{-\alpha(l-x)}\, \epsilon^{-j\frac{2\pi}{n\lambda}(l-x)(1+n\cos\theta)}}{\alpha + j\dfrac{2\pi}{n\lambda}(1 + n\cos\theta)} \right\} \quad (35)$$

Fig. 59 shows the current distribution in an ideal antenna one wave length long, calculated by equation (**35**). Thus the building-up curve found by measuring the current at various points in the line, is of different form from that found by changing the length of the antenna and measuring the end currents. The latter shows a continuous increase as shown in Fig. 29.

A measurement was made in May 1921 of the intensity of the received signals on the Riverhead antenna. Mr. Weinberger of the Research Department of the Radio Corporation, brought to Riverhead a calibrated oscillator, by which a known voltage at the desired frequency could be supplied to a circuit. By this means a voltage of signal frequency was introduced in series with the damping resistance at the north east end of the antenna, and adjusted to give as loud a tone in the receiver as the European signal which was being measured. The results of Mr. Beverage's and Mr. Weinberger's observations were, P. O. Z. Nauen, Germany, 80 millivolts; M. U. U., Carnarvon, Wales, 54 millivolts.

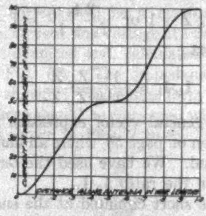

FIG. 59—CALCULATED CURRENT DISTRIBUTION IN WAVE ANTENNA

These correspond to about 9 and 4 microwatts respectively of received energy on the antenna.

Since the antenna is 14.5 kilometers long the voltage readings indicate a horizontal potential gradient of 5.5 millivolts per kilometer for Nauen and 3.7 millivolts per kilometer for Carnarvon. These values represent normal receiving conditions. During fading periods the signals are much weaker.

ANTENNA CONSTANTS

The electrical constants of an antenna or line which are of most immediate interest, are the wave velocity *u*, attenuation constant α, and surge impedance **Z**. These may be ascertained by measuring the input impedance of the line through a sufficient range of frequency, first with the far end of the line open, and then with it short-circuited, (or grounded, if we are dealing with a ground return circuit). The ground connection must be of low resistance, for the equations which follow are based on the assumption of a short-circuit reflection, and all losses will therefore be attributed to the line attenuation.

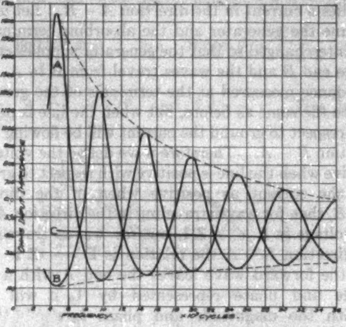

FIG. 60—INPUT IMPEDANCE OF TWELVE KILOMETER ANTENNA. A—FAR END GROUNDED. B—FAR END OPEN. C—GEOMETRICAL MEAN OF A AND B.

As the frequency of the current supplied to the line is varied, a series of maximum and minimum current values are observed, corresponding to standing wave conditions which cause current loops and current nodes. A current maximum corresponds to an impedance minimum and a current minimum to a maximum impedance. The impedance may be determined from the supplied voltage and current or by the substitution method, which will be described. Only the maximum and minimum values of impedance are required for the present purpose.

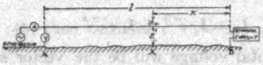

FIG. 61—REFERENCE FIGURE FOR DISCUSSION OF LINE IMPEDANCE

Fig. 60 shows the input impedance of a 12-kilometer two-wire antenna as a function of frequency. A maximum input impedance when the far end is open is seen to occur at the same frequency at which the input impedance is a minimum when the far end is short-circuited. The frequencies at which the maxima and minima occur indicate the line velocity, while the ratio of minimum to maximum impedance gives a basis for calculating the attenuation.

Determination of Constants from Input Impedance. Fig. 61 shows a generator (or oscillator) supplying alternating current to a line. This results in waves which travel from A to B. Assuming for the time being that there is no reflection at B and no return waves, the relation of voltage to current is $\mathbf{E}/\mathbf{I} = \mathbf{Z}$ in which $\mathbf{Z}$ is the surge impedance of the line. When the generator makes the point A positive with respect to ground it supplies a current in the direction $A - B$. Let us now imagine the generator and the absorbing terminal circuit interchanged so that the generator supplies current to the end B and the waves travel from B to A. The impedance measured at the terminals of the generator will be the same as before, but when the line is positive with respect to ground the current will be flowing in the direction $B - A$. Therefore if we define the voltage as positive when the line is positive with respect to ground, and the current as positive when in the direction $A - B$, then for waves traveling in the direction $A - B$ the voltage is $\mathbf{E} = + \mathbf{Z}\,\mathbf{I}$, but for waves traveling in the direction $B - A$ the relation is $\mathbf{E} = - \mathbf{Z}\,\mathbf{I}$.

If the forward wave leaves the end A with a magnitude and phase represented by $\mathbf{I}_0$ it will reach B with the magnitude and phase $\mathbf{I}_0\,\epsilon^{-(\alpha + j\beta) l}$ or $\mathbf{I}_0\,\epsilon^{-\gamma l}$ in which $\gamma = \alpha + j\beta$. We shall represent the forward wave as it reaches B by the symbol $\mathbf{I}_1$. Then since $\mathbf{I}_1 = \mathbf{I}_0\,\epsilon^{-\gamma l}$, $\mathbf{I}_0 = \mathbf{I}_1/\epsilon^{-\gamma l} = \mathbf{I}_1\,\epsilon^{\gamma l}$. Likewise at any point X, Fig. 61, x kilometers from B measured toward A, the current of the forward wave is $\mathbf{I}_1\,\epsilon^{\gamma x}$. And the voltage of the forward wave at X is $\mathbf{Z}\,\mathbf{I}_1\,\epsilon^{\gamma x}$.

If the reflected wave leaves B with a magnitude and phase represented by $\mathbf{I}_2$, it will be $\mathbf{I}_2\,\epsilon^{-\gamma x}$ when it reaches X, and $\mathbf{I}_2\,\epsilon^{-\gamma l}$ when it reaches A. And the voltage of the return wave is $- \mathbf{Z}\,\mathbf{I}_2$ at B, $- \mathbf{Z}\,\mathbf{I}_2\,\epsilon^{-\gamma x}$ at X and $- \mathbf{Z}\,\mathbf{I}_2\,\epsilon^{-\gamma l}$ at A.

When waves traveling in both directions are present, the total current in the line at any point X will be the vector sum of the currents due to the two trains of waves or

$$\mathbf{I}_x = \mathbf{I}_1\,\epsilon^{\gamma x} + \mathbf{I}_2\,\epsilon^{-\gamma x} \tag{36}$$

And the total voltage will be the vector sum of the voltages $\mathbf{Z}\,\mathbf{I}_1\,\epsilon^{\gamma x}$ and $- \mathbf{Z}\,\mathbf{I}_2\,\epsilon^{-\gamma x}$, or

$$\mathbf{E}_x = \mathbf{Z}\,\mathbf{I}_1\,\epsilon^{\gamma x} - \mathbf{Z}\,\mathbf{I}_2\,\epsilon^{-\gamma x} \tag{37}$$

The ratio of voltage to current, or the impedance at X is

$$\mathbf{E}_x/\mathbf{I}_x = \mathbf{Z}\,\frac{\mathbf{I}_1\,\epsilon^{\gamma x} - \mathbf{I}_2\,\epsilon^{-\gamma x}}{\mathbf{I}_1\,\epsilon^{\gamma x} + \mathbf{I}_2\,\epsilon^{-\gamma x}} \tag{38}$$

or at A the impedance is

$$\mathbf{Z}_A = \mathbf{Z}\,\frac{\mathbf{I}_1\,\epsilon^{\gamma l} - \mathbf{I}_2\,\epsilon^{-\gamma l}}{\mathbf{I}_1\,\epsilon^{\gamma l} + \mathbf{I}_2\,\epsilon^{'\gamma l}} \tag{39}$$

This becomes equal to Z if $I_2 = 0$, and becomes $- Z$ if $I_1 = 0$. The negative sign means that power is being supplied from the line to the terminal circuit by waves which travel in the direction $B - A$, instead of from the terminal circuit to the line as is the case for waves leaving A. If the waves leaving A are larger than those arriving at A (*i. e.* if $\mathbf{I}_1\,\epsilon^{\gamma l} > \mathbf{I}_2\,\epsilon^{-\gamma l}$) there is a net power supply to the line.

If the line is made very short $\mathbf{Z}_A$, the impedance at A, of the line plus the terminal circuit, must eventually become equal to that of the terminal circuit alone or equal to Z_t. Setting $l = 0$ in (**39**) and equating to $\mathbf{Z}_t$, we have

$$\mathbf{Z}_{A\ (\text{for } l = 0)} = \mathbf{Z}\,\frac{\mathbf{I}_1 - \mathbf{I}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \mathbf{Z}_t \tag{40}$$

The value of $\mathbf{Z}_t$ relative to $\mathbf{Z}$ determines the relation of the reflected wave $\mathbf{I}_2$ to the oncoming wave $\mathbf{I}_1$ at the reflection point B. From (**40**)

$$\frac{\mathbf{I}_1 - \mathbf{I}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \mathbf{Z}_t/\mathbf{Z} \tag{41}$$

Adding 1 to each side of this equation,

$$\frac{2\,\mathbf{I}_1}{\mathbf{I}_1 + \mathbf{I}_2} = 1 + \mathbf{Z}_t/\mathbf{Z} \tag{42}$$

Subtracting 1 from each side of (**41**)

$$\frac{-2\,\mathbf{I}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \mathbf{Z}_t/\mathbf{Z} - 1 \text{ or } \frac{2\,\mathbf{I}_2}{I_1 + \mathbf{I}_2} = 1 - \mathbf{Z}_t/\mathbf{Z} \tag{43}$$

From **42** and **43**

$$\mathbf{I}_2/\mathbf{I}_1 = \frac{1 - \mathbf{Z}_t/\mathbf{Z}}{1 + \mathbf{Z}_t/\mathbf{Z}} \tag{44}$$

This is the general vector expression for reflection, which was given on page 56, where an illustrative problem was worked. We have used the term "reflection coefficient" for the ratio $\mathbf{I}_2/\mathbf{I}_1$.

If the line is short circuited at the end, $Z_t = 0$ and (**44**) becomes $\mathbf{I}_2/\mathbf{I}_1 = 1$ or $\mathbf{I}_2 = \mathbf{I}_1$. The total current $\mathbf{I}_1 + \mathbf{I}_2$ then becomes $2\,\mathbf{I}_1$ which is the familiar case of the current doubling at a short-circuit reflection.

If $Z_t = Z$, $\mathbf{I}_2/\mathbf{I}_1 = 0$, or $\mathbf{I}_2 = 0$ or there is no reflection.

With an open circuit at the end $Z_t = \infty$, $\mathbf{I}_2/\mathbf{I}_1$ becomes $- 1$, or $\mathbf{I}_2 = - \mathbf{I}_1$, the total current at the reflection point being $\mathbf{I}_1 + \mathbf{I}_2 = \mathbf{I}_1 - \mathbf{I}_1 = 0$. The total voltage at the reflection point becomes $\mathbf{Z}\,(\mathbf{I}_1 - \mathbf{I}_2) = Z\,(\mathbf{I}_1 + \mathbf{I}_1) = 2\,\mathbf{Z}\,\mathbf{I}_1$ or twice that due to the oncoming wave alone. This is the case of doubling of voltage at an open circuit reflection.

Fig. 62 shows the current and voltage vectors at 0, 1, 2, 3, 4, 5 and 6 kilometers from a short-circuit reflection, the attenuation constant being taken as 0.05 per kilometer, the wave length 15 kilometers and the velocity ratio $n = 0.8$ so that the length of a wave on the wire is 12 kilometers. Since in the case of a short circuit reflection $I_2 = I_1$, their vectors coincide at the reflection point. The vector $\mathbf{I}_1\,\epsilon^{\gamma x}$ of the forward wave, is advanced in phase and increased in magnitude as we proceed toward the source, or away from the reflection point, while the vector of the reflected wave, $I_2 \epsilon^{-\gamma x}$, becomes smaller and retarded in phase. At 3 kilometers from the reflection end, the current vectors have rotated 90 deg. each, in opposite directions, and are therefore 180 deg. apart, and the resultant current is seen to have a minimum value equal to the difference in the lengths of the two vectors. Referring next to the voltage diagram, the vectors $\mathbf{Z}\,\mathbf{I}_1$ and $-\mathbf{Z}\,\mathbf{I}_2$ at the reflection point are equal and opposite giving zero resultant, for there is no voltage across the line at the short circuit. At 3 kilometers from the short circuit the voltage vectors have rotated into coincidence, giving a maximum resultant equal to the sum of the lengths of the two vectors. Since the current reaches a minimum and the voltage a maximum value at this point, the impedance or ratio of voltage to current will be a maximum. Fig. 63 shows the total current and voltage at various distances from the short-circuited end, calculated by vectors as indicated in Fig. 62. The impedance and phase angle are also shown. The general form of the current distribution curve should be compared with the experimental curves shown in Figs. 7 to 9.

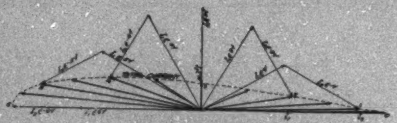

FIG. 62A—CURRENTS AT VARIOUS DISTANCES FROM SHORT CIRCUIT (HEAVY ARROWS ARE VECTORS OF TOTAL CURRENT)

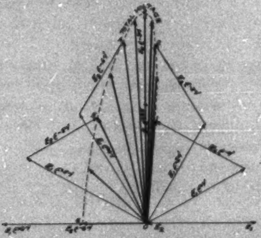

FIG. 62B—VOLTAGE AT VARIOUS DISTANCES FROM SHORT CIRCUIT (HEAVY ARROWS ARE VECTORS OF TOTAL VOLTAGE)

The three kilometers, or distance at which the impedance minimum was reached in Fig. 62, is a quarter of the length of a wave on the wire. Let us compare this result with that given by equation (**38**). Restating the equation.

$$\mathbf{E}_x/\mathbf{I}_x = \mathbf{Z}\,\frac{\mathbf{I}_1\,\epsilon^{\gamma x} - \mathbf{I}_2\,\epsilon^{-\gamma x}}{\mathbf{I}_1\,\epsilon^{\gamma x} + \mathbf{I}_2\,\epsilon^{-\gamma x}} \tag{38}$$

In the case of a short-circuit reflection $I_2 = I_1$. Making this substitution in (**38**) and multiplying numerator and denominator by $\epsilon^{-\gamma x}$, which simplifies the expression, we get

$$\mathbf{E}_x/\mathbf{I}_x = \mathbf{Z}\,\frac{\mathbf{I}_1 - \mathbf{I}_1\,\epsilon^{-2\gamma x}}{\mathbf{I}_1 + \mathbf{I}_1\,\epsilon^{-2\gamma x}} = \mathbf{Z}\,\frac{1 - \epsilon^{-2\gamma x}}{1 + \epsilon^{-2\gamma x}}$$

$$= \mathbf{Z}\,\frac{1 - \epsilon^{-2\alpha x}\,\epsilon^{-j2\beta x}}{1 + \epsilon^{-2\alpha x}\,\epsilon^{-j2\beta x}} \tag{45}$$

which is the impedance of the line x kilometers from a short circuit reflection

In the case we have been considering $x = 3$ kilometers and the wire wave length $= n\,\lambda = 12$ kilometers, so that $x = \frac{1}{4}\,n\,\lambda$. Since $\beta = \frac{2\,\pi}{n\,\lambda}$, $\beta\,x = \frac{2\,\pi}{n\,\lambda} \times \frac{n\,\lambda}{4} = \pi/2$, and $2\,\beta\,x = \pi$. The factor $\epsilon^{-j2\beta x}$ in (**45**) then becomes $\epsilon^{-j\pi} = \cos\pi - j\sin\pi = -1$ so that the impedance a quarter of a wire wave length from a short-circuit reflection is

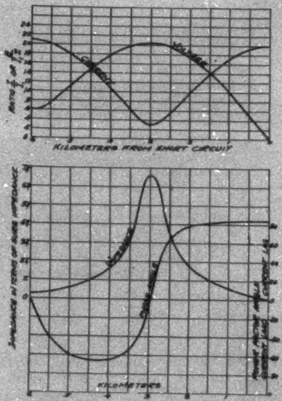

FIG. 63—CURRENT, VOLTAGE, IMPEDANCE AND POWER FACTOR ANGLE ON SHORT-CIRCUITED LINE. (CURRENT AND VOLTAGE EXPRESSED AS RATIO TO THOSE OF WAVE REACHING REFLECTION POINT)

$$\mathbf{Z}_{max} = \mathbf{Z}\,\frac{1 + \epsilon^{-2\alpha x}}{1 - \epsilon^{-2\alpha x}} \tag{46}$$

At six kilometers or a half wire wave length from the short-circuit reflection, we find in Fig. 62 that the total voltage is a minimum and the total current a maximum, so that the line impedance is at a minimum value. Here $x = \frac{1}{2}\,n\,\lambda$, $\beta\,x = \frac{2\,\pi}{n\,\lambda} \times \frac{n\,\lambda}{2} = \pi$, and $2\,\beta\,x = 2\,\pi$. Therefore $\epsilon^{-j2\beta x} = \epsilon^{-j2\pi} = +1$. Using this in (**45**) we find that the impedance a half wire wave length from a short-circuit reflection is

$$\mathbf{Z}_{min} = \mathbf{Z}\,\frac{1-\epsilon^{-2\alpha x}}{1+\epsilon^{-2\alpha x}} \tag{47}$$

The factor $\epsilon^{-j2\beta x}$ again reaches the value -1, when $2\beta x$ becomes 3π, 5π, 7π, 9π, ..., or when the distance x is equal to any odd number of quarter wave lengths, and it becomes $+1$ when $2\beta x$ equals 2π, 4π, 6π, 8π, ..., or when x is an even number of quarter wave lengths. Therefore the impedance of a short-circuited line is expressed by (**46**) if x is an odd number of quarter wave lengths and by (**47**) if x is an even number of quarter wave lengths. (It being understood that "wave length" in this connection refers to the wire wave length, $n\,\lambda$)

In the case of an open-circuit reflection $I_2 = -I_1$, and the general expression (38) for impedance becomes

$$\mathbf{E}_x/\mathbf{I}_x = \mathbf{Z}\,\frac{\mathbf{I}_1\,\epsilon^{\gamma x} - \mathbf{I}_2\,\epsilon^{-\gamma x}}{\mathbf{I}_1\,\epsilon^{\gamma x} + \mathbf{I}_2\,\epsilon^{-\gamma x}} = \mathbf{Z}\,\frac{I_1\,\epsilon^{\gamma x} + \mathbf{I}_1\,\epsilon^{-\gamma x}}{\mathbf{I}_1\,\epsilon^{\gamma x} - \mathbf{I}_1\,\epsilon^{-\gamma x}}$$

$$= \mathbf{Z}\,\frac{1+\epsilon^{-2\gamma x}}{1-\epsilon^{-2\gamma x}} = \mathbf{Z}\,\frac{1+\epsilon^{-2\alpha x}\,\epsilon^{-j2\beta x}}{1-\epsilon^{-2\alpha x}\,\epsilon^{-j2\beta x}} \tag{48}$$

In Fig. 62 the minimum current or voltage is seen to be the difference between the lengths of the vectors of the forward and return waves. If these are very nearly equal, the impedance becomes very high at a current node, and very low at a voltage node. On the other hand if the return wave is small compared with the forward wave, the impedance varies through a much smaller range. Thus the ratio of minimum to maximum impedance shows the magnitude of the return wave as compared with the forward wave, or in other words it shows the attenuation which the waves undergo in traveling to the end of the line and back.

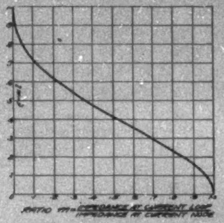

Fig. 64—Curve for Determining Line Attenuation from Impedance Ratio

TABLE VIII
CONDITIONS FOR MAXIMUM OR MINIMUM IMPEDANCE OF LINE

Distance from point of reflection, wave lengths..................	1/4	2/4	3/4	4/4	5/4	6/4	7/4	8/4	9/4
Impedance, with short-circuit reflection........................	max.	min.	max.	min.	max.	min.	max.	min.	max.
Impedance, with open circuit reflection........................	min.	max.	min.	max.	min.	max.	min.	max.	min.

When $\epsilon^{-j2\beta x} = -1$, or when x is an odd number of quarter wave lengths, the impedance (**48**) takes the minimum value shown in (**47**), and when $\epsilon^{-j2\beta x} = +1$ or x is an even number of quarter wave lengths the impedance x kilometers from the end of the open-circuited line has the maximum value given by (**46**).

Summarizing, the impedance of the line takes the minimum value (**47**) or the maximum value (**46**), depending on the type of reflection and the distance from the reflection point, as shown in Table VIII. Since in the present case the impedance is to be measured at the end of the line we substitute l for x in (**46**) and (**47**) giving for the line input impedance.

$$\mathbf{Z}_{max} = \mathbf{Z}\,\frac{1+\epsilon^{-2\alpha l}}{1-\epsilon^{-2\alpha l}} \tag{46a}$$

$$\mathbf{Z}_{min} = \mathbf{Z}\,\frac{1-\epsilon^{-2\alpha l}}{1+\epsilon^{-2\alpha l}} \tag{47a}$$

In the foregoing discussion we have thought of the wave length as constant and the distance from the point of reflection as being varied, but a series of maxima and minima as indicated in Table VIII is also obtained if the length of the line is constant and the wave length, or frequency is changed. This is what is done when we take the measurements for plotting the input impedance curve like that shown in Fig. 60.

If the frequency of the current being supplied to the line, (in Fig. 61) is such that there are a whole number of quarter waves on the line, and we take measurements of the input impedance, with the far end open, and also with it short-circuited, we obtain a maximum impedance (**46a**) and a minimum impedance (**47a**) for the same frequency. Letting m stand for the ratio of the minimum to maximum impedance, we may calculate the attenuation as follows:

$$m = \mathbf{Z}_{min}/\mathbf{Z}_{max} = \frac{\mathbf{Z}\,\dfrac{1-\epsilon^{-2\alpha l}}{1+\epsilon^{-2\alpha l}}}{\mathbf{Z}\,\dfrac{1+\epsilon^{-2\alpha l}}{1-\epsilon^{-2\alpha l}}}$$

$$= \left(\frac{1-\epsilon^{-2\alpha l}}{1+\epsilon^{-2\alpha l}}\right)^2$$

$$\sqrt{m} = \frac{1-\epsilon^{-2\alpha l}}{1+\epsilon^{-2\alpha l}} \tag{49}$$

$$\sqrt{m}+1 = \frac{2}{1+\epsilon^{-2\alpha l}}$$

$$\sqrt{m} - 1 = \frac{-2\epsilon^{-2\alpha l}}{1 + \epsilon^{-2\alpha l}} \text{ or}$$

$$1 - \sqrt{m} = \frac{2\epsilon^{-2\alpha l}}{1 + \epsilon^{-2\alpha l}}$$

$$\frac{1 - \sqrt{m}}{1 + \sqrt{m}} = \epsilon^{-2\alpha}$$

$$\sqrt{\frac{1 - \sqrt{m}}{1 + \sqrt{m}}} = \epsilon^{-\alpha l} \qquad (50)$$

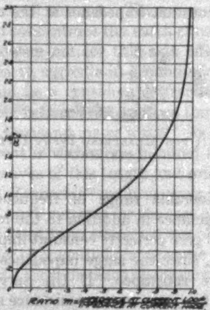

Fig. 65—Curve for Determining Attenuation Constant

Fig. 64 shows $\epsilon^{\alpha l}$ as a function of the impedance ratio m, and Fig. 65 shows $\alpha\, l$ as a function of m.

If the frequency at which it is desired to determine the attenuation, is not such as to make the line an exact number of quarter waves long, the values of maximum and minimum impedance may be found by interpolation, using the envelopes of the curves as shown in Fig. 60. For example, in Fig. 60 the impedance ratio at 25,000 cycles is $m = 220/740 = 0.297$. Using this in Fig. 64 we find $\epsilon^{-\alpha l} = 0.54$.

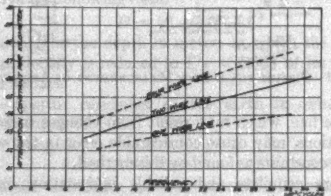

Fig. 66—Attenuation Constants of Antennas

Then $-\alpha\, l = \log_\epsilon 0.54$ and $+\alpha\, l = \log_\epsilon \dfrac{1}{.54} = \log_\epsilon 1.85 = 2.3 \log_{10} 1.85 = 2.3 \times 0.2672 = 0.617$ which may also be found directly from Fig. 65.

In this case $l = 12$ kilometers so that $\alpha = 0.617/12 = 0.0513$ Fig. 66 shows the values of the attenuation constant α, corresponding to the input impedance curves shown in Fig. 60.

Referring again to equations (**46**) and (**47**) we see that if we multiply the two together we have (**Z** max.) (**Z** min.) = **Z**² or

$$\mathbf{Z} = \sqrt{(\mathbf{Z}\text{ max.})\ (\mathbf{Z}\text{ min.})} \qquad (51)$$

Using the values of impedance for 25,000 cycles obtained from Fig. 60 we find that the surge impedance of the line at this frequency is $\mathbf{Z} = \sqrt{740 \times 220} = 435$ ohms. The surge impedance or geometrical mean of the maximum and minimum impedance values is indicated in curve *C* of Fig. 60.

For finding the line velocity it is necessary to know the mode of oscillation (or number of quarter waves on the line) for each frequency at which a maximum or minimum impedance occurs. A curve like the one marked *A* in Fig. 67 can then be plotted showing the number of quarter waves on the line, as a function of frequency. A similar curve *B* is plotted on the same sheet for a light velocity line of the same length. Thus if the length of the line is 12 kilometers and its wave velocity were equal to that of light, it would show a 4/4 wave oscillation at $\lambda = 12$, or $f = \dfrac{3 \times 10^5}{12} = 25{,}000$ cycles. Since the line *B* is straight and passes through the origin, it may be drawn by calculating and plotting a single point. The ordinate to the line *B* is l/λ and the ordinate to curve *A* is $l/n\,\lambda$, whence the velocity ratio for any frequency is

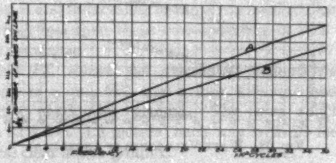

Fig. 67—Mode of Oscillation of Twelve Kilometer Antenna. A—Actual Line. B—Ideal Line

$$n = \frac{\text{Ordinate to } B}{\text{Ordinate to } A}$$

Fig. 68 shows the velocity ratio u/v or n corresponding to the input impedance curves of Fig. 60. The values shown represent a fair average of those so far observed on antennas consisting of two bare 0.102 inch (0.26 cm.) diameter wires in multiple, seven to nine meters above ground. Curves are also shown which give an idea of the effect of using a different number of wires. The line constants depend not only on the type of construction but on the character and moisture of the soil and therefore will vary from place to place and change somewhat with the season.

Fig. 69 shows the apparatus which the writers have used for measuring line input impedance. At either a voltage or current node the line is substantially a unity power factor load and its impedance may be determined by finding the non-inductive resistance which will give the same current. The pick-up coil should preferably have a low reactance compared with the impedance to be measured. When a current maximum is observed, the resistance is substituted for the line and the condenser C_2 adjusted to tune out the reactance of the pick-up coil. The circuit is then switched back to the line and the oscillator frequency readjusted to give current maximum. If the change of oscillator frequency has been considerable a repetition of the process will be needed. In the case of a low loss line it is desirable to use a relatively high-reactance pick up coil and high sensitivity meter for measuring the impedance maxima and a lower impedance coil and less sensitive meter for measuring the low values of line impedance. Fig. 70 shows such an arrangement, indicating coil values which have been found suitable for a 10,000 to 20,000 meter range of wave lengths. Current transformers are permissible. Owing to the rapid change from low to high-current values precautions should be taken to avoid meter burn outs. Oscillator harmonics should be minimized by using large capacity C, and high-efficiency, low-inductance coils in the oscillator circuit, with loose coupling to the pick up coil.

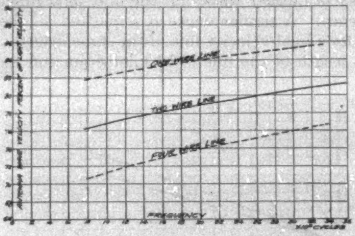

FIG. 68—WAVE VELOCITIES OF ANTENNAS

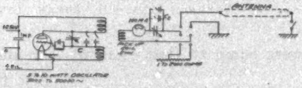

FIG. 69—CIRCUIT FOR MEASURING ANTENNA INPUT IMPEDANCE FOR LONG WAVES

It is not in all cases necessary to take the impedance characteristics of the line, both open and short-circuited. With a good set of readings for either condition, the envelope of the curves can be drawn in and the surge impedance and attenuation determined approximately.

It is not necessary to use a damping resistance at the oscillator end of the line with a view to preventing reflection. For a given frequency the line presents a definite impedance at its terminals, and adding resistance in the supply circuit merely reduces the current and voltage supplied to the line without altering the ratio of voltage to current at the line terminals.

It is desirable in some cases to check the values of line constants as determined from the input impedance, by direct measurement. Arrangement is made for telephone communication over the line as shown in Fig. 71 the circuits being designed to have negligible effect on the radio frequency currents. Various resistances are tried at the far end of the line, until a value is found which gives constant impedance at the oscillator end, over a considerable range of frequency. A small amount of reactance in addition to the resistance may be required to give perfectly constant impedance at the oscillator end, since the surge impedance is not necessarily a pure resistance. If the surge impedance changes with frequency a new resistance setting will be required for a different frequency range. Leaving the surge impedance as found in this way, in the far end of the line, simultaneous readings of the currents at the two ends are taken at a number of different frequencies, and the average ratio of received to supplied current gives the attenuation, $\epsilon^{-\alpha l}$. Measurements at different frequencies of the impedance of the line at the oscillator end give a check on the surge impedance as found by trial at the far end. The presence of partial reflections on the line, which may result from changes in ground conditions, may cause considerable error in the values of the constants as found by this method.

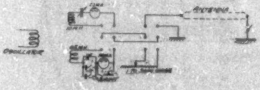

FIG. 70—CIRCUIT FOR MEASURING INPUT IMPEDANCE OF LOW LOSS LINES

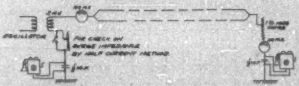

FIG. 71—CIRCUIT FOR DIRECT MEASUREMENT OF ATTENUATION

More complete and reliable information on the behavior of the line is obtained by supplying one end with current of constant frequency and amplitude, and measuring the current at intervals along the line, with different circuit conditions at the far end. If the end is damped with the true surge impedance, and there are no points of partial reflection on the line itself, the current will show a continuous decrease, following the exponential law. If the current is plotted as a function of distance on "semi log" paper, the points will fall on a straight line and the slope of the line will show the attenuation constant. If reflections occur either at the end or at any other point on the line there will be humps or hollows in the curve. With the end open-circuited or grounded, this method of study shows the standing waves on the line, from which the velocity and attenuation can be calculated. Curves of this kind are shown in Figs. 3, 4, 5, 7, 8 and 9.

Interpretation of Observed Line Constants. The explanation of the manner in which velocity and attenuation are affected by frequency, is to be found in the varying depth of penetration of the return currents into the ground. Fig. 72 shows the general shape of the path of the ground current. There is a "skin effect" which tends to concentrate the earth currents near the surface. If it were not for this skin effect the mean depth of the earth currents in ground of uniform conductivity would be a considerable fraction of a wave length, probably between one and two thousand meters with a twelve thousand meter wave. As it is, most of the earth current is within one hundred meters of the surface, with waves of this length and soil of moderate conductivity. Zenneck's[19] analysis gives the depths of penetration of earth currents for the case of space waves of plane wave front from which we can obtain a rough idea of the order of magnitude for the case of waves on a wire supported a short distance above earth.

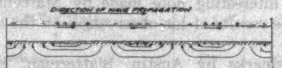

FIG. 72—DISTRIBUTION OF GROUND CURRENTS UNDER WIRE CARRYING WAVES

If the earth carries current almost entirely by conduction rather than by capacity, which is true for all except short wave lengths or extremely high-resistance ground, Zenneck's formula for penetration may be stated in the following form.

$$D = 50\sqrt{\rho/f}$$

in which D is the depth in meters at which the earth current density is reduced to $1/\epsilon = 0.368$ of its value at the surface.

ρ = specific resistance of the earth in ohms per centimeter cube.

f = frequency

This gives for a 12,000 meter wave ($f = 25{,}000$) and a specific resistance $\rho = 10^5$ ohms, a penetration $D = 100$ meters.

In Fig. 72 it is seen that there are both vertical and horizontal earth currents, but for the vertical currents the distance is relatively small and the cross section great. If the drawing were more nearly to scale this difference between the vertical and horizontal earth currents would be still more apparent. The conditions may be approximately represented by Fig. 73 in which a small resistance is shown in series with the line to ground capacity and a higher resistance in the horizontal return conductor, which is at a depth corresponding to the mean depth of the earth currents. The capacity of such a line would be substantially the same as for a wire of the same height, over a perfectly conducting earth. The inductance would be that corresponding to a wire $H + D$ meters above a conducting plane which forms the return conductor. There would be a small added charging current loss due to the resistance in series with the capacity and a much larger loss due to the resistance of the horizontal return conductor. Since the depth of penetration increases with wave length we should expect greater inductance and therefore lower velocities on long waves. The greater the penetration the lower the resistance to the earth currents. Therefore the losses are less and the attenuation less on long waves. High-ground resistance increases the penetration and loss at the same time, and therefore reduces the velocity and increases the attenuation.

Beverage found for the sandy soil near Eastport, L. I. a specific resistance of about 2×10^6 ohms per centimeter cube. Since ground water occurs at a depth of something less than 100 feet (30 meters) the excess resistance and inductance of the Riverhead antenna are materially less than those corresponding to this value of soil resistance.

Table IX shows the calculated inductance and capacity of a one, two, and a four wire line based on perfectly conducting ground. The wire spacing is taken as 4 feet (1.3 meters) each way with 20 feet (6.6 meters) clear above ground.

TABLE IX.

CONSTANTS OF LINE OVER PERFECT GROUND

	1 wire	2 wires	4 wires
L_0 inductance, $\frac{m\,h}{k\,m}$, perfect ground	1.86	1.14	0.8
C_0 capacity, $\frac{m\,f}{k\,m}$, perfect ground	0.006	0.0098	0.0139
D-C. resistance of wires, $\frac{\text{ohms}}{km.}$	3.3	1.64	0.82
R_w = A-C.[20] resistance of wires			
at f = 12,000 ~	4.44	2.22	1.11
20,000	5.50	2.75	1.38
30,000	6.6	3.3	1.64

The observed attenuation and velocity shown in Figs. 66 and 68, give a basis for calculating the effective inductance and resistance provided the capacity is known. The actual capacity is somewhat greater than C_0 as given above owing to insulators, poles, and the proximity of trees. An audio-frequency bridge measurement showed 0.011 microfarads per kilometer for two wires at a height of thirty feet. As this value seems high and the accuracy of the bridge was not checked, we shall assume the capacity to be 10 per cent greater than the calculated value C_0. This gives the constants shown in Table X for a two wire line.

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[19] Translation given in Fleming "Principles of Electric Wave Telegraphy and Telephony" Third Edition P. 812.

[20] Calculated by formula for skin effect given on page 135 of Principles of Electric Wave Telegraphy and Telephony, J. A. Fleming, 1916.

TABLE X.

	Two Wire Line		
Frequency	12,000	20,000	30,000
u = observed velocity, km. per sec.	2.33×10^5	2.37×10^5	2.42×10^5
$n = \frac{u}{3 \times 10^5}$ = velocity ratio	77.4	79	80.6
$L = \frac{1}{u^2\,(1.1\,C_0)}$ = effective inductance	1.73	1.66	1.59
$L_g = L - L_0$ = added inductance for ground, mh. per km.	0.59	0.52	0.45
D = mean depth of earth currents (meters)	118	82	56
$Z = \sqrt{\frac{L}{1.1\,C_0}}$ surge impedance	400	393	384
α = observed attenuation per kilometer	0.033	0.044	0.056
$R = 2\,\alpha\,Z$ = resistance to give observed attenuation, ohms per k. m.	26.5	34.5	43
$R_g = R - R_w$ = equivalent resistance of ground	24.3	31.75	39.7
$\omega\,L$ = Inductive reactance per kilometer	131	208	300

The ground losses and penetration would not be materially different for the one and four wire lines. Taking the values of R_g and L_g for the two wire line as applicable to the one and four wire lines, we may calculate the attenuation and velocity to be expected on the latter as follows:

The formulas $Z = \sqrt{L/C}$, $\alpha = \frac{R}{2\ Z}$, and $u = \frac{1}{\sqrt{L\ C}}$ are approximations applicable when the resistance is small compared with the inductive reactance. In this instance $\omega\,L$ is more than four times R for all the cases calculated and the errors in the magnitudes of α, u, and Z, due to the use of these abbreviated formulas are small. There is however an appreciable phase angle to the vector of the surge impedance Z, which it is of interest to estimate. If the losses were equally divided between dielectric and vertical ground current losses on the one hand and wire resistance and horizontal ground current losses on the other, or to put it differently if a short section of line with ground return had the same power factor, considered as an inductance as it has when treated as a condenser, then the surge impedance, would have zero phase angle or be equivalent to a pure resistance. In the present case the ground losses are for the most part equivalent to the effect of an added resistance in the line. If we assume all the losses to be the result of line resistance we shall obtain a maximum value for the phase angle of the surge impedance. The full expression for surge impedance in vector terms is

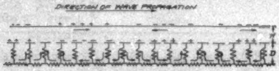

Fig. 73—Circuit Approximately Equivalent to Ground

$$\mathbf{Z} = \sqrt{\frac{R + j\,\omega\,L}{G + j\,\omega\,C}}$$

in which G represents α shunt conductance of such value as to give the equivalent of all the dielectric loss.

The phase angle of Z is $1/2\left(\tan^{-1}\frac{\omega\ L}{R} - \tan^{-1}\frac{\omega\ C}{G}\right)$

which becomes $-\ 1/2\tan^{-1}\frac{R}{\omega\ L}$ when $G = 0$. From the values of L, C, and R given in the above tables we find the values for the surge impedance shown in Table XII. The figures preceded by $-j$ are ohms of capacity reactance. The actual amount of reactance would be slightly less than this, since G is not actually zero.

DIRECTIVE RECEPTION AND STRAY RATIO

Direction of Static. The general opinion that, on the average, summer static comes from a definite direction, for example from a southwesterly direction on our North Atlantic coast, seems to have been brought to a focus by Pickard's classical paper[21] on "Static Elimination by Directional Reception." In the discussion Pickard cites an interesting note made by Marconi in 1906, giving the results of his observation on the directivity of static.

The remarks by Austin, Blatterman, Hoxie and Beverage are of particular interest in showing the trend of opinion as based on observation. The early work of Taylor[22] is also of considerable interest in this connection. Taylor found that atmospherics were very strong on the high vertical antennas contemplated for European reception. After describing tests of various heights and lengths of antenna with the result that the signal to static ratio remained constant, he goes on to say—"One of the first questions that naturally arises in connection with these sturbs is, 'where do they come from?' and it was considered that if their office of origin could be located this might help solve the problem of their elimination. For this purpose an investigation has been conducted at the Belmar Station, where a directive antenna of the Bellini-Tosi type was erected." The conclusions from his observations are that static at Belmar has an average direction of south to southwest, as later observations have shown. Alexanderson[23] had also observed the marked directivity of static with his Barrage Receiver. More recently a great deal of very valuable systematic experimental work has been done by Austin[24] on this subject.

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[21] Institute of Radio Engineers 1920 Vol. 8, P. 397.

[22] C. H. Taylor, Direction of Maximum Atmospheric Disturbances on Wave Range 6000 to 12000 Meters "Belmar," N. J. Sept., Oct. 1915 and Nov., Jan. 1916, Yearbook of Wireless Telegraphy and Telephony 1917 P. 726-743.

[23] E. F. W. Alexanderson, I. R. E. Aug. 1919.

[24] Louis Austin Jour. Franklin Inst. May 1921, page 619.

TABLE XI

	One Wire Line			Four Wire Line		
Frequency	12,000	20,000	30,000	12,000	20,000	30,000
L_0 — inductance, perfect ground, mh. per km	1.86	1.86	1.86	0.8	0.8	0.8
L_g — inductance due to penetration	0.59	0.52	0.45	0.59	0.52	0.45
$L = L_0 + L_g$ = total inductance	2.45	2.38	2.31	1.39	1.32	1.25
$C = 1.1\ C_0$ = Capacity	0.0066	0.0066	0.0066	0.0153	0.0153	0.0153
$u = \frac{1}{\sqrt{LC}}$ = velocity	2.48×10^5	2.52×10^5	2.56×10^5	2.16×10^5	2.22×10^5	2.28×10^5
$n = \frac{u}{3 \times 10^5}$ = velocity ratio	0.827	0.84	0.853	0.72	0.74	0.76
R_g = ground resistance, $\frac{\text{ohms}}{k\ m}$	24.3	31.75	39.7	24.3	31.75	39.7
R_w = wire resistance $\frac{\text{ohms}}{k\ m}$	4.4	5.5	6.6	1.1	1.4	1.6
$R = R_g + R_w$ = total resistance	28.7	37.25	46.3	25.4	33.15	41.3
$\alpha = \frac{R}{2\ Z}$ = attentuation constant	0.0235	0.031	0.039	0.042	0.0565	0.072
$Z = \sqrt{L/C}$ = surge impedance	610	602	592	301	294	286
$\omega\ L$ = inductive reactance per km	185	299	435	105	166	236

TABLE XII

Surge Impedance

	1 Wire Line			2 Wire Line			4 Wire Line		
Frequency	12,000	20,000	30,000	12,000	20,000	30,000	12,000	20,000	30,000
$\omega\ L$	185	299	435	131	208	300	105	166	236
R	28.7	37.2	46.3	26.5	34.5	43	25.4	33.1	41.3
$\text{Tan}^{-1} \frac{R}{\omega\ L}$	8.8°	7.0°	6.0°	11.4°	9.4°	8.2°	13.6°	11.3°	10°
Angle of $Z = -1/2\ \text{Tan}^{-1} \frac{R}{\omega\ L}$	−4.4°	−3.5°	−3.0°	−5.7°	−4.7°	−4.1°	−6.8°	−5.65°	−5°
Z = Total Impedance	610	602	592	400	393	384	301	294	286
Components, R	608	601	591	398	392	383	290	292	285
X	$-47j$	$-37j$	$-31j$	$-40j$	$-32j$	$-27j$	$-36j$	$-29j$	$-25j$

Austin's conclusions on the directivity of static for the wave length range of 8000 to 18,000 meters are briefly as follows:

1. *U. S. Atlantic Coast* static mainly southwest.
2. *Gulf Coast* roughly southwest.
3. *Seattle* (vicinity) roughly east.
4. *San Francisco and San Diego* sharply east.
5. *Porto Rico* two marked directions, namely, west and south.

Gain from Directive Receivers. Even at times when static shows no marked predominating direction, a directive receiving system will obviously reduce or eliminate that fraction of the static which comes from directions to which the receiving system is insensitive. When static is sharply directional the possibilities of improving the stray ratio through the use of a suitable directive receiving system are still greater.

An important step in the improvement of stray ratios was taken when loops superseded static antennas for long wave reception. A further improvement resulted from combining a loop with a static antenna to give a unidirectional antenna system. The loop is more directive than the static antenna. Fig. 74 shows the directive curve of a loop, the large circle being drawn to show the relative sensitiveness of a static antenna for various directions as compared with the loop. The area of the directive curve of the loop is one-half that of the static antenna when both have the same sensitiveness for signal. This means that if disturbances come equally from all directions, the loop will receive just half the energy from the disturbing waves which the static antenna receives.

The combination of loop and static antenna while no more sharply directive than the simple loop from the standpoint of the area of its directive curve, can be adjusted so that reception from certain directions is prevented. Fig. 75 shows the directive curve of the combination of loop and static antenna when the intensities of the two are adjusted to equality for signal. By using less energy from the static antenna than from the loop the directive curve can be made to assume any form such as Fig. 76, intermediate between the cardioid of Fig. 75 and the lemniscate of Fig. 74. This is an exceedingly useful property, particularly if static or other disturbances comes largely from a certain direction.

At times static has a predominating direction, while at other times it appears to be widely distributed, so that both features are important—directive curve of small area, and ability to prevent reception entirely from certain directions. If static, while not confined to a specific direction, comes largely from a certain quarter, it is important to have an antenna system whose directive curve has a small area within the angle from which the heaviest static comes.

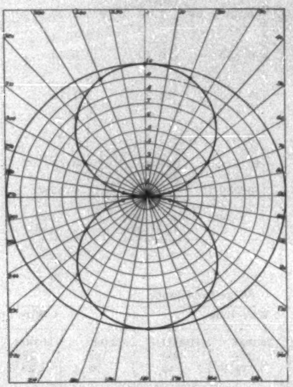

Fig. 74—Directive Curve of Loop

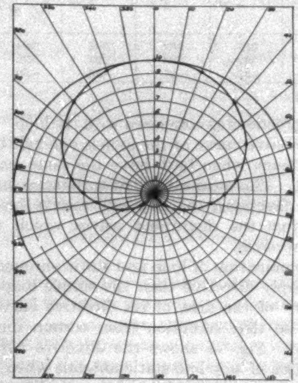

Fig. 75—Directive Curve of Combined Loop and Vertical Antenna (Equal Intensities)

A number of antenna arrangements have been used which are considerably more directive than the loop and vertical. The calculated directive curves of some of these are very similar to that of the one-wave-length, full-velocity wave antenna. Fig. 77 shows the directive curve of a pair of loops spaced apart in the direction of signal propagation, the currents from the two loops being combined in the receiving circuit in such phase as to neutralize for disturbances coming from a direction opposite to the desired signal. The spacing of the loops is preferably between an eighth and a quarter wave length. Compared with systems which obtain their directivity in a small space, the full length wave antenna has the advantage mentioned in connection with the discussion of short wave antennas, namely, that the signal currents developed are strong in comparison with residuals, and therefore there is a better chance of realizing in practise the directive properties predicted by calculation.

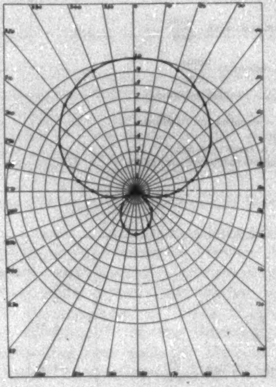

Fig. 76—Directive Curve for Loop and Vertical (Vertical Giving Half Intensity of Loop)

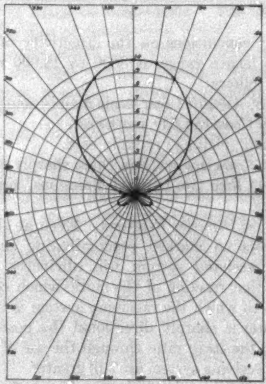

Fig. 77—Directive Curve for Two Loops an Eighth Wave Length Apart

Reference to Figs. 35 to 41 and 44 shows that the areas of the directive curves of wave antennas are small, that the areas of the back and lobes of the curves are very small so that disturbances coming from any direction more than 90 deg. from the signal have relatively small effect, and we have shown in the discussion of the reflection balance, that a blind spot can be produced for any direction more than 90 deg. from that of the signal. The benefit resulting from reduction in directive curve area is illustrated by the fact that in a number of comparisons the full wave antenna has always shown noticeably better stray ratios than an antenna half the length, although the difference in calculated area is comparatively slight.

Directive Effects for Impulses. The question will naturally arise whether directivity curves calculated for continuous waves, are applicable to the steep wave fronts and pulses of static. The experimental evidence is that they are applicable. It is clear that in the case of the wave antenna, waves on the wire will build up in the direction of travel of the space wave, and relatively feeble waves will reach the opposite end of the antenna, whatever the wave shape or number of waves in the train. As applied to antennas or circuits in which a balance of some sort is employed, to give zero reception for continuous waves from certain directions, the explanation of our experience with static is to be found in the great frequency selectivity of our receiving sets. Harmonic analysis of a pulse would show it to be equivalent to the sum of a large number of trains of waves of different frequencies. Of these the receiving set rejects all but the waves of signal frequency. Another view of the problem is the following:

Any circuit which produces a balance or zero reception for continuous waves of a certain frequency will react to a single pulse in such a way as to cause a second pulse in the opposite direction, simultaneously or a whole number of cycles later, or else a second pulse in the receiver in the same direction as the original, but an odd number of half cycles later. The receiving set has a tuned circuit and a detecting system which integrates over many cycles the effects of the oscillation of the tuned circuit. If the neutralizing pulse is simultaneous with the original, the tuned circuit is unaffected; otherwise the tuned circuit is set into oscillation by the initial pulse, but immediately stopped by the neutralizing pulse. The integrated effect of the brief oscillation is comparatively slight. If there were no neutralizing pulse the tuned circuit would (assuming a reasonably low loss circuit), execute something like a hundred oscillations before the amplitude is reduced to half the initial value.

While we have discussed static as if it consisted of single pulses or of waves of very high decrement, it may well be that some of our static consists of trains of many waves of a fairly constant frequency. The wave antenna, being aperiodic, provides a means of studying some of these disturbances without altering their character. If we insert an ordinary telephone receiver in the ground lead, at one end of a wave antenna, we hear a variety of "crackling" and "sputtering" noises, some of which coincide with the static disturbances in the radio receiving sets. Among these noises is an occasional "ping," or sound of definitely musical character, resembling the sound given out when a bare telephone or telegraph wire is struck a sharp blow. No such sound in the receiver is heard, however, when the outside wire is actually struck.

The manner in which such continuous trains of waves might originate is not evident, but the following analogy is of interest. If you throw a stone into the water, you will note two or three circular waves as soon as the splash has subsided. Three or four seconds later you can count seven or eight waves, of substantially uniform size with a calm area inside, and after some ten seconds there may be a dozen to twenty waves. This analogy may have no significance in connection with ether waves, but it suggests a possibility. If it is true that static contains trains of waves of moderately low decrement, this would in part explain the failure of attempts to improve stray ratio by interior circuits (apart from the frequency selectivity obtainable by highly tuned circuits) and point to the conclusion that increased directivity must be our main reliance for further improvements in receiving through static.

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Type of Construction. It is brought out in the discussion of the theory that, so far as collecting signal energy is concerned, there is no object in placing the wires of a wave antenna higher than is required for security and to pass obstructions. A high line will show slightly greater wave velocity and less attenuation than a low line, and be less affected by changes in ground conditions or proximity of trees, which sometimes cause sufficient changes in the line constants to give rise to slight reflections. The differences in favor of the higher line, however, are so small that they would rarely warrant the expense of taller poles.

Apart from the importance of a straight line and avoiding proximity of other conductors, the specifications for wave antenna construction might be taken bodily from those written for an open wire copper telephone circuit. Any change in construction or material which will appreciably alter the line impedance and give rise to reflections, should be avoided. Special care should be given to obtaining clean surfaces for making joints, since we are dealing with voltages which, on the average, are hardly a tenth of those developed in ordinary telephone circuits. Sleeve joints are recommended for permanence. The smallest copper wire which will stand the storms will make as satisfactory an antenna as a heavier wire. Good balance, where two wires are used, is important, and for this reason first-class insulation should be provided. We have seen that a single-wire antenna shows lower attenuation and higher velocity than a two wire antenna. For the same reasons, although in less degree, the use of small wire, and placing the wires near together (if the antenna consists of several wires in multiple) is conducive to high velocity and small attenuation.

Except for temporary or experimental purposes, two-wire antennas are practically always desirable, since they permit adjustments in the station for putting out "back end" disturbances. The use of more wires will, in some cases, collect slightly more energy, but has no other advantage.

Whether a minimum of attenuation and a high velocity are desirable depends primarily on the length of the antenna. For an antenna a wave-length long, the best directive properties are obtained with a velocity between 0.7 and 0.8 of that of light. Higher velocities are desirable for longer antennas and lower velocities for shorter antennas. These considerations may govern the choice of number of wires, or other features of the design. It may even be desirable, in some cases, to reduce the wave velocity of the antenna below the natural value, by loading. The loading may be done by adding series inductance, with the effect of raising the line impedance and reducing the attenuation, as well as reducing the velocity. Slowing down the line by adding capacity to ground will lower the impedance and increase the attenuation. The amount of attenuation experienced on antennas consisting of bare wires on poles, does not appear to affect their directive properties adversely.

Location of Antenna. The land chosen for a wave antenna should be as flat and uniform as possible. The desired location of the receiving station need not control the selection of the location for the antenna. Parallels with other wire lines are to be avoided as far as possible, since the foreign lines, acting as antennas, pick up disturbance from various directions and introduce these into the antenna by induction. There is no simple way of balancing out this induction, for both the lines and the antenna are acting as ground return circuits. It is possible to prevent detrimental effects from adjacent lines by loading them so as to prevent their carrying radio frequency currents. The importance of a straight right of way depends in part on the desired over-all velocity, which, in turn, depends on the length employed. Considerable deviations from a straight line affect the directive properties of the antenna, not only by reducing its wave velocity, but by altering the electromotive forces induced in it. For example, the antenna shown in Fig. 78A would receive disturbances from a direction at right angles to the mean line of the antenna, since the electromotive forces in the sections *a-b* and *d-c* would by no means neutralize. On the other hand, in Fig. 78B, the sections *a-b* and *c-b*, which are affected by disturbances at right angles to the antenna, are a small fraction of a wave length apart, and the effects of the disturbance would nearly neutralize each other in the two sections.

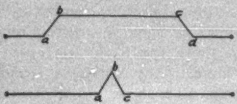

FIG. 78—EQUALLY CROOKED ANTENNAS WHICH ARE NOT EQUIVALENT

It is important to provide grounds at the ends of the antenna which will not change sufficiently to upset adjustments. A body of water in which several hundred feet of copper wire can be laid is the most desirable terminal for the antenna, but fairly satisfactory grounds can be made by burying sufficient lengths of wire. Beverage found on Long Island that, for grounds of this type, it was best to lay the wire in the sod rather than bury it deeper, since the humous of the surface soil retains its moisture better than the sand below. A star consisting of ten or a dozen 100 foot radii of 0.081 in. copper wire laid in this way usually gives a resistance as low as 20 to 40 ohms.

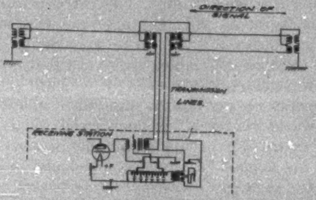

FIG. 79—ARRANGEMENT PERMITTING LOCATION OF RECEIVING STATION AT A DISTANCE FROM THE ANTENNA

Location of Receiving Station. In the cases of the wave antennas which have so far been built, the receiving station has been located at the end nearest the transmitting station and the signals sent back over the transmission line as illustrated in Fig. 18. Figs. 79 and 80 show arrangements by which the receiving station may be located some distance from the antenna. Effective damping must be provided at the end *A*, and this can be done either by wasting most of the energy in a resistance at *A* and transmitting to the station only so much as is necessary for compensation, or by using close coupled transformers of proper ratio to fit the impedance of the circuits which they connect together, and effectively damping the transmission lines in the station. Experience has shown that signals can easily be transmitted a number of miles over open wire lines, with comparatively little loss of intensity, and if we start with signals of such intensity[25] as is usually obtained from a full wave length antenna, there is no perceptible impairment of stray ratio. For the transmission lines which are not a part of the antenna, there is no object in avoiding parallels with other circuits, but they should be transposed frequently enough to prevent picking up radio frequency currents by induction from other circuits.

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[25] The quietness and balance of a two-wire transmission line are not absolute, and if we attempted to transmit very weak signals, they would obviously suffer in stray ratio.

Transmission lines will inevitably act as antennas and waves will be built up on them by disturbances traveling parallel with the direction of the transmission lines. If the lines are balanced, these waves will cause no difference of potential between wires, but only potentials to ground. Balanced transformers at the ends, with electrostatic shielding between primary and secondary, will in general suffice to prevent any effects of the parasitic waves from entering the receiver. Cases may arise, however, in which the waves built up on the transmission line are especially strong, making adequate balance difficult. There are several possible measures for reducing the antenna effects of the transmission lines, such as

1. Loading to give low velocity.
2. Sectionalizing with transformers.
3. Draining.

If the loading coils in the two wires are inductively coupled, they may be made to introduce comparatively little inductance in the transmission line and a higher inductance in the ground return line. A short-circuited secondary of suitable resistance will cause high loss to radio frequency currents in the two wires in multiple, but have no effect on the currents in the metallic circuit.

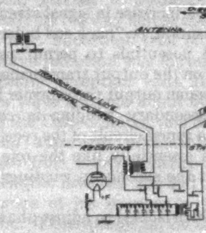

Fig. 80—Alternative to Fig. 79

Sectionalizing as shown in Fig. 81 is one of the most effective means of preventing antenna effects in a transmission line. Drains from the neutrals of the transformer are also shown, to dissipate energy and prevent standing waves from building up.

Use of Existing Wires for Antenna. Existing copper wire lines, if uniform throughout the required length, and having the proper bearing, may be utilized as wave antennas. If the wires are in use for telegraph service, coils of 0.075 to 0.1 henry inductance may be used to isolate the part of the line which is to be used as an antenna. The remainder of the line may be drained through condensers if objectionable disturbances get past the coils into the antenna section. Unless the antenna ground is of very low resistance the ground for the drain should be separate, in order that disturbances shall not be carried through to the antenna, owing to common resistance in the ground. Fig. 82 shows a circuit designed to isolate a portion of a telephone line for antenna purposes. This provides both chokes and a drain, and the coils and condensers are proportioned to cause minimum interference with the passage of the telephonic currents. If the grounding of the wires through the 0.3 microfarad condensers makes the telephone line noisy, a 0.2 microfarad condenser in the ground lead will reduce this tendency.

All parallel wires which are not used as part of the antenna should be sectionalized for radio frequency currents, either by coils, or by links of artificial line like that shown in Fig. 82, at intervals of a quarter or a third wave-length or less, of the shortest waves for which the antenna is to be used.

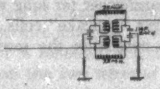

Fig. 81—Sectionalizing Transformer for Transmission Lines, with By-Pass for Direct Current

The principal disadvantages of using telegraph or telephone wires for an antenna are the compromise antenna design which is likely to result, and the difficulty of making tests on the antenna for balance or leakage. The latter applies especially to telegraph lines. If the wires are used for telephony only, large stopping condensers which will permit the telephone ringing currents to pass, may be introduced in series with the wires, at the ends of the antenna, thus permitting direct current tests to be made on the antenna.

Antenna Testing. In any permanent receiving system it should be possible, from the station, to test the continuity and insulation of the antenna and the balance or quietness of the transmission line. Fig. 83 shows an arrangement which Beverage applied to the Riverhead antenna. With the switch S_1 open, the line is insulated from ground and may be tested for leakage by voltmeter or megger. Throwing the switch S_2 to the battery side operates the relay at the far end of the antenna. This cuts out the reflection transformer, and if the transmission line is balanced the receiving set becomes almost entirely quiet. Fig. 84 shows the application of the same method of testing to the case where the receiving station is connected through transmission lines to the antenna. Arrangements for more complete tests are obviously possible, employing polarized relays or

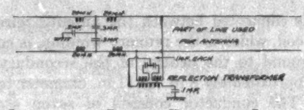

Fig. 82—Isolation of Part of Telephone Line for Use as Antenna

selector switches, but the need of any more elaborate testing system has not yet arisen.

Protection. Potentials of several hundred volts are not uncommon on the wave antenna, even with no storm in the immediate vicinity. All coils which may have to stand these voltages should therefore have substantial insulation. Condensers rated at 1000 volts have been used in the installations at Belmar, Chatham, and Riverhead. Vacuum tube lightning arresters rated at 350 volts are connected between antenna wires and ground at both ends. The switch S_1, in Fig. 83, is closed except during tests, to prevent static potentials from accumulating.

Apparatus Used with Wave Antenna. Some idea of the design of the essential pieces of apparatus which go with the wave antenna, may be of interest.

The reflection transformer for long wave work consists of three 84 turns "pancake" coils. The two outside coils in series constitute the secondary winding which is connected between the antenna wires. The middle coil, which is the primary winding, is connected from ground to the neutral of the secondary. Each coil is approximately 8 in. by 11 in. diameter by 1/2 in. thick. The conductor consists of seven strands of 0.010 in. wire with double cotton covering and a cotton braid, thus giving a loose, low-capacity winding. The whole transformer is placed in a wooden box which is filled with paraffin to exclude moisture. The following readings show the inductance:

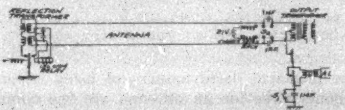

FIG. 83—ARRANGEMENTS FOR INSULATING AND BALANCE TESTS

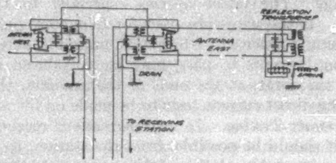

FIG. 84—RELAY CIRCUIT FOR TESTING ANTENNA OVER TRANSMISSION LINES

Primary (84 turns) with secondary open	2.4 millihenrys
Primary with secondary short-circuited	0.56 "
Secondary (168 turns) with primary open	7.5 "
Secondary with primary short-circuited	1.2 "

An arrangement suggested by Kellogg eliminates the reflection transformer. This consists in grounding one wire of the antenna and leaving the other open-circuited. The signal wave currents built up on the antenna flow in the same direction in the two wires. When the waves reach the end of the antenna, reflections occur in opposite phase on the two wires, so that the waves which travel back toward the receiving station are of opposite sign, and are received by a transformer in the station connected across from wire to wire, exactly as is the case when a reflection transformer is used.

Iron core transformers have been used for the antenna output, or between the transmission line and the receiving set. The core is of 0.0015 in. enamelled iron, and is approximately a square inch in cross-section. The primary, or line winding, is in two equal coils of 60 turns each, placed symmetrically on the core and symmetrically with respect to the secondary windings. There are four secondary windings of 160 turns each with a grounded tinfoil shield between each winding and the next to prevent electrostatic coupling. The secondary windings are connected to the grids of pliotron tubes. A greater step-up ratio might have been employed, with consequent increase in signal strength, but the method of compensation, or "back end balance," did not provide sufficient potentials to permit using any more secondary turns on the output transformer.

Another type of antenna output transformer has been used in which the secondary winding is connected directly in series with a tuned circuit of the receiving set. This transformer was essentially like the one already described, except that the secondary windings were of ten turns each.

The artificial line which is used to adjust phase for the back end compensation, consists of a wooden cylinder 4 inches (10 cm.) in diameter, on which is wound a single layer 36 in. long of 0.0126 enamelled copper wire spaced 46 turns per inch. A tap is brought out, 3/4 in. from the end, and every 1½ inches thereafter, giving a total of 24 taps. A 0.005 microfarad condenser is connected from each winding tap to the common conductor, which forms the other side of the "line." A damping resistance of about 400 ohms, wound on a card, is connected across one end of the line. The line has an electrical length of about 16,000 meters and an intensity loss of about 5 per cent from end to end. Four sliders are provided, with double-contact phosphor bronze springs which bear lightly on the wires of the solenoid.

A trap to prevent interference from the transmitting station at Marion, Mass., which is directly in front of the Riverhead antenna, was used to advantage by Beverage. This consisted in a low-resistance series tuned circuit (about 15 millihenrys and a 0.005 microfarad variable air condenser) connected between the wires of the antenna in the station, thus shunting the primary of the output transformer. This formed such a low-impedance shunt when tuned to Marion's wave length as to practically extinguish his signals. Subsequently, with receiving sets in which additional frequency selectivity was provided, this shunt trap was omitted, but it has a field of usefulness, and the high power factor of the circuit, where it is applied, makes its operation simple and satisfactory.

Application to Short Waves.[26] We have discussed the wave antenna as applied to long wave reception only; that is, to the reception of waves ranging from 7000 to 25,000 meters, used in transoceanic communication. It was in this field that the need of greater directivity in reception seemed most urgent, and in this field that the wave antenna was developed. The writers early demonstrated, in the short wave tests at Schenectady, that the wave antenna functioned in the same way on short waves as on long waves.

The first commercial application to shorter waves was the construction of a 2000-meter antenna at Chatham, Mass., for ship reception. This antenna was built in the summer of 1921 and is used for receiving traffic from ships having 1800 to 3000 meter continuous wave transmitting sets. It was in no wise a disappointment, for it resulted in a great improvement in reception, making it possible to receive ships from practically all the way across the Atlantic.

The next important trial of the wave antenna for short wave reception was during Mr. Paul F. Godley's transoceanic reception tests at Ardrossan, Scotland. Using a wave antenna about 400 meters long, pointed toward the United States, and using the best short wave receiving apparatus obtainable, Mr. Godley copied messages from many American amateur stations, on wave lengths between 200 and 300 meters. He attributed much of his success to his directive receiving antenna. Descriptions of the tests and of the antenna, written by Mr. Godley, were published in the February, 1922, Q. S. T., and the March, 1922, Wireless Age.

Recently the writers have done some experimenting with wave antennas for wave lengths in the 300 to 400 meter range. The advantage of the wave antenna on long waves in giving especially strong signals, is less apparent on short waves. The principal reason for this is that, for short waves, the static antennas or loops which we use as a basis of comparison are much larger in proportion to the wave antenna than is true in the case of antennas used in long wave reception.

The advantage, then, of the wave antenna for receiving waves of 450 meters or less, lies in its directive properties. Many amateurs wish to hear all the stations within range, but where the object is to receive from a certain direction only, and to exclude as much else as possible, the wave antenna will perform its function as well as on long waves.

The form of antenna best for short wave reception is practically the same as for long waves, although it will in general be desirable to reduce the height, in order to lessen the effect of the vertical conductors at the ends. Fig. 85 shows suitable arrangements for short wave reception. The surge impedance for a given type of construction will be slightly less on short waves than on long waves. The double wire antenna with reflection transformer has a decided advantage in convenience compared with a single-wire antenna. The equivalent of the reflection transformer—namely, grounding one wire and leaving the other open-circuited, will, as a rule, be preferred for its simplicity. Rear end compensation by means of the reflection balance is desirable and easily applied. This calls for a series-tuned circuit in series with the surge impedance, as shown in Fig. 85. The resistance should be variable and the capacity reactance and inductive reactance should preferably not exceed about 500 ohms each. For output a coil of about 0.1 millihenry in the ground lead of a single-wire antenna, or, if the reflection transformer system is followed, a 0.2 millihenry coil connected between the two wires of the antenna is suitable. The first tuned circuit of the receiving set may then be coupled to this output coil.

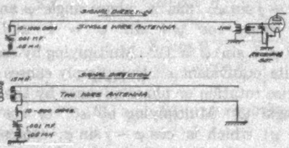

FIG. 85—ARRANGEMENTS FOR SHORT WAVE RECEPTION

APPENDIX A

Typical Operations with Vector Quantities

For additions or subtractions, vectors must be expressed in terms of their components,

$$(a + j\,b) + (c + j\,d) = (a + c) + j\,(b + d)$$

$$(a + j\,b) - (c + j\,d) = (a - c) + j\,(b - d)$$

The vector $a + j\,b$ has a length $\sqrt{a^2 + b^2}$ and an angle $\tan^{-1} b/a$.

Letting φ stand for the angle $\tan^{-1} b/a$, $a = \sqrt{a^2 + b^2} \cos \varphi$ and $b = \sqrt{a^2 + b^2} \sin \varphi$ whence $a + j\,b = \sqrt{a^2 + b^2} (\cos \varphi + j \sin \varphi)$.

There are several other ways of expressing a vector in terms of its length and angle $a + j\,b = (\sqrt{a^2 + b^2})\, \epsilon^{j\varphi}$ if φ is expressed in radians or $a + j\,b = (\sqrt{a^2 + b^2})\, \epsilon^{j(\varphi^\circ/57.3}$ if φ° is expressed in degrees. $a + j\,b = \sqrt{a^2 + b^2} \underline{/\varphi}$ if we define the symbol $\underline{/\varphi}$ as meaning that the quantity $\sqrt{a^2 + b^2}$ after which it appears is to be multiplied by $\cos \varphi + j \sin \varphi$.

The identity of $\epsilon^{j\varphi}$ and $\cos \varphi + j \sin \varphi$ is most readily shown by expending $\epsilon^{j\varphi}$, $\cos \varphi$, and $\sin \varphi$ in power series, and replacing j^2 by -1. This relation is shown in a number of text books.

The product of two vectors is found by multiplying their lengths together and adding their angles.

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[26] For more detailed discussion of the application to short wave reception, see article by H. H. Beverage in Q. S. T., Nov. 1922.

Thus $(A\,\underline{/\varphi_1})\,(B\,\underline{/\varphi_2}) = \overline{A \times B}\,\underline{/\varphi_1 + \varphi_2}$

$$(a + j\,b)\,(c + j\,d) = (\sqrt{a^2 + b^2})\,(c^2 + d^2)\;\underline{/\tan^{-1} b/a + \tan^{-1} d/c}$$

To divide one vector quantity by another we take the quotient of their lengths and the difference of their angles, $\dfrac{A\,\underline{/\varphi_1}}{B\,\underline{/\varphi_2}} = A/B\,\underline{/\varphi_1 - \varphi_2}\,.$

Multiplication and division may also be performed with the vectors in the form $a + j\,b$, thus $(a + j\,b)\,(c + j\,d) = a\,c + j\,a\,d + j\,b\,c + j^2\,b\,d = (a\;c\; - \;b\;d) + j\,(a\,d + b\,c)$

$$\frac{a + j\,b}{c + j\,d} = \frac{(a + j\,b)}{(c + j\,d)} \times \frac{(c - j\,d)}{(c - j\,d)}$$
$$= \frac{(a\,c + b\,d) + j\,(-\,a\,d + b\,c)}{c^2 + d^2}$$

In the foregoing any component or any angle may have a negative sign.

In the expression $a + j\,b = \sqrt{a^2 + b^2}\,\underline{(\cos\varphi} + j \sin \varphi)$ where $\varphi = \tan^{-1} b/a$ the quantity $\sqrt{a^2 + b^2}$ taken by itself would have zero phase angle, and the quantity $(\cos\varphi + j \sin\varphi)$ has a phase angle φ and a length unity (since $\cos^2\varphi + \sin^2\varphi = 1$ and therefore $\sqrt{\cos^2\varphi + \sin^2\varphi} = 1$). Multiplying by $\cos\varphi + j\sin\varphi$, or its equivalent $\epsilon^{j\varphi}$ thus simply causes a counter-clockwise rotation or phase advance by the amount of the angle φ. Multiplying by $\epsilon^{-j\varphi}$ or $\cos(-\,\varphi) + j \sin(-\,\varphi)$ which is $\cos\varphi - j\sin\varphi$, causes clockwise rotation or phase retardation by the amount of the angle φ.

From the rule for multiplication it follows that to square a vector quantity we square its length and double its angle. Thus $(A\,\underline{/\varphi})^2 = A^2\,\underline{/2\,\varphi}$ and conversely to find the square root of a vector we take the square root of its length and divide its angle by two, $\sqrt{A\,\underline{/\varphi}} = \sqrt{A}\,\underline{/1/2\,\varphi}$.

Differentiation of a vector quantity **A**, with respect to some variable X, on which **A** depends, gives the vector change in **A** per unit change in X.

$$\frac{d\,\mathbf{A}}{d\,x} = \frac{\mathbf{A}_2 - \mathbf{A}_1}{x_2 - x_1}$$

in which $\mathbf{A}_2 - \mathbf{A}_1$ is a vector difference, $\mathbf{A}_1$ is the value of **A** corresponding to X_1 and A_2 that corresponding to X_2, and X_2 differs from X_1 by an infinitesimal amount.

In Fig. 62A a vector connecting the ends 1 and 2 of the total current vectors, corresponding to $x = 1$ and $x = 2$, would represent $\dfrac{d\,\mathbf{I}}{d\,x}$ corresponding to a value 1.5 for X.

Integration of a vector quantity gives the vector sum of an infiite number of infinitesimal vectors. This process is illustrated, using finite numbers, in Figs. 28 and 33.

APPENDIX B

Analysis of Action of Wave Antenna

The following treatment of the problem of the wave antenna, was worked out by Mr. Ivar Herlitz or Kellogg's request. Mr. Herlitz was at the time pursuing graduate studies in electrical engineering at Union College as exchange student of the American Scandinavian Foundation. The treatment is given here partly by way of acknowledgment for the substantial assistance derived from Mr. Herlitz's solution of the problem, at an early date in the evolution of the theory of the antenna, and partly because the equations are derived in a radically different manner, and provide a valuable means of checking results as calculated by the expressions given in the body of the paper.

Referring to Fig. 26, x is a distance measured along the antenna from the end A, nearest the source of signals, and l is the total length of the antenna.

As in the discussion given in the paper, the induced voltage per unit length of wire at the point X, is taken as

$$\mathbf{E}_x = \mathbf{E}_0 \cos\theta\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v}$$

The following new symbols will be used.

i = current in wire at X

e = potential of wire at X, with respect to ground.

The potential gradient along wire is the resultant of that due to the passage of current through the line inductance and resistance, and the voltage induced in the wire by the space wave, or

$$\frac{d\,\mathbf{e}}{d\,x} = -\,(R + j\,\omega\,L)\,\mathbf{i} + E_0 \cos\theta\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v} \tag{52}$$

The current in the wire changes from point to point by the amount of the leakage and charging currents, or

$$\frac{d\,\mathbf{i}}{d\,x} = -\,(G + j\,\omega\,C)\,\mathbf{e} \tag{53}$$

Differentiating (**53**) and solving for $\dfrac{d\,\mathbf{e}}{d\,x}$ gives

$$\frac{d_2\,\mathbf{i}}{d\,x^2} = -\,(G + j\,\omega\,C)\,\frac{d\,\mathbf{e}}{d\,x}$$
$$\frac{d\,\mathbf{e}}{a\,x} = -\,\frac{1}{(G + j\,\omega\,C)}\,\frac{d_2\,\mathbf{i}}{d\,x^2} \tag{54}$$

Substituting this in (**52**) and multiplying through by $-\,(G + j\,\omega\,c)$ gives

$$\frac{d_2\,\mathbf{i}}{d\,x^2} = (R + j\,\omega\,L)\,(G + j\,\omega\,C)\,\mathbf{i}$$
$$-\,(G + j\,\omega\,C)\,\mathbf{E}_0 \cos\theta\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v} \tag{55}$$

or since $(R + j\,\omega\,L)\,(G + j\,\omega\,C) = \gamma^2$

$$\frac{d_2\,\mathbf{i}}{d\,x^2} = \gamma^2\,\mathbf{i} - (G + j\,\omega\,C)\,\mathbf{E}_0 \cos\theta\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v} \tag{56}$$

Use trial solution of the form

$$\mathbf{i} = \mathbf{A}\,\epsilon^{\gamma\,x} + \mathbf{B}\,\epsilon^{-\gamma\,x} + \mathbf{D}\,\epsilon^{-j\,\omega\,x\,(\cos\theta/v)} \tag{57}$$

Differentiating (**57**)

$$\frac{d\,\mathbf{i}}{d\,x} = \gamma\,\mathbf{A}\,\epsilon^{\gamma x} - \gamma\,\mathbf{B}\,\epsilon^{-\gamma x}$$

$$- j\,\frac{\omega\cos\theta}{v}\,\mathbf{D}\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v} \qquad \textbf{(58)}$$

$$\frac{d_2\,\mathbf{i}}{d\,x_2} = \gamma^2\,\mathbf{A}\,\epsilon^{\gamma x} + \gamma^2\,\mathbf{B}\,\epsilon^{-\gamma x}$$

$$- \frac{\omega^2\cos^2\theta}{v^2}\,\mathbf{D}\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v} \qquad \textbf{(59)}$$

We next substitute the value of **i** given in (**57**) and the value of $\frac{d_2\,\mathbf{i}}{d\,x^2}$ given in (**59**) in equation (**56**)

$$\gamma^2\,\mathbf{A}\,\epsilon^{\gamma x} + \gamma^2\,\mathbf{B}\,\epsilon^{-\gamma x} - \frac{\omega^2\cos^2\theta}{v^2}\,\mathbf{D}\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v}$$

$$= \gamma^2\,\mathbf{A}\,\epsilon^{\gamma x} + \gamma^2\,\mathbf{B}\,\epsilon^{-\gamma x} + \gamma^2\,\mathbf{D}\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v}$$

$$- (G + j\,\omega\,C)\,\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v}$$

from which

$$\mathbf{D}\left(\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}\right) = (G + j\,\omega\,C)\,\mathbf{E}_0\cos\theta$$

$$\mathbf{D} = \frac{(G + j\,\omega\,C)\,\mathbf{E}_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}} \qquad \textbf{(60)}$$

Using this value of D in (**57**), we have as the general expression for current

$$\mathbf{i} = \mathbf{A}\,\epsilon^{\gamma x} + \mathbf{B}\,\epsilon^{-\gamma x}$$

$$+ \frac{(G + j\,\omega\,C)\,\mathbf{E}_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v} \qquad \textbf{(61)}$$

The constants **A** and **B** depend on the terminal impedance and we must find an expression for the voltage **e**, as well as the current **i**, in order to evaluate them.

From (**53**)

$$\mathbf{e} = -\frac{1}{(G + j\,\omega\,C)}\,\frac{d\,\mathbf{i}}{d\,x}$$

in which we may substitute the value of $\frac{d\,\mathbf{i}}{d\,x}$ given in (**58**) using the value given in (**60**) for **D**

$$\mathbf{e} = \frac{-\gamma\,\mathbf{A}\,\epsilon^{\gamma x}}{G + j\,\omega\,C} + \frac{\gamma\,\mathbf{B}\,\epsilon^{-\gamma x}}{G + j\,\omega\,C}$$

$$+ \frac{j\,\frac{\omega\cos\theta}{v}\,\mathbf{E}_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v}$$

Since $\frac{\gamma}{G + j\,\omega\,c} = \mathbf{Z}$, the expression for **e** becomes.

$$\mathbf{e} = -\mathbf{Z}\,\mathbf{A}\,\epsilon^{\gamma x} + \mathbf{Z}\,\mathbf{B}\,\epsilon^{-\gamma x}$$

$$+ \frac{j\,\frac{\omega\cos\theta}{v}\,\mathbf{E}_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}\,\epsilon^{-j\,\omega\,x\,(\cos\theta)/v} \qquad \textbf{(62)}$$

For an antenna with non-reflecting ends, or in other words, with the surge impedance at each end,

$$\mathbf{e} = -\mathbf{Z}\,\mathbf{i} \text{ for } x = 0$$

$$\mathbf{e} = +Z\,\mathbf{i} \text{ for } x = l$$

Using the expresssions for **i** and **e** given in (**61**) and (**62**) we set $x = 0$ and equate $-\mathbf{Z}\,\mathbf{i}$ to **e**, and have

$$-\mathbf{Z}\,\mathbf{A} - \mathbf{Z}\,\mathbf{B} - \mathbf{Z}\,\frac{(G + j\,\omega\,C)\,\mathbf{E}_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}$$

$$= -\mathbf{Z}\,\mathbf{A} + \mathbf{Z}B + \frac{j\,\frac{\omega\cos\theta}{v}\,\mathbf{E}_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}$$

whence

$$-2\,\mathbf{Z}\,\mathbf{B} = \frac{\mathbf{E}_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}\left\{ j\,\frac{\omega\cos\theta}{v} + \mathbf{Z}\,(G + j\,\omega\,C) \right\}$$

and since $\mathbf{Z}\,(G + j\,\omega\,c) = \gamma$

$$\mathbf{B} = -\frac{\mathbf{E}_0\cos\theta}{2\,\mathbf{Z}}\;\frac{\gamma + j\,\frac{\omega\cos\theta}{v}}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}$$

$$= -\frac{\mathbf{E}_0\cos\theta}{2\,\mathbf{Z}\left[\gamma - j\,\frac{\omega\cos\theta}{v}\right]} \qquad \textbf{(63)}$$

Next setting $x = l$ in (**61**) and (**62**) and equating $+\mathbf{Z}\,\mathbf{i}$ to **e** we have

$$\mathbf{Z}\,\mathbf{A}\,\epsilon^{\gamma l} + \mathbf{Z}\,\mathbf{B}\,\epsilon^{-\gamma l}$$

$$+ \mathbf{Z}\,\frac{(G + j\,\omega\,C)\,E_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}$$

$$= -\mathbf{Z}\,\mathbf{A}\,\epsilon^{\gamma l} + \mathbf{Z}\,\mathbf{B}\,\epsilon^{-\gamma l}$$

$$+ \frac{j\,\frac{\omega\cos\theta}{v}\,\mathbf{E}_0\cos\theta}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}$$

$$2\,\mathbf{Z}\,\mathbf{A}\,\epsilon^{\gamma l} =$$

$$\frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}\left[j\,\frac{\omega\cos\theta}{v} - \mathbf{Z}\,(G + j\,\omega\,C) \right]$$

$$= -\frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{\gamma^2 + \frac{\omega^2\cos^2\theta}{v^2}}\left[\gamma - j\,\frac{\omega\cos\theta}{v} \right]$$

$$= -\frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{\gamma + j\,\frac{\omega\cos\theta}{v}}$$

$$\mathbf{A} = \frac{-\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{2\,\mathbf{Z}\,\epsilon^{\gamma l}\left(\gamma + j\,\frac{\omega\cos\theta}{v}\right)}$$

$$= -\frac{\mathbf{E}_0\cos\theta\,\epsilon^{-(\gamma + j\,\omega\,l\,(\cos\theta)/v}}{2\,\mathbf{Z}\left(\gamma + j\,\frac{\omega\cos\theta}{v}\right)} \qquad \textbf{(64)}$$

With these values for **A** and **B** and with $\gamma/\mathbf{Z}$ substituted for $G + j\,\omega\,C$, equation (**61**) for current becomes

$$\mathbf{i} = \mathbf{E}_0 \cos\theta \left\{ \frac{-\epsilon^{-(\gamma + j\,\omega\,l\,(\cos\theta)/v}}{2\,\mathbf{Z}\left(\gamma + j\,\dfrac{\omega\cos\theta}{v}\right)}\,\epsilon^{\gamma\,x} - \frac{1}{2\,\mathbf{Z}\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)}\,\epsilon^{-\gamma\,x} + \frac{\gamma\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{\mathbf{Z}\left(\gamma^2 + \dfrac{\omega^2\cos^2\theta}{v^2}\right)} \right\} \qquad (65)$$

The receiver end current is found by letting $x = l$ in (**65**) which gives

$$\mathbf{i}_b = \mathbf{E}_0 \cos\theta \left\{ \frac{-\epsilon^{-(\gamma + j\,\omega\,l\,(\cos\theta)/v}}{2\,\mathbf{Z}\left(\gamma + j\,\dfrac{\omega\cos\theta}{v}\right)}\,\epsilon^{\gamma\,l} - \frac{\epsilon^{-\gamma\,l}}{2\,\mathbf{Z}\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)} + \frac{\gamma\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{\mathbf{Z}\left(\gamma^2 + \dfrac{\omega^2\cos^2\theta}{v^2}\right)} \right\}$$

$$= \frac{E_0\cos\theta}{2\,\mathbf{Z}} \left\{ \frac{-\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{\gamma + j\,\dfrac{\omega\cos\theta}{v}} - \frac{\epsilon^{-\gamma\,l}}{\gamma - j\,\dfrac{\omega\cos\theta}{v}} + \frac{2\,\gamma\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{\left(\gamma + j\,\dfrac{\omega\cos\theta}{v}\right)\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)} \right\}$$

$$= \frac{\mathbf{E}_0\cos\theta}{2\,\mathbf{Z}}\,\frac{\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)} \times \left\{ -\frac{\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)}{\gamma + j\,\dfrac{\omega\cos\theta}{v}} - \epsilon^{-\gamma\,l}\,\epsilon^{+j\,\omega\,l\,(\cos\theta)/v} + \frac{2\,\gamma}{\gamma + j\,\dfrac{\omega\cos\theta}{v}} \right\}$$

$$= \frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{2\,\mathbf{Z}\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)} \times \left\{ \frac{\gamma + j\,\dfrac{\omega\cos\theta}{v}}{\gamma + j\,\dfrac{\omega\cos\theta}{v}} - \epsilon^{-(\gamma - j\,\omega\,l\,(\cos\theta)/v} \right\}$$

$$= \frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{2\,\mathbf{Z}\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)} \{1 - \epsilon^{-(\gamma - j\,\omega\,l\,(\cos\theta)/v}\}$$

$$= \frac{\mathbf{E}_0\cos\theta\,\epsilon^{-j\,\omega\,l\,(\cos\theta)/v}}{2\,\mathbf{Z}\,[\,\alpha + j\,\beta\,(1 - n\cos\theta)\,]} \times \{1 - \epsilon^{-[\alpha + j\,\beta\,(1 + n\cos\theta)\,]\,l}\} \qquad (66)$$

The back end current is found by setting $x = 0$ in (**65**) which gives

$$\mathbf{i}_a = \mathbf{E}_0 \cos\theta \left\{ \frac{-\epsilon^{-[\gamma + j\,\omega\,l\,(\cos\theta)/v}}{2\,\mathbf{Z}\left(\gamma + j\,\dfrac{\omega\cos\theta}{v}\right)} - \frac{1}{2\,\mathbf{Z}\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)} + \frac{\gamma}{\mathbf{Z}\left(\gamma + j\,\dfrac{\omega\cos\theta}{v}\right)\left(\gamma - j\,\dfrac{\omega\cos\theta}{v}\right)} \right\}$$

$$= \frac{\mathbf{E}_0\cos\theta}{2\,\mathbf{Z}\left(\gamma + j\,\dfrac{\omega\cos\theta}{v}\right)} \left\{ -\epsilon^{-[\gamma + j\,\omega\,(\cos\theta)/v\,]\,l} - \frac{\gamma + j\,\dfrac{\omega\cos\theta}{v}}{\gamma - j\,\dfrac{\omega\cos\theta}{v}} + \frac{2\,\gamma}{\gamma - j\,\dfrac{\omega\cos\theta}{v}} \right\}$$

$$= \frac{\mathbf{E}_0\cos\theta}{2\,\mathbf{Z}\left(\gamma + j\,\dfrac{\omega\cos\theta}{v}\right)} \{1 - \epsilon^{-[\gamma + j\,\omega\,(\cos\theta)/v\,]\,l}\}$$

$$= \frac{\mathbf{E}_0\cos\theta}{2\,\mathbf{Z}\,[\alpha + j\,\beta\,(1 + n\cos\theta)\,]} \times \{1 - \epsilon^{-[\alpha + j\,\beta\,(1 + n\cos\theta)]\,l}\} \qquad (67)$$

APPENDIX C

List of Symbols

Symbols in heavy faced type, as **I**, **E**, **Z** stand for vector quantities.

L = Series inductance of antenna conductors, henrys per kilometer.

C = Shunt capacity of antenna, farads per kilometer

R = Effective series resistance of antenna, ohms per kilometer

G = Leakage conductance to ground, mhos for one kilometer, (effective value at high frequency)

Z = Characteristic or surge impedance of antenna

$\mathbf{Z} = \sqrt{\dfrac{R + j\,\omega\,L}{G + j\,\omega\,C}}$ or for most purposes at radio frequency $Z = \sqrt{L/C}$

γ = Propagation constant for antenna

$\gamma = \sqrt{(R + j\,\omega\,L)\,(G + j\,\omega\,C)}$

α = Attenuation constant per kilometer = real part of γ. If $\omega\,L$ is large compared with R, and $\omega\,c$ large compared with G, $\alpha = R/2\,\sqrt{C/L} + G/2\,\sqrt{C/L}$ approximately

$j\,\beta$ = Imaginary component of γ, or β = Wave length constant of Antenna. $\gamma = \alpha + j\,\beta$. For most purposes at radio frequency $\beta = \omega\,\sqrt{L\,C}$ approximately

u = Antenna wave velocity, kilometers per second $u = \omega/\beta$ or $u = 1/\sqrt{LC}$ approximately

v = Velocity of space waves = Velocity of light = 3×10^5 kilometers per second

n = Velocity ratio of antenna = u/v

f = frequency of signal waves, cycles per second

λ = Signal wave length = $\dfrac{3 \times 10^5}{f}$ kilometers

$\omega = 2\pi f$

ϵ = Base of natural logarithms = 2.718 $\epsilon^x = 10^{0.4343\,x}$

l = Length of antenna in kilometers

θ = Angle between direction of signal and direction of antenna

E_0 = Measure of signal intensity = Induced volts per kilometer in horizontal conductor parallel to signal direction

I_b = Current at receiver end of antenna (both ends of antenna assumed to be damped by surge impedance)

I_a = Current at back end of antenna (both ends assumed damped)

ρ = Specific resistance of earth, ohms for one centimeter cube.

Bibliography on Antennas

DIRECTIONAL, GROUND, UNDERGROUND AND UNDERWATER

E. Rotheman. Telegraphie ohne metallische Leitung. *E. T. Z.* Vol. 15. Page 616-618. 1894.

K. Strecker. Ueber die Ausbreitung starker elektrischer Strome in der Erdoberflache. *E. T. Z.* Vol. 17. Page 106-110, 1896.

F. Braun. Drahtlose Telegraphie durch Wasser und Luft. (Book). Leipzig. 1901.

E. Lecher. Uber drahtlose Telegraphie. *Phys. Zeits. 3.* 273, 274. 1902.

F. Braun. Notizen über drahtlose Telegraphie. *Phys. Zeits. 4.* 361-364. 1903.

E. Lecher. Uber kunstliche Elektrisierung der Erdkugel. *Phys. Zeitschrift, 4.* 320-321. 1903.

L. Zehnder. German Patent No. 17886. 1904.

L. Zehnder. English Patent No. 10601 of 1905. German Patent 178861 of 1906.

F. Braun. Directed Wireless Telegraphy. *Electrician, 57.* 222, 244. 1906.

R. A. Fessenden. Stream of Water as Antenna. *Elektrotechnische Zeitschrift.* 1906. P. 690.

J. A. Fleming. The Theory of Directive Antenna or Unsymmetrical Hertzian Oscillators. *Proc. Royal Soc. A, 78.* 1-8; 1906. *Sci. Abs. A.* No. 2072. 1906.

G. Marconi. Directive Antenna. *Proc. Roy. Soc.* London, *77A.* 413. 1906.

E. Bellini and A. Tosi. A Directive System of Wireless Telegraphy. *Electrical Engineering* (London) *2.* 771-775 (1907) *3.* 348-351 (1908); *Sci. Abs. B.* Nos. 83 and 466 (1908). *Jahrb. d. drahtl. Tel. 1.* 598-608; 1907.

F. Braun. Gerichtete drahtlose Telegraphie. *Jahrb. drahtl. Tel. 1.* Page 1. 1907.

G. Eichhorn. System einer gerechteten drahtlosen telegraphie noch, E. Bellini and A. Tosi. *Jahrbuch 1,* 598-608. 1908. *Electrical Engineering 2,* 771-1907.

Pickard, G. W. *Elec. Rev. 50,* 963, 985; *52,* 262; *53,* 494; *Electrician 59,* 563, 1907.

Bellini-Tosi. Telegraphie et Telephonie sans fil dirigeables. Tosi. *Bull. Soc. Int. des Electriciens 8,* 707-730, second series. 1908.

E. Bellini. Uber einige Luftgebilde fur gerichtete drahtlose Telegraphie. *Jahrbuch 2,* 381-396, 1908.

Bellini and Tosi. Methods of Obtaining Directed Waves. *Electrician 60,* 748, 1908; *61,* 598, 1908; *62,* 381, 1909; *62,* 608, 1909.

Bellini and Tosi. Directive Wireless Telegraphy. *Proc. Phys. Soc. London, 21,* 305, 1908; *Elec. Engineering 3,* 348, 1908; Bellini, *Jahrbuch 2,* 381, 808, 1909-1910.

Blondel, A. Uber die Bestimmung der Richtung von Schiffen Vermittels der Hertzchen Wellen. *Jahrbuch 2,* 190, 1908.

F. Kiebitz. Gerichtete drahtlose Telegraphie. *Verh, d. deutsch. Physik, 10.* 934-944, 1908. *Mitt, Tel. Ver. 5,* 22, 1910.

J. Zenneck. Uber die Wirkungsweise der sender fur gerichtete drahtlose Telegraphie. *Phys. Zeits, 9,* 553-556, 1908.

E. Bellini et A. Tosi. La Portee et les Avantages des Aeriens Dirigeables et le Radio-goniometre Bellini-Tosi. *Lumiere Elec.* 2nd series, *5,* page 263-268. Feb. 27, 1909.

G. Eichhorn. Das Radio Goniometer von Bellini und Tosi. *Jahrbuch d. drahtl. Tel. 2,* 511-525. 1909.

T. Karrass. Geschichte der Telegraphie. (Book). Page 43. Brunswick, Germany, 1909, Vieweg. In Bureau of Standards library.

C. Montu. Recent Progress in Directive Wireless Telegraphy. Elettricita (Milan), *33,* 291-293, Nov. 25, 1909. Describes Bellini-Tosi system.

Greenleaf W. Pickard. Antennae. *Proceedings of the Wireless Institute of America.* Vol. 1, No. 2, May 1909. (Meeting in New York City, May 5, 1909.)

A. Sommerfield. Uber die Ausbrettung der Wellen in der drahtlosen Telegraphie. *Ann d. Phys.* page 665-736, 1909. *Jahrb. d. drahtl. Tel. 4,* 157-176. 1911.

H. Behnken. Eine Method zur Messung der Wirksamen Kapazitat von Antennen. *Phys. Zeit. 14,* page 430-433, 1913. *Mitt. Tel. Ver., 7,* 173, 1910.

E. Bellini. Constancy of the Coupling in the Bellini-Tosi Radiogoniometer. *Jahrbuch 3,* 571-580, 1910; *Sci. Abs. B.,* p. 420, 1911.

W. H. Eccles and A. Arton. Directive Wireless Telegraphy. *Electrician 64,* 834, 899, 947, 1910.

C. Fischer. Strahlung von Antennen. *Phys. Zeit. 12,* page 295-303, 1911. *Mitt. Tel. Ver. 6,* 103, 1910.

F. Kiebitz. Versuche uber drahtlose Telegraphie mit verschiedenen Antennen formen. *Ann. d. physik. 337,* pages 941-998. 1910.

F. Kiebitz. Neuere Versuche uber gerichtete drahtlose Telegraphie mit Erdantennen. *Verh. d. Deutsch. Physik Ges. 13,* 876-889. *Mitt. Tel. Ver. 6,* 158, 1910.

L. H. Walter. Radiation from Directive Aerials. *Electrician, 64,* 790, 875, 899, 945, 1910; *Sci. Abs. B,* p. 160, 1910.

L. H. Walter. Directive Wireless Telegraph Station at Boulogne. *Elec. Eng. 6,* 547-549, July 14, 1910; *Sci. Abs. B,* p. 390, 1910. See also *Sci. Abs.* B, No. 465, 1910.

Bellini and Tosi. Le Compas Azimutual Hertzien. *La Lumière Elec. 14,* 227, 1911.

A. Blondel. Directed Aerials in Wireless Telegraphy. *Lumière Elec. 16,* 7, 131, 1911; *Comptes Rendus 153,* 593, 661, 1911; *Sci. Abs.* B, p. 149, 1912.

A. Blondel. Notes sur les aeriens d'orientation en radiotélégraphie. *Lumière Elec. 26,* 131. Oct. 7, 1911.

Elec. World. Antenna Investigations by Cornell Students. *Elec. World, 58,* 948, Oct. 14, 1911.

G. Leimbach. Drahtlose Telegraphie im Erdinnern. *E. T. Z. 32,* 237-238. 1911.

G. E. Petit. Method of Determining the Bearing of a Radiotelegraphic Station. *Lum. Elec. 13*, 227-230, 1911; *Sci. Abs. B*, p. 173, 1911.

L. H. Walter. Accuracy of the Bellini-Tosi Wireless "Compass" for Navigational Purposes. *El. 67*, 749-751, August 18, 1911.

F. Addey. Directional Wireless Telegraphy. *Electrician 70*, 568-588, Dec. 27, 1912. Abstract of paper read before the Institute P. O. Elec. Engr. *Electrotechnische Zeitsch 34*, 534-535, May 8, 1913. *Rev. Electrique 19*, 392-395, April 18, 1913.

F. Braun. Earth Antennas. *Jahrbuch 5*, 586, 1912.

W. Burstyn, Die Wirkungsweiss der Erdantennen. *E. T. Z. 33*, 613, 1912. *Jahrbuch d. Tel. 6*, 10, 1912.

W. Burstyn. The Action of Earth Antennae. *Jahrbuch 6*, 10-13, 1912. *Sci. Abs. B*, p. 518, 1912. *E. T. Z. 33*, 615. June 13, 1912.

W. Burstyn. Die Wirkungsweise der Erdantennen. *Jahrbuch 6*, 1013, July 1912. *Elektrotechnische Zeitsch, 33*, 615-616, July 13, 1912. *Sci. Abs. B*, 1912. page 518.

J. A. Fleming. Earthed Directive Antennae. *El. 68*, 1020, March 29, 1912.

F. Kiebitz. Uber die Geschichte der Erdantennen. *Jahrbuch 5*, 360, March 1912; *6*, 1 and 554, 1912.

F. Kiebitz. Neuere Versuche uber gerichtete drahtlose Telegraphie mit Erdantennen. *Jahrb, d. drahtl. Tel. 5*, 349-360, March 1912. *Sci. Abs. B*, No. 321, 1912.

F. Kiebitz. Versuche uber gerichtete drahtlose Telegraphie, *Jahrbuch d. drahtlosen Tel. 6*, 1-9, July 1912. *Sci. Abs. B*, 1912, page 519. *Mitt. Tel. Ver. 7*, 71.

F. Kiebitz. Recent Experiments on Directive Wireless Telegraphy with Earth Antennae. *Electrician 68*, 868, 1912.

Mosler, Burstyn and Hausrath. Earth Antennas and Underwater Antennas. *Jahrbuch 6*, 359-366, 570-573, 1912.

M. Reich. Relation between the Radiation of an Antenna and its Shape. *Phys. Zeits, 13*, 228, March 15, 1912. *Sci. Abs. B*, p. 300, 1912.

A. Rothé. Reception of Radiotelegraphic Signals with Diminutive Aerials. *Comptes Rendus* Vol. 154, Jan. 27, 1912. *Sci. Abs. B*, p. 203, 1912. See also *Sci. Abs. B*, No. 321, 1912.

E. Rothé. Reception by Earth Antennae. *Comptes Rendus, 155*, 1075, Nov. 25, 1912. *Sci. Abs. B*, p. 144, 1913.

J. E. Taylor. Earthed Directive Antennae. *Electrician, 68*, 978, March 22, 1912.

A. Turpani. Antennae of the Horizontal and Vertical Type. *Rev. Electrique, 18*, pp. 207-208. Sept. 13, 1912. *Sci. Abs. B*, p. 517, 1912.

L. Zehnder. Directive Antennae. *Jahrbuch 5*, 594, June 1912

Earth Antennas. *Elektrotechnische Zeitschrift, 33*, 139, 1912.

Earth Antennas. (Discussion of Kiebitz' paper). *Electrician* 68, 936, 978, 1020, 1912.

The Telefunken "Compass". *Electrician 69*, 269, 1911; *Sci. Abs. B*, p. 304, 1912.

F. Kiebitz. Further Experiments on Earth Antennae. (*Jahrbuch d. drahtlosen Telegraphie 6*, 554-569, March 1913. *Electrician 71*, 608-611, July 18, 1913). *Sci. Abs. B*, No. 1013, 1913.

H. Mosler. Uber die Wirkungsweise der Erdantennen. *Jahrb. d. drahtl. Tel. 6*, 570, 1913.

Marconi Patent for Closed Aerials. British Patent 4514 of 1913. *Engineer 116*, 641, Dec. 12, 1913. *Sci. Abs. B*, p. 52, 1914.

The Wireless Direction Finder. (Marconi) *Yearbook of Wireless Telegraphy.* pp. 306-317, 1913. Historical and descriptive.

Direction Finding by Wireless Telegraphy. *Elek. u. Masch. 31*, 1029-1030, Nov. 30, 1913.

E. Bellini. The Possibility of Sharp Directive Wireless Telegraphy. *Electrician 74*, 352, 1914; *Lum. Elec. 28*, 203, 1915; *Elettrotechnica 2*, 89-90, Feb. 5, 1915; *Sci. Abs. B*, p. 207, 1915.

H. Brand. Latest Developments in Wireless Telegraphy. *Verein z. Beförd des Gewerbfleisses Verh.* No. 4, pp. 225-295, April 1914. *Sci. Abs. B*, p. 449, 1914.

F. Braun. Use of Closed Circuits in Wireless Telegraphy in Place of Open. *Jahrbuch 8*, 1-34, Jan. 1914.

A. Artom. Directivity of Electric Waves. (*Acad. Lincei. Atti. 24*, 42-47, Jan. 21, 1915). *Electrotecnica 2*, 202-204, March 25, 1915.

E. Bellini. La Portée des Aérion à ondes Dirigeés. *La Lum. Elec. 30*, 1, 1915.

E. Bellini. Some Details of the Direction Finder. *Electrician 75*, 776-778, 1915; *Sci. Abs. B*, p. 493, 1915.

E. Bellini. Range of Directive Aerials. *Lum. Elec. 30*, 1-6, 1915; *Sci. Abs. B*, p. 382, 1915.

H. Brand. Experiments in Sending with Low Horizontal Antennae. *Jahrb. d. drehtl. Tel. 9*, 431-438, 1915. *Sci. Abs. B*, p. 491, 1915.

C. A. Culver and J. A. Riner. Radio Telegraphy without Elevated Antennae. *El. World, 65*, 723-726, March 20, 1915.

Arthur R. Hinks. Wireless Telegraphy in Survey. Yearbook Wireless Telegraphy. Pages 628-636. 1915.

R. B. Woolverton. Effectiveness of Ground Antenna on Long Distance Reception. *Proc. I. R. E. 3*, 371, December 1915.

J. Zenneck. An Arrangement for Directive Wireless Telegraphy. *Jahrbuch 9*, 417-424, April 1915. *Sci. Abs. B*, 1915, dage 450.

J. Zenneck. Wireless Telegraphy. (Book). 1915. p. 364.

C. A. Culver. A Few Experiments with Ground Antennae. *Proc. I. R. E., 4*, 455, October 1916.

C. A. Culver. Notes on Radiation from Horizontal Antenna. *Proc. I. R. E. 4*, 449, October 1916.

B. F. Meissner. Radio Dynamics. (Book). 1916. p. 186.

M. Siegbahn. Electromagnetic Waves in Coils with Distributed Capacity. *Archiv. f. Elektrotechnik, 4*, 305-313, (1916). *Sci. Abs. A*, No. 1067, 1916. Refers to article by K. W. Wagner. *Sci. Abs. B*, No. 659, 1915. See also Fleming *Sci. Abs. A*, No. 2683 (1904).

A Unilateral Directive Aerial, British Patent 24098 of 1914. *Engineer 121*, 165, F. 18, 1916. *Sci. Abs. B*, No. 244, March 1916. Aerials differing in phase.

E. Bennett, Univ. of Wisconsin. High Versus Low Antennae in Wireless Telegraphy and Telephony. *Bull.* (No. 810 of Eng. Series). Vol. 8, Nov. 4, *Electrician 79*, pp. 18-20, April 6, 1917.

Marley Fotheringham Hay. Secrets of the Submarine. (Book). Dodd, Mead & Co., N. Y. 1917.

John H. Morecroft. Some Experiments with Long Electrical Conductors. (With particular reference to radio antennas). *Proc. I. R. E. 5*, 389, 1917.

Rushmore. Bibliography of the Literature of Submarines. *G. E. Review*, August 1917.

Alexandre Arton. A New Kind of Directive Antenna. French Patent 485622. *Rev. Gen. d'El. 4*, 99D., Sept. 28, 1918.

E. Bennett. Feasibility of the Low Antenna in Radio Telegraphy. *Proc. I. R. E.* Vol. 6, page 238, 1918.

J. Joly. A Method of Avoiding Collision at Sea. *Proc. Roy. Soc. 94*, 547, 1918.

J. B. Pomey. Capacité d'une Antenne Horizontale. *Rev. Gen. d'El. 4*, 790-792, Nov. 23, 1918. (Derivation of Pederson formula by use of complex quantities).

L. Zehnder. Horizontal Antennas. *Elektrotechnische Zeitschrift, 52*, 515-516, Dec. 26, 1918.

Max Abraham. Die Strahlung von Antennen Systemen. *Jahrbuch 14*, 146-152, July 1919.

E. F. W. Alexanderson. Simultaneous Sending and Receiving, *Proc. I. R. E. 7*, 363, August 1919.

Thomas G. Alvord. Radio Messages to Submarines from Shore or Sea Discovered by Accident in War Experiment. *New York Herald*, August 10, 1919.

L. W. Austin. Quantitative Experiments with Coil Antennas.

Jnl. Wash. Acad. Sci. 9, 335-339, June 19, 1919. *Rev. Gen. d'El. 6*, 551, October 25, 1919. *El. Wld. 74*, 311, August 9, 1919.

E. Bellini. Un Radiogoniométre Electrostatique, *R. G. E. VI 122D*, Nov. 1, 1919. *Electrician*, Sept. 12, 1919. LXXXIII, p. 273-275.

A. Blondel. On the Goniometric Functions Applicable to Directive Aerials. *Radio Review I*, 1-10, Oct., 1919.

A. Blondel. On the Electric Functions Applicable to Directive Aerials. *Tech. Review, Eng. Index 5*, 198, Nov. 25, 1919. *Radio Review*, Nov. 1919.

A. Blondel. On the Goniometric Functions Applicable to Directive Aerials. *Radio Reviews I*, 58-66, November 1919.

A. S. Blatterman. Theory and Practical Attainments in the Design and Use of Radio Direction Finding Apparatus Using Closed Coil Antennas. *Journal of the Franklin Institute 188*, 289-362, Sept. 1919.

A. S. Blatterman. Radio Direction Finding Apparatus. *Electrical World 73*, 464, 1919.

P. H. Boucheron. Guarding the Ether During the War—the Radio Compass. *Radio Amateur News, 1*, 104, Sept. 1919.

David S. Brown. Loop Antennae and Direction Finders for Amateur Use. *Radio Amateur News, 1*, 60, August 1919.

J. S. Brown. Underground Antennas. *Radio Amateur News* July 1919.

J. Stanley Brown. Design of Rogers "Ground" Antenna. *Radio Amateur News, 1*, 10, July 1919.

W. Burstyn. Die Strahlung und Richtwirkung einiger Luftdrahtformen in freien Raume. *Jahrb. d drahtl. Tel. 13*, 362-378, Jan. 1919.

W. Burstyn. Die Schleife als Empfänger. *Jahrbuch 13*, 378-385, Jan. 1919. *Elec. World, 73*, 1177, May 31, 1919. *Sci. Abs. B*, No. 481, June 1919.

Capt. C. A. Culver. Transatlantic Radio Reception. *Journal of the Franklin Institute*, May 1919, p. 578.

J. H. Dellinger. Radio Transmission Formulas for Antenna and Coil Aerials. *Journ. Franklin Institute 188*, 95-96, July 1919.

J. H. Dellinger. Principles of Radio Transmission and Reception with Antenna and Coil Aerials. *Scientific Paper of the Bureau of Standards*, No. 354, December 11, 1919.

J. H. Dellinger. Bureau of Standards Radio Work. *The Federal Employes 4*, 590, Oct. 11, 1919. Reprinted in *Cleveland Plain Dealer*, Nov. 23, 1919.

A. K. Dyall. Workings of the Radio Compass. *Popular Science Monthly 95*, 116, Sept. 1919.

W. H. Eccles. Wireless Telegraphy and Telephony. (Book). p. 134.

R. A. Fessenden. The Fessenden Pelorus. (Wireless Compass). A Caution as to its Use. *Electrician 83*, 719, Dec. 19, 1919.

A. O. Gibbon. Wireless Telephony and Direction Finding Demonstrations. Post Office *El. Eng*, July 12, 1919, 101-104.

John Hays Hammond, Jr. Aero Radio Surveying and Mapping. *Flying 8*, 160, 184, March 1919.

Walter J. Henry. A Loop of Wire. *Sci. American 120*, 362, Oct. 11, 1919.

W. J. Henry. The Priess Loop Set. *Radio Amateur News, 1* 270, Dec. 1919.

Charles D. Herrold. Experiments on Ground Antenna with their Relation to Atmospherics. *Radio Amateur News, 1*, 11, July 1919.

E. T. Jones. Underground Radio Made Possible for the Amateur. *Radio Amateur News, 1*, 274, Dec. 1919.

E. T. Johnstone. A Radio Goniometer. *Radio Amateur News, 1*, 179, October 1919.

L. M. Knoll. Concentrated or Loop Aerials. *Electrical Experimenter 7*, 328, August 1919.

Jerome Lachenbruch. The Wireless Compass. *Sci. Am. 120*, 291, March 22, 1919.

Jerome Lachenbruch. Radiogoniometer. *Elec. Rev.* (London) April 25, 1919.

Marconi and Franklin's Directive Radio System. *Wireless Age, 6*, 20, Sept. 1919.

A. Meissner. Uber die Bestimmung der Eigenschwingung von Antennen. *Jahrbuch 14*, 269-270. August 1919.

A. Meissner. Uber die Bestimmung der Eingeschwingung von Antennen . *Phys. Ztschft. 20*, 130-131, March 15, 1919.

L. A. Pollock. Value of Radio Compass Invented by F. A. Kolster. *Radio Am. News 1*, 136, Sept. 1919.

Capt. J. Robinson. Directional Wireless with Special Reference to Aircraft. *Radio Review 1*, 39, Oct. 1919.

J. Robinson. Direction Finders for Aircraft. (British Association Paper). *Wireless World 7*, 475, Nov. 1919.

Robinson's Manual of Radio Telegraphy and Telephony. Fifth edition (1919) p. 173.

J J. Rogers. International Submarine and Radiocommunication as Related to American Commerce. *Telegraph & Telephone Age*, Dec. 16, 1919, p. 614-417.

H. R. S. Sankey. Wireless Direction Finders. *Engineering 108*, 523, Oct. 17, 1919. Lecture at Olympia Ex. Oct. 9, 1919. *Tech. Rev. 5*, 182, Nov. 25, 1919.

H. R. S. Sankey. Direction Finding by Wireless Telegraphy. *Elec. Rev. 85*, 509, Oct. 17, 1919. Abs. lecture, Shipping Exhibition.

H. R. Shankey. Direction Finding by Wireless Telegraphy. *Elec. Rev. 85*, 539, Oct. 24, 1919. Abst. lecture, Shipping Exhibition.

H. R. Sankey. Wireless Direction Finders. *Engineering* (London). 108, 523, Oct. 17, 1918.

H. Winfield Secor. The Rogers Underground Wireless. *Electrical Experimenter, 6*, March 1919.

Martin D. Stevers. Working Radio Underwater and Underground. *Illustrated World*, June 1919.

H. de F. Stevers. Submarine Radiotelegraphy. *Armour Eng. 11*, 12-16, Nov. 1919.

Geo. O. Squier. (Editorial) Airplane Direction Finder a Development of the War. *Electrical World 73*, 222, Feb. 1, 1919.

Martin D. Stevers. Working Radio Underwater and Underground. *Illustrated World, 31*, 539, June 1919.

A. Hoyt Taylor. The Possibilities of Concealed Receiving System. *Proc. I. R. E. 7*, 261-266. June 1919.

A. Hoyt Taylor. Subterranean and Submarine Antennae. *Elec. Review 85*, 620, Nov. 14, 1919. Proc. I. R. E.

A. Hoyt Taylor. Short-Wave Reception and Transmission on Ground Wires. (Subterranean and Submarine). *Proc. I. R. E. 7*, 337, August 1919.

A. Hoyt Taylor. Long-Wave Reception and the Elimination of Strays on Ground Wires. (Subterranean and Submarine). *Proc. I. R. E. 7*, 559, December 1919.

Roy A. Weagant. Reception Through Static and Interference. *Proc. I. R. E. 7*, 207-256, June 1919.

J. A. Willoughby and P. D. Lowell. Development of Loop Aerials for Submarine Radio Communication. *Phys. Rev. 14*, 193, August 1919.

Unidirectional Transmitting System. *Wireless Age 6*, 19-20, June 1919.

Directional Effect of Radio Useful in Transatlantic Aircraft Flight. *Wireless Age, 6*, 9, June 1919.

Loop Antenna Used Thirteen Years Ago. *Everyday Engineering 8*, p. 111, Nov. and Dec. 1919.

The Loop Fallacy. *Q. S. T. 3*, 23, September 1919.

Navigation by Direction Finding Apparatus. *Wireless World, 7*, 105. May 1919.

The Marconi Direction Finder. *Electrician 83*, 142-143, Aug. 9, 1919. Describes standard Marconi apparatus. *Elec. World 74*, 770, Oct. 4, 1919.

Field Wireless Telephony and Direction Finding. *Wireless World 7*, 199-205, July 1919.

Direction Finder Locates Ship with Great Accuracy. *Wireless Age, 6*, 10, June 1919.

Direction Finding by Wireless. *London Times Eng. Sup.* June 1919.

Artom's Visual Receiver for Directive Wireless Telegraphy. *Wireless Age, 21*, August 1919.

Richtungsbestimmung im Flugeug mittels drahtlosen Telegraphie, *Luftfahrt 23*, April 1919.

Marconi Radiogoniometer. *Electrician 83*, 142, August 8, 1919.

La Radiogoniometrie. *R. G. E. 6*, 112D, Nov. 1, 1919. *La Nature*, June 28, 1919. p. 404-414.

The U. S. Navy Radio Compass. *Everyday Engineering, 8*, 109-11, Nov. and Dec., 1919.

Radiogoniométre. *La Nature*. No. 2361. 404-414, June 28, 1919.

Rogers' Underground Aerials. *Wireless Age, 6*, 31, August 1919.

The United States Naval Communication Service. *Journal of the Franklin Institute, 188*, 751, Dec. 1919.

How Bureau of Standards Helped Win the War. *Washington Star*, August 24, 1919.

The Submarine's Under-Water Radio. *Electrical Experimenter 7*, 539, October 1919. November 1919, 7, p. 661

Ralph R. Batcher. Loop Antenna for Submarines. *The Wireless Age*, 7, 28, Jan. 1920.

A. Hoyt Taylor. The Use of Ground Wires at Remote Control Stations. *Proc. I. R. E.*, 1920.

Discussion

H. H. Beverage: Since the wave antenna paper was written, more quantitative measurements have been made on the effective height of the wave antenna. The measurements mentioned in the paper gave the ratio of horizontal to vertical voltage gradient as 30 per cent. This measurement was based on comparatively few observations and was not checked simultaneously against an antenna of known effective height. Furthermore, the measurements were based on vertical gradients calculated by Austin's formula, which gives values much lower than observed values at certain periods of the day.

Recently, a much more accurate quantitative method has been developed for measuring signal intensity, and many measurements have been made on the wave antenna at Belmar. In this case, the effective height of the wave antenna was determined by simultaneous observations on a vertical antenna of known effective height. Measurements made by H. H. Beverage and H. O. Peterson indicated that the average effective height of the Belmar long wave antenna was 200 meters. Since the horizontal length was 12,500 meters, the horizontal potential gradient is 1.6 per cent of the vertical potential gradient. At 11,000 meters, the effective height was somewhat greater than this average value, and at 19,000 meters the effective height was lower than the average value. These observations give the same order of magnitude as the calculations by Zenneck's formula for wave tilt.

On short wave lengths, that is, 200 to 600 meters, the horizontal gradient appears to be around 5 per cent to 6 per cent of the vertical gradient over soil of moderately low conductivity such as is found at Belmar. This increase in tilt with shorter wave length is in accordance with Zenneck's formula.

A further check on the wave tilt theory has been noted by comparing the effective height of short wave antennas over earth of high and low conductivity. These antennas were about 400 meters long, and measurements made by Peterson on 400 meters indicated that the signal strength on an antenna over moderately conducting ground was three to four times as great as on a similar antenna partly over salt water and partly over a marsh. In this case, comparisons were made directly with a local loop in order to check the field intensity in the vicinity of the wave antennas.

Dissymmetrical Electrical Conducting Networks

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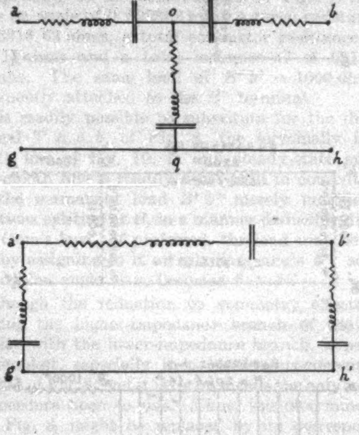
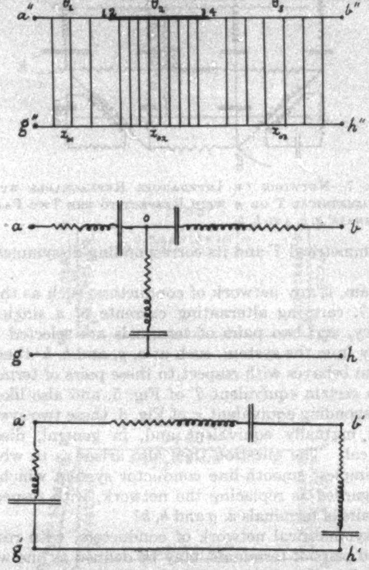
It is well known that a symmetrical T-connection of three simple impedances $a\,o$, $o\,b$, and $o\,q$, Fig. 1, in which the two arms $a\,o$, and $b\,o$ are electrically equal, and with their common terminal o connected to the ground or return conductor $g\,h$ through the impedance $o\,q$, can be completely replaced,* at any single assigned alternating-current frequency, by a certain symmetrical π, $a'\,b'\,g'\,h'$, Fig. 2, in which the two pillars $a'\,g'$ and $b'\,h'$, have equal impedances; also that either of these two symmetrical systems can be completely replaced, at the same frequency,† by a single smooth line conductor $a''\,b''$, Fig. 3, having uniformly distributed series impedance in its conductor and also uniformly distributed lateral leak admittance between the conductor and the return conductor $g''\,h''$. Such a line conductor may be regarded as subtending or possessing, at the frequency in question, a certain complex hyperbolic angle θ, and also a certain single surge impedance or "characteristic impedance," z_0 ohms. Consequently, both the symmetrical T conducting system of Fig. 1, and the symmetrical π of Fig. 2, may be regarded as possessing the same complex angle θ and surge impedance z_0, by virtue of equivalence. In any network of conductors, carrying alternating currents of the assigned frequency in the steady state, any of the three equivalent systems of Figs. 1, 2 and 3 may be interchanged, without disturbing the distributions of potential, current and power, at and outside the terminals of the system.



FIGS. 1, 2 AND 3—SYMMETRICAL T, π AND SMOOTH LINE, EQUIVALENT AND INTERCHANGEABLE



FIGS. 4, 5 and 6—COMPOSITE LINE OF THREE SECTIONS WITH ITS EQUIVALENT DISSYMMETRICAL T AND π

Moreover, if two or more simple smooth line conductors, of the type indicated in Fig. 3, are connected in series to form a composite line as in Fig. 4, it is known that the composite line system $a''\,b''\,g''\,h''$ (Fig. 4) can be replaced by its equivalent T of Fig. 5,‡ or by its equivalent π of Fig. 6, and that these equivalent conductors are, in general, dissymmetrical. For a given composite-line system of Fig. 4 operated in the steady state at a single frequency, there can be only one equivalent T and also only one equivalent π. On the other hand, neither the equivalent T of Fig. 5, nor the equiva-

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* Bibliography (1).

† Bibliography (3).

‡ Bibliography (4).

lent π of Fig. 6 can determine the elements of smooth line section in the equivalent composite-line system of Fig. 4. In the absence of specific information concerning the number and characters of the component-line sections, an infinite number of different composite-line systems like that of Fig. 4, could be found as equivalents for the dissymmetrical T or π of Figs. 5 and 6. The question thus arises as to what is the simplest smooth-line system which can replace a given dissymmetrical T and its corresponding dissymmetrical π?

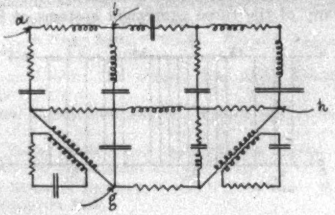


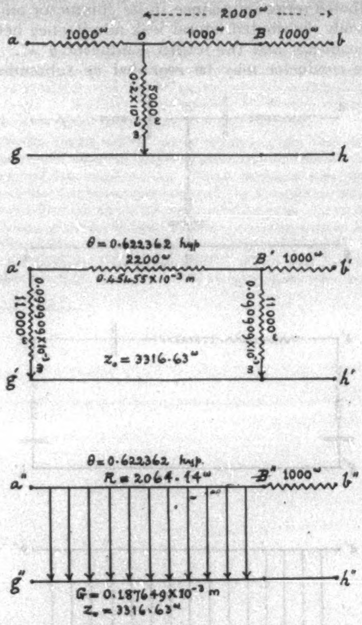
Fig. 7—Network of Impedances Replaceable by One Dissymmetrical T or π with Respect to the Two Pairs of Terminals, a, g, and b, h.

Again, if any network of conductors, such as that of Fig. 7, carrying alternating currents of a single frequency, and two pairs of terminals are selected arbitrarily from the system, such as a, g, and b, h; then the system behaves with respect to these pairs of terminals like a certain equivalent T of Fig. 5, and also like the corresponding equivalent π of Fig. 6, these two systems being mutually equivalent and, in general, dissymmetrical. The question then also arises as to what is the simplest smooth-line conductor system which can be regarded as replacing the network, with respect to the pairs of terminals a, g and b, h?

A symmetrical network of conductors, with respect to two pairs of terminals, may be defined as one whose equivalent T and π are symmetrical, like Figs. 1 and 2. A dissymmetrical network is then one which, with respect to the two pairs of terminals, has a dissymmetrical equivalent T and π, like Figs. 5 and 6. In general, networks are dissymmetrical, and symmetrical networks are particular cases.

Although we always discuss a network with reference to two pairs of terminals, such as a, g and b, h in Fig. 7; yet the number of terminals involved may be reduced, in particular cases, from four to three, by the merging of two terminals. Thus, we might select the two pairs of terminals, a, g and b, g, which employ the terminal g in common.

As has already been pointed out, a network which is symmetrical with respect to two pairs of terminals becomes defined either by a certain symmetrical T and π, or by a certain hyperbolic angle θ and accompanying surge impedance z_0. We can then readily compute what changes in alternating-current potential difference, current and power will be brought about at one of the pairs of terminals by any assigned electric change steadily impressed on the other pair. When, however, the network is dissymmetrical, we shall find that it requires for its definition one additional characteristic. A convenient additional characteristic is the "inequality ratio" q; so that the network acquires three characteristics θ, z_{ab} and q, which are capable of being measured in any given case, by following a suitable technique at each pair of terminals. Having ascertained these three determining characteristics for the network, we can either draw immediately its equivalent T and π; or we can compute what change in potential difference, current and power will be produced at one pair of terminals, say a, g, by a given change established steadily at the other pair b, h.



Figs. 8, 9 and 10—Dissymmetrical T Reduced to Symmetry by the Retention of a Permanent Series Load at the b Terminal with the Correspondingly Loaded π and Smooth Line.

The computation is effected by following the same process as in the symmetrical case; but with a slight modification.

The subject can best be studied with reference to continuous-current lines and networks; *i. e.*, with reference to the frequency of zero. The results so obtained are immediately applicable to alternating-current systems, by the substitution of complex numbers for real numbers, in the well known way.

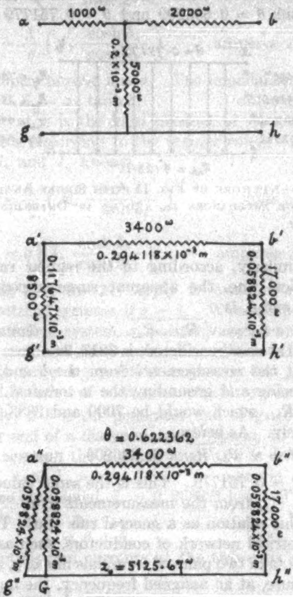
Fig. 8 shows a simple dissymmetrical *T*. In this particular case the branch *a o* has 1000 ohms and the branch *b o* 2000 ohms. Two methods suggest themselves for dealing with it; namely, (1) by reducing it to a terminally loaded symmetrical *T*, and (2) by retaining the dissymmetry, but making measurements from each end in turn. We may consider these two methods successively.

(1) *Method of Reduction to Symmetry*. The simplest way to reduce the *T a, o, b* of Fig. 8 to symmetry, is perhaps to cut off the portion *B, b* from the preponderating arm, so as to leave the remainder *o, B* equal to the branch *a, o*. We thus obtain symmetrical *T a, o, B* with a load *B, b*, in this case 1000 ohms, permanently attached to the terminal *B*. The corresponding loaded symmetrical π is shown in Fig. 9; where the same load *B′, b′*, is permanently attached to the *B′* terminal. Fig. 10 gives the terminally loaded smooth line *a″ B″*, corresponding to the combinations of Figs. 8 and 9. It has an angle of 0.622362 hyp., a surge resistance of z_0 = 3316.63 ohms, a total conductor resistance *R* of 2064.14 ohms and a total leakance *G* of 0.187649 millimho. The same load of *B″ b″* = 1000 ohms is permanently attached to the *B″* terminal.

It is readily possible to substitute for the dissymmetrical *T a, o, b*, of Fig. 8, the terminally loaded smooth line of Fig. 10, in any steady-state system. The smooth line is readily dealt with in computation, and the permanent load *B″ b″* merely modifies the conditions existing at *B*, in a manner deducible directly from Ohm's law. If preferred, the load may be dealt with by assigning to it an auxiliary angle θ'; so that the position angle at *a*, becomes $\theta + \theta'$.

Although the reduction to symmetry effected by reducing the higher-impedance branch of the *T* to equality with the lower-impedance branch, is perhaps the simplest, especially in a continuous-current case like that of Fig. 8; yet it is by no means the only method of procedure open to use. Thus, the dissymmetrical *T* of Fig. 8, might be replaced by its corresponding dissymmetrical equivalent π, as in Figs. 11 and 12. The π, *a′ b′ g′ h′* of Fig. 12 may be reduced to symmetry, by cutting off part of the leak *a′ g′* as a terminal load, and leaving the remainder at *a″ G*, Fig. 13, equal to the leak *b″ h″*. This leaves the symmetrical π, *a″ b″ G h″*, of angle θ = 0.622362 and surge impedance z_0 = 5125.67 ohms, but with the leak *a″ g″* of 0.058824 millimho permanently attached to *a″*. This symmetrical π might now be replaced either by a leak-loaded symmetrical *T*, or by a smooth line with the same terminal leak load.

Again it would be possible to reduce the *T* of Figs. 8 and 11 to symmetry, by employing permanent pairs of unequal terminal series loads, as for instance 100 ohms at *a*, and 1100 ohms at *b*, so as to produce a symmetrical *T* with 900 ohms in each branch. This could be done in an infinite number of ways, each giving a different θ, a different z_0, and a different pair of permanent terminal loads. It would also be possible to reduce the π of Fig. 12 to symmetry, by employing pairs of unequal terminal leak loads. There would be an infinite number of such possible pairs, with corresponding values of θ and z_0. Consequently, a dissymmetrical network admits of being reduced to symmetry for computation, in one way with a single terminal series load, in one way with a single terminal leak load, in an infinite number of ways with a pair of opposite terminal series loads, and also in an infinite number of ways with a pair of opposite terminal leak loads. In practise, the two single-load methods are preferable and easier to use.



Figs. 11, 12 and 13. Dissymmetrical *T* Replaced by its Correspondingly Dissymmetrical π, Which is Reduced to Symmetry by the Retention of a Permanent Leak at the *a″* Terminal.

(2) *The Method of Applying the Inequality Ratio q.* In this method, the system is left in its dissymmetrical state; but measurements are made from each pair of terminals in turn, to determine the apparent surge impedance from each end.

Returning to the dissymmetrical *T* of Figs. 8 and 11, we can measure the resistance R_{af} offered between the terminal *a* and the return conductor or ground *g*, when the system is freed or opened at the opposite terminals b, h. We may also measure the resistance R_{ag} offered between the same terminals at a, when the end b is grounded, or the terminals b, h are short-circuited. Following the usual rule for finding the angle of a symmetrical smooth line, the apparent angle of the system is expressed by:

$$\tanh \theta = \sqrt{R_{ag}/R_{af}} \qquad \text{numeric} \angle \qquad (1)$$

In the particular case of Fig. 8, the resistance R_{ag} would be 2428.57 ohms, and $R_{af} = 6000$ ohms, from which $\tanh \theta = 0.636209$ and $\theta = 0.751779$ hyp. radians. Similarly, according to the regular rule for a symmetrical line, the apparent surge impedance z_{0a} from the end a, is

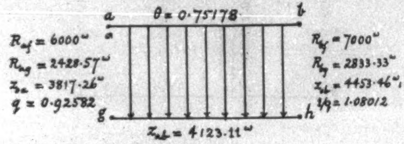


FIG. 14—NETWORK OF FIG. 11 WITH SINGLE ANGLE θ, BUT WITH SURGE IMPEDANCES z_{0a} AND z_{0b} AS DETERMINED FROM EACH END.

$$z_{0a} = \sqrt{R_{af} \,.\, R_{ag}} \qquad \text{ohms} \angle \qquad (2)$$

which in the case considered, is 3817.26 ohms. If now we repeat the measurements from the b end, successively freeing and grounding the a terminal, we find R_{bf} and R_{bg}, which would be 7000 and 2833.33 ohms respectively. As before,

$$\tanh \theta = \sqrt{R_{bg}/R_{bf}} = 0.636209 \qquad \text{numeric} \angle \qquad (3)$$

whence $\theta = 0.751779$. This is the same value as was found by (1), from the measurements at a. We may express this relation as a general rule thus: If in any dissymmetrical network of conductors, such as that of Fig. 7, we take two pairs of terminals like a, g, and b, h, and measure, at an assigned frequency, the impedance offered by the network between each pair, with the opposite pair first opened and then shorted, we may denote these impedances by R_{af}, R_{ag} and R_{bf}, R_{bg}, ohms $\angle$ respectively. Then the ratios R_{ag}/R_{af} and R_{bg}/R_{bf} will be equal to each other, and to the square of the tangent of the angle subtended by the network with respect to those four terminals.

On the other hand, if the network is dissymmetrical, the apparent surge impedance from b, or

$$z_{0b} = \sqrt{R_{bf} \,.\, R_{bg}} \qquad \text{ohms} \angle \qquad (4)$$

will not be the same as z_{0a}. In the case considered, $z_{0b} = 4453.46$ ohms. The geometrical mean z_{ab} of the two opposite apparent surge impedances is defined by

$$z_{ab} = \sqrt{z_{0a} \,.\, z_{0b}} \qquad \text{ohms} \angle \qquad (5)$$

is a characteristic property of the network, in this case 4123.11 ohms. It may be called the "geomean surge impedance" of the network.

The ratio of the a-end surge impedance z_{0a} to the geomean surge impedance z_{ab} may be called q, the "inequality ratio" of the system. It will be defined by:

$$q = z_{0a}/z_{ab} = \sqrt{R_{ag}/R_{bg}} = \sqrt{R_{af}/R_{bf}} = \sqrt{z_{0a}/z_{0b}}$$

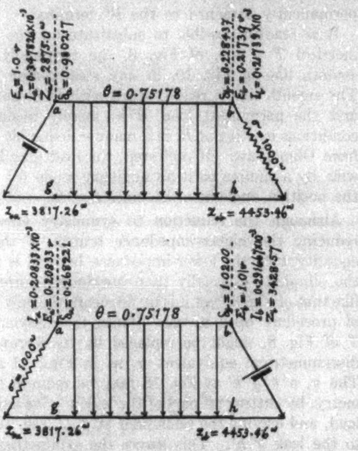
$$= \sqrt{\frac{\rho_1 + \mathcal{R}}{\rho_2 + \mathcal{R}}} = \sqrt{\frac{g_2 + \nu}{g_1 + \nu}} \qquad \text{numeric} \angle \qquad (6)$$

In this case the inequality ratio is $q = 0.92582$.

We may thus represent either the dissymmetrical T of Fig. 11, or its corresponding π of Fig. 12, by the dissymmetrical line system of Fig. 14. Here the line a, b, is to be regarded as a composite line, offering the same angle from each end, but different surge impedances.

Terminal Potentials and Currents in a Dissymmetrical System. If we have any physically consistent distributions of potential, current and power at the a and b terminals of a known dissymmetrical system, and we know the values of potential and current at one end, we can readily ascertain the corresponding values at the other, for one and the same frequency.

In the case of Fig. 15, we have the same dissymmetrical system as in Figs. 11, 12 and 14, for which θ, z_{0a} and z_{0b} are given. A potential of 1.0 volt and an entering current of $I_a = 0.347826$ milliampere, are found at the generating end a. Required the corresponding values at the motor end b. The value of the load σ at b need not be known. We have then:



FIGS. 15 AND 16—NETWORK LOADED WITH A TERMINAL IMPEDANCE σ AT EACH END ALTERNATELY

$$E_b = \frac{1}{q}\,(E_a \cosh \theta - I_a\, z_{0a} \sinh \theta) \quad \text{volts} \angle \qquad (7)$$

and $I_b = q\,(I_a \cosh \theta - E_a\, y_{0a} \sinh \theta)$ amperes $\angle$ (8)

where $y_{0a} = 1/z_{0a}$, is the surge admittance, as measured from a. If $q = 1$; so that the dissymmetry is made to disappear, the above formulas are those which apply to any ordinary smooth line of θ and z_0. In the case considered, $E_b = 0.21739$ volt, and $I_b = 0.21739$ milliampere.

If on the contrary, we know E_b and I_b only, without necessarily knowing the physical conditions at the motor terminal b, we may proceed to find the corresponding values at the generator end a. We obtain:

$$E_a = q\,(E_b \cosh\theta + I_b\, z_{0b} \sinh\theta) \qquad \text{volts } \angle \quad (9)$$

and
$$I_a = \frac{1}{q}\,(I_b \cosh\theta + E_b\, y_{0b} \sinh\theta) \qquad \text{amperes } \angle \quad (10)$$

where $y_{0b} = 1/z_{0b}$ is the surge admittance as measured at b. If we make $q = 1$, these formulas become identical with those pertaining to a simple smooth line and symmetrical system. In any such smooth-line case, formulas like (7), (8), (9) and (10) can be employed for any point along the line; but in the case of a dissymmetrical system, they apply only to the terminals a and b.

Again, if b is the generator end, and a the motor end of the dissymmetrical system, (see Fig. 16), and we know the potential E_a and current I_a, at a, we can find the corresponding values at b as follows:

$$E_b = \frac{1}{q}\,(E_a \cosh\theta + I_a\, z_{0a} \sinh\theta) \qquad \text{volts } \angle \quad (11)$$

$$I_b = q\,(I_a \cosh\theta + E_a\, y_{0a} \sinh\theta) \qquad \text{amperes } \angle \quad (12)$$

Similarly, if we know the generator-end values, E_b and I_b, the corresponding motor-end values E_a and I_a become:

$$E_a = q\,(E_b \cosh\theta - I_b\, z_{0b} \sinh\theta) \qquad \text{volts } \angle \quad (13)$$

$$I_a = \frac{1}{q}\,(I_b \cosh\theta - E_b\, y_{0b} \sinh\theta) \qquad \text{amperes } \angle \quad (14)$$

Here again, if $q = 1$, the formulas are those which apply at the terminals of a smooth line and symmetrical system. Hence we may formulate the following rule for any dissymmetrical case. Having given the potential and current at one end, write the corresponding values for the other, with regard to the direction of power transmission, as though the line were smooth and the system symmetrical. Use the surge impedance or admittance belonging to the end at which the values are known. Then an unknown potential E_a at a, will be $q = \sqrt{z_{0a}/z_{0b}}$ times that found by the regular formula, and likewise an unknown current I_b at b. On the other hand, an unknown current I_a at a, or a potential E_b at b, will be $1/q$ times that found by the regular smooth-line formula. The power products $E_a I_a$, and $E_b I_b$ will therefore not differ from those given by the corresponding smooth-line formulas.

Position Angles at the Terminals of a Dissymmetrical System. If we load the motor end b of a dissymmetrical system, like that of Fig. 15, with an impedance σ ohms ∠, we may find the position angle of that end in the same manner as though the load were applied to the motor end of a simple smooth line; *i. e.*

$$\tanh\delta_b = \sigma / z_{0b} \qquad \text{numeric } \angle \quad (15)$$

Thus in Fig. 15, the position angle δ_b at b is 0.228437 hyp. The position angle δ_a at the generator end a is then obtained by the usual rule:

$$\delta_a = \theta + \delta_b \qquad \text{hyps } \angle \quad (16)$$

which in this case is 0.980217 hyp. The potentials and currents at the ends of the dissymmetrical system are then related as follow:

With E_b and I_b known,

$$E_a = q\left(E_b\,\frac{\sinh\delta_a}{\sinh\delta_b}\right) \qquad \text{volts } \angle \quad (17)$$

$$I_a = \frac{1}{q}\left(I_b\,\frac{\cosh\delta_a}{\cosh\delta_b}\right) \qquad \text{amperes } \angle \quad (18)$$

The impedance beyond b is σ. The impedance at a is:

$$Z_a = z_{0a}\tanh\delta_a \qquad \text{ohms } \angle \quad (19)$$

The power at a is the local product of E_a and I_a in vector watts, according to the regular rule.

With E_a and I_a known,

$$E_b = \frac{1}{q}\left(E_a\,\frac{\sinh\delta_b}{\sinh\delta_a}\right) \qquad \text{volts } \angle \quad (20)$$

$$I_b = q\left(I_a\,\frac{\cosh\delta_b}{\cosh\delta_a}\right) \qquad \text{amperes } \angle \quad (21)$$

Formulas (17) to (21) apply to ordinary smooth-line symmetrical systems, if $q = 1$. Whereas, however, in a smooth-line system, a position angle can be assigned to each and every point of the line, we are only justified in assigning position angles to two points—the a and b ends—of a dissymmetrical system.

Similarly, if b is the generator end (Fig. 16) and a the motor end of a dissymmetrical system, to which a load σ is applied, the position angle at a is defined by:

$$\tanh\delta_a = \sigma / z_{0a} \qquad \text{numeric } \angle \quad (22)$$

In the case considered, $\delta_a = 0.268221$ hyp. The position angle at b is:

$$\delta_b = \theta + \delta_a \qquad \text{hyps } \angle \quad (23)$$

The potentials and currents at the two ends are thus related:

With E_b and I_b known,

$$E_a = q\left(E_b\,\frac{\sinh\delta_a}{\sinh\delta_b}\right) \qquad \text{volts } \angle \quad (24)$$

$$I_a = \frac{1}{q}\left(I_b\,\frac{\cosh\delta_a}{\cosh\delta_b}\right) \qquad \text{amperes } \angle \quad (25)$$

With E_a and I_a known,

$$E_b = \frac{1}{q}\left(E_a\,\frac{\sinh\delta_b}{\sinh\delta_a}\right) \qquad \text{volts } \angle \quad (26)$$

$$I_b = q\left(I_a\,\frac{\cosh\delta_b}{\cosh\delta_a}\right) \qquad \text{amperes } \angle \quad (27)$$

The impedance at the generator end is:

$$Z_b = z_{0b}\tanh\delta_b \qquad \text{ohms } \angle \quad (28)$$

Formulas (24) to (27) are identical with (17), (18) and (20), (21).

We may formulate the deductions from the last 13 formulas as follow: In any dissymmetrical system for which θ, z_{0a} and z_{0b} are given, and which is loaded at the motor end with an impedance σ, find the position angle at the motor end, with reference to that end's surge impedance. If the potential and current are given at one end of the system, the values at the other end are found by using the ordinary position-angle formulas for a smooth line. An unknown E_a, or I_b, will then be $q = \sqrt{z_{0a}/z_{0b}}$ times the value so obtained. On the other hand, an unknown E_b or I_a, will be $1/q$ times the value obtained by the ordinary formula. This rule agrees with that given above for use with formulas (**7**) to (**14**).

In view of the above mentioned reciprocal factors for potential and current at each terminal, leaving the power products unchanged, it becomes possible to reduce a dissymmetrical system to a corresponding symmetrical system plus a terminal ideal transformer, without losses, and with a transformation ratio of q. It is doubtful, however, there would be any advantage in this plan over the method above outlined.

In the case of an actual composite line, like that of Fig. 4, with definite sections, having respective angles $\theta_1, \theta_2, \theta_3 \ldots$ and surge impedances $z_{01}, z_{02}, z_{03}, \ldots$, it is advantageous to work up the position angles at the successive terminals commencing with the motor end. The terminal position angle at a would thus become δ_1, with a terminal surge impedance z_{01}; while working from the other end, the terminal position angle would be δ_n, with a terminal surge impedance of z_{0n}. The impedance R_{ag} as measured at a would be

$$R_{ag} = z_{01} \tanh \delta_1 \qquad \text{ohms} \angle \qquad (\mathbf{29})$$

while that measured at b would be

$$R_{bg} = z_{0n} \tanh \delta_n \qquad \text{ohms} \angle \qquad (\mathbf{30})$$

These values are not inconsistent with (**19**) and (**28**), which pertain to a general network; whereas (**29**) and (**30**) pertain to a particular composite line with detailed sections.

Receiving-End Impedance of a Dissymmetrical Network. If we define this quantity as the ratio of the e. m. f. impressed at the generator terminals to the current received through a load connecting the motor terminals, we find that for a load σ at b, (Fig. 15), the receiving end impedance is:

$$Z_{lb} = q\,(z_{0b} \sinh \theta + \sigma \cosh \theta) \qquad \text{ohms} \angle \qquad (\mathbf{31})$$

In the case represented in Fig. 15, $Z_{lb} = 4600.0$ ohms. When $q = 1$, this expression reduces to the ordinary value $z_0 \sinh \theta + \sigma \cosh \theta$, for a simple smooth line or symmetrical system.

When $\sigma = 0$, or the b terminals are short-circuited, (**31**) becomes:

$$Z_{lb} = q\, z_{0b} \sinh \theta = z_{ab} \sinh \theta \qquad \text{ohms} \angle \qquad (\mathbf{32})$$

which is the architrave impedance in Fig. 12, or 3400.0 ohms.

Similarly, for a load of σ ohms $\angle$ at the motor end a, the receiving-end impedance becomes (Fig. 16.)

$$Z_{la} = \frac{1}{q}\,(z_{0a} \sinh \theta + \sigma \cosh \theta) \qquad \text{ohms} \angle \qquad (\mathbf{33})$$

In the case of Fig. 16, this becomes 4800.0 ohms. When $q = 1$, the expression (**33**) again coincides with the ordinary smooth-line formula. Moreover, when $\sigma = 0$,

$$Z_{la} = \frac{1}{q}\,(z_{0a} \sinh \theta) = z_{ab} \sinh \theta \qquad \text{ohms} \angle \qquad (\mathbf{34})$$

or 3400.0 ohms, the architrave impedance of the equivalent dissymmetrical π.

We may formulate these results as follows. The receiving-end impedance of a dissymmetrical network loaded at the receiving end, is found by the usual rule for a terminally loaded smooth line, using the surge impedance of the motor end. The result must then be multiplied by q, when the generator is at a, and by $1/q$ when the generator is at b. In case the load is short-circuited, the receiving-end impedance reduces to $\rho'' = z_{ab} \sinh \theta$, the architrave impedance of the equivalent dissymmetrical π.

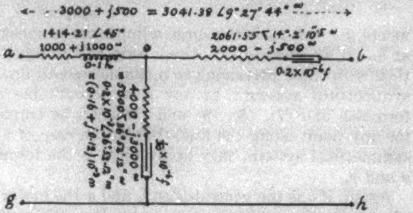


FIG. 17—DISSYMMETRICAL *T* OF IMPEDANCE REPRESENTING A NETWORK OF ALTERNATING-CURRENT CONDUCTORS

Technique of Measurements. We have assumed that the measurements made on the network are the four terminal impedances R_{af}, R_{ag}, R_{bf}, and R_{bg}. In cases where there is but little difference between R_{af} and R_{ag}, or between R_{bf} and R_{bg}, the value of θ may not admit of being determined with satisfactory precision; although the value of z_{ab} will usually be acceptable. In such a case, if the two ends of the network can be brought to the same testing table, it may be preferable to adopt the following technique:

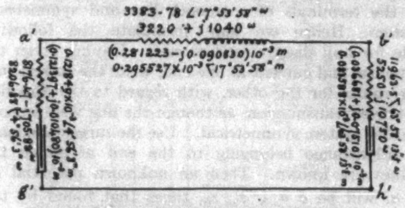


FIG. 18—DISSYMMETRICAL EQUIVALENT π OF SAME

1. Measure, with b open, the impedance R_{af} ohms at a, and the entering current I_a at a. At the same time measure the vector p. d. e_b volts thereby produced at the b terminals. From these data, ρ_1 and $\mathfrak{R}$ can be deduced.

2. Measure R_{ag} ohms $\angle$ at a. With this value and R_{af}, find z_{0a}, by (2).

3. Repeat the measurements (1) and (2) reciprocally with the ends of the network reversed, so as to obtain R_{bf}, e_0 and R_{bg}. These will give ρ_2 and z_{0b}, together with check values of θ and $\mathcal{R}$.

Example of a Dissymmetrical Alternating-Current System. Fig. 17 offers a particular dissymmetrical T system, which might be the equivalent of a certain network between the two pairs of terminals a, g and b, h. The impedance $a\,o$, is $1000 + j\,1000$ ohms, such as might be produced by an inductor of 0.1 henry and 1000 ohms, at an angular velocity of $\omega = 10000$ radians per second, (1591.5 ~). The impedance $o\,b$ is $2000 - j\,500$ ohms, such as might be produced at the same frequency by a condenser of 0.2 microfarad, in series with 2000 ohms. The impedance $o\,g$ is $4000 - j\,3000$ ohms. The equivalent π of this system is given in Fig. 18, and the equivalent composite line network in Fig. 19. The angle subtended by the network is $\theta = 0.705768 + j\,0.305124$ hyps; or quadranting the imaginary part $\theta = 0.705768 + j\,\underline{0.194248}$ hyps. The functions of this angle are:

$$\sinh\theta = 0.822652 \angle 27°\,23'\,05''$$
$$\cosh\theta = 1.223218 \angle 10°\,50'\,28''$$
$$\tanh\theta = 0.672531 \angle 16°\,32'\,37''$$

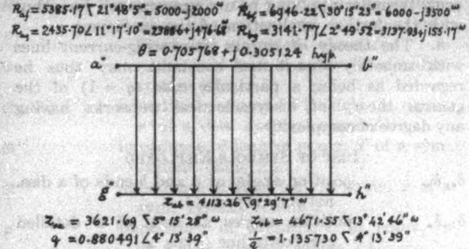

Raf = 5385.17 ⦨ 21°48'5" = 5000 − j2000 ω
Rbf = 6946.22 ⦨ 30°15'23" = 6000 − j3500 ω
Rag = 2435.70 ∠ 11°17'10" = 2388.6 + j476.68 ω
Rbg = 3141.77 ∠ 2°49'52" = 3137.93 + j155.17 ω
θ = 0.705768 + j0.305124 hyp.
a"
b"
g"
h"
zab = 4113.26 ⦨ 9°29'7" ω
zoa = 3621.69 ⦨ 5°15'28" ω
zob = 4671.55 ⦨ 13°42'46" ω
q = 0.880491 ∠ 4°13'39"
1/q = 1.135730 ⦨ 4°13'39"

FIG. 19—NETWORK OF FIG. 17 WITH SINGLE ANGLE θ, BUT WITH SURGE IMPEDANCES z_{0a} AND z_{0b} AS DETERMINED FROM EACH END.

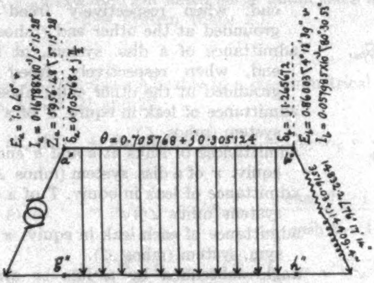
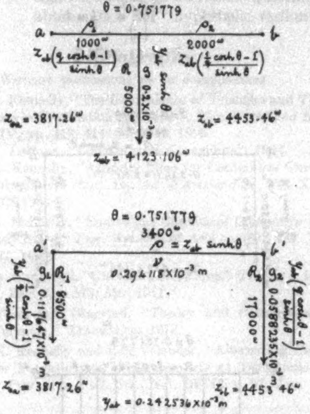

Ea = 1.0 ∠0° v
Ia = 0.167788×10⁻³ ∠5°15'28"
Za = 5956.48 ⦨ 5°15'28"
δa = 0.705768 + jπ/2
θ = 0.705768 + j0.305124
δb = j1.265672
Eb = 0.86003 ⦨ 4°13'39" v
Ib = 0.0579687×10⁻³ ⦨ 80°30'55"
a"
b"
g"
h"
14832.2 ∠76°17'14"
3516.03 + j14409.4 ω

FIG. 20—SAME WITH TERMINAL LOAD AT b

Many of the formulas in Appendix A can be checked from Figs. (17), (18) and (19) by an inspection of the slopes of the various complex quantities presented.

In Fig. 20, the system of Figs. (17), (18) and (19) is represented as being loaded at the b end with $\sigma = 3516.03 + j\,14409.4$ ohms, and supplied at the a end with an e. m. f. of $1.0 \angle 0°$ volt at the frequency of reference. The terminal position angles, potentials and currents are marked on the Figure, in accordance with formulas (15) to (21).

In Appendix A, a number of formulas have been collected relating to dissymmetrical networks and their equivalent circuits. The notation is in accordance with Figs. 21 and 22, and the numerical values pertain to the particular case of Figs. 14, 15 and 16.

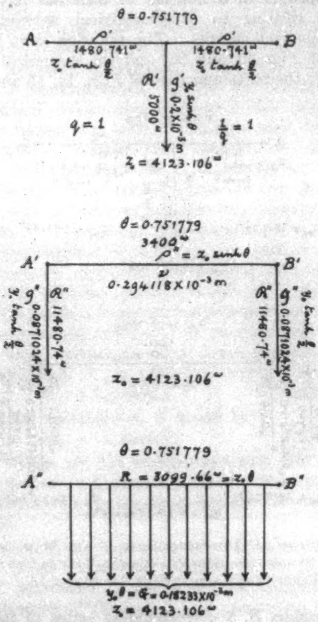

θ = 0.751779
a
b
ρ₁
ρ₂
1000 ω
2000 ω
zab((q cosh θ − 1)/sinh θ)
zab((1/q cosh θ − 1)/sinh θ)
𝓡
g
yab sinh θ
5000 ω
0.2×10⁻³ m
zoa = 3817.26 ω
zob = 4453.46 ω
zab = 4123.106 ω
θ = 0.751779
3400 ω
ρ = zab sinh θ
a'
b'
ν
0.294118×10⁻³ m
g₁
𝓡₁
8500 ω
yab((1/q cosh θ − 1)/sinh θ)
0.117647×10⁻³ m
𝓡₂
g₂
17000 ω
yab((q cosh θ − 1)/sinh θ)
0.0588235×10⁻³ m
zoa = 3817.26 ω
zob = 4453.46 ω
yab = 0.242536×10⁻³ m

FIGS. 21 AND 22—DISSYMMETRICAL T AND π WITH THEIR ELEMENTS SYMBOLIZED, ALSO NUMERICAL VALUES CONFORMING WITH THE PARTICULAR CASE OF FIGS. 11, 12 AND 14.

In Appendix B, a corresponding series of formulas has been collected, relating to symmetrical networks and their equivalent circuits; *i. e.*, for the case $q = 1$. The notation is in accordance with Figs. 23, 24 and 25. The numerical values pertain to the system having the same θ as in Figs. 14, 15 and 16 and with its surge impedance z_0 equal to the geomean surge impedance z_{ab} of the dissymmetrical system. The values of R_f and R_g in this case are respectively the geomean values of R_{af}, R_{bf} and R_{ag}, R_{bg}.

We have considered, in all of the cases represented by the figures, that in any dissymmetrical T, all of the series impedance is lodged in the conductor a, o, b, and that the return conductor g, h, has negligible impedance. According to the known theory of T conductors, however, any desired share of series impedance can be transferred from a, o, b, to g, h, without disturbing the e. m. fs., currents and powers at and beyond the system terminals, provided that the total series impedance on the a side of the leak $o\ q$ is constant, and likewise the total series impedance on the b side. In other words a T-system may be converted into a H-system.

Similarly, although in all the π-systems indicated in the Figures, all the series impedance is lodged in the conductor a', b'; yet any desired share of this impedance may be transferred to g', h', without disturbing the e. m. fs., currents and powers at and beyond the



FIGS. 23, 24 AND 25—SYMMETRICAL T, π AND SMOOTH LINE HAVING THE SAME ANGLE FEEDER AS IN FIGS. 14, 21 AND 22, ALSO WITH $z_0 = z_{ab}$ OF THE DISSYMMETRICAL CASE.

system terminals. In other words, the dissymmetrical π s may be converted into corresponding O-systems.

SUMMARY OF CONCLUSIONS

1. Any dissymmetrical conducting network, each element of which obeys Ohm's law generalized for alternating currents, may be reduced to symmetry, with respect to two pairs of terminals, and at an assigned frequency, either by finding its equivalent T and equating the two arms, through the medium of a permanent series terminal load; or, by finding its equivalent π and equating the two pillars, through the medium of a permanent terminal leak load. The system then offers a certain angle θ, a surge impedance z_0, and a single terminal load, which may be treated as having an auxiliary angle θ'.

2. Any such dissymmetrical network may also be regarded as possessing and being defined by (a) an angle θ, (b) a geomean surge impedance z_{ab}, and (c) an inequality ratio q. The electric behavior of the network, at the assigned frequency, with respect to the two pairs of terminals, may then be computed by the usual formulas applying to a simple smooth line; but with the further application of the coefficient q.

3. The theory of simple alternating-current lines with smoothly distributed constants may thus be regarded as being a particular case ($q = 1$) of the general theory of dissymmetrical networks having any degree of complexity.

LIST OF SYMBOLS EMPLOYED

δ_a, δ_b,	position angles at a and b ends of a diss. network (hyps ∠).
δ_1, δ_n	position angles at ends of a detailed composite line (hyps ∠).
E_a, E_b	e. m. fs. applied to a and b ends of a diss. network (volts ∠).
e_a, e_b,	p. d. s at open a and b ends of diss. network by application of a testing e. m. f. at the opposite end (volts ∠).
f	frequency applied to network (cycles per second).
G	total conductance of a smooth sym. line (mhos ∠).
G_a, G_b,	admittance of a sym. system, when respectively freed and grounded at the other end (mhos ∠).
G_{af}, G_{ag},	admittance of a diss. system, at the a end, when respectively freed and grounded at the other end (mhos ∠).
G_{bf}, G_{bg},	admittance of a diss. system, at the b end, when respectively freed and grounded at the other end (mhos ∠).
g	admittance of leak in equiv. T of a diss. system (mhos ∠).
g_1, g_2,	admittances of leaks at a and b ends in equiv. π of a diss. system (mhos ∠).
g'	admittance of leak in equiv. T of a sym. system (mhos ∠).
g''	admittance of each leak in equiv. π of a sym. system (mhos ∠).
θ	angle subtended by a line or system (hyps ∠).
θ'	angle subtended by a terminal load (hyps ∠).
$\theta_1, \theta_2, \ldots \theta_n$	angles subtended by successive sections of a composite line (hyps ∠).
I_a, I_b,	current strengths at ends of a network (amperes ∠)
$j = \sqrt{-1}$	
ν	series adm. of a sym. or diss. π (mhos ∠).
$q = \sqrt{z_{0a}/z_{0b}}$	inequality ratio of a diss. network (numeric ∠).

R	total conductor resistance of a smooth line (ohms ∠).
$R_f, R_g,$	impedance at one end of a sym. system, when the other end is freed and grounded respectively (ohms ∠).
$R_{af}, R_{ag},$	impedance at the a end of a diss. system when the other end is freed and grounded respectively (ohms ∠).
$R_{bf}, R_{bg},$	impedance at the b end of a diss. system, when the other end is respectively freed and grounded (ohms ∠).
$\mathcal{R}$	impedance of leak in equiv. T of a diss. system (ohms ∠).
$\mathcal{R}_1, \mathcal{R}_2,$	impedances of a and b leaks in the equiv. π of a diss. system (ohms ∠).
$\mathcal{R}'$	impedance of leak in equiv. T of a sym. system (ohms ∠).
$\mathcal{R}''$	impedance of each leak in equiv. π of a sym. system (ohms ∠).
ρ	impedance of architrave in equiv. π of a diss. system (ohms ∠).
$\rho_1, \rho_2,$	impedance of a and b arms in equiv. T of a diss. system (ohms ∠).
ρ'	impedance of each arm in equiv. T of a sym. system (ohms ∠).
ρ''	impedance of architrave in equiv. π of a sym. system (ohms ∠).
σ	impedance of a load at motor end of a system (ohms ∠).
y_0	surge admittance of a sym. system (mhos ∠).
$y_{0a}, y_{0b},$	surge admittance at a and b ends of a diss. system (mhos ∠).
$y_{ab} = \sqrt{y_{0a} y_{0b}}$	geometrical mean surge admittance of a diss. system mhos ∠).
$Z_a, Z_b,$	generator-end impedances of a diss. system (ohms ∠).
$Z_{1a}, Z_{1b},$	receiving-end impedances of a diss. system, including load (ohms ∠).
$z_0,$	surge impedance of a sym. system (ohms ∠).
$z_{0a}, z_{0b},$	surge impedance at a and b ends of a diss. system (ohms ∠).
$z_{ab} = \sqrt{z_{0a} z_{0b}}$	geomean surge impedance of a diss. system (ohms ∠).
$z_{01}, z_{02}, \ldots z_{0n}$	surge impedances of successive sections in composite line (ohms ∠).
$\omega = 2\pi f$	angular velocity of frequency impressed on a system (rad. per sec.)
~	sign for "cycles per second."
∠	sign for a complex numerical quantity.
hyp.	abbreviation for "hyperbolic radian."

Bibliography

Without pretensions as to completeness

1. A. E. Kennelly, "The Equivalence of Triangles and Three-Pointed Stars in Conducting Networks", *Elec. World and Engr.*, Vol. XXXIV, pp. 413, 414, Sept. 16, 1899.
2. J. L. LaCour, "Leerlauf und Kurzchluss", 1904.
3. A. E. Kennelly, "Artificial Lines for Continuous Currents in the Steady State," *Proc. Am. Ac. of Arts and Sc.*, Vol. XLIV, No. 4, p. 97, 1908
4. A. E. Kennelly, "Equivalent Circuits of Composite Lines in the Steady State," *Proc. Am. Ac. of Arts and Sc.*, Vol. XLV, No. 3, p. 31, 1909.
5. G. A. Campbell, "Cisoidal Oscillations," TRANS. A.I.E.E. Vol. XXX, Part 2, p. 873, Apr. 1911.
6. La Cour and Bragstad, "Theory and Calculations of Electric Currents," Translation, 1913.
7. A. E. Kennelly and Edy Velander, "Alternating Current Planevector Potentiometer Measurements at Telephonic Frequencies," *Proc. Am. Phil. Soc.*, Vol. LVIII, p. 97, Apr. 1919.

APPENDIX A

List of formulas applicable to a dissymmetrical T, π or Network. The numerical values pertain to the case of Figs. 17, 18, 21 and 22.

$$R_{ag}/R_{af} = R_{bg}/R_{bf} = G_{af}/G_{ag} = G_{bf}/G_{bg} = \frac{\mathcal{R}(\rho_1+\rho_2)+\rho_1\rho_2}{(\mathcal{R}+\rho_1)(\mathcal{R}+\rho_2)} = \frac{\nu(g_1+g_2)+g_1 g_2}{(\nu+g_1)(\nu+g_2)} = \tanh^2\theta = 0.404762 \quad \text{numeric } \angle \qquad (35)$$

$$\rho_1/\rho_2 = g_2/g_1 = \mathcal{R}_1/\mathcal{R}_2 = \frac{q\cosh\theta-1}{\frac{1}{q}\cosh\theta-1} = 0.5 \quad \text{numeric } \angle \qquad (36)$$

$$q = \sqrt{R_{ag}/R_{bg}} = \sqrt{R_{af}/R_{bf}} = \sqrt{G_{bg}/G_{ag}} = \sqrt{G_{bf}/G_{af}} = \sqrt{z_{0a}/z_{0b}} = \sqrt{\frac{\rho_1+\mathcal{R}}{\rho_2+\mathcal{R}}} = \sqrt{\frac{g_2+\nu}{g_1+\nu}} = z_{0a}/z_{ab} = 0.92582 \quad \text{numeric } \angle \qquad (37)$$

$$1/q = \sqrt{R_{bg}/R_{ag}} = \sqrt{R_{bf}/R_{af}} = \sqrt{G_{ag}/G_{bg}} = \sqrt{G_{af}/G_{bf}} = \sqrt{z_{0b}/z_{0a}} = \sqrt{\frac{\rho_2+\mathcal{R}}{\rho_1+\mathcal{R}}} = \sqrt{\frac{g_1+\nu}{g_2+\nu}} = z_{0b}/z_{ab} = 1.08012 \quad \text{numeric } \angle \qquad (38)$$

$$\frac{\frac{1}{2}(\rho_1+\rho_2)}{\mathcal{R}} = \frac{\frac{1}{2}(g_1+g_2)}{\nu} = \left(\frac{q+1/q}{2}\right)\cosh\theta - 1 = 0.3 \quad \text{numeric } \angle \qquad (39)$$

$$\frac{\sqrt{\rho_1\rho_2}}{\mathcal{R}} = \frac{\sqrt{g_1 g_2}}{\nu} = \sqrt{(q\cosh\theta-1)\left(\frac{1}{q}\cosh\theta-1\right)} = 0.28284 \quad \text{numeric } \angle \qquad (40)$$

$$\rho_1/g_2 = \rho_2/g_1 = \rho_1\,\mathcal{R}_2 = \rho_2\,\mathcal{R}_1 = \rho\,\mathcal{R} = \rho/g = \mathcal{R}/\nu = \sqrt{\frac{\rho_1\,\rho_2}{g_1\,g_2}} = z_{0a}\,z_{0b} = z_{ab}^2 = R_{af}\,R_{bg}$$

$$= R_{ag}\,R_{bf} = \rho_1\,\rho_2 + \mathcal{R}\,(\rho_1 + \rho_2) = \sqrt{R_{af}\,R_{ag}\,R_{bf}\,R_{bg}} = \sqrt{\rho_1\,\rho_2\,\mathcal{R}_1\,\mathcal{R}_2}$$

$$= \sqrt{\{\,\rho_1\,(\rho_2 + \mathcal{R}) + \rho_2\,\mathcal{R}\,\}\ \{\,\rho_2\,(\rho_2 + \mathcal{R}) + \rho_1\,\mathcal{R}\,\}} = 17{,}000{,}000 = (4123.106)^2 \qquad \text{ohms}^2\ \angle \qquad (41)$$

$$g_1/\rho_2 = g_2/\rho_1 = \nu\,g = g/\rho = \nu/\mathcal{R} = \sqrt{\frac{g_1\,g_2}{\rho_1\,\rho_2}} = y_{0a}\,y_{0b} = y_{ab}^2 = G_{af}\,G_{bg} = G_{ag}\,G_{bf}$$

$$= g_1\,g_2 + \nu\,(g_1 + g_2) = \sqrt{G_{af}\,G_{ag}\,G_{bf}\,G_{bg}} = \sqrt{\{g_1\,(g_2 + \nu) + g_2\,\nu\}\ \{g_2\,(g_1 + \nu) + g_1\,\nu\}}$$

$$= 0.058824 \times 10^{-6} = (0.2425355 \times 10^{-3})^2 \qquad \text{mhos}^2\ \angle \qquad (42)$$

$$z_{0a} = \sqrt{R_{af}\,R_{ag}} = \sqrt{\rho_1\,(\rho_1 + \mathcal{R}) + \rho_2\,\mathcal{R}\,\frac{\rho_1 + \mathcal{R}}{\rho_2 + \mathcal{R}}} = q\,z_{ab} = q^2\,z_{0b} = 3817.26 \qquad \text{ohms}\ \angle \qquad (43)$$

$$z_{0b} = \sqrt{R_{bf}\,R_{bg}} = \sqrt{\rho_2\,(\rho_2 + \mathcal{R}) + \rho_1\,\mathcal{R}\,\frac{\rho_2 + \mathcal{R}}{\rho_1 + \mathcal{R}}} = \frac{1}{q}\,z_{ab} = \frac{1}{q^2}\,z_{0a} = 4453.46 \qquad \text{ohms}\ \angle \qquad (44)$$

$$\sinh\theta = \sqrt{\frac{R_{ag}}{R_{af} - R_{ag}}} = \sqrt{\frac{R_{bg}}{R_{bf} - R_{bg}}} = \frac{\sqrt{\mathcal{R}\,(\rho_1 + \rho_2) + \rho_1\,\rho_2}}{\mathcal{R}} = \frac{\sqrt{\nu\,(g_1 + g_2) + g_1\,g_2}}{\nu}$$

$$= \sqrt{\rho/\mathcal{R}} = \sqrt{\rho\,g} \qquad = z_{ab}/\mathcal{R} = y_{ab}/\nu = 0.82462 \qquad \text{numeric}\ \angle \qquad (45)$$

$$\cosh\theta = \sqrt{\frac{R_{af}}{R_{af} - R_{ag}}} = \sqrt{\frac{R_{bf}}{R_{bf} - R_{bg}}} = \frac{\sqrt{(\mathcal{R} + \rho_1)(\mathcal{R} + \rho_2)}}{\mathcal{R}} = \sqrt{1 + \rho\,g}$$

$$= \frac{\sqrt{(\nu + g_1)(\nu + g_2)}}{\nu.} = \frac{\sqrt{R_{af}\,R_{bf}}}{\mathcal{R}} = \frac{\sqrt{G_{ag}\,G_{bg}}}{\nu} = 1.296148 \qquad \text{numeric}\ \angle \qquad (46)$$

$$\tanh\theta = \sqrt{R_{ag}/R_{af}} = \sqrt{R_{bg}/R_{bf}} = = \sqrt{\frac{\mathcal{R}\,(\rho_1 + \rho_2) + \rho_1\,\rho_2}{\mathcal{R}^2 + \mathcal{R}\,(\rho_1 + \rho_2) + \rho_1\,\rho_2}}$$

$$= \sqrt{\frac{\nu\,(g_1 + g_2) + g_1\,g_2}{\nu^2 + \nu\,(g_1 + g_2) + g_1\,g_2}} = \frac{z_{ab}}{\sqrt{R_{af}\,R_{bf}}} = \frac{y_{ab}}{\sqrt{G_{ag}\,G_{bg}}} = 0.636209 \qquad \text{numeric}\ \angle \qquad (47)$$

$$\rho_1/\mathcal{R} = \rho_1\,g = g_2/\nu = g_2\,\rho = \rho/\mathcal{R}_2 = q\cosh\theta - 1 = 0.2 \qquad \text{numeric}\ \angle \qquad (48)$$

$$\rho_2/\mathcal{R} = \rho_2\,g = g_1/\nu = g_1\,\rho = \rho/\mathcal{R}_1 = \frac{1}{q}\cosh\theta - 1 = 0.4 \qquad \text{numeric}\ \angle \qquad (49)$$

$$\rho_1 = z_{ab}\left(\frac{q\cosh\theta - 1}{\sinh\theta}\right) = 1000 \qquad \text{ohms}\ \angle \qquad (50)$$

$$\rho_2 = z_{ab}\left(\frac{\frac{1}{q}\cosh\theta - 1}{\sinh\theta}\right) = 2000 \qquad \text{ohms}\ \angle \qquad (51)$$

$$\mathcal{R} = z_{ab}\,\text{cosech}\,\theta = \sqrt{R_{bf}\,(R_{af} - R_{ag})} = \sqrt{R_{af}\,(R_{bf} - R_{bg})} = 5000 \qquad \text{ohms}\ \angle \qquad (52)$$

$$g = y_{ab}\sinh\theta = 0.2 \times 10^{-3} \qquad \text{mhos}\ \angle \qquad (53)$$

$$\rho = z_{ab}\sinh\theta = \sqrt{\frac{R_{bg}}{G_{ag} - G_{af}}} = \sqrt{\frac{R_{ag}}{G_{bg} - G_{bf}}} = R_{ag}\sqrt{\frac{R_{bf}}{R_{af} - R_{ag}}} = R_{bg}\sqrt{\frac{R_{af}}{R_{bf} - R_{bg}}}$$

$$= \frac{R_{af}\,R_{bg}}{\mathcal{R}} = \frac{R_{ag}\,R_{bf}}{\mathcal{R}} = 3400 \qquad \text{ohms}\ \angle \qquad (54)$$

$$\nu = y_{ab}\,\text{cosech}\,\theta = \sqrt{G_{bg}\,(G_{ag} - G_{af})} = \sqrt{G_{ag}\,(G_{bg} - G_{bf})} = \frac{G_{af}\,G_{bg}}{g} = \frac{G_{ag}\,G_{bf}}{g}$$

$$= 0.294118 \times 10^{-3} \qquad \text{mhos}\ \angle \qquad (55)$$

$$g_1 = y_{ab}\left(\frac{\frac{1}{q}\cosh\theta - 1}{\sinh\theta}\right) = 0.117647 \times 10^{-3} \qquad \text{mhos}\ \angle \qquad (56)$$

$$g_2 = y_{ab}\left(\frac{q\cosh\theta - 1}{\sinh\theta}\right) = 0.058824 \times 10^{-3} \qquad \text{mhos} \angle \qquad (57)$$

$$R_{ag} = z_{0a}\tanh\theta = q\, z_{ab}\tanh\theta = q^2 z_{0b}\tanh\theta = 2428.57 \qquad \text{ohms} \angle \qquad (58)$$

$$R_{bg} = z_{0b}\tanh\theta = \frac{1}{q} z_{ab}\tanh\theta = \frac{1}{q^2} z_{0b}\tanh\theta = 2833.3 \qquad \text{ohms} \angle \qquad (59)$$

$$R_{af} = z_{0a}\coth\theta = q\, z_{ab}\coth\theta = q^2 z_{0b}\coth\theta = 6000 \qquad \text{ohms} \angle \qquad (60)$$

$$R_{bf} = z_{0b}\coth\theta = \frac{1}{q} z_{ab}\coth\theta = \frac{1}{q^2} z_{0b}\coth\theta = 7000 \qquad \text{ohms} \angle \qquad (61)$$

APPENDIX B

List of formulas applicable to a symmetrical T, π, or network, corresponding to the formulas of Appendix A. The numerical values pertain to the case of Figs. 23, 24 and 25.

$$R_g/R_f = G_f/G_g = \frac{\rho'(\rho' + 2\,\mathcal{R}')}{(\rho' + \mathcal{R}')^2} = \frac{g''(g'' + 2\,\nu)}{(g'' + \nu)^2} = \tanh^2\theta = 0.404762 \qquad \text{numeric} \angle \qquad (35a)$$

$$\rho'/\mathcal{R}' = g''/\nu = \rho' g' = \rho'' g'' = \cosh\theta - 1 = \operatorname{versh}\theta = 2\sinh^2(\theta/2) = 0.296148 \qquad \text{numeric} \angle \qquad (39a)$$

$$\rho''/g' = \rho''\,\mathcal{R}' = \mathcal{R}'/\nu = \rho'(\rho' + 2\,\mathcal{R}') = R_f R_g = z_0^2 = (4123.106)^2 = 17{,}000{,}000 \qquad \text{ohms}^2 \angle \qquad (41a)$$

$$g'/\rho'' = \nu g' = \nu/\mathcal{R}' = g''(g'' + 2\,\nu) = G_f G_g = y_0^2 = (0.2425355 \times 10^{-3})^2 = 0.058824 \times 10^{-6} \qquad \text{mho}^2 \angle \qquad (42a)$$

$$\sinh\theta = \sqrt{\frac{R_g}{R_f - R_g}} = \frac{\sqrt{\rho'(\rho' + 2\,\mathcal{R}')}}{\mathcal{R}'} = \frac{\sqrt{g''(g'' + 2\,\nu)}}{\nu} = z_0/\mathcal{R}' = y_0/\nu = \sqrt{g'\rho''} = \sqrt{\rho''/\mathcal{R}'} = 0.82462 \qquad \text{numeric} \angle \qquad (45a)$$

$$\cosh\theta = \sqrt{\frac{R_f}{R_f - R_g}} = \frac{\rho' + \mathcal{R}'}{\mathcal{R}'} = \frac{g'' + \nu}{\nu} = R_f/\mathcal{R}' = G_g/\nu = \sqrt{1 + g'\rho''} = 1.296148 \qquad \text{numeric} \angle \qquad (46a)$$

$$\tanh\theta = \sqrt{R_g/R_f} = \frac{\sqrt{\rho'(\rho' + 2\,\mathcal{R}')}}{\rho' + \mathcal{R}'} = \frac{\sqrt{g''(g'' + 2\,\nu)}}{g'' + \nu} = z_0/R_f = y_0/G_g = 0.636209 \qquad \text{numeric} \angle \qquad (47a)$$

$$\rho' = z_0\left(\frac{\cosh\theta - 1}{\sinh\theta}\right) = z_0\tanh(\theta/2) = 1480.741 \qquad \text{ohms} \angle \qquad (50a)$$

$$\mathcal{R}' = z_0\operatorname{cosech}\theta = R_f\operatorname{sech}\theta = \sqrt{R_f(R_f - R_g)} = \frac{(\mathcal{R}'')^2}{\rho'' + 2\,\mathcal{R}''} = z_0^2/\rho'' = \frac{R_f R_g}{\rho''} = 5000 \qquad \text{ohms} \angle \qquad (52a)$$

$$g' = y_0\sinh\theta = G_f\cosh\theta = \sqrt{\frac{G_f}{R_f - R_g}} = \frac{g''(g'' + 2\,\nu)}{\nu} = y_0^2/\nu = \frac{G_f G_g}{\nu} = 0.2 \times 10^{-3} \qquad \text{mhos} \angle \qquad (53a)$$

$$\rho'' = z_0\sinh\theta = R_g\cosh\theta = R_g\sqrt{\frac{R_f}{R_f - R_g}} = \frac{\rho'(\rho' + 2\,\mathcal{R}')}{\mathcal{R}'} = z_0^2/\mathcal{R}' = 3400 \qquad \text{ohms} \angle \qquad (54a)$$

$$\nu = y_0\operatorname{cosech}\theta = G_g\operatorname{sech}\theta = \sqrt{G_g(G_g - G_f)} = \frac{g''(g'' + 2\,\nu)}{g'} = y_0^2/g = \frac{G_f G_g}{g} = 0.294118 \times 10^{-3} \qquad \text{mhos} \angle \qquad (55a)$$

$$g'' = y_0\left(\frac{\cosh\theta - 1}{\sinh\theta}\right) = y_0\tanh(\theta/2) = 0.0871024 \times 10^{-3} \qquad \text{ohms} \angle \qquad (56a)$$

$$R_g = z_0\tanh\theta = \frac{\rho'(\rho' + 2\,\mathcal{R}')}{\rho' + \mathcal{R}'} = 2623.157 \qquad \text{ohms} \angle \qquad (58a)$$

$$R_f = z_0\coth\theta = \rho' + \mathcal{R} = 6480.741 \qquad \text{ohms} \angle \qquad (60a)$$

$$G_g = y_0\coth\theta = g'' + \nu = 0.38122 \times 10^{-3} \qquad \text{mhos} \angle \qquad (62)$$

$$G_f = y_0\tanh\theta = \frac{g''(g'' + 2\,\nu)}{g'' + \nu} = 0.154149 \times 10^{-3} \qquad \text{mhos} \angle \qquad (63)$$

Discussion

F. S. Dellenbaugh, Jr.: The extreme simplicity with which complex circuits may be worked down to a simple circuit is really extraordinary. As you will notice in the bibliography, Dr. Kennelly was the first to publish "The Equivalence of Triangles and Three-pointed Stars," in other words, star-delta transformations in 1899, and it has been a little over 23 years that this process has been known and published, and yet, commercially, it appears to be used very little.

I know a number of commercial cases that I have met with or heard of, that required a very long time, solving networks by the older methods and where, by the use of a star delta transformation, it could be solved immediately, without much effort. In one case, it took an engineer the better part of a day to calculate a circuit by the net work method, that is the older method of Kirchoff's Laws, and so forth, and a young engineer without much experience, but who happened to know this method of Dr. Kennelly's, did this same job in half an hour. The other method made a few approximations as well, while the latter method was accurate.

I think perhaps one trouble is due to the fact that when you have these formulas, the average man does not always know just what to do with them, and while the formulas come down to a very simple looking result, you cannot find out how just to get the numerical answer. If you look it up you find that there are tables of these complex angles and you attempt to use the tables, and find for accurate work, that you have to interpolate. With angles of complex quantities in two dimensions, it is necessary to make six averages to get the final answer, which is laborious.

But in addition to those tables, Dr. Kennelly has prepared some very beautiful charts, and I never think of using the tables any more, because the charts are as accurate as ordinary slide rule work, and you have the very great advantage of seeing the way in which the functions change. Very often, in interpolating from the tables, there is a rapid rate of change, which may make considerable error in a straight line interpolation. Of course, you can use more accurate methods of interpolation, but it requires more time.

I want to make a plea for the use of this method. For those who find it difficult to get results, let them look at Dr. Kennelly's charts of hyperbolic functions, and once the system of going through those charts to find the functions is learned, you will find it is no more difficult to use hyperbolic functions of complex angles than ordinary formulas for the solution of triangles. It involves putting down a few more figures, but it is so much simpler and very much quicker than any method of Kirchoff's Laws, if you have more than the simplest kind of network.

Physical Interpretation of Complex Angles and Their Functions

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Review of the Subject.—*A child's idea of an angle is a corner or edge; the sharper the corner or edge, the smaller is the angle, and hence expressions like "acute (sharp) angle," "obtuse (blunt) angle," etc. With a little schooling his idea of an angle is broadened into that of turning or rotation and finds a geometrical expression in a circle. The amount of turning is represented by the area of the sector of the circle like a piece of pie gone over by the turning radius. This broadened idea is still in agreement with the earlier idea because the sector is a surface having a corner at the center of the circle, and the sharpness or bluntness of this corner varies with the area. With further progress, he also learns to associate the angle with the arc of the circle, for, evidently, the length of the arc is a measure of the amount of turning and is proportional to the area of the sector. The angle when represented by the arc of the circle loses all resemblance to a corner, which latter idea however is discarded as unnecessary, emphasis now being laid on "turning" which is well represented by the arc of the circle. Mathematical analysis is then applied which still further broadens the scope of angles and gives rise to the so called "imaginary" angles which are called "hyperbolic" in contrast with ordinary "real" angles which are called "circular," combinations of these two kinds of angles being called "general" or "complex." At this stage, any physical interpretation or geometrical representation of an angle practically vanishes, for, a hyperbolic angle can not be conceived of as a rotation, not even as a rotation around an "imaginary" axis, for, how can one conceive of an "imaginary" axis? If the "imaginary" axis be defined as a new axis at right angles to the old axis, rotation around it can easily be conceived of, but it does not correspond to facts, because a hyperbolic angle implies no change in direction, and hence no rotation, but merely affects the magnitude of a quantity. Inasmuch as problems can be solved mathematically without any visualization, the mathematician discards visual interpretations and bases his conceptions on formulas. However, the engineering type of mind finds it both difficult and distasteful to be dependent on symbolic definitions, and craves for visualization. He feels that if physical problems lead to "imaginary" or "complex" angles, these angles must stand for some concrete physical facts and must therefore be capable of a physical interpretation.*

A broad and yet simple physical interpretation applicable to both circular and hyperbolic angles is developed below, which also applies to their trigonometric functions, as follows:

A quantity may be expressed or specified either by its own dimensions or as a percentage of another quantity. The first kind of specification is its "absolute" measure, the second kind is its "angular" measure. Thus "angle" and "percentage" are synonymous terms. If a quantity changes by a certain percentage (compound percentage as in compound interest) that "percentage" is the "angular" change in the quantity in radians. According to this interpretation, neither corner, edge, turning nor rotation are essential characteristics of an angle; they are the characteristics of a limited class only. If the percentage change in a quantity is like itself, geometrically parallel to itself, the per cent change, that is, the angle, is called a hyperbolic angle, and there is no rotation. If the quantity varies without any change in magnitude but only with a change in direction, geometrically perpendicular to itself, the per cent change, that is, the angle, is called a circular angle, and there is rotation. Thus, hyperbolic and circular angles are at right angles to each other, the first being a per cent change parallel to the quantity, and the second perpendicular to it. "Imaginary" thus means a quadrature. However, it is not rotation around a quadrature axis but per cent change in a quadrature direction.

This new interpretation is in agreement with the representation by arcs in a more extended vectorial sense in which one is not limited to the arc of a circle but arcs of any shape have an angular interpretation. Thus, consider a portion of an arc of any arbitrary shape, and resolve it into two components, one circular (at right angles to the radius drawn to the point) and the other straight and radial (parallel to the radius at that point). These two arcs expressed as a percentage of the radius represent circular and hyperbolic angles, respectively, and being in quadrature with each other their vector sum is the "complex" angle of the arc. In this interpretation a hyperbola has nothing to do with a hyperbolic angle, and the name is unfortunate. It would have been more intelligible to speak of the ordinary circular angle as a "quadrature" angle, and of the so called hyperbolic angle as a "linear" or a "radial" angle.

This new interpretation of an angle is then utilized to interpret the physical significance of sines and cosines of angles, whether circular or hyperbolic. The common idea of a cosine as the projection of the radius vector on the X axis, and of sine as the projection on the Y axis, is found inadequate, this idea being limited to circular angles and not being of much value in explaining their occurrence in physical problems. A cosine is then interpreted as the mean vector value of a unit quantity for a given per cent increase and an equal per cent decrease, or a given value of rotation positive and negative. Examples are given illustrating the application of this vector idea in physical problems, as in the resolution of a harmonic wave into two oppositely rotating vectors, solution of differential equations, etc. A geometrical construction is given for the sines and cosines of complex angles, which is not merely a representation but photographically true, so to speak.

INTRODUCTION

THE student of trigonometric functions finds by some dark analytical transformations that the sines, cosines, etc., of imaginary angles are not impossible or absurd quantities but resolve themselves into hyperbolic functions of those angles looked upon as real angles. Thus if θ is an "ordinary" angle, that is, a circular angle,

$$\cos (j\,\theta) = \cosh (\theta) \tag{1}$$

$$\sin (j\,\theta) = j \sinh (\theta) \tag{2}$$

$$\tan (j\,\theta) = j \tanh (\theta) \tag{3}$$

These identities indicate that a hyperbolic angle is j with respect to circular angles, and the questions naturally arise: (a) Wherein is a so-called hyperbolic angle an angle at all? (b) Wherein does it differ from ordinary angles? (c) What is the physical significance of its being j with respect to the ordinary angles?

---

1. All percentages, in this paper are supposed to carry their decimal points explicitly. Thus, ten per cent is to be given as 0.10, not merely as 10.

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The usual representation of hyperbolic angles and functions by means of a hyperbola, as circular angles and their functions are represented by circles (See Figs. 1 and 2), completely fails to throw any light on these questions; the representation merely serving the purpose of giving an idea of their numerical values but not of their physical nature. Certainly, it is not evident from these figures wherein the area of the hyperbolic sector representing the hyperbolic angle is j with respect to the circular sector representing the circular angle. Furthermore, it is not evident wherein the hyperbolic sine and tangent are j with respect to the circular sine and tangent, while the hyperbolic cosine is "real" like the circular cosine. These points not being explainable on the basis of representation by a hyperbola, it is not to be expected that an explanation can be given on that basis of the physical significance of the so-called imaginary periodicity of hyperbolic functions, as in the equation

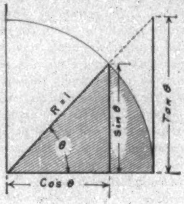

Fig. 1

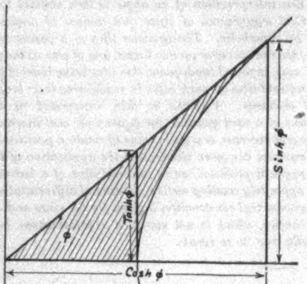

Fig. 2

$$\cosh(\theta) = \cosh(\theta + 2\pi n j) \qquad (4)$$

Evidently, we need first to develop a simple physical interpretation of what an angle is in its most general sense, a physical conception which will apply to hyperbolic angles as well as to circular angles and will show their natural relationship. Then we must develop a physical interpretation of trigonometric functions, broad enough to include both types, and clear enough to show their relationship.

It will be shown below that an angle, in its broadest sense, is nothing more or less than a *percentage*, more specifically a percentage change in a quantity, the percentage being *compound* not simple percentage, effective at every point or instant, as for instance in compound interest. It will, then, be shown that an imaginary angle is nothing more or less than a quadrature angle, that is, one in quadrature with the circular angle. This is in accordance with the use of j in engineering as a quadrature operator, is consistent with the mathematics of hyperbolic trigonometry and leads to an extremely simple physical interpretation of the imaginary periodicity of hyperbolic functions, *viz.*, that their periodicity is not imaginary or unthinkable but in quadrature with themselves. The point of view is evidently vectorial. Since the arc represents the change in a vector vectorially, the component of the arc in quadrature with the vector represents the circular angle, the component parallel to the vector represents the hyperbolic angle, and the arc as such represents the complex angle. From this novel point of view all imaginariness disappears from the trigonometry of complex angles, and the various relationships usually established by tedious mathematical transformations become almost self-evident physical facts. We must therefore analyze this new interpretation in somewhat greater detail.

PHYSICAL MEANING OF AN ANGLE IN ITS BROADEST SENSE

Let the radius vector R move and trace an arbitrary curve, as shown in Fig. 3. On this curve consider any two adjacent points p_1 and p_2. The arc between these two points represents the vector change in R and we shall designate it in the usual way as dR. Now, a radius vector can move or change in only two essentially different ways, *viz.*, (1) parallel to itself, stretching or shrinking, and, (2), at right angles to itself, rotating. Any other motion can be resolved into two such components, one parallel to the radius vector, the other in quadrature with it. In Fig. 3, the arc dR is resolved into two such components, dR_1 and dR_2. It will be noted that the component at right angles to the radius vector is curved being the arc of a circle, as it must be to keep perpendicular to the radius vector at every successive position of the latter (See Fig. 4). However, if the path dR is taken short enough, dR_2 will be a straight line.[2] It is highly important to note that the two components, dR_1 and dR_2 of dR, are at right angles to each other.

The distance dR which the radius vector covers in its sweep along an arbitrary curve may be given in either one of two ways: (a) it may be given by its actual value without any reference to the radius vector itself or, (b) it may be given as a percentage of the radius vector. The first method of measuring distances is the absolute way, the second method is the angular way. That is, the length and direction of a path traced by a moving radius vector is the absolute (vector) value of the change; the same path expressed as a percentage of the radius vector is the corresponding angular (vector) value.

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[2] At a later point in the discussion of angles in three dimensional space, a rigid physical and mathematical justification will be given why the vector angle is represented by the curved line and not by its chord.

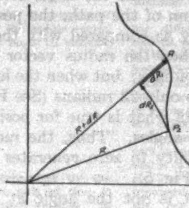

Fig. 3

According to the foregoing, we may write,

Absolute value of path $= d\,R$ (Inches or cm.) **(5)**

Angular value of path $= \dfrac{d\,R}{R}$ (Radians or percentage) **(6)**

Both the absolute and the angular values of the path are vectors: thus, treating the radius vector as the zero angle axis,

$$d\,R = (d\,R_1) + j\,(d\,R_2) \tag{7}$$

$$\frac{d\,R}{R} = \frac{d\,R_1}{R} + j\,\frac{d\,R_2}{R} \tag{8}$$

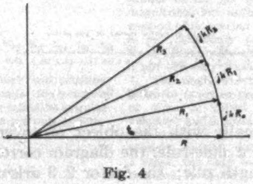

Fig. 4

It is commonly known that $(d\,R_2/R)$ is a small ordinary circular angle; call it $(d\,\theta_2)$. The reader must have already surmised that $(d\,R_1/R)$ is a small hyperbolic angle; call it $(d\,\theta_1)$. Calling the complete vector angle $(d\,\theta)$, we have,

$$d\,\theta = (d\,\theta_1) + j\,(d\,\theta_2) \tag{9}$$

$$= (\text{hyperbolic angle}) + j\,(\text{circular angle}) \tag{10}$$

In these equations θ_1 and θ_2 have been considered as "real" numbers, and hence the necessity for attaching j to θ_2. However, if θ_1 and θ_2 were considered vector symbols, as is really logical to do, no separate j would have been necessary. The same statement applies to equations (7) and (8).

To summarize what has been said: a moving or changing radius vector traces a path. This change or path expressed vectorially in per cent of the radius vector is the angular measure of the change or path (in radians) through which the radius vector sweeps. If the radius vector moves perpendicular to itself at every instant, it rotates and traces a circle, and the length of its path in per cent of the radius vector is called a circular angle. If the radius vector moves parallel to itself at every instant, it stretches or shrinks and traces a straight line and this path expressed as a per cent increase (or decrease) in the radius vector is called the hyperbolic angle (in radians) of the path.

If angles are defined by the area which the radius vector covers, as is sometimes done, there would be no physical bond between circular and hyperbolic angles. But, defining an angle by an arc of a path, circular and hyperbolic angles are equally included. If the path is perpendicular to the generating line at every point, it is called circular; if parallel to the radius vector, it is called hyperbolic; if at an intermediate direction with respect to the generating line, it is called a *general* or *complex* angle. The arbitrary path in Fig. 3 represents a complex angle. It is physically evident that circular and hyperbolic angles (as arcs or paths) are j with respect to each other, that is, they are in quadrature. Furthermore, it is more correct to call the circular angle and not the hyperbolic angle (arc) the j angle, because the circular arc is in quadrature with the reference line, the radius vector; while the hyperbolic angle (arc or path) is in phase with it.

In Fig. 3, if $d\,R_2$ is rotated through 90 degrees, into parallelism with $d\,R_1$, it becomes a hyperbolic angle. That is, $j\,\theta$ circular angles are equal to θ hyperbolic angles.

When the radius vector moves perpendicular to itself, the term "circular" for the resulting angle may be appropriate because the path is circular, but it does not appear at first very clear why, when the radius vector moves parallel to itself, only stretching or shrinking, the resulting angle should be called hyperbolic. Historically, the reason for this nomenclature has been the fact that by certain graphical constructions a hyperbola is made use of to evaluate hyperbolic angles and their functions in the way a circle is utilized for circular angles as illustrated in Figs. 1 and 2. This, however, is likely to lead to the erroneous idea that when the radius vector is tracing a hyperbolic angle it is tracing a hyperbola, for it traces a straight line not a hyperbola. Hence, it appears that the two kinds of angles could be designated more appropriately as "circular" angle for one and "linear" angle for the other, or, "quadrature" angle for one and "in phase" angle for the other. Furthermore, so far as graphical constructions are concerned, we are not limited to the hyperbola, but more direct, and perhaps simpler, constructions are available in which the circular and hyperbolic angles and their functions are shown in their true physical nature and relationship.

It is to be observed that according to this new interpretation the characteristic which constitutes an angle is not *turning*, but just a *per cent change*, and therefore, it is not necessary to assume that hyperbolic angles are produced by the rotation of the radius vector around an "imaginary" axis in an "imaginary" plane as is sometimes suggested as explanation.

HOW TO CALCULATE THE ANGLE OF A PATH

When the radius vector R moves at right angles to itself, at every instant, its length is unchanged, and therefore, if we start with a radius vector of unit length, and let it trace a circle, the angle $(d\,R/R)$ will always be equal to $(d\,R)$ numerically. Therefore, in circular angles, the length of the arc is also a direct measure of the angle, but in hyperbolic angles, where the radius vector is moving parallel to itself and continually changing in length, the hyperbolic angle, $(d\,R/R)$, can not be represented by $(d\,R)$ numerically. Hence, the scale of a hyperbolic angle continually changes.

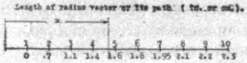

Fig. 5

Consider now a unit radius vector R along the X axis, (Fig. 5), and let it move along this axis generating a pure hyperbolic angle. Some time the radius vector will reach the point X. The question is, "What is the measure of hyperbolic angle which the path of the radius vector covers?" Or, in plainer language, what is the present change in the radius vector? Since the length of R has been changing every instant, we can not divide the length of path by the initial value of the radius vector, that is, unity, nor by the final value, but we must calculate the angle for very short intervals and add them. In calculus language, the hyperbolic angle described by the radius vector in its travel from 1 to X is

$$\theta = \int_{1}^{x} \frac{d\,x}{x} = L\,n^{3}\,(x) - L\,n\,(1) = L\,n\,(x) \qquad (11)$$

This shows that the hyperbolic angle subtended by a straight line based on its initial portion of unit length is equal to its natural (or hyperbolic) logarithm, (and there is thus one reason why such an angle may be called hyperbolic), although, in referring to this feature, "logarithmic" angle may be a better name.

A few points of interest are to be noted in connection with equation (11) and Fig. (5).

(1) It will be seen that "natural log of path," "angle covered by path" and "per cent length of path" are all equal and synonymous.

(2) The angles indicated are for a stretch of the radius vector from the point 1, *i. e.*, from the end of the unit radius vector, with which we started (not from O), to any desired point along its length. At the starting point, *i. e.*, at R equal to 1, the angle is zero. If the radius vector shrinks, instead of stretching, starting with a unit value, it generates a negative angle, which also is to be seen in Fig. 5.

(3) Due to the fact that the angle of the path is equal to the logarithm of the path, the positive angle increases very slowly as compared with the length of the path. Thus, when the radius vector or path is 10, the angle is 2.3 radians; but when the length is ten millions, the angle is only 23 radians (See Fig. 6).

(4) The reverse of what is true for positive angles is true for negative angles. Thus, the radius vector in shrinking from unity to zero generates an infinite negative angle (See Fig. 5).

(5) Ordinarily, it is not the angle at a point X (that is, between 1 and X) but between two points X_1 and X_2 which is desired, and which evidently is given by $L\,n\;(x_2) - L\,n\,(x_1)$.

(6) The radius vector is essentially positive, and, therefore, no matter in what direction it is drawn, it traces a positive angle if increasing, negative angle if decreasing, and therefore, it is impossible to get a negative radius vector or quantity, which also explains in a physical way the mathematical fact that "ordinary" logarithms do not apply to negative numbers. However, it is possible to transcend the "ordinary" even in the matter of logarithms as will be shown at a later point.

(7) Considering Fig. 6 it will be observed that the angle is graduated uniformly or arithmetically, whereas the radius vector or its path is graduated exponentially or geometrically with the object of covering a wide range, as in a slide-rule, the diagram corresponding to 2.3 unit length rule; the factor 2.3 arising from the ratio of ordinary logs to the natural logs. Though convenient for the purpose indicated, this scheme is not serviceable in every case, but the scheme of Fig. 5, in which the radius vector is graduated uniformly and the angle logarithmically, is applicable universally, as will be noted particularly in deriving the trigonometric functions of complex angles.

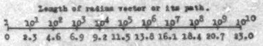

Fig. 6

Before proceeding to the analysis of somewhat more involved aspects of the subject, it may be profitable at this point to tabulate the substance of the foregoing analysis both in every day and in mathematical language in two parallel columns for convenient comparison and greater intelligibility.

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3. We shall use $L\,n$ to designate natural logarithms, and "log" to designate ordinary decimal logarithms.

An Exposition of Angles in General

In Mathematical English	In Every Day English
1. Let the radius vector R describe an arbitrary path.	1. Let a quantity R undergo any arbitrary change.
2. The path divided by the radius vector is the angle in radians subtended by the path at the radius vector.	2. The change in the quantity R divided by R is a fraction and gives the change as a percentage of R. A *per cent* change is called *angular* change in mathematics.
3. Any portion of the path may be resolved into two components, one parallel to the radius vector, the other perpendicular to it.	3. The change, or per cent change, in R may be one of magnitude only, or it may be one of direction only. A general change may imply a certain per cent change in magnitude and a certain per cent change in direction.
4. The "in phase" component of path divided by the radius vector is the hyperbolic angle in radians.	4. A per cent change in magnitude has different names in practise, such as, *per cent* (*compound*) *interest* in banking; *per cent discharge* in hydraulics; *per cent decay* in electrical and radioactive problems; and, in general, as "per cent gain," or *per cent loss*. These percentages are called "hyperbolic angles," in mathematics. One hundred per cent is considered the unit of angles and is called one radian.
5. The angle generated by the radius vector per unit of time is called its "angular velocity" in radians. This applies to all kinds of angles, whether hyperbolic, circular, or the combination of the two, that is, general or complex.	5. The per cent change which is produced per unit of time is called the "per cent rate of change" in general, when the change is one of magnitude only, as for instance, "per cent *rate* of (*compound*) interest," "per cent *rate* of discharge" of a reservoir or of a condenser, "per cent *rate* of decay" of a radioactive substance, etc. A "per cent rate" is called "angular velocity" in mathematical language.
6. If the path of the radius vector is at right angles to itself the path is then circular and expressed as a fraction of the radius vector is called a "circular angle" in radians. The circular angle (path) being perpendicular to the radius vector, is therefore perpendicular to the hyperbolic angle (path) which is parallel to the radius vector.	6. If the quantity under consideration has direction as well as magnitude, it can change its value without changing its magnitude, that is, it can rotate. The change is then called "angular" in ordinary language as well as in mathematics and is specified in a number of ways, such as revolutions, or right angles, or degrees, or "radians" the latter term meaning the length of the arc (along which the rotation takes place) in per cent of the length of the rotating line, very similar to the per cent change in magnitude, but unfortunately this method of measuring circular angles is not commonly appreciated although it is a very natural method, if not the only natural method.
7. The angle which the rotating radius vector tends to produce per unit of time is called its angular velocity in radians per unit of time, as covered in art. 5.	7. The angle which the rotating quantity tends to produce per unit of time is called its "angular velocity" and is ordinarily measured in revolutions per minute, although a more natural way (in fact, in a sense, the only natural way) to measure or specify it would have been in "per cent radius" *e. g.* (that is, radians, for, radian means per cent radius) per unit of time. For instance, to say that the angular velocity is five radians per second means that a point at the end of any assumed radius covers a distance equal to five radii in one second. In other words, the natural way of specifying an angular velocity is to specify the tangential velocity as a percentage of the length of the radius. Although the popular mind has grasped the fundamental concept of percentages in linear changes, unfortunately it has not grasped the idea of percentages in quadrature or rotary changes.
8. In speaking of arcs as subtending angles, the angles are not subtended at the origin or at any other *point*, but at the radius vector. Hence is the necessity for dividing the arc by the radius vector to obtain the angle.	8. In speaking of change (of either kind) as a per cent change, the percentage is based on the value of the quantity in which the change is taking place. This point so clear in ordinary language is not so clear in mathematical definitions.
9. When the radius vector is varying in magnitude, the angle of an arc is obtained by the summation $\int_{R_1}^{R_2} \frac{d\,R}{R} = L\,n\;R_2/R_1$ Let the two limits of integration be 20 and 30 respectively. $L\,n\,(1.5) = 2.3 \times \text{Log}\,(1.5)$ $= 2.3 \times 0.176$ $= 0.405$ hyperbolic radians.	9. When the quantity is varying in magnitude, the change must be expressed not in per cent of the initial value of the quantity, nor of the final but of a mean equivalent value. When only a very small change is considered, the mean value of the quantity is sufficiently approximated by either the initial or the final value or the arithmetic mean of the two. For considerable changes an arithmetic mean value of R will not do, but the percentage may then be calculated in a number of steps as in compound interest. For instance, if the quantity changes from 20 to 30, the percentage of the first unit gain is 1/20.5, the second 1/21.5, the third 1/22.5, etc. and the sum of ten such terms gives the total percentage gain (approximately) which in this case adds up to 0.405, (what is called forty and a half per cent) and happens to check the correct answer in this case to the third significant figure.

The Radius Vector

When the radius vector is given, the angle θ between itself and the unit radius vector R_1 of zero angle is,

$$\theta = L\,n\,R - L\,n\,R_1 = L\,n\,R/R_1 \qquad \textbf{(12)}$$

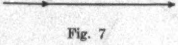

Fig. 7

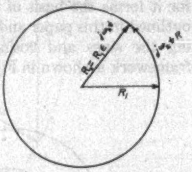

Fig. 8

$$\text{Hence}\quad R = R_1\,\epsilon^{\theta} \qquad \textbf{(13)}$$

This last result gives a physical interpretation to the exponential function as follows. If a quantity, with any initial value R_0, changes (θ) radians, *i. e.*, (θ) per cent (compound percentage), its value becomes,

$$R_\theta = R_0\,\epsilon^{\theta} \qquad \textbf{(14)}$$

The percentage (θ), however, may have any complex value. It may be a "real" number, so called, in which case the quantity has varied in magnitude only; or it may be a quadrature number, in which case the quantity has varied at right angles to itself, that is, rotated; or again, it may be a complex number, in which case the quantity has both rotated and changed in length. In the foregoing, (θ) is taken as the total complex angle.

If the quantity is varying continuously, it traces a path or curve. Assuming the velocity constant:] if it is "real," that is, like itself or in-phase, as in

$$R = R_1\,\epsilon^{w_1 t} \qquad \textbf{(15)}$$

the path is a straight line (Fig. 7). If the angular velocity is imaginary, that is, quadrature, as in

$$R = R_1 \epsilon^{j w_2 t} \tag{16}$$

the path is a circle (Fig. 8); and, if the angular velocity is general or complex, with in phase and quadrature components as in

$$R = R_1 \epsilon^{(w_1 + j w_2) t} \tag{17}$$

the path is an exponential spiral[4] (Fig. 9). The circle applies to undamped harmonic waves; the straight line, with exponential scale, (usually plotted to rectangular coordinates as increasing or decreasing exponential), applies in general to the uniformly damped d-c. or non-periodic quantities; and, the exponential spiral applies to uniformly damped harmonic quantities like the oscillatory discharge of a condenser, etc. The increasing exponential spiral and the decreasing exponential spiral are the images of each other as from a mirror, or an increasing exponential spiral may be considered as decreasing by reversing the direction of rotation, and vice versa.

The exponential spiral of Fig. 9 is of particular interest, for it gives the value of the radius vector both in magnitude and phase angle for any given complex angle. This is the only direct physical representation of the radius vector as it traces a complex angle. In fact, we might speak of it as the *photograph* of the radius vector as it traces a complex angle, in contrast with other graphical constructions which may in some sense be considered as a *representation.* The frame of the exponential spiral deserves special consideration, for it forms the basis of the physical interpretation as outlined in this paper and will be made use of in obtaining the sines and cosines of complex angles. This framework is shown in Fig. 10 as two systems of mutually perpendicular lines; a system of radial lines and a system of circular lines, with common center or origin. The radial lines are graduated logarithmically and measure the hyperbolic angles in radians; the circular lines are graduated uniformly and measure the circular or quadrature angles in radians. The two sets of angles are at right angles to each other as they ought to be. Given the circular and hyperbolic components of a complex angle, the corresponding point is at once located. The vector from the origin to this point gives the radius vector in magnitude and phase angle. The magnitude of the radius vector is measured linearly to some scale. Example: find the radii vector R_1 and R_2 which have the angles $+ (0.5 + j\,\pi/6)$ and $- (0.5 + j\,\pi/6)$ respectively (Fig. 10). It will be observed that the hyperbolic angle determines only the magnitude, the circular angle the direction of the radius vector. Thus,

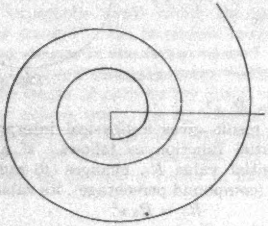

Fig. 9

$$R_1 = \epsilon^{+0.5} (\cos \pi/6 + j \sin \pi/6) \tag{18}$$

$$R_2 = \epsilon^{-0.5} (\cos \pi/6 - j \sin \pi/6) \tag{19}$$

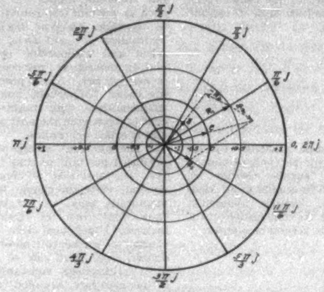

Fig. 10

LOGARITHMS OF NEGATIVE NUMBERS

In the expression for the radius vector R, having an initial value R_1 of plus unity with zero angle,

$$R = R_1 \epsilon^{\theta} \tag{20a}$$

or

$$R/R_1 = \epsilon^{\theta} \tag{21b}$$

if we assume (θ) as equal to (πj), the radius vector rotates through π circular radians, that is, 180 deg. and becomes minus unity, and we have,

$$R/R_1 = \epsilon^{\pi j} = -1 \tag{21}$$

But in the general expression,

$$R/R_1 = \epsilon^{\theta} \tag{22}$$

(θ) is the logarithm of (R/R_1) and, therefore, from

$$R/R_1 = \epsilon^{\pi j} = -1 \tag{23}$$

we must conclude that (πj) is the logarithm of (-1). This solves the problem of what the logarithm of a negative number could be. Thus,

$$L\,n\,(-x) = L\,n\,(-1)\,(x) \tag{24}$$

$$= L\,n\,(-1) + L\,n\,(x) \tag{25}$$

$$= \pi j + L\,n\,(x) \tag{26}$$

Physically, it means this:

In the discussion of hyperbolic angles especially in connection with Fig. 5, it was seen that "angle," "percentage" and "logarithm" were synonymous terms. Since, as it has already been shown that "angle" and "percentage" are synonymous terms for circular angles as well as for hyperbolic angles, therefore, we may extend our idea of a "logarithm" and identify it with "circular" angles as well as with "hyperbolic" angles. The foregoing mathematical transformation can then be obtained by an extremely simple physical reasoning. Thus, considering $(+1)$ as a unit radius vector at zero angle.

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[4] Ordinarily called a "logarithmic spiral." Exponential and logarithmic functions are merely the inverse of each other.

(1) What is the angle of $(+1)$? 0 **(27a)**
" " " logarithm of $(+1)$? 0 **(27b)**
(2) What is the angle of $(+j)$[5] ? $j\,\pi/2$ **(28a)**
" " " logarithm of $(+j)$? $j\,\pi/2$ **(28b)**
(3) What is the angle of (-1) ? πj **(29a)**
" " " logarithm of (-1)? πj **(29b)**
(4) What is the angle of $(-j)$[6] ? $3\,\pi j/2$ **(30a)**
" " " logarithm of $(-j)$? $3\,\pi j/2$ **(30b)**
(5) What is the angle of $(+1)$[7] ? $2\,\pi j$ **(31a)**
" " " logarithm of $(+1)$? $2\,\pi j$ **(31b)**

These examples not only illustrate the case of the logarithms of (j) besides that of (-1) but, comparing item No. 1 with No. 5, it is seen that a logarithm is periodic with a quadrature periodicity. That is, adding or subtracting $2\,\pi j$ or an integral multiple of it to a logarithm leaves its value unaffected. This, however, need not startle us, for any kind of quantity is periodic for that matter; that is, rotation through $2\,\pi$ circular angles brings a quantity back to its original value. This, then, is the physical meaning of the imaginary, that is quadrature or circular, periodicity of hyperbolic functions, a characteristic true not exclusively of hyperbolic functions but of circular functions and of every other function as well. The reason why this appears like a mathematical mystery to many is simply due to an incorrect trigonometric nomenclature by which, instead of making a distinction between real (that is, linear or in-phase) and circular (or quadrature) angles, calling linear angles (θ), and circular angles $(j\,\theta)$, and having only one kind of sine and one kind of cosine, we have two kinds of angles and two kinds of trigonometric functions. It is probably not too much to think that the early workers who established these nomenclatures had only a hazy conception of these physical relationships. Ordinarily, a circular angle is written as (θ) instead of $(j\,\theta)$, and the periodicity of the trigonometric functions is given by an equation like

$$\cos(\theta) = \cos(\theta + 2\,\pi\,n) \qquad \textbf{(32)}$$

which if written as

$$\cosh(j\,\theta) = \cosh(j\,\theta + 2\,\pi\,n\,j) \qquad \textbf{(33)}$$

exactly as

$$\cosh(\phi) = \cosh(\phi + 2\,\pi\,n\,j), \qquad \textbf{(34)}$$

shows that the periodicity of circular angles is also imaginary that is, quadrature, because equation **(33)** is identically the same thing as equation **(32)** except that it is in hyperbolic garb.

Physical Meaning of Sine and Cosine

Let (θ) be a complex angle, and let the value of the radius vector for $(+\theta)$ be R_1, and for $(-\theta)$ be R_2. The vector mean value of R_1 and R_2 is called the cosine of angle (θ): hyperbolic cosine, if (θ) is a real or in-phase angle; circular cosine, if (θ) is a quadrature angle.

The value of the radius vector for $(+\theta)$ is $\epsilon^{+\theta}$, and for $(-\theta)$ it is $\epsilon^{-\theta}$, and the mean value of the two gives the cosine, as

$$\cosh(\theta) = \frac{R_1 + R_2}{2} = \frac{\epsilon^{+\theta} + \epsilon^{-\theta}}{2} \qquad \textbf{(35)}$$

We prefer to write "cosh," instead of "cos," because we are treating the hyperbolic angle as real or linear, and the circular angle as quadrature, as this view is much more natural.

The foregoing formula agrees with the usual geometric representation of the cosine of a circular angle as the projection of the radius vector on the $X =$ axis as shown in Fig. 11.[8]

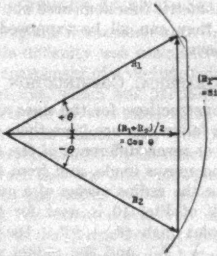

Fig. 11

For circular angles the numerical value and projection of the radius vector is the same for $(-\theta)$ as for $(+\theta)$, and therefore the definition of the cosine could be given either way, as mean sum or as projection, but not so with the hyperbolic functions. Assuming (θ) as an in-phase angle, $\epsilon^{+\theta}$ and $\epsilon^{-\theta}$ are not equal in magnitude neither are their projections, for their projections are identical with themselves. We could, therefore, define a cosine, if we pleased, either as the mean projection or as the (vectorial) mean value of the radius vector for $(+\theta)$ and $(-\theta)$. The cosine then may be looked upon as a vector parallel to the mean radius vector, as is clear from Figs. 11 and 12.

Turning to the sine of (θ), it is the mean (vector) difference of the radius vector for $(+\theta)$ and $(-\theta)$. Thus,

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[5] j must not be ignored, so that the circular, that is, quadrature, angle may not be confused with the hyperbolic angle.

[6] Meaning 270 deg.

[7] Meaning 360 deg.

[8] This vectorial point of view is the physical basis of the treatment of a harmonic flux as the resultant of two constant fluxes rotating in opposite directions in single-phase induction motors, as will be better evident in section on "Characteristics of Sines."

$$\sinh(\theta) = \frac{R_1 - R_2}{2} = \frac{\epsilon^{+\theta} - \epsilon^{-\theta}}{2} \qquad \textbf{(36)}$$

If (θ) is a real angle, the sine is hyperbolic, if a quadrature angle, the sine is circular.

The idea of projection does not hold very well for the sine of both trigonometric and hyperbolic angles without undue straining, but the idea of "one half vector sum" for cosine, and "one half vector difference" for sine, holds equally for circular as well as for hyperbolic angles.

For a hyperbolic angle, the sine is a vector in phase with the radius vector; but for a circular angle, the sine is perpendicular to the mean radius vector as will be seen from Figs. 11 and 12. From this it follows that, if θ is an in-phase or hyperbolic angle,

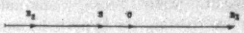

Fig. 12

$$\sinh(j\,\theta) = j\sin(\theta) \qquad \textbf{(37)}$$

whereas

$$\cosh(j\,\theta) = \cos(\theta) \qquad \textbf{(38)}$$

Other trigonometric functions need not be considered here, because they can all be expressed in terms of sines and cosines.

GRAPHICAL CONSTRUCTION

Graphical constructions for the sines and cosines of complex angles follow at once from their definitions as the mean sum or mean difference of the radius vector for the plus and minus angle, and from the graphical construction for the radius vector of a complex angle. The framework of Fig. 10 is used for this purpose. Given the complex angle $(\theta_1 + j\,\theta_2)$; the radius vector R_1 for plus $(\theta_1 + j\,\theta_2)$, and the radius vector R_2 for minus $(\theta_1 + j\,\theta_2)$ are located. Then, half their vector sum and difference give the trigonometric functions:

$$\cosh(\theta_1 + j\,\theta_2) = \frac{R_1 + R_2}{2} \qquad \textbf{(39)}$$

$$\sinh(\theta_1 + j\,\theta_2) = \frac{R_1 - R_2}{2} \qquad \textbf{(40)}$$

The addition and subtraction of R_1 and R_2 are carried out vectorially as shown in Fig. 10. Calculated analytically, we have

$$\cosh(\theta_1 + j\,\theta_2) = \frac{\epsilon^{+\theta_1}}{2}(\cos\theta_2 + j\sin\theta_2) + \frac{\epsilon^{-\theta_1}}{2}(\cos\theta_2 - j\sin\theta_2) \qquad \textbf{(41)}$$

$$\sinh(\theta_1 + j\,\theta_2) = \frac{\epsilon^{\theta_1}}{2}(\cos\theta_2 + j\sin\theta_2) + \frac{\epsilon^{-\theta_1}}{2}(\cos\theta_2 - j\sin\theta_2) \qquad \textbf{(42)}$$

CHARACTERISTICS OF SINES AND COSINES

The graphical constructions for the trigonometric functions give a true physical picture but can not naturally fully convey the various characteristics of these functions at sight. The solution of many physical problems involves differential equations, that is, requires a knowledge of the "rate of change" characteristics of functions. We may therefore analyze such characteristics of the radius vector as it traces circular or hyperbolic or complex angles to see how and why sines and cosines of these angles come in.

If a quantity is varying at a constant percentage, its absolute rate of change is proportional to itself. If the function is increasing, its rate of change, that is, its velocity, is positive; if decreasing, its velocity is negative; if rotating without change of length, its velocity is j, that is, quadrature; and, if both rotating and changing in length, its velocity is general or complex. If the rate of change or velocity is proportional to the quantity, the rate of change of the rate of change, that is, the rate of change of the velocity, or, in other words, the acceleration, is also proportional to the quantity. If the velocity is in-phase, the acceleration is also in-phase and has the same direction as the original quantity, regardless of whether the quantity is increasing or decreasing. If the velocity is in quadrature, the acceleration is in quadrature with the velocity and thus negative with respect to the original quantity. Thus, if a quantity is rotating, (*i. e.*, varying at a constant quadrature percentage of itself at every point), its acceleration is proportional to itself but negative, regardless of whether it is rotating positively or negatively, that is, clockwise or counter-clockwise. If then, we know by any physical consideration that the acceleration of a quantity R is proportional to itself and has the same sign as itself, it must be either a positively rotating quantity (equation **43**); or a negatively rotating quantity (equation **44**); or the resultant of two quantities rotating in opposite directions (equations **45** and **46**), that is, a harmonic quantity. Conversely, a harmonic quantity may be considered as the resultant of two constant vector components rotating in opposite directions as is assumed in the theory of the single-phase induction motor.

$$R = A\,\epsilon^{+jwt} \qquad \textbf{(43)}$$

or

$$= A\,\epsilon^{-jwt} \qquad \textbf{(44)}$$

or

$$= A\,(\epsilon^{+jwt} + \epsilon^{-jwt}) = 2\,A\cos(w\,t) \qquad \textbf{(45)}$$

or

$$= A\,(\epsilon^{+jwt} - \epsilon^{-jwt}) = 2\,A\sin(w\,t) \qquad \textbf{(46)}$$

From this it follows that the sine and the cosine functions are more general than the simple exponential functions, and include the common characteristics of both the increasing and the decreasing exponentials. This applies to the hyperbolic functions as well as to the circular. Thus, if the acceleration of the quantity

R is proportional to itself and in the same direction, then, the equation of the quantity is of the form.

$$R = A\,\epsilon^{+wt} \tag{47}$$

$$\text{or} \quad = A\,\epsilon^{-wt} \tag{48}$$

$$\text{or} \quad = A\,(\epsilon^{+wt} + \epsilon^{-wt}) = 2\,A \cosh{(w\,t)} \tag{49}$$

$$\text{or} \quad = A\,(\epsilon^{+wt} - \epsilon^{-wt}) = 2\,A \sinh{(w\,t)} \tag{50}$$

Thus, again, the sine and cosine functions are more general than the simple exponential, and include the acceleration characteristics of both the increasing and the decreasing exponential.

Just as the trigonometric functions can be compounded from the simple exponentials, so, the simple exponentials can be given in terms of the trigonometric functions. Thus,

$$\epsilon^{\pm wt} = \cosh{(w\,t)} \pm \sinh{(w\,t)} \tag{51}$$

$$\epsilon^{\pm jwt} = \cos{(w\,t)} \pm j \sin{(w\,t)} \tag{52}$$

In these equations, the exponential form is of course to be preferred to the trigonometric form on account of its simplicity.

The application of the characteristics of sines and cosines may be illustrated by a few familiar examples:

An object is displaced against an elastic force which varies directly with the displacement but is directed opposite to it. If the body is released, the force accelerates it and since the force and therefore the acceleration are proportional to the displacement but opposite to it, the motion will be like a circular sine or cosine function, that is, harmonic.

A condenser is discharged through an inductance. The rate of change of the current produces voltage across the inductance equal and opposite to that of the condenser. That is, the condenser voltage is proportional and opposite to the rate of change of the current. However, a condenser current is proportional to the rate of change of the condenser voltage, and since the condenser voltage is proportional (but negative) to the rate of change of the current, the rate of change of the condenser voltage and, hence, the current are proportional to the acceleration of the current. Thus, the current and its acceleration are proportional to each other and opposed, and therefore the current is a circular sine or cosine function, that is, harmonic.

A leaky transmission line is carrying current. Assume it direct current for simplicity. As we advance along the line the current will become less and less, due to leakage to ground. The rate at which the line current is changing is equal to the leakage current, and this rate is negative because the current is decreasing. The rate at which the leakage current changes along the line is proportional to the line current and is negative with respect to it in its turn because the leakage current also decreases. Thus the line current and its second rate of change are proportional to each other and have the same sign, and therefore, the distribution of current along the line must be like a hyperbolic sine or cosine.

Analytically,

If $\dfrac{d^2\,y}{d\,t^2} = +\,w^2\,y$, then,

$$\begin{aligned} y &= A \sinh{(w\,t)}, \text{ or,} \\ &= B \cosh{(w\,t)}, \text{ or their combination} \end{aligned}$$

If $\dfrac{d^2\,y}{d\,t^2} = -\,w^2\,y$, then,

$$\begin{aligned} y &= A \sin{(w\,t)}, \text{ or,} \\ &= B \cos{(w\,t)}, \text{ or their combination} \end{aligned}$$

And, if, $\dfrac{d^2\,y}{d\,t^2} = (w_1 + j\,w_2)^2\,y$, then,

$$\begin{aligned} y &= A \sinh{(w_1 + j\,w_2)}\,t, \text{ or} \\ &= B \cosh{(w_1 + j\,w_2)}\,t, \text{ or} \\ &= C \sin{(j\,w_1 - w_2)}\,t, \text{ or} \\ &= D \cos{(j\,w_1 - w_2)}\,t. \end{aligned}$$

These solutions are of course the simpler solutions, and still more general solutions are obtained by combining the simpler solutions, but that is another subject.

COMPLEX ANGLES IN SPACE

All the foregoing discussions were limited to a plane and one little difficulty was encountered in having the quadrature component of a vector angle a curve instead of a straight line, which was partially excused on the ground that if the arc was taken short enough it would be straight. The reason why the arc as a vector could not be represented by its chord was stated to be due to the fact that any finite chord could not be perpendicular to the radius vector at every point along that chord. In other words, the arc was curved with respect to the X-axis but straight with respect to the rotating radius vector, having a constant angle with respect to it at every position, for what is the fundamental definition of a straight line (in terms of differential equations) if not this, that its slope (therefore angle) is constant? The argument is admittedly a trifle strained; the reason is that it is limited to a plane. However, if we take a three-dimensional view of the matter, these arguments become unnecessary, for we then find that the quadrature angle is really to be represented by the axis of the arc. It then is easy to see that no matter how curved the arc, its axis will be straight. In plane analysis, however, there is no room for an axis, the third dimension being barred, and therefore the arc itself is taken to represent the angle. Representing the quadrature (circular) angle by the axis of the arc does not alter the fact that the circular and hyperbolic angle will be at right angles with each other, because hyperbolic angle will still be represented by the component of the path parallel to the radius vector, and, therefore, the axis of the quadrature component perpendicular to the plane of the path will also be perpendicular to the in-phase component of the path.

In treatises on vector analysis it is shown that the product, or quotient, of two vectors in quadrature is another vector and is perpendicular to the plane of the two factors. Referring now to Fig. 3, the elementary circular angle is

$$d\,\theta_2 = \frac{d\,R_2}{R}$$

which is the quotient of two vectors in quadrature, and is therefore, a vector perpendicular to the plane of $(d\,R_2)$ and R. Of course, it is difficult to show this normal in a flat diagram and therefore it is rotated into the plane of the diagram to make it visible, in which case it coincides with the circular arc, not with its chord. That is, when $d\,R_2/R$ is represented by $(d\,R_2)$, it is equivalent to rotating the normal through 90 deg. around the radius vector as axis into the plane of $(d\,R_2)$ and (R) in infinitesimal steps.

Considering now the elementary hyperbolic angle,

$$d\,\theta_1 = \frac{d\,R_1}{R} \qquad \text{Fig. 3.}$$

This is the quotient of two parallel vectors, and is therefore a scalar. In order that the representation may be consistent and show the true relationship between the two kinds of angles, if the circular angle is multiplied by (R) so must the hyperbolic angle be multiplied by R. The result is that this multiplication by R rotates the vector circular angle into the plane of the diagram and identifies it with the circular arc $d\,R_2$, and changes the scalar hyperbolic angle into vector and identifies it with the radial vector component $d\,R_1$ of the path in-phase with the radius vector. Looking at this from another point of view, if we are interested in showing the relationship of circular and hyperbolic angles, then,

$$\frac{d\,R_1}{R} : \frac{d\,R_2}{R} = d\,R_1 : d\,R_2$$

Hence, the representation of the circular and hyperbolic angles by the vectors $d\,R_2$ and $d\,R_1$ is justified.

It may be evident from the foregoing critical analysis that a complex angle is a vector in the same sense and limitation as an impedance; the per cent resistance component of an impedance corresponding to the hyperbolic angle, the per cent reactance component to the circular angle. In fact, the per cent vector impedance of a circuit is the complex angle of the circuit in radians. Strictly, of course, the percentage must be compound percentage not simple percentage. However, in small percentages, the distinction is negligible. When the percentages are large, or when the circuit has distributed constants the distinction can not be ignored.

If the radius vector traces a three dimensional curve, the quadrature component of the path may be in any one of an infinite number of planes and its determination requires two mutually perpendicular vectors in quadrature with the radius vector, of the form,

$$\theta = \theta_0 + j_1\,\theta_1 + j_2\,\theta_2$$

where j_1 and j_2 are the two reference vectors. As electrical problems are as a rule in one plane, and those which are three-dimensional have such symmetry that they can be treated as problems in one plane, the three dimensional complex angle has little application for the engineer. However, it has been mentioned above to indicate that the physical interpretation developed in this paper covers even this more generalized angle, and for the guidance of those who may wish to make a more extended study of the subject.

Discussion

A. E. Kennelly: I think we are all indebted to Mr. Boyajian for bringing to our notice that we shall have to modify our views concerning angles, and their treatment by engineers.

I think that has been evident to a great number of us, but Mr. Boyajian has been the first to say so definitely. He has shown us how complex angles may be worked. I think that the method he gives is workable and reliable, but I would like to make one reservation in regard to the treatment that he offers.

I think we all agree in regard to what he says about circular angles, and that a hyperbolic angle is an imaginary circular angle, also that a circular angle is an imaginary hyperbolic angle, but he takes the position, if I understood him correctly, that for a hyperbolic angle, it is not necessary to involve any kind of rotation, that it is simply a one-dimensional movement, or a one-dimensional quantity.

I think that while for ordinary purposes that may be true, and that processes of calculation may be conducted on that basis, but to say that the hyperbolic angle is of one-dimensional quantity, is I believe, limiting it unnecessarily. It seems to me that we can use Mr. Boyajian's method without necessarily confining ourselves to that point of the doctrine. I still think that we can maintain the idea of rotation through an area in connection with a hyperbolic angle, and one reason for that is presented in these slides of a model for projecting the cosines and sines of a complex hyperbolic angle, and which attributes rotation of a radius vector to both the hyperbolic and circular components of the complex angle.

A. Boyajian: As for the reservation which Dr. Kennelly wishes to make so as to maintain the idea of rotation in connection with hyperbolic angles, may I submit the following considerations:

When the radius vector traces a hyperbolic sector, it certainly also traces a circular angle by virtue of its rotation. According to usual schemes, the area of this hyperbolic sector is made use of to represent the numerical value of the hyperbolic angle, and the circular angle which the radius vector has traced is ignored. As I have stated in my paper this is permissible representation but is not completely illustrative of the essence of a hyperbolic angle. If we were called upon to make a representation of a hyperbolic angle, in which there shall be no trace of a circular angle, I think we shall have to fall back on the straight line diagram given in my paper in accordance with the newer point of view.

Dr. Kennelly feels that to make the hyperbolic angle a one-dimensional quantity, that is, to eliminate rotation from the representation of hyperbolic angles, limits them unnecessarily. I feel however that the idea of rotation which has been forced on hyperbolic angles in the past has unnecessarily handicapped the interpretation of the true physical nature of hyperbolic angles. The new point of view, by eliminating this unnecessary condition of rotation, gives greater freedom to the hyperbolic angles and to their interpretation.

Radiation from Transmission Lines*

BY CHARLES MANNEBACK

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Review of the Subject:—*The aim of this investigation was to contribute something to our knowledge of traveling electromagnetic waves. Among the transient phenomena that occur along transmission lines, these are still little known, especially as far as their "attenuation" and "distortion," or change in shape near the "wave front" are concerned.*

It is first seen that the "classical theory" of the propagation of electrical disturbances along lines, as it has been chiefly developed by Heaviside and Poincare, does not give a correct representation of the facts near the wave front, because it assumes an instantaneous penetration of the current in the wires. It is shown that this theory is an "unidimensional" one, as it considers only one space variable, the coordinate along the line, and, from an electromagnetic point of view, amounts to identifying the traveling waves with plane wave phenomena.

Steinmetz's theory of the radiation from traveling waves (TRANS. *A. I. E. E. Feb., 1919) is then examined, and, as Carson pointed out* (JOUR. *A. I. E. E., Oct. 1921), found based on a misconception of the propagation of the electromagnetic field near the wires of a line. It is remarked that this theory amounts to propagating longitudinal electric waves, a conception in conflict with the basis of Maxwell's theory. This latter proves very easily that, along a perfect line, i. e., without ohmic or leakage losses, plane electromagnetic traveling waves are propagated without distortion and without attenuation; hence that there is no radiation. Losses do not change anything from the radiation point of view (Mie. Ann. Phys. 2, 1900). In the theory of Steinmetz, the radiation was the controlling factor at high frequencies.*

Although the question could be considered as settled, in so far as Steinmetz raised it, it is felt that the conditions under which traveling waves are started must be elucidated, in order to decide whether, during the transient process of establishing a plane electromagnetic traveling wave, any loss of energy can occur by radiation in free space. This would reduce the radiation to a transient phenomenon instead of to a steady one, as assumed by Steinmetz, i. e., to an effect produced when the condition of plane wave is departed from, as at the origin or the end of a line ("end effect").

The problem, considered in its broad aspect, i. e., from the electromagnetic point of view of Maxwell, appears to be of great complexity. It involves a study of what might be called time-three-dimensional space transient, while the most complicated transients considered in electrical engineering are time-one-dimensional space transients, as was already noticed. It was found possible, however, to decide whether the radiation is a factor of enginering importance in attenuating and distorting waves.

There, first, the distribution of current along the conductors of a line is shown to be a very close approximation to the actual, unknown, distribution, with regard to the possibly existing radiation. Then, from this assumption, the radiation of the system is calculated at a corresponding approximation, and its amount found to be negligible compared to the heat dissipated in the line during the same time. From this it is concluded that the effect of the radiation upon the attenuation and the distortion of the waves along the line must also be negligible, compared to the effect of the joulian losses, and in this result a proof "a posteriori" of the correctness of the initial assumption is seen. The first idea of that procedure, but limited to stationary waves, is probably to be credited to M. Abraham (Phys. Zeit. 2-p. 329-1901) who applied it to the calculation of the radiation from a single isolated wire (oscillator), and found it in practical agreement with a more elaborate theory, based on Maxwell's equations. As in this latter application the radiation is larger than the ohmic dissipation, the conclusions of this investigation are even strengthened.

An application to the steady radiation from a transmission line oscillating freely at one of its natural frequencies shows, even at very high frequencies, that the power radiated is negligible compared to the heat dissipated. For instance, a 100-kilometer transmission line, No. 00 B. & S. wire, oscillating at three million cycles per second, wastes by radiation only 1/3600 of what is wasted by heat. This is in accordance with the results given for steady a-c. traveling waves along a line by Carson, (loc. cit.).

In the case of a traveling wave suddenly started at the origin of a perfect line, it is found that, when the wave has become plane, the amount of energy

$$\frac{3}{2}\, d\, 10^{-9}\, I^2 \text{ joules}$$

is wasted by radiation. d is the distance in cm. between the two wires of the line and I the constant current suddenly flowing in the line, in amperes. This energy is carried to infinity by an electromagnetic field limited to a thin spherical shell of variable depth, (not thicker than d), expanding at the speed of light. A reflection of the wave at a free end of a line is shown to add to the preceding an amount of radiated energy

$$4\, d\, 10^{-9}\, I^2 \text{ joules}$$

and only half that amount for a grounded line. A complete transposition causes a radiation four times as large as the sudden starting of the wave. Compared to the joulian dissipation during the time the wave takes to travel once along the line, the radiated energy is of the order of 1/10,000th.

The influence of the radiation on the attenuation and distortion of traveling waves is thus found entirely negligible in engineering practise. Other possible factors are suggested.

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INTRODUCTION

ONE of the chief reasons at present limiting the development and interconnection of high-power and long-distance electrical transmission lines is the trouble due to the occurrence of abnormal voltages and abnormal currents in the line and the connected apparatus. Whenever any change occurs in the steady state conditions of an electrical system, *e. g.* by connecting or disconnecting a load, or by producing or interrupting a short circuit, the new steady state conditions are generally reached after a very short time, during which so-called "transient phenomena" or "transients" take place. As the flow of effective and reactive power are not the same in the different parts of the system during the first and the second steady state, the necessary adjustment is made by transient electromagnetic "traveling waves," *i. e.* waves of both voltage and current, carrying energy from one part of the system to another.

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*This paper is the result of an investigation submitted in partial fulfillment of the requirements for the degree of Doctor of Science from the Massachusetts Institute of Technology, 1922. It has been undertaken under the direction of Professor V. Bush, to whom the author wishes to express his fullest appreciation.

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The values of voltage and current in these waves, or as we shall say the amplitude of the waves, may reach abnormally high values and become destructive to the line. The steepness of the front of the wave also, *i. e.*, the high rate of change of the voltage and current with the time may be harmful. Therefore, a knowledge of the "attenuation" or diminution of amplitude of the wave with the time, and of the "distortion" or change in the wave shape, is of engineering importance. Our present knowledge of the subject is still far from being complete. Joulian and dielectric losses produce both attenuation and distortion, but there may be other causes. C. P. Steinmetz[1] considered the possible influence of radiation, but his results may be questioned.

From this began the present investigation, aiming towards the determination of the radiation effect upon the decay of traveling waves along transmission lines. It was found that the radiation effect is entirely negligible compared with the effect of joulian dissipation, and hence that, from an engineering point of view radiation can be ignored in connection with transmission lines.

CLASSICAL THEORY OF THE PROPAGATION OF WAVES ALONG ELECTRIC LINES

The classical theory of the propagation of waves along a line, *i. e.*, a system of straight parallel outgoing and return conductors, rests upon two assumptions: the displacement currents in the dielectric, parallel to the conductors are negligible, and the so-called "four distributed constants" of the line can be determined. They are the resistance, inductance, leakance and capacity per unit length of the line (R, L, G, C). These quantities are functions of the frequency for sinusoidal steady alternating current; for transient currents, they are somewhat indeterminate, and an average value must be assumed.

The fundamental equation for the propagation of the voltage V or the current I is under these assumptions

$$L\,C\,\frac{\partial^2 V}{\partial t^2} + (R\,C + L\,G)\,\frac{\partial V}{\partial t} + R\,G\,V = \frac{\partial^2 V}{\partial x^2} \qquad (1)$$

where t denotes the time and x the space coordinate along the line. From this equation, well known conclusions can be drawn, of which the following only need be considered here:

1. An electric disturbance, *i. e.*, a discontinuity of voltage or current, is always propagated along any line at the constant speed $v = 1 : \sqrt{L\,C}$, whether there is resistance and leakance or not.

2. The disturbance is attenuated exponentially with the time according to

$$\exp\left[-1/2\,(R/L + G/C)\,t\right].$$

3. The distortion of the disturbance depends on the value of the term

$$1/2\,(R/L - G/C)\,.$$

In the case of transmission lines, the attenuation is the controlling factor, and the wave has died out before any appreciable distortion has taken place. Thus a steep wave front will very nearly keep its shape during the propagation.

This conclusion, however, is based on the two assumptions made; the first amounts to saying that the variations of V and I along the line will not be too rapid, and the second, that the penetration of the current in the wires is instantaneous, neither of which is true at the very front of the wave. From Maxwell's point of view, the classical theory amounts to a plane wave theory, *i. e.*, the electromagnetic field around the wires lies in planes; this implies that the dissipation is taking place everywhere in the plane wave, instead of being localized in the wires. This has been shown very clearly by O. Heaviside in his "Electrical Papers."

We thus consider the question of change in the wave shape near the front of the wave, due to the resistance of the line, still as an open question.

DR. STEINMETZ'S THEORY OF THE RADIATION

Dr. Steinmetz's theory of the radiation from transmission lines deals only with the electromagnetic field. The electric and the magnetic fields are supposed to propagate from the wires perpendicularly into space, thus implying a flow of energy from the wires into space, and also non-transversal electromagnetic waves, two conceptions in conflict with Maxwell's theory. It is shown[2], as an immediate deduction of Maxwell's equations that plane electromagnetic transversal waves are propagated undistorted and unattenuated, at the speed of light in the surrounding medium, along a pair of infinitely long straight perfect conductors, *i. e.*, along a line without joulian or leakage losses; thus there is no radiation. Non-perfect conductors, as it has been shown, introduce more complexity, because the waves are no more plane waves, but nothing is changed from the radiation point of view.[3] The flow of energy in the dielectric is evidently parallel to the wires, with a small component directed into the wires if there are losses. This settles entirely the question, in so far as Dr. Steinmetz raised it: *traveling plane* waves along a line are *not* attenuated nor distorted by radiation, *i. e.*, by escape of energy in a direction perpendicular to the line. Plane waves were an *implicit* assumption in the criticized theory; however, very near the source of a disturbance, or near the end of the line where reflection occurs, or in general near any place where there is a change in the geometrical or electrical behavior of the line, the electromagnetic field, of course, cannot be plane.

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[1] C. P. Steinmetz—PROC. A. I. E. E., Feb. 1919, p. 249. —Transient Electric Phenomena, 1920; Sec. III, Chap. 8, 9; Sec. V.

[2] H. Hertz, Wiedemann's Annalen, 36, p. 1, 1889. Die Ausbreitung der elektrischen Kraft, 1892. J. R. Carson, JOUR. A. I. E. E., Oct. 1921, p. 789.

[3] G. Mie, Annalen der Physik, 2, p. 201, 1900.

This raises a new question: If plane traveling waves are only a limiting state, is there not any radiation taking place when the electromagnetic wave is initiated? Or, otherwise expressed, is radiation not an "end effect" with respect to the line? This involves the consideration of the whole surrounding space as the seat of the phenomena, and we need to start from a broader point of view than we did.

POSITION OF THE PROBLEM FROM MAXWELL'S POINT OF VIEW—OUTLINE OF AN ENGINEERING SOLUTION

According to a classification made by Steinmetz, we can say that the transients considered in electrical engineering are either simple time transients, as in machinery, or simple space transients, like voltage and current distribution along a transmission line under steady a-c. conditions, or both time and space transients, as in the case of a line under transient conditions. We have already noticed that the notion of space involved in these transients is unidimensional or depending on one space variable only (equation 1); this supposes implicitly that the electromagnetic field of these transients is in planes. We are led, therefore, to consider more extended classes of transients; three dimensional space transients, as occuring near the ends of a steady a-c. line or near a wireless c. w. antenna and time and three dimensional transients, when steady conditions in such systems are changing. The only means of studying such transients is to resort to Maxwell's equations of the electromagnetic field, connecting the electric force E and magnetic force H. From these equations, follows the same equation for E and H, *viz.*:

$$\frac{\epsilon\,\mu}{c^2}\,\frac{\partial^2 E}{\partial t^2} + \frac{4\,\pi\,\mu\,\gamma}{c^2}\,\frac{\partial E}{\partial t} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} \qquad (2)$$

E and H are space vectors, C represents the velocity of light in vacuo, ϵ and μ are the numerical values of the dielectric constant and magnetic permeability of the medium and γ the conductance of the medium expressed in electrostatic units. In air, ϵ and μ are unity and γ zero; in conductors, ϵ is to be taken as zero. From (2) is shown that a disturbance, *i. e.* a local discontinuity of E or H is propagated in the medium at the speed $c : \sqrt{\epsilon\,\mu}$. Equation (2), of which (1) is a particular case, is the "damped wave equation," well known in mathematical physics. Although much work has been done on this equation even in electromagnetics, no solution has been obtained that seems directly available for the present problem, as far as we know. They generally deal with steady state sinusoidal waves along wires, without considering the source producing them, nor how they are initiated. (Sommerfeld, *Ann. Phys.* 67, 1899. Mie, Abraham, *Ann. Phys.* 2, 1900, Hondros. *Ann. Phys.* 1909, etc.) What we want is a knowledge of the whole electromagnetic field proceeding from a disturbance of voltage or current initially localized at a point of the line; the values of the field near the wires will give the amplitude and shape of the traveling wave, the values of the field at great distances from the line will give its radiation.

The mathematical difficulties involved in a general solution are indeed very great, but we believe that the position of the problem as it was sketched is the only one that can lead to a full answer to the question of the attenuation and distortion of traveling waves along wires.

We found it, however, possible to decide whether the radiation is of engineering importance in the attenuation of the waves, either traveling or stationary. By *radiation*, we mean, as usual, the constant limit S towards which tends the flow of energy, as defined by Poynting's theorem, through a sphere enveloping the whole electrical system, the transmission line in the present case, when the radius of the sphere is increasing above any fixed value. The time integral of the preceding limit S will be called the "total radiation" and denoted by W. We will show first that the classical theory of waves (neglecting radiation) gives a "theoretical" distribution of current along the line that is a very close approximation to the "true" distribution, *i. e.*, the unknown distribution taking place when the radiation is considered. Knowing the current in the line, one can calculate the corresponding radiation that must exist; an approximate value will be obtained by substituting the "theoretical" current for the "true" current. If, now, the amount of radiated energy is found to be quite negligible, say one to a hundred, compared to the joulian energy dissipated in the wires, we get an *a posteriori* proof of the rectitude of our premises, that is, that the true distribution of current, is given at, say, one per cent by the classical theory. As attenuation and distortion of the waves are produced by the dissipation of energy in the system, it is logical to conclude that the radiated energy, a cause hundred times less in magnitude than the energy dissipated by joulian heat, will produce a correspondingly small effect in attenuating and distorting the waves, *i. e.*, the radiation will be a negligible factor from an engineering standpoint.

The method is one of successive approximations, as used in experimental physics. It has already been used for calculating the steady radiation of given types of antennas, generally without being substantiated.

(Pierce, El. waves, Chapter IX. F. Cutting, *Proc. I. R. E.*, p. 129, 1922). In the case of a straight oscillator, Abraham found the present method in quantitative agreement with the more elaborate solution based on the full solution of Maxwell's equation. (*Phys. Zeit*, 2, p. 329, 1901). It is to be noticed that in the case of antennas, the radiation is the controlling factor and the joulian dissipation only secondary, while in the case of transmission lines the conditions are reversed. This strengthens our conclusions even more.

We first consider perfect conductors, and later take into account the influence of joulian losses.

When any kind of disturbance of current is propagated along a perfect thin wire, Heaviside shows how the corresponding electromagnetic field is set up by means of spherical traveling waves. (El. Theory, Vol. I, art. 54, etc., where what he calls the "motion of a charge" is entirely equivalent to the propagation of a "current along a wire.") The reflection of current and the corresponding electromagnetic field generated by spherical waves originating at the point of reflection are indicated. (in El. Theory, Vol. II, art. 393, 394.) This theory neglects the diameter of the wires, compared to the length of the disturbance. At that order of approximation, the current (or traveling wave) is propagated undistorted and unattenuated at the speed of light in the air, and reflected as usual, even with regard to possible radiation.

We thus take the distribution of current along the line, either a stationary or traveling sinusoidal one in the steady a-c. case, or a traveling rectangular one in the case of a sudden impulse.

STATIONARY RADIATION FROM A TRANSMISSION LINE

We mean the radiation that takes place under steady a-c. conditions by means of stationary waves, such as the natural oscillations of a line, without joulian losses.

Different modes exist, according to the terminal conditions to which the line is subjected. We consider, as an example, a line free at both ends, thus capable of half-wave length oscillations and multiples thereof.

We assume spherical polar coordinates, with the origin O_1 at the middle of the length l of the wire No. 1, and the polar axis $+ z$ along the same wire.

A length along the wires will be z, r_0 is the distance between a point P and the origin O_1, θ_0 is the angle between $O_1 P$ and the $+ z$ axis; r and θ are the same quantities referred to any point of the wires instead of to O_1; ϕ is the angle (longitude) that the plane $P\, O_1\, z$ makes with the plane of the wires, measured from the direction opposite to the second wire No. 2; d is the distance between the wires. (Fig. 1.)

The distribution of the current along wire 1 is from the preceding:

$$I = \imath \cdot \cos \frac{\pi\, n\, z}{l} \cdot \sin \frac{\pi\, n}{l}\, c\, t \tag{3}$$

with $- l/2 \leqq z \leqq + l/2$, and with reversed sign for the current along wire 2. I is positive, if directed along $+ z$; n is an odd integer giving the "order" of the oscillation; c represents the velocity of light; $\imath$ is the maximum current.

We have for the electromagnetic field due to the rate of change of a current element $I\, d\, z$, at distances large compared to the wave length of the oscillation, the values first given by Hertz under an equivalent form (*Wied. Ann.* 36, p. 1, 1889; or *Ausbr. Elekt. Kraft.*, p. 153, 1892).

$$E = H = 1/c \frac{\sin \theta}{r} \frac{d}{d\, t} (\hat{I}\, d\, z) \tag{4}$$

where $I\, d\, z$ is the same as the time derivative of the moment of Hertz's doublet. $\hat{I}$ means I, where the time t has been replaced by $t - r/c$; the current I and magnetic force H are expressed in electromagnetic units; the electric force E is expressed in electrostatic units. At a point P, E lies in the plane of P and the wire, and is perpendicular to r; at the same point, H is perpendicular to this plane. The positive directions are shown on the diagram. We consider the whole transmission line as a large sum of such current elements. At the point P, the elementary $H'\,s$ add themselves, but all the $E'\,s$ do not fall in the same direction. They will do so, however, and will add, provided we recede far enough from the line, because all differences in direction then vanish, *i. e.* θ can be taken as a constant. We can write neglecting quantities of the second order, *i. e.*, $1/r^2$ as compared to $1/r$:

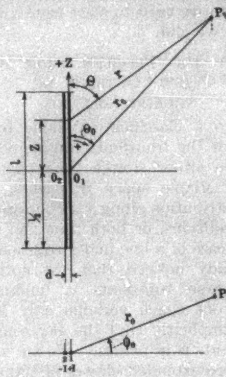

FIG. 1

$$E = H = 1/c \,.\, \frac{\sin \theta_0}{r_0} \,.\, \frac{a}{d\, t} \int_{-\, l/2}^{+\, l/2} (\hat{I}_1 - \hat{I}_2)\, d\, z \tag{5}$$

where I_1 and I_2, the currents in the wires 1 and 2 are to be given their proper time phase $t - r/c$.

At great distances, we have with the same approximation

for wire 1 $\quad r = r_0 - z \cos \theta_0$

for wire 2 $\quad r = r_0 - z \cos \theta_0 + d \sin \theta_0 \cos \phi_0 \qquad (6)$

Still with the same approximation, and neglecting the higher powers of d/l, which is of the order of 10^{-3} to 10^{-5}, (3) gives

$$I_1 - I_2 = i \, . \cos \frac{\pi n z}{l} \, . \cos \frac{\pi n}{l} (c\,t - r_0 + z \cos \theta_0) \, . \frac{\pi n d}{l} \sin \theta_0 \cos \phi_0 \qquad (7)$$

In this expression, z cannot be neglected as compared with r_0, because these quantities figure as argument of the function cos. Substituting in (5), performing the time[4] derivation and the integration with respect to z, we get

$$E = H = (-1)^{\frac{n+1}{2}} \; 2\,i \, . \frac{\pi n d}{l} \; \frac{\sin \frac{\pi n}{l} (c\,t - r_0)}{r_0} \; \cos \left(\frac{\pi n}{2} \cos \theta_0 \right) \cos \phi_0 \qquad (8)$$

where n is an odd positive integer, and the signs agree with the assumed convention. This formula represents the steady electromagnetic field from an a-c. transmission line of length l, at a distance r_0 from the origin so large that the ration l/r_0 is very small. The electric and magnetic forces E and H lie on pure spherical waves, traveling at the speed c from the origin on the line. We consider the intersection of the $+ z$ axis with a sphere as a positive pole (origin of the angles θ) of the sphere. Then, the electric force is directed along meridians and the magnetic force along parallels; the sign in formula (8) is in accordance with the original conventions. (Fig. 2).

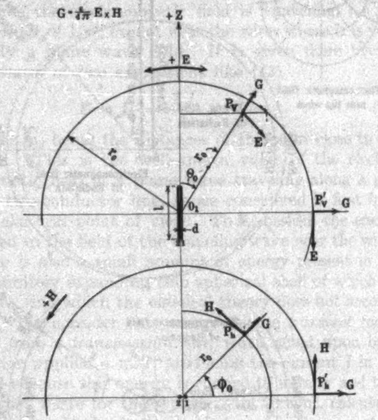

FIG. 2

The main difference between the "field of radiation" of a transmission line and that of a single wire line (antenna), is that, in the first case, the forces are only of the order of d/l, compared to the second case, which is very small.

It is seen that the electric and magnetic forces are perpendicular to the direction of propagation, (the radius of the sphere), as it must be in the case of pure transversal waves; and thus Poynting's vector $E \times H$ is perpendicular to the sphere. Hence, we have for the flow of energy or energy radiated per unit of time

$$S = \frac{c}{4\pi} \int E\,H\,d\,\Sigma \qquad (9)$$

integrated over the area of the sphere of radius r_0, with $d\,\Sigma$ as element of area.[5]

Substituting the values (8) for E and H, and performing the integrations, and taking the time average, we obtain:

$$S = C \, . \, \pi^2/2 \left(\frac{n\,d}{l} \right)^2 . \, i^2 \text{ ergs/sec.} \qquad (10)$$

or

$$= 15 \, . \, \pi^2 \left(\frac{n\,d}{l} \right)^2 I^2 = 15 \left(\frac{2\,\pi\,d}{\lambda} \right)^2 I^2 \text{ watts} \qquad (11)$$

where I is here the maximum current in amperes and λ the wave length. The *power radiated* by an oscillating transmission line, open at both ends, is thus, for a given current, numerically *proportional to the square of the ratio of the distance between the wires to the wave length of the oscillation.*

We compare the power radiated to the power dissipated by heat, taking as an example the line whose constants are

No. 00 B & S
d = 183 cm.
l = 100 km.
Total d-c. resist. line = 48 ohms
Total external inductance = 0.238 henry

We have, for the power radiated S
for $n = 1$ $\lambda = 2 \times 10^7$ cm.
$S = 0.5 \times 10^{-7}$ watts per max. ampere
f = 1500 cycles/sec.
$n = 2001$ $\lambda \cong 10^4$ cm.
S = 0.2 watts per max. ampere
$f \cong 3 \times 10^6$ cycles/sec.

Due to the sinusoidal distribution of current, the power dissipated by heat is only 48/4 watts per max. ampere at low frequency; it is about 60 times more at three million cycles.

Thus, even at three million cycles, the power radiated by the line is only 1/60 or 1.7 per cent of the power wasted in heat at low frequency, or 1/3600 of the total heat wasted at three million cycles. *Hence, the radiation of oscillating transmission lines is entirely negligible in engineering practise.*

Dr. Steinmetz's formulas amounted to writing the radiated power proportional to $d \, . \, l/\lambda^2$ instead of d^2/λ^2.

Similar results are obtained in the case of forced oscillations of a line or impressed a-c. and load to Carson's formula. (JOUR. A. I. E. E., Oct. 1921, p. 789.)

Instead of considering an ideal sinusoidal current distribution along the line, as when there are no losses, we might introduce the actual distribution given by the hyperbolic theory. There would be only more complication in the integrations, but the order of magnitude of the result would remain the same, *i. e.* proportional to d^2/λ^2; thus quite negligible.

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[4] See note 1.

[5] See Note 2.

TRANSIENT RADIATION FROM A TRANSMISSION LINE

We mean the radiation that accompanies the production of traveling waves. When a disturbance of current is suddenly produced at the end O of a semi-infinite, thin, perfectly conducting wire, it is propagated in first and very close approximation as seen above, unattenuated and undistorted, at the speed of light, along the wire. The wire is supposed to be the only conductor in the empty space. The accompanying electromagnetic field is propagated, at sufficient distance from the source, by spherical waves, also at the speed of light. Its expression at a point P is, when r is great, *i. e.*, such as to make $1/r^2$ negligible compared to $1/r$:

$$E = H = \hat{I}/r \cdot \frac{1 + \cos\theta}{\sin\theta} \qquad (12)$$

r denotes the distance PO; θ is the angle that PO makes with the wire; $\hat{I}$ the current at the origin O, given as a function of time t, where t has been replaced by $t - \frac{r}{c}$: c is the speed of light. I and H are expressed in electromagnetic units; E is in electrostatic units. I is positive when flowing along the positive direction of the wire. The electric force E is directed along meridians; the magnetic force H along parallels.

This formula is substantiated[6] by noticing that it verifies both Maxwell's equation for r large, and that for θ very small, it represents plane electromagnetic waves traveling at the speed c along an infinitely thin wire;

$$E = H = \frac{2\,\hat{I}}{l} \qquad (13)$$

with $l = r \sin\theta$ being the distance of a near by point close to the wire. When the traveling disturbance of current reaches the end of the wire, it can be absorbed by the terminal apparatus without reflection; this is equivalent to starting at the end a new disturbance $-\hat{I}$, of opposite sign, but of same direction of propagation as the first, so that each cancels the other along the wire supposed to be extended. If there is partial or total reflection, a new superimposed disturbance of current $K\,\hat{I}$ is supposed to be sent along the wire from the end towards the origin, thus propagating in opposite direction, the value and sign of K, which is the coefficient of reflection, depend on the terminal conditions.

We shall consider, just for simplicity, as only disturbance of current the sudden starting or stopping of a constant current at the origin, *i. e.*, "rectangular waves" of current. The stopping will be considered as the starting of a current of opposite sign, in the same direction of propagation.

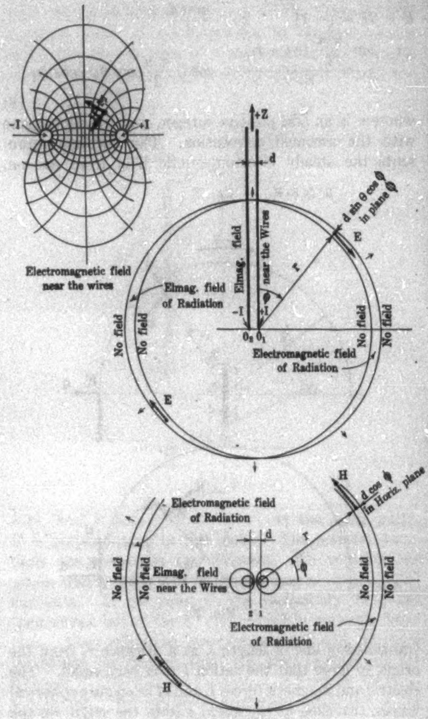
-I
+I
Electromagnetic field near the wires
+Z
d
d sin θ cos φ in plane φ
E
r
Elmag. field near the Wires
Elmag. field of Radiation
φ
-I
+I
No field
No field
O_2 O_1
No field
No field
Electromagnetic field of Radiation
E
H
d cos φ in Horiz. plane
Electromagnetic field of Radiation
d
r
φ
No field
No field
Elmag. field near the Wires
No field
No field
2 1
Electromagnetic field of Radiation
H

FIG. 3

The case of a *transmission line* reduces itself to the preceding case. We consider a line composed of two parallel semi-infinite wires originating in O_1 and O_2 and at the distance d apart. The coordinates used are the same as before, and are shown on the diagram, Fig. 3. A disturbance is produced at the origin, consisting of a suddenly flowing constant current $+I$ at O_1 and $-I$ at O_2. At *great* distances from the source, the electromagnetic field of the disturbance will be given by the sum of two expressions like (**12**) where due regard is given to the position of the sources O_1 and O_2 and their influence upon the "retardation" $t - \frac{r}{c}$, and also to the sign of the disturbances. This supposes that the presence of one thin wire does not interfere with the electromagnetic field of the other wire at great distances; the linearity of the electromagnetic equations permits the addition of the fields.

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[6] See Note 3.

When a sufficient time has elapsed after the disturbance started at the origin, *i. e.*, when its spherical front will have reached a great distance, in the sense we gave to that expression, the electromagnetic field of the disturbance can be considered as composed of two parts. (See Fig. 3.) One, in the space common to both "front spheres" O_1 and O_2, centered upon the sources O_1 and O_2; the other in the space between the two spherical surfaces, which can be considered as a very thin spherical shell of variable depth

$$d \sin \theta \cos \phi$$

this being equal to the distance $O_1 O_2$ projected in the direction considered (θ, ϕ). The electromagnetic field outside both spheres is obviously zero. In the shell, it is given by (12), with the proper sign for I, and lasts at a fixed point in the space during the time

$$| d/c \sin \theta \cos \phi |$$

the bars meaning an absolute value. Inside both spheres the electromagetic field is vanishing, *i. e.*, of the order of $1/r^2$, except near the wires where it is very nearly a plane wave field. It is given there by the difference of two expressions like (**13**)

$$E = H = \frac{2\,I}{\rho_1} - \frac{2\,I}{\rho_2} \qquad \textbf{(14)}$$

ρ_1 and ρ_2 being the distances of the point close to the wires. This is the well-known value of the electromagnetic field of a plane wave traveling along a perfect two-conductor line, as we considered it first from the classical point of view. Thus, besides the energy stored in the field of the traveling wave near the wires, there is also a small amount of energy present in the indefinitely expanding thin spherical shell of which we spoke, and which the classical theory does not account for. We consider this energy as the *transient radiation from a transmission line*, when acted upon by a sudden applied e. m. f. producing the current I in the line, because this energy is carried to infinity and thus is a net waste for the system. Its amount, calculated in a way entirely similar to that used in the case of the steady radiation above, is[7]

$$W = 3/2\, d \cdot 10^{-9} \cdot I^2 \text{ joules} \qquad \textbf{(15)}$$

where d is the distance between wires in cm. and I the sudden started constant current in amperes.

We consider as an example a *rectangular pulse* of current, *i. e.*, a traveling disturbance of constant current, extending over a length λ cm. The total waste by radiation will be twice as much as given by (**15**), because the same transient radiation occurs when suddenly stopping the current at the origin as when suddenly starting it. The total energy in Joules carried by the pulse along the wires, when it is far enough from the source, is given by $\dfrac{L\,.\,\lambda\,.\,I^2}{2}$ where L is the self induction in henries per cm. of the line, I the current in amperes. For the line already considered, L is equal to 23.8×10^{-9}. Thus the ratio of the radiated energy to the energy contained in the plane electromagnetic field of the pulse is given by

$$\frac{3\,d}{23.8}\,1/\lambda = .126\, d/\lambda$$

or, for a pulse extending over a length λ one hundred times the distance d between the wires, only 0.1 per cent of the total energy of the pulse. It is to be noticed that the fundamental frequency of the pulse in this example is as high as 820,000 cycles. Taking as effective resistance the resistance at one million cycles (17.3 ohms per km. of line, Steinmetz loc. at p. 202), it is found that when the pulse travels only one third of its own length, it wastes in heat an amount of energy equal to the energy radiated. This shows how negligible is the effect of radiation.

When a rectangular wave of current reaches the end of a single wire, its stopping generates a new spherical wave, expanding at the velocity of light and centered upon the end R; (Fig. 4) and which is tangent to the first spherical wave originating from O.

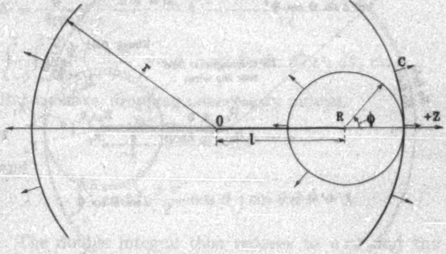

FIG. 4

Similarly, in the case of a two wire line, a spherical shell of electric and magnetic field is generated, centered upon the two ends R_1 and R_2, and tangent to the first shell originating from O_1 and O_2. (Fig. 5.) Thus the total radiation will be the energy carried into infinite space by the two expanding spherical shells. As the two shells have the part $X_1 X_2$ partly in common, where their fields add to each other, the total energy radiated will not be exactly that obtained by adding the energies of both shells separately, but will be a little less. It is shown that taking this sum an error is committed that will not exceed the ratio of the angle $\dfrac{X_1 X_2}{r}$ to 2, *i. e.*, of the order of d/l,[8] or between 10^{-3} to 10^{-5} or less. Hence, to that approximation, we can say that *the starting current and successive reflections will each add a given amount to the energy lost by radia-*

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[7] See note 4.

[8] See note 6.

tion. The solution of specific cases is thus directly given by the preceding. We give the following results as illustrations of the method.

1. The *sudden* establishment from one end of a line of a sustained continuous current I, without reflection at the other end, wastes

$$3\,d\,10^{-9}\,I^2 \text{ joules} \tag{16}$$

by radiation; this result is independent of the length of the line or of its material; it depends only on the distance d between the two wires of the line. In this formula and in all the following, d is in centimeters and I in amperes.

Taking the same 100 km. transmission line as previously, and assuming only the d-c. resistance even during the transient conditions, it is seen that the energy wasted by radiation is only 1/10,000th of the energy wasted in heat during the time in which the line is being charged, and the steady d-c. conditions are reached. For a 1-km. line, the radiated energy would still be only one per cent of the energy wasted in heat during the transient conditions. The same remark as to the order of magnitude of the radiation compared to the joulian losses applies to all following examples.

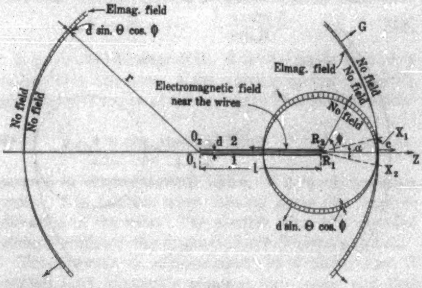

FIG. 5

2. A total *reflection* of a "rectangular wave" of current I at an *open* end of a line radiates[9]

$$4\,d\,10^{-9}\,I^2 \text{ joules,} \tag{17}$$

at a *grounded* end,

$$2\,d\,10^{-9}\,I^2 \text{ joules.} \tag{18}$$

In general, we have a *partial reflection* when the amount of current $K\,I$, (in magnitude and sign), is reflected and thus the rest of the wave is absorbed in the terminal apparatus. The radiated energy there is given by

$$3/2\,(1 - 2/3\,K + K^2)\,d\,10^{-9}\,I^2 \text{ joules} \tag{19}$$

of which the three preceding formulas are particular cases. ($K = 0, K = -1, K = 1$)

3. A complete transposition, *i. e.* a sudden rotation of 180 deg. of the plane of the wires of the line, produces a radiation of

$$6\,d\,10^{-9}\,I^2 \text{ joules,} \tag{20}$$

or four times as much as given by formula (**15**), because a transposition amounts to a sudden stopping of the current plus a sudden starting of the same current with opposite sign, in the same direction of propagation or to a sudden starting of twice the same current in the same direction, with opposite sign.

Hitherto, we considered a current flowing suddenly in the line at maximum value or a "rectangular wave" as we called it. Any other law for the current as a function of the time at the origin, as for example a sinusoidal law, can be assumed, with only changes in the calculations, but yielding the same conclusions as to the magnitude of the radiation effect.

We also neglected the influence of the ground and supposed a transmission line isolated in space. A perfectly conducting ground can be accounted for by taking with the line its image with respect to the ground; the resulting radiation is obviously less.

If, instead of considering disturbances between the two wires of a line, we consider disturbances between line and ground, the distance d between the wires would have to be replaced by twice the height of the line above the ground, in the case of a good conducting ground. This can magnify the radiation effect about twenty times, but keeps it still negligible compared to the effect of heat dissipation in the conductors.

CONCLUSIONS

The present conclusions settle the question of the relative unimportance of radiation in the attenuation and distortion of traveling waves along lines. They, however, leave the way open for the investigation of other more important causes of attenuation and distortion, among which we suggest the transient penetration of the current into the wire at the front of the disturbance, as mentioned above; the dissipation of energy in the ground due to the fact that some lines of force of a two-wire transmission line reach the ground and cause an attenuation as in the case of wireless waves; the non-homogeneity of the transmission line for high-frequency waves, *i. e.*, the slight periodic variations that occur in the constants of the line due to presence of poles and curved spans. The influence of changes in direction in the line, as far as radiation is concerned, can be found by the method employed here and is negligible.

SUMMARIZING

1. A criticism of the classical theory of propagation of traveling waves along a two-wire transmission line has been given showing that, as far as the shape of the wave near the front is concerned, this theory is still unsatisfactory.

2. Dr. Steinmetz's theory of the effect of radiation on traveling waves has been examined.

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[9] See note 5.

3. The position of the problem of radiation of lines has been sketched from Maxwell's electromagnetic point of view, in connection with the accepted engineering theories of transients.

4. An engineering solution of the problem is offered, showing that under steady a-c. as well as under transient conditions, *the effect of radiation upon electromagnetic waves propagating along transmission lines is quite negligible as compared to the effect of heat dissipation in the wires.*

The procedure used here in the case of steady a-c. conditions is to be considered an extension of a method due to Abraham;[10] it is different in the case of transients, and thought to be new.

5. Other possible causes of attenuation and distortion of traveling waves are suggested.

NOTE 1

The integration to be performed is obtained by substituting the time derivative of $I_1 - I_2$, given by (7) into (5); so

$$E = H = - \imath \frac{\pi n}{l} \cdot \frac{\sin \theta_0}{r_0} \cdot \frac{\pi n d}{l} \cdot \sin \theta_0$$

$$\cdot \cos \phi_0 \int_{-l/2}^{+l/2} \cos \frac{\pi n z}{l} \cdot \sin \frac{\pi n}{l} (c t - r_0 + z \cos \theta_0) \cdot d z$$

But

$$\sin \frac{\pi n}{l} (c t - r_0 + z \cos \theta_0) = \sin \frac{\pi n}{l} (c t - r_0) \cdot \cos \frac{\pi n}{l} (z \cos \theta_0) + \cos \frac{\pi n}{l} (c t - r_0) \cdot \sin \frac{\pi n}{l} (z \cos \theta_0)$$

The second term contributes nothing to the integration, for $\sin \frac{\pi n}{l} (z \cos \theta_0)$ changes its sign with z. We thus have only to perform the integration

$$\frac{\pi n}{l} \int_{-l/2}^{+l/2} \cos \frac{\pi n z}{l} \cdot \cos \frac{\pi n}{l} (z \cos \theta_0)\, d z$$

$$= \int_{-\frac{n\pi}{2}}^{+\frac{n\pi}{2}} \cos x \cdot \cos a x \cdot d x$$

$$= 1/2 \int_{-\frac{n\pi}{2}}^{+\frac{n\pi}{2}} \cos (1 + a) x \cdot d x + 1/2 \int_{-\frac{n\pi}{2}}^{+\frac{n\pi}{2}} \cos (1 - a) x \cdot d x$$

$$= \frac{\sin (1 + a)\, n \pi/2}{1 + a} + \frac{\sin (1 - a)\, n \pi/2}{1 - a}$$

$$= \frac{2}{1 - a^2} \cdot \sin \frac{\pi n}{2} \cdot \cos a \frac{\pi n}{2} \ (n \text{ odd})$$

$$= \frac{2}{\sin^2 \theta_0} \cdot (-1)^{\frac{n-1}{2}} \cdot \cos \left(\frac{\pi n}{2} \cos \theta_0 \right)$$

which, multiplied by the constant factors that we did not write through, gives formula (8).

NOTE 2.

The flow of energy S, given by (9), is

$$S = \frac{c}{4 \pi} \int E H d \Sigma$$

integrated over the area of the sphere of radius r_0; the values of E and H are to be taken from formula (8), and $d \Sigma$ is the element of area in spherical polar coordinate, *i. e.*,

$$r_0^2 \sin \theta_0\, d \theta_0\, d \phi_0$$

We thus have to integrate

$$S = \frac{c}{4 \pi} \imath^2 \left(\frac{2 \pi n d}{l} \right)^2 \frac{\sin^2 \frac{\pi n}{l} (c t - r_0)}{r_0^2}$$

$$\int_{\theta_0 = 0}^{\pi} \int_{\phi_0 = 0}^{2\pi} \cos^2 \left(\frac{\pi n}{2} \cos \theta \right) \cos^2 \theta_o \sin \theta_o\, r^2_o\, d\theta_o\, d\phi_o$$

But we have, dropping unnecessary indices,

$$\int_0^{2\pi} \cos^2 \phi \cdot d \phi = \pi$$

and

$$\int_0^{\pi} \cos^2 \left(\frac{n \pi}{2} \cos \theta \right) \sin \theta\, d \theta = 1$$

The double integral thus reduces to πr_0^2 and the value of S becomes

$$S = c\, \imath^2 \left(\frac{\pi n d}{l} \right)^2 \sin^2 \frac{\pi n}{l} (c t - r_0)$$

of which the time average is given by replacing $\sin^2$ by 1/2. Hence formula (10)

NOTE 3.

The equations (12) for E and H can be substantiated by proving that they satisfy both Maxwell's equations for r large, *i. e.* when $1/r^2$ is negligible compared to $1/r$. Maxwell's equations are, in air

$$1/c \frac{\partial E}{\partial t} = \operatorname{curl} H$$

$$- 1/c \frac{\partial H}{\partial t} = \operatorname{curl} E$$

where E and H are space-vectors, E being expressed in electrostatic units and H in electromagnetic units.

The component of the curl of a vector, at a given point, in a given direction is defined as the line integral of that vector taken around a unit area located at the given point, of which the positive normal coincides with the given direction.

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[10] *Phys. Zeitschrift*, 2, p. 239, 1901.

The curl components of E and H in the direction of the radius (see Fig. 3) are seen to be zero or of the second order, (*i. e.*, of the order of $1/r^2$), thus negligible. The components of curl E along a meridian line, and of curl H along a parallel line are obviously zero, because the vector is then perpendicular to the line of integration.

The component of curl H in the positive direction of a meridian line, (*i. e.*, towards the increasing θ), is given by

$$\operatorname{curl}_\theta H =$$

$$\frac{-H(r+\Delta r)\,.\,(r+\Delta r)\sin\theta\,.\,\Delta\theta + H(r)\,.\,r\,.\,\sin\theta\,.\,\Delta\theta}{\Delta r\,.\,r\sin\theta\,\Delta\theta}$$

where $H(r)$ means the scalar value of H at the distance r. But we have, neglecting quantities of the second order, and using the value of H given by (1):

$$H(r+\Delta r) = H(r) + \frac{\frac{\partial I}{\partial r}}{r}\,.\,\frac{1+\cos\theta}{\sin\theta}\,.\,\Delta r$$

$$= H(r) - 1/c\,\frac{\partial E}{\partial t}\,.\,\Delta r$$

Substituting in $\operatorname{curl}_\theta H$, at the same approximation, we find the first equation of Maxwell.

The same procedure with regard to the signs leads to the second equation of Maxwell.

NOTE 4.

Formula (**15**) represents the energy carried to infinity by the spherical shell of variable depth

$$d\sin\theta\cos\phi$$

in which the value of the electromagnetic field is given, for $-\pi/2 \leqq \phi \leqq +\pi/2$, by formula (**12**)

$$E = H = I/2\,.\,\frac{1+\cos\theta}{\sin\theta}$$

where I is the constant value of the suddenly impressed current. For $\pi \geqq 2/\phi \leqq \frac{3\pi}{2}$, the sign of both E and H is to be changed.

At a given point on the sphere of (large) radius r, this electromagnetic field lasts during the time

$$|\,d/c\sin\theta\cos\phi\,| \qquad (c \text{ speed of light})$$

the bars meaning a quantity that is positive.

The total energy radiated W will be given by the time integral extended over the sphere of radius r of Poyting's vector, which is directed everywhere along the radius outwards the sphere; *i. e.*

$$\frac{c}{4\pi}E \times H\,|\,d/c\sin\theta\cos\phi$$

integrated over the sphere of which the element of area is

$$r^2\sin\theta\,d\theta\,d\phi$$

We thus have to perform

$$W = \frac{c}{4\pi}I^2/r^2\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\left[\frac{1+\cos\theta}{\sin\theta}\right]^2.$$

$$|\,d/c\sin\theta\cos\phi\,|\,r^2\sin\theta\,d\theta\,d\phi$$

But we have

$$\int_0^{2\pi}|\cos\phi|\,d\phi = 4$$

and

$$\int_0^{\pi}[1+\cos\theta]^2\,d\theta = \int_0^{\pi}(1+2\cos\theta+\cos^2\theta)\,d\theta$$

$$= \pi + 0 + \pi/2 = \frac{3\pi}{2}$$

The value of W is thus

$$W = 3/2\,d\,I^2 \qquad \text{elmag C. G. S. units}$$

which is equivalent to (**15**).

NOTE 5.

The total reflection "with change of sign" of a rectangular wave of current at the free end of a line can be considered as the result of the stopping of the incoming wave $+I$ at the end, by starting at the end a new wave $-I$ along the same direction of propagation as that of the incoming wave, plus starting a new wave $-I$ in the opposite direction, *i. e.*, back into the line. This last wave is the wave "reflected with change of sign." The sign of a wave bears no relation at all to the direction of propagation.

The reflection at a free end of a line is thus equivalent to starting simultaneously two waves $-I$ from the end in two opposite directions. The electromagnetic field of such a double disturbance along a single wire is easily deduced from that given in formula (**12**), for a single disturbance. It is, with regard to the signs

$$E = H = \frac{(-I)}{r}\,\frac{1+\cos\theta}{\sin\theta} + \frac{(-I)}{r}\,\frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{(-2I)}{r}\,\frac{1}{\sin\theta}$$

$(-I)$ is the current started in both directions at the free end, equal in absolute value to the incoming current, and where the time t, of which I is a function in the general case, has been replaced by $t - r/c$, r being taken from the free end.

The case of a two-wire (transmission) line reduces to the preceding one as was seen. Thus, in the case of a rectangular wave of current along a line, the electromagnetic field due to the sudden starting at the end of the line of a current $(-I)$ in both directions is:

$$E = H = -\frac{2\hat{I}}{r}\frac{1}{\sin\theta}$$

This field exists in a spherical shell of variable depth, $d\sin\theta\cos\phi$, expanding at the speed of light, as it has been shown. It lasts at a fixed point in space during the time

$$|\, d/c \sin\theta\cos\phi \,|$$

the bars meaning a positive quantity. The amount of radiated energy W can be found exactly as in the preceding note. We have:

$$W = \frac{c}{4\pi}\frac{4I^2}{r^2}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{1}{\sin^2\theta}\,|\, d/c\sin\theta\cos\phi \,|\; r^2\sin\theta\, d\theta\, d\phi$$

what reduces immediately to:

$$W = 4\,d\,I^2 \qquad \text{elmag c. g. s. units}$$

which is equivalent to formula (**17**).

In general, when the amount of current $K\,I$ (in sign) is reflected into the line, the rest being absorbed in the terminal apparatus, we have for the electromagnetic field, with regard to the sign,

$$E = H = \frac{-\hat{I}}{r}\cdot\frac{1+\cos\theta}{\sin\theta} + \frac{(K\hat{I})}{2}\cdot\frac{1-\cos\theta}{\sin\theta},$$

which exists in a spherical shell of variable depth.

$$d\sin\theta\cos\phi$$

expanding at the speed of light.
The radiated energy is, as above,

$$W = \frac{c}{4\pi}I^2/r^2 \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\left[\frac{(1-K)+(1+K)\cos\theta}{\sin\theta}\right]^2\cdot\,|\, d/c\sin\theta\cos\phi \,|\; r^2\sin\theta\, d\theta\, d\phi$$

The value for W follows

$$W = 3/2\,(1 - 2/3\,K + K^2)\,d\,I^2 \text{ elmag. c. g. s. units}$$

which is equivalent to (**19**).

NOTE 6.

The position of the point X_1, limiting the length $X_1 X_2$ (Fig. 5) along which the two spherical shells can have parts in common is determined by the intersection of a circle of radius r centered upon O_1, and of a circle of radius $r - l$ centered upon R_2, r being very large compared to l. This allows to approximate the circles by parabolas of the same curvature, *i. e.*, of which the parameter p is equal to the radius of the circles; the axis of the first parabola will coincide with the wire $O_1 R_1$, the axis of the second with the wire $O_2 R_2$ We so have

$$y - d = \sqrt{2\,(r-l)\,x} \qquad \text{Par. 2}$$
$$y = \sqrt{2\,r\,x} \qquad \text{Par. 1}$$

with obvious axis of coordinates. Calling $(\xi_1\,\eta)$ the coordinates of the point X_1, we deduce

$$\eta = \sqrt{\frac{r}{r-l}}\cdot\sqrt{2\,(r-l)}\,\xi = \sqrt{\frac{r}{r-l}}\,(\eta - d)$$

As ξ is very small, η is the distance of X_1 to wire 1—We have thus, from the preceding equation, neglecting the square and higher powers of r/l:

$$\eta/r = 2\,d/l\,.$$

which proves that, viewed from a point of the line, the length $X_1 X_2$ does not cover an angle larger than $\frac{4\,d}{l}$, provided l is small compared to r.

Hence, calculating the energy radiated by the two spherical shells in contact as if they were not in contact, *i. e.*, integrating over π instead of π minus the angle η/r, will introduce only an error of the order of d/l, thus quite negligible.

Discussion

V. Karapetoff: On the second page of Dr. Manneback's paper, on the top of the second column, three conclusions are drawn. The first conclusion is that: "An electric disturbance, *i.e.*, a discontinuity of voltage or current, is always propagated along any line at the constant speed $V = 1:\sqrt{L.C.}$" And then he adds: "whether there is resistance and leakance or not". I am not sure that this is correct; it seems to me that the velocity of propagation depends on the presence of resistance and leakance and is thereby reduced.

Then, coming to the page on which is Fig. 2, in the second column there is an integral which represents the energy radiated, and this energy is expressed by a finite formula. In Peirce's book on "Electric Waves and Oscillations," in a simpler case of an antenna, a similar result is expressed in the form of an infinite series; and I should like to know just where the difference lies and why in this case a finite expression is possible.

In the treatment of transient radiation on the page on which Fig. 3 is placed, the principle of super-position is used, and I wonder if much of the mathematics in the preceding case of static conditions could not be eliminated by also using the principle of super-position in this form. We already have a formula for energy radiation from a single antenna. Since in this article the effect of the ground is neglected, it does not make any difference whether the transmission line is horizontal or vertical, so suppose we place it vertically. Then the transmission line is analagous to two antennae placed side by side and electrically subjected to disturbances at 180 degrees in time lag. Therefore, if one should write equations for radiation from one antenna and then take a finite difference for the other antennae, one should get the correct result. Since Dr. Manneback does this for the transient condition, I should like to know why it could not be for the static condition as well.

Joseph Slepian: I was very much interested in the paper by Mr. Manneback because there has been in the last year something of a dispute in the pages of our JOURNAL between such eminent authorities as Dr. Steinmetz and Dr. Carson as to whether there is really any radiation from a traveling wave on a transmission line. This paper I believe supports Dr. Carson's contention that there is no such radiation. A casual reader might believe that the conclusion of this paper is that there is a radiation but only a small one. However, Dr. Manneback shows that this small radiation is due to ohmic loss, reflections, or anything which prevents the wave from freely traveling and that a freely traveling wave does not radiate energy.

I would like to give an argument for this last conclusion based on a principle of relativity. The principle I shall use is this: that an observer moving with an object both at uniform and the same velocities sees nothing different in that object over what he would see if both he and the object were at rest. This is true whether the body is electrically charged or not. Now a freely traveling wave on a transmission line consists of a charge moving with uniform velocity. So far as electrical effects go it is the same as if a charged material object were moving with uniform velocity. An observer moving with this charged object would see nothing different from what he would see if both he and the object were at rest. If both he and the object were at rest we know that there would be no change and no loss of energy from the object by radiation. We must therefore conclude that in the case of the uniformly moving charged object there is also no radiation of energy.

V. Bush: Recent progress in radio communication has focused the attention of engineers upon the radiation characteristics of electrical circuits. All a-c. circuits radiate to a certain extent; that is there is a loss in any such circuit, which is caused by the production of waves carrying energy off into space. The filment of an incandescent lamp has a certain small amount of input, over and above that consumed by ohmic losses, which is necessary to provide the energy radiated as heat and light. The radiation from a lamp is caused by the temperature of the filment; that is by the rapid oscillatory heat motion of its electrons. When an alternating current flows in a circuit, the electrons move comparatively slowly back and forth along the conductor. They radiate in each case in exactly the same manner. The waves from the lamp are very short, while those from an alternating or oscillating circuit are very long; but otherwise they are the same sort of waves. When a circuit is designed to radiate a large fraction of its input, it is called an antenna. The length of the waves produced from a circuit depends upon the frequency existing in the circuit. From a 60-cycle circuit, the waves will be 5,000,000 meters or 3100 miles long. The radiation per volt or per ampere from a given circuit varies as the fourth power of the frequency. Hence, at commercial frequencies radiation is ordinarily entirely negligible. An antenna which radiates 100 kw. at a radio communication frequency of 30,000 cycles per second would radiate only 1.6 microwatts if excited to the same potential at 60 cycles. Thus we can ordinarily disregard the effect in our power networks as far as loss of power is concerned.

At high enough frequency, a transmission line may radiate appreciable energy. It becomes a sort of loop antenna. This has been made use of in radio telephone systems for transmission lines, where the line is caused to act as a carrier of the very high-frequency waves which serve the telephone. These waves may be placed on and taken from the transmission line by isolated terminal radio apparatus making use of the radiation characteristics of the line.

It is natural, therefore, to inquire what part radiation plays in the progress of transients of various sorts over transmission lines. Is the abruptness of a switching transient greatly modified by radiation of part of its energy from the line? We know that the steep wave front of such a transient involves rates of current and voltage change corresponding to the rates found at very high frequencies. Is the radiation large as a consequence?

The attack on this problem is attended by one prominent difficulty. Much has been written, particularly of late and by radio engineers and physicists, concerning the radiation from circuits of various sorts, and some of this analysis has been experimentally checked. Practically all of it, however, refers only to the steady state, after transient conditions in the system have entirely died away. It is the effect during the transient period that we wish to examine. Radiation during transients has been little treated except in the classics, and there not for the cases which most interest us from the practical standpoint.

In fact, the very definition of the radiation during a transient is a matter of some concern. Radiation in the steady state is easily defined. When steady alternating potential and currents exist in a circuit, they are accompanied by steady electrostatic and electromagnetic fields surrounding the circuit. These fields constitute a storage of energy. When a field collapses, the stored energy largely returns to the circuit, but not all of it returns. Some of the energy stored in one half-cycle does not return during the next half-cycle. This residual of energy is the energy of the radiated wave, which proceeds out from the circuit at the speed of light. The power radiated from a circuit in the steady state is the energy lost per second in this manner.

In the transient state, we suddenly establish, or remove, potentials and currents. This means the establishment, or removal, also of electrostatic and electromagnetic fields. What portion of the energy represented by the fields thus set up or destroyed shall be considered as radiated energy? When a current is suddenly caused to flow in a circuit, the accompanying magnetic field appears first in the immediate vicinity of the wires. This field then rapidly spreads outward at the speed of light, and finally a field will be produced to indefinite distances. If, while this field is spreading, we examine it mathematically, we find that there is a flow of energy outward. The front of the disturbance carries outward the energy, and spreads it over space as it goes. But if we examine the expression for the total energy in this front of disturbance, we find that it does not tend to zero as the disturbance proceeds to infinity, but approaches a definite finite value. Over and beyond the energy stored in the field, there is thus carried outward an additional amount which moves off indefinitely and is lost forever from the circuit. It is this energy which is the energy radiated during the transient.

In this paper, Dr. Manneback examines the amount of energy thus radiated from a transmission line of usual size and form, where steep transients are involved. He treats the progress of rectangular waves during reflections and at transpositions of the line. The results may be very simply summed up. To engineering accuracy the effect of this radiation may be, without error, ignored in analysis, where the progress of a single steep wave is concerned.

This does not mean, of course, that radiation is always negligible in connection with a transmission line. It indicates only that we cannot depend upon this factor for assistance in reducing its effect of single steep transients. When, for instance, we close a switch at one end of a long line producing a wave which travels down the line and impinges upon the transformer at the distant end, radiation will very little diminish the severity of the impact. If, on the other hand, an arc to ground under certain conditions sets up a steady state of very high-frequency waves, then the effect of radiation may be considerable and very welcome.

The transmission engineer does not deal, after all, simply with a few long wires in which current flows. The effects which he produces and utilizes are spread all over surrounding space. A comprehensive analysis must then deal not only with effects in and between the wires, but also with those fields which are propagated outward from the system. Circuits are, strictly speaking, always three dimensional.

CORRESPONDENCE

PROPAGATION OF DISCONTINUITIES OF THE FIRST DERIVATIVE OF VOLTAGE AND CURRENT ALONG AN ELECTRIC LINE

To the Editor:

IT has been proved by the writer[1] that a discontinuity of the magnitude of voltage or current is always propagated along any electric line at the velocity of light in the surrounding medium ($v = 1 : \sqrt{LC}$), whether or not there is resistance and leakance present. It has been asked by Dr. Bush if it is not true that this discontinuity of voltage or current may be either a discontinuity of magnitude or simply of derivative. Thus, if a sinusoidal voltage is applied to a long transmission line at the zero of voltage, the discontinuity which travels has no steep wave front; what is propagated in this case is a discontinuity of derivative only of voltage and current. There is no doubt about this discontinuity being also propagated along any line at the velocity of light, whether or not there is resistance and leakance. The following may be a proof for it.

The voltage V and the current I along a line are connected together by the well-known equations, where R, L, G, C represent the four constants of the line as usual:

$$L \frac{\partial I}{\partial t} + R I = - \frac{\partial V}{\partial x} \qquad \textbf{(a)}$$

$$C \frac{\partial V}{\partial t} + G V = - \frac{\partial I}{\partial x} \qquad \textbf{(b)}$$

By eliminating V or I between these equations, the same equation of propagation is obtained for V and I. (equation **1** in paper.)

Let us consider a point $M(x, t)$ along the line, at which V and I are continuous functions of x and t, but where $\partial V / \partial x$ is a discontinuous function of x. Let this discontinuity be propagated at an unknown velocity v. We will show that v must necessarily be equal to $1 : \sqrt{LC}$. After the time Δt has elapsed, the discontinuity will have traveled from $M(x, t)$ to $M'(x + v \Delta t,\ t + \Delta t)$. Let us express the fact that the voltage V, which is continuous at M, is still continuous at M'. Calling V_1 and V_2 the values of the voltage left and right of M and infinitely near this point, we have

at M $\qquad V_1 = V_2$

at M' $\qquad V_1 + \dfrac{\partial V_1}{\partial x} . v \Delta t + \dfrac{\partial V_1}{\partial t} \Delta t$

$$= V_2 + \frac{\partial V_2}{\partial x} . v \Delta t + \frac{\partial V_2}{\partial t} \Delta t$$

hence, $\qquad v \bar{V}_x + \bar{V}_t = 0 \qquad$ **(c)**

denoting by $\bar{V}_x$ the discontinuity of the space-derivative of the voltage at $M(x, t)$, *i. e.*, $\partial V_2 / \partial x - \partial V_1 / \partial x$, and by $\bar{V}_t$ the discontinuity of the time-derivative of the voltage at the same point $M(x, t)$, *i. e.*, $\partial V_2 / \partial t - \partial V_1 / \partial t$. It is seen from ($c$) that when a discontinuity $\bar{V}_x$ exists at a point $M(x, t)$, it is necessarily accompanied by another discontinuity, $\bar{V}_t$, at the same point $M(x, t)$.

Let us now assume that V is continuous at M. We have as above $V_1 = V_2$. By means of (b), this becomes

$$C \frac{\partial V_1}{\partial t} + \frac{\partial I_1}{\partial x} = C \frac{\partial V_2}{\partial t} + \frac{\partial I_2}{\partial x}$$

where I_1 and I_2 denote the values of I left and right of M and infinitely near this point. Using for the derivatives of I notations similar to those used for V, we get from the preceding equation

$$C \bar{V}_t + \bar{I}_x = 0 \qquad \textbf{(d)}$$

which shows that a discontinuity of V/t at the point $M(x, t)$ entails a discontinuity of I/x at the same point $M(x, t)$.

Similarly, assuming that I is continuous at M and using equation (a), we obtain

$$L \bar{I}_t + V_x = 0 \qquad \textbf{(e)}$$

which shows that a discontinuity of $\partial V / \partial x$ at the point $M(x, t)$ entails a discontinuity of $\partial I / \partial t$ at the same point $M(x, t)$.

Expressing finally, as we did for the voltage, that the current I, which is continuous at $M(x, t)$, is still continuous at $M'(x + v \Delta t,\ t + \Delta t)$, we get similarly

$$v \bar{I}_x + \bar{I}_t = 0 \qquad \textbf{(f)}$$

which connects the two discontinuities of I at the same point.

There are thus four homogeneous linear equations (c, d, e, f) connecting the four discontinuities $\bar{V}_x$, $\bar{V}_t$, $\bar{I}_x$, $\bar{I}_t$. In order that the system of equations may be compatible, the following relation must be verified

$$\begin{vmatrix} v & 1 & o & o \\ o & C & 1 & o \\ 1 & o & o & L \\ o & o & v & 1 \end{vmatrix} = 0$$

or, developing the determinant,

$$v^2 L C - 1 = 0.$$

This shows that the velocity of propagation v of a discontinuity of the derivative $\partial V / \partial x$ is necessarily equal to $1 : \sqrt{LC}$, which is the velocity of light in the surrounding medium. Moreover the four above equations prove that, V and I being continuous functions of x and t, the mere existence of a discontinuity of any one of the derivatives of V and I at a point $M(x, t)$ entails the existence of discontinuities of the three other derivatives at the same point $M(x, t)$. They are all propagated together at the same velocity v.

The discontinuity of voltage which is produced when a sinusoidal voltage is impressed on a long transmission line at the zero of voltage is a particular case of the preceding.

CHARLES MANNEBACK

Brussels, Belgium, November 18, 1923.

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[1] Manneback, Charles; *Radiation from Transmission Lines*, JOURNAL of the A. I. E. E., February, 1923. p. 95.

A New Equation for the Static Characteristic of the Normal Electric Arc

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COMMERCIALLY the electric arc is now extensively used, and yet after more than a hundred years of study, there has been little crystallization of opinion as to certain phases of its fundamental theory. Absolute contradictions are frequently found in the literature, and premature generalizations have given rise to more disagreement among the investigators of the equation of the static characteristic of the normal[1] arc than the experimental difficulties warrant.

The static characteristic is the relation between the current (i) flowing in the arc, the length (L) of the arc, and the difference in potential (E) across the arc. This relation can be represented approximately by simple expressions of the form of the Ayrton[2] equation

$$E = a + b \cdot L + \frac{c + d \cdot L}{i} \qquad (1)$$

or the Steinmetz[3] equation

$$E = a + \frac{c\,(L + d)}{i^{0.5}} \qquad (2)$$

(a, b, c, and d are constants)

if the range in variation of the current and arc length, for which the constants are calculated, is the closely defined.

In particular, these equations are recognized as approximations, because *the difference in potential* (E) *is not a linear function of the arc length* (L), and although the difference in potential does decrease when the current increases, *the current* (i) *enters neither as the first power* (as in the Ayrton equation) *nor as the square root* (as in the Steinmetz equation). Duddell[4] was among the first to demonstrate by the use of the curve shown in Fig. 1, that, for arcs less than 15 mm. in length, the difference in potential cannot possibly be truly represented by a linear function of the arc length. As far as I know, Steinmetz[5] was the first to advance the thesis that, "while the stream voltage varies with the current, and in the opposite direction to it, its variation is much less than the inverse proportion to the current."

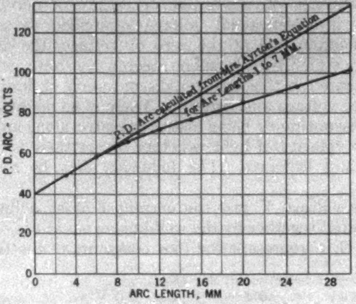

FIG. 1

This paper attempts to make these two criticisms constructive or positive instead of negative.

THE EXPONENT OF THE CURRENT IS DIRECTLY PROPORTIONAL TO THE TEMPERATURE OF THE BOILING POINT OF THE ANODE

If the arc length is assumed to be constant, the Ayrton and the Steinmetz equations can be written in the form of equation (3)

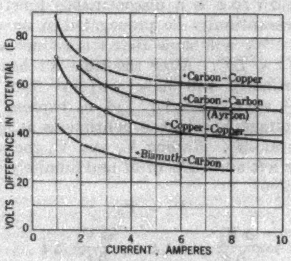

FIG. 2—VOLT-AMPERE CHARACTERISTICS WITH THE ARC LENGTH CONSTANT $L = 3.0$ MM.

$$E = A + B / i^n \qquad (3)$$

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[1] Each stage of the electric arc from the glow arc to the hissing arc has its own physical, electrical and chemical characteristics. Therefore any equation of the electric arc must necessarily be limited to the characteristics of one stage. The normal arc has two general requirements: (1) the current intensity must be greater than the maximum for the glow arc, and less than the minimum for the hissing arc; and (2) the arc must be free from external electrical, magnetic, atmospheric, and physical disturbances.

[2] Hertha Ayrton, The Electric Arc, "The *Electrician*," Printing and Publishing Company, London.

[3] C. P. Steinmetz, TRANS. A. I. E. E., 1906, p. 802.
C. P. Steinmetz, "Radiation, Light, and Illumination," McGraw Hill, New York, 1909, p. 139.
C. P. Steinmetz, "Theory and Calculation of Electric Circuits," McGraw Hill, New York, 1917, p. 35.
C. P. Steinmetz, "Chem. and Met. Eng.," 22, p. 455, (1920).

[4] W. Duddell, *Phil. Trans.*, 203 (A), p. 338, (1904).

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y. February 14-17, 1923.

[5] See foot-note 3.

in which A and B are constants dependent on the arc length and the electrode material and in which n is a constant dependent only on the electrode material. The value of n for the characteristic curves of an arc between any electrode materials can be calculated directly from curves such as those in Fig. 2. For example, the calculation of the constants of equation (**3**) for an arc 3.0 mm. long between copper electrodes, indicates that n is equal to 0.665. (See Fig. 2, Table I, and Equation (**3a**)). This study with copper electrodes was extended to include many characteristic curves for arcs ranging in length between 1.0 and 10.0 mm. The average value of n was found to be 0.670.

TABLE I

THE DETERMINATION OF n FOR AN ARC BETWEEN COPPER ELECTRODES

Anode—Copper Cathode—Copper

Arc Length 3.0 mm.

$A = 27.5 \quad B = 44.0$

i Amperes	E Volts	$n = \dfrac{\log \dfrac{44.0}{E - 27.5}}{\log i}$
1.0	71.5	
1.5	61.0	0.677
2.0	55.5	0.652
2.5	51.5	0.668
3.0	49.0	0.653
4.0	45.0	0.660
5.0	42.5	0.670
7.0	39.5	0.669
10.0	37.0	0.667

Average $n_3 = 0.665$

Average for curves of 1.0 to 10.0 mm.
arc length.......................... $n_{ave.} = 0.670$

$$E = 27.5 + \frac{44.0}{i^{0.665}} \tag{3a}$$

It is generally agreed that n is equal to 1.0 for the characteristic curves representing the arc between carbon electrodes[6], while it has just been demonstrated that n is 0.670 in the case of the arc between electrodes of copper. The natural question arising from these observations is: what will be the value of n characteristic of an arc between *unlike* electrodes? The outcome of the first investigation was striking; it showed n to be 0.985 when the *anode* was *carbon* and the cathode was copper. This result indicates that the anode material predominates in the determination of n.

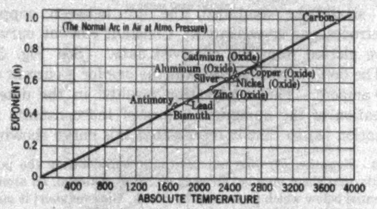

FIG. 3—THE EXPONENT (N) AND THE ABSOLUTE TEMPERATURE OF THE BOILING POINT OF THE ANODE MATERIAL

TABLE II

THE DETERMINATION OF n FOR AN ARC BETWEEN UNLIKE ELECTRODES

Anode—Carbon Cathode—Copper

Arc Length 3.0 mm.

$A = 56.0 \quad B = 31.0$

Current Amperes	E Volts	$n = \dfrac{\log \dfrac{31.0}{E - 56.0}}{\log i}$
1.0	87.0	
1.5	77.0	0.960
2.0	72.0	0.955
2.5	68.5	0.992
3.0	66.5	0.985
4.0	64.0	0.997
5.0	62.5	0.970
7.0	60.5	0.993
10.0	59.5	0.950

Average $n_3 = 0.975$

Average for curves of 1.0 to 10.0 mm.
arc length................ $n_{ave.} = 0.985$

A continuation of this investigation of arcs between unlike electrodes not only supplied convincing evidence to substantiate the above thesis, but it opened the way to the discovery of the exact property of the anode material upon which n is dependent. It is apparent from Table III that there is a definite relationship between the value of n for each element represented and its boiling point. Furthermore, if the absolute temperature of the boiling point for each element is plotted as the abscissa and the corresponding value of n as the ordinate in a system of coordinates, the exact nature of the relation is at once evident (See Fig. 3.):

TABLE III

THE RELATION BETWEEN n AND THE BOILING POINT OF THE ANODE

Anode	Cathode	Oxide[7] or Non Oxide on Anode	n	Abs. Temp. of Boiling pt. or Sublimation pt.	Authority
Carbon	Copper	..	0.985	3770	Van der Waals
Cadmium	Carbon	Oxide	0.720	2770	Approximation
Copper	Copper	Oxide	0.670	2580	Greenwood
Aluminum	Carbon	Al_2O_3	0.650	2480	Ruff—Schmidt
Nickel	Carbon	Oxide	0.640	2450	Hagenbach—Langbein
Silver	Carbon	Non oxide	0.624	2370	v. Wartenberg
Zinc	Carbon	Oxide	0.570	2170	Approximation
Lead	Carbon	Non oxide	0.480	1850	v. Wartenberg
Antimony	Carbon	Non oxide	0.460	1710	Greenwood
Bismuth	Carbon	Non oxide	0.445	1690	Greenwood

The exponent (n) *of the current* (i) *in the equation*

$$E = A + B/i^n \tag{3}$$

is directly proportional to the absolute temperature of the

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[6] Calculations from Mrs. Ayrton's observed data indicate that n is slightly less than 1.0 for the arc between carbon electrodes.

[7] G. Shulze, *Ann. d. Phys.* (4), 12, p. 828, (1903).

A. Hagenbach and K. Langbein, "*Archives des Sciences Physiques et Naturelles*," Vol. 5, Part 1, p. 48, (1919).

***boiling point or the sublimation point, as the case may be, of the anode*[8].**

E is not a Linear Function of L when the Current is Constant

Both Mrs. Ayrton and Steinmetz have assumed the difference in potential (E) to be a linear function of the arc length (L) when the current is constant, but this is not even approximately true if the arc is less than 15.0 mm. in length. In spite of the fact that Duddell[9] made this criticism in 1904, as far as I know, no substitute has been offered for this much discredited assumption. A cursory examination of a typical constant current curve (which shows the relation between the difference in potential (E) across the arc and the length of the arc (L)) and its component parts will reveal the steps to be taken in the formulating of a more accurate equation to supplant the old. (See Fig. 4.)

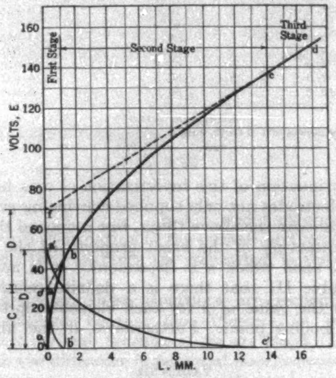

FIG. 4—A TYPICAL CONSTANT CURRENT CURVE AND ITS COMPONENT PARTS

The arc goes through three distinct stages as it increases from an infinitesimal length to 15.0 mm. or more. These changes are in turn reflected in the constant current curves, and although the dividing line between the stages is not sharp, the first modification takes place when the arc length reaches 0.7 mm. to 1.0 mm. and the second when it is 13.0 mm. to 15.0 mm.

Arcs of the third stage can be represented easily, since the constant current curve is straight, by

$$E = D + C + \gamma \cdot L \qquad (4)$$

Furthermore, curve $a' c'$ (the difference between the straight line $f\,d$ and the constant current curve extrapolated) is so nearly a pure exponential of the form of the equation

$$e_2 = D \cdot \epsilon^{-\delta \cdot L} \qquad (5)$$

that the second and the third stages may be approximated very closely by the equation

$$E_{23} = C + D \cdot (1 - \epsilon^{-\delta \cdot L}) + \gamma \cdot L \qquad (6)$$

Curve $o' b'$, which is the deviation of $a\,b\,c\,d$ from the constant current curve, can be represented by the equation

$$e_1 = C \cdot \epsilon^{-d \cdot L} \qquad (7)$$

Therefore the equation of the constant current curve taken in its entirety[10] is

$$E = C \cdot (1 - \epsilon^{-d \cdot L}) + D \cdot (1 - \epsilon^{-\delta \cdot L}) + \gamma \cdot L \qquad (8)$$

In the above equations

$C + D$ = the intercept of line $f\,d$ on the E axis,
C = the intercept of curve $o' b'$ on the E axis,
D = the intercept of curve $a' c'$ on the E axis,
γ = the slope of line $f\,d$,
$-\delta$ = the slope of the straight line represented by the equation $\log e_2 = \log D - \delta \cdot L$,
$-d$ = the slope of the straight line represented by the equation $\log e_1 = \log C - d \cdot L$.

Since equation (8) represents the curves in which the current is constant and equation (3) represents the curves in which the arc length is constant, a combination of these two equations will show the relation between the current (i) flowing in the arc, the length (L) of the arc, and the difference in potential (E) across the arc, without limitation. In other words, a new equation will have been derived to represent the static characteristic of the normal arc.

$$E = A + B/i^n \qquad (3)$$

THE NEW EQUATION

In order to discover just how equations (3) and (8) should be combined, the arc between copper electrodes was investigated. Constant current curves were plotted from the observed data. (See Fig. 5.)

The first curve analyzed was the one for a current of 1.0 ampere. This analyzation resulted in the determination of the numerical values of the constants in equation (8) so that it represented this curve showing

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[8] Although this law is being published at the present time, its confirmation is pending the investigation of a greater number of electrodes.

[9] See footnote 4.

[10] Equation (8) does not take into consideration the possibility that there may be, for each electrode material, an arcing potential below which no arc can be struck, however short it may be. This potential seems to be about 5.0 volts in the case of the arc between copper electrodes. If this observation is correct, equation (8) can be modified by the introduction of the constant ρ in the first term, so that the minimum potential for arcing is taken into consideration.

$$E = C \cdot (1 - \rho \,.\, \epsilon^{-d \cdot L}) + D \cdot (1 - \epsilon^{-\delta \cdot L}) + \gamma \cdot L$$

Where:

$$E_{min} = C \,.\, (1 - \rho) \,.$$

Column Δ_a in Table IV shows the error between the observed and the calculated values of E when the modified form of equation (8) is used. The examination of the minimum potential for arcing would offer an interesting field for additional investigation.

the relation between the arc length and the difference in potential The substitution of the values:

$$C = 28.0, \quad D = 42.5, \quad \gamma = 5.0, \quad d = 6.0, \quad \delta = 0.373,$$

in equation (8) gave (9).

$$E = 28.0\,(1 - \epsilon^{-6 \cdot L}) + 42.5\,(1 - \epsilon^{-0.373 \cdot L}) + 5.0 \cdot L \tag{9}$$

TABLE IV
THE COMPARISON BETWEEN THE OBSERVED AND THE CALCULATED VALUES OF E

Electrodes—Copper; Current—1.0 Ampere
E_c Calculated with equation (9)

Arc Length L mm.	Observed E_o Volts	Calculated E_c Volts	Error Δ Volts	Error[11] Δ_a Volts
0.02	6.0	3.20	+2.40	−1.50
0.04	10.0	6.20	+3.80	−0.20
0.10	17.0	14.64	+3.36	−0.44
0.20	26.0	23.60	+2.40	+0.60
0.30	31.0	31.50	−0.50	−1.30
0.50	37.0	36.32	+0.68	+0.58
0.70	42.0	40.90	+1.10	+1.10
1.0	46.0	45.97	+0.03	+0.03
2.0	60.0	60.30	+0.30	+0.30
3.0	71.5	71.60	−0.10	−0.10
4.0	81.0	81.00	0.00	0.00
5.0	89.0	88.90	+0.10	+0.10
6.0	96.0	95.90	+0.10	+0.10
7.0	102.0	102.20	−0.20	−0.20
8.0	108.0	108.20	−0.20	−0.20
10.0	120.0	119.50	+0.50	+0.50

[11] See footnote 10.

The difference in potential as observed and that as calculated by equation (9), have been placed in parallel columns in Table IV. The error (Δ) between the observed and the calculated values, is well within that attributable to experimental variations.

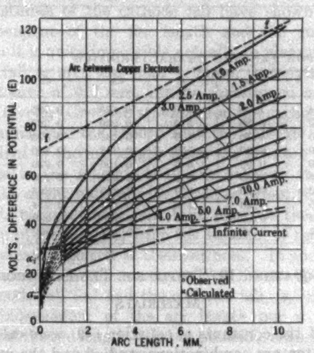

Fig. 5—Volt-Arc Length Characteristic for Different Current Values

If the current is infinite equation (3) becomes

$$E = A = f\,(L) \tag{10}$$

Therefore, the curve showing the relation between the arc length and the constant A can be called the "infinite current" curve. (See Fig. 5.) Equation (8) represents this curve if the constants are assigned the following values:

$$C = 14.5, \quad D = 15.5, \quad \gamma = 1.7, \quad d = 6.0, \quad \delta = 0.235.$$

Hence, the second curve studied can be approximated very closely by equation (11). (See Table V.)

$$E = 14.5\,(1 - \epsilon^{-6 \cdot L}) + 15.5\,(1 - \epsilon^{-0.235 \cdot L}) + 1.7\,L \tag{11}$$

TABLE V
THE COMPARISON BETWEEN THE VALUES OF A AS DETERMINED FROM EQUATION (3) AND THAT CALCULATED BY EQUATION (11)

Electrodes—Copper Current—Infinite
A Determined by Equation (3)
A_c Calculated by Equation (11)

Arc Length L mm.	Determined A Volts	Calculated A_c Volts	Error Volts
1.0	19.6	19.43	+0.17
2.0	23.7	23.71	−0.01
3.0	27.5	27.48	+0.02
4.0	30.8	30.74	+0.06
5.0	33.8	33.74	+0.06
6.0	36.5	36.40	+0.10
7.0	39.0	38.90	+0.10
8.0	41.3	41.23	+0.07
10.0	45.5	45.52	−0.02

The numerical values of C, D, γ, δ, and d were also determined for each of the remaining curves in Fig. 5, and these values were plotted as functions of the current. In this way, it was discovered that d is independent of the current and that C, D, γ, and δ are functions of the current of the form of equation (12)

$$f\,(i) = a + \frac{b - a}{i^n} \tag{12}$$

In equation (12), a is the value of the constant when the current is infinite and b, the value when the current is 1.0 ampere. Therefore, in the case of the arc between copper electrodes, the constants of equation (8) may be expressed as functions of the current as follows:

$$C = 14.5 + \frac{28.0 - 14.5}{i^{0.67}} \tag{13}$$

$$D = 15.5 + \frac{42.5 - 15.5}{i^{0.67}} \tag{14}$$

$$\gamma = 1.7 + \frac{5.0 - 1.7}{i^{0.67}} \tag{15}$$

$$\delta = 0.235 + \frac{0.373 - 0.235}{i^{0.67}} \tag{16}$$

$$d = 6.0 \tag{17}$$

The substitution of the above functions in equation (8) gives (18), the desired final equation for the arc between copper electrodes.

$$E = \left(14.5 + \frac{28.0 - 14.5}{i^{0.67}} \right) (1 - \epsilon^{-6 \cdot L})$$

$$+ \left(15.5 + \frac{42.5 - 15.5}{i^{0.67}} \right) \left(1 - \epsilon^{-\left(0.235 + \frac{0.373 - 0.235}{i^{0.67}}\right) L}\right)$$

$$+ \left(1.7 + \frac{5.0 - 1.7}{i^{0.67}} \right) L \qquad \textbf{(18)}$$

As an indication that equation (**18**) is the characteristic equation sought, eighty-eight calculated values of the difference in potential have been placed beside the corresponding observed values in Tables IV and VI. Tables IV and VI indicate that equation (**18**) does fit these data without systematic error. However, if apparatus had been available for the study of arcs more than 10.0 mm. in length, the constants of equation (**18**) could no doubt have been determined with a higher degree of accuracy.

TABLE VI

A COMPARISON BETWEEN OBSERVED AND CALCULATED VALUES OF E

Electrodes—Copper

E_0 Observed Value of E

E_c Calculated by Equation (18)

Arc Length L	1 mm.		2 mm.		3 mm.		4 mm.		5 mm.		6 mm.		7 mm.		8 mm.		10 mm.	
Current 1 amp.	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c
1.5	40.0	39.73	52.0	51.81	61.0	60.83	69.0	68.64	75.5	75.34	82.0	81.65	87.5	87.30	93.0	92.43	102.0	101.82
2.0	36.0	36.23	47.0	47.11	55.5	55.36	62.0	62.04	68.0	67.87	74.0	73.70	79.0	78.75	84.0	83.33	92.0	92.30
2.5	34.0	33.73	43.5	43.74	51.5	51.38	58.0	57.74	63.0	63.29	68.5	68.40	73.0	72.85	77.5	77.13	85.0	85.30
3.0	32.5	32.15	41.5	41.38	49.0	48.58	55.0	54.69	60.0	59.84	65.0	64.75	69.0	69.00	73.5	73.13	80.0	80.47
4.0	30.0	29.98	38.0	38.21	45.0	44.88	50.5	50.52	55.5	55.56	60.0	59.90	64.0	63.85	68.0	67.65	74.5	74.60
5.0	28.5	28.51	36.5	36.26	42.5	42.38	48.0	47.83	53.0	52.46	57.0	56.75	61.0	60.70	64.5	64.18	70.5	70.64
7.0	26.5	26.58	33.5	33.71	39.5	39.41	44.0	44.19	48.5	48.54	52.5	52.55	56.0	56.10	59.5	59.43	65.5	65.60
10.0	25.5	25.15	31.5	31.61	37.0	36.89	41.5	41.44	45.5	45.52	49.5	49.18	52.5	52.45	55.5	55.53	61.0	61.46

TABLE VII

THE CONSTANTS OF EQUATION (19) DETERMINED FOR TEN ARC CHARACTERISTICS

Anode	Cathode	n	α_∞	α_1	β_∞	β_1	γ_∞	γ_1	δ_∞	δ_1	d
Carbon	Copper	0.985	24.5	39.0	35.0	41.4	1.25	5.0	0.535	0.535	7.0
Cadmium	Carbon	0.720	10.0	15.0	9.2	37.5	0.80	5.0	0.220	0.229	7.0
Copper	Copper	0.670	14.5	28.0	15.5	42.5	1.70	5.0	0.235	0.373	6.0
Aluminum	Carbon	0.650	8.0	17.5	6.0	35.0	0.80	5.0	0.258	0.410	5.0
Nickel	Carbon	0.640	10.0	28.0	8.0	38.0	0.90	5.0	0.166	0.270	7.0
Silver	Carbon	0.624	13.0	27.5	7.0	37.5	0.80	6.0	0.229	0.246	7.0
Zinc	Carbon	0.570	8.6	24.0	2.9	41.0	0.65	5.0	0.210	0.263	7.0
Lead	Carbon	0.480	8.2	16.5	4.8	17.5	0.80	5.0	0.180	0.214	7.0
Antimony	Carbon	0.460	12.2	24.0	4.3	19.0	0.60	5.0	0.190	0.230	7.0
Bismuth	Carbon	0.445	7.8	17.5	6.2	24.5	0.80	5.0	0.187	0.200	7.0

Although equation (**18**) is an empirical equation derived as a result of the study of the arc between copper electrodes, it can be expressed in general terms and used to represent the static characteristic of any arc. It will be remembered that n was the first constant in equation (**18**) to be determined. This determination also yielded the values of A from which the curve for infinite current was plotted. (See equation (**3**) and Fig. 5.) The constants of equation (**8**) were then determined for the curves in Fig. 5 for 1.0 ampere and for infinite current. For the curve for 1.0 ampere, let

$C = \alpha_1$, $D = \beta_1$,
$\gamma = \gamma_1$, $\delta = \delta_1$, and
$d = d$

and for the curve for infinite current, let

$C = \alpha_\infty$, $D = \beta_\infty$,
$\gamma = \gamma_\infty$, $\delta = \delta_\infty$, and
$d = d$

These constants incorporated in a general equation give equation (**19**).

$$E = \left(\alpha_\infty + \frac{\alpha_1 - \alpha_\infty}{i^n} \right) (1 - \epsilon^{-d \cdot L})$$

$$+ \left(\beta_\infty + \frac{\beta_1 - \beta_\infty}{i^n} \right) \left(1 - \epsilon^{-\left(\delta_\infty + \frac{\delta_1 - \delta_\infty}{i^n}\right) \cdot L}\right)$$

$$+ \left(\gamma_\infty + \frac{\gamma_1 - \gamma_\infty}{i^n} \right) \cdot L \qquad \textbf{(19)}$$

EQUATION (19) TESTED FOR TEN ELECTRODE MATERIALS

All of the constants of equation (**19**) were calculated for the ten arc combinations studied and the resulting equations were checked graphically against the observed data. (See Table VII.)

The agreement between the observed and the calculated values was exceedingly close in every case. Therefore, *equation* (**19**) *is undoubtedly a truly general equation* *representing the static characteristics of normal arcs between any electrode materials.*

THE CATHODE, ANODE, AND ARC FALLS AS COMPONENTS OF E

The three components of the difference in potential between the electrodes have long been recognized as the anode fall, the cathode fall, and the arc fall (See Fig. 6), but there is still some doubt as to the exact dependence of each on the current intensity and the arc length. The simplest of these, the arc fall, is generally thought to be a linear function of the arc length and a hyperbolic function of the current. If this is true, the arc fall (e_a) must be represented by the last term of equation (**19**).

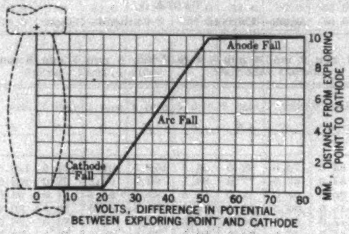

FIG. 6

$$e_a = \left(\gamma_\infty + \frac{\gamma_1 - \gamma_\infty}{i^n} \right) \cdot L \tag{20}$$

Some studies of the cathode fall have shown it to be independent of the arc length and the current. Although it is quite impossible actually to measure the cathode fall in arcs less than 1.0 mm. in length, the fact that arcs of 0.1 mm. can be maintained on less than 20.0 volts (Grotrian's[12] value for the cathode fall alone) invalidates the theory that the cathode fall is independent of the arc length when the arcs are short. A change in the magnitude of the cathode and the anode falls might naturally be expected when the arcs are short, because these falls take place in the space a fraction of a millimeter from the surfaces of the electrodes instead of exactly at their surfaces. In addition, it is apparent from the curves in Fig. 5 that the sum of the anode fall (e_b) and the cathode fall (e_c) can be neither independent of the arc length nor a linear function of the arc length. It is a plausible assumption that the cathode fall reacts upon the anode fall as the arc length becomes shorter until finally the two falls merge, and each rapidly cancels the effect of the other. This interaction may be measured roughly by the deviation of the slope of the curves for constant current from the minimum slope of these curves. The relation between the slope and the arc length for any particular current is expressed by equation (**21**).

$$\frac{dE}{dL} = \left(\alpha_\infty + \frac{\alpha_1 - \alpha_\infty}{i^n} \right) \cdot d \cdot \epsilon^{-d \cdot L}$$

$$+ \left(\beta_\infty + \frac{\beta_1 - \beta_\infty}{i^n} \right)$$

$$\left(\delta_\infty + \frac{\delta_1 - \delta_\infty}{i^n} \right) \cdot \epsilon^{-\left(\delta_\infty + \frac{\delta_1 - \delta_\infty}{i^n}\right) \cdot L}$$

$$+ \left(\gamma_\infty + \frac{\gamma_1 - \gamma_\infty}{i^n} \right) \tag{21}$$

The curve in Fig. 7 is the graphical representation of equation (**21**) for an arc between copper electrodes when the current is 3.0 amperes.

Since the cathode fall is independent of the arc length for arcs longer than 1.0 mm., the second term of equation (**19**) must be a function of the anode fall. Therefore, this term represents either the *increase in* the anode fall as the arc length increases and the *change* in this increase as the current varies, or the *total anode fall* for any arc length and current. If the latter is the case, the first term of equation (**19**) represents the cathode fall for any current and arc length. Such conclusions would be in entire agreement with Mrs. Ayrton's[13] observations of the arc in its component parts.

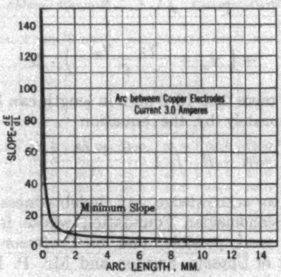

FIG. 7—A MEASURE OF THE INTERACTION OF THE CATHODE AND THE ANODE FALLS

It is evident from this examination of the equations (**19**) and (**21**) that arcs of less than 15.0 mm. can not possibly be represented accurately by an equation less complex than equation (**19**). However, arcs of a length greater than 15.0 mm. can be represented by equation (**22**) which is not unlike Mrs. Ayrton's in simplicity.

$$E = A + B \cdot L + \frac{C + D \cdot L}{i^n} \tag{22}$$

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[12] W. Grotrian, *Ann. d. Phy.*, 47, p. 141, (1915).

[13] The Electric Arc, p. 238.

In equation (**22**),

$A = \alpha_\infty + \beta_\infty$, $B = \gamma_\infty$,

$C = \alpha_1 - \alpha_\infty + \beta_1 - \beta_\infty$, and $D = \gamma_1 - \gamma_\infty$

Although the results of this study are illuminating, they only point the way. A more extensive investigation of the arc in different atmospheres and at different pressures should be undertaken in hope that this equation, or some modification of it, will offer a method by which the arc characteristic can be studied in its component parts without the introduction of an exploring electrode. The equation might then be used to solve the many problems involving the distribution of the power consumption in the arc and the problem of determining the temperature in the arc, which now confronts the investigators of commercial arcs employed for the fixation of nitrogen, arc lighting, the electric furnace, etc.

CONCLUSIONS

1. The exponent n of the current i in the equation

$$E = A + B/i^n \qquad (\mathbf{3})$$

is directly proportional to the absolute temperature of the boiling point or the sublimation point, as the case may be, of the anode.

2. Equation (**19**) can be made to represent the static characteristic of the normal arc without apparent systematic error.

$$E = \left(\alpha_\infty + \frac{\alpha_1 - \alpha_\infty}{i^n}\right)(1 - \epsilon^{-d \cdot L}) + \left(\beta_\infty + \frac{\beta_1 - \beta_\infty}{i^n}\right)\left(1 - \epsilon^{-\left(\delta_\infty + \frac{\delta_1 - \delta_\infty}{i^n}\right) \cdot L}\right) + \left(\gamma_\infty + \frac{\gamma_1 - \gamma_\infty}{i^n}\right) \cdot L \qquad (\mathbf{19})$$

3. Arcs more than 15.0 mm. in length can be represented accurately by the equation

$$E = A + B \cdot L + \frac{C + D \cdot L}{i^n} \qquad (\mathbf{21})$$

I wish to acknowledge my indebtedness to the American-Scandinavian Foundation whose fellowship made this work possible, and to Professor Gustav Grandqvist of Uppsala, Sweden and Mr. P. P. Cram for their assistance in the preparation of paper.

APPENDIX

The Description of the Apparatus. The electrodes were 12 mm. in diameter and made of the very highest grade of materials. After each reading the electrodes were refaced to insure precision in arc length measurements and to reduce the meter fluctations to a minimum. The upper electrode was the anode except in the cases of the cadmium, antimony, bismuth and lead arcs.

Loosely fitting jackets supplied by independent water systems, were used to carry the excessive heat from the electrode supports. Readings taken with and without these jackets showed that this slight cooling had no appreciable effect on the arc characteristic.

For the longer arcs, the length was determined by the measurement of the image of the arc produced by a lens system (magnification 15 diameters) on a scale calibrated to read arc length directly in millimeters. The short arcs were adjusted by a micrometer screw.

The arc was freely open to the air at all times.

The Observed Data. The principal data from which the constants of the equations were calculated, are recorded in the following tables, *I* to *X* inclusive.

TABLE I

Anode—Carbon Cathode—Copper

Arc length *L*	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	61.0	76.0	87.0	95.5	102.5	108.5	114.5	120.0	130.0
1.5	54.0	67.5	77.0	84.5	90.0	94.5	99.0	103.0	110.5
2.0	50.5	63.0	72.0	78.5	83.5	87.5	91.0	95.0	101.0
2.5	48.5	60.5	68.5	74.5	79.5	83.0	86.5	90.0	95.5
3.0	47.0	59.0	66.5	72.5	77.0	80.0	83.0	86.0	91.5
4.0	45.3	56.5	64.0	69.5	73.5	76.5	79.5	82.0	86.5
5.0	44.2	55.5	62.5	67.5	71.5	74.5	77.0	79.5	84.0
7.0	43.0	54.0	60.5	65.5	69.0	72.0	74.2	76.5	80.0
10.0	42.0	52.5	59.5	64.0	67.5	70.0	72.5	74.5	78.0

TABLE II

Anode—Cadmium (oxide) Cathode—Carbon

Arc length *L*	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	27.5	38.5	48.5	57.5	65.5	73.0	80.0	86.5	98.0
1.5	24.0	32.5	40.5	47.5	53.5	60.0	65.0	70.5	80.0
2.0	21.5	29.0	36.5	42.5	48.0	53.0	57.0	62.0	67.5
2.5	20.5	27.0	33.5	39.0	43.5	48.0	52.0	56.0	63.0
3.0	19.5	25.5	31.0	36.5	40.5	45.0	48.5	52.0	58.0
4.0	18.0	23.5	28.5	32.0	36.5	40.5	44.0	47.0	52.5
5.0	17.5	22.5	27.0	31.0	34.0	37.5	41.0	44.0	49.0
7.0	16.3	20.5	24.5	28.0	31.5	34.0	37.0	39.0	43.5
10.0	15.5	19.5	23.0	26.5	29.0	31.5	34.0	36.0	39.5

TABLE III

Anode—Copper (oxide) Cathode—Copper

Arc length *L*	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	46.0	60.0	71.5	81.0	89.0	96.0	102.0	108.0	120.0
1.5	40.0	52.0	61.0	69.0	75.5	82.0	87.5	93.0	102.0
2.0	36.0	47.0	55.5	62.0	68.0	74.0	79.0	84.0	92.0
2.5	34.0	43.5	51.5	58.0	63.0	68.5	73.0	77.5	85.0
3.0	32.5	41.5	49.0	55.0	60.0	65.0	69.0	73.5	80.0
4.0	30.0	38.0	45.0	50.5	55.5	60.0	64.0	68.0	74.5
5.0	28.5	36.5	42.5	48.0	53.0	57.0	61.0	64.5	70.5
7.0	26.5	33.5	39.5	44.0	48.5	52.5	56.0	59.5	65.5
10.0	25.5	31.5	37.0	41.5	45.5	49.5	52.5	55.5	61.0

TABLE IV

Anode—Aluminum (oxide) Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	34.0	47.0	56.0	65.5	73.0	79.5	85.5	91.0	102.0
1.5	28.0	38.5	45.5	53.0	58.5	64.0	69.0	74.0	82.0
2.0	25.0	34.0	40.0	46.0	51.0	55.5	60.0	64.0	71.5
2.5	22.5	30.5	36.0	41.5	46.0	50.0	53.5	57.0	64.0
3.0	21.0	28.5	33.0	38.0	42.5	46.0	49.5	52.5	59.0
4.0	19.0	27.5	29.5	34.0	38.0	41.0	44.0	47.0	52.0
5.0	18.0	23.5	27.5	31.0	34.5	38.0	40.5	43.0	47.5
7.0	16.0	21.0	24.6	28.0	31.0	33.0	36.0	38.0	42.0
10.0	15.0	19.0	22.5	25.0	28.0	30.0	32.0	34.0	38.0

TABLE V

Anode—Nickel (oxide) Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	42.0	53.5	64.0	73.0	81.0	88.5	95.0	101.5	113.5
1.5	35.5	44.0	53.5	60.0	67.0	73.5	78.0	83.0	93.0
2.0	32.0	39.5	47.0	53.0	59.0	64.0	69.0	73.0	82.0
2.5	29.0	36.0	42.5	48.0	53.0	58.0	62.0	66.0	74.0
3.0	27.0	34.0	39.5	45.0	49.5	54.0	57.0	61.0	69.0
4.0	24.5	30.0	35.5	40.0	44.5	48.5	51.5	55.0	61.0
5.0	23.0	28.0	33.0	37.0	41.0	45.0	48.5	51.0	57.0
7.0	21.0	25.5	29.0	33.5	37.0	40.0	42.5	45.5	50.5
10.0	19.0	23.0	27.0	30.0	33.0	36.0	38.5	41.0	46.0

TABLE VI

Anode—Silver Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	41.5	54.0	65.0	75.0	84.0	92.5	100.5	108.0	121.5
1.5	36.0	46.0	54.5	63.0	69.5	78.0	83.5	90.0	99.5
2.0	32.5	42.5	49.0	55.5	62.0	69.0	74.0	79.5	88.5
2.5	30.0	38.0	45.0	51.0	56.5	62.5	67.5	72.0	81.0
3.0	28.5	36.0	42.5	48.0	53.0	58.0	62.5	67.0	74.5
4.0	26.0	32.5	38.5	43.5	48.0	52.0	56.5	60.0	66.5
5.0	25.0	30.5	36.0	40.5	45.0	49.0	51.9	55.5	61.0
7.0	23.0	28.0	32.5	36.5	40.0	44.0	48.0	50.0	55.0
10.0	22.5	26.0	30.0	33.5	36.5	40.0	43.0	45.0	50.0

TABLE VII

Anode—Zinc (oxide) Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	38.5	51.0	61.0	71.0	79.0	86.5	93.5	100.0	112.0
1.5	33.0	43.5	51.5	59.5	66.5	72.5	78.0	83.0	92.0
2.0	30.0	39.0	45.5	53.0	59.5	64.0	69.0	74.0	81.0
2.5	27.5	36.0	42.0	48.0	53.5	58.0	63.0	67.5	74.0
3.0	24.5	33.5	39.0	44.5	49.5	54.0	58.0	62.0	69.0
4.0	23.5	30.0	35.0	40.0	44.0	48.0	52.5	55.0	61.0
5.0	22.5	28.0	32.5	37.0	41.0	44.0	47.0	50.0	55.0
7.0	20.0	25.0	29.0	33.0	36.0	39.0	41.5	44.0	48.0
10.0	18.0	22.5	26.0	29.0	32.0	34.5	36.5	39.0	42.0

TABLE VIII

Anode—Lead Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	25.0	32.5	40.0	46.5	53.0	59.0	65.0	71.0	82.0
1.5	22.5	29.0	35.0	41.0	46.0	51.0	56.0	61.5	71.0
2.0	21.0	26.5	32.0	37.5	42.5	47.0	51.0	56.0	64.5
2.5	19.5	25.0	30.0	35.0	39.5	44.0	48.0	52.0	60.0
3.0	18.8	23.5	28.5	33.0	38.5	41.5	45.0	49.0	56.0
4.0	17.5	22.5	26.5	30.5	34.5	38.0	41.5	45.5	52.0
5.0	17.0	21.0	25.0	29.0	32.5	36.0	39.5	42.5	49.0
7.0	15.8	19.5	23.5	26.5	30.0	33.0	36.0	39.0	44.5

TABLE IX

Anode—Antimony Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	33.0	41.0	48.5	55.5	62.0	68.0	74.0	80.0	91.0
1.5	29.5	36.5	43.0	48.5	54.0	59.0	64.0	70.0	79.0
2.0	27.5	33.5	39.5	45.0	49.5	54.0	58.5	63.0	71.5
2.5	26.0	31.5	37.0	42.0	46.0	50.0	54.5	58.5	66.5
3.0	25.0	30.0	35.0	40.0	44.0	48.0	51.5	55.5	63.0
4.0	23.5	28.0	33.0	37.0	40.5	44.5	48.0	51.5	57.5
5.0	22.5	27.0	31.0	35.0	38.5	42.0	45.0	48.5	54.5
7.0	21.0	25.0	29.0	32.0	35.5	38.5	41.5	45.0	50.0

TABLE X

Anode—Bismuth Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	27.0	35.5	43.5	51.0	58.0	64.5	71.0	77.0	88.5
1.5	24.0	31.5	38.5	45.0	50.5	56.5	62.0	67.0	78.0
2.0	22.5	29.0	35.5	41.5	47.0	51.5	57.0	61.5	71.0
2.5	21.0	27.5	33.5	38.5	44.0	48.5	52.5	57.0	66.0
3.0	19.5	26.0	32.0	37.0	41.5	46.0	50.0	54.0	62.5
4.0	19.0	24.5	29.5	34.0	38.5	42.5	46.5	50.0	57.0
5.0	18.0	23.0	28.0	32.0	36.0	40.0	43.5	47.0	54.0
7.0	16.5	21.5	25.5	29.5	33.0	37.0	40.0	43.0	49.0

Discussion

W. N. Eddy: As a supplement to his detailed analysis of the voltage-length-current characteristics of short arcs Mr. Nottingham might be interested in the results of some arc tests that were published in 1922.[1] These tests were made with the idea of (1) trying out Dr. Steinmetz' equation at longer arc lengths than used by him in 1906 and (2) finding the effect on his equation of different pressures on the arc stream.

While developed from data on arcs no longer than 2 in. the Steinmetz equation was found to hold true for arcs as long as 6 in. the maximum length of arc tested.

While the Nottingham equation satisfactorily represents the short arcs the longer arcs show a straight line relation between length and voltage and therefore are better represented by the Steinmetz equation. Thus it would seem that a combination of the two equations should satisfactorily represent the volt-ampere-length characteristics of normal arcs throughout the whole range of length.

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[1] Length-Voltage-Current Pressure Characteristics of Normal Arcs for Different Electrode Materials, *G. E. Review*, March 1922.

The effect of the pressure on the three different constants of the Steinmetz equation was shown empirically in the form of curves for various electrode materials. By the use of these curves the Steinmetz equation can be used to show the relation of the five variables current, voltage, length, electrode material and pressure on the arc stream.

W. B. Nottingham: In answer to Mr. Eddy, I wish to call attention to the statement in my paper that "It is evident from this examination of the equations (**19**) and (**21**) that arcs of less than 15 mm. (length) can not possibly be represented accurately by an equation less complex than equation (**19**). However, arcs of length greater than 15 mm. can be represented by equation (**22**), which is the limiting form of (**19**) for large arc lengths; *viz.*

$$E = A + B\,L + \frac{C + D\,L}{i^n} \qquad (22)$$

E = Difference in potential across the arc.
L = Length of the arc.
i = Current in the arc.
A, B, C, D, and n are constants depending upon the electrode material, the surrounding medium, etc.

Equation (**22**) shows the difference in potential (E) across the arc to be a linear function of the arc length (L), and therefore it agrees with Mr. Eddy's observations. When equation (**19**) is used for long arcs (15 mm. or more) the only differences between it and the Steinmetz equation are that the current enters as i^n instead of $\sqrt{i}$ and that there is a term $\gamma \infty\, L$ (or $B\,L$ in equation (**22**)) which is omitted from the Steinmetz equation. *In both equations the voltage is a linear function of the arc length.*

Fig. 1 has been prepared to show that equation (**19**), or its limiting form equation (**22**), holds quite well over a range far greater than that for which its constants were originally calculated. The three curves shown in the figure are; (A) the characteristic curve for an arc 5.0 cm. long between copper electrodes as observed by Grau and Russ[1]; (B) the characteristic curve for a copper arc of the same length calculated by equation (**19**) (or equation **22**), using the constants given in Table VII[2]; and (C) the calculated characteristic for the same arc using the Steinmetz equation with constants given by Mr. W. N. Eddy[3] for the normal arc between copper electrodes in air at atmospheric pressure.

Steinmetz Equation:

$$E = a + \frac{c\,(l + \delta)}{\sqrt{i}}$$

a = 48.8 volts
c = 38.5
δ = 0.28 cm.
l = Arc length in cm.

W. N. Eddy's constants for arc between copper electrodes in air at atmospheric pressure.

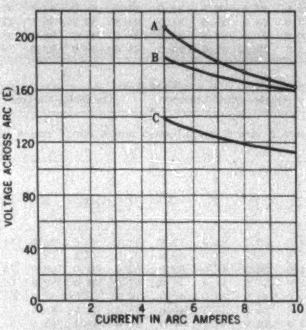

Fig. 1.—A Characteristic Curve for an Arc Between Copper Electrodes Arc Length = 5 cm.

A Grau and Russ Observed.
B Nottingham Calculated.
C Steinmetz Equation. Constants from W. N. Eddy.

It was the aim of my paper to present an equation to represent satisfactorily "the volt-ampere-length characteristics of normal arcs throughout the whole range of length." Equation (**19**) meets this requirement in every way that I have been able to test it.

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[1] Grau and Russ, *Phys. Zeitschr.*, 9, p. 107, 1908.

[2] A. I. E. E. Journal, Vol. XLII, p. 16, Jan. 1923.

[3] W. N. Eddy, *G. E. Review*, March 1922, p. 191.

Electromagnetic Forces; A Search for More Rational Fundamentals; a Proposed Revision of the Laws

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Reasons are given why it is desirable to revise some of our older laws regarding electromagnetic forces and motions which are the basis of all electromotive devices, in order to conform better to more modern developments. Researches with high-current densities in such mobile conductors as liquids and arcs, have brought out some heretofore unnoticed forces, the existence of some of which had been denied. Some of our older laws are claimed to mislead, to be inaccurate, incomplete, to involve unnecessary complications such as the forced definitions of a sliding contact, are based on the wrong fundamentals, specify results contrary to the facts, and are not universal, thereby checking possible progress if accepted as universal.

A new and simple general law is proposed, based on one of the fundamental universal laws of physics. It is shown how this might also be made the basis of a much desired universal law of induction. It leads to the existence of a force longitudinal to the conductor, which our older laws deny. Numerous experiments are described illustrating and bearing out the arguments. Suggestions are made showing how the laws and the present usual methods of mathematical treatment of such forces might be revised in order to make them more satisfactory, easier for the student to understand, and for the engineer to use. If the alleged improper restrictions imposed by former laws are removed, developments in new fields may become possible. In conclusion a tentative plan for revision is suggested.

THE foundations of a building erected some 50 or 100 years ago may have served their purpose perfectly, but when story after story is being added to that building reaching skyscraper heights a conscientious architect will do well to examine these foundations more carefully to find out whether they really rest on a firm rock bottom or whether they may not need some altering or strengthening to make them absolutely safe and trustworthy for the additional load.

Similarly, the fundamental laws of electromagnetic forces and motions formulated some 50 or 100 years ago may have served their purpose perfectly for a time, and some are still very serviceable if limited to special cases, but in the rapid development of the various applications of electricity more and more weight has been rested on these original laws and they are now being held responsible for applying equally correctly to new and much more severe conditions than were even thought of by those who framed them. The writer's experience with the application of electromagnetic forces in practise has convinced him that the time has come when they should be reexamined and tested, to find whether they can still safely stand the weight of the much greater responsibilities which we are now placing on them, or which may in the future be placed on them. As these forces are the basis of all motive devices using electrical energy it is of the utmost importance to the engineer to be able to place absolute reliance on them. Not only should the laws be so stated that he will not waste time and money trying to do what is really impossible, but it is quite as important, if not even more so, that they should not forbid what may be possible, as that may check progress in new, untrodden fields; if for instance a law is considered to be universal (that is, without exceptions) when as a fact it is not, it has improperly forbidden any developments in new fields in which there might have been progress. It is folly to violate nature's laws, but it should not be forgotten that our versions of them are man-made. Besides being absolutely correct these laws should also be as simple, and direct as possible; when we instruct people how best to go from New York to London we could, though should not, tell them to go via Japan and India; nor should the laws be burdened with complicated explanations necessitated by trying to make them fit apparent exceptions. Moreover it is our duty to the student and to the rising generation to give preference to that one of two otherwise equal forms of laws, statements or explanations, which is most easily grasped, understood and retained.

PURPOSE OF THIS PAPER

The purpose of the present paper is to show by means of experiments and arguments how certain of our older laws and explanations of electromagnetic forces and motions, as now taught in many text books and colleges, sometimes mislead or lead away from, rather than toward the truth, how they sometimes fail to apply, even specifying results directly contrary to the facts, and how they have practically forbidden us new fields in which progress might perhaps be made. Also to offer interpretations of these experiments and to suggest how the laws might be revised to bring them up to date, including a proposed new, simple, general law of the motions produced by these forces, which it seems to the writer applies correctly to all cases, old and new, and if so it is universal as far as we now know. Also to show how this new law may be made the logical basis of a new and general law of induction, which has the very great advantage of avoiding all the present complications due to the many involved and forced explanations of a sliding contact. In general the object of the proposed revision is to endeavor to simplify the whole subject for the benefit of the engineer and the student, to make the laws more reliable and more directly applicable; also to open up new fields, if any, which may have been

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forbidden to us before. The development of a revised mathematical treatment may thereby however become far more difficult, as the simple short cuts of Maxwell may then no longer be applicable.

The paper is intended merely to open up the subject and to make suggestions for a revision; it is not intended to be a completed and finished treatise on the subject. It is hoped some others with better facilities for research will find enough of interest and value in the experiments and in the suggestions to continue the investigations more thoroughly and perhaps to offer different and better interpretations and deductions from them.

If no unquestionable errors can be found in the proposed revision, the chief subject of discussion should be, whether the proposed revision would or would not have any advantages over the older system and this of course is a matter of opinion. It is of no consequence that it may be possible to explain some of the writer's experiments by means of the old laws, the important question is, which explanation is the more rational one and which is the better one for the student to grasp. The fact that the development of a corresponding mathematical treatment may then become far more difficult, ought not to be an insurmountable objection. Some of the points involved were recommended by Faraday and by Ampere, but they were discouraged by Maxwell; they did not fit in well with his mathematical short cuts.

THE PHYSICIST AND THE ENGINEER

It is the province of the physicist to discover and formulate the laws of nature concerning matter and energy and to develop the mathematical relations; and it is the province of the engineer to then take these laws and apply them to the benefit of mankind; the present subject therefore more properly belong under physics. During the past fifteen years the writer has repeatedly called the attention of physicists to the experimental evidence of the present unsatisfactory state, and showed how some of our laws have misled and even deceived the engineer when he tried to apply them; they were repeatedly appealed to by the writer to revise them so that the engineer could use them and depend on them as being correct. But not only was there no response (with one notable exception, the development of the quantitative relations by Northrup, concerning the "pinch effect," which has been of great value) but there was a surprising lack of interest in correcting alleged mistakes and shortcomings, and even a determined effort to prevent the publication of the writer's investigations. In one case publication was at first refused on the ground that if the experimental evidence was correct, which was easily demonstrated, it was so serious a matter to change one of the older laws, that it ought to be kept a secret! In another case the refusal was because it was, "so subversive of long established principles," the age of a law being considered more important than its correctness. Both papers however have been published.

The invention of a new and special definition of a sliding contact, to make an exception still fit an old law, seemed to be quite satisfactory to many physicists. Their attitude was that the older laws "were good enough for them." To make misfits fit, some have alleged that a circuit which was in fact open must be considered as being closed; others, that one which was in fact closed must be considered as being open. Their students have the sympathies of the writer, especially as one of the first things they learn in the laboratory is the sometimes very serious physical difference between an open and a closed circuit.

On account of this strange neglect, lack of interest, and even opposition of the physicist, which was not a credit to his profession, to provide the engineer with laws that are reliable and up to date, the writer must now appeal to the electrical engineers among whom there are some physicists of a more progressive class. The laws of physics are the engineer's most important tools; if their makers refuse to make proper tools for him he must make them himself.

MOTIONS OF MOBILE CONDUCTORS

By passing currents, especially at high current densities, through such very mobile metallic conductors as mercury or molten metals in some types of electric furnaces, the writer many years ago noticed the existence of some heretofore unrecognized electromagnetic forces which tended to move the conductors, and being mobile liquids they responded much more readily to such forces than solid conductors do.

Some of these new forces were very formidable, for like most of such forces they presumably increase with the square of the current. The writer then made use of them in electric furnaces many of which are in daily use, these new forces being the absolutely essential factor, showing their industrial importance.

Arcs also respond readily to some of these forces.

LAWS NEED REVISION

Not only were there these new forces, but that gospel faith in the reliability of laws which the engineer ought to be able to have, led the writer in two notable cases to waste time and money before finding out that the laws had not been correctly stated. These experiences gave rise to the present investigation which developed similar results with other laws.

It will be shown below how some of the older laws sometimes mislead, leading us away from rather than toward the true goal; others apply ambiguously, or even specify results quite the contrary to the facts. Some supposedly universal ones (that is, having no exceptions) were found not to be so, thereby having checked possible progress. Some were not based on their true fundamentals but on mere incidental factors; some which apply correctly to special cases have been applied to the more general case, which process may lead to serious errors. In a mathematical treatment it is important to select the best fundamental, which has not always been done; thus if the mathematics of something circular, like a soap bubble film, is based on the radial pressure, the formula fails for the tension of a plane film, giving infinity, which is not true; while if based on the tension of a plane film the internal pressure in a bubble may be determined from it. Our unit of current is based on the magnetic force of a circular circuit. A closed finite line had better be made a special case of an infinite straight line, than the reverse.

In going from the general to a special case some factors generally drop out and can never again reappear in any subsequent deductions from the mathematical treatment of this special case. Or if a mathematical treatment is started for a case which is really a special and not the fundamental one, it is an error to apply it to the more fundamental one, or to other special cases in which the dropped factors may be different. Yet it seems such mistakes have been made. In a complicated system it is therefore important to find or select the best and most basic fundamentals.

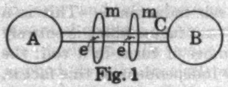

Fig. 1

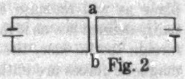

Fig. 2

A revision of our former laws is also desirable now that it is known that every electric current is merely a movement of electrons in the conductor. In Fig. 1, if negative electrons pass from one body *A* to another *B*, through a conductor *C*, what we call a current flows through that conductor during that passage, creating simultaneous magnetic effects *m*, *m*, around it, each caused by its moving electron *e*, *e*; after the operation the body *A* is left charged positively and *B* negatively. Telegraph messages have been sent by currents in such a conductor, after an aurora borealis. It is therefore no longer an absolute necessity to consider every electric circuit as being a "complete circuit;" in many cases it is a great convenience to do so, a short cut, but in some it misleads and involves great difficulties to make it fit the facts; a law to be universal should therefore not be based on the circuit being a complete one. The complete circuit should better be made a special case of a unit length of a straight circuit. Ampere advocated basing the mathematical treatment on elemental parts of circuits, but Maxwell did not approve it, preferring the easier method or short cut based on the complete circuit, which however is a special case and not the most general one.

When two current carrying conductors are in some parts quite near to each other as at *a b* Fig. 2, and at other parts far apart, the force between them will reside almost entirely in the near parts *a b;* in practise this force would be practically the same whether the rest of the circuits are a foot or a mile long. In engineering work the consideration of the "rest of the circuit" is generally quite unnecessary; the leads to a motor do not electromagnetically affect the mechanical forces in the motor.

The present tendency to abandon that useful stepping stone, the ether, also makes it desirable to revise some of our former conceptions. A magnetic field in space was thought to be a storage of energy in the form of a sort of stress or strain in the surrounding ether, which was and is still a very convenient and useful picturization. It seems to be much better, however, to consider the magnetic field around a current-carrying conductor to be merely an action at a distance, as Faraday proposed, the seat and source of the force being in the material of the conductor and not in the space around it; no assumption of any ether is then necessary. The reaction of these external forces on each other or on a foreign force, must then manifest itself in the material of the conductor through which the electrons are passing, and at the places where these forces are originated by them.

It is therefore the conductor itself, that is, the matter in it, which had better be referred to in these laws and not the more subtle currents, circuits, fields or ether. Experiments illustrating this will be described below.

It is also our duty to the student and the rising generation to point out any errors and exceptions and to revise the laws, conceptions and explanations so as to make them the simplest and clearest possible in order that they may be grasped and retained as easily as possible. Complicated conceptions and explanations such as many of those for sliding contacts, are objectionable. Above all, the laws should be strictly correct, should not mislead and should if possible be universal.

MISTAKES IN SOME LAWS

The elementary law given in every text book, that like currents attract and unlike repel, is wrongly worded; it is not on the currents but on the material of the conductor that these forces act. If they acted on the currents then the current density would have to be greater in the center of a conductor than near the outside, and Ohm's law would no longer apply to large conductors; or when two conductors are close to each other the current densities in different halves of the same conductor would have to be different; or the current in the middle of a cathode or anode in an electrolytic bath would have to be greater than elsewhere. But we know these are not the facts. Moreover like cathode rays repel. The industrially important pinch effect[1] would not exist, yet we know it does. Maxwell (Art. 501) recognized and called attention to this important distinction between the current and the conductor, but many of his followers did not. Nor did Maxwell continue to make a similar distinction in his law of induction (Art. 531) as he ought to have done.

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[1] Considering a conductor as consisting of a bundle of filamentary ones, this force tends to make them move to the center. When the conductor is a liquid the material is free to move and these radial forces may then become great enough to cause rupture. This phenomenon has been called the pinch effect. See bibliography at end of this paper.

In a paper before this Institute (TRANS. Vol. 27, Part 2, 1908 p. 1341) the writer described a simple experiment showing conclusively that a similar distinction must also be made in Maxwell's law of induction. It is the conductor, that is, the matter and not merely the circuit, which must cut the magnetic flux in order to induce a current; if the circuit alone cuts it, the conductor remaining stationary with respect to the flux, there will not be the slightest induction even though exactly the same linkages of the flux and the circuit have taken place. (See also a modification of Faraday's experiment in the *Jour. Frank. Inst.* Nov. 1, 1921 p. 605). This important law therefore also requires revision. It moreover also fails to show in what part of the circuit the induction actually takes place and therefore what parts are dead or inactive, which is sometimes of importance to know.

The writer maintains that just as in the two cases mentioned above, it is necessary to make a similar distinction between the conductor itself and the magnetic field surrounding it. The forces in this field have their seat and origin in the material of the conductor where the electrons are passing through it; which should therefore be their anchorage or abutment; any actions of these forces on each other or on a foreign force should, therefore, have their reaction in the conductor. If a large heavenly body passed through the field of force of gravity between this earth and the sun, the abutments of the new gravity forces would be in the earth and the sun, not in the space between them. It is difficult to conceive how the abutment of an electromagnetic force could be in the ether; the energy in the cases here referred to is not a radiated wave motion.

In other words the flux around a conductor or magnet should be pictured as though it were in some way elastically attached to the material in which its source lies. Faraday entertained this idea (*Exp. Res.*, Vol. 2; p. 293; Vol. 3; p. 447), as pointed out by Maxwell (Art. 529) when the latter says: "He even speaks of lines of force belonging to a body as in some sense part of itself, so that in its action on distant bodies it cannot be said to act where it is not." Maxwell however does not favor the idea of action at a distance (Art. 552).

A distinction should here be made between a field of force or a storage of energy, which remains with the source, such as in most electromagnetic machinery, and one which leaves it completely as by radiation, such as in wireless transmission. In the latter case the forces acting in such fields of energy do not react on the source; when a bullet hits a target or is deflected, the forces on it do not react on the gun. The present discussion refers only to fields of force or energy which remain with their source.

One of the most common of the older laws concerning the production of motions by a circuit is to the effect that a circuit will produce such motions as will make it include more flux; Maxwell (Art. 489 and 490) states it in equivalent terms. It will be shown below that this law, though often true, is not based on the correct fundamental principle, the increase in flux so often noticed being an incidental and indefinite secondary consequence which may or may not take place, and is not the true primary cause, and therefore not the proper crucial factor in a law. In the most usual case of a motor, the flux does not increase but remains constant, hence the law fails even in the most usual case; complicated and involved definitions of sliding contacts must be called to the rescue; the Faraday unipolar motor eliminates the element of commutation. But more than that, in the simplest and most fundamental case, which is described below, (the proof of which is given in the Appendix) the flux is actually *less* in amount after the motion.

This law has led many to draw wrong conclusions, as was illustrated by the writer receiving from many physicists a positive answer to a certain case which was directly contrary to the facts. Moreover it is the flux energy and not the flux which is concerned when a circuit does work in producing motion; the flux energy may increase or diminish while the flux can remain constant.

Another favorite law is that only such motions will take place as will increase the self inductance. This is a badly defined law as it involves a factor which is entirely foreign to the case, the number of turns in a coil; the magnetic forces are entirely independent of this factor, at least for direct currents; they depend on the ampere-turns. The self inductance is a purely geometric quantity and is independent of any current or energy. In every motor, and in certain electric furnaces in which powerful motion exists, the self inductance remains constant, as also in the experiment Fig. 17 described below. In the one described in Fig. 16 it will be generally admitted that it actually is *less* after the self produced motion. Another case in which it is less after the motion is referred to in the Appendix. It may be claimed that these cases are unusual ones; but a law to be depended upon as being really universal must not have a single exception. The magnetic energy stored in a coil depends only on the ampere-turns, the reluctance and the flux, it is entirely independent of the self inductance in the sense that the latter may be greater or less in different cases in which the former are the same. The fact that this energy is readily calculated from the self inductance of any particular coil and the current, misleads one to believe it to be a crucial factor in matters concerning energy, but it is not, as the energy is dependent on the ampere-turns but not on the turns by themselves.

Another law is to the effect that anything that increases the stored energy will give rise to a force. There is no such increase in a running motor; and in the most fundamental case described below the stored energy becomes *less* (as it should) when the force acts.

Another is that such motions will take place as will increase the permeance. This also fails for motors unless rescued by forced definitions of sliding contacts. A noted physicist recently explained to the writer that in a certain experiment the motion was due to an "increased reluctance" the exact reverse of an increase in permeance.

With such ambiguities and inaccuracies of the older laws, a new law seems very desirable. Above all a law should not forbid what may be possible.

A General Law of Motions

In view of these descrepancies, and the fact that when forces act to produce motion it involves energy, the writer concluded to look for a general, universal law among those for energy.

There is a well recognized general law in physics applied generally to a system of mechanical forces and energy, to the effect that *in any system such motions will tend to take place as will reduce the potential energy of the system*; this potential energy is thereby transformed into the kinetic energy of the motion; water flowing from an elevated storage reservoir is a simple example.

It is believed that this is a universal law of nature and therefore should apply in quite as general a way to any and all electromagnetic systems in which potential energies are involved and mechanical motions are produced. To make it more convenient to be applied directly to electromagnetic systems, it seems preferable to redraft it and state it in such electrical terms that it will not only be correct and easily understood but that it cannot be misunderstood, nor will deny what may be possible. What the best wording is, is a matter of opinion.

The writer suggests the following version for the usual conditions: *in an electromagnetic system in which the current is being maintained by a source, any and only such mechanical motions of the conductor will tend to take place as will generate a counter e. m. f. somewhere in the circuit.* The actual motions which this tendency will produce will depend, of course, on the particular freedom of motion which the various parts of the circuit may have. The counter e. m. f. is not necessarily generated in the part that moves as will be shown below. It is believed that this is a universal law of all such electromagnetic systems, that is, that there are no exceptions.

The reason for introducing the clause about the current and the source is to exclude certain very unusual cases, like condenser discharges, and others in which there is (as with permanent magnets) no outside source of supply of additional energy when some is withdrawn from the system; to also include such unusual cases the statement of the law would seem to have to suffer in simplicity. The universality of the law is, of course, claimed only for the cases specified in it, which however are the usual ones.

Magnetic material such as iron in such a system can store and give off some energy and thereby may introduce a correction factor in some cases. General laws are supposed to be exactly correct only under theoretically perfect condition, and just as the element of friction does not invalidate the general laws of mechanics, so the presence of iron should not be considered as invalidating this law.

It will be noticed that, as in the general law, there are no further restrictions as to any particular directions of the forces which cause these motions; nor are there any apparent reasons why there should be; according to this version of it any and all motions whatsoever are therefore justified, no matter what the direction of the forces may be which cause them.

The chief reason for proposing this electrical version of the general law is that it is believed to be without exception and in fact a necessary consequence, that when any current-carrying system which is connected to a source produces mechanical motions of any kind, that is, gives off energy, there is always produced a counter e. m. f., which when multiplied by the current (in phase) is the quantitative equivalent of the mechanical energy produced. This alone might justify this version, but there are further reasons and explanations.

The Counter E. M. F.

In a system consisting of current-carrying conductors, it will doubtless be conceded that at least in the sense of this general law, the potential energy in an electromagnetic system resides in the magnetic flux energy which is stored when the current was started and is somewhat like that of inertia. It will probably also be conceded that any self-produced mechanical forces which tend to produce motions, are the direct results of the action of the energy stored in this flux, which may for this purpose be considered as a sort of stress or strain or inertia; the greater this flux energy the greater the forces are likely to be. It is believed to be true also that when such flux energy actually produces mechanical motion, that is, kinetic energy, it disappears as such, just as when a mechanical stress of strain relieves itself by producing motion it disappears as such; but such a disappearance is quite different in kind from that occurring when a current-carrying circuit is broken, as the potential energy of the flux in the latter case is transformed into energy in the form of the momentary current caused by the direct e. m. f. which the disappearing flux produces; the e. m. f. then is a direct and not a counter one.

When the potential energy of the flux is transformed into the kinetic energy of mechanical motion, it seems evident that no counter e. m. f. should be produced directly thereby, though there will be when this lost energy is subsequently replaced again by the current. This is shown by the following example of a more fundamental case.

Assume two parallel copper rings at 0 deg. absolute temperature at which their resistance is zero; let like and equal currents be started in both by drawing a magnet pole through them; the energy of this process will then be stored as magnetic energy and the currents which maintain it would then continue to flow indefinitely. Now let the rings move toward each other by their attraction over a certain distance and then stop, thereby converting part of their potential energy into kinetic; there will then be less magnetic energy and therefore less current flowing after the motion has ceased; this cannot be due to a counter e. m. f. as the motion has ceased.

If two unlike poles of two permanent magnets are permitted to move toward each other, an amount of the flux energy must have disappeared exactly equal in amount to the kinetic energy produced by this motion; this is restored again when the two poles are subsequently forced apart.

In both of these cases the flux itself as distinguished from the flux energy, will also be found to be less, the proofs of which by Prof. R. E. Brown are given in the Appendix.

These two illustrations were chosen because there is no source of new energy connected to them to replace that which was transformed into mechanical energy. They are therefore more basic or more fundamental than the usual cases, as showing what actually takes place physically in the simplest case, and the order in which things take place; they both show that when the potential energy of the flux is converted into the kinetic energy of motion the flux energy (and in fact the flux also) disappears as such, and in the former case without generating any counter e. m. f.; the process here is not masked by any addition of energy from an outside source. The immediate, practically simultaneous, replacing of this lost flux energy by the usual source is therefore hereby shown to be a secondary consequence, and not the primary cause of the motion, and it is this supply of new energy which really generates the counter e. m. f., just as in the starting or increase of a current. This seems to be an explanation of the reasons why a counter e. m. f. is always generated when such a system generates mechanical motions.

Concerning the above proposed experiment with the two rings at absolute zero (the result of which can be definitely calculated, as shown in the Appendix, easier than it could be measured), it has been variously claimed by some that with zero resistance it is not possible to induce any current at all, and by others that even with the slightest induced e. m. f. the current would then be infinite; these are rather large differences of opinions. The facts are that at nearly 0 deg. Onnes induced a finite and limited current which persisted for a long time. For the purpose of the above argument it may be assumed that the conditions are like those in the Onnes experiment, therefore possible, and that the current was not decreased appreciably by the very small resistance, during the very short time of the motion.

That the current would not be infinite at absolute zero resistance nor enormous near to zero, is shown by the fact that the finite amount of energy spent in drawing the pole through the rings, is stored in the system as magnetic energy of an exactly equal amount; it could not possibly be any greater. Hence the current which would flow continuously is only that which by its ampere-turns and with the existing reluctance, would produce and maintain exactly that amount of flux energy. It is analogous to the energy of inertia stored in a spinning top, under frictionless conditions.

THE POTENTIAL ENERGY

One of the objections to using the term "potential energy" in the above electrical equivalent of the general physical law, is that it may not always be clearly and definitely understood. But a greater objection is that it may sometimes involve necessary and perhaps complicated explanations of apparent contradictions and ambiguities; the diminution and disappearance of the potential energy (the energy in the flux) is not always as readily apparent as it is in mechanics; especially as the process is generally masked by an immediate replacement of this energy by the source; in a motor the ultimate quantity of the flux energy remains constant; in some cases it is even greater after the motion than before. There is a generally accepted law (though as shown above it is not always correct) that a circuit always tends to move so as to embrace the largest possible amount of flux, therefore apparently directly contradicting this general law; the amount of the flux and of the flux energy however, by no means necessarily vary in the same ratio, the energy is the product, of the flux and the m. m. f.

A careful and unbiased analysis will, however, show that these contradictions and anomalies are only apparent and not real; it can be shown that any additional or new flux energy has in those cases been added by the source of current after the motion has taken place or while it is taking place; the motion itself may have made room for more flux or have decreased the reluctance. This increase of flux energy is a secondary result and not the primary cause of the motion, as seems to have been claimed; it may or may not take place. Similar anomalies and apparent contradictions occur in the application of the more general law to purely mechanical conditions, yet they do not in fact violate that law; for instance, the potential energy in the water in an elevated tank which is being kept full by a pump, will according to the general law, cause it to flow into and fill another neighboring tank and at the end of the operation there will be more potential energy than there was at the start. To avoid such anomalies and apparent contradictions it seems preferable to use a different term than potential energy in the electrical equivalent of this general law.

Maxwell (Art. 568) defines potential energy as a tendency to a change of relative position, and he sometimes treats the energy of magnetization as potential. He says furthermore (Art. 638) that he "assumed that the energy of a magnetic system is potential energy, and that this energy is *diminished* when the parts of the system yield to the magnetic forces which act on them." (The Italics are his).

To the writer it therefore seems that the generation of a counter e. m. f. is the best crucial condition in the electrical equivalent of that general physical law, to take the place of the reduction of the potential energy. That they are the equivalents of each other, or at least necessary accompaniments of each other, when there is a source, seems evident; the general physical law calls for such motions as will reduce the potential energy, the flux energy in the electrical case; this flux energy then must disappear as such when it is transformed into the kinetic energy of motion; as the current necessarily always generates as much flux as the reluctance of the surrounding space permits, it will at once replace any flux which has thus disappeared and, as is well known, the generation of any flux in a circuit is always accompanied by a counter e. m. f. The writer has applied this simple law to every case he could think of, old and new, and it never fails to apply correctly. If no exceptions can be found to it then it seems to be a universal law for the class of phenomena specified in it.

The Hall effect, like the skin effect, seems to deal with the shifting of the current paths in a conductor, as distinguished from a movement of the conductor itself. They therefore would have no direct bearing on this law except in so far as the changes of current paths may change the flux distribution which in turn may affect the forces.

Deductions From This Law

If this law is sustained, a number of deductions follow. It specifies no limitations to any particular directions of the forces which produce the motions, hence any directions are possible, provided only that the motion generates a counter e. m. f. somewhere in the circuit. It has been strenuously maintained that these forces could be only perpendicular to the conductor and never otherwise; it therefore opens up a formerly forbidden field. If for instance any particular motion is desired, it follows from this law that if that motion is such as would produce a counter e. m. f. then it ought to be possible to make the system produce it by providing the necessary freedom of motion; this has for instance been made good use of for giving molten metals a desired motion in some types of electrical furnaces.

Lengthening a current-carrying conductor (the stretch effect), reducing its cross section (the pinch effect), straightening it (the corner effect), winding it into a coil, etc., all produce a counter e. m. f. in a constant-current circuit and the circuit could therefore (theoretically at least) be made to produce them. According to Lenz' Law reversing any of these motions should generate current. Some of these deductions are further discussed below.

Longitudinal Force

One of the deductions from this law is of special interest because it has been most hotly contested; there is however also another proof of this deduction which is entirely independent of this law (and therefore aids in confirming the law) as will be shown in connection with Fig. 9. If a current-carrying conductor be lengthened (as for instance by stretching, sliding contacts, mercury troughs, etc.) new flux will be produced around the added part and the production of this will generate a counter e. m. f., as in any conductor in which a current is started. Hence it would follow from this law that the circuit itself could produce such a lengthening if the necessary freedom of motion exists. In other words, a current tends to stretch or lengthen its conductor, or more generally, there are longitudinal forces tending to move the conductor in the direction of its axis.

This force, the existence of which can readily be shown experimentally as described below, the writer years ago colloquially termed the "stretch effect," it being a complementary phenomenon to the "pinch effect" which tends to crush the conductor radially, both seem to follow the same laws and lead to the same ultimate result in liquid conductors by rupturing them, when strong enough.

This longitudinal force is also a consequence of conceiving that these electromagnetic forces have their real origin and abutments in the material of the conductor at the seat of the moving electron which causes them; that is, the forces in the field around a conductor are actions at a distance (as Faraday maintained). In Fig. 1 the lines *m m* represent the disks of flux around those parts of a conductor; such like lines of force are known to repel each other, and if the seats or abutments of these repelling forces are in the material of the conductor where the electrons *e*, *e*, which produce these lines are moving, the conductor will tend to stretch. The greater the density of this self repelling flux the greater this force.

Physicists have emphatically denied and many of them do so still, that such a longitudinal force exists or could exist.[2] Maxwell (Art. 507) described an experiment, due to Ampere, made under certain conditions, from which he generalized that no such force existed or could exist; the writer admits that it would not make its appearance under the conditions in that particular test, (in which the two longitudinal forces are equal and opposite), but that under different conditions to be described below, (Figs. 3 to 11), this force becomes very evident. Recognizing both forces means that a resultant in *any* direction is then possible.

This longitudinal force does not appear in the usual mathematical treatments of electromagnetic forces; the reason seems to be that such treatments were originally based on a special case in which this longitudinal component consists of two opposite forces which are exactly equal and therefore drop out of the mathematics; having once dropped out this force can of course never again reappear in any subsequent mathematical deductions which are based on this original case even when they are unequal, as they are in some of the experiments described below.

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[2] Northrup's Laws of Physical Science (1917), p. 152, par. 3. "If a wire carries a current no external magnetic force can so act upon the wire as to tend to make it move in the direction of its length."

These two new forces, the pinch effect and the stretch effect, are generally not great enough to crush or stretch solid conductors, (though they might perhaps act on the softened filaments of forced incandescent lamps), but with liquid or gaseous conductors like those in some electric furnaces or in the arc, they may become of vital importance, being sometimes quite formidable, and should therefore be recognized, studied, and included in mathematical treatments; to deny their existence might check possible progress and developments. That they are of practical importance is shown by the fact that hundreds of electrical furnaces are at present in daily successful use in which this longitudinal movement of the liquid conductor by its own current, is the absolutely essential factor.

Some physicists who declined to recognize this longitudinal force have endeavored to explain the very evident longitudinal movement in the writer's earlier experiments with liquids, as a hydrodynamic action of the liquid due to the pinch effect. But as some of the experiments described below show that this force will also move solid conductors lengthwise, they are a positive proof that the action cannot be primarily hydrodynamic in the liquid; in the writer's opinion the hydrodynamic action resides in the field itself.

Unlike the stretching of a compressed helical spring, this stretching theoretically continues indefinitely and is constant if the current and the cross section of the conductor remain constant, as in a mercury trough. As in all these phenomena, it will no doubt be found to increase as the square of the current. It is independent of the direction of the current or the length of the conductor. For the same current it becomes greater the smaller the cross section and can rupture the circuit by tearing it. It no doubt is greatest at the center of a round conductor and is least at the circumference. In a liquid conductor the motion is therefore greater along the central axis than near the circumference. Like a compressed helical spring a flexible circuit secured only at the two ends may form itself into the shape of the letter *C* as is often seen in the arc, or even like the letter *S*, if thereby it can lengthen itself (see Fig. 8). It has been suggested by a noted physicist that this might explain the sinuous character of a bolt of lightning.

A circuit making an angle at a hinged joint tends to straighten itself, due to this stretching force. This is usually explained by the action of the component of a force perpendicular to the moving part, and when it is straight this force which straightened it becomes zero. It can be shown however, that if the necessary freedom of motion exists at the joint the force which straightened it, namely the repulsion of the disks of flux, still continues and will then stretch it, showing the older explanations to have been wrong. A flexible circuit is known to expand; why should this same force cease when the conductor becomes a straight line; it does not; our former limitation to a perpendicular force has misled us. A straight soap bubble film (therefore having no radial pressure) has a tension just as the curved one has.

A straight conductor leading down into a large liquid conductor tends to be moved upward out of the liquid, if the proper freedom of motion exists, and new flux will appear around the lengthened part, causing the counter e. m. f. In general, where there exists any difference of flux density near a conductor there will be a tendency to produce a motion to equalize this flux density if that motion would generate a counter e. m. f. In experiments care must of course be taken that the force is not counterbalanced by an equal and opposite force somewhere else, which is generally the case. As will be shown below, the longitudinal force may be made very evident by making the two opposing forces unequal.

Ampere had referred to such a possible longitudinal force and gave some formulas; this is briefly discussed by Maxwell (Art. 526) and is not then denied by him, though he denies it elsewhere. In Art. 687 however he described the Ampere trough experiment (see Fig. 5) saying "This experiment is sometimes adduced to prove that two elements in the same straight line repel one another" (this is what the present writer calls the stretch effect) "and thus to show that Ampere's formula, which indicates such a repulsion of collinear elements, is more correct than that of Grassman, which gives no action between two elements in the same straight line." Maxwell adds that the experiment does not favor one more than the other; it is significant that he here does not deny such longitudinal forces, though he does elsewhere.

According to Maxwell (Art. 527) Ampere had claimed that the force between two elements of the circuit, (evidently meaning limited parts of a circuit, or of circuits), is along the line joining them, which Maxwell (Art. 527) considers the best assumption. For conductors inclined to each other, such lines joining their parts must necessarily be inclined to them, hence such forces *must* have longitudinal components. (See Fig. 9).

THE CORNER EFFECT

Many years ago the writer noticed that in a wide mercury trough which turned a right angled corner, there was great agitation of the mercury at that corner when a large current flowed, showing the presence of decided local forces; the current was not great enough to show a marked pinch effect in the straight parts; he called it the "corner effect." The writer's explanation was that as the amount of flux must of course be the same on the inside and outside of the corner, the density must be far greater on the inside part of the corner, therefore the forces on the mercury there must be far greater than those on the outside part, hence the agitation.

Recognizing the existence of the longitudinal force, this experiment can also be described as the mutual action of two currents whose conductors are inclined toward each other (see Fig. 9); the forces will of course be greatest where they are nearest to each other, in this case at the corner. When these two currents are unlike in direction (as in the above case) there should be a longitudinal flow of the conductor away from the corner, and for like currents toward the corner. An electric furnace based on this, for obtaining a unidirectional flow of the liquid conductor, operated as predicted, the motion being quite strong and rapid (*Jour. Frank. Inst.* Nov. 1921, Fig. 9 and p. 612).

QUANTITATIVE VALUE OF THE PINCH EFFECT

In the c. g. s. system the writer found (*Met. & Chem. Eng.* Vol. 9, Feb. 1911, p. 86) that the quantitative value of the pinch effect at the central axis of a circular conductor was $P = I^2/S$ in which P is the pressure in dynes per square centimeter, I the current in c. g. s. units, and S the cross section in square centimeters. This pressure is of course greatest at the center and least at the circumference. The fact that this is a unit relation in the c. g. s. system is of interest. It will be seen that the pressure is not proportional to the current density. As the current in a given liquid conductor is increased, an unstable state is ultimately reached at which this force increases automatically by contracting the conductor enough to rupture the circuit.

HYDRODYNAMIC ACTION IN A MAGNETIC FIELD

No one has yet deduced the quantitative formula for the longitudinal force, this force having dropped out of our present mathematical treatment of such forces. But the writer has reasons to believe it will ultimately be shown that these electromagnetic forces in a magnetic field act as though the medium, the ether, were like a liquid or gas, that is, they act hydrodynamically and therefore can turn corners, as it were. As they act on solid conductors also, the conductor itself need not be this hydraulic medium, as was formerly claimed by others.

If this is true, the quantitative formula for the stretching should be exactly the same as that for the pinch effect. Some years ago Northrup made a very careful and accurate measurement of what he claimed was the pinch effect, but as the force he measured was a longitudinal one (claiming that the liquid conductor, mercury, acted, as the hydraulic medium) the writer maintains that what he really measured was this stretching force. His measurements confirmed the formula of the pinch effect, hence the writer's conclusion that they are numerically equal. Maxwell (Art. 109) in discussing so-called static electricity says "the numerical magnitude of the pressure" (between lines of force of static electricity) "being equal to that of the tension" (along these lines) "and both varying as the square of the resultant force at the point." Many parallels between electrostatic and electrodynamic phenomena exist.

If these forces act as though the medium (not necessarily the conductor) were a fluid, the whole subject is simplified as only one of the two magnetic forces need then be recognized as the primary, either and preferably the tension along a line of force, the other, the repulsion of like lines, being a hydrodynamic resultant, just as the radial pressure on the side of a cylindrical pump is produced by and is exactly equal to the longitudinal pressure of the piston.

POLE MOVING ALONG A LINE

A third electromagnetic force is sometimes included with these two, namely the force which tends to move a single pole along a line of force; Faraday showed this in a very ingenious experiment (see Fig. 12). This force is however not a property of the field itself, but of a foreign body which is introduced into it, that is, the force is that between two independent fields.

There is a deduction from this experiment which seems to bear out one of the writer's contentions. It will be conceded that when a single pole moves along a line of force encircling a conductor, the equal and opposite reaction required by Newton's third law should tend to rotate the conductor around its axis in the opposite direction, which it does, as will be shown below. As there is no physical connection between the pole and the conductor it seems that the flux around a conductor must be considered as though mechanically connected with its conductor, or in other words that it is a case of action at a distance from the moving electron.

THE "SLIDING CONTACT"

The most unsatisfactory and most confusing element in our present system, and the most troublesome and discouraging one to the student, is the inevitable "sliding contact." Every book-writer and teacher seems to try to invent a new definition.[3] When new experiments conflict with the older laws, a new definition is at once invented to try to make them fit, and this seems to be quite satisfactory to many physicists. To the writer however the responsibility placed on the sliding contact has always appeared as a tacit admission that something is wrong, an admission of our ignorance or of some mistake.

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[3] As a good illustration the following is from a letter recently received from a professor of physics in a well known college, who is responsible for teaching many young men. "I think the only satisfactory way to treat such cases, and the way in which they usually have been tacitly reasoned out, is to imagine the sliding contact replaced by a very thin layer of conducting liquid in which the motion varies continuously from one surface of the layer to the other. Then everything becomes clear (!). . . Sliding contacts are to be treated as the limiting cases of a thin transition layer in which the motion varies continuously from one side of the layer to the other." He speaks of "what happens to the flux-linkage when the filament is broken by the sliding at a sliding contact. . . If you do not like my conducting fluid (in the above definition), then put in a short wire across the break and consider what happens during the thousandths of a second before the wire breaks." The circuit was never broken in the case referred to.

A well-known electrical engineer and able physicist defined a sliding contact as a "commutator with an infinite number of sections." Maxwell (Art. 491) says that in the case of a sliding contact the circuit "must be regarded as a system of two or of some greater number of circuits of variable strength," currents of even opposite directions.

All this could be avoided by the revision suggested below. The adequate definition of a sliding contact then reduces itself as it should, merely to "a contact which slides," and nothing more.

GENERAL LAW OF INDUCTION

Our laws of induction (Faraday's law of cutting lines of force and Maxwell's law of change of linkages in a complete circuit) were based on special cases. They have been invaluable and perfectly satisfactory when limited to those special cases. But trouble and misfits generally arise when the laws for one special case are applied to another special case; it is unwise to attempt it. Moreover as both are special cases can we depend upon it that together they cover all possible cases, without exception, that is, are they universal.[4] Has not our belief that they are universal perhaps checked development in other special cases. A single pole will move around a conductor, hence the reverse motion should generate a current. A magnet will rotate a conductor (Fig. 14) hence rotating the conductor near a magnet should generate a current. Contracting a current-carrying conductor, or expanding its cross section, or unwinding it from a coil, and perhaps many other odd motions, ought to generate current. Some of these cases of induction are difficult to explain by our present laws, except perhaps by means of special forced and involved definitions of a sliding contact. Our older laws would surely not have led us to them, though no claim is made here that those above mentioned are of any industrial importance.

The writer maintains that this state of affairs is not satisfactory and that we ought to look for a more fundamental general law of induction, of which the present ones are then special cases, to which they should then be limited. Such a general and presumably universal law could be based on a combination of the new law given above for motions and Lenz' law, that is, if any of these self produced motions are reversed, there will be induction. Lenz' law has presumably never been questioned, and the law of motion is nothing more than an electrical version of a well established and noncontested law of physics concerning energy; both are believed to be universal. In such devices as transformers the relativity principle is then also involved, in that it is immaterial whether the conductor moves across the flux or the flux (originating with the moving electron) moves across the conductor; there is relative motion in either case.

INTENSITY AND DIRECTION OF FIELD NOT SPECIFIC ENOUGH

It has been claimed that a magnetic field is completely specified by its intensity and direction. This means that if the source, either a magnet or a current-carrying conductor, moves in such a way that the field at any outside point does not alter its intensity or direction, then the field does not move, in the sense that it would not cause induction in a fixed conductor. Experiments seem to indicate that each line is attached to its source, the conductor, as the rim of a wheel is attached to its hub, except that the attachment is elastic. If so, the above definition of a field is incomplete and should include a reference to the motion of the source, even if such motion does not alter the intensity or direction at any specific point. See also Figs. 15 and 17.

PRESENT MATHEMATICAL TREATMENT NEEDS REVISION

Our usual mathematical treatment has misled us in creating a very positive and nearly universally accepted belief that the only force which can possibly exist is one perpendicular to the conductor, like the one in the definition of the unit of current. This has checked progress. The usual mathematical treatment should not have been based on a special case in which the longitudinal force happens to fall out in the mathematics.

A new mathematical treatment should now be devised in which the longitudinal force is recognized. The mathematical short cut based on integrating around the "complete circuit" which often involves quite unnecessary complications, should be limited to those many cases in which it is useful. It should be accompanied by a mathematical treatment (far more difficult to devise) based on limited parts or elements of circuits (as proposed by Ampere) and on their relative inclination and distance apart. When a desired force is like that between two inclined conductors, by far the greatest part is where they are nearest together, and we should therefore be able to determine how long it is economical to make such conductors.

It is believed that such a system could be based on the assumption of only a single magnetic force, the tension along a line of force; then deduce the other, the repulsion of like lines, on the basis that there is an action analogous to that in hydrodynamics and that therefore this one force can produce motions in *any* other direction depending only on the freedom of motion which exists. It should also be based on the forces being actions at a distance, the abutments of these forces being in the material of the conductors and in that part in which the corresponding electron is moving.

EXPERIMENTS

The following experiments, mostly original with the writer, illustrate some of the facts, opinions and departures expressed above. The results in many of them were in accordance with the predictions based on these opinions and contrary to opinions based on the older laws. Gospel faith in the older laws would not have led to them. Some of them have been published before and are reproduced here (by request) to illustrate the deductions from them. Many of them have been discussed for some time with able physicists whose comments have been considered, though generally when they could not justify them with the older laws they ignored the experiments.

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[4] That Maxwell considered his law to be universal is indicated in Art. 541 in which he says we can "enunciate completely the true law of magneto-electric induction" in this way.

The apparatus was mostly crude and improvised though the results were always quite definite and decided. No claim is made that this is a thorough and completed research, nor were any quantitative measurements made. The experiments should be considered merely as a preliminary investigation.

As it is often not possible to multiply these forces by using coils of many turns, the currents required are generally rather large and cannot be kept flowing long on account of the heat. About six storage batteries of the usual portable laboratory type, and a low volt transformer giving up to about 2000 amperes, were generally used; in some cases a dry cell current suffices.

In Fig. 3, *W* is a long horizontal wire suspended so that it can move lengthwise. One end dips into the center of a large dish of mercury *O*, in which, therefore, there was practically no change in the flux when the wire moved slightly. The other end dips into the narrow mercury trough *M*, in which a force to stretch or lengthen that electrically elastic part of the conductor could manifest itself. When current was passed the wire moved decidedly to the right showing the tendency of the part *M* to stretch. Assuming a constant current, new flux was produced in the part which was added to the mercury trough part of the circuit by this lengthening; and it is in this part that the counter e. m. f. must have been generated. Moderately large currents must be used unless the apparatus is light and sensitive, though they should not be large enough to cause an appreciable pinch effect in the part *M*. The vertical parts were made very short and can be neglected (see Fig. 6). Care must be taken to have the whole circuit in a vertical plane, or there will be a disturbing side motion of *W*.

Fig. 4 described over a decade ago is of interest because without exception every one of numerous physicists gave the wrong answer as to the direction of motion, showing how badly our older law had misled us. A horizontal wire *W* is pivoted at *P*, the other end being free to move in a curved mercury trough *M*. It was placed in the position as shown before the current was started; with current it moved in the direction shown, which is the contrary one to that called for by the older law that a circuit tends to expand. The enclosure of the circuit *decreased* instead of increasing. The writer's explanation is that the only part of the circuit which has a freedom of motion to lengthen itself, namely the mercury part, did so. When started on the other side of the "dead point" it of course moved in the opposite direction, for the same reason.

Fig. 5 is a modified form of the classical Ampere trough experiment. *B* is a bridge wire floating on the two mercury troughs *M M*. In the original experiments by Ampere the battery circuit was placed at the left as shown in dotted lines, and the resulting movement of the wire *B* to the right was attributed to the old law that the circuit tends to enlarge itself; also to the repulsion due to that part of the battery wire which is parallel to the bridge wire. The writer attributes the motion to the tendency of the mercury parts of the circuit to lengthen, owing to the lateral pressures of the disks of flux around them; they are the only parts in which this force is free to act. This was shown to be the correct explanation, and the older explanations were shown to be wrong, by reversing the position of the battery circuit as shown in full lines, as the motion was then in the *same* direction, as it should be, while according to both the older laws it should have been reversed. The battery wires should not be too near the troughs.

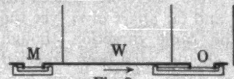

Fig. 3

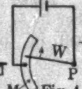

Fig. 4

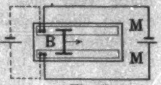

Fig. 5

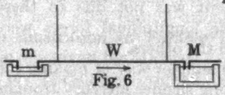

Fig. 6

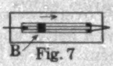

Fig. 7

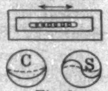

Fig. 8

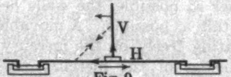

Fig. 9

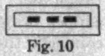
Fig. 10

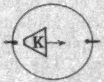

Fig. 11

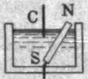

Fig. 12

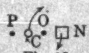

Fig. 13

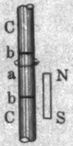

Fig. 14

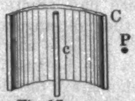

Fig. 15

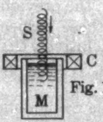

Fig. 16

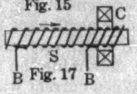

Fig. 17

It is of interest that Ampere himself thought there was some such longitudinal force in this experiment, but his followers, notably Maxwell, were not convinced; it did not fit in with Maxwell's mathematics.

An interesting feature about this experiment is that, assuming a constant current, the counter e. m. f, is evidently generated in the parts which are added to the lengths of the trough part of the circuit, as these are the only parts around which new flux has been generated, hence it is not essential that the counter e. m. f. must be generated in the part which moves, namely *B*; a motion may therefore be produced if a counter e. m. f. is thereby generated in *any* part of the circuit. While *B* is the part which moves, the forces which move it reside entirely in the mercury trough parts, and not in B itself, as is generally claimed.

In Fig. 6, a long horizontal wire W suspended so that it has freedom of motion lengthwise, has one end dipping into a mercury trough M the cross section of which is large, and the other end into one (m) in which it is small. For the same current the flux encircling the large cross section in M is of course less in amount than that around m hence the lateral repulsion of this flux in m should be greater than that in M; the wire W should therefore move to the right, which it does quite decidedly.

It was, of course, claimed by the upholders of the old law that this was due to the repulsion of the vertical parts entering the troughs, even though they had been made very short. These were therefore placed as shown, close together in M and far apart in m, yet the movement was still in the same direction though slightly less in intensity; it should have been reversed if caused by those vertical parts. This shows conclusively that the movement was not due to these vertical parts and that the longitudinal forces were greater as they overpowered the others.

Fig. 7 shows a better and simpler way of making the same experiment. Three long slender mercury troughs were made as close together as possible, the walls between them being made as thin as possible. A movable copper bridge piece B floating freely on the mercury, connects them. The meniscus of the mercury projected slightly above the troughs so that the bridge piece had practically no vertical component. Both of the right-angular parts of the bridge piece were therefore made so short that they could be neglected; this was done because it is these right angular parts that the upholders of the older theory depend upon; the previous experiment in Fig. 6, however, in which these perpendicular parts were much longer showed conclusively that even when very much longer than in Fig. 7, they did not determine the motion.

When current was passed into the middle trough and out of the outer two (or all three) the bridge piece moved very decidedly and quite rapidly to the right. The motion appeared to have a constant velocity. The explanation is the same as in Fig. 6, that the flux around the smaller cross section was much greater than around the larger one, hence the longitudinal force of the former overpowered that of the latter. This apparatus is in a more convenient form for a lecture room experiment.

In Fig. 8, a well amalgamated copper chain was laid in a much longer mercury trough in a contracted position so that it could stretch. When the current was passed it immediately and decidedly stretched to its extreme length. Copper being a far better conductor than mercury most of the current passed through the copper, which having a far smaller cross section had much more flux around it than there was around the ends of the mercury trough, hence the longitudinal force overpowered that at the ends. As the copper is likely to be sucked under by the pinch effect (where the stretch effect is also stronger) and disappear while the current is flowing, it is well to float the ends with some light non-conductor.

This experiment also meets the claims sometimes made that in some of the movements in these experiments it is the pinch effect which by its hydrodynamic action in the mercury causes the motion. If the pinch effect were the cause, it would act to push the ends of the chain towards the middle, but the fact is that the movement is in the *opposite* direction. This experiment which requires large currents, might perhaps be more conveniently carried out with much smaller currents by suspending the chain from floats in an electrolyte.

When this chain was laid in a contracted form in circular mercury dishes C and S with its ends secured near the terminals, and in the positions shown by dotted lines, it stretched itself into the positions shown in full lines when current was passed, as a compressed helical spring would do. Repeated short applications of the current seemed to act better.

Fig. 9 is a modified form of an old experiment attributed to Faraday or perhaps to Ampere. It furnishes a different and independent proof of the longitudinal force and one which it is difficult if at all possible to meet by the older laws. In the original a vertical conductor V was mounted so that it could move to the right or left parallel to itself. It contacted with a horizontal wire H which was stationary. When currents were passed in the directions indicated the movable wire V moved to the left.

The writer maintains that as the movement of V was caused by the current in H, then if the apparatus be reversed so that V is fixed and H has a freedom of motion in the opposite direction, the same force would move H in the direction of its length, which it did, thus showing the existence of this strongly denied longitudinal force. This must follow from Newton's third law, that for every action there is an equal and opposite reaction. It also must follow from the view of Ampere and others, apparently endorsed by Maxwell (Art. 527), at least not denied by him, that the force between two elements is along the line which joins them, as shown by the diagonal line. If so, such a direction must have components in the directions of the lengths of both conductors.

It seems strange that although this experiment, the law of Newton, and the views of Ampere, have been known for the past hundred years, this method of proving the existence of the longitudinal force has apparently not been considered before, or if it has it has certainly not been generally known, or had been forgotten, and is still being strongly contested.

In the writer's modification the wire V was fixed and the long wire H was suspended so that it had a freedom of motion lengthwise. When the currents were passed in the directions shown, the wire H moved to the right, or when one of the currents was reversed, then to the left. The contact between the two was made with a small mercury trough carried by *H*. When *H* was fixed and *V* allowed (by means of a mercury trough) to move in the direction of its length, it so moved away from *H*. Before the experiment a current was passed through *H* alone to make sure that no motion was caused by the very short vertical parts that dip into the mercury dishes; moreover the final motion was again in the opposite direction to what it would be if it had been caused by these vertical parts as most physicists will claim, because the current in one pair of ends is necessarily greater than in the other.

It will probably not be denied that the forces involved are concentrated almost entirely at or near the corner where the conductors are nearest together; it is therefore a weak argument to make a crucial point, as has been done, of where the "rest of the circuit" is. Beyond a few inches from the corner the circuits have probably an entirely negligible effect on the forces, and it therefore does not matter where they are. Some physicists have "grabbed at straws" to uphold the older laws, instead of being helpful in trying to improve them.

This same test was also made and exhibited by the writer some years ago, in a different way, resembling more closely the apparatus existing in many physical laboratories to show the original experiment of Faraday. *H* was a stationary circle and *V* moved around this circle. In the modified form *V* was fixed and the circular part moved. Sliding contacts were used to replace the usual liquid conductor and this caused much friction, but still the movement was quite decided, and was witnessed by many.

In Fig. 10 three short pieces of well amalgamated copper wire were floated on a mercury trough in a straight line with their ends close together. When alternating current was passed they separated a short distance as shown, but would move no farther even with repeated applications of the current. This seemed to indicate that the denser field around the copper pieces was, as it were, elastically connected to its conductor and that the repulsion of like lines made it extend itself a slight distance beyond the ends.

Fig. 11 shows an interesting difference between a direct and an alternating current. A kite shaped piece of well amalgamated copper was floated as shown on a circular dish of mercury. The idea was that the denser flux around the main path near the small end would produce a greater longitudinal force than the less dense flux around the much larger main path at the larger end, thus causing the piece to move to the right. With direct current it moved every time the circuit was closed but only for a very short distance and then stopped. An alternating current ought therefore to move it continuously, which it did, across the whole bath and at apparently a constant velocity, always in the direction of the large end forward. This experiment showed that the motion was not due to the pinch effect, as had been claimed, though if the current becomes very great the alleged hydrodynamic resultant of the pinch effect in the liquid conductor will contribute to the motion.

Fig. 12 represents Faraday's ingenious experiment to show that a single pole *N* of a magnet moves along the circular lines of force around a fixed current-carrying conductor *C*. This experiment does not seem to be described in his well known Experimental Researches, but was published by him in the *Quart. Jour. of Sciences*, XII, 186, and again with an illustration in the next issue 283; the date seems to be about 1822, just 100 years ago; this description is included in a later collection of his works. Maxwell (Art. 486) refers to Faraday's experiments and shows a figure (Art. 491) which however is totally different from the original and the explanation accompanying it is difficult if at all possible to understand, owing apparently to Maxwell's efforts to include an involved explanation of a sliding contact, necessitated by his mathematical treatment. Faraday's own explanation is very simple and is not burdened with any such complications. The writer could find no illustration in Faraday's works corresponding to the one given by Maxwell.

In Faraday's original experiment a permanent magnet *N S* was floated in a cup of mercury in a slanting position as shown in Fig. 12, with one pole tied with a string to the center of the dish at the bottom. The current entered through the central fixed conductor *C* and left through the center of the bottom. When current flowed the free pole moved in a circle around the wire *C*.

The writer repeated the experiment, and found that apparently the chief force was a couple, as shown in Fig. 13, which represents a top view of the upper part of Fig. 12; the conductor *C* and magnet pole *N* each endeavored to revolve around the other, as Faraday showed. If linked together and pivoted at *O* they should therefore revolve around *O*; if the link is pivoted at *P* they should revolve less forcibly around *P*, due to the difference between the two torques; these were not tried.

When the conductor is revolvably mounted on its axis it ought to revolve when the magnet is stationary, according to Newton's third law. The revolving force is then tangential, like the pull on a string wound around the wire, and its effect therefore becomes very small when the diameter is small. By making its diameter large relatively to the distance to the magnet, especially when it is made hollow, the writer obtained decided rotation.

It is of interest that according to Lenz' law when any of these motions are reversed by an external force, a current should be generated.

A result of some interest is that when this experiment is arranged as shown in Fig. 14 in which a large (preferably hollow) fixed conductor *C C* has a part *a* mounted so that it can revolve on its axis, (*b b* being sliding contacts like a layer of mercury), it revolves when the magnet is placed as shown, parallel to the conductor.

If the magnet were mounted so that it could move around the conductor keeping parallel to itself, it would seem to have to follow that when the part *a* is forced to rotate there would be some movement of the magnet; this was not tried.

Another deduction is shown in Fig. 15. We are told that for the same current in the small conductor *c* and in the large hollow one *C* the magnetic effect on a point *P* is the same, being the same as though the current were all concentrated at the axis. Yet, as shown above, if a magnet pole be placed at the point *P*, it will easily rotate *C*, but *c* only with great difficulty. And if the current was concentrated at the axis, it would take an infinite force to revolve its conductor. Hence there is some physical difference between the fields around those two conductors and our older law leads us away from rather than toward this difference.

In Fig. 16, *S* is a solenoid mounted so that it can move down into a mercury cup *M*. *C* is a stationary coil. When like, constant, currents were passed through both, *S* moved downward decidedly. It thereby cuts out some of the ampere-turns, which reduces the flux generated by it; the reluctance of the combination will also be decreased somewhat but presumably in a less proportion, and if so the total flux of the combination would be *less* after the motion than before, thereby contradicting the older law that the flux increases, which it surely does not do in this case. Moreover, the self inductance is surely *less* after the motion than before; the writer had no means for measuring it. The two coils may be assumed to be in series. As actually carried out the mercury was replaced by a fixed brush, as in Fig. 17.

In Fig. 17 the conditions are similar except that the current is now led into the solenoid *S* by a fixed brush, *B*; the other brush *B* at the coil was the same as before. To avoid cutting out and in a whole turn at a time, these brushes may form a complete circle around the solenoid, therefore acting like the surface of the mercury. In this arrangement the magnetic field of the combination remains exactly the same everywhere in intensity and direction, and there is no contraction of the flux as a whole, yet there was very decided motion. The self-inductance evidently also remains constant. There is no commutation. If the lines of force are not considered to be in some way attached to the moving electrons which produced them the troublesome sliding contact must again be called to the rescue to explain it. In the writer's opinion new flux is generated at the distant end of the solenoid as the current is started in new parts of the conductor, this is strongly attracted by the coil *C*; the flux at the near end which is being cut out, exerts little or no attraction to *C*; a part of the flux which was generated by both coils together disappears as such by being converted into the kinetic energy of the motion. The flux therefore seems to move with the coil.

Fig. 18 represents a top view of a mercury trough *M* ending in two large reservoirs of mercury, to which the current connections were also made. When currents is passed the mercury in the trough should stretch thereby moving into the end reservoirs, the flux around it being far denser than around the end reservoirs. It did so, as was shown by a rapid flow of mercury from the middle reservoir *R* through the short connecting channel and into the trough. In this case there was absolutely no perpendicular component of the circuit on which the upholders of the older laws depend. That this motion was not due to the pinch effect, as claimed by others, is conclusively shown by the fact that if it were, the mercury in the small connecting channel should have flowed in the *opposite* direction; the action of the pinch effect would be a compression of the liquid in the trough, which would cause it to flow into all three reservoirs.

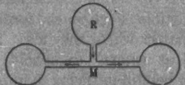

Fig. 18

GENERAL

It has been claimed that some of these experiments are "tricky" and therefore are not worthy of notice. The writer however maintains that a law to be universal must not have a single exception; if it has then possible progress in new fields is forbidden by it.

The writer desires to express his thanks to the departments of Physics and of Electrical Engineering at the University of Pennsylvania for the courtesy of extending to him the use of their laboratories for demonstrating some of the experiments. Also to Prof. R. E. Brown of Philadelphia for his encouraging cooperation in some parts of these researches. Also to the many physicists and electrical engineers whose antagonistic attitude during the past decade led the writer to a number of new and apparently significant experiments.

CONCLUSION

As a basis for discussion the writer suggests tentatively the following general outline of a proposed revision, in logical sequence, of the older laws, and of their mathematical treatment, assuming that the correctness of each step has been, or will first be, sufficiently conclusively demonstrated.

Starting with the tension along a straight line of force (the pull between magnets) as the most basic fundamental, deduce from it the repulsion of like lines and the radial force on the conductor, from the principles of hydrodynamics; then the two forces in a single conductor, the pinch effect and the stretch effect, which are independent of the length; then those between unit lengths of conductors, and those between magnets and unit lengths of conductors. Omitting the ether, recognize that the forces in a field are an action at a distance, having their abutments in the material of the conductors or magnets, and that therefore the imaginary lines of force must be picturized as having some physical elastic connection with their source, the moving electron. Formulate in electrical terms a general law of motion (like the one suggested above) based on the long established general law in mechanics concerning the self-conversion of potential into kinetic energy. Finally on this law, combined with the Lenz law, base the general law of generators, that is of induction.

Appendix

The following is the mathematical solution by Prof. Richard E. Brown of Philadelphia, of the problem referred to above. The problem is: two like copper rings on the same axis, are reduced to 0 deg. absolute, at which their resistance is zero; like and equal currents are induced in them by drawing a magnet through them once, which currents will then continue to flow indefinitely (there is no outside source of supply); the energy put into the system is stored in the magnetic field. Let their attraction move them nearer to each other, thereby setting free kinetic energy at the expense of their potential energy; their flux energy and the currents will thereby have been reduced; (the reverse motion would increase the currents.) The question is, will the total flux of the combination have been reduced thereby.

The combined flux energy of two like coils carrying the same current, as though connected in series, is $1/2\, L\, i^2 = 1/2\, l\, i^2 + 1/2\, l\, i^2 + M\, i^2$ in which i is the current, L the self-inductance of the combination, l that of each coil separately, and M the mutual inductance; the sign of $M\, i$ is plus because the fluxes are in the same direction.

The total flux linking any coil is: self-inductance multiplied by the current and divided by the number of turns; there is only one turn per coil in this case.

Assume the current carrying rings to be so far apart that there is no mutual inductance, and then to move toward each other until they coincide (theoretically), the current i being at first assumed to be maintained constant during this motion by some outside source, hence at the end there will be one coil of two turns; they are at first assumed to be filamentary, having no thickness.

When the rings are far apart (no mutual inductance):
magnetic energy $= 1/2\, l\, i^2 + 1/2\, l\, i^2 = l\, i^2$
self inductance of combination $= L = 2\, l$ because the energy $1/2\, L\, i^2 = l\, i^2$
total flux of combination $= l\, i + l\, i = 2\, l\, i$

When the rings coincide:
magnetic energy $= 1/2\, l\, i^2 + 1/2\, l\, i^2 + l\, i^2 = 2\, l\, i^2$
self inductance of combination $= L = 4\, l$ because $1/2\, L\, i^2 = 2\, l\, i^2$
total flux of combination $= 4/2\, l\, i = 2\, l\, i$.

Hence when the current is assumed to be maintained constant, the total flux remains constant during this motion, and the magnetic energy would be doubled, being supplied from the assumed source which maintained the current constant. But in the above problem there is no such source and there must of course be less magnetic energy in the combination after the motion than before by the amount of kinetic energy given off by the self produced motion. Hence the current must have *decreased*, and as the total flux was shown above to remain the same when the current is kept constant, it follows that with a decreased current the total flux will have *decreased*.

The writer here calls attention to the fact that in the above the self inductance of the combination has been doubled by this motion, on the assumption that at the end there is one coil of two turns. The flux and flux energy, however, and therefore the forces, are quite independent of whether there are then two turns with a current i or one turn with a current $2\, i$, as for instance if the rings were of mercury or of liquids in a furnace. In the latter case, the self inductance would be halved at the end of the motion instead of doubled. This shows again that the self inductance is a wrong term to use as a crucial one in such a law; also that in practise the self inductance may actually decrease. The fact that in practise when two such material rings coalesce (the single ring having double the original current per ring) the flux will be somewhat less than double that of a single ring, (because both the cross section and the current have been doubled) shows that the self-inductance may be even still less than a half.

In the case of the attraction of two permanent magnets, it seems self-evident that not only the total flux energy but the total flux itself, of the combination, is *less* after the motion than before, thus disproving the older law that it always increases. At the neutral part of such a magnet, if it is a good one, it is presumably saturated, hence after they are in contact the flux in that part of each one cannot be assumed to have practically doubled, which would have to be the case if the total flux did not change, or more than double if it increased with the motion.

Prof. Brown's tests confirmed this in the following way though he says the tests were somewhat crude. By the total flux in a *U*-shaped permanent magnet is here meant the flux in the magnet at its netural point (at the curved part). This was measured approximately by means of a galvanometer and a coil around the keeper, while it was being attracted; call this *F*. The change of flux at the neutral point of one of the two like magnets was then measured while the other was moved into contact with the first; call this *f*. This quantity *f* was found to be less than *F*, showing that the total flux of the combination was less after the motion than before, that is, less than double that of one magnet. Some flux as well as flux energy has therefore disappeared.

Bibliography

The following is a list of the writer's papers and articles bearing on parts of the subject matter of the present paper.

A Practical Limitation to Resistance Furnaces; The "Pinch Phenomenon." *Trans. Amer. Electrochem. Soc.*, Vol. 11, May 1907, p. 329.

An Imperfection in the Usual Statement of the Fundamental Law of Electromagnetic Induction. TRANS. A. I. E. E., Feb. 1908, Vol. 27, Part 2, p. 1341.

A new Factor in Induction; the "Loop" vs. the "Cutting Lines of Force" Laws. *Elec. World*, Vol. 51, March 14, 1908, p. 559.

The Working Limit in Electrical Furnaces due to the "Pinch Phenomenon." *Trans. Amer. Electrochem. Soc.*, Vol. 15, May 1909, p. 255.

The Stretching of a Conductor by its Current. *Jr. Frank. Inst.* Jan. 1911, p. 73.

Formulas for Pinch Phenomenon. *Met. & Chem. Eng.*, Vol. 9, Feb. 1911, p. 86.

A New Type of Electric Furnace. *Trans. Amer. Electrochem. Soc.*, Vol. 19, April 1911, p. 255.

Electromagnetic Forces in Electric Furnaces. *Trans. Amer. Electrochem. Soc.*, Vol. 39, April 1921, p. 313.

Revision of some of the Electromagnetic Laws. *Jour. Franklin Inst.* Nov. 1921, p. 599.

Longitudinal Electromagnetic Forces. (Note). *Science.* Nov. 18, 1921, p. 492.

A Neglected Electromagnetic Force, *Chem. & Met. Eng.*, Mar. 15, 1922, p. 513.

A New System of Electromagnetic Forces Needed. JOUR. A. I. E. E. April 1922, p. 305.

Electromagnetic Motions in Electric Furnaces. Paper read before the *Amer. Electrochem. Soc.* in April 1922; to be published in Vol. 41.

Discussion

ELECTROMAGNETIC THEORY AND ITS BEARING ON THE PINCH AND STRETCH PHENOMENA

C. O. Mailloux: The features of commanding interest in Dr. Hering's paper are the "pinch" and "stretch" phenomena, both discovered by him; and the difficulty of explaining these phenomena by the existing theories of electromagnetic fields is what has led Dr. Hering to urge the revision of these theories.

The discussion of the paper must, at the outset, recognize the polemic situation due to the very interesting and significant circumstance of the apparent antagonism to the paper, which has resulted from the necessity of questioning the completeness and adequacy of certain theories, notably those of Maxwell, in regard to electromagnetic fields, because, as will be seen, the limitations of Maxwell's theories are of the greatest importance in this case. It is as if it was not known that flaws have been found, even in great numbers, in Maxwell, years and years ago, and by men who were Maxwell's peers,—Hertz, Henri Poincaré, Helmholtz, Heaviside, Bjerkness, etc. When facts and theories fail to agree, the theories usually have to yield ground in the end. It is the speaker's opinion that the facts have the best of it in this case.

It seems hard to believe that any teacher could expect unquestioning faith in Maxwell today, in the light of the commentaries and revisions made by the men just named, to mention no others. For over twenty-five years Maxwell's treatise, though a great classic, and a valuable work of reference, has been obsolete as a text book, and as a means of presentation of Maxwell's own theories. The reasons for this were made very clear by the very man—Hertz—whose research work on electric waves supplied experimental evidence of the accuracy of Maxwell's electromagnetic theory of light, and helped to prove that theory.[2] Hertz's two papers on the "Fundamental Equations of Electromagnetics," published in *Wiedemann's Annalen*, in 1890, (and which constitute Chapters XIII and XIV of the English translation published in 1893), were an attempt to simplify and clarify Maxwell's equations and to make them more consistent with Faraday's fundamental idea that the "field of action" of energy was an intervening medium known as the ether. As Hertz put it, Maxwell frequently wavered between the conceptions that he found in existence and those at which he arrived himself. Hertz points out that between the concept of direct action at a distance, springing across space, and the concept of an action which is propagated from point to point in a hypothetical medium, four distinct methods of reasoning and mathematical treatment can be distinguished, and that Maxwell, in different parts of his treatise, leans towards two of these modes of thought, each involving a different conception of "electricity." He says: "And so, unfortunately, the word 'electricity,' in Maxwell's work, obviously has a double meaning." After giving the two meanings, he adds: "If we read Maxwell's explanations and always interpret the meaning of the word 'electricity' in a suitable manner, nearly all the contradictions which at first are so surprising can be made to disappear. Nevertheless, I must admit that I have not succeeded in doing this completely or to my entire satisfaction."

Poincaré expressed a similar opinion in his discussion of Maxwell's theories in Vol. I of his "Electricité et Optique." Also see on this point Poincaré's "La Science et l'Hypothése," page 250 and page 259.

While Maxwell is always given the highest credit by all authorities for the marvelous gigantic work done by him as a pioneer in the development, formulation, and demonstration of new theories and principles, it is admitted generally that his work was not so complete and perfect as to require no ulterior revision or modification.

The work of revision of Maxwell's equations was begun as early as 1885 by Heaviside, and the importance and value of Heaviside's contributions are admitted by all those who have participated in the work since then. For twenty years or more it has been considered necessary, in order to obtain a complete understanding and a satisfactory interpretation of Maxwell's theories, to discard Maxwell's own methods, and to resort to more modern methods of presentation which include the ideas of Heaviside, of Hertz, Poincaré and the other eminent authorities who have gone into the subject exhaustively. The study of Maxwell without the guidance of these authorities is apt to be misleading and harmful. Admiration of the magnificent structures erected by the genius of Maxwell does not make it a duty for his admirers to close their eyes to certain defects in these structures, or to insist upon their being also acknowledged as perfect anyhow. We would not proceed far before realizing the absurdity of such a policy.

Thus, Hertz has called attention to rudimentary ideas of a physical and of a mathematical nature ("vector-potential," being cited as an example of the latter), used by Maxwell, which are now regarded as being superfluous and really encumbering scaffolding. He says, significantly, that there is no object in replacing the *forces* themselves by *potentials* unless a mathematical advantage is gained thereby. He sees no such advantage in the introduction of the vector-potential in the fundamental equations; and he adds:

"Furthermore, one would expect to find, in these equations, relations between the physical magnitudes which are actually observed, and not between magnitudes which serve for calculation only."

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[2] Although Hertz was one of the first workers in this field, his first experimental determinations of the velocity of propagation of electromagnetic waves gave erroneous results—only about 200,000 km. per second (instead of 300,000)—which threw serious doubt upon the correctness of of Maxwell's equations. Blondlot, in France, was the first physicist to obtain experimental results by two independent methods that were in complete accordance with Maxwell's theory. Hertz' experiments confirmed these results subsequently.

It is for such reasons that vector-potential has been eliminated altogether, and that other concepts have been modified in the modernized form of Maxwell's equations.

Having thus brought out the now recognized fact that Maxwell's work was *not* invulnerable, but that it *was* weak in spots, and has had to be retouched, we may next consider the fundamental limitations of the Maxwellian theory, to see if perhaps too much has not been expected of it.

Poincaré, the peer of all his contemporaries in pure and applied mathematics, and the greatest genius of all time in mathematical philosophy and logic, said:

("La Science et l'Hypothèse," page 249):

"Maxwell does not give a mechanical explanation of electricity, and of magnetism.

He merely shows that such an explanation is possible."

The notes obtained from some courses of lectures on Maxwell's theories presented in the revised form above mentioned, which the speaker attended many years ago, as a graduate student, contain the following interesting commentary:

"It should be stated and understood at the outset that the Maxwellian theory does not tell us anything of the true ultimate nature of electricity and magnetism, or of electromagnetic processes. What it tells us is how certain vectors, namely, vectors of electric and of magnetic force and flux, vary in space and in time. It furnishes a fairly complete *geometrical* theory of the space and time relations of these vectors and of the forces or fluxes which they represent. While it tells us of these things to a satisfactory degree of completeness, it gives us only limited information—satisfactory as far as it goes, but not going far enough—regarding the *dynamic* theory of these forces and of the phenomena in which they are involved."

These views were confirmed and corroborated in convincing manner by Prof. V. F. K. Bjerknes, in his lectures on "Fields of Force" delivered at Columbia University in 1905, which the speaker attended. In his introductory remarks (See the printed account of these lectures, Publication One of the E. K. Adams Fund for Physical Research, *Columbia University Press*, 1906), Prof. Bjerknes said:

"The idea of electric and magnetic fields of force was introduced by Faraday to avoid the mysterious idea of an action at a distance. After the victory which Maxwell's theory gained through the experiments of Hertz, the idea of these fields took its place among the most fruitful of theoretical physics. And yet if we ask, what *is* an electric or a magnetic field of force? No one will be able to give a satisfactory answer. We have theories relating to these fields, but we have no idea whatever of what they *are* intrinsically, nor even the slightest idea of the path to follow in order to discover their true nature. Above all other problems which are related to fields of force, and which occupy investigators daily, we have, therefore, the problem of fields of force, namely, the problem of their true nature."

The method of research followed by Prof. Bjerknes consisted in generating what he termed "*hydrodynamic*" fields of force in masses of water by means of motion of vibratory nature, produced by pulsating and oscillating bodies, and then tracing analogies between the phenomena of attraction and repulsion produced in these "hydrodynamic" fields and those produced in electric and magnetic fields. By means of beautiful experiments he showed that the lines of magnetic force in a magnetic field can be represented in perfectly analogous manner by lines of oscillation in a hydrodynamic field. The hydrodynamic field experiments produced phenomena resembling magnetic attraction and repulsion so closely as to suggest a very close analogy, if not an identity, between the physical principles involved in both cases; and explanations of electric and magnetic phenomena were sought by the aid of these analogies. It was found, however, that the analysis and interpretation of the phenomena was much less difficult in the case of the hydrodynamic field than in the case of the electric and magnetic fields. Prof. Bjerknes says:

"The extent of our knowledge of the different kinds of fields differs greatly. All the properties of the hydrodynamic fields follow directly from the most trustworthy laws of nature, that is from the principle of the conservation of the mass, and from the principles of dynamics. With reference to electric or magnetic fields, on the contrary, we have only formal theories. First we have an extensively developed *geometric theory of the distribution in space of the vectors*, which describe the field. And then, in a more or less superficial connection to this geometric theory, we have a very much less developed theory of the dynamic properties of the fields.

Taking the facts as they lie before us, we shall be obliged, therefore, to give to our theory a dualistic form, comparing separately the geometric and the dynamic properties of the two kinds of field. It may be reserved for the future to penetrate to the central point, where the geometry and the dynamics of the question are perfectly united, and thus make the comparison of the two kinds of field perfectly easy."

In reference to the dynamics of the electric or the magnetic field he says:

"Our knowledge of the dynamics of the electric or magnetic field is very incomplete, and will presumably remain so as long as the true nature of the fields is unknown to us.

"What we know empirically of the dynamics of the electric or magnetic field is this:—bodies in the fields are acted upon by forces which may be calculated when we know the geometry of the field. Under the influence of these forces the bodies may take visible motions. But we have not the slightest idea of the hidden dynamics upon which these visible dynamic phenomena depend."

Faraday's idea, for instance, of a tension parallel to, and a pressure perpendicular to the lines of force, as well as Maxwell's mathematical translation of this idea, is merely hypothetical. And even though this idea may contain more or less of the truth, investigators have at all events not yet succeeded in making this dynamical theory a central one, from which all the properties of the fields, the geometric, as well as the dynamic, naturally develop, just as, for example, all properties of hydrodynamic fields, the geometric as well as the dynamic, develop from the hydrodynamic equations. Maxwell himself was very well aware of this incompleteness of his theory, and he stated it in the following words:

"It must be carefully borne in mind that we have only made one step in the theory of the action of the medium. We have supposed it to be in a state of stress but have not in any way accounted for this stress, or explained how it is maintained. I have not been able to make the next step, namely, to account by mechanical considerations for these stresses in the dielectric."

In spite of all formal progress in the domain of Maxwell's theory, these words are as true today (1905) as they were when Maxwell wrote them.

In reference to the geometric properties of electromagnetic fields according to Maxwell's theory, Prof. Bjerknes says:

"Our knowledge of electromagnetic fields is contained in what is generally called Maxwell's theory. This theory does not tell us what electromagnetic fields are in their true nature. It is a formal theory, bearing upon two aspects of the properties of the fields. What are called Maxwell's equations give a very full description of the variation from time to time of the geometric configuration of electromagnetic fields. To this geometric theory is only feebly linked the much less developed theory of the dynamical properties of these fields."

In reference to the geometric description of electromagnetic fields, Prof. Bjerknes says:

"To give this description, a series of special electric and special magnetic vectors has been introduced.

"We believe that these vectors represent real physical states existing in, or real physical processes going on in the medium which is the seat of the field. But the nature of these states or processes is perfectly unknown to us. What still gives them, relatively speaking, a distinct physical meaning is, as we shall show more completely in the next lecture, that certain expressions formed by the use of these vectors represent quantities, such as energy, force, activity, etc. in the common dynamical sense of these words. These quantities can be measured in absolute measure. But their expressions as functions of the electric or magnetic vectors contain always two quantities of unknown physical nature. When once the discovery of a new law of nature allows us to write another independent equation containing the same unknown quantities, we shall be able to define perfectly the nature of the electric and magnetic vectors, and submit them to absolute measurement in the real sense of this expression. Provisionally, we can only do exactly the same as does the mathematician in problems where he has more unknowns than equations; viz. content ourselves with relative determinations, considering provisionally one or other of the unknown quantities as if it were known. But we retain the symbols for the unknown quantities in all formulas bearing upon the pure theory of electromagnetic phenomena, for this will be the best preparation for the final solution of the problem."

"This imperfect knowledge is, of course, also the reason why our theory of electromagnetic fields is split into two different, loosely connected, parts; first the geometric theory of the fields, where the relation of the vectors to time and space is considered independently of every question of the physical sense of the vectors; and, second, the dynamical theory of the fields, where the question of the nature of the vectors is taken up, but only imperfectly solved."

If we bear in mind the obviously electrodynamic nature of the new phenomena mentioned in Dr. Hering's paper, and for which the classic laws of electromagnetism fail to give an adequate explanation even when they are strained by more or less sophistic expedients (sliding contacts), we can appreciate the force of the preceding comments of Prof. Bjerknes upon the limitations of Maxwell's theories from the electrodynamic point of view. Indeed, in the opinion of the speaker, the burden of proof may be considered as having been thrown back upon the Maxwellites, meaning those of the orthodox school, because the Maxwellites of the reformed school would have been less militant, precisely because they are informed in regard to the flaws, the gaps, and the weak joints in electromagnetic theory as it stands today. It is dangerous in physics, as in other things, to claim more than the physical facts will warrant. The true office of physical laws is to explain physical facts and not to contradict them or deny their possibility.

The reference made by Bjerknes to Faraday's idea of a tension parallel to, and a pressure perpendicular to, the lines of force in a magnetic field, indicates that he attached much importance to further development of a dynamic theory based upon that idea, in a manner that would make it possible, as he says, for *all* the properties of magnetic fields, both *geometric* and *dynamic*, to be developed *naturally*. Dr. Hering comes very close to Bjerknes when he expresses the belief that a satisfactory theory could be based on the assumption of only a single magnetic force, the tension along a line of force, the other property, the repulsion of like lines, being then deduced from the first.

Unfortunately, this is easier said than done. We may see the goal that it is desired to reach, but as Bjerknes well said: "We have not even the slightest idea of the path to be followed in order to attain it."

The trouble appears to be that the specification of the physical characteristics and properties of magnetic fields is not yet complete. That is why, as Bjerknes says, we have more unknown quantities than equations. Nevertheless, Dr. Hering's paper contains many suggestions and cautions that will prove of great value, as a means of keeping on the right track whenever we do succeed in finding an entrance to the right path. The paper is filled with ideas, the complete discussion of which, in some cases, would lead us far into physical and philosophical problems of more or less abstruse character.

With regard to the stretch phenomenon, which seems to be the principal cause of objection to Dr. Hering's views, it may surprise those who dispute its existence and even its possibility on Maxwellian or other theoretical grounds, to learn that two very distinguished disciples of Maxwell, Mascart and Joubert, disagree with it very seriously. This may be due to the fact that the treatment of electromagnetism in their own celebrated classical treatise on "Electricity and Magnetism" does not follow Maxwell very closely, but follows more the French School of Physics. In Volume I, Part IV, Chapter I, Paragraph 455, after the statement that "the action of a current upon itself, can also produce deformations or continuous motions," some experiments with a movable portion of circuit are described; and the discussion ends with the following significant remark: "If the wire is elastic it will *stretch* until its elasticity balances the electromagnetic forces."

This may be considered expert opinion of highly qualified and competent character in defense of Dr. Hering's views.

Dr. Hering's suggestion that the mathematical theory should avoid integrations around complete circuits and should, instead, be based upon limited portions or elements of circuits, is of great importance. He sees the same dangers in going too far, or beyond proper limits, that occurs in the case of the mathematics of the soap bubble film, mentioned in the paper, in which a formula that is quite rational for finite values of the radius of the buble, becomes "insane" when the radius is increased to infinity. This is a matter of such importance, and similar cases of mathematical insanity are so frequent, that it is worth while to call attention to the simple means of preventing it, namely: "*Beware of absolute infinity,*" "*No thoroughfare here.*" That is perhaps the first thing that every student of mathematical physics should learn by heart. It is the one and only unfailing preventive of that form of insanity. It may be consoling to learn that many of the greatest mathematicians that the world has known were not free from attacks of that form of insanity; but that is no excuse for not taking measures to stamp out the disease.

Ask any student to draw two right lines, one perpendicular to the other, at or near its middle, and then to imagine both lines to be prolonged indefinitely; and ask him if he can possibly conceive that as the two lines were increased in length, there is at any stage even a faint resemblance of the *T* to a sector of circle. He will say "no." Let him, now, start with a small sector of circle, and let him imagine the radius of the circle to be increased indefinitely; and then ask him if, ultimately, the arc of the circle will become a right line. The chances are that he will answer "yes;" although the correct answer should be: "There can be no circle whose periphery is a perfectly right line." What is the reason for this error? In the first example, the mind was left free to see and to reason; in the second example, the mind was hypnotised by a question which assumed the possibility of doing something that is impossible physically, metaphysically, and mathematically, for human beings, namely, the crossing of the abyss that extends from the limits of the finite to the infinite. That impossible kind of circle *is the hypothetical* "circle of Pascal" whose center is everywhere and whose circumference is nowhere. The limit of the finite is only a *relative* infinity. The other is an *absolute* infinity. The relation between these two kinds of infinities has been treated exhaustively in the work of Henry Fleury ("Théorie rationelle de l'Infini mathématique et du Calcul Infinitesimal," Paris 1899, pp. 324). Under the heading of "Properties of Infinity" we read: "Infinity has no other property than that of being *impossible. Any calculation based upon absolute infinity, or upon any function whatever of absolute infinity, is itself absurd.*"

The following quotations are of interest:

"In analysis as in geometry, I distinguish between two infinities; one which I term absolute infinity, is impossible; the other, which I term relative infinity, consists of a very large fictitious value which is assigned to a variable."

"To tell the truth, *there is no infinity in mathematics; Because absolute infinity is impossible, and relative infinity is essentially finite,* since it is a very large value assigned to a variable. Hence, mathematics, like the infinitesimal calculus, could get along without it perfectly. Just as the harm does not consist so much in believing in witches as in burning them, so the harm does not consist so much in admitting the existence of infinite quantities as in attributing fantastic properties to them. I concede to absolute infinity only the property of being impossible. *As for relative infinity, since it is essentially finite, it will have all the properties of finite quantities, and it will have no other properties.*"

"Whenever, in mathematics, we come up against a stumbling-block, a scandal, or a paradox, you can ask *where the infinity is*: I mean to say that the error is the result of attributing some false property to infinity, which, I repeat, has no other property than that of being impossible. . . . . .

Since absolute infinity is impossible, any property assigned to it, any operation performed upon it, will be absurd. On the contrary, since relative infinity is essentially finite, it will have the same properties and it will be subject to the same operations as finite quantities. It follows that any property which does not apply to finite quantities cannot apply to either absolute infinity or relative infinity. I therefore declare absurd any reasoning which assigns to infinity a property that does not also apply as well to finite quantities."

"The distinction between the two infinities (absolute and relative) exists in geometry as in mathematical analysis; hence, if by an infinite right line we mean, a right line having no ends, it is absurd and impossible; but if we mean a right line which is extremely long with respect to others that are considered finite, it will still have ends and it will in reality be finite; it will be a relative infinity.

Likewise, since every plane necessarily ends at a line which is its contour, then if by the term plane, we mean a plane deprived of this contour, or perimeter, it is absurd and impossible. But if we mean an extremely large plane, bounded like any other plane, by its perimeter, it will, in reality, be a finite plane. If we call it infinite, it will be a *relative infinity*. An infinite right line which has ends can be doubled or tripled in length. A right line having no ends, being an impossibility, it cannot be lengthened or shortened.

In geometry as in mathematical analysis, the general properties of finite magnitudes apply to infinite magnitudes, when we mean relatively infinite. But if we mean an absolutely infinite magnitude, or one without limits, it is impossible and absurd, and so is any property that may be assigned to it.

The *circle of Pascal*, whose center is everywhere and whose circumference is nowhere, is an absurd infinity. The same would be true of a square whose center would be everywhere, and whose perimeter is nowhere. It is not necessary to inquire which is the most round or the most square of these figures. Any comparison would be absurd."

We can now see clearly enough why the formula for the tension of a plane soap bubble film gives infinity, which is not true, as Dr. Hering says. According to Fleury, the very idea of attempting to use such a formula with any infinite values is absurd, and the alleged result is simply impossible from every point of view.

It is easy to reach equally absurd and impossible conclusions in electrodynamics, and, for example, to ascribe infinite values to the magnetic flux around a conductor under conditions where we know and can demonstrate experimentally that the flux decreases with the distance from the conductor. In one case, it was actually contended that the magnetic flux, though decreasing until it was less than any assignable value at "relative" infinity, suddenly jumped to an infinitely great value when "jockeyed" across the abyss between relative and absolute infinity. This wonderful acrobatic performance was indeed taken, in some cases brought to the speaker's attention, as evidence that Hering must be wrong and Maxwell must be right. It is to prevent such fool jumps that the signs "Beware of absolute infinity," "No thoroughfare here," should be ever present in the mind's eye of mathematical physicists, especially those of the amateur class.

Dr. Hering's idea of using a counter e. m. f. generated somewhere in the circuit as a criterion and a measure of mechanical motions appears acceptable, but it does not tell *where* or *how* the motions are produced; it gives the end-result without definite information in regard to the path, agency and process by which this result is reached.

His idea that "the flux around a conductor or magnet should be pictured as though it were in some way elastically attached to the material in which its source lies," would be acceptable if there was not some doubt left as to his precise meaning by the statement previously made to the effect that "it seems to be much better to consider the magnetic field around a current-carrying conductor to be merely an action at a distance, *as Faraday proposed*, the seat and source of the force being in the material of the conductor and not in the space around it." Now the very thing that Faraday denied consistently was the possibility of action at a distance under any circumstances. The reference to hydrodynamic action made further on by Dr. Hering shows that he himself has realized the necessity of an intervening medium having the physical properties of a fluid. In a previous paper on "A New System of Electromagnetic Forces Needed" (A. I. E. E. April, 1922, Vol. XLI, No. 4, pp. 305-307), reference was made to "hydraulic action in the ether" as a possible explanation of the "pinch" and "stretch" phenomena.

The speaker finds himself at variance here with Dr. Hering's views.

The speaker has believed for more than forty years, and he is still of the opinion that actions and reactions of elastic character play a role of paramount importance in electromagnetic phenomena. In his opinion, a liquid having merely *hydraulic* properties and characteristics can exhibit some of the mechanical or dynamical, (let us rather say *physical*) properties and characteristics of magnetic strains and stresses, but it cannot exhibit them all. Indeed, some of the phenomena observable in magnetic fields indicate *rigidity*, *stiffness* and *springiness*, which are properties that only certain metals, (e. g. tempered steel), can exhibit, and which no liquid of any sort can even approximate.

Just forty years ago, the speaker, then a very young man, delivered an experimental lecture before the New York Electrical Society on "Action and Reaction in Magnetic Fields." The condensed report of the lecture, which appeared in the *Electrical World* of March 10th, 1883, was prepared by the lecturer himself, who was then the editor of that publication. The lecturer mentioned, among other things, an interesting new property of lines of magnetic force observed by him a few years before (1879) in the course of an experimental study of magnetic fields, namely, the property of *resiliency* or *rebound*, under certain conditions.

The following is an extract from a reference to the lecture and to the property in question contained in the A. I. E. E. TRANSACTIONS (Vol. VI, 1889, pp. 288, 289):

"I have believed in the theory of the elasticity of lines of force for some years. I employed the term resiliency in my lecture, and I still adhere to it as being, in my opinion, somewhat more appropriate, and I have believed, for a long time, that when the mathematical theory of elasticity shall have been developed to a sufficient point, we shall have all of the data necessary for working out a perfect theory of magnetism. In other words, I believe that magnetism is nothing more than a manifestation due to the elasticity of the surrounding medium, that it is an elastic disturbance, and I may state, in passing, that it is here that I differ with the conception advanced by Faraday and Prof. Thomson as to the shortening of the lines of force. I think that the idea of shortening is rather vague. I hardly think that we can say that lines of force shorten as such. It would probably be more appropriate to express it in some other way. What seems to us to be shortening is probably merely the manifestation of some other phenomenon. Another point is the snapping of the lines of force. I shall endeavor to present some experiments which illustrate the fact that lines of force do snap and break, as I did in the lecture of 1883, and that the snapping is not simultaneous."

These remarks would need very little modification today, so far as their *substance* is concerned. They might be amplified somewhat so as to make clear what were (and are still) the speaker's views in regard to the so-called "shortening" of lines of magnetic force. There was also a slight error in saying that lines of magnetic force "break" without explaining more fully what was meant. The fact is that lines of magnetic force do not "*break*" in the sense of being severed or parted like a string or a wire that is broken into two pieces; but they may *break away* and be parted from an "anchorage" such as from an iron armature which is moved in front of the poles of a magnet or in any magnetic field. That "break away" movement is sudden or "snappy," but it does not occur simultaneously for all the lines of force in a given portion of magnetic field; it occurs for each "vortex" by itself when the conditions are right for that vortex or line of force.

The additional details promised were not sent in, partly from lack of time, but mainly from a realization of the importance of reviewing the whole subject fully before adding to the extempore discussion referred to.

The earliest conjectures of the speaker, as outlined in 1879 (in the unpublished paper mentioned in the previous A. I. E. E. discussion) are not without interest on this occasion, when it is desirable to gather together all ideas that may prove helpful. These conjectures were:

That lines of magnetic force were elastic like steel rings or hoops; that they tended, normally, in a homogeneous diamagnetic medium, to take the form (figure) of perfect concentric circles; that any departure from the circular concentric form was, in a sense, abnormal, being due to disturbing conditions and forces, and that it was always attended by reactions which tend to restore the circular form; that there was a limit to the extent to which they could be swerved or distorted from the circular form; and that, at the limit of distortion, they would "break away" and assume a more circular ("normal") form, and would do so in a sudden, snappy manner as if they were *resilient*, and recoiled like an elastic ring that has been distorted and is suddenly released or else suddenly breaks loose from the distorting force that was acting upon it.

The process of reasoning followed was as follows:

If a hoop made of tempered steel wire forming a perfect circle is distorted in any way and then released suddenly, it will instantly recover the form of a perfect circle. A perfectly elastic steel hoop would be a good physical model of a line of magnetic force or of a bundle or coil of such lines, in regard to the manner in which it is influenced by, and reacts upon, outside forces; but such a model is incomplete and inadequate for the representation of the reactions which the magnetic field itself exerts upon the forces producing it. According to Newton's principle, dynamic action and reaction are always equal and opposed to each other. On the theory that, in this case, an elastic disturbance is produced in the ether surrounding an electric conductor, a line of magnetic force may be regarded as being a dynamic *reaction* against some dynamic *action* which is exerted by the electric current passing through the conductor If we picture the action at each line of force as comparable to that of a conical wedge which is forced into a rubber ring, then the *reaction* is made intelligible to us as a state of tension which causes the ring to be stretched into a larger, thinner ring; and the tension thus produced is the complete measure of both the reaction and the action. If the action is increased, the reaction will increase enough to balance the action and the rubber ring will be expanded into a still larger circle. If the action decreases, the ring will shrink in size until the decreased reaction in the new state of stretch balances the reduced action. This is what the speaker had in mind when he said (in the A. I. E. E. discussion in 1890) that we can hardly say that lines of force shorten as such, and that the shortening is probably the manifestation of some other phenomenon. The phenomenon which he had in mind was the *elastic reaction of the medium* against the action of the current producing the lines of force. In this case, the "action" is, in all probability, different from that represented by a mechanical pressure exerted radially like the pressure exerted by the conical wedge above mentioned; and, consequently, the reaction is entirely different from that represented by the tension of a stretched rubber ring. The speaker early acquired the habit, which he has never lost, of using elastic steel rings as a mechanical model or analogy in explaining magnetic action. Thus in a note-book under date of August 11, 1889, we have:

Theory of Magnetic Attraction and Repulsion (based on experiments of 1879), (with two diagrams):

"Taking two conductors parallel to each other and carrying currents, as the simplest illustration, we have:

1st, attraction when the currents are in the same direction;

2nd, repulsion when the currents are in contrary directions.

(1) In the first case, I regarded the attraction as due to the blending of the magnetic circuits into one circuit, as shown in the sketch, and the effort made by the resultant circuit to assume the concentric circular form. In other words, when there is only one conductor it tends to maintain itself in the center of the vortex· when there are two or more they tend to go to the center of a combined or resultant vortex, and consequently to come together.

In such a case I can conceive that, instead of being shortened, some of the lines immediately surrounding the conductors, might be on the contrary actually stretched or distended. In fact, they would all be in a sense stretched as long as the vortex did not resume the perfect circular form.

(2) The second case (of repulsion), I regarded as that of the compression of a steel hoop or ring, by contact with another steel hoop or ring, with the conductor impelled as before to maintain itself central with respect to the rings, and being consequently, urged in a direction opposite from attraction. In this case, we might consider that the lines of force act rather by expanding than by contracting or shortening."

In a note made the next day, the concept of a vortex-like disturbance of the ether is introduced.

The vortex-motion theory always was interesting to the speaker largely because it is a *kinetic* theory. He believes, with Maxwell, Shelford-Bidwell, and others, that kinetic theories of magnetism are the only ones that will stand serious scrutiny. Even in a medium which is infinitely tenuous like the ether, kinetic energy can still be developed in measurable quantities, since the decrease in mass can be made up by an increase in velocity. In the formula $E = m\,v^2/2$, the value of E will remain constant as long as the product $m\,v^2$ remains constant, and when the mass (m) is reduced to $1/n$ of its value, the velocity (v) does not have to be increased n times but only the *square root* of n times. Now the elastic properties that make lines of magnetic force act as they do are just such as might be expected to result from kinetic reactions in which the mass is very small but the velocity very great. It is interesting to note in this connection that the speaker succeeded in making vortex rings of the kind known as "*smoke rings*" exhibit the phenomena of attraction and repulsion which are illustrated in the two diagrams (in the note of August 11, 1889). He has made two smoke rings coalesce into a single larger ring which tended immediately to assume the figure of a larger circle; and he has made smoke rings repel each other exactly in the manner indicated in the note. Although the real physical character of the "action" and of the "reaction" are not yet (and may never be) definitely known to us, we are, nevertheless, justified in describing the characteristics of the reaction as comparable to those of an elastic disturbance. Things which are equal to the same thing are equal to each other, in mathematics; and in mathematical physics, they can be treated as if they were equal to each other, and they can be substituted for each other. Therefore, even if, in the end, we should have to modify our views as to the *causes*, we are not likely to be obliged to revise them in regard to the *effects*.

The "pinch" and "stretch" phenomena were regarded at the outset by the speaker as being merely natural and logical consequences of the "theory" of magnetic action which he had built up for his own private use, so to speak. To his mind, the pinch effect was an elastic *reaction*, *equal* and *opposed* in its effect to the *action* of a force produced by the electric current passing through the conductor. He looked upon the action as being comparable to a pressure applied to the inside of a cylinder made of an elastic substance possessing some peculiar properties, such as he conceived might be exhibited by vortex motion of quasi-infinitely great velocity, in matter of quasi-infinitely small mass; and he pictured the reaction as comparable to the stress or tension produced in that elastic substance. He did not picture the substance, as Dr. Hering did, as one having the characteristics of a liquid, because his study of the subject had taught him that the characteristics implied physical properties like *stiffness*, *strength*, *elasticity*, *resilience*, which recall *steel* and other metals. Kelvin, years ago, in presenting the undulatory theory of light, postulated the existence of a transmission-medium having, at times, the properties of *jelly*, and, at other times, the properties of a rigid but elastic solid like steel.

The speaker's views in regard to the nature of magnetic action makes him feel warranted in reasoning about the outward physical effects produced in a magnetic field as if these effects were not only *analogous* but *equivalent* to an elastic disturbance in a medium where kinetic energy is developed which is due less to *mass* than to *velocity* (as in the case of a vortex-ring). (A vortex-ring illustrates a condition where elastic and resilient properties are due to velocity-effects). To his mind, the "stretch" effect seems as natural and as reasonable as the "pinch" effect, being merely the same "reaction" manifesting itself against the "action" in an axial direction. In other words, it seems reasonable to him that the reaction should manifest itself physically by a tendency not only to *pinch* but also to *stretch* the conductor carrying the current which produces the magnetic field around that conductor.

The preceding considerations show why the author does not support the idea of hydraulic action in the ether, while believing strongly in the idea of elastic actions and reactions. The author's "conjectures" of 1879, have rendered good service on many occasions as the basis of a working theory, in making clear many phenomena and facts about magnetic fields. Seeing the difficulties that seem to confront all other working theories in regard to "action and reaction in magnetic fields" in spite of the efforts made in the last forty years, the speaker has been wondering *if*, perhaps, the "conjectures" could not be utilized to a good purpose in other attempts to revise and extend the present theories.

V. Karapetoff: In discussing this paper it is necessary to use certain well established laws of mathematical electricity. I will write them down, so as to make it clearer as to what the controversial matter is. One of the established laws of electro-magnetism is that the stored energy in a circuit without iron is equal to one-half the square of the current times the co-efficient of inductance, or $W = 0.5\, i^2 L$. This may be taken as the definition of inductance as well, so that if you know the energy and the current in the circuit, you can compute L. The same coefficient L is sometimes defined from the induced e. m. f. as $E = - L\, d\, i/d\, t$. All this is not a controversial matter, but that of definitions. Now we come to the fundamental law of mechanical forces which law is somewhat assailed in the paper. In the first place, Maxwell himself gives credit for this law to Lord Kelvin, referring to Nichols *Cyclopaedia of Physical Science*, edition of 1860, so that that is not Maxwell's Law, but Lord Kelvin's Law. It is rather surprising that in the paper under discussion, Kelvin, the originator of the law of electro magnetic forces is not mentioned at all. This law applies specifically to a constant current and no iron must be present. Consider a small deformation in space and impose the condition that the current during this deformation remains constant. The circuit is such that a part of it is flexible, or has a sliding contact. What are the energy relations during this small displacement? Lord Kelvin's answer is that the energy brought from the external source, let us say from a battery, is twice as great as is necessary to perform the mechanical work. The excess goes to increase the stored energy of the circuit. I have repeatedly called attention to this law in some previous volumes of our TRANSACTIONS, for instance, in the discussion on the opening of disconnecting switches on short circuit, on stresses in bus-bars, and so forth. The last chapter of my "Magnetic Circuit" is devoted to the subject of mechanical forces in a magnetic field, to which chapter I refer for proofs.

With a constant current, the infinitesimal energy brought from the source of current is $d\, W_1 = i^2\, d\, L$, since it is equal to the change in electromagnetic linkages. The increase in the stored energy is $0.5\, i^2\, d\, L$. Hence, the energy converted into mechanical work, equal to the difference of the two, is

$$F\, d\, s = 0.5\, i^2\, d\, L$$

Therefore, the mechanical force is

$$F = 0.5\, i^2\, (d\, L/d\, s)$$

where s is the direction of the virtual displacement. It may also be of interest to state that the coefficient of self-inductance may be conveniently represented in the form

$$L = n^2\, P_{eq}$$

where n is the number of turns and P_{eq} is the equivalent permeance of the magnetic circuit (Magnetic Circuit, p. 184).

I shall limit myself to one feature of Dr. Hering's paper, namely that of mathematical relationships which are entirely independent of our ideas or pictures as to what electro-magnetic phenomena are. Even if our physical conceptions should be radically changed tomorrow, we could still define the co-efficient of inductance and use the foregoing formulas just the same.

Those of us who know at what personal sacrifice and with what persistence Dr. Hering's experiments have been performed realize the debt of gratitude that we owe him. I say this, even though I do not believe that his experiments have in any way disproved the accepted laws. The experiments are purely qualitative, and in none of them, as far as I can see, the coefficient of self-inductance is smaller at the end of the motion than it is at the beginning. The derivative $d\, L/d\, s$ is positive, as it should be according to Kelvin's law. As to whether or not the actual mechanical force observed checks with the theoretical formula, the burden of the proof is entirely with Dr. Hering, or with someone who wishes to defend his views. The experiments in the present inaccurate form can hardly be used as a basis for a scientific discussion on such a fundamental law of nature.

John H. Morecroft: Dr. Hering's paper is rather difficult to discuss, as he makes so many ambiguous and general criticisms of our present laws, trying to show their inadequacy, but never apparently doing so, when one analyzes his methods. Such ideas as conveyed by the expressions "reactions of these forces on each other, or on a foreign force" and "abutments or anchorage of the forces in a field" certainly do not tend to clarify his arguments.

When one conceives of the electric current as moving electrons, as he is evidently beginning to do, it seems rather inappropriate to speak of the energy of the magnetic field as potential energy, as he does; this energy is due entirely to the motion of the electrons and so would naturally be classified as kinetic energy, while the energy of the electric fields of the electrons, present when the electrons are stationary, is better called potential energy.

The whole paper is so full of ambiguities and misleading statements that a complete discussion is out of the question, but an analysis of a few points Dr. Hering brings up is attempted in the following notes.

In discussing Fig. 1, in his paper he apparently thinks that the idea of current flowing in such a circuit is a new one; it seems well to point out therefore, that every time he telephones he is probably sending his voice current through just such a circuit, and that the telephone engineers use circuits of this make-up probably more than any other kind. The circuit of his Fig. 1 is closed through a condenser, the two plates of which consist principally of the two spheres A and B; the ordinary telephone circuit is closed in just the same manner, but there the condensers have a somewhat larger capacity than is the case in Fig. 1. In spite of this statement to the contrary, it is necessary to consider every circuit closed, if we wish to calculate how much current will flow in it; what seems to constitute an "open circuit" in his Fig. 1 is rather a capacitive reactance between plates A and B, a reactance so high for ordinary frequencies that negligible current can flow and so it is classed by engineers dealing with ordinary power circuits, an open circuit. But there is no real distinction between an open circuit and a closed circuit, from the theoretical viewpoint; current will flow in any circuit in which there is electromotive force acting, especially if this is an alternating e. m. f.; the amount of current may be so small that its effects are negligible and the circuit is therefore called open. Actually the same law of current flow holds good for all circuits.

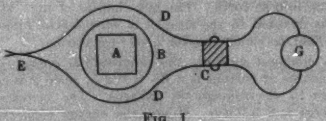

FIG. 1

In such a circuit as that of his Fig. 1, this current would be so small in the ordinary test as not to be measureable and so the circuit is said to be open; if however a frequency of one hundred million cycles is used the capacity reactance of the condenser A-B is sufficiently low to permit the passage of measurable current and so the circuit would not be called open; the writer sees no reason at all therefore for any argument about this circuit; it follows the ordinary laws of circuits, as do all the rest of his circuits, one or two of which will now be discussed.

To clarify somewhat (even though only a little) some of the material presented in the paper, and to show how the author of the paper thinks and reasons, I will recall an argument he presented about fifteen years ago; I hope he will bear with me for doing so as the experiment he then described contains what seems to the writer a fallacy, which fallacy is present in some of the experiments contained in the present paper.

Can a circuit, being linked with some magnetic flux, be unlinked from this flux without having a voltage induced in it? That was the question Dr. Hering put to himself, and his answer was—Yes, arrived at as a result of the experiment now to be described.

His experiment was arranged about as shown in Fig. 1. An electric circuit consisting of the galvanometer G, two flat brass springs D, D, fastened to a wooden block C, and in contact at E, is linked with all of the flux through a bar magnet, shown in section at A. Around the bar magnet is fitted a smooth brass ring B.

By pulling block C (towards the right in Fig. 1) the springs D-D come in contact with ring B, further motion in the same direction opens the contact at E, and still further motion carries them around the ring, until they finally slide over the ring and again come in contact on the right side of the ring. This sequence of events is suggested in Fig. 2. In Fig. 1 the circuit evidently is linked with the magnet's flux, and in condition c of Fig. 2 it evidently does not, and hence, as the galvanometer has shown no deflection during the progress of this ingenious experiment, the author would have us conclude that a circuit can be unlinked from magnetic flux without having an e. m. f. induced in it. The results of this experiment, however, constitute no reason at all for the conclusion; our old ideas about changing interlinkages generating e. m. f's. are just as trustworthy after the experiment as before.

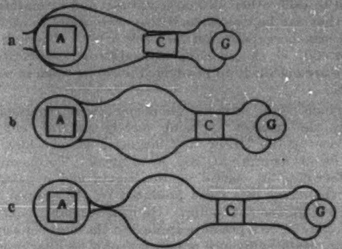

FIG. 2

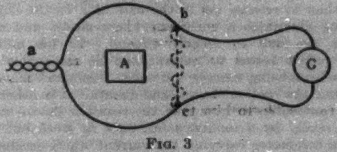

FIG. 3

It will be seen that in position *a* Fig. 2 there is no longer *a* circuit involved in the argument, but there are *two* circuits, one of which (springs and left hand side of ring) still links with the flux and another (springs and right side of ring) which does not link with the flux. So, the experimenter has replaced *a* circuit by *two* circuits, *one of which does not link with the flux at all*.

I suggested at the time that the audience would understand the experiment better if it were performed as suggested in Fig. 3, the sequence of events being exactly the same as it was before, but the various steps being somewhat more evident. A circuit shown by the full lines of Fig. 3 links with the flux of magnet A; the wire of which the circuit is comprised being twisted together at *a* to complete the circuit. This is the same circuit as the experimenter started with in Fig. 1. Now a wire is connected from *b* to *c*, the wires are untwisted and separated at *a*, the magnet is moved through the opening thus created, the wires are again twisted together at *a*, the wire from *b* to *c* is now removed, and lo, the circuit has been unlinked and the galvanometer has not budged! Has the circuit been unlinked without generating an e. m. f., as indicated by the quiescent galvanometer? Certainly not—the circuit has been opened at *a*, the magnet has been carried through the opening, and then the gap in the circuit has been repaired. The fallacy of such an experiment is evident to every one yet the author wished the teachers of physics to revise the theory they had been professing, on the basis of it.

This fallacy occurs repeatedly in the paper under discussion. When trying to show that a circuit does not have to move so as to include more flux in the circuit involved (a principle we still teach) he cites the case of the motor—"In the most usual case of the motor the flux does not increase but remains constant, hence the law fails even in the most usual case..." Here the brushes sliding on the commutator continually change one circuit for another just *because the law does hold good;* if the circuit involved were not continually changed for another the motor would not turn. Let Dr. Hering solder two wires on opposite sides of the commutator of a bipolar motor and try how far the armature will turn. He will find that in this case *a* circuit (ensured in this case by soldering the wires in place) the armature will turn just as long as further movement will make the total interlinkages of the system a maximum, and *there it will stop*, just because the law does hold good in this case.

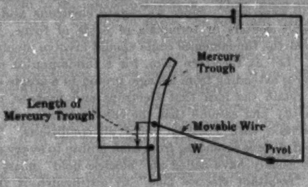

Fig. 4

Although it would undoubtedly prove a fascinating pastime it is obviously impossible for one man to carry out all the experiments suggested by the author, unless he devotes considerable time to the task; one easy experiment he suggests however, which seems a typical one, so I took the time to carry it out more carefully than he had apparently done. The experiment is the one indicated in Fig. 4; the question is—which way will the wire *W* move? The results of the test are supposed to disprove the theory we teach, namely that the wire will so move as to make the included flux increase.

The older law (which presumably is at fault because predicting the wrong answer in this case) says that the circuit should so re-arrange itself as to make the flux interlinkages increase—yet according to the author the wire *W* moved in such a way as to decrease the enclosure, and so presumably disproves the law. (It is to be noted that here also the circuit is not a definite one, the motion of the wire making the length of the circuit greater by the amount of mercury trough introduced into the circuit by its motion). It is a fact that if one tries (and understands the old fashioned law) he can so fashion the circuit that the wire *W moves in*, as stated by the author, but that same knowledge will give him the idea as to how he can fashion the circuit so that it will *move out*. The phenomenon needs no new laws for its explanation although such is apparently believed by the author. The explanation of the result of the experiment is this—*the circuit will so rearrange itself so as to include the maximum flux*, the current presumably being held constant. But *this is the older law.*

I made up several circuits with mercury troughs and movable wires and tried the motions of the wires under various conditions. After eliminating the effects of the earth's field (this can be done by using very large currents in the circuit) I found certain combinations which would make the wire *W* move out, and others which would make it move in. In Fig. 4, for example, if the wire *W* is put a sufficient distance inside the rectangle to start with, it will move inward and find a position of stable equilibrium; if the wire is pushed farther into the rectangle the forces exerted on it push it outward; there is a definite position of equilibrium for a definite circuit. If the back wire of the circuit (on the side of the rectangle opposite to *W*) is too close to *W*, that is, the rectangle is a narrow one, the wire *W* will always move outwards. All of these effects can be calculated by the older laws but as it seems that the author is more amenable to experiment than to the calculus, I made some quantitative determinations on some of the circuits. As the interlinkages of a circuit are measurable by determining the self induction of the circuit I used this method for finding out if the circuit did always so move as to increase the inter-linkages. One turn of wire, a few inches in dimensions, has a rather low value of self induction so it is necessary to use fairly high-frequency current to make the determination. I used a frequency of four million cycles a second.

A heavy current was sent through a circuit essentially like that given in Fig. 4, letting the movable wire come to its position of stable equilibrium. By a resonance method I then determined the self induction of the circuit for various positions of the movable wire, obtaining results in a typical case as recorded herewith.

Length of mercury part of circuit	Self induction in microhenries
0 inches	3.36
2 "	3.36
4 "	3.43
6 "	3.47
8 "	3.50
10 "	3.46
12 "	3.43
14 "	3.41

The position of stable equilibrium for the movable wire was when the length of mercury trough was eight inches, thus showing indisputably that this experiment, which the author of the paper thought disproved the older laws, actually vindicates them. He evidently jumped to the conclusion that if the enclosed area became smaller the self-induction must also necessarily be smaller. This is only the case if the length of the wire comprising the circuit is constant in length; here the circuit becomes actually longer as the movement of the wire includes more of the mercury trough in the circuit; *the circuit is thus indefinite, sufficiently so that this very thing led the originator of the experiment to jump to the wrong conclusion.*

In a similar way the motion of the conductors in all of the combinations shown in the authors Figs. 3 to 11 so take place that the flux inter-linkages increase. For example, in his Fig. 6 the more that trough *M* (of large cross-section, and hence low *L* per unit length) can be replaced by trough *m* (of smaller section, hence greater *L* per unit length) the higher will be the self induction of the circuit.

In such an experiment as that shown in Fig. 5 of the author's paper we have to bear in mind that we do not have *a* circuit to deal with, length being continually added to the circuit conductor as the bridge moves. From what has been said and shown above it seems evident to the writer that we may still use the old laws in our teaching, without running the risk of teaching any fallacies. We must, of course, try to instill into the student's mind that he must think accurately, not confusing one circuit with another—he must not be willing to apply a law to a case which involves premises which the law does not. It seems to the writer that the older laws, about which doubt was supposed to be cast by these experiments, are rather substantiated by them.

John Mills: I have had no opportunity unfortunately to try out any of these experiments. I approach the problem very much from the standpoint of the remarks of past President Mailloux and I wish to use the occasion to arouse a further interest in the mechanisms which underlie the phenomena Mr. Hering describes.

I think Mr. Hering's interesting paper should be approached in the spirit of one who recognizes that the past twenty-six years, since J. J. Thompson identified the electron, have been revolutionary and scientific concerts. He calls attention to a much needed reconsideration of our formal expression and exposition of electro-magnetic phenomena. Today, we have electronic mechanisms which Maxwell and his immediate successors did not have. Today the ether is the most debatable assumption and energy and an electrical matter are the two entities with which the electronic physicist deals most consciously. With Mr. Hering's fundamental appeal for recognition in our expositions of underlying mechanisms I am most heartily in accord. I have tried some missionary work along that line myself but with relatively small success.

Without any regard to the criticism which can be directed towards the experiments which Mr. Hering has presented let us consider his Fig. 4, and look at it without reference to the inductance. Imagine billions of electrons rushing along from the negative terminal of the battery, along the connecting wire and plunging into the liquid of the trough. Then remember that each of these electrons is about 1/1845 of the mass of a hydrogen atom. Remember that for the currents used there would be about a thousand billion billion of these electrons taking that plunge each second. Now I simply raise the question: Is it inconceivable that, as they do plunge, they should kick back on the slider? I do not believe it is inconceivable,—and I cannot see—and here is where I differ and am unpopular—I cannot see that flux and other Maxwellian concepts enter into the problem at all.

Whether or not, however, these electrons, in their obedience to Newton's third law of equal action and reaction will result in a perceptible motion of any given slider is a question that the electronic physicists can answer for us, either from data at hand or by simple but refined researches.

I admit our knowledge is incomplete, but it seems to me our knowledge has to be expressed in terms of the electron of which we have heard for twenty-six years. We may apply the same ideas to other figures of the paper by Mr. Hering and reach similar conclusions. In each case, however, the problem is one of electrons and is not a problem of flux nor a problem of "stretch effect." If there is a stretch effect it must be explainable on an electronic basis.

Two of the experiments, namely those of Figs. 3 and 6 are conceivable upon the electronic basis mentioned above but require a recognition of the fact that because the shape and sizes of the vessels, into which the moving wire dips, are dissimilar there will be possible dissimilar conditions of turbulence due to thermal effects. One could have wished that Mr. Hering had approached this investigation more directly from the view point of electronics and obtained quantitative results.

H. G. Brinton: In connection with the subject of fundamentals, I would like to say a few words about fundamental electric fields and the electron theory.

The electron theory is not completely perfected. There are some phenomena which it has not yet explained, although it is accepted as our most fundamental theory and deals with the constitution of matter as well as electric and magnetic phenomena. The phenomena of gravitation and of the propogation of electromagnetic waves through space are two of the most interesting cases of unexplained phenomena. There is also need for more clear and fundamental explanations of other phenomena, as pointed out by Dr. Hering. These unsolved problems make a consideration of the fundamental theory still more fascinating. I have made a study of this subject as a matter of personal interest and have worked out a development of the electron theory which affords a basis for the explanation of gravitational as well as electromagnetic phenomena.

The point of view of this discussion is based on the idea or fact that we have no conception of the ultimate essence of electrons, protons, and fields of force. We only know how they act. The human mind has not been capable of going further than that. What is spoken of as "knowing the ultimate nature of matter" can only consist, under present conditions at least, of a detailed knowledge of the nature of the behavior of the smallest units. We consider matter as made up of a large number of fundamental units. From this point of view, an explanation of complex phenomena involving fundamental units, will be a statement of the actions taking place, in terms of the actions of fundamental units and in accordance with fundamental laws. The things we now consider most fundamental cannot now be explained in terms of anything more fundamental, but our ideas in regard to what is most fundamental are subject to change. Our fundamental ideas must be judged by their consistency with each other and by their utility as a basis for explanation.

The fundamental units of matter should involve electrons and protons in some way. Since we regard electrons and protons as primarily centers or terminals of fields of force, it seems logical that the fundamental units should involve electric fields of force. Let us therefore take, for a fundamental unit, the field of force extending from a single proton to a single electron. We will retain the idea that electrons and protons maintain their identity and assume that the fundamental unit fields of force maintain their identity under all conditions. There is such a field of force extending from each proton to every electron. What we usually have called an "electric field" must now be called a "resultant electric field" to distinguish it from the fundamental unit electric fields.

We have to consider next the inter-action of these fundamental fields. The action of a single field needs no description here. Let us consider the case of two positive particles with fields directed outward from each particle, or the case of two negative particles with fields directed inward to each particle. In these cases the adjacent fields are in the same direction or have components in the same directions, and we know that there is a resultant force tending to move the two particles apart. We may, therefore, say that two adjacent fundamental fields in the same direction tend to move apart. This is the same action that takes place within a single field. If one field is superimposed on another field having components in the same direction, but of varying intensity, the first field tends to move away from the stronger part of the second field as a result of the combined actions of adjacent parts of the fields. This resultant action is what determines the direction of the resultant force tending to move two positive particles or two negative particles away from each other.

In the case of two particles, one of which is positive and one negative, we have fields directed outward from the positive particle and fields directed inward to the negative particle. In this case the two adjacent fields have components in opposite directions and we know that the two particles tend to move together. To be consistent, we must say that two fields having components in opposite directions tend to move together. This is the direct opposite of the action between two adjacent fields in the same direction. There is, however, another action to consider in the case of a positive and a negative particle. In addition to the fields referred to above extending to and from the surroundings, there are fields extending from the positive particle to the negative particle. These fields, which are an inherent part of this particular combination, make the resultant attractive force between a positive particle and a negative particle slightly greater than it would otherwise be.

Now consider the case of two neutral particles. In this case the field directed outward from each particle is exactly equal and opposite to the field directed inward to that particle. The field directed outward from one particle tends to move away from the outwardly directed field, but toward the inwardly directed field of the other particle. The resultant of such actions in this case is zero. There is, however, a resultant force of attraction, which is due to the fields extending from the protons of each body to the electrons of the other body. This particular resultant force is called "the force of gravitation." We know that a gravitational field of force is not affected by the presence of any other gravitational or electric field. This is due to the fact that a gravitational field is composed of two equal and oppositely directed fields and the resultant action upon other electric fields is zero.

From this point of view, we may easily show why we have electromagnetic waves but do not have gravitational waves. A gravitational field, being composed of two equal and oppositely directed electric fields, cannot set up any magnetic field by its motion in one direction. On the other hand, the movement of a magnetic field does result in actions upon gravitational fields, moving one electric field in one direction and the oppositely directed field in the opposite direction. Thus, when an electromagnetic wave is moving in space, the resultant electric fields are due to the separation of the component electric fields which form the gravitational field. When the electric fields separate, energy is transferred to the electric fields. When the electric fields recombine, this energy is transferred from the fields at that point. Thus, energy is transferred from point to point as the wave moves through space. The magnetic field must have been set up in the first place by the movement of a resultant electric field. For example, the movement of electrons sets up magnetic fields and results in electro-magnetic waves.

The fact that electrons play a more active part than protons play in electrical phenomena may be explained as follows: The number of electrons in this universe is much greater than the number of protons. There is a fundamental field of force extending from each proton to every electron. The number of fundamental fields of force terminating on a single proton is, therefore, much greater than the number terminating on a single electron. That is, the total field centering on a proton is much greater or more intense than the field centering on an electron. For this reason an electron can be moved by electromagnetic actions much more easily than a proton.

Gravitational actions are comparatively simple and the explanation is therefore correspondingly simple. In the case of the actions and interactions of electric and magnetic fields, the phenomena are very complex in most cases and the explanation is correspondingly difficult and complicated. We will consider at present two of the more simple cases.

We know that the effect of inductance is to oppose the increase or decrease of a current in a conductor. Consider the case of a current flow in a wire. When such a current starts to decrease, the magnetic field around the wire contracts and the movement of the magnetic field results in a pressure upon the inwardly directed field, causing such fields and associated electrons to move in one direction along the wire. We need not consider, for present purposes, the action upon protons, because it is principally the electrons that move as stated above.

In the case of a loop of wire rotating in a uniform magnetic field about an axis at right angles to the field, we know that there is flow of current and an induced voltage in the loop. In this case we have to consider the action of the magnetic field upon the fundamental fields between the two sides of the loop which are parallel to the axis of rotation. There are two equal and oppositely directed sets of fundamental fields to consider. These fields extend from the protons of each side of the loop to the electrons at the other side. As a result of the motion of these fields relative to the magnetic field, the electrons (referring to those that move) in one side of the loop are moved in one direction and the electrons in the other side of the loop are moved in the opposite direction. There is thus a tendency for the circulation or flow of electrons around the loop. The direction of motion may be found by the well-known three finger rule for induced e. m. f. taking the direction of movement of electrons as opposite to the direction of the e. m. f.

The electron theory is our fundamental theory and should be used for the explanation of electromagnetic phenomena. The electron theory has, however, been in need of some development and improvement before it could form a satisfactory basis for explanation.

R. E. Doherty: I wish to discuss in particular the statements made in the appendix of this paper, since this is the only place where either a numerical value, or an equation appears. I submit that although a number of our so-called physical laws may possibly be wrong, including the one attacked by the author, it is not a sound policy to conclude that the law is wrong just because the force on some part of a complicated electromagnetic system does not happen to be in the direction one would expect, from a knowledge of the law and a visual inspection of the apparatus. From a tilted railway car, a stream may appear to run up hill, but in fact, of course, it does not. And just as in the railway car a proper set of measurements could show whether the law of gravity had ceased to operate, so in the present case, quantitative tests could show whether the law is actually or only apparently wrong. This may well be left to physicists.

But the ideas developed in the Appendix can be discussed in a decisive way. Starting with a neutral copper ring of zero resistance, it would be very interesting to learn the details of the author's process of establishing a magnetic field linked with this ring, and a current in it which will flow indefinitely. The current will flow indefinitely, of course, if a magnetic field linking the ring is established; but establishing such magnetic linkage in the given neutral ring is the first point which I cannot fathom. If there is a single law which can be used as a starting point, it is that in a closed ring

$$\frac{d}{d\,t}\,(L\,i) + r\,i = 0$$

If r is zero, then

$$\frac{d}{d\,t}\,(L\,i) = 0$$

That is, constant magnetic linkages. Hence, if the linkages are initially zero, they must remain zero.

Or for two closed rings,

$$\frac{d}{d\,t}\,(L_1\,i_1) + r_1\,i_1 + \frac{d}{d\,t}\,(M\,i_2) = 0$$

and
$$\frac{d}{d\,t}\,(L_2\,i_2) + r_2\,i_2 + \frac{d}{d\,t}\,(M\,i_1) = 0$$

If $r_1 = r_2 = 0$, as assumed, then

$$\frac{d}{d\,t}\,(L_1\,i_1 + M\,i_2) = 0$$

and
$$\frac{d}{d\,t}\,(L_2\,i_2 + M\,i_1) = 0$$

But the first of these gives the magnetic linkages of the first ring, and the second, the linkages of the second ring. That is, the linkages must be constant in each ring. And if they are zero initially then they must remain zero, whatever else happens, so long as the rings are closed and have zero resistance.

Therefore, the deductions in the appendix are wrong. The magnetic linkages cannot decrease or increase in a closed ring without resistance.

Inasmuch as the equations used by the author assume the validity of the above equations, the conclusion would seem to be clean cut.

It may be of interest to further point out that the author's proposed law is merely a specialized re-statement of the commonly applied principle that the electromagnetic force *acting on* any part of a circuit is

$$f = 1/2\, i^2 \frac{d\,L}{d\,x}$$

where, $\frac{d\,L}{d\,x}$ = the rate against distance, not time, at which the total inductance of the circuit would change if motion of that part of the circuit occurred in the direction of the force.

i = instantaneous value of current.

The author's proposed law is: "in an electromagnetic system in which the current is being maintained by a source, any and only such mechanical motions of the conductor will tend to take place as will generate a counter e. m. f. somewhere in the circuit."

The counter e. m. f. which the author defines here is,

$$e = i \frac{d\,L}{d\,t}$$

Of course if there are circuits in inductive relation to the one in question, the force is given by

$$f = 1/2\, i_1^2 \frac{d\,L_1}{d\,x} + \sum i_1\, i_s \frac{d\,M_{1s}}{d\,x}$$

and the counter e. m. f. referred to, is given by

$$e = i_1 \frac{d\,L_1}{d\,t} + \sum i_s \frac{d\,M_{1s}}{d\,t}$$

Furthermore, using the above familiar expression for force, it is easily demonstrated that it is not necessary to assume, as the author does, the special case of constant current. The expression gives the force existing for the particular current, whether the latter is rapidly varying or constant.

In other words, the proposed law is not new, and hardly merits the name of a *law*, for it is merely a re-statement, with an unnecessary limiting condition, of a long established corollary of the Law of Conservation of Energy, from which the above mathematical expression at once follows.

Thus to summarize, all of us will probably agree that, for the profession, it is a wholesome and commendable thing to review and question, as the author has done, the validity of our fundamentals, and to demand an explanation of certain observations which appear to him to be in contradiction to the so-called *laws*. However, I do not agree that he has demonstrated the invalidity of the law of electromagnetic induction in the particular application of determining forces. On the other hand, the fundamental assumptions involved in the equations given in the Appendix, contradict the conclusion there arrived at. Also, the proposed new law is merely an incomplete re-statement of a well-known corollary of the Law of Conservation of Energy.

E. H. Kennard: Dr. Hering's paper arouses in me very mingled feelings. With his criticisms of the textbooks I feel compelled to sympathize to a considerable degree. The writers of textbooks ought, I think, to remember that their books are sure to be employed more or less as reference books and the text ought to be so written that it cannot be misunderstood by any attentive and thoughtful reader. Every exception or restriction upon the scope of a law ought to be mentioned, at least in a footnote. And then, besides imperfections of statement, many books contain a distressing number of genuine errors. But to err is human and we shall never see perfect books.

Let us take up Dr. Hering's chief points in order.

1. *Do circuits tend always to increase their flux?* I think that they do; Dr. Hering's own experiments confirm this law. Of course, in calculating the change in flux one must imagine the current to be kept constant. This assumption will be found to underlie all deductions of the law, although I am afraid that textbook writers do not sufficiently emphasize the point.

In Dr. Hering's Fig. 5, a careful study of the flux shows that the motion increases the flux no matter on which side the battery is placed, for when we flop the circuit over to the other side of the mercury troughs we reverse the direction in which a line of flux must be called positive as it passes through the plane of the paper, and this reversal compensates for the change in the location of the area enclosed by the circuit.

What this experiment really shows is that a circuit does not always *enlarge* under the action of its own flux—and I must admit that some textbooks make wrong statements about this. I think that any plane circuit so shaped that it bends always toward the same side as we go round it, will always tend to enlarge under its own electromagnetic forces, whereas this may not hold if the circuit contains a part shaped like an *S* or a *Z*.

The law holds also for Dr. Hering's imaginary experiment in which two rings of no resistance have currents induced in them and are then brought toward each other; for the currents in the rings do not remain constant. We can write for each ring $L\,d\,i/d\,t = -\,d\,\phi/d\,t$ where ϕ is the flux through the ring due to the current in the other one; this equation gives by integration $L\,i + \phi = \text{const.}$, which shows that the flux through each ring remains fixed forever. Hence as the rings approach and the mutual flux increases, the flux of self-induction, $L\,i$, must decrease, and so must the current, i. Thus the change in flux produced by the motion, which determines the mechanical force, is exactly compensated for by another change caused by the change in the current, and this is why the flux through each ring remains constant.

2. *A General Law of Motion.* Dr. Hering emphasizes two principles, (1) that potential energy tends to become kinetic, and (2) that no motion can be produced by electric currents, at least in the absence of iron, without the generation somewhere of an e. m. f. These two principles are, of course, well known both to physicists and to engineers, although the proposal to make them the foundation of the theory of electromagnetic forces is, perhaps, new. The merits of this proposal are a matter of opinion; for myself, I cannot feel that the proposal is a good one, but the subject is too large for discussion here.

3. *A longitudinal force* is not shown to exist by Dr. Hering's experiments, which are entirely explained by the accepted laws, as I shall show presently.

4. *Sliding Contacts are a Nuisance.* So they are. To the mathematical theorist every discontinuity or abrupt change in conditions is a nuisance and requires a special explanation. The usual practise among physicists (except, perhaps, among the French) seems to be to state the general laws of physics as if such discontinuities did not exist, and then to replace the discontinuous cases by an equivalent continuous one.

The law of induction I like to state in three steps as follows:

(a) *Definition.* A circuit means a closed filament of matter.

(b) The *E. M. F. induced* in any circuit (measured in electromagnetic units) equals the time rate of decrease of the flux that is linked with the circuit.

(c) At a *sliding contact* one may imagine a thin layer of conducting liquid to be inserted between the sliding surfaces, so that there will be a continuous gradation of motion through this layer and permanent elementary circuits can be drawn through the layer.

There are other good ways of treating sliding contacts, of course; the essential thing is to have some sound method of doing it that one understands. My own opinion is that such a transition layer really exists, in the form of interlocking electron atmospheres if not in the form of a layer of pulverized metal.

This law refers only to closed circuits but has, I think, no exceptions. Open circuits are a different proposition.

Of course a statement can be given in terms of the cutting of the magnetic lines by the conductor which is mathematically equivalent to what I have just stated. But I do not think that physicists will ever wish to substitute this statement of the law in terms of line-cutting for their own formulation, because in many cases, such as a transformer, one has to introduce a motion of the lines themselves that is imaginary and arbitrary, or at least not susceptible of observation, and then in order to make any calculations one passes right back from the line-cutting conception to the variation of flux that is referred to in the physicists' statement of the law.

If it were possible to state a simple general law for the electromotive force induced in each part of the circuit, this sliding-contact difficulty would disappear. For the dynamo type of induction, where the conductor moves and cuts the flux, such a law can be given, and is, in fact, well known. But in the transformer type of induction, where the flux itself moves or changes, the exact location of the induced e. m. f. can be found only by solving the Maxwell equations for the electromagnetic field, and such a solution has to be worked out anew for each special case. Under these circumstances the simplest plan would seem to be the one actually adopted, to state the value of the e. m. f. in the whole circuit in terms of the total flux.

5. *The experiments* described by Dr. Hering are interesting but are quite inconclusive. Every one of his results is easy to explain in terms of the familiar "motor rule" for the transverse force on a conductor, although in the absence of quantitative measurements one cannot always be sure which of two or more factors is actually effective.

Thus in Figs. 3 to 7 inclusive the magnetic field produced by the *liquid* part of the circuit has in all cases the right direction to account for the motion, which is due to the transverse forces thereby exerted upon the moving piece. In Figs. 3 and 6, this force must be greater upon the vertical part that dips into the narrow trough than upon the wire dipping into the broad one because of the difference in concentration of the current. I feel pretty sure that this explanation is correct but cannot be quite certain of it in the absence of exact data which would make possible a numerical check.

Figs. 16-17 are unusual only in the presence of a sliding contact, which has to be treated here in the same way as one treats it in applying the law of induction if one wishes to deduce the motion from the law that a circuit seeks to increase its flux. Or, the usual "motor rule" for the force on a conductor is also easy to apply and gives the correct results.

In Figs. 9 and 12 the argument turns upon an application of Newton's Third Law. But, owing to experimental difficulties, the validity of this law has never been established for current elements. As a matter of fact, if the modern theory of the electron is correct, Newton's Third Law does not hold, in general, for the mutual actions of two moving electrons (although by modifying its meaning the law can be made to hold for the electrons and the ether taken together), and it follows that action and reaction are not usually equal for two current elements (or for a current element and a magnet), although the law *is* true for an element and a closed circuit. Thus, before Dr. Hering can infer from the existence of a force upon V in Fig. 9 that there will be a longitudinal force upon H, he must prove by experiment that Newton's Third Law holds for parts of circuits. Otherwise he is himself guilty of that blind dogmatic faith in old laws of which he accuses the physicists.

The observed motion of H in Fig. 9 arises, I think, from forces at the ends, where the currents are unequal.

As to the motion in Fig. 18, I shall associate myself with those who, according to the author, ascribe it to the pinch effect. The magnetic field at the mouth of the tube leading to R pushes the mercury toward the axis of the other tube and so causes the flow. That is, the mercury just at the end of the side-tube is itself squeezed toward the axis of the main tube and so is prevented from escaping into R, as Dr. Hering thinks it should. I have no desire to be presumptuous, but in reading Dr. Hering's discussion of such cases I am sometimes led to wonder whether he has not underestimated the training in habits of exact and consistent thought which is necessary in order to reason correctly about hydrodynamical phenomena.

The more complicated motions in Figs. 8, 10, 11, I should also ascribe to the same action as that which produces the pinch effect or to analogous actions upon solid conductors.

Thus none of these experiments points unmistakably to the existence of a longitudinal force.

Carl Hering: The present discussion, the far more extended though unfortunately unrecorded personal discussions for some years, and the republication of a translation of the entire paper in a French journal, in many of which there are a number of quite favorable comments, are far more encouraging than was to be expected from the strenuous efforts which were made against the acceptance, publication, and reading of this paper. It is no credit to our profession that modest attempts, based on experimental researches and discussions extending over years, to bring some of the acknowledged antiquated factors in our branch of science up-to-date in order to correspond better with modern developments and to open up new fields, if any, which orthodox teachings had closed to us,—should be discouraged by being met with such strenuous efforts chiefly by book writers and teachers, to suppress their publication, even including public attempts at personal ridicule, instead of being welcomed and encouraged by co-operation and constructive criticism. A true scientist is characterized as one who is grateful instead of angry to have pointed out to him any shortcomings, imperfections, inadequacies, omissions, possible errors, etc., in the orthodox views, and suggested improvements open for criticisms.

Apart from attacks on things I did not say, the discussion did not prove a single error in any important point, and it is safe to assume that if the paper had contained any actual error it would surely have been brought out and used as ammunition by the opponents to the suggested reforms. Hence the statements in the paper must be allowed to stand on whatever merits they may have. Differences were entirely on matters of opinion and not on matters of fact; attacks on things I did not say are not proper discussions. Many of the unorthodox statements were not attacked at all, which may be significant.

The proposed general law of electromagnetic motions was not only not attacked but was upheld and proved by others; engineers and students may, therefore, now feel safe to depend on it; I have for years found it far more useful, clear and direct in practise than the older laws which it is intended to replace. Students will be pleased that they need no longer burden their brains with complicated and confusing definitions of that subterfuge "sliding contacts."

The strenuously opposed and long neglected longitudinal force was strongly upheld by some able discussers and disproved by none; the worst that opponents could say about it was to the effect that they could get along without it. The verdict of the opponents is to the effect that the old laws are good enough for them; their students and followers may fall into the same pitfalls that I did years ago while I still had gospel faith in our old laws, and they will not be able to use some useful available new tools, the lack of which may handicap them in competition with those who are not hidebound to traditions, and in the developments in new, formerly forbidden fields which now appear to be open to us.

Practically the only attack of importance by several discussers, concerns those old laws which are to the effect that there must always be an increase of flux and of self inductance when a system does mechanical work. These critics claim that I have denied the corrections of these laws. As there is no such denial in the paper, these critics in this case as also in several other cases in which I did not say what they are attacking, have merely set up a straw man in order to have something to knock down. Discussers ought to read a paper carefully before they attack it.

I had even admitted that in many cases these laws apply correctly, although in some a highly involved, very artificial, confusing definition of sliding contacts (like the very old one used in Prof. Morecroft's discussion), or other limitations, must be included to make them apply. Many misfits can be made to fit, and our ignorance be made to appear to be wisdom (at least to the unsuspecting student), by resorting to an involved definition of sliding contacts, especially when the student cannot understand the definition.

What I did claim, and do so still, now even stronger than before, as my opponents could not deny it, is that these laws are special case laws, and not the most fundamental or universal ones. A universal law is one which can have no exceptions, but these older laws have, as was clearly shown. To exclude these exceptions (and probably others also) it is absolutely necessary to read some limitations, restrictions or qualifications into the law, so that it will fit and not mislead; these, of course, make it a special case law, as it then admittedly does not apply outside of these limitations. This, of course, leaves us *without any law* for all cases outside of those limitations, which is the best proof that it is not a universal law. There may perhaps be very useful cases which are outside of these limitations. One of these proposed restrictions is "provided the m. m. f. does not change;" another is "provided there are no structural changes," yet in every motor there are continual structural changes of the circuit at the commutator. Moreover it is surely no crime to change the m. m. f. or make structural changes, if something useful may be gained thereby, and for these cases we had no laws. One of the chief purposes of this paper is to help those who desire to work in new and formerly forbidden fields.

In my opinion taking refuge behind a "sliding contact" to make misfits fit, is unscientific and a tacit admission that there is something wrong somewhere, either in the subject or in the teacher or book writer, when that old fashioned subterfuge becomes necessary. Sliding contacts used as a subterfuge in explanations can now be (as I have shown), and should now be, ruled out as obsolete, if they are not, the students have my sympathies. Moreover these laws seem to be taught so as to leave the impression in the mind of the student that such increase of flux and self induction were in some mysterious way the *cause* of the motion (which they are not, though in many cases they are incidental to it, due to the connected outside source of energy, as I showed), and that without such increase there could not possibly be any self produced motion; this is not true and ought not to be taught.

The self inductance (a purely geometric quantity) depends greatly on the number of turns, while the flux and flux energy (on which the forces depend) are quite independent of the number of turns being a function only of the ampere-turns; for the same ampere-turns and a given reluctance, hence for the same flux and flux energy, the self inductance may have very greatly different values when the number of turns is changed; to give the student the impression that the self inductance and the flux and flux energy always go hand in hand, is positively wrong. To define A in terms of B when B varies greatly with a factor that does not affect A, is *not* a proper definition. By combining the self inductance and the current as one quantity (representing flux and flux energy) useful laws and undoubtedly correct calculations may be made, as admitted in the paper, but the student should by all means be told (though apparently he is not) that this is limited to special or restricted cases, and is not universally true; this was shown in the paper.

The presence of iron moreover increases the self inductance and the flux very greatly, yet the flux in the iron may not always be free to produce motions; an iron pipe around a centrally located conductor would presumably not increase either the pinching or the stretching forces, notwithstanding the greater flux and self inductance. Hence formulas based on self inductance may sometimes give wrong results.

The chief other point in the paper which the opponents attacked, though ineffectively, is the one concerning the two rings at nearly 0 deg. absolute, though no one has shown the statements made in the paper to be incorrect; this case shows clearly that the flux and flux energy *diminish* when there is self produced motion. In this case the two rings were, and must of course be, considered together as forming one system; to treat one ring and its linkages by itself, as some have done, leads to nothing new and useful, and is improper as one ring does not constitute the whole system.

The real purpose of this example was apparently lost sight of or was not grasped by the opponents, namely to completely separate an electromagnetic system, containing stored energy, from any outside source of new energy, in order to be able to study this stored energy alone and by itself without any interference by new energy. This can easily be done with electrostatic systems, but it is believed that this is the first time an experiment was devised in which this could be done with an electromagnetic system. Our old book laws (like the very old Kelvin law) were always based on the *united action* of *two* things, the stored energy and a source of new energy inseparably connected; never before (I believe) could the stored energy be experimentally studied separately, and this is very necessary in order to get at the real fundamentals. A single equation with two unknowns cannot give the values of each by itself, it is incomplete information; so is an experiment or law involving the inseparable joint action of two factors, as in the old Kelvin law.

The above answers the discussion in general. The constructive discussion of Dr. Mailloux is a contribution of value and importance. His remarks about infinity are especially important, as they explain the absurdities, inconsistencies and even serious fallacies arrived at by even able expert mathematical physicists when they have recklessly jumped into that dangerous abyss, infinity; and still worse, have then tried to force their unacceptable conclusions on to the more cautious thinker or on to the trusting, unsuspecting student, thereby tending to muddle his brain and to check progress. The bringing out of his discussion on infinity has been one of the useful fruits of the paper.

His references to the able work of some of the older French physicists who sometimes disagreed with Maxwell, will no doubt surprise the orthodox, hidebound, Maxwellians who seem to consider it a crime to even question anything which that admittedly brilliant mathematical physicist said 50 years ago, as though progress in science was not possible. The writer, who has discussed these subjects for years with many, has found that there are many progressive physicists who do not consider all the 50-year-old Maxwellian views to be incontestible today, and who admit that progress has since been made. Able physicists have admitted that the Maxwellian "complete circuit" restriction has long ago been abandoned as the most fundamental case and is now out of date and behind the times; it refers to a special case only (though the most usual one) and when so restricted is very useful. Today we must admit that an electron can start from rest at a point A, move to a point B and stop there, that is, it is no longer necessary to involve all the other electrons in the world or universe in this localized action. The old conception that the "complete circuit" is the most basic fundamental one has checked progress and is now antiquated; students should be told this.

Dr. Mailloux's statement that Mascart and Joubert recognized the stretching force, is interesting in view of the strong opposition from lesser lights. That Bjerknes thought all the actions in his experiments could be explained with a single force, is of interest as his experiments were made in liquids which we know act hydro-dynamically. Dr. Mailloux's criticism of the clause "action at a distance" seems to be due merely to different uses of the term; moreover it is used only incidentally in the paper.

Prof. Karapetoff lays great weight on the old Kelvin law. The pioneer dynamo and motor builders, who were no doubt brought up on that law, as I was, soon had to unlearn it in order to construct dynamo and motor installations having a greater efficiency than 50 per cent, which is the greatest that this law proclaims. Practical men must sometimes set aside old theories that mislead. His insistance on a "constant current" shows that this law does not apply to my two-ring example in which there is intentionally no source attached to the system. He evidently did not read the paper carefully as the paper *does* show several cases in which "the coefficient of self inductance is smaller at the end of the motion than it is at the beginning." To exclude these exceptions so-called "patches" (limitations) must be put on the old law to cover the dents made in it by these cases; the law then is admittedly no longer a universal law.

He condemns the paper because it does not also give quantitative results and for that reason calls the experiments "inaccurate," which seems improper criticism as the results in all cases were very decided and left no doubts. In every new investigation qualitative experiments are, and should always be, conducted before the quantitative ones, and the fact that they must be made first should not condemn them. To condemn a paper because it does not contain what it was not intended to contain, is not fair criticism.

But in this criticism he (like others) ignores completely a most important and crucial *quantitative* result shown in connection with Fig. 6, namely that the longitudinal force so strongly opposed by the orthodox physicist, actually overpowers the standard perpendicular forces on which the orthodox physicist depends entirely for the support of his theories; this surely is decidedly a quantitative result, not only in amount but in sign also.

Prof. Morecroft admits he does not understand the paper; as many others have understood it, it seems that the fault is not mine. Yet he discusses at great length what he admits he has not understood! As he differs from many others in his views, his free and unqualified use of the term "fallacy" is not a serious matter; it expresses only his opinion. The difference between kinetic and potential energy is a relative one; the pressure of a confined heated gas often and properly considered as potential energy, is a bombardment of the molecules, hence kinetic, in a sense. Maxwell has also treated magnetic energy as potential. The kinetic energy in a spinning gyroscope may be considered as potential in some problems; it is a case of relativity again. There is nothing in the paper which warrants his saying that I thought "that the idea of a current flowing in such a circuit (Fig. 1) is a new one," in fact I said "it is known."

The conception in Fig. 1, is as old as the electron theory. In saying "it is necessary to consider every circuit closed" he is not in accord with many of his more progressive colleagues.

His long and very complicated description of my 15-year-old experiment, is a mere republication of an old argument published fifteen years ago, which I thought by this time was considered out of date and behind the times. It well illustrates the desperate efforts which many make to seek refuge in the "sliding contact" subterfuge. There are at least several other cases (*Jour. Frank. Inst.* Nov. 1921 Figs. 4 and 6) and probably many more, of which he does not appear to be informed, in which the linking and unlinking of flux with a *closed* circuit induces no e. m. f., and for each of these he would have to invent a new theory of the "sliding contact." The alternative to his long, complicated description is the simple statement that to induce an e. m. f. the flux must cut the material of the conductor and not merely the circuit. This I understand also agrees with the more modern electron theories. It can safely be left to the students and engineers which of the two explanations they prefer. If they are not taught the latter, they have my sympathies, as they may then fall into the same pitfalls that I did before I found this important difference between the circuit and the conductor, which I believe is now generally recognized by progressive men, as it is easily proved. It has now been known for about 15 years and no one has shown that it is not correct, although strenuous attempts have been made for years by theorists to disprove it, or failing in this, to suppress it, merely because it interferes slightly with their 50-year-old theories and their present teachings.

Prof. Morecroft says that it seems that I am "more amenable to experiment than to calculus." Yes, this is true when the calculations are based on man-made laws; in experiments nature talks to us.

His explanation of the constancy of the flux in a motor rests on the sliding contact subterfuge, hence involves a complicated description, which the student could be spared. Concerning his very extended remarks about my (and his) Fig. 4, he again accuses me of saying what is not in my paper and which I purposely avoided saying, as I knew long ago and published it, that the flux does increase; he evidently is not posted on what has been published, and his experiments were not necessary. What had misled so many physicists to giving the wrong answer, was the impression which the old law gives, and which I think is even stated in those words in some books (as is admitted above by Prof. Kennard) and by some teachers, is that "a circuit tends to expand" which are the words used in my paper. It was therefore not I but others who had "jumped to the wrong conclusions." That a circuit can "so arrange itself" that the flux *diminishes*, was clearly shown in the paper, as he will see if he reads it. His law needs a "patch" to make it fit; as he states it it is *not* a universal law.

The remarks of Mr. Mills, as also unpublished discussions of other progressive men who are versed in the new electron theory, are very encouraging. It seems that many of the proposed reforms suggested in the paper, fit in with the electron theory. "Thermal effects" do not necessarily enter into the experiments Figs. 3 and 6; the action is instantaneous.

Mr. Doherty, like others, has overlooked the decidedly "quantitative test" shown in Fig. 6, in which the longitudinal force was greater and opposite to the perpendicular force in which many physicists rely. He is evidently unfamiliar with the now classic Onnes experiment; the explanation which he says he "cannot fathom" is that the current was generated by drawing a single magnetic pole through the ring, as described in the paper. He has not proved that the deductions made in the paper "are wrong." I have replied above to the discussion of this experiment. To avoid the dangerous "jump to infinity" so ably discussed above by Dr. Mailloux, I have assumed that the temperature was only very near zero (as it actually was) and that the experiment was performed quickly. In reference to the general law proposed in the paper, he says I had assumed a "constant current;" he will not find this in the paper; he ought to have read it more carefully before making that statement.

Prof. Kennard says "Of course, in calculating the change in flux one must imagine the current to be kept constant." This is just what I claim should not be done when one is seeking the fundamentals. In my two ring experiment I have succeeded in separating the source (which ordinarily keeps the current constant) from the stored energy of the flux, and this shows (what no one has or can deny and what he himself proves) that the current (and therefore the flux also) diminishes when the flux energy is used up in doing work.

He refers to one ring; the "system" consists of two rings and they must, of course, be considered together as one system, as I did. I am pleased that he admits that "some text books make wrong statements" about a circuit tending to enlarge (my Figs. 4 and 5). In saying that "sliding contacts are a nuisance" he ought to have added, to the theorist who tries to make misfits fit (and to whom they are really a blessing); starting with better laws they drop out completely as a subterfuge; sliding contacts are extremely useful in practise. Apparently in order to meet the well known exceptions to the linkage law, he says "a circuit means a closed filament of matter." I differ with him decidedly, as I have shown in the paper some extremely important differences, which every student ought to be taught, between a "circuit" (as usually considered) and the "material" of the conductor. Moreover an e. m. f. can be induced in an open circuit, a straight bar for instance. To me it seems clear that a line of flux can vanish by converting its energy into mechanical energy, hence be unlinked (in effect) without inducing an e. m. f.; but this view may not yet be generally accepted. In his thought that physicists will never adopt the line-cutting conception for that based on linkages, he is mistaken as there are prominent ones who long ago have done so; students ought to be taught both.

I am quite willing to leave it to students whether my experiments are quite "as easy to explain" by the old laws as by the proposed new ones. Surely no one but a "heretic" would ever have thought of making these experiments (which even opponents admit are interesting) and in every case the results were as predicted. I repeat here what I have often said before, that if a teacher, who is responsible for what his students have learned, were to ask them what would happen in some of these cases, they would probably give the wrong answer or none at all. In questioning the validity of Newton's third law, I think Prof. Kennard has few if any followers. I admit having "blind dogmatic faith" in this law. His explanation of the forces in Fig. 9, are contradicted by the facts. In not accepting the longitudinal force he differs with many others, including more especially some noted French physicists, like Ampere, Mascart, Joubert, etc. When he says that Fig. 8 can be explained by the pinch effect, he shows that he has not read the paper or does not understand the pinch effect, as the observed motion of that chain is in the *reverse* direction to that which would be produced by the pinch effect. In conclusion he seems to intimate that I have not been trained "in habits of exact and consistent thought." As I have also been accused of exactly the reverse (being too exact and consistent in interpreting the old laws), I leave the verdict to the students and engineers who may read the paper.

In conclusion, now that these proposed reforms have been thoroughly discussed, some of them for years, the engineer and the student may form their own opinion as to whether they find the new or the old more useful, direct, correct, reliable and less misleading. In some respects, the choice is a matter of opinion. With liquid conductors, like in electric furnaces and in electrolysis, in which these forces have greater freedom of action than in solids, the new method will be found to be far more helpful and reliable. The open minded progressive student may find interesting departures in new fields formerly closed to us.

Those interested in the subject of this paper will find some additional matter in a note on "Magnetic Flux Around a Conductor," by Carl Hering, A. I. E. E. JOURNAL, May, 1923, p. 519.

Permeability

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INTRODUCTION

WHEN we use the term magnetic permeability we ordinarily think of it as defined by the ratio of $B/H = (\mu)$, where B is expressed in gausses and H in gilberts per centimeter. It is the ratio of the flux in a magnetic circuit produced by a given magnetizing force to the flux which would exist if there were no magnetic material present. If we are considering permeability values for inductions less than that at which maximum μ occurs the true permeability is given only when the material has been carefully demagnetized by applying gradually decreasing reversals of magnetizing force. Referring to the magnetization curve of Fig. 1 the permeability of the material at the induction P is given by the slope of the dotted straight line $P\,O$.

There are, however, other less well known types of permeability. For instance, we have the so-called differential permeability (μ_d) which may be defined as $d\,B/dH$ for any point on the magnetization curve. It is the rate of change of flux with respect to magnetizing force. At the point P the differential permeability is the slope of the tangent drawn through this point as represented by $T\,P$. The differential permeability may be greater or less than the ordinary perme-

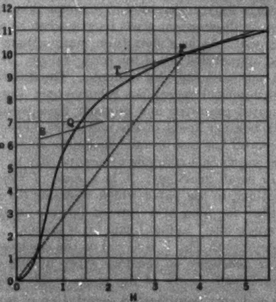

FIG. 1—MAGNETIZATION CURVE SHOWING DIFFERENT KINDS OF PERMEABILITY

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y. February 14-17, 1923.

ability depending upon the induction at which it is taken. Another type of permeability is the reversible permeability (μ_r) of Gans[1]. If at any point on a hysteresis loop we apply a very small reverse magnetizing force, the induction will be slightly changed. If now, we return to the original magnetizing force the induction will return to approximately the original value with the production of a minute hysteresis loop. The change of flux with respect to magnetizing force under these conditions is termed by Gans' reversible permeability. The slope of the line through O indicates the magnitude of this type of permeability. We shall have more to say about this later.

INCREMENTAL PERMEABILITY

The type of permeability which we wish to consider here will be termed incremental permeability (μ_Δ)[2] and will be defined as $\Delta B / \Delta H$ where the change of B may range from the small values required by Gans' reversible permeability up to large values. We shall attempt to give a means of calculating its value at least approximately for any magnetic conditions regardless of the previous magnetic history of the material and for any known types of ferro-magnetic substance. A better conception of the nature of this incremental permeability may be had from Fig. 2. For any minor displaced loops superimposed on a major loop, as for instance, $a\,b$, $c\,e$, or $d\,f$, the μ_Δ will be the ratio of the difference of inductions for the two tips of the minor loops to the difference of the H values for these tips. For minor loops one of whose tips coincides with one tip of the major loop the incremental permeability is represented by the slopes of the dotted lines shown at the bottom of the major loop.

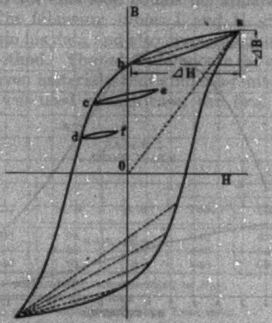

FIG. 2—MAJOR AND MINOR HYSTERESIS LOOPS SHOWING RELATION BETWEEN ORDINARY AND INCREMENTAL PERMEABILITY

The mean slopes (μ_Δ) of these minor hysteresis loops is a function of two variables, *i. e.*

1. For a given ΔB the greater the displacement of the minor loop from its normal position the less the incremental permeability (μ_Δ).

2. For a given displacement of a minor loop the greater the ΔB the greater the incremental permeability.

There is a number of cases where a method of estimating this type of permeability is desirable. For instance, suppose we wish to insert an iron core choke in a circuit carrying direct current. The effective reactance with respect to small alternating currents in the same circuit can not be calculated even approximately from a knowledge of the permeability of the core at the induction corresponding to the known d-c. magnetizing force as may be seen from an inspection of Fig. 2. Suppose the a-c. component of current produces a displaced hysteresis loop $a\,b$. Then the effective permeability of the core is not the slope of the line $o\,a$, but the line $b\,a$, which may be very much less. In radio circuits, where we have an iron-core choke coil in the plate circuit, for instance, we have exactly this condition.

Similarly in the case of a transformer with a d-c. component of current in one of the windings a knowledge of the incremental permeability is essential if we wish to estimate the value of the a-c. magnetizing current.

If it is desired to calculate the magnitude of the skin effect due to the high-frequency flux pulsations in the laminations which make up the teeth of an induction motor, for instance, a knowledge of the incremental premeability is essential since here we have the case of displaced minor hysteresis loops superimposed on a major loop.

Again take the case of the permanent magnet of a d-c. voltmeter or magneto. According to the usual practise this magnet is magnetized with a yoke across the air gap. The magnetizing force and yoke are then removed and due to the demagnetizing effect of the poles the magnet is partially demagnetized, for instance, to a point d (Fig. 2). Now when the regular pole pieces are applied this demagnetizing effect is decreased and we have an increased flux (point f). In order to estimate this increased flux and calculate the available magnetic energy for a given type of magnet steel a knowledge of the incremental permeability is essential.

In order to devise a method of estimating this type of permeability we have principally made use of the data in two previous published articles.[3,4] From the data of the first article we calculated the average permeability of various unsymmetrical loops and plotted these values against different variables. It was found that when plotting against pulsating amplitude a series of approximately straight lines was obtained as shown by Fig. 3. The values on the curve marked B_m indicate the maximum inductions of the minor loops. The short cross lines indicate the value at which the pulsating amplitude equals one-half the pulsating flux for the major loop. Now these curves can obviously be represented by a simple formula, thus,

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[1] Die Reversible Permeabilitat auf der idealen Magnetesierungskurve, Richard Gans. *Annalen der Physik.* Vol. 61, No. 4, p. 379 (1920.

[2] Term suggested by Mr. Yensen.

[3] The Effect of Displaced Magnetic Pulsations on the Hysteresis Loss of Sheet Steel, L. W. Chubb and T. Spooner. PROC. A. I. E. E., Vol. 34, 1915.

[4] Tooth Frequency Losses in Rotating Machines, T. Spooner. JOURNAL A. I. E. E., Vol. XL, Sept. 1921.

$$\mu_\Delta = a + b \times \Delta B,$$

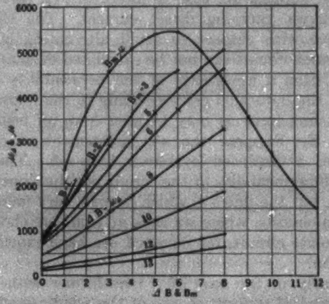

FIG. 3—INCREMENTAL PERMEABILITY AS A FUNCTION OF B_m AND ΔB

where

a = intercept on the vertical axis,
b = slope of the line, and
ΔB = amplitude of pulsation for the minor loop.

From these data and other data largely taken from the results of the second above mentioned paper, the average curves for "a" and "b" are drawn as shown by Fig. 4. A further examination of the various test results showed that the values of μ_Δ were also a function of the ordinary permeability (μ) of the material corresponding to the induction of the tip of the minor loops farthest from zero induction. The formula for incremental permeability is, therefore,

$$\mu_\Delta = \mu_{Bm} (a + b \times \Delta B), \text{ where}$$

μ_Δ = the incremental permeability or the average permeability for a displaced loop,
μ_{Bm} = the ordinary permeability corresponding to the induction of the tip of the loop farthest from zero induction,
a and b = constants as given by Fig. 4, and
ΔB = amplitude of flux pulsation of the minor loop expressed in kilogausses.

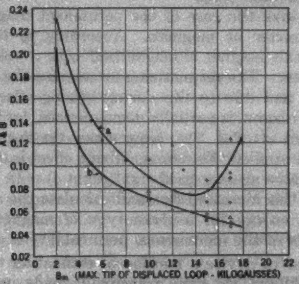

FIG. 4—CONSTANTS "a" AND "b" FOR INCREMENTAL PERMEABILITY EQUATION $\mu_\Delta = \mu_m (a + bx\, \Delta B)$

The value of "a" when B_m equals zero is one and approaches one again for high values of B_m: "b" approaches zero for high values of B_m.

In order to illustrate the difference between normal permeability and incremental permeability see Fig. 5. Here we have shown a normal permeability curve plotted against B_{max} and an incremental permeability curve also plotted against B_{max} where ΔB = 1 kilogauss for the latter curve. As ΔB becomes smaller the difference between the two curves would increase and as ΔB becomes larger the difference would decrease.

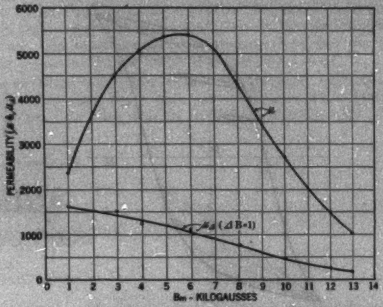

FIG. 5—CURVES SHOWING RELATION BETWEEN ORDINARY AND INCREMENTAL PERMEABILITY ($\Delta B21$) AS A FUNCTION OF B_m

DISCUSSION OF RESULTS

It must be understood that this method of calculating μ_Δ is merely an approximate empirical one and while it holds very well for most commercial ferromagnetic materials of very different properties ranging from permanent magnet steel to very high permeability vacuum iron, it is subject to considerable error for certain special types of material as will be pointed out later. If an attempt is made to use the formula for pulsating amplitudes (ΔB) much greater than one-half B_m the calculated results will in general be somewhat low. This can be seen for instance by an inspection of the $B = 3$ or $B = 5$ curves of Fig. 3. The average slope decreases at the higher pulsating amplitudes.

Now we need to give some consideration to the case of the minor loops which do not occur at the tips of the major loops. In the investigation of tooth pulsation losses in induction motors referred to above[4], a large number of minor loops were obtained located in various positions with a reference to the major loops of various magnitudes. So far as we could determine the average permeability for the minor loops is very largely independent of the magnitude of the major loops with which they are associated, but is chiefly determined by the amplitude and the displacement of the minor loops. A minor loop for a given amplitude and displacement will have a slightly higher μ_Δ if it occurs on the downward branch of a major loop above zero induction than if it occurs below zero induction, but the calculated values will in general be between the two.

Mr. Searle[5] has published some data which may be used for checking our formulae. He applied a certain magnetizing force to his sample bringing it up to an induction, say of "a" Fig. 2. He then applied a certain demagnetizing force bringing his induction down to such a point as "o". He then reversed this demagnetizing force for two hundred times and determined his minor hysteresis loop. The μ_Δ or average permeability for this loop equals his apparent permeability. The following tables I and II give results deduced from his data and also gives calculated results using our method. The only difference in the various values for a given material is that a different initial magnetizing force was used in each case, the first horizontal line corresponding to the highest magnetizing force. This means that the first horizontal line corresponds to the largest major loop.

TABLE I
IRON WIRE

B_m' Minor Loop	μ_m	ΔB	ΔH	$\mu\Delta$ Test	$\mu\Delta$ Cal	H_m
4.4	1760	1.03	3	340	474	2.5
3.65	1590	1.38	3	460	550	2.3
1.82	1040	1.93	3	645	694	1.75

μ_m is normal permeability corresponding to B_m'.
H_m is maximum H for major loop.
B_m' is the tip of the minor loop farthest from zero.

TABLE II
TAGGER PLATE

B_m' Minor Loop	μ_m	ΔB	ΔH	$\mu\Delta$ Test	$\mu\Delta$ Cal	H_m
6.8	4110	3.67	2	1840	1800	1.65
4.8	3870	4.71	2	2350	2500	1.24
*2.78	2780	5.56	2	2780	2960	1.00

*Normal loop.

We have obtained similar data on ring samples of commercial sheet with the results as shown by Tables III and IV.

TABLE III
0.014" 2¼ PER CENT SILICON STEEL

B'_m Minor Loop	μ_m	μ	ΔB	ΔH	$\mu\Delta$ Test	$\mu\Delta$ Cal.	Major Loop H_m	Major Loop B_m
4.77	3670	3260	6.07	2	3030	2930	30.0	14.8
5.33	3600	980	0.73	1	730	756	30.0	14.8
4.1	3630	3330	6.3	2	3150	2960	4.4	10.0
4.77	3670	1020	0.77	1	770	842	4.4	10.0

μ = normal permeability corresponding to $\frac{\Delta B}{2}$

There are some interesting points in connection with these loops.

a. The first minor loop has 10 to 20 per cent higher permeability than the 100th loop.

b. This change is nearly complete at the end of ten double reversals and is quite complete (within the limits of accuracy of test) at 50 double reversals.

TABLE IV
0.028" BESSEMER

B_m' Minor Loop	μ_m	μ	ΔB	ΔH	$\mu\Delta$ Test	$\mu\Delta$ Cal.	Major Loop H_m	Major Loop B_m
6.25	2030	400	0.50	2	250	357	6.15	10.0
7.65	1960	370	0.45	2	225	294	30.	15.45

c. After 100 double reversals the minor loop has shifted down towards the horizontal axis to a very slight amount only (perhaps 2 or 3 per cent on the average) as shown by measuring the change in induction when going to the lower tip of the major loop.

It will be seen from these data that the low values of permeability obtained from a magnetic test sample which has been insufficiently demagnetized is simply a case of displaced loops and if we know the displacement we may calculate the corresponding effective permeability (μ_Δ), which will be the test value under these conditions. The test μ_Δ should be compared with the normal permeability (μ) for the undisplaced loop of the same B amplitude. It will be seen that the μ_Δ values are always lower.

TABLE V
BALL'S DATA

B_m	μ_m	ΔB	ΔH	$\mu\Delta$ Test	$\mu\Delta$ Cal.
12	2310	4	5.45	735	775
10	3510	4	3.35	1190	1330
8	4450	4	2.4	1670	1900
6	4610	4	1.9	2100	2290
4	4060	4	1.6	2500	2550
*2	2700	4	1.48	2700	2840
11.5	2880	3	4.18	718	786
9.5	4130	3	2.65	1130	1290
7.5	4690	3	2.05	1460	1670
5.5	4660	3	1.68	1780	1980
3.5	4110	3	1.37	2190	2320
*1.5	2310	3	1.30	2210	..
11	3140	2	3.3	607	698
9	4190	2	2.3	870	1040
7	5000	2	1.68	1200	1430
5	4420	2	1.41	1420	1550
3	3750	2	1.15	1740	1800
*1	1670	2	1.14	1750	..
10.5	3690	2	2.15	465	580
8.5	4720	1	1.52	658	850
6.5	5000	1	1.12	892	1050
4.5	4280	1	1.05	952	1130
2.5	3330	1	0.85	1180	1270
*0.5	1320	1	0.76	1320	..

*Symmetrical Loop.

As a further check on our formulae we took Ball's data[6] and determined the μ_{Δ} for his test results. We then compared them with the values obtained from our formula. Table V gives the results. One tip value of the minor loop coincides with the tip value of the major loop in all cases.

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[5] Studies in Magnetic Testing, G. F. C. Searle. I. E. E. Vol. 34, page 55 (1904).

Now to come to the limiting case of Gans of very small pulsating amplitudes, namely, the case of reversible permeability. This should be given by the first part of our formula, thus

$$\mu_{\Delta} = \mu_{Bm}\ a,$$

since ΔB and therefore b are negligible. It will be found by comparing our results with those of Gans that the values do not check well. This is due to the fact that as shown by Gans' reversible premeability is a function of the initial permeability at low and moderate inductions which varies quite considerably for various materials. For pulsating amplitudes of any appreciable magnitude when "b" becomes an important factor the results will be much more accurate since "b" is not subject to much variation for different materials (see Fig. 4), where the circle and triangle points are "b" values and the dots and crosses are "a" values. If we were to take into account the variations of the initial permeability we would have to use a different curve for each material.

TABLE VI
GAN'S DATA

Material	$K_0 = \left(\frac{\mu_0 - 1}{4\pi}\right)$	B	$\mu\ B_m$	$\mu\Delta\ (\mu r)$	
				Cal.	Gans
Hard Steel........	41.8	12.9	99.2	7.55	15.0
" "	41.8	7.54	130.0	14.3	34.0
" "	41.8	2.51	68.0	13.9	43.0
Annealed Steel.....	71.0	15.7	131.0	11.0	8.0
" "	71.0	7.54	471.0	51.9	59.0
" "	71.0	2.51	209.0	42.7	70.0
Soft Iron..........	92.0	17.6	116.0	15.8	10.0
" "	92.0	7.54	1680.0	185.0	72.0
" "	92.0	2.51	838.0	176.0	89.0
Nickel...........	11.2	4.06	24.0	3.89	6.8
"	11.2	2.51	50.2	10.2	10.1

Mr. Gans has given test results for four samples as shown by one of the previously mentioned articles[1]. The following table VI gives his results as compared with our calculated results.

K_0 and μ_0 are respectively the initial susceptibility and initial permeability for the samples (equals the susceptibility and permeability as H approaches zero). If the reversible permeability is required accurately, it may be calculated by the following formulas due to Gans:

$$\frac{K_r}{K_0} = \frac{1}{X^2} - \frac{1}{\sin^2 X}$$

$$\frac{I}{I\,\alpha} = \cot X - \frac{1}{X}$$

$K_r = \left(\frac{\mu_r - 1}{4\pi}\right)$ is the susceptibility for any given intensity of magnetization $I_\alpha = \left(\frac{(B-H)}{4\pi}\right)$

K_r is the initial susceptibility corresponding to zero H. $I\,\alpha$ is the saturation value of the intensity of magnetization.

The difficulty in using these formulas is that ordinarily we do not know either the initial susceptibility or the saturation induction since both are rather difficult to obtain experimentally.

The data of table VII were obtained from tests of various experimental alloys having a wide range of properties, and serves to show the accuracy and limitations of our formula.

The data of sample *A* refers to a major loop having a maximum induction of 14 kilogausses. The minor loops therefore start at a considerable distance down on the demagnetization curve of the material. For all the other samples the maximum tip of the minor loop coincides with the maximum tip of the major loop and the permeability, therefore, corresponds to the slopes of the dotted lines of Fig. 2.

Sample *B* is a hard chromium magnet steel, and sample *C* is a much softer specimen of the same sample.

Samples *D*, *E*, *F* and *G* are nickel-iron alloys supplied by Mr. Yensen. Samples *E* and *G* have very low retentivity. Sample *F* has a low saturation value of the order of 5 kilogausses.

Sample *H* is a very pure iron having a very high retentivity value.

Samples *I*, *J* and *K* are approximately 4 per cent silicon-iron alloys having very high maximum permeability and low hysteresis.

These samples have been selected because of their very wide variations in magnetic properties. It is seen that with very few exceptions the checks between calculated and test values are quite good, certainly good enough for any ordinary commerical calculations. When the retentivity is exceptionally low or exceptionally high (see samples *E* and *G* for low B_r values and *I* for high) the checks are not so good, but for materials having anything like normal B_r values regardless of the permeability whether 20,000 or 20 the calculated results check the test results very well. No check results in addition to those already presented are given for commercial electrical sheet, but in all cases that we have examined the calculated results may be relied on to within 10 or 20 per cent even in the completed apparatus.

When an air gap is present in the magnetic circuit a little different procedure is necessary in order to calculate the effective permeability. First from the known characteristics of the material calculate μ_{Δ}, then $\Delta H = \frac{\Delta B}{\mu_{\Delta}}$. Now calculate or determine experimentally the gilberts per centimeter necessary to overcome the reluctance of the air gap, namely, the magnetizing force used up in the air gap to produce a change

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[6] The Unsymmetrical Hysteresis Loop, John D. Ball. TRANS. A. I. E. E., page 2693 (1915).

TABLE VII
MISCELLANEOUS SAMPLES

Sample	Material	B_m	H_m	ΔB	ΔH	μB_m	μ_Δ	
							Test	Cal.
A	Cr Magnet Steel Hard	8.8	—10.5	1.8	36.5	165.0	49.4	40.1
	" " " "	7.2	—7.7	2.2	41.3	147.0	53.3	44.2
B	" " " "	14.0	200.0	1.0	85.0	70.0	11.8	9.9
	" " " "	14.0	200.0	4.0	200.0	70.0	20.0	22.0
	" " " "	14.0	200.0	14.0	250.0	70.0	56.0	62.6
C	" " " Soft	14.9	200.0	1.0	95.0	74.6	10.5	10.4
	" " " "	14.9	200.0	5.55	200.0	74.6	27.7	29.7
	" " " "	14.9	200.0	14.9	214.9	74.6	69.4	69.5
D	50 Per Cent Nickel Steel	10.0	4.1	1.0	1.35	2440.0	740.0	412.0
	" " " " "	10.0	4.1	6.0	3.9	2440.0	1540.0	1279.0
E	34.5 " " " "	10.0	29.5	2.0	16.7	340.0	120.0	82.0
F	20 " " " "	4.0	21.0	1.0	14.5	190.0	69.0	54.0
	" " " " "	4.0	21.0	1.8	21.0	190.0	86.0	73.0
G	14.9 " " " "	10.0	27.7	2.0	14.0	361.0	143.0	85.0
H	Pure Iron	10.0	0.085	1.0	0.085	11750.0	1175.0	1980.0
	" "	10.0	0.085	5.0	0.119	11750.0	4200.0	5320.0
	" "	10.0	0.085	10.0	0.130	11750.0	7690.0	9500.0
I	Silicon Steel	10.0	0.435	1.0	0.0335	23000.0	2960.0	3880.0
	" "	10.0	0.435	5.0	0.0535	23000.0	9350.0	10400.0
	" "	10.0	0.435	10.0	0.0585	23000.0	17100.0	18600.0
J	" "	10.0	0.638	1.0	0.478	15700.0	2090.0	2660.0
	" "	10.0	0.638	5.0	0.748	15700.0	6690.0	7110.0
	" "	10.0	0.638	10.0	0.798	15700.0	12500.0	12670.0
K	" "	10.0	0.75	1.0	0.53	13300.0	1890.0	2250.0
	" "	10.0	0.75	5.0	0.82	13300.0	6100.0	5760.0
	" "	10.0	0.75	10.0	0.88	13300.0	11400.0	10740.0

in the iron induction of ΔB. Let this be ΔH_g. If the air gap is very short this will be approximately ΔB times the length of the gap in centimeters. Then we have μ_Δ (actual) $= \dfrac{\Delta B}{\Delta H + \Delta H_g}$.

The "a" and "b" curves of Fig. 4 have not been carried below B_m values of 2 kilogausses because the effect of displacement is very small for small displacements and it is just as well or better under these conditions to use the ordinary permeability corresponding to ΔB. For values above B_m equals 18 kilogausses the hysteresis loop practically collapses and the values of μ_Δ can just as well be calculated from the ordinary magnetization curve and will be equal approximately to the differential permeability for the mean pulsating induction.

In using this method of calculating μ_Δ there are a few precautions to take, for instance, if we have the case of an iron core with a magnetizing winding carrying both alternating and direct currents, the maximum induction in the iron can not be obtained with certainty from the d-c. magnetization curve of the material and the value of the d-c. magnetizing component, but for moderate values of induction it will be higher and possibly very much higher due to two causes. In the first place the maximum magnetizing force is the arithmetical sum of the d-c. and maximum a-c. components of the magnetizing force and second the magnetic particles may be more readily oriented since they are shaken up by the a-c. magnetizing force. If the a-c. component of magnetization is sufficiently large the apparent d-c. magnetization curve will be approximately a straight line passing through the induction point corresponding to the maximum permeability and through the origin. Above the point of maximum permeability the a-c. has little effect on the d-c. magnetization curve.

Again if the a-c. frequency is high enough or the laminations thick enough we shall have skin effects which may make the effective permeability much less than the normal d-c. value[7] and will make μ_Δ less than that calculated by the above formula when a d-c. component of magnetizing force is present. This was evidently a large factor in the results obtained by A. W. Smith[8], where he found even for symmetrical conditions an effective a-c. permeability of about 1/5 the d-c. value at 500 cycles. As an illustration of the use of the μ_Δ values we had occasion recently to put an autotransformer in a d-c. circuit as a choke for a small high-frequency component which was also present in the circuit. The effect of this choke could of course be determined from a knowledge of the inductance which might be calculated by the following well known formula

$$L = \frac{4\pi N^2}{R},$$

where

$$R = \frac{l}{\mu A}$$

Now taking the permeability corresponding to the maximum induction due to the d-c. component as determined from a magnetization curve of the core material we would obtain a certain value for L. If, however, instead of using μ_{max} as determined above, we

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7. Losses in Sheet Steel at Radio Frequencies, Marius Latour Institute Radio Engineers, Feb. 1919, page 61.

8. Effect of a Superimposed Constant Field upon the Alternating Current of Permeability and Energy Loss in Iron, A. W. Smith, *Physical Review*, March 1921, page 416.

use the correct value μ_Δ, the value of the inductance for the particular conditions considered is about 1/20 the value obtained by using μ_{max}.

CONCLUSIONS

We have given an approximate method of calculating the ratio $\frac{\Delta B}{\Delta H} = (\mu_\Delta)$ usually with an error of not greater than 10 per cent or 20 per cent for any class of ferro-magnetic materials regardless of the magnitude of ΔB and ΔH and of the previous magnetic history. The results will be more accurate if the amplitude of B is considerable. For very small values of ΔB the result may be rather unreliable. In order to calculate μ_Δ for any value of ΔB and any value of magnetic induction B_m we need simply to know the ordinary permeability (μ) of the material corresponding to B_m and the constants "a" and "b" as given by Fig. 4. It is rather surprising that this method checks as well as it does due to the fact that the curves of Fig. 4 are based on rather meager data involving only a few samples. We might have increased the accuracy by using a different "a" curve for different classes of material but this would have complicated the method and is not necessary except when considerable accuracy is required or when the ΔB values are very small. It is hoped that this simple method may find application for the solution of various electrical problems as suggested in the early part of this paper.

Discussion

E. L. Bowles: Mr. Spooner has presented some very interesting data concerning an involved and difficult subject. The analysis of circuits containing what are rather equivocally called "variable constants," is a difficult and sometimes an impossible problem. At present we are faced with a very serious situation no matter which way we turn. Variable coefficients of resistance are met with in the case of gaseous or thermionic conduction, and they are met with also in the case of circuits containing magnetic materials. In fact, even dielectrics suggest the consideration of variable coefficients. In view of these conditions, it seems that we should talk of the *Coefficients* of an electric circuit rather than the *Constants*, for after all, in developing the subject, one must overcome the handicap which results from a treatment of the electric circuit on the basis of constants.

Thus far, circuits with variable coefficients have required the use of Fourier series, Fourier integrals and integral equations. Many analyses are so involved that one loses sight of the very principle or purpose. Unless a great simplification is made in our treatment of such problems, we will have to rely almost solely upon empirical, or else cut-and-try methods. Perhaps it will be possible to develop a number of graphical solutions wherein the volt-ampere or other characteristics are used as the foundation. Graphical methods have been applied in the determination of the operating characteristics of generators and motors. To a certain limited extent, they have been applied to thermionic problems, and it is the writer's feeling that this method of attack may ultimately become a very fruitful one.

It is hard to place general confidence in Mr. Spooner's special results, since the assumptions made are likely to mislead one rather seriously. The question of what happens in a choke coil used for radio frequency is as yet unanswerable, owing to the complexity of the conditions involved. Conclusions based on low frequency analyses can hardly hold in such cases. The reactance of an iron circuit depends not only upon the phenomenon of hysteresis but also upon the eddy current loss. The separation of these two is difficult, and in some cases it is impossible.

Another point to be brought up is the question of the inductance of a circuit containing variable coefficients. In a circuit containing a magnetic material of variable permeability, it is not possible to speak of such a thing as a constant inductance L, but rather to speak of another variable or instantaneous inductance l for the induced voltage is by fundamental concept:

$$e = -N\frac{d\phi}{dt} \times 10^{-8} \text{ volts}$$

$$= -N\frac{d\phi}{di}\frac{di}{dt} \times 10^{-8}$$

or

$$e = -l\,\frac{di}{dt} \qquad \text{volts}$$

where

$$1 = -N\frac{d\phi}{di} \times 10^{-8} \text{ henrys}$$

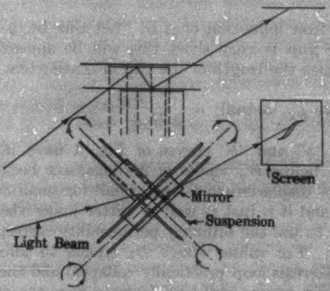

FIG. 1

If now the permeability is constant, then the derivative of flux with respect to current is constant, or:

$$L = \frac{N\Phi}{I} \times 10^{-8} \text{ henrys}$$

which is the familiar expression for inductance ordinarily used, or

$$e = -L\frac{di}{dt} \text{ volts}$$

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[*] The Research Department of the Massachusetts Institute of Technology has spent considerable time during the last three years in attempting to exactly determine the dynamical conditions existing in a magnetic circuit containing iron. Much work has been done in the development of a special bi-vibrator-quadrantal oscillograph. This idea is not new, nor was its application new in this particular case. This oscillograph consists of two vibrators facing each other, and at right angles. A beam of light striking one of the mirrors is reflected to the other, and then reflected once more, as shown in Fig. 1. In this way, a compounding of the motions of the two mirrors will produce the well known Lissajous figures in the case where the two motions are harmonic. If the deflection of one of the vibrators is made porportional to the magnetizing force, and the deflection of the

other proportional to the flux, such an oscillograph is capable of tracing hysteresis loops visually so that they can be directly photographed.

The parts of the completed oil-damped oscillograph are shown in Fig. 2. One of the vibrators is capable of rotating about an axis parallel to the axis of the suspension, and the other vibrator is capable of rotating about an axis lying in and perpendicular to the mirror. Thus the beams from the two mirrors can be made to trace orthoganal lines on a screen or negative. The completed oscillograph, with its magnets, is shown in Fig. 3.

Fig. 2

Fig. 3

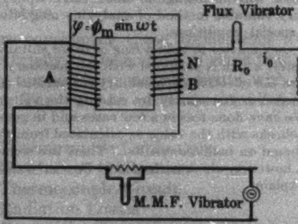

Fig. 4

The research problem has resolved itself into the determination of a circuit which would carry a current proportional to the flux. An example of a particular circuit with the vibrators inserted is shown in Fig. 4. Neglecting the leakage reactance of the coil *B*, we have that

$$e = N \frac{d\phi}{\phi t} = R_o i_o + L \frac{d i_o}{\phi t} \quad \text{volts} \qquad (1)$$

And the solution of this equation, neglecting the transient term, is:

$$i_o = K \epsilon^{-\frac{Rt}{L}} \int \epsilon^{-\frac{Rt}{L}} d B \quad \text{amperes} \qquad (2)$$

If now R_o is negligible, we have the result:

$$i_o = K \int d B \quad \text{amperes} \qquad (3)$$

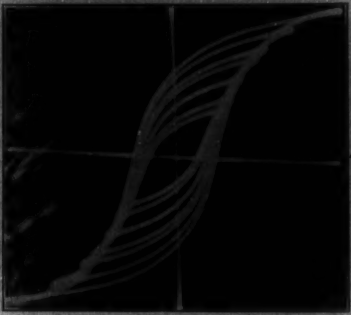

Fig. 5

So that if an oscillograph vibrator were inserted, as in Fig. 1, at any instant its deflection would be proportional to the flux in the iron.

Some idea of the validity of the assumption of negligible resistance can be obtained if certain conditions are taken. For example, if a sinusoidal voltage is impressed on the magnetizing coil *A* of Fig. 1, then the flux will be a sinusoidal. With this condition, the exact steady state solution of equation (1) is:

Fig. 6

$$i_e = K_1 \frac{\omega}{R_e^2 + L^2 \omega^2} \left[B \pm \frac{R_e}{L \omega} \sqrt{B_m^2 - B^2} \right] \text{ amperes} \quad (4)$$

Or by expansion in which all terms except the first are neglected, equation (4) becomes:

$$i_e = K_2 \left[B \pm \frac{R_e}{\omega L} \sqrt{B_m^2 - B^2} \right] \text{ amperes} \quad (5)$$

From which the per cent deflection error, η_e, for a major loop is:

$$\eta_e = \frac{\dfrac{R_e}{\omega L} \sqrt{B_m^2 - B^2}}{B} \text{ numeric} \quad (6)$$

It is seen that the error is zero when $B = B_m$, and a maximum when $B = 0$, or:

$$(\eta_e)_{max} = \frac{R_e}{\omega L} \text{ numeric} \quad (7)$$

The area represented by the error term is of the form of an ellipse. In order that the area of the ellipse be negligible, R_e must be very low, or L very high, or both. For frequencies in the power range, L would need to be impractically large. It was attempted to make R_e small by the use of a dynatron, but difficulties were encountered which at that time, at least, could not be overcome. With no corrections for resistance, J. P. Putnam, in his Bachelor's Thesis (1921), secured some very interesting oscillograms which are shown in Figs. 5 and 6. (These were obtained with an experimental instrument as first used.) In Fig. 5, a very interesting effect is noticeable. The tips of the successive loops do not follow the magnetization curve as one ordinarily imagines. Fig. 6 shows the effect of a direct component of current in an iron core. These curves suggest points of considerable interest which can only be investigated by dynamical methods of hysteresis loss determinations.

Another method of securing a current through the flux vibrator proportional to the flux is to replace the inductance L, shown in Fig. 1, by a high resistance in series with a condenser. The difference in potential between the terminals of the condenser is:

$$v_c = 1/c \int i \, d \, t \quad \text{volts}$$

But if R_e is large, then

$$v_c = \frac{1}{R_e \, C} \int e \, d \, t \text{ volts}$$

And since the integral of voltage is flux:

$$v_c = \frac{1}{R_e \, C} \, \phi \text{ volts}$$

where ϕ is the instantaneous flux. The error is of the same nature as in the case of the inductance, and is expressible by:

$$(\eta_e)_{max} = \frac{1}{\omega \, R_e \, C} \text{ numeric}$$

The two methods mentioned are impractical as they stand, but a third method is now being worked upon, which promises to yield results which will be accurate, under the condition that the laminations making up the magnetic material under test are so thin that the effect of eddy currents is negligible.

G. H. Cole: Mr. Spooner's paper is of value to the designing engineer rather than to the mathematical physicist. It is quite probable that in the majority of design calculations involving incremental permeability, its value is simply guessed at. Fig. 5 of Mr. Spooner's paper shows that the ordinary permeability may be several times as large as the incremental permeability and hence the use of this value instead of incremental permeability may result in large errors, while Mr. Spooner's simple formula quickly gives the incremental permeability of sufficiently good accuracy for most practical purposes.

For the benefit of engineers who are accustomed to measuring electrical quantities with errors of less than a per cent, it may be well to mention that the standard method of measuring ordinary permeability of sheet steel may introduce errors, if the permeability is very high, of the same magnitude as the difference in the incremental permeability given by the Spooner formula and the test values recorded in Table VII covering a variety of materials of quite different magnetic properties. Even though the ordinary permeability may be accurately determined on specimens, the variation throughout the lot of steel from which the samples are chosen may be of the same order as those of the incremental permeability expected from the Spooner formula. This formula therefore, seems to be of sufficient accuracy for most applications and enables the designer to quickly estimate this characteristic of steel concerning which so little is generally known.

T. Spooner: With reference to the calculation of incremental permeability where the frequency is sufficiently high or the thickness of the laminations sufficiently great so that skin effect is appreciable Mr. Bowles points out that this method does not give reliable results. This is, of course, true as we have taken pains to mention in the paper. We hope to go into this aspect of the subject at a later date.

With reference to this new oscillographic arrangement mentioned by Mr. Bowles I had the pleasure a year or so ago of seeing some hysteresis loops taken with this apparatus. These loops were rather far from true hysteresis loops due to the fact that the resistivity of the secondary circuit could not be made negligible. This, of course, introduced very appreciable errors. It is to be hoped that this apparatus can be developed so that the resistance factor will be negligible since there would then be available a very valuable method of analyzing magnetic circuits under these special conditions.

It is a very easy matter to take a ring or other suitable sample of laminated steel which is provided with the necessary windings and to measure the effective permeability for various conditions of superimposed d-c. flux simply by reading the voltmeter and ammeter. We have done this in a few cases and in general have obtained fair checks with the value as calculated from the curves and formula based on ballistic results. There are some marked discrepancies, however, which perhaps Mr. Bowles' oscillographic method can explain.

Applications and Limitations of Thermocouples for Measuring Temperatures

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THE increasing application of temperature measurements to central station control arises primarily from the fact that the limiting factor of safe operation is temperature rather than current or energy output. Another and closely related factor is that of efficiency. With these two influences bearing upon the management there is demanded a knowledge of temperature conditions in the boiler, the engine, the generator and the distributing system. Hence the wide range of temperatures to be measured and the varying circumstances under which they are made often place them beyond the capacity of mercurial or other expansion types of thermometers. Moreover, a permanent and continuous record of temperature may be necessary or it may be desirable to automatically control or limit the temperature of some apparatus or process. The thermocouple has definitely established itself as a convenient and precise instrument for such purposes. We wish, therefore, to consider some of the difficulties to be overcome and precautions to be taken in applying the thermocouple to such measurements.

From the viewpoint of the station engineer we may conveniently classify temperature measurements under the following heads:

1. Feed Water
2. Boiler Water
3. Economizer
4. Flue Gas
5. Superheated Steam
6. Bearings
7. Generator Windings
8. Transformer Windings
9. Cables

Before considering these applications it is desirable to refer briefly to the sources of error that may arise in making temperature measurements with the thermocouple. They may be outlined as residing in the following:

1. Thermocouple Calibration
2. Instrument Calibration
3. Thermocouple Circuit
4. Radiation Losses
5. Conduction Losses
6. Parasitic e. m. f.
7. Temperature Lag
8. Cold Junction Temperature
9. Measured Temperature Variable
10. Measured Temperature Not Representative.

I. THERMOCOUPLE CALIBRATION

A thermocouple may be in error because its original calibration was wrong or because its calibration may have changed from use. Manufacturers are accustomed to furnish thermocouples with either a table or a curve giving the relation between e. m. f. and temperature. It is not safe to employ such couples or to secure wires and make up couples without either the manufacturer's or one's own check on the calibration. This applies particularly to base metal couples but it should not be ignored even with noble metal couples. The writer received some reports recently on checks of *Pt-Pt Rh* couples that were apparently standard when checked by the wire manufacturer but were out 6 deg. cent. at some points when checked by the pyrometer maker. This was due in part to the use of different standard curves and in part to errors in cold junction measurements.

For temperature measurements below 320 deg. cent. it is customary to use couples of copper and constantan. They may be relied upon to remain constant to within 0.5 deg. cent. Even up to 500 deg. cent. they will be found accurate and reliable although the copper element oxidizes rapidly at such a high temperature and therefore must be renewed frequently. It is the writer's experience that electrolytic copper taken from various sources and at different times will not vary 0.05 deg. cent. when checked against the same piece of constantan. With the constantan, however, despite its name, one must use caution and not assume that any material supposedly constantan or its equivalent will have the same temperature e. m. f. relation against copper. In fact there is a sufficient variation in a single melt of constantan to demand on the part of the manufacturer that careful checks be made on all coils of wire and even at intervals along a single coil. To show to what extent a single coil of constantan may vary, ten couples were made from the same coil cutting them off in succession from the coil. They were checked with great care so that errors of checking were less than 0.05 deg. cent. The couples differed amongst themselves by 0.1 of 1 per cent at 38 deg. cent., and 0.2 of 1 per cent at 320 deg. cent. Because of the great variation in the e. m. f. of constantan when got from different sources, pyrometer manufacturers have adopted different standards of e. m. f. against copper. For example, the standards of two American makers differ 15.6 deg. cent. at 93 deg. cent. 19 deg. cent. at 200 deg. cent., and proportionately more at higher temperatures. Of course no error results from the use of any couple if the proper e. m. f. temperature relation

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is known and used but it emphasizes the fact that couples should be used only with either the maker's or one's own check to insure that no error results from this source.

Up to 320 deg. cent. thermocouples may be checked against a mercurial thermometer by placing both in a well stirred oil bath keeping them close together but away from the sides or bottom of the bath. Practically the only difficulty is to obtain a reliable and accurate mercury thermometer available up to 320 deg. cent. When employing a mercurial thermometer it is necessary to avoid stem errors by immersing in the bath just up to the point where it was immersed at the time of calibrating or by applying stem corrections. Failure to do this may result in errors as great as 14 deg. cent. at 320 deg. cent. and errors proportionately great at lower temperatures (Scientific Paper No. 170 (1911) Bureau of Standards). Nearly as large errors may also result from using mercurial thermometers that have not been properly annealed. In view of these facts, the writer considers the mercurial thermometer less desirable as a standard than a thermocouple. It is found both convenient and reliable to reserve two or three copper constantan couples of about No. 22 B. & S. gage wire as standards, sending them occasionally to the Bureau of Standards for check and using them only for the purpose of checking working thermocouples. Checking one couple against another is easily and quickly accomplished. No particular precaution need be observed to avoid errors due to the cooling effect of the emergent portion of the couples. The writer immersed fine wire couples to a depth of about one half inch in a bath of mercury which was stirred and electrically heated. The couple wires were drawn down to diameters varying from 0.013 in. to 0.031 in. While the mercury was heated to 149 deg. cent. and then cooled, a continuous record was made of the differences in temperature registered by the various couples. They differed at no time by so much as 0.25 deg. cent. This difference was not increased measurably by maintaining the bath at a constant temperature and heating the extended wires to a red heat. Nor was it increased by placing the wires in ice. A similar procedure using an oil bath but immersing the couples to a depth of six inches produced like results. As indicated in Fig. 1, an oil bath had a light asbestos sheet cover ½ in. above the oil and the thermocouple wires extended through the cover. Heating or cooling the wires was done just above the cover. It appears, therefore, that from the standpoint of cooling by conduction along the couple wires, there need be no fear of errors of any appreciable magnitude when the couple wires are of No. 22 B. & S. gage or smaller, providing the hot junction is in intimate contact with the bath. When, however, the couples are of heavy wire or are not intimately in contact with the bath, error from this source may be considerable in magnitude as well as uncertain in amount. For example, a No. 8 B. & S. at 149 deg. cent. by immersing to a depth of six inches in oil had an apparent error of 10 deg. cent. due to conduction along the wires. By so coiling the wire that there was an amount under the oil equivalent to a depth of immersion of eighteen inches the couple showed no measurable error.

While twists and sharp bends in the couples set up strains which may be the seat of parasitic e. m. f., such disturbances are too small to be a source of trouble at temperatures no higher than those to be measured with copper-constantan couples when the wires of which the couples are made are small. The writer has taken couples of No. 22 B. & S. gage wire and twisted and bent them sharply at the point of emersion from the oil bath without introducing errors as great as 0.05 deg. cent.

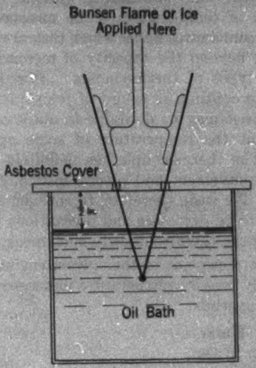

Fig. 1

In checking a thermocouple, it is sufficient, if a potentiometer method of measurement is employed, simply to twist the couple wires together, no precaution such as soldering or welding the wires being necessary. In fact, couples are often used in actual tests by merely twisting them together. While this procedure would be unsafe when measuring with a millivoltmeter, it is not conducive to error when using a potentiometer type of indicator. If, due to twisting the wires together, there results a high resistance at the joint, the only effect is to reduce the sensitivity of measurement when employing a potentiometer. As a rule the ample sensitivity will still permit of precise measurements. The writer has made accurate temperature measurements in this manner when the couple contact resistance had risen to two hundred ohms, a value so high that a millivoltmeter method of measurement would have given a totally incorrect result.

For measuring temperatures above 320 deg. cent. it is usual to employ a couple made of iron and constantan. These couples are serviceable up to 1000 deg. cent. Here also care must be taken to insure that the couple is in correct calibration. A fairly large error may result from the use of iron wire not standard. In a check on a lot of iron constantan couples the iron elements were found to arrange themselves in two distinct groups, one group centering around + 0.5 deg. cent. departure from standard and the other around − 3 deg. cent. In similar manner the constantan elements grouped themselves about − 3 deg. cent. and + 5 deg. cent. These were all checked at 820 deg. cent. While the use of this material would account for an error of 8 deg. cent. at 820 deg. cent. the amount of the error at lower temperatures would be proportionately less. Iron constantan couples are very reliable in maintaining their calibration. Even though the iron is oxidized to a point of nearly opening the circuit the e. m. f. temperature relation maintains and no error is introduced if the measuring instrument is of the potentiometer type.

The life of such couples is satisfactorily long for temperatures below 1000 deg. cent. excepting however, under atmospheric conditions where oxidization is very rapid. To lengthen their life under such service conditions, they have sometimes been calorized. This materially adds to the life but changes their e. m. f. temperature relation. Twenty-four iron-constantan couples after calorizing were found to have changed by varying amounts ranging from 55 deg. cent. to 111 deg. cent. at 820 deg. cent. This variation from the original calibration was found to be wholly in the constantan. The iron wire had not changed. Making up couples using the calorized iron but new constantan, not calorized, resulted in securing the normal e. m. f. temperature relation of uncalorized couples. This, therefore, offers a means for adding to the life of iron constantan couples when they are to be used continuously in oxidizing atmospheres and at temperatures around 900 to 1000 deg. cent.

Above 1000 deg. cent. the so-called chromel alumel couple has been found serviceable. They are useful up to 1400 deg. cent. Above this temperature *Pt-PtRh* couples must be used. As we are not, however, interested in temperatures of this magnitude, no further consideration will be given to these useful thermocouples.

II. Instrument Calibration

The e. m. f. of a thermocouple is best determined by means of a potentiometer. The millivoltmeter method of measurement is for several reasons one of less precision and reliability but it has the advantage of being direct reading and of being more readily comprehended by the layman. Either type of instrument may, of course, be out of calibration and therefore should be checked occasionally. A millivoltmeter whether calibrated in millivolts or directly in temperature cannot as a rule be checked excepting in conjunction with its proper lead wires as the calibration includes the potential drop along the leads as well as the e. m. f. set up at the hot junction. On the other hand a potentiometer may be checked independently of its leads whether it reads directly in temperature or is calibrated in millivolts since the leads do not enter into the calibration. As a rule a check on the accuracy of the standard cell is all that is needed with a potentiometer to warrant one in relying upon the correctness of the instrument as a whole.

III. Thermocouple Circuit

Any change in the resistance of the thermocouple circuit causes an error in a millivoltmeter type of pyrometer or thermometer proportional to such change. The circuit resistance may vary due to changes in temperature or oxidation of the leads or due to changes in the material, size or length of the leads. Oxidation of the hot junction also may greatly change the resistance of the circuit. With modern types of millivoltmeter of high resistance the error due to changes in lead temperature may be kept low but this source of error limits the length of leads which may be used.

With instruments of the potentiometer type no error results from the above sources excepting a small one arising from a change in sensitivity of the instrument due to a change in resistance of the circuit. If, for example, for some reason, perhaps oxidation of the hot junction of a couple, the resistance of the circuit has doubled, then a potentiometer capable at first of balancing to say 0.5 deg. cent. would now be of such sensitivity as to balance to 1 deg. cent. thereby introducing an uncertainty of 1 deg. cent. in the measurement from this cause. On the other hand a millivoltmeter would show an error at say 100 deg. cent. of nearly 50 deg. cent. This source of error is not readily determined with a millivoltmeter unless the whole be checked at some known temperature. Such error in a potentiometer, however, may be readily checked by simply throwing the dial somewhat off of the balance position and noticing from the deflection of the instrument to what extent the sensitivity is impaired by the unusually high circuit resistance.

On the whole the necessity of making a proper check of a millivoltmeter instrument so as to take account of changes in couple and leads as well as in the instrument is one of the chief drawbacks of this type. It demands removing the couple to some location where a definite and known temperature may be imposed and at the same time necessitates employment of the proper leads in the check to insure that no error resides in them.

IV. Radiation Losses

Errors due to radiation may be positive or negative, depending upon whether the radiant energy is being transferred to or from the thermocouple. The errors are reduced by making the couple of fine wires and by not enclosing them in protecting tubes. This will be considered in greater detail when dealing with particular measurements.

V. CONDUCTION LOSSES

In measuring temperature with a thermocouple there is, in general, an error introduced due to the fact that the wires of which the couple is composed conduct heat to or from the hot junction. With a good thermal connection between a couple and a good heat conducting solid or liquid no trouble is experienced with couples of small dimensions. Under other conditions, however, large errors may result and are often difficult of elimination or estimate. If, for example, one were to attempt the measurement of the temperature of a cylinder of baked porcelain, even a couple inserted in a hole in the porcelain would be likely to show a large error in its measurement. Heat conducted along the wires of the couple would result in cooling appreciably the temperature of the hot junction. To avoid error from this source it is usual where practical to place fine wire couples with their wires leading away from the point, the temperature of which is to be measured, along a thermally equipotential surface. This eliminates the conduction errors and permits of accurate measurement but of course is a procedure which can be followed only under special circumstances. One might naturally think that a couple of wires so fine as 0.031 in. diameter (No. 22 B. & S. gage) inserted in a fairly deep hole in a porcelain block would closely assume the temperature of the porcelain. But this is by no means true even when the block is of some good conducting material such as metal. A No. 22 gage couple was hammered flat and placed in intimate contact with a brass block at a temperature of 150 deg. cent. See Figure 2. The couple registered 16 deg. cent. low. Pressing the couple firmly against the brass block with a strip of hard fibre, *F*, Figure 2, resulted in reducing the error to 8 deg. cent. This could be reduced still more by using a plastic insulator covering the surface of the couple and the sides but there is always the danger that such a procedure will cause a change in the heat distribution and to that extent give an incorrect measurement. A similar couple with ends well twisted but not flattened and inserted to a depth of ½ in. in a ⅛ in. hole in the block registered 11 deg. cent. low. Figure 3 indicates the arrangement. The couple made contact with the bottom of the hole. Porcelain insulating tubes slipped over the thermocouple wires failed to reduce the error below 8 deg. cent. When the hole was filled with mercury, however, the couple registered correctly to within 0.2 deg. cent. The mere contacting of a couple against a surface whose temperature is to be measured may result in large errors. A No. 22 couple bearing end on against a brass block at 150 deg. cent. registered 67 deg. cent. low. This way of applying a thermocouple is plainly not a practical method but the writer has on several occasions seen couples contacting in this manner. It is quite apparent that care must be taken if a surface temperature of a few hundred degrees *C* is to be measured to an accuracy of 10 per cent.

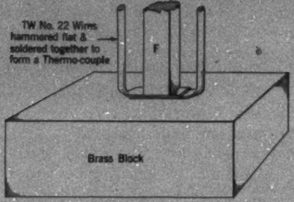
TW No. 22 Wires hammered flat & soldered together to form a Thermo-couple
F
Brass Block

FIG. 2

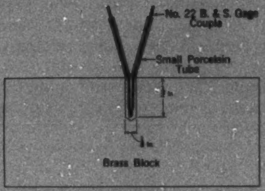

FIG. 3

A couple of small dimensions embedded in the material, preferably by peening each couple wire separately into a hole in the body whose temperature is to be measured, is the best approach to ideal conditions and in many cases is a practical means of eliminating conduction errors. Under such conditions conduction errors are negligible. Small couples ranging from 0.013 in. to 0.031 in. diameter when employed in this manner showed errors of less than 0.5 deg. cent. when the extended wires were heated to a dull red and when cooled with ice.

VI. PARASITIC E. M. F.

The use of iron constantan thermocouples at high temperatures creates a region of inhomogeneity where the steepest temperature gradient exists. This becomes the seat of local e. m. f. To avoid error from this source in checking or in future use the couples should be immersed to at least an equal depth.

As stated previously, parasitic currents arising from bends and twists are small enough to neglect when using small wires and measuring temperatures of but a few hundred degrees cent.

VII. LAG

In the use of thermocouples it is sometimes required that they be protected from contact with their surroundings by lagging them. This lagging may be for the purpose of protecting them against the chemical action of gases, molten metals or liquids or it may be for the purpose of electrically insulating them from their surroundings. The errors introduced thereby may be the result of radiation losses or simply due to the thermal resistance introduced. In the latter case the losses due to conduction are accentuated. This subject will be further considered when the various applications are taken up in detail. It might be stated, however, that under some conditions lag may be found of value in steadying the temperature readings from a varying source of heat.

VIII. Cold Junction Temperature

Perhaps the one source of error most often referred to is that arising from the temperature of the cold junction of the thermocouple. It is now well understood that if a thermocouple temperature indicator is calibrated when the cold junction is at one temperature, it will not read correctly when the cold junction is at another temperature unless the instrument is so constructed as to eliminate error from this source. It is not correct, as some makers have assumed, to add to the instrument reading the excess or deficit of cold junction temperature over the calibrated value. The error from this procedure is not large when measuring temperatures of but a few hundred degrees cent. By far the best way, however, is to employ an instrument having a correct automatic cold junction compensator. There is great satisfaction in knowing that when making measurements this source of error is satisfactorily and automatically eliminated by the apparatus itself. This will be particularly appreciated by those who have taken a series of measurements, only later to make the discovery that the cold junction temperature had not been recorded or the position of the hand operated compensator was unknown. When temperatures are to be recorded an automatic cold junction compensator is essential, or otherwise interpretation of the record becomes impossible or burdensome.

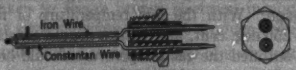

Fig. 4

IX. Measured Temperature Variable

When the measured temperature is varying it is perhaps incorrect to attribute the error to the thermocouple but, as will be shown later, a consideration of the thermal couple design and location will offer a means for lessening any error from this cause. Cases have arisen where lag in the couple has been conducive to better results by offering an average temperature more nearly representative of the mean than would otherwise be secured. Where a recording instrument is employed it may often be so adjusted as to damp out many variations and secure a record more representative and useful through being less subject to constant variation. And where a control mechanism is in use such lagging of the recorder or damping out of its motion may be essential for proper control.

X. Measured Temperature Not Representative

The measured temperature may vary in place rather than time. For example, flue gas temperatures may vary across the section of the boiler passes or down take. Here again the error is not properly an instrument error but one which may be in part, at least, eliminated by giving due consideration to the design of the couple, for example, by using a multiple couple. This will be more fully considered later.

The various applications previously enumerated will now be considered and the methods of dealing with the problems arising will be detailed.

Classification of Temperature Measurement

I. Feed Water

The determination of feed water temperatures is usually made with a mercurial or other type of expansion thermometer. The mercury thermometer placed in a mercury or oil well is in general a sufficiently accurate method of measurement but it often entails placing the thermometer in a more or less inaccessible place and does not permit of recording such temperatures. If a record is desired or if a distant indication is deemed necessary the thermocouple will be found convenient. For this and similar applications a thermocouple has been developed which is capable of withstanding the boiler pressure and convenient to install. When supplied with an automatic cold junction compensator it leaves nothing to be desired in the way of reliability and accuracy, convenience of installation, and ability to have indications or records or both at distant points. This type of thermocouple is in design quite similar to a gas engine spark plug. As indicated in Figure 4, it is so constructed with tapered and insulated leads that the internal pressure acts to prevent rather than produce leaks. For this service no particular care need be taken to avoid errors as when once properly calibrated they should hold their calibration indefinitely. The intimacy of contact with the water whose temperature is measured insures against errors. The relativly low temperatures are within the range of copper constantan couples. There may be cases where stray e. m. f. will demand unusual care in insulating the circuit and instrument but this is a precaution which should be taken in practically any installation.

II. Boiler Water

This problem is similar to that of determining feed water temperatures but in this case we have to contend with both high temperatures and high pressure. The same type of thermocouple plug has been found effective. The plug insulation is of mica and as stated before, the design is such as to preclude the liklihood of leaks starting due to the pressure and temperature to which the plugs are subjected. In fact, due to improper workmanship some plugs of this type when they were first being experimentally produced did leak when first installed. The temperature and pressure to which they were subjected soon closed up the leak. It has been found practical and safe to tap such plugs directly into a boiler. There has been some discussion as to the temperature distribution throughout the tubes and drums of water tube boilers. Plugs of this type may be tapped into drums and headers and extended into tubes a distance sufficient to enable a study to be made of the relative steaming efficiency of various sections of a boiler. As in the determination of feed water temperatures no serious errors are encountered in this application.

III. ECONOMIZER

The determination of the temperature of the gases entering and leaving economizers has been made by means of thermocouples arranged in series and so disposed as to secure an approximation to the average temperature of the gas. In this case the various cold junctions are brought out to a convenient point where a compensator is applied.

When such couples are placed both before and after the economizer a double curve drawing recorder may be employed indicating at each instant the temperature of the entering gas, the temperature of the exit gas and the drop in temperature through the economizer.

IV. FLUE GAS

A continuous record of flue gas temperatures may be required or it may be desired to control some other element of operation automatically in terms of flue gas temperature. The thermocouple controller recorder offers a means of accomplishing either or both results.

Kreisinger in Bulletin No. 145 of the Bureau of Mines calls attention to the fact that the major error in the determination of flue gas temperatures is that due to the radiation from the couple to its cooler surroundings unless precautions are taken to prevent such error. He states that under some conditions the error may amount to several hundred degrees in some passes of the boiler. Even in the down take an error of 25 deg. cent. may result from this cause.

In the last pass of the boiler or in the down take the error is always in a direction indicating temperatures lower than actual, owing to the fact that the tubes in the last pass are at a much lower temperature than the gases. For this reason the couple radiates to the tubes and is therefore lowered in temperature. Kreisinger pertinently calls attention to two points: first, that the degree of accuracy of gas temperature measurements depends more upon the judgment of the user of the instrument than upon its correct calibration and second, because an instrument is correctly calibrated, the readings are too often assumed to accurately indicate the temperature of the gas.

The important point to note in gas temperature measurements is that the larger the diameter of the couple or its protecting tube the greater the radiation error will be. While it is usually assumed that a heavy couple or a heavy protecting tube markedly lowers the couple temperature by reason of conduction along the wires or protecting tube such is not generally the case. The conduction error can be made small. The real source of error lies in the fact that a large diameter couple offers a large surface for gain or loss of heat by radiation. With a couple projecting into the hot gases a distance of eighteen inches, conduction error may be ignored. Hence a properly constructed couple for gas measurements should be placed in a substantial steel tube for mechanical strength but the hot junction of the couple should be exposed for a distance of perhaps six inches. The couple wires should be unprotected for this distance and should be not larger than No. 22 B & S. gage. With the low temperature of flue gases 200 to 250 deg. cent. and with a couple constructed as above outlined the gas temperature should be measurable to an accuracy of 3 to 5 deg. cent.

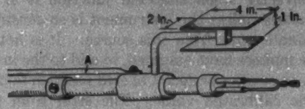

Fig. 5

In the last pass of a boiler where the temperature of the gas may be 650 deg. cent. to 750 deg. cent. this type of couple should give a reading not more than 25 deg. cent. low. Experience seems to indicate that for the latter measurement a correction approximating 3 per cent will leave only a small outstanding error and for temperatures in the down take a correction of about 1 per cent will allow for the lowering of temperature due to radiation.

In some tests made for the writer for the purpose of determining the error in flue gas measurements due to radiation, screens or baffles were so located and constructed as to shield the couple from radiation to the boiler tubes. The arrangement is illustrated in Fig. 5. The rod *A* could quickly slide the baffles away from the couple. They could be quickly withdrawn and replaced without disturbing any part of the couple and without loss of time. When readings were made in rapid succession both with and without the baffles, differences of temperature of not over 1 deg. cent. resulted. In this case the couples were of No. 22 gage wire. While this method is a satisfactory means of measuring gas temperature at a point in the flue it does not insure that the measured temperature is a representative one. For this reason multiple couples have been used to secure an average in the same manner as in averaging the temperature of economizer gases.

V. SUPERHEATED STEAM

The thermocouple because of its rapid response to changes in temperature, has found an unexpected application in connection with the measurement of temperature of superheated steam. If water is carried along with the steam and strikes the thermocouple it lowers its temperature suddenly. A continuous record of the thermocouple temperature, therefore, shows sudden breaks, the cause of which is readily recognized. The application to this service was primarily for the purpose of determining the degree of superheat in making acceptance tests of superheaters. It proved very useful in this respect in that it was immediately responsive to any changes due to change in load and was free from suspicion of error due to lead conduction.

For this service the plug type couple will be found useful. Experience, however, shows that its construction must be carefully attended to in order that it may have sufficient rigidity to withstand the rather high pressure due to the velocity of the steam and the impact of the water.

VI. BEARINGS

Thermocouples embedded in bearings operate under conditions that insure reliable measurements providing care is taken to secure proper electrical insulation in order that stray currents may not produce errors.

The couple should be set in a hole of such depth that the hot junction is near the running surface and should be made of small wires, otherwise conduction along the wires may result in indicating too low a temperature. Couples constructed and installed as indicated in Fig. 6. have been suggested although the writer cannot say whether they have been tried out.

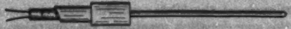

FIG. 6

This construction grounds the thermocouple since the copper sheath is one element of the couple. Perhaps a better construction would be to use silver for a sheath for only about one inch of length the balance being made of porcelain or light wall german silver. In this case the couple can be made of fine wire and insulated from the sheath.

It is practical to keep not only a continuous record of bearing temperatures but to have the recorder furnish a light or bell signal if the temperature rises above a safe limit.

VII. GENERATOR WINDINGS

After a careful analysis of the design of a machine has indicated the probable hottest region, there remains the problem of so locating the couple as to enable the temperature of the *copper* at this point to be determined.

In service, the couple has to be placed outside the insulation and therefore a correction is necessary for the temperature drop through the insulation. Standard corrections for this drop have been adopted in the Standardization rules of the A. I. E. E.

In a shop test constantan wires can be substituted for some of the small strands in the armature conductors and these constantan leads welded to the armature copper at the hot spots. A potentiometer connected between the constantan lead at the neutral point, if grounded, and the neutral copper will permit determination of the copper temperature. If at the same time additional copper constantan couples are placed outside the copper insulation adjacent to the first couples, simultaneous measurements will give the correction for the latter couples when used in service.

After the test the leads of the inside couple can be cut off.

Thermocouples installed in generators are subject to strong a-c. fields hence they should be so designed as to prevent heating due to eddy currents.

Thermocouples used in generators are always grounded as a protection to the operator but of course only one ground is permissible since the difference in potential between two grounded points may be large in comparison with e. m. f. generated in the couple. Couples should not have common return leads as this gives rise to trouble and also makes it more difficult to locate trouble. Furthermore, trouble in any one couple effects the whole system and not simply its own circuit.

It is almost unnecessary to point out that the leads should be individually insulated with good rubber. The fact that the e. m. f. of a thermocouple is but a few millivolts may lead to the inference that a low insulation is sufficient.

VIII. TRANSFORMER WINDINGS

Thermocouples have not been used to any great extent in this country for measuring the temperature of transformer windings. The resistance thermometer has the advantage that it may be placed in an a-c bridge and insulated by means of a special insulating transformer. This is the arrangement used by the General Electric Co.

Large transformers are usually of higher voltage than generators and consequently the insulation problem is more difficult. Even if the "hot spot" could be reached, it would not be safe from the standpoint of life hazard to use undergrounded thermocouples, and it would cause dangerous stresses in the insulation to run grounded thermocouples into the windings in the ordinary manner.

The Westinghouse Company employs an "artificial hot spot" located in the oil of the transformer and supplied with current from an auxiliary transformer. In this "artificial hot spot" it is safe to place a thermocouple or any convenient type of temperature detector.

Very many high-voltage transformers have one winding connected in *Y* with the neutral solidly grounded. Accordingly it is possible to place thermocouple leads in the main winding with the instrument connected at the neutral point. This method is now being investigated by one of the transformer manufacturers.

IX. Cables

In the last year or two the thermocouple has been used extensively for temperature measurement in cable systems. The thermocouple is valuable in this work because of its extreme simplicity and robustness.

Where it is possible to solder the couples to the lead sheath of a cable before drawing in, the problem is rather simple. The errors to be avoided are those due to stray e. m. fs. leaking on to the leads, and due to e. m. fs. produced by electrolytic action on the thermocouple leads. The ducts are usually very wet and excellent insulation is required to avoid the errors mentioned.

Most of the measurements have to be made in empty ducts as it is not possible to pull a couple for any great distance into a duct occupied by a cable. However, thermocouples can be pulled into occupied ducts up to limited distances, but there is considerable uncertainty in the measurements as it is quite impossible to tell whether the couple is in contact with the sheath, the duct wall or neither.

Whether the couples are pulled into empty or occupied ducts it is necessary to guard against conduction along the leads and it has been found advantageous to place a small mass at the hot junction in order to reduce conduction errors. This makes it necessary to leave the couple at each point to be measured for a considerable length of time (15 minutes to ½ hour) to allow the mass to assume the local temperature.

Of course the results are affected by radiation from and to the duct walls, but this error is probably small and usually in the direction to show a temperature more nearly that of the cables in adjacent ducts.

Long leads are used with thermocouples for cable measurements and these leads are subjected to varying temperatures along their length. The temperature of leads may be above or below the junction temperature but is not very different. It is necessary to select the lead wire in order to avoid errors due to parasitic e. m. fs. but it has not been difficult to obtain lead wire which is apparently very satisfactory.

The writer is indebted to Mr. I. M. Stein of the Leeds & Northrup Company for the substance of the remarks pertaining to generators, transformers and cables.

Discussion

E. D. Tanzer: Not so long ago, a thermocouple was solely a physicists instrument. At the present time, however, it has a wide field of application among the operating companies. At least, we of the Philadelphia Electric Company, during the past three years have used it extensively in field determinations of the operating temperatures of cable duct lines, cable sheaths and transformer manholes, as well as in research problems pertaining to the operation of an extensive underground cable system.

Mr. Smith has defined very nicely some of the limitations inherent in the use of various types of couples in the field. We have experienced in our own investigations, some of these same factors, and accordingly I believe that it may be of interest to show some ways in which we have found the thermocouple particularly useful in the investigation of temperatures in our underground system.

In measuring temperatures in duct lines, carrying several cables, it is obviously impossible to get the thermocouple into a duct occupied by a cable. The only thing that we can do is to devise a means of readily locating a couple along an adjacent empty duct.

In the device utilized for this purpose a mandril carries the thermocouple. In the center of this mandril, the thermocouple junction, imbedded in a small copper block, is protected so far as possible from any mechanical injury that might result from pulling the mandril through a duct. The first of these devices made up to investigate a duct run of about 400 feet length, was made with these mandrils spaced at approximately 50 foot intervals. The little block of copper in which the couple junction is mounted, is for the purpose of providing a small amount of thermal capacity just at that particular point and so obviate the possibility of heat radiation or conduction away from the couple junction itself since this would mitigate against the accuracy of its indication.

In one particular duct line in which a large number of measurements were made we had a duct length of about 400 feet badly congested with cables. Accordingly, a great deal of heat was to be radiated and the operating temperatures were rapidly becoming large. Thermocouples made possible the exact determination of the existing temperature within this duct run and so gave the loading that could be used to advantage for the cables involved. The configuration of the duct run was such as to cause difficulty in determining temperatures at midlength locations. The leads from each of the 50-ft. spaced couples were all brought back to the one location enabling the operator to read the temperatures along the duct run with a minimum of effort.

Couples have also been used to determine the location of a leak beneath the street level in a steam service main. In this case steam was present in adjacent manholes but the exact location of the leak was not readily determinable from the surface of the street by ordinary methods. However, by checker-boarding the surface of the pavement and locating a couple at the center of each one of these squares, the temperatures attained by the couples served to indicate the particular point in the steam main which was giving trouble, to within a distance of approximately 3 ft.

I believe, therefore, that in the future, there will be more, of what I might call "out of the ordinary," applications of thermocouples to the problems of the operating companies. Accordingly, Mr. Smith's paper is very timely as it discusses some of the things which we cannot expect thermocouples to do. For instance, although the couple, by itself, is very accurate, yet it may be so mounted as to nullify a portion, or even a large part, of its accuracy. Particular attention, therefore, must be devoted to not only the couple itself, but to the mounting with which the couple is to be used.

G. H. Cole: Thermocouples, because of their simplicity are frequently used under conditions which have been given little thought. For example, even with somewhat more thermocouple experience than the average engineer, it is improbable that I would have hesitated about measuring the temperature of a brass block by putting a thermocouple junction at the bottom of a small hole ½ in. deep, as shown in Fig. 3 of Mr. Smith's paper, unless very good results were desired. Yet this procedure resulted in nearly as great a difference in the temperature desired and that read as between the mean temperature of May and December in New York City.

While it is well known that the temperature drop at surfaces of solids through which heat flows is relatively large, as for example at the junction of bricks in a furnace wall, Mr. Smith's paper has been of value in calling attention to the need for using this knowledge in thermocouple applications.

Due to the numerous surfaces to be crossed by heat flowing across a laminated transformer core, the thermal drop may 50 to 100 times greater in this direction than perpendicularly to it for the same distance in one lamination. The foregoing indicates the importance of making intimate contact with the surface whose temperature is to be measured.

I would like to ask Mr. Smith if surface drop and conduction errors can be largely eliminated by having the two thermocouple wires make contact with the body to be measured at different points.

R. P. Brown: Mr. Smith, in his paper, makes the following statements:

"Any change in the resistance of the thermocouple circuit causes an error in a millivoltmeter type of pyrometer or thermometer proportional to such change." He further states:

"With instruments of the potentiometer type, no error results from the above sources excepting a small one arising from change in sensitivity of the instrument due to a change in resistance of the circuit"—which he explains, with doubling the resistance of the circuit, might cause an error of 1 deg. cent. Mr. Smith further adds:

"On the other hand, a millivoltmeter would show an error at say 100 deg. cent. of nearly 50 deg. cent. and that this source of error is not readily determined with a millivoltmeter unless the whole be checked at some known temperature."

In making a comparison of the errors encountered in using a millivoltmeter pyrometer as compared to a potentiometer pyrometer, under conditions wherein a thermocouple circuit resistance is doubled, it is, of course, necessary to compare on the basis of the same thermocouple circuit conditions for each meter.

In the case of a millivoltmeter pryometer normally of about 600 ohms resistance and used with a thermocouple circuit of, say, one ohm resistance, it would be necessary for the thermocouple circuit to be increased to 600 ohms in order to halve the deflection of the millivoltmeter pyrometer.

For comparison,—in the case of a potentiometer pryometer, the same thermocouple circuit conditions must necessarily exist. In other words, the same change from one ohm to 600 ohms must take place. Mr. Smith, in his paper, states that an uncertainty of ½ deg. cent. normally exists. He then states that by reason of the circuit resistance being doubled, the uncertainty in reading becomes 1 deg. cent. instead of ½ deg. cent. This is not possible under the conditions encountered in the usual potentiometer furnished for such measurements. The slide wire resistance of such a potentiometer is normally approximately 30 ohms and the galvanometer resistance is approximately 20 ohms. It is therefore apparent that a change in the resistance from one ohm to 600 ohms in the couple circuit would far more than double the uncertainty existing in measuring with a potentiometer. Since the total circuit resistance increases from approximately 51 ohms to 651 ohms, it is evident that the uncertainty of measurement must increase in ratio of the resistance. In other words, the uncertainty of the balance will then be 13 times as great as originally, thus amounting to approximately 6½ deg cent., instead of 1 deg. cent. claimed by Mr. Smith. From this it is apparent that the comparison as made is incorrect, in that the same condition of thermocouple circuits have not been applied in making the comparison.

The only condition under which the comparison as stated would be substantially correct, is to assume that the thermocouple circuit, as referred to by Mr. Smith, applies only to the external circuit, including the thermocouple and its lead wires.

The average millivoltmeter pyrometer of today, put out by the majority of manufacturers, has an internal resistance of some 600 ohms. I have seen an instrument made by one of our competitors having a resistance of 1200 ohms and we frequently build instruments of this resistance ourselves, but to take an average condition, we can assume 600 ohms for the instrument.

A thermocouple most commonly used, 36 in. long, with 15 ft. of extension leads to convey the cold junction from the thermocouple to the instrument, has a resistance of ¾ of an ohm. Lengthening the extension leads from 15 to 30 ft. and increasing the thermocouple from 3 to 6 ft. long, would double the resistance of this circuit and would produce an error of only 1.3 deg. cent. in the readings of an instrument graduated, for example, to 1000 deg. cent. This is equivalent to only 0.13 of 1 per cent error (instead of nearly 50 per cent claimed by Mr. Smith.)

It is misleading to have a statement appear broadly implying that a millivoltmeter pyrometer would show an error at, say, 100 deg. cent. of nearly 50 deg. cent. on account of doubling the thermocouple circuit resistance.

The variation in temperature of the lead wires, referred to by Mr. Smith, will also produce exceedingly small errors in a modern millivoltmeter pyrometer. Assuming a length of lead wire of 100 ft., the change in resistance due to a change in temperature of 100 deg. cent, will amount to 0.1 ohm in a circuit of 600 ohms, or about 0.02 per cent. On a pyrometer reading to 1000 deg. cent. the error would amount to only 0.2 deg. cent. It is quite evident, therefore, that errors in a high resistance millivoltmeter, due to lead wire changes in temperature, are immaterial.

The only remaining possible cause for change in resistance in a millivoltmeter pyrometer, is the change produced by oxidation of the thermocouple. Claim is made that oxidation of the thermocouple results in an increase of thermocouple resistance, thus affecting the millivoltmeter pyrometer and not the potentiometer. The truth of the matter is that resistance errors due to oxidation are exceedingly small. Before the deterioration of a thermocouple can be carried to such an extent that the resistance errors are appreciable, the e.m.f. of a thermocouple changes so badly as to render the thermocouple useless for measurements either with a potentiometer or a millivoltmeter.

When we compare the relative merits of the millivoltmeter and potentiometer for temperature measurements, let us not take for comparison a type of millivoltmeter which is not produced and compare it with a standard potentiometer.

I recognize the valuable features in both instruments—the millivoltmeter for its simplicity, the ease with which an unskilled operator can use it, its absence from standard cells, dry batteries and slide wires required in the potentiometer.

In the potentiometer I recognize its valuable feature in the entire absence, for practical purposes, of any errors due to changes in circuit resistance, and its value for extremely accurate measurements of temperature. Both types of instruments have their field and both have many valuable features.

I. B. Smith: The calculations made by Mr. Brown and myself may be carried out in a variety of ways. It seems hardly worth while, however, to go into details since Mr. Brown and myself appear to be in agreement as evidenced by his last remark; that one valuable feature of the potentiometer is "the entire absence for practical purposes, of any errors due to changes in circuit resistance."

The Measurement of Power in Polyphase Circuits[*]

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***Review of the Subject.**—The object of this paper is to point out that the existing methods of measuring power when applied to unbalanced three-phase systems are not equitable for symmetrical polyphase machinery. On the other hand, unsymmetrical loads on polyphase systems are not sufficiently penalized for the trouble which they create in the system.*

It is first of all shown that a symmetrical generator cannot deliver power except through the balanced components of current. The unbalanced currents are capable of resolution into two balanced systems of currents, one of which is of the same phase sequence as the generator e. m. f., and the other component is of reversed or negative phase sequence. The generator cannot deliver power through the medium of this latter component of the currents, because the instantaneous product of the generated voltage and these currents in the three phases is always zero. However, the volt-ampere product per phase is of great significance, because it is a measure of the effect of the current unbalance on the system.

The generator therefore delivers power only through the medium of the positive phase sequence currents. Any power that appears in the system through the negative phase sequence currents is positive phase sequence power which has been supplied by the generator and degraded through unbalanced loads and fed back to the system in the form of negative phase sequence power. This power is always additional loss in all rotating machines on the system, and with the present method of charging, the consumer having symmetrical machines is charged with this additional power, which serves him no useful purpose but reduces the output of his machine and decreases his load power factor.

In the paper it is proposed that the positive phase sequence power output only be measured, and the power charges be made on the basis of this measurement. It is further proposed that the unbalanced k-va., which is the product of the positive sequence voltage and the negative sequence current be measured either by means of a negative sequence ammeter, indicating or recording, or a k-va. meter, and a charge made for the amount of unbalance. The user of symmetrical polyphase rotating machinery should then be given a lower rate, based on the estimated cost of unbalance, and the consumer having unbalanced loads should be charged directly for the amount of unbalance he creates, or else should have his positive phase sequence power rate increased, based upon the estimated cost of unbalance.

It is pointed out that the unbalanced kv-a. is a factor of the same order of importance as reactive kv-a. and in any system subject to unbalanced conditions this factor should be considered and the unbalanced factor, as well as the power factor, should be measured. The unbalanced factor is the ratio of the negative phase sequence kv-a. and the positive phase sequence kv-a., the former being obtained by taking the product of the positive phase sequence voltage and the negative phase sequence current.

Devices for measuring these quantities are being developed and the outfit for making these measurements will be no more complicated than the present existing measurement devices. In fact the tendency is towards greater simplicity.

In presenting this subject the author has no intention of suggesting how rates should be made, but merely wishes to point out what factors enter into the question of equitable rates when the polyphase system is subject to unbalance.

GENERATORS

FOR the purpose of this discussion, it will be sufficient to consider symmetrically wound machines. Such machines when excited by a direct-current machine, give balanced three-phase voltages at their terminals of sine wave form. If currents are supplied from these terminals to a symmetrical load, they will be in balanced three-phase relation to one another, and the sequence of the maxima of the current waves will be the same as that of the generated e. m. f. waves. Generators are usually built with the field the rotating member; to avoid confusion we shall refer to this member as the rotor and the rotor field will in general be the main field of the machine, although as we shall see later, the main field is, in reality, a field set up by the resultant of the m.m. fs. of both stator and rotor.

A symmetrical system of currents flowing in the stator winding produces a resultant synchronously rotating m. m. f. which in the case of balanced currents having the same sequence of phases as the generated e. m. fs. will rotate in the same sense as the main field, and its phase position with respect to the main field will depend upon the time phase of current with respect to the e. m. f. Where the relation between m. m. f. and field in linear, the former may be resolved into two components at right angles with each other, and the field that would be produced by these m. m. fs. may be compounded with the field produced by the exciter currents in the stator to give the actual field of the machine under load. Since the e. m. fs. produced by the components will be proportional to them, we may consider these components in terms of the e. m. fs. they produce in the stator winding and compound these instead. Thus, in each phase an e. m. f. will be produced by the resultant magnetomotive force due to the symmetrical currents flowing in the stator windings and will lead it in time phase by one quarter of a cycle. In addition to this e. m. f., there will be a resistance drop due to the current in its windings. A complete theoretical discussion of these problems is given in the appendix. It is shown there that if the resistance drop which is negligible is ignored, the increase in exciting current required to maintain a given e. m. f. is proportional to the wattless current, and the phase angle of the rotor and armature adjust themselves, so that there is equilibrium of induced voltages. In other words, the presence of wattless lagging current in the winding necessitates a proportional increase in the exciting current. This entails larger conductors in the exciting winding and larger parts for the complete machine. The effect of saturation is not so important as might be supposed; for since the field itself does not change in value during a cycle, the value of the average mutual and self inductance will not be affected during a cycle, but will depend upon the average degree of saturation present. In most machines this does not vary much from no-load to full-load on account of the long air gaps.

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Let us suppose the generator to be provided with a winding on the rotor of similar character to the stator winding, but short-circuited on itself. It will be quite obvious that the various component fields discussed above, since they are stationary with regard to the rotor, will have no inductive influence on this winding and that, therefore, so long as the machine is rotating at uniform speed, and the currents in the winding of the stator are symmetrical three-phase of the same sequence as the terminal e. m. fs., no current will flow in this winding. Let us now consider the effect of a system of balanced three-phase current, the sequence of whose maxima is opposite to that of the generated e. m. fs., that is to say, the maxima of the currents in phase *A*. *B*. *C*. instead of following in time the order *A* leading *B* and *B* leading *C*, will instead follow the order *C* leading *B* and *B* leading *A*. It will be evident that the sequence of these currents will correspond with the normal sequence of currents for the same machine with the direction of rotation reversed, consequently following the same general reasoning for the case of normal sequence given above, this system of currents will set up a m. m. f. rotating synchronously in the opposite direction to the m. m. f. of the exciting current of the rotor. This m. m. f. will, therefore, rotate relatively to the distributed winding on the rotor at double synchronous speed and the field it will tend to produce will set up double frequency-balanced polyphase currents, which will in turn set up a m. m. f. opposing that due to the stator currents.

The field actually produced will be the resultant of these two opposing m. m. fs., and will be such as to produce e. m. fs. of sufficient value to overcome the impedance of the rotor winding due to its resistance and leakage reactances. We have here an action similar to that of a short-circuited transformer, the difference being that there is in addition a tranformation of frequency that is always accompanied by a transformation of mechanical power into electrical power. In this particular case, the transformation of mechanical power into electrical is such that one-half the secondary losses are directly supplied through rotation by the prime mover. The e. m. f. at the generator terminals required to force this current through the stator windings will be determined by an impedance which will be similar to the short-circuited impedance of a transformer. Similarly, the relation between the double-frequency rotor current and the stator current will be similar to that between the secondary and primary currents of short-circuited transformers. For practical purposes, we may say that the ampere-turns of stator and rotor winding will be equal. These relations are independent of the actions accompanying the flow of balanced currents of normal frequency in the windings.

It will now be evident that the presence of current of opposite phase sequence in the generator e. m. f. requires something in the generator which was not necessary before, that is, damper capacity, and this is a further requirement in addition to the additional capacity required in the stator winding as a result of the current.

The product of the negative phase sequence current and the normal terminal voltage is a measure of the amount of phase conversion taking place in the generator and the presence of the negative phase sequence current requires this phase converting capacity in the machine. The mean heating in the stator will be proportional to the sum of the squares of the positive phase sequence current and the negative phase sequence current; and the heating in the rotor will be proportional to the sum of the squares of the exciting current and the double-frequency current if they have a common winding.

It has been shown elsewhere and the solution has been repeated in the appendix that the operation of a generator depends upon two fundamental inpedances, namely, the impedance to the normal balanced current or positive phase sequence impedance, and the impedance to the negative phase sequence current, the action of which we have just discussed; this latter is the negative phase sequence impedance.

In this discussion we are leaving entirely out of consideration currents flowing through the winding by way of the neutral of the three-phase winding. These currents while important in questions of inductive interference are of little importance in commercial power circuits. It seems hardly necessary to say that a solution of any circuit condition may be obtained without the use of positive and negative sequence currents and impedances by using self and mutual impedances. These impedances are, however, difficult to evaluate and are not the natural constants of the machine.

A discussion of the equations of a generator using self and mutual impedances and the derivation of their relation to the positive and negative phase sequence impedance will be found in the appendix. This method of solution might be regarded as the single-phase method of treating a polyphase problem, each phase being considered as a single-phase branch circuit related to the other circuits through mutual impedance.

In the symmetrical coordinate method, the symmetrical polyphase component system are the units, and these have definite impedances and are independent of each other in a symmetrical machine. This method, therefore, leads to considerable simplification. It is interesting to note that a similar relation holds between the phase impedances and the positive and negative sequence impedance, as exists between the phase currents and the positive and negative sequence components of current.

We are not concerned, however, with the mere solution of the three-phase circuit, but what we desire is a measure of the requirements of the machine.

(1) How much true power it must carry—kw. capacity.

(2) How much reactive power it must supply—exciter capacity, field copper, additional stator copper.

(3) How much unbalanced kv-a. it must supply—its phase-converter capacity—damper windings on rotor—additional copper on stator.

The first two requirements are well known, the first being determined by the demand for power current or active component of current, the second being determined by the demand for magnetic energy which is furnished by the reactive component of current. The third requirement is determined by the magnitude of current unbalance, and is measured by a balanced system of currents, the sequence of the maxima of whose phases is the reverse of that of the generated voltage.

If we take I_1 for the total current due to the first two, then if I_p, I_Q be the two components of power current and wattless current respectively, $I_1^2 = I_p^2 + I_Q^2$. If I_2 be the third current, which measures the magnitude of the unbalanced flow, the total average heating in the stator alone will be determined by $I_1^2 + I_2^2 = I_p^2 + I_Q^2 + I_2^2$, so that in its relation to the generator capacity, I_2^2 is a factor of the same order as I_Q^2, and the three quantities as regards the average heating are independent of one another, and the necessity for supplying I_2 calls for additional copper in the stator winding as well as the addition of a suitable damper winding. To sum up: The character of a generator is such that it has three functions:

(a) To deliver true or active power.

(b) To deliver reactive power or supply magnetic energy.

(c) To maintain balance at its terminals (phase converter capacity.)

The quantities that are measures of these three characteristics combine as far as average total heating is concerned as if they were three vectors in space mutually at right angles to one another.

In the preceding paragraphs, we have concerned ourselves only with the average heating, due to the presence of unbalance currents. We have shown that a machine subjected to such operating condition must have damper capacity and additional copper in the stator; but this is not the whole story, the effect of unbalance is to cause unequal dissipation of energy in the three phases, as a result any phase may have to withstand a loss several times the average loss according to the average loss expression given in one of the preceding paragraphs. Since the phase of the negative phase sequence component current is not definitely fixed, any one of the three phases may have to stand this high loss. In a single-phase machine, the unbalance is definite and may be provided for, but in a three-phase machine, each winding must be designed for the maximum possible loss it may incur under unbalanced loads.

It appears to be a reasonable deduction that the simplest system and the most fundamental one in considering a rotating machine, is one in which the reactions in the magnetic circuit will appear in terms of fields of the same character as that set up by the rotor. We see that balanced currents flowing in the stator will set up a field of this character which may be combined simply with similar fields set up by other balanced systems of currents. The balanced systems of currents in the stator setting up these component fields will differ from one another in magnitude, phase-sequence and frequency. Any particular component fields may be the resultant of two balanced systems of current, one of which flows in the stator winding and the other in the rotor. A balanced system of currents which produces a m. m. f. rotating in the same direction as the main field is termed a positive phase-sequence system. A system of balanced currents flowing in the stator producing a field rotating in the opposite sense is termed negative phase sequence. It is convenient to refer all the quantities to one phase "*A*" which is termed the principal phase.

In dealing with the generator in these terms, we are considering the field as resolved into component fields of exactly the same character as that set up in the field magnetic circuit by the exciter. In the complete theory the field may be compounded of a number of fields of different numbers of poles rotating at different multiples or submultiples of the speed of the main field. Rotating positively or negatively each field is associated with a symmetrical system of currents in both rotor and stator, each having a definite frequency depending on the speed of rotation of the field and a definite phase sequence depending on the direction of rotation.

The instantaneous value of the field set up by a symmetrical system of currents of other than sine wave form in a symmetrically wound generator may be expressed in the form of a Fourier series consisting of only odd harmonics of the fundamental, the electric angle for a complete pole pitch being 2π. In the three-phase symmetrical generator, there may be present in the field form, due to the symmetrical currents, the 5th, 7th, 11th, 13th, etc., harmonics of which the 5th, 11th, 17th, etc. will be fields rotating in the negative direction while the 7th, 13th, 19th will be positively rotating fields.[1] On account of the counter m. m. fs.

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[1] The reason of this may be explained very simply as follows: If we take the wave form of say phase *A* and displace it through the angle $\frac{2\pi}{3}$ we obtain the wave form of phase *B* for a symmetrical machine. For the *r*th harmonic which has $1/r$th the wave length, this displacement corresponds to the harmonic

due to currents set up in the damper winding and in the pole faces, these harmonic fields are largely damped out. The harmonic fields due to the flow of symmetrical positive phase-sequence currents of fundamental frequency do not produce harmonic e. m. fs. in the stator, but the e. m. fs. produced are all of fundamental frequency and of symmetrical positive phase sequence. The field forms so produced are of the nature of submultiple rotating fields, having numbers of poles related to the main poles in the inverse ratio of the relative rotational speeds. Their values are small, due to the counter reactions set up by the induced currents in the rotor.

The fields set up by the flow of negative sequence current of fundamental frequency in the stator are of similar character, but the direction of rotation is reversed, consequently, all these fields are opposed by those of the currents set up in the damper windings. All the e. m. f. set up in the stator is used up in overcoming the resistance and the internal reactance of its windings and in addition a certain amount of mechanical power is required to produce a portion of this e. m. f. which is dissipated in the form of $I^2 R$ in the rotor windings. The e. m. fs. set up in the stator are all dissipative and reactive e. m. fs.

A symmetrical rotating machine driven below synchronous speed is incapable of delivering power when excited in the primary alone; an additional exciting agent in the secondary circuit is required. Any mechanical power necessary to drive such a machine is used up in heat losses in the windings. The above includes machines driven in a negative direction at any speed. There is an apparent exception to this in the case of standard generators; due to the saliency of the poles, they are capable of delivering a certain amount of power when excited from the primary. This characteristic is the result of the variation of inductance of the stator winding with varying position of the rotor, and may be regarded as a form of dissymmetry. The amount of power a machine is capable of delivering in this way is a fixed fraction of the total exciting kv-a. Steam turbine generators have very little of this characteristic.

The generator relative to the negative phase-sequence current is being driven backwards at synchronous speed, and, therefore, as we have shown, if it is a symmetrical machine, it cannot deliver any useful work at its terminals through the medium of the negative phase-sequence currents. The mechanical power at the shaft, due to the negative phase-sequence currents is completely used up in losses in the damper windings. In short, a generator cannot deliver useful power at its terminals, except through the system of e. m. fs. set up through its main field excitation. It will now be apparent that a symmetrical generator cannot deliver any other than positive phase-sequence power. All the negative phase-sequence current flowing therein represents losses; the flow of these currents requires also the application of a certain amount of mechanical torque at the driving shaft which must be supplied from the true source of power which in the case of a generator is the prime-mover.

One special form of generator should be mentioned, in which the field windings have high external reactance and the field is thoroughly laminated and is not provided with damper windings. In such a machine the impedance to negative phase-sequence current is the same as that to positive phase-sequence current. Such a machine delivering a single-phase load will have a pulsating field. Moreover, the harmonics due to the field form will not be damped out as in the machine provided with damper winding. The unbalance in voltages due to unbalanced load in such a machine will be very marked.

It is possible that a generator may give perfectly balanced voltages at no load and will be inherently unbalanced under load. The fact that a symmetrical generator cannot deliver power through the negative phase-sequence currents affords a simple check to determine whether an unbalance is wholly due to external sources or is partly inherent in the generator. The total negative sequence power output may be measured, and if it is negative, the unbalance is due mainly to external conditions; if it is positive, a great part of the unbalance is inherent.

THREE-PHASE INDUCTION MOTORS: SYNCHRONOUS MOTORS: ROTARY CONVERTERS:

The action of polyphase motors is quite similar to that of polyphase generators. Thus, if the machine is

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angle $= \frac{2 r \pi}{3}$. If r is a multiple of 3, we see that the angle $-\frac{2 r \pi}{3}$ is always a multiple of 2π or a complete electrical revolution, so that in a three-phase machine, all harmonics which are multiples of 3 give voltages in phases A B C in phase with one another. The two groups of numbers $(6 n - 1)$ obtained by giving n all integral values from 1 to infinity give all the odd integers from 5 to infinity, except the multiples of 3. If we take r equal to $(6 n - 1)$ this group of harmonics will be displaced by the angle $- 4 n \pi + \frac{2 \pi}{3}$ or what is the same thing by the angle $\frac{2 \pi}{3}$ that is to say phase B will be advanced by this angle; similarly if r is taken equal to $(6 n + 1)$ the angular displacement for the harmonic will be $-\frac{2 \pi}{3}$, that is, this group of harmonics will be displaced in phase B by the angle $-\frac{2 \pi}{3}$. Hence the first group will be positive phase sequence, while the second will be negative phase sequence. In $(6 n - 1)$ giving all integral values, we obtain the harmonic group 5, 11, 17, etc., and similarly, the group $(6 n + 1)$ gives the harmonics 7, 13, 19, etc.

an induction motor, at no load, the rotor, since it is rotating at near synchronous speed, is traveling at approximately the same angular velocity as the field set up by its primary currents; there is, therefore, practically no current set up in the secondary winding. The effect is similar to that in an open-circuited transformer. As a load is thrown on the motor, the speed decreases and sufficient current flows in the secondary to produce the required torque. In the case of negative phase-sequence currents in the primary, the rotor is traveling, relative to the field set up by these currents, at double synchronous speed, approximately; the effect, therefore, is similar to that in a short-circuited transformer. For a slight unbalance of the impressed voltage, a relatively larger unbalanced current will, therefore, flow. If the negative sequence input is measured, it will always be positive if the motor is tending to maintain a balance. If it should read negative, the motor is unsymmetrical, and is causing unbalance. The above analysis applies equally well in the case of synchronous motors. In the case of rotary converters, the effects of unbalance may be quite serious for the negative phase-sequence current is not commutated, but circulates in the winding and produces secondary double-frequency currents. Both primary and secondary currents may be quite large, and since the former is not compensated by the flow of direct current, it has its full heating effect in the windings. Moreover, the negative phase-sequence current in the primary sets up a double-frequency voltage between commutator segments, which increases the tendency to flash over. A certain amount of double-frequency current flows through the brushes into the d-c. system, heating the brushes and causing inductive disturbances.

In the measurement of power supplied to a symmetrical motor, we are chiefly concerned with its operation considered with reference to a balanced positive phase-sequency system which is the standard polyphase system of reference in the same way that the sine wave is the standard wave form of current and e. m. f. As pointed out in a preceding paragraph, in a symmetrical motor, the total power input which is the sum of the positive and negative sequence power, is greater than the positive phase-sequence power input and the difference constitutes loss in the motor, due to the unbalance. But this is not the only loss due to the unbalance; there is a counter torque required to assist in driving the double-frequency current through the secondary windings, and this is supplied by the positive phase-sequence power which is also increased as a result of the unbalance; the increase is, however, very small, being approximately equal to one-half the secondary loss due to the double-frequency current flowing therein, and it may practically be ignored.

It will be apparent from the above statement that the existing methods of measuring power input to motors and rotary converters are not quite equitable and penalize the machines that assist in balancing the system while it favors those which inherently have characteristics that increase unbalance.

The simplest procedure to obtain the input of power to polyphase motors is to ignore the small additional loss due to the back torque set up by the negative phase-sequence currents in the motor, and to take the positive phase-sequence power and kv-a. or power factor as a measure of the performance of the machine. This method is a great deal more accurate than the present method of measuring the performance of the machine by the total power input and the effective power factor.

The product of negative phase-sequence current and positive phase-sequence volts, is a measure of the phase balancing or unbalancing effect of the motor on the circuit. When it is known that the motor is symmetrical, this will always measure the balancing effect of the motor on the system. When there is doubt as to the symmetry of the motor, the question as to whether its operation is favorable or detrimental to the balance of the circuit may be answered by obtaining negative sequence power input. If the value is positive, the operation is favorable; if it is negative, the operation is detrimental. There are certain cases where these criteria fail, but the number of cases in which it will be necessary to apply the test is small, and the number of cases in which the test fails is an unimportant figure.

Single Phase Motors

Considered from the point of view of symmetrical coordinates, single-phase motors are only a special case of three-phase motor unbalanced operation. The positive phase-sequence current and the negative phase-sequence currents are equal in magnitude; the unbalancing effect of such a motor is, therefore, quite definite and is equal to its positive phase-sequence kv-a.

It would appear reasonable to charge a consumer, using single-phase motor, on the basis of this unbalanced kv-a. whether other consumers using similar machines, when operating simultaneous with his, are connected, to jointly produce a balance or not. The cost of single-phase service on polyphase systems should be based on the cost of unbalance, compiled from the previous year.

This method of charging could be extended to all single-phase power loads, and would be equitable except in the case of consumers having mixed loads, who should be encouraged to balance up the single-phase portion of their loads as far as practicable. It should be remembered, however, that single-phase loads even when perfectly balanced are inferior to balanced polyphase rotating loads, because the latter have high inherent balancing characteristics which the single-phase balanced loads do not have. That is to say, the admittance to negative phase-sequence current for a balanced single-phase load is the same as its admittance to positive phase-sequence current, whereas in the case of a polyphase synchronous or induction motor or rotating converter carrying the same load, admittance to negative phase-sequence current may be many times that to positive phase-sequence currents.

SINGLE-PHASE LIGHTING

The same comments apply to this type of load as to single-phase motors, except that the power factor being near unity, the lighting load becomes less objectionable from this standpoint. When it is possible to balance the loads on all phases, it should be done by the consumer as closely as practicable, and he should be charged on the basis of the polyphase power consumed and the unbalance he creates. At the present stage, it will be sufficient to charge him for his unbalanced kv-a. without considering whether he is helping or hindering the balance. This charge may be considered the penalty attached to single-phase service, which is reduced in proportion to the care with which the consumer arranges his circuits. This method of charging holds equally well for all single-phase loads where they can be arranged to give good approximate balance.

THREE-PHASE FURNACES

The three-phase furnaces may be balanced by introducing a properly designed control. With proper control, it should be balanced both as to current and e. m. f. between neutral and terminals. Under these conditions of balance such harmonics of current as occur will be a minimum. A practical method of obtaining balance, is with the use of a series phase balancer which will be small as compared with the output of the machine. This should be combined with a voltage regulator to control the depth of the electrodes; in this way perfect balance as to the current as well as voltage may be obtained. Furnace loads when balanced have no inherent balancing capacity whatever, and therefore all unbalanced kv-a. should be charged for. In addition if the wave form of current is very much distorted, some account should be taken of this distortion when the operating conditions warrant it.

EQUITABLE SYSTEM OF RATES BASED ON SEQUENCE MEASUREMENTS

The positive-phase sequence power delivered to a load may be measured by a single-element wattmeter with the proper form of network constraint which can be incorporated in the meter. It has been pointed out that this power is the only power that a generating station is capable of delivering. The total power delivered measured at the terminals of a symmetrical generator which carries an unbalanced load is always less than the positive phase-sequence power. The difference is absorbed by the generator in the form of losses. In the case of a load causing unbalance, the positive phase-sequence power input will always include the negative phase-sequence power delivered to the system by the load as a result of the unbalance. Balanced polyphase loads, such as motors and balanced polyphase networks in general, do not contribute to the unbalance, but assist in maintaining balance. They absorb negative phase-sequence power, but do not transform positive phase-sequence power into negative phase-sequence power. Their operation as motors has nothing to do with their phase-balancing characteristics and is dependent solely on the positive phase-sequence power taken. There is a certain amount of positive phase-sequence power, so small that it may be neglected in the motor power, absorbed by reason of the balancing effect which is dissipated in the secondary windings as $I^2 R$ loss. The motor is performing an important service as a phase balancer for which, with the present method of charging, it is penalized. If only the positive phase-sequence power delivered is measured and charged for, this penalty will be removed and placed, where it should be placed, on the unbalanced loads on the system. The consumer who uses polyphase motors should get a polyphase rate somewhat lower than that for other kinds of polyphase load depending upon the estimated cost of unbalancing.

In the case of the consumer having a mixed load, if his single-phase load is of sufficient importance, it may be metered separately. If it is small as compared with the polyphase motor load, it may be connected so as to produce a minimum unbalance and the charge should be based on the positive phase-sequence meter reading and a rate obtained by taking the weighted average of the polyphase rate for single-phase balanced networks and that for polyphase motors, based on the respective value of the wired single-phase and polyphase loads.

In the case of single-phase loads connected so as to minimize unbalance, the power charges should be made on the basis of the positive phase-sequence power. A charge should also be made for negative phase-sequence kv-a. obtained by means of a recording negative phase-sequence current device, in the same manner as reactive power is charged for, the cost of unbalance being computed from previous results.

To sum up: Positive phase-sequence power is the only commodity manufactured by the power plant; phase balance is a service to which all consumers using polyphase motors contribute and with which users of single-phase devices interfere. Users of balanced polyphase power other than motors should pay for this service in the regular polyphase rates. Users of single-phase devices should pay over and above the regular polyphase rate depending upon the amount of unbalanced current they create. Users of polyphase motors should obtain a lower polyphase rate, based on the average amount of service they contribute to the maintaining of balance.

The meters required for assigning the proper charges are as simple or simpler than those used at the present time for polyphase power measurements and the results should prove more satisfactory and equitable all round and are based on correct theoretical grounds.

CHARACTERISTICS OF TRANSMISSION AND DISTRIBUTION SYSTEMS REQUIRED TO MAINTAIN BALANCED POLYPHASE SERVICE

It is essential to the maintenance of good balanced polyphase service that all banks of transformers should be symmetrically grouped that all transformers in one bank shall have the same performance and characteristics. It was not unusual in the past to find rotary converters connected to polyphase systems through a bank of open delta-connected transformers; such practises should not be tolerated except for short intervals while starting or under emergency conditions.

All feeders used to supply mixed loads should be loaded as symmetrically as possible. They should have as low impedance as possible for two reasons: (1) So that the effect of starting and stopping motors will not affect the lighting service unreasonably. (2) In order to reduce the voltage unbalance caused by any given value of unbalanced load they may incur.

Where a polyphase circuit is called upon to supply a large single-phase load, special provision should be made for the service such as heavier feeders and in many cases balancers or phase converters should be installed.

In selling power to a consumer of this kind the cost of the converter station will naturally enter into the rates charged for power, otherwise, the rates used will be the regular polyphase rates for power.

Dissymmetry in transmissions, feeders, etc. should be avoided with careful transpositions, dissymmetry in conductor arrangements may be compensated. In long transmission systems when synchronous condensers for regulating purposes are distributed over the line, they will also serve the purpose of phase balancers.

The fact that normal overloads have the characteristics of remaining balanced offers great possibilities in the way of protection on feeders selective operation and so forth. For if a loaded system is normally balanced, we may employ a positive phase-sequence relay for load control having the proper time element, and for short-circuit control, we may use the negative phase-sequence principle, and since negative phase sequence current flows only under short circuit, complete selective operation may be obtained.

There are many more applications of this principle to apparatus protection and control, too many to enumerate, that will suggest themselves to anyone who takes the trouble to study this subject.

Appendix

There appears to be a certain amount of misunderstanding in regard to the derivation of power quantities from the vector quantities used in alternating-current problems. In a recent letter, a copy of which was sent to the writer, the statement was made: "The quantity $\Sigma E_K I_K \sin \theta_K$ seems to me without physical significance, and as far as I am aware cannot be measured except when there are no harmonics present."

This quantity, however, has this physical significance, for example, in a constant voltage transmission system it is a measure of the magnetic energy that must be supplied to the system in order to maintain the voltage constant. It is also a measure of the capacity of the synchronous condensers required to supply the magnetic energy. In certain cases it would be impossible to supply more than a very limited amount of power, over a transmission line without supplying the necessary amount of this quantity which is stated to have no physical significance.

The fact that when harmonics are present the sum of these products cannot be measured with any known device is not of any great importance, as engineers are not greatly concerned with harmonic power quantities except in their elimination. It may be safely said that as far as the regulation of the lines and the supply thereto of magnetic energy is concerned these harmonic quantities have no practical significance.

Current and e. m. f. are from the point of view of linear circuits scalar quantities, but it is convenient to represent them by means of vector quantities. Similarly, power is a scalar quantity, and it is found equally convenient to represent it by means of a vector.

When the current and e. m. f. are simple harmonic quantities they are directly resolvable into two vectors rotating in opposite senses at uniform angular velocity. The power however, will not be a simple harmonic quantity but will consist of a mean term and a double-frequency term. The instantaneous power is therefore represented by two fixed vectors combined with two vectors rotating at double normal angular velocity in opposite directions. In both cases the two systems of vectors are conjugate and it is found convenient to ignore one system and represent those quantities by the positively rotating system. In the case of simple harmonic quantities such as current and e. m. f. a simple vector, rotating positively at an angular velocity, giving one complete revolution for each cycle is used. Power due to simple harmonic quantities is represented by a stationary vector which gives the mean power and a vector of the same magnitude rotating positively at double synchronous speed, or making one complete revolution for each half-cycle.

REPRESENTATION OF SINGLE-PHASE POWER

The representation of harmonic scalar quantities by vectors is not in any sense artificial, the relations follow at once when we express the harmonic quantities in their exponential form. The ignoring of one of the two conjugate quantities naturally follows from expediency, as it simplifies matters and makes the interpretation of formulas and equations easier.

If e and i are the instantaneous values of e. m. f. and current in a circuit and $\bar{E}$ and $\bar{I}$ are the corresponding symbolic expressions for these quantities the following relations hold good:

$$e = \frac{\breve{E} + \acute{E}}{\sqrt{2}} \qquad (1)$$

$$i = \frac{\breve{I} + \acute{I}}{\sqrt{2}} \qquad (2)$$

where $\acute{E}$ is conjugate to $\breve{E}$ and $\acute{I}$ is conjugate to I, and all the symbolic terms are expressed as root mean square quantities.

e and i being simple harmonic $\breve{E}$ and $\breve{I}$ take the form $E\, e^{j\omega t}\, I\, e^{j\omega t}$ when ω is 2π times the frequency.

In carrying out linear operations on i and e it is unnecessary to consider the conjugate terms $\acute{I}$ and $\acute{E}$ as the same operation on these terms yield a result which is the conjugate of that obtained by operation on $\breve{I}$ and $\breve{E}$. We may say therefore that i and e are fully represented by the symbolic terms $\breve{I}$ and $\breve{E}$, and that their actual value may be determined by either adding the conjugate term to each of these quantities and dividing by $\sqrt{2}$ or by taking the real part of these quantities multiplied by $\sqrt{2}$.

The instantaneous power delivered to the circuit is $e\, i$, and when the values of e and i are expressed in terms of $\breve{E}$ and $\breve{I}$ and their conjugates we obtain

$$e\, i = \frac{\breve{E}\acute{I} + \acute{E}\breve{I}}{2} + \frac{\breve{E}\breve{I} + \acute{E}\acute{I}}{2} \qquad (3)$$

e and i being both simple harmonic

$$\breve{E} = E\, e^{j\omega t} \qquad \acute{E} = E\, e^{-j\omega t}$$
$$\breve{I} = I\, e^{j\omega t} \qquad \acute{I} = I\, e^{-j\omega t}$$

so that the term $\frac{\breve{E}\acute{I} + \acute{E}\breve{I}}{2}$ is constant and represents the mean power. The term $\frac{\breve{E}\breve{I} + \acute{E}\acute{I}}{2}$ is a harmonically varying quantity of double normal frequency and consequently its mean value is zero.

The mean power input is defined in the usual manner by the integral

$$P = \frac{1}{2\pi} \int_{o}^{2\pi} e\, i\, d\,\theta$$

where θ is equal to $\omega\, t$ at any instant t. If we substitute from (3) for $e\, i$ and carry out the integration we obtain

$$P = \frac{\breve{E}\acute{I} + \acute{E}\breve{I}}{2} \qquad (4)$$

since the second pair of quantities in equation (3) being of double frequency becomes zero when integrated between the limits o and 2π.

If we have an electric system consisting of linear circuits having self-induction and mutual induction, its electromagnetic energy W is a quadratic function of the currents or $W = 1/2\, L\, i_1^2 + M_{12}\, i_1\, i_2 + 1/2\, L\, i_2^2 + \ldots$ Taking into account the dissipative forces we have

$$e_1 = R\, i_1 + \frac{d}{d\,t}\, \frac{\partial W}{\partial i_1}$$

we find that

$$1/2\, R\, i_1 \int^{t} i_1\, d\,t + W = 1/2\, i_1 \int^{t} e_1\, d\,t$$

$$\int^{t} e_1\, d\,t = j\, \frac{\acute{E} - \breve{E}}{\omega \sqrt{2}} \qquad \text{for a sine wave}$$

t being measured from any convenient datum. Therefore

$$i/2 \int_{o}^{t} e_1\, d\,t = -j\, \frac{\breve{E}\acute{I} - \acute{E}\breve{I}}{4\,\omega} - j\, \frac{\breve{E}\breve{I} - \acute{E}\acute{I}}{4\,\omega} \qquad (5)$$

The mean value of this integral is therefore given by

$$-j\, \frac{\breve{E}\acute{I} - \acute{E}\breve{I}}{4\,\omega}$$

and this is equal to W the electromagnetic energy since the mean value of $1/2\, R\, i_1 \int^{t} i_1\, d\,t$ is zero. We thus have

$$W_{mean} = -j\, \frac{\breve{E}\acute{I} - \acute{E}\breve{I}}{4\,\omega}$$

The "reactive power" is the average flow of electromagnetic energy throughout one half-cycle from the instant that $W = o$ to the next instant that $W = o$ and since W is a double-frequency pulsating quantity whose average value is one half the maximum, the "reactive power" Q is given by $2\,\omega\, W$ or

$$Q = -j\, \frac{\breve{E}\acute{I} - \acute{E}\breve{I}}{2} \qquad (6)$$

multiply (6) by j and add to (4) we then have

$$P + J\, Q = \breve{E}\, \acute{I}$$

This product therefore defines both the true and reactive power of the circuit.

The second terms of (3) and (5) are the double-frequency portions of the power product and we find if we take

$$P_{\text{n}} = \frac{\breve{E}\breve{I} + \acute{E}\acute{I}}{2}$$

$$Q_{\text{n}} = -j\, \frac{\breve{E}\breve{I} - \acute{E}\acute{I}}{2}$$

$$P_{\text{n}} + j\, Q_{\text{n}} = \breve{E}\, \breve{I}$$

The complete symbolic expression for the power is therefore

$$(P + j\, Q) + (P_{\text{n}} + j\, Q_{\text{n}}) = \breve{E}\, \acute{I} + \breve{E}\, \breve{I}$$

The expression $P_{\text{n}} + J\, Q_{\text{n}}$ however is superfluous as we are only interested in the mean values of power and furthermore, when the values of P and Q are given, the values of P_{n} and Q_{n} may be determined. It may be shown that the vector $P_{\text{n}} + j\, Q_{\text{n}}$ which is rotating positively at double synchronous speed will be equal in magnitude to $P + j\,Q$ and coincide with it at the same instant that the vector $\check{E}$ is in coincidence with it, and since it rotates in the positive direction with twice the angular velocity of $\check{E}$, it will always be in advance of $\check{E}$ by the same angle that $\check{E}$ is in advance of the stationary vector $P + j\,Q$ (See Fig. 1).

In the case of wave-forms having harmonics it seems reasonable, since the heating effect of the harmonics is independent, to treat each frequency independently as to both true and reactive power so that if

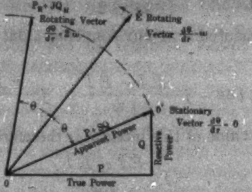


FIG. 1—VECTOR REPRESENTATION OF RELATION BETWEEN ZERO FREQUENCY POWER PRODUCT $(P + SQ)$ AND DOUBLE FREQUENCY POWER PRODUCT $(P_H\, SQ_H)$ IN SINGLE-PHASE CIRCUIT

$$\check{E} = \Sigma\, \check{E}_n$$
$$\check{I} = \Sigma\, \check{I}_n$$
$$P + j\,Q = \Sigma\, \check{E}_n\, \check{I}_n = \Sigma\, (P_n + j\,Q_n)$$

and apparent power might then be defined by

$$\sqrt{P^2 + Q^2} = \sqrt{(\Sigma\, P_n)^2 + (\Sigma\, Q_n)^2}$$

MEASUREMENT OF POWER IN SINGLE-PHASE CIRCUITS

True power may of course be measured by a wattmeter in the usual way. To measure reactive power a wattmeter having a large inductance in series with the voltage coil may be used, connected into the circuit in the same way as the ordinary wattmeter. Assuming the resistance of the voltage coil to be negligible and taking i_0 for the current flowing in it, φ for the angle of the voltage coil relative to the current coil in which the current i is flowing (the current and voltage coils of a balance type instrument shouldbe in non-inductive relation and therefore $M = o$) we have

$$i_0 = 1/L \int^t e\, d\, t$$

The instantaneous torque is

$$F = \frac{\partial M}{\partial \varphi}\, i_0\, i$$

$$= 1/L\, \frac{\partial M}{\partial \varphi}\, i \int^t e\, d\, t$$

and the mean value of the torque is

$$F_{mean} = 1/L\, \frac{\partial M}{\partial \varphi}\, \frac{1}{2\,\pi} \int_0^{2\pi} i\, d\, \theta \int e\, d\, t$$

$$= \frac{1}{\omega\, L}\, \frac{\partial M}{\partial \varphi}\, Q$$

This arrangement therefore gives the mean magnetic energy, and when the frequency is known, the mean reactive power.

When the wave form of current and e. m. f. has harmonics this instrument gives the value of $2\, W_{mean}$ or the mean magnetic energy. If Q_n be the reactive power for the nth harmonic the quantity measured by the instrument is equal to $\Sigma \left(\frac{Q\, n}{n\, \omega} \right)$. There will therefore be a frequency error in the instrument.

MEASUREMENTS REQUIRED FOR COMPLETE DETERMINATION OF SINGLE-PHASE CIRCUIT

The following measurements or their equivalent are required to completely determine a single-phase current:

(1) Magnitude of impressed e. m. f.
(2) Phase of impressed e. m. f.
(3) Magnitude of current in circuit
(4) Phase of current in circuit.

For the last two, measurements of true and reactive power may be substituted. The second measurement is not required if the particular circuit only is under consideration. If we have measurements of e. m. f. kv-a. and power-factor, the circuit is completely determined and no other measurements are required.

MEASUREMENTS REQUIRED FOR COMPLETE DETERMINATION OF BALANCED POLYPHASE CIRCUIT

In a balanced system if e. m. f. and current are determined in one phase they will also be known in the other phases. The requirements are therefore the same as for single-phase and apply to one phase only which we shall call the "principal" phase.

REPRESENTATION OF POWER IN UNBALANCED THREE-PHASE CURRENTS

Confining ourselves to three-phase systems, we find that an unbalanced four-wire system required twelve measurements to define it completely or if expressed in symbolic form six quantities are required. If $A\, B\, C$ are the three phases these symbolic quantities may be

$$\left.\begin{matrix} \check{E}_A & \check{I}_A \\ \check{E}_B & \check{I}_B \\ \check{E}_C & \check{I}_C \end{matrix}\right\} \qquad (7)$$

In symmetrical coordinates the six quantities will be

$$\left.\begin{matrix} S^0\, \check{E}_{A0} & S^0\, \check{I}_{A0} \\ S'\, \check{E}_{A1} & S'\, \check{I}_{A1} \\ S^2\, \check{E}_{A2} & S^2\, \check{I}_{A2} \end{matrix}\right\} \qquad (8)$$

These are related to the quantities (7) by the simple relation

$$\left.\begin{aligned} S^0\, E_{A0} &= S^0\, \frac{\check{E}_A + \check{E}_B + \check{E}_C}{3} \\ S^1\, \check{E}_{A1} &= S^1\, \frac{\check{E}_A + \alpha\, \check{E}_B + \alpha^2\, \check{E}_C}{3} \\ S^2\, \check{E}_{A2} &= S^2\, \frac{\check{E}_A + \alpha^2\, \check{E}_B + \alpha\, E_C}{3} \end{aligned}\right\} \qquad (9)$$

where $\alpha = -1/2 + j\sqrt{3/2}$ $\alpha^2 = -1/2 - j\sqrt{3/2}$ exactly the same relations exist between $S^0 \check{I}_{A0}$ $S' \check{I}_{A1}$ $S^2 \check{I}_{A2}$ and $\check{I}_A \check{I}_B \check{I}_C$. We have also the reciprocal relations

$$\left.\begin{aligned} \check{E}_A &= \check{E}_{A0} + \check{E}_{A1} + \check{E}_{A2} \\ \check{E}_B &= \check{E}_{A0} + \alpha^2 \check{E}_{A1} + \alpha \check{E}_{A2} \\ \check{E}_C &= \check{E}_{A0} + \alpha \check{E}_{A1} + \alpha^2 \check{E}_{A2} \end{aligned}\right\} \tag{10}$$

and corresponding relations between line currents and symmetrical coordinate currents. We sum the above up by the short hand expression

$$S^r(\check{E}_A) = S^0 \check{E}_{A0} + S^1 \check{E}_{A1} + S^2 \check{E}_{A2}$$

$$S^r(\check{I}_A) = S^0 \check{I}_{A0} + S^1 \check{I}_{A1} + S^2 \check{I}_{A2}$$

The terms prefixed by S^0 S^1 and S^2 are respectively called the zero-phase sequence, the positive phase sequence, and the negative phase sequence components of e. m. f. or current. The sequence symbol S prefixing each of the quantities, indicates that the quantity in question is the principal one of a group of three quantities constituting a symmetrical three-phase system.

For each current or e. m. f. sequence there is a conjugate current or e. m. f. sequence which is distinguished from it by having a negative exponent

$$\left.\begin{aligned} &S^0 \check{E}_{A0} \text{ conjugate seq } S^{-0} \hat{E}_{A0} \\ &S^1 \check{E}_{A1} \text{ conjugate seq } S^{-1} \hat{E}_{A1} \\ &S^2 \check{E}_{A2} \text{ conjugate seq } S^{-2} \hat{E}_{A2} \end{aligned}\right\} \tag{12}$$

The relation between instantaneous values in the symmetrical coordinate system and the vector values is exactly the same as for single-phase. Thus the system of e. m. fs. e_a e_b e_c is given by

$$(e_a, e_b, e_c) = \frac{S^0 \check{E}_{a0} + S^{-0} \hat{E}_{a0}}{\sqrt{2}} + \frac{S^1 \check{E}_{a1} + S^{-1} \hat{E}_{a1}}{\sqrt{2}} + \frac{S^2 \check{E}_{a2} + S^{-2} \hat{E}_{a2}}{\sqrt{2}}$$

The conjugate symbol S^{-1} is as it implies, a sequence which is conjugate to S^1. Thus, if $S^1 \check{E}_{a1}$ signifies the system

$$\left.\begin{aligned} \check{E}_{a1} &= \check{E}_{a1} \\ \check{E}_{b1} &= \alpha^2 \check{E}_{a1} \\ \check{E}_{c1} &= \alpha \check{E}_{a1} \end{aligned}\right\} \tag{13}$$

The conjugate system $S^{-1} \hat{E}_{a1}$ is

$$\left.\begin{aligned} \hat{E}_{a1} &= \hat{E}_{a1} \\ \hat{E}_{b1} &= \alpha \hat{E}_{a1} \\ \hat{E}_{c1} &= \alpha^2 \hat{E}_{a1} \end{aligned}\right\} \tag{14}$$

If we sum the terms of a sequence other than zero sequence the value will be always zero or

$$\Sigma (S^1 P) = 0$$

$$\Sigma (S^2 P) = 0$$

But $\Sigma (S^0 P) = 3P$ for three phase

Consider the system of e. m. fs. and currents e_a, e_b, e_c i_a, i_b, i_c. These two systems have been shown to be equal to

$$(e_a, e_b, e_c) = \frac{S^0 \check{E}_{a0} + S^{-0} \hat{E}_{a0}}{\sqrt{2}} + \frac{S^1 \check{E}_{a1} + S^{-1} \hat{E}_{a1}}{\sqrt{2}} + \frac{S^2 \check{E}_{a2} + S^{-2} \hat{E}_{a2}}{\sqrt{2}}$$

$$(i_a, i_b, i_c) = \frac{S^0 \check{I}_{a0} + S^{-0} \hat{I}_{a0}}{\sqrt{2}} + \frac{S^1 \check{I}_{a1} + S^{-1} \hat{I}_{a1}}{\sqrt{2}} + \frac{S^2 \check{I}_{a2} + S^{-2} \hat{I}_{a2}}{\sqrt{2}}$$

If we take the product of these two quantities treating the sequence symbols as if they represented actual quantities we shall obtain the instantaneous power in each of the phases. The zero-frequency portion of this quantity will be considered first as it represents the mean power in each phase. It will consist of two terms which are conjugate to each other. These two terms are those obtained by taking one half the product of $S^r(\check{E}_a)$ and $S^{-r}(\hat{I}_a)$ or $1/2\,(S^0 \check{E}_{a0} + S^1 \check{E}_{a1} + S^2 \check{E}_{a2})(S^{-0} \hat{I}_{a0} + S^{-1} \hat{I}_{a1} + S^{-2} \hat{I}_{a2})$. From this, following the method of proof used for single-phase, we find that the mean power per phase may be represented by the vector sequence quantity $S^r(P_A + J\,Q_A)$. The mean power per phase obtained by taking the products of instantaneous values of current and e. m. f. is one half the sum of this term and its conjugate. Therefore the real part of this term represents the mean true power per phase and the imaginary part the mean flow of magnetic energy or the reactive power per phase. Using currents i_A i_B i_C and e. m. fs. e_A e_B e_C instead of i_A i_B i_C etc.

$$\begin{aligned} S^r(P_A + j\,Q_A) &= (S^0 \check{E}_{A0} + S^1 \check{E}_{A1} + S^2 \check{E}_{A2})(S^{-0} \hat{I}_{A0} + S^{-1} \hat{I}_{A1} + S^{-2} \hat{I}_{A2}) \\ &= S^0 (\check{E}_{A0} \hat{I}_{A0} + \check{E}_{A1} \hat{I}_{A1} + \check{E}_{A2} \hat{I}_{A2}) \\ &\quad + S' (\check{E}_{A0} \hat{I}_{A2} + \check{E}_{A1} \hat{I}_{A0} + \check{E}_{A2} \hat{I}_{A1}) \\ &\quad + S^2 (\check{E}_{A0} \hat{I}_{A1} + \check{E}_{A1} \hat{I}_{A2} + \check{E}_{A2} \hat{I}_{A0}) \end{aligned} \tag{15}$$

The symbol S^{-1} being in the three-phase system the same as $S_1{}^2$ and S^{-2} being the same as S^1. The total mean power is $P + j\,Q = \Sigma\, S^r(P_A + J\,Q_A)$ or

$$(P + j\,Q) = 3\,(\check{E}_{A0} \hat{I}_{A0} + \check{E}_{A1} \hat{I}_{A1} + \check{E}_{A2} \hat{I}_{A2}) \tag{16}$$

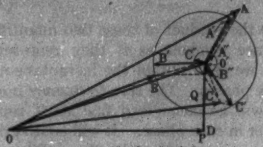
$\Sigma\, S^1(\;)$ and $\Sigma\, S^2(\;)$ being both equal to zero represent instantaneous power interchanges among the three phases, and are caused by unbalance. The double-frequency power product $S^r(P_{HA} + J\,Q_{HA})$ is given by

$$\begin{aligned} S^r(P_{HA} + j\,Q_{HA}) &= S^0 (\check{E}_{A0} \check{I}_{A0} + \check{E}_{A1} \check{I}_{A2} + \check{E}_{A2} \check{I}_{A1}) \\ &\quad + S^1 (\check{E}_{A0} \check{I}_{A1} + \check{E}_{A1} \check{I}_{A0} + \check{E}_{A2} \check{I}_{A2}) \\ &\quad + S^2 (\check{E}_{A0} \check{I}_{A2} + \check{E}_{A1} \check{I}_{A1} + \check{E}_{A2} \check{I}_{A0}) \end{aligned} \tag{17}$$

which is seen to be the product

$(S^0 \check{E}_{A0} + S^1 \check{E}_{A1} + S^2 \check{E}_{A2})(S^0 \check{I}_{A0} + S^1 \check{I}_{A1} + S^2 \check{I}_{A2})$

and therefore the double-frequency product is obtained for three-phase in exactly the same way as it is obtained for single-phase, and furthermore, the amplitude and three-phase angle of the double-frequency term for the principal phase is related to the e. m. f. of which it is composed in exactly the same way as in the case of single-phase. That is the double-frequency power vector is as much in advance of the corresponding e. m. f. vector as this vector is in advance of the corresponding mean power vector. The sequence of the double-frequency power vector is obtained from (**17**), that is the exponent of the sequence symbol or order of the sequence is the sum of the exponents of the sequences of current and e. m. f.



$P + SQ = \Sigma\,\{S^0 \check{E}_{A1}\,\hat{I}_{A1} + \check{E}_{A2}\,\hat{I}_{A2}\} = 00'$

$(0'A'',\ 0'B'',\ 0'C'') = S^1 \check{E}_{A2}\,\hat{I}_{A1}$

$(0'A',\ 0'B',\ 0'C') = S^2 \check{E}_{A1}\,\hat{I}_{A2}$

$(0A,\ 0B,\ 0C)$ = Load per phase

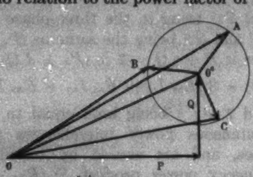
FIG. 2—POWER VECTOR DIAGRAM

25 per cent current unbalance, 5 per cent voltage unbalance

The significance of the expression (**15**) is that it gives not only the mean power for each phase, but it also determines the magnitude and phase of the double-frequency power product which results from unbalance. The portions of (**17**) whose instantaneous sums are zero are of no significance. The important double-frequency quantities are given by

$$P_H + J\,Q_H = \Sigma\, S^r\,(P_{HA} + Q_{HA})$$
$$= 3\,(\check{E}_{A0}\,\hat{I}_{A0} + \check{E}_{A1}\,\hat{I}_{A1} + \check{E}_{A2}\,\hat{I}_{A2})$$

and are seen to be double-frequency power vectors of like value and phase in each phase. These power vectors are present only when there is unbalance and they have no relation to the power factor of the system.



$P + SQ = 3\,\check{E}_{A1}\hat{I}_{A1} = 00'$

$(0'A',0'B',0'C') = S^r\,\check{E}_{A1}\hat{I}_{A2}$

$0 = S^2\check{E}_{A2}\hat{I}_{A1}$

$(0A,0B,0C)$ = Load per Phase

FIG. 3—POWER VECTOR DIAGRAM

25 per cent current unbalance. No unbalance in voltage. Illustrates the load condition at a distribution point at which a phase balancer is installed.

In a balanced system the instantaneous sum of the double-frequency power vectors is always zero.

A power vector diagram for the zero frequency power products may be drawn. This vector diagram represents the resolution of the mean power per phase into symmetrical coordinate vectors. For each of these coordinates there is a corresponding double-frequency power vector which gives the actual value of the double-frequency power at each instant.

Fig. (3) illustrates a system having about 25 per cent unbalance in current and about 5 per cent unbalance in voltage. The relative distribution of load among the three phases will depend upon the position of the triangle *A B C* which may be rotated about its centroid without changing the total mean power but producing a marked difference in the values of mean power per phase. The unbalance factor for current is the ratio $\hat{I}_{A2}/\hat{I}_{A1}$ which is a complex quantity. The conjugate of this quantity gives the magnitude and position of the unbalanced power vector $S^2\,\check{E}_{A1}\,\hat{I}_{A2}$ with respect to $S^0\,\check{E}_{A1}\,\hat{I}_{A1}$ the quantity which gives equal values per phase and is the total mean power product or kv-a. per phase.

The vector method of representing power quantities affords a simple means of obtaining a physical picture of the heating effect due to unbalanced currents. In many cases the e. m. f. unbalance is small enough to be ignored in rough approximations. Take the case of a small induction motor having a negative phase sequence impedance of 15 per cent which is about an average figure; 3¼ per cent unbalance in voltage at the terminals of this motor will cause 25 per cent unbalance in current. From a practical standpoint in estimating the heating effect of the unbalance the voltage unbalance may be ignored. The mean power load will be practically the same as for balanced voltages with the addition of the kv-a. per phase due to the unbalanced current which will be added vectorially to the mean power; this will be obtained by taking the square of the unbalanced current multiplied by the negative sequence impedance. The end of this vector obtained in this manner serves as a centroid for an equilateral triangle *A′ B′ C′* such that the lines drawn from the centroid to *A*, *B*, and *C*, are equal to 25 per cent

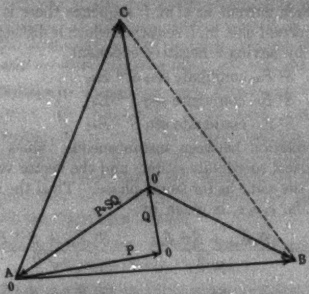


FIG. 4—POWER VECTOR DIAGRAM

Illustrates a single-phase load on a three-phase system, the balance of which is 100 per cent due to a phase balancer. $(O'A, O'B, O'C)$ represent the load on the balancer due to the single-phase load of the mean power per phase or the vector $S^0 \check{E}_{A1} \hat{I}_{A1}$. By rotating this triangle about its centroid all the different conditions of load to which the motor may be subjected on a line having this degree of unbalance are obtained. The triangle A', B', C' is drawn negative sequence. If we wish also to include the unbalance due to the voltage we may represent this by a positive phase sequence equilateral triangle whose distance from centroid to apex is 3¼ per cent of the mean power per phase. The angle this vector makes with the mean power vector is arbitrary and is the same as the angle $\check{E}_{A2}$ makes with E_{A1}. Let us denote it by θ_1. Let θ_2 be the angle that the negative sequence current lags behind the negative sequence voltage, this is given by the power factor of the negative phase sequence impedance. Then the principal phase or apex A'' will be in advance of the datum line by the angle $\theta_2 - \theta$. To obtain a complete solution we should take

(1) mean power per phase $P/3 + j\,Q/3$ leads datum line by angle θ_2

(2) neg. seq. power per phase $\check{E}_{A2} \hat{I}_{A2}$ length of vector compared with $(P/3 + j\,Q/3)$ is given by product of current unbalance factor and voltage unbalance factor leads datum line by angle θ_3.

(3) voltage unbalance volt. amp. $S^1 \check{E}_{A2} \hat{I}_{A1}$ length of vector compared with $(P/3 + j\,Q/3)$ same percentage as $\check{E}_{A2}$ is of $\check{E}_{A1}$ leads datum line by angle $\theta_2 + \theta_1$

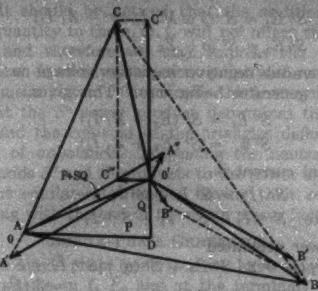
(4) current unbalance volt amperes $S^2 \check{E}_{A1} \hat{I}_{A2}$ length of vector compared with $P/3 + j\,Q/3$ or $\check{E}_{A1} \hat{I}_{A1}$ is same percentage as $\hat{I}_{A2}$ is of $\hat{I}_{A1}$ or unbalance factor leads datum line by angle $\theta_2 - \theta_1$.

Each of the systems of vectors $S^2 \check{E}_{A1} \hat{I}_{A2}$, $S^1 \check{E}_{A2} \hat{I}_{A1}$ will give an equilateral triangle whose centroid is the end of the vector $S^0 (\check{E}_{A1} \hat{I}_{A1} + \check{E}_{A2} \hat{I}_{A2})$. The two combine to form a triangle not equilateral which may be rotated about its centroid at the end of the mean power vector to give the power per phase for all possible conditions of load, thus the complete range of possible values of kv-a. per phase may be obtained by the lengths of the vectors $O\,A$, $O\,B$, $O\,C$ drawn from the origin to the apices of the composite triangle.

A study of the diagram will show that the quantities measuring unbalance do not affect the true mean power and power factor, and therefore any attempt to deduce a factor which will simultaneously take account of both power factor and unbalance is useless. The positive and negative sequence current, voltage, true power, reactive power, etc. may all be measured with single element meters and a suitable constraining network. Mr. Evans has kindly undertaken to describe in the discussion some of those which have been developed.

Applying the principles outlined above it is easy to prove that a symmetrical generator can deliver power only through the positive phase sequence current it delivers to the system, and that any power due to negative phase sequence current is absorbed as losses in the generator. This is also true of all symmetrical rotating machines on the system, any power due to negative phase sequence current is absorbed in the form of additional losses.

In the body of the paper the task has been undertaken of showing that unbalance is of the same order of importance as power factor. If we denote by $\check{E}_1$ the terminal e. m. f. of one winding of a generator and by $\check{E}_0$ the e. m. f. that would appear at the terminals if the current were zero, and if $r_1\,I$ be the resistance drop $j \times I_1$ the e. m. f. due to the component of the field set up by the stator m. m. f. including the internal leakage field



$P + SQ = (\check{E}_{A1} \check{I}_{A1} - \check{I}_{A1} Z_1)\, \hat{I}_{A1} - I_{A1}^2 Z_2$

$0A',\ 0B',\ 0C' = -S^2(\check{E}_{A1} \check{I}_{A1} - \check{I}_{A1} Z_1)\, \hat{I}_{A1}$

$(0A'',\ 0B'',\ 0C'') = S^1 I_{A1}^2 Z_2$

$\check{I}_{A1} = -\check{I}_{A2}$

FIG. 5—POWER VECTOR DIAGRAM

Single-phase generator supplying power ($O\,A$, $O\,B$, $O\,C$) represent loads on phase A, B and C respectively

$$\check{E}_0 = \check{E}_1 + (r_1 + j\,x_1)\, I_1$$

The phase and magnitude of $\check{E}_0$ will depend upon the time phase of I_1. Since $\check{E}_0$ is the component due to the exciting current I_0 we may use the coefficient of mutual induction M_0 to give the generated voltage in terms of the exciting current

$$\check{E}_0 = j\,\omega\,\dot{M}_0\, I_0\, e^{j\omega t}$$

and therefore

$$j\,\omega\,\dot{M}_0\, I_0\, e^{j\omega t} = \check{E}_1 + (r_1 + j\,x_1)\, I_1$$

In particular if we take E_1 as datum and if I is lagging in time phase behind E_1 by a quarter cycle

$$j\,\omega\,\dot{M}_0\, I_0\, e^{j\omega t} = E_1\, e^{j\omega t} + (r_1 + j\,x_1)\,(-\,j\,I_1)\, e^{j\omega t}$$

$$j\,\omega\,\dot{M}_0\, I_0 \quad = (E + x_1\, I) - (j\,r\,I)$$

Thus if we ignore the value of $r_1\, I_1$ which is small, the increase in the exciting current required to maintain a given terminal e. m. f. under zero power factor load is proportional to I

$$\Delta\, I_0 = \frac{x_1}{j\,\omega\,M_0}$$

showing that the presence of wattless lagging current in the winding necessitated a proportional increase in the exciting current. The cyclic changes in saturation will not affect the value of M_0 which is the average value. But the average value will change with the strength of the resultant field, but on account of the long air gaps, these changes are not so important as might be supposed.

When the terminal voltages are unbalanced they may be resolved into their positive and negative phase sequence components $S' \check{E}$ and $S^2 \check{E}_2$ so that we have

$$S \check{E} = S^1 \check{E}_1 + S^2 \check{E}_2$$

we have further

$$S^1 \check{E}_1 = S' (j \omega M_0 I_0 e^{j\omega t} - Z_1 \check{I}_1)$$

$$S^2 \check{E}_2 = S^2 (o - Z_2 \check{I}_2)$$

the generated negative sequence voltage in a symmetrical generator being zero. This gives

$$S^1 \check{E}_0 = S^2 (\check{E}_1 + Z_1 \check{I}_1)$$

$$S^2 o = S^2 (\check{E}_2 + Z_2 \check{I}_2)$$

The total current is

$$S^r (\check{I}) = S^1 \check{I}_1 + S^2 \check{I}_2$$

conjugate $S^{-r} (\hat{I}) = S^{-1} \hat{I}_1 + S^{-2} \hat{I}_2$

The power generated

$$P_0 + j Q_0 = \Sigma [S^1 \check{E}_0 + S^2 (o)] [S^{-1} \hat{I}_1 + S^{-2} \hat{I}_2] = 3 (\check{E}_0 \hat{I}_1)$$

which shows that no power is generated through negative sequence current, but such as appears at the generator terminals is power that was originally generated by the machine through positive sequence voltage and current which has been degraded through some unbalanced load and fed back into the system in the form of negative phase sequence voltage and current, producing increased losses and heating.

The total heating due to $\check{I}_2$ in the stator windings will be obtained correctly by considering $\check{I}_2$ as entirely independent of $\check{I}_1$. The losses due to the double-frequency current in the rotor which arises from $\check{I}_2$ will also cause independent heating so that the total copper loss in the machine is obtained by the power product $3 (I_1^2 Z_1 + I_2^2 Z_2)$, the real part being the true copper loss. This may be obtained by considering the output of the machine at its terminals; this is given by

$$P + j Q = \Sigma (S^r \check{E}_A \times S^{-r} \hat{I}_A)$$

$\check{E}_A$ being the terminal voltage and $\hat{I}_A$ the conjugate of the terminal current with particular reference to phase A as principal phase

$$S^{-r} \hat{I}_A = S^1 \hat{I}_1 + S^2 \hat{I}_2$$

$$\check{I}_A = \check{I}_1 + \check{I}_2$$

$$\check{I}_B = \alpha^2 \check{I}_1 + \alpha \check{I}_2$$

$$\check{I}_C = \alpha \check{I}_1 + \alpha^2 \check{I}_2$$

and the terminal e. m. f.

$$S^r \check{E}_A = S' (\check{E}_0 - \check{I}_1 Z_1) - S^2 \check{I}_2 Z_2$$

$$\check{E}_A = \check{E}_0 - Z_1 \check{I}_1 - Z_2 \check{I}_2$$

$$\check{E}_B = \alpha^2 \check{E}_0 - Z_1 \alpha^2 \check{I}_1 - Z_2 \alpha \check{I}_2$$

$$\check{E}_C = \alpha \check{E}_0 - Z_1 \alpha \check{I}_1 - Z_2 \alpha \; {}_2$$

where $\alpha = - 1/2 + j \sqrt{3/2}$ $\alpha^2 = - 1/2 - j \sqrt{3}/2$ $\alpha^3 = 1$

$$\begin{aligned} \Sigma (S^r \check{E}_A \, . \, S^{-r} \hat{I}_A) &= \Sigma [S' (\check{E}_0 - Z_1 \check{I}_1) - S^2 Z_2 \check{I}_2] (S^{-1} \hat{I}_1 + S^{-2} \hat{I}_2) \\ &= \Sigma [S^0 (\check{E}_0 \hat{I}_1 - Z_1 I_1^2) - S^0 I_2^2 Z_2 \\ &\quad + S^3 (\check{E}_0 - \check{I}_1 Z_1) \hat{I}_2 - S' \check{I}_2 \hat{I}_1 Z_1 \\ &= 3 (\check{E}_0 \hat{I}_1 - I_1^2 Z_1 - I_2^2 Z_2) \end{aligned}$$

$3 \check{E}_0 \hat{I}_1$ is the output of the generator not including the copper losses. The real part of $(I_1^2 Z_1 + I_2^2 Z_2)$ therefore gives the total copper loss.

The solution of the system may of course be obtained by the methods used in ordinary linear circuits having mutual inductances. If we have a three-phase machine with winding impedances $Z_{11} Z_{22} Z_{33}$ and mutual impedances $Z_{12} Z_{23} Z_{31}$ etc. we have the following relations in a symmetrical machine

$$Z_{11} = Z_{22} = Z_{33} = Z_1 + Z_2$$

$$Z_{12} = Z_{23} = Z_{31} = \alpha^2 Z_1 + \alpha Z_2$$

$$Z_{13} = Z_{21} = Z_{32} = \alpha Z_1 + \alpha^2 Z_2$$

Conversely we have for Z_1 and Z_2 in terms of the impedances $Z_{11} Z_{12}$ etc.

$$Z_1 = \frac{Z_{11} + \alpha Z_{12} + \alpha^2 Z_{13}}{3}$$

$$Z_2 = \frac{Z_{11} + \alpha^2 Z_{12} + \alpha Z_{13}}{3}$$

The equations of the generator in terms of the impedances $Z_{11} Z_{12}$ etc. are

$$\left. \begin{aligned} \check{E}_A &= \check{E}_0 - Z_{11}/3 \, \check{I}_A - Z_{21}/3 \, \check{I}_B - Z_{31}/3 \, \check{I}_C \\ \check{E}_B &= \check{E}_0 - Z_{12}/3 \, \check{I}_A - Z_{22}/3 \, \check{I}_B - Z_{32}/3 \, \check{I}_C \\ \check{E}_C &= \check{E}_0 - Z_{13}/3 \, \check{I}_A - Z_{23}/3 \, \check{I}_B - Z_{33}/3 \, \check{I}_C \end{aligned} \right\} \quad (18)$$

The relations

$$\left. \begin{aligned} \check{I}_A + \check{I}_B + \check{I}_C &= 0 \\ Z_{11} + Z_{12} + Z_{13} &= 0 \\ \check{E}_A + \check{E}_B + \check{E}_C &= 0 \end{aligned} \right\} \quad (19)$$

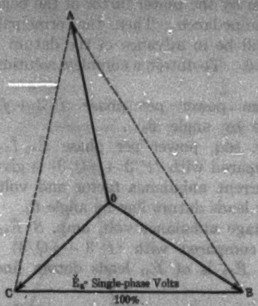


FIG. 6—VECTOR DIAGRAM OF E. M. FS.
Single-phase generator supplying power. Obtained from Fig. 5

reduce the number of independent variables in these equations to two but there is not much simplification introduced.

The above solution illustrates the difference between the method of solution which consists in considering each phase as an independent single-phase circuit and that in which the effects of the currents in all phases in symmetrical groups are considered simultaneously. Equations (18) and (19) are the forms to which the differential equations reduce on solution for steady conditions. The impedances Z_{11} Z_{12} Z_{1}, etc. are compounded of two impedances Z_1 and Z_2 which are fundamental characteristic impedances of the machine and it is interesting to note that the form of these impedances bears the same relation to Z_1 and Z_2 that the currents $\hat{I}_A$ $\hat{I}_B$ $\hat{I}_C$ bear to I_1 and I_2.

THREE-PHASE POWER MEASUREMENTS

It is convenient to represent line currents, and, e. m. fs. measured from the neutral point, by $\hat{I}_A$ $\hat{I}_B$ I_C, $\check{E}_A$ $\check{E}_B$ $\check{E}_C$ respectively and the corresponding sequence quantities by $S^0 \hat{I}_{A0}$ $S^1 \hat{I}_{A1}$ $S^2 \hat{I}_{A2}$ etc. Delta currents and e. m. fs. between lines are represented by I_a I_b I_c $\check{E}_a$ $\check{E}_b$ $\check{E}_c$ as shown in Fig. (7) the positive direction being indicated by an arrowhead. This gives the following relations between the line currents, delta currents, Y voltages and delta voltages

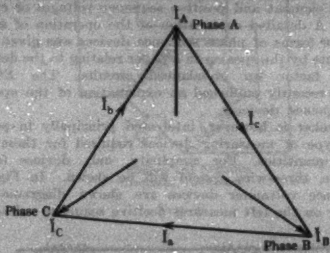


FIG. 7—DIAGRAM ILLUSTRATING CONVENTION USED IN STAR AND DELTA CURRENTS BY SYMMETRICAL COORDINATES

$$S^r \check{E}_A = S^1 \check{E}_{A1} + S^2 \check{E}_{A2}$$

$$S^r \check{E}_a = S^1 j \sqrt{3}\, \check{E}_{A1} - S^2 j \sqrt{3}\, \check{E}_{A2}$$

So that

$$S^1 \check{E}_{a1} = S^1 j \sqrt{8}\, \check{E}_{A1}$$

$$S^2 \check{E}_{a2} = S^2 - j \sqrt{3}\, \check{E}_{A2}$$

With these relations the various methods of measuring power in three-phase circuits may be easily proved.

Three-Phase Power with Two Wattmeter

$$\begin{aligned} P + j\,Q &= \Sigma\, (S^1 \check{E}_{A1} + S^2 \check{E}_{A2})\,(S^{-1} \hat{I}_{A1} + S^{-2} \hat{I}_{A2}) \\ &= \Sigma\, [\, S^0 (\check{E}_{A1} \hat{I}_{A1} + \check{E}_{A2} \hat{I}_{A2}) + S^2 \check{E}_{A1} \hat{I}_{A2} \\ &\qquad + S^1 \check{E}_{A2} \hat{I}_{A1}] \end{aligned}$$

The sum of the instantaneous values of all but the S^0 group is zero. We may therefore add to the e. m. f. any quantity which will give S^1 and S^2 groups with no S^0 group without changing the value of the S^0 term. Thus if we add to $S^r E_A$ any S^0 term we shall not change the mean power. If we add $- S^0 (\check{E}_{A1} + \check{E}_{A2})$; this will make the power in phase A zero. The wattmeter in this phase may therefore be dispensed with. This will add $- S^2 (\check{E}_{A1} \hat{I}_{A2} + \check{E}_{A2} \hat{I}_{A1})$ and $- S^1 (\check{E}_{A1} \hat{I}_{A2} + \check{E}_{A2} \hat{I}_{A2})$ to the power product. The voltages will be

$$\check{E}_A = \check{E}_{A1} + \check{E}_{A2} - \check{E}_{A1} - \check{E}_{A2} = 0$$

$$\check{E}_B = \alpha^2 \check{E}_{A1} - \check{E}_{A1} + \alpha\, \check{E}_{A2} - \check{E}_{A2} = \check{E}_C$$

$$\check{E}_C = \alpha\, \check{E}_{A1} - \check{E}_A + \alpha^2 \check{E}_{A2} - \check{E}_{A2} = - \check{E}_B$$

We have therefore the line currents $\hat{I}_B$ $\hat{I}_C$ in the series coils and the voltages E_B and $- E_C$ across the shunt coils. It should be noticed that the addition of S^0 in any quantity to the e. m. f. will not affect the mean power, and therefore we may impress the voltage to the lines from any convenient point without changing the mean power. Graphically this is shown by the fact that the S^0 power product determines the mean power and the centroid of the triangles defining the amount of unbalance. Displacing the neutral point merely adds S' and S^2 products to the power and these represent equilateral triangles having their centroids coinciding with the end of the mean power factor.

SINGLE-PHASE GENERATOR

In the single-phase generator we have a generated symmetrical e. m. f. $\check{E}_0$ and at the terminals e. m. fs. $S_1 \check{E}_{A1}$ and $S^2 \check{E}_{A2}$. Furthermore if $S^1 I_{A1}$ be the positive phase sequence current $- S^2 I_{A1}$ will be the negative phase sequence current and therefore

$$S^1 (\check{E}_0 - Z_1 \hat{I}_{A1}) = S^1 \check{E}_{A1}$$

$$S^2 \check{E}_{A2} = S^2 Z_2 \hat{I}_{A1}$$

Then

$$S^1 \check{E}_{A1} + S^2 \check{E}_{A2} = S^1 (\check{E}_0 - Z_1 \hat{I}_{A1}) + S^2 Z_2 \hat{I}_{A1}$$

$$\begin{aligned} S\,(P_A + J\,Q_A) &= S^0 (\check{E}_0 - Z_1 \hat{I}_{A1})\, \hat{I}_{A1} \\ &\quad - S^0 (Z_2 I_{A1}^2) - S^2 (\check{E}_0 \\ &\quad - Z_1 \hat{I}_{A1})\, \hat{I}_{A1} + S^1 (Z_2 I_{A1}^2) \end{aligned}$$

The mean output per phase $P + j\,Q$ is therefore $S^0 (\check{E}_0 - Z_1 \hat{I}_{A1})\, \hat{I}_{A1} - S^0 (Z_2 I_{A1}^2)$ so that the addition of the vectors $- S^2 (\check{E}_0 - Z_1 \hat{I}_{A1})\, \hat{I}_{A1} + S^1 (Z_2 I_{A1}^2)$ makes the power in the A phase zero as it should be.

The vector diagram may be drawn as follows: Draw from the origin O the line $O\,O'$ equal to $P + j\,Q$. The value of I_{A1} is known it is $P + j\,Q$ divided by the Y voltage corresponding to the value of the single-phase terminal voltage. Through the point o' draw the vectors $S^1 I_{A1}^2 Z_2$ giving $O'\,A''$ $O'\,B''$ $O'\,C''$ in positive phase sequence relation. Draw $O'\,A'$ parallel and equal to $A''\,O$, $O'\,A'$ will then be the principal vector of the system $- S^2 (\check{E}_0 - Z_1 \hat{I}_{A1})\, \hat{I}_{A1}$. Complete the system by the lines $O'\,B'$ $O'\,c'$ in negative sequence relation with $o'\,A'$. The resultant of the systems $S'\,I_{A1}^2 Z_2$ and $- S^2 (\check{E}_0 - Z_1 \hat{I}_{A1})\, \hat{I}_{A1}$ represented respectively by the equilateral triangles $A''\,B''\,C''$ and $A'\,B'\,C'$ combined give the triangle $A\,B\,C$ of which the apex A lies on the point O and the lines $O\,B$ and $O\,C$ represent the power delivered by the phases B and C and these vectors are also proportional to the voltage to neutral of these two phases under load. The datum of $P + j\,Q$ represents also the phase of $\hat{I}_B$ and $\hat{I}_C$ in drawing the diagram. Fig. 6.

Discussion

R. D. Evans: The proper method of making measurements on polyphase circuits has been the subject of considerable discussion within the last few years. With balanced circuits, the methods were rather generally accepted, but this was not the case with unbalanced circuits. It is recognized that if the definitions and methods of measurement for polyphase circuits were to have any assured standing, they should cover the unbalanced circuit. Unbalanced polyphase circuits were not well understood, and consequently there has been a lag in the development of corresponding measuring systems.

A method for analyzing unbalanced polyphase circuits was presented before the Institute by C. L. Fortescue in his paper on "Symmetrical Coordinates". Subsequently, it is found that the various quantities used in the analysis of the unbalanced circuit could be measured. In the present paper, Mr. Fortescue has applied this method to the question of the measurement of power for the general case of the polyphase circuit, viz., the unbalanced circuit. Three conclusions of this study may be stated as follows: 1. The present basis for the energy charge is not fundamentally correct, and a new basis is proposed. 2. Power factor should be defined as being distinct from unbalance. 3. The use of an unbalance factor is desirable in metering polyphase circuits when there is appreciable unbalance.

That the present basis for the energy charge for electrical power is open to criticism may be surprising, the explanation however, lies in the fact that the special, rather than the general case of a polyphase circuit has hitherto been given consideration. Mr. Fortescue has pointed out that a customer with rotating machinery on the same line with a customer drawing a large single-phase load such as an electric furnace load, will have an energy charge in excess of that which would occur if the loads were balanced. Instead of employing the present basis for the energy charge, it would appear more equitable to charge the furnace customer with the energy which he consumes and also with the energy which he causes to be dissipated in the rotating machinery, and at the same time, not to charge the rotating machinery customer with the energy dissipated in the damper windings of his machines, due to unbalance. Mr. Fortescue proposes to do this very thing by basing the energy charge on what he terms "positive sequence power". This charge is in reality based on a component of the total power in the system, which quantity may be measured by a wattmeter of the ordinary type with suitable supplementary net work. As has been pointed out, this method would not appreciably modify the aggregate energy charge, but would distribute it more equitably. The success of the system proposed, will depend in a large measure on the reliability, simplicity and cost of the necessary metering equipment. Some of these metering devices will be described and their operation will be explained in a general way.

The method proposed by Mr. Fortescue for analyzing unbalanced circuits is based on the fact that any unbalanced system may be resolved into two or more balanced or symmetrical systems. For example, in a three-phase three-wire circuit an unbalanced system of current may be resolved into two symmetrical three-phase systems, in which the currents of one system reach their maxima in the different phases in a positive sequence as A, B, C, and the currents of the other system reach their maxima in the negative sequence C, B, A,. The method used here of resolving currents into components, positive and negative sequence, is similar to the method now in general use for resolving currents into components in phase or active, and out of phase or reactive. The reason for the resolution of the currents into components for the two cases are the same, viz., that the different components have different effects, thus simplifying the solution of the problem. Each of these phase sequence quantities if present on a polyphase circuit may be measured. The method of separating out these components is accomplished by the use of suitable supplementary net works. In general, the method for measuring these phase sequence quantities is based upon a separation of positive and negative sequence components of voltages and currents, which are used separately for the measurement of sequence voltages or currents, or which are used in combination for the measure of sequence watts, reactive watts, power factor or unbalance. The method may be described for a positive sequence device as involving a meter element in connection with a net work such that positive sequence voltage or current will cause current flow through the meter element and negative sequence voltages or currents will not. Similarly, the negative sequence device consists of a meter element in connection with a net work such that negative sequence voltage or current will cause current to flow through the meter element and positive sequence voltages or currents will not. A detailed explanation of the operation of some of the simpler forms of phase sequence devices was given before the Institute by the speaker in a paper relating to the definition of power factor on unbalanced circuits. The *Electrical World* has recently published an explanation of the operation of the sequence devices.

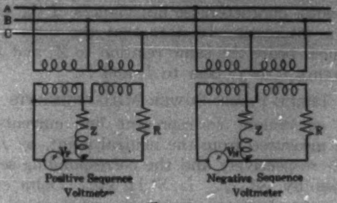


FIG. 1

The speaker is however, interested principally in pointing out the type of measuring devices required for these phase sequence quantities. For simplicity, only devices for the three-phase, three-wire system will be shown. In Figure 1, the sequence voltmeter devices are shown diagramatically. The meter on the left measures positive sequence voltage and consists of a meter element, a resistor and a reactor which constitute the supplementary net-work, and the potential transformers. The negative sequence voltmeter is shown on the right and the same apparatus is involved only a difference in connection is required. In Figure 2, the sequence current devices are shown diagrammatically. The positive sequence ammeter is shown on the left with ammeter element, resistor and reactor, which constitute the supplementary net work and the current transformers. If both positive and negative sequence voltages of a circuit are desired, they may be obtained from one set of potential transformers as shown in Figure 3. The corresponding devices for measuring positive and negative sequence current are shown in Figure 4. The schemes shown in Figs. 1 to 4 inclusive indicate how positive and negative se-

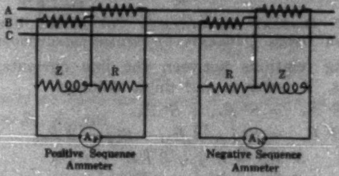
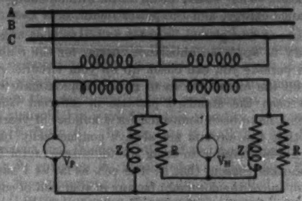


FIG. 2

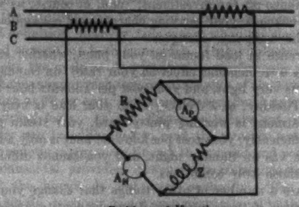
quence voltages may be separated and measured independently. These net work arrangements may be used to separate the sequence voltages and currents which may be combined to measure for example, positive sequence watts. In Figure 5 is shown positive and negative sequence wattmeters. The positive sequence wattmeter is shown on the left with negative sequence wattmeter on the right. *R* and *Z* represent resistors and reactors which consitute the supplementary net work.

With reference to the sequence devices in general, the speaker wishes to emphasize that standard types of meter elements or relays and standard types of instrument transformers may be employed. The supplementary net work consists only of resistors and reactors. By these means, reliability and accuracy are insured. It is to be pointed out that the form of net work restraint here employed is of a type permitting higher accuracy, as reactors without resistance or resistors without reactance are not required.



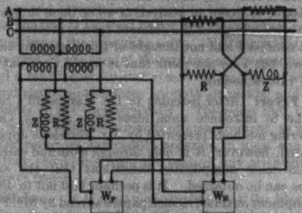
Positive and Negative Sequence Voltmeters

Fig. 3



Positive and Negative Sequence Ammeters

Fig. 4



Positive and Negative Sequence Wattmeters

Fig. 5

With the proposed sequence system, there will result either more complete indication of circuit conditions or fewer metering elements. With the sequence measuring system, there are particular advantages resulting from the use of the quantities thus made available. For example, Mr. Fortescue has proposed that the energy charge be measured by the positive sequence wattmeter. Another example is the application of the negative sequence current device as a relay to provide protection against single-phase operation of motors.

In conclusion, the speaker wishes to emphasize the simplicity of the sequence measuring system. In considering the application of the proposed system of measuring devices, there are two cases to be considered. First, with no unbalance present, in which case, the proposed system and the present systems give the same result, and the proposed system has the advantage of employing fewer metering elements than used with the present system. Second, with unbalance present, in which case, the two systems will give different results, and it is probably desirable to give special consideration to unbalance, which is best measured by the system proposed.

V. Karapetoff: I fully realize the importance of the principle of resolution of an unbalanced polyphase system of currents or voltages into two balanced or symmetrical systems of opposite rotation; and this method has been and will be in the future of inestimable value in numerous problems. However, when it comes to a proposal to use this system for charging customers for energy consumed or even for the maximum demand principle, I believe there will be difficulties which are well-nigh insurmountable. The very necessity of convincing regulating bodies, public service commissions, of the soundness of this principle would be a big job. Besides, the average consumer will have to understand what is meant by two systems of opposite rotation, and that the energy of each system has to be charged for, and how much he ought to be charged.

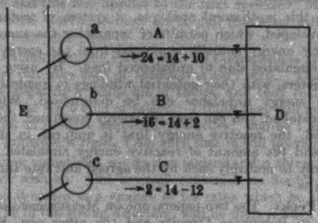


Fig. 6

This matter has been presented and discussed at length at one of the preceding Annual Meetings, and as a result of that discussion, I tried to evolve a different principle of metering, perhaps not so sound theoretically, but one that may be much more easily understood by an average consumer of energy, and by an average member of a public service commission. I shall illustrate this principle by means of a hydraulic analogue.

The three horizontal lines *A*, *B*, *C*, in Fig. 6 represent pipes through which water is being delivered to a common reservori *D* at the following rates: 24 gallons per minute in *A*; 16 gallons per minute in *B*, and 2 gallons per minute over the pipe *C*. The pumps are denoted *a*, *b*, *c*, and the common intake is *E*.

Here is an analogue of an unbalanced polyphase system. The consumer needs 24+16+2=42 gallons per minute; but instead of opening the three valves uniformly, the operator opens one valve wide, another less wide, and the third one just a little until he gets a total of 42 gallons per minute. It is an easy problem to explain to the manager that it makes a considerable difference from the point of view of the pump and pipe equipment whether 42/3=14 gallons per minute is delivered through all the three pipes, or 24, 16 and 2 respectively. We surely will not say, "What difference does it make to you as long as you deliver 42 gallons per minute." We, as engineers, feel that the consumer of that careless type ought to be penalized.

Now, the question is how much should a consumer be penalized who opens one valve wide, and another a little, as compared to what he would pay had he opened the three valves to the same extent? This, and not two opposite vector rotations, is a right kind of approach to the man to whom we sell electricity, and to those who govern the rates at which it is sold. In the figure, 24 can be replaced by 14 plus 10; 16 can be replaced by 14 plus 2; and 2 can be replaced by 14 minus 12. That is, in each pipe we resolve the actual discharge in each pipe into the uniform flow 14, plus or minus the excess or deficiency as compared to that flow. We can explain to the consumer that what he does is this: He takes 14 gallons per minute through each pipe; that makes 42. In addition to this, he creates a *circulation of water*. This circulating water flows at the rate of 10 gallons through A, 2 gallons through B, with a common return through C. If the pumps a, b, c, are good only for 14 gallons per min., then additional pumps have to be installed because of the unbalance. Of course, these pumps will merely churn water, without lifting it to his reservoir, but any practical man will agree that if in addition to taking water at a uniform rate, he forces the producer to install pumps to circulate water, he has to pay for it. He can also see that the rate for this additional unbalanced water should be different from the rate which water is delivered to his reservoir for consumption.

From this mechanical analogue, a system of metering has been developed, which permits of separating the average flow of electric energy actually consumed from the energy that is merely circulated due to unbalanced load. By using another set of meters, with their potential windings in quadrature, the reactive energy unbalance can be determined. With such meters installed, you can read the energy consumed, the energy circulated, the reactive energy that is uniform, in the three phases and the amount of reactive energy circulated. It remains only to multiply each by the agreed unit rate and to add the results.

E. P. Peck: The two papers, one on Measurement of Power Balance in Polyphase Circuits and the other one, Kilovolt-ampere Demand Measurement, are very distinct technically, but in their application they merge, because they both work back into the question of measuring something that you are going to sell and the determination and application of a rate for what you sell.

The rate, to be workable, must be understandable, and it should be simple. As brought out in one of the papers, there were 418 rates investigated; 59 of them had power factor clauses, and a relatively few of those 59 power clauses were actually enforced in the collection of the bills.

We made an investigation somewhat similar to this in which we found that a good many companies were not putting in power factor rates, and very few of the companies that have power factor rates were enforcing them. Apparently this is for the reason that a business man or a manufacturer could not be made to understand what power factor was. Horse power means something to him, but even the terms "kilowatt" and "kilowatt hour" give him considerable trouble.

One of the men in the Company came down to my office a few months ago, and he said: "I have been with the Company for years and years and I have heard of power factor for years and years. Now, can you tell me in about five minutes what power factor is?" I talked about fifteen minutes, and he said he understood what power factor was in a rather general way.

You cannot, in general, use a factor in your rate which cannot be understood by a man who is paying money for his service; he must understand what he is paying money for.

For that reason, the more recent proposal of changing to a kv-a. rate, particularly for demand, is appealing. You can explain, so that a man can understand what you are talking about, what the kv-a. is, and if he can understand and see the justice of paying on a kv-a. basis, you do not have the objection that you would have otherwise.

Now, in the proposal of including another element—the unbalance in the rate, I think, as Dr. Karapetoff just brought out, you are bringing in something that is still less understandable, and therefore still less applicable. I do not believe it could be applied at all. That would set up a rate with an energy charge, an unbalance charge, and the power factor, or a kv-a. charge.

It appears at first glance, or as far as I have been able to see, unnecessary to apply an unbalance charge, for the reason that at certain periods you can check up on your circuits, readjust the taps on the circuit, so as to get an approximate balance. That has been the standard practise in different companies with which I have been connected for a number of years. The result is that the circuits are always commercially balanced.

It appears inadvisable to bring the unbalance factor into the rate for another reason, that is, it is not, as far as I have been able to see, one of the dominant factors in your cost of supplying service, and, of course, your rate is quite largely based on the element of cost of supplying that particular customer.

The distance from the generating station to the customer has a good deal to do with your investment in transmission and distribution; the voltage that you serve your customer at has a large influence on the investment; and, of course, as your investment changes, your carrying charges, and consequently the cost to the company, changes.

A difference in load factor on your plant, whether steam or hydro, has a tremendous effect on your rate; on Sunday night your load is very light, and your cost per kilowatt hour is away up. On Tuesday, at 2 o'clock, when your load is very heavy, your investment is working pretty hard, your steam plant is working efficiently, your cost per kilowatt hour is only a fraction of what it was on Sunday night, and you cannot differentiate between those two conditions.

There are a number of other things that change your costs, not in a ratio of three-quarters of one per cent to one per cent, but change it several hundred per cent.

Since so many of those things have to be absolutely neglected, it would not seem advisable to include a factor in the rate which represented a relatively minor element of the cost. Of course it is advisable to make an analysis of all of these things, so that you have a knowledge of the different factors involved.

I must confess I had not thought of this particular element in cost before, but I do not think that it would work out well in a practical rate schedule.

R. C. Fryer: After listening to Dr. Karapetoff's remarks I would like to introduce some ideas from another viewpoint. The metering methods presented by Mr. Fortescue appear to be complicated, however, it is only through close and careful analysis of the most complicated conditions that the simplified conditions can be obtained. It is perhaps best not to discourage the development outlined in such papers as read by Mr. Fortescue but to encourage this development since the actual use of such methods will never take place until the economy of the situation so dictates.

It is quite certain that these fundamentals which are being worked out are not going to be put into use until they are tried and approved. Would it not then be better to encourage such studies knowing that they will lead to a similar analysis which may be the simplified condition so desired?

W. V. Lyon: It seems to me that Mr. Fortescue has outlined the effects of power factor and unbalanced factor both from the standpoint of the producer and the consumer of electric energy exceptionally well. There is no doubt that in time, both of these factors will be considered when metering electric energy. In commercial circuits, the effects of harmonics in the electromotive force and current may be disregarded ordinarily and then the readings of recording, active and reactive wattmeters and positive and negative sequence ammeters and wattmeters, may be used in determining equitable charges.

I am also wholly in accord with Mr Fortescue's statement in the first sentence of the appendix. It seems to me that there is need to go no farther than the next two paragraphs for evidence of this belief. It should be said that the quoted sentence had reference, not to the metering of energy in commercial circuits, but to a proper definition of power factor. The aspects of the question may be entirely different when considered from a commercial rather than from a scientific point of view. Definitions and modes of measurements that are good practise may be entirely unsuited to form the basis of scientific definitions.[1] Thus it is with the quantity $\Sigma E_k, I_k \sin \theta_k$. In commercial circuits where the electromotive force and current are sufficiently near the sinusoidal in wave form this quantity is approximately equal to $E_1 I_1 \sin \theta_1$. In this case it is all that Mr. Fortescue claims except that the description of the derivation of its value as shown in equation (6) is somewhat ambiguous. $E, I \sin \theta$, omitting the subscripts (1), is ordinarily called the reactive power. It is the maximum rate at which energy flows into the magnetic field. It is also equal to the average value of the stored magnetic energy multiplied by 2ω, where $\omega = 2\pi$ times the frequency of the electromotive force and current. If the resistance and inductance of the circuit do not vary throughout the cycle, the second of these definitions is still correct even though the electromotive force and current are non-sinusoidal. The first definition, however, fails except when the electromotive force and current are both sinusoidal. If the inductance varies throughout the cycle as does that of a synchronous motor, in which the maximum variation on either side of the average may be as much as 30 per cent, neither of these definitions is in accord with physical facts. As Mr. Fortescue points out, a reactive wattmeter can be calibrated to read the average value of the stored magnetic energy even though the current and voltage may not be sinusoidal, provided, however, the resistance and inductance of the circuit do not vary cyclically. The quantity $\Sigma E_k I_k \sin \theta_k$, however, is not even proportional to the average value of the stored magnetic energy, nor to the reading of a reactive wattmeter, except when the subscript k is limited to the value 1, that is, when the current and the electromotive force are both sinusoidal. Furthermore, the product of the electromotive force and current is not equal to the square root of the sum of the squares of the real power and the reactive power, as defined in either of the preceding ways. Neither is this product the square root of the sum of the squares of the readings of an active and a reactive wattmeter except in those cases where the current and electromotive force are both sinusoidal and the circuit constants do not vary cyclically. The point that I wish to bring out is that while the methods of measurement, as sketched by Mr. Fortescue are sufficiently accurate in most commercial practise, they should not form the basis of any scientific definition of power factor.

F. B. Silsbee: It is most unfortunate that the ideas which electrical engineers have inherited from the single-phase circuit, have become so fixed along the lines of a separate consideration of the several phases of a polyphase circuit, that a very great deal of mental inertia has to be overcome, before the simpler methods of analysis here presented can come into general use. I feel that several more papers similar to the present one will be required before the metering systems of central stations are changed over to register positive and negative sequence power.

Several years ago a committee was appointed to consider the possibilities of obtaining a definition of balance factor in polyphase systems, but the inherent complexity of the subject, the lack of realization of the importance of unbalance in some cases, and the lack of familiarity with the symmetrical coordinate method of analysis all conspired to produce a general apathy among engineers which, so far, has prevented the adoption of any definite method for treating unbalanced conditions.

In this and earlier papers Mr. Fortescue has suggested the ratio of negative sequence to positive sequence currents as a definition of unbalance, which would solve this problem. While this definition works out perfectly for the case of 3-wire, 3-phase circuits which, of course, is by far the most frequent case in practise, it seems desirable to have a more general definition of which this might be a special case, but which would cover the more complex cases of 4-wire circuits and those of other than three phases. It seems to me certain that such a generalized definition will ultimately be based upon the symmetrical method of analysis, and I trust that Mr. Fortescue will supply us with such a definition in the near future.

Charles Fortescue: I want to express my appreciation of the discussions that I have received this afternoon on my paper. Professor Karapetoff always adds lucidity to any subject which he discusses. However, I do not agree with him that the commercial application of this principle presents insurmountable obstacles, and this also applies to the next speaker. Mr. Peek seems to think that consumers are so dull that they will not be able to absorb this idea.

Now, I think that the Public Service Commissions and large consumers can take care of themselves very well. They seem to be investigating a whole lot of things, and they seem to be able to get the right kind of talent. I think they will make out pretty well on positive sequence quantities; and as I am quite sure the principle is right, I am sure that it will prevail.

You know sometimes we do not understand a thing because we do not wish to. The average consumer no doubt is quite dull when it comes to a change in his rates which he thinks is going to be in the wrong direction, and he finds it very hard to understand such things as power factor and unbalance factor.

However, if you ride on any train, you will hear them talking there glibly about detectors, multi-stage amplifiers, feed back regenerative circuits, and so forth—they seem to understand those things very thoroughly, so that I think they must be quite capable of understanding such a simple thing as unbalanced kv-a.

However, should there be any possibility that this method will not come into commercial use, nevertheless it provides a means to enable the rate man to arrange his rates so that they are more equitable; so that no doubt you will find it very convenient to make use of positive sequence devices in the laboratory, and in investigating various kinds of loads.

I believe that there is a real application of these principles, and as Mr. Evans has shown, the principle can be applied with standard instruments, commercial instruments that are used every day.

It does not add any complication; in fact, the tendency is towards simplicity, and moreover, certain conditions do occur from time to time in circuits which demand protection for motors, and this principle affords the best means to give that protection.

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[1] Trans. A. I. E. E., Vol. 39, p. 1515.

Volt-Ampere Meters*

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THE point is well known and generally accepted that many things can be done which do not become general practise until the economics governing the practise show that the difference between "cost of doing" and "earnings through doing" is on the proper side of the balance sheet.

The engineer, being a technician, may be a little slow to admit, even to himself, this fundamental; yet engineering development, in terms of actual progress for the good of all concerned, is as fully subject to the immutable law based on this fundamental as is human progress in general subject to other economic and psychological laws.

We in the engineering profession perhaps now, more than ever before, are less ready to admit that any task is impossible, yet we will have to admit that in general our research and development follow the cause of the necessity rather than precede it. This in some respects is as it should be, yet the question arises "Would not this necessity be more often clearly recognized and the solution more promptly offered if it were known that means were at hand for meeting the situation or condition brought about by that necessity?"

The first recognition of the necessity of metering or measuring kilovolt-amperes of alternating currents came through experience to manufacturers of electrical machinery. Many who do not consider themselves as aged can remember when it was common to purchase generators, motors, transformers etc. rated in kilowatts. Due either to the fact that the above machines were often required to carry inductive loads or to the fact that under these conditions controversies regarding the capacity ratings of electrical apparatus often arose the manufacturers took a wise step and changed their a-c. apparatus rating from a kilowatt to a kilovolt-ampere basis.

At the time this took place the central station industry was carrying a load in which highly inductive apparatus was the exception rather than the rule, and to a large number of central station engineers the full significance of the change was not at once apparent.

During the few years following, which really constituted the "youth" of the central station industry, the effort of the central station was to secure more load, and in the majority of cases very little thought was given to the character of the load with the exception perhaps, of its load factor. Following these years, however, some of the more progressive central stations carrying industrial loads realized that they were purchasing electrical equipment rated upon a kv-a. basis and apportioning the use of same upon a kw. basis. It became apparent that increased investment was necessary to carry an inductive load over the investment necessary to carry an equivalent non-inductive load. This not having yet become a matter of much importance to many central stations it did not attract general interest and the result was that where steps were taken to remedy conditions, such action was local in application and generally took the form of penalization of the customer operating an inductive load. Being largely local the methods and degree of penalization were varied and largely arbitrary. Perhaps we can not be sure, but it seems that, while these attempts were a step in the right direction, the "penalization idea" was not very constructive. Perhaps these attempts secured a better income on the demand charge of customers operating inductive loads, yet no basic attempt was made by manufacturers in general or by central stations to remove the infection, the result being that with each additional industrial load the condition became worse.

About the beginning of the World War many central stations generated a peak load approaching their kv-a. capacity. Finding it difficult to carry their kilowatt load without exceeding their kv-a. capacity these stations were faced with the necessity of not being able to care for all load offered or of having to enlarge generating and distributing capacity at a time when the capacity was badly needed and yet at a time when capital for same was very difficult to obtain.

This condition stimulated study of the "low power factor" or inductive load question and it was generally admitted that some action was necessary.

Study of the situation brought out the fact that about 75 to 80 per cent of the central station investment lay in the generating and distributing apparatus. It was pointed out that the carrying capacity of the apparatus covered by this portion of the investment was directly limited by the kilovolt-ampere load. From this it became apparent that while no unit of measurement defined central station capacity in terms of 100 per cent investment the unit of capacity applying directly to the greater percentage of the investment was and is the kilovolt-ampere.

Many utilities had given a power factor clause trial in their rates, but finding that this complicated their contracts; finding that many customers could not or did not understand its application; finding that most customers did not see the justice of that which they termed "penalization for low power factor;" finding they were unable to directly meter the factors involved and that the human element largely entered into calculation methods and finding that the return given a customer under "power factor correction clauses" was

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not a direct ratio to his investment in necessary correction apparatus, many utilities allowed the "power factor clause" to become a "dead letter" in the contract.

Results of the attempts cited above are given space in the *T*-2-'22 report of the Subcommittee on Power Factor and kv-a. of the National Electric Light Association Meter Committee, which states that of 418 utilities in cities of 20,000 and over in the United States and Canada, 59 show clauses relating to charges in billing low power factored customers. Among these 59 companies there are more than 20 different clauses.

During the last two years this condition of affairs has added interest to the development of

(1) Induction motors to operate at better power factors.

(2) Corrective apparatus (static and synchronous condensers).

(3) Kilovolt-ampere meters.

This has led to more study in proportioning the size of the motor to the load carried, and in obtaining both induction and synchronous motors of the proper voltage rating.

Even isolated stations generating and using their own power have found it economical to employ synchronous apparatus.

The writer recently witnessed a 20,000-kw. load in a large rubber mill near Akron, Ohio, operating at 98 per cent power factor. This was secured largely through the unusual practise of using synchronous motors specially designed for operation on rubber mill drive.

It is of course understood that at unity power factor the ordinary induction watt or watt-hour meter will measure volt-amperes or volt-ampere-hours.

It follows that if both the current and voltage fluxes in the meter be held at their proper value and in proper phase relation the meter will measure volt-ampere hours for any power factor, leading or lagging.

Your attention is also called to the fact that if the in-phase and reactive components of a circuit are properly metered the volt-amperes may be known through calculation.

In general, then, any volt ampere meter or volt ampere hour meter must, of necessity, obtain its operation from one or both the above principles.

I desire therefore, to more tersely cover the present state of volt-ampere metering art to divide the types and makes of meters as follows:

(1) Proper phase relation type meters.	**Full Range**	**The Bodi Meter (Westinghouse)** **The Angus Meter (Esterline-Angus)**
	Fixed Range	**General Electric Meter** **The Lincoln Meter**
(2) Resultant type meters	**Full Range Indicating**	**The Lincoln Meter** **The Sangamo Meter**
	Full Range Integrating	**The Sperti-Blackmith Meter (Westinghouse)**

In the above outline only meters involving the *principle* and not the practise are mentioned. Other meters involving the above principles may now be made or may be made in the future.

The Bodi (Westinghouse) meter involves the principle of operation of the polyphase power-factor meter. It has been found that moving coils carrying the current of the circuit measured (or a portion of same) or carrying the potential of the circuit when suspended in the potential field (in one case), or in the field of the current (in the second case) will take such positions that current taken from them may be in phase with the field in which they are placed.

If then such an arrangement of coils is provided the current from it may be led through the current element of a watt or watt-hour meter and if this current is of the proper value the watt or watt-hour meter is caused to register volt amperes or volt-ampere hours. To supply such a current to the potential element of the meter involves some difficulty since the torque tending to change the position of the movable coils with a slight change in power factor is rather small. More torque for such an arrangement may be had by connecting these movable coils in series and thus supplying current of proper phase relation to the current element of the standard watt or watt-hour meter. The difficulty encountered with this arrangement however is that the volt-ampere load of such an arrangement is rather above the secondary volt-ampere rating of standard current transformers. It must be remembered however, that with this arrangement the standard induction watt or watt-hour meter may be used.

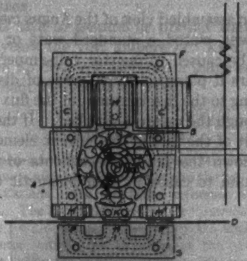

Fig. 1—One Element of the Angus 3-Phase Meter

The Angus meter employs somewhat the same principle with a slightly different arrangement. Instead of using the standard watt or watt-hour meter a special meter comprising in the case of three-phase circuits two elements (one of which is shown in Fig. 1) are used. The frame *F* is magnetized by the current coils *C* and *C*1. The potential element *A* is so mounted that it may rotate, changing its position with variation in power factor of the circuit. The polyphase winding on potential element *A* produces a rotating field about its periphery which reacts against and with the current flux, a portion of which cuts the disk D at the points $K\,N$. This reaction causes the potential element, therefore, to assume a position such that its resultant flux is in phase with the field produced by the coils $C\,C\,I$, therefore, the torque on the disk is proportional to volt amperes. With a rotating disk and a register the meter becomes a volt-ampere-hour meter, since the usual type of full-load adjustment can be supplied.

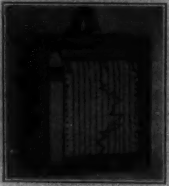

FIG. 2—ANGUS METER

Fig. 2 is an assembled view of the Angus meter.

As has been stated a standard watt or watt-hour meter may be caused to register volt-amperes or volt-ampere hours if potential of a degree of displacement corresponding to the displacement of the flux due to the current through the meter is supplied. If the degree of displacement of the flux in the current elements of the meter is known the potential elements of the meter may be wound to cause in their magnetic circuit the same degree of displacement. Under such conditions the meter will measure volt-amperes or volt-ampere hours over about 22 degrees angular range with acceptably good accuracy. The General Electric kv-a. hour meter of this type is shown in Fig. 4. Since the meter is not a standard watt-hour meter its range is increased by the addition of another shaft and necessary special potential coils on the required elements.

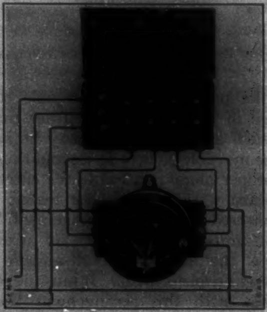

FIG. 3—KILOVOLT-AMPERE-HOUR METER

Both shafts are connected to one register through an overrunning clutch in such a way that the shaft which is running fastest is carrying the register. Such an arrangement practically doubles the range of the single shaft meter.

To change the range of operation beyond the 22 degrees angular range for which one set of elements is built it is necessary to add another rotor and elements or change the windings of the potential elements in use.

In Fig. 3 is shown the Lincoln type $V\,A\,D$ transformer and the Lincoln thermal, volt ampere demand meter. This is a fixed range instrument, but unlike the instrument just previously described the displacement of the potential flux does not take place in the meter itself, but is secured by picking up the desired taps supplied on the small auto transformer provided for the meter. The range of the meter is thereby increased since the change necessary in widely varying power factors can be more readily made, although it must be remembered that the change is to be manually accomplished.

FIG. 4—3-ELEMENT AMPERE HOUR METER

Mr. Paul M. Lincoln has also worked out other ideas in volt-ampere measurement which have been proved, but which are not on the market. It is understood that patents on these are held by the Westinghouse Company for the United States. In one of these meters Mr. Lincoln provides for the movement of two arms—one proportionate to kilowatts and the other proportionate to reactive volt-amperes.

These two arms are attached to a common point, the resultant movement of which is proportionate to the square root of the sum of the squares of the movements of the kw. and r. kv-a. arms, therefore, the movement of the common point is closely proportionate to volt-amperes.

In one arrangement Mr. Lincoln carries the resultant motion to a rotating member engaging a disk rotating at a constant speed. The value of the resultant determines the distance from center at which the rotating member engages the disk, thus the volt-ampere value is integrated. If this were a practical arrangement it would seem to be ideal, since he obtains a value proportional to the integrated square root of the sum of the squares.

The Sangamo Company is producing in Canada its watt-hour meter combined with the Lincoln volt-ampere demand meter as described for use with phase shifting transformer. Combining as this instrument does a registration of kilowatt-hours and volt-ampere demand it is said to be meeting with hearty approval in Canada. Last years report shows some 1500 volt-ampere-demand meters in use in Canada.

FIG. 5—KV-A DEMAND AND KILOWATT HOUR METER

Possibly the latest volt-ampere-demand meter on the market is the Sperti-Blecksmith meter being manufactured by the Westinghouse Company. The actual construction of this meter is shown in Fig. 5. As shown the meter is designed to register kilowatt-hours and 15 minute block interval volt-ampere-demand. The meter uses standard watt-hour current and potential elements, —no special windings being required.

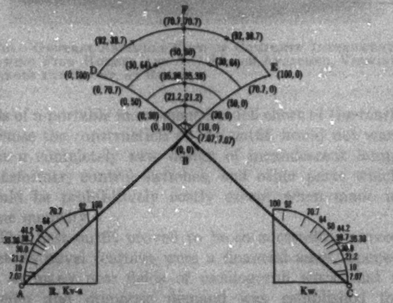

FIG. 6—DIAGRAM SHOWING ACCURACY OF KV-A. METER

The rotation of the two right disk sare proportional to kilowatt-hour and the rotation of the two left disks are proportional to r. kv-a-h. The shafts are connected to two arms operating a pantograph, the centre point of which is connected through a flexible lead to the kv-a. demand indicator which indicates the resultant value of the in phase and reactive components—*i. e.* the square root of the integrated kw. and r. kv-a. for the interval of time for which the meter may be designed. Further details of operation are shown in Fig. 6.

The reactive meter is so connected to the pantograph that its integrated value is constant in direction for all ranges of power factor, leading or lagging; therefore the standard meter elements are operative over 100 per cent range in power factor. The inherent errors are small, and being constant in progression have easily been cancelled so that the meter compares in accuracy well with the standard watt-hour meter.

It is seen from the foregoing that quantity production on volt-ampere, volt-ampere-hour or volt-ampere-demand meters merely awaits a consistent demand.

Several large utilities have made, are making and are arranging to make contracts on a volt-ampere-demand basis.

It seems safe to assume or predict that during the next few years the volt ampere or "kv-a." meter will have taken its deserved place on a plane equal at least to the present kw. demand meter.

Discussion

P. A. Borden: I am pleased to see Mr. Fryer's reference to the advances which have been made in the use of volt-ampere meters in Canada, with particular reference to the Lincoln instrument. The Hydro-Electric Power Commission of Ontario, with which I am connected, is probably the largest user of the Lincoln meter, either with or without the so-called V.A.D. transformer. The Commission and the 300 municipalities to which it supplies power for resale use a large percentage of the number of these meters manufactured in Canada, and I may say that both those who sell and those who buy power as metered by this method seem satisfied with the principle as well as with its practical working out.

The combination meter, to which Mr. Fryer has referred as being developed by the Sangamo Company in conjunction with the Lincoln Company is, to date, I believe, a wholly Canadian product. A sample of this device is at present undergoing tests in the Commission's Laboratories. It combines a polyphase watthour meter and a Lincoln demand element, phase shifting for the latter being arranged for three different ranges of power factors, covering everything from about 40 per cent to unity with a theoretical error of less than 2 per cent. In this meter no attempt is made to integrate the volt-amperes. The integration is performed solely on a watt basis; while demand may be determined on either a watt or a volt-ampere basis, as desired.

A brief description of an ingenious volt-ampere meter developed some years ago by Mr. H. S. Baker may here be of interest. In some points this meter is not unlike the present volt-ampere meter produced by the Esterline Company, but it was in regular use some years before anything was published in regard to the latter instrument. Referring to Fig. 1, (which for purposes of clearness is shown as for a two-phase circuit,) we have first an element, consisting of the fixed coils, 1,1, and 2,2, together with the movable coils 3 and 4, thus constituting a power-factor meter. The rotatable element is normally in neutral equilibrium, and, consisting, as it does, of a polyphase winding in a polyphase field, will tend to take a position about its axis, representing the power-factor of the circuit, according to the "Second Definition" of the Institute.

The moving part, however, has another function. It will be noted that the equatorial plane of this part does not coincide with that of the fixed part, and that the moving part is free, not only to rotate about its axis but to shift in the direction of that axis. It may easily be worked out that, the element having rotated into the position which represents the power factor of the load, the vertical thrust on the shaft will represent the value of the volt-amperes.

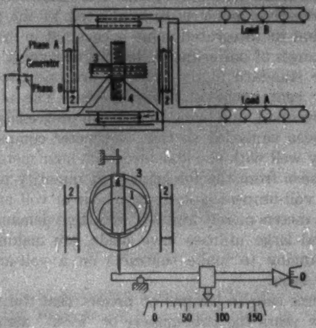

FIG. 1

As shown in the diagram, this thrust is balanced by a counterweight on a scale beam. In actual practise the beam carries contacts, which, through a relay mechanism, similar to that of the Westinghouse graphic instrument, produce an indication on a scale or chart. At the same time, by attaching a pointer to the moving element it is possible to directly read the power factor on a horizontal scale concentric with its axis. And if the moving element be locked from rotating, in the position corresponding to any chosen power factor, the instrument reads volt-amperes at that power factor; so that if it be locked in the unity position the device becomes a wattmeter. This scheme has been in use on the circuits of the Ontario Power Company since 1917, measuring some very important loads, and has proved of great value.

Charles L. Fortescue: An accurate volt-ampere meter is very necessary for the actual determination of unbalanced factor; and I have been watching this development of volt-ampere meters with a great deal of appreciation.

I think that the balance method of measuring power, the balance factor will require a simpler device than volt-amperes on a polyphase circuit, as measured in the ordinary way, so that ought to help a little bit.

R. C. Fryer: I think the conclusions that might be drawn are these: That considerable progress has been made in the solving of the relation of power factor to the rate question. That considerable progress is being made towards the correction of poor power factors, and that the metering engineers in the industry stand prepared to offer practically any device that is desired for the measurement of volt amperes, whether it is taken by the first or second definition. The actual placing of the apparatus in use is now taking place. Several companies are actually using kv-a. meters in their billing department. We will have to recognize the fact, of course, that the practise will not, for a long time, become general. The reason I think is this: That a great many contracts are made on a five or ten-year basis, especially for the interchange of power. The result is that these contracts must run their life, before they can be changed, and those that are in a position to change contracts now are so doing. Canada, it seems for some reason, has been a little bit more forward than the United States, in this direction at least.

Expansion of Oscillography by the Portable Instrument

BY J. W. LEGG
Associate, A. I. E. E.
Westinghouse Electric & Mfg. Co.

THE oscillograph has emerged from the obscurity of scientific laboratories and from the test departments of great electrical industries so that today it is occasionally seen on a rapidly accelerating elevated train, in a soaring air-plane, in a destroyer dashing through a rough sea, on the polished top of a conference table, and, in each case, operating as perfectly as though it were in a well-equipped laboratory.

In order that the oscillograph may act properly in these new surroundings and respond favorably to the touch of new masters, who have not received extensive training in the art, it was necessary not only that the outfit be transformed into a portable instrument but also that it should be more complete in its equipment, more permanent in its adjustments, more reliable in its results and quite independent of power supply.

The outfit described in the JOURNAL of July 1920 was the first big advance made towards a portable, *commercial oscillograph.* This outfit contained the essentials of a portable instrument but fell short of the mark because the construction of one outfit would not warrant a completely new design of incandescent lamp, transformer, control switches, and other parts which would be prohibitively costly except when made in some quantity.

FIG.—GENERAL ASSEMBLY VIEW OF COMPLETE INSTRUMENT, SHOWING FILM HOLDER AND A-C. MOTOR ATTACHED, VIEWING MIRRORS AND SHUNT, AT RIGHT

This first outfit proved to be so successful, proved that its novel features were a financial asset, opened up so many new fields of oscillograph work that a foreign and domestic demand was established for additional outfits. This made it possible to re-design the oscillograph along the same general principles, but by specially designing nearly every part, it was possible to reduce the total bulk fifty per cent. At the same time new features were added, refinements were made, the case was constructed wholly of micarta, and the outfit became an instrument.

This instrument is *complete in one unit,* 11 inches wide, 11½ inches high and 25 inches long, except for the film holder and driving motor. See Fig. 1, and Fig. 2. The optical system and the vibrating system, of the three-elements, are no different in principle from that of predecessors using the instantly responding moving-coil galvanometer.

The internal equipment is so complete that the oscillograph may be used to study the recurrent or transient phenomena of almost any commercial or laboratory circuit without additional external resistors. The exception to this is the 3000-volt d-c. railway circuits, those operated from high-voltage rectifiers, etc. In such cases a water-tube resistance has been used up to over 100,000 volts.

Like the first portable outfit described, this instrument utilizes the great intrinsic brilliancy of an *incandescent filament on momentary abnormal voltage.* This intense light is focused on the tiny vibrating mirrors, of the three elements of the galvanometer, so that the reflected beams may give three instantaneous records of current, or voltage, on a photographic film passing at right-angles to the vibrating beams. The shutter mechanism, which gives but a single revolution exposure to the film, is mechanical in its action, and in closing knocks open the lamp switch, thus preventing the early destruction of the filament.

FIG. 2—ELEMENT-CONTROL AND LAMP-HOUSE SIDE OF OSCILLOGRAPH

As was the case with the first outfit, this instrument is designed to be supplied from any lighting or power circuit of 110 or 220 volts at 25, 50 or 60 cycles.

An additional feature of this instrument is that it is designed to *operate from any standard 6-volt storage battery,* such as is used in automobile starting and lighting, by the substitution of a small shunt motor in place of the back-geared induction motor regularly supplied. The more constant speed of the a-c. motor and the more constant potential supplied to the lamp from the secondary of the transformer, make the a-c. operation more desirable. The 6-volt battery operation is used

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Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

only where no a-c. is available as would be the case with a test on a moving electric car.

The operator may quickly change from correct connections for a-c. operation of lamp, trip-magnet, motor, etc. to 6 volt d-c. operation.

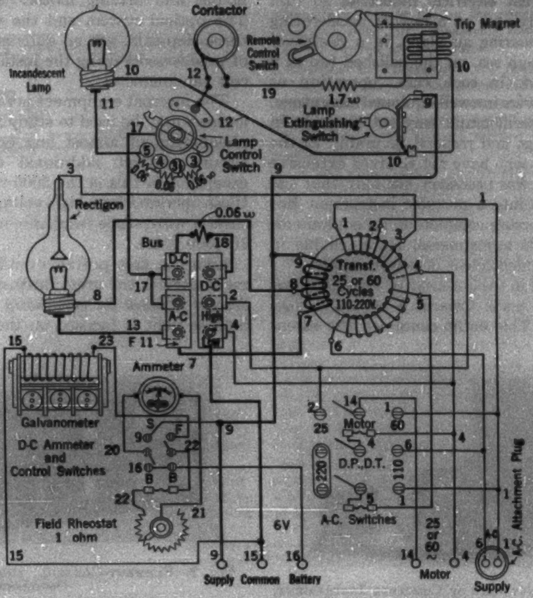
Contactor
Trip Magnet
Remote Control Switch
Incandescent Lamp
Lamp Extinguishing Switch
1.7 ω
Lamp Control Switch
0.06
0.06
0.06 ω
Rectigon
0.06 ω
Bus
D-C
D-C
A-C
High
Low
F 11
Transf. 25 or 60 Cycles 110-220V.
Ammeter
Galvanometer
S
F
B
B
D-C Ammeter and Control Switches
Field Rheostat 1 ohm
6V
Motor
25
60
D.P., D.T.
220
110
A.C. Switches
25 or 60 ~
A.C. Attachment Plug
A.C.
Supply
Common
Battery
Motor
Supply

Fig. 3—Diagram of Supply Circuit for Lamp, Motor, etc.

A complete *diagram of circuits* and a key for operating the same is moulded into the inside surface of the element-control panel-cover. This is shown in Fig. 3.

The three complete sets of *element resistors* and dial switches (Fig. 2) are extremely convenient for adjusting the elements so as to give any desired deflection. The upper dial, of each set, has a range from 0 to 100 ohms for e. m. f. below 10 volts and for use as a deflection adjuster when the element is used, with a shunt, as an instantaneous ammeter. The lower dial is used to limit the deflection when it is desired to record the instantaneous values of e. m. f. above 10 volts. This dial has a range from 100 to 10,000 ohms. This permits the recording of a d-c. e. m. f. of 3000 volts, peak. Since the sensitivity of the standard element is 0.10 ampere per inch deflection, this would mean a 3 inch deflection on the ground glass, or on the film. The *super-sensitive element* requires but 0.02 ampere per inch deflection and hence will give about 3 inches deflection, with 10,000 ohms series resistance, on 600 volts, d-c. This element will give nearly 3 inches deflection on a 100-millivolt shunt. This makes it possible to use a standard meter-shunt, when taking slow films, and thus obviates carrying a 500 millivolt oscillograph shunt for tests on huge reversing motors, etc. The super-sensitive element will not respond to as high frequencies as the standard element. The former has a natural period in air of approximately 3000 cycles per second, while the standard element has a natural period of approximately 5000 cycles, at usual tension and sensitivity. The ultimate tensile strength of the vibrator ribbons is very high (approximately 160,000 pounds per sq. inch) and hence permits a greater tension on the ribbons when it is imperative to have a higher natural period, at a sacrifice in sensitivity.

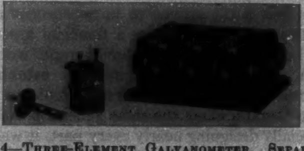

Fig. 4—Three-Element Galvanometer. Separate Element and Cell to One Side

The refinements in the *three element galvanometer*, Fig. 4 have made it most reliable. The overall dimensions have been reduced and the test increased to over *10,000 volts between each element*. The increase in sensitivity is not due to sacrifice in natural period but to the efficient design of the magnetic system.

The novel method of controlling transients, on the first portable outfit, proved to be so reliable that it was incorporated in the final instrument. Further advantage was taken of this method by designing a daylight-loading film-holder. Such would have been useless with former controls, which do not place the transient on a predetermined part of the film, for the daylight-loading roll-film cannot cover the full circumference of the rotating drum.

FILM HOLDER

The daylight-loading film-holder, Fig. 5 is the latest development of the whole outfit and now operates reliably with all standard No. 3-A "Kodak" roll-films. The novel features of this film holder proved to be most satisfactory. A light-tight door in the stator gives access to two doors in the rotor. One rotor-door carries the unexposed film-spool, between clips. This door is snapped in place with the unexposed film within the rotor. The initial opaque paper is drawn two-thirds the way around the rotor and its end inserted in the empty spool which is to receive the exposed film. The second inner door is then closed and the stator door also. By the simple twist of a cap, as it is removed, the rotor may be locked in proper position for turning the spool, by an external winding-key. To perfect the action of this film-holder, it was found necessary to turn the spool so as to reverse the curvature of the film and thus keep the film, as well as the opaque paper, tight on the drum. The exposure number, on the opaque paper of the standard film, may be observed through the opening in the stator and rotor made by the removal of the cap. After each setting of the film, the cap is replaced and the rotor is free to turn at any speed desired. After the oscillogram has been taken, the cap is again removed and the film turned to position for the next exposure. The rotor is 4¾ inches in diameter and is so designed that the film comes very close to the optical slot of the main oscillograph case. This permits a much greater angle of convergence of light from the cylindrical condensing-lens. This has made possible an optical efficiency 250 per cent of that of former outfits. With a standard No. 3-A (3¼ in. x 5½ in.), 6 exposure, roll-film, one may take six oscillograms 3⅝ in. wide and 5 in. in effective length, or three oscillograms 10 in. in effective length. The total length of film after cutting, is more than eleven inches, for the standard oscillogram. Thus prints are correct in length for filing with standard 8½ in. x 11 in. report paper.

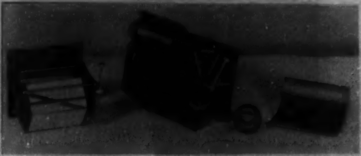

FIG. 5—ROTATING VIEWING, MIRRORS AND DAYLIGHT-LOADING. ROTATING FILM HOLDER.

VIEWING ATTACHMENT

A rotating-polygon of mirrors, Fig. 5, is added to the outfit for viewing recurrent a-c. phenomena. The mounting of these mirrors is so designed that they are operated from the standard driving-head at the proper speed to cause the waves to float slowly by, or stand still, as the tension on the belt of the induction motor is slackened. Since the incandescent lamp requires no attention, and since the optical efficiency of this instrument has been increased 150 per cent over former outfits, it is possible to operate this visual attachment continuously (without attention) and observe any chance momentary change in wave-shape, due to harmonics introduced to the line. The waves are so bright that several people may observe them at the same time, in an undarkened room. A mirror may be placed above the attachment at 45 deg. and thus shift the plane of vision from the vertical to the horizontal. These features make the attachment particularly valuable in classroom work.

For commercial work the photographic record is always required. The simplicity of operation, with the incandescent lamp and daylight-loading film-holder, makes it easy for the student, as well as for a trained operator, to take oscillograms. An oscillogram must always be taken to show transient (non-recurrent) phenomena. With this instrument it is very easy to obtain excellent records of transient phenomena, well spread out on the film.

PHOTOGRAPHIC OPERATION

The general scheme of photographic operation is best understood by reference to the diagram, Fig. 3. For actual construction see Fig. 1 and Fig. 6 The *contactor*, on the film driving-head, is set so as to operate the *trip-magnet* a fraction of a revolution ahead of the opening of the *mechanical shutter*. This fraction of a revolution is timed to be equal to the lag in operation of the remote controlled apparatus. (The setting of this contactor is immaterial when recurrent a-c. phenomena are being photographed). The *lamp-control switch stop* is set at the 3½, 4 or 5-volt position for slow, medium or fast films, respectively. The control switch is first brought up to the 3-volt position, allowing the lamp to come up to normal brilliancy and is then thrown against the stop. This makes a connection through the contactor to the *trip-magnet* thus operating the *remote-control switch*. After the remote control, of the apparatus under test, has started to function, the shutter opens at the beginning of the active film. The transient starts, immediately afterward, and makes its record on the film, by the proper functioning of the galvanometer and optical systems. One revolution after the shutter opens, it closes and knocks open the *lamp-extinguishing switch*, thus saving the lamp for hundreds or thousands of such operations. Thus the lamp may be on 3½ volts for many minutes taking a slow film, or may be on 5 volts for 0.02 second when taking a film running at a speed of 1000 feet per minute. (0.005 sec. per inch). A good film can be taken, of 60-cycle waves, with the film running at 1300 feet per minute, using the incandescent lamp and the induction motor, in the second belt position.

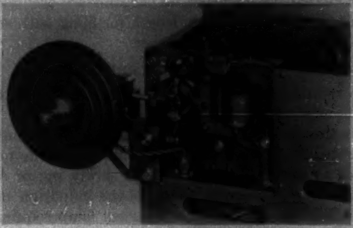

FIG. 6—DRIVING HEAD, SHUTTER MECHANISM, REMOTE-CONTROL SWITCH, AND LAMP-EXTINGUISHING SWITCH

MOTOR

The instrument is supplied with an *induction-motor* for driving the photographic-film and viewing-mirrors. This gives most satisfactory results equipped, as it is, with step pulleys and back gears. For any one belt position, the *film speed is very constant* and can be relied upon to within a fraction of one-per cent, when operated at a given frequency of supply. Where no a-c. supply is available it is easy to obtain a *small six-volt shunt-motor to operate the film.* Fig. 7 shows such a motor equipped with step pulleys and a field rheostat. The field is normally supplied with 6 or 12 volts and the armature with 2 or 6 volts, depending on the range of speed desired. An additional pulley may be supplied to go on the end of the a-c. motor-shaft so the backgears may be used for great reductions in film speed. This is convenient for *studying actual operating conditions* of electric elevators, subway trains, steel-mill motors, etc. This range of 0.2 foot per minute to over 1000 feet per minute, film speed, covers practically all requirements for the standard film and is a big advance over previous outfits.

FIG. 7—SIX-VOLT SHUNT MOTOR WITH FIELD RHEOSTAT AND PULLEYS FOR OPERATING FILM ON ISOLATED TESTS

A detailed description of the shutter mechanism and of the variable shunt (with a range of 20 to 1000 amperes continuous and 25,000 momentary) will not be repeated here since it would be the same as for the original outfit.

LONG FILM ATTACHMENT

A slow-speed, long film, daylight-loading attachment, Fig. 8 has been developed for certain fields of works. This is readily attached to the standard outfit and makes it possible to take any number of oscillograms, 58 inches in length, *without resort to a darkroom.* The film speed is remarkably constant over the whole length. On the first test-run, a slow belt-position, and the back-gears of the induction motor, were used to give a total exposure of over five minutes. With the belt tight, the record was so uniform that each individual cycle could be counted on a 25 cycle wave, even though they were less than one hundredth of an inch apart. This special holder was repeatedly loaded and unloaded on a destroyer while the latter was rolling and pitching as it plowed through a stormy sea.

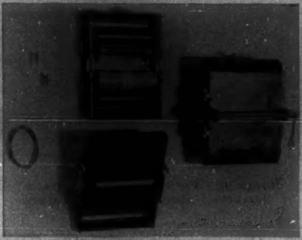

FIG. 8—LOW-SPEED, LONG-FILM HOLDER, OPENED UP AND ASSEMBLED

This film-holder is quite novel in its *operation.* A standard 10 exposure *#3-A* "kodak" film is used. Split, flanged pins are plugged into the ends of the spools so these extensions fit into vertical slots in the holder with the unexposed film below a main driving drum, 1½ inches in diameter. The film raps half way around this drum and is wound up on another spool just above. Two rollers, one below the unexposed spool and one above the exposed spool, are drawn tight against the opaque paper, on the spools, by flexible (spiral-spring) belts working in pulleys beyond the ends of each roller. The lower roller tends to hold back the unexposed film while the upper roller ensures the proper winding of the exposed film. Due to the fact that the upper pulleys are slightly smaller than the lower pulleys, all creepage is compensated for, and the film keeps the same peripheral velocity as the one driving drum. A red observation window permits the operator to note the numbers on the film as the exposure progresses. The holder is easily loaded and unloaded on the oscillograph without fogging the film. The films may be developed by any photographer or by the operator at his convenience, either in a dark-room or in a "kodak" tank-developer. This film holder is much better for low-speed work than the standard rotating-drum, as it gives nearly six times as long a record and thus shows up the details of operation of reversing-motors, electric trains, electric-arc furnaces, maneuvering of submarines, etc. Long films have been taken with a total time of 45 minutes. This may be made still greater if the case demands.

Fig. 9—Typical Oscillograms Taken with Portable Instrument

Ventilation

Even though the various parts occupy a very small space adequate ventilation is provided. The resistance units are wound on thin cards of micarta so as to be very nearly non-inductive and so as to present a relatively great radiating surface. Over 1700 square inches of radiating surface is provided for the 30,000 ohms of resistance. Openings below and above these cards allow good circulation of air. Practically the only continuous operation that they would be subjected to would be the voltage from the secondary of a potential transformer. This would mean but 10 watts per element resistor. At ten times this input no appreciable heating is noticed.

The galvanometers in the first units were of an improved electromagnet type and were very satisfactory for an electromagnet equipment, occupying less space, consuming less energy and being insulated for a higher voltage between elements than any previous 3-element unit. The very latest developments, however, have brought out a superior magnet, of a *permanent* type, *which gives even better sensitivity* to the same vibrator elements, and eliminates: the field coil, rheostat, ammeter, battery, rectifier, bus-bars, etc., which were formerly required for the electromagnet galvanometer. This simplifies the diagram shown in Fig. 3 and greatly improves the instrument. These features insure cool operation of the damping fluid in the vibrator well and thus minimize the errors due to a change in viscosity of the damping liquid. This cool operation, together with the fact that the walls of the wells are made of micarta (with no coloring matter), minimizes discoloration of the damping fluid and thus prolongs the optical efficiency without attention.

Automatic Operation

Any number of these instruments may be operated *simultaneously*. Each oscillograph may be in a different substation and show the local effect of the same short-circuit. The novel features, of this instrument make it possible to connect its supply (for motor, lamp and trip-magnet) to a quick-acting relay, in the secondary of a current transformer in a high-voltage power-system, so that the oscillograph will function perfectly on a *chance short-circuit* (or heavy over-load). The record will show the unreduced value of the short, the arcing voltage, the current interrupted by the breaker and the restored line conditions. This remarkable action is made possible by the automatic, quick-acting features of this oscillograph. With no series resistance in the lamp circuit, the filament will come up to abnormal brilliancy in less than one-tenth of a second due to the great initial rush of current through the filament (the cold resistance of the filament being less than one-tenth the hot resistance). The lamp-extinguishing switch is re-connected, for automatic operation, so that it breaks the supply circuit, at the end of the exposure, and thus cuts off the motor, lamp and trip-magnet.

Fig. 10

New Fields of Oscillograph Activity

From the foregoing description of the new instrument it is easily seen that only a few of its many new possibilities have been utilized. Engineering schools may use such an instrument to solve the many problems arising in small electrical industries, in the mills of the district, in the local electric power plants and transmission lines. This would be good training for post-graduate students and might result in lessening the financial burdens of the institution.

The whole outfit, Fig. 1, may be taken on a Pullman car in a special carrying case (for the main unit) and a handbag (for the motor, film-holder, viewing mirrors and shunt). An operator, assisted at times by the local porter, may personally conduct the outfit to almost any part of the country. A lower Pullman berth gives ample room for both operator and outfit. This is in considerable contrast to the bulk of express crates required to properly transport any previous, truly commercial, oscillograph outfit. In addition to its portability, its independence of any source of power supply greatly increases its range of operation, and its complete independence of a dark room makes its application still more simple. Such features enabled an operator to take two-hundred oscillograms in two days without changing a single optical adjustment. In another case an operator set up the apparatus in an electric car, in a distant city, and took over three dozen oscillograms in one day. The apparatus was set up in the morning with suitable switches to give a score of different combinations of current and voltage, without cutting off the power from the car. Calibrations were made and then the car was put through many maneuvers, in and out of thick traffic. Oscillograms were taken while the car was at full speed, during reversing, braking, acceleration, heavy overload, short-circuit, etc. Each record was of a transient condition and most of the records were made when the car was jerking or jolting, yet the records were as undisturbed as though they were made in a quiet laboratory. Not a single film was developed until after the outfit had been removed and the car put back into service, yet there was not a poor record in the lot, as far as the oscillograph was concerned. During the whole test the outfit was supplied by a battery borrowed from the starting and lighting system of an automobile.

Fig. 11

The problems which are studied with an oscillograph are not always fundamentally electrical. Many purely mechanical movements can be detected and studied only through the medium of electricity and the oscillograph. Undesirable vibrations in machinery, noises, minute movements, momentary pressures, disturbances in the atmosphere, properties of materials and many other non-electrical functions have been studied with the oscillograph. Force or movement has to be transformed into a change in electric current through the medium of the carbon microphone, a special generator, a varying capacity or reactance, or the action of the piezo-electric crystal. The resulting current may be recorded directly by the oscillograph, or first amplified to suitable strength by the three-electrode vacuum tube. It is necessary to study these mechanical problems under operating conditions, and usually far from any electrical laboratory.

The instrument makes easy the solving of inductive interference troubles in distant telephone lines, and, in fact, makes it easy to study the actual conditions under which any piece of electrical apparatus is operating no matter whether that apparatus is in a well equipped laboratory or in some other part of the country, far from any laboratory convenience.

SUMMARY OF THE SPECIAL FEATURES OF THIS OSCILLOGRAPH INSTRUMENT

1. Complete in one micarta case, 11 x 11½ x 25 inches over all, except for film holder and motor.
2. May be supplied by 110 or 220 volts at from 20 to 70 cycles a-c., or from a 6 volt storage battery, for field operation.
3. Shutter mechanism and remote control switch which place transient phenomena on desired part of fast film.
4. Improved optical efficiency and an incandescent lamp illuminant, operated at momentary abnormal voltage, so as to take fast oscillograms.
5. Included transformer, for supplying lamp and motor.
6. Very compact and well insulated 3-element galvanometer.
7. Element resistances and control to cover the broadest range of commercial testing.
8. Film, holder using standard roll film requiring no dark room.
9. Weight of complete outfit, 100 pounds.

Discussion

J. R. Craighead: The oscillograph shown represents a combination of elements which have been considered necessary for oscillographic work under most conditions. There is a distinct advantage in getting as many of these as possible into a single device, and making that device as nearly portable as possible.

The statement made by Mr. Legg, that by the development of this portable device, Oscillographs can now be used everywhere, rather implies that they could not be used everywhere before.

Now, "everywhere" is a pretty broad term, and I do not happen to have any personal experience that goes quite that wide; but I have known of satisfactory work done with oscillographs in factories, foundries, mines, battleships, destroyers, out-doors in the rain, out-doors in tents, and in various conditions where it would seem that a limited laboratory device would be impracticable. The gain from the development under discussion, is not so much a gain in the field where the oscillograph can be used, as it is a gain in the convenience with which it can be transported to that field, and applied, when it is brought there.

In developing this device, considerable attention has been applied to diminishing weight. I have checked up as nearly as I could the elements contained in this device, which is stated to weigh 100 pounds. The latest oscillograph, with full-sized corresponding elements separate from the oscillograph, weighs about 145 pounds. That is a very distinct gain. It is worth while, provided it is not accomplished at a sacrifice of any necessary functions.

In looking through the paper, I find on page 382 certain statements regarding the application of 3000 volts on the 10,000-ohm resistors of the oscillograph. Now, 3000 volts on 10,000-ohms produces 900 watts, and if this element carries one-third of the 1700 sq. in. of radiating surface referred to, this means, roughly, a watt and a half per square inch of radiating surface, evidently an impractical value for continuous service.

I should like to ask the author what is the highest practicable voltage for which this resistance is designed, on a reasonable continuous basis? That is, where the voltage might be on for half an hour, or something of that kind.

On page 108, in reference to the remote control of transients, I note that apparently the transients have been taken at a film speed of 1000 feet per minute, operating a remote control from the oscillograph. This would seem to imply very close timing of heavy switches controlling the circuits through which the transients are obtained. It has been our experience that it is very easy to obtain in connection with the oscillograph, an actuation of an operating circuit to heavy switches, to get transients; but it is not very easy to make the switches work to the limit of two or three one-hundredths of a second, which is absolutely necessary to place the transient in the position you select on the film. Will Mr. Legg give his experience on this point?

On page 385, in respect to operation on chance short circuits: That is a very important field, and if the improvements as designed will enable an oscillograph to catch chance short circuits, a very important addition will be made to our knowledge.

The paper states that the record will show the unreduced value of the short circuit. In most important short circuits, the unreduced value occurs at the actual moment of commencing any short circuit, or at latest, within the first half cycle. Accompanying the above is the statement that the lamp requires something like 0.02 sec. to reach full brilliancy. Without raising the question as to how long it takes to accelerate the motor, in order to get the film up to the proper speed, I should like a little further explanation as to whether it is really meant that the actual beginning of the short-circuit is caught at its full value by this means. If so, we would have added a very remarkable thing to our collection of traps; that is, we would have a trap for short circuits which could be set to catch them, and which would catch the maximum value.

P. A. Borden: The shipping weight of 100 pounds as mentioned for this oscillograph particularly impresses me by its comparison with a weight of from 500 to 800 pounds for the portable (?) outfit which it is customary to use when oscillographic tests are required at points remote from the laboratories. In my experience, we have been required to take such apparatus to parts of the country where the services of neither taxicab drivers nor Pullman porters were available. In one instance the equipment was taken to a plant miles back in the woods in the dead of winter, over rough wood-roads where the passage was difficult, even for a lumber sled. In such cases as these, a conveniently portable instrument such as Mr. Legg has described would have proved eminently suitable.

I am particularly interested in the subject of mirrors. We have had trouble in using the ordinary silvered glass mirrors in that they loose their surface, not only when immersed in the damping oil, but when they are in storage. Occasionally we get one that will last for years, even in service, and outlive others that have not been removed from their original package. I understand that the Bureau of Standards, some time ago, developed a mirror made of aluminum; but I have not been able to obtain information as to its practical success. If Mr. Legg or any of the other gentlemen present who are interested in the subject of the oscillograph know anything about the non-silvered mirror I should consider it a great favor if they would tell their experience.

H. L. Curtis: There are two points which I should like to discuss, viz., the light source and the mirrors for the moving elements. Mr. Legg uses incandescent lamps, and without doubt they are the most suitable light source for work that he has in mind. However, there is a source which may be used when it is necessary to have a very high brilliancy, provided it is only for a short time. A tungsten wire heated in air to a very high temperature will not burn out for several hundredths of a second. By using such a wire, a higher brilliancy can be obtained than from an incandescent lamp unless a lamp is to be used for each exposure. We have tried this and found one very serious disadvantage in that it leaves a cloud of tungsten oxide over surrounding objects. I hope sometime to have an opportunity of using a metallized carbon filament in place of the tungsten wire. This should be free from the above objection.

The question of aluminum mirrors has been raised. We have undertaken to use aluminum mirrors on the oscillograph, and have successfully made mirrors for that purpose. The means of making them is comparatively simple though the percentage of good mirrors that you get in the process is comparatively small. Cut pieces of sheet aluminum a little smaller than you want the mirror and press these pieces between blocks of hardened steel whose faces have been ground to an optically flat surface. When removed from the press, a reasonable portion of the aluminum pieces will have an optical surface. We are working on the method and hope to improve it.

The real reason for using these mirrors is not because they reflect any better than the mirrors used at present, but we hope that, by the use of aluminum mirrors, we will be able to get sufficient damping by means of the eddy currents in the mirror itself. This will do away with the oil damping and greatly increase the portability of the oscillograph. I cannot say that it is accomplished as yet but it is something to which we are looking forward.

F. S. Dellenbaugh, Jr.: We have had a good deal of experience with the portable oscillograph at Massachusetts Institute of Technology. It might be interesting to mention that soon after this machine was delivered we put it in the back of an automobile, drove 1300 miles from Boston to Alabama in six days and put it into operation the next day without any difficulty. Those who have driven two hundred miles a day over southern roads in the Spring will realize that the instrument was extremely strong. In the work we were doing very high-speed oscillograms were necessary and photographs were taken of a transient wave having a period of approximately one thousand cycles and having amplitude of somewhat over one inch upon the film. For this extreme speed it was impossible to use the incandescent light but it was found a very simple matter to substitute an arc lamp. With some device such as that developed by Professor Turner of Yale for repeating transients it would be possible to use the incandescent light and obtain satisfactory results. The day light loading film holder is a very great advantage for work in the field. I would like to suggest to Mr. Legg that this holder be more nearly balanced so that it could be used at higher speeds without so much vibration. Of course perfect balance is impossible due to the unequal distribution of the film. Another convenience, although not a necessity, would be to have a hood over the viewing mirror since when working in the field reflections from the sky make it difficult to see the curve at times. In general I am extremely glad to see some one developing the oscillograph into a practical instrument which will compare with other metering equipment in ease of operation and reliability. With the large number of graduate students now present at M. I. T. a large number of oscillographs are necessary in order to carry out thesis work of advanced nature. We have three oscillographs in the research laboratory at present and could easily keep three or four more quite busy. The present cost of such equipment makes a large number prohibitive for an educational institution and we would like to see a somewhat simpler model developed which could be bought at a much lower price. For laboratory work it is not necessary to have the control elements self-contained and a very simple outfit consisting of a single vibrator with the simplest kind of shutter and film drum device would meet the requirements in many cases. There is no such instrument upon the market at present and it would appear to me as though it would be extremely useful for smaller institutions and as an auxiliary to the more complete oscillographs for the larger institutions.

J. W. Legg: The latter part of this discussion has, to a considerable extent, cleared up some of the questions brought out in the first part.

The paper shows that expansion of oscillography requires more than portability; it requires greater completeness of equipment, greater permanency of adjustments, greater reliability, and greater independence of power supply. The instrument described has all these improvements, and may be transported, as shown by Mr. Borden's discussion, at from one-fifth to one-eighth the weight of previous apparatus of comparable value.

I believe the 145 pounds weight, claimed for previous apparatus, does not include any such resistance as 30,000 ohms. The figure is nearer 4000 ohms. Neither does it include a two-kw. motor-generator set, or bulky 110-volt storage battery, one of which must be obtained if no constant voltage, d-c. power-circuit is available to supply the older oscillograph outfit. The new outfit will operate from any a-c. lamp socket, or from a six-volt storage battery.

The paper states that the 3000 volt d-c. circuit is the exception which cannot be handled by the oscillograph continuously, without external resistor. A 3000 volt peak is very common during switching operations in a 600-volt inductive circuit. The internal resistances of the oscillograph will handle this peak, very nicely, for electric railway tests. The whole 30,000 ohms may be used with one element, without overheating, for recording 3000 volts applied for half an hour or more. Long application of high voltage is seldom required, as control switches may be used to throw on the 3000 volts shortly before taking an oscillogram, and disconnect it immediately after.

Another reason why heating will not be as great as expected, in this oscillograph, is that the vibrator element requires less current per unit deflection, than previous makes, and hence there is less heat given off in the resistors. After four years experience with the included resistors, no undue heating has ever been encountered in commercial testing.

Formerly a day's delay was often caused by the necessity of fixing up a suitable dark room for developing, and reloading film holders. A portable dark room was sometimes carried into the field for these film holders. This added to the already large shipment of carefully packed apparatus. Now we take as many as a hundred oscillograms without resort to a dark room, and then turn them in to the town photographer to develop after the test is over. This daylight-loading feature of the new film holder, surely helps to expand oscillography.

In regard to the remote control of transients; high-speed films are generally required only with quick-acting switches, and slow-acting switches generally require longer exposure films. Such cases are very easily handled with the remote-control device incorporated in the oscillograph. High-speed closing switches may operate in a few thousandths of a second. With the film traveling at 1000 feet per minute the control mechanism may be started seven-hundredths of a second before the opening of the shutter, if desired. Even an ordinary contactor will close in this time, if it is set somewhat closer than usual. Heavy switches, such as oil breakers in high voltage lines, are more sluggish in action, both electrically and mechanically. In order to show the complete electrical transient, the photographic drum must be run much slower. Thus the oscillograph mechanism has ample time to operate the large switch and still cause the transient to appear on the proper part of the film. In case a higher-speed film is required, to study the first part of the short-circuit transient, then the large oil switch may be blocked up so as to be nearly closed, and thus greatly reduce the time required for the magnet to completely close the switch.

Automatic operation, on chance short-circuits, will show the a-c. wave before it is reduced by any breaker action. An actual record is not made of the first a-c. peak, but this maximum peak may be estimated by projecting back the wave according to general laws of transients. In most such tests, the manufacturer is most interested in the characteristics of arc rupture in the oil breaker. These are clearly shown on the oscillogram. The first discernable cycle may be somewhat crowded, due to the fact that the film motor may not be fully up to speed, but this is no material handicap in studying the film.

The problem of keeping the mirrors bright and clear was taken up ten years ago by the Westinghouse company. This company has made mirrors ever since, with considerable improvement in optical efficiency and permanence. One of the vibrators in the first portable oscillograph was not taken from its well in over a year of travel and testing, yet it was just as good, at the end of that time, as a freshly applied mirror on the adjacent vibrator. This is due, partly to the fact that the galvanometer remains at room temperature and hence does not discolor the damping-fluid, partly because the well is made of clear duck-micarta and does not react with the damping-fluid, and partly because a certain clearer damping-fluid may be used since it remains at room-temperature.

At greatest abnormal voltage, the incandescent-filament lamp is operated at the melting point of tungsten, but practice has shown that it is not materially harmed at this temperature provided the application of excessive current is short enough. It is fully up to temperature, as shown by the fact that the beginning of the oscillograph record is just as intense as the end. A continuous half second at this maximum temperature would destroy the filament. Unless a complete quartz optical system is to be used, the writer cannot see any great advantage to a tungsten wire in open air.

A vibrator mirror is not very large in comparison with the vibrator ribbon itself. Hence, as the vibrator ribbon does not introduce appreciable damping, by eddy-currents, it would not seem reasonable that an aluminum mirror would add damping enough to replace the damping-fluid. When the wells are made slop tight, the damping-fluid detracts little from the portability of the outfit.

Mr. Dellenbaugh greatly exceeded the advertised speed limit by taking transient phenomena with a film speed of 2500 feet per minute, with the help of the great optical efficiency of this instrument, and the application of a d-c. arc-lamp. Such a speed is not recommended, as it required special adjustment of the shutter mechanism for this unparalled film-speed with high-speed transients. A film speed of 1250 feet per minute is often used by the manufacturer. At this speed the incandescent lamp gives abundant light to show up transient phenomena with the main amplitude varying from 3000 to 6000 cycles per second. Only an experienced operator could expect good results at these speeds.

To prevent reflection from the sky, or white ceiling, it is well to place a mirror above the viewing attachment at an angle of 45 deg. This will enable many people to observe simultaneously if they are seated in line with the inclined mirror.

The writer has designed a six-element oscillograph which is even smaller than the three-element model just described. This is to be used for three-phase tests on transmission lines and oil switches. Hence four sets of 5000-ohm resistances and four of 50 ohms is sufficient. The outfit is to be operated on six-volt storage-battery only, and hence has no included transformer. These modifications, and the extremely compact construction of the new permanent-magnet galvanometer-elements, has made it possible to get this complete six-element outfit in a case 9½ by 11 by 25 inches over all, except for the six-volt motor and the film-holder (taking a seven-inch width of standard roll film).

E. E. F. Creighton: (Communicated) I have in mind one point that was brought up by Mr. Legg, namely the credit to those who have been involved in the development of the oscillograph. I do not want to take advantage of this occasion to impose on you any of my personal history, but there are incidents apropos to this discussion, in that they relate to the early development of the oscillograph.

In 1898, I was with Mr. Duddel in London, and saw the progress that he had made in the development of the oscillograph. Shortly afterwards I went to work with M. André Blondel and remained two years. When I arrived in Paris in 1898 he had his oscillograph completely developed. He had both the magnet and bifilar types of vibrators. Without question M. Blondel was the father of the oscillograph. Mr. Duddel did a great deal to improve it.

I brought the first Blondel oscillograph to America in 1900 and used it first at Stanford University and for several years while with the Stanley Company. But when I arrived in Schenectady in 1904 I found a very fully developed oscillograph there. So I have always given Mr. L. T. Robinson full credit for the work that he has done, which is greater than Duddel's in making a practicable, useful device out of the oscillograph,—it is actually the oscillograph as we know it today.

I have used the oscillograph every year since 1898 and having taken many thousands of oscillograms I can answer one question about the mirrors. The mirrors are stuck on to the bifilar with shellac. When the shellac dries it naturally shrinks and cracks, and thereby breaks the continuity of the silver on the back of the glass. Therefore we renew these mirrors periodically, without stopping to make extensive photographic tests to determine whether or not they are in first-class condition. It can be done in just a few minutes,—when you get used to it.

Measurement of Transients

BY FREDERICK E. TERMAN

Leland-Stanford University

Review of the Subject.*—The transient crest meter was developed to investigate several pressing problems in high-voltage transients. It consists of a gold leaf electroscope in series with a rectifying device, an electron tube, which enables the electroscope to charge, but prohibits discharge. The charge that is captured in this manner is proportional to the maximum impressed potential, and where adequate insulation is possible, will remain substantially undiminished for many seconds or even minutes after the transient has passed by.*

The instrument described is thus seen to be essentially a crest voltmeter. By the proper use of inductance, resistance, and capacitance it is possible to make the applied voltage dependent upon the maximum peak, maximum slope, or maximum area of current and voltage waves, thus giving information regarding the character as well as the size of the transient.

The method of calibration, and the precautions that must be taken in measuring transients with this instrument are pointed out in detail in the paper. In particular, care must be taken to carefully insulate the gold leaf. The minute charge stored at several hundred volts makes it necessary to employ special vacuum bulbs with the filament and plate leads coming through opposite ends of the tube.

The crest meter has already served successfully in several investigations. The first application was the measurement of the potential of the sphere electrode of a high-voltage oscillator. The method followed consisted in measuring the charging current to the sphere by means of a shielded electroscope resting within this electrode. When combined with a second gold leaf acting as relay contact to give an alarm at a given voltage, the instrument is admirably suited for the study of transmission system disturbances, for the observer need be tied no closer to the test than remain within hearing distance of an electric bell. Such an application was made during four months of the past summer on a 110 kv. line belonging to the Pacific Gas & Electric Company, in an effort to discover the cause of apparently unprovoked flashovers which this particular line experienced from time to time. Other similar applications become evident as their need arises.

PRINCIPLE

TRANSIENT phenomena occur continually in electrical practise, and are often of the most practical importance because of the disturbances they give rise to in electrical circuits. Due to the momentary nature of transients, direct experiment and measurement of such oscillations is unusually difficult, and the choice of methods available for any investigation is very limited.

While studying certain high-voltage phenomena, Prof. Harris J. Ryan of Stanford University found it necessary to develop a new type of transient crest voltmeter. The instrument that was evolved to fill this want can be used to advantage in the investigation of many types of transient oscillations, to determine their nature and magnitude. The following paragraphs give an outline of this meter with the hope that others will find it of assistance in solving their problems.

Some years ago, Dr. Clayton Sharp[1] described a crest meter used to determine the form factor of low-frequency alternating-current waves. This indicator consisted of an electron tube in series with a condenser, and made use of the principle that the vacuum tube will allow a condenser to charge, but not discharge. The condenser will hence charge to the crest voltage of the wave and hold this potential.

A consideration of the principles involved in such an instrument brings forth the remarkable possibilities of this type of indicator in reference to high-frequency transients. From elementary electrostatics, the voltage across a condenser is:

$$E = (1/C) \int i \, . \, dt$$

It is apparent that the factors controlling the time required for the condenser voltage to attain a certain value are the rate of current flow and the amount of current required, that is to say, upon the size of con-

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Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

[1] Sharp and Doyle, "Crest Voltmeters," A. I. E. E., 1916.

denser used, and the space current of the vacuum tube. In practise the charging rate cannot be greatly increased, since the saturation current of vacuum tubes is at best only a few hundredths of an ampere. Where the time element is very small, as in the case of radio frequencies, it is evident that this low rate of charge lasting only a small part of a second requires a very minute capacitance to build up to the voltage of the transient.

The requirement of a small capacitance led to the adoption of a gold leaf electroscope as the basic unit of the instrument. This arrangement was chosen since a gold leaf electroscope not only supplies a very small capacitance, but also reads the potential of the condenser by the deflecting gold leaf. The capacitance can be very small, in the neighborhood of two millionths of a microfarad being the value for the instruments constructed. This quantity is such that one one hundredth of an ampere flowing for a ten millionth of a second is sufficient to charge the electroscope to five hundred volts. It is obvious that with such minute amounts of energy stored at potentials of hundreds of volts, insulation of the very best must be provided.

Throughout the instrument particular care must be taken in this regard. The gold leaf and its electrode should preferably be isolated by a piece of cast sulphur. Sulphur is chosen, since it can be readily handled, and is an almost perfect non-conductor. In fact the resistance between the gold leaf and the case of one electroscope constructed was several trillion ohms, and a large part of this conductivity could undoubtedly be attributed to leakage arising from surface contamination rather than by conductivity through the sulphur.

A great amount of trouble is experienced in obtaining adequate insulation between elements of the vacuum tube. It is well known that glass gathers a surface film of moisture, a film varying in thickness with conditions, and practically impossible to remove. Due to this factor, a long leakage distance between elements of the bulb is necessary, the ordinary tube with all connections coming out of the same press being practically useless. It was found necessary after some experimentation to employ two element bulbs built especially for the crest meter by the Moorhead laboratories. These tubes have the leads coming from opposite ends, and hence give a one hundred per cent utilization of the possible leakage distance.

Even with such construction certain atmospheric conditions cause the leakage rate to be excessive. To reduce the thickness of the moisture film on the glass to a minimum it is desirable to keep the walls of the electron tube slightly warmer than the surrounding air. In practise this can be gained by covering the bulb with a celluloid hood that prevents convection currents carrying away the heat radiated by the filament. Under normal conditions the rate of leakage per minute will not exceed ten to fifteen per cent of the total charge. For most kinds of work it is possible to provide a relay that will disconnect the electroscope from the bulb after the transient has passed. Action by the relay makes it possible for the leakage to take place only over the superior sulphur insulation, and hence reduces the rate of loss of charge to something less than one per cent a minute.

The appearance of the electroscopes that have been built is shown in Fig. 1. The method of construction illustrated is both simple and very satisfactory. The case was formed on four sides by bending a long strip of sheet tin into a cube. The edges were flared out as shown, and small wires run across the open ends and soldered in place, forming a complete electrostatic cage with perfect visibility to the interior. To protect the gold leaf from air currents it is necessary to shield the open ends with thin glass plates held in place by heavy paper pasted over the edge. The sulphur insulation is in the form of a slab fitting on the top of the cube, and supporting the gold leaf on a rod extending through a hole in the metal case. Other details of construction, as well as the celluloid hood, are clearly shown in the photograph, so will not be described further.

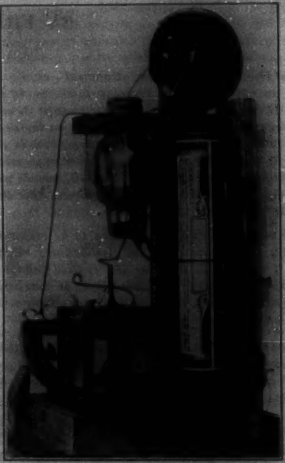

Fig. 1—The Transient Crest Voltmeter

The deflection of the gold leaf is read by lining up a thread with the deflected leaf, and determining the thread angle by means of a protractor that is mounted on the electroscope as shown in Fig. 9. An accuracy of about plus or minus a half of a degree can be readil obtained with a little practise. The personal equation enters strongly into the readings of the instrument, and it is preferable, though not necessary, that calibration curves and observations during tests be made by the same investigator.

APPLICATIONS OF THE CREST METER

The crest meter that has been described is essentially a crest voltmeter, but in this capacity its use can be extended to the investigation of other quantities. By utilization of the proper circuits, the crest meter will indicate the *maximum* peak, *maximum* steepness of wave front, and *maximum* area of both voltage and current waves.

A series of such connections for the investigation of current waves is shown in Figure 2. The measurement of peak currents is almost self explanatory. It must be remembered that R should be absolutely non-inductive where radio frequencies are present. Replacing the resistance by an inductance as shown in Fig. 2B makes the voltage indicated by the crest meter equal to the maximum value of $L\,d\,i/d\,t$, that is, proportional to the steepness of the wave front. Using a capacitance in place of the inductance gives information regarding the area of the wave, since the e. m. f. across a condenser is the time integral of the current flowing in the condenser. Extending the same principles to the study of voltages is a simple matter. Peak voltages can be read either directly or by the use of a potentiometer. Wave fronts and areas can be determined by the applications of the principles illustrated in Fig. 3B and 3C. In these circuits if the impedance offered by R greatly exceeds that offered by L or C, the current through the shunt circuit containing R is an exact replica of the voltage wave in the main line.

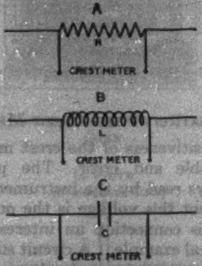

FIG. 2—CONNECTIONS FOR MEASURING PROPERTIES OF CURRENT WAVES

In the study of transmission line transients, a potentiometer of the water filled hose type[2] serves admirably. Fig. 4 shows such a potentiometer used in an investigation conducted on a 110,000-volt line. The power consumption in the hose is about two kilowatts for the complete loop of two hoses in parallel, so by circulation of the water through a wash tube as reservoir, the necessary dissipation of energy can be obtained with a limited amount of water.

The equivalent circuit of the crestmeter is shown in Fig. 5. The electroscope acts as a pure capacitance, the electron tube as a capacitance shunted by a variable resistance. In the proportions existing in the instrument the condenser furnished by the bulb is the smaller of the two by only a slight amount. This brings into consideration certain complications that can affect the accuracy of the results obtained.

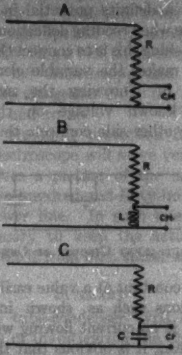

FIG. 3—CONNECTIONS FOR MEASURING PROPERTIES OF VOLTAGE WAVES

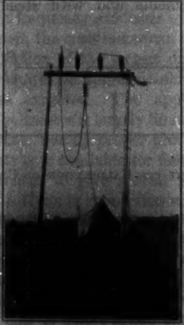

FIG. 4—HIGH-VOLTAGE POTENTIOMETER USED WITH CREST METER

An examination of Figure 5 will make this evident. At the crest of a transient the potential of the two elements of the electron tube is the same. As soon as the voltage begins to die away R becomes infinite, the potential of the filament is no longer equal to that of the plate, and an electric field is set up between them. The energy represented by this field comes from the charge the electroscope possesses, and hence must affect the indication of the gold leaf. Where the final potential of the filament is not controlled, serious errors will be introduced in this fashion.

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[2] H. J. Ryan, "The High Voltage Potential Meter," A. I. E. E., 1916.

The remedy is simple, being merely to always keep the filament at a definite potential in respect to the electroscope case when reading deflections. The easiest way of accomplishing this is to connect the two together. This operation makes the variable element otherwise shown in Fig. 6. Throwing the switch one way impresses the known voltage on the instrument, throwing to the other side connects the case and filament present become constant at a value easily reproducible. Where connections such as shown in Fig. 2A are used, and there is no current flowing when the instrument is being read, it is obvious that this condition is attained without the special switch necessary under other circumstances.

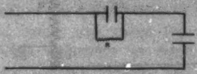

Fig. 5—Equivalent Circuit of Crest Meter

Calibration

Calibration of the instrument may be readily carried out using direct current, keeping in mind the considerations that have just been presented by using the circuit ment. If the potential is supplied by a generator it is preferable to use a machine with a large number of commutator segments and with steady brushes to reduce the ripple to a minimum. One must remember at all times that the crest meter reads the peak voltage, while the direct-current voltmeter does not.

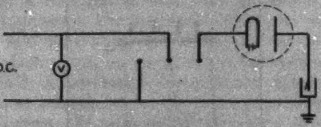

Fig. 6—Circuit for Calibration of Crest Meter

A calibration of the crest meter incorporating the electroscope shown in Fig. 1 is given in Fig. 7. It is seen that the useful range of this particular instrument is between two hundred fifty and one thousand volts. Above this the curve changes too rapidly to be of very much use. It had been thought that the intense field at the edges of the gold leaf might start ionization of the air at low potentials, but investigation of this point up to twenty-two hundred volts gave no evidence that such was the case. From a comparison of the various electroscopes built, the size of the gold leaf appeared to be the principle factor determining the sensibility, the general over-all dimensions in other respects having very little direct influence.

The calibration of the electroscope changes from day to day. This variation extends in a similar fashion to the entire range of the gold leaf, and will raise and depress the calibration curve as much as seven or eight per cent at times. Where relative or approximate values are desired this factor does not introduce such serious trouble, but when absolute values are being determined, frequent recalibration is necessary.

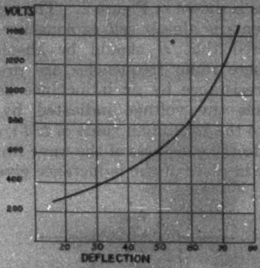

Fig. 7—Typical Calibration Curve

Operation of the Crest Meter

The very sensitiveness of the crest meter is often a source of trouble and error. The peak value of voltage is always read by the instrument, irrespective of whether or not this voltage is the one being investigated. In this connection an interesting discovery serves as a typical example. A circuit such as shown in Fig 8. was being used to study damped oscillations. From the connections it would appear that the voltage read by the crest meter after charging the condenser and closing its switch would be equal to or less than the charging potential, but the result was actually three to four times this value. Analysis of the factors involved gave the explanation of this unexpected result. The circuit comprising the electroscope, electron tube, and leads, made a series circuit of inductance and capacitance to which was applied an instantaneous voltage upon switching in the condenser. It can be shown that under these conditions large voltages of extreme frequencies will exist over parts of the circuit, exactly as the crest meter indicated. This is merely cited to show how apparently unreliable results merely verify the integrity of the instrument as an indicator

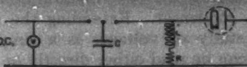

Fig. 8—A Circuit Which Set Up Unexpected Transients That Were Observed by the Crest Meter

of crest voltages. It was found that such resonance phenomena did not occur when the instantaneous voltage was supplied by generators instead of condensers, apparently the inductance of the machine windings preventing the rapid mobilization of energy possible in the condenser discharges.

A number of possibilities of the crest meter idea will occur to the engineer. An electrostatic voltmeter can be used to replace the electroscope, although a considerable increase in capacitance results which will render the instrument unfit for the most rapid transients. It must be remembered, also, that the crest meter as described can only work on one-half of the wave being investigated, due to the unilateral conductivity of the vacuum tube. Where it is suspected that positive and negative crests are different, two such instruments connected back to back must be used. Fig. 9 shows a useful form of the instrument for the investigation of currents flowing in parts of a high-voltage circuit. Being self-contained, and completely shielded by copper gauze, the crest meter may be raised to any potential above ground that is convenient.

Fig. 9—Shielded Crest Meter, With Window to Interior Open

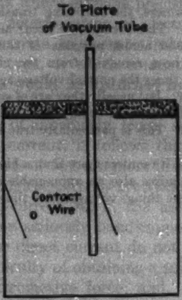

Fig. 10—Showing Additional Gold Leaf Acting to Close a Relay Circuit

In the investigation of infrequent surges it is advisable to incorporate a relay in the test outfit. By placing an additional gold leaf in the electroscope arranged as shown in Fig. 10 in relation to an adjustable contact, a relay capable of handling sufficient current to throw a polarized relay results. The polarized relay can operate any number of circuits, which may perhaps ring a bell, short-circuit the filament of the tube to the electroscope case, and then disconnect the tube altogether from the electroscope to reduce leakage, and so on.

By carefully providing protection from dust, dirt, and dew, the electroscope will serve very well for field use. Fig. 4 gives a picture of the arrangement for a ten-day, two-hundred-and-forty-hour test on a 110,000-volt power line. In such open conditions it is always necessary to keep the actual crest meter inclosed to protect it from dirt, spider webs, etc.

SUMMARY

The electron tube-condenser type of crest meter affords a means of investigating the character as well as the size of transients of extremely short duration. In order to accomplish this, certain precautions in construction and operation must be observed. Of particular importance is the capacity and insulation of the condenser, and the filament potential at the time of making a reading. The final instrument is suitable for the reading of *maximum* height, area, and steepness of current and voltage waves with considerable accuracy. Its readings are readings of maximum disturbances occurring, and if unsuspected oscillations of a greatly different frequency exist side by side with those being investigated, the crest meter can only be expected to give an indication of the largest disturbance. For this reason it requires an apprenticeship before reliable results can be expected. The application of the transient crest voltmeter is largely limited to laboratory and research work, but by proper protection from disturbing elements it is available for field service.

The writer wishes to give special acknowledgment to Prof. Harris J. Ryan in connection with the transient crest meter. He is the originator of the instrument, and took a constant and most helpful interest in the work that was performed with it.

Discussion

E. E. F. Creighton: We have had so much difficulty with large spark lags in vacuums that I should like to ask Mr. Terman a question about his measurements. We have found in all sorts of vacuums a considerable dielectric spark lag and he has, I know, tested this with an oscillator where the duration of discharges can be expressed in thousandths of a second. But I should like to know if he has tried out the spark lag, using a frequency say of about five million cycles per second, and a single discharge.

F. S. Dellenbaugh, Jr.: Mr. Terman's device for measuring transients should prove a very valuable addition to the instruments for measuring electric conditions which exist for extremely small periods of time. In the Massachusetts Institute of Technology Research Laboratory a good deal of work is being done investigating transient conditions upon smooth artifical transmission lines. If the artificial line, or any line for that matter, is long, the duration of the transient occupies sufficient time to be measured by an oscillograph with a vacuum tube repeater where drawing power would distort the conditions. However, a great many cases arise where short lines with extremely rapid transient oscillations require investigation. Usually the maximum voltage is the point of most interest as one of the chief dangers of transients lies in the rupture of insulation. For this type of investigation the Terman crest meter is ideal. Under those conditions it would also appear possible to use an instrument such as the Compton electrometer instead of the goldleaf electroscope which would somewhat increase the accuracy of the readings providing the transient existed for a long enough time to charge the electrometer.

In Figs. 2 and 3 of Mr. Terman's paper he shows the crest meter connected to an inductance in case *B*. This would not always give the steepness of the wave front, since the penetration of a wave into the inductance depends upon the distributed capacity present. If the inductance were free from distributed capacity then a vertical wave front would be completely reflected and the crest meter would show no reading at all, while the rate of change of current immediately adjacent to the inductance would be really infinite.

F. E. Terman: Intergrity tests of the transient crest voltmeter under severe laboratory conditions have given no indications of a dielectric lag which Mr. Creighton refers to as playing an important part in many vacuum discharges. Considering the mechanism of current flow in a vacuum tube it is evident that the conditions are hardly comparable to those involving a spark. The ever present electrons surrounding the filament begin to move as soon as the electrostatic field is established between filament and plate. This occurs with the velocity of light, making the only lag that arising from the time required by the electron in its travel to the plate, which a short computation shows to be entirely negligible, even for frequencies in the order of many millions.

Certain experimental results verify this reasoning. Thus with the transients referred to in connection with Fig. 8, which were in the order of tens of millions of cycles per second, it was found that when the filament temperature was sufficiently low the electroscope was unable to charge fully on one transient, in contrast with the result accompanying high filament emissions. All evidence obtained pointed to limitations in space charges as the important factor governing speed of operation. Some other experiments not described here, in which travelling waves on a twenty foot pipe were partially reflected by a water column potentiometer gave results for the voltage at the water column which indicated that the crestmeter indications were reliable under these conditions.

Prof. Dellenbaugh points out an important fact that is too frequently overlooked. Where nearly vertical wave fronts exist, reflections, inductance, and minute amounts of distributed capacitance take on exaggerated importance compared with their effects under the more usual conditions. All results given by the crest meter under these conditions must be interpreted accordingly. Where vertical wave fronts are present this is a difficult task, but in the more usual case where the wave front has been rounded off the crest meter is still serviceable.

C. L. Fortescue: Referring to Mr. Frederick Terman's paper on Measurement of Transients, this question of the Measurement of Transients of transmission lines and in apparatus connected to transmission lines, is becoming of great importance. However, there are surges of several kinds; we may have surges of very high voltage, which last for a very short time, so short that they do not do any damage.

It takes more than the application of a high voltage to puncture insulation. Insulation requires time as well as voltage in order to be injured, so that what we really need in the measurement of these transients, is something which will weigh the effect of the surge or transient, so as to give some measure of its real destructiveness.

The mere recording of the maximum values does not appear to be sufficient and may be quite misleading. We ourselves in the Westinghouse Company are working on this problem of developing a device that will record transients, and we are at the same time considering some device that will also give a measure of the destructiveness of the transients.

I have had occasion to do some work on transients in connection with transformers, the switching transients that occur in the transformers. These transients occur across the terminals of transformers and across portions of the lining after the transformer has been removed from the circuit. They may reach four or five times the normal voltage across those portions of the transformer; but usually they are of such short time, and have such a small amount of energy, that they have little effect on the insulation. This is particularly true in connection with oil transformers.

As I said before, it requires more than a high voltage to injure insulation. It requires also an appreciable amount of energy, or what is the same thing, voltage, plus time.

Balance Methods in Alternating Current Measurement

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INTRODUCTION

It is the purpose of this paper to describe a system whereby many of the advantages of balance methods of measurement may be made applicable to the study of alternating-current quantities which do not lend themselves to easy measurement with ordinary indicating instruments. Convenience, rather than extreme precision is aimed at. These tests are carried out without the use of vibration galvanometers, synchronous contactors or the need of precise frequency control. The asynchronous detector, using the "heterodyne" principle at commercial frequencies is described. In current measurement use is made of the three-coil transformer or "test ring." There is illustrated the application of a simple method of plotting the vector positions of measured currents; and in conclusion there are shown a number of practical applications of null and semi-null methods of measurement. A bibliography dealing with kindred problems or cases where the methods here described might be applicable in whole or in part, is appended.

The advantages of opposition or balance methods of measurement, except where speed is the primary requisite, are too well known to require elaboration in this paper. They may be used, not only for measurements where extreme precision is required, but also in the determination of electrical quantities which are too minute to operate instruments of the direct-deflecting type, or which emanate from sources where conditions would be disturbed by the introduction of such instruments. Since alternating-current instruments are of inherently lower sensitivity than those for direct current, it follows that the need for opposition methods of test are more pressing here than where meters of high sensitivity are available.

In alternating-current work, the use of opposition methods has been seriously handicapped by two conditions which for direct current do not apply. These are, first, the difficulty of obtaining a suitable opposing current or electromotive force; and, secondly, that of detecting a condition of true balance between the opposing quantities. While these troubles have been fairly well overcome for tests where the most perfect conditions of wave form, frequency regulation and freedom from outside disturbances are available, the expense of the equipment, and the delicate conditions of operation seriously interfere with the use of these methods in ordinary commercial laboratories and testing shops. The value of opposition methods in alternating-current work is probably best exemplified in the Drysdale[1] potentiometer; but, as is well known by all who have studied this device, any of the interfering conditions mentioned above will seriously limit its usefulness.

The methods of test described below are not in any sense those of the extreme precision of the standardizing laboratory; but are intended to illustrate how, with comparatively simple and common instruments, and apparatus which can be built up by any good mechanic there may be made measurements on alternating-current quantities of small value or of poor regulation which will be far more satisfactory than those directly obtainable with indicating instruments.

Opposition methods of measurement may be classified under the two heads of "null" and "semi-null," or "balance" and "balance-deflection." *Null* methods are those in which the quantity to be measured is opposed by a similar quantity both adjustable and measurable, the difference reduced to zero, and the measurement performed upon the latter quantity. In *semi-null* measurements, the opposing quantity, while measurable, is not of necessity adjustable. The final measurement is performed upon that quantity which represents the difference between the other two. Referring to Fig. 1, if $O\,A$ represents a vector quantity which it is desired to measure, and $O\,B$ be a vector of adjustable and measurable magnitude and phase position, the null method of measurement would consist in reducing $A\,B$ to zero and measuring $O\,B$. The semi-null method would call for a measurement of $A\,B$, which, combined with $O\,B$ would make possible the derivation of the value and position of $O\,A$. Each of these classes of measurement has an important place in alternating-current work; and in some cases they may be combined in the one test. In this discussion they will, as far as possible, be studied separately, the null method receiving the first consideration.

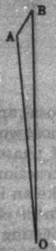

FIG. 1

Null Methods

There are three requisites to the use of the null method in electrical measurement, first, the opposing quantity, against which is balanced the quantity under investigation; second, a means of detecting when a condition of balance exists; and, third, a means of measuring the opposing quantity as to magnitude, and possibly as to phase position. These three requisites are dealt with in order.

SOURCE OF OPPOSING CURRENT OR E. M. F.

In direct-current null method work, (probably best

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[1] See Bibliography.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

exemplified in the simple potentiometer), it is necessary only to oppose the current or voltage being measured by a similar quantity derived from a separate battery and adjust the magnitude of the latter quantity, until a balance is obtained. With alternating currents it is necessary to derive the opposing force from a source synchronous with the quantity under investigation, and to provide means whereby not only the magnitude but the phase position may be adjusted with accuracy. There are several ways in which this may be accomplished. It may be done by a combination of rheostats fed from a polyphase source or by a similar combination of tapped transformers, or by the use of inductances and capacities but these schemes are awkward and do not lend themselves to fine adjustment unless elaborate interconnections are used. Probably the best method lies in the use of a separate alternator driven synchronously with the supply and capable of having its field structure shifted about the armature. Such machines, however, are not very common, and unless one has the facilities of the electrical manufacturing plant at his disposal, are likely to be disproportionately expensive. The most workable method is in the use of the induction type of phase shifting transformer. This is similar in construction to the feeder voltage regulator, being built like an induction motor with a wound rotor, having its stator energized from a polyphase source to produce a rotating field. The secondary winding may be either single-phase or polyphase; and the rotor is adjustable to any desired position. Phase-shifting transformers manufactured especially for the purpose are available, and are made with a smooth core, practically eliminating wave distortion. The apparatus used by the writer, however, was constructed from a small three-phase motor, having the rotor bars removed and replaced by a well distributed single winding, and the rotor fitted with a worm wheel and graduated dial. While a slight wave distortion was visible when the primary and secondary voltages were opposed and an oscillogram made of the residual voltage, this was not found to affect results to any appreciable extent; and while the graduated dial was used merely as an indicator of the rotor position, its readings were usually found to be in remarkable agreement with phase positions as determined by precise methods. By means of a small auto-transformer and a potentiometer arrangement of resistances supplied from the secondary of the phase shifter it was possible to obtain voltages and currents which could be adjusted to a very nice degree of precision. The general arrangement of the phase shifter is shown in the wiring diagrams illustrated in Figs. 4 and 5.

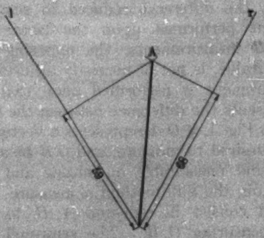

FIG. 2

DETERMINATION OF BALANCE

Probably the greatest drawback in alternating-current methods of potentiometery, and null methods in general has been the determination of a condition of balance. For direct current there are available numerous galvanometers and detectors, the most popular doubtless being the D'Arsonval galvanometer. Failing this, it has been possible to use the telephone with some form of interrupter introduced into its circuit. On alternating current we have not the same flexibility of choice. High sensitivity instruments having permanent magnets are, of course, not directly applicable; and instruments of the moving iron or single-circuit electrodynamic type, operating on a law of squares, lose their sensitivity as the zero point of the scale is approached. The telephone may, of course, be used; but as it has a low sensitivity on commercial frequencies, it is almost as necessary to use an interrupter as on direct currents. To this difficulty is added the effect of harmonics; for, while the fundamental may be balanced out, the unbalanced harmonics, to which the telephone is much more sensitive than to the fundamental, have a very disturbing effect. A galvanometer of the permanent magnet type may be used, the balancing current being led to it through a synchronous contactor; and while in many tests this arrangement has proved most satisfactory, there remains the drawback that if a complete balance is desired, observations must be made with more than one position of the contacting points relative to the field of the driving motor. Similarly, a detector of the dynamometer type with a separately excited field may be used, the excitation being obtainable from two out-of-phase sources, synchronous with the supply. With either of these detectors it is possible to obtain a balance by alternately using the two brush positions, or field excitations, as the case may be; and balancing, first on one and then on the other, until no deflection is obtained with either, which, of course, is the condition of true balance. This method, while quite workable, has been found slow and cumbersome, as well as very confusing unless extreme care is exercised in the sequence of adjustments. There remains, of course, the vibration galvanometer, which is undoubtedly the most sensitive detector we have for this work. It requires, however, a closely regulated frequency, which is often not obtainable; and it must be carefully protected from outside mechanical vibrations. These features seriously limit its use except under the most exacting conditions.

As an outcome of these methods of detection, there has been developed by the writer a system whereby the detection of balance is greatly facilitated; and this, to borrow the terminology of the radio engineer, may be termed the "heterodyne" method; and the detector the "heterogalvanometer" or "heterodynamometer," according to the equipment used. This consists, in brief, in replacing the synchronous contactor which would ordinarily be used with the D'Arsonval galvanometer by an asynchronous contactor, or the synchronous field of the electrodynamic instrument by a field derived from a source slightly differing in frequency from the supply. If a galvanometer is being used, the change to a "heterogalvanometer" is made by driving the contactor at an asynchronous velocity, when the galvanometer will oscillate with a periodicity corresponding to the difference between the frequency of the supply and that represented by the speed of the contactor. It is at once evident that this may be very simply accomplished by direct attachment of the contactor to the shaft of an induction motor driven from the supply, when the oscillations of the detector will represent the slip of the motor. In balancing, it then becomes only necessary to adjust until the oscillations of the detector cease, when it is known that the current flowing in its coil has a value of zero, and that consequently there is a true balance between the opposed quantities. If an electrodynamometer is to be used as a "heterodynamometer," the field is excited from a separate source, having a frequency differing slightly from that of the supply. If a separate motor generator is available this may be very conveniently done; and by slight adjustments of its frequency, an excellent control obtained of the period of swing of the detector. If an alternator is not at hand, a suitable off-frequency supply may be obtained by passing direct current through a "rectifying" commutator attached to the shaft of an induction motor. All these methods have been used by the writer with good success. Excellent results have been obtained by the heterogalvanometer method, using a Leeds and Northrup portable indicating galvanometer as a detector; while for the heterodynamometer method, use was made of a re-modelled Weston Model 18 voltmeter. In this instrument the fixed and movable circuits were separated and brought out to different sets of terminals, the windings being left unchanged, with the exception of an auxiliary field circuit of a comparatively few turns, used for "feeling out" conditions. The zero point was brought to the center of the scale, and a new calibration made, using a constant field and a varied current in the moving coil.

MEASURING AND LOCATING THE OPPOSING QUANTITY

When a condition of balance between the measured and opposing quantities has been obtained, there remains the problem of determining the magnitude, and possibly the phase position of the latter, from which to derive the former. If the quantity under measurement be a current, and its value be beyond the range of ordinary indicating instruments, it is necessary to make use of a three-coil transformer, which is described under the head of "Measurement of Current." If a small voltage is being investigated, a potentiometer arrangement is made up, and a measurable voltage obtained. These quantities would then be determined as to magnitude with direct indicating instruments of any reliable type. As it is frequently desirable to find the vector position of the current or voltage, as well as its magnitude, there is described below a method developed by Mr. H. S. Baker, and capable of almost unlimited application.

Assuming a sinusoidal wave and a constant voltage, the indication of a wattmeter or of any instrument of the electrodynamic type is in proportion to the projection of the vector representing the current in one circuit upon that representing the field produced by the other; or, to be more lucid, though perhaps not so precise, the reading of a wattmeter represents the projection of the current upon the voltage. Conversely, the locus of the end of the current vector will be a perpendicular to the voltage vector drawn through a point on the latter representing, in its distance from the common origin, the indication of the instrument. Now, if an equal voltage of different phase position be applied to the instrument, the current remaining as before, the end of the current vector will lie also on a perpendicular drawn through the new voltage vector at a point representing the new indication. Since these results are obtained with the one current, it follows that the intersection of the perpendiculars locates the end of the current vector, thus definitely fixing the position of the current in relation to the two voltages. In a similar way any number of current vectors may be plotted by projecting them against the same reference voltages.

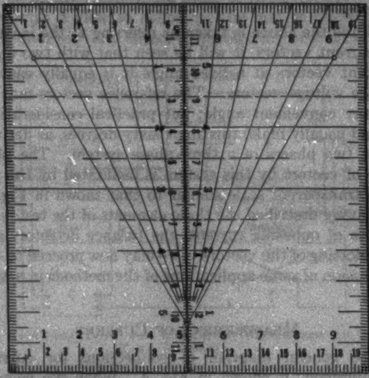

FIG. 3—TRANSPARENT SCALE USED IN PLOTTING VECTORS

This method of plotting vectors is illustrated in Fig. 2. The vectors "*l*," (left), and "*r*," (right), are the two reference vectors, representing the field excitations which are used in the wattmeter or dynamometer. The indications of the instrument as read with a single current projected against first one and then the other of these are scaled off along the respective vectors. The position of the end of the current vector will then lie at the intersection of perpendiculars drawn through these points. Suppose the readings with the left and right hand vectors had been 59 and 68 respectively; the procedure would be as follows: On vector *l*, using any convenient scale, measure off a distance of 59 units, and erect a perpendicular. On vector *r*, using the same scale measure off 68, and erect a perpendicular. The intersection of these perpendiculars gives the point *A*, the end of the current vector. In a similar way any number of current vectors may be plotted by projecting them in a dynamometer, against the same reference vectors.

While the above is taken as referring to the location of current vectors, it will be seen that with two equal current vectors at a fixed angle it is equally easy to locate voltage vectors. The reference vectors may be at any convenient angle; but practical considerations would usually make this angle sixty degrees, as derived from two phases of a three-phase system. The plotting of vectors by this system is facilitated by the use of a transparent scale similar to that shown in Fig. 3.

Having described the three elements of the test—the source of opposing current, the balance detector, and the plotting of the vectors,—we may now proceed to the discussion of some applications of the methods in actual work.

MEASUREMENT OF CURRENT

The general diagram of connections for current measurement is shown in Fig. 4. When the current investigated is not of a magnitude conveniently measurable on the indicating instruments at hand, use is made of a three-coil transformer or "test ring." Great flexibility of design is here possible, almost any small transformer with three separate windings about its core being adaptable to the work. Excellent satisfaction has been obtained by the writer by making use of a "Baker test ring," designed for current transformer ratio and phase-angle measurements. This ring carries three windings, which may be referred to as "primary," "secondary," and "tertiary," though here these terms are not in strictly correct usage. As employed in these tests, the primary consists of two turns wound on the ring and carrying the opposition current derived from the phase-shifter. The secondary winding, set to 200 turns carries the current which was under measurement. The tertiary winding of some 7000 turns is led to the detector operating as a heterogalvanometer. The opposing current being adjusted until the detector indicates a condition of balance, it follows that the flux in the magnetic circuit of the ring has a value of zero; and hence that the ampere-turns of the primary and secondary windings are equal and opposite. Under these conditions the transformer is operating at its true turn-ratio; so that the measured current will be a definite and known multiple of the opposing current, and in exact phase opposition. The opposing current then being measured as to magnitude and vector location, the same information is obtainable in regard to the measured current by applying the turn ratio of the transformer. With the set-up described above, the writer has been able to measure and definitely locate currents of the order of one tenth of a milliampere; and there does not appear to be any obstacle in the path of far greater refinement if found desirable. The same general scheme, with a reversal of ratio arrangement would be suitable for measurement of very large currents. In cases where the current is of a magnitude measurable by instruments at hand, the test ring may be omitted and a direct opposition connection used. A cut of the actual apparatus used is shown in Fig. 7.

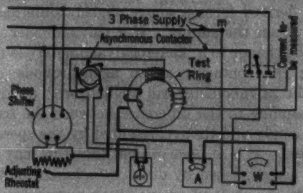

FIG. 4—MEASURING SMALL CURRENT, USING "HETEROGALVANOMETER" DETECTOR

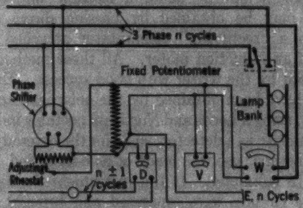

FIG. 5—MEASURING SMALL VOLTAGE, USING "HETERODYNAMOMETER" DETECTOR

MEASUREMENT OF VOLTAGE

The connections for voltage measurement are shown in Fig. 5. It will be seen that, with exception of the replacement of the test ring by a non-inductive tapped resistance of the potentiometer type, the arrangement is practically the same as that for the determination of current. If a non-inductive adjustable potentiometer is at hand, it may be possible to apply the true potentiometer principle to the test; when the method closely approaches the system developed by Dr. Drysdale.[2]

MEASUREMENT OF POWER

The application of the null method to measurement of power suggests itself, and requires little explanation.

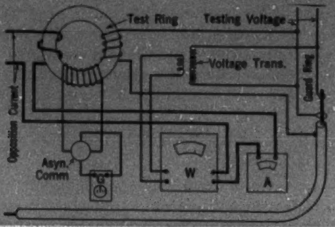

FIG. 6—SIMPLIFIED DIAGRAM SHOWING MEASUREMENT OF DIELECTRIC LOSS IN CABLE

In the measurement of small wattages or of power in circuits which would be disturbed by the introduction of instruments, it is seldom that both current and voltage would require a null method of measurement. Such being the case, the directly measurable quantity is led in the usual way to a suitable wattmeter, and the other quantity transferred by the null method to the instrument; when the proper ratios being taken into consideration, the wattmeter will indicate the actual power in the circuit. Here, of course, if the power be at low power factor, as is frequently the case, careful allowance must be made for phase angle of the instrument transformer (if any) used in the direct measurement. A system of connections for power measurement is shown in Fig. 6.

FIG. 7—SET-UP FOR MEASURING TRANSFORMER EXCITING CURRENT AT 1/150 VOLTAGE

A: Transformer under test, in shipping crate; B: Phase-shifter; C: Adjusting rheostat; D: Three-coil transformer, or "test-ring," with two turns in primary; E: Asynchronous contactor; F: Ayrton shunt; G: Portable indicating galvonometer used as a "heterogalvanometer"; H: Standard non-inductive resistance, used in examining opposing current; I, J: Ammeter and wattmeter used in examining opposing current; K: Portable motor-generator set, used with "heterodynamometer" method.

DETERMINATION OF INDUCTANCE AND CAPACITY

Inductance measurements may be made by plotting the voltage across the circuit investigated, and comparing it with the drop across a non-inductive resistance carrying the same current. This will give the phase angle, from which the inductance may readily be derived. In an exactly similar way capacity may be determined. If more convenient, the voltage may be taken as the fixed quantity, and measurement made upon the currents flowing through the circuits.

Semi-Null Methods

While semi-null methods of opposition are commonly used in alternating-current measurement, it is thought well to briefly recapitulate some of these applications, more with a view to collecting the matter under one head, than with any object of introducing new material.

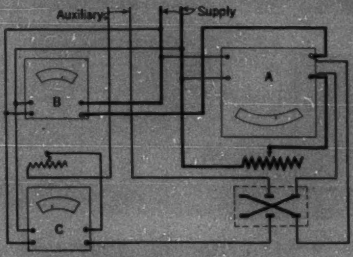

FIG. 8—CONNECTIONS FOR CHECKING WATTMETER BY SEMI-NULL METHOD

A: Standard Instrument; B: Instrument being checked; C: Low reading instrument, indicating error.

TRANSFORMER RATIOS

A considerable number of methods of determining instrument transformer ratios by opposition methods are in vogue; and the principal differences among these lie in the way of dealing with the differential current or voltage representing the amount by which the transformer departs from its intended ratio. While it will at once be evident that the null methods described above may be applied in the study of these small vector differences, it is frequently possible to dispense with this refinement; and, by means of the system of reference vectors, directly examine the differential current or voltage, whence may be derived the constants of the transformer.

In testing current transformers with the "Baker test ring,"[3] previously referred to, the primary and secondary currents of the transformer under test are made to circulate in opposition about the ring, (the respective windings having first been set to the nominal ratio of the transformer) and the vector difference of the two sets of ampere-turns as derived from the tertiary winding examined against reference vectors in a dynamometer. By introducing known errors into the turn ratio of the ring, a number of vectors are obtained, from which a diagram may be constructed, giving graphically the constants of the transformer to a high degree of precision. A similar method may be applied to the testing of voltage transformers against a standard potentiometer; but the possible errors due to current flowing in the tap impose limitations which make desirable the use of a null method or the employment of a standard transformer.

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[2] See Bibliography.

[3] See Bibliography.

INSTRUMENT CHECKS

A method has been published by the writer[4] whereby increased precision in the comparison of indicating instruments with standards is made possible by a semi-null method. This is particularly applicable to wattmeters, but with certain limitations to ammeters and voltmeters also. Here, an auxiliary current is passed through the circuit of one of the instruments under test, as shown in Fig. 8, both instruments thus being brought to an identical reading. This auxiliary current being then measured in an instrument of comparatively high sensitivity, the difference in accuracy of the two instruments under comparison is thus shown in a greatly magnified form.

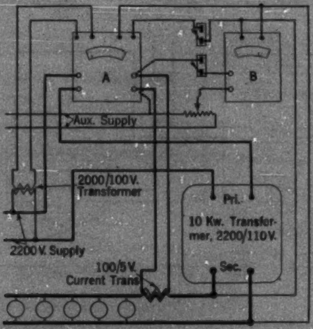

Fig. 9—Determining Transformer Losses by Semi-Null Method

A: Two-element (polyphase) wattmeter; B: Low reading wattmeter, indicating losses.

EFFICIENCY MEASUREMENT

A semi-null method of efficiency measurement has been developed and used with considerable success on pieces of apparatus wherein both the input and output are electrical quantities[5]. This can be applied in the case of transformers and some motor-generator sets. The system, as illustrated in Fig. 9, consists in passing the input and output in opposite senses through the circuits of a two-element wattmeter such as is commonly used for polyphase measurements, and adding an amount of power derived from a separate source to the output side of the instrument, to balance the two, and bring the wattmeter to zero indication. This comparatively small quantity may then be measured on a separate wattmeter of suitable range, giving a magnified indication of the losses in the apparatus tested.

CONCLUSIONS

The tests which have been described above cannot in any sense be said to exhaust the possibilities of opposition methods of measurement as applied in alternating-current work. The instances which have been cited are given as thoroughly practical examples, and as cases which are backed by the author's experience. It is felt that all who have studied this branch of electrical measurement or who are in their work confronted by problems of measurement which seem to surpass the capabilities of the instruments and equipment of the ordinary commercial laboratories will be able to find in this study many helpful suggestions. In the course of the paper, a number of references are made to the investigations and developments of Mr. H. S. Baker of the Ontario Power Company, whose earlier work along the lines here set out has been of inestimable assistance to the writer, and whose inspiration has been a continual incentive to carry the development of balance methods of testing to a further degree of perfection than has heretofore been published; and the writer takes this opportunity to pay a part at least of the debt of gratitude which he owes to Mr. Baker's infinite genius and kind cooperation in the working out of these methods of testing.

SUMMARY

(1) Owing to the comparatively low inherent sensitivity of alternating-current instruments, balance or opposition methods of measurement are more desirable and of wider application in alternating-current testing than in direct-current work.

(2) The application of balance methods to alternating-current tests is a greater problem than in direct-current work, because of the necessity of having to oppose the measured quantity in phase position as well as in magnitude; and because of the difficulty of precisely detecting when a condition of balance is obtained.

(3) A suitable source of current or voltage for opposition tests consists in the phase-shifting transformer, to which is added a potentiometer type of rheostat.

(4) A satisfactory detector is found in the D'Arsonval

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[4] See Bibliography.

[5] See Bibliography.

galvanometer or the electrodynamometer, so arranged that the indicator oscillates at an easily observable periodicity. A condition of balance exists when the oscillations have been reduced to zero.

(5) In current measurement, use is made of a transformer having three windings, the measured and opposing current flowing in two of these windings, and the difference, as derived from the third, reduced to a zero value, when the ratio between the active currents becomes the turn ratio of the transformer. With this arrangement it is equally easy to measure extremely small or extremely large currents.

(6) In voltage measurement a simple potentiometer arrangement of resistance is used, the ratio being set to a suitable value at the beginning of the test.

(7) The opposing current or voltage, after a balance has been obtained, may be measured by indicating instruments, and if desired, its vector position plotted, by means of a pair of readings obtained with an instrument of the wattmeter type against two "reference vectors."

(8) Null methods of test may be applied to measurement of quantities of current, voltage or power which are either too small, (and in some cases too large) for instruments of ordinary types, or are derived from sources wherein conditions are so unstable that they would be disturbed by the introduction of these instruments into the circuit.

(9) Semi-null methods, where a complete balance is not obtained, are applicable to transformer ratio measurements, instrument checks and, in some cases, to the measurement of efficiencies.

Bibliography

Agnew, P. G. and Fitch, T. T. The Determination of the Constants of Instrument Transformers. *Bull. Bureau Stds.* P. 281, Vol. VI, 1909.

Agnew P. G. and Silsbee, F. B. The Testing of Instrument Transformers. Trans. A. I. E. E., June 28, 1912.

Agnew, P. G. A Study of the Current Transformer. *Bull. Bureau of Stds.* P. 423, Vol. VII, 1911.

Baker, H. S. Current-Ratio and Phase-Angle Test of Series Transformers. *Electrical World*, P. 234, Vol. LVII, 1911.

Baker, H. S. Ohmic and Inductive Resistance of Meter-Current Circuits. *Electrical World*, P. 565, Vol. LVIII, 1911.

Baker, H. S. Current Transformer Ratio and Phase-Error by Test-Ring Method. Proc. A. I. E. E., P. 1173, Sept. 1918.

Bennett, Edward. A Milliampere Current Transformer. Trans. A. I. E. E., P. 571, Vol. XXXIII, 1914.

Borden, Perry A. Method of Ascertaining Error in Indicating Instruments. *Electrical World*, p. 495, Vol. LXVII, 1916.

Borden, Perry A. Determination of Transformer Losses. *Electrical World*, P. 716, Vol. LXVIII, 1916.

Brooks, H. B. A New Potentiometer for the Measurement of Electromotive Force and Current. *Bulletin Bureau Stds.*, P. 225, Vol. II, 1906.

Brooks, H. B. A Deflection Potentiometer for Voltmeter Testing. *Bulletin Bureau Stds.*, P. 275, Vol. IV, 1907-08.

Brooks, H. B. Deflection of Potentiometers for Current and Voltage Measurements. *Bulletin Bureau Stds.*, P. 395, Vol. VIII, 1912.

Brooks, H. B. Design of Deflection Potentiometers. *Bulletin Bureau Stds.*, P. 419, Vol. VIII, 1912.

Campbell, A. On the Measurement of Very Large and Very Small Alternating Currents. *Phil. Mag.* 52, P. 271, 1896.

Cone, D. I. Bridge Methods for Alternating Current Measurements. Trans. A. I. E. E., P. 1743, Vol. XXXIX, 1920.

Cotton, H. Measuring of Power at Low Power Factors. *Electrician*, P. 554, May 6, 1921.

Craighead, J. R. Potential Transformer Testing. Trans. A. I. E. E., June 8, Vol. XXXI, 1912.

Dawes, Chester L. Phase-Angle of Current Transformers. Trans. A. I. E. E., P. 1585, Vol. XXXIV, 1915.

De La Corce, P. Determining Transformation Ratio and Phase-Lag in Current Transformers. *Bulletin de la Soc. Internat des Elect.*, July 1916.

Electrician, Jan. 12, 1917.

Elect. World, P. 327, Vol. LXIX, 1917.

Drysdale, C. V. Measurement of Phase Differences. *Electrician*, Pp. 726 & 782, Vol. LVII, 1906.

Drysdale, C. V. Some Measurements on Phase Displacements in Resistances and Transformers. *Electrician*, P. 160, Vol. LVIII, 1906.

Drysdale, C. V. The Use of the Potentiometer on Alternating Current Circuits. *Phil. Mag.*, P. 402, Vol. XVII, 1909.

Drysdale, C. V. The Alternating Current Potentiometer as a Standard Instrument. *Electrician*, P. 157, Vol. LXXV, 1915.

Drysdale, C. V. Measuring Instruments. *Electrician*, P. 853, Vol. LXXVII, 1916.

Farmer, F. M. Measurement of Power Loss in Dielectric of Three-Conductor Cable. Trans. A. I. E. E., P. 221, Vol. XXXVII, 1918.

Fortescue, Chas. The Calibration of Current Transformers by Means of Mutual Inductances. Trans. A. I. E. E., P. 1599, Vol. XXXIV, 1915.

Harned, Mark L. Operating Characteristics of Current Transformers. *Electrical World*, P. 869, Vol. LXVII, 1916.

Kennelly, A. E. and Velander, E. Potentiometer Measurements at Telephone Frequencies. *Amer. Phil. Soc. Proc.* 58, P. 97, 1919.

Knopp, Otto A. The Commercial Standardization of Instrument Transformers. *Electrical World*, P. 92, Vol. LXVII, 1916.

Knopp, Otto A. New Current Balance for Calibration Work. *Electrical World*, P. 993, Vol. LXXV, 1920.

Laws, Frank A. Application of the Potentiometer to Alternating Current Measurements. *Electrical Measurements* (Mc Graw Hill, 1917) Chap. V.

Makower, A. J. Measurement of Phase Difference, *Electrician*, 58, P. 695, 1907.

Newman, M. G. Measurement of Alternating Current of Low Value. Trans. A. I. E. E., P. 1489, Vol. XXXI, 1912.

Northrup, E. F. A new Instrument for the Measurement of Alternating Currents. Trans. A. I. E. E., P. 263, Vol. XXIV, 1905.

Northrup, E. F. Null Methods. Resistance Measured by Differential Instruments. *Method of Measuring Electrical Resistance.* (Mc Graw Hill, 1912) Chap. III.

Pesterini, Signor. Apparatus for Measuring Very Strong or or Very Weak Currents. *Electrician*, P. 154 Nov. 2, 1917.

Robinson, L. T. Electrical Measurements on Circuits Requiring Current and Potential Transformers. Trans. A. I. E. E., P. 1905, Vol. XXVIII, 1909.

Rosa, E. B. The Compensated Two-Circuit Electrodynamometer. *Bull. Bureau Stds.*, P. 43, Vol. III, 1907.

Rosa, E. B. and Lloyd, M. G. The Determination of the Ratio of Transformation and of the Phase Relations in Transformers. *Bulletin Bureau Stds.*, P. 281, Vol. VI, 1909.

Sharp, C. H. and Crawford, W. W. Recent Developments in A. C. Measurements. Trans. A. I. E. E., P. 1517, Vol. XXIX, 1910.

Sharp, C. H. and Farmer, F. M. Measurements of Maximum Values in High Voltage Testing. Trans. A. I. E. E., P. 1617, Vol. XXXI, 1912.

Silsbee, F. B. Improved Method of Testing Current Transformers. *Electrical World*, P. 667, Vol. LXX, 1917.

Sumpner, W. E. Measurement of Small Differences of Phase. *Phil. Mag.*, P. 155, Vol. IX, 1905.

Thomas, Benj. F. Notes on Wiping-Contact Methods for Current and Potential Measurements. TRANS. A. I. E. E., P. 263, Vol. IX, 1892.

Weibel, Ernest. A Study of Electromagnet Moving-Coil Galvanometers. *Bulletin Bureau Stds.*, P. 23, Vol. XIV, 1918-19.

Wenner, Frank. Characteristics and Applications of Vibration Galvanometers. TRANS. A. I. E. E., P. 1243, Vol. XXXI, 1912.

Wescott, E. C. Differentially Wound Watthour Meter for Testing Current Transformers. *Electrical World*, P. 433, Vol. LXXV, 1920.

Whitehead, J. B. and Isshiki, T. Notes on the Synchronous Commutator. TRANS. A. I. E. E., P. 407, Vol. XXXIX, 1920.

Discussion

C. L. Fortescue: This question of Null measurement is one that has been discussed in the Institute in the past quite a good deal. I think that Null methods of calibrating current transformers are in general use now. We ourselves have used a method of calibrating current transformers which was described by me in the Institute JOURNAL some time ago, in which variable mutual inductances were used. At that time we used d'Arsonval portable type galvanometer, and a synchronous contactor. This method was quite satisfactory so long as the contactor was properly maintained; but as we were using this apparatus right on our test floor, with green operators from time to time, who had to be trained to use the device, it was difficult to get them familiar enough with it to be able to tell when it was working properly, and the results were sometimes erratic.

We therefore changed over from the D'Arsonval instrument and synchronous contactor and used instead a dynamometer type instrument, which was excited from a synchronously driven motor-generator set through a phase-shifter. This could be adjusted to give maximum sensitivities for a change in e. m. f. in quadrature to the current or a change of e. m. f. in phase with the current. Thus we had two sensitive positions, and in adjusting our circuit, so as to get zero reading we alternated these two positions making the necessary adjustments until zero reading was obtained.

Now, this hetero-galvanometer or hetero-dynamometer scheme passes through both those positions during the cycle, and instead of getting steady measurement to be reduced to zero, we get slow cyclic measurement to be reduced to zero, so that it amounts to the same thing, but the method used by us is much quicker and simpler to adjust.

However, Mr. Chubb and I made use of this method of a commutator driven at slightly below synchronous speed, a number of years ago, in connection with the calibration of the sphere-gap voltmeter and we found it quite effective. Of course, we were not using it with a Null method, but for direct measurement of crest voltage.

We are using the Null method device which I have described for calibrating current transformers right in our transformer test floor and we have very little trouble now. We are able to detect differences down to a small fraction of a micro-ampere.

I. M. Stein: Mr. Borden has stated that a-c. instruments are of *inherently* lower sensitivity than those for direct current. I believe that the lower sensitivity to which he refers is not inherent but is brought about by the conditions imposed by the use of the instruments. If the conditions are such that an iron-core electrodynamometer can be used you can obtain as good sensitivity with an a-c. instrument as you can with the best d-c. instruments.

On page 36, Mr. Borden indicates that high sensitivity instruments having permanent magnets are not directly applicable. I do not believe that he intends to exclude the vibration galvanometer because this instrument is mentioned later on but from the statements made one would be led to believe that vibration galvanometers were not made with permanent magnets. Where the vibration galvanometer is mentioned, the paper states that the vibration galvanometer "is undoubtedly the most sensitive detector" for such work. This is not true because the iron-core electrodynamometer is capable of much greater sensitivity. However, the vibration galvanometer is an excellent instrument for such work and is of particular value in exploring the circuits for stray field effects before starting measurements. In d-c. measurements it is usually satisfactory to shield the measuring instrument from stray fields but in a-c. measurements it is necessary to shield the whole circuit. The fact that the vibration galvanometer responds to all currents of the fundamental frequency regardless of phase relations makes it an ideal instrument for preliminary stray field explorations.

I do not agree with Mr. Borden that the use of the separately excited dynamometer is "slow and cumbersome." That is a matter which depends very largely upon the design of the instrument and its accessories; when properly designed and applied. The use of the separately excited dynamometer is very rapid and convenient.

Referring to the asynchronous commutator and galvanometer to which Mr. Borden has given some new names, I do not believe that the device can be called new. An instrument using this principle and called the "Ondograph" has been in use for more than 10 years. A similar arrangement for measuring a-c. wave form was described in the Institute TRANSACTIONS several years ago. About 3 years ago Mr. Doyle of the Electrical Testing Laboratories described to me a device which he had built and tried out in connection with instrument transformer testing. This device was identical with the one which Mr. Borden describes.

Concerning a-c. opposition methods Mr. Borden has stated that such methods are best exemplified in the Drysdale potentiometer and goes on to say that this instrument is seriously limited in its usefulness. I am afraid that nearly all a-c. measuring instruments are seriously limited but that should not be discouraging. I believe that opposition methods in a-c. work are best exemplified by some of the very useful and convenient methods which have been developed for a specific purpose. I refer to the opposition methods which have been developed for testing instrument transformers. I believe that a mistake is often made in designing an instrument to make too many measurements; accuracy, rapidity and convenience are thereby sacrificed. Concerning the use of the word "Null" I cannot agree with the definition which Mr. Borden has given. A "Null method" is defined in Funk and Wagnall Dictionary as follows:

Null Method: "A method of measurement in which the thing to be observed is not the degree or extent of a thing but merely whether the thing occurs at all, as when the equality of an electrical resistance, with another against which it is balanced, is indicated by the absence of deflection of a galvanometer needle."

I should not consider this source of information a final authority on such matters except that the definition expresses very clearly the generally accepted idea.

There is one point in Mr. Borden's paper which is not brought out as clearly as it deserves to be. This point is in connection with the arrangement shown in Fig. 4. While I do not agree that this arrangement should be called a null method, I believe that the circuit has considerable merit, at least for certain kinds of measurements.

If my interpretation of Fig. 4 is correct, the outstanding advantage of this circuit is that the current in which you are interested can be measured without materially impeding its flow. When the adjustments are made so that a balance obtains, it would appear that the only impedance to the flow of the current is the resistance of the secondary winding.

In measuring small alternating currents, the resistance of this winding would probably be very much lower than the impedance of any instrument winding which you would have to insert in order to obtain the desired sensitivity.

If the circuits in which the current is to be measured were located some distance from where the measurements are to be made, then by placing the ring at the remote point, the impedance of the lead wires between the two locations would be eliminated. Of course, it would be necessary to run four wires instead of two, but in many cases there would be no objection to this, and the elimination of the impedance of the lead wires would be a distinct gain.

H. B. Brooks: Mr. Borden's paper is one of a number of evidences of the great potential usefulness of the potentiometer or balance principle, and it reminds us of the fact, often overlooked, that deflection instruments disturb conditions in the circuit to which they are applied.

In using the two-element wattmeter as suggested by Mr. Borden, it should be kept in mind that the two elements are in general not exactly balanced and that the amount and sign of the unbalance differ from point to point over the scale. For careful work the amount and direction of unbalance at the zero point should be determined, and a suitable correction applied.

Chester L. Dawes: Mr. Borden mentions the Drysdale potentiometer as illustrative of one of the null methods. At Harvard University, we require of each senior electrical engineering student four or five experiments involving the use of this apparatus, and we find it highly satisfactory. It requires perhaps two experiments to give the students skill and confidence in the manipulation of the apparatus. Two, and sometimes three experiments are given in which the students measure the current and voltage relations along artificial lines. As a rule, we use a Campbell bifilar vibration galvanometer as a detector, since it is not affected by stray fields as is the Tinsley magnetic-vane type. I have found that after some practise in the manipulation of the apparatus, balances can be rapidly and accurately made, provided the frequency and voltage are maintained constant. Our power is supplied by a 12 kv-a. alternator, driven by a 20 h. p. d-c. motor. Since we are able to control the field current of either machine from any laboratory, we have little difficulty with variations of frequency or voltage.

I have also used this potentiometer in the investigation of losses occurring in the leads, bus-bars, and adjacent iron supports of large brass furnaces. The potentiometer was set up in a small laboratory adjacent to the furnace room in a large manufacturing establishment. The commercial, 60-cycle power system was used for supply. Currents as high as 5000 amperes were involved, and e. m. fs. of the order of a few millivolts were measured. Balances could usually be obtained rapidly and accurately, even under these commercial conditions. Twice during those times when the main power plant was not in operation, and a small, lightly-loaded, turbo-alternator was carrying the load, some difficulty was experienced in obtaining balances, owing to the slight hunting of the turbine. The fact that the governor was not holding the turbine speed absolutely constant was not appreciated at the power plant until this time.

From my own personal experience, I feel that this type of potentiometer is very useful, and in many cases may be adapted to commercial measurements. Personally, I prefer the simple, vibrating galvanometer to the "heterogalvanometer" method described by Mr. Borden, since the asynchronous contactor adds considerably to the complication of the apparatus.

There is another type of a-c. potentiometer, called the rectangular[1] or two-dimensional potentiometer, which perhaps is not so well known as the Drysdale, because it has not been generally described in textbooks and engineering literature. A simple diagram is shown in Fig. 1.

R is a non-inductive potentiometer wire, which may be subdivided by any of the various methods employed with d-c. potentiometers. The working current *i* is maintained at its correct value by a rheostat *r*, this current being measured by some type of a-c. milliammeter *A*. This milliammeter is preferably of the dynamometer type, since this type may be calibrated with a standard cell, using reversed d-c. readings. We have also found a vacuum thermocouple, used in conjunction with a d-c. milliammeter, very satisfactory. A hot-wire milliammeter is also satisfactory, if sufficient sensitivity is obtainable. The primary *P* of a mutual inductance *M* is connected in series with *R*. *D* is a detector. The component of the unknown e. m. f., which is in phase with *i*, is balanced along the resistance *R*. The component which is in quadrature with *i* is balanced by an e. m. f. $M\,\omega\,i$ supplied by the secondary of the mutual inductance M ($\omega = 2\,\pi\,f$), where f is the frequency in cycles per second. The unknown e. m. f. is therefore determined as two quadrature components, one along the axis of reals and the other along the axis of imaginaries. By using a reversing switch and positive and negative values of *M*, e. m. fs in any of the four quadrants may be measured. This type of apparatus is simple, does not require a phase-shifter, and may be made of apparatus usually to be found in the measuring-laboratory equipment.

I have found many instances where mutual inductances may be used for making e. m. f. measurements by null methods.

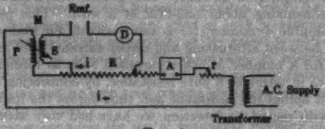

Fig. 1

I have also used the separately-excited dynamometer as a detector. Instead of obtaining two balances, one with the dynamometer across one phase of a polyphase system, and then across another phase, I have excited the dynamometer from the secondary of a small phase-shifter. In using a separately-excited dynamometer, one or two precautions are necessary. If the moving coil has a metal bobbin, and its plane does not coincide with the direction of the magnetic field, the induced currents in the metal will cause deflection, even though the moving coil is open-circuited. Even with a non-conducting bobbin, this same effect may occur, but to a lesser degree, due to the fact that a current flows in the moving coil windings, because of the capacitance between turns. Therefore, care must be taken when using this type of detector.

I have also used a Drysdale[2] polyphase phase-shifter to supply the current to the Drysdale potentiometer. This phase-shifter does not differ in principle from that described by Mr. Borden. It is more satisfactory than the phase-shifter supplied with the potentiometer, since the rather tedious task of splitting the phase accurately is eliminated, and also it is independent of the frequency.

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[1] "Alternating-Current, Planevector, Potentiometer Measurements at Telephonic Frequencies," Kennelly & Velander, *Proc. Am. Phil. Soc.*, Vol. 58, April 1919, pp. 97-132.

"A Rectangular-Component, Two-Dimensional, Alternating-Current Potentiometer," Kennelly & Velander, JOURNAL. Franklin Institute, July, 1919.

[2] "Electrical Measurements," F. A. Laws, page 502.

I have used rotating synchronous contactors of the commutating type, and in one or two cases have experienced difficulty due to frictional electricity being generated. For this reason, I have found that contacts which are opened and closed synchronously by some type of cam are more satisfactory.

I have not as yet had an opportunity to test Mr. Borden's heterodyne methods, and therefore cannot compare them with the methods which I have just described. Undoubtedly when there are slight but continuing fluctuations of voltage and frequency, more rapid balances can be obtained by the heterodyne method.

E. D. Doyle: Referring to Mr. Borden's paper (Fig. 8), it would appear to me that this is by no means a null or even a semi-null method. Every instrument there shows a deflection. With regard to the accuracy with which the checking can be done, one has the observational errors in both meters, *A* and *B*, and the only purpose that meter *C* can serve, is to read the difference between *A* and *B* when they are brought to a common indication. Of course, a meter can be held on a point with much greater accuracy than an observation can be made in between the cardinal points.

Referring to Fig. 9, the same observational error comes in as in Fig. 8. However, if the polyphase meter *A* were provided with a very weak suspension, that is, weak compared to the torques of the two elements, you may obtain very great sensitivity. We have applied this particular principle to a double dynamometer of the reflecting type, for comparing a-c. voltages with each other. We have an instrument which we built ourselves, and which enables us to compare a-c. voltages with an error of less than two-hundredths per cent. This is done with a meter which is only consuming about one-fifth of a watt. This particular type of double dynamometer is also available for making comparisons between alternating and direct currents, and may also be used for a number of other purposes, such as measuring an impedance in terms of a pure resistance, by connecting one element first across the resistance and then across the impedance.

Now with regard to detectors, we have had very satisfactory results from a synchronous rectifier and d-c. galvanometer. The rectifier we are now using has brass segments and butt copper brushes. When it is lubricated rather freely with ordinary machine oil we have no trouble at all from thermal e. m. fs., or bad contacts. We have also provided two sets of d-c. brushes, and a double throw switch, we can quickly throw from a position where we are sensitive in-phase voltages, to the reactive position and vice versa thereby making it much more convenient from the operating standpoint.

One other point which I would like to bring up in connection with the various methods of detection is the influence of harmonics in the wave on the particular apparatus. Of course, the vibration galvanometer is sensitive only to the fundamental frequency. The rectifier has a greater sensitivity for fundamental frequency, but any harmonics in the wave, will be detected to a greater or less extent, depending on the phase of rectification, and also on the order of the harmonic.

At a maximum the third harmonic will influence the galvanometer to one-third of its value; because in rectifying two of the loops will balance out, leaving only the third. In a similar way, the fifth harmonic will appear only to one-fifth of its value and so on. The double dynamometer, which has its two elements entirely separate from each other is affected only by the effective value of the wave irrespective of any harmonics which may be present.

P. A. Borden: Both Mr. Fortescue and Mr. Stein have compared the use of the synchronous with the asynchronous dynamometer, the latter gentleman being inclined to differ from my statement that the synchronous use of the instrument was "slow and cumbersome," as compared with the other. I have made many tests using the electrodynamometer in a-c. work, both the indicating type, having about the sensitivity of an ordinary wattmeter, and the reflecting type with a sensitivity comparable to that of a d'Arsonval galvanometer. In these tests I have found. I think I may safely say, without exception that the slow swing introduced by asynchronous excitation is much easier to reduce to zero than the two separate deflections necessary where excitation is derived from the same source as the test. This, of course, may lie partly in the accessory equipment used; and I may here say that practically all the apparatus employed in the work described in my paper is that intended for general use in a laboratory called upon to perform an immense variety rather than a great volume of tests, so that the primary object in the equipment is flexibility rather than specialized fitness for one type of test.

I am pleased to learn that use has been made of the asynchronous commutator in conjunction with a galvanometer. Upon trying this scheme, I was surprised that anything of such evident usefulness had not previously been employed; but being unable to find any published record of its use, by Mr. Fortescue or others, I was forced to refer to my own development of the idea.

Mr. Stein has taken issue with my statement that a-c. instruments are of *inherently* lower sensitivity than direct, and states that if you can use iron in an a-c. instrument you can get the same degree of sensitivity as in a direct. You can, *if you can use iron*; but conditions are unfortunately such that the use of iron is generally out of the question. Again, on reading the context, as it appears in the first paragraph of my paper, I think it will be quite evident that I was here referring to indicating instruments rather than to detectors as used in balance work; and it is a well known fact that even the best makers have not attempted to market a-c. instruments which for smallness of energy consumption could compare with instruments of the permanent-magnet moving-coil type. In the use of an instrument of the electrodynamic type, incorporating iron in its magnetic circuit, my experience is confined to a certain type of a-c. galvanometer, in which I found that before the damping resistance could be reduced to a value to cut out swinging on 25-cycle work, the circulating currents in this circuit, due to e. m. fs. set up in the moving coil completely obliterated the quantities under investigation. In my opinion, it would have been necessary in this case to fit the instrument with a torsion head and make all observations with the moving coil in the position of zero mutual induction with the field.

I think I may be pardoned for having overlooked the Hospitalier Ondograph as an instrument making use of the asynchronous commutator, in view of the fact that the use of that device as a wave tracer is so radically different from my application of the principle as a voltage detector that I had neglected to associate the two ideas. As to the use of the word "Null" in my work, I am quite ready to yield the point to Mr. Stein; and if my usage is not in strict accord with the dictionary definition of the word, admit the fact that I sought no higher authority than the generally accepted technical works and writers. As to whether in the particular instance to which Mr. Stein refers, I have properly classified the test under the heading of "Null," I leave to those interested, to decide. And at the same time I thank Mr. Stein for pointing out the wide uses to which may be put the circuit shown in Fig. 4.

I wish to thank Mr. Brooks for calling attention to the fact that the elements of a polyphase wattmeter may not be in exact balance, at zero or at other points of the scale; and while this was recognized by the writer, it is well to repeat that in tests of this class it is assumed that a high grade instrument is used, in which this unbalance is very small. At the same time; as the deflection of the wattmeter is by the method used reduced to zero, it is usually sufficient that the strengths of the respective elements be equal only at the zero point.

Mr. Doyle has called attention to the possibility of observational errors in the additional meter used in the calibration described in Fig. 8. True, the error of the instrument C, as well as that of A comes in, but in a negligible degree. In the first place, A, the standard is read when indicating on a cardinal point, which is much more definite than the interpolation which would ordinarily be necessary. At the same time, this would usually be a point whereon the accuracy of the standard was determined in comparison with a primary standard. In the second place any error in the calibration of C, appearing as an *error in the error*, would be of the second order of magnitude, and hence usually negligible. The possibility of observational errors, too, is much reduced; for once a setting has been obtained, it may be maintained indefinitely until the tester is assured that the principal meters are in agreement on the point at which the observation is being made, after which he may read the error, as indicated on C, at his leisure. What I have said in regard to observational errors in this test applies equally to Mr. Doyle's comment upon similar errors in the transformer test in Fig. 9, with the additional feature that the wattmeter when finally balanced, is read only upon the zero point of the scale. A two-element meter with a weak spring would, without doubt, greatly increase the sensitivity of this test; but I have found the ordinary type of polyphase instrument, when used on the zero-deflection principle, to have sufficient sensitivity to prove itself of great value in this work.

In demonstrating this method of efficiency determination, I have often made use of a small motor-generator set, driven from a d-c. supply, and giving a single phase output. Such a set has, of course, comparatively large losses; and when the output and input are opposed in a two-element wattmeter, the convenience of the method becomes very evident. This method is particularly useful in determining the efficiency of single-phase rectifiers, and does away with much controversy as to the type of instruments used in the measurement of the rectified output.

General Considerations in Grounding the Neutral of Power Systems

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Review of the Subject.—*In the early days of power transmission, there was no consistent practise in respect to operating with neutral isolated or with neutral grounded. The rapid growth of transmission systems with their extensive networks soon began to show disastrous results from arcing grounds on isolated neutral systems and now most power transmissson networks have their neutrals grounded in some manner.*

The discussion of general considerations of neutral grounding is divided into two parts, that of overhead line systems and underground cable systems. He brings out the fact that while most overhead systems are grounded there is some difference in practise as to the extent to which they are grounded, that is as to whether they are grounded solidly or through resistance. Prevailing practise tends toward little or no resistance.

Attention is called to different possible methods of grounding a system and shows by diagrams the flow of short-circuit current with the different methods.

Underground cable systems are consistently operated with neutral grounded but general practise tends toward the use of resistance in neutral. General considerations as to protection from the voltage strains due to arcing grounds on cable systems are similar to overhead line systems and the author analyzes briefly the character of cable breakdowns and general effect of such breakdowns with a view to determining the importance of the extent to which a cable system should be grounded. The conclusion in regard to cable systems indicates about the same limitations as those found for overhead systems and no very good reasons are found for a distinctive difference in practise.

The paper considers the use of grounding resistors of different types and gives some cost figures to show the effect of time and current in the design of metallic resistors.

The general conclusion arrived at is that, on either overhead or underground transmission systems, high-voltage strains are more to be feared than high-current strains and that resistance to limit ground current, if used at all, should be of very low value.

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THE question of grounded neutral has been under discussion for many years and many papers have been written covering various phases of the subject from time to time, each with the purpose in view of presenting data tending to show the logic of the author's conclusions on the particular phase of the subject which has interested him. The general practise of the country has undergone certain changes during the past ten or fifteen years in the methods and extent in which systems have been grounded. These changes of practise have been brought about partly from the influence of individual writers' ideas as presented in the various papers mentioned and partly by an actual change in the problem due to the rapid growth of our transmission systems during that period.

In the early days of power transmission, there was little to base one's judgment upon in determining the advantages or otherwise, of operating the system with the neutral grounded. Such experiences as had been obtained by the pioneers of transmission were complicated by conflicting evidence due largely to the fact that where trouble had occurred, it was often accompanied by the presence of many uncertain factors, and no direct comparison could be made between the isolated neutral and grounded neutral arrangement.

From the beginning of power transmission in 1888 or 1890 to 1910 or 1912, it is safe to state that there was

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no general practise as regards the question of operating transmission systems either with grounded neutral or with the neutral isolated. During that period, transmission voltages increased at a very rapid rate, the length of individual lines for straight-a-way transmission greatly increased, and the number and length of interconnected lines forming networks handling thousands of kilowatts of power where hundreds had been handled before, presented an entirely new phase of the question.

A glance at the transmission problem of today seems to divide the question into two natural classes, that of overhead lines which have grown in extent and voltage to cover very wide areas handling large blocks of power, and that of underground transmission systems which cover smaller areas but with a much higher density of power per square mile and much lower transmission voltage.

The overhead systems generally speaking, vary in voltage from 13,200 volts to 220,000 volts and involve an aggregate of some 50,000 miles of line. There are several systems having a metallic connection of circuits extending well above 3000 miles at voltages around 44,000 volts and several systems having 1200 or 1500 miles of line at voltages of 110,000 volts or above.

The underground cable systems have an aggregate cable length of perhaps 10,000 miles and handle close to the same amount of power that the overhead lines do. An adequate discussion of grounded neutral versus isolated neutral, requires a careful consideration of the operating characteristics of the two types of systems and an analysis of the results of practical operating conditions from the early days of transmission to the present time.

OVERHEAD TRANSMISSION SYSTEMS

It probably can be truthfully said that in the early stages of transmission practise where the extent of the systems was small and the voltage low, there was no decided difference in the results of operating either with neutral grounded or ungrounded. As they grew in extent, voltage surges began to make themselves felt over wide areas, resulting in breakdown of insulation at various points. In seeking a remedy, many engineers grounded the neutral of their generators or transformers.

So many systems were installed simultaneously and at ever increasing voltage that there was not time to make an analysis of the advantages of either plan and such practise as was shown in the development of our American systems was the result of individual engineer's ideas and experiences during a period when development was too rapid to compare notes thoroughly and to build up a practise based upon weighted judgment and experience.

It has been the natural result then, that during the years from about 1904 to 1912 or 1914, many systems were put into service, some arranged for isolated neutral and some for grounded neutral operation as well as others for what might be called a compromise between the two, that is, neutral grounded through extremely high resistance. During this period, then, it might be said that there were two schools of engineers making an impress on the practise of the country. Neither had very much past experience on which to base its judgment and their conclusions were drawn largely from theoretical considerations.

In the early days of high-voltage overhead line transmission development, certain principles of design and factors of safety of insulation were followed which results seemed later to conclude to be inadequate. Line insulators in particular showed too small a factor of safety and too short a life under operating conditions. The overhead transmission systems during the period mentioned have passed through many vicissitudes and few of the line insulators in use as far back as 1904 are still in service. We have gone to higher insulation and have vastly improved the design of porcelain insulators to make them stand up under operating conditions.

In 1912 or 1914 there were thousands of miles of overhead lines in operation on systems using isolated neutral and grounded neutral as well as some systems with the neutral grounded through high resistance. Sufficient experience had been obtained to enable engineers contemplating the design of new systems to take stock of past experience and the result of this stock taking seemed to indicate that the grounded neutral systems had certain advantages in most cases over the isolated neutral system. The evidence was not conclusive at that time and perhaps is not conclusive today, but there are certain fundamental principles that seem to be reasonably definite and most of them tend to show important advantages in favor of the grounded neutral systems.

The arguments in favor of the isolated neutral system, especially with delta connection, were that a single broken insulator would not cause a short circuit and consequent loss of service over an individual line and that the breakdown of a single transformer did not completely interrupt the service of an entire bank, as it was possible to operate with one line grounded or with two transformers out of three connected in open delta and maintain at least partial service. There was a third theoretical consideration that with delta-connected transformers, any disturbance coming in over one conductor, such as lightning or a steep wave front surge, would have its energy expended over two transformers and be less likely to cause serious damage. In the early days of transmission when the mileage and voltage were low, many systems operated for hours at a time and in some cases days, with one phase grounded and there was considerable use made of open delta connection of transformers on such systems.

It was argued that line insulators were designed to have the necessary factor of safety with full potential to ground and that while the grounding of one phase increased the strain on a given insulator to 73 per cent above normal, it should entail no hardship as the insulator was designed for continuous operation on this basis. Practise, however, showed that most grounds of this nature were not solid grounds but were accompanied by severe arcing which produced oscillations tending to build up high-frequency and high-voltage disturbances which placed the whole system under severe strain and often resulted in breakdown of apparatus at different points on the system.

As the systems grew in extent and increased in voltage, the severity of the disturbances due to these arcing grounds, became a very serious menace to service, not only from the fact that breakdowns often occurred at widely separated points on a network as a result of an arcing ground at a given point, but these disturbances resulted in the breakdown of apparatus such as bushings, transformers, lightning arresters, etc.

As the amount of power transmitted grew, the importance of isolating cases of trouble to the section of line actually in trouble, became apparent. The great exposure to lightning, varying weather conditions, etc. of these extensive systems considerably increased the actual number of unavoidable breakdowns in spite of the greatest vigilance and the question of continuity of service became more and more important as the territory served, increased, and much energy was brought to bear on the study of protection for these systems.

Up to this time, the relay situation was not all that could be hoped for but improvements were going on constantly and there was every hope that if the distribution of current during breakdowns could be predetermined, satisfactory relays for selection of trouble, would be available.

An analysis of the character of breakdowns with this in view, gave some pretty definite indications that on isolated systems the problem was an extremely complicated one. Nearly all transmission line breakdowns occur initially as the failure of some weak insulator from lightning or other cause. The current flowing in the arc caused by this breakdown, is the charging current to ground of the other two conductors of the entire interconnected system. This current may be considerable, depending upon the amount of interconnected line, but is almost entirely wattless and shows up in such an uncertain manner as to be useless for selective relaying. During the time that this arc is playing over the weak insulator, the other two phases of the whole network are subjected to the strains due to this arcing ground until a second breakdown occurs, producing a short circuit from phase to phase and causing sufficient power current to flow to trip the relays.

It is evident that the location of the second breakdown will be uncertain and one that should not have occurred if the line in trouble could have been tripped out by the relay before the arcing ground surge had time to spread the trouble. This second breakdown often occurs on an entirely different line, perhaps miles from the origin of the trouble and it is entirely within the range of experience to have several secondary breakdowns on different lines or different parts of the same line following an arcing ground at a given point. Such occurrences make selective relaying out of the question and a general shut-down of the system may well occur under such conditions.

The frequent occurrence of arcing ground troubles on isolated neutral systems led to the development of the so-called arcing ground suppressor which was brought out and advocated by Mr. Creighton and also by Mr. Nicholson, with the hope that a dead ground immediately placed on the phase in trouble would eliminate the destructive effects of the arcing ground and make it unnecessary to use relays to take the line out of service until inspection and necessary repairs could be made. The arcing ground suppressor however, was an expensive device and only a few were installed.

Another remedy for the elimination of a multiplicity of breakdowns was to ground the neutral of the circuit and thus make all initial breakdowns short circuits, which positively defined the flow of current and made possible the selective action of relays. As most systems feed their important stations by more than one circuit the loss of a line due to trouble is not prohibitive and the system of protection that has grown up on high-voltage networks has been to ground the neutral either solidly or through resistance and to make the maximum use of selective relays to cut off the section of line in trouble in the shortest possible time.

The good results obtained from grounding high-voltage networks and increasing the efficiency of protective relays has been such that the practise has extended to practically all overhead line work and the great majority of transmission line networks at all voltages from 22,000 to 220,000 volts are laid out with some plan of grounding the neutral. The usual practise is to establish a neutral by grounding the transformers at the source of power, that is, the generating station or generating stations as the case may be, employing delta-*Y*, step-up transformers at these stations. A definite flow of short-circuit current away from the generating stations is thus established and on straight-away transmission lines, particularly, leaves it optional as to the type of transformer connections that may be used at the step-down substations. Grounding at the generating stations and not the substations allows the use of delta-*Y*, step-down transformers at the substation, thus establishing the neutral point at the source of power for the secondary network emanating from these substations. This general practise may be followed on to the ultimate distribution system in several steps if desirable, each network in itself having its neutral grounded at the source of power.

There has been some difference in opinion as to the desirability of several neutral grounds on an overhead network at distant points, on account of the possibility of interchange current between neutrals which may affect telephone communication. There are, however, in operation, many extensive transmission systems with neutrals grounded at generating stations as well as at many substations with little, if any, telephone interference. The normal interchange of current between neutrals is usually so small as to be negligible and there have been in fact, only a few isolated cases where telephone interference due to circulating current in neutrals, has manifested itself. The conclusion from operating experience to date, indicates that no serious telephone interference under normal operating conditions need be anticipated even though the neutral is grounded at many points.

For the effective application of selective relays on a complicated network, a calculation of the short-circuit current with its distribution as to amount, direction and source, is of great value. Such calculations are comparatively simple with only one neutral grounded but where several are involved, each contributes to the ground current modifying the amount and direction, depending upon the location of these ground points. It is sometimes very difficult to predetermine this ground current with sufficient definiteness as to allow accurate selection and adjustment of relays so that in studying the question of grounding the neutral of a system, it is well to have this point in mind and to attempt a determination in advance as to the complications that may arise from the addition of neutral grounding points.

There is a number of possible combinations both in the connections and kind of transformers that may be used for grounding a system, and the different possible locations of ground points and an analysis of some of these combinations may be of interest.

The simplest arrangement is as shown in Fig. 1, with one generating station transmitting power over a single three-phase line to a substation. In such a case, delta-Y transformers would be used at the generating station with the neutral grounded at this point. The substation transformers may be connected in any manner, if there is no generating equipment at this point, without affecting the distribution of current. In such an arrangement with the ground at G, the flow of current will be in one conductor only, direct to the ground point and back to the neutral, appearing in one phase only in the transformer and two phases only in the generator. The calculation of this current is simply taking into account the reactance of the generator, transformer and line to the ground point.

Fig. 2, shows a similar arrangement of transmission line but with the substation transformers connected Y-delta with the neutral grounded at the substation only. The flow of current in this case is quite different from that in Fig. 1, and will be as indicated by the arrows on Fig. 2. It will be noted that the current in all three legs of the substation transformer Y are equal and so far as the generating station is concerned, there are also heavy currents in all three phases; the current in the grounded phase, however, being double that in the other two phases. The reason for the equal distribution of current in all three legs of the step-down transformer will be apparent, in that, if there is current in the primary of the transformer of the grounded phase it must appear in the secondary and if it appears in the secondary, it must flow, and the only path is around the delta. This puts the same current in the other two legs which must again reappear in the primary giving balanced current in the substation transformer which acts as a grounding transformer for the system.

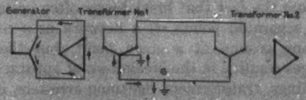

FIG. 1—TRANSMISSION SYSTEM WITH NEUTRAL OF GENERATING STATION TRANSFORMER GROUNDED

Line ground at point "G". Arrows indicate flow of current

Fig. 3, shows a case similar to Figs. 1 and 2 with the exception that both the generating station transformer and the substation transformers have their neutrals grounded. In this case the distribution of current will be similar to a combination of Figs. 1 and 2, current flowing as in Fig. 1, directly to the ground G from the generating station and appearing on one phase of the transformers and two phases of the generators. There will also be a current back from the substation neutral to the ground G, which current will appear in all three phases of the substation transformers. The flow of current to the ground from the generating station direct and the generating station by way of the substation, will depend upon the relative impedance of the various circuits and its calculation is somewhat complicated. For a comprehensive analysis of the calculation of single-phase, short-circuit currents on complicated networks, see article by W. W. Lewis, entitled, "Short-Circuit Currents on Grounded Neutral Systems" in the *General Electric Review* for June 1917.

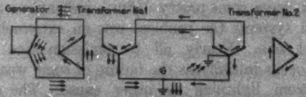

FIG. 2—SAME SYSTEM AS SHOWN IN FIG. 1, EXCEPT SUBSTATION TRANSFORMER NEUTRAL GROUNDED

Fig. 4, shows a similar arrangement of circuits but with two substations, each having Y-delta, step-down transformers with their neutrals grounded as well as the neutral grounded at the generating station. The distribution of current on this system will be similar to that shown in Fig. 3, except that still greater complication due to the division of current among all three neutrals, will follow, the substation transformers having equal current in all three legs but the grounded phase in the generating station carrying the heaviest current. This again will divide up with relation to the impedance of the various circuits.

Fig. 5, shows an arrangement with two generating stations each having the neutral grounded, which introduces a combination of Figs. 1 and 3. The generating station "A" will feed direct to the short circuit. The generating station "B" will feed direct to the short circuit. Generating station "A" has a tendency to feed over the other two lines to the transformers in generating station "B" which will act similarly to the substation transformers in Fig. 3. Generating station "B" has a tendency to feed to the transformers in Station "A" and back to the short circuit in a similar manner. The distribution of current under such circumstances is problematical, depending upon the location of the short circuit with respect to the two stations assuming they are of the same size. If these stations are of different size, the relative impedances of the various paths to the short circuit will influence the division of current. The above examples will serve to call attention to the possible complication in relay equipment that may be encountered in systems grounded promiscuously. There are many other possible combinations that might be mentioned but they are nearly always only variations of the cases cited above.

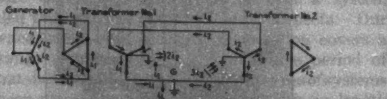

FIG. 3—SAME SYSTEM AS SHOWN IN FIG. 1

Neutrals of transformers at both generating station and substation grounded.

It will be noted that in the suggestions above, delta-*Y* or *Y*-delta transformers have been used in all cases. Transformers connected in this manner are most usually employed for establishing neutral points on a system. There are a few other combinations that have been used occasionally, such as transformers connected *Y*-*Y* with a delta tertiary winding. Where this combination is used, a neutral may be established on both primary and secondary networks, the delta tertiary winding acting as a secondary for either high-tension or low-tension *Y*, depending on which circuit the fault occurs.

A three-phase, core type transformer connected *Y*-*Y* will act as a partial grounding transformer, as the interlinked magnetic circuit, allowing the circulation of flux, performs somewhat the same function as the delta which allows circulation of currents. It, however, is a rather inefficient grounding transformer as its reactance with one phase grounded is of the order of 50 per cent. That is, it will allow only about two times full-load current of the transformer to flow.

Another form of transformer that may be used to establish a neutral and allow heavy currents to flow in case of a fault to ground, is the so-called zig-zag or interconnected *Y* transformer which has no secondary winding as such. The winding on each core leg consists of two equal parts. The upper part of one leg is connected to the lower part of the adjacent; the three phases thus formed, being connected in *Y* and the neutral grounded. When a ground occurs on the line, this transformer acts similarly to a *Y*-delta grounding transformer as indicated in Fig. 2, equal currents appearing in all three phases. One-half the winding on each leg acts as a primary and the other half as a secondary winding, the currents in each half being equal and opposite in direction. Such a transformer is equal in efficiency as a grounding transformer, to a *Y*-delta connected transformer and generally speaking, will cost slightly less to build. Transformers of this type have been used to ground the neutral of systems where no *Y*-delta connection is available to establish a neutral point for no other purpose than for protection.

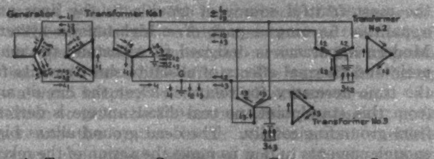

FIG. 4—TRANSMISSION SYSTEM WITH GENERATING STATION AND TWO SUBSTATIONS

Transformer neutrals at all stations grounded

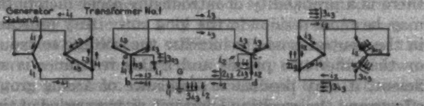

FIG. 5—TRANSMISSION SYSTEM WITH TWO GENERATING STATIONS TRANSFORMER NEUTRALS AT BOTH STATIONS GROUNDED

Current i_4 flows as shown if voltage *c d* is greater than voltage *a b*. If *a b* is greater than *c d*, then i_4 flows in reverse direction.

The use of the *Y*-*Y* transformers is very limited and confined almost exclusively to three-phase core type units as *Y*-*Y* connected single-phase transformers invariably have circulating triple harmonic currents between their neutral and the neutral of any other grounded transformers on the system thus tending to cause telephone interference. If there are no other neutrals, the lack of a triple frequency exciting current for these transformers tends to produce triple frequency voltage between line and ground and to put excessive voltage strains on the system. Transformers connected *Y-Y* or auto transformers *Y*-connected, perform no function as grounding transformers unless their neutrals are connected to the neutral of a generator or other *Y*-connected transformer which would in itself, act as a grounding transformer.

The question of the desirability of limiting the current in the neutral on overhead systems is one on which there is a considerable difference of opinion. There is little question that the greatest protection from high-voltage strains comes from the use of a solidly grounded neutral and that in the long run high-voltage disturbances are the most fruitful source of trouble to the lines and apparatus forming a part of a high-tension system. Modern apparatus is designed to withstand the magnetic and thermal effects of absolute short circuits for the time necessary for relays to clear the circuit and from this standpoint, no real disadvantage is derived from grounding solidly. The dead ground allows high enough currents to flow to make the action of the relays definite and a tendency to greater selectivity results. Where the proper relays are used and short time settings employed, the jolt to the system tending to trip synchronous apparatus out of step, is not a serious matter. In any case, many phase to phase and three-phase short circuits are bound to occur and must be provided for.

The main disadvantage claimed by the advocates of limited currents to ground, are almost entirely those of telephone interference. This is a difficult point to pass upon for all cases, as the relative exposure varies as does the relative activity of the engineers of the telephone company in different territories. There is no question that with a short circuit to ground of heavy current, the acoustic shock on an exposed telephone line is considerable and is to be avoided if possible unless there is a probability of introducing other factors that may be more serious in the long run. Any resistance in the neutral tends to increase the voltage to ground on the other two phases and if this resistance is sufficient, it may produce a semblance of arcing ground effects and the resulting operation over a period of time may cause more acoustic shock trouble than would be the case with a solidly grounded neutral system.

There are different methods employed for limiting the ground current, such as neutral resistance, grounding only one transformer of limited capacity or by the use of a small grounding transformer bank. The grounding resistance for high-voltage transformers is an expensive device and requires considerable attention and maintenance. Various types of resistance have been used such as metallic grid or wound tube resistors, water resistance, concrete and graphite columns, etc. The most satisfactory resistance, however, is either the grid or tube type. Fig. 6 shows a 45,000-volt neutral grounding resistance of the tube type designed for two-minute duty. The individual resistance tubes are insulated by porcelain between sections and post type porcelain insulators to ground. This completed resistor is insulated for line to neutral voltage and is not only very costly to build but requires considerable installation expense from the fact that it takes up a large amount of room and must be housed.

At the present time, the greater number of high-voltage systems are operating with the neutral solidly grounded at their important generating stations. As an example of such systems, the following may be mentioned:

Alabama Power Company, Georgia Power Company, Southern Power Company, Mississippi River Power Company, Montana Power Company and the Utah Power & Light Company and many others. There are a few high-voltage systems operating with neutral resistance such as the Hydro-Electric Power Commission of Ontario, Niagara Falls Power Company, Niagara & Lockport Power Company and Public Service Company of New Jersey and a few others. The Pennsylvania Power & Light Company, Shawinigan Water & Power Company and the Georgia Power Company operated for several years with a neutral resistance but have since taken them out of service.

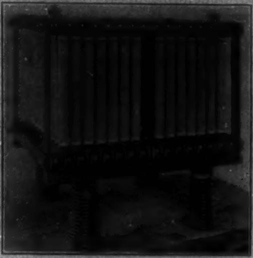

FIG. 6—NEUTRAL GROUNDING RESISTANCE FOR 45,000-VOLT SERVICE

The effect on methods of relaying high-voltage systems with limited ground current for the selection and isolation of faults, varies with the extent to which the current is limited.

A ground resistance is often selected of such value as to allow sufficient current to flow to trip overload relays on any feeder and in such cases the current amounts to two or three times full-load current of the largest one. There are no complications with such an arrangement and any type of relay that could be used with solidly grounded neutral, will give equally good results with such a neutral resistance. Systems using high resistance in the neutral and consequent low ground current must resort to special methods of relaying for selective action and there is usually a narrow margin of error allowable in these special cases. The overload relay has no function in clearing a fault on such a system except to back up the special selective relays in case a phase-to-phase short circuit ensues. In case of a ground on a line, the small amount of current, where a high resistance in the neutral is employed, may be indefinite in its action, due to the fact that it is a small percentage of the normal load current on the various circuits and this action is complicated by the action of charging current to ground, and it is doubtful if as positive results can be obtained as in the case where comparatively heavy currents flow and their amount and direction can be reasonably closely determined.

We have today, high-voltage systems operating up to 110,000 volts with the neutral grounded solidly in many cases and in a few cases with the neutral grounded through varying values of resistance.

It is difficult to make a comparison of the operation of any two systems with respect to the effect of any one feature. Generally, results are influenced by so many factors that equivalent continuity of service on two dissimilar systems would give little proof of the advantages of one protective scheme, over another. It is, therefore, difficult to say from the experience to date that the system with the solidly grounded neutral or with the neutral grounded through comparatively low resistance shows distinct advantages over a system grounded through high resistance in all respects. Our experience under both schemes of operation covers a wide range of voltage over a considerable period of time, but during this period of time both types of systems have been constantly increasing in capacity, in length of line and complication of network, and have been under the handicap of passing through a transition period in insulation, relaying, maintenance methods, etc., all of which upset the conclusions which might otherwise be drawn.

The generally accepted primary object in grounding the neutral of high-voltage systems is to avoid the possibility of trouble due to arcing grounds. An arcing ground produces a resonant condition, building up voltages of high value that subjects the entire system to abnormal strains likely to cause serious damage. These strains bear a relation to the character of the current in the arc, that is, whether it is lagging, leading or of high power factor. The current in the arc of an isolated system is almost entirely leading current and produces oscillations giving much more severe disturbances than will a lagging current arc of the same value. Where solidly grounded neutral systems are involved, the current in the arc is usually of comparatively high value and of very low power factor lagging. The power factor of the current in the arc is influenced by the voltage and extent of the system, the power factor being a combination of the low power factor lagging current due to the short circuit and the leading current flowing to ground from the entire system and this leading current is of practically the same value that it would be on an isolated system of the same extent. In all ordinary cases of dead grounded neutral, the short-circuit current will easily predominate. The oscillatory character of the arc will, therefore, be small in such a case.

With an extensive high-voltage system grounded through a comparatively high resistance, it is easily within the bounds of reason to expect the charging current to ground to be as high or higher than the short-circuit current when limited by resistance. Under such circumstances the arc will tend to be oscillatory and the strain on the system will approach arcing ground strains on an isolated system. We have 110,000-volt systems operating with an interconnected network of more than 1500 miles. The charging current on such a system to ground would be in the neighorhood of 750 amperes. It will be evident that a high resistance in the neutral of this system which limited the current to 100 amperes or so, would have little effect in the way of stabilizing the arc and keeping down the high-voltage strains. In addition to this, the 750-ampere charging current goes back into the system in an uncertain and variable manner and makes selective action of relays most difficult.

Above 110,000 volts we have no systems operating as yet with neutrals grounded through resistance. At higher voltage the charging current feature becomes more important for the same mileage of interconnected lines. The Japanese power systems have adopted 154,000 volts as their standard transmission voltage and several extensive networks are under construction at this voltage. It is their plan to ground the neutral through high resistance limiting the current to low value and in some cases the extent of the system will be such as to make the charging current to ground predominate. The relay problem of such a system is complicated and expensive and the safety of operation under these conditions is yet to be proved.

There are other methods of limiting the current to ground as mentioned before, such as only grounding one generating station or only part of the transformers in one generating station. With such an arrangement the power factor of the current in the arc would be of lower lagging value from the short circuit itself than in the case where the current is limited by grounding resistance. It is doubtful if a materially different result would be obtained, however. Many systems originally laid out with delta-connected transformers have since been grounded by the use of what is commonly called a grounding transformer, such as a *Y*-delta transformer or a zig-zag transformer, discussed previously. In selecting the size of such a transformer bank the amount of current required for the kind of protection decided upon, determines the bank capacity. If sufficient current to trip overload relays on any part of the system is required, the bank capacity must be such that under short-circuit conditions it allows this current to flow through its own reactance and through the reactance of lines to the grounding transformer and away from the grounding transformer to the farthest point on the system. As an example, a grounding transformer on a 110,000-volt network, would be required to furnish sufficient current to trip an overload relay on any 110,000-volt line. As an average value the normal line capacity may be taken as 25,000 kv-a. Under abnormal conditions this line might be required to carry as high as 50,000 kv-a. The overload relay settings should be something above this value, say 60,000 kv-a. The grounding transformer then should be able to pass 60,000 kv-a. equivalent current over the grounded phase. Assuming no reactance in the circuit to the grounding transformer, a Y-delta bank of 6000 kv-a. having 10 per cent reactance would be required, or a 4800 kv-a. bank having 8 per cent reactance. If the longest line from the substation should have a reactance based upon this 60,000 kv-a. of 10 per cent a transformer bank 10 per cent larger of the same reactance would be required.

Such a grounding transformer would give protection equivalent to a system grounded through a neutral resistance of low enough value to allow the same current to flow. If more effective grounding is desired, such as afforded by the solidly grounded neutral system through a large percentage of its generating capacity, a still larger bank must be used or a bank of lower reactance, thus allowing more current to flow.

The advantage in a high-current flowing would seem to be the assurance of a minimum increase in the voltage strain of any part of the system to ground above its normal value. A heavy current flowing from the generator tends by armature reaction to not only reduce the voltage in the grounded phase but also in the other two phases. Full-load current on the generating station flowing to ground through a circuit consisting of step-up transformer, transmission line and grounding transformer would have a fairly low lagging power factor which would have a tendency to drop the voltage of the entire system and in consequence the voltage to ground from the two ungrounded phases to a value little in excess of normal and insure no excessive voltage strains on any part of the system in case of trouble.

It has seemed to the writer that a rule to the effect that ground current at least equal to the full-load current of all the generators should flow. This would give adequate grounding protection to the system and would be a surer rule to follow than merely to allow sufficient current to flow to trip the relays. In some cases this rule would give about the same results as the method of arriving at the size of grounding transformer by a determination of the short-circuit current necessary to trip all the overload relays taking into account multiple circuits, etc.

In case the engineer determines that he wishes to create the same conditions by use of a grounding transformer as he would on a new system by the use of high resistance in the neutral, the grounding transformer would be, of course, correspondingly smaller in capacity as the requirements would be much less.

It will be evident from the foregoing discussion that the extent to which a system is grounded is usually dependent upon the relation of ground current during a short-circuit to the full-load current of the total generating capacity and not necessarily on whether resistance is used in the neutral or not. We normally speak of a system as grounded solidly if no resistance is used, but many so-called dead grounded neutral systems are grounded through such a small part of their total generating station transformer capacity as to allow less current in terms of a percentage of their total generating capacity to flow, than in some other systems grounded through resistance. The impedance from the generators to the point of short circuit is really the measure of the extent of the grounding.

Adequate grounding for the maximum voltage protection of a system will always be obtained by grounding all the generating station transformers and usually by only one half or less. The grounding ot any system at its source of power through all the transformers feeding that system will give ample assurance of no arcing ground effects tending to produce high-voltage strains. There is not as a rule, much object in grounding the receiving stations, practically the only advantage being to insure a neutral point on a station normally acting as both a receiving station and generating station should this be split apart from the main system. There is in fact, some disadvantage in grounding the neutral of several substations fed from the same line or network as a ground at any point on the system draws short-circuit current from every one of these as illustrated in Fig. 4, tending to trip their overload relays or blow their fuses as the case may be, while there is no reason why these stations should be involved in the short circuit at all except for a slight dip in voltage on one phase.

Another method of neutral grounding is by use of a reactance coil between neutral and ground. Such a reactor when designed with an ohmic value such as to allow a current to flow equal and opposite in phase to the charging current to ground of the entire line on which it is connected, is called a Petersen Coil. Papers are being presented at this meeting by Mr. W. W. Lewis and Messrs. J. M. Oliver and W. W. Eberhardt, discussing the action of such a reactor and some operating results from its use. It is probable that such a coil has a very limited field of application and that on comparatively small low-voltage systems or very short lines up to perhaps 110,000 volts where the charging current is of low value and a reasonable percentage of error in tuning will not be likely to allow sufficient current to flow over the insulator to maintain an arc.

CABLE SYSTEMS

The practise in grounding underground cable systems which are usually supplied direct from the generators without the interposition of transformers has in the main, been somewhat different from the practise on overload transmission systems. The greater number of such systems are operating with the neutral grounded as is the case on overhead lines but a much smaller proportion are operating solidly grounded. It has been quite common practise on such systems to use a small amount of grounding resistance designed to limit the fault current to a value calculated to trip overload relays. A few are operating with solid grounds and a few with comparatively high resistance but the latter are so few in number as to require little comment. No difficulty from telephone interference need be provided for and the object of extremely low fault current being absent accounts for the difference in practise in this respect as the same difficulties are met in selective relaying from the low short-circuit current complicated by high ground charging current.

The most usual method of grounding the neutral of an underground system is to ground one generator through a resistor at each generating station, provision being made to ground any one unit at a time for the sake of flexibility. This practise grew from the fact that in the early days generator wave shapes differed widely and large third harmonic currents were likely to circulate between neutrals if more than one unit were grounded at a time. Modern generators have only a slight third harmonic and such circulating currents will be negligible in most cases. There is, however, no great object in grounding all the generators in a station where a resistor is used as in most cases one unit can provide all the short-circuit current that can pass through the resistor.

Underground cable systems have grown in extent and amount of power handled at a rate that is comparable with the growth of overhead systems and there is a marked tendency just at this time for even more rapid growth. The generating stations being constructed by the larger companies are making the distribution of this power more and more difficult to handle and the voltage of cable distribution is increasing at a rapid rate. At this time, there is a considerable amount of 22,000 to 24,000-volt cable in operation and some at 26,400 volts and one or two installations in the United States of 33,000-volt, 3-conductor cable. There are also one or two installations of 66,000-volt, single-conductor cable being seriously considered. Abroad they have been operating 33,000-volt, 3-conductor cable for sometime, have a 50,000-volt, 3-conductor system about to go in operation and two or three 60,000-volt, single conductor cable systems working. In all of the higher voltage cable installations the neutral has been grounded and up to date, everything above 26,000 volts, has had this neutral solidly grounded. Grounding in this manner definitely insures a limitation of the voltage strain to ground, during faults, not exceeding the leg voltage, which in the present state of the art at least would seem to be extremely desirable.

There seems to be a preponderance of evidence from operation of underground cable systems to indicate that practically all cable faults occur initially as breakdowns from one conductor to the sheath rather than from conductor to conductor. The limitation of the ground current thus caused to flow by means of resistance in the neutral of the generator or transformer feeding the system and the use of selective relays with extremely short time setting gives a good chance of clearing the trouble before the fault can develop into a phase-to-phase short circuit. Of course the greater proportion of underground cable troubles have developed in the the cable joints rather than the cable itself and there is probably considerably greater tendency for such breakdowns to go from conductor to conductor in the joint than in the cable itself, so that we are probably getting a large number of short circuits on our cable system that are not limited by the neutral resistance.

In higher voltage cable such as 26,400 or 33,000 volts, it is problematical as to whether cable breakdowns from conductor to sheath may not develop so rapidly as to produce phase-to-phase short circuits before the switches can open on both ends of the cable even with the minimum time of operation possible with relays. If this proves to be the case the advantage of neutral resistance for limiting the current will be lost and it is probable that better operation will follow from dead grounding of such systems.

Many of the systems now in operation have been laid out for radial distribution and there is a marked tendency toward the use of reactors to limit the current that can be fed into a short circuit on any cable. With such an arrangement it is common practise to use reactors of about 3 per cent reactance based on the rating of the table. Generating stations of the large capacity now becoming general are not seriously jolted by a short circuit of even 33 times the capacity of a given feeder, especially as the use of bus reactors to partially segregate the station are in quite common use. This again probably precludes the necessity of neutral resistance to limit the short-circuit current.

Several recent large generating stations have been built with a radically new bus arrangement generally referred to as "isolated phase" construction. The main object in such an arrangement is to preclude the possibility of a bus short circuit as all parts of each phase are isolated from the other two phases. The only way in which a short circuit can occur will be to ground and here we obtain a definite advantage from the use of neutral resistance which will limit the possible short circuit in the station to the value of current that will pass through the resistor. In such stations it is common practise to use reactors on all feeders, and in many cases reactors in the generator leads, omitting the use of bus reactors for partial isolation of sections of the station. The only manner in which a station short circuit can occur is through two simultaneous breakdowns to ground which should be an extremely rare occurrence. If neutral resistance is used with such an arrangement of a value sufficient to limit the current to that necessary to trip overload relays or to a value from 1000 to 1500 amperes, it is probable that the heaviest arc that could be expected would be of this value. If neutral resistance were not used, and the practise maintained of grounding one generator at a time, the current in this arc might be as high as 25,000 or 30,000 amperes to ground in some of the larger stations.

At the present time there seems to be little agreement among engineers as to the best method of grounding the neutral on cable systems. There is some tendency toward reducing the resistance in the neutral to a very low value or to leaving it out altogether. The writer hesitates to give an opinion as to which is the better method, but is inclined to think that the omission of resistance is not out of the question, from an operating standpoint, and to feel that lower voltage strains on the complete system would ensue from such a practise, for reasons similar to those discussed for overhead line practise.

Underground cable systems may be grounded in similar ways as discussed for overhead lines and the different methods have a similar influence on the amount and distribution of short-circuit current. The only distinctive difference lies in the tendency for circulating currents between neutrals of generators where it is usually of less importance than between the neutrals of transformers. It is quite general practise to wind high-voltage generators for Y connection and a ready means of grounding is thus provided.

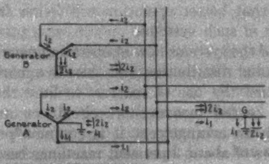

FIG. 7—TWO GENERATORS IN MULTIPLE, NEUTRAL OF ONE ONLY GROUNDED

Line ground at "G". Current flows to line ground from both generators, as illustrated.

It was shown in Fig. 5, that a delta-Y transformer at a generating station not only furnishes current direct to a fault to ground from its own generators but in addition may furnish extra current from its function as a grounding transformer from other generating stations. A generator with a grounded neutral will act in a similar manner furnishing current generated internally in the ground phase and also extra current coming from other ungrounded neutral generators. This current flows from the ungrounded neutral generators to ground on the phase in trouble and returns through the neutral of the grounded neutral generator and thence back over the other two phases to the ungrounded neutral generators. (See Fig. 7).

Should it be impracticable or undesirable to obtain a neutral ground by the direct grounding of generator neutrals, the system may be grounded through Y-delta transformers Y-Y transformers with tertiary delta, zigzag connected transformers, etc., as in the case of overhead systems and the calculation of the rating of such transformers would follow the same general method as outlined.

The effect of different methods of grounding the neutral of a cable system on the function of various types of relays is similar to that on overhead lines although there are in use in some cases certain relay schemes on underground networks not generally considered applicable to overhead line use.

Overload relays on radial feeders are used very extensively and are only affected by the method of grounding to the extent that sufficient current must be available to cause them to trip with a fault to ground. For this purpose provision is usually made in grounding for a current of two or three times the rating of the feeder. As an alternative scheme for radial feeders residual current relays are sometimes used in which case lower ground current is possible as residual current relays may be made more sensitive and set to trip at a lower current.

Relay schemes such as the Merz-Hunter or Cole scheme using split conductor cable for differential protection or using pilot wires, are peculiarly applicable to cable systems and are only affected by the method of grounding by the relation of the sensitiveness of their setting to the current available in case of a fault to ground. While it would seem that lighter ground currents would be possible with these relay schemes, it is probable that more accurate and consistent results would be obtained by working with heavier currents with more available energy for the relays.

Relay schemes such as the case of balanced current relays at the source of power and directional relays at substations require practically as much current as overload relays for selective action.

It has sometimes been argued that higher resistance could be used to ground the neutral of a cable system than a high-voltage overhead line system for the same protection against voltage strains, on the basis that a large amount of energy is stored in the electrostatic capacity of the latter system which tends to produce an oscillatory arc when discharged to ground. Such an argument can hardly be substantiated as the electrostatic capacity of a cable system is much greater at the same voltage than an overhead line system. A 13,200-volt, three-conductor cable at 60 cycles has a charging current per mile to ground of about one and eight tenths amperes, while a mile of 110,000-volt line at 60 cycles has a charging current of only about one half an ampere. It is evident that a 60-cycle system consisting of 13,200-volt cable of the same total mileage as a 110,000-volt overhead line system discharged to ground, would have more than three times the amount of leading current in the arc and a consequent greater destructive tendency.

The presence of the high charging current in an arc from conductor to sheath in an extensive cable system tends to nullify the effect of an unduly high resistance in the neutral for the purpose of limiting the current to a low value, as the charging current does not return through the neutral and hence is not limited by it. A 13,200-volt, 60-cycle cable system with 600 miles of cable would have a charging current to the sheath of over 1000 amperes and on account of the indefiniteness of its path in the system tends to make relaying unreliable. Such a large leading current is almost sure to produce a destructive oscillatory arc unless the short-circuit current combined with it, is of high enough value to stabilize it.

The relay protection of generators is influenced to some extent by the method of grounding the neutral of the system. As units are becoming so large and represent such a heavy investment, it is important that an adequate relay scheme be employed to preserve them from excessive damage resulting from electrical breakdown. It is necessary to open the oil circuit breakers, disconnecting them from the bus and to open their field switches, as well, in the minimum possible time after a breakdown to prevent excessive burning. The relay scheme almost universally employed is to use sensitive quick acting relays, differentially connected to current transformers in the neutral and line end of each phase. A breakdown to ground causes current to flow back through the neutral current transformer and not through the line current transformer, thus producing differential current to operate the relays. Should such a ground develop close to the neutral end of the winding, this unbalance will be small and if limited by a neutral resistance of high value may delay the action of the relays and extend the damage to the generators.

Fig. 8—Neutral Grounding Resistance for 13,200-Volt Service

The most usual type of neutral resistor used for grounding generator neutrals consists of the grid type similar to Fig. 8. All parts are of standard construction in general use for starting resistors for induction motors and show a high efficiency as to space, first cost, reliability and minimum of maintenance cost. Such resistors can be safely operated for short time duty at temperatures up to 400 degrees cent. and thus absorb a great amount of energy for the space taken up. When it is realized that a 1200-ampere, 6.3-ohm resistor in the neutral of a 13,200-volt generator must absorb 9100 kw., the importance of working materials to their limit, will be apparent.

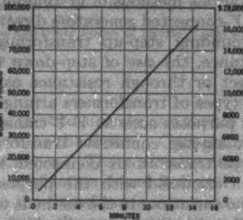

Fig. 9—Relation of Weight and Cost to Duty in Time of a 1200-Ampere, 5-Ohms, 13,200-Volt Resistor

A factor of great importance in selecting a neutral resistor is the length of time to be specified for safe operation. The size, weight and cost of such a resistor increases very rapidly with increase in the time for which it is designed. Curve Fig. 9, shows the relation of cost and weight to duty in time of a 5-ohm, 1200-ampere resistor.

Practise varies widely in respect to the time specified for such resistors but the tendency is to make it as short as possible on account of the great saving in first cost. There are many resistors in operation designed for 30-second duty and they range from that all the way up to 10 minutes.

The principles on which the rating should be based are not entirely clear and the issue is usually decided on more or less a compromise basis. The primary function of such a resistor is for the purpose of limiting the current flow to ground for the time necessary to clear the short circuit. Most relay schemes are laid out with the expectation of tripping the oil circuit breakers in a few seconds at most and if they work properly a resistor designed for 30-second duty should have ample margin of safety. If the circuit breaker or relay fails to operate, either due to being out of order or unforeseen distribution of current over several feeders, time must be allowed for the operator to act and 30 seconds is an insufficient time for him to locate the trouble and clear it. The assumption may be made that failure of relay operation is very remote and that in such a case the loss of a resistor may not be serious. This of course, will depend on the damage that might be expected outside of the loss of the resistor itself.

CONCLUSION

The author has attempted to outline the major problems that present themselves for consideration in laying out a large power system, having in mind the maximum protection feasible for good continuity of service. It is evident that future practise in the design and operation of power systems, both overhead and underground, will be with some form of neutral ground.

There are various methods of grounding in use, such as grounding *Y*-connected generators direct on through resistance, grounding step-up transformers direct or through resistance, the use of step-down transformers grounded direct or through resistance as well as the use of special types of transformers arranged especially for grounding purposes such as *Y-Y* transformers with delta tertiary, zig-zag connected transformers, three-phase core type transformers connected *Y-Y*, etc. all of which were discussed with the purpose of calling attention to the variations in the methods of grounding, which the author has tried to make, leads to the conclusion that effective grounding for desired results can be obtained by any of the methods outlined. The choice of method, then becomes one of convenience and economics.

The one important consideration that the author has laid the most stress on, is the question of the extent to which a system should be grounded for best all around protection to equipment and service. Systems have been loosely spoken of generally as being grounded solidly or grounded through resistance, irrespective of the fact that a system grounded solidly through a relatively small part of its total step-up transformer capacity may show less effective grounding than another system where low resistance in the neutral is used. The real measure of the extent to which a system neutral is grounded depends upon the relation of ground current flowing to the full-load current of the system.

The important function of the grounded neutral is to limit voltage strains on the system at the time of faults to ground, as experience has shown that such strains are the most fruitful source of extended interruption to service. Modern apparatus is designed to withstand the magnetic and thermal stresses of high current and modern relays will protect service under short-circuit conditions. Voltage strains, however, are difficult to predetermine and expensive to protect against and the neutral ground should be selected with this in mind.

The author has attempted to show that effective grounding for limitation of voltage strains can best be obtained by allowing comparatively heavy currents to flow which means either solid grounding of an appreciable part of the system capacity or a larger part through fairly low resistance.

It has been brought out that from the standpoint of continuity of service, which involves the protection of apparatus and equipment, the successful functioning of relays, etc., that the system with heavy ground current, has many advantages.

Discussion

For discussion of this paper see page 464.

The Neutral Grounding Reactor

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Review of the Subject.—*A neutral grounding reactor (Petersen Coil) has been in operation on the system of the Alabama Power Company since October 12, 1921. This is the first of these devices in this country, although there are perhaps several hundred of them in operation in Europe. The present paper discusses the tests made shortly after installation of this reactor. A companion paper by Messrs. J. M. Oliver and W. W. Eberhardt, discusses the operation of the reactor for a period of about a year.*

It is believed that this device has a somewhat limited use in this country on account of the prevalence and popularity of the solidly grounded neutral. Nevertheless, there are certain installations favorable to the use of the reactor and it is partly the purpose of the present paper to outline the field of this device as well as to show how its rating may be determined for a particular system.

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ISOLATED neutral systems are disadvantageous from the standpoint of abnormal voltages produced by arcing grounds and the difficulty of relaying against the short circuits frequently produced by such abnormal voltages. The grounded neutral has eliminated the overvoltages and allowed the relaying to become positive. Systems with neutrals grounded solidly or through a low resistance predominate in this country, and their operation has been very satisfactory.

However, there are some systems for which grounding is not desirable for one reason or another: For example, the apparatus will not stand short circuit, or there being but a single circuit, the frequent interruption of this circuit cannot be tolerated. For such systems, various schemes have been proposed with the object of eliminating the dangerous arcing grounds, thereby allowing the system to continue in operation. Among these are the arcing ground suppressors of Creighton and Nicholson, the neutral grounding reactor of Petersen and kindred devices.

The neutral grounding reactor has been used a great deal in Europe, but very little in this country, owing largely to extensive and satisfactory use of the grounded neutral system. However, there has been a reactor in operation on the Alabama Power Company system since October 12, 1921, and it is the purpose of the present paper to describe this installation and some tests made thereon. Messrs. J. M. Oliver and W. W. Eberhardt will discuss the operation of the reactor in service.

THEORY OF OPERATION

Suppose that on a three-phase system, of which the Y of Fig. 1 represents the generator, or more commonly the high-tension side of the step-up transformer bank, there is an arc from phase 3 to ground. Then charging currents will flow through the capacitances C_1 and C_2 from phases 1 and 2 to ground, through the arc, back on conductor 3 to the transformer and thence to conductors 1 and 2 as shown on the figure. Capacitance C_3 is short-circuited by the arc. There are also charging currents passing from conductors 1 and 2 directly to conductor 3 and back to the transformer over conductor 3, but as these do not enter the arc circuit, they are immaterial in this discussion and will be ignored.

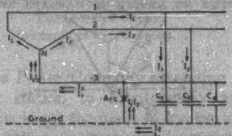


FIG. 1—THREE-PHASE SYSTEM WITH AN ARC FROM PHASE TO GROUND. FIGURE SHOWS FLOW OF CURRENTS

In Fig. 2 are represented assumed positive directions for the various voltages, and it is also shown how voltages E_{13} and E_{23} are formed by horizontal and vertical components.

If we let $E_{12} = E$ and let the direction of E_{12} be the line of reference, then E_{13} and E_{23} may be expressed vectorially as follows:

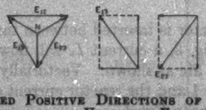


FIG. 2—ASSUMED POSITIVE DIRECTIONS OF VOLTAGES, AND FORMATION OF VOLTAGES E_{13} AND E_{23}

$$E_{13} = -E/2 - j\,E/2\,\sqrt{3}$$
$$E_{23} = -E/2 + j\,E/2\,\sqrt{3}$$

Also the current I_1 and I_2 may be expressed:

$$I_1 = +j\,E_{13}\,\omega\,C_1$$
$$I_2 = -j\,E_{23}\,\omega\,C_2$$

Let $$C_1 = C_2 = C_G$$

Presented at the Spring Convention of the A. I. E. E., Pittsburgh, Pa., April 24-26, 1923.

Then
$$I_1 + I_2 = j\,\omega\,C_G\,(E_{13} - E_{23})$$
$$= j\,\omega\,C_G\,(-\,2\,j\,E/2\,\sqrt{3})$$
$$= \omega\,C_G\,E\,\sqrt{3} \qquad (1)$$

But
$$E = \sqrt{3}\,e$$

in which e = voltage to neutral,

Then
$$I_1 + I_2 = 3\,\omega\,C_G\,e \qquad (2)$$

That is, by equation (1) the current through the arc is $\sqrt{3}$ times the charging current from one of the two ungrounded conductors to ground when the third conductor is grounded, or by the equation (2) it is three times the normal charging current from one of the three conductors to ground, when none of the conductors is grounded.

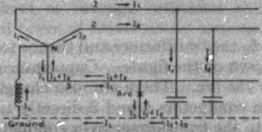


Fig. 3—Three-Phase System with Reactor Between Neutral and Ground. Figure Shows Flow of Currents with Arc from Phase 3 to Ground

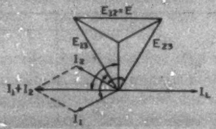


Fig. 4—Vector Relation of Currents Flowing in Circuit of Fig. 3. All Angles Marked by Arrows Equal 90 Deg.

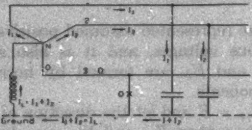


Fig. 5—Flow of Current in Circuit of Fig. 3 when There is Perfect Neutralization of Lagging and Leading Currents

Now if a reactor is inserted between the neutral and ground as in Fig. 3, a current I_L will flow through the reactor and the arc as shown. Vectorially the charging currents of Fig. 1 and the reactor current of Fig. 3 may be represented as in Fig. 4.

If we so proportion the reactor that the current I_L is numerically equal to $I_1 + I_2$, as it is directly opposite in phase, the resultant current through the arc will be zero, and the final flow of current will be as shown in Fig. 5.

The inductance necessary to give this relationship is found as follows:

$$I_L = \frac{e}{\omega\,L}$$

Let
$$I_L = I_1 + I_2$$

Then
$$\frac{e}{\omega\,L} = 3\,\omega\,C_G\,e$$

From which
$$L = \frac{1}{3\,\omega^2\,C_G}$$

In actual practise there is a watt component of current due to the losses in the reactor and lines, and this cannot be balanced out, but exists as a residual current through the arc. (See Fig. 6). In this figure $I_1 + I_2$ is the line charging current and I_L the reactor current, both with a watt component, which add up to give the total watt or resultant current I_R. The reactive components I_A and I_B neutralize each other.

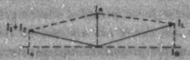


Fig. 6—Typical Relation of Currents in a Practical Circuit, with Watt as Well as Reactive Components

The next question that arises is how to determine C_G, the capacitance from line to ground. The writer has done this for three typical arrangements by means of Maxwell's coefficients. The arrangements chosen are shown in Fig. 7. In each case conductor 3 was grounded. The charging currents found, in terms of the normal three-phase charging current, are as follows:

Case A.

Normal charging current........ = 1.00
Conductor 1.................... I_1 = 1.31
Conductor 2.................... I_2 = 1.29

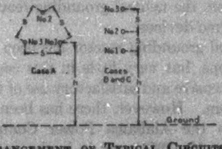


Fig. 7—Arrangement of Typical Circuits for Calculating the Capacitance to Ground by Means of Maxwell's Coefficients

Case A
Spacing.. s = 0.8 ft.
Height lowest conductor........................ h = 46.5 ft.
Diameter Conductor............................. d = 0.4 in.

Case B
Spacing.. s = 6 ft.
Height lowest conductor........................ h = 30 ft.
Diameter conductor............................. d = 0.365 in.

Case C
Spacing.. s = 10 ft.
Height lowest conductor........................ h = 43.5 ft.
Diameter conductor............................. d = 0.47 in.

Conductor 3 (beyond ground)..... I_3 = 0.51
Ground.......................... I_G = 1.45
Conductor 3 (generator side of ground)................... $I_3 + I_G$ = 1.96

Case B.

Conductor 1.................... $I_1 = 1.36$
Conductor 2.................... $I_2 = 1.41$
Conductor 3 (beyond ground)...... $I_3 = 0.52$
Ground.......................... $I_G = 1.51$
Conductor 3 (generator side of ground).................... $I_3 + I_G = 2.03$

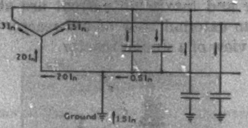
Case C.

Conductor 1.................... $I_1 = 1.32$
Conductor 2.................... $I_2 = 1.35$
Conductor 3 (beyond ground)...... $I_3 = 0.48$
Ground.......................... $I_G = 1.51$
Conductor 3 (generator side of ground).................... $I_3 + I_G = 1.99$

Average

Conductor 1.................... $I_1 = 1.33$
Conductor 2.................... $I_2 = 1.35$
Conductor 3 (beyond ground)...... $I_3 = 0.50$
Ground.......................... $I_G = 1.49$
Conductor 3 (generator side of ground).................... $I_3 + I_G = 1.99$

Grounding a conductor other than No. 3 may change the results slightly, and other configurations may give somewhat different results. The presence of a ground wire will increase the capacitance of the conductors to ground and thereby probably increase the current to ground, and consequently increase I_G.



13In
13In
20In
20In
0.5In
Ground
1.5In

Fig. 8—Calculated Flow of Current with One Conductor Grounded.

Current Values Expressed in Terms of Normal Three-Phase Charging Current.

The results of the preceding tabulation, somewhat rounded off, are shown in Fig. 8. From these figures it will be seen that the capacitance from the ungrounded conductors to ground is about 75 per cent of the normal three-phase capacitance from one conductor to neutral.

Now it is simply necessary to calculate the normal three-phase charging current of the line, and multiply this by 1.5 in order to get the current to ground, in case of a ground on one conductor. Then the neutral grounding reactor should be designed for this current. That is, the ohmic reactance of the neutral reactor is found from the relation:

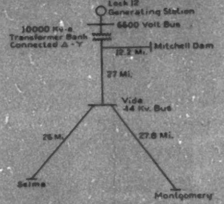
$$X = \frac{e}{1.5\, I_n}$$

in which X = reactor ohms
e = voltage line to neutral
I_n = normal line charging current.

The writer has recently read Technical Report No. 52 of the Report of the Joint Committee on Inductive Interference of the California Railroad Commission and from this has learned that Mr. L. P. Ferris of that committee worked out a similar relationship between the charging currents of a line with one conductor grounded. Mr. Ferris also refers in his report to some work done a number of years ago by Mr. Geo. S. Humphrey. The figures of both Mr. Ferris and Mr. Humphrey check fairly well with those of the writer as given in the preceding tabulations and in Fig. 8.

The Alabama Power Co. Installation

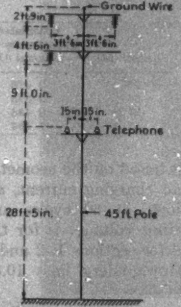
The system chosen for a trial installation of the neutral grounding reactor is the 44-kv., 60-cycle system of the Alabama Power Company extending from Lock 12 generating station to Montgomery and Selma, Alabama. This is not the main 44-kv. network of this company, but only a small, though important, isolated section. This line is subjected to frequent and severe lightning disturbances and has given considerable trouble from this cause in the past.



Lock 12
Generating Station
6600 Volt Bus
10000 Kv-a
Transformer Bank
Connected Δ - Y
Mitchell Dam
12.2 Mi.
27 Mi.
Vida
44 Kv. Bus
26 Mi.
27.8 Mi.
Selma
Montgomery

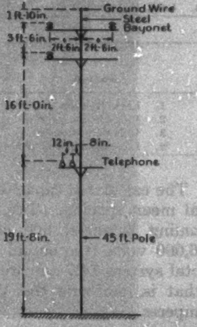
Fig. 9—One Line Diagram of System on which Reactor Tests were Made.

System is 44 kv. 60 Cycles.



Ground Wire
2ft-9in.
4ft-6in
3ft-6in
3ft-6in.
5ft 0in.
15in
15in.
Telephone
28ft-5in.
45ft Pole

Fig. 10 (a)—Arrangement of Conductors on Lock-12-Vida Selma Section.



Ground Wire
Steel
Bayonet
1ft-10in.
3ft-6in.
2ft-6in
2ft-6in.
16ft-0in.
12in.
8in.
Telephone
19ft-8in.
45ft Pole

Fig. 10 (b)—Arrangement of Conductors on the Vida-Montgomery Section.

Fig. 9 shows a simplified one-line diagram of the line. Fig. 10a shows the arrangement of conductors on the Lock 12-Vida-Selma section, Fig. 10b the Vida-

Montgomery section and Fig. 10c the Lock 12-Mitchell Dam section.

Following are the data as to the transmission line conductors, lengths of line, etc.:

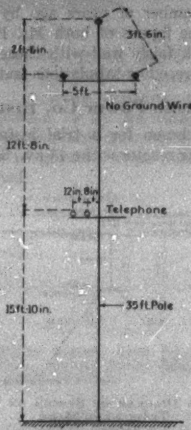


FIG. 10 (c)—ARRANGEMENT OF CONDUCTORS ON THE LOCK 12-MITCHELL DAM SECTION

Section	Conductor	Spacing Ft.
1. Lock-12-Vida	3/0 Alum. steel core	5-3.5-6.1
2. Vida-Selma	No. 4.Copper	5-3.5-6,1
3. Vida-Montgomery	3/0 Alum. steel core	7-4.5-8.3
4. Lock 12-Mitchell Dam	No. 4 Aluminum	3.5-3.5-5

Section	Length Mi.	Calculated capacitance to Neutral μf.
1. Lock 12-Vida	27	0.476
2. Vida-Selma	26	0.375
3. Vida-Montgomery	27.8	0.434
4. Lock 12-Mitchell Dam	12.2	0.184

The calculated capacitance is based on the geometrical mean spacing. The normal charging current, assuming balanced line conditions, at 60 cycles and 46,000 volts (the actual operating voltage) is for the total system 14.7 amperes, and for sections 1, 2 and 4 (that is, omitting the Vida-Montgomery line) 10.35 amperes.

In order to take care of various combinations of the present line, and some future proposed extensions, as well as to compensate for possible errors in calculating the current to ground with one conductor grounded, the reactor was designed with taps in approximately 5 per cent steps from 2015 ohms to 811 ohms. The change from one tap to another is made by means of two ratio adjusters with control handles at the top of the cover. Following is a list of the taps as found by test with the corresponding amperes flowing through the reactor:

Tap	Reactance Ohms	Amperes at 46,000 volts.
1	2015	13.2
2	1915	13.9
3	1825	14.5
4	1735	15.3
5	1630	16.3
6	1567	16.9
7	1492	17.8
8	1410	18.8
9	1357	19.6
10	1276	20.8
11	1207	22.0
12	1140	23.3
13	1088	24.4
14	982	27.0
15	920	28.6
16	870	30.5
17	811	32.7

The reactor has no core. It is oil immersed and water cooled, and designed to operate continuously at 795 kv-a. with 55 deg. cent. rise. In order to prevent eddy losses in the steel tank, the cooling coils are connected as a short-circuited winding in accordance with the Kierstead Patent.[1] The total losses in the reactor at full voltage and maximum current are about 21 kw. Fig. 11 is an external view of the reactor and Fig. 12 an internal view of a similar reactor.

FIG. 11—EXTERNAL VIEW OF NEUTRAL GROUNDING REACTOR

Occasionally there may be a ground on a conductor which is in the nature of a dead ground, for example ground wire against a conductor, conductor down on ground, baling wire connecting conductor and ground wire or conductor and tower, shattered or punctured insulator, etc. In these cases, while the grounding reactor would neutralize the current at the fault, still there would be a large ground current flowing and the system would be in a state of voltage unbalance. It therefore would be desirable to be able to cut out the section of the system in trouble. This is accomplished as follows:

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[1] U. S. Patent 1,394,910.

In series with the reactor at the ground end is placed a current transformer of suitable ratio. To the secondary of the current transformer is connected a definite time limit relay, set so that with the minimum current through the reactor the relay will operate in a predetermined time and energize the closing coil of an oil circuit breaker, which closes around and short-circuits the reactor. The system now has a dead or solidly grounded neutral and short-circuit current flows from the power source to the point of line ground. The regular protective relays respond to the short circuit and the section in trouble is tripped out. This scheme was suggested by Mr. E. W. Pragst.[2]

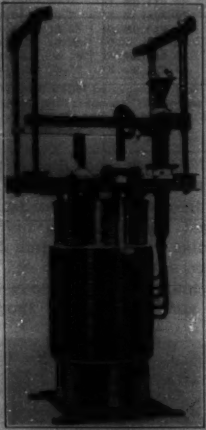

Fig. 12—Internal View of Neutral Grounding Reactor

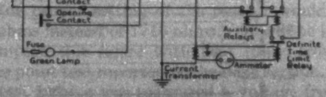
High Tension Side of
Power Transformer
Disconnecting
Switch
125 Volt D C Control Bus
+
−
Fuse
Fuse
Reactor
To Bell
Voltmeter
Closing
Coil
Control
Relay
Trip
Coil
Oil
Circuit
Breaker
Fuse
Red Lamp
Closing
Contact
Opening
Contact
Auxiliary
Relays
Fuse
Green Lamp
Definite
Time
Limit
Relay
Current
Transformer
Ammeter

Fig. 13—Diagram Showing Connection of Reactor and Auxiliary Devices in Circuit

Connected to the secondary of the current transformer is a recording ammeter, which gives an indication when current passes through the reactor, and across the red lamp of the oil circuit breaker is a recording voltmeter. By these devices a record may be kept of the operation of the reactor and the short-circuiting breaker. A bell alarm also warns the operator that the circuit breaker has closed.

Fig. 14—View of Reactor Installed

Fig. 15—View of Oil Circuit Breaker and Disconnecting Switches Used with Reactor

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[2] U. S. Patent 1,378,557.

TABLE I.
Summary of Test Results
All Readings Changed to 46,000 Volts Equivalent

		Voltage				Current					
Test	Date	Line to Line	Line to Ground	Line to Neutral	Neutral to Ground	1	2	3	N	G	L
1	11/27	46,000			369	9.88	11.05	10.82			
	12/4	46,000	26,800	26,700	793	10.1	11.02	11.07			
	Av.	46,000	26,800	26,700	581	9.99	11.04	10.95			
2	11/27	46,000			26,800	21.1		14.35		17.7	
	12/4	46,000			26,950	21.4	14.55	14.2		17.77	3.85
	Av.	46,000			26,875	21.3	14.55	14.28		17.74	3.85
3	11/27	46,000			29,100	21.3		14.33			
	12/4	46,000			28,000	21.72	14.9	14.1		18.3	
	Av.	46,000			28,550	21.51	14.9	14.22		18.3	
4	11/27	46,000				9.6		10.7	0.52		
	12/4	46,000		26,600		10.35	11.1	10.8	0.59		
	Av.	46,000		26,600		9.98	11.1	10.75	0.56		
5	11/27	46,000			1,050	9.22		11.43	1.58		
	12/4	46,000		26,400	3,613	9.53	11.08	11.7	2.46		
	Av.	46,000		26,400	2,332	9.38	11.08	11.57	2.02		
6	11/27	46,000			27,600	6.29		14.98	15.3	3.04	
	12/4	46,000			26,400	6.28	13.9	14.2	14.86	2.6	3.81
	Av.	46,000			27,000	6.29	13.9	14.59	15.08	2.82	3.81
7	11/27	46,000			27,250	5.14		14.28	14.95		
	12/4	46,000			26,300	6.1	14.54	14.15	15.02	0 +	
	Av.	46,000			26,775	5.62	14.54	14.22	14.99	0 +	

Fig. 13 is a wiring diagram of the auxiliary connections and shows how the reactor is connected in circuit.

Figs. 14 and 15 show views of the apparatus in service.

TESTS

Steady Conditions.

A series of tests was made under steady conditions (no arcs). The Vida-Montgomery section was disconnected and tests made on the remaining sections, *i. e.,* Lock 12-Vida, Vida-Selma and Lock 12-Mitchell Dam. The line was not loaded and the step-down transformers at the ends of and along the line were disconnected.

Fig. 16 shows a diagram of connections for these tests. The readings were taken by means of current and potential transformers connected directly to the high-tension lines. In Table I are summarized and averaged the results of the tests of November 27th and December 4th, 1921, the results in all cases being changed to equivalent figures at 46,000 volts. In tests 5, 6 and 7 the reactor was set at 1735 ohms.

In Table II are summarized the calculated currents for the various conditions. These calculations are based on a capacitance of 1.035 microfarads for the lines in circuit. This would give a normal three-phase charging current of 10.35 amperes, and with one conductor grounded the following amperes in accordance with the previous discussion and as shown in Fig. 8.

I_1 (grounded conductor).... $2 \times 10.35 = 20.7$
I_2 (ungrounded conductor).. $1.3 \times 10.35 = 13.5$
I_3 (ungrounded conductor).. $1.3 \times 10.35 = 13.5$
I_L (beyond ground)......... $0.5 \times 10.35 = 5.2$
I_G (current to ground)...... $1.5 \times 10.35 = 15.5$

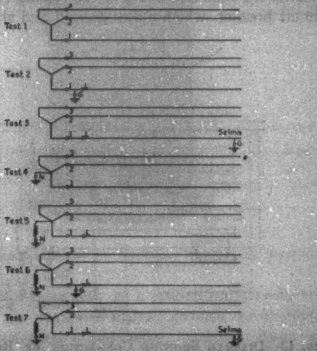

FIG. 16—DIAGRAM OF CONNECTIONS FOR TESTS 1 TO 7 INCLUSIVE MADE UNDER STEADY CONDITIONS

Based on a current to ground of 15.5 amperes, the reactor setting should be 1710 ohms. The nearest tap 1735 ohms was used in the tests. Comparing the cal-

TABLE II.
Calculated Currents
Lock 12—Vida, Vida—Selma, Lock 12—Mitchell Dam Lines

	Voltage		Current					
Test	Line to Line	Line to Neutral	1	2	3	N	G	L
1	46,000	26,540	10.35	10.35	10.35			
2	46,000	26,540	20.7	13.48	13.48		15.53	5.18
3	46,000	26,540	20.7	13.48	13.48		15.53	20.7
4	46,000	26,540	10.35	10.35	10.35			
5	46,000	26,540	10.35	10.35	10.35			
6	46,000	26,540	5.18	13.48	13.48	15.3	0.23	5.18
7	46,000	26,540	5.18	13.48	13.48	15.3	0.23	5.18

These values are what would be expected if the three conductors were symmetrically arranged with respect to each other and ground. This not being the case in practise, the actual values should differ somewhat from the above.

culated currents with those measured in tests 2 and 3 we find that the current to ground averages about 18 amperes against a calculated current of 15.5 amperes. Also the current in conductor 1 beyond the ground measures 3.85 amperes against a calculated current of 5.2 amperes. In other words, the current to ground is about 16 per cent greater and that to the grounded conductor about 26 per cent less than calculated.

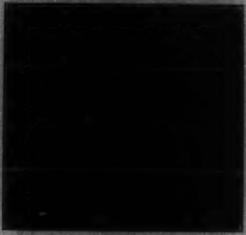

FIG. 17—C D 57279, TEST 4, LINE CHARGED, NO LOAD, NEUTRAL SOLIDLY GROUNDED

Curve A, Current in Conductor 2.
Curve B, Current in grounded neutral.
Curve C, Voltage between conductors 2 and 3.

This is very likely due to the ground wire, which increases the capacitance of the ungrounded conductors and thereby their charging current to ground. All the ground current, whether it goes directly to ground or first to the ground wire, must eventually return by way of the fault or grounded point on the conductor.

Using the figure of 18 amperes for ground current, then the proper setting of the reactor would be about 1470 ohms instead of 1710 ohms. It will be interesting to compare this figure with that found from the resonance point test reported in the next section.

In test 1 there was measured a voltage from neutral to ground of about 600 volts. This is evidently the residual voltage due to the unbalanced capacitance from the three conductors to ground. If the neutral is grounded a current should flow, due to this voltage. In test 4 (neutral directly grounded) there is a neutral current of about 0.6 amperes and in test 5 (neutral grounded through reactor) there is a neutral current of about 2 amperes.

From the oscillogram Fig. 17 it is interesting to note that the neutral current of Test 4 consists of a fundamental and large fifth and seventh harmonics, *i. e.*, the same harmonics as contained in the line charging current. However, in the neutral current of Test 5 (Fig.

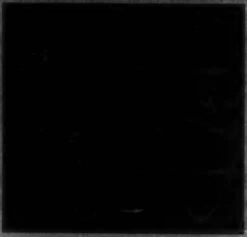

FIG. 18—C D 57281, TEST 5, LINE CHARGED, NO LOAD, REACTOR BETWEEN NEUTRAL AND GROUND

Curve A, Current in Conductor 2.
Curve B, Current in Neutral Reactor.
Curve C, Voltage between Conductors 2 and 3.

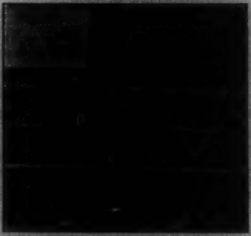

FIG. 19—C D 57282, TEST 5, LINE CHARGED, NO LOAD, REACTOR BETWEEN NEUTRAL AND GROUND

Curve A, Voltage between Neutral and ground.
Curve B, Current in Conductor 1.
Curve C, Voltage between Conductors 2 and 3.

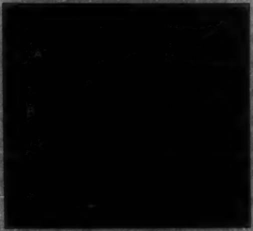

FIG. 20—C D 57293, TEST 7, LINE CHARGED, NO LOAD, REACTOR BETWEEN NEUTRAL AND GROUND. CONDUCTOR 1 SOLIDLY GROUNDED AT SELMA

Curve A, Current in Conductor 2.
Curve B, Current in Neutral Reactor.
Curve C, Voltage between Conductors 2 and 3.

18) the harmonics have disappeared, leaving the fundamental only, although the line charging current is practically identical with that of Test 4. The voltage across the reactor (Curve A, Fig. 19,) also contains a fundamental only. The impedance of the reactor to harmonics other than the fundamental is apparently so high that they are wiped out. This conclusion is confirmed by numerous other oscillograms, for example, Fig. 20 curve *B* shows the current through the reactor for Test 7. This is again a pure sine wave of fundamental frequency.

Determination of Resonance Point

The resonance point was determined in two ways:

a. Under normal conditions with no conductors grounded. In this case the reactor ohms are varied and readings taken of the reactor current, which current in this case is merely a residual capacitance current due to unbalanced capacitance from the three conductors to ground. The resonance point is indicated by the maximum current.

b. When one conductor is grounded. The reactor ohms are again varied and readings taken of current flowing into the ground. This current is the resultant of the leading charging current and the lagging reactor current, combined with the loss current. At the point of resonance this current should be a minimum and consist only of the loss current.

In Table III are given the readings taken with the system unloaded and the Vida-Montgomery section of line disconnected. The results are plotted in Fig. 21 and show that the resonance setting in this case is about

TABLE III.
Test 12. Determination of Resonance Point.
System Unloaded. No Line Grounds.
Vida—Montgomery Line Out.
Jan. 20, 1922.

Reactor Ohms	Amp. N Direct	Amp. N at 46,000 V
1207	3.15	3.2
1276	4.2	4.27
1357	6.4	6.5
1410	7.65	7.78
1492	6.15	6.25
1567	4.3	4.37
1630	3.1	3.15
1735	2.27	2.31

Readings at about 113 × 400 = 45,200 volts
Raining during test.
See Fig. 21.

TABLE IV.
Test 12. Determination of Resonance Point.
System Normal, Carrying Load
Vida—Montgomery Line in Dry Weather Jan. 18, 1922.

Reactor Ohms	Volts 2—3		Volts N G		Current N Direct
	Read	Actual	Read	Actual	
811	114.7	45,880	43	2580	2.13
		46,000		2590	2.14
870	114.5	45,800	52	3120	2.92
		46,000		3140	2.94
929	114.6	45,840	70	4200	4.35
		46,000		4210	4.37
982	114.5	45,800	104	6240	6.05
		46,000		6250	6.08
1088	114.5	45,800	67	4020	3.46
		46,000		4040	3.48
1140	114.7	45,880	54	3240	2.31
		46,000		3250	2.32
1207	114.6	45,840	49	2940	1.68
		46,000		2950	1.69
1276	114.6	45,840	43	2580	1.35
		46,000		2590	1.36
1357	113.5	45,400	40	2400	1.07
		46,000		2440	1.09
1410	113.8	45,520	38	2280	0.91
		46,000		2300	0.92
1492	113.5	45,400	36	2160	0.79
		46,000		2190	0.80
1567	113.8	45,520	33	1980	0.69
		46,000		1995	0.70
1630	113.6	45,440	32	1920	0.60
		46,000		1950	0.61

Pot. Trans. Ratio for Volts 2-3 = 400
" " " " " NG = 60
See Fig. 22.

TABLE V.
Test 12. Check on Resonance Point
Wet Weather Conditions Jan. 19, 1922.

Reactor Ohms	Current N Direct	Current N at 46,000 *V.*
811	2.35	2.44
870	3.35	3.48
929	4.9	5.09
982	5.55	5.75
1088	2.95	3.06
1140	2 1	2.18
1207	1.6	1.66

Rain all along line for several hours previous to, and during this test.
See Fig. 22.

TABLE VI.
Test 13. Determination of Resonance Point
System Carrying Load. One Conductor Grounded.
Vida—Montgomery Line In. Jan. 18, 1922.

React'r Setting	Volts	2—3	Volts	N G	Cur.	G	Cur.	N
	Av. Reading	Actual	Av. Reading	Actual	Av. Reading	Actual	Av. Reading	Actual
1630	111.5	44,600	107	26,800	4.9	9.8	2.53	15.2
		46,000		27,600		10.1		15.7
1567	111.2	44,500	106.1	26,600	4.63	9.26	2.63	15.8
		46,000		27,400		9.57		16.4
1492	111.5	44,600	106.3	26,600	4.23	8.46	2.77	16.6
		46,000		27,400		8.74		17.1
1410	111.5	44,600	105.9	26,400	3.82	7.64	2.9	17.4
		46,000		27,200		7.87		17.9
1357	111.0	44,400	104.9	26,200	3.34	6.68	3.05	18.3
		46,000		27,200		6.94		19.0
1276	111.0	44,400	104.9	26,200	2.84	5.68	3.23	19.4
		46,000		27,200		5.80		20.1
1207	111.0	44,400	104.5	26,150	2.37	4.74	3.4	20.4
		46,000		27,100		4.91		21.2
1140	111.0	44,400	104.25	26,100	1.9	3.8	3.59	21.6
		46,000		27,050		3.94		22.4
1088	111.0	44,400	104.5	26,150	1.45	2.90	3.8	22.8
		46,000		27,100		3.0		23.6
982	111.5	44,600	104.6	26,150	1.19	2.38	4.2	25.2
		46,000		26,900		2.45		26.0
929	111.5	44,600	104.65	26,150	1.53	3.06	4.46	27.4
		46,000		26,900		3.16		28.2
870	111.5	44,600	104	26,000	2.23	4.46	4.74	28.4
		46,000		26,800		4.6		29.2
811	111.5	44,600	103.8	25,950	3.1	6.2	5.02	30.1
		46,000		26,700		6.39		31.0

Pot. Trans. ratio Volts 2-3 = 400
Volts N-G = 250
Cur. Trans. ratio Cur. G = 2
Cur. N = 6
See Fig. 23.

1410 ohms, which should be compared with the 1470 ohms found from Tests 2 and 3 and 1710 ohms in the original calculation.

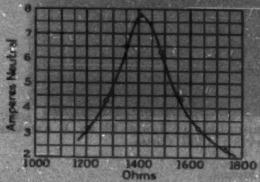

FIG. 21—RESONANCE CURVE, SYSTEM NOT LOADED, VIDA-MONTGOMERY SECTION OUT, NO GROUNDS ON LINE. WET WEATHER CONDITIONS.

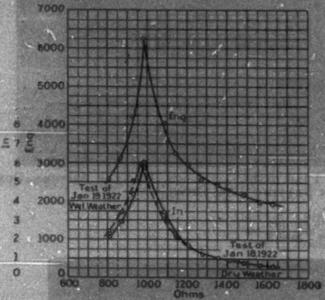

FIG. 22—RESONANCE CURVE, SYSTEM LOADED, ALL CIRCUITS IN. NO GROUNDS ON LINE. DRY AND WET WEATHER CONDITIONS

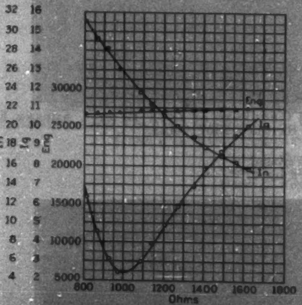

FIG. 23—RESONANCE CURVE, SYSTEM LOADED, ALL CIRCUITS IN. CONDUCTOR 1 GROUNDED AT LOCK 12. DRY WEATHER CONDITIONS

In Table IV are given the readings taken with the system carrying load and all lines in circuit including the Vida-Montgomery section. There are no grounds on the conductors and the weather is dry. In Table V are similar readings for wet weather conditions, the two sets of readings having been taken respectively on January 18th and 19th, 1922. The results are plotted in Fig. 22. The resonance point is at about the 982 ohm setting and is apparently shifted to a slightly lower value for the wet condition.

In Table VI are shown the readings for the case in which conductor No. 1 was grounded at Lock 12. The results are plotted in Fig. 23. The resonance point is again at about 982 ohms. This figure should be compared with the 1204 ohms calculated from 1.5 times the normal charging current of the system.

ARCING TESTS

Several series of arcing tests were made, totalling about 50 arcovers. These were made over horn gaps, pin-type insulators and suspension insulators, with various settings of the reactor and with the line loaded and unloaded. Most of the arcs were made at Lock 12 but one each was made at Selma and Montgomery. A typical set of tests is shown by Figs. 24 to 32 inclusive. These show the result of arcs over a 50,000-volt pin-type insulator at Lock 12 with the system loaded and all lines in service. A one-ampere fuse was placed over the insulator and then an oil circuit breaker closed, thus grounding conductor No. 1 through the fuse. Upon the fuse blowing there would result an arc across the insulator. The reactor setting was varied from 811 to 1630 ohms, the normal setting being 982 ohms. Thus the current was off balance anywhere from 40 per cent leading to 21 per cent lagging, *i. e.*, at one extreme the resultant or residual current at the arc was leading and 40 per cent of the total ground current, and at the other extreme the resultant current was lagging and 21 per cent of the total ground current. The results are summarized in Table VII.

In this series of tests the arc was extinguished in from 5½ to 14 cycles. There was only one arc which resembled a power arc, that at the 811 ohms setting. Practically all the other arcs were extinguished with a sharp snap. In some later tests power arcs were produced on two other occasions, both at the 811 ohm setting.

Figs. 33 and 34 are typical photographs of arcs over pin-type insulators, with the reactor set at 1630 and 811 ohms respectively. Figs. 35 and 36 show views of arcs over three suspension insulator disks with the 1630 and 811 ohm settings.

Now we have found by test that a reactor setting of 982 ohms gives resonance with the total system in circuit. The capacity reactance of the system must also be 982 ohms. From this we can work back and find the capacitance of the system by the relation $X_c = \dfrac{1}{2\,\pi\,f\,C}$

At a frequency of 60 cycles this gives a capacitance of 2.7 μf. from each of the two ungrounded conductors to ground. Using this value of capacitance and the various reactor ohms, we can now figure the natural

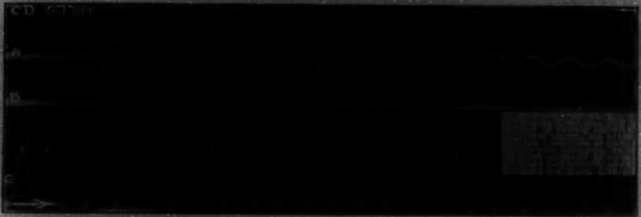

Fig. 24—C D 57311, System Loaded, All Lines in Service. Conductor 1 Grounded at Lock 12 by 1 Ampere Fuse Over Pin Type Insulator. Neutral Reactor Setting 1630 Ohms.

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor No. 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

Fig. 25—C D 57310, System Loaded, All Lines in Service. Conductor 1 Grounded at Lock 12 by 1 Ampere Fuse Over Pin Type Insulator. Neutral Reactor Setting 1492 Ohms.

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

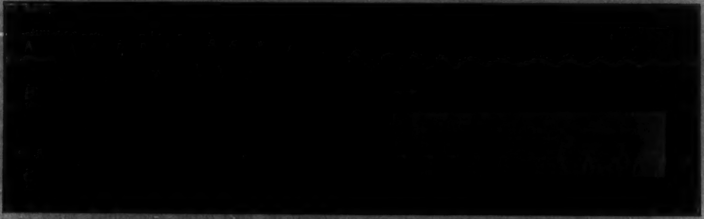

Fig. 26—C D 57309, System Loaded, All Lines in Service. Conductor 1 Grounded at Lock 12 by 1 Ampere Fuse Over Pin Type Insulator. Neutral Reactor Setting 1357 Ohms.

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

Fig. 27—C D 57303, System Loaded, All Lines in Service. Conductor 1 Grounded at Lock 12 by 1 Ampere Fuse Over Pin Type Insulator. Neutral Reactor Setting 1207 Ohms.

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

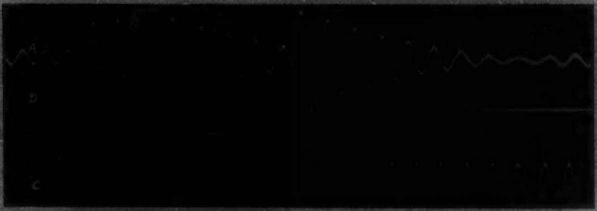

FIG. 28—C D 57304, SYSTEM LOADED, ALL LINES IN SERVICE. CONDUCTOR 1 GROUNDED AT LOCK 12 BY 1 AMPERE FUSE OVER PIN TYPE INSULATOR. NEUTRAL REACTOR SETTING 1068 OHMS

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

FIG. 29—C D 57305, SYSTEM LOADED, ALL LINES IN SERVICE. CONDUCTOR 1 GROUNDED AT LOCK 12 BY 1 AMPERE FUSE OVER PIN TYPE INSULATOR. NEUTRAL REACTOR SETTING 982 OHMS

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

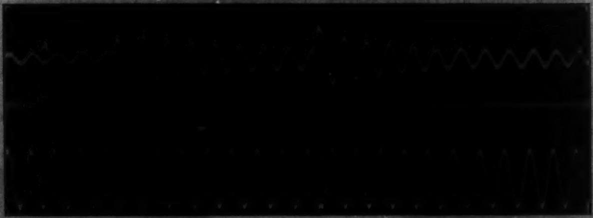

FIG. 30—C D 57306, SYSTEM LOADED, ALL LINES IN SERVICE. CONDUCTOR 1 GROUNDED AT LOCK 12 BY 1 AMPERE FUSE OVER PIN TYPE INSULATOR. NEUTRAL REACTOR SETTING 929 OHMS

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

FIG. 31—C D 57307, SYSTEM LOADED, ALL LINES IN SERVICE. CONDUCTOR 1 GROUNDED AT LOCK 12 BY 1 AMPERE FUSE OVER PIN TYPE INSULATOR. NEUTRAL REACTOR SETTING 870 OHMS

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

TABLE VII.
Arc-Over Tests with Neutral Reactor
Pin Type Insulator

Fig.	C D	Ohms	Amperes	Per cent Unbalance	Arc Lasts
24	57,311	1630	16.3	40 lead	10.5 cycles
25	57,310	1492	17.8	34 "	7.5 "
26	57,309	1357	19.6	27 "	8.5 "
27	57,303	1207	22.0	19 "	7.0 "
28	57,304	1088	24.4	10 "	9.5 "
29	57,305	982	27.0	Zero	6.5 "
30	57,306	929	28.6	6 lag	8.5 "
31	57,307	870	30.5	13 "	5.5 "
32	57,308	811	32.7	21 "	14.0 "

frequency of the circuit consisting of reactor and line capacitance for different settings of the reactor by the equation $f = \frac{1}{2\pi\sqrt{LC}}$. In Table VIII are summarized the results for the reactor settings corresponding to Figs. 24 to 32 inclusive, both as calculated and as measured from the oscillograms.

FIG. 32—C D 57308, SYSTEM LOADED, ALL LINES IN SERVICE. CONDUCTOR 1 GROUNDED AT LOCK 12 BY 1 AMPERE FUSE OVER PIN TYPE INSULATOR. NEUTRAL REACTOR SETTING 811 OHMS

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

Assume a reactor setting of 811 ohms. Then currents will flow somewhat as shown in Fig. 37 (neglecting loss current). Fig. 38 represents an equivalent circuit. A current of 27 amperes circulates in the resonant circuit consisting of reactor and line capacitance, and a residual current of 6 amperes circulates through the reactor and arc in series. The resonant circuit has a natural frequency of 66 cycles.

FIG. 33—PHOTOGRAPH OF ARC OVER PIN TYPE INSULATOR. REACTOR SETTING 1630 OHMS. TAKEN WITH C D 57311 (FIG. 24)

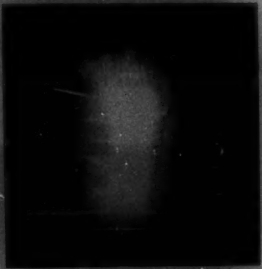
FIG. 34—PHOTOGRAPH OF ARC OVER PIN TYPE INSULATOR. REACTOR SETTING 811 OHMS. TAKEN WITH C D 57308 (FIG. 32)

Now suppose that the arc breaks. Then the 6 amperes through the arc cease to flow. The 27 amperes in the reactor-condenser circuit tend to die away by an oscillation at the natural frequency of the circuit, 66 cycles. The potential of conductors 1 and 2 decreases from 1.732 times normal to normal potential (e) above ground. The potential of conductor 3 increases from zero to normal potential (e) above ground. The charges on conductors 1, 2 and 3 adjust themselves correspondingly. This adjustment takes place through a compound oscillation, consisting of a superposition of the normal frequency and the frequency of the resonant circuit.

Fig. 39 illustrates the adjustment of the voltage of conductor 3 to ground when the normal frequency is 60 cycles and the frequency of the resonant circuit greater than, equal to, and less than 60 cycles.

From this figure it will be noticed that there is a tendency for the voltage from conductor to ground to over-shoot in the adjustment to normal value after the arc is extinguished. Mr. L. F. Blume has shown this in another way by plotting, in polar coordinates, the voltage from neutral to ground (voltage across reactor). Three of his plots are reproduced in Figs. 40, 41 and 42, these being respectively for the 49, 60 and 66-cycle resonance frequencies. The data for these tests are taken directly from the oscillograms, Figs. 25, 29 and 32 as follows:

Fig. 35—Photograph of Arc Over Three Suspension Insulator Disks. Reactor Setting 1630 Ohms

Fig. 36—Photograph of Arc Over Three Suspension Insulator Disks. Reactor Setting 811 Ohms

TABLE VIII.

Natural Frequency of Resonant Circuit

Fig.	*C D*	Ohms	*L* Henry	*C* Mf.	Natural Frequency Calculated	Natural Frequency Measured
24	57,311	1630	4.32	2.7	47 cycles	47 cycles
25	57,310	1492	3.95	2.7	49 "	49 "
26	57,309	1357	3.6	2.7	51 "	51 "
27	57,303	1207	3.2	2.7	54 "	54 "
28	57,304	1088	2.88	2.7	57 "	57 "
29	57,305	982	2.6	2.7	60 "	60.75 "
30	57,306	929	2.46	2.7	62 "	62.5 "
31	57,307	870	2.3	2.7	64 "	63 "
32	57,308	811	2.15	2.7	66 "	66 "

Curve *A* of the oscillograms shows the current in the neutral reactor. This may equally well be taken for the voltage across the neutral reactor, this voltage having the same shape and being at all times the product of the current in the reactor times the ohms. The amplitude of the reactor voltage may therefore be measured on the oscillograms and the angle of lag or lead determined by comparison with the reference voltage, Curve *C*, remembering that the reactor voltage is 90 deg. ahead of the current. After plotting these to scale it is then possible to determine at any instant the voltage from any point to ground, the ground being considered to occupy successive positions along the spiral. Thus in Fig. 40 at the instant the arc breaks, the voltage from conductor 2 to ground is the distance 2-*G*, conductor 3 to ground 3-*G*, and neutral to ground 0-*G*, conductor 1 being at ground potential. At the instant represented by the first positive peak after the arc breaks the voltages are respectively from conductor 3 to ground 3-G_1, from conductor 2 to ground 2-G_1, from conductor 1 to ground 1-G_1 and from neutral to ground 0-G_1, etc. The voltage from neutral to ground does not finally go to zero but remains at some value such as 0-G_{11} on account of the residual ground current which flows through the reactor at all times.

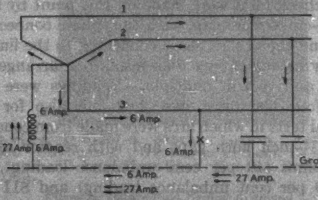

Fig. 37—Three-Phase Circuit with Neutral Grounding Reactor Ohms Off Balance. Assumed Flow of Currents

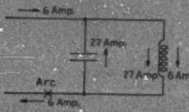

Fig. 38—Equivalent Diagram, Showing Resonant Circuit

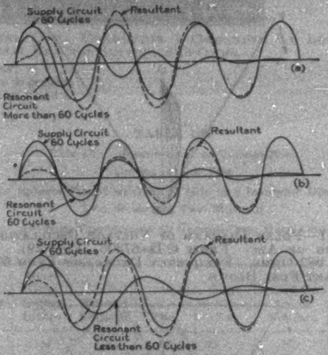

Fig. 39—Adjustment of Voltage From Faulty Conductor to Ground After Arc Breaks

a—Natural frequency of resonant circuit greater than that of supply circuit.
b—Natural frequency of resonant circuit same as that of supply circuit.
c—Natural frequency of resonant circuit less than that of supply circuit.

It will be noted that on some of the oscillograms the curve of current in the neutral reactor after the arc breaks apparently decreases to a minimum value and then increases, in other words has an anti-nodal point. Fig. 43 illustrates this nicely. This is because the current in the reactor consists of the oscillation current at the natural frequency of the oscillation circuit, superimposed on the 60-cycle residual ground current. At times their maxima coincide and at other times oppose and the anti-nodes appear at the points of opposition.

We have thus seen that there is the theoretical possibility when the arc-breaks of a voltage building up from the faulty line to ground greater than the normal line to neutral voltage and from the sound lines to ground greater than the normal line to line voltage, in case there is a difference between the resonant and supply frequencies, *i. e.*, providing the reactor setting is not in exact resonance. Let us see what the tests show in this respect.

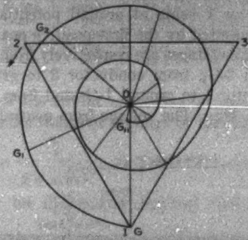

Fig. 40—Polar Diagram of Voltage Oscillation After Breaking of Arc. From C D 57310 (Fig. 25). Reactor Setting 1492 Ohms. Frequency Decreases From 60 to 49 at Instant Fuse Blows

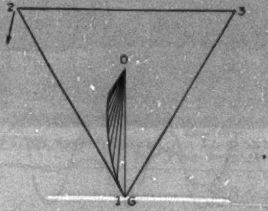

Fig. 41—Polar Diagram of Voltage Oscillation After Breaking of Arc. From C D 57305 (Fig. 29). Reactor Setting 982 Ohms. Frequency Decreases From 60 to 59.4 at Instant Fuse Blows

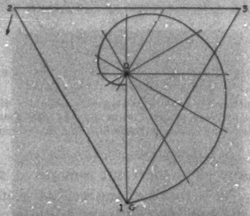

Fig. 42—Polar Diagram of Voltage Oscillation After Breaking of Arc. From C D 57308 (Fig. 32). Reactor Setting 811 Ohms. Frequency Increases From 60 to 66 at Instant Fuse Blows

Two tests were made at the 982 ohm setting (resonance setting), arcing over a ¾-inch horn gap from conductor 1 to ground. The oscillograms showed the following: (a) the wave of voltage line 1 to ground went abruptly to zero at the making of the arc and at the breaking of the arc slowly increased to normal. (b) The voltage from line 3 to ground jumped from normal to about 40 per cent above normal (should theoretically go to 73 per cent above normal for a dead ground). (c) The voltage from neutral to ground goes to a value of about 26,800 root-mean-square volts (theroetically should be 26,500 volts) and remains at this value throughout the arc, except that at the first half cycle after the arc is made there is a peak 5 to 20 per cent greater than the steady peak value.

With the 1207 ohm setting (19 per cent unbalance leading) three tests were made, one with an arc over a ¾-inch gap and two over a 3½-inch gap from conductor 1 to ground. The oscillograms showed (a) wave of voltage line 1 to ground, after the arc broke, went through a slow oscillation increasing above normal then decreasing to less than normal and finally increasing to normal. The maximum was about 35 per cent above normal. (b) The voltage from line 3 to ground increased from normal to a steady value of 50 per cent over normal and for a half cycle to about 70 to 75 per cent above normal. (Should go to 73 per cent above normal as a steady value for a dead ground). (c) The voltage from neutral to ground goes to a steady value of about 26,800 volts, but at the first half cycle after the arc is made there is a peak 15 to 25 per cent greater than the steady peak. Fig. 44, Fig. 45 and Fig. 46 are typical.

As far as the oscillograms are concerned for the limited number of settings tried, there is no overvoltage from any conductor to ground beyond the 73 per cent excess voltage expected from two conductors to ground when the third conductor is grounded. From neutral to ground there is a half-cycle peak about 25 per cent above the line to neutral voltage expected with one conductor grounded. Possibly a further unbalance in the reactor setting would increase these figures somewhat.

Some check tests were made on this point by means of a 12½-cm. sphere gap, with one sphere connected to ground and the other sphere connected to the line conductor by means of four disk insulators, arranged two in series and two in multiple. Readings were taken for the four tests just described and also for three additional tests in which arcs were made from conductor 1 over 3½-inch horn gaps and with reactor settings of 1630 ohms (40 per cent unbalance leading), 1207 ohms (19 per cent unbalance leading) and 811 ohms (21 per cent unbalance lagging). The insulator and gap were arranged in some of the tests between conductor 1 and ground and in others between conductor 2 and ground. The sphere gap was set above the expected arcing distance and then screwed down until it arced. The results are given in Table IX.

The figures of Table IX may be discussed as follows: With the sphere located between conductor 1 and ground and the 1207 ohm setting, the sphere gaps arced over in one case at 9.5 kv. and in another case at 14.5 kv. With the sphere from conductor 2 to ground and the 1207 ohm setting, the sphere gap arced over at 25 kv. in one case and 30 kv. in another case. With the 1630 ohm setting the spheres arced over at 30 kv. and with the 811 ohm setting at 25 kv. Now in order to interpret these results, it would be necessary to know the division of voltage between the spheres and the insulators. This division of voltage depends upon the relative capacitances of the sphere gap and insulators, and on whether the spheres are grounded or not. This is further complicated by the fact that the capacitance of the spheres varies with the setting, being greater with smaller settings. It is possible that the division of voltage is such as to give rather high momentary voltage from line to ground. However, if so, this is not sufficiently prolonged to show itself on the oscillograms, and therefore is probably relatively harmless.

Fig. 43—C D 57257-B. System not Loaded. Vida-Montgomery Line Out. Conductor 1 Grounded at Lock 12 by 1 Ampere Fuse Over 3.5 Inch Horn Gap. Neutral Reactor Setting 2015 Ohms

Curve A, Current in Neutral Reactor.
Curve B, Current from Conductor 1 to ground.
Curve C, Voltage between Conductors 2 and 3.

Fig. 44—C D 57367-A. System Loaded. All Lines in Service. Conductor 1 Grounded at Lock 12 by 1 Ampere Fuse Across 3.5 Inch Horn Gap. Neutral Reactor Setting 1207 Ohms

Curve A, Voltage from Neutral to ground.
Curve B, Voltage from Conductor 1 to ground.

In considering the question of overvoltages, we must remember that all transformers (except those with graded insulation, which at present are not very common) are tested at 3.46 times normal (line to ground) voltage and that the line insulation has this much or greater factor of safety. Also it must be remembered that high-voltage switching operations on grounded neutral and ungrounded neutral systems commonly give overvoltages 70 to 80 per cent above normal (theoretically twice normal), also that single-phase short circuits from line to neutral on grounded neutral systems will so distort the voltage triangle as to give an overvoltage between the two unshort-circuited conductors of approximately 80 per cent. Overvoltages less than these values should therefore not be considered excessive.

Short Circuit from Line to Line

Some tests were made to determine whether or not a

TABLE IX.

Test 15. Measurement of Voltage Rise
During Arcing Test

12½ Cm. Spheres Placed on Ground Side of 4 Insulator Discs Arranged Two Series and Two Multiple.

Arc from Conductor 1 to Ground Over Horn Gap.

Jan. 19, 1922

Reactor Ohms	Horn Gap Setting Inches	Spheres Located	Sphere Gap		Sphere Gap Arc or Not Arc	Oscillo *C D*
			Cm.	Kv.		
982	¾	1-*G*	1.65	35	No	57,360-1
			1.2 to	26 to	No	57,362-3
			1.0	21		
1207	¼	1-*G*	1.2 to	26 to		
			0.51	10	No	57,364-5
			0.5	9.5	Yes	
1207	3½	1-*G*	0.5 to	9.5 to		
			0.72	14.5	Yes	57,366-7
			0.73	15	No	
1207	3½	2-*G*	2.45	50	No	57,368-9
			2.1	44	No	
			1.65	35	No	
			1.0	20	Yes	
			1.15	25	Yes	
			1.4	30	No	
1630	3½	2-*G*	1.4	30	Yes	
			1.65	35	No	
			1.65	35	No	
1207	3½	2-*G*	1.4	30	Yes	
			1.65	35	No	
811	3½	2-*G*	1.4	30	No	
			1.15	25	Yes	

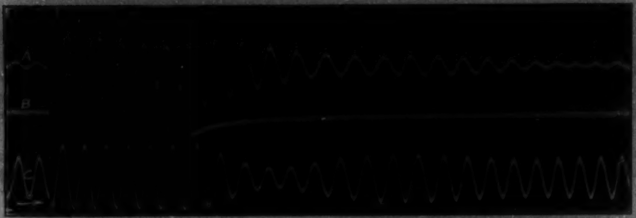

Fig. 45—C D 57368. System Loaded. All Lines in Service. Conductor 1 Grounded at Lock 12 by 1 Ampere Fuse Across 3.5 Inch Horn Gap. Neutral Reactor Setting 1207 Ohms

Curve A. Current in Neutral Reactor.
Curve B. Current from Conductor 1 to ground.
Curve C. Voltage from Conductor 3 to ground.

Fig. 46—C D 57369. System Loaded. All Lines in Service. Conductor 1 Grounded at Lock 12 by 1 Ampere Fuse Across 3.5 Inch Horn Gap. Neutral Reactor Setting 1207 Ohms

Curve A. Voltage from Neutral to ground.
Curve B. Voltage from Conductor 1 to ground.

Fig. 47—C D 57372. System Not Loaded. Vida-Montgomery Line Out. Neutral Grounded Through Reactor Set at 1410 Ohms. Short Circuit Between Conductors 2 and 3 at Vida

Curve A. Voltage from Conductor 1 to ground.
Curve B. Voltage between Conductors 2 and 3.

Fig. 48a

Fig. 48b

Fig. 48—C D 57375-A, C D 57375-C. System not Loaded. Vida-Montgomery Line Out. Neutral Solidly Grounded. Short Circuit Between Conductors 2 and 3 at Vida.

Curve A. Current in grounded neutral.
Curve B. Voltage from Conductor 1 to ground.
Curve C. Voltage between Conductors 2 and 3.

single-phase line to line short circuit would cause a voltage rise from the unshort-circuited line to ground. The system was not loaded, and the Vida-Montgomery line was out. A single-phase short circuit was placed between conductors 2 and 3 at Vida and the switch closed at Lock 12. Voltage was measured from Line 1 to ground and between lines 2 and 3. Tests were made with the neutral grounded through the reactor, set at 1410 ohms, and also with the neutral solidly grounded. Fig. 47 illustrates the case with reactor, Fig. 48 the case with the neutral solidly grounded. It is interesting to note that in Fig. 47 there is a momentary rise at the make and break of the short circuit of 57 to 58 per cent in the voltage from line 1 to ground. In Fig. 48 there is a voltage rise of about 63 per cent at the break of the short circuit. In other words there is about the same voltage rise whether the neutral is grounded solidly or through a reactor, and this is probably solely due to the distortion of the voltage triangle by the short-circuit current acting on the generator voltage.

SOLID GROUNDS ON LINE

As previously explained when a solid ground occurs on a conductor, means are provided to cut off the section in trouble at a predetermined time. Fig. 49 and Fig. 50 illustrate the action of this device.

Fig. 49—C D 57262. System Not Loaded. Vida-Montgomery Line Out. Neutral Reactor Setting 1735 Ohms. Conductor 1 Solidly Grounded at Lock 12 and Reactor Automatically Short Circuited. Thus Placing Short Circuit on System

Curve A. Current in Neutral Reactor.
Curve B. Short circuit current.
Curve C. Voltage between Conductors 2 and 3.

In Fig. 49 the top wave is the current through the reactor due to the solid line ground on conductor 1 at Lock 12. About the middle of the oscillogram the oil circuit breaker closes and short-circuits the reactor, the reactor current going slowly to zero. Short-circuit current then flows (middle wave) until the line is disconnected by the overload relays. The bottom wave is the voltage between conductors 2 and 3 and it is interesting to note that this increases in value to a marked extent, the highest peak being about 90 per cent greater than normal.

In Fig. 50 the middle wave is the reactor current, which is residual current only until the line ground is thrown on about ⅝ of the distance from the left end of *C D* 57302-*A*. At the end of the two seconds the oil circuit breaker short-circuits the reactor and the reactor current goes to zero (just beyond the center of *C D* 57302-*B*.) The short-circuit current then appears in the top wave and is cut off by the line oil circuit breaker near the end of the oscillogram. The bottom wave is the voltage from conductor 1 to conductor 3, which increases during the time of short circuit at the maximum peak about 70 per cent above normal voltage and about 30 per cent during the sustained period of the short circuit.

The relay connected to the secondary of the current transformer in series with the reactor, in this case was set to operate in two seconds and on about 12 amperes, and in Fig. 50 it will be noted that the reactor oil circuit breaker closed in just 120 cycles after the ground was placed on the circuit.

Fig. 50A

Fig. 50B

Fig. 50—C D 57302-A, C D 57302-B. System Not Loaded. Vida-Montgomery Line Out. Neutral Reactor Setting 1735 Ohms. Conductor 1 Solidly Grounded at Lock 12 and Reactor Automatically Short Circuited, Thus Placing Short Circuit on System

Curve A. Short circuit current.
Curve B. Current in Neutral Reactor.
Curve C. Voltage between Conductors 2 and 3.

LIMITATIONS IN USE AND OTHER FORMS OF DEVICE

From the writer's study of this device he is of the opinion that its use will be limited for the present to comparatively low-voltage lines of moderate length, *i. e.*, perhaps up to 66,000 volts, or in a few cases up to

100,000 volts if the length is short. The reason for this limitation is as follows: The current through the reactor is affected by the line reactance. In the case of a short line or low-voltage line this is comparatively small. For example, the Alabama line we have been considering has about 20 ohms reactance, which added to 982 ohms makes an insignificant change in the reactor current. On the other hand a typical 154,000-volt line may require 200 ohms in the reactor and reactance of the line itself may be 150 ohms. It is probable that the residual current in this case would maintain the arc.

In this country the device appears most applicable to those systems which are now operating isolated neutral, and which for some reason connected with the operation or apparatus, do not wish to go to the solidly grounded neutral.

If more than one point is grounded, a reactor must be provided for each point.

If there are no points suitable for grounding, an artificial neutral may be provided by means of a grounding transformer connected *Y*-delta, the *Y* being connected to the high-tension line and the reactor placed between its neutral and ground, or the reactor may be connected in one corner of the low-tension delta, or the neutral of the transformer may be grounded solidly and the necessary reactance placed inherently in the transformer between high-tension and low-tension windings.

It has been suggested that with each section of the line a reactor be provided, which may be switched on and off with the line section, thus providing a good balance at all times. Such a reactor may take any of the forms previously mentioned or it may consist merely of a reactor between each of the three conductors and ground, this reactor being designed to balance the capacitance to ground of that section of the line. If one conductor becomes grounded, then the reactor on that phase is short-circuited as well as the capacitance to ground, while on the other two phases the reactor and capacitance as well are raised to the same voltage above ground, and therefore still neutralize each other. By this method, the reactors being in circuit at all times, tend to counteract the normal line charging current, which is sometimes useful for line regulation.

CONCLUSION

Calculations show that the current to ground when one conductor of a three-phase system is grounded is approximately 1.5 times the normal three-phase charging current. Tests on one system show that if the system has a ground wire the ground current is increased about 20 per cent.

About 50 arcover tests were made with the neutral grounding reactor in service, and various settings of reactance both in and out of balance. The reactor satisfactorily performed its function as an arc extinguisher. From the tests it would appear that considerable tolerance is permissible in the setting of the reactor.

From the standpoint of voltage stresses, the system with reactor is more like an isolated neutral system than a grounded neutral system, except that the excessive voltage rises due to arcing grounds which occur on an isolated neutral system are eliminated. There is a possibility of overvoltage when the arc breaks due to a difference between the resonant and supply frequencies, providing the reactor setting is not in exact resonance.

The system with reactor has an advantage over the grounded neutral system in that arcs are eliminated without short circuit, which is desirable since short circuits cause mechanical stress to apparatus as well as frequent interruption of service.

In the case of a dead ground on the line, the reactor is short-circuited, the neutral grounded and the circuit cleared by line relays. The device designed to take care of this operation satisfactorily performed its function in test.

In the writer's opinion the device will be limited to comparatively low voltage and short systems for the present, owing to the expense of installing on high voltage and long systems and the difficulty of obtaining a current balance at the arc. Owing to the popularity of and satisfaction achieved with grounded neutral systems in this country, the use of the grounding reactor will probably be limited mainly to isolated neutral systems, whose operation is not satisfactory, but on which for some reason it is not wished to solidly ground the neutral.

On the system tested there was a small current flowing at all times to the solidly grounded neutral, due to unbalanced line capacitance. This current contained fifth and seventh harmonics. The neutral reactor caused this current to increase appreciably but the fifth and seventh harmonics were totally eliminated.

The writer wishes to acknowledge the hearty cooperation of Mr. J. M. Oliver, Operating Engineer of the Alabama Power Company, and his assistants, also that of Messrs. M. A. Rusher and C. L. Schermerhorn of the General Engineering Laboratory of the General Electric Company in making the tests.

Bibliography

1. Limitation of Earth Current and Suppression of Earth Fault Arcs by the Earthing Coil, by W. Petersen, *Elektrotechnische Zeitschrift* Jan. 2 and 9, 1919. The fundamental paper on the subject of the neutral grounding reactor.

2. Grounding Through Earth Coils as a Means of Protection Against Accidental Grounds and the Resultant Overvoltages, by R. Bauch, *Elektrotechnische Zeitschrift*, June 2 and 9, 1921. Discusses the use of grounding transformers in lieu of the Petersen Coil.

3. The Petersen Earth Coil, by R. N. Conwell and R. D. Evans, JOURNAL, A. I. E. E., Feb. 1922. Discusses the results of some tests made on the system of the Public Service Company of New Jersey.

Discussion

For discussion of this paper see page 464.

Operating Performance of a Petersen Earth Coil

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Review of the Subject.—*This paper is a report on the Alabama Power Company's operating experience with a Petersen Earth Coil installed between the neutral and ground of a 120-mile, 44,000-volt, 3-phase, star-connected, 60-cycle system.*

A Petersen coil is essentially an inductive reactance of such value as to maintain resonance with the capacitance of the system to ground at the fundamental system frequency. With a ground on one wire the current through the fault is reduced to such a low value as to prevent maintenance of an arc. Therefore, on the assumption that the majority of phase-to-ground short circuits start as insulator flashovers, the installation of such a device as a Petersen coil which would snuff out flashover arcs should considerably reduce the number of interruptions to the line.

By means of proper relaying, cases of trouble outside of the operating sphere of the Petersen coil, such as phase-to-phase short circuits and solid grounds have also been successfully taken care of.

Previous to the installation of this coil numerous interruptions to service were experienced on this system during lightning storms, which are unusually severe in the territory covered by these lines. This system, therefore, offers an ideal location for a trial installation of the coil.

Since the installation of the coil the number and duration of interruptions due to lightning flashovers have been reduced by 83.5 per cent and 94 per cent respectively. Several doubtful actions, however, have occurred during switching operations, indicating the presence of unusual phenomena. By proper relaying it is hoped to prevent the recurrence of such action. Further tests are also contemplated to investigate the unusual phenomena accompanying certain operating conditions.

CONTENTS

Introductory Remarks. (150 w.)
Description of Lines. (375 w.)
Description of Petersen Coil. (325 w.)
Control and Relay Connections. (450 w.)
Operation. (125 w.)
Flashovers. (250 w.)
Faulty Operations. (225 w.)
True Flashovers in which Grounding Switch Closed. (100 w.)
Neutral Current Indications on Switch Operations. (85 w.)
Grounded Phase Operation. (150 w.)
Comparison of Operation with and without the Petersen Coil. (350 w.)
Conclusions. (125 w.), (4 tables and Log of Operation).

A PETERSEN Earth Coil, or neutral grounding reactor was installed on the Alabama Power Company's Lock 12-Vida 44,000-volt system on October 12, 1921, and, with the exception of short intervals during testing and construction periods, has been in continuous service to date. Although Petersen coils have been in use in Europe for sometime, the Lock 12 installation is perhaps the first one in this country which has been in actual continuous operating service for a considerable length of time, and it has consequently aroused a great deal of interest.

The purpose of this paper is to give a description of the Lock 12 installation and a complete account of its operating performance to date. The theory of operation of the Petersen coil together with a report of tests made at Lock 12 on December 3-4, 1921, and January 18-19, 1922, in conjunction with the General Electric Company, are presented in a companion paper by Mr. W. W. Lewis.

DESCRIPTION OF LINES

The Lock 12-Vida 44,000-volt system is a three-phase, star-connected, 60-cycle system, consisting of a main line from Lock 12 to Vida having tap lines to Mitchell Dam and Clanton, an east branch from Vida to Montgomery, and a west branch from Vida to Marion as shown in the single-line diagram Fig. 1.

These lines are not a part of the main 44,000-volt network, but form a separate and very important system, as they are the main source of supply to the cities of Montgomery, Selma, Marion and Clanton, having a combined population of over 60,000. The maintenance of service to these lines consequently is of prime importance, and the application of any device such as a Petersen coil, which would reduce the number of interruptions could be well demonstrated here. Furthermore, the territory covered by this system is subjected to lightning storms of unusual frequency and intensity resulting in numerous line outages due to insulator flashovers, the very trouble from which the Petersen coil promises relief. These lines were, therefore, selected as the most ideal location for a trial installa-

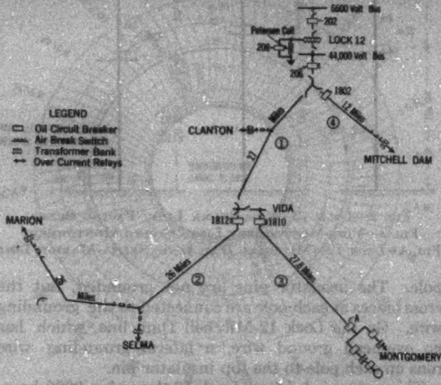

FIG. 1—LOCK 12—VIDA 44,000-VOLT SYSTEM

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[1] So named after its inventor Prof. W. Petersen, of Darmstadt, Germany.

Presented at the Spring Convention of the A. I. E. E., Pittsburgh, Pa., April 24-26, 1923.

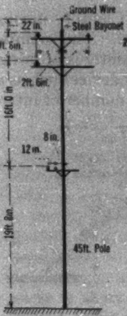
tion of the Petersen coil on the Alabama Power Company's system.

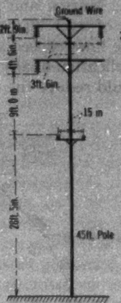
The physical characteristics of the lines comprising the Lock 12-Vida system are as follows:

Line	Length miles	Number of circuits	Conductor	Spacing	Insulation
Lock 12-Vida	27.0	1	(168,000 cm. Al. steel core)	5-3.5-6.1	50-kv. pin type
Mitchell dam tap	12.0	1	No. 4 Alum.	3.5-3.5-5	50-kv. pin type
Clanton tap	1.3	1	No. 4 Cu.	5-3.5-6.1	50-kv. pin type
Vida-Montgomery	27.8	1	(168,000 cm. Al. steel core)	7-4.5-8.3	4 O. B. suspension units
Vida-Marion	52.0	1	No. 4 Cu.	5-3.5-6.1	50-kv. pin type

These lines are all of pole construction with wooden cross arms giving conductor spacings as shown in Figs. 2, 3 and 4 of the various lines. There are no transpositions in any of the lines.

All the lines, with the exception of the Lock 12-Mitchell Dam line, are equipped with one ⅜-in. 7-strand steel, overhead ground wire grounded at each pole. The insulator pins are not grounded, but the cross braces at each pole are connected to the grounding wire. On the Lock 12-Mitchell Dam line, which has no overhead ground wire, a lateral grounding wire runs up each pole to the top insulator pin.


Ground Wire
Steel Bayonet
22 in.
3ft. 6in.
2ft. 6in.
16ft. 0 in.
8 in.
12 in.
19ft. 8in.
45ft. Pole


Ground Wire
2ft. 9in.
4ft. 6in.
3ft. 6in.
9ft. 0 in.
15 in.
26ft. 5in.
45ft. Pole

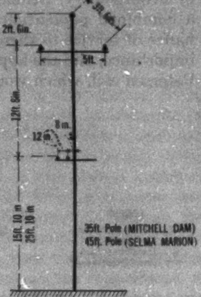

3ft. 6in.
2ft. 6in.
5ft.
12ft. 8in.
8 in.
12 in.
15ft. 10 in.
25ft. 10 in.
35ft. Pole (MITCHELL DAM)
45ft. Pole (SELMA MARION)

FIG. 2—LOCK 12—VIDA-SELMA LINE, FACING SELMA

FIG. 3—VIDA-MONTGOMERY LINE, FACING MONTGOMERY

FIG. 4—LOCK 12—MITCHELL DAM LINE, SELMA-MARION LINE

This system is fed from Lock 12 through a 9000-kv-a. transformer bank consisting of three 3000-kv-a. single-phase, step-up transformers, connected 6600 volts delta on the low-tension side to 44,000 volts star on the high-tension side. The Petersen coil is connected between the neutral of the 44,000-volt winding and ground, as shown in Fig. 5.

DESCRIPTION OF PETERSEN COIL

The Petersen coil is an oil-immersed, water-cooled, reactor without core having a continuous rating of 795 kv-a. at 26,540 volts, 60-cycle, with a 55 deg. cent rise. The reactor is designed to have a reactance of 2015 ohms with 5 per cent taps ranging down to 811 ohms. External and internal views of the reactor are shown in Figs. 6 and 7 respectively.

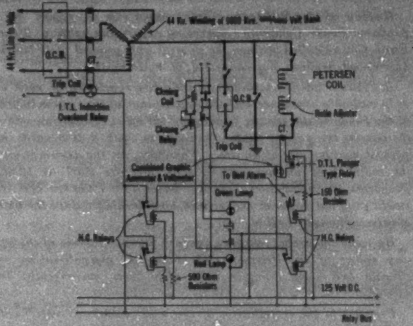

44 Kv. Line to Vida
44 Kv. Winding of 9000 Kva. Volt Bank
O.C.B.
C.T.
Trip Coil
I. T. L. Induction Overload Relay
Closing Coil
Closing Relay
O.C.B.
Trip Coil
PETERSEN COIL
Ratio Adjuster
C.T.
Combined Graphic Ammeter & Voltmeter
To Bell Alarm
D.T.L. Plunger Type Relay
Green Lamp
150 Ohm Resistor
H.G. Relays
H.G. Relays
Red Lamp
500 Ohm Resistors
125 Volt D.C.
Relay Bus

FIG. 5—LOCK 12—PETERSEN COIL INSTALLATION—SCHEMATIC DIAGRAM OF CONNECTIONS

The numerous taps on the reactor are equipped to take care of various combinations of the present lines in service and proposed future extensions. The Petersen coil operates on the principle of an inductive reactance resonated at the fundamental system frequency with the capacity reactance of the system to ground—hence, for every setup on the system involving different combinations of lines in service, a corresponding value of inductive reactance will be required equivalent to the capacity reactance of lines in service.

FIG. 6—OIL-INSULATED, DISK-COIL, WATER-COOLED REACTOR

The setting of the reactor to maintain resonance is conveniently accomplished by means of two ratio adjusters, with control handles extending through the tank cover, enabling adjustments to be made in but a few minutes time.

Following is a table showing the adjustments corresponding to various combinations of lines in service:

Lines in Service (See Fig. 1)	Reactance ohms	Adjuster "A"	Adjuster "B"
1-2-3-4-5	1088	IV	V
1-2-3-5	1140	IV	III
1-2-3-4	1207	IV	II
1-2-4-5	1357	I	V
1-2-3	1410	I	III
1-2-5	1567	I	I
1-3-4	1630	III	I
1-2-4	1735	II	V
1-3	1915	II	II
1-2	2015	II	I
4	Reactor out of service.	Neutral solidly grounded.	

CONTROL AND RELAY CONNECTIONS

Since the Petersen coil functions only in cases of insulator flashover, which experience shows are the cause of the majority of line troubles, protection must nevertheless, also be provided for the occasional cases of solid grounds, and phase-to-phase short circuits.

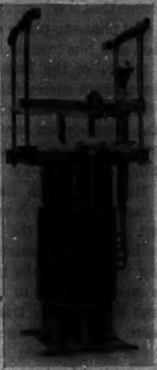


FIG. 7—DISK-COIL, OIL-IMMERSED, WATER-COOLED REACTOR

Phase-to-phase short circuits can be cleared by regular overload relays installed on the line switch. A solid ground on a Petersen coil system, however, will have no effect on the line relays and other means must be provided to clear the trouble, unless it is desired to continue operation with a phase grounded. In the Lock 12 installation an oil circuit breaker is connected across the reactor, as shown in Fig. 5. This breaker is normally open, but closes automatically in cases of solid grounds on the lines, thereby grounding the neutral solidly, after which the overload relays on the line switch will operate as in a normal grounded system and clear the trouble.

The control scheme is shown in detail in the schematic diagram Fig. 5. A definite time limit overcurrent relay is connected in the secondary of a current transformer, the primary of which is in series with the Petersen coil and ground. This relay is given a time setting of approximately two seconds. Normally, flashovers are cleared by the Petersen coil in 5 to 15 cycles, consequently, when the reactor current holds on for several seconds it is evident that the reactor is not functioning, and it can be taken as a safe indication that a solid ground has developed on the line. At the end of two seconds the relay contacts will close, thereby energizing two auxiliary relays, one of which causes the grounding switch to close and the other ringing a bell alarm to warn the operator that the line is in trouble. The grounding switch on closing then, short circuits the reactor and the regular overcurrent relay scheme comes into action to clear the trouble. This short-circuiting switch must be opened by hand to again place the reactor in service. Five disconnect switches are provided, as shown in Fig. 5, whereby the reactor or grounding switch can be isolated for clearance or for testing.

For a permanent daily record of the operations, a curve drawing ammeter and voltmeter combined in one instrument is provided; the ammeter indicating the current passing through the reactor, and the voltmeter indicating when the grounding switch is open or closed. A typical chart showing both ammeter and voltmeter actions is shown in Fig. 8. The scale on the chart applies only to the inner or current curve, and on the particular day shows an indication of 21 amperes at 10:40 p. m. No scale applies to the outer or voltage curve; it shows only a notched indication when the grounding switch operates. On this chart, such a record is shown slightly ahead of the 21-ampere current indication. In reality, these indications occurred only two seconds apart; the 15 minute separation on the chart being necessary to prevent interference between the two pens.

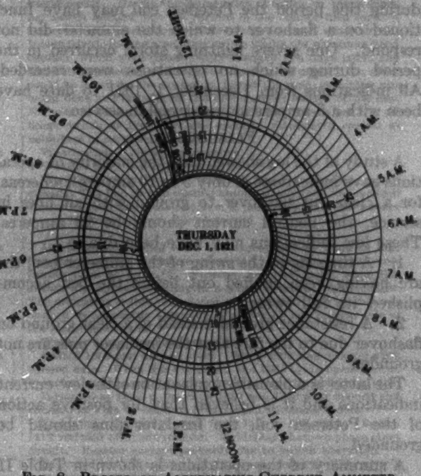


FIG. 8—RECORD OF ALTERNATING-CURRENT AMMETER

After the reactor had been in operation for some time, several flashovers occurred (July 7, August 25, 1922 and Feb. 23, 1923) on the 44,000-volt bus insulators at Lock 12, when the line switch was opened automatically or by hand. At these times the Petersen coil was in service. Whether it was in any way responsible for the flashovers, is not known, but, as a precaution against similar troubles in the future, the line relays have been electrically interlocked with the control circuit of the grounding switch as shown in Fig. 5, insuring that the grounding switch is first closed before the line switch can open on relay action. This interlocking, however, has the disadvantage of slowing up the relay operation on phase-to-phase short circuits. The operators likewise have instructions to always close the grounding switch before doing any line switching.

OPERATION

Table I is a log of all the Petersen coil operations, from October 12, 1921, to March 1, 1923, as read from the charts of the combined graphic ammeter and voltmeter. From October 12th to November 30, 1921, the graphic ammeter was equipped with a 30-ampere coil instead of a 5-ampere coil, which would give only 1/16 inch deflection on the chart for 22 amperes. Since flashovers last only 5 to 15 cycles, it is thought that during this period the Petersen coil may have functioned on a flashover to which the ammeter did not respond. One severe lightning storm occurred in the period during which no operations were recorded. All indications from December 1, 1921 to date have been with a 5-ampere ammeter coil in service.

FLASHOVERS

A study of the log showed that the majority of operations gave indications of only 5 to 10 amperes, whereas, for a clear cut flashover to ground with all lines in service, the neutral current should be 22 amperes. These low indications may have been due to

1. A time lag in the swing of the ammeter pen, the arc having been snuffed out before the pen accomplished its full swing.
2. A resistance ground instead of a clear ground on flashover due to the fact that the insulator pins are not grounded.

The latter is a plausible explanation of the low-current indications and it is believed that, for positive action of the Petersen coil, the insulator pins should be grounded.

A summary of the operations is shown in Table II where it is noted that there have been 168 flashover indications, which seems rather high for the period covered. It may be that all of the low-current indications are not the result of flashovers, but if such is the case it would be difficult to draw a dividing line between the true flashover indications and those due to other causes. The ammeter scale is considerably contracted from 0 to 5 amperes whereas above 5 amperes the unit scale divisions are more uniform, making an ampere division above 5 amperes on the chart nearly equal to the division representing 0-5 amperes. Since the 5 ampere indications are comparatively small, a tentative classification has been made in Table II of the flashover indications of 6 amperes and above, and those below 6 amperes.

FAULTY OPERATIONS

In all there were only two operations which could be strictly classed as faulty operations. These occurred on December 26th at 5:34 p. m., and on August 8 at 4:55 p. m. when the flashovers held on long enough to close the grounding switch and cause the overcurrent relays to open the line. The line in each case went back in service properly, indicating that the trouble was not a solid ground, but a flashover which the Petersen coil evidently failed to clear.

SOLID GROUNDS AND PHASE-TO-PHASE CIRCUITS

The summary shows that there were a total of 29 cases of solid grounds and phase-to-phase short circuits due to line, transformer, and bus trouble, which are outside of the operating sphere of the Petersen coil. All of these cases, however, were successfully cleared by the line relays. Of these operations, however, three are considered doubtful—namely, on July 7th, August 25th and February 23rd when, after the line switch opened, a string of insulators on the 44-kv. bus between the transformer and the line switch flashed over. The flashover in each case was across 5 units of O. B. suspension insulators, indicating the presence of an unusually high voltage. The insulator units all meggered well on being tested after the trouble, indicating that the troubles were not due to insulator failures. It seems that some high-voltage transient was introduced in these cases due to the disconnection of the line capacitance. It is proposed to make further tests to verify this point. As stated before, all switching, both hand and automatic, is now being done with the grounding switch closed—that is, with the reactor out of service.

TRUE FLASHOVERS IN WHICH GROUNDING SWITCH CLOSED

There were seven operations in which the grounding switch closed, but no line switch opened, and no disturbance occurred on the system. As the closing of the grounding switch seemingly indicated a solid ground on the system, the only explanation for these operations is that a flashover or solid ground held on long enough (two seconds) to cause the definite time limit overcurrent relay to operate, but cleared before the contacts of the grounding switch closed. These operations are all considered correct as there were no interruptions to service.

NEUTRAL CURRENT INDICATIONS ON SWITCH OPERATIONS

There were twenty cases of reactor current indications coincident with switching operations on the lines under conditions when the neutral current must have been zero. These indications may have been due to the

TABLE I

ALABAMA POWER COMPANY—LOCK 12 GENERATING STATION

Log of Petersen Coil Operation

Date	Flashover to ground	Solid grounds	Phase-to phase short circuits	Automatic switch operations	Reactor current amperes	Operation	Nature of trouble and remarks
1921							
Nov. 16			11:28 a.m.	#206 opened	0	Correct	A surge occurred on the Montgomery line, and Line Switch #206 at Lock 12 opened automatically. Cause of trouble was the failure of a transformer at Prattville approximately 15 miles northwest of Montgomery. The fuses at transformer failed to clear trouble, and a single-phase line-to-line short circuit developed. The Petersen coil is not supposed to function under such conditions, so the line switch operation was correct.
Dec. 1	10:40 p.m.			208 closed	21	Correct	Weather cloudy. Cause unknown. No surge or outage occurred. Evidently a flashover or solid ground occurred and held on for two seconds at which time closing relay of the grounding switch was energised, but the flashover cleared before the grounding switch closed its contacts.
Dec. 2					7		Occurred 12:40 p. m. when switch #1802 was closed. Evidently due to switch contacts not closing together.
Dec. 8	5:18 p.m.				13	Correct	Weather cloudy. No outage or surge occurred.
Dec. 8	5:20 p.m.			208 closed	22	Correct	Weather cloudy. No outage or surge. Evidently a flashover or momentary ground occurred and held on long enough (2 sec.) to cause grounding switch to close, but cleared before grounding switch completed its closing operation.
Dec. 8	5:22 p.m.				15	Correct	Weather cloudy. No outage or surge occurred.
Dec. 8	6:00 p.m.				15	Correct	" " " " " " "
Dec. 8	6:12 p.m.				15	Correct	" " " " " " "
Dec. 11	9:18 a.m.			208 closed	18	Correct	Cause unknown. No lightning or rain occurred during this period. The ammeter pen would swing up to values noted, the grounding switch would close, but not latch, and then operation would repeat. On the fourth operation at 9:30 a. m. the grounding switch latched, but the current in the reactor held on at 6 amperes. No surge, however, occurred due to a short circuit. It is believed that the grounding switch blades did not make good contact, which were consequently changed two days later, and carefully checked to see that good contact was made. No cause could be found for the operation, but at the time, switching was being done at Prattville on the Montgomery section of the line. No arcs of unusual length were noted, however, when the air break switch at Prattville was opened. Considered correct as no outage or surge occurred.
	9:20 a.m.			208 closed	17	Correct	
	9:25 a.m.			208 closed	20	Correct	
	9:30 a.m.			208 closed	10	Correct	
Dec. 17	8:55 a.m.				8	Correct	Weather fair. No outage or surge.
Dec. 17	9:00 a.m.				12	Correct	Weather fair. No outage or surge.
Dec. 26	5:34 p.m.			208 closed 206 opened	25	Faulty	Weather cloudy. Surge occurred when grounding switch closed. The line Switch opened automatically, but was reclosed 2 minutes later and remained in o. k. indicating that disturbance was a flashover and not a solid ground.
1922							
Jan. 6		1:10 a.m.		208 closed 202 opened	21	Correct	Disturbance occurred when 206 was opened by hand to take line out of service to change a bad 44-kv. bushing on the transformer bank at Lock 12. Bushing, however, failed when line switch was opened resulting in a phase-to-ground short circuit. The grounding switch then closed automatically, and trouble was cleared by automatic operation of switch on low tension side of the transformer bank.
Jan. 16	2:35 p.m.				11	Correct	Weather fair. No outage or surge.
Mar. 3			1:05 p.m.	1802 opened	2	Correct	Switch 1802 first opened auto. When 1802 was reclosed #202 opened auto. Then 202 was closed and 1802 again opened auto. Patrolman later reported a tree down across Mitchell Dam Line.
			1:35 p.m.	202 "	10	"	
			1:40 p.m.	1802 "	7	"	
Mar. 8					8		Recorded at 12:25 p. m. when 1810 and 1812 were opened and reclosed at Vida by hand on prearranged switching orders.
Mar. 14			5:35 p.m.	1812 opened	5	"	Lightning. Voltage surged from 115 to 108 volts. Switch 1812 reclosed after one minute. Evidently phase-to-phase short circuit. Reactor current probably due to switch operation.
Mar. 26					7		Recorded at 7:55 a. m. when 1810 was closed at Vida after a prearranged interruption to Montgomery.
Mar. 31	5:00 a.m.				19	"	Severe lightning. No outage or surge.
	5:07 a.m.				23	"	" " " " " "
Apr. 11	2:40 a.m.				17	"	Lightning. No outage or surge.
	4:50 a.m.				8	"	" " " " "
	5:05 a.m.				6	"	" " " " "
Apr. 23	5:28 a.m.				6	"	Weather fair. No outage or surge.
	7:30 p.m.				5	"	" " " " " "
Apr. 27	3:10 p.m.				5	"	Weather cloudy. No outage or surge.
Apr. 28			8:17 a.m.	202 opened	10	"	Lightning at Lock 12. Voltage surged from 118 to 101 volts. Switch 202 was reclosed o. k. two minutes later.

TABLE I (Continued)

Date	Flashover to ground	Solid grounds	Phase-to-phase short circuits	Automatic switch operations	Reactor current amperes	Operation	Nature of trouble and remarks
Apr. 28	9:10 a.m.				5	Correct	Lightning at Lock 12. No outages or surges. Operator at Lock 12 reported telephone noises at the times of these Petersen Coil actions. No disturbances were recorded on any of the other lines of the system at the time.
	9:25 a.m.				5	"	
	9:30 a.m.				5	"	
	9:40 a.m.				5	"	
	9:55 a.m.				5	"	
	10:20 a.m.				5	"	
	10:35 a.m.				5	"	
	10:50 a.m.				5	"	
	11:30 a.m.				5	"	
	11:50 a.m.				6	"	
	12:00 Noon				5	"	
	12:10 p.m.				5	"	
	12:20 p.m.				5	"	
	12:35 p.m.				5	"	
	12:40 p.m.				5	"	
	12:50 p.m.				5	"	
	1:00 p.m.				5	"	
	2:05 p.m.				6	"	
May 1	4:30 p.m.				5	Correct	Weather fair. No outage or surge.
May 2	3:35 a.m.				5	Correct	Raining. No outage or surge.
May 6	11:35 a.m.				5	Correct	Weather cloudy. No outage or surge.
May 8	8:28 p.m.				5	Correct	Lightning at Vida. No outage or surge.
May 9	10:35 a.m.				5	Correct	Cloudy. No outage or surge.
	7:30 p.m.				5	Correct	Lightning around Lock 12. No outage or surge.
May 10	4:20 p.m.				5	Correct	Weather cloudy. No outage or surge.
May 16	1:15 a.m.				5	Correct	Weather fair. Line was patrolled in the forenoon and an insulator found off its pin on pole #38 between Vida and Lock 12. A man was stationed at this point to watch and in case of trouble he was to place the insulator back on its pin. No further trouble occurred at this point. Considered correct operation as a ground was evidently prevented. Line was operated with loose insulator until 12:05 a. m. May 17 when an interruption was prearranged to place insulator back on pin.
	1:20 a.m.				3	Correct	
	1:30 a.m.				3	Correct	
	2:40 a.m.				3	Correct	
	2:55 a.m.				3	Correct	
	3:05 a.m.				3	Correct	
May 16			5:02 p.m.	1812 opened "A" opened	7	Correct	Lightning was general over territory covered by line. Voltage surged from 116 to 106 volts. Grounding switch 208 did not close, indicating a phase-to-phase short circuit. Switch 1812 reclosed at 5:03 p. m. and Switch "A" at 5:07 p. m. O. K. Switch "A" opened because Montgomery Steam Plant was on at time.
May 16	5:17 p.m.			"A" opened	11	Correct	General lightning. Voltage surged from 113 to 108 volts. No outage at Lock 12. Evidently a flashover. Cause of Switch "A" opening not known.
May 17					10		Recorded when switch 206 was opened to take line out of service.
					5		Recorded when switch 206 was closed to put line back into service.
May 17	9:50 a.m.				3	Correct	Rain and lightning.
	10:05 a.m.				3	Correct	No outage or surge.
	10:55 a.m.				3	Correct	" " " "
	11:10 a.m.				3	Correct	" " " "
	12:02 p.m.				3	Correct	" " " "
	12:20 p.m.				3	Correct	" " " "
May 18	10:15 p.m.				5	Correct	Weather fair. No outage or surge.
May 19	9:55 a.m.				5	Correct	Lightning at Montgomery. No outage or surge.
May 20	9:16 a.m.				5	Correct	Weather cloudy. No outage or surge.
	8:05 p.m.				5	Correct	Lightning. No outage or surge at Lock 12. Montgomery reported surges at these times.
	8:20 p.m.				5	Correct	
	10:30 p.m.				5	Correct	Lightning. No outage or surge.
May 21					6		Recorded when Switch #1802 was opened to take the Mitchell Dam line out of service. Current held on at 6 amperes until switch was reclosed.
May 21	2:32 p.m.				9	Correct	Rain and lightning. No outage or surge.
May 24	12:20 a.m.				5	Correct	Weather cloudy. No outage or surge.
	12:29 a.m.				5	Correct	" " " " " "
	12:44 p.m.				5	Correct	Lightning in vicinity. No outage or surge.
	2:03 p.m.				5	Correct	" " " " " " "
May 25	10:50 a.m.				5	Correct	Rain and lightning. No outage or surge.
	10:56 a.m.				5	Correct	" " " " " " "
	11:05 a.m.				5	Correct	" " " " " " "
	11:25 a.m.				6	Correct	" " " " " " "
	3:55 p.m.				5	Correct	" " " " " " "
	7:00 p.m.				5	Correct	" " " " " " "
May 27	6:30 a.m.				5	Correct	Raining. No outage or surge at Lock 12. Operator at Vida reported leaky insulator. Insulator was changed 12:00 o'clock that night.
	8:15 a.m.				5	Correct	
	9:40 a.m.				5	Correct	
	10:20 a.m.				5	Correct	
	3:05 p.m.				5	Correct	
	6:30 p.m.				5	Correct	
	7:40 p.m.				5	Correct	
	8:00 p.m.				5	Correct	

TABLE I (Continued)

Date	Flashover to ground	Solid grounds	Phase-to phase short circuits	Automatic switch operations	Reactor current amperes	Operation	Nature of trouble and remarks
May 28	9:18 a.m.				5	Correct	Lightning at Lock 13. No outage or surge.
	9:50 a.m.				5	Correct	" " " " " " " "
	7:20 p.m.				5	Correct	" " " " " " " "
May 29	5:52 p.m.				5	Correct	Weather fair. No outage or surge.
May 30	10:25 a.m.				5	Correct	Raining. No outage or surge.
	11:20 a.m.				5	Correct	" " " " "
	8:25 p.m.				4	Correct	" " " " "
May 31	11:55 a.m.				5	Correct	" " " " "
	5:15 p.m.				5	Correct	" " " " "
	7:48 p.m.				5	Correct	" " " " "
	10:52 p.m.				5	Correct	" " " " "
June 1	11:00 a.m.				5	Correct	Raining. No outage or surge.
	11:28 a.m.				5	Correct	" " " " "
	12:15 a.m.				5	Correct	" " " " "
	1:20 p.m.				5	Correct	" " " " "
June 4	1:28 p.m.				4	Correct	Lightning at Lock 12. No outage or surge.
June 6	1:20 p.m.				5	Correct	Raining at Lock 12. No outage or surge.
	1:40 p.m.				5	Correct	" " " " " " " "
	2:00 p.m.				5	Correct	" " " " " " " "
June 7	4:30 p.m.				5	Correct	Lightning. No outage or surge.
			4:57 p.m.	1812 opened	10	Correct	Lightning. Voltage surged from 115 to 112 volts. Grounding switch 208 did not close. Switch 1812 was reclosed o. k. one minute later. Probably a phase-to-phase short circuit, although no evidence of such was found upon patrolling line.
June 12	11:20 a.m.				5	Correct	Weather fair. No outage or surge.
June 13	7:05 a.m.				5	Correct	" " " " " "
June 16	6:00 a.m.				10	Correct	Lightning. No outage or surge.
June 17			1:47 p.m.	206 opened	5	Correct	Lightning. Voltage surged from 118 to 108 volts. Grounding switch 208 did not close. Probably phase-to-phase flashover. Switch 206 was reclosed o. k. 2 minutes later.
			2:35 p.m.	1812 opened	11	Correct	Lightning. Voltage surged from 113 to 105 volts. Grounding switch 208 did not close. Probably a phase-to-phase flashover. Switch was reclosed o. k. one minute later.
June 19	12:00 Noon				7	Correct	Sparrow caught between horns on lightning arrester causing flash to ground. No outage or surge.
June 20	5:40 p.m.				7	Correct	Lightning at Lock 12. No outage or surge.
	6:10 p.m.				5	Correct	" " " " " " " "
	6:25 p.m.				5	Correct	" " " " " " " "
June 21	4:15 p.m.				5	Correct	Weather cloudy. No outage or surge.
June 26			11:02 p.m.	206 opened	6	Correct	Lightning. Voltage surged from 0-120 volts. Grounding switch 208 did not close. Switch 206 was reclosed o. k. 2 minutes later. Trouble evidently a phase-to-phase flashover. Reactor current probably recorded when 206 opened due to switch contacts not opening together.
June 28	4:15 a.m.				5	Correct	Weather fair. No outage or surge.
	11:12 a.m.				5	Correct	" " " " " "
June 29	3:20 p.m.				4	Correct	Lightning at Lock 12. No outage or surge.
July 3					9	Correct	Recorded at 12:05 a. m. when 206 was opened on prearranged basis.
					18	Correct	Recorded at 12:07 a. m. when 206 was reclosed.
	10:15 a.m.				5	Correct	Raining. No outage or surge.
	10:25 a.m.				5	Correct	" " " " "
	2:10 p.m.				3	Correct	Lightning. " " " "
			3:10 p.m.	1812 opened 202 "	6	Correct	Lightning. Voltage surged from 116 to 96 volts.1 812 opened first but trouble developed in switch which caused 202 to open to clear trouble. Line switch 206 found in half open position showing it attempted to clear trouble, which it should have done as its relays are faster than those on 202. Trouble either a phase-to-phase short circuit on line, or started as a phase-to-ground short circuit on line and then developed into a phase-to-phase short circuit at switch 1812. Line was out only 2 minutes.
	10:00 p.m.				3	Correct	Lightning. No outage or surge.
July 4	3:32 p.m.				8	Correct	Fair weather. No outage or surge.
July 7		6:12 p.m.		208 closed 202 opened	19	Doubtful	Trouble started when operator opened line switch 206 by mistake. A flashover then developed on the 44-kv. bus between transformers and line switch which flashed over 2 strings of 5-O.B. suspension unit insulators supporting 44-kv bus. A short circuit to ground and from phase to phase developed. Switch #208 closed and trouble was cleared by switch 202 on low tension side of transformer bank. Although relays cleared trouble o. k. the action of the Petersen coil is considered doubtful as trouble did not start until after the line switch was opened. Line was out 1 hour and 19 minutes.
July 9					9		Recorded at 4:15 a. m. when switch 206 was opened to obtain clearance to work on transformer bank. Line remained charged from Montgomery Steam Plant.
					5		Recorded at 3:20 p. m. when line switch was closed and Lock 12 paralleled with Montgomery Steam Plant.

TABLE I (Continued)

Date	Flashover to ground	Solid grounds	Phase-to-phase short circuits	Automatic switch operations	Reactor current amperes	Operation	Nature of trouble and remarks
July 11	6:28 p.m.				6		Weather cloudy. No outage or surge.
July 12		7:56 a.m.		#208 closed	16	Correct	A tree fell on Mitchell Dam line bringing all three phases to ground. The four additional operations were obtained when line was plugged four times to sectionalize line in locating the trouble. Line was out 47 minutes.
				206 opened			
		7:58 p.m.		208 closed			
				206 opened	25	Correct	
		8:00 a.m.		208 closed			
				206 opened	25	Correct	
		8:12 a.m.		208 closed			
				206 opened	25	Correct	
		8:18 a.m.		208 closed			
				206 opened	26	Correct	
July 12					11		Recorded when 206 was opened by hand prior to switching Mitchell Dam back on line.
July 13	12:18 p.m.				6	Correct	Lightning at Montgomery. No outage or surge.
	12:25 p.m.				6	Correct	" " " " " " "
July 14			2:01 p.m.	206 opened	11	Correct	Lightning. Voltage surged from 118 to 99 volts. Grounding switch 208 did not close. Line switch 206 reclosed o. k. after 2 minutes interval. Trouble was either phase-to-phase short circuit and the 11 ampere reactor current was the result of 206 opening, or trouble started as flashover to ground and developed into a phase-to-phase short circuit.
July 16		1:34 a.m.		208 closed	18	Correct	Failure of Potential Transformer Bushing at Vida causing dead ground on system.
				206 opened			
		1:36 a.m.		208 closed	19	Correct	Operation repeated three times while plugging line and sectionalizing for trouble. Reactor current each time rose to 19 amperes. Line was out 32 minutes.
				202 opened			
		1:45 a.m.		208 closed	19	Correct	
				202 opened			
		1:47 a.m.		208 closed	21	Correct	
				202 opened			
July 16					6		Recorded at 4:06 a. m. when line switch 206 was opened by hand to obtain clearance on the line.
	1:35 p.m.				5	Correct	Weather cloudy. No outage or surge.
	1:47 p.m.				4	Correct	" " " " " "
July 18	1:55 p.m.				5	Correct	Weather fair. No outage or surge.
July 18	2:00 p.m.				5	Correct	" " " " " "
July 19	4:28 p.m.				11	Correct	Weather stormy. No outage or surge.
July 23	12:20 p.m.				4	Correct	Weather cloudy. " " " "
July 24	10:15 a.m.				5	Correct	" " " " " " .
July 26					8		At 6:30 a. m. Mitchell Dam operator reported that they were only receiving single-phase power. The chart showed a reactor current of 3 amperes which held on until 7:00 a. m. when switch 1802 was opened by hand when a current of 8 amperes was recorded due to switching. Upon patrolling line, a lug on a test loop was found burned in two and remaining in the clear but opening the line.
	1:40 p.m.				6	Correct	Lightning. No outage or surge.
	2:05 p.m.				6	Correct	" " " " "
July 27			5:28 p.m.	1802 opened	7	Correct	Stormy. Started as a phase short circuit, which caused #1 phase conductor to burn in two and fall on ground on Mitchell Dam line at pole 98. Line was plugged back four times in sectionalizing for trouble. At 5:55 p. m. it was decided to operate with the one phase grounded with Petersen coil in service. Grounding switch was blocked so it couldn't close and short out the coil. At 8:12 p. m. a bushing failed on switch 1802 due, no doubt to full line voltage having been imposed on the two ungrounded phases. The Mitchell Dam line was then taken out of service.
		5:32 p.m.		208 closed			
				1802 opened	22	"	
		5:32½ p.m.		208 closed			
				1802 opened	22	"	
		5:33 p.m.		208 closed			
				1802 opened	22	"	
		5:33½ p.m.		208 closed			
				1802 opened	22	"	
July 27		10:40 p.m.		208 closed			
				1802 opened	22	"	When Mitchell Dam was closed back in after above switch trouble was repaired, it again opened automatically. This time a line conductor (phase 2) was found burned in two and on the ground at pole #98. This is the same place where #1 phase burned in two at 5:32 p. m. Conductor on Phase #2 was evidently badly burned on the line to line short at 5:28 p. m.
July 28	2:10 p.m.				9	"	Lightning. No outage or surge.
	2:30 p.m.				5	"	" " " " "
July 30					6		Recorded at 10:50 a. m. while switching was being done at Vida.
July 31					5		Recorded at 11:10 a. m. when #1802 was opened by hand. Current held on until 1802 was closed again at 11:32 a. m.
			12:12 p.m.	206 opened	5	"	Lightning. Voltage surged from 114 to 103 volts. Probably a phase-to-phase short circuit as 208 did not close. Reactor current probably due to opening of 206. Switch 206 closed back in o. k. after interval of 2 minutes.
			1:28 p.m.	1802 opened	6	"	Lightning. Voltage surged from 115 to 110 volts. Grounding switch 208 did not close, therefore, probably a phase-to-phase short circuit. Reactor current probably due to opening of switch 1802. Line was closed back in o. k. 12 minutes later.
					5		Recorded at 1:40 p. m. when 1802 was closed back in.

TABLE I (Continued)

Date	Flashover to ground	Solid grounds	Phase-to phase short circuits	Automatic switch operations	Reactor current amperes	Operation	Nature of trouble and remarks
July 31			1:50 p.m.	1802 opened	9	Correct	Lightning. Voltage surged from 120 to 108 volts. Grounding switch 208 did not close, therefore, probably a phase-to-phase short circuit. Line closed back in o. k. 20 minutes later. This time taken by operator to inspect switch.
			1:59 p.m.	1812 opened	11	"	Lightning. Grounding switch 208 did not close. Probably phase-to-phase short circuit. Line was closed back in o. k. one minute later.
Aug. 1					5		Recorded at 11:15 a. m. when switch 1802 was opened for clearance. Reactor current held on at 5 amperes until 1802 was closed back in at 12:05 p. m.
	12:28 p.m.				6	Correct	Weather fair. No outage or surge.
Aug. 2					5		Recorded at 10:55 a. m. when switch 1802 was opened for clearance and held on until 1802 was closed again at 11:35 a. m.
	8:48 p.m.				5	Correct	Lightning. No outage or surge.
	8:55 p.m.				5	Correct	" " " " "
	9:05 p.m.				5	Correct	" " " " "
Aug. 3					5		Recorded at 11:00 a. m. when switch 1802 was opened for clearance. Current held on at 5 amperes until 12:00 Noon when 1802 was closed back in.
Aug. 5	10:10 a.m.				5	Correct	Weather fair. No outage or surge.
Aug. 8		4:55 p.m.		208 closed 202 opened	24	Faulty	Lightning. Switch 206 failed to trip due to mechanical trouble. Switch 206 was opened by hand and line charged from Montgomery Steam Plant and tested o. k.
					9		Recorded at 7:50 p. m. when 206 was closed back in.
Aug. 9	1:42 p.m.				5	Correct	Weather fair. No outage or surge.
Aug. 10	10:40 a.m.				3	Correct	" " " " " "
	11:20 a.m.				3	Correct	" " " " " "
Aug. 12	10:10 p.m.				5	Correct	Weather cloudy. No outage or surge.
	10:20 p.m.				4	Correct	
	10:30 p.m.				5	Correct	
	10:40 p.m.				4	Correct	
	11:10 p.m.				3	Correct	
Aug. 14	1:55 p.m.				4	Correct	Weather cloudy. No outage or surge.
Aug. 17	9:20 a.m.				5	Correct	Weather fair. No outage or surge.
	9:55 a.m.				3	Correct	" " " " " "
Aug. 18	6:50 p.m.				4	Correct	" " " " " "
Aug. 25			8:35 a.m.	1802 opened	4	Correct	Phase to phase short circuit on low side of transformers at Mitchell Dam.
		3:18 p.m.		208 closed 206 opened	22	Correct	Tree fell across telephone line causing broken conductors to fall across the 44-kv. line between Vida and Lock 12.
		3:20 p.m.		206 opened 208 closed 202 opened	26	Doubtful	Switch 206 on being closed after above trouble at 3:18 p. m. did not latch and on dropping out a flashover occurred across a five unit string of O. B. Insulators on 44-kv. bus between switch 206 and transformer bank. Grounding switch then closed and trouble was cleared by switch 202 on low side of transformer bank. The O. B. Insulators were meggered after taken down and showed up well, indicating that trouble was not caused by insulator failure. Bus conductor was badly burned.
Aug. 26			12:04 a.m.	1802 opened	9	Correct	Lightning. Grounding switch did not close therefore flashover must have developed into a phase-to-phase short circuit. Switch 1802 was closed back in o. k.
Aug. 27	11:35 a.m.				4	Correct	Weather cloudy. No outage or surge.
Sep. 1	12:32 p.m.				5	Correct	Weather fair. No outage or surge.
Sep. 2	4:10 p.m.				6	Correct	" " " " " "
Sep. 4	1:00 p.m.				5	Correct	" " " " " "
Sep. 5	3:25 p.m.				5	Correct	" " " " " "
Sep. 14	4:32 p.m.				18	Correct	Lightning. " " " "
Sep. 17			1:18 p.m.	206 opened	3	Correct	Rain and lightning. Grounding switch did not close. Voltage surged from 116 to 96 volts. Switch 206 was reclosed o. k. after one minute interval.
Sep. 21	12:28 p.m.				6	Correct	Rain and lightning at Lock 12. No outage or surge.
Oct. 7	10:54 a.m.				11	Correct	Lightning. No outage or surge.
Oct. 8	6:52 p.m.				6	Correct	Weather cloudy. No outage or surge.
Oct. 29	5:25 p.m.				5	Correct	" " " " " "
Nov. 5	1:35 p.m.				10	Correct	Weather fair. " " " "
		The Peterson Coil was out of service from November 16, 1922 to January 13, 1923, while the Transformer Bank was being moved to a new location in the Lock 12 Power House.					
1923							
Jan. 14					8		Occurred at 8:50 a. m. when switch 1810 was opened to cut out Vida-Montgomery line. Current held on at an average of 7 amperes until switch 1810 was reclosed. It is possible that pen may have been stuck.
	6:26 p.m.				10	Correct	Cloudy. No outage. Surge 108 to 104 volts.
Jan. 19	4:20 a.m.				6	Correct	Cloudy. No outage or surge.
Jan. 23		9:04 a.m.		208 closed	16	Correct	Raining. No outage or surge. Seems that flashover held on long enough to cause 208 to operate, but cleared before contacts of 208 closed.

TABLE I (Continued)

Date	Flashover to ground	Solid grounds	Phase-to-phase short circuits	Automatic switch operations	Reactor current amperes	Operation	Nature of trouble and remarks
Feb. 23		6:56 p.m.		#206 opened 208 closed 202 opened	27	Doubtful	A light surge was noticed on the line one minute earlier—at 6:55 p. m. At 6:56 p. m. a flashover occurred on 44-kv. bus between transformer and line switch #206. One unit of a five unit string of O. B. disk insulators was shattered but not punctured.* Insulators were meggered afterwards and found to be in good condition indicating that trouble was not an insulator failure. The bus conductors near the point of flashover were badly pitted on all three phases indicating that a phase-to-phase short circuit developed. Sequence of switch operations is not known, but as there was no cause for line switch 206 to open on bus trouble, it is thought that 206 opened first on some initial case of line trouble, as indicated by surge at 6:55 p. m. Then due to perhaps a switching transient, the bus flashed over and this secondary trouble was then cleared by the low side transformer switch #202. It is thought that grounding switch #208 did not close until after the 44-kv. bus trouble developed. The line was patrolled afterwards, but no evidence of trouble was found. Line was closed back in o. k. 1 hour and 26 minutes later.

*This unit was next to the bus conductor.

TABLE II
SUMMARY OF PETERSEN COIL OPERATIONS

	Flashover Indications				Line Trouble		Trans. Trouble		Cir. Break. Tble.		Bus Trouble		Other Indications	
Month	Under 6 amperes	Over 6 amperes	Faulty. Line Sw opened	208 closed but no interruption	Phase to ground	Phase to phase	Phase to ground	Phase to phase	Solid ground	Phase to phase	Solid ground	Phase to phase	Opening switches	Closing switches
1921														
Nov.	..	..	..	..	..	..	..	1						..
Dec.	..	6	1	6	..	..	..	..	..	..	..	..	..	1
1922														
Jan.	..	1	..	..	..	..	1	..	..	..	..	..	..	..
Feb.	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Mar.	..	2	..	..	..	2	..	..	..	..	..	..	1	1
Apr.	18	6	..	..	..	1	..	..	..	..	..	..	..	..
May	53	3	..	..	..	1	..	..	..	..	..	..	2	1
June	17	3	..	..	..	4	..	..	..	..	..	..	..	..
July	11	8	..	..	3	5	1	..	1	1	1	..	6	4
Aug.	18	1	1	..	1	1	..	1	..	..	1	..	2	1
Sep.	3	3	..	..	..	1	..	..	..	..	..	..	..	..
Oct.	1	2	..	..	..	..	..	..	..	..	..	..	..	..
Nov.	..	1	..	..	..	..	..	..	..	..	..	..	..	..
Dec.	...	..	..	..	..	..	..	..	..	..	..	..	..	..
1923														
Jan.	..	2	..	1	1	..	..	..	..	..	..	..	1	..
Feb.	..	..	..	..	..	..	..	..	..	..	1	..	..	..
Total	121	38	2	7	5	15	2	2	1	1	3	..	12	8

failure of all three poles of the oil circuit breaker to open or close simultaneously, thereby causing the system neutral to shift somewhat due to a momentary unbalanced electrostatic condition on the line. It is proposed to make further tests to verify this point.

GROUNDED PHASE OPERATION

As noted in Table I operation with one phase conductor on the ground was attempted on July 27, 1922, in the effort to continue service until the line could be more conveniently taken out for repairs later on. After 17 minutes of grounded phase operation a switch bushing failed on one of the ungrounded phases, due no doubt to the imposition of full line voltage to ground on these phases. Our experience in this one case indicates that, although the Petersen coil may prevent arcing grounds on grounded phase operations, it is hardly safe to take the risk, as the imposition of full-line voltage to ground on the ungrounded phase for a considerable length of time will tend only to aggravate the trouble.

The Petersen coil was designed for continuous service to take care of possible grounded phase operations in emergency. With grounded phase operation eliminated, the Petersen coil need only have approximately a 2-minute rating, and could be built at a cost much less than that of a continuously rated coil.

COMPARISON OF OPERATION WITH AND WITHOUT THE PETERSEN COIL

In the ultimate analysis the value of the Petersen coil will be determined by its service record, and no better conclusions can be drawn than by a direct comparison of the interruption records of similar periods and conditions, with and without the Petersen coil in service.

For such a comparison we have taken the period, January through September 1921, as covered by Table III before the Petersen coil was installed, and a corresponding period in 1922, as covered by Table IV, when the Petersen coil was in service.

In 1921 there were 43 interruptions due to lightning causing the line to be out of service a total of 230 minutes. In 1922 there were only 7 cases of interruptions due to lightning, or a reduction of 83.5 per cent, with the line out of service a total of only 14 minutes, or a reduction of 94 per cent.

TABLE III
INTERRUPTIONS TO LOCK 12-VIDA LINE—1921
(Petersen Coil not in Service)
(Exclusive of interruptions due to trouble on other parts of Alabama Power Company's system)

Month	Lightning		Transformers		Conductor		Oil Switch		Unknown		Total	
	No cases	Time out in minutes	No cases	Time out in minutes	No cases	Time out in minutes	No. cases	Time out in minutes	No cases	Time out in minutes	No cases	Time out in minutes
January	..	..	..	..	..	..	..	..	..	..	..	..
February	1	3	..	..	..	..	..	..	..	..	1	3
March	4	9	..	..	..	..	..	..	..	..	4	9
April	1	3	..	..	..	..	..	..	..	..	1	3
May	2	28	..	..	..	..	..	..	..	..	2	28
June	3	18	..	..	..	..	..	..	..	..	3	18
July	13	121	..	..	..	..	..	..	..	..	13	121
August	5	20	..	..	..	..	..	..	..	..	5	20
September	14	28	..	..	..	..	1	72	1	2	16	102
Total	43	230	..	..	..	..	1	72	1	2	45	304

TABLE IV
INTERRUPTIONS TO LOCK 12-VIDA LINE—1922
(Petersen Coil in Service)
(Exclusive of interruptions due to trouble on other parts of Alabama Power Company's system)

Month	Lightning		Transformers		Conductor		Insulator		Oil Switch		Total	
	No cases	Time out in minutes	No cases	Time out in minutes	No cases	Time out in minutes	No cases	Time out in minutes	No cases	Time out in minutes	No cases	Time out in minutes
January	..	..	1	343	..	..	..	..	..	..	1	343
February	..	..	..	..	..	..	..	..	—	..	..	..
March	..	..	..	..	1	2	10	88	..	..	11	90
April	1	2	..	..	..	..	..	..	..	..	1	2
May	..	..	..	..	..	..	..	..	..	..	..	..
June	2	4	..	..	..	..	..	..	..	..	2	4
July	3	6	1	13	1	47	1	79	2	11	8	156
August	..	..	..	..	..	..	1	62	1	172	2	234
September	1	2	..	..	..	..	..	..	..	..	1	2
Total	7	14	2	356	2	49	12	229	3	183	26	831

In 1922 there were, however, more cases of trouble due to other causes, but that the majority of them were independent of the Petersen coil performance can be shown as follows:

Transformers. The interruption of 343 minutes due to transformer trouble occurred on January 21 while short-circuit tests were being made at Lock 12. A transformer failed due to short-circuit stresses. The Petersen coil was out of service during the tests.

Conductors. The interruptions of 2 minutes and 47 minutes respectively in March and July charged to conductors, was caused in each case by a tree falling down on the line.

Insulators. The ten interruptions in March all occurred on the 7th when an insulator pulled loose from its pin and swung into the phase above. This resulted in a phase-to-phase short circuit and consequently gave no indications on the Petersen coil chart for this day. The interruptions of 79 and 62 minutes respectively in July and August were due to insulator flashovers on the 44 kv. bus at Lock 12. These flashovers seemed to develop after the line switch opened, so there is some doubt as to the origin of the trouble.

Oil Switches. The interruptions charged to Oil Switches in July and August were all due to mechanical trouble in the switch mechanisms.

CONCLUSIONS

The Petersen coil has decidedly reduced the number of interruptions due to insulator flashovers during lightning storms. It has, however, produced several actions which need further investigation, namely, the bus insulator flashovers which occurred when line switching was done with the Petersen coil in service. It is proposed to make further tests on such actions to see if high voltages are actually produced.

Points, which operating experience with the Petersen coil has thus far proven are:

1. All insulator pins should be grounded in order to secure the best positive action on the coil.
2. All switching, both hand and automatic should be done with the coil out of service—namely, with the system neutral solidly grounded.
3. Grounded phase operation is not advisable.

Discussion

For discussion of this paper see page 464.

Present Day Practises in Grounding of Transmission Systems

First Report of Subcommittee on Grounding of Protective Devices Committee A. I. E. E.

THE Subcommittee on Grounding of Systems of the Protective Devices Committee was created on October 14th, 1921, "To Study the special problems and applications of grounding of systems." In order to fulfill its function, the subcommittee prepared an inquiry on the general subject of grounding which was sent out through its members to representative operating companies throughout the United States and Canada.

Thirty-six replies were received to this inquiry covering 6,371,850 kv-a. of generating capacity and 31,408 miles of transmission lines. The data obtained in this manner form the basis of this first report of the subcommittee.

On account of the complexity of the various systems which are not only operated at various voltages but also inter-connected with each other, it has been found necessary to establish some sort of arbitrary classification. Accordingly in the analysis following each transmission section of each company which operated at one voltage has been designated as a system, *i. e.*, each company is said to operate as many transmission systems as it uses different voltages. On this basis the 36 companies reporting operated a total of 111 systems. This definition of a system will be used throughout the report.

With respect to the grounding of neutral problems, transmission systems divide themselves into two main classes, each of which has its own peculiar characteristics radically different from those of the other. The two classes included are:

I—Systems Transmitting at Generated Voltage
II—Systems Transmitting at Higher Than Generated Voltage

In the following report these two classes of systems will be treated in detail separately:

I—Systems Transmitting at Generated Voltage

BY W. W. WOODRUFF,

Associate, A. I. E. E., Field Engineer, Philadelphia Electric Co.

Twenty companies reported on 31 systems transmitting power at generated voltage, the total generating capacity of which companies was 2,387,500 kv-a. and the total mileage was 8627.

In analyzing the information received, it is intended to point out the features upon which there is general agreement and to indicate those that require further discussion. On account of the complexity of the various systems considerable difficulty has been experienced in getting them on a comparable basis. In order to arrive at some definite means of comparing the different systems, it was endeavored to segregate the actual kv-a. generator capacity of each property, in accordance with the method of grounding employed in the generating station, grouping all capacity of each system, regardless of voltage, in accordance with whether it was dead-grounded, grounded through resistance or ungrounded.

In Fig. 1 the result of this classification is shown very clearly.

It is evident from inspection that by far, the greater portion is grounded through resistance, that the ungrounded system comes next in importance, while the dead grounded system is in the minority. The figures are:

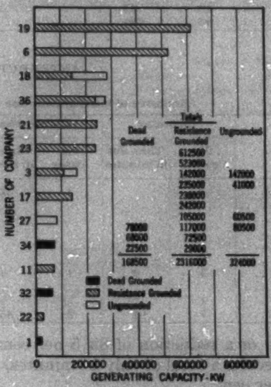


FIG. 1

Grounded through resistance	1,901,700 kv-a.
Ungrounded	371,300 kv-a.
Dead Grounded	114,500 kv-a.

These figures do not include those systems which transform practically all the power generated to a higher voltage.

This analysis leaves no doubt that the preponderance of opinion is in favor of operating with a neutral grounded through resistance. This generalization must

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Presented at the Spring Convention of the A. I. E. E., Pittsburgh, Pa., April 24-26, 1923.

be understood to apply only where the generated voltage is used on the distribution system.

The main points brought out by the analysis of the information received are as follows:

a. The tendency is toward operation with some form of grounded neutral.

b. The abolition of the free neutral and adoption of the grounded neutral has decreased cable troubles.

c. The general opinion is in favor of grounding the neutral through a resistance, in preference to the dead-grounded.

d. The relation between voltage and value of neutral resistance as indicated by calculation of the "volts per ohm" shows that a fairly consistent policy has been followed in proportioning the neutral resistance for the various voltages.

e. Where ground relays are used the neutral resistance is generally higher than for systems relaying on phase relays.

f. The cast iron grid resistance box mounted on insulators appears to be giving satisfactory service.

In going over the information submitted, it is interesting to note the changes that have come about with the growth of the properties, and the relative importance of certain engineering features as compared with the amount of generating capacity.

In general, the systems when comparatively young and with small generating capacity were operated without grounds—and the tendency seems to have been to avoid the grounded neutral wherever possible. This policy seems to have been dictated by the desire to prevent accidental grounds from developing into short circuits.

It should be borne in mind that at the time these systems were built, the protective devices for opening breakers and the circuit breakers themselves had not reached their present state of development and that consequently a short circuit was very likely to be accompanied with quite unpleasant results in the stations.

So long as the systems remained comparatively small, and the greater part of the distribution system consisted of aerial feeders, there seems to have been very little reason to complain of the ungrounded system. However, with the increase in the amount of underground work, and consequently the electrostatic capacity of the system, it began to appear that such grounds as did occur had a very disturbing effect, causing surges and high-frequency oscillations that were communicated to otherwise unaffected parts of the system, and furthermore, that the actual location of the circuit in trouble was very difficult.

As the systems continued to increase in size it began to appear that the ground current, or system charging current, had reached such a magnitude that the arc formed at an accidental ground was very destructive and would burn the cable at the fault sufficiently to establish a short before the faulty section could be isolated.

With the advent of this contingency the supposed advantages of the isolated system, in so far as they pertained to the prevention of short circuits disappeared and the tendency in several instances was to go to the other extreme and dead ground the neutral without resistance.

In general, it may be said that dead grounded systems operated quite successfully for a time, but it became evident with the increase of generating capacity that such grounds and short circuits as occurred began to cause disturbances of increasing severity.

While these data do not prove the point, it is known that the first step taken to minimize the effects of the severe short circuits was to place reactors first in the generator leads and then in the outgoing feeders. Eventually, however, the increasing severity of the disturbances caused by grounds led to the insertion of resistance in the neutral connection.

The question naturally arose as to the amount of resistance required and apparently the first tendency was towards the insertion of so great a resistance, that the ground current was decreased to the neighborhood of the normal current of the feeders. Under this condition, it developed that very frequently when a ground occurred, the combined ground and load current was insufficient to operate the overload relays. In consequence, the tendency was again towards decreasing the resistance to a point where the ground current would be sufficient to operate the relays regardless of the extent to which the particular feeder happened to be loaded.

This policy naturally brought the conditions back towards the point where ground currents were of such magnitude that considerable disturbance resulted from them, but the general opinion seemed to be that this condition with the certainty of the ground being automatically isolated, was preferable to the conditions of the less certain operation of the breakers. However, attention naturally turned towards methods of decreasing of the current disturbances, so that at the present time the tendency appears to be towards the increasing of this neutral resistance with the addition of various types of ground detecting, and reverse power relays to automatically isolate any feeder that is defective.

In comparing the replies from the different companies, it is quite interesting to note the relation between the magnitude and nature of the system and the amount of attention which has been given to the matter of grounding. Even with several of the larger companies, where conditions are such that a considerable portion of the construction is aerial, no very careful attention seems to have been given to the system of grounding, or at least to a comprehensive scheme of relay protection, while those companies whose systems are almost entirely underground, and which have a very high standard of operating performance, have gone into the matter in considerable detail.

As to the systems of protections used there seems to be very little standardization of practise except that there may be said that at present the majority of properties rely on the ground current being of sufficient magnitude to trip out the circuit breakers of the affected feeder. This, of course, applied mainly to those systems where there is no provision for ground detecting relays.

However, where ground relays are provided, there is wide divergence in the methods employed, both for the operation of feeder switches, and the making of the ground connections.

Coming to the methods of operating protective relays, a very wide divergence of practise is again noticeable. In some cases, one company may have several different arrangements on various parts of this system.

It does not appear that any of the companies have developed an arrangement which meets with its entire approval as in practically all cases where efforts have been made to develop a system of protection, there are either several different arrangements of apparatus in use, or the arrangement that is in use has not been made universal.

The principal points about which the committees inquiry brought out information are as follows:

Value of Ground Resistance

The value of the ground resistance must be adjusted in accordance with the voltage, generating capacity, number of points at which the system is grounded, system of relays, size of reactors used, electrostatic capacity of lines, and several other features.

In an effort to determine if there was any relation between the voltage of the system and the neutral resistance, a calculation of the volts per ohm, for each system has been made. That is, the potential from line to ground is divided by the value of the neutral resistance. Consequently, the final values obtained are the "volts per ohm" used in proportioning the resistance. This value—"volts per ohm"—is identical with the "maximum ground fault current" which could exist, assuming that the resistance and reactance of the return circuit, from fault to ground connection was zero, and that the voltage at the fault be held at normal.

In the accompanying tabulation it will be observed that of the thirteen values calculated, eight show a fairly consistent agreement, varying from a minimum value of 908 to a maximum of 2080 and averaging 1500. Of the exceptions, one value, is very much below the average, being 423, and one very much above this average being 3800.

VOLTS PER OHM
of
SYSTEMS GROUNDED THROUGH RESISTANCE

Number of Company	Capacity kv-a.	Voltage	Neutral Resistance	Volts per Ohm	Remarks
3	105,000	13,200	4	1910*	(*) Indicates values showing very close agreement
6	119,500	12,000	3	2310	
	331,500	9000	2½	2080*	
11	46,000	11,000	5	1270*	(*) Indicates ground relays in service
17	117,000	13,200	3	2540	
18	241,700	13,200	2	3800	
21	242,250	12,000	5–10	1065*	
22	29,000	13,200	5	1765*	
23	112,000	11,000	15	423	
20	49,900	7800	2	2250	Capacity grounded by zig-zag transformers not included
	157,400	13,200	8	955	
35	95,000	13,200	4	1910*	
	102,000	13,200	7	1090*	

Number of Points at Which it is Necessary to Ground a System

It appears to be a well established practise to ground at each generating station. The majority of systems find it advisable to ground only one generator on each section of bus. Naturally, the number of points at which it is found necessary to ground will be dependent on the size and extent of the system and to the transformer connections used at substations.

Arrangement of Switches for Protecting Neutral Resistance and Controlling Ground Current

As to this point there seems to be two schemes of operation diametrically opposed to each other. One of these supplies a switch between the generator and ground, this is sometimes automatic and sometimes hand-controlled, but in either event the operation is to break the ground connection. The other places a short-circuiting switch in parallel with the resistance. Both of these arrangements seems to be designed for the protection of the ground resistance grids, rather than for the protection of the system. In general it appears that the main reliance is placed on the operator locating and clearing the feeder in trouble before the resistance is destroyed. While it is evident that the most desirable practise will depend upon the kind of relay protection in use, there still seems to be considerable room for discussion as to which method is preferable, and the operating phenomena and experience gained in connection with each method.

In Table I is found a tabulation of data covering the above points for a number of the most important systems. These data were very generously furnished by the Electrical Apparatus Committee of the National Electric Light Association.

Forms of Relay Protection

There are several forms of relay protection in use and all seem to give more or less satisfactory service, although, as mentioned before, the fact that there does not appear to be even two duplicate arrangements, indicates that the operating engineers have not reached any unanimity of opinion as to the merits of the different arrangements.

The systems of most importance are as follows:

a. Ground relay operated from a sheath type current transformer on each feeder.

GROUNDING OF GENERATOR NEUTRALS AND FEEDER GROUND RELAY PROTECTIVE SYSTEMS
THE SYSTEMS INCLUDED TRANSMIT THE GREATER PART OF THEIR OUTPUT AT GENERATED VOLTAGE

No. of company	System voltage and frequency	No. of generators arranged for grounding	No. of generators in each station operated with neutral grounded	Arrangement of neutral ground connection	Rating of neutral resistors	Feeder ground relays	Settings of feeder ground relays	Number of stations involved	Remarks
6	9000 volts, 25 cycles 13,000 volts, 60 cycles	All	One for each section of 25-cycle bus and one for each section of 60-cycle bus.	One resistor between common 25-cycle neutral bus and ground and one between 60-cycle neutral bus and ground.	2.5 ohms, 2000 amperes for 30 seconds.			3	
19 and	6600 volts, 25 cycles,	None, bus grounded through zig-zag transformers.	None	One 3 ph. resistor in series with 3 ph. zig-zag transf. connected to each main bus.	57 ohms, 67 amp. per phase, 200 amperes total	ITL G. E. Bellows Relays operated from sheath current transformers.	140 amp. min. 180 amp. 1/2 sec.	2	The sheath current transformer operates a telephone drop which gives a visual indication of the ground.
20	11,600 volts, 25 cycles	"	"	"	48 ohms 133 amp. per phase 400 amp. total	ITL Special sensitive type induction relays.	125 amp. min. 300 amp. 1/4 sec.	2	"
	13,200 volts, 60 cycles	All	One	One resistor between common neutral bus and ground.	8 ohms 1000 amperes for 30 seconds.	West. Type in one station. No ground relays on feeders in other station.	480 amp. min. 1000 amp. 1.1 sec.	2	30 sec. resistor replaced with one of 10 min. rating, in one station.
18	13,200 volts, 60 cycles	All	One	One resistor between common neutral bus and ground.	2 ohms, 3750 amperes for 2 minutes. (See remarks.)	Westinghouse type CO 0.5 to 1.5 amp.	Tapered time settings. All set for same value of current.	1	
8	13,200 volts, 60 cycles 4000 volts, 60 cycles	Station No. 1 All Station No. 2 One	Station No. 1 Station No. 2	Solidly grounded				2	
	13,200 volts, 60 cycles	All	One	One resistor between common neutral bus and ground in each station.	4 ohms, 200 amperes for 1 minute.	G. E. Type M. G.	64 amperes, instantaneous	3	Resistor is made on / w
36	13,200 volts, 25 cycles	All	One		7 ohms, 1100 amperes for one minute.			1	
23	11,000 volts, 60 cycles	Station No. 1 None Station No. 2 All	One	One resistance between common neutral bus and ground. (See remarks.)	15 ohms, 868 amperes for 30 seconds.	West. Type CO 0.5 to 1.5 amperes	25 per cent of full-load current.	1	Delta Star transformer bank at other station neutral grounded through 15 ohm resistance.
37	12,000 volts, 25 cycles	All	One	One resistor between common neutral bus and ground.	5 ohms				
17	13,200 volts, 60 cycles 13,200 volts, 25 cycles	None	None	Resistance between transformer neutral and ground.	3 ohms, 2000 amperes, 1 minute.	3		2	
21	12,000 volts, 25 cycles		One	One resistance between generator neutral and ground.	5 to 10 ohms—water.	Installed	25 per cent of full-load current.	3	
3	13,000 volts, 60 cycles	5 generators, 3 bus tie transformer banks.	One	One resistor for each generator and one for three transformer banks connected to common ground bus.	4 ohms, 600 amperes for one minute with temp. not more than 750 deg. fahr.	None installed		1	
22	13,200 volts, 60 cycles	Two	One	Resistance between neutral of each machine and ground.	5 ohms on one generator 0.5 ohms on one generator.			1	
1	13,200 volts, 25 cycles	All	All have neutrals solidly grounded.	Solidly grounded					

b. Ground relay inserted in the common connection of three current transformers.

c. A similar system operated in connection with a differential relay interconnected with a current transformer in the grounded neutral.

In connection with relaying it is worthy of note that the sheath type transformer is not affected by such triple harmonic currents as may be induced in the common connection of current transformers when a heavy load occurs on the feeder, due to slight difference in the transformers and consequently may be adjusted at a lower setting. It would appear that this point should prove of considerable interest and have a very decided bearing on the selection of the most satisfactory type of relay.

SPECIAL GROUNDING ARRANGEMENTS

Two special arrangements were reported. One was zig-zag connected transformer in series with a three-phase resistance connected to the bus bars as shown in Fig. 2. In the other arrangement the neutral of the system was obtained from the middle point of the transformer primary winding of a 5000 kv-a. bank of transformers; the secondary winding being connected in closed delta. The middle point of the star was connected to ground through a 15-ohm resistance. No load was handled by this transformer bank its sole function being to provide a system ground. See Fig. 3.

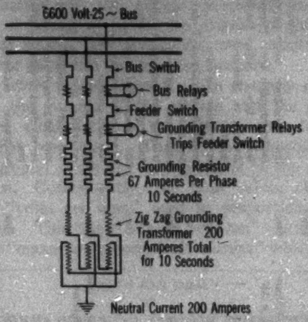

FIG. 2—BUS GROUNDING TRANSFORMERS, 6600 VOLTS, 25 CYCLES

ARC SUPPRESSORS

The use of arc suppressors on generated voltage systems does not appear to have any adherents. Apparently, none are, or have been, in service at these voltages. The suppressors have been used at higher voltages but generally have been abandoned even for this service.

REACTORS

The use of reactors is hardly mentioned in any of the replies, however, it is known that many are in use. As these have considerable effect on the short-circuit current, their influence on the operation of the system and of the protective apparatus should be considered.

NEUTRAL RESISTANCE CONSTRUCTION

The cast iron grid type of resistance mounted on insulators seems to be almost universally adopted, and apparently is giving satisfactory service. The rating of the resistance is usually sufficient to carry the full current to ground for from one-half to two minutes.

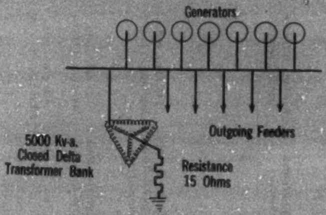

FIG. 3

SURGE ABSORBERS

One company operating 29 miles of underground cable at 11,000 volts ungrounded reports the use of surge absorbers for protection of the cable. Although these have not been in operation long enough to give conclusive experience they are reported as satisfactory so far.

The above paragraphs have outlined the present general trend of practise on the various points covered, constituting all companies reporting. However, there is a wide divergency of practise and a thorough discussion of each of the points above presented should prove very valuable. In future reports of the subcommittee it is hoped to offer more detailed information relative to the various methods used and results obtained.

All data obtained from the inquiry sent out by the subcommittee are summarized in Tables IA, IIA and IIIA following.

There is also presented in the appendix a series of excerpts from the various reports received which contain much valuable information supplementing the summarized data.

II—Systems Transmitting at Higher Than Generated Voltage

BY E. C. STONE
Member, A. I. E. E., Chief of Electrical Planning Divisions, Duquesne Light Co.

In this class, thirty-two companies reporting eighty systems with a total generating capacity of 3,984,350 kv-a. responded to the subcommittee's inquiry.

Dead grounding the neutral is clearly the prevailing practise as shown by the following figures:

	No. of Systems	Total Mileage	Average Miles per System
Ungrounded	31	5377	176
Dead Grounded	39	13,037	334
Grounded through resistance	9	2626	285
Grounded through reactance	1	100	100

Although the above figures show the trend to be in favor of the dead grounded neutral, the mileage ungrounded is very considerable. The average mileage per system of the ungrounded system is however, less than one half that of the grounded systems. Furthermore a number of the companies reporting ungrounded systems report unsatisfactory operation—generally trouble through high voltage disturbances—and state they are planning to change to grounded neutral operation.

There are several well defined reasons for the prevalence of the dead grounded neutral in this class of systems among which are the following:

a. Higher transmission voltages where a dead ground stabilizes operating conditions and simplifies the insulation problem.

b. Lower maximum currents to ground both on account of the higher reactance in all parts of the system, especially the step-up transformer and of small generating capacities on the average.

c. Greater use of outdoor installations at the higher voltages where line damage is possible from heavy ground currents.

The limitations to the dead grounded neutral is of course the maximum short circuit developed in case of a grounded phase and the damage which may result therefrom.

On systems with dead grounded neutral the usual practise is to ground at fairly frequent intervals; the average number of miles per ground on 34 such systems is 32 as against 136 miles per ground on 9 systems with neutral grounded through resistance.

On long transmissions at high voltage both transmitting and receiving ends are almost invariably grounded, while in high or medium voltage networks the neutrals of the transformer banks are frequently grounded; not only at generating stations but also at all or nearly all step-down systems. This practise is undoubtedly due in part to the simplicity of making dead ground connections. One company reported very frequent grounding on account of soil conditions which makes the conductivity of the individual grounds uncertain.

The only company in this class reporting over one hundred miles of cable reports on three hundred and fifty nine miles at 24,000 volts. The neutrals of the step-up transformers are dead grounded at two generating plants. Automatic switching is prevalent on this system and is operated in a network the same as similar lower voltage systems. Above 30,000 volts, only one company reported operation of cable. This system consists of 28 miles at 33,000 volts constituting two tie lines between principal power plants; each line is supplied with its own step-up and step-down transformers. The high-voltage windings on both being connected in star with neutrals dead grounding. Switching is done on the low-tension side.

Most of the systems in this class operating with resistance in the neutral are of comparatively recent development. The magnitude of resistance used shows wide variation. The prevailing type of resistance is the cast iron grid similar to that used in generating stations, described in the first section of this report.

Two systems, however, are using water rheostats with apparently entirely satisfactory results.

Dominating influences leading to the insertion of resistance in the grounding neutral of high-voltage systems are the rapidly increasing generating capacities of these systems, leading to dangerous currents with dead ground and increasingly rigid demands for high grade service, requiring the elimination of voltage disturbances which would result from heavy ground currents. Conditions making such operation practicable are the high reliability of insulators; development of automatic circuit breakers for heavy duty, at high voltages and improvement in relay systems which make selective relaying possible on ground current of less than full load of the lines protected.

One company reports the use of a Petersen Coil on a 44,000-volt system of one hundred miles extent, all overhead. The installation is a recent one and no comments were made as to the results attained.

Two companies reported former use of the arc suppressor. Both however, regarded this equipment as outgrown and expressed themselves in favor of automatic circuit breakers and selective relays rather than the arc suppressor as protective equipment.

III—PROTECTION OF HIGH-VOLTAGE UNDERGROUND CABLES

While seven companies reported operating 525 miles of cable above 20,000 volts only one of these reported cable operating above 30,000. The subcommittee has, however, also secured data on four British installations of cable which are now in operation at 33,000 volts. The following is a summary of the protective arrangements used on the one American installation and on the four British installations which were reported on:

It will be noted that four out of five systems have neutral dead grounded and operate each cable as a separate unit isolated at both ends by transformers. Only one company attempts anything like a transmission net work with sectionalizing switches.

Inquiry among cable designers which has been made by the subcommittee brings out a definite and uniform opinion that in the present state of the art, cables

No. of company	Number of lines	Miles each line	Arrangement of neutral	Lightning arresters	Switching		Scheme of operation
					Location	Method	
6	2	14-1/2	Dead grounded on both lines at both ends	None	Low-tension side	Automatic	Separate step-up and step-down transformers for each cable at both ends
38	1	15-1/2	Dead grounded generating end only	None	Same	Same	Same
39	2	9	Same	None	Same	Same	
40	2 1	2-3/4 9	Grounded at step-up transformers through resistance 200 amperes 2 min.	None	High-tension side	Same	All operated through one 33-kv. bus
41	1	4-1/2	Dead grounded generator end only	None	Low-tension side	Same	Separate step-up and step-down transformers at both ends

operated at 33,000 volts or above should have the neutral dead grounded at as many points as possible.

With reference to switching practise, the high-voltage, high duty switches now available should prove entirely adequate for switching on both routine and automatic on failure on high-voltage cables. If such cables are to be used to any great extent in future transmission systems the present practise of isolated operation must necessarily give way to operation in net works,—in many cases connected with overhead lines—with the usual automatic sectionalized switches.

Even if greater voltage disturbances are produced by switching at cable voltage rather than on the low-tension side—and opinion is more or less divided on this point—such operation cannot be avoided.

One manufacturer writes as follows with reference to these matters:

"Testing and research have indicated that the low-tension switching generally produces less voltage disturbance than high-tension switching. However, the voltages produced by switching are only a part of the disturbances to which cables are subject. If low-tension switching is used, therefore, there will be a slight probability of fewer troubles in the cable, and where individual transformer banks are used for the cables it is easy to use low-tension switching. There will be many places, however, where it is necessary to use a high-tension bus and high-tension switching. We believe this will be the case in the future and that the system should be made in some way proof against the disturbances resulting from high-tension switching. Future experience will indicate whether this can be done best by increasing the insulation of the cable or by installing protective devices, such as resistance in shunt with reactance or resistance in series with condensers.

While cables are less subject to lightning disturbances than overhead lines, yet, it appears desirable to protect such cables by lightning arresters whether they are connected to overhead lines or not. Since the cost of arresters is low compared to that of the cable, that is, the insurance is cheap, they should without doubt be protected by lightning arresters where they are connected with overhead lines."

IV—Conclusions and Summary

BY E. C. STONE

Duquesne Light Company, Chairman of Sub-Committee

From general considerations fundamental to the problem and analysis of all data received by the Sub-committee, the following conclusions are reached:

OPERATION WITH UNGROUNDED NEUTRAL

The only argument in favor of operating a system with ungrounded neutral is the possibility of continuing in operation with one phase grounded. In the early days of power transmission, when mileages were small and voltages low, this was possible not infrequently, and on systems in undeveloped territory, especially with lines running radially out from a central power station, such operation became of distinct economic advantage. However, as the size of these systems increased it was soon found that in a majority of cases that the increased charging currents on a grounding phase not only made continued operation impossible but developed excessive voltages at other points of the system which resulted in one or more break downs, thus causing serious interruptions to considerable portions of the system as well as material damage to valuable equipment.

WHEN TO GROUND THE NEUTRAL

The grounded neutral of a transmission system performs two definite functions:

a. It acts as an anchor to hold the generator windings, transformer windings and line wires at potentials to ground which are within the values for which a system is designed;

b. It completes, in case of a grounded phase, a closed circuit through the generator which short circuits the electrostatic capacitance, of a system to ground and prevents the building up of arcing grounds involving disasterous voltages.

The dead grounded neutral limits the voltage stresses that may occur at normal frequency to a value of approximately 50 per cent of that which may occur with any other operating arrangement.

The creation of excessive voltages in transmission systems, both from internal surges and arcing grounds depends largely upon the amount of energy storage in the electrostatic capacity of a system which increases directly as the number of miles of line and as the square of the voltage. Hence the increasing mileage and increasing voltage of transmission systems have been the definite factors leading to grounding of the neutral.

These conclusions have been amply proved by operating experiences. In many cases the excessive voltages have become increasingly frequent and disastrous as ungrounded neutral systems have grown and have subsequently been practically eliminated by the grounding of the system neutral.

From the data available it is not possible to indicate, except in a very general way, at just what stage in the growth of a system the neutral should be grounded. When however, the magnitude of any system has become so great that evidences of excessive voltages become frequent and insulation failures develop simultaneously on different parts of the system, it is time to consider grounding of the neutral.

In this connection, it should be remembered, that because of the higher electrostatic capacity of cables, one mile of cable is equivalent to approximately 25 miles of overhead lines. On all systems reporting only 6385 miles out of a total of 31,408 are operating with free neutral. Mileage of systems is the determining factor in grounding and is shown by the fact that the average mileage per ungrounded system is 152 against a mileage per system with dead grounds of 441 and with resistance grounds of 248. The latter being chiefly on systems consisting largely of underground cable.

HOW TO GROUND

Four factors must be considered when designing a neutral ground for transmission systems:

a. Excessive voltages must be taken care of.

b. Excessive currents must be prevented.

c. Due consideration must be given to conditions imposed by selective relaying.

d. Due consideration must be given to reducing to a minimum the effect of system disturbances on interference with operation of motors and other load.

As previously stated operating experiences show conclusively that dangerous voltage disturbances will become more and more prevalent as *mileage* and *voltage* of systems increase, unless suitable grounding of the neutral is resorted to.

Several companies grounding through a resistance report gradual reduction of resistance as the magnitude of the system increases. With this premise, let it be assumed that for the same degree of protection on various systems, the grounding resistance should vary inversely as the total system charging current. Inductance and electrostatic capacity per mile of overhead line and cable do not vary very widely within the ranges of construction used by the systems under consideration; let them be assumed constant with the electrostatic capacity of cable taken as 25 times that of overhead line. Expressed as a formula these assumptions give the following results:

$$(L\,a + 5 + 25\,L\,b) \times f \times R = K.$$

where:

$L\,a$ = Miles of overhead line
$L\,b$ = Miles of cable
f = Frequency of system in cycles per second
R = Resistance of combined neutral ground circuit, in ohms.

A calculation of the value of K for those systems on which sufficient information is available shows interesting results. If K came out a constant for all systems it would mean that a definite relation had been established between systems charging current and ohmic value of grounding resistance. As a matter of fact however very wide variation is shown as indicated in the following summary:

	No. of Systems	Value of $K/10^6$		
		Minimum	Average	Maximum
Transmitting at generated voltage..............	8	0.008	1.04	3.55
Transmitting at higher than generated voltage.	7	0.022	0.52	2.00

Petersen, inventor of the Petersen grounding coil, has given a value of K of 6.2×10^6 as suitable for calculating a grounding resistance of proper ohmic value for preventing the accumulative voltage variations from arcing grounds. This would give a value of resistance from 6 to 12 times the average, as shown by the above data, and two or three times the maximum values in actual use.

In view of the fact that some of the later grounded neutral installations have been installed with the highest resistance values it is of interest that one company with a relatively very high grounding resistance reports occasional insulation failures simultaneously at different points on the system, similar to those which previously occurred on the ungrounded system.

So much for the higher limit of resistance value. The lower limit is determined by the maximum permissible current flow in case of a grounded phase and this in turn is determined by the possible damage to equipment and disturbance to service, resulting from such current flow. The limits have however, a considerable range depending on service conditions, system connections, outdoor and indoor bus structures, etc.

The data collected show that of the 49 systems with grounded neutral transmitting at higher than generated voltage and operating 24,926 miles of line 39 systems with 18,478 miles are dead grounded, while only 9 systems with 2626 miles are grounded through resistance.

The reason for the predominance of the dead grounded neutral on the higher voltage systems seems clear when it is remembered that the reactance of a system goes up at least in proportion to the voltage, with the result that relatively small currents flow in case of a grounded phase at the higher voltages. For example—assume that a 200,000-kv-a. station is designed for 11,000-volt operation and that the generators have 6 per cent internal reactance, the maximum current which could then flow to ground if neutral of all generators were dead grounded would be:

$$\frac{200{,}000 \times 1.73}{11{,}000 \times 0.06} = 525{,}000 \text{ amperes}$$

If however, the generators step up their entire output to 132,000 volts through transformers of 6 per cent reactance and all transformer neutrals are dead grounded, the maximum possible current at the high-tension bus would be reduced to one twenty-fourth of the above value or 20,000 amperes.

From another point of view if it was found that 25,000 kv-a. was the maximum permissible station capacity at 11,000 volts for dead grounded neutral, then a 600,000-kv-a. station at 132,000 volts might have its neutral dead grounded without any increase in maximum possible ground currents. The resistance of a ground circuit through a fault is so low that it is practically negligible compared to the impedance of the rest of the circuit. Hence the greatest currents arising from grounding the neutral may occur at the large stations at the lower voltages and it is at these stations that limitation of ground current by insertion of resistance in the neutral becomes compulsory.

From the point of view of relay operation, the dead grounded neutral is the most satisfactory arrangement, since it allows ample currents for positive selective action of relays and permits using the same relays for protection against grounds and short circuits. A low internal resistance of the order of two or three ohms does not materially alter this condition, since ground currents will be still much in excess of normal full-load line currents. The information received indicates that selective relay requirements, have been a limiting influence in 12 cases on the maximum value of resistance used in neutral grounds. See Tables 6A and 6B.

On the other hand, the demands of a rapidly growing motor load—much of it synchronous—for relief from inconvenient disturbance due to drop of voltage has created a tendency toward the use of greater values of ground resistance particularly on high-voltage overhead systems where most break downs are grounds, rather than short circuits, and individual lines have very large carrying capacities so that a single fault to ground may effect a very large block of service.

In the last few years there has been a fairly marked trend toward the use of higher resistance in grounded neutrals especially in the higher voltage systems. This has led to the development of a number of relay schemes in which selective operation of breakers is obtained with ground currents less than the full-load current of the line. Six companies are using schemes of this nature on parts of their system.

In studying the effects of heavy ground currents on the operation of synchronous machinery a series of tests was made by one of the manufacturers under conditions approximating those of a certain transmission system. The results of this test indicated that the average voltage at the motor could not drop more than 30 per cent without interference with operation. With greater drop in voltage than this motor generators may be expected to drop out of step and rotaries to flash badly. The average drop of thirty per cent at the load, means of course, materially less than this on transmission systems since when the voltage begins to drop, motors immediately draw more current, thus adding to the drop due to ground currents.

Insertion of resistance in the neutral reduces the distortion to the voltage triangle in case of ground, as well as the average drop in voltage.

Calculations of the maximum current which will flow in case of a grounded phase on systems with neutral grounded through resistance have been made by dividing the rated system voltage, from neutral to phase wire, by the resistance, at the grounding point, assumed to be in parallel. On 17 systems for which this information is available the following figures for maximum ground current have been obtained:

Maximum	Average	Minimum
5950 Amperes	2080 Amperes	102 Amperes

Under 700 amperes, 5 systems: 1100 to 2600 amperes, 8 systems: 4000 amperes or over, 4 systems.

TYPE OF RESISTANCE

Details of the data obtained by the Subcommittee will be found in Tables 3A, 3B, 6A and 6B. Prevailing practise calls for grounding resistances made up of cast iron grids for either indoor or outdoor operation.

These grids are designed for a maximum temperature of 350 deg. cent. to 600 deg. cent. The average being about 400 deg. cent. On 30 grids now in service, the time required to reach this temperature runs from 10 seconds to two minutes with an average of 45 seconds. One grid has a time rating of 10 minutes having replaced a 30-second grid on account of burning out of the latter.

In general, these grids are giving satisfactory service. Of 11 systems for which information was furnished on this point, 8 reported no trouble, while 3 using thirty second resistances and one having a 10 second resistance reported trouble from burnouts on sustained grounds. Of two of these using 30 second grids, one has gone to dead grounding and the other to 10 minute rating, while the company using the 10 second resistance on a 66-kv. system has installed a circuit breaker to automatically short-circuit the grids on maximum ground current sustained for 10 seconds.

Two companies are using water rheostats for grounding resistance, the ohmic values are five to one hundred and fifty ohms. One of these reported satisfactory operation and one burnout due to low capacity. One company reports resistance of 60 to 100 ohms made of

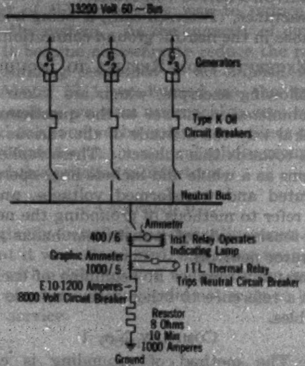
13200 Volt 60 ~ Bus
G #1
G #2
G #3
Generators
Type K Oil
Circuit Breakers
Neutral Bus
Ammeter
400/6
Inst. Relay Operates
Indicating Lamp
Graphic Ammeter
1000/5
I T L Thermal Relay
Trips Neutral Circuit Breaker
E-10-1200 Amperes
8000 Volt Circuit Breaker
Resistor
8 Ohms
10 Min
1000 Amperes
Ground

Fig. 4—Grounding of 13,200-Volt, 60-Cycle System

Monel metal, the ratings being 67 amperes for 10 seconds and 40 amperes for 20 seconds respectively.

Grounding Through Reactance

The most recent development in methods of grounding of neutrals is the use of a reactance instead of

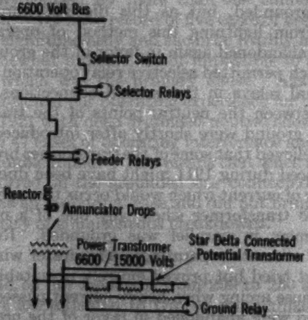
6600 Volt Bus
Selector Switch
Selector Relays
Feeder Relays
Reactor
Annunciator Drops
Power Transformer
6600 / 15000 Volts
Star Delta Connected
Potential Transformer
Ground Relay

Fig. 5—Showing Method of Grounding and Protecting 15,000-Volt, 25-Cycle Station Tie

resistance. If for any given system, the reactance is properly proportioned it will exactly neutralize the capacitance of the transmission system to ground, so that in case of a grounded phase no current will flow through the fault. It is claimed that arcing grounds are absolutely prevented by this device which is called the Petersen coil, after its inventor.

Although a number of his schemes of grounding through reactance have been in operation for some time in Europe, only one company in the United States was reported to the subcommittee to be using it. A thorough analysis of the theory of the grounding reactance was presented by Messrs. Conwell and Evans in the A. I. E. E. Journal for February, 1922, and a report on practical results obtained from operation of such a coil over a period of eighteen months is presented by Messrs. Oliver and Eberhardt coincidently with this report.

Grounding the neutral of a transmission system in a suitable manner is thoroughly standardized in the United States as the best protective measure against internal voltage disturbances. This conclusion is definitely proved by the data received by the subcommittee.

Protective Schemes Other Than Grounded Neutral

Several companies have used various forms of arc suppressors with more or less success, for a number of years. Outlines of their experiences will be found in the "excerpts" in the appendix of this report.

In general the arc suppressor has proved only reasonably successful as it has many limitations. The concensus of opinion seems to be that with the high duty circuit breakers and selective relaying now available the arc suppressor should be abandoned. One company reports the use of surge absorbers on their ungrounded cable systems with satisfactory results. This device consists of reactance and capacitance in series in the form of a static condenser and this connected into the line in a manner similar to lightning arresters. The use of this device is suggested in addition to the dead grounded neutral for protection of very high-voltage cables.

In Europe it appears that considerable success in protecting against excessive voltages has been obtained by the use of horn gaps in series with the resistance connected to the line, the same as lightning arresters.

One case has come to the attention of the subcommittee where the neutral of a transformer bank was connected through a spark gap to ground.

In conclusion the present situation as to grounding the neutral of transmission systems is summarized in the following paragraphs:

1. Grounding the neutral has proved successful to a very high degree in limiting trouble from excessive voltage on transmission systems. Case after case is on record where frequent and destructive insulation break downs on systems operating ungrounded have been practically eliminated after the neutral is grounded.

2. The trend of present practise is definitely toward grounding the neutral of large transmission systems.

Increasing mileage and voltage have been the factors necessitating this practise:

a. On systems transmitting at generated voltage, the neutral usually is grounded at each generating station with resistance of low value.

b. On systems transmitting at higher than generated voltage the neutrals of the transformers are grounded at each generating station and some or all substations.

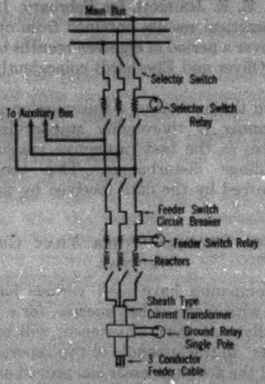

FIG. 6—6600 AND 11,000-VOLT RADIAL FEEDER RELAYS, 25 CYCLES

c. Where resistances are used they are generally made up of cast iron grids with time ratings at maximum current of 30 seconds to 60 seconds.

3. The use of arc suppressors has been very generally abandoned. This is not because such apparatus was without merit since some fairly good results were obtained with its use, but rather because of the development of high interrupting capacity breakers and selective relays which are found to be a better solution to the problem.

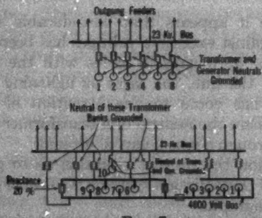

FIG. 7

These two plants are operated in parallel.

4. Certain departures from present standard practise in neutral grounding as outlined above are taking place which are apparently proving successful and are likely to become increasingly used:

a. High resistances in the neutral ground connection of the order of one to one and one half ohms per thousand line volts. One company is using a resistance of 150 ohms on a 26,400-volt system.

b. Reactance, (Petersen coil), is being tried instead of resistance in the neutral ground connection.

EXCERPTS FROM REPLIES TO INQUIRY

The following excerpts, which are taken from the replies submitted in answer to the questionnaire, give in a general way the attitude of the various operating engineers towards this subject. These replies refer to the systems as a whole and include lines operated both at generated and transformed voltages, and, consequently, refer to methods of grounding the neutrals on the high-tension side of transformer banks as well as the generator neutrals.

The company number at beginning of each excerpt furnishes a reference to other data on same company in the tables.

COMPANY NO. 1

(1-A). The method of grounding is considered entirely satisfactory. The only difficulty which has been experienced is the burning up of the cast iron rheostat in case the ground, due to failure of some other protective equipment, should hang on too long.

When the main power house started in operation the neutral of one of the transformers connected to each of the two high-tension circuits was connected directly to ground without any resistance. As the summer of 1911 proved that these lines were subject to considerable lightning trouble it was tried for a while to run the system ungrounded, but as this did not minimize the effects from lightning this method of operation was quickly abandoned again in favor of the grounded system which permitted selective relay operation by means of neutral relays in the ground connections. Resistances between the neutral points of the transformers and the ground were shortly after introduced because it was believed that some of the transformer breakdowns experienced during 1911 might have been due to excessive flow of current which would occur through the dead grounded transformer when only one of a number of transformers in parallel was grounded. Resistances made of concrete separating electrodes of wire netting were first tried but proved entirely unsuitable as they would cause arcing when heavy current passed through them and were abandoned in favor of the present cast iron grid rheostats.

Arc suppressors and a field destroying device have been used a number of years on the transmission system with fair success. Both have been abandoned, however, due to the fact that they were found unnecessary, as four parallel circuits have been constructed and an elaborate relay system of overload and neutral relays at the generating station and reverse power relays at the terminal station have been brought to such perfection that the other devices were found unnecessary on the system.

(1-B). We originally operated without neutral grounded. About 1907 we grounded the neutral of one generator through a grid resistance having 10 ohms. The object of this grounding was to reduce voltage strains on the system. As the capacity of our system increased it became necessary to reduce the resistance of the grounding resistance to accomplish the same results. About 1915 we burned up the grounding resistance and as we were approaching zero resistance we decided to abandon the resistance. Since that time we have operated with the neutral solidly grounded and have found this practise satisfactory.

Company 5

We originally installed resistance in series with the neutral but it was abandoned, and since we have been operating with a dead-ground we had an arc suppressor a number of years ago, but it did not seem to be of any particular service.

Company 7

The operating condition during the period from February to August was such, however, that for all practical electrical considerations the system was operated as a purely grounded system. On ungrounded operation, from the beginning of March to the end of August 1920, 61 cases of trouble interfered with the operation of the system. Upon investigation it was found that there were actually 120 cases of trouble; that is, usually each case of trouble brought with it a reflected case of trouble; *i. e.* interference with operation was usually caused by more than one sumultaneous breakdown. We have record of five cases of breakdown occurring simultaneously and as can be readily imagined it was somewhat difficult, if not impossible to actually establish which was the primary cause. Against the before-mentioned our operating experience for the same period in 1921 with dead grounded neutral showed that there were 47 cases of operating interference and but 48 cases of trouble found. This means that when a breakdown occurred on the grounded star-connected system it stayed as an individual case and did not bring with it a multiplicity of simultaneous breakdowns.

Company 9

Full load on our feeders is from 200 to 250 amperes. The relays are adjusted to trip on approximately 120 amperes ground current.

Company 11

Before grounding neutral, simultaneous cable failures did occur and in one case five faults appeared. Since grounding neutral only one has occurred when two feeders failed at once.

Company 18

The 150-ohm resistances in the two 26,400-volt sections have not been in service long enough to supply sufficient data regarding the suitability of this value of resistance. The ungrounded sections in are not entirely satisfactory in comparison with grounded operation.

Company 20

On four occasions the 1000-ampere, 30-second resistor through which the neutral bus of the 60-cycle, 13,200-volt system was grounded failed in service because of sustained grounds on other station busses with the ground current divided over several feeders. In each case the resistor burned apart leaving the neutral bus ungrounded.

A new resistor rated at 8 ohms, 100 amperes for 10 minutes has been installed. A thermal relay with current time characteristics similar to those of the resistor opens an oil circuit breaker in series with the resistor when the grids reach a dangerous temperature.

Company 21

Our system was first changed from free neutral to solid grounded neutral. This eliminated the trouble due to surges but caused our lead cable sheaths to be burned when the line became grounded. To remedy this, we changed from solid grounded neutral to neutral grounded through resistance. After trying various values of resistance, we have found that approximately 5 to 10 ohms gives very satisfactory results on our systems.

Company 24

No trouble has been experienced with the resistance used in grounding the neutral of our transmission system, while in service. Inspection of the grids has shown broken grids at various times. However, as the units are built up with two grids in parallel, this at no time has opened up the circuit to ground.

Company 29

As our 11-kv. system has increased in extent and consequently in electrostatic capacity, surges due to arcing grounds have become increasingly violent. They are particularly destructive to lightning arresters, cable terminals, transformers etc., we therefore propose when one new 15,000-kw. 11-kv. turbo generator is installed to ground the neutral of the system, but whether through resistance or not has not yet been determined.

Company 36

The 60-cycle system was originally operated with a free neutral but after failure of the insulation of a large generator, cause of which was attributed to voltage disturbances which would very likely not have occurred had there been a neutral ground connection, plans were made for connecting this neutral to ground through an 8-ohm resistor.

Experience obtained during the next few years indicated that there was a certain number of faults to ground through which the current was not of sufficient value to cause the circuit breakers to open, and it, therefore, appeared advisable to reduce the resistance with the ground connection. This was done in such a manner as to obtain a resistor of 4-ohm resistance designed to carry 2000 amperes for one minute.

At the present time there is a number of trial installations of low-current ground relays connected in the common return wire of the current transformer secondaries which operate alarms to indicate to the operator which line is grounded. Owing to the fact that nearly all transmission is through underground cables, the ground faults usually develop into short circuits before the operator can clear the line manually. It is possible that in the future installations will be made of ground relay protective equipments which will automatically insolate a grounded section.

COMPANY 10

At the time the plant was built, the manufacturers did not feel capable of building 110,000-volt oil switches which could be relied on to open short circuits safely with the power of the plant behind them. Instead, therefore, of relieving short circuits which might occur on the 110,000-volt lines by automatically opening the supply with oil switches, all of these oil switches were made non-automatic and provision made for lowering the voltage on the line in trouble by cutting in resistance of the exciter fields. After the arc was extinguished in this manner the voltage was again brought up on the units and service re-established.

You will see that the above method of operation meant that every time a short circuit occurred on the circuits service was interrupted. We therefore looked around for some suitable way of taking care of such service interruptions and installed the arc suppressor for that purpose.

The arc suppressor has very satisfactorily performed the work for which it was intended and has taken care of about 80 per cent of the line flashovers, which are practically all caused by lightning, without the loss of load. On the other hand, severe short circuits at receiving end caused by cable failures, etc., also operate the arc suppressor and under such conditions the load is generally shaken off, so you will see that there are times when we would be better off without the arc suppressor.

In addition to the above, there is no doubt in my mind but what the short circuits which the arc suppressor places across the lines at the power house is very severe on the transformers and while we have had no failures of such apparatus, it has never seemed to me just right to place such severe short circuits on the generating equipment if it could be avoided.

The state of the art at present is such that I believe if we were building a new plant at the present time, all of the outgoing lines would be equipped with suitable oil switches which would be made automatic and short circuits would be taken care of by promptly opening the circuit in trouble. For this reason I believe our arc suppressor equipment is out of date and I doubt very much whether a story of the operation of the arc suppressor would be of very much value, except as a matter of history.

SUMMARY OF DATA

	Systems at generated voltage	Systems at higher than generated voltage	All systems
Number of Companies	20	32	36*
Number of Systems:			
Ungrounded	11	31	42
Dead Grounded	3	39	42
Resistance Ground	17	9	26
Reactance Ground	0	1	1
Total	31	80	111
Total Mileage:			
Ungrounded	1008	5377	6385
Dead Grounded	3797	14,678	18,475
Resistance Ground	3822	2626	6448
Reactance Ground	0	100	100
Total	8627	22,781	31,408
Average Mileage:			
Ungrounded	92	176	152
Dead Grounded	1270	377	441
Resistance Ground	225	292	248
Reactance Ground		100	100
Resistance Factor K:			
Minimum	0.008	0.022	0.008
Average	1.04	0.52	0.80
Maximum	3.55	2.00	3.55
Maximum Ground Current:			
Minimum	425	102	102
Average	2300	2000	2080
Maximum	4620	5950	5950
Miles per Ground:			
Dead Ground	24	32	..
Resistance Ground	110	146	..
Reactance Ground	..	100	..
Total Generating Capacity	2,387,500	3,984,350	6,371,850

*Some companies operate both types of system.

TABLE I-A—TRANSMISSION AT GENERATED VOLTAGES UNGROUNDED SYSTEMS

Voltage	No. of Company	Miles			Protection Other Than Ground
		Overhead	Underground	Total	
11,000	13	69	7	76	
	18	..	..	63	Arc suppressor
	27	26	29	55	High-frequency absorber
	29	315	0	315	
13,000	5	133	..	133	
13,200	18	..	..	115	Arc suppressor
	25	5	0	5	
	26	70	6	76	
	31	0	50	50	
	34	92	0	92	
	35	28	0	28	

TABLE I-B—TRANSMISSION AT HIGHER THAN GENERATED VOLTAGES UNGROUNDED SYSTEMS

Voltage	No. of company	Miles Overhead	Miles Underground	Miles Total	Voltage	No. of company	Miles Overhead	Miles Underground	Miles Total
15,000	15	28	0	28		25	23	0	23
	26	45	0	45		26	185	0	185
17,500	12	88	0	88		27	90	0	90
19,000	34	68	0	68		33	115	0	115
20,000	7	80		80	44,000	2	80	0	80
22,000	2	60		60		12	305	0	305
	11	22	0	22		34	89	0	89
	12	122	0	122	50,000	4	895	0	895
	27	94	0	94	66,000	3	21	0	21
	30	217	0	217		4	355	0	355
	31	580	0	580		13	29	0	29
24,000	21	135	0	135		33	467	0	467
	33	70	0	70		36	31	0	31
26,400	31	240	0	240	75,000	12	171	0	171
33,000	7	93	0	93	140,000	12	518	0	518
	13	61	0	61					

TABLE NO. II-A—SYSTEMS DEAD GROUNDED TRANSMISSION AT GENERATED VOLTAGE

Voltage	No. of Company	Frequency	Miles Overhead	Miles Underground	Miles Total	No. of Grounds	Location of Grounds
11,000	10	60	3596	0	3596	?	At all company substations
13,200	1	25	5	75	80	3	Neutrals of generators and transformer banks
13,200	32	25	89	32	121	1	Generator neutral grounded on condenser piping

TABLE NO IIB—SYSTEMS DEAD GROUNDED TRANSMISSION AT VOLTAGES HIGHER THAN GENERATOR VOLTAGES

Voltage	No. of Company	Frequency	Miles Overhead	Miles Underground	Miles Total	No. of Grounds	Neutral of transformers grounded at
15,000	16	50	385	4	389	5	Generating and substations
16,500	20	25	0	13	13	1	
17,000	30	60	..	..	327	?	
20,000	6	25	13	80	93	4	Step-up substations
22,000	6	60	10	17	27	4	Step-up substations
24,000	8	60	578	359	937	2	Generating stations
25,000	3	60	7	14	21	3	Step-up substations
26,400	1	25	30	10	40	2	Step-up substations (auto transformers)
30,000	10	60	105	0	105	3	Step-down substations
30,000	16	50	69	0	69	4	Generating and substations
30,000	16	60	110	0	110	7	Generating and substations
33,000	6	60	0	28	28	4	Step-up substations
33,000	7	60	657	0	657	4	Substations
33,000	14	50	..	..	105	5	Substations
44,000	2	60	240	0	240	2	Substations
44,000	5	60	709	0	709	1	Substations
48,000	8	60	212	0	212	?	Step-up substations
50,000	16	50	204	0	204	?	Generating and substations
55,000	28	60	344	0	344	?	Generating and substations
60,000	16	50	901	0	901	21	Generating and substations
60,000	16	60	99	0	99	2	Generating and substations
60,000	30	60	2042	0	2042	129	Generating and substations
66,000	10	60	910	0	910	43	All generating and substations
66,000	24	60	26	0	26	1	Generating station only
66,000	29	60	72	0	72	2	
66,000	33	60	180	0	180	1	Generating station only
66,000	34	60	460	0	460	?	
67,000	28	60	224	0	224	1	All points
100,000	4	60	11,000	0	1100	3	
100,000	5	60	1489	0	1489	8	
100,000	28	60	298	0	298	?	All points
110,000	2	60	564	0	564	3	
110,000	10	60	193	0	193	6	Generating and substations
110,000	13	25	144	0	144	1	Generating stations
110,000	14	50	90	0	90	1	Generating stations
110,000	30	60	515	0	515	12	Generating stations and substations
110,000	31	60	75	0	75	1	Generating stations
132,000	33	60	186	0	186	2	Generating stations
150,000	16	50	480	0	480	3	Generating stations

TABLE III-A—SYSTEMS GROUNDED THROUGH RESISTANCE TRANSMISSION AT GENERATED VOLTAGES

	No. of		Miles			No. of	Resist-ance		Resistance			
									Rating			
Voltage	Company	Frequency	Overhead	Underground	Total	Grounds	Per Ground	Type	Amps	Time-Sec.	Temp.	Location of Ground Connection
6000	36	25	..	..	10	2	7	Grid	..	..	..	Generator neutral
6600	19	25	0	400	400	2	57* per phase	Zig Zag Transf. and resistor	67 per phase	10	..	Station Bus
7800	20	60	0	91	91	1	2	Grid	2300	30	..	Generator neutral
9000	6	60	0	396	396	?	2-¼	Grid	2000	..	..	Generator neutral
11,000	11	60	46	31	77	1	4	Grid	1800	60	350° C	Generator neutral
	19-20	25	0	182	182	2	48*	Zig Zag Transf. and Resistor	150 per phase	60	..	Station Bus
	23	60	160	151	311	1	15	Grid	868	30	..	Transformer neutral at power stations
12,000	6	60	22	353	375	4	3	Grid	2000	..	..	Generator
	21	25	27	143	170	?	5 to 10	Water	..	..	..	One generator in each group
13,200	17	25	65	33	98	1	?	Grid	2000	60	..	Generator neutral
	17	60	146	85	231	1	3	Grid	2000	60	..	Generator neutral
	18	60	..	..	?	1	2	Grid	3750	120	450° C	Generator neutral
	19-20	60	0	198	198	2	8	Grid	1000 1000	30 600	..	One generator in each power station
	22	60	25	0	25	1	5	Grid	1600	60	400° C	Generator neutral
	36	25	..	..	33	1	7	Grid	..	..	..	Generator neutral
	36	60	..	..	170	1	4	Grid	..	..	..	Generator neutral
13,800	3	60	71	353	424	2	4	Grid	2000	..	..	Neutral of one generator and one transformer

*Includes impedance of Zig zag transformer.

TABLE III-B—TRANSMISSION AT HIGHER THAN GENERATED VOLTAGE SYSTEMS GROUNDED THROUGH RESISTANCE

	No. of		Miles			No. of	Resist-ance	Resistance				
									Rating			
Voltage	Company	Frequency	Overhead	Underground	Total	Grounds	Per Ground	Type	Amps	Time Secs	Temp.	Neutral of transformer grounded at
22,000	23	60	172	0	172	7	15	Grid	868	30	..	Feeding substations
23,000	9	60	0	25	25	1	10	Grid	1325	30	..	Generating stations
25,000	24	60	886	0	886	3	28.8	Grid	200	60	..	Generating stations
26,400	17	60	277	18	295	2	6	Grid	2000	60	..	Generating stations
	18	60	..	..	?	2	150	Water	200	2	..	Generating stations
66,000	1	25	200	0	200	1	58	Grid	200	30	..	Generating stations
	17	25	85	0	85	1	?	Grid	2000	60	..	Generating stations
	23	60	161	0	161	2	63.5	Grid	600	10	600° C	Generating stations
110,000	31	25	802	0	802	1	100.0	Water	..	..	..	

TABLE IV-B—TRANSMISSION AT HIGHER THAN GENERATED VOLTAGE SYSTEMS GROUNDED BY SPECIAL METHODS

	No. of		Miles			No. of	
Voltage	Company	Frequency	Overhead	Underground	Total	Grounds	Description of Ground
44,000	3	60	100	0	100	1	Peterson coil placed in neutral of transformer bank at generating station.

TABLE V-A. CALCULATIONS TO ANALYZE DESIGNING OF GROUNDING RESISTANCE

					Calculation of Factor k				
Voltage	No. of company	Frequency	Number of grounds	Ohms per ground	$25 L b$	$L a + 25 L b$	$R f$	$k/10^6$	Maximum current all grounds. Connections in parallel
7500	19	60	1	2	3,775	3,775	120	0.45	2250
11,000	11	60	1	4	..	..	..	..	1590
	23	60	1	15	3,775	3,935	900	3.55	425
12,000	6	60	3	2.75	18,725	18,747	55	1.03	4620
13,200	17	60	1	3	2,120	2,266	180	0.41	2540
	19-20	60	2	8	4,150	4,150	240	0.99	1910
	22	60	1	5	0	25	300	0.01	1520
	36	60	1	4	4,250	4,702	240	1.13	1900
13,800	3	60	2	4	8,725	8,796	120	1.06	4000

TABLE V-B. CALCULATIONS TO ANALYZE DESIGN OF GROUNDING RESISTANCE

Voltage	No. of company	Frequency	No. of grounds	Ohms per ground	Calculation of Factor k				Maximum current all grounds. Connections in parallel
					$25\,L\,b$	$L\,a + 25\,L\,b$	$R\,f$	$k/10^6$	
22,000	23	60	7	15	0	172	129	0.02	5950
23,000	9	60	1	10	625	625	600	0.38	1330
25,000	24	60	3	28.8	0	886	576	0.51	1500
26,400	17	60	2	6	450	727	180	0.13	5100
66,000	1	25	1	58	0	200	1450	0.29	660
	23	60	2	63.5	0	161	1900	0.31	600
110,000	31	25	1	100	0	802	2500	2.00	1100

TABLE VI-A. TRANSMISSION AT GENERATED VOLTAGES
DATA ON CHANGES IN GROUNDING METHOD AND COMPARATIVE PERFORMANCE

Voltage	Company	Present method of grounding	Change in type of ground	Is present method of grounding satisfactory	Factors determining magnitude of resistance, if used	Trouble experienced with resistance
6000	36	7 ohm resistance in gen. neutral	..	Yes	Operation of relays	None
6600	19	Zig Zag Transformer and resistance	Ungrounded to Zig Zag Transf. & resist. ground	Yes	Operation of relays and limited disturbance	None
7800	20	2 ohm resistance	Ungrounded to 2 ohm resistance	Yes	Current equal to 125 per cent of full load on generator	None
9000	6	2-1/2 ohm resistance in generators	Free to resistance	Yes	Limitation of damage	None
11,000	11	Dead grounded at substations	Free neutral to resistance ground	Relays do not operate as quickly as desired	Severity of faults Operation of breakers	None
	23	15 ohms at source	Ungrounded to dead ground to resistance ground	Fairly lower resistance may be installed	Minimum current to operate relays. Elimination of damage to equipment	
	27	Ungrounded		Ungrounded system is unsatisfactory due to increased capacity	..	
	29	Ungrounded		Propose to change from free to grounded neutral	..	
12,000	6	3 ohms at generator		Yes	Resistance in generator to limit current to 2000 amperes	
	21	5 to 10 ohms—water	Free neutral to dead ground to resistance	Yes	Elimination of surges Limitation of Damage High resistance found unsatisfactory to prevent surges	
13,200	1	Dead ground at power and substations	Ungrounded to 10 ohm resistance, 10 ohms by gradual steps to dead dead ground	Not entirely	Capacity of system Current necessary for proper breaker operation Voltage strains on system	Resistance burnt up
	17	3 ohms on generator neutral	..	Yes	Operation of relays	
	18	2 ohms on generator neutral	Ungrounded to 2 ohm resistance	Yes	Severity of faults Operation of relays	
	20	8 ohms on generator neutral	Ungrounded to 8 ohm resistance 30 second resistance to 10 min. resistance	Yes	Severity of faults Operation of relays	Grid resistance burnt up four times because of divided ground current
13,200	22	5 ohms in generator neutral	Ungrounded to resistance ground	Yes	Protection of machine Operation of relays	None
	25	Ungrounded		Ungrounded neutral unsatisfactory	..	..
	32	Dead grounded at generator	Ungrounded to dead ground	Yes	..	..
	36	Resistance in generator neutral	Free to 8 ohms to 4 ohms	Yes	Operation of relays	None
13,800	3	4 ohms at generator	Dead to resistance	Yes	Magnitude of current in case of ground	..

TABLE VI-B. TRANSMISSION AT HIGHER THAN GENERATED VOLTAGES
DATA ON CHANGES IN GROUNDING METHODS AND COMPARATIVE PERFORMANCE

Voltage	Company	Present method of grounding	Change in type of ground	Is present method of grounding satisfactory	Factors determining magnitude of resistance, if used	Troubles experienced with resistances
22,000	27	Ungrounded	..	Ungrounded neutral unsatisfactory due to increased capacity of system	..	
23,000	9	10 ohms at generator	Dead ground to resistance	Yes	Severity of faults Operation of breakers	None
25,000	24	28.8 ohms at generator stations	Ungrounded to resistance ground	Not satisfactory with present relay installation	Elimination of surges Protection of equipment	Broken grids
26,400	17	6 ohms at generating stations	..	Yes	Operation of relays	
	18	150 ohms water at generating stations	Dead grounded to 150 ohms resistance	Not sufficient information to decide	Minimum ground current giving satisfactory relay operation	
33,000	7	Dead grounds at substation	Free neutral to dead ground	Yes		
	25	Ungrounded	..	Ungrounded neutral unsatisfactory because of surges		
33,000	27	Ungrounded	..	Ungrounded neutral unsatisfactory due to increased capacity		
44,000	2	Reactance coil	Dead ground to reactance	Expect to ground all lines		
	5	Dead ground at substations	Resistance to dead ground	Yes		
66,000	1	58 ohms at power station	Dead ground to free neutral to resistance ground. Concrete resistance to grid resistance	Yes	Current required to operate relays but not cause overload of transformers	Concrete resistance caused arcing upon passing current. Grid resistances melt when breakers fail to clear trouble
	17	Resistance grounded at power stations		Yes	Operation of relays	
	23	63.5 ohms at power plants		Yes		Grid resistance burnt up by hanging on of ground current
	24	Dead ground at power plant		Ungrounded neutral unsatisfactory due to increase capacity		
	36	Ungrounded				
100,000	4	Dead grounded	Ungrounded to dead ground			
	5	Dead grounded	Resistance to dead ground	Yes		
110,000	31	Dead grounded at power station	Ungrounded to 1000 ohm resistance to 100 ohm resistance	Yes	Elimination of surges and limitation of duty on breakers and transformers	High resistance of low capacity burnt up

TABLE VII. GENERAL DATA—SUMMARY

No. of company	Generating capacity kv-a.	Ungrounded			Grounded					
					Dead		Resistance		Special Schemes	
		Voltage	Miles	Protection other than ground	Volts	Miles	Volts	Miles	Volts	Miles
1	130,000				13,200	180	66,000	200		
					26,400	40				
2	153,500	22,000	60		44,000	240			44,000	100
		44,000	80		110,000	564				
3	173,000	66,000	21		25,000	20	13,800	424		
4	252,000	50,000	895		100,000	1100				
		66,000	355							
5	315,850	13,000	133		44,000	709				
					100,000	1489				
6	523,000						9000	306		
					20,000	93	12,000	375		
					22,000	27				
					33,000	28				
7	102,265	20,000	80		33,000	657				
		33,000	93							
8	293,000				24,000	937				
					48,000	212				
9	14,000						23,000	25		
10	124,750				11,000	3596				
					30,000	105				
					66,000	910				
					110,000	193				
11	72,500	22,000	22				11,000	77		

TABLE VII. GENERAL DATA—SUMMARY (Continued)

No. of company	Generating capacity kv-a.	Ungrounded			Grounded					
		Voltage	Miles	Protection other than ground	Dead		Resistance		Special Schemes	
					Volts	Miles	Volts	Miles	Volts	Miles
12	174,300	17,500	88							
		22,000	122							
		44,000	305							
		75,000	171							
		140,000	518							
13	139,000	11,000	69		110,000	144				
		33,000	61		Arc suppressors also used on 110 kv. system					
		66,000	29							
14	69,160				33,000	105				
					110,000	90				
15	30,800	15,000	28							
16	288,750				15,000	389				
					30,000	179				
					50,000	204				
					60,000	1000				
					150,000	480				
17	117,000						13,200	320		
							26,400	295		
							66,000	85		
18	241,700	11,000	63	Arc suppressor	26,400	?	13,200	661		
		13,200	115	" "			26,400	?		
19-20	657,550						6600	400		
							7800	91		
							11,000	182		
							13,200	198		
	277,800						15,500	13		
21	302,250	24,000	135				12,000	170		
22	38,400						13,200	25		
							11,000	311		
23	238,000						22,000	172		
								172		
							66,000	161		
24	145,000				66,000	26	25,000	886		
25	16,600	13,200	5							
		33,000	23							
26	16,000	13,200	70							
		15,000	45							
		33,000	185							
27	80,500	11,000	55	Frequency absorber on cable system						
		22,000	94							
		33,000	90							
28	60,000				55,000	344				
					87,000	224				
					100,000	208				
29	17,700	11,000	315		66,000	72				
30	326,200	22,000	217		17,000	327				
					60,000	2042				
					110,000	515				
31A	267,000	13,200	50				110,000	802		
		26,400	240							
31B	21,200				110,000	75				
31C	9950	22,000	580							
32	92,000				13,200	121				
33A	12,000	66,000	175		132,000	76				
		24,000	70							
33B	35,700	66,000	292							
33C	145,000				66,000	180				
					132,000	110				
33D	34,000	33,000	115							
34	78,000	13,200	92		66,000	460				
		19,000	68		Experimenting with arc suppressor					
		44,000	89							
35	55,600	13,200	28							
36	278,000	66,000	31		6,000	10				
					13,200	203				

SUMMARY

Number of Companies 36 | Total Capacity 6,371,315 kv-a.

Number of Systems	108	Mileage	31,308	Ave. Mil. per System	290
" " " Ungrounded	43	" Ungrounded	6,385	" " " Ungrounded	148
" " " Dead Grounded	41	" Dead Grounded	18,475	" " " Dead Grounded	450
" " " Resistance Ground	25	" Resistance Ground	6,448	" " " Resistance	258
" " " Specially Grounded	1	" Specially Grounded	100	" " " Specially Grounded	100

TABLE VIII—KEY TO COMPANIES SUBMITTING INFORMATION

Number of Company	Name of City	Name of Company
1	Baltimore, Md.	Consolidated Gas, Elec. Light and Power Co. and Pennsylvania Water and Power Co.
2	Birmingham, Ala.	Alabama Power Co.
3	Boston, Mass.	Edison Electric Illuminating Co.
4	Butte, Mont.	Montana Power Co.
5	Charlotte, N. C.	Southern Power Co.
6	Chicago, Ill.	Commonwealth Edison Co.
7	Chicago, Ill.	Public Service Elec. Co. of Northern Illinois
8	Detroit, Mich.	Detroit Edison Co.
9	Fall River, Mass.	Fall River Electric Lighting Co.
10	Fresno, Cal.	San Joaquin Light & Power Co.
11	Hartford, Conn.	Hartford Electric Light Co.
12	Jackson, Mich.	Consumers Power Co.
13	Keokuk, Iowa	Mississippi River Power Co.
14	Los Angeles, Cal.	Department of Public Service
15	Los Angeles, Cal.	Los Angeles Gas and Electric Corp.
16	Los Angeles, Cal.	Southern California Edison Co.
17	Milwaukee, Wis.	Milwaukee Electric Railway and Light Co.
18	Newark, N. J.	Public Service Electric Co.
19	New York, N. Y.	New York Edison Co.
20	New York, N. Y.	United Electric Light & Power Co.
21	Niagara Falls, N. Y.	Niagara Falls Power Co.
22	Pawtucket, R. I.	Blackstone Valley Gas and Electric Co.
23	Pittsburgh, Pa.	Duquesne Light Co.
24	Pittsburgh, Pa.	West Penn. Power Co.
25	Port Arthur, Tex.	Eastern Texas Electric Co.
26	Poughkeepsie, N. Y.	Central Hudson Gas and Electric Co.
27	Providence, R. I.	Narragansett Electric Lighting Co.
28	Riverside, Cal.	Nevada-California Power Co.
29	San Diego, Cal.	San Diego Cons. Gas and Electric Corp.
30	San Francisco, Cal.	Pacific Gas and Electric Co.
31	Toronto, Ont.	Hydro-Electric Power Commission
32	Washington, D. C.	Potomac Power Co.
33	Wheeling, W. Va.	American Gas and Electric Co.
34	Worcester, Mass.	New England Power Co.
35	Worcester, Mass.	Worcester Electric Light and Power Co.
36	Philadelphia, Pa.	Philadelphia Electric Co.
37	Buffalo, N. Y.	Buffalo General Electric Co.

Discussion

GENERAL CONSIDERATIONS IN GROUNDING THE NEUTRAL OF POWER SYSTEMS (Dewey); THE NEUTRAL GROUNDING REACTOR (Lewis) OPERATING PERFORMANCE OF A PETERSEN EARTH COIL (Oliver and Eberhardt); PRESENT DAY PRACTISE IN GROUNDING TRANSMISSION SYSTEMS (Committee Reports; (a) SYSTEMS TRANSMITTING AT GENERATED VOLTAGE, (Woodruff); (b) SYSTEMS TRANSMITTING AT HIGHER THAN GENERATED VOLTAGE (Stone),

Pittsburgh, Pa., April 24, 1923.

R. W. Atkinson: I want to say a word with regard to the comparison between the condition of a cable system and an overhead open-wire system with regard to the grounding of the neutral. There is a large and important difference for two reasons. The most important reason is the relative cost of insulation of the cable system as compared with the overhead cable for high voltage. The insulation is a very important part of the transmission system, whereas it is a relatively minor part in the overhead system. You can afford in the overhead system to put on insulation to take care of the voltage rise due to abnormal conditions, and you can afford to put on the insulation to take care of service conditions; whereas the same difference in service conditions costs you a whole lot more in insulation of cable systems. Another difference from the standpoint of operation, is the very much greater electrostatic capacity for the cable system.

Another thing that we have often overlooked, not only the operating people but some of engineers connected with the manufacturers, is that when you have the same thickness of insulation between conductor and sheath in a cable, as you have between conductors, you really haven't the same voltage strength.

The best reason perhaps is the fact that the ratio between the maximum stress and the average stress is much higher where you have the stress concentrated only on one side, as you do from the conductor to the sheath, as compared with where you have it concentrated in two places, and your stress subdivided. Your belt insulation is probably not equal in its voltage resisting strength to the insulation on the conductor, and as a general proposition a cable with equal insulation between copper and sheath as between conductor and conductor; has very much less actual strength from copper to sheath than from copper to copper. Therefore, if we only got the same amount of voltage disturbance or the same potential between copper and lead, as between copper and copper, we would be expecting more trouble.

As a matter of fact, with the underground system, it has been shown to proceed to higher ground as in cases of arcing grounds. Coming to the other side of the case, the neutral grounded through resistance; there we have removed the most serious source of possible disturbance, but we still have the condition that any conductor of the system may be subjected suddenly to a potential, exceeding line potential. In view of the less apparent strength of the insulation from conductor to sheath, we want to have more insulation there than we would have applied otherwise.

Another point to consider in comparing systems is the matter of the size of the system. A very few cable systems operate without grounded neutrals at voltages above 90 kv. and a few have operated with resistance in the neutral. I believe all of those systems above 20 kw. which have not operated with solidly grounded neutral are very small or the actual part of the system itself is small. That is, if we have a bus at the generator voltage and a step-up transformer to the cable, and then through the cable to another transformer, without going to a bus, then only the one cable is involved. That system is radically different in its operating condition from the other system, whereby the voltage is stepped up before you get to the bus, and you have a bus voltage above generator voltage. Now, in that case, all of the cable on the system is subjected to the abnormal condition resulting from a single voltage failure, which means that we have the difference between the large and small system.

The large transmission system may have a number of small units each having the same characteristic, as far as the effect on grounding neutral is concerned, as a small transmission system, and the effect is this.

A system, of course, without grounded neutral acts just exactly like one with the grounded neutral as long as nothing happens. As soon as something happens, you put strains on the system, and one thing causes something else, and that causes something else again, and troubles will be multiplied in proportion to the square of the amount of cable involved, perhaps, if they are troubles due to incomplete grounding.

There is another thirty-three thousand volt cable in operation in this country, (it is a very small amount in addition to the one which is mentioned here) in Los Angeles, and that is operating with a solidly grounded neutral system, and has been for nearly two years.

F. C. Hanker: Mr. Dewey has given us a very comprehensive review of the problems involved in grounding the neutral of transmission and distribution systems. I do not believe that too much emphasis can be placed on the advantages of operating a system with the neutral stablizer. The principal argument for the free neutral, that it permits continuity of supply even when one phase is grounded, is misleading in that it is a dangerous practise as well. The possibility of arcing grounds with their resultant recurring oscillations are so likely to cause extremely high voltages as to be dangerous. It is far better policy to remove a circuit that has failed from service rather than risk more general and widespread breakdowns due to high-voltage disturbances.

While it is true that the smaller systems can generally operate successfully with the neutral free, it is also true that is a limit to the growth of a system. One of the systems referred to by Mr. Dewey, the Montana Power Co., was initially a delta system operating with the neutral free. As the system grew the effect of failures became more serious and widespread with the result that the engineers practically reached the conclusion that there was a definite limit to the size of a transmission system that could be operated successfully. The solution of the problem was in the stabilization of the neutral. This has proved so successful that, based on the experience of several years operation, the present conclusion is that there is no limit to the size of a transmission system.

The strongest advocates of the stabilized neutral are the Pacific Coast systems. In 1898 a transmission system operating at 60,000 volts was planned by the Standard Elec. Company. They adopted the 60-cycle, grounded system and the same plan was adopted later by the Bay Counties Power Company. The grounded system has been generally used on the Pacific Coast from the beginning and engineers, prominent on the coast have expressed the opinion that their success is largely due to the fact that the neutral was fixed.

An analysis of the relative magnitude of possible surge voltages on the free neutral and grounded neutral systems will show to the advantage of the grounded system. Advantages of this feature can be considered in two ways, first by retaining the greater factor of safety, in insulation strength and second, by grading the insulation. On lower potentials up to 110,000 volts the advantage in cost is not great and it may be desirable to retain the higher factor of safety. On higher voltage systems the saving is appreciable and we are justified in taking advantage of the reduction. Very appreciable savings have resulted in apparatus purchased for operation on 220 kv. and have been accepted by operating engineers after careful comparative analysis of various factors affecting the conditions.

J. B. Taylor: A few years ago there would have been little difficulty in securing a number of papers on this general topic in which perhaps the majority would have favored working without grounded neutral. The absence of any paper today in which the author definitely advocates running in the old method is quite obvious; it is not necessary to comment on it. The practise is fairly well crystallized and those who haven't swung into line, as far as I can discover, are holding back either for reasons of expense or because their systems may be in such good shape that their troubles with the free neutral are not serious.

Of course, if there were no causes of trouble, there would be no point in deciding whether to work one way or the other, and the whole question could be left open. Consequently, it isn't possible for me at any rate to dispute much of what is said here, and a discussion is necessarily limited to commenting and questioning a few of the points which boil down really to comparatively minor differences in practise.

We might emphasize this best by calling your attention to the fact that Mr. Dewey in his conclusions advocates grounding heavily so that voltage rises will be kept down at the expense of currents. Mr. Lewis, in his conclusions, says the Petersen coil is a good thing where it is desirable to keep down the rises in current at the expense of voltage. In the report of the committee, I think you will find on one page a statement that the tendency is to cut out the resistance and elsewhere a statement that the tendency is to increase the resistance.

Now, this state of affairs is not disturbing; all systems are not alike. They operate at different voltages, have different lengths of line; the cables are from different makers and loaded to different extents. The ducts are laid differently, so that there are plenty of good reasons why the prevailing kind of trouble which one system has is different from the prevailing kind of trouble which another system has.

One thing to which the engineers who have presented the papers, at any rate, agree and probably a great many more, is that grounding in some form is desirable.

Now, the particular form seems to come down to a matter of accumulated experience on the particular system and engineering judgment on how to meet that particular condition.

I am inclined to think that if you can clear your faulty cable without a big rush of current, even though your apparatus will stand that rush, there is particular virtue in doing it that way. Consequently, continued experience in designing selective relays and these various differential arrangements will perhaps be the tendency.

I am also inclined to make a general comment on some of the statements in the committees reports. It seems to me there is a little confusion in spots between presenting the present practise and stating the trend.

Now, a census does not always show the trend of things at all. It may show the amount of previous mistakes compiled, so we must discover the trend in more subtle ways instead of counting up what you already have on your hands. I would draw a line through the word trend, because it appears to have been put in where they should have said the practise is thus and so.

There is only one more point. In the committee's reports where there is a discussion of what determines the value of resistance in the neutral, where resistance is used, on the third page, the middle of the first column. My comment is that the most important controlling factor in determining this resistance is included under the several other features not mentioned. The current-carrying capacities of the cable determines the setting of the oil switch, very often set on triple, double and normal load.

The neutral resistance, to work properly, must take into account the setting of the switches. I believe that this has been the controlling factor in determining the value of this resistance.

L. P. Ferris: In presenting these remarks I want to emphasize that I do not wish to suggest that the features which I will mention are necessarily to be considered controlling. All I wish to do is to call attention to some of the effects which manifest themselves and that these effects should be given their due consideration and nothing more.

On the third page of his paper Mr. Dewey states as a conclusion, that no serious telephone interference under normal operating conditions need be anticipated even though the neutral is grounded at many points. I believe that operating experience does not justify such a generalization, although there are doubtless instances where this conclusion holds. There is just this difference, under normal operating conditions, between the grounded neutral system and the isolated neutral system: In the former you give opportunity for triple harmonic voltages to be impressed upon the line with respect to the ground, and thereby affect the neighboring communication circuits, whereas in the isolated systems these triple harmonic voltages and currents cannot get out on the line. There are certain minor exceptions to that which we won't go into.

Sometimes this effect is emphasized by multiple grounding of the neutral although you can create a disturbance with only a single ground. The capacity of the system to ground presents a path for any triple harmonic current, the circuit being completed via the neutral. Such current would in general be negligible from the standpoint of the operation of the power system, however, there are cases where it is important in its effect on neighboring circuits.

I am glad to note on page 5 of Mr. Dewey's paper his conclusion that the use of Y-Y transformers is very limited in view of the interference which they tend to cause in the neighboring communication lines.

Obviously from the standpoint of telephone interference it is an advantage to limit the short-circuit current to ground which if not limited may give severe acoustic shocks to operators listening on parallel circuits. Mr. Dewey, evidently inadvertently, stated the contrary at the bottom of his fifth page.

Will Mr. Dewey please explain what is meant by a semblance of arcing ground effects.

At the midwinter convention of a year ago, I called attention to some of the properties of the Petersen coil from the standpoint of its effects on induction into neighboring circuits. It is very gratifying to note from the paper by Messrs Oliver and Eberhardt that operating experience with this device shows that its use reduces the interruptions due to flashovers from lightning by some 83 per cent. Without that device, it is fair to conclude that in those cases in which the coil cleared the fault there would otherwise, in general, have been a direct short circuit to ground on the system, and had there been a telephone line from Lock 12 extending toward Vida, it is reasonable to expect that in a good many of those cases there would have been induced in the telephone circuits high voltages, some producing acoustic shock. In such a situation the use of this coil would have saved about 83 per cent of the disturbances in the neighboring circuits.

This installation uses in combination with the Petersen coil a short-circuiting switch which solidly grounds the system in those cases where the Petersen cannot be expected to clear the line. In this instance there in no particular reason why a resistance would offer advantages over the direct grounding of the system. However, had there been a telephone system exposed to this power line, in those cases where it was necessary to resort to other means than the Petersen coil to clear the line, if instead of grounding the neutral solidly the switch had connected the neutral to ground through a moderate resistance of such value as to permit reliable operation of a ground relay you could have accomplished the desired result and at the same time prevented a condition which might give rise to a disturbance in the telephone system. The combination of this coil and a moderate resistance, may, therefore, offer advantages in situations where we have disturbances in telephone lines to contend with.

In closing, I would make one suggestion for the consideration of the authors of the Petersen coil papers. The network of the Alabama Power Company with which the coil is associated is in the form of a Y, and Mr. Lewis presents curves which show that at resonance the current in the neutral reaches a maximum as we should expect, amounting in this case to about six amperes. That is due, of course, to the unbalanced voltages of the system produced by the unbalanced line capacitances acting in the series resonant circuit. There is an unusual opportunity at the junction point of this Y to connect conductors of high, low and medium capacitance to ground, one from each branch of the Y, and so to improve the balance of the system without the expense of transpositions. Of course, if the branches of the Y are very different in length, this scheme would not work so well, unless the excess lengths of two branches over the third were transposed. At any rate, a considerable improvement would seem to be practicable at slight expense. Improved balance would, of course, enable the coil to be operated closer to the resonance adjustment without excess current under normal conditions.

H. W. Smith: In regard to the question of grounding the neutral of power systems, it should be pointed out that the engineers of the Westinghouse Company have consistently advocated this from the early days of power transmission.

For instance, P. M. Lincoln read a paper in 1907 on "The Grounded Neutral With and Without Series Resistance in High-Tension Systems," which he followed in 1909 with a paper on "Protection of Electrical Equipment" in which he stated "It is my opinion that so far as protection to apparatus is concerned, the advantages of grounding the neutral very much outweigh the disadvantages and if protection to apparatus alone were to be considered I would have no hesitation in recommending a solidly grounded neutral system."

The experience of transmission companies is recorded in the files of A. I. E. E. in such papers as: 1914—The Experience of the Pacific Gas & Electric Company with Grounded neutral by Messers. Jollyman, Downing & Baum; 1916—Report of of the Transmission Committee; 1919—Grounding The Neutral of Generating and Transmission Systems by H. R. Woodrow; Grounded Neutral Transmission Lines by W. E. Richards. They have led towards a general adoption of the grounded neutral.

I believe the operating companies, as regards this question of grounded neutral, should be divided into three classes: 1. Large metropolitan companies transmitting at generator voltage mostly through underground cables. 2. Medium voltage systems transmitting at 22, 33 or 44 kv. These systems often do not involve a large kw. capacity, but there is a large mileage of lines covering large districts. These systems often distribute at these voltages, rather than transmit; 3. High-voltage systems 66 kv., 88 kv., 110 kv., 132 kv., 154 kv., 165 kv., and 220 kv.

The general practise in class 1, is to ground the neutral. This was done quite early in the art, for example on the Manhattan Railway in New York about 1904.

A severe cause of trouble which was theoretically investigated by Dr. Steinmetz in his paper "High Power Surges in Electric Distribution Systems of Great Magnitude" (Trans. 1905), led to the grounding of the neutral through a 6-ohm resistance (1000 amperes for two minutes).

As mentioned by Mr. Dewey, these systems can be:

(1) Grounded solidly; (2) Grounded through low resistance to pass currents in case of ground of such magnitude to operate relays set for short current protection; (3) Grounded through higher resistance to limit the current and prevent excessive burning of cable in case of ground.

This latter system involves a special system of ground relays the expense of which many companies have not seen fit to incur.

Satisfactory systems of ground relays have been developed for this use and are available for those companies which desire to use them.

Exact data showing the proportion of cable faults which are grounds and those which start as short circuits would be a welcome contribution to our information and help us to decide whether such ground relays should be used.

As pointed out, if it is desired to apply differential protection to generators to disconnect them in case of a ground on winding, it is necessary to ground the neutral of one or more generators.

In regard to the design of grounding resistors I believe it is safe to work up to a maximum temperature of 600 deg. cent. It should be pointed out that working through this range there is a considerable increase in the value of the resistance probably around 50 per cent, so that a resistor which was 5 ohms when cold would be around 7½ ohms at the end of the time interval for which it was designed.

In regard to the time for which the grounding resistor should be designed there are a number of points to be considered. If there are a number of feeders in parallel feeding a substation, in the case of a ground occurring in that substation or beyond, the ground current will divide up so that the relays will probably not trip out the lines carrying the ground current and the total ground current through the resistor may burn it out. The resistor, therefore, in a case like this should have a long time element or should be protected by thermal relays which will either short-circuit or open-circuit the resistor when it reaches a dangerous temperature.

In regard to systems of the second class. In many cases the neutral is grounded at several points through a resistance. There is no doubt that as these systems grow they will reach a point where it is essential to ground the neutral. There are however a number of systems with single-loop transmission lines having customers tapped at intervals which hesitate to ground the neutral on account of interruption of service. In many cases these systems are not properly equipped with relays and oil circuit breakers. All extensions to such systems should be made so that adequate relay and circuit breaker protection are installed with duplicate lines to important load centers so that a grounded neutral system can be installed as soon as possible.

In regard to considerations fixing the size of resistance. If no resistance or a low resistance is used to ground the neutral, then any ground will seriously distort the voltage triangle, and affect sychronous apparatus on the system.

Tests have been made which show that the factor to be considered is the positive phase sequence component of the voltage at the terminals of the apparatus considered. If this voltage is less than 70 per cent of normal voltage it is probable that no trouble will be experienced from apparatus dropping out of step provided they are not overloaded and have full excitation. Tests have shown that the value of ground resistance to give this drop in voltage is approximately a resistance which allows double full-load current to flow to ground, that is $\frac{\text{phase voltage}}{R} = 2 \times \text{F.L. current}$, so that if the ground current does not exceed twice full-load current, synchronous apparatus should generally stay in step.

H. M. Trueblood: I have merely a brief suggestion to offer with regard to the three cases of doubtful operation of the Petersen coil which are mentioned in the paper by Messrs. Oliver and Eberhardt. They are referred to on page 907 of that paper and further in Table I, under the dates of July 7 and August 25, 1922, and February 23, 1923. It is rather interesting, I think, that in each of these three cases what was observed was a flashover on the 44-kv. bus, and, connected with that disturbance in some way, was an operation of line switch 206 between the line and the bus.

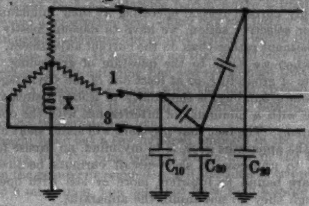

Fig. 1

In the diagram, the Fig. 1, 1, 2, and 3 represent the three poles of a switch and the condensers C_{10}, C_{20} and C_{30} represent the direct capacities of the three phases to ground. X is the Petersen coil. Let us suppose that one of the three switch contacts, *e. g.*, No. 3 opens up while the other two remain closed, as might occur, for instance, if, through some defect in the switch mechanism, the three poles are not operated simultaneously. In this case the capacity in what has sometimes been called the "series resonant circuit", consists of C_{10} and C_{20} in parallel, plus an additional capacity equivalent to the direct capacities of phases 1 and 2 to phase 3 in parallel, in series with C_{30}. Now, if the reactance in X is larger than is correct for resonance with all three poles of the switch closed, a condition of resonance with pole No. 3 open may be approached. This will tend to produce high voltages across the inductance and also across the capacity of the series circuit. If the reactance of the Petersen coil exceeds the correct value for resonance under normal conditions by the appropriate amount, it will, of course, resonate exactly with the capacity of the system to ground with one pole of the switch open. In this case there will be nothing to limit the rise of voltage except the system losses.

In comparing the data in the table on page 906 of the Oliver-Eberhardt paper with Figs. 21 and 22 of Mr. Lewis' paper, I have been led to the conclusion that the system described has been operated with a reactance on the Petersen coil in excess of the proper value for resonance under normal conditions. *E. g.*, the curves of Fig. 22 of Mr. Lewis' paper indicate that for exact resonance about 980 ohms is required in the reactor. From the table in the other paper, however, it appears that the reactance actually used for the system arrangement referred to is 1207 ohms. The excess reactance is thus about 23 per cent in this case. The same condition may be traced from a comparison of Fig. 21 of Mr. Lewis' paper with the data in the Oliver and Eberhardt paper. The figure shows that about 1410 ohms in the reactor would be correct for lines 1, 2 and 4. The value used, however, is apparently 1735 ohms. The excess is again about 23 per cent.

If I am correct in inferring from these data that the system has been operated with excess reactance of this magnitude in the Petersen coil, I think that the explanation of the excessive voltages that have been observed may be as I have indicated. The condition could, of course, be corrected by decreasing the reactance used.

R. D. Evans: Mr. Oliver and Mr. Eberhardt have presented the results of the operating experience with the Petersen coil installed on the Alabama Power Company's system. In comparison with the previous year's operation as a solidly grounded neutral system, the Petersen coil installation resulted in decreased interruptions due to line insulator flashovers caused by lightning, and increased interruptions in the remainder of the system, apparently due to excess voltages causing bus flashover and transformer trouble. Both of these results might have been anticipated, as previous discussions have pointed out their probability. That excess voltages may occur with the Petersen coil is borne out by the flashovers on the 44-kv. bus at Lock 12, which followed the opening of a circuit breaker between the coil and the transmission line. These flashovers may have been due to the non-simultaneous opening of the 3 poles of the circuit breaker, which condition is one that may produce excessive voltages with the Petersen coil system.

The operating record shows that in a number of cases an insulator flashover produced an arc lasting for two seconds, when the neutral was grounded by the circuit breaker. All of these conditions were not produced by dead grounds, as in some cases the line could be closed back in service o.k. The operating log for these cases is marked "Correct Operation" though the Petersen coil did not suppress the arc, and it is doubtful if it would after the arc had persisted for two seconds. In the arcing test made by Mr. Lewis, it is to be noted that the arc was maintained for 14 cycles for a reactor setting of 80 per cent normal, and that the arc resembled "a power arc". The arcing test and the maintenance of an arc for two seconds under service conditions indicates that the limits for the satisfactory operation of the Petersen coil are being approached for a system as extensive as this section of the Alabama Power Company's 44-kv. system.

On account of the high voltages produced by switching operations, which led to interruptions, Mr. Oliver and Mr. Eberhardt have concluded that all switching operations should be performed with the neutral solidly grounded. In view of this fact, and because of the number of conditions where the arc was not suppressed by the Petersen coil, it appears very desirable to supplement the Petersen coil with equipment to solidly ground the neutral.

Mr. Lewis is of the opinion that the field of the Petersen coil will probably be limited mainly to isolated neutral systems, whose operation is not satisfactory, but on which for some reason it is not wished to solid-ground the neutral. In view of the reasons given above for supplementing the Petersen coil with grounding equipment, it would appear that a change-over from a free neutral system would require that the system be designed for solidly grounded neutral operation, including the installation of relays and circuit breakers, in addition to the Petersen coil, auxiliary grounding circuit breakers and relay. It is to be noted that Mr. Lewis concludes that for the present, at least, the Petersen coil is not of general application, but limited to comparatively low-voltage lines of relatively short length.

C. L. Fortescue: In 1914 I presented a paper (PROC. May 1914 p. 771) grounding of the neutrals of sytems, above 44,000 volts. At that time the General Electric made an announcement of their new policy with respect to grounding neutrals of high-voltage systems. It is interesting to see that about a decade later we are investigating the results of this agreement in the engineering policy of the two large electric manufacturing companies and the results are quite gratifying.

Mr. Dewey seems to imply that at that time there was a certain amount of uncertainty in the minds of engineers as to whether to ground or not to ground. However, my impressions at that time were that the evidence in favor of grounding was quite convincing. The basis of the evidence on which this conclusion was based was well borne out by the experimental work of Mr. Faccioli. The theory on which most of us worked at that time was that an ungrounded system was free to oscillate at its natural period with very little damping, and therefore in extensive systems surges set up would be reflected, superimposed, etc., causing high-voltage disturbances. At the present time with the very extensive high-voltage systems that are in contemplation, there are additional very weighty reasons for grounding, and also for grounding at many points.

For example, let us consider a 750-mile transmission line of 440-kv. I have taken this length because it is not outside future possibilities, and happens to be about one-quarter wave length of 60 cycles. If such a line were ungrounded a ground near the generating station would be equivalent to a potential of 127,000 volts impressed on an open line 750 miles long, and having exceedingly low loss. The result would be, to quote from a recent paper by Mr. Peek, "Instability will be noticed at distances considerably less than this. At no load and very low generator voltages, the power of such a line would be limited, in fact, only by the losses." Therefore, if a high-voltage line of the kind in contemplation were not grounded, we would have in the case of ground on one line exceedingly high-voltages at the further end.

A transmission system grounded at many points is on the other hand equivalent to a number of short-circuited transmission lines, and the surge potentials that can be obtained due to disturbance at immediate points are very limited in value.

In regard to the effect on telephone systems, these will best be taken care of by keeping as far away from such high-voltage transmission lines as possible. Quick operating relays may afford a measure of relief, but the power companies and the telephone will have to depend much on co-ordination.

The Petersen coil seems to have found the place which was predicted for it. Personally the writer believes that the use of grounding resistance, of value below the critical value, will be found more satisfactory.

As to how to ground, high-voltage systems above 110,000 should be solidly grounded. In general this should apply for still lower voltages, say, down to 66,000 volts. On very high-voltage systems, all distributing points and the substation at the end of all feeders of high voltage should be grounded.

Of course, no system can actually be solidly grounded, but as far as the transformers are concerned if their neutral points are solidly connected to ground, the potential stresses they will be subjected to will be a minimum. This will also permit graded insulation which in the case of very high-voltage transformers makes a very substantial reduction in cost.

J. A. Johnson: There are two or three points in connection with this matter of grounding which I feel should be perhaps brought out a little more in detail than have been mentioned.

In the first place, systems differ very greatly in characteristics, and I very much doubt whether any one system of grounding will ever be found satisfactory for all systems; systems which have large exposure to communication systems may always require high-resistance grounding for the limitation of interference. In that connection engineers I think, as well as other people, are prone to follow the line of least resistance and when they run against a problem which requires an immediate solution, they usually jump to something that looks pretty obvious and if it works, why, they stick to it until something else comes up that causes them to change, and possibly some situation of that sort may be responsible for some of the systems using high-resistance grounding.

Now, in connection with systems, more particularly those operating at generator voltage: I think that we sometimes lose sight of how the tools which we have to work with influence our practise, and in that connection I want to bring to your notice the connection between the characteristics of our modern relays, referring especially to the so-called induction type, having the inverse-time characteristic, and the question of grounding.

In using these relays for short-circuit protection, on account of the fact that short-circuit currents usually exceed normal currents by so great an amount as they do, it is very diffcult to apply these relays in such a way as to take advantage of their inverse-time-current characteristic, and they become in most cases simply current relays, operating at their minimum time, so that for selective action, we have to depend mostly on the time adjustment rather than on the current adjustment.

Now, when we come to the question of grounding, one of the objects of grounding, or at least one of the advantages that we get from grounding, is the ability to isolate a circuit that is in trouble with a minimum of interference with the operation of the system. The fact that a system will stand short circuits, so far as the apparatus goes, is to my mind, no excuse for subjecting it to unnecessary short-circuit currents because large currents are bound to interfere more or less with operation, even though they do not injure the apparatus.

Now, suppose you have a simple radial system, consisting of some sort of a center of distribution, with various generating sources feeding into it, and various lines feeding out of it. And let us suppose that we get a ground on a feeder. If we ground our neutrals at all of these generating sources through moderately high resistances, we will then get a moderate ground current going over each of these circuits, but all concentrating into the feeder that is in trouble, and if we use a ground relay separate from our overload protection; we can choose that relay of such a capacity or we can choose our resistances of such a value as to produce a ground current which comes within the normal range of the relay and take advantage not only of the current characteristic but also of the time characteristic of the relay. We can thereby trip out the faulted circuits without danger to the rest of the system better than we can if we depend solely on the over-current relays for tripping out ground faults.

L. F. Blume: The excellent performance of the Petersen earth coil, as recorded in the two papers presented today is a gratifying confirmation of the sound theory upon which it is based. Although it was clear theoretically that such a coil should function to reduce the current flowing in a line fault to ground, it nevertheless could not be foretold that it would prove so effective in disposing of the line fault and thereby materially lessen interruptions in service.

The flashover which resulted in the breakdown of the busses is probably very excellently explained by Mr. Trueblood and shows the effect of extreme dissymmetry in the lines, dissymmetry of the three lines being very extreme due to the breaking of the line at the point of the operation of the switch.

It is perfectly feasible by having fifty per cent untuning, combined with this combination of circumstances, to get serious resonance of this voltage to ground, and at normal frequency. Of course, under those conditions very excessive voltage will be obtained. The remedy for that however is very simple. All that one need do is to be sure that this coil is untuned on the other side of resonance, by making the resistance a little less than for the normal resonating conditions. The voltages which you will get under this particular condition of operation will be only moderate, so if that is the explanation of the flashovers from the busses, I think that a simple adjustment of the coil will take care of the trouble.

It seems to me that the records given in the two papers show that the operation of the coil could be improved by introducing more resistance. The coil as designed has very little resistance, because it was designed to operate continuously with a fork on the line.

Now, if it is to be operated for intermittent duty only, it is perfectly feasible to introduce considerable inherent resistance in the coil which will be very effective in preventing danger due to serious over-voltage, resonance, dissymmetry. It will reduce the residual current under such conditions as shown by the oscillograms considerably.

H. L. Wallau: We have been operating for about eighteen years a moderate voltage system with a grounded neutral and the results of our experience, confirms Mr. Johnson's statements to a very wide degree. I wanted to comment in that connection on a statement made by Mr. Smith: If I understood him rightly he said that the thermal capacity of the resistance might be a considerable factor where a ground took place beyond a substation bus, which bus was fed by a number of lines in parallel, because the ground current would be divided between these lines. It seems to me that would only be the case if the breaker in the outgoing line was set so high that it could not trip at a reasonable speed with the total ground current that must flow through it, due to the current flowing into the fault beyond the substation bus.

You might have but a few hundred amperes in the individual supply lines, but the total current would flow in the final line on which the ground developed, and if the time of tripping is reduced to a mimimum, you need not have such a large thermal capacity.

Another point brought up by Mr. Atkinson was that the cables with equal thickness of insulations, conductor insulation equal to belt insulation were not quite as effective as cables would be designed with a greater belt thickness. It has been our experience that since we changed over from a six by six thirty-seconds insulation on 11-kv. cables to eight by two, thereby increasing the insulation beween phases and decreasing from phase to ground, we have had considerably less cable trouble, and I believe although I am not positive, less trouble from phase to ground.

W. I. Slichter: There is a practical condition in connection with the tuning and detuning of the circuit which seems to me has been overlooked, namely that the step-down transformers at the receiving end of the transmission line will probably remain in circuit and they afford a circuit between phases which it would seem would prevent the detuning. I wish to ask whether this is not the case in practical operation.

W. W. Lewis: The discussion has brought out a number of constructive suggestions. Mr. Ferris suggests transposing the three banks of the Y at Vida; Mr. Trueblood gives an explanation of the possible cause of the three cases of over-voltage which resulted in bus flashover and suggests a reduction in the reactor ohms to remedy the situation; Mr. Blume suggests increasing the resistance of the coil to reduce the residual current and increase the damping effect. All of these suggestions will probably be acted upon within the present year and will no doubt be reflected in an improved operating record.

As to Mr. Evans' statement that the use of the coil resulted in increased interruptions in the remainder of the system, apparently due to excess voltages causing bus flashover and transformer trouble: I would call attention to the fact that the 343 minutes outage due to transformer trouble in January was the result of a burned out transformer coil. This transformer had been subjected to repeated short circuits during the progress of switch tests in November and December 1921 and January 1922, which finally broke down the transformer winding. The Petersen coil was not in circuit during the switch tests but the neutral of the transformer bank was dead grounded. The Petersen coil can only be charged with the three insulator flashovers which caused a total outage of 229 minutes.

W. W. Eberhardt: Mr. Evans has obviously made a misinterpretation of Tables III and IV of our paper in stating that "The Petersen coil installation increased interruptions in the remainder of the system apparently due to excess voltage causing bus flashover and transformer trouble." As pointed out in the explanation of the 1922 interruptions, the 343 minute interruption charged to transformers occurred during a short-circuit test when the Petersen coil was out of service; the interruptions charged to conductors were caused by trees falling on the line; and the interruptions charged to oil switches were caused by mechanical trouble in the switches. These interruptions were obviously, therefore, not caused by any action of the Petersen coil. The only interruptions strictly chargeable to the Petersen coil are the bus insulator flashovers of 79 and 62 minutes in Table IV. Eliminating the cases of trouble not chargeable to the Petersen coil, the total for 1922 would be 9 cases of trouble with a total outage of 155 minutes compared with 45 interruptions in 1921 with a total outage of 304 minutes.

In order to make the comparison between Tables III and IV complete, we wish to present the following tabulation of lightning storms occurring over the Lock 12-Vida line in the years 1921 and 1922.

	1921	1922
January	0	3
February	7	7
March	8	12
April	2	8
May	6	17
June	11	16
July	18	16
August	19	10
September	11	8
Total	82	97

From this tabulation it is seen that during the period when the Petersen coil was in service the lightning storms were more numerous than before the installation, thereby making the comparison even more favorable for the Petersen coil.

Mr. Trueblood in his discussion has pointed out that the Petersen coil has been operated with approximately 23 per cent excess reactance. The object in operating above the resonant point is to avoid the continuous flow of the neutral current of 6 amperes at the point of resonance. As pointed out by Mr. Trueblood, the same results could be obtained by operating below the resonant point, with the added advantage of perhaps eliminating the excessive voltages obtained in the past. Experiments will probably be carried out along this line.

Mr. Blume's suggestion of improving the operaiton of the Petersen coil by the introduction of more resistance, thereby, preventing danger due to serious overvoltage and resonance is well taken. As pointed out in our paper, our experiments with the Petersen coil will be continued along the lines suggested, the results of which we hope to have available at some future date.

Frequency Conversion by Third Class Conductor
and
Mechanism of the Arcing Ground and other Cumulative Surges

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Review of the Subject.—*In high-voltage power circuits, such as transmission lines and the high-voltage coils of large power transformers, not infrequently disturbances are observed of a frequency differing from, and usually very much higher than that of the power supply, and differing from the typical transient of energy readjustment, in that they do not gradually die out, but increase in intensity until either destruction occurs, or they finally limit themselves. Such cumulative oscillations or arcing grounds derive their energy from the machine power of the system, and so constitute a frequency transformation, of which the mechanism has been little understood.*

Physically they may be derived from the typical condenser discharge by the conception of a ***negative resistance,*** *in combination with a* ***source of power,*** *which supplies the energy given out by the negative resistance.*

Attention is drawn to a class of conductors—to which arcs and gas discharges belong—the so-called "third-class conductors," in which the voltage decreases with increase of current, and it is shown that these conductors can be considered as a combination of a negative resistance with a source of power, and as such are capable of transforming the low machine frequency into a high oscillation frequency of alternating currents, and their presence in an electric system thereby may produce cumulative oscillations.

The general equations are then derived of a system comprising a third-class conductor shunted by an inductive circuit containing capacity, and supplied with voltage over an inductive circuit from an alternating low-frequency source, and it is shown that in such a system currents and voltages of two distinct frequencies may continuously exist, of which the one is the machine frequency, the other a high oscillation frequency. It is further shown that the voltage of the latter is limited only by the resistance of the oscillating circuit, and in low-resistance circuits may build up to very high values. Furthermore, the high oscillation frequency is essentially limited to the circuit shunting the third-class conductor and but little of it enters the supply circuit, while the supply frequency enters the shunt circuit to a limited extent only, and both frequencies are superimposed in the third-class conductor as the frequency converter.

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A. Physical Explanation

I. THE THIRD CLASS CONDUCTOR

In a first class or metallic conductor, the voltage increases slightly more than proportional to the current, due to a positive temperature coefficient of resistance.

In a second class or electrolytic conductor, the voltage increases slightly less than proportional to the current, due to a slightly negative temperature coefficient of resistance.

As third class conductor may be defined a conductor in which, at least within a certain range of current, the voltage *decreases* with *increase* of current. Third class conductors comprise different types, such as

(a) Electronic or vacuum, gas or Geissler tube (spark) and vapor or arc conduction. In these the decrease of voltage with increase of current is due to the change of the conducting path by the current.

(b) Most of the so-called insulators probably are third class conductors. In these the decrease of voltage with increase of current is a temperature effect. That is, the negative temperature coefficient of resistance is so large that—at least in a certain range—the increase of the conductor temperature with increasing current decreases the resistance more than the current increases.

In Fig. 1, *I* gives the volt-ampere characteristic of a metallic conductor, *II* of an electrolytic conductor, and *III* of an arc as third class conductor. Fig. 2, gives the volt-ampere characteristic of a pyroelectric conductor, a Nernst lamp glower, which in the range above 0.02×10^{-3} ampere is a third class conductor

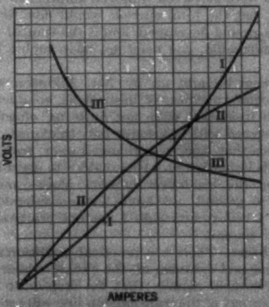

FIG. 1—VOLT-AMPERE CHARACTERISTICS OF DIFFERENT CONDUCTOR TYPES

1. First class or metallic conductor
2. Second class or electrolytic conductor
3. Arc as third class conductor

Presented at the Spring Convention, Pittsburgh, Pa., April 24-26, 1923.

(plotted for convenience with the fourth root of current and voltage as coordinates).

Such a volt-ampere characteristic of a third class conductor can be considered as a combination of a constant e. m. f. E_1 and an effective negative resistance r_1:

$$e = E_1 - r_1 i$$

The effective negative resistance r_1 decreases with increase of current, as $r_1 i$ must always remain smaller than E_1.

II. TIME LAG

If the current in the third class conductor varies periodically, between i_1 and i_2, the voltage also will vary periodically, between e_1 and e_2, and if the variation is slow enough so that at every value of current stationary condition is reached, the variation of voltage is inverse to that of current. That is, if i_1 is the minimum value of current, the corresponding value of voltage e_1 is the maximum value, and inversely. If however, the variation is sufficiently rapid, a lag of the voltage occurs behind the current, that is, the maximum value of voltage is not reached at the minimum value of current, but a little later, and so the minimum value of the voltage is reached a little later than the maximum value of current, and correspondingly, the amplitude of the variation is reduced, the more so, the more rapid the pulsation.

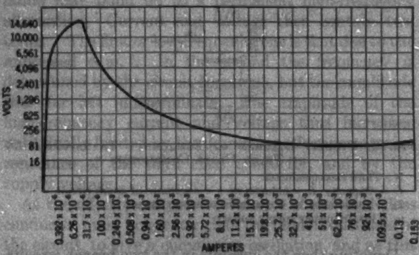

FIG. 2—VOLT-AMPERE CHARACTERISTICS OF NERNST LAMP GLOWER

As Third Class Conductor

This time lag of the third class conductor, and the reduction of amplitude of variation resulting from it, depends on the nature of the conductor. It is extremely small in ionic conduction, but may be very considerable where the variation of the effective resistance of the third class conductor is a temperature effect.

As first approximation, the periodic variations of current and voltage in a third class conductor thus may be expressed by:

$$i = I_0 - I \sin 2 \pi f t$$
$$e = E_0 + E \sin (2 \pi f t - \varphi)$$

where the lag angle φ increases with increasing frequency f of the pulsation. It is negligible for very low frequency, and becomes 90 deg., and the amplitude of the resistance variation therefore negligible, for very high frequency.

In ionic conduction, φ is still small at ratio frequencies; in the Nernst lamp glower (temperature effect) φ is already practically 90 deg. at 60 cycles.

The volt-ampere characteristic of the third class conductor under rapidly varying current (its "transient characteristic") thus differs more or less from its "permanent" characteristic as shown in Figs. 1 and 2.

III. THE CONDENSER DISCHARGE

If a condenser of capacity C discharges through an inductance L and (constant) ohmic resistance r, and the latter is below the critical value, the discharge current is oscillatory, that is, consists of successively alternating half waves of constant frequency and uniformly decreasing amplitude. The decrease of the current, or the "attenuation" is given by the exponential factor:

$$A = \epsilon^{-\frac{r}{2L} t}$$

If then the discharge circuit of the condenser contains, besides the ohmic resistance r, a third class conductor of the effective negative resistance r_1, the attenuation of the condenser discharge is:

$$A = \epsilon^{-\frac{r - r_1}{2L} t}$$

Thus, if the effective negative resistance r_1 is greater than the ohmic resistance r, the exponent

$$-\frac{r - r_1}{2L} t$$

is positive, and A increases with the time t, thus is not an attenuation but an accumulation. The amplitudes of successive half waves of the condenser discharge current then progressively increase, that is, we get a cumulative surge.

As seen in *I*, the effective resistance r_0 of the third class conductor decreases with increase of current. In such a cumulative surge produced by the presence of a third class conductor in the inductive condenser discharge circuit, the successive half waves of current will progressively increase, until by the increase of current the effective negative resistance r_1 has decreased to equality with the ohmic resistance r, and the exponent of the exponential term A has become zero, $A = 1$, and the successive half waves of the condenser discharge current become equal, that is, an alternating current results.

Thus the final result of a condenser discharge through a third class conductor of sufficiently high effective negative resistance is an alternating current of a frequency determined by the circuit constants.

IV. ENERGY RELATIONS

A current i through the effective negative resistance $- r_1$ consumes a voltage $- r_1 i$ which is in phase with the current, thus represents electric power generation, Hence an effective negative resistance can exist independently only in the presence of some other source of energy, supplying the power (mechanical drive for instance). Thus an electric generator may be considered as an effective negative resistance. The most typical negative resistance is the induction machine driven above synchronism, since (below saturation) it generates a voltage proportional to the current, thus is a constant negative resistance.

As seen in *I*, a third class conductor may be considered as the combination of a counter e. m. f. E_1, and an effective negative resistance $- r_1$: $e = E_1 - r_1 i$. As $r_1 i$ must always remain smaller than E_1, therefore the power generated by the effective negative resistance: $- r_1 i^2$ always is less than the power consumed by the counter e. m. f. E_1: $E_1 i$ and the counter e. m. f. E_1 thus abstracts from the electric circuit the power which is returned by the effective negative resistance r_1.

In the condenser discharge through a third class conductor, the current wave builds up cumulatively by the effective negative resistance r_1, but the condenser voltage is lowered discontinuously by the counter e. m. f. E_1, at every reversal of current, and with it the current wave is lowered, and as E_1 is greater than $r_1 i$, the discharge oscillation gradually dies out, (though by a different law than in the standard condenser discharge equation,) as discussed in a previous paper.[1]

If however the counter e. m. f. E_1 of the third class conductor is supplied by some outside source of electric power, and therefore does not abstract energy from the condenser discharge, then the cumulative effect of the negative resistance of the third class conductor r_1 on the condenser discharge current i would continue until limited by the decrease of r_1 with increasing i, and an alternation results.

Now the importance of this phenomenon is, that the character of the supply voltage giving the counter e. m. f. E_1 has no necessary relation to the frequency of the condenser discharge; the latter is determined by the values of the capacity and inductance, and may be a high or very high frequency, while E_1 may be of machine frequency, 25 or 60 cycles, or even continuous voltage. The power supplied by the effective negative resistance to the high-frequency condenser discharge is derived from the counter e. m. f. E_1 and the latter is fed from the low-frequency machine power. We therefore have here a frequency transformation giving a steady power supply to the high-frequency oscillation, so that the latter are not any more limited energy transients, but unlimited power permanents, of corresponding destructiveness. As such they have been frequently observed in electric power systems, as arcing grounds in transmission lines, and as high-frequency cumulative surges in the high-voltage coils of large power transformers.

As the exact mechanism of this frequency transformation from the low machine power frequency to the high frequency of the cumulative surge is not generally familiar, the following may be of interest, though a similar problem has been studied in a different manner in radio engineering in the theory of the vacuum tube as oscillator.

V. FREQUENCY

Suppose a condenser of capacity C discharges through an inductance L, an ohmic resistance r and a third class conductor N, and upon this third class conductor, a constant alternating voltage e_0 is impressed through a supply circuit of resistance r_0 and inductance L_1, as shown diagrammatically in Fig. 3. We can consider the arrangement as a divided circuit consisting of a third class conductor N in shunt to the circuit C, L, r, and energized over the circuit L_0, r_0 by voltage e_0.

We can assume that r is sufficiently small to make the condenser discharge oscillatory, and that the frequency f of this oscillation is high compared with the frequency f_0 of the supply voltage e_0.

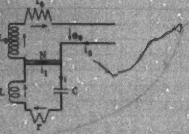

FIG. 3—CIRCUIT DIAGRAM OF FREQUENCY TRANSFORMATION BY THIRD CLASS CONDUCTOR

N. Third class conductor
C = Capacity, L = inductance, r = resistance of shunt circuit
L_0 = Inductance, r_0 = resistance of supply circuit
e_0 = low-frequency supply voltage

Denoting then the currents in the two branch circuits by i and i_1, the voltage by e_1, the differential equations can be written as those of a circuit containing resistance, inductance and capacity.

As seen therefore, if the effective negative resistance of the third class conductor is sufficiently high, the oscillatory starting transient of the condenser C does not die out, but cumulatively increases in amplitude, until limited by the decreasing negative resistance, and so reaches an alternating sine wave as final value, i, i_1 and e_1 then must consist of sine waves.

The differential equations are therefore integrated by representing i, i_1 and e_1 by summation of sine waves.

$$\Sigma B \sin (q t - \beta)$$

substituting these into the differential equations, and from the identities thus produced, calculate the constants B, β etc.

One of the terms must be of the supply frequency $q = 2 \pi f_0$, and the amplitude and the phase angle of its currents and voltages are given by the terminal condition, that the supply voltage is e_0.

This term merely gives the permanent low-frequency alternating current and voltage distribution in the divided circuit Fig. 3.

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[1] Condenser Discharge Through a Gas Circuit A. I. E. E., Feb. 1922.

For all other terms in the solution of the differential equation, the impressed e. m. f. equals zero. This condition gives an equation, which determines the frequency $q = 2\pi f$, of the term.

It follows therefore, that besides the low-frequency term of the supply frequency f_0, only one second term can exist, in the equations of current and voltage, of an oscillation frequency f of the condenser discharge circuit.

The frequency of local oscillation f, which from the viewpoint of frequency transformation may be called the "secondary frequency" depends on the constants of the circuit, and therefore has no numerical relation to the supply or "primary" frequency, is not a multiple or a higher harmonic of it, as in most other frequency transformations.

VI. AMPLITUDE

If an alternating supply voltage is impressed upon a third class conductor contained in an inductive condenser discharge circuit, the transient condenser oscillation may build up to a permanent high-frequency alternation. Current and voltage then contain two terms. The first or low-frequency one is that due to the impressed e. m. f. In the second the frequency is determined by the circuit constants, but the amplitude thus far left indeterminate. The amplitude of the high-frequency current and voltage thus increases, until finally limited by the decreasing effective negative resistance of the third class conductor, and by the supply voltage.

As the effective negative resistance of the third class conductor must be greater than the ohmic resistance of the circuit, to give a cumulative oscillation, and as this effective negative resistance decreases with increase of current, the limit, to which the high-frequency alternating current can build up, is that value, at which the effective negative resistance of the third class conductor had decreased to equality with the ohmic resistance of the circuit, as discussed in III.

As the only source of power is the low-frequency supply by the impressed voltage e_0, this must always be positive. That is, in the voltage e_1 and the current i_1 of the third class conductor, the high-frequency term —which absorbs power—if of opposite sign of the low-frequency term—which supplies the power—must always be less than the low-frequency term, to maintain positive power. This means, the maximum value of the high-frequency term must be less than the instantaneous value of the low-frequency term at the moment when the maximum value of the high-frequency term occurs. Therefore the amplitude of the high-frequency term is not constant, but periodically rises and falls, with the frequency of the low-frequency term and the latter thus is the envelope of the high-frequency term, as illustrated in Fig. 4.

The difference between this type of condenser discharge, and the standard form is that in the latter the envelope of the periodic discharge current is exponential, and the current thus gradually dies out, as transient, while in the present case, with power supply through a third class conductor, the envelope of the condenser discharge current is a low-frequency sine wave, and the discharge current thus periodically rises and falls, but persists as permanent.

Such a periodically rising and falling alternating wave can be considered as the superposition of two waves, differing in frequency from each other by the frequency of the beats, that is, in the present case, twice the frequency of the supply voltage; hence of the frequency $f - f_0$ and $f + f_0$. However, as at very low currents the third class conductor usually ceases to represent an effective negative resistance, the periodic high-frequency oscillation usually dies out in a short range at the reversal of the low-frequency wave, and then starts again at the rise of the latter, as indicated diagrammatically in Fig. 4, and the phases of the groups of the high-frequency waves in the successive low-frequency half waves therefore are independent of each other. The resolution of the high-frequency waves into two components of different frequency therefore has no physical significance.

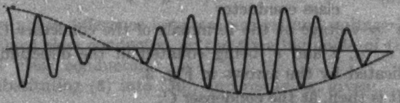

Fig. 4—High-Frequency Secondary and Low-Frequency Primary Wave of Frequency Transformation by Third Class Conductor

As the maximum value of the high-frequency wave is limited to the instantaneous value of the low-frequency wave, the mean square of the maximum value of the high-frequency wave is limited to the mean square of the instantaneous value, and therefore to half the square of the maximum value of the low-frequency wave. This means, that the power of the high-frequency alternation is limited to half the power of the low-frequency wave; in other words, under the conditions herein considered, the efficiency of the frequency transformation cannot exceed 50 per cent.

If the secondary frequency f is much higher than the primary frequency, very little low-frequency enters the supply circuit, since it is kept out by the high capacity reactance, at the low-frequency of the condenser C, and very little high-frequency current enters the supply circuit, since it is kept out by the high inductive reactance, at the high frequency, of L_0. Thus the capacity shunt is essentially a high-frequency circuit, the supply a low-frequency circuit, and both frequencies concur in the third class conductor, in which the transformation takes place.

B. Mathematical Calculation

I. General Equation

Let a third class conductor be shunted by a circuit of capacity C, inductance L, and ohmic resistance r and supplied by a constant alternating e. m. f. of machine frequency, over a resistance r_0 and inductance L_0.

We may assume that r is below the critical value, so that the current in C, L, r is oscillatory, and that the frequency of oscillation of this circuit is high compared with the machine frequency, and that L_0 is large compared with L.

Let the alternating supply e. m. f. be a sine wave:

$$e_0 = E_0 \sin (q_0 t - \alpha) \tag{1}$$

where:

$q_0 = 2 \pi f_0 t$,

and,

f_0 = frequency of power supply.

Let:

e_1 = voltage at the terminals of the third class conductor

i_1 = current in the third class conductor

i = current in the circuit C, L, r shunting the third class conductor

e' = voltage at the terminals of the condenser C.

e_0, e_1, i_1, i and e' being counted in the direction as indicated by the arrows in Fig. 3.

It is then, at the condenser C:

$$i = C \frac{d e'}{d t} \tag{2}$$

It is, in the closed circuit between the third class conductor and the capacity inductance shunt:

$$e_1 = e' + r i + L \frac{d i}{d t} \tag{3}$$

Hence, differentiating (3) and substituting (2) therein, gives:

$$\frac{d e_1}{d t} = i/C + r \frac{d i}{d t} + L \frac{d i^2}{d t^2} \tag{4}$$

It is, in the supply circuit:

$$e_0 = e_1 + r_0 (i_1 + i) + L_0 \frac{d (i_1 + i)}{d t} \tag{5}$$

and the voltage across the condenser terminal is, by (3):

$$e' = e_1 - r i - L \frac{d i}{d t}$$

Assuming that the circuits have been closed sufficiently long to reach stationary conditions, that is, that the starting transient has passed and any cumulative oscillation which may occur has built up to its final value.

e_1, i_1 and i then must be periodic functions, and as such may be represented by a sume of sine waves:

$$\left. \begin{aligned} e_1 &= \Sigma E_1 \sin (q t - \beta) \\ i_1 &= \Sigma I_1 \sin (q t - \gamma) \\ i &= \Sigma I \sin (q t - \omega) \end{aligned} \right\} \tag{7}$$

Substituting (7) into (3) then must give an identity, that is, the coefficient of $\cos q t$ and $\sin q t$ must individually disappear. This gives:

$$\left. \begin{aligned} E_1 \cos \beta &= I (r \cos \omega - R \sin \omega) \\ E_1 \sin \beta &= I (r \sin \omega + R \cos \omega) \end{aligned} \right\}$$

where

$$R = \frac{1}{q C} - q L \tag{8}$$

Herefrom follows:

$$E_1 = z I \tag{9}$$

and

$$\tan \omega = \frac{r \sin \beta - R \cos \beta}{r \cos \beta + R \sin \beta} \tag{10}$$

where:

$$z = \sqrt{R^2 + r^2} \tag{11}$$

Substituting (1) and (7) into (5) then gives:

$$E_0 \sin (q_0 t - \alpha) = \Sigma E_1 \sin (q t - \beta) + \Sigma z_0 \{I_1 \cos (q t - \gamma - \zeta) + I \cos (q t - \omega - \zeta) \} \tag{12}$$

where:

$$z_0 = \sqrt{q^2 L_0^2 + r_0^2} \tag{13}$$

$$\tan \zeta = \frac{r_0}{q L_0} \tag{14}$$

II. Low Frequency Terms

(12) also must be an identity. That is, the coefficients of $\cos q t$ and $\sin q t$ must individually vanish. This gives, for: $q = q_0$; denoting the terms by the index O:

$$\left. \begin{aligned} E_0 \sin \alpha &= E_1 \sin \beta_0 - z_0 \{I_1 \cos (\gamma_0 + \zeta_0) + I \cos (\omega_0 + \zeta_0) \} \\ E_0 \cos \alpha &= E_1 \cos \beta_0 + z_0 \{I_1 \sin (\gamma_0 + \zeta_0) + I \cos (\omega_0 + \zeta_0) \} \end{aligned} \right\} \tag{15}$$

Let:

$$r_1 = E_1/I_1 \tag{16}$$

= effective resistance of the third class conductor.

Substituting (16) and (9) into (15), gives two equations in I and α, which resolved give:

$$I = \frac{E_0}{\sqrt{\left[z_0 \left\{\frac{z_0}{r_1} \cos(\gamma_0 + \zeta_0) + \cos(\omega_0 + \zeta_0)\right\} - z_0 \sin \beta_0\right]^2 + \left[z_0 \left\{\frac{z_0}{r_1} \sin(\gamma_0 + \zeta_0) + \sin(\omega_0 + \zeta_0)\right\} + z_0 \cos \beta_0\right]^2}} \qquad (17)$$

$$\tan \alpha = \frac{z_0 \sin \beta_0 - z_0 \left\{\frac{z_0}{r_1} \cos(\gamma_0 + \zeta_0) + \cos(\omega_0 + \zeta_0)\right\}}{z_0 \cos \beta_0 + z_0 \left\{\frac{z_0}{r_1} \sin(\gamma_0 + \zeta_0) + \cos(\omega_0 + \zeta_0)\right\}} \qquad (18)$$

These expressions can be very much simplified by neglecting terms of secondary order, and choosing the phase of the current in the third class conductor as base line, that is, $\gamma_0 = 0$.

For the low frequency $q = q_0$, the time lag of the third class conductor can be neglected, giving $\beta_0 = \gamma_0 = 0$. By (10), this gives

$$\tan \omega_0 = -R_0/r \qquad (19)$$

Neglecting the resistance r_0 against the reactance of the supply circuit $x_0 = q_0 L_0$, gives by (14) $\zeta_0 = 0$, and by (13):

$$z_0 = q_0 L_0 = x_0 \qquad (20)$$

For the low-frequency $q = q_0$, $q_0 L$ is negligible against $\frac{1}{q_0 C}$, and, denoting the latter, the capacity reactance, by

$$k = \frac{1}{q_0 C} \qquad (21)$$

it is by (8), (11), (9) and (19):

$$\left.\begin{aligned} R_0 &= k \\ Z_0 &= k \\ E_1{}^0 &= k\,I \\ \tan \omega_0 &= -k/r \end{aligned}\right\} \qquad (22)$$

Thus approximately:

$$\omega_0 = -90 \text{ deg.}$$

and we now get from (17) and (18):

$$I^0 = \frac{E_0}{k\sqrt{\left(\frac{x_0}{r_1}\right)^2 + \left(1 - \frac{x_0}{k}\right)^2}} = \frac{E_0}{k\sqrt{\left(\frac{x_0}{r_1}\right)^2 + 1}} \qquad (23)$$

$$\tan \alpha = -\frac{x_0}{r_1}\left(1 + \frac{x_0}{k}\right) = -\frac{x_0}{r_1} \qquad (24)$$

and herefrom the approximate values of E^0, $I_1{}^0$, etc.

III. HIGH-FREQUENCY TERMS

In the identity (12), it is for:

$$q \neq q_0$$

$$\left.\begin{aligned} 0 &= E_1 \sin \beta - z_0 \left\{I_1 \cos(\gamma + \zeta) + I \cos(\omega + \zeta)\right\} \\ 0 &= E_1 \cos \beta + z_0 \left\{I_1 \sin(\gamma + \zeta) + I \sin(\omega + \zeta)\right\} \end{aligned}\right\} \qquad (25)$$

Substituting (9) into (25) and rearranging; gives:

$$\left.\begin{aligned} z_0 I_1 \cos(\gamma + \zeta) &= I \left\{z \sin \beta - z_0 \cos(\omega + \zeta)\right\} \\ z_0 I_1 \sin(\gamma + \zeta) &= -I \left\{z \cos \beta + z_0 \sin(\omega + \zeta)\right\} \end{aligned}\right\} \qquad (26)$$

Thus

$$\tan(\gamma + \zeta) = \frac{z_0 \sin(\omega + \zeta) + z \cos \beta}{z_0 \cos(\omega + \zeta) - z \sin \beta} \qquad (27)$$

If L_0 is large compared with L, then z is small compared with z_0, and can be neglected against it, and it thus is:

$$\tan(\gamma + \zeta) = \tan(\omega + \zeta) \qquad (28)$$

$$\gamma = \omega \qquad (29)$$

Substituting (29) into (10) and rearranging, gives:

$$R/r = \tan(\beta - \gamma) = \tan \delta \qquad (30)$$

where

$$\delta = \beta - \gamma \qquad (31)$$

is the phase difference between current and voltage in the third class conductor, or the time lag angle of the third class conductor.

Substituting (8) into (30), and resolving, gives:

$$q = -\frac{r}{2L} \tan \delta + \sqrt{\frac{1}{LC} + \frac{r^2}{4L^2} \tan^2 \delta} \qquad (32)$$

[If the angle of time lag of the third class conductor vanishes: $\delta = \beta - \gamma = 0$, that is, current and voltage are in phase with each other, then

$$q = \frac{1}{\sqrt{LC}}$$

the same is the case, if $r = 0$, that is, the ohmic resistance of the condenser inductance shunt is negligible.]

As seen, only two terms can exist in the equation (6) the one with q_0, of the machine frequency, due to the impressed voltage e_0, and the one with q, (32), the frequency of oscillation of the inductive condenser circuit.

From (30), (11) and (9) follows:

$$R = r \tan \delta$$

$$z = \frac{r}{\cos \delta} \qquad (33)$$

$$I = \frac{E_1 \cos \delta}{r}$$

and by substituting into (26), and neglecting r against z_0, gives:

$$I_1 = -I = -\frac{E_1 \cos \delta}{r} \qquad (34)$$

IV. Amplitude

Combining the low and high-frequency terms then gives the total expressions of current and voltage:

$$\left.\begin{aligned}
e_0 &= E_0 \sin (q_0 t + \alpha_0) \\
\tan \alpha_0 &= + \frac{x_0}{r_1} \\
e_1 &= \frac{E_0}{\sqrt{1 + \left(\frac{x_0}{r_1}\right)^2}} \sin q_0 t + E_1 \sin (q t - \delta) \\
q &= - \frac{r}{2 L} \tan \delta + \sqrt{\frac{1}{L C} + \frac{r^2}{4 L^2} \tan^2 \delta} \\
i_1 &= \frac{E_0}{\sqrt{r_1^2 + x_0^2}} \sin q_0 t - \frac{E_1 \cos \delta}{r} \sin q t \\
i &= \frac{E_0}{k \sqrt{1 + \left(\frac{x_0}{r_1}\right)^2}} \cos q_0 t + \frac{E_1 \cos \delta}{r} \sin q t \\
i_0 &= i_1 + i \\
&= \frac{E_0}{\sqrt{r_1^2 + x_0^2}} \{\sin q_0 t + r_1/k \cos q_0 t\}
\end{aligned}\right\} \tag{35}$$

where:

$$\left.\begin{aligned}
q_0 &= 2 \pi f_0 \\
x_0 &= q_0 L_0 \\
k &= \frac{1}{q_0 C}
\end{aligned}\right\} \tag{36}$$

r_1 = resistance of third class conductor.

Substituting into equation (6), gives the terminal voltage e' of the condenser C.

$$e' = \frac{E_0}{\sqrt{1 + \frac{x_0^2}{r_1^2}}} \sin q_0 t = E_1 \left(\sin \delta + \frac{q L}{r} \cos \delta \right) \cos q t \tag{37}$$

As seen, the high-frequency term of e' indefinitely increases with decreasing r. That is, the cumulative oscillation builds up to the resonance voltage of L and C, or until limited by the ohmic resistance.

Thus, where r is small, as in the high-voltage coils of power transformers, very high voltages may be produced.

As discussed in *A III* the second term in i_1 and e_1 must always be smaller than the first term. That is, it must be, in i_1:

$$\frac{E_1 \cos \delta}{r} < \frac{E_0}{\sqrt{r_1^2 + x_0^2}}$$

Thus we may substitute:

$$\frac{E_1 \cos \delta}{r} = p \frac{E_0}{\sqrt{r_1^2 + x_0^2}} \tag{38}$$

where:

$$p < 1 \tag{39}$$

Equations (35) thus become:

$$\left.\begin{aligned}
i_1 &= \frac{E_0}{\sqrt{r_1^2 + x_0^2}} \sin q_0 t \{1 - p \sin q t\} \\
&\text{and:} \\
e_1 &= \frac{E_0}{\sqrt{1 + \frac{x_0^2}{r_1^2}}} \sin q_0 t \left\{ 1 + p \frac{r}{r_1 \cos \delta} \sin (q t - \delta) \right\}
\end{aligned}\right\} \tag{40}$$

As the second term in e_1 must be less than the first, it must be:

$$\frac{p r}{r_1 \cos \delta} \leqq 1$$

or:

$$r = \leqq \frac{r_0 \cos \delta}{p}$$

or:

$$r_1 \cos \delta \geqq p r$$

Herefrom follows:

(1) With a given resistance r of the oscillating circuit, the oscillation increases, that is, current and voltage build up, until they reach the values, at which the transient effective negative resistance of the third class conductor, $r_1 \cos \delta$, has dropped down to equality with the resistance r, or rather slightly lower.

(2) When in the inductive condenser shunt to a third class conductor, the ohmic resistance r is increased, the cumulative high-frequency oscillation still occurs—though of an amplitude decreasing with increase of the resistance r—until the ohmic resistance becomes equal to the transient negative resistance of the third class conductor, $r_1 \cos \delta$, or rather slightly larger. Then the oscillation stops.

(3) The amplitude e' of the voltage oscillation produced by the third class conductor, is approximately inverse proportional to the resistance of the oscillating circuit. (As seen by equation (37)).

V. Instances

(1) Spark discharge on high-voltage coil of 60,000 volt 3000-kw. 60-cycle power transformer.

Let the constants be:

r = 6 ohms, or about 0.5 per cent resistance of the coil
L = 0.1 h, or about 3 per cent reactance
C = 0.0001 mf.
L_0 = 0.6 h, or about 20 per cent reactance of supply circuit
r_0 = 40 ohms, or about 3⅓ per cent resistance of supply circuit
f_0 = 60 cycles, thus $q_0 = 2 \pi f_0 = 377$
E_0 = $60{,}000 \sqrt{2} = 85{,}000$ volts.

Assuming $\delta = 60$ deg. as time lag at the oscillation frequency.

It is then, by equations (35) to (37):

$q = 316{,}000 = 837\, q_0$

$f = 50{,}300$ cycles

$x_0 = 226$ ohms

$q\,L = 31{,}600$ ohms

$k = 26{,}600{,}000$ ohms

Assuming the effective resistance of the oscillating circuit increased to 10 times the ohmic resistance, by the energy losses in the insulation etc. at the high frequency, that is:

$r = 60$ ohms.

and assuming stationary conditions reached by the transient effective resistance of the spark discharge having dropped to equality with the ohmic resistance, that is:

$$r_1 \cos \delta = 60, \text{ and } r_1 = 120 \text{ ohms}$$

this gives:

$\alpha_0 = 62$ deg.

and, denoting:

$\varphi = q_0\, t$, gives:

$e_0 = 85{,}000 \sin (\varphi + 62.)$

$e_1 = 47{,}000 \sin \varphi \,\{1 + \sin (837\, \varphi - 60.)\}$

$i_1 = 392 \sin \varphi \,\{1 - \sin 837\, \varphi\}$

$i = 392 \sin \varphi \sin 837\, \varphi$

$i_0 = 392 \sin \varphi$

$e' = 47{,}000 \sin \varphi \,\{1 - 263 \cos 837\, \varphi\}$

or a maximum value of the condenser voltage of over 12 million volts.

Obviously, no such voltage is reached, but the insulation destroyed before the cumulative oscillation is built up to full value.

(2) Arcing ground on 30 mile three phase transmission line supplied with 30,000 volts, 60 cycles from generator and step up transformer.

Let the constants be:

$r = 4$ ohms

$L = 12$ mh.

$C = 0.3\ \mu f$

$L_0 = 0.2$ h.

$r_0 = 6$ ohms

$f_0 = 60$ cycles, thus $q_0 = 377$

$E_0 = 30{,}000 \sqrt{2} = 42{,}500$ volts.

Assuming $\delta = 45$ deg. as time lag at the oscillation frequency.

It is by equations (35) to (37):

$q = 16{,}500 = 44\, q_0$

$f = 2640$ cycles

$x_0 = q_0\, L_0 = 75.4$ ohms

$q\,L = 198$ ohms

$$k = \frac{1}{q_0\, C} = 8800 \text{ ohms}$$

Assuming as effective resistance of the oscillating circuit at 2640 cycles, twice the ohmic value, or:

$r = 8$ ohms.

gives as limiting value of the transient effective negative resistance.

$$r_1 \cos \delta = 8, \text{ and } r_1 = 11.3 \text{ ohms}$$

This gives:

$\alpha_0 = 81.5$ deg.

and, denoting:

$\varphi = q_0\, t$; gives

$e_0 = 42{,}500 \sin (\varphi + 81.5 \text{ deg.})$

$e_1 = 6300 \sin \varphi \,\{1 + \sin (44\, \varphi - 45 \text{ deg.})\}$

$i_1 = 560 \sin \varphi \,\{1 - \sin 44\, \varphi\}$

$i = 560 \sin \varphi \sin 44\, \varphi$

$i_0 = 560 \sin \varphi$

$e' = 6300 \sin \varphi \,\{1 - 19.2 \cos 44\, \varphi\}$

or approximately:

$e' = 120{,}000 \sin \varphi \cos 44\, \varphi$

That is, the arcing ground in this case produces a continuous oscillation which builds up to nearly three times the normal line voltage.

It is interesting to note that the cumulative surge of the arcing ground in a transmission line does not reach such excessive values as that in the high-voltage coil of the power transformer, but may be of moderate intensity, of the magnitude of the normal line voltage.

Discussion

For discussion of this paper see page 490.

Voltages Induced by Arcing Grounds

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Description. (430 w.)
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THE subject of arcing grounds in transmission systems is one of the greatest interest to operators of power systems of any extent. The almost universal grounding of the neutral in this country is done primarily to alleviate the destructive effects produced by arcing grounds. However, in spite of its great importance, a clear understanding of what happens in an arcing ground in not general. There is no agreement as to the magnitudes of voltages and surges produced, and the various theories proposed call for different properties of the arc. The authors therefore considered it well worth while to attempt in the laboratory to duplicate the conditions of an arcing ground on a transmission system and by spark gap determinations of voltages and by oscillograms to determine the maximum voltages developed and to discriminate between the various theories proposed.

TRANSIENT CONDITIONS IN ELECTRICAL SYSTEMS

Any sudden change in the constants in an electrical system in general produces transient oscillations which superimpose themselves upon the normal voltage and current distributions in the system. A simple example may be given in the closing of a switch in a circuit containing inductance and capacity in series, such as is indicated in Fig. 1A. Fig. 1B shows the manner in which the condenser acquires the voltage of the power supply, the battery. The voltage on the condenser does not at once come to that of the battery but there is a transient oscillation about this voltage provided the resistance in the circuit is not too great. The initial amplitude of the oscillation will be equal to the change in voltage which the condenser must undergo in passing from its initial condition to the final steady condition. Therefore, in Fig. 1B the maximum voltage which the condenser momentarily takes on is approximately twice the voltage of the supply.

In respect to the effect upon the insulation of electrical apparatus two factors must be considered in these transient oscillations. One is the absolute value of the potential reached in the oscillation and the other is the rapidity or steepness with which the potential changes during the transient condition. The latter factor plays an important part in determining the distribution of potentials within the apparatus during the transient condition, whereas the former determines the stress upon the insulation to ground. In transmission systems rapidly changing or steep wave front surges are impressed upon the apparatus as a result of the distributed character of the inductance and capacity as will be explained in more detail later in the paper. The type of oscillation considered in the first part of the paper, which will deal entirely with lumped inductances and capacitances, will involve changes of potential which are so slow that practically only the absolute value to ground will be important.

It is obvious that an arc when it starts constitutes just such a sudden change in the constants of a circuit as has just been described so that when it strikes, an arcing ground must necessarily set up oscillations similar to those set up by closing a switch. However, there is a concensus of opinion that an arcing ground has in it more elements of danger than would correspond merely to this effect. It is generally believed that short-circuiting a line permanently to ground will not produce nearly the same disastrous results as arcing grounds are known to do.

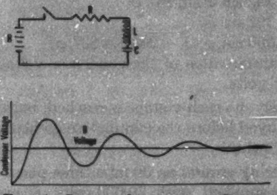

FIG. 1—SIMPLE OSCILLATING CIRCUIT

THEORIES OF THE ARCING GROUND

Three theories to explain the high voltages induced by arcing grounds have been proposed. To explain these three theories in a similar manner the authors prefer to describe the action going on in an arc by an artifice of properly manipulated switches. The resistance of an arc is not a constant quantity but may vary from a practically infinite value when the current through it is very small, to an almost zero value when the current

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through it is very large. Thus when alternating or pulsating currents traverse the arc the resistance of the arc may undergo changes which may keep pace with these pulsations or alternations of current. A switch also may be made to vary the resistance in a circuit from very high to very low values. The three theories may be classified according to how the change of resistance of the arc from low to high values or vice versa or the equivalent switching is controlled.

THEORY I. SWITCHING CONTROLLED BY THE HIGH-FREQUENCY OSCILLATION

This is usually known as the theory of generation of high-frequency by the "negative resistance" of the arc. The Poulsen arc generator of radio frequency currents operates on this principle. The manner in which oscillations are built up when a switch is operated synchronously with the natural period of an oscillating circuit which it shunts may be seen from Fig. 2. Fig. 2A shows the circuit diagrammatically and Fig. 2B shows the building up of voltage on the condenser as the switch is operated. Beginning with the switch open, the condenser *C* is charged to the voltage of battery *B* with zero current flowing through the inductance *L* and resistance *R*. On closing the switch the condenser begins to discharge through the inductance giving an oscillation, such as has just been described, about the zero voltage value to which the condenser would ultimately come if the switch remained closed. However, suppose at the end of a half cycle of this oscillation at the moment the current from the condenser is zero, the switch is opened. At this moment the condenser has a negative voltage equal in magnitude to the positive voltage which was first impressed upon it. An oscillation then begins in the circuit consisting of the condenser, inductance, resistance and battery. At the start of this oscillation the voltage of the condenser will add to the voltage of the battery in building up current through the inductance *L* and the resistance *R*. If the switch were left permanently open after the transient oscillation, the condenser would again come to have the voltage of the battery *B*. The amplitude of the oscillation will therefore, as Fig. 2B shows, be equal to twice the battery voltage so that the maximum potential which the condenser attains will be after the first half cycle of this oscillation equal to three times the battery voltage.

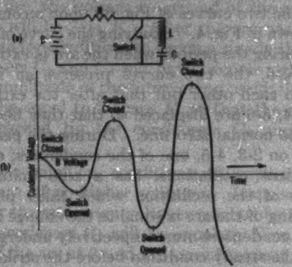

FIG. 2—GENERATION OF OSCILLATIONS BY SYNCHRONOUS SWITCH

Now suppose at this moment the switch is again closed. The condenser, charged to three times battery voltage, again undergoes an oscillation and after a half cycle has a negative voltage three times that of the battery. We may again suppose the switch opened.

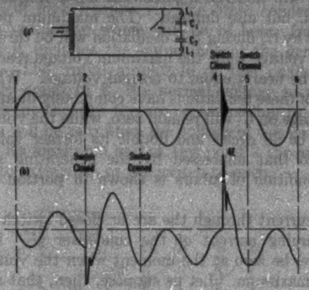

FIG. 3—SWITCHING CONTROLLED BY 60-CYCLE CONDITIONS IN SINGLE-PHASE CIRCUIT

The oscillation which then takes place in a half cycle will bring the condenser to a positive voltage of five times that of the battery. In this way the voltage on the condenser may build up to indefinitely large values.

Of course the resistance of an actual arc may not range from zero to infinite values. A closer approximation to the action of an arc building up oscillations, due to its "negative resistance" characteristic may be obtained by using one or more switches shunted by and in series with properly chosen resistances.

To apply this theory to an arcing ground on a transmission system, Fig. 3A will more nearly represent actual conditions. The oscillating circuit will then be the inductance and the capacitance of the grounded phase, L_1 and C_1, or the inductance of the generating system and the inductances and capacitances, L_2 and C_2, of the ungrounded phases, or both.

THEORY II. SWITCHING CONTROLLED BY 60-CYCLE CONDITIONS

Again using Fig. 3 we may follow the oscillations and the resultant development of potentials which ensue when the changes in the resistance of the arc or the opening and closing of the equivalent switch are controlled entirely by 60-cycle conditions. Let us suppose that the striking of the arc or closing of the switch takes at the moment when the 60-cycle voltage across it is a maximum and that the extinction of the arc or opening of the switch takes place when the 60-cycle current through it is passing through zero. Assume that the two capacitances in Fig. 3A are equal or that a single-phase transmission line is being considered. Just before the first closing of the switch the voltages on the condensers will be as shown in portion 1-2, in Fig. 3B. Now, when the arc strikes or the switch closes at 2, L_1 C_1 undergoes a rapidly damped oscillation, damped because we are supposing that the resistance of the arc does not follow or synchronize with this high-frequency oscillation. C_2 L_2, together with the inductance of the supply system, will undergo a somewhat lower frequency oscillation, but also damped. The maximum potential reached by C_2 during this oscillation will be three times normal voltage while the maximum voltage reached by C_1 will be nearly equal to normal voltage. When the effects of these oscillations have completely disappeared the voltage on C_1 will remain zero, while the voltage on C_2 will be 60 cycles and double its former value and equal to that impressed by the generating system. This condition of affairs is shown in portion 2-3, in Fig. 3B.

The current through the arc or closed switch is now the charging current of the condenser C_2. It will, therefore, be zero at the moment when the voltage on C_2 is a maximum. Let us suppose, then, that at such a moment the arc extinguishes or that the switch is opened. Opening the switch at this point evidently leaves a charge trapped on the condensers. From this point on the variation in supply voltage will again divide equally between the condensers. The course of of these voltages will then be as shown in portion 3-4, Fig. 3B; the charge which is retained upon the condensers causes their respective voltage waves to be displaced from the former zero line by an amount equal to their normal voltage.

If the leakage of the condensers is small this condition will persist until the arc strikes or the switch closes for its second time. Again we assume that this closing of the switch takes place at the maximum of the voltage wave. Immediately after the striking of the arc transient oscillations take place, as shown at 4, in Fig. 3B, just as took place at 2. However, it is obvious that now the amplitude of oscillation of C_2 will be twice as great as that at 2 so that the maximum voltage reached will be four times normal voltage. The maximum voltage reached by C_1 will be twice normal voltage. Now if we suppose the switch stays closed so that these oscillations completely damp out we will again be back in the condition shown in portion 2-3. Again, during the period 4-5, the voltage on C_1 will remain zero and that on C_2 will be 60-cycle and of double normal value. Again the arc will extinguish or the switch will open when the 60-cycle current through it is zero, which will be at a maximum of the voltage wave of C_2, as shown at 5. When the switch does open we will then be exactly in the condition as shown at 3 so that should the arc strike again an oscillation will again take place exactly as shown at 4 with the development again of voltage four times normal on C_2 and twice normal on C_1. Thus, however often the arc may restrike and re-extinguish, conditions will not become any worse than that shown at 4.

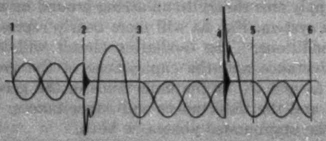

FIG. 4—CURVES OF FIG. 3 REFERRED TO COMMON ZERO

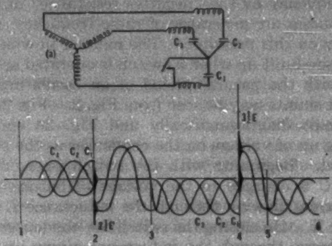

FIG. 5—SWITCHING CONTROLLED BY 60-CYCLE CONDITIONS IN THREE-PHASE CIRCUIT

To follow conditions more clearly in the three-phase case, which will now be considered, it is desirable to combine the two curves of Fig. 3B upon a common zero line as shown in Fig. 4. Studying the curve of Fig. 4 we see that during the periods when the arc is extinguished, 1-2, 3-4, 5-6, the two curves preserve an invariable relation to each other but that after the extinction of the arc 3-4, 5-6 are displaced so that they become tangent to the normal zero line. During the periods that the arc is on 2-3, 4-5, one of the condensers, except for the transient oscillations, has a zero voltage. The amplitude of the oscillation which takes place upon each striking of the arc is equal to the change in voltage which the condensers must respectively undergo in passing from the steady condition before the striking of the arc to the steady condition after the striking of the arc.

Being guided by Fig. 4, we may now readily draw the course of voltages under this theory for the case of a three-phase transmission system. Fig. 5A is the diagram of the three-phase circuit and 5B shows the voltages on the condensers. Before the striking of the arc in the portion 1-2 of Fig. 5B the three voltage waves are in three-phase relation and symmetrical about the zero line. After the first striking of the arc and after the initial oscillations have died down, just as in Fig. 4, one of the voltages will be reduced to zero while the other two will take delta voltage symmetrical about the zero line. This is shown in portion 2-3. When the arc extinguishes the three voltage waves will will have the same position relative to each other as they had in the portion 1-2 but will now be displaced so that they are tangent to the zero line. After the arc extinction again at 4 and after the oscillations have damped out one of the voltages will again be zero and the other two will be delta voltages about the zero line. After the extinction of the arc at 5 the voltages will be relative to each other again as in portion 1-2, but again will be displaced so as to be tangent to the zero line as in 3-4. The steady conditions of voltage before and after the striking of the arc being known, the amplitudes of the oscillations are readily determined. The amplitude will, of course, be equal to the amount by which the voltage on any one condenser must change to get from the value it had in the steady state it was leaving to the value it will have in the steady state it is reaching.

At the moment 2 in Fig. 5, condensers C_2 and C_3, just before the striking of the arc, have the voltage ½ *E*, where *E* is the normal voltage to neutral. After the arc has been on long enough for steady conditions to be reached the voltage will be 1½ times normal voltage to neutral. The amplitude of the oscillation which takes place in going from the one condition to the other will therefore be equal to the difference of 1½ *E* and ½ *E*, or equal to *E*. Hence, it is obvious that the maximum of voltage reached by the condensers C_2 and C_3 will be 2½ times *E*.

Upon the second striking of the arc at 4, conditions are different because of the fact that the voltage waves are displaced in the portion 3-4. Here, before the striking of the arc, the voltage to ground of the condensers C_2 and C_3 will be −½ *E*. After the arc has been on long enough for conditions to become steady the voltage of condensers C_2 and C_3 will have become +1½ times *E*. The amplitude of the intervening oscillation, therefore, will be equal to 2 times *E*, so that the maximum voltage reached by the condensers C_2 and C_3 will be 3½ *E*.

From this point on, conditions will repeat with the development of no higher voltages. Inspection of 5B shows that the maximum voltages reached by the short-circuited condenser C_1 will be equal to *E* on the first striking of the arc at 2, and 2 *E* on subsequent strikings, as at 4.

THEORY III. SWITCH CLOSING CONTROLLED BY 60 CYCLES AND OPENING BY HIGH FREQUENCY

This is a theory which has been advanced and elaborated by Petersen[1] in recent years. This theory assumes that the arc strikes at or near the maximum of the 60-cycle voltage wave but that the arc extinguishes at a moment when the arc current passes through zero as a result of that high-frequency oscillation which is of intermediate frequency.

Referring to Fig. 6, we may follow the development of voltages consequent upon this hypothesis. Again considering the equivalent of a single-phase line with equal condensers C_1 and C_2, we have before the striking of the arc equal voltages symmetrical about the zero line in the portion 1-2 of Fig. 6B. The arc is then supposed to strike at 2 when the voltages are maximum. The short-circuited condenser undergoes a damped oscillation of high frequency. The other condenser begins an oscillation which will be of lower frequency because it includes the inductance of the generating system. The initial amplitude of this oscillation is evidently equal to *E*, the normal voltage to neutral. The current through the arc corresponding to this oscillation will be zero when the voltage of this oscillation reaches its maximum and at this point 3 the arc is assumed to extinguish or the switch to open.

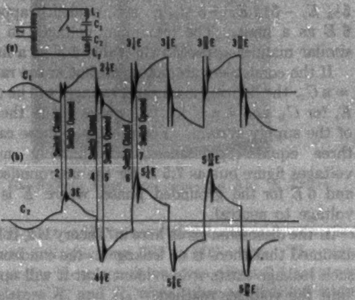

FIG. 6—SWITCHING CONTROLLED BY HIGH-FREQUENCY CONDITIONS

The voltage on condenser C_2 at this moment is 3 times *E*, whereas that on condenser C_1 is zero. The supply voltage, however, can only maintain a difference of potential of 2 *E* between the condensers C_1 and C_2. There is, therefore, an equalization of charge between the condensers C_1 and C_2 until the voltages on C_1 and C_2 come to differ by 2*E*. Thus there is at 3 a charge equalizing oscillation and after this is damped out the voltages of C_1 and C_2 are, respectively, +½ *E* and +2½ *E*. The arc being extinguished, the course of voltage change of the two condensers C_1 and C_2 is as it was before in portion 1-2 except that they are displaced from the zero line.

We now suppose that the next striking of the arc or closing of the switch takes place when condenser C_1 reaches its maximum voltage to ground. This will be one-half cycle later at 4 in Fig. 6B when the voltage of C_1 will be +2½ *E* and that of C_2, +½ *E*. The arc strikes; condenser C_1 loses its charge completely in a rapidly damped high-frequency oscillation and condenser

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[1] E. T. Z. 38 Heft 47, Nov. 22, 1917, pp. 553-555.
E. T. Z. 38 Heft 48, Nov. 29, 1917, pp. 564-566.

C_2 begins a lower frequency oscillation. The steady state value of voltage to which condenser C_2 strives at this point is $-2\,E$. The voltage which it has at the moment it starts the oscillation is $+\frac{1}{2}\,E$. The amplitude of the oscillation is the difference between these two values, or $2\frac{1}{2}\,E$. The maximum of voltage which it reaches, therefore, will evidently be $4\frac{1}{2}\,E$. At the moment it reaches this maximum voltage by the hypothesis as to the nature of the arc, the arc extinguishes or the equivalent switch is opened. Again there is an oscillation which equalizes the charge upon the two condensers. Since one condenser must rise in voltage through this equalizing oscillation by the same amount that the other condenser falls in voltage and since 60-cycle equilibrium will be obtained when the two condensers differ in voltage by $2\,E$, it is evident that after the equalization of charge the voltages of C_1 and C_2 will be respectively $-1\frac{1}{4}\,E$ and $-3\frac{1}{4}\,E$.

We may repeat this process at succeeding half cycles. The series of maximum voltages reached by C_2 will be $5\frac{1}{4}\,E$, $-5\frac{5}{8}\,E$, $+5\frac{13}{16}\,E$, etc., evidently approaching $6\,E$ as a limit. The maximum voltages on C_1 in a similar manner approach the value $4\,E$ as a limit.

If the condensers are not equal but in the ratio n, $C_2 = n\,C_1$, the limiting maximum voltages will be $(4n + 2)\,E$, for C_2, and $4\,n\,E$ for C_1, where $2\,E$ is the voltage of the supply system. For the three-phase case, with three equal capacitances, the limiting maximum voltages figure out as $7.5\,E$ for the ungrounded phases and $6\,E$ for the grounded phase, where E is normal voltage to neutral.

In the discussion given here in Theory III, it has been assumed that there is no leakage in the condensers. If such leakage exists it is evident that it will operate to limit the voltages reached.

PROPERTIES OF ARC REQUIRED BY THE THREE THEORIES

The three theories discussed are all similar in depending upon the property of an arc of undergoing large changes in resistance under various circumstances. The theories differ in their hypothesis as to what these circumstances are. It seems desirable, then, to examine the three theories critically in the light of what is generally known of the properties of arcs in the air.

The familiar volt-ampere characteristic of an arc is given in Fig. 7. Interpreting in terms of resistance the figure states that the resistance of the arc is large for small values of the current and small for large values of the current. This seems to be exactly the characteristic required by Theory 1, for in the discussion of Theory 1 the open switch or high resistance was introduced in the circuit at the time the current from the condenser was opposing the power current through the switch, whereas the switch was closed or low resistance introduced when the current from the condenser was adding to the power current through the switch. However, the curve of Fig. 7 is the relation obtained between volts and amperes of an arc when the current through it varies only slowly. If the current in the arc is varied rapidly quite different characteristics may be obtained. As an illustration of the change in the characteristic of an arc when the current is varied rapidly we may refer to Fig. 8, taken from "Fleming Electric Wave Telegraphy & Telephony," 3d edition, 1916. The striking contrast between Fig. 8B and Fig. 7 indicates that it is not permissible to assume that high frequency can be sustained by an arc because it has a static characteristic such as Fig. 7. In fact, in order to sustain high frequency by means of arcs it has been found necessary, as in the Poulsen arc radio frequency generator, to adopt measures to prevent the altering of the static characteristic such as by the use of powerful magnetic fields and of special gases. Fig. 8A, for example, shows a 60-cycle arc between carbon electrodes in coal gas and gives a characteristic more nearly like that of the static characteristic of Fig. 7. These considerations rather remove the possibility of Theory 1 of the development of high voltage by an arc.

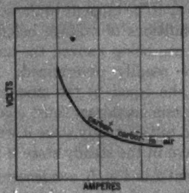

FIG. 7—STATIC CHARACTERISTIC OF ARC IN AIR

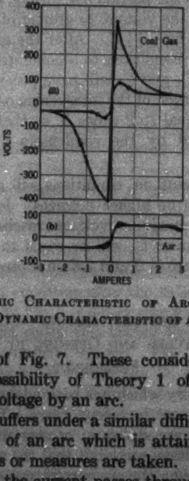

FIG. 8A—DYNAMIC CHARACTERISTIC OF ARC IN COAL GAS
FIG. 8B—DYNAMIC CHARACTERISTIC OF ARC IN AIR

Theory III suffers under a similar difficulty in requiring a property of an arc which is attained only when special methods or measures are taken. In Fig. 8B, for example, when the current passes through zero rapidly the voltage necessary to reignite the arc is only of the order of 40 or 50 volts, so that it is hardly likely that an arc in air will extinguish when a current of the order of a few amperes is passing through zero at a rate corresponding to a thousand cycles or more. The second hypothesis, therefore, of Theory III, which requires the arc to extinguish on the first passing through zero does not seem to be well founded on the actual properties of arcs.

Theory II would seem to have the best substantiation in the known properties of arcs. The arc, if it strikes at all, will of course be most likely to strike at the peak of a voltage wave. Again the arc, if it goes out at all, will go out as the current is passing through zero and the current will be passing through zero with the least rapidity and therefore with the closest approach to the static characteristic when it is the 60-cycle or lowest frequency current.

EXPERIMENTAL WORK

Apparatus: A photograph of the assembled apparatus used in these experiments is shown in Fig. 9. The transformer used as a supply was 250 kv-a., single-phase, 2200-13,200 volt, 60-cycle. A 228-millihenry air cored reactance was arranged in the primary side of the transformer so that it could be short-circuited or left in, as desired. Various air cored reactors were used in the high-voltage side and values are tabulated in the experimental results. The condensers consisted of four units each of 0.2 microfarads and were capable of withstanding 100,000 volts. They were made up of fuller-board in oil as the dielectric with sheet iron for plates.

FIG. 9—SET-UP OF TEST EQUIPMENT

Condenser bushings were also used where smaller values of capacitance were desired. The spark gaps for measuring the voltage developed were 12½ cm. sphere gaps. The gap used for producing the arc is shown in Fig. 10.

The lower electrode was provided with a screw to change the length of the gap and could be operated from a distance by means of a string. The upper electrode was on a swinging arm which could be moved by means of an insulating rod. A pipe connecting with the compressed air supply was brought up bearing on the gap so that the arc could be subjected to an air blast at will.

In order to take oscillograms without disturbing the voltage conditions it was necessary to adopt special measures to reduce the current required by the oscillograph to a negligible value. This was done by the use of three electrode vacuum tubes. A diagram of connections is shown in Fig. 11. The tubes were experimental 50-watt radio transmitting tubes and were used with plate voltages from 750 to 900 volts. The plate current under these conditions was about 0.1 amperes. The resistors R_1 and R_2 were tubes of distilled water which were frequently measured and varied from 4.5 to 8.8 megohms. The resistors r_1 and r_2 were wire wound resistors of relatively low values. No effort was made to eliminate slow time changes in the values of the water resistances.

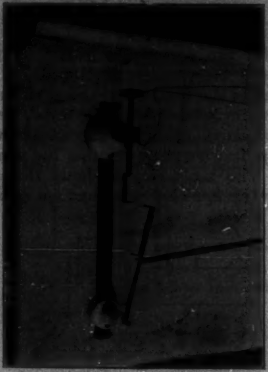
FIG. 10—SWITCH USED FOR STRIKING ARC IN TESTS

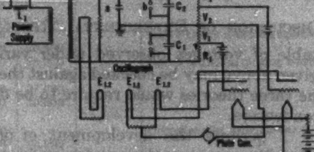

FIG. 11—CONNECTION DIAGRAM OF OSCILLOGRAPHIC TESTS

EXPERIMENTS TO DETERMINE THE INFLUENCE OF THE METAL IN ARC ELECTRODES

First experiments were undertaken to determine whether the nature of the metal making up the elec-

TABLE I

	Iron and alum.	Copper and iron	Copper and copper	Copper and alum.	Brass and brass	Brass and iron	Remarks
Volts at *b*..............	10,000	10,300	9500	8500	9400	11,000	Without air blast
Volts at *c*..............	23,400	24,200	23,000	22,000	..	..	Without air blast
Volts at *b*..............	11,500	11,500	11,000	11,000	..	..	With air blast
Volts at *c*..............	23,600	24,500	24,900	24,700	..	..	With air blast
With C_1 of Circuit above short-circuited							
Volts at *b*..............	22,500	23,200	21,200	22,500	..	..	Without air blast
Volts at *b*..............	28,800	29,500	33,000	30,500	..	..	With air blast

During the above tests voltages were measured as follows:

E_{sec} with voltage transformer = 12,960
E_b with gap *b* = 7700
E_c with gap *c* = 7050

trodes of the arc played any particular part in the magnitudes of the voltages developed.

Fig. 12 gives the diagram of the circuit and Table I gives the results obtained on these tests. In these tests the upper electrode of the arc gap was kept stationary and the gap drawn out by rotating the threaded lower electrode. The table clearly shows that the metals making up the arc electrodes had no great influence upon the voltages developed and therefor in the subsequent tests only copper electrodes were used.

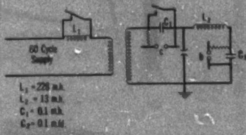

FIG. 12—CONNECTIONS USED IN TESTING EFFECT OF DIFFERENT ELECTRODE MATERIALS

EXPERIMENTS TO DETERMINE VOLTAGES DEVELOPED IN ARCING GROUND

The circuits used are shown in Fig. 13 and the results are given in Table II. During the progress of this work various oscillograms were taken of which characteristic ones will be given and described in the discussion of results following.

DISCUSSION OF EXPERIMENTAL RESULTS

The table of voltages observed under various conditions given above may be checked against the voltages which the three theories would require to be developed on the condensers.

Theory I permits the development of practically unlimited voltages upon the condenser in the oscillating circuits. Theory II, however, if the condensers are equal, limits the maximum voltage which can be obtained on the short-circuited condenser to twice its normal voltage and the maximum voltage which can be obtained on the other condenser to four times its normal voltage. When the short-circuited condenser is much smaller than the other, Theory II limits the voltage which can be developed in it to twice its normal voltage, or, that is, twice the voltage of the supply and the voltage of the other condenser to three times the voltage of the supply. Theory III, for the case of equal condensers, will permit the development of four times normal voltage on the short-circuited condenser and six times normal voltage on the other condenser. If the short-circuited condenser is small compared to the other, then Theory III permits of practically unlimited voltages.

Examining Table II we find that the results are entirely in agreement with the predictions of Theory II.

In the case of equal condensers, as shown in tests 1-20 and 76-90, inclusive, with the exception of the first four tests, no voltages exceeding twice normal or 13,000 were observed on the short-circuited condenser; although voltages were observed exceeding this value, all the voltages were nearly equal to this value. For the voltage across the other condenser in no tests was more than three times normal voltage observed and in many cases this value three times voltage was nearly reached. The falling short of the maximum values predicted by the theory is to be expected because of the leakage from the condensers during the relative long time between successive drawings of the arc.

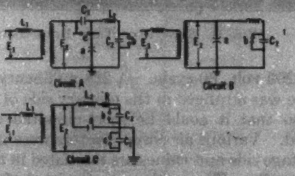

FIG. 13—CONNECTIONS USED IN ARCING GROUND TESTS

Unequal condensers were used in tests 30-44 and 51-63. The unlimited voltages called for by Theories I and III were not observed at all and even the maximum voltage of three times the supply voltage required by Theory II was not observed. However, more than twice supply voltage was usually observed and the smallness of the condenser with the attended rapid leakage of charge between strikings of arc readily explains the inability to obtain higher voltages.

TABLE II

Test	Cir-cuit	C_1 μ f	C_2 μ f	L_2 mh.	L_1 mh.	E_1 volts	E_2 volts	E_3 volts	E_c volts	Kind of arc	*b* in volts	*c* in volts	Film No.	Remarks
1	*A*	0.1	0.1	13	228	..	12,960	6500	6400		16,000	22,500	..	The min. length of gap *a* = 1/8″
2	"	"	"	"	"	..	"	"	"	Air	19,000	26,500	..	" " " " " " " "
3	"	"	"	"	"	..	"	"	"		14,800	25,500	..	" " " " " " " 3/16″
4	"	"	"	"	"	..	"	"	"	Air	15,000	25,000	..	" " " " " " " "
5	"	"	"	"	0	..	..	..	..		11,600	22,800	..	No *R* across C_1 & C_2 for making oscillograms
6	"	"	"	"	"	..	..	..	..		11,300	21,000	..	With *R* across C_1 & C_2 (see sketch for oscillograms)
7	*C*	"	"	"	228	2300	..	6800	7100		11,800	24,200	..	No *R* across C_1 & C_2 (checking change in circuit for taking oscillograms)
8	"	"	"	"	0	..	..	..	..		11,500	20,000	..	
9	"	"	"	"	"	..	..	..	..		11,400	21,000	72,579-a, b, c & d	
10	"	"	"	"	"	..	..	..	..		..	..	72,579-e	
11	"	"	"	"	"	..	..	..	..	Air	..	..	72,579-f	
12	*A*	"	"	"	"	..	13,130	..	..		12,500	..	..	
13	"	"	"	"	"	..	"	..	..	Air	12,000	21,000	..	
14	"	"	"	"	"	..	"	..	..	Air	..	19,600	..	
15	*C*	"	"	"	"	..	..	..	..	Air	..	..	72,579-g	
16	"	"	"	"	"	..	..	..	..	Air	..	..	72,579-h	Thin pointed electrodes at *a*
17	"	"	"	"	"	..	..	..	..	Air	..	..	72,579-i	One tube out. Object to check regulation of plate generator.
18	..	..	..	..	..	..	..	..	..	Open circuit voltage of transformer			72,579-j	Object: to find cause of h. f. ripple in wave supply voltage.
19	*C*	0.1	0.1	13	0	..	13,440	5850	5600		10,400	19,800	..	To check conditions before making oscillograms.
20	"	"	"	"	"	..	..	..	..	Air	10,500	19,100	..	To check conditions before making oscillograms.
21	*B*	..	"	"	228	..	14,065	..	..		21,000	..	..	The min. length of gap *a* = 1/16″
22	"	..	"	"	"	..	"	..	..	Air	23,500	..	..	" " " " " " " "
23	"	..	"	"	"	..	"	..	..		21,200	..	72,579-g	" " " " " " " "
24	"	..	"	"	"	..	"	..	..	Air	32,000	..	..	" " " " " " " "
25	*B*	..	0.1	13	228	..	14,065	..	..	Air	33,500	..	..	The min. length of gap *a* = 1/16″
26	"	..	"	"	"	..	"	..	..		23,500	..	..	" " " " " " " 3/16″
27	"	..	"	"	"	..	"	..	..	Air	27,000	..	72,579-ff	" " " " " " " "
28	"	..	"	"	"	..	..	..	..		17,800- 19,000	..	..	Gap *b* broke when closing breaker.
29	"	..	"	"	"	..	..	...	..	Air	30,500- 29,300	..	..	
30	*A*	0.1	0.00016	90	0	2370	13,940	12,060	..		24,600	31,000	..	
31	*C*	"	"	"	"	..	..	..	..		..	..	72,579-k	
32	"	"	"	"	"	..	..	..	..		..	..	72,579-l	
33	*A*	"	"	"	"	2370	13,940	12,060	..		24,600	30,000	..	
34	"	"	"	"	"	"	"	"	..		24,000	30,000	..	
35	"	"	"	"	"	2300	..	..	..	Air	24,000	31,400	..	
36	*C*	"	"	"	"	..	..	..	..	Air	..	..	72,579-m	
37	*A*	"	"	"	"	2300	..	..	..	Air	22,100	29,600	..	
38	"	"	"	"	"	"	..	..	..	Air	19,500	29,600	..	
39	"	0.1	"	"	228	"	12,420	..	..		19,000	29,200	..	
40	"	"	"	"	"	..	..	..	..		19,500	30,000	..	
41	"	"	"	"	"	..	..	..	..		20,800	31,500	..	
42	"	"	"	"	"	..	..	..	..	Air	20,700	27,700	..	
43	"	"	"	"	"	..	..	..	..	Air	20,800	26,200	..	
44	"	"	"	"	"	2180	..	..	..	Air	18,600	26,300	..	
45	*B*	..	"	"	"	2190	..	..	..	Air	11,200	..	..	Gap *b* broke down on closing breaker
46	"	..	"	"	"	"	..	..	..		16,800 once only	..	..	" " " " " " "
47	"	..	"	"	"	"	..	..	..		11,000	..	..	
48	"	..	"	"	"	"	..	..	..	Air	12,000	..	72,579-n	
49	"	..	"	"	"	"	..	..	..	Air	13,500	..	..	
50	"	..	"	"	"	"	..	..	..	Air	13,000	..	..	
51	*A*	0.1	0.00008	90	0	2190	13,140	..	..		25,200	31,500	..	
52	"	"	"	"	"	"	"	..	..		30,600	..	..	Gap *b* broke down on opening breaker
53	"	"	"	"	"	"	"	..	..	Air	24,500	30,500	..	
54	"	0.4	0.0008	"	"	2184	..	11,266	Too low for gap		24,800	38,300	..	Gap *b* broke on closing breaker
55	"	0.05	0.00032	"	"	2180	12,984	..	..		23,500- 23,200	27,500- 30,000	...	
56	*C*	"	"	"	"	"	"	..	..		..	..	72,579-o	
57	*A*	"	"	"	"	"	"	..	..		30,000	..	..	Occasionally, when breaker opened
58	"	"	"	"	"	"	"	..	..		20,000	25,200	..	
59	*C*	"	"	"	"	"	"	..	..		..	..	72,579-p	
60	*A*	"	"	"	"	"	"	..	..		26,000	30,500	..	
61	*C*	"	"	"	"	"	"	..	..	Air	26,000	30,500	72,579-q	
62	*A*	"	"	"	"	"	"	..	..	Air	24,000	..	..	
63	"	"	"	"	"	"	"	..	..	Air	..	26,700	..	
64	*B*	..	"	"	228	"	..	..	..		11,500	..	..	Gap *b* broke down when breaker closed
65	"	..	"	"	"	"	..	..	..	Air	11,300	..	72,579-r	
66	"	..	"	"	"	"	..	..	..	Air	11,700	..	..	
67	*C*	0.2	0.1	13	0	..	..	..	..		15,200	23,800		
68	"	"	"		"	..	..	..	..	Air	16,100	23,800	72,579-aa	
69	"	0.1	0	0	0	2250	13,380	..	..		24,000	..	..	
70	"	"	"	"	"	"	"	..	..		21,000	..	..	
71	"	"	"	"	"	"	"	..	..		29,000	..	..	
72	"	"	"	"	"	"	"	..	..	Air	26,800	..	72,579-gg	
73	"	"	"	"	"	"	"	..	..	Air	30,000	..	..	
74	"	"	"	"	"	"	"	..	..	Air	28,500	..	..	
75	"	"	"	"	"	"	"	..	..	Air	21,500	..	..	

TABLE II—*Continued*

Test	Circuit	C_1 μ f	C_2 μ f	R_d ohms	L_2 mh.	L_1 mh.	E_1 volts	E_2 volts	E_b volts	E_e volts	Kind of arc	*b* in volts	*c* in volts	Film No.	Remarks
76	*C*	0.1	0.1	99_c	13	0	2170	..	..	..	Air	11,000	18,000	72,579-*y*	When film was made L_1 = 228 mh.
77	"	"	"	53_c	"	"	"	..	..	..		10,500	18,700	72,579-t	" " " " " " "
78	"	"	"	53_c	"	228	2160	..	..	..	Weak air Insulator flashover	..	..	72,579-v, w, x	See note *a* for kind of arc
79	"	"	"	0	"	0	..	..	..	..	Weak air	..	19,000	..	
80	"	"	"	0	"	"	..	..	..	..	Strong air	..	19,500	..	
81	"	"	"	90_c	"	"	..	..	..	..	Air	..	19,500	..	
82	"	"	"	35_c	"	"	..	..	..	..	Air	..	19,500	..	
83	"	"	"	5_w	"	"	..	..	..	..	Air	..	20,100	..	
84	"	"	"	0	"	"	..	..	..	..	Air	..	20,100	..	
85	"	"	"	0.5_w	"	"	..	..	..	..	Air	..	20,500	..	
86	"	"	"	0.25_w	"	"	..	..	..	..	Air	..	20,500	..	
87	"	"	"	0	13	"	..	..	..	..	Air	..	20,500	..	
88	*C*	"	"	"	0	"	..	..	..	..	Air	..	19,000	72,579-u	When film was made L_2 = 228
89	"	"	"	0.5_w	"	"	..	..	..	..	Air	..	19,000	..	
90	"	"	"	0.25_w	"	"	..	..	..	..	Air	..	19,000	..	
91	"	0.2	"	"	"	"	..	..	..	..	Air	..	24,500	..	
92	"	"	"	0	"	"	..	..	..	..	Air	..	24,000	..	
93	"	"	"	5_w	"	"	..	..	..	..	Air	..	24,000	..	
94	"	"	"	50_w	"	"	..	..	..	..	Air	..	25,000	..	
95	"	"	"	200_w	"	"	..	..	..	..	Air	..	23,200	..	
96	"	"	"	"	13	"	..	..	..	..	Air	..	23,800	..	
97	"	"	"	50_w	"	"	..	..	..	..	Air	..	24,000	72,579-bb	When film was made R_d = 42 carbon
98	"	"	"	53_c	"	228	2160	..	..	..	Air	..	..	72,579-s	
99	"	"	"	5_w	"	0	..	..	..	..	Air	..	23,500	..	
100	"	"	"	115_w	"	"	..	..	..	..	Air	..	..	72,579-cc	
101	"	"	"	115_w	"	"	..	..	..	..	Air Flash over insulator	..	..	72,579-dd	See note *b*
102	*B*	..	0.1	"	13	228	..	..	..	..	Air	..	..	72,579-ee	

(*a*) R_c means resistance was carbon rod.
(*b*) R_w wire on tube 1 in. in diameter.

DISCUSSION OF OSCILLOGRAMS

With the results of Table II indicating the probable correctness of Theory II, it is interesting to study the various oscillograms for evidences of the action called for by each of the three theories. In the oscillograms the zero lines are shown which represent zero current through the oscillograph element but do not represent the zero lines of the voltage waves. This is due to the fact that when zero voltage is impressed upon the grid there is still the plate current flowing through the oscillograph element.

THEORY I

Theory I would require the appearance on the oscillogram oscillations of increasing amplitude during the periods that the arc was on. However, no oscillograms show this effect. Oscillograms 72,579-*d*, *o* and *z*, which are typical of those obtained with circuits *A* and *C*, show the oscillations through the inductance of the supply which were produced by the striking of the arc very clearly and these oscillations are clearly damped. An oscillation of the short-circuited condenser may be inferred by the slight breaks following each discharge in the curve for voltage across C_2. We may infer that these oscillations were highly damped because the curve becomes continuous again very shortly after. Oscillogram 72,579-*r*, taken with circuit *B* with a larger and lagging arc current, also shows only slight breaks in the curve for voltage across C_2.

THEORY III

Theory III requires the extinction of the arc to take place at the peak of the high-frequency voltage wave so that when the arc extinguishes a charge is left on the condenser in excess of the charge corresponding to the 60-cycle voltage at that time. In no oscillogram was this observed. One would expect that the greatest chance for the Petersen theory to be valid would be when the short-circuited capacitance was small, for then the heating due to the dissipating of the energy of

(72579-*O*)

(72579-*Z*)

the short-circuited condenser in the arc would be small, in fact, oscillogram 72-579-*o* does show the arc extinguish for a short time in nearly every half cycle. But even here the arc, after striking, held on long enough for the high frequency to damp out so that at the time of extinction the charge retained on the condenser did not exceed that corresponding to the instantaneous 60-cycle voltage. Thus, the displacement of the voltage waves was never in excess of the normal voltage to neutral as is required by this theory.

THEORY II

Theory II requires the striking of the arc at the peak of the 60-cycle voltage wave and its extinction also near the peak of a 60-cycle voltage wave. When the striking of the arc took place after the arc had been out for a little time, that is, more than a half cycle, the striking always did take place at the peak of the voltage but when an air blast was used so that the arc would be blown out before the gap had become very long, reignition frequently occurred within the next half cycle before the peak of the voltage wave as shown at 1, oscillogram 72,579-*z*. Also when an air blast was used extinction sometimes occurred before the peak of the voltage, that is, before the 60-cycle charging current would normally pass through zero. However, these extinctions and reignitions give rise to voltages less than those called for by Theory II since the retained charge under these circumstances would be less than that corresponding to peak voltage and the amplitude of oscillation would be less because of the smaller total voltage to ground at the moment of striking. We may say then that the voltages called for by Theory II do not develop with each individual extinction and reignition of the arc, but we can say that usually when the arc extinguishes for a longer time it will do so at the peak of voltage with a retained charge equal to normal voltage to neutral and that the next striking of the arc will be at the peak voltage so that the maximum voltages called for by this theory will nearly always be ultimately developed. The final extinction of the arc at the peak of the voltage wave and the displacement of the voltage waves from the zero line are clearly seen in oscillograms 72,579-*d* and *o*.

(72579-*B*)

(72579-*D*)

The spark gap voltages obtained and the oscillograms appear to substantiate Theory II as to the maximum voltages which may be developed by an arcing ground. However, it must be pointed out that the experimental work was limited to a 13,200-volt circuit and it is conceivable that for higher voltages with greater arc lengths in which heating of the electrodes might play a smaller part, Theory III calling for the extinction of the arc at zero of high-frequency current might find application. The authors' opinion is that this will seldom prove to be the case. As to Theory I calling for the development of high voltages due to the "negative resistance" of the arc they are even more strongly of the opinion that it will fail to be realized even with higher voltages.

GROUNDED VS. UNGROUNDED NEUTRAL SYSTEMS

In the experiments described above circuits *A* and *C* approximate conditions on a transmission system with an isolated neutral and circuit *B* approximates conditions on a grounded neutral system. The predictions of Theory II applied to circuit *C* were substantiated both by spark gap voltage determinations and by oscillograms. This theory applied to a three-phase system calls for displacement of neutral relative to ground and the development as a result of transient oscillations of maximum voltage of 3½ times normal voltage to neutral on the sound phases and two times normal voltage to neutral on the grounded phase. In a grounded neutral system if the impedance between neutral and ground is not too large the charge left on the system after the extinction of the arc will be drained off before the next striking of the arc. Hence the maximum voltage which will be developed will be that corresponding to 2, Fig. 5, that is, 2½ times normal voltage to neutral on the sound phases and 1 times normal voltage to neutral on the grounded phase.

Thus the maximum voltages to ground developed by arcing grounds on isolated neutral and grounded neutral systems, respectively, are in the ratio 3½ to 2½. However when the distribution of potentials within apparatus is considered, the magnitude of sudden changes in voltage becomes important and these sudden voltage changes are very much greater in the isolated neutral case than in the case of the grounded system. We must, therefore, consider the surges set up by arcing grounds which are propagated due to the distributed capacitance and inductance in transmission systems.

TRAVELING WAVES SET UP BY GROUNDING PHASE OF TRANSMISSION SYSTEM

The transient oscillations generated by grounding a phase in a transmission system were described above in terms of lumped capacitances and inductances. When the capacitances and inductances are distributed, oscillations take place but are not simple sine waves as were described above. These oscillations are best pictured in terms of traveling waves. Referring to Fig. 14, when a ground suddenly occurs on a phase of the transmission line the capacity of the line is not discharged as a whole like a single condenser but those parts of the line nearest to the grounded point discharge first. Since the inductance to the grounded point is less for these nearest parts, they discharge almost at once while the farthest parts of the line still retain their charge. There are thus set up waves traveling in both directions from the point of ground. These traveling waves reduce the potential from V, the value just before the ground occurred to zero. When the traveling wave reaches the end of the line where a change of impedance is encountered, reflection occurs and at the point of reflection a sudden change of voltage takes place amounting to $\frac{2\,Z_2}{Z_1 + Z_2}\,V$, where Z_1 and Z_2 are respectively the surge impedance of the line and the surge impedance of the apparatus at the end of the line. The mathematical derivation of this expression may be found in the appendix.

The oscillations of the ungrounded phases are also accompanied by traveling waves but owing to the inductance of the supply system which is in series, the wave fronts will be of much less steepness and therefore much less disastrous as regards voltage distribution within the apparatus at the ends of the line.

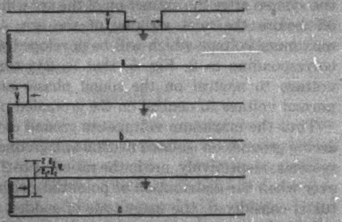

FIG. 14—PROPAGATION AND REFLECTION OF TRAVELING WAVE

We may now compare the magnitudes of the traveling waves set up by an arcing ground in the two cases of the isolated neutral and the grounded neutral systems. For the isolated neutral system we have seen that the arc will usually strike at the maximum of the 60-cycle voltage wave and at that moment there will also be a retained charge on the system after the previous extinction which further displaces the voltage of the line from ground. The voltage of the line to ground is then two times the normal voltage to neutral. The magnitude of the sudden change in voltage which occurs at the end of the line is therefore

$$\frac{2\,Z_2}{Z_1 + Z_2} \times 2\,E,$$

or nearly four times normal voltage to neutral. For the grounded neutral system there is no charge retained by the system at the moment the arc strikes and therefore we have $V = E$. Hence the sudden change in voltage due to reflection at the end of the line will be approximately $2\,E$ or only one-half as great as that for the isolated neutral system. Most observations of abnormal voltages on isolated neutral systems due to arcing grounds have been not of voltages to ground but of voltages across parts of apparatus. The above discussion of the traveling waves set up by arcing grounds on isolated neutral systems show that this is what should be expected.

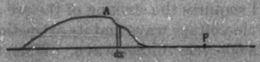

FIG. 15—TRAVELING WAVE ON TRANSMISSION LINE

Appendix

REFLECTION OF TRAVELING WAVES

Consider a traveling wave started by any abrupt change in potential on a system containing distributed inductance and capacitance. The wave may start with an abrupt or sheer front such as results when closing a switch on a dead line or grounding an energized line or it may start with a tapered front such as may be induced by the release of an electrostatic charge on a cloud. As the traveling wave proceeds on its course it gives up energy in Joulian heat and radiation which causes a progressive tapering of its front. When this traveling wave strikes a point on the line where the impedance of the line makes an abrupt change the wave undergoes a reflection. The theory of such reflection follows. In Fig. 15 let A represent a traveling wave of voltage of any shape and let v be its potential at any point x.

Let L = inductance per unit length of line.
Let C = capacitance per unit length of line.

Then $$\Delta v = -L\,\Delta x\,\frac{d\,i}{d\,t} \qquad (1)$$

$$\Delta i = -C\,\Delta x\,\frac{d\,v}{d\,t} \qquad (2)$$

from (1) $$\frac{\partial v}{\partial x} = -L\,\frac{\partial i}{\partial t} \qquad (3)$$

from (2) $$\frac{\partial i}{\partial x} = -C\,\frac{\partial v}{\partial t} \qquad (4)$$

from (3) $$\frac{\partial^2 v}{\partial x^2} = -L\frac{\partial^2 i}{\partial t\,\partial x} \tag{5}$$

from (4) $$\frac{\partial^2 i}{\partial t\,\partial x} = -C\frac{\partial^2 v}{\partial t^2} \tag{6}$$

from (5) and (6) $$\frac{\partial^2 v}{\partial t^2} = \frac{1}{LC}\,\frac{\partial^2 v}{\partial x^2} \tag{7}$$

also $$\frac{\partial^2 i}{\partial t^2} = \frac{1}{LC}\,\frac{\partial^2 i}{\partial x^2} \tag{8}$$

It is readily checked by substitution that

$v = f(x + \alpha t)$ and

$v = g(x - \alpha t)$ are solutions of (8)

f and g denoting arbitrary functions of $(x + \alpha t)$ and $(x - \alpha t)$, respectively, and

$$\alpha^2 = \frac{1}{LC}$$

For, substituting $v = f(x + \alpha t)$ in (7) we get

$$\alpha^2 f''(x + \alpha t) = \frac{1}{LC} f''(x + \alpha t)$$

or $$\alpha^2 = \frac{1}{LC}$$

$v = f(x + \alpha t)$ evidently represents a voltage wave traveling with the velocity α in the direction of negative x; for if we increase t by unity and at the same time decrease x by α the value of v is unchanged; but increasing t by unity and changing x by $-\alpha$ is equivalent to shifting the wave through the distance $-\alpha$ in unit time. Similarly, $v = g(x - \alpha t)$ represents a voltage wave traveling in the direction of positive x.

Let us now determine the traveling wave of current which accompanies the voltage wave $v = f(x + \alpha t)$, substituting, $v = f(x + \alpha t)$ in (3) and (4), we get

$$-L\frac{\partial i}{\partial t} = f'(x + \alpha t) \tag{9}$$

$$\frac{\partial i}{\partial x} = -C\alpha f'(x + \alpha t) \tag{10}$$

Integrating (9) $i = -\dfrac{1}{\alpha L} f(x + \alpha t) + F_1(x)$ (11)

Integrating (10) $i = -C\alpha f(x + \alpha t) + F_2(t)$ (12)

since $\dfrac{1}{\alpha L} = C\alpha, F_1(x) = F_2(t)$

$F_1(x)$ and $F_2(t)$ therefore represent a constant current and can be neglected in considering the traveling wave. We have then, finally

$$i = -\frac{f(x + \alpha t)}{\alpha L} = -\frac{f(x + \alpha t)}{\sqrt{L/C}} = -v/Z \tag{13}$$

where $Z = \sqrt{L/C}$ = surge impedance.

In the same way we may determine the current wave corresponding to $v = g(x - \alpha t)$ and find

$$i = \frac{f(x - \alpha t)}{\alpha L} = v/Z$$

Consider now a traveling wave $v = f(x + \alpha t)$ reaching a point P at $x = 0$ where the line constants change from a surge impedance of Z_1 to surge impedance Z_2. There will then appear a transmitted and reflected wave in addition to the oncoming wave. Before reaching P we have only the advancing wave

$$v = f(x + \alpha t) \tag{14}$$

$$i = -1/Z_1 f(x + \alpha t) \tag{15}$$

After reaching P we have on one side a transmitted wave.

$$v_2 = f_2(x + \alpha_2 t)\ x < 0 \tag{16}$$

$$i_2 = -1/Z_2 f_2(x + \alpha_2 t)\ x < 0 \tag{17}$$

And on the other side of P the oncoming plus reflected wave

$$v_1 = f(x + \alpha t) + g(x - \alpha t)\ x > 0 \tag{18}$$

$$i_1 = -1/Z_1 f(x + \alpha t) + 1/Z_1 g(x - \alpha t)\ x > 0 \tag{19}$$

At point P where $x = 0$ the voltage from either side must be the same and the current flowing to P on one side must equal the current flowing from P on the other side. Therefore from (16), (17), (18) and (19)

$$f_2(\alpha_2 t) = f(\alpha t) + g(-\alpha t) \tag{20}$$

$$-1/Z_2 f_2(\alpha_2 t) = -1/Z_1 f(\alpha t) + 1/Z_1 g(-\alpha t) \tag{21}$$

multiplying (21) by Z_1

$$-Z_1/Z_2 f_2(\alpha_2 t) = -f(\alpha t) + g(-\alpha t) \tag{22}$$

subtracting (22) from (20)

$$(1 + Z_1/Z_2) f_2(\alpha_2 t) = 2 f(\alpha t)$$

$$f_2(\alpha_2 t) = \frac{2}{1 + Z_1/Z_2} f(\alpha t)$$

or $$v_2 = \frac{2v}{1 + Z_1/Z_2} = \frac{2Z_2}{Z_1 + Z_2} v \tag{23}$$

Therefore the transmitted voltage and also the oncoming plus reflected voltage has a ratio to the oncoming voltage of $\dfrac{2Z_2}{Z_1 + Z_2}$ irrespective of the nature or shape of the wave.

Discussion

FREQUENCY CONVERSION BY THIRD CLASS CONDUCTOR AND MECHANISM OF THE ARCING GROUND AND OTHER CUMULATIVE SURGES

(Steinmetz), and

VOLTAGES INDUCED BY ARCING GROUNDS

(Slepian and Peters)

Pittsburgh, Pa., April 24, 1923

H. R. Woodrow: In both papers, the maximum surge potentials have been given neglecting the absorption of our systems, and I believe experience would show that the actual potential rise is less than that given in either one of the two papers particularly on underground systems. This factor has been brought out in connection with some tests which have recently been conducted and which I hope someone will discuss later.

Our surges are not all due to arcing grounds, but some of them are due to switching operations, and on some tests which have recently been made on the Brooklyn Edison system, it appears that these surges may be as high as two and a half times normal voltage.

I have felt in a good many of our cases we say our trouble is due to arcing grounds and surges, and in many cases our deductions are simply the result of our inability to connect the trouble to anything else.

A particular case of this type has come to my attention recently and has caused a great deal of worry and annoyance as to the source of the trouble. We did find some surges of small magnitude and short duration, but none of them seemed to properly account for the trouble. However, after two or three months' study, and the loss of several 5000 kv-a. transformers, we found it due to nothing more than a little piece of iron placed in the wrong position and without ventilation.

J. Slepian: It is brought out very clearly in the first part of Dr. Steinmetz's paper that the generation of the high frequency is due to the properties of arcs. Some peculiar properties are necessary in arcs in order to produce high frequency. One would expect in a mathematical treatment, that somewhere in the treatment a definite statement would be made as to what is assumed about the arc. An equation or some definite statement should be made as to what is assumed as to the nature of the arc so that from it the currents and voltages can be calculated. However, you look in vain for any such statements in the mathematical work. You have to go through it carefully and look for points here and there. Sometimes it is a definite statement, and sometimes in a rather concealed way a property of the arc is assumed.

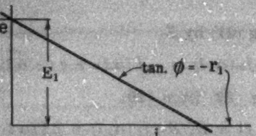

Fig. 1

Now, of course, I was helped considerably going through the mathematical work by the previous discussion. The previous discussion gives some idea of the simplified characteristic that Dr. Steinmetz takes for the arc.

Taking on page 273, the first equation given, he assumes that the arc has a straight line characteristic, as given by the equation $e = E_1 - r_1 i$.

The equation given there would give a straight line as shown in accompanying Fig. 1, the slope r_1, being the negative resistance. This applies only for the low frequency. When it comes to the high frequency, he brings in the idea of time lag between current and volts. That is, the high-frequency volts and high-frequency amperes are not in phase. This is expressed in the equations at the bottom of the same column on page 273.

These equations plotted with volts and amperes as coordinates give an ellipse as shown in accompanying Fig. 2, so we guess that probably the characteristic that Dr. Steinmetz makes use of in his mathematic work is something of that nature.

Now, to find where he brings in quantitatively the magnitude of the negative resistance and axes of the ellipse, and so on, we have to hunt carefully. The first assumption we find is in equation 16 on page 276, which says, Let $r_1 = E_1/I_1$ = effective resistance of the third class conductor.

In this section, however, he is dealing only with the low-frequency terms. In calling this resistance r_1, he makes the assumption that volts and amperes of the low frequency are in the same ratio as for the high frequency. This means that the low-frequency volt-ampere curve is a straight line which has the same positive slope as the negative resistance with which he is going to work. As the low-frequency volts and amperes rise and fall, the high-frequency volts and amperes go round ellipses, and these ellipses come in and out, swell and retract, so that you get a curve of volts and current similar to that given on page 275 Fig. 4.

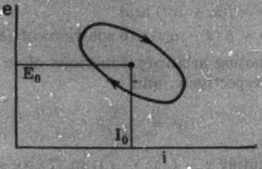

Fig. 2

The high-frequency volts and amperes rise and fall with the low frequency but for some reason he has chosen to take the two in equal ratio. There is no reason why that should be the case.

The next assumption he makes as to the magnitudes in the arc is rather hard to illustrate in the mathematical work, so I will go to the first numerical case on pages 278 and 279. By assuming that delta equals sixty degrees as the time lag between high-frequency current and volts, an arbitrary assumption is made as to the shape of the ellipse. However, I don't quarrel with that. Some assumption must be made and that seems to be reasonable.

We next come to the assumption whereby a numerical value is given to the negative resistance, r_1. Near the top of page 279, taking the high-frequency resistance of the oscillating circuit as $r = 60$ ohms, Dr. Steinmetz says, "and assuming stationary conditions reached by the transient negative resistance of the spark discharge having dropped to equality with the ohmic resistance, that is:

$$r_1 \cos \delta = 60, \text{ and } r_1 = 120 \text{ ohms," etc.}$$

Thus, from the resistance of the oscillating circuit he has calculated the negative resistance of the arc.

Now, this is not always a correct procedure and may as in this example lead to absurd results. In the text of the paper itself, it is explained very clearly that we may not calculate the negative resistance in this way. In the last paragraph of Section III on page 273 we find, "Thus the final result of a condenser discharge through a third class conductor of *sufficiently high* effective negative resistance is an alternating current of a frequency determined by the circuit constants."

I have italicized the words *sufficiently high* because they indicate that there are two possibilities as to the steady state. If the negative resistance is *sufficiently high* there will be sustained oscillations with the negative resistance reduced to equality with the ohmic resistance of the oscillating circuit. But if the negative resistance is not *sufficiently high*, the steady state will be one with no oscillation and the negative resistance of the arc will not be equal to the ohmic resistance of the oscillating circuit.

Now, in applying these ideas to a practical case, we must not forget that there are these two possibilities for the steady state. Dr. Steinmetz assumes that the first possibility holds, namely, that the negative resistance is *sufficiently high*, so that oscillations will build up until there is equality between the negative resistance and the ohmic resistance. Thus he is able to calculate the negative resistance from the resistance of the transformer coil, and using the other assumptions, he calculates the various other currents and voltages of the circuit. But this does not prove that there will be oscillations, but only that if there are sustained oscillations, and if the other assumptions are valid, the arc must have a negative resistance $r_1 = 120$ ohms, as indicated on page 279. The thing to do now is to consider whether it is reasonable to expect the arc to have so large a negative resistance. If it is unreasonable, then we must conclude that the other possibility holds, namely, that the negative resistance is not *sufficiently great*, and that oscillations will not be built up.

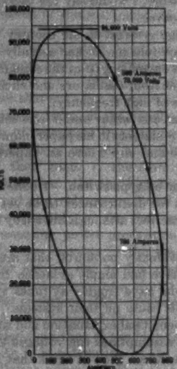

FIG. 3

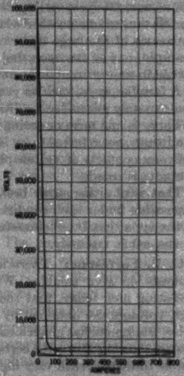

FIG. 4

I have taken the currents and voltages of the arc as calculated by Dr. Steinmetz, on page 279,

$$e_1 = 47,000 \sin \phi \{1 + \sin (837 \phi - 60.)\}$$

$$i_1 = 392 \sin \phi \{1 - \sin 837 \phi\}$$

and plotted them as a volt-ampere diagram. (Fig. 3). The ellipse shown is for ϕ in the neighborhood of $\frac{\pi}{2}$. Now, this is a most extraordinary characteristic for an arc. For definiteness consider the point marked in Fig. 3. Here with 500 amperes flowing through it, the arc must have a voltage of 78,000 volts. There never was such an arc!

An actual arc characteristic plotted on the same scale would look as in accompanying Fig. 4, full line. When the current exceeds a few amperes, the voltage falls to well under a kilovolt. If we wish to approximate to the dynamic characteristic by means of an ellipse, we must make the axis of the ellipse nearly horizontal, dotted curve, Fig. 4. That is, the negative resistance of such a heavy current arc is extremely small, and could not sustain alternating current in a 60-ohm oscillating circuit.

We can assure ourselves that heavy current arcs do not have high voltages and large negative resistance by considering the Poulsen arc generator of radio frequency currents. Here it is desirable to make the negative resistance as large as possible, but using all the means known to us, we cannot make a fifty or one hundred kilowatt arc consume more than a few hundred volts, or have a negative resistance greater than ten ohms or so.

The conclusions which Dr. Steinmetz arrives at in the numerical example also show that the arc required by the mathematics is not such a one as really exists. Thus he finds that the magnitude of the low-frequency current is considerably affected by the arc, whereas the common expectation which is probably based on experience, is that an arc-over of a transformer is practically a short circuit of the 60-cycle current.

In closing my discussion I would like to repeat that what criticisms I have felt it necessary to make apply only to part B, Mathematical Calculation. Part A, Physical explanation, is a masterly work. I maintain, however, that the description of the phenomena in an arc given in Part A does not supply sufficient data for the calculations carried out in Part B, and that in Part B tacit assumptions are made which lead to erroneous conclusions.

E. P. Peck: The arcing ground on an isolated system is a definite occurrence, and quite often a very prominent one. I do not doubt that every old operating man who has dealt with large isolated systems has had as much trouble and as serious trouble from arcing grounds as from any other one thing.

The papers this morning brought out some of the early arguments and discussions on the relative merits of the isolated delta and the grounded neutral systems. It happens that down in the Georgia Company we were going through the argument among ourselves a number of years ago, and we could not reach in practise the conclusions that some of the highly theoretical engineers reached. We found that we had to establish grounded neutrals on our systems in order to keep the long transmission lines in service. There was one particular 22,000-volt line 63 miles long that developed voltages which we knew were higher than 180,000 volts. A large series of troubles which had extended over several months was cured entirely by putting a grounded neutral on the line. The value of the grounded neutral had been proved previous to that time on other lines.

Rather recently, on the Utica system, we had some troubles which started locally and spread to other points in which the secondary troubles were much more serious than the originating troubles. These troubles were also cured, and apparently permanently, since we have had no recurrence of them, by establishing a solid ground on the neutral, which, in this case, is on the generator.

Going back into the old history, it is very interesting indeed to see that the consensus of opinion has now come to agree with what we found the requirements to be a number of years ago. It is true, as one of the speakers said to-night, we are quite likely to blame a good many troubles on arcing grounds simply because they are mysterious. However, a great many of the troubles that occur are so evidently tied up with and follow so closely an arcing ground that there is no question as to the cause of them

L. C. Nicholson: I was interested in Dr. Slepian's remark that an arc may be 780 in. long. I do not think that 780 in. is longer than an arc can be. I have definite evidence of an arc drawing out for approximately 400 feet. This was a short circuit of a 60,000-volt transmission system. The arc carried about 3000 amperes, 25-cycle current. It started by flashover of a pin type insulator, and a strong wind blew one end of it out along the conductor and the other end down the tower leg. The total length of the arc was about 400 feet. Its passage along the conductor could easily be traced by burns on the power wire.

A side-light on the conductivity of arcs is the length of time necessary for an arc to cool sufficiently to prevent its reforming. Exhaustive study of this subject shows that a 25-cycle arc of several hundred or several thousand amperes, will re-form, that is, strike again, if it is not interrupted for say one-fifth second. This minimum time seems to be dependent to some extent upon the length of the arc. This being the case, it appears that a power arc is a good conductor when thought of in half-cycle intervals, and that the resistance of the arc is low throughout the cycle. Otherwise, suppression of the voltage at the arc for so long a time as indicated would not be required to stop it.

Arcing grounds on ungrounded systems certainly cause disastrous voltages. It has been my experience that if the neutral of a star-connected system is grounded through a resistance low enough to carry, say fifty amperes, the surges which are produced on delta or ungrounded systems do not occur to any dangerous degree. Accidental grounds produce no dangerously high-voltage effects on the ungrounded phases throughout the system, as indicated by the absence of lightning arrester discharges at various connected stations. For this reason, I do not consider that a high-resistance neutral is objectionable from the voltage standpoint; on the other hand, it is very desirable, from a current standpoint, since the flow of ground current is limited to a moderate amount.

Herman Halperin: Mr. Woodrow made some remarks about surges on underground cables, and I thought he was referring to some tests concerning which I was telling his committee which had taken place the last month in Chicago. I will show in brief, Fig. 5, what we did and what was found on some of these surge tests.

A kenotron set with one side grounded was used and the d-c. voltage charged a factory condenser. After that condenser was charged up in a few seconds or so, it went over the sphere gap into some three-phase paper-insulated lead-covered cables. In these three-phase cables we usually discharged into one conductor and the other two conductors were grounded; and at the end of the lines, the one conductor was grounded solidly or through a resistance. Then at various points we measured the voltages to ground by means of sphere gaps. In one case the first length was 350 feet and the remaining cable was about 15,000 feet long or more. These traveling waves that were put in at the sphere gap had a steep front and a gradually decreasing tail. The length of the main portion of the wave would be of the order of twenty micro seconds in time, which is a wave of very high frequency. In these tests we found that the voltage was decreased to somewhere around two-thirds of its initial value after it had gone through 350 feet of cable, and that it went down to a fifth or so at the end of the line, depending upon the cable and connections. Plotting the voltages against feet from the station the waves of this nature decreased in voltage rapidly at first and gradually towards the end. Of course when resorting to waves corresponding to very high frequency, the voltage decrease was very rapid at the beginning. The wave after it got down to the end of the cable was considerably flattened out.

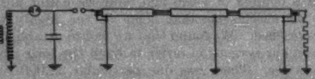

Fig. 5

It just indicates that traveling waves such as are shown in this second paper on page 791 will be considerably reduced in voltage after traveling a very short distance in underground cables, due to the damping qualities of the cable.

W. A. Hillebrand: Some of the largest of the high-power radio stations in the world utilize the negative temperature coefficient of the arc in order to transform the energy from a d-c. generator into the energy of a high-frequency oscillation. This is known as the Poulsen arc.

Dr. Steinmetz, in Fig. 3 sets up the circuits of the Poulsen arc and the equations thereof naturally follow. The identity between the fundamental relations that he derives and the Poulsen arc is rather striking.

With respect to some of the discussion on this subject, I would say that in order to generate oscillations by means of an arc, the magnetic field is not necessary, neither is it necessary to have a field of hydrogen. The magnetic field is used in the field with the Poulsen arc to get uniform arc conditions, so the conditions will repeat with a fair degree of uniformity. The hydrogen is used because of the high mobility of the hydrogen, and because of the high amount of radiation that is obtained. It is equivalent to a very large increase in the quantity minus R_1 that Dr. Steinmetz uses. In my opinion, in this discussion of the arcing ground, the matter of voltage is not of fundamental importance. One of the most important factors, it seems to me, has been overlooked and that is the matter of frequency. The moment you puncture the dielectric, you will get a streamer that will travel practically indefinite distance due to the high current that results from high frequency, and the high conductivity of the resulting arc. The relations between the strike distance of such an arc, once the dielectric is punctured, and those of, for instance, the normal 60-cycle variety, or rather, I would say the two bear no relation to one another whatsoever.

On one of the large high-voltage systems of the country, for example, one of the operating men told me he has actually observed in time of trouble streamers emanating from the conductor of a familiar type, such as you get from oscillators and such as you can only get under conditions of high frequency.

There is one form or arcing that can occur under one set of conditions, not referred to by Dr. Steinmetz in his paper, namely, between contacts of switch plane and clip where an imperfect contact is made.

That is of either an air break or oil type switch. I believe it is possible under such conditions for disturbances to be set up of the type that Dr. Steinmetz describes although obviously not of the same intensity.

C. L. Fortescue: There is evidently no doubt that arcing grounds do give trouble. We have had ample evidence from operating men today, that they do have trouble at times very high voltages are obtained by this cause on their systems.

Dr. Steinmetz has given a mathematical explanation and some figures showing that with certain assumptions the voltages that can be obtained reach very high values. Now, I think that Dr. Steinmetz' idea of the negative resistance might be carried out so as to give possibly more plausible values for the voltages reached.

I think that the principal thing these studies show is that our knowledge of these phenomena is very limited and that there is need for much further study.

Now, one of the difficulties of the study of these problems is that it is impossible to study them on a large scale because of the danger of causing damage. They have to be studied in the laboratory and in studying them in the laboratory we are so apt to get conditions that we do not get outside; it is so difficult to approximate conditions in the laboratory to those in actual service and theoretical analysis without corroborative experimental work is not very convincing when the problem is so complicated as in this case.

Charles P. Steinmetz: It is regrettable that Mr. J. Slepian rather misunderstood the bearing of my paper, as this led him to guess at assumptions made in the paper, and as I did not make these assumptions, his discussion, being based on these assumptions, becomes non-pertinent to the paper. For instance, I never made the assumption of a linear arc characteristic, which Mr. Slepian imputes by a misinterpretation of the equation on page 273, and his discussion based on this assumption does not apply. The purpose of the paper is not to prove that an arc produces high-frequency oscillations. It has been well known for years, that an arc may produce undamped and cumulative high-frequency oscillations, or continual oscillations, or isolated damped wave trains—such as once more described in Messrs. Peters' and Slepian's present paper—depending on circuit conditions. For instance, in my book on "Theory and Calculation of Electric Circuits" which was published in 1917, an entire chapter of 40 pages is given to this subject. There for a number of different arcs, the stationary characteristics are given and the transient characteristics—called "dynamic characteristics" by Mr. Slepian—as derived by oscillogram, and discussion of the arc characteristics giving the various types of arc oscillations and their limitations, and oscillograms of undamped high-frequency waves produced by the arc. All this being well known, its rehash obviously has no proper place in the paper, especially as it has no direct bearing on it.

The purpose of my paper, as indicated by the title, is to show the mechanism by which a third class conductor—which may or may not be an arc—can act as frequency converter. None of the assumptions, which Mr. Slepian tries to find in the paper, are made, nor are they necessary, but the only assumption is that the circuit, Fig. 3, contains a third class conductor, as defined in the paper, that is, a conductor in which, over a certain range of current, the voltage decreases with increase of current. The argument of the mathematical part of the paper then is: Whatever may be the particular characteristic of the third class conductor, as univalent functions of time, current and voltage can be represented by a Fourier series with constant coefficients, equation (7). Integration of the differential equation then shows that only two terms can exist in this Fourier series, a low-frequency term, of the supply frequency, and a high-frequency term of resonance frequency. In the low-frequency term, the coefficients of current and voltage are found of the same sign, and their ratio r_1 therefore is positive, representing power consumption. In the high-frequency term, the coefficients of current and voltage are found of opposite signs, and their ratio (approximately r, the load resistance) therefore is negative, representing power production. The mechanism of frequency conversion consists in the low-frequency component acting as motor circuit, due to the positive r_1, and the high-frequency component as generator circuit, due to the negative $-r$, as seen in the equations (33). To make oscillation possible, the power of the fundamental or motor component r_1 must be equal or greater than that of the high-frequency or generator component r. As r_1 is a function of the amplitude, the amplitude of the oscillation adjusts itself at that value of r_1, where the motor and generator components are equal.

Here Mr. Slepian seems to have been badly mixed up, and to mistake for assumptions the mathematical conclusions derived from the equations. There is no assumption whatever involved in the term r_1, but r_1 is merely defined as the constant ratio of the two constant coefficients of the fundamental terms of the Fourier series of current and voltage respectively, equation (7). Being of the dimension of resistance it is called "effective resistance." As conclusion from the equations and not as assumption as Mr. Slepian seems to think, the relation between the load resistance r of the high-frequency component, and the effective resistance r_1 of the low-frequency term is derived.

As regards to the numerical values, and the probability of high power oscillating arcs in industrial circuits, I may best quote from my book of 1917.

"In transmission lines, usually the resistance is too high to produce a cumulative oscillation; in underground cables, usually the inductance is too low and thus no cumulative oscillation results,—in the high-potential windings of the large high-voltage power transformers, however, as circuits of distributed capacity, inductance and resistance, the resistance commonly is below the value through which a cumulative oscillation can be produced and maintained, and in high-potential transformers, destruction by high voltages resulting from the cumulative oscillation of some arc in the system, and building up to high stationary waves, have frequently been observed."

J. F. Peters: With reference to Mr. Woodrow's and Mr. Halperin's remarks, there is no doubt but what considerable absorption of surges occur, preventing the full value of possible surges neglecting absorption. This is particularly true for cable systems.

It is very interesting to note the large degree of absorption obtained by tests described by Mr. Halperin. Since energy varies with voltage squared after the surges have traveled 350 feet, the energy was reduced to less than one-half of its original value.

With reference to Mr. Hildebrand's discussion concerning an arc as generator of high frequency, he states that a gas is not necessary but is used on account of the large amount of radiation that it produces. The increased cooling of the arc is the big factor in making an arc generate high frequency. This is shown clearly by the difference in characteristics of an arc in air and in gas, as shown in Figs. 8A and 8B. With reference to the streamer following puncture of dielectric, mentioned by Mr. Hildebrand, I assume he means the gaseous vapors produced by the large current resulting from shorting the electrolytic charge. This, of course, may reach a considerable distance. High-frequency voltage of short duration, such as accompany surges, will not jump any further and when solid insulation is involved will not jump as far as low-frequency voltage.

Some Problems in Electric Furnace Operation

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Review of the Subject.*—Based upon the theory that the reactance of a three-phase electric furnace does not depend upon the load but is a constant depending on the arrangement of the conductors, the general equations of the three-phase furnace are developed and discussed.*

It is shown that, due to the fact that the main conductors carrying the current from the interlaced bus bars to the furnace are more or less in one plane, a transformer action is introduced between phase 1 and phase 3, taking power away from 1 and transporting it to 3. The phase rotation being 1, 2, 3. This power may be considerable.

When voltages between phases and currents are balanced the power delivered by the three electrodes is quite unbalanced, a fact that is very apparent in large furnaces, electrode 1 seeming dead while electrode 3 takes most of the power.

Nevertheless, when only voltage, current and total power are concerned, very simple expressions can be used which are entirely satisfactory in practise.

A few methods are suggested to reduce the unbalancing of the load.

Finally the two-phase connection of the load is studied to show that the unbalancing of the currents which is inherent in this connection does not check with observations on the furnace under operation.

The load must be considered as connected in star, the bottom of the furnace being the neutral. The electric energy is transformed into heat energy in three separate zones situated under the three electrodes.

The load resistance is probably furnished by an arc in an atmosphere of vapors under pressure.

* * * * *

WHEN operating electric furnaces of the ordinary ferro-alloy type it is often necessary to know in advance what a furnace will do under changed conditions, and it has been tried in the following to develop and discuss the general equations of the electric furnace. Most of the large furnaces are of the three-phase type and only such have been considered here.

A large number of tests made on different furnaces manufacturing various products showed that the reactance per phase, measured on the low-tension side, was practically constant over the whole range of loads. This reactance depends upon the disposition of the bus bars and other conductors carrying the current into the furnace.

The load circuit, that is, this part of the circuit where the electric energy is transformed into heat does not show any reactance and acts like a pure resistance.

In some of the tests mentioned above, the current was increased from 6000 to 24,000 amperes per phase, at constant voltage without the reactance changing its value.

Through a large number of observations the writer is led to believe that the transformation of electric energy into heat energy, in the furnace of the type considered, occurs in an arc passing through an atmosphere of vapors under pressure. The properties of the arc will naturally vary with different materials. Other conditions remaining the same the current will increase when the arc is shortened under constant voltage, and for constant current, the voltage across the arc will increase with increasing length.

It has been suggested that the heat energy was generated by the current passing through the slag and the molten metal.

To this view it may be objected that, if the electrode tips were touching the molten bath, the path through the bath to the bottom of the furnace would offer much less resistance to the current, due to its short length and large section, than the path across the bath to the nearest electrode, which is long and of narrow section and the current would flow directly to the bottom. The resistance of the short length of molten metal and slag is much too small to account for the power which is dissipated according to the $i^2 R$ rule. It can further be shown that the electrodes are far above the bath.

If, on the other hand, the pasty charge above the bath were the resistor, the current would pass mostly to the nearest electrode without going to the bath. The conditions are here reversed; the larger resistance would be toward the bath.

In a three-phase furnace such a distribution of load would correspond to an open delta connection. When a load is connected across two phases only of a three-phase system, the center conductor carries a current which is about 1¾ times the current in the outer conductors.

Such an unbalancing does not appear in our furnaces and in following the movements of the electrodes and their effect upon voltage, current and load one is led to the conclusion that the load is star-connected with the bottom of the furnace acting as the neutral point. As for the cold charge, it can be considered as a perfect insulator.

The heat energy, therefor, is generated in three small zones situated between the ends of the three electrodes and the bath; most of that energy is used up by the chemical reactions and to bring the products to their final temperature. The balance flows through the charge and the walls of the furnace, from points of higher to points of lower temperature.

If the electrodes are too far apart it might happen that the zone half way between two electrodes is too cold to keep the charge in fusion and we get a wall in the furnace.

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To raise the temperature at this point we have to increase the input of energy. For constant current this is done by increasing the voltage, so that indirectly the distance between electrodes is a function of the voltage.

Based on the above conclusion, that the reactance per phase was a constant, independent of the load, the general equations of the three-phase furnace were developed.

To avoid any undue complications and obtain equations which could be easily discussed, the following assumptions were made:

The furnace is fed by three parallel and similar conductors situated in one plane and symmetrically disposed in relation to the middle one. Constant voltage across phases is impressed to these conductors by the transformers and their resistance is neglected, unless otherwise specified. As can be seen the impedance of the transformer is neglected also, unless otherwise specified.

GENERAL EQUATIONS

The voltage drop due to the inductance in three parallel conductors is:

$$\left.\begin{aligned} I_1 X_a &= - I_2 X_{1\cdot 2} - I_3 X_{1\cdot 3} \\ I_2 X_b &= - I_1 X_{2\cdot 1} - I_3 X_{2\cdot 3} \\ I_3 X_c &= - I_1 X_{3\cdot 1} - I_2 X_{3\cdot 2} \end{aligned}\right\} \text{ where } \qquad (1)$$

$$X_{1\cdot 2} = X_{2\cdot 1}$$
$$X_{2\cdot 3} = X_{3\cdot 2}$$
$$X_{3\cdot 1} = X_{1\cdot 3}$$

and where I_1, I_2, I_3 designate the currents in phase 1, 2 and 3 respectively in amperes

$X_{1\cdot 2}$ is the effective reactance of phase 1 due to current I_2 in ohms

$X_{2\cdot 3}$ is the effective reactance of phase 2 due to current I_3

$X_{3\cdot 1}$ is the effective reactance of phase 3 due to current I_1

for reasons of symmetry $X_{12} = X_{23} = X$, X_{13} can be written αX where $\alpha > 1$. A dot above a quantity indicates that it must be taken as a vector. $\dot{I}_1$, $\dot{I}_2$, $\dot{I}_3$ being star currents must add up to 0.

$$\dot{I}_1 + \dot{I}_2 + \dot{I}_3 = 0 \qquad (2)$$

Using the complex notation and taking I_2 as the origin we have

$$\left.\begin{aligned} \dot{I}_1 &= I\,(-\,\gamma - j\,\beta) \\ \dot{I}_2 &= I \\ \dot{I}_3 &= I\,(-\,\epsilon + j\,\beta) \end{aligned}\right\} \qquad (3)$$

where $\gamma + \epsilon = 1$.

If we call R_a, R_b, R_c the resistance of the load in phase 1, 2 and 3 respectively. $I_1 R_a$, $I_2 R_b$, $I_3 R_c$ are the voltage drops due to load.

The total voltage drop in the three phases, from the point where the voltage is impressed to the neutral is:

$$\left.\begin{aligned} \dot{I}_1 Z_1 &= \dot{I}_1\,(R_a - j\,X_a) = \dot{I}_1 R_a + j\,\dot{I}_2 X + j\,\dot{I}_3\,\alpha\,X \\ \dot{I}_2 Z_2 &= \dot{I}_2\,(R_b - j\,X_b) = \dot{I}_2 R_b + j\,\dot{I}_3 X + j\,\dot{I}_1 X \\ \dot{I}_3 Z_3 &= \dot{I}_3\,(R_c - j\,X_c) = \dot{I}_3 R_c + j\,\dot{I}_1\,\alpha\,X + j\,\dot{I}_2 X \end{aligned}\right\} \qquad (4)$$

Combining (3) and (4):

$$\left.\begin{aligned} \dot{I}_1 Z_1 &= I\,\{-\,(\gamma\,R_a + \beta\,\alpha\,X) - j\,[\,(\alpha\,\epsilon - 1)\,X \\ &\qquad + \beta\,R_a]\,\} = \dot{e}_1 \\ \dot{I}_2 Z_2 &= I\,[R_b - j\,X] = \dot{e}_2 \\ \dot{I}_3 Z_3 &= I\,\{-\,(\epsilon\,R_c - \beta\,\alpha\,X) - j\,[\,(\gamma\,\alpha - 1)\,X \\ &\qquad - \beta\,R_c]\,\} = \dot{e}_3 \end{aligned}\right\} \qquad (5)$$

The impedances of the three phases are:

$$\left.\begin{aligned} Z_1 &= R_a + \frac{(\alpha - 1)\,\beta\,X}{\beta^2 + \gamma^2} \\ &\quad - j\,X\left(\alpha - \frac{(\alpha - 1)\,\gamma}{\beta^2 + \gamma^2}\right) = R_1 - j\,X_1 \\ Z_2 &= R_b - j\,X \qquad\qquad = R_2 - j\,X_2 \\ Z_3 &= R_c - \frac{(\alpha - 1)\,\beta\,X}{\beta^2 + \epsilon^2} \\ &\quad - j\,X\left(\alpha - \frac{(\alpha - 1)\,\epsilon}{\beta^2 + \epsilon^2}\right) = R_3 - j\,X_3 \end{aligned}\right\} \qquad (6)$$

We see that in phase 1 a resistance $\dfrac{\alpha - 1}{\beta^2 + \gamma^2}\,\beta\,X$ is added to the load resistance R_a, in phase 3 a resistance $\dfrac{\alpha - 1}{\beta^2 + \epsilon^2}\,\beta\,X$ is subtracted from R_c. Multiplying these two resistances by I_1^2 and I_3^2 respectively we have

$$P_{t_1} = I_1^2 \cdot \frac{\alpha - 1}{\beta^2 + \gamma^2}\,\beta\,X$$

$$P_{t_3} = I_3^2\;\frac{\alpha - 1}{\beta^2 + \epsilon^2}\,\beta\,X$$

but $I_1^2 = I^2\,(\beta^2 + \gamma^2)$ and $I_3^2 = I^2\,(\beta^2 + \epsilon^2)$ so that

$$P_{t_1} = P_{t_3} = P_t = I^2\,(\alpha - 1)\,\beta\,X \text{ Watts} \qquad (7)$$

P_t represents an amount of power which is subtracted from phase 1 and transmitted to phase 3 by a transformer action.

If E_1, E_2. E_3 are the voltages impressed across phase 2 and 3, phase 3 and 1 and phase 1 and 2 respectively, we have in combination with (5)

$$\left.\begin{aligned} \dot{E}_3 &= \dot{e}_1 - \dot{e}_2 = I\,[-\,(\gamma\,R_a + R_b + \beta\,\alpha\,X) \\ &\qquad - j\,[\,(\epsilon\,\alpha - 2)\,X + \beta\,R_a)\,] \\ \dot{E}_1 &= \dot{e}_2 - \dot{e}_3 = I\,[\,(R_b + \epsilon\,R_c - \beta\,\alpha\,X) \\ &\qquad - j\,[\,(2 - \alpha\,\gamma)\,X + \beta\,R_c)\,] \\ \dot{E}_2 &= \dot{e}_3 - \dot{e}_1 = I\,[-\,(\epsilon\,R_c - \gamma\,R_a - 2\,\beta\,\alpha\,X) \\ &\qquad - j\,[\,(\alpha\,\gamma - \alpha\,\epsilon)\,X - \beta\,R_a - \beta\,R_c)\,] \end{aligned}\right\} \qquad (8)$$

$$\dot{E}_1 + \dot{E}_2 + \dot{E}_3 = 0$$

The total power consumed by phases 1, 2 and 3 respectively are:

$$\left.\begin{aligned} W_1 &= I_1^2 R_1 = I^2\,[\,(\beta^2 + \gamma^2)\,R_a + (\alpha - 1)\,\beta\,X] \\ W_2 &= I_2^2 R_2 = I^2 R_b \\ W_3 &= I_3^2 R_3 = I^2\,[\,(\beta^2 + \epsilon^2)\,R_c - (\alpha - 1)\,\beta\,X] \end{aligned}\right\} \qquad (9)$$

The power delivered to the furnace by electrodes 1, 2 and 3 respectively are:

$$\left.\begin{aligned} W_a &= I_1^2 R_a = I^2 [(\beta^2 + \gamma^2) R_a] \\ W_b &= I_2^2 R_b = I^2 R_b \\ W_c &= I_3^2 R_c = I^2 [(\beta^2 + \epsilon^2) R_c] \end{aligned}\right\} \qquad (10)$$

Equations (8) which include voltage, current, resistance and reactance are the desired general equations of the three phase furnace.

DISCUSSION

Taking first the general case where the three impressed voltages are equal, and the three currents are balanced: the conditions for balanced currents are,

$$\gamma = \epsilon = 1/2, \beta = 1/2 \sqrt{3}, \gamma^2 + \beta^2 = \epsilon^2 + \beta^2 = 1 \qquad (11)$$

$$\left.\begin{aligned} \dot{I}_1 &= - I (1/2 + j\, 1/2 \sqrt{3} \\ \dot{I}_2 &= I \\ \dot{I}_3 &= - I (1/2 - j\, 1/2 \sqrt{3}) \end{aligned}\right\} \qquad (12)$$

if $\dot{E}_3 = E (\pi + j \zeta)$ the condition for equal voltage is

$$\left.\begin{aligned} \dot{E}_1 &= E (\pi + j \zeta) (- 1/2 + j\, 1/2 \sqrt{3}) \\ &= E [(- 1/2\, \pi - 1/2\, \zeta \sqrt{3}) \\ &\qquad - j (1/2\, \zeta - 1/2\, \pi \sqrt{3})] \\ \dot{E}_2 &= E (\pi + j \zeta) (- 1/2 - j\, 1/2 \sqrt{3}) \\ &= E [(- 1/2\, \pi + 1/2\, \zeta \sqrt{3}) \\ &\qquad - j (1/2\, \zeta + 1/2\, \pi \sqrt{3})] \\ \dot{E}_3 &= E (\pi + j \zeta) \end{aligned}\right\} \qquad (13)$$

Combining (8), (11) and (13) we have:

$$\left.\begin{aligned} R_a &= \frac{R_a + R_b + R_c}{3} - 2/3\, (\alpha - 1)\, X \sqrt{3} \\ &= R_{av} - 2/3\, (\alpha - 1)\, X \sqrt{3} \\ R_b &= \frac{R_a + R_b + R_c}{3} = R_{av} \\ R_c &= \frac{R_a + R_b + R_c}{3} + 2/3\, (\alpha - 1)\, X \sqrt{3} \\ &= R_{av} + 2/3\, (\alpha - 1)\, X \sqrt{3} \end{aligned}\right\} \qquad (14)$$

if we call $\dfrac{R_a + R_b + R_c}{3} = R_{av}$.

The three resistances of total power consumed by each phase are:

$$\left.\begin{aligned} R_1 &= R_{av} - 1/6\, (\alpha - 1)\, X \sqrt{3} \\ R_2 &= R_{av} \\ R_3 &= R_{av} + 1/6\, (\alpha - 1)\, X \sqrt{3} \end{aligned}\right\} \qquad (15)$$

The total power input is $3\, I^2 R_{av}$, but we see that the total power consumed by phase 1 is smaller than that consumed by phase 2 by an amount $I^2/6\, (\alpha - 1)\, X \sqrt{3}$. The total power consumed by phase 3 is larger than that consumed by phase 2 by the same amount.

Subtracting from phase 1 and adding to phase 3 the transformation power $\dfrac{I^2 (\alpha - 1)\, X \sqrt{3}}{2}$ we have finally for the power delivered to the furnace by phase 1, 2 and 3

$$\left.\begin{aligned} W_a &= I^2 (R_{av} - 2/3\, (\alpha - 1)\, X \sqrt{3}) \\ W_b &= I^2 R_{av} \\ W_c &= I^2 (R_{av} + 2/3\, (\alpha - 1)\, X \sqrt{3}) \end{aligned}\right\} \qquad (16)$$

$$W_a + W_b + W_c = 3\, I^2 R_{av}.$$

if $\dot{E}_3 = E (\pi + j \zeta) \quad E_3^2 = E^2 (\pi^2 + \zeta^2)$

combining this last equation with (8) and (11) we have

$$\begin{aligned} E_3^2 = E_2^2 = E_1^2 &= 3\, I^2 (R_{av}^2 + (\alpha + 2)^2 X^2/9) \\ &= 3\, I^2 (R_{av}^2 + X_{av}^2) \end{aligned}$$

where $$X_{av} = X \frac{(\alpha + 2)}{3}$$

If we replace in (6) R_1, R_2, R_3, X_1, X_2, X_3 by their respective values we have

$$Z_1 = R_{av} - 1/6\, (\alpha - 1)\, X \sqrt{3} - j \frac{(1 + \alpha)}{2} X$$

$$Z_2 = R_{av} - j X$$

$$Z_3 = R_{av} + 1/6\, (\alpha - 1)\, X \sqrt{3} - j \frac{(1 + \alpha)}{2} X$$

X_{av} represents the average of the three reactances just as R_{av} represents the average of the three load resistances, and $3\, I^2 X_{av}$ is the total reactive power as would be registered, for instance, by the reactance side of a double *R A* Max. demand meter.

If we are only interested in the total power without paying any attention to the unbalancing of the individual phases, the general equations are reduced to the equations of a three-phase balanced circuit. Since the currents are balanced we can introduce without error, the resistance of the conductors and if desired the impedance of the transformer.

In this case

$$\begin{aligned} R_{tot} &= \frac{R_a + R_b + R_c}{3} + R_{cond} + R_{transf.} \\ &= R_{av} + R_{losses} \end{aligned}$$

and $$X_{tot} = \frac{(\alpha + 2)}{3} X + X_{transf.}$$

$$Z_{tot} = \sqrt{R_{tot}^2 + X_{tot}^2}$$

Table I gives the values of the resistance per phase R_{tot}, the reactance X_{tot}, the power factor $\cos \varphi = R_{tot}/Z_{tot}$, the power input $W_{tot} = 3\, I^2 R_{tot}$, the current I and the voltage E when three of these quantities are given. Multiplying W_{tot} by R_{av}/R_{tot} gives the power delivered into the furnace.

TABLE I
(The Symbol $_{tot}$ is Omitted)

	Given	E = in volts	I = in amperes	X = in ohms	R = in ohms	W = in watts	$\cos\varphi$ =
1	$E\,I\,X$	E	I	X	$\sqrt{\frac{E^2}{3I^2} - X^2}$	$I\,\sqrt{3\,E^2 - 9\,I^2\,X^2}$	$\sqrt{1 - 3\,\frac{I^2\,X^2}{E^2}}$
2	$E\,I\,R$	E	I	$\sqrt{\frac{E^2}{3I^2} - R^2}$	R	$3I^2\,R$	$\frac{R\,I\,\sqrt{3}}{E}$
3	$E\,I\,W$	E	I	$\sqrt{\frac{E^2}{3I^2} - \frac{W^2}{9I^4}}$	$\frac{W}{3I^2}$	W	$\frac{W}{E\,I\,\sqrt{3}}$
4	$E\,I\cos\varphi$	E	I	$\frac{E}{I\,\sqrt{3}}\,\sqrt{1-\cos^2\varphi}$	$\frac{E\cos\varphi}{I\,\sqrt{3}}$	$E\,I\,\sqrt{3}\,.\,\cos\varphi$	$\cos\varphi$
5	$E\,X\,R$	E	$\frac{E}{\sqrt{3\,(X^2+R^2)}}$	X	R	$\frac{E^2\,R}{X^2+R^2}$	$\frac{R}{\sqrt{R^2+X^2}}$
6	$E\,X\,W$	E	$\sqrt{\frac{E^2 \pm \sqrt{E^4-4W^2X^2}}{6\,X^2}}$	X	$\frac{2\,W\,X^2}{E^2 \pm \sqrt{E^4-4W^2X^2}}$	W	$\frac{W\,X\,\sqrt{2}}{E\,\sqrt{E^2 \pm \sqrt{E^4-4W^2X^2}}}$
7	$E\,X\cos\varphi$	E	$\frac{E\,\sqrt{1-\cos^2\varphi}}{X\,\sqrt{3}}$	X	$\frac{X\cos\varphi}{\sqrt{1-\cos^2\varphi}}$	$\frac{E^2\,\sqrt{1-\cos^2\varphi}.\cos\varphi}{X}$	$\cos\varphi$
8	$E\,R\,W$	E	$\sqrt{\frac{W}{3\,R}}$	$R\sqrt{\frac{E^2}{W\,R} - 1}$	R	W	$\frac{\sqrt{W\,R}}{E}$
9	$E\,R\cos\varphi$	E	$\frac{E\cos\varphi}{R\,\sqrt{3}}$	$\frac{R\,\sqrt{1-\cos^2\varphi}}{\cos\varphi}$	R	$\frac{E^2\cos^2\varphi}{R}$	$\cos\varphi$
10	$E\,W\cos\varphi$	E	$\frac{W}{E\cos\varphi\,\sqrt{3}}$	$\frac{E^2\cos\varphi\,\sqrt{1-\cos^2\varphi}}{W}$	$\frac{E^2\cos^2\varphi}{W}$	W	$\cos\varphi$
11	$I\,X\,R$	$I\,\sqrt{3\,(R^2+X^2)}$	I	X	R	$3I^2\,R$	$\frac{R}{\sqrt{X^2+R^2}}$
12	$I\,X\,W$	$I\sqrt{3\,X^2 + \frac{W^2}{3I^4}}$	I	X	$\frac{W}{3I^2}$	W	$\frac{W}{\sqrt{9I^4\,X^2+W^2}}$
13	$I\,X\cos\varphi$	$\frac{I\,X\,\sqrt{3}}{\sqrt{1-\cos^2\varphi}}$	I	X	$\frac{X\cos\varphi}{\sqrt{1-\cos^2\varphi}}$	$\frac{3I^2\,X\cos\varphi}{\sqrt{1-\cos^2\varphi}}$	$\cos\varphi$
14	$I\,R\,W$	undeterm.	I	undeterm.	R	W	undeterm.
15	$I\,R\cos\varphi$	$\frac{I\,R}{\cos\varphi}\,\sqrt{3}$	I	$\frac{R}{\cos\varphi}\,\sqrt{1-\cos^2\varphi}$	R	$3I^2\,R$	$\cos\varphi$
16	$I\,W\cos\varphi$	$\frac{W}{I\cos\varphi\,\sqrt{3}}$	I	$\frac{W}{3I^2\cos\varphi}\,\sqrt{1-\cos^2\varphi}$	$\frac{W}{3I^2}$	W	$\cos\varphi$
17	$X\,R\,W$	$\sqrt{\frac{W\,(X^2+R^2)}{R}}$	$\sqrt{\frac{W}{3\,R}}$	X	R	W	$\frac{R}{\sqrt{R^2+X^2}}$
18	$X\,R\cos\varphi$	undeterm.	undeterm.	X	R	undeterm.	$\cos\varphi$
19	$X\,W\cos\varphi$	$\sqrt{\frac{W\,X}{\cos\varphi\,\sqrt{1-\cos^2\varphi}}}$	$\sqrt{\frac{W\,\sqrt{1-\cos^2\varphi}}{3\,X\cos\varphi}}$	X	$\frac{X\cos\varphi}{\sqrt{1-\cos^2\varphi}}$	W	$\cos\varphi$
20	$R\,W\cos\varphi$	$\sqrt{\frac{W\,R}{\cos^2\varphi}}$	$\sqrt{\frac{W}{3\,R}}$	$\frac{R\,\sqrt{1-\cos^2\varphi}}{\cos\varphi}$	R	W	$\cos\varphi$

Numerical Example. A 3000-kv-a. transformer bank with taps for 75, 85 and 100 volts with 44,000 volts high-tension voltage feeds a three-phase electric furnace. The bus bars are connected delta, so that the transformer voltage, is also the voltage between phases. The transformer losses at full-load are 2 per cent, its reactance 6 per cent at the highest tap. We assume that the magnetizing current is negligible.

The following quantities have been measured by instruments:

High-tension line current: 40 amperes per phase

High-tension voltage: 44,000 volts between phases

The high tension is connected to give 100 volts on the low tension

Total power input high tension 2600 kw.

Using Case No. 3 of Table I, we find

$$R = \frac{2600 \times 10^3}{3 \times 40^2} = 542 \text{ ohms}$$

The resistance per phase on the secondary side is

$$R_{tot}\ \frac{542 \times 100^2}{44000^2} = 0.00280 \text{ ohms}$$

The low-tension current is

$$\frac{40 \times 44000}{100} = 17600 \text{ amperes}$$

$$X_{tot} = \sqrt{\frac{100^2}{3 \times 17600^2} - \frac{2600^2 \times 10^6}{9 \times 17600^4}}$$

$$= 0.00171 \text{ ohms}$$

The full-load current of a 3000 kv-a. transformer bank for 100 volts is 17,300 amperes in the furnace conductors, so that with 2 per cent resistance and 6 per cent reactance

$$R_{transf.} = \frac{3000 \times 10^3 \times 2}{3 \times 17{,}300^2 \times 100} = 0.000067 \text{ ohms}$$

$$X_{transf.} = \frac{3000 \times 10^3 \times 6}{3 \times 17{,}300^2 \times 100} = 0.0002 \text{ ohms}$$

assume that R_{cond} has been figured to be 0.000133 ohms so that $R_{transf.} + R_{cond} = 0.0002$ ohms.

$R_{av} = 0.00280 - 0.0002 = 0.0026$ ohms
$X_{av} = 0.00171 - 0.0002 = 0.00151$ ohms

$$\text{Efficiency} = \frac{0.0026}{0.0028} = 92.8 \text{ per cent}$$

$$\text{Aver. power factor} = \frac{2600 \times 10^3}{100 \times 17{,}600 \times \sqrt{3}} = 0.853$$

$$\text{Power input into furnace} = 2600 \times \frac{92.8}{100} = 2412 \text{ kw.}$$

After operating a while the furnace at 100 volts it is decided to change the taps to 85 volts. What will be the new conditions for the same high-tension current?

At 85 volts with 40 amperes high-tension current, the low-tension current will be $\dfrac{40 \times 44{,}000}{85} = 20{,}720$ amperes.

Using Case No. 1 of Table I:

$E = 85$ volts $I = 20{,}720$ a. $X_{tot} = 0.00171$ ohms

$$R_{tot} = \sqrt{\frac{85^2}{3 \times 20{,}720^2} - 0.00171^2} = 0.001643 \text{ ohms}$$

$R_{av} = 0.001443$
$W = 3 \times 20{,}720^2 \times 0.001643 = 2114$ kw.

$$\text{Power input into furnace} = \frac{2114 \times 0.001443}{0.001643} = 1856 \text{ kw.}$$

Efficiency = 87.9 per cent

Power factor

$$= \sqrt{1 - 3 \,.\, \frac{20{,}720^2 \times 0.00171^2}{85^2}} = 0.693$$

It can easily be shown that with increasing current the power input passes through a maximum for $R_{tot} = X_{tot}$. After this point any further increase in current means a decrease in power.

Taking, in case 5,

$$W = \frac{E^2 R}{X^2 + R^2},$$

where E and X are constant.

The condition for maximum power is

$$\frac{dW}{dR} = 0 = \frac{E^2 (X^2 + R^2 - 2R^2)}{(X^2 + R^2)^2}$$

$$0 = X^2 - R^2 \qquad X^2 = R^2$$
$$X = R$$

$$\text{Power Factor} = \frac{R}{\sqrt{X^2 + R^2}} = \sqrt{2}/2 = 0.707$$

$$I = \frac{E}{\sqrt{3\,(X^2 + R^2)}} = \frac{E}{X\sqrt{6}}$$

In our case, the power input is a maximum for $R = 0.00171$ ohms.

$$I = \frac{85}{0.00171 \,.\, \sqrt{6}} = 20{,}310 \text{ amperes}$$

$$W = \frac{3 \times 0.00171 \times 20{,}310^2}{1000} = 2116 \text{ kw.}$$

$$\text{Efficiency} = \frac{0.00171 - 0.00020}{0.00171} = 88.3 \text{ per cent}$$

Power factor = 0.707. Power delivered to furnace = 2116 × 0.883 = 1867 kw.

The power delivered to the furnace passes also through a maximum but at a lower current as the total power input

$$W_{furn} = \frac{W \,.\, R_{av}}{R_{tot}} = \frac{E^2 (R_{tot} - R_{losses})}{R_{tot}^2 + X_{tot}^2}$$

$$\frac{d W_{fce}}{d R_{tot}}$$

$$= E^2 \frac{R_{tot}^2 + X_{tot}^2 - 2\,(R_{tot} - R_{losses})\,R_{tot}}{(R_{tot}^2 + X_{tot}^2)} = 0$$

$$0 = X_{tot}^2 - R_{tot}^2 + 2\,R_{tot} \,.\, R_{losses}$$

$$R_{tot} = R_{losses} + \sqrt{R_{losses}^2 + X_{tot}^2}$$

In our case:

$$R_{tot} = 0.0002 + \sqrt{0.0002^2 + 0.00171^2}$$
$$= 0.0002 + 0.00172 = 0.00192 \text{ ohms}$$

$$I = \frac{85}{\sqrt{0.00192^2 + 0.00171^2} \,.\, \sqrt{3}}$$

19,120 amperes

$W_{input} = 3 \times 19{,}120^2 \times 0.00192 = 2104$ kw.
$W_{fce} = 3 \times 19{,}120^2 \times 0.00172 = 1885$

$$\text{Efficiency} = \frac{0.00172}{0.00192} = 89.6 \text{ per cent}$$

Power factor, (High-tension side) = 0.747.

It will therefore be more economical in our case to drop the high-tension current to $\dfrac{19{,}120 \times 85}{44{,}000} = 37$ amperes when we change the voltage from 100 to 85 volts on the low-tension side. This current will give us maximum power into the furnace at that voltage.

Problems like the above can easily be solved by the analytical method; nevertheless it is generally simpler to prepare sets of curves giving for each available low-tension voltage the power input in transformer and furnace, the power factor and if desired the low-tension resistance in functions of the low or high tension-current, depending on the manner in which the measuring instruments are connected, and whether high or low-tension current is mostly used for computations.

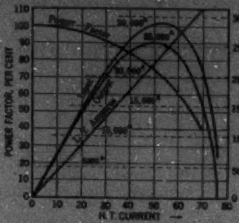

FIG. 1—CHARACTERISTICS OF ELECTRIC FURNACE FROM SIMPLIFIED CIRCLE DIAGRAM (SEE FIG. 2)

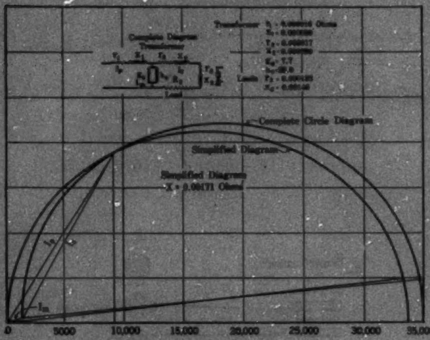

FIG. 2—COMPARISON BETWEEN SIMPLIFIED AND COMPLETE CIRCLE DIAGRAM

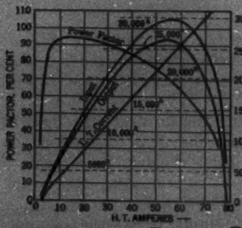

FIG. 3—CHARACTERISTICS OF ELECTRIC FURNACE FROM COMPLETE CIRCLE DIAGRAM

The two circle diagrams are so arranged that for 40 amperes high-tension current the kw. input and the power factor are the same. In comparing Figs. 1 and 3 one can see the error which is introduced by using a simplified diagram which neglects the magnetising current.

Fig. 1 gives a set of curves for 100 volt low-tension voltage and $X = 0.00171$ ohms. One can naturally, if greater accuracy is desired, draw an exact circle diagram similar to the one used for induction motors, but it will generally be quite sufficient for practical purposes to use curves as shown above.

Fig. 2 shows a comparison between the simplified circle diagram corresponding to the curves on Fig. 1 and the theoretical diagram which takes the magnetizing current of the transformer into account. This last circle has been drawn so as to give the same values as the simplified circle for 40 amperes high-tension current.

INFLUENCE OF UNBALANCING

As will be shown later the value of α, which is the ratio between $X_{1\text{-}3}$ and $X_{1\text{-}2}$, can be taken as being between 1.2 and 1.3 according to the dimensions and arrangements of the leads. In the case of balanced current $\beta = 1/2\sqrt{3}$.

The value of X_{av} will vary greatly, depending on the arrangement of the leads and the frequency. At 60 cycles, with well designed furnaces, values as low as 0.0015 were found, but in the same plant were furnaces with X_{av} as high as 0.0021. At 25 cycles these furnaces would have shown values of 0.000625 and 0.000875 ohms respectively.

In the previous example, we had found $X_{av} = 0.00151$

but $X_{av} = X \dfrac{(\alpha + 2)}{3}$, and if we take $\alpha = 1.25$

$$X = \frac{0.00151 \times 3}{3.25} = 0.001394 \text{ say } 0.00140 \text{ ohms}$$

introducing these values in the equation (7) we find that

$$P_t = I^2 (\alpha - 1)\, \beta X = I^2 (0.25 \times \sqrt{3/2} \times 0.00140) = 0.000303\, I^2$$

for $I = 17{,}600$.

$$P_t = \frac{17{,}600^2 \times 0.000303}{1000} = 94.5 \text{ kw.}$$

The total power consumed by each phase is

$$W_1 = I^2 (R_{av} + R_{losses} - {}^1/_6 (\alpha - 1)\, X \sqrt{3})$$

$$= \frac{17{,}600^2}{1000} (0.0028 - 1/6\, (1.25 - 1)\, 0.0014\, .\, \sqrt{3} = 836 \text{ kw.}$$

$$W_2 = I^2 (R_{av} + R_{losses})$$

$$= \frac{17{,}600^2}{1000} \times 0.0028 = 867 \text{ kw.}$$

$$W_3 = I^2 (R_{av} + R_{losses}) + 1/6\, (\alpha - 1) \times \sqrt{3})$$

$$= \frac{17{,}600^2}{1000^2} (0.0028 + 1/6\, (1.25 - 1)\, 0.0014 \sqrt{3} = 898 \text{ kw.}$$

Total power input 2601 kw.

The losses per phase are

$$\frac{17{,}600^2}{1000} \times 0.0002 = 63 \text{ kw.}$$

The power delivered by each electrode to the furnace will be

Phase 1,
$W_a = 886 - 63 - 94.5 = 678.5$ kw. or 28.1 per cent
Phase 2,
$W_b = 867 - 63 = 804$ kw. or 33.3 per cent
Phase 3,
$W_c = 898 - 63 + 94.5 = 929.5$ kw. or 38.6 per cent
Total power delivered
to furnace 2412.0 kw.

The difference between the phases is therefore quite large, and would still be larger for furnaces with higher phase reactance.

Since it appears that the arc acts as a straight resistance, the arc voltage on each phase is easily calculated:

$$\text{Arc voltage phase 1 } \frac{678{,}500}{17{,}600} = 38.55 \text{ volts}$$

$$\text{phase 2 } \frac{804{,}000}{17{,}600} = 45.70 \text{ volts}$$

$$\text{phase 3 } \frac{929{,}500}{17{,}600} = 52.8 \text{ volts.}$$

Taking now the point where the power input on the high-tension side is a maximum, with the low-tension at 85 volts and the low-tension current 20,310 amperes, we have

	Total power consumed	Losses	Power due to transformer action	Power delivered to furnace	In per cent	Arc voltage
Phase 1......	664	− 82.5	− 124.5	= 457	24.44	22.5 volts
Phase 2......	705.5	− 82.5		= 623	33.38	30.7 volts
Phase 3......	747	− 82.5	+ 124.5	= 789	42.28	38.9 volts
Total....	2116.5 kw.			1869 kw.	100.00	

The lower the average power factor the larger are the differences between phases.

In the case studied above the value of X in equation (7) was 0.001394, this is rather low, and furnaces with larger values will be found, also α may be larger than 1.25. It follows that the values calculated above are rather favorable and that much larger differences between phases are possible.

This difference of voltage and power input between the phases is detrimental to the good operation of the furnace. The stock will move much more rapidly around electrode No. 3, and have a tendency to stick around No. 1.

The difference in voltage may affect the uniformity of the product. It is therefore necessary to design the furnace and arrange the leads in such manner as to make the difference between phases as small as possible.

SUGGESTIONS TO REDUCE UNBALANCING

Equation (7) gives the value of the power transmitted from phase 1 to phase 3.

$$P_t = I^2 (\alpha - 1) \beta X$$

For balanced currents $\beta = 1/2 \sqrt{3}$

$$\alpha = X_{1.3}/X \quad \text{where} \quad X = X_{12} = X_{2.3}$$

For three parallel conductors, one of which carries the return current of the other two.

$$X_{12} = X_{23} = X = 2 \pi f \times 2 l \left(\log_e d/a + 1/4 - d/l\right) \times 10^{-9} \text{ in ohms}$$

If the three conductors are in the same plane, as assumed.

$$X_{13} = 2 \pi f \,.\, 2 l \left(\log_e \frac{2d}{\alpha} + 1/4 - \frac{2d}{l} \right) . 10^{-9} \text{ in ohms}$$

f is the frequency
l is the length of the circuit in cm.
d the distance between center lines of adjacent phases in cm.
a the geometrical radius of the conductor in cm.

This formula is only an approximation and assumes that d/l and a/d are small.

$$\alpha = X_{13}/X = \frac{\log_e \frac{2d}{a} + 1/4 - \frac{2d}{l}}{\log_e d/a + 1/4 - d/l} = 1 + \frac{\log_e 2 - d/l}{\log_e d/a + 1/4 - d/l}$$

$$\alpha = \text{approx. } 1.43 - 0.34\, d/l - 0.016\, d/a$$

$$\text{for } d/l < 0.25 \text{ and } 6 < d/a < 10$$

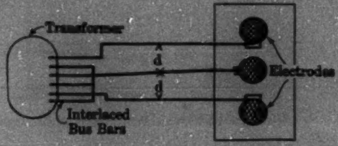

FIG. 4

P_t can be reduced by reducing X or α

(1) X being proportional to the frequency, the same furnace will be more satisfactory at 25 cycles than at 60.

(2) The closer the furnace is brought to the transformer the better, since X is proportional to the length of the leads. In case the distance is given, the bus bars should be carefully interlaced as close to the furnace as possible.

(3) The leads should be as close together as possible.

(4) The conductors should be of large dimensions.

The use of extra flexible cables will often permit to reduce l.

To reduce the distance between leads the method shown in Fig. 4 is suggested. The distance between the leads is smaller than the distance between electrodes.

Where two water-cooled pipes are used to carry the current they should be placed in a vertical rather than in a horizontal plane (Fig. 5).

The use of hollow conductors will help to increase α. When using cables, they should form a hollow cylinder, each cable making a complete turn to avoid unequal current distribution (Fig. 6). The only way to reduce α without at the same time increasing X is to make l small.

If the conductors could be placed so as to form the 3 edges of an equilateral prism α would be equal to 1. Even if the ideal case cannot be attained, it would be possible to raise or lower conductor 2 out of the plane 1 3, and it will often prove advisable to bring electrode 2 forward so as to make tapping easier.

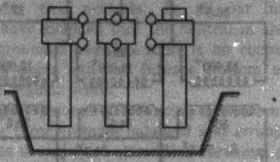

Fig. 5

It is interesting to study under which conditions the powers delivered by the three electrodes are balanced.

The condition for equal delivered power is:

$$I_1^2 R_a = I_2^2 R_b = I_3^2 R_c \qquad (17)$$

1st *Case.* $I_1 = I_2 = I_3$ that is balanced currents. From equation (17) it follows that $R_a = R_b = R_c = R$, the three load resistances are equal

The conditions for balanced currents is

$$\gamma = 1/2 \quad \epsilon = 1/2 \quad \beta = 1/2\sqrt{3} \quad \gamma^2 + \beta^2 = 1$$

Introducing these conditions in equation (8), we get

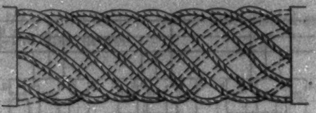

Fig. 6

$$\left.\begin{aligned} \dot{E}_3 &= I\left[-\left(\frac{3R}{2} + \alpha/2 \times \sqrt{3}\right)\right. \\ &\qquad \left. - j\,(R/2\sqrt{3} + (2 - \alpha/2)\,X)\right] \\ \dot{E}_1 &= I\left(\frac{3R}{2} - \alpha/2 \times \sqrt{3}\right) \\ &\qquad - j\left(\frac{R\sqrt{3}}{2} + (2 - \alpha/2)\,X\right)\Big] \\ \dot{E}_2 &= I\,(\alpha \times \sqrt{3}) - j\,(-R\sqrt{3}) \end{aligned}\right\}$$

$$\left.\begin{aligned} E_3^2 &= I^2\,(3R^2 + X^2\,(\alpha^2 - 2\alpha + 4) \\ &\qquad + 2XR\sqrt{3}\,(\alpha - 1) \\ E_1^2 &= I^2\,(3R^2 + X^2\,(\alpha^2 - 2\alpha + 4) \\ &\qquad - 2XR\sqrt{3}\,(\alpha - 1) \\ E_2^2 &= I^2\,(3R^2 + X^2\,(3\alpha^2) \end{aligned}\right\}$$

$$\left.\begin{aligned} E_3^2 &= I^2 X^2\,(3R^2/X^2 + 3\alpha^2 \\ &\qquad + 2\,(\alpha - 1)\,[R/X\sqrt{3} - (\alpha + 2)\,] \\ E_1^2 &= I^2 X^2\,(3R^2/X^2 + 3\alpha^2 \\ &\qquad - 2\,(\alpha - 1)\,[R/X\sqrt{3} + (\alpha + 2)\,] \\ E_2^2 &= I^2 X^2\,(3R^2/X^2 + 3\alpha^2) \end{aligned}\right\}$$

Table II gives the value of the square root of the factor in parenthesis for different values of α and R/X. To obtain the respective voltage multiply by $I\,X$.

TABLE II

R/X		$\alpha = 1$	1.1	1.2	1.3	1.4	ϵ
	E_3	1.7321	1.7349	1.7436	1.7578	1.7776	
0	E_1	1.7321	1.7349	1.7436	1.7578	1.7776	0
	E_2	1.7321	1.9053	2.0785	2.2517	2.4249	
$\frac{1}{2\sqrt{3}}$	E_3	1.8028	1.8330	1.8682	1.9079	1.9519	$\frac{1}{3}$
	E_1	1.8028	1.7776	1.7578	1.7436	1.7349	
	E_2	1.8028	1.9698	2.1378	2.3065	2.4759	
$\frac{1}{\sqrt{3}}$	E_3	2.00	2.0518	2.1071	2.1656	2.2271	$\frac{2}{3}$
	E_1	2.00	1.9519	1.9079	1.8682	1.8330	
	E_2	2.00	2.1517	2.3065	2.4637	2.6230	
$\frac{2}{\sqrt{3}}$	E_3	2.6458	2.7221	2.8000	2.8792	2.9597	$\frac{4}{3}$
	E_1	2.6458	2.5701	2.4980	2.4269	2.3580	
	E_2	2.6458	2.7623	2.8844	3.0116	3.1433	
$\frac{3}{\sqrt{3}}$	E_3	3.4641	3.5511	3.6387	3.7269	3.8158	
	E_1	3.4641	3.3779	3.2924	3.2078	3.1241	2
	E_2	3.4641	3.5539	3.6497	3.7510	3.8575	
$\frac{5}{\sqrt{3}}$	E_3	5.2915	5.39	5.48	5.58	5.67	$\frac{10}{3}$
	E_1	5.2915	5.20	5.10	5.01	4.92	
	E_2	5.2915	5.35	5.42	5.48	5.56	
$\frac{10}{\sqrt{3}}$	E_3	10.1489	10.25	10.34	10.44	10.54	$\frac{20}{3}$
	E_1	10.1489	10.05	9.95	9.85	9.76	
	E_2	10.1489	10.18	10.21	10.25	10.29	
$\frac{100}{\sqrt{3}}$	E_3	100.0	100.1	100.2	100.3	100.4	$\frac{200}{3}$
	E_1	100.0	99.9	99.8	99.7	99.6	
	E_2	100.0	100.0	100.0	100.0	100.0	

These factors multiplied by $I\,X$ give the voltage to be applied between phases on the low tension, to get balanced loads in the furnace with balanced currents.

Since the currents are balanced R may include the resistance of the conductors.

Example:

Take $X = 0.00140$ ohms, $I = 19{,}000$ amperes, $\alpha = 1.3$ and $R/X = 2/\sqrt{3}$ we shall include in R the resistance of the leads which is taken as 0.000133 ohms.

In this case

$$E_3 = 2.8792\,I\,X = 2.8792 \times 26.6 = 76.5 \text{ volts}$$
$$E_1 = 2.4269\,I\,X = 2.4269 \times 26.6 = 64.5 \text{ volts}$$
$$E_2 = 3.0116\,I\,X = 3.0116 \times 26.6 = 80.1 \text{ volts}$$

The power delivered by each electrode is

$$W_a = W_b = W_c = \frac{19{,}000^2}{1000}\,(0.00140 \times 2/\sqrt{3} - 0.000133) \text{ kw.} = 537 \text{ kw.}$$

The losses in the leads are

$$= \frac{19{,}000^2}{1000} \times 0.000133 \text{ kw. per phase} = 48 \text{ kw.}$$

The power transmitted from phase 1 to phase 3 is

$$P_t = \frac{19{,}000^2}{1000} (\alpha - 1)\, \beta\, X \text{ kw.}$$

$$= \frac{19{,}000^2}{1000} (0.3 \times \sqrt{3/2} \times 0.0014) \text{ kw.} = 131 \text{ kw.}$$

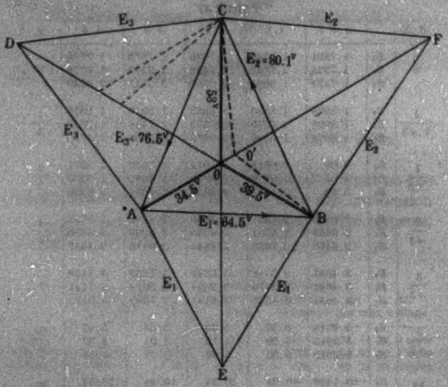

FIG. 7

So that the total power consumed by each phase will be

		In Per Cent	Voltage
$W_1 = 537 + 48 + 131 =$	716 kw.	40.8	28.3 volts
$W_2 = 537 + 48 =$	585	33.3	28.3 volts
$W_3 = 537 + 48 - 131 =$	454	25.9	28.3 volts
	1755 kw.		

If the transformers were connected in delta on the low-tension, the voltages calculated above are the voltages which the three transformers should give at the chosen load. However since we assume balanced voltage on the high-tension, that is voltages which are 120 deg. apart, it is impossible to have 3 low-tension voltages which do not form an equilateral triangle. It follows that we cannot use the proposed method with a delta on the low tension.

If the transformers are connected in star on the low-tension one can always find three transformer voltages, 120 deg. apart, which will give the desired voltage across the star using special taps if necessary. The diagram in Fig. 7 will show how to find the three voltages.

Draw 3 equilateral triangles with E_1, E_2 and E_3 as bases. Join *A F*, *B D*, *C E*; these 3 lines meet in one and the same point 0 and the angles *A O B*, *B O C*, *C O A* are equal to 120 deg.

O A, *O B*, *O C* are the desired transformer voltages. The no-load voltage should be selected so as to give this voltage at the chosen load. The kv-a. on the high and low side of the transformers are equal if we neglect the magnetizing current; the three high-tension voltages are equal and 120 deg. apart, while the three low-tension currents are equal and 120 deg. apart. It follows that in each transformer high-tension current and low-tension voltage are proportional. The three low-tension transformer voltages do not necessarily form a closed triangle so that the only possible connection on the high side must be a delta, where the currents do not necessarily form a closed triangle.

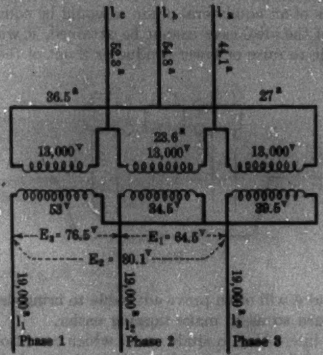

FIG. 8—DISTRIBUTION OF CURRENTS AND VOLTAGES

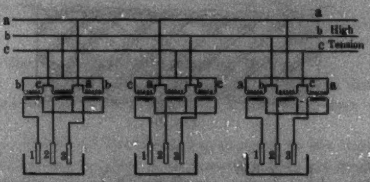

FIG. 9—DISTRIBUTION OF FURNACES TO KEEP HIGH TENSION BALANCED

We see therefore that the only combination which will permit us to use the proposed method of balancing the powers delivered to the furnace is star on the low side and delta on the high side.

The three line currents on the high side will be unbalanced, being proportional to the low-tension volts between phases. Fig. 8 gives a distribution of voltage and current in our case, assuming a high-tension voltage of 13,000 volts between phases. It is evident that any power company would object to any such unbalancing.

In a plant where there are several furnaces in operation this unbalancing can be corrected by connecting each high-tension line in succession to a different low-tension phase as shown in Fig. 9 being careful not to change the direction of phase rotation.

It may be objected that any change in load requires another value of the voltage. Since electric furnaces are mostly run at constant load, and changes are only of short duration, it would be quite satisfactory to keep the voltages unchanged.

If we plot in Fig. 7, the neutral point 0, in the furnace we see that it does not coincide with the neutral in the bus bars. There is a slight difference of voltage between these two neutrals. In all our investigations, we have assumed that the neutral in the bus bars is not grounded and that there is no connection between this point and the furnace bottom except through the regular leads.

Second Case. Instead of balancing the currents we can keep the three voltages balanced and arrange the currents so as to satisfy the condition for equal power on the three electrodes.

We have again $I_1^2 R_a = I_2^2 R_b = I_3^2 R_c$

The conditions for balanced voltage is

$$\dot{E}_1 = \dot{E}_3 \left(-1/2 + j\, 1/2 \sqrt{3}\right)$$

$$\dot{E}_2 = \dot{E}_3 \left(-1/2 - j\, 1/2 \sqrt{3}\right)$$

introducing these conditions in (8), remembering that $I_1^2 = I^2(\beta^2 + \gamma^2)$, $I_2^2 = I^2$ and $I_3^2 = I^2(\beta^2 + \epsilon^2)$, and solving for β and γ, we get two equations of the third degree.

$$y^2 (4 - \alpha + 3\alpha z) - 6 g y (z - 1) + (3 \alpha z - 4 + \alpha)(1 + 3 z^2) = 0$$

$$3 z^2 (\alpha y + g) - 2 z [(4 - \alpha) y + 3 g] + \alpha y - g y^2 - \alpha y + 3 g = 0$$

where $g = \dfrac{2 R \sqrt{3}}{3 X}$, $y = 1 - 2\gamma$ and $z = \dfrac{2\beta}{\sqrt{3}}$

Solving for y and z leads to rather complicated expressions of the fifth degree.

Keeping α constant and varying g from 0 to ∞ gives one set only of continuous values for γ ranging from $\gamma = 1$ for $g = 0$ and tending towards $\gamma = 1/2$ for $g = \infty$, the corresponding values of β being 0 for $g = 0$ and $1/2 \sqrt{3}$ for $g = \infty$.

For the special case where $\alpha = 1$, that is when the three reactances are equal, there are three sets on values forming continuous curves from $g = 0$ to $g = \infty$; the general solution is $\gamma = \epsilon = 1/2$ and $\beta = 1/2 \sqrt{3}$ for every value of g, it is the case of balanced currents.

But it is interesting to know that for every value of g there are besides this general solution, two other solutions which give equal powers under the three electrodes, but with unbalanced currents. For certain values of g there are two more additional solutions but the curves on which they lie extend only over a small range of g. Similarly when $\alpha \neq 1$ there are values of g for which there are three and sometimes five solutions.

Of practical interest is only the one set of solutions which gives a continuous series of values from $g = 0$ to $g = \infty$.

Table III gives the values of β, γ and ϵ for different values of g, and for $\alpha = 1$ and $\alpha = 1.25$, while Table IV gives the corresponding values of $\sqrt{\beta^2 + \gamma^2}$ and $\sqrt{\beta^2 + \epsilon^2}$.

Take as an example, the case of $g = 2$, $\alpha = 1.25$ when $X = 0.00150$ ohms and $E = 110$ volts.

$$g = \frac{2 R \sqrt{3}}{3 X} \quad R = \frac{3 g X}{2\sqrt{3}} = \frac{2.3 \times 0.0015}{2 \times \sqrt{3}}$$

$$= 0.0026 \text{ ohms}$$

From Table III we find $\beta = 0.513$, $\gamma = 1.0825$, $\epsilon = 0.9175$. Introducing these values in (8), we find

$$\dot{E}_3 = I\left(-0.005517 + j\,(0.0022245)\right)$$

$$\dot{E}_1 = I\left(+0.000836 - j\,(0.0059205)\right)$$

$$\dot{E}_2 = I\left(+0.004681 + j\,(0.0036960)\right)$$

TABLE III

g	γ_1	ϵ_1	β_1	γ_2	ϵ_2	β_2	γ_3	ϵ_3	β_3	γ_4[1]	γ_5[1]
$\alpha = 1$											
0	+ 1.0	0	0	0.5	0.5	0.866	0.0	1	0	..	..
1	+ 1.23075	− 0.23075	0.295	0.5	0.5	0.866	0.2712	0.7288	0.288	..	..
2	+ 1.1425	− 0.1425	0.550	0.5	0.5	0.866	0.323	0.677	0.451	..	..
4	+ 0.9325	+ 0.0675	0.746	0.5	0.5	0.866	0.3766	0.6234	0.592	..	..
10	+ 0.6948	+ 0.3052	0.843	0.5	0.5	0.866	0.4333	0.5667	0.682	0.3025	− 3.25
∞	+ 0.5	+ 0.5	0.866	0.5	0.5	0.866	0.5	0.5	0.866	..	..
$\alpha = 1.25$											
g	γ_1	ϵ_1	β_1	γ_2	ϵ_2	β_2	γ_3	ϵ_3	β_3	γ_4[1]	γ_5[1]
0	1	0	0	0.5	0.5	0.635	0	1	0	..	..
1	1.1675	0.8325	0.302	0.4694	0.5306	0.620	0.3193	0.6807	0.34	..	..
2	1.0825	0.9175	0.513	..	..	..	..	..	..	..	..
3.168	0.9613	0.0387	0.634	..[2]	..	..	..	..	..	..	..
4	0.897	0.103	0.678	..	..	..	..	..	..	..	..
10	0.701	0.299	0.7625	..	..	..	..	..	..	− 0.023	− 2.15
∞	0.5	0.5	0.866	..	..	..	..	..	..	..	..

Values of γ, ϵ and β for various values of g and $\alpha = 1$ and $\alpha = 1.25$.

1. Due to the negligible importance of these values the corresponding figures for B and E have not been calculated.
2. Where no figures are shown the values are not real.

TABLE IV

g		α = 1			α = 1.25		
		1	2	3	1	2	3
0	I_1	1	1.0	0	1.0	0.808	0
	I_2	1	1.0	1	1.0	1.0	1.0
	I_3	0	1.0	1	0	0.808	1.0
1	I_1	1.266	1.0	0.3607	1.206	0.777	0.467
	I_2	1.0	1.0	1.0	1.0	1.0	1.0
	I_3	0.374	1.0	0.767	0.345	0.815	0.761
2	I_1	1.268	1.0	0.555	1.198	..	..
	I_2	1.0	1.0	1.0	1.0	..	..
	I_3	0.568	1.0	0.813	0.519	..	..
	I_1				1.152	..	..
3.168	I_2	not calculated			1	..	..
	I_3				0.635	..	..
4	I_1	1.194	1.0	0.702	1.125	No values	No Values
	I_2	1.0	1.0	1.0	1.0		
	I_3	0.750	1.0	0.859	1.686		
10	I_1	1.093	1.0	0.808	1.025	..	..
	I_2	1.0	1.0	1.0	1.0	..	..
	I_3	0.897	1.0	0.887	0.818	..	..
∞	I_1	1.0	1.0	1.0	1.0	..	..
	I_2	1.0	1.0	1.0	1.0	..	..
	I_3	1.0	1.0	1.0	1.0	..	..

Multiply by I to obtain I_1, I_2, and I_3.

It follows that $E = I \times 0.00596$. Since we have assumed E to be 110 volts we have

$$I = \frac{110}{0.00596} = 18{,}470 \text{ amperes}$$

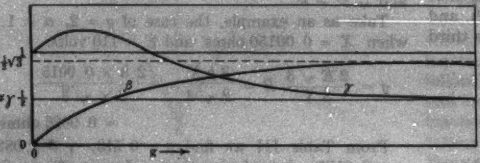

FIG. 10

from Table IV we find:

$$I_1 = 18{,}470 \times 1.198 = 22{,}140 \text{ amperes}$$

$$I_2 = 18{,}470 \times 1.0 = 18{,}470 \text{ amperes}$$

$$I_3 = 18{,}470 \times 0.519 = 9580 \text{ amperes}$$

Power delivered to furnace by each phase:

$$W_a = W_b = W_c = \frac{18{,}470^2}{1000} \times 0.0026 = 887 \text{ kw.}$$

$$P_t = I^2 (\alpha - 1)\, \beta\, X = \frac{18{,}470^2}{1000} (0.25) \times 0.513 \times 0.0015 = 65.6 \text{ kw.}$$

$$W_1 = W_a + P_t = 887 + 65.6 = 952.6$$

Power consumed by Phase 1

$$W_2 = 887.0$$

Power consumed by Phase 2

$$W_3 = W_c - P_t = 887 - 65.6 = 821.4$$

Power consumed by Phase 3

$$2661.0 \text{ kw.}$$

The arc on electrode 1 will have a voltage of

$$\frac{887{,}000}{22{,}140} = 40.1 \text{ volts}$$

The arc on electrode 2 will have a voltage of

$$\frac{887{,}000}{18{,}470} = 48.0 \text{ volts}$$

The arc on electrode 3 will have a voltage of

$$\frac{887{,}000}{9580} = 92.5 \text{ volts}$$

As in the previous case, but in a larger degree the high tension currents will be unbalanced.

When the high tension, and the low tension are both in star or both in delta the high-tension line currents will be proportional to the correspondent low-tension currents.

If the high tension is in star and the low tension in delta, the high tension line currents can be found according to the diagram of Fig. 12, and in the case of high tension in delta and low in star according to Fig. 14.

In Fig. 12 the low-tension transformer currents I_a, I_b, I_c pass through the center of gravity of the triangle formed by the low-tension line currents. Dividing I_a, I_b, I_c by the voltage ratio of the transformers gives the high tension line currents.

In Fig. 14, i_1, i_2, i_3 are proportional to I_1, I_2, I_3. The lengths $A\,B$, $B\,C$, $C\,A$ represent I_a, I_b, I_c respectively.

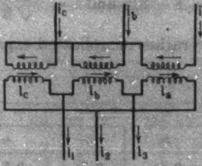

FIG. 11

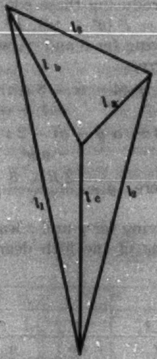

FIG. 12

Although this method of balancing the power delivered by the electrodes is easier to apply than the previous one,—balanced currents—it has two big disadvantages: First the currents on the high-tension side are very unbalanced, and secondly the 3 arc voltages are very different so that although the powers are the same, the conditions under the electrodes are different with their bad effect on the product.

Notwithstanding this rather negative result the investigation of this second case will show in which direction to go to correct partly for the unbalancing of the loads. In a furnace under observation the difference in the loads was very large under balanced currents and showed badly on the electrode consumption, phase No. 3 using many more electrodes than 1 and 2. An increase of 10 per cent in the current of phase 1 and a somewhat larger decrease of 3 proved very successful.

INVESTIGATION OF OPEN DELTA CONNECTIONS

In order to study the distribution of currents in case the materials between electrodes should act as a resistor, and the energy should be consumed in that part of the furnace we can use equation (8)—setting $R_b = 0$. For reasons of symmetry $R_a = R_c = R$. Solving for γ and β we get:

$$\gamma = \frac{3\,R^2 + 3\,\alpha^2\,X^2 + 2\,X\,R\,(2-\alpha)\sqrt{3}}{6\,(R^2 + \alpha^2\,X^2)}$$

$$= 1/2 + \frac{2-\alpha}{3}\;\frac{R/X\,\sqrt{3}}{R^2/X^2 + \alpha^2}$$

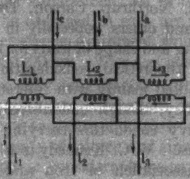

FIG. 13

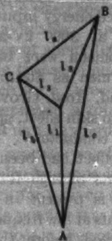

FIG. 14

$$\epsilon = \frac{3\,R^2 + 3\,\alpha^2\,X^2 - 2\,X\,R\,(2-\alpha)\sqrt{3}}{6\,(R^2 + \alpha^2\,X^2)}$$

$$= 1/2 - \frac{2-\alpha}{3}\;\frac{R/X\,\sqrt{3}}{R^2/X^2 + \alpha^2}$$

$$\beta = \frac{R^2 + X^2\,(4\,\alpha - \alpha^2)}{6\,(R^2 + \alpha^2\,X^2)}\sqrt{3} = \sqrt{3}/6$$

$$+ \frac{2-\alpha}{3}\;\frac{\alpha\,R^2/X^2\,\sqrt{3}}{R^2/X^2 + \alpha^2}$$

$$\gamma^2 + \beta^2 = 1/3 + \frac{2-\alpha}{3}\;\frac{2 + R/X\,\sqrt{3}}{R^2/X^2 + \alpha^2}$$

$$\epsilon^2 + \beta^2 = 1/3 + \frac{2-\alpha}{3}\;\frac{2 - R/X\,\sqrt{3}}{R^2/X^2 + \alpha^2}$$

$$I_1^2 = I^2\,(\gamma^2 + \beta^2)$$

$$I_2^2 = I^2$$

$$I_3^2 = I^2\,(\epsilon^2 + \beta^2)$$

$$I^2 = \frac{3\,E^2}{R^2 + X^2\,(4-\alpha)^2} \qquad I = \frac{E\,\sqrt{3}}{\sqrt{R^2 + X^2\,(4-\alpha)^2}}$$

Table V, gives the values of $\sqrt{\gamma^2 + \beta^2}$ and $\sqrt{\epsilon^2 + \beta^2}$ for different values of R/X and α. The factors given in the table when multiplied by I give us I_1, I_2 and I_3.

TABLE V.

$\frac{R}{X}$		$\alpha = 1$	1.1	1.2	1.3	1.4	e
0	I_1	1.0	0.910	0.839	0.780	0.733	0
	I_2	1.0	1.000	1.000	1.000	1.000	
	I_3	1.0	0.910	0.839	0.780	0.733	
$\frac{1}{2\sqrt{3}}$	I_1	1.051	0.956	0.878	0.814	0.760	$\frac{1}{3}$
	I_2	1.000	1.000	1.000	1.000	1.000	
	I_3	0.892	0.825	0.771	0.728	0.698	
$\frac{1}{\sqrt{3}}$	I_1	1.041	0.957	0.885	0.824	0.771	$\frac{2}{3}$
	I_2	1.000	1.000	1.000	1.000	1.000	
	I_3	0.764	0.727	0.695	0.669	0.648	
$\frac{2}{\sqrt{3}}$	I_1	0.951	0.897	0.847	0.801	0.759	$\frac{4}{3}$
	I_2	1.000	1.000	1.000	1.000	1.000	
	I_3	0.577	0.577	0.577	0.577	0.577	
$\frac{3}{\sqrt{3}}$	I_1	0.866	0.826	0.796	0.763	0.731	2
	I_2	1.000	1.000	1.000	1.000	1.000	
	I_3	0.500	0.512	0.522	0.532	0.541	
$\frac{5}{\sqrt{3}}$	I_1	0.764	0.744	0.724	0.704	0.685	$\frac{10}{3}$
	I_2	1.000	1.000	1.000	1.000	1.000	
	I_3	0.475	0.489	0.501	0.513	0.524	
$\frac{10}{\sqrt{3}}$	I_1	0.671	0.661	0.652	0.643	0.633	$\frac{20}{3}$
	I_2	1.000	1.000	1.000	1.000	1.000	
	I_3	0.505	0.514	0.522	0.529	0.537	
$\frac{100}{\sqrt{3}}$	I_1	0.586	0.585	0.584	0.583	0.582	$\frac{200}{3}$
	I_2	1.000	1.000	1.000	1.000	1.000	
	I_3	0.569	0.569	0.570	0.571	0.572	
∞	I_1	0.577	0.577	0.577	0.577	0.577	∞
	I_2	1.000	1.000	1.000	1.000	1.000	
	I_3	0.577	0.577	0.577	0.577	0.577	

Multiply by I to obtain I_1, I_2 and I_3.

The table shows that the currents are quite unbalanced, especially for large values of R. Such an unbalancing does not appear in a furnace under normal operation. The assumption that in alloy furnaces the current passes from one electrode to the nearest one through the charge is not correct.

CONCLUSIONS

The above study shows that the general performance of a three-phase electric furnace can easily be determined in advance with a large degree of accuracy by using the simple equations of a balanced three-phase system. The only thing that is necessary is to have previously determined X, or the low-tension reactance per phase, which is an important characteristic of the furnace.

On the other hand, due to the particular construction of the furnaces the individual phases show large differences which may affect the quality of the products. These differences can be calculated and once their cause and importance are well understood methods can be devised for their correction.

Discussion

For discussion of this paper see page 511.

Improvements in Ferro-Alloy Electric Furnaces of High Power Input

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***Review of the Subject.**—The paper deals with the subject of efficiency of power input in ferro-alloy furnaces and discusses the electrical factors to be considered in the design of the leads for achieving such efficiency. It also describes a new system of regulation, whereby the furnace temperature is kept constant by keeping the energy input constant by means of true watt regulation. Also several factors in design and construction, are discussed, which help to keep the load-factor of the operation as close to 100 per cent as possible.*

* * * * *

ALL metallurgical furnace operations can be divided into two general classes, the one consisting of reducing ores by means of a reducing agent, commonly carbon, and the other consisting of remelting and refining already reduced metals. The electric furnace has been adapted to both classes of operations. To the latter class belong the steel and brass, and to the former the ferro-alloy furnaces. As the heat energy required for reduction of ores is much greater than that required for the mere melting and refining of metals the ferro-alloy furnaces are fed by comparatively very much larger amounts of power. Consequently, the structure and arrangement of these furnaces have presented more interesting problems from the standpoint of the electrical engineer.

Since the classical experiments of Moissan more than thirty years ago to reduce metals in the electric furnace the application of electrical energy has constantly widened in this field. The furnaces have not only been constructed for a constantly increasing number of products but also have increased in their size and power input, so that today furnaces having a 12,000 or even more kv-a. capacity are commercially operating. Undoubtedly in the United States the Niagara district can be called the cradle of this industry. It was there that the carborundum, aluminum, graphite, carbide, and ferro-alloy furnaces had their inception and commercially established by the efforts of such men as Hall, Acheson and Price, and from there with the development of power projects spread to other industrial locations.

In the early days little was known of the electrical phenomena met with in alternating currents of higher than 25-cycle frequency and little attention was given to the purely electrical phases of the problem. Practically the only consideration given was to have sufficient conductor cross-section for a predetermined current. The pressure to be employed was arbitrarily chosen. It was considered dangerous to go over 100 volts on a furnace. The control of the arc and of the bath resistance, and the spacing of the electrodes were considered not practical with high voltages. Generally the pressure decided on was between 40 and 80 volts. As the size of the furnaces and their power inputs increased it was found that the conductors heated up excessively in spite of the fact that the current density was no greater than in the smaller initial installations. Addition of extra copper was helpful but did not effect a cure. The conductors insisted on heating up with a density as low as 400 amperes per square inch. Undoubtedly other factors had to be taken into consideration, especially as frequencies higher than 25-cycle had to be encountered. As 60-cycle current became more and more standardized it appeared for a time that furnace energy input at this frequency had reached its limit at 3000 kv-a. Meanwhile alternating-current phenomena were being studied and investigated by such men as Steinmetz, Northrup, Carson, Dwight, Roland, Rosa, Grover, and others, especially in reference to skin effect, spacing and shape of leads for heavy duty conductors. The theoretical considerations of skin effect had been studied mainly with reference to currents of high frequency, though undoubtedly this effect is present in heavy conductors carrying large currents at low frequencies. W. C. Kennedy in a paper presented before the Association of Iron and Steel Electrical Engineers in 1917 stated that the resistance loss due to skin effect on a 3750-kv-a., three-phase, furnace, at 12,000 amperes per phase, was by actual measurements about 11 volts per lead, the drop without skin effect being 0.15 volt for 10,000 amperes.

Numerous suggestions for cutting down impedance by methods of distribution of current, by proper spacing and dimensioning of bars, looping the circuit, so-called interlacing, etc., were forthcoming and eagerly investigated. Putting them into actual practise had, however, been deferred. It was, therefore, the desire of the authors to practically carry out these theoretical considerations in the design of ferro-alloy furnaces they were responsible for.

It appears to have been common practise for some time to employ tubular conductors for high-tension bus structures. However, they do not seem to have found use as conductors for large currents.

It appears to be generally accepted that the skin effect resistance factor of flat strip conductors is less than that of equi-sectional solid round conductors. This holds good only under certain conditions at com

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mercial frequencies, the effect being influenced by the relative position of parallel conductors the current values of which are of opposite sign. Surveying all available data it indicated that copper tubes would have much less skin effect than equi-sectional conductors of any other form.

The ideal a-c. installation would have a total impedance equal to that of a d-c. installation of equi-sectional conductors. This would mean that the factors of skin effect, reactance and inductance in a-c. leads must be eliminated. As the elimination or neutralization of all these factors is very difficult, if not impossible, it follows that the best solution of the problem in a practical installation, would be the use of conductors of such shape, spacing and arrangement that the total impedance or a-c. resistance ratio to the d-c. resistance would approximate unity.

In the design under consideration, preliminary experiments from a metallurgical standpoint had proved the advisability of high voltages and also high-current density. As 220 was a standard pressure it was chosen. In order to secure high-current density, graphite was selected as electrode material and the size chosen was 12 in., the largest commercially available. Keeping the current density on the electrodes at about 100 amperes per square inch, ten to twelve thousand amperes could be obtained in each arc which for a three-phase three electrode equipment would represent a total energy of 4000 kw. This was adapted as a standard unit.

Since prevailing electric furnace practise was to use woven standard conductors or multiple straps in the endeavor to secure increased surface for a given carrying capacity of copper, and since this practise was found to result in considerable resistance loss in the conductors, also since many instances were found where no attempt had been made to investigate these phenomena it was thought that an effort to obtain the relation between factors influencing the passage of large current through such conductors might point the way to better practise.

Accordingly, a current density of 800 amperes per square inch of copper section was decided on with conductors 40 feet long. It was decided to investigate mathematically 8 different types of secondary transformer lead construction. These methods together with the tabulated results are shown in Table 1. The first three columns show different arrangements of 4 in. by ⅛ in. copper strips; fourth column an arrangement of 5 in. diam. copper tubes; the fifth and sixth columns 4½ in. diam. copper tubes; the seventh and eighth 3 in. copper tubes interlaced.

The results thus obtained indicated two promising methods. The first being from column No. 3 consisting of 72 strips of 4 in. by ⅛ in. copper interlaced and spaced ⅛ in. apart. The second in column No. 8 consisting of 12 copper tubes of 3 in. diameter spaced 7 in. center to center. For the two systems considered, calculated values as tabulated indicate the ratio of the a-c. resistance to the d-c. resistance and the total line loss for the tubes to be practically ½ of that of the strips.

THEORETICAL ELECTRICAL CONDITIONS IN ELECTRODE LEADS OF VARIOUS TYPES
FOR 4,000 KV-A. ELECTRIC FURNACE

Electrical Properties									Numerical Relation to Property
Sizes	[illegible]	[illegible]	[illegible]	[illegible] Tubes per Phase	[illegible] Tubes per Phase	[illegible]	[illegible] Tubes Interlaced	[illegible] Tubes Interlaced	
Inductive Ckt. Type	Middle Phase	Middle Phase	[illegible]	Middle Phase	Middle Phase	[illegible]	Phase Balanced	[illegible]	When unbalanced
D.C. Resistance	[illegible] Ω	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] Ω	Phase 1 [illegible]
[illegible]	2.18	2.88	2.61	1.13	1.09	1.09	1.02	1.02	Phase 2 R Ω
A.C. Resistance	[illegible] Ω	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] Ω	Phase 3 [illegible]
Inductance	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			Phase 1 X [illegible]
Ind. Reactance	[illegible] Ω	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			Phase 2 X Ω
									Phase 3 X [illegible]
IR drop	2.50 V.	0.34	[illegible]	[illegible]	0.27	0.27	0.275	0.275 V.	
IX drop	52.0 V.	51.0	4.3	51.0	47.0	47.0	[illegible]	[illegible] V.	
% IR drop	[illegible] %	0.426	0.426	[illegible]	[illegible]	[illegible]	0.217	0.217 %	
% IX drop	41.0 %	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible] %	at Unity Power Factor
% Volts Loss	8.5 %	8.0 %	0.6 %	7.8 %	6.5 %	6.5 %	2.3 %	2.0 %	Total Volts Lost
Watts Line Loss	3×[illegible] W.	3×5400	3×5400	3×[illegible]	3×2700	3×2700	3×2700	3×2700 W.	[illegible] V. in
% Watts Loss	0.46 %	0.41 %	0.41 %	0.23 %	[illegible] %	[illegible] %	[illegible] %	[illegible] %	neutral

DATA

No. Electrodes-3-Y-Connected, [illegible] neutral
Transformer-1-3 Phase, Y Connected
Primary Voltage-22,000
Secondary Voltage-220
Amperes per Phase 10,000
Kva. Rating 4,000
Current Density in Electrode Leads-800 Amp. per □"
Frequency = 60~ Values calculated for Conductors [illegible] Ft. Long Each

TABLE I

The reactance volt-drop for all of the systems was then calculated for varying power factors at full load and results plotted as shown in Fig. 1 for power factor-voltage loss. Here the 4 in. by ⅛ in. strip interlaced showed up to much better advantage than the others for power factors of less than 99 per cent. On the other hand, as a preliminary experimental 750-kw. furnace was found to operate at a power factor very close to unity, it was decided that the same precaution taken to insure a high power factor could be utilized for the 4000-kw. unit. Moreover, the indicated reactance volt-drop for the interlaced 3-in. tubes would serve as a protection in case of sudden heavy overloads or possible short circuits, which, probably on very rare occasions only, are balanced in a 3-phase, 3-electrode system — the greater the state of unbalance the greater being the drop in voltage.

To summarize the tabulated calculations it is found that:

1. The larger the cross-section of the conductor the larger the skin-effect. This effect is appreciable at frequencies of 60 cycles for conductors greater than ½ in. diameter.
2. Skin-effect is higher in straps than in tubes of equi-sectional area with subsequent higher power loss.
3. Distribution of the current over the sectional area of a conductor is not affected by the inductive disposition of the conductor, since the internal inductance is unaffected by the mutual inductances.
4. Inductive reactance may cause considerable voltage drop in the electrode leads.
5. The larger the diameter of the conductors the less the inductance.

6. The farther apart the conductors the larger the inductance.

7. When inductive reactance expressed as per cent reactance drop amounts to 20 per cent or more, the actual volts drop in the line increases rapidly with decreased power factor.

Considering these factors, namely, the calculated voltage drop and line loss, the probable advantage of a progressively increasing voltage drop with decreasing power factor, and the fact that a bus structure of large copper tubing gave promise of a simple and rugged construction, it was decided to install 12 copper tubes of 3 in. diameter, interlaced and spaced 7 in. center to center, fixing the current density at approximately 800 amperes per square inch.

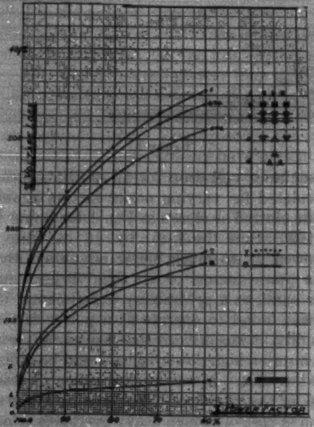

Fig. 1—Power Factor—Voltage Loss Curve for Various Types of Conductors

Next in order was to find a transformer so constructed that its secondary terminals could be brought out to allow the installation of the tubes as planned. The one purchased is wound with two secondary coils per phase, making four terminals per phase and the adjacent leads are brought out plus and minus. In the 12 terminals thus brought out the alternate plus and minus arrangement is continued over the series.

The transformer delivers a 4000-kv-a. output at 40 deg. cent. temperature rise and 5000-kv-a. at a rise of 55 deg. cent. It is water cooled, 3-phase, 60-cycle. The high-tension voltage is 22,000, low-tension, 224 volts at no-load which drops to 220 volts at full-load with 100 per cent power factor and 209 volts at a power factor of 90 per cent. The actual transformer impedance is 11.3 per cent.

The nearest approach to the tubular form of conductor for flexible leads for connecting the movable electrodes to the bus structure is the familiar asbestos cored extra flexible cable developed expressly for furnace operation. Consequently this type of cable of 1,000,000 circular mils diameter is used. There are 16 cables per phase, making 4 cables for each of the 3-in. tubes. The other end of the cable is connected to a 5-in. copper tube, about 8 feet long running across the furnace to the electrode holder. Directly underneath the terminals of the 3-in. tubes, which terminate at different lengths to allow placing of cables, are 3 slabs of slate each drilled with a hole just large enough to pass a cable. The 16 holes per phase thus drilled are in the form of a square, the spacing being such that each cable is 2 in. away from its neighbor. This spacing is maintained throughout the cable length by means of grooved blocks, one block clamping a row of 4 cables on both sides. These blocks also serve to prevent swaying motion of the cables without interfering with the up and down motion.

The general scheme of arrangement for adjacent tubes was applied to the cables. The "delta" connection is formed at the end of the 5-in. tube projecting over the back of the furnace and the cables are so placed that those carrying current of opposite sign are adjacent both with respect to cables in the same square and neighboring cables of adjacent squares.

This method of construction with respect to arrangement of the 3-in. transformer lead tubes and the flexible cables results in a very material reduction of impedance due to a-c. effects, as total resistance under full-load tests is increased but slightly over the measured d-c. resistance, the excess being less than 2 per cent with phases balanced. The absence of material reactive and inductive influences, and the absence of any iron forming magnetic fields in the circuit, naturally cause little if any disturbance in the phase relation between the impressed e.m.f. and the current, allowing the system to operate at a very high power factor.

Slight unbalancing of current values in the several leads does not materially affect the power factor. Heavy unbalancing however sets up impedances at which the power factor had been observed to drop below 0.6 and the voltage at the furnace, due to reactive inductance, drops approximately in accordance with the calculated power factor-voltage drop curve.

The phenomena of inductance and reactance are here utilized for the purpose of protecting the complete electrical equipment against heavy overloads, yet allowing it to operate at maximum efficiency when under control. It may be here pointed out that this effect could be further augmented by substituting for the 5-in. copper tubes over the furnace roof, a continuation of the interlacing system employed for the cables by extending the cables to the electrode holder, or by means of tubes or strips.

In order to control and keep up an electrical balance it is essential that the control mechanism employed be sensitive to slight changes in power and rapid restoration of balance. Moreover, for this particular installation on account of metallurgical reasons it is necessary that the effective watt input to the furnace be controlled. Accordingly a control of this type was developed as shown in Fig. 2. The principal elements of the control consist of a watt measuring device and means for reversing rotation of motors to operate the electrodes. The watt element selected is that on the principle of the well known "Kelvin Balance" in which the movable or floating element coils receive the impressed e.m.f. of the circuit and the fixed coils a current proportional to that in the conductors. Each phase is regulated by its own watt element. The fixed coils are fed from a current transformer on the high-tension side corresponding to its low-tension component. The movable potential coils receive their energy from the low-tension side of the phase to which they are connected. The circuits operating the reversing switches of the electrode motors are closed and opened by relay method. The movable coil beam is provided with two-way contact points placed one above the other connected to a circuit of very small amperage (0.02 ampere), taken direct from the low-tension lines, energizing magnets placed in the motor switch cabinet.

Fig. 2

At a predetermined wattage, obtained by means of a spring calibrated against the pull exerted by the movable coils, the beam is floating in balance and out of contact. As soon as any watt fluctuation occurs the beam is inclined and makes contact with one of the two-way points, closing the circuit operating the relay magnet in the switch cabinet. The point at which the beam makes contact depends on whether the pull is exerted by the spring overcoming the magnetic pull, in event of under load or vice versa for overload. The actuated relay magnet in the switch cabinet closes a circuit energizing another larger coil containing a plunger which lifts contact arms into the path of other contact arms in constant to-and-fro rocking motion. The meeting of the contact arms closes the circuit operating the electrode motors. The motion of the nonracking arms is controlled by means of very fine adjustments which allow them to meet the rockers during the desired part of the arc described by them, the speed of the rockers at the same time being variable. The rocking arms also make and break contact of the circuit operating the coils energized from the watt balance, which eliminates unnecessary closing of circuits, which in turn very materially cuts down the wear on the contacts affected.

Further to facilitate quick regulation and restoration of balance in the furnace a "selective regulation" device is applied. This consists of a set of 2 plunger magnets and a lamp in series per phase, one terminal of each set is connected to its respective phase, the other terminal being connected to the furnace. The lamps serve as pilot lights visually indicating the relative resistance between electrodes and furnace bath. The plunger magnets, one each for the up and down side of the motor reversing switches are so arranged that a change in current values causes the plungers to change positions from a predetermined neutral point adjustable by means of weights. Magnetic field exerts an upward pull on one plunger and a downward pull on the other. When the furnace load is balanced and resistance under electrodes equal, the plungers are all in the neutral position. A disturbance of potential balance in the furnace between the bath and electrodes causes a corresponding unbalance in the magnet coils and one or the other magnet of each set allows its plunger to drop. The dropping plunger opens the watt balance relay circuit for the up or down switches, depending on the nature of the disturbance, rendering one or the other of all the motor switch sets inoperative. For example: Should the resistance under one of the electrodes suddenly drop it would cause an increase in the furnace load and set all of the switches to work to raise the electrodes, however, as the decreased potential between the bath and the particular electrode causing the trouble decreases the current in the magnet controlling downward motion of the electrode decreases and allows its plunger to drop thus opening the relay circuit, but as the other magnet in the same set does not pull its plunger down the upward motion of the electrode is left undisturbed and the electrode is raised to restore balance. At the same time the current in the other sets is increased and causes the respective plungers to be pulled down thereby opening the relay circuit controlling the upward motion of the electrodes. This renders the switches for raising electrodes inoperative except for the electrode which caused the trouble. Reversed condition reverses the operation of the magnets. Thus when one of the electrodes is out of balance this device throws the other two electrodes out of circuit and holds them in position while the unbalanced electrode finds its adjustment thereby restoring operating condition in much quicker time. By means of these regulations the period of contact of the arms and duration of circuit closing on the motors can be regulated to a great degree of nicety avoiding unnecessary movement of electrodes and wear and tear on the regulating equipment.

The general arrangement of the electrical equipment consists of two incoming 3-wire, 3-phase 22,000-volt feeder lines, equipped with lightning arresters and choke coils, automatic oil switches opening on overload and reverse current. A set of disconnecting switches are placed on each side of the oil switches. The lines feed parallel into a common set of busses. Each furnace transformer is connected to the bus through disconnecting switches and automatic overload and no-voltage release oil switch. All high-tension conductors within the building are of copper tubing. The secondary leads of the transformer are equipped with a set of disconnecting switches operated by hand or motor for "deadening" the portion of equipment situated in the furnace room, leaving the transformer alive to furnish power for motor drive, etc. The control room, containing all instruments, regulators, relays, etc., is a totally enclosed compartment. All instruments are placed on pedestals and the regulators in cabinets. This allows ready and easy access to all parts of the wiring and facilitates repairs. The low-tension disconnecting switches are operated from the control room. Switches of the same phase are levered to a common shaft and these shafts extend through the wall to the control room.

Each furnace is equipped, beside the regulators, with graphic recording watt meter, watt hour meter, voltmeter, ammeter for each phase and power factor meter. All of the instruments for the first unit installed are of the recording type to facilitate the study of the operation. The control room also contains a frequency meter. The furnaces may be regulated automatically or by hand — remote control. All switching, except high and low-tension disconnecting switching, is by means of remote control. An automatic change-over switch is located directly under the control room. This switch serves to connect the main factory feeder lines with a "live" transformer. It automatically shifts from a dead to a live line. It is operated, when desirable, by remote hand control from the control room. A spare set of regulator switches is provided that may be used on either furnace as may be required. A small auxiliary control room is located in the furnace building. This room contains switches for raising and lowering electrodes, ammeters and voltmeters. It is installed for the convenience of the furnace operators to enable them to change electrodes quickly and other work connected with the operation. While using these switches the main control room is automatically cut out and the motor speed automatically doubled. All high-tension leads and equipment are contained in bus structure and partitioned off from the rest. One transformer is placed in a vault, the other is of the out-door type.

It is highly desirable that an electric furnace operation of this nature be made continuous with interuptions as few and short as possible. With this view in mind facilities for rapid change of furnaces and of electrodes have been provided. The furnaces are placed on a double set of rollers at right angles to each other, so that when the furnace needed relining it was rolled out of its position, then rolled aside and a spare one already lined rolled into its place. This method of change can be effected in less than two hours time.

For the same reason an electrode holder was designed to curtail the time of electrode changes. The holder used works on the friction-contact principle, eliminating all bolts, nuts or other tightening devices. It consists essentially of three members, (1) the outer casting of copper, forming a hollow water-cooled ring having a slight taper on the inside; (2) a copper ring fitting inside of the water-cooled ring and having its outside surface tapered to conform to the inside surface of the first member; (3) bronze wedges fitting between the second member and the electrode. In operation the second member is applied to the electrode by means of the wedges, this taking place on the furnace room floor and the electrode thus provided is lifted with the crane and simply dropped into the first member, making a friction contact by its own weight. The arrangement is shown in Fig. 3.

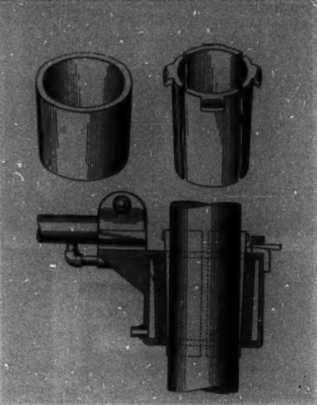

Fig. 3

The advantage of this device, besides saving time, is that it obviates the necessity of loosening or tightening bolts or other devices on top of the furnace where working conditions are disagreeable. Further, should the holder get hot, as the water-cooled outer ring and electrode do not expand, the only material that can expand would be the second member ring and the wedges, which would produce a tighter contact, instead of a looser one, as in the case of a holder of the ordinary type. Also, as the electrodes are prepared for service between changes, under agreeable working conditions and at more leisure, it follows that, in preparing the wedges and ring they are properly cleaned and adjusted, insuring good contacts. This holder permits the change of an electrode in 1 to 2 minutes.

This furnace equipment operates at an average power factor of 99 per cent and an over-all electrical efficiency — for transformer and low-tension leads to electrode —of 98.5 per cent. Repeated tests in operation check well with calculated values.

The practical experience and data gathered from this installation indicates the advantage of application of the principles involved to large input furnaces for various metallurgical operations. Also they indicate the feasibility of larger installations probably with twice the power input operating with equal high efficiencies.

Discussion

SOME PROBLEMS IN ELECTRIC FURNACE OPERATION (Andreae); IMPROVEMENTS IN FERRO-ALLOY ELECTRIC FURNACES OF HIGH POWER INPUT

(Saklatwalla and Anderson);
Pittsburgh Pa., April 25, 1923.

J. A. Seede: I remember distinctly a large smelting furnace making ferro-alloys in which the condition existed that Mr. Andreae points out, *i.e.*, the charge surrounding the left hand electrode, looking at the furnace from the transformer, indicated that the temperature was too low to produce the required metallurgical action while the charge around the other electrodes indicated that the temperature was high and the metallurgical conditions satisfactory. Such unbalanced metallurgical conditions never produce satisfactory operation and no one had made, or has made to my knowledge, such a thorough analysis as the author has made in this paper.

One plausible reason has been suggested why the electrodes in electric furnaces do not operate equally and that has to do with the manner in which the furnace is charged. It is evident that if the furnace is not charged properly and an excessive amount of coke happens to be thrown around one of the electrodes the result is equivalent to inserting a resistance between the electrode and the charge which prevents the power being concentrated around the end of the electrode and while the same amount of energy may appear as heat around that electrode it will be diffused over a large area and the temperature gradient will be so low that no metallurgical action may result.

I am quite sure we are all much interested to read his analysis of the conditions in the electric furnace and it is astonishing to note the difference, where in a furnace taking a total of 2600 kw. there was a difference of 620 kw. between the input to electrodes No. 1 and No. 3. The suggestions for balancing the power input to the furnace are very interesting but it is not quite evident how a practicable arrangement can be developed as is shown in Fig. 6.

Edward T. Moore: With reference to the second paper by Mr. Saklatwalla the means of regulation by the watt meter principle on the furnaces upon which he is working, I think are necessary, but I would not want it inferred, when it comes to steel furnaces in general, that this is necessary or even desirable. The regulation by the current method seems to be working out very satisfactorily; in fact, experiments conducted by using a Kelvin Balance alongside of the common regulator does not show any particularly improved results.

F. V. Andreae: I am aware that some of my suggestions for balancing the loads of the three electrodes are rather unsatisfactory from the point of view of the power companies.

My aim was to put the whole problem on a solid mathematical basis, and to show how the equations could be interpreted. Once the operator has a clearer view of his difficulties he is better able to attack them.

I wish to point out that the unbalancing of which I am speaking is not momentary but continuous, and is due to the disposition of the bus bars. In a ferro-alloy furnace of low power factor, say around 75 or 80, the operator has to choose between two evils. If he tries to keep his high-tension circuit balanced to please the power company his furnace will be quite unbalanced. If, on the other hand, he tries to balance his load in the furnace by equalizing the power under the three electrodes he unbalances his high-tension circuit. He will therefore have to compromise and if he has several furnaces he can easily connect them in such a way that the disturbance of the power company's line will be kept within satisfactory limits.

B. D. Saklatwalla: In the discussion the matter of uneven balance of the electrodes in a three-phase furnace has been brought out. The conditions of the bath at the electrodes can be kept uniform, avoiding an unbalance, by proper feeding of raw materials. In the case of the ferro-vanadium furnace this is achieved by a continuous feeding of the material between the electrodes at a rate commensurate with the electrical load.

Another point brought out by the discussion is the possibility of the use of the induction furnace in competition with the arc furnace. Besides circulation of the bath and other mechanical, or rather hydro-dynamical, effects, the passage of current through the molten metal may have effects of a molecular or even electronic nature depending on the electro-magnetic forces set up within the fluid metal. A study of such electromagnetic influences may prove some superiority of induction furnaces over that of the arc type, achieving a more perfect refinement of the metal.

A. N. Anderson: The regulation of the power input in an electric furnace by means of a true watt control offers several advantages over current regulators as there are several factors entering the problem. Some of these factors lie outside of actual furnace operations but are nevertheless worthy of consideration as they form very important factors in the problem as a whole.

The use of a control mechanism in which the current alone is kept constant can not govern or regulate other than the current consumption.

The parties interested in the power consumption of an electric furnace may be divided into three groups:

1. The manufacturer, selling the product of the furnace, who wants maximum production at minimum expense.
2. The operator responsible for the product.
3. The central station furnishing the power and interested in a high power factor, balanced load and freedom from excessive fluctuations.

The control of the energy input is therefore a factor of prime importance.

Because of the prevailing methods of transmitting power a large majority of electric furnaces now in operation are of the three-phase three-electrode type whether used for the smelting of ores or the production and refining of steel.

It is not possible to control and keep constant with any degree of accuracy the effective power input in these furnaces by means of current regulation alone or the combination of current and voltage regulation, where the voltage regulation consists of keeping a balanced potential between the electrodes and furnace charge.

All that a central station can do to deliver reliable power is to guard against overloaded lines and to keep, the voltage constant at the station. It cannot materially regulate the load factor.

On lines supplying power to customers using electric furnaces of various types and of more or less intermittent energy consumption it is very difficult, if not impossible, to keep up a steady voltage and power factor. There are often variations in voltage between phases of the same line and the power factor in phases may also vary.

These conditions are not noticeable where power is supplied to motors or apparatus having balanced circuits, but is noticeable where devices such as electric furnaces are used.

It is highly desirable both from a technical as well as commercial standpoint to be well supplied with measuring instruments to keep a check on the various fluctuating factors.

A properly metered electric furnace installation, employing current regulators, will often show an erratic behavior of the furnace and irregular power consumption in spite of the control indicating a steady amperage flowing, as in addition to the comparatively small fluctuations in the incoming power lines those of the transformer secondary vary to a much greater degree, and it is not at all surprising to find such conditions as Mr. Andreae has investigated.

On the other hand, a furnace installation operating on a true watt control principle automatically corrects fluctuations due to variations in voltage and power factor so that the actual predetermined energy is controlled, measured and delivered to the furnace, and conditions such as Mr. Andreae describes cannot exist.

Assuming that volts times amperes times power factor are the factors that produce the heat and temperature required for the successful operation of an electric furnace of the type under consideration, it is apparent that the control of but one of these is insufficient unless one assumes that the others are constant, which unfortunately is very seldom the case. Often a heat gone wrong, a tap or power "off" on analysis, has been blamed on the operator or laid to some other cause when the real culprits were the current regulators.

To the manufacturer who pays the bill from the control station and collects from his customers for his product, the watt regulator serves to prevent peaks much more efficiently than a current regulator, thereby keeping costs at a minimum; it regulates the actual power input into the furnace and thereby maintains the predetermined energy at which the furnace is set to operate, thus helping to produce the desired product uniformly and at maximum efficiency.

The watt regulator benefits the central station insofar as it has a tendency to more effectively balance the load on the feeder lines and to prevent prolonged duration of peaks.

Selective Relay System of the 66,000-Volt Ring of the Duquesne Light Company

BY H. P. SLEEPER
Associate, A. I. E. E.
Protection Engineer, Duquesne Light Co., Pittsburgh, Pa.

Review of the Subject.—*The selective relaying of a high-voltage transmission system gives rise to many interesting problems. This is particularly the case when the system is grounded through a high resistance, as the amount of energy available on ground faults is limited and the ground protection introduces a problem entirely separate from that of the short-circuit protection of the system.*

It is also interesting to note that it is only in recent years that the design of high-voltage large capacity control equipment has been perfected to such a state as to place the protection of high-voltage systems on a par with that of comparatively lower voltage systems. The relay problems involved in the protection of low or high-voltage systems are not materially different except as affected by the equipment available, or by the ever present problem of economics.

The system in question presented no unusul engineering features except as presented by the problem of securing circuit breakers of the required voltage and with sufficient capacity to rupture the large blocks of power presented by short-circuit conditions. This problem was successfully solved by the manufacturer.

A ring system of duplicate feeders lends itself to various relay schemes to secure protection. A number of these schemes was thoroughly investigated and a summary of the conclusions is given, showing the advantages and disadvantages of the schemes considered. These studies showed the advisability of using the scheme of balanced directional relays for short-circuit protection, and selective differential current relays for ground protection, on the 66,000-volt ring of the Duquesne Light Company.

Before putting the relays into service on this system it was decided to make a series of actual service tests on the lines. Accordingly a series of dead grounds were put on the ring and the relays allowed to function to clear the grounded section of line. The dead ground was replaced by a simulated fallen line wire lying on the surface of the ground. This was in turn replaced by a series of arcing grounds accomplished by fusing over a suspension insulator to ground. A total of 28 grounds were thrown on the 66,000-volt ring and successfully cleared by the relays. A description of the tests and a summary of the results are given.

THE 66-kv. transmission system of the Duquesne Light Company surrounds the City of Greater Pittsburgh, enclosing an area of approximately 250 square miles, and supplying electric light and power to an area of approximately 1000 square miles. The ring is somewhat elliptical in shape, one end circle being about twice the diameter of the other. The total circumference of the ring is about 80 miles, and there is one radial tap of 11 miles length. The geographical locations and lengths of the lines may be seen in Fig. 1.

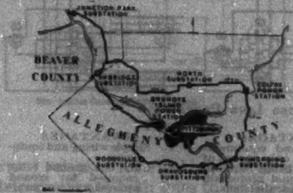
JUNCTION PARK SUBSTATION
BEAVER COUNTY
AMBRIDGE SUBSTATION
NORTH SUBSTATION
COLFAX POWER STATION
BRUNOTS ISLAND POWER STATION
ALLEGHENY COUNTY
PITTSBURGH
WOODVILLE SUBSTATION
DRAVOSBURG SUBSTATION
WILMERDING SUBSTATION

FIG. 1—THE DUQUESNE RING

The ring is made up of parallel lines and is sectionalized at seven points, two of which are generating stations, the remaining five being step down substations. Power is fed into the ring at two points which are approximately diametrically opposite, being located at the two end circles of the ellipse, namely at Colfax and Brunots Island. The number and arrangement of lines in each section are shown in Fig. 2.

The standard section of line consists of seven strands of No. 6 copper wire and has the characteristics of 4/0 copper wire. The lines are carried on steel ltaticed towers as shown in Fig. 3, each tower carrying two three-phase circuits. The phase wires of each circuit are hung vertically, the spacings being approximately seven feet, eight feet, and fifteen feet respectively.

As previously stated the transmission ring is sectionalized at seven points. Since this sectionalizing involved a heavy cost for switching equipment it was very carefully considered before being decided upon. The high capacity, 30,000 kv-a., of each circuit and the heavy investment involved led to the decision that such sectionalizing was justified.

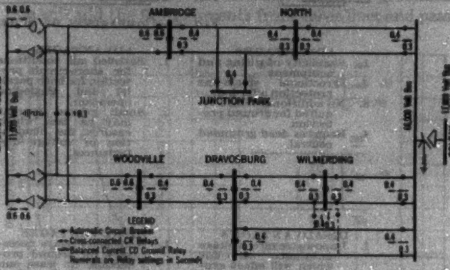
AMBRIDGE
NORTH
JUNCTION PARK
BRUNOT'S ISLAND
11,000 Volt Bus
66,000 Volt Bus
11,000 Volt Bus
COLFAX
WOODVILLE
DRAVOSBURG
WILMERDING
0.6
0.4
0.3
LEGEND
Automatic Circuit Breaker
Cross-connected CR Relays
Balanced Current CD Ground Relay
Numerals are Relay settings in Seconds

FIG. 2—SCHEMATIC DIAGRAM OF 66 KV. RING SYSTEM SHOWING RELAY PROTECTION

The greater part of the load carried on this system is industrial motor and synchronous railways load. Voltage disturbances in any event lead to unsatis-

Presented at the Spring Convention of the A. I. E. E., Pittsburgh, Pa., April 24-26, 1928.

factory operation to such load and may cause complete interruption. It was therefore decided, since most failures on the 66 kv. system would undoubtedly be grounds, that the neutral of the system should be grounded through a high resistance to minimize disturbance resulting from such failures.

FIG. 3—TYPICAL 66 KV. LINE TOWER

The conditions imposed introduced certain problems in the relaying as follows:

1. On account of the high-resistance ground, relaying must be provided which would trip out a line on ground current less than full-load current.

2. On account of the heavy industrial motor and synchronous railway load, faults must be eliminated in a minimum time, thus barring out selective time systems.

3. Single line operation must be possible, preferably without additional relays over the number required for parallel line operation.

4. It must be possible to take care of future growth by the addition of parallel lines between any existing stations, or to add radial lines from any ring station, or to introduce additional sectionalizing stations to the present loop, without destroying the existing protection, either schematically or materially.

A study of all of the above factors showed conclusively that the general principle of balanced protection must be used. Such protection always possesses the inherent advantage that it is absolutely unaffected by conditions outside of its own section. This was a fundamental requisite in the problem at hand, which when viewed with the proper perspective, consisted of the proper selective relaying of a basic system frame work. The next problem presented was to choose

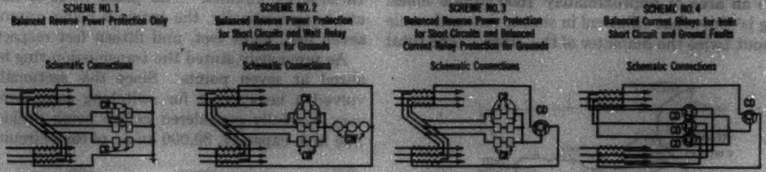
SCHEME NO. 1
Balanced Reverse Power Protection Only
Schematic Connections
CR
CR
SCHEME NO. 2
Balanced Reverse Power Protection for Short Circuits and Watt Relay Protection for Grounds
Schematic Connections
CR
CR
CW
SCHEME NO. 3
Balanced Reverse Power Protection for Short Circuits and Balanced Current Relay Protection for Grounds
Schematic Connections
CR
CR
CO
SCHEME NO. 4
Balanced Current Relays for both Short Circuit and Ground Faults
Schematic Connections
CO
CO
CO
CO

FIG. 4—COMPARISON OF SCHEMES OF BALANCED LINE PROTECTION

ADVANTAGES	ADVANTAGES	ADVANTAGES	ADVANTAGES
1. Simplicity of wiring and equipment. 2. Directional single-line protection afforded. 3. No additional relays required for ground protection. 4. Requires dead grounded neutral.	1. Satisfied all conditions for short-circuit protection for both parallel and single-line operation. 2. Small ground current may be used which ensures the elimination of voltage disturbances.	1. Wiring and equipment simple, only one set of relays being required for ground protection of two parallel lines. 2. Satisfies all short-circuit conditions, for both parallel and single-line operation. 3. Small ground current required to operate ground relays. 4. Ground relays very positive in action.	1. Simple wiring and equipment. 2. Only one ground relay required for ground protection of two parallel lines. 3. No potential connections required. 4. Ground relays very positive in action.
DISADVANTAGES	DISADVANTAGES	DISADVANTAGES	DISADVANTAGES
1. Imposes severe voltage unbalance on line which will effect synchronous machines. 2. Severe damage may result as heavy ground current must pass to operate relays.	1. Additional relays necessary for ground protection, at least one per circuit being required. 2. Wiring is complicated. 3. Extra grounding potential transformer required for one scheme using the watt ground relay. 4. Another scheme used three double contacting watt relays per pair of lines but this scheme does not have a dependable selectivity.	1. Ground protection is inoperative when ring is open. 2. Ground relays are non-directional for single-line operation.	1. Neither short circuit nor ground protection is operative if ring is open. 2. Single-line operation is not directionally selective.

the proper form of balanced protection. This matter was gone into to some length and Fig. 4, shows a comparison of the schemes considered.

After considerable deliberation Scheme 3 was finally decided upon as fulfilling the most important requirements; namely, the ability to give complete short-circuit protection for both double and single line operation, with the ring either open or closed, with the use of but one set of short-circuit relays; the ability under normal conditions to selectively disconnect grounded sections of line with a minimum of disturbance to the system itself.

The relay equipment as installed at the various stations is shown diagramatically in Fig. 5 and a typical station relay panel is shown in Fig. 6. The short-circuit relays are standard Westinghouse single-contact, type *C R*, 4-12 ampere, reverse power relays, and the ground relays are single-contact 1-3 ampere, selective differential current relays. Interlocking relays are installed in the trip circuits of both lines to render the good line non-automatic for a few seconds after the relays have tripped out one end of the defective line. This locks out the trip circuit of the good line until the relays at the opposite end of the section have had time to clear the defective line, thereby preventing the balanced relays from improperly disconnecting the good line of the pair on the end which clears first. On standard line sections the short-circuit relays are set to operate at 0.4 second definite minimum. The 7-ampere current tap is being used on the *C. R.* relays, and the 1-ampere tap on the *C. D.* relays. One 63-ohm grounding resistor is installed at each of the two generating plants. It is possible that this value of resistance may be increased at some later date.

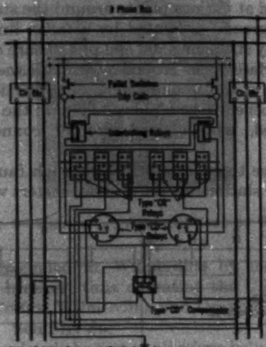

Fig 5

A graphical representation of a typical relay operation on a ground fault is shown in Fig. 7. Sketch No. 7A shows the division of ground currents for a ground fault on line 2 at a point just outside station *M*. With a ground resistance of 63 ohms at each generating station and a neutral voltage of 38,100 volts, this will give between 500 and 600 amperes ground current supplied by each generating station depending upon the location of the fault. Using the value of 600 amperes for convenience, it will be noted as shown in Fig. 7A that these currents divide equally until the faulty section is reached, where the shortest section of grounded line will add the fault currents from three lines and pass 900 amperes to ground, the remaining portion of the grounded line supplying 300 amperes into the fault. This will supply the line relays at the fault with a total of 1200 amperes, since the cross connection of the *C. R.* relays accumulates the currents in the two lines as

Fig. 6—Typical Relay Panel at Ring Stations

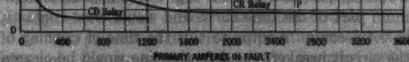

Fig. 7—Load Time Curves of *C R* and *C D* Relays

shown in Fig. 7B. The ground relays will receive a differential current of 900 − 300 = 600 amperes. Thus both sets of relays will start to close contacts but it will be noted that the line relays receive only sufficient current to cause them to operate on the inverse portion of their time curves, requiring approximately 0.6 second to close contacts. The ground relays, however, operate on the definite minimum portion of their time curves and close contacts in 0.3 second. Assuming 0.4 second for the breaker to open, this makes a total time of 0.7 second to clear one end of the defective line. The opposite end of the grounded line will then start to clear on a similar cycle of operation and thus after 1.4 seconds the grounded line will be entirely disconnected from the remainder of the system. See Fig. 7c.

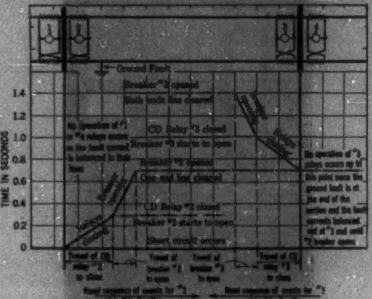

FIG. 7C—CHART SHOWING SEQUENCE OF EVENTS IN CLEARING GROUNDED LINE

If for any reason the ground relays fail to respond to the ground fault, provision has been made that the grounding resistors will be short-circuited after a period of approximately 10 seconds following the occurrence of the ground on the system. This places a dead ground on the system which will draw short-circuit current through the line *C. R.* relays and cause them to operate on the definite minimum part of their time curves.

If the fault had been between phase wires instead of to ground, the *C. D.* relays would not have operated but the *C. R.* relays would receive the short-circuit current. For the minimum fault current these relays would operate at a point *P* Fig. 7 on the definite minimum part of the time curve and would close contacts, in 0.4 second. Adding to this the time of opening of the circuit breaker, the defective line would be cleared at one end in 0.8 second. Thus a maximum of 1.6 seconds would isolate the defective line from the system in case of short circuit.

Before putting this relay equipment into operation and in order to prove the effectiveness of the relay scheme, as well as to detect any defects that might exist in the apparatus which would jeopardize service, it was decided to give service tests to the entire installation. The following procedure was accordingly carried out:

One pole of an out-door type *G*-11 circuit breaker was mounted portably and placed as near as possible to the point where it was proposed to apply the ground to the system. The point chosen because of its convenience was just outside the station and a connection was made to the line between the line breaker and the last disconnection point on the line side of the station. Control cables were brought into the station where the grounding apparatus could be manipulated safely and the relay operations observed simultaneously. The ground connection to the grounding breaker was made by two different methods:

1. A connection directly to the grounded side of the 11,000-volt ground resistor, which ground consisted of five interconnected copper plates 2 ft. 6 in. by 3 ft. 4 in. buried five feet under ground and surrounded by charcoal, located at various points around the station.

2. The ground presented by a fallen line wire was simulated by stretching 800 feet of 4/0 bare copper wire over the surface of the ground and this connected to the ground side of the test breaker. The particular point chosen happened to be a plowed field, in the case where it was tried, and a good ground connection was obtained.

To observe the effects of the through fault currents at the other stations on the system, meters were placed in the ground and relay circuits of those stations. Thus when the ground was applied, the readings on these meters would give evidence of the ability of these stations to balance out the through fault currents in the cross connection of the current transformers. Prior to the tests the relays were checked for proper operation by means of phantom loads.

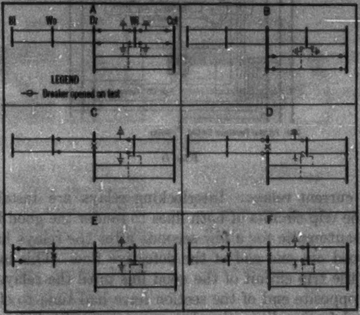

FIG. 8—SYSTEM CONNECTIONS FOR TESTS

In order to test all of the relays on the system with a minimum of test apparatus and connections the tests were made in the following manner:

A grounding breaker was installed at Wilmerding substation. With all bus sections paralleled at all ring stations, a ground was placed successively on lines 1 and 2 on both sides of the Wilmerding Bus as in Fig. 8A. This operated the *C.R.* and *C.D.* relays on both sides of Wilmerding, on the Wilmerding side of Colfax, and on the Wilmerding side of Dravosburg. In this, and in all following tests, the two sets of relays were tested separately, the one set tripping and the other set being rendered temporarily inoperative. The emergency taps to Wilmerding from the two direct lines between Colfax and Dravosburg, Nos. 3 and 4, were then successively grounded at Wilmerding by means of the grounding breaker, thus operating the *C.R.* and *C.D.* relays at Colfax, Wilmerding and Dravosburg on these two lines. See Fig. 8B.

FIG. 9—GROUNDING BREAKER USED ON TESTS

This completed the test of all relays between Dravosburg and Colfax. The relays between Dravosburg and Brunots Island were then tested as follows:

The bus was split at Dravosburg between lines 1 and 2, thus giving two parallel lines between Wilmerding and Woodville, as in Fig. 8C. The trip circuits of all relays on lines 1 and 2 at Dravosburg were opened. The ground was then applied to lines 1 and 2 successively at Wilmerding and the *C. R.* and *C. D.* relays on the Dravosburg side of Woodville operated, as well as the *C. R.* and *C. D.* relays on the Dravosburg side of Wilmerding. The *C. D.* trip circuits only at Dravosburg were then closed on the Woodville side, and all trip circuits at Woodville opened. See Fig. 8D. The ground at Wilmerding was then applied successively to lines 1 and 2 and the *C. D.* relays only at Dravosburg operated; there was not sufficient feed back from Colfax to test the *C. R.* relays at this same point but their operations was tested by inspection of the movement of the disks of the directional elements. The relays at Brunots Island were then tested by splitting the busses at Dravosburg and Woodville and opening all trip circuits at these stations. See Fig. 8E. By grounding the two lines successively at Wilmerding the *C. R.* and *C. D.* relays at Brunots Island operated. This left only the *C. R.* relays and the *C. D.* relays on the B. I. side of Woodville without test. This test was made by opening the trip circuits on these lines at Brunots Island and closing the trip circuits of the *C. D.* relays only on the Brunots Island side of Woodville which operated when the grounds were applied at Wilmerding. The *C. R.* relays at this point were inspected for proper operations of the directional elements. It was thus possible to test all relays on that side of the ring with the one test set-up at Wilmerding. Some of the apparatus used in these tests are shown in Figs. 9 and 10.

The result of these tests were extremely satisfactory and very conclusive. Several incorrect operations were obtained before correct adjustments were made and defective apparatus detected, but following the correction of these matters perfect operation was recorded.

A total of 28 grounds were applied to the ring in these tests and every operation was correct, both the ground *C. D.* and the line *C. R.* relays being tested as explained previously. By means of operation indicators installed in the relays the operations were checked very exactly. A summary of the test results may be made as follows:

1. The complete scheme of protection was proved to be theoretically correct.

2. Both the short circuit and ground relays, as set up, for final test, operated correctly in every case. No difference in performance was observed or recorded by the operation obtained by grounding the line breaker dead or through the simulated fallen line wire.

FIG. 10—EFFECT OF GROUND ON 66-KV. LINE WIRE

3. The ability of all sections to properly pass through short-circuit current was demonstrated.

4. The presence of defective apparatus was detected although the equipment apparently operated satisfactorily on phantom load tests.

5. It is interesting to record that these tests resulted in no injurious effects whatever to either apparatus or service.

Discussion

For discussion of this paper see page 532.

Ground Selector for Ungrounded Three-Phase Distribution Systems

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Review of the Subject.—*Ever since three-phase distribution has been known, a controversy has been going on with respect to the relative merits of grounded and ungrounded neutral. The inability of locating and isolating a ground fault is the chief disadvantage of the ungrounded neutral system, this being the reason why lately the grounded neutral has been given preference.*

The purpose of the so-called ground selector is to provide means to permit the selective clearance of a ground fault in an ungrounded neutral system. The device thus eliminates the mentioned disadvantage and raises the ungrounded neutral system to the same operating standard as the grounded neutral system, retaining at the same time the many other advantages inherent to the ungrounded system.

The paper describes the functioning of the ground selector and compares in detail the two systems. Two ground selector installations on largely different systems are described and the practical operating results are reported and discussed.

The conclusion is reached that the ungrounded neutral system equipped with a ground selector combines the advantages of the ungrounded and grounded neutral systems without being afflicted with their respective disadvantages.

INTRODUCTION

The question of the advisability of operating three-phase distribution systems grounded or ungrounded has, within the past ten years, formed one of the liveliest subjects for discussion among transmission engineers. The reason for the greatly diversified opinion on this subject is chiefly due to the fact that either principle has its pronounced advantages and disadvantages.

In the early days of three-phase transmission the ungrounded system was used almost exclusively except where it was developed as a four-wire system.

With the increase in transmission and distribution voltages, insulation troubles started to multiply. Grounds started to develop quite frequently. It was then discovered that the ungrounded system was at an extreme disadvantage, particularly in case of extensive distributing systems, because there was no means available to detect and select the faulty feeder. Moreover the grounded phase produced a 73 per cent voltage rise between ground and the two sound phases. Such dynamic overvoltage, sometimes aggravated by resonance effects from an unstable arc at the point of fault, usually resulted in secondary breakdowns of weak insulation at entirely different parts of the system. Thus a short circuit developed disturbing the whole system and extending the damage unnecessarily to other parts of the system. A ground, left on the system for any appreciable time, was found to also have its weakening effect on the whole metallically interconnected system and would thus prepare for future troubles. It was soon discovered, therefore, that continuing to operate on a ground fault was detrimental.

Improvements were suggested such as Creighton's arc suppressor to render the ungrounded system safer. These improvements however failed chiefly in view of the fact that they aimed at the less vital disadvantage of the ungrounded system, namely the suppression of an arcing ground. They offered no solution, however, for detecting and selecting a faulty feeder.

As a result of this weakness of the ungrounded system the operating engineer's opinion gradually expressed in favor of the grounded system despite many disadvantages in regard to transformer connections. In the grounded system every ground meant a short circuit thus supplying the current necessary to operate relays and assure clearance of the faulty feeders. Thus the grounded system offered a natural solution for the most troublesome feature of the ungrounded system.

Today the stage has been reached where the general opinion is overwhelmingly in favor of the grounded system. Many operating companies, which have operated ungrounded in the past, have changed in recent years to grounded systems by devising various means of grounding.

The writer, being connected with a large distribution system with delta-connected transformers was, some years ago, confronted with the same problem of overcoming the serious conditions created by a ground on the ungrounded system as described above. Circumstances were such that grounding, either by changing to star-connected transformers or by providing an artificial grounding scheme, was not readily possible. He was forced therefore to the development of another scheme which is hereafter called "ground selector."

The purpose of this paper is to give a brief description of this scheme, to point out its advantages substantiated by actual operating records and to show how the ungrounded system equipped with a ground selector retains practically all inherent advantages of the ungrounded system and at the same time assures the same

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Presented at the Spring Convention of the A. I. E. E., Pittsburgh, Pa., April 24-26, 1923.

effectiveness of clearing grounded lines as would be the case with the grounded system.

Description of the Ground Selector[1]

A ground, occurring on a non-grounded distribution system, will be felt over the whole system. There is normally no means available to locate the faulty line except by disconnecting alternately individual sections of the system and observing whether the ground disappears. Such cut-and-try method is rather slow and always necessitates the momentary interruption of a larger part of the system than necessary, since many of the sound feeders may first be disconnected until the faulty feeder is located. The ground will cause a 73 per cent increase of voltage on the two sound phases to ground, and if left on for any length of time may cause the breakdown of another weak spot at an entirely different part of the system.

To reduce to a minimum the damaging effect of a ground, it is essential to provide means to have the grounded section cut out selectively and within the shortest possible time.

The ground selector has been developed to accomplish such selective and immediate clearance of a grounded feeder.

The fundamental idea of the ground selector is to produce intentionally a short circuit immediately after a ground develops on the system. This is accomplished by automatically grounding one of the sound phases at the main station bus by means of closing an oil switch, immediately upon the occurrence of a ground. Thus a complete phase-to-phase short circuit is produced and as a result overload current will flow into the faulty line, as if the line were on short circuit. Accordingly the ordinary overload protection will enter into action and disconnect the faulty line. As soon as the faulty feeder has cleared itself, the ground switch will open automatically and thus restore the system to normal condition.

Figure 1 shows schematically the principal features of the device. $P\,1$-$P\,2$-$P\,3$ are single-phase potential transformers with their primary connected in star and with the star point grounded. The secondary of these transformers is connected to overvoltage relays $R\,1$, $R\,2$ and $R\,3$, which latter, two in joint action, close the ground switches. The so-called transfer relays $T\,1$ and $T\,2$ are provided to assure an automatic clearance of the ground switch after the faulty line has cleared, as will be explained later.

Normally the three phases of the system will be balanced to ground, and as a result the potential on the transformer $P\,1$, $P\,2$ and $P\,3$ will be the same and equal 58 per cent of the voltage between wires. A ground on one phase will tend to lower the voltage on the transformer, which corresponds with the grounded phase, and at the time the voltage on the other two phases will tend to rise toward a value equal to line voltage, that is 1.73 times normal voltage to ground. This overvoltage on the two sound phases is made use of to trip the two corresponding overvoltage relays and thus cause, by their joint closure, the automatic closing of one of the ground switches.

For instance a ground on phase (1) will cause a high voltage on phases (2) and (3). The joint trip of relays $R\,2$ and $R\,3$ will close the ground switch phase (3) thus causing a complete short-circuit between phases (1) and (3) and causing the faulty line to clear on overload. After the faulty feeder is cleared from the system a ground would remain due to the ground switch still being in closed position. In our particular example the ground would remain on phase (3) due to ground switch (3) remaining closed. The overvoltage relays of phase (1) and (2) would therefore trip and close ground switch (1) thus causing a bus bar short circuit if no special provision were made as by the installation of a transfer relay, to prevent such action.

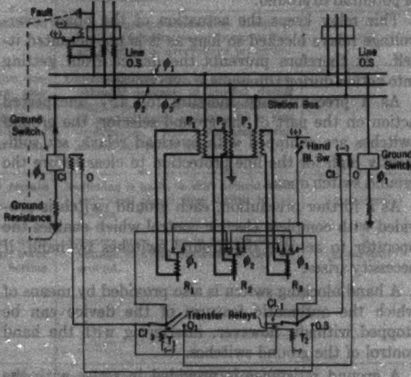

Fig. 1

The transfer relay blocks the closing circuit of the second ground switch immediately upon the closing of the first ground switch, and at same time prepares for the tripping of the ground switch which has been closed. When ground switch (3) closes, it energizes the coil of the transfer relay ($T\,2$) throwing the lever over into the other contact position. In this position of the lever high voltage on phase (1) and (2) due to ground switch (3) being in closed position, will trip the ground switch (3) and thus re-establish normal conditions instead of closing wrongly the ground switch (1). Transfer relay (1) will in a similar manner prevent the closing of ground switch (3) and assure the opening of ground switch (1) when overvoltage relays $R\,2$ and $R\,3$ will trip after a ground on phases (2) or (3) has been cleared, leaving ground switch (1) closed.

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[1] Original description in "Relay Protective Features of the Toronto Power Co's. Transmission and Distribution System" by P. Ackerman, *Journal, Engineering Institute of Canada*, April, 1921.

The above is a description of the essential features of the ground selector.

Various other features had to be embodied to make the device practical. These additional features are not shown in the schematic diagram but may be briefly discussed in the following:

The sketch shows three overvoltage relays only, and no provision to prevent these relays tripping on three-phase overvoltages, as may occur from various reasons. Such action would not be harmful in itself, since none of the ground switches would be able to close. The closing circuits would be ruptured instantly by the transfer relays upon the initial energizing of the circuits. However, their action would unnecessarily alarm the operator and necessitate the re-setting of the device. For this reason a balance potential relay has been introduced which only responds to an unbalance of potential to ground.

This relay keeps the actuation of the phase overvoltage relays blocked so long as it is not actuated itself. It therefore prevents the device from getting into action during three-phase overvoltage.

As a precautionary measure for any unexpected action on the part of the ground selector, the ground switches are equipped with overload relays, set sufficiently high for the line protection to clear before the ground switch opens.

As a further precaution, each ground switch is provided with complete electric control which enables the operator to actuate the ground switches by hand, if necessity arises.

A hand blocking switch is also provided by means of which the automatic features of the device can be stopped without, however, interfering with the hand control of the ground switches.

A ground resistance is provided in series with the ground switches. This resistance is purposely made low so as to influence only slightly the magnitude of distant short circuits, but having a tendency to hold up the bus voltage for grounds very close to the station.

The ground selector will operate once only and will have to be reset by hand by moving the transfer relay into the original position before it is ready for new action. This, however, does not cause any inconvenience, and has been found preferable to an automatic resetting which latter might cause trouble should a ground develop into a short circuit and clear the feeder before the ground switch has been closed completely.

From this description it will be noticed that the name "ground selector" is not quite correct, since it is actually the relay protection that selects the ground, whereas the device is a means only to create the condition which will enable the relays to act and select the grounded line. Nevertheless the name was adopted as being the most descriptive.

COMPARISON OF UNGROUNDED AND GROUNDED SYSTEMS

To fully appreciate the problem involved and the advantage derived from the ground selector it will be desirable to compare the characteristic features of grounded and ungrounded system under various operating conditions.

The following table tries to cover in a concise form, the behavior of the various possible operating methods, showing in the last column for each operating condition the scheme which is considered advantageous.

In going over Table I, it will be noticed that most of the items indicate that the ungrounded system equipped with ground selector proves in practically every respect advantageous or equal to the grounded system.

One item only may prove pronouncedly advantageous for the grounded system that is cost of grounding where Y = connected transformers are in existence (see item 26). In this respect it may be pointed out however, that in many instances this saving compared with the cost of a ground selector may be more than offset by other inconveniences such as ineffective relay protection or loss of flexibility in the use of different transformer connections.

Some other items indicate that the grounded system appears to offer some advantage over the ungrounded system equipped with the ground selector.

It will be worth while to further analyze these items.

Item 7 points out the possibility of line switching causing a momentary shifting of the neutral should the three phases not close simultaneously. It is well realized today that damage to insulation is a function of magnitude of voltage and its duration; in other words, it does not only require high voltage, but the voltage must also last a certain time before it actually is able to do damage. A switching surge is of such brief duration and of rather limited voltage magnitude, so that the possibility of serious voltage stresses from switching surges is more problematic than real. Practical experience also plainly shows that troubles are hardly ever experienced at times of switching except in cases of very depreciated insulation which may barely be able to stand normal voltage.

Item 8 points out the fact that solidly grounded systems can have much more sensitivity set arresters than ungrounded systems. This is no doubt an advantage, but in view of the fact of the standard practise of a test voltage of two times line voltage for one minute on all equipment, the general improvement in design of transformer windings to withstand steep wave front impulses and the possibility of sensitive adjustment of arresters close to line voltage, this drawback of the ungrounded system should not be so serious.

Item 12 shows how, in case of the ground selector on the ungrounded system, the overvoltage on two phases created by a ground on the third phase cannot be avoided. The duration of this overvoltage is, however, so short and the overvoltage of such limited nature that any

TABLE I

COMPARISON OF GROUNDED SYSTEM WITH UNGROUNDED SYSTEM EQUIPPED WITH GROUND SELECTOR

Reference No.	Operating Condition	A Grounded System	B Ungrounded System with Ground Selector	Advantageous Scheme
	I Normal conditions (a) Transformer connections.			
1	Flexibility of connections.	Requires Y/y transformation to enable grounding of all systems. Danger from third harmonics which can only be overcome by tertiary delta winding.	Permits use of any kind of transformer connection. Y/Δ . Δ/Y . Δ/Δ . Λ/Λ No danger from third harmonic.	B
2	Complications involved.	Delta systems have danger from third harmonic eliminated, but require special grounding transformer.	Ground selector equipment adds complication.	
3	Advantage of Y connection.	Star transformer will have larger conductors making trf. mechanically stronger and providing larger electrostatic capacity between turns which improves high frequency stresses. This, however, only important at voltages higher than 60,000 volts.	By using delta connection on low-tension side and star on high-tension, same advantages can be obtained from star winding.	
4	(b) Inductive interference.	Residuals can be reduced but never completely eliminated.	Residuals can be eliminated by proper transpositions.	B
5	(c) Effect from static charges.	Static charges on wires are drained off through ground connection.	Static charges can accumulate. Should effectively be taken care of by corona leakage or effective overvoltage protection.	
6	(d) Voltage to ground.	Phase voltage to ground remains constant, but increased leakage may cause increased local heating and thus rapidly work toward a complete breakdown, since there is not only limited capacity current flowing, but actual dynamic current.	Unbalanced capacity to ground or unbalanced leakage to ground during foggy weather may cause slight shifting of neutral point adjusting to a new equilibrium. With fair insulation this should not prove serious.	
7	(e) Effects from switching.	Phase voltage to ground will remain constant.	Switching is liable to shift neutral when individual phases close slightly in succession.	A (apparently)
8	(f) Setting of arresters.	Arresters can be set in relation to fix ground voltage, which will be lower than on ungrounded system. Setting therefore, more effective.	Arresters must be set in relation to line voltage to avoid discharge in case of ground.	A (apparently)
	II Abnormal Conditions.			
	(a) Effect on system.			
9	Permanent ground (1 wire to ground.)	Develops instantly into short. Faulty feeder is cleared by relay causing partial interruption.	Ground operates ground selector. As a result short circuit develops. Faulty feeder is cleared by relay causing partial interruption.	
10	Self-clearing momentary ground. (1 wire to ground).	Develops instantly into short. Will actuate relays and cause partial interruption. Interruption may be saved by arc extinguisher, but short-circuit disturbance cannot be avoided.	If ground clears before ground selector switch closes, no dynamic voltage disturbance will result and in consequence no interruption to any part of the system.	B
11	Short circuit between phases.	Short circuit will be cleared by relays.	Short circuit will be cleared by relays.	
12	(b) Voltage stresses in case of ground. (1 wire to ground).	With solidly or low-resistance grounded neutral no extra voltage stress is exerted on any of the windings.	The ground causes a voltage rise to full line voltage on sound phases for about half second until ground selector switch is closed. After the short-circuited feeder is cleared, the ground left on the system by the closed ground selector switch, will cause again opposite phases to rise to full voltage for ¼ sec. until ground switch has opened.	A (apparently)
	(c) Inductive interference.			
13	Ground (1 wire to ground)	Short circuit with ground return will create heavy electro-magnetic interference to telephone lines. This cannot be prevented, but damage can be reduced by clearing short circuit by quick relay action.	The electrostatic interference, preceding the short until ground selector switch closed, will be small. The short circuit which follows will have the same serious effect as in case of grounded system.	

Reference No.	Operating Condition	A Grounded System	B Ungrounded System with Ground Selector	Advantageous Scheme
14	Short between phases without touching ground.	Inductive interference will be small because of the neutralizing effect of feed and return wire.	Same condition as on grounded system.	
15	Shorts of two phases through ground.	Will constitute two single phase shorts 120° out of phase with resultant current passing back through ground to grounded neutral. Will result in heavy electro-magnetic interference.	Will constitute a single-phase short with metallic return so that inductive interference becomes neutralized.	B
	(d) Permissible emergency conditions.			
16	Disabled single phase transformer.	Y/△ Transformers can be operated with one trf. disconnected, if step-up and step-down transformers have grounded neutral. This condition, however, will create serious electrostatic and electro-magnetic interference.	△/△ Transformers, if using single-phase units, will permit operation on open delta, with one single-phase transformer disconnected.	B
17	Disabled line wire.	Y Permits operation with two wires only, provided step-up and step-down transformers have grounded neutral. Creates, however, serious electro-static and electro-magnetic interference.	△ Could be operated with two wires if ground was used as third wire. Voltage between wires and ground would be increased by 73 per cent. Arrangement would create serious electrostatic and electro-magnetic interference.	A Advantage in exceptional cases only.
18	(e) Danger of secondary breakdowns in case of a ground.	No such danger because of fixed voltage to ground.	Hazard of cross short circuit very small because of ground condition lasting only for a very brief period.	A (apparently)
19	(f) High-resistance ground.	Occasionally high resistance grounds develop. Only 58 per cent of normal line voltage will be across the fault causing danger of insufficient current to actuate relays.	Ground selector when closing will throw full 100 per cent line voltage across fault, so that current will be 1.73 times greater, under similar conditions than in case of grounded neutral. As result more definite relay action is assured.	B
20	(g) No. of troubles.	Larger number of voltage disturbances and partial interruptions is to be expected since every ground represents a short circuit.	Smaller number of voltage disturbances and partial interruptions is to be expected since momentary self-clearing grounds not affecting system.	B
21	(h) Adaptability to existing systems.	Y-Connected transformers permit easily grounding of neutral point.	Ground selector can be easily installed on bus-bar of one of main stations irrespective of existing transformer connections.	
22		△ Systems require special kind of grounding scheme, which will be particularly expensive if to be of low impedance to assure sufficient current for relay action.	Ground selector can be easily installed on bus-bar of one of main stations irrespective of existing transformer connections.	B
23	(i) Limiting effect on system in case of ground.	Disturbance can be limited by ground resistance or high impedance of grounding transformer as far as relay protection will permit.	Disturbance can be limited at liberty by ground resistance in circuit with the ground selector switches.	
24	(k) Short circuit stresses on equipment.	In case of all transformer neutrals of parallel operated transformers being grounded, the short-circuit stress will be equally distributed between all the transformers.	Ground selector providing a direct short circuit across phases of the system will divide the short-circuit stress equally among parallel operated transformer and generator equipment.	
25		Where only one generator or one transformer is grounded or where special grounding transformer is used, the total short-circuit stress will have to be taken up by the respective unit.	Ground selector providing a direct short circuit across phases of the system will divide the short-circuit stress equally among parallel operated transformer and generator equipment.	B
26	(l) Cost.	Where transformers are star-connected so that system can be grounded by grounding the transformer neutral, the expense will be very small.	The ground selector will require oil switches, potential transformers, ground resistances and some relay equipment which will involve a smaller or larger expenditure depending on the operating voltage.	A
27		Where a special grounding equipment must be provided, the expense may become considerable.	The ground selector will require oil switches, potential transformers, ground resistances and some relay equipment which will involve a smaller or larger expenditure depending on the operating voltage.	

fair insulation of the system should be able to resist this stress without harm. Nevertheless the fact remains that this condition is liable to invite cross short circuits to some weak point of the system (see Item 18). The insulation giving thus away, however, would have to be so weak that such cross short circuit and the weeding out of such point would only be desirable. Operating results of the ground selector bear out the above contention.

Regarding the danger of high-frequency surges created by an arcing ground at the point of fault, it may be pointed out that in the writer's opinion the existence of the arcing ground should be extremely rare.

Puncture of insulation, be it puncture of an insulator or a cable, most frequently cause permanent grounds. In either case the current passes through a very occluded and short path, so that the heat concentration is enormous and in consequence the local resistance of the path must be zero and paramount to a solid ground. In case of a punctured cable the heat must also instantly char the organic insulation so that a carbonized short-circuit path will be formed.

The only conceivable conditions which may create arcing grounds would be a puncture under oil or an open arc over a broken insulator or any gap the breakdown point of which would just about correspond to the line voltage or surge voltage. Under such condition the arc would naturally become very unstable and would thus be able to form an arcing ground. The experience with the ground selector does not indicate any danger from this source, not even in cases of swinging grounds which sometimes are causing arcs at intervals of a few minutes, and which no doubt create the most unstable arc.

Operating experience with the ground selector indicates that the disadvantage of momentary overvoltage on the sound phases is practically negligible except in case of very depreciated insulation.

Item 17 indicates the advantage of the star grounded system of permitting maintaining operation over two wires using ground as return wire. Inductive interference from such method of operation will be so serious that this method could only be applied in special cases where the lines are running in isolated territory far away from telephone and telegraph lines.

From the above it will be noticed that the few items which appear to speak for the grounded system are of minor importance and turn out to offer in actual practise a very small advantage over the ungrounded system equipped with ground selector.

From the foregoing it follows that the most important advantages of the ungrounded system combined with ground selector, compared with grounded system, are:

a. Flexibility in the use of transformer connections.

b. Reduced number of system disturbances and partial interruptions because of the ability of momentary grounds to be self clearing.

c. Easier adaptability of the ground selector, particularly to existing systems, with delta-connected transformers.

d. Prevention of concentrated short-circuit stresses on individual apparatus, such as might be the case with special grounding transformers or in systems where only individual apparatus are grounded.

e. Reduced inductive interferences.

f. Increased voltage across the point of fault which is particularly advantageous in case of long lines to supply sufficient ground current despite the increased reactance due to the ground return circuit; in case of high-resistance grounds the higher dynamic voltage will also assist in breaking down the fault sufficiently to supply current for effective relay action.

OPERATING RESULTS

Hereafter two cases are described where the ground selector has been effectively in service for a few years. The two cases are particularly interesting in that they represent about the most extreme cases of type of distribution and voltage where the ground selector may be applied. A brief description of the respective systems and the reasons for adopting the ground selector are given in each case.

12,000-VOLT TORONTO DISTRIBUTION OF THE TORONTO POWER COMPANY

Fig. 2 shows a single line diagram of the distributing system. The power is received at 90,000 volts and is stepped down to 12,000 volts by Y/Δ transformer banks, the low-tension system being operated ungrounded.

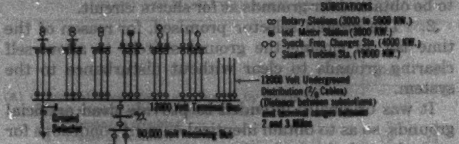
SUBSTATIONS
Rotary Stations (3000 to 5000 KW.)
Ind. Motor Station (3000 KW.)
Synch. Freq. Changer Sta. (4000 KW.)
Steam Turbine Sta. (19000 KW.)
12000 Volt Underground Distribution (2/0 Cables) (Distance between substations) and terminal ranges between 2 and 3 Miles
12000 Volt Terminal Bus
Ground Selector
Y/Δ
90,000 Volt Receiving Bus

FIG. 2

The 12,000-volt distribution consists of about 80 miles undergrounded cables and about 80 miles overhead lines. All overhead lines are wooden pole construction with wooden cross-arms and ungrounded steel pins and some of them are of considerable length, passing out into rural districts. Grounds were quite frequent, many of them being self clearing without causing any disturbance to the system. Permanent grounds, however, always caused serious disturbances and interruptions to the system until located. On several occasions the system was completely paralyzed due to secondary breakdowns developing on account of the overvoltage on the opposite phases created by a ground.

In fact such secondary breakdowns or so-called cross-short circuits were so much feared, that the practise established itself that the whole station was cleared immediately upon indication of a permanent ground, and then the system was started again by closing one feeder at a time, leaving out the feeder, which made the ground reappear. With this procedure secondary breakdowns could usually be avoided although it meant a complete shutdown of the system.

It was realized that something had to be done to improve this situation and it was then that the idea of the ground selector developed. This was as far back as 1916. It was realized, however, that the scheme would only be beneficial if the overload protection of the distribution were effective and dependable. In an endeavor of putting first the short-circuit protection of the distribution into shape, the installation of the ground selector was delayed until September, 1918.

The main reasons in this particular case for adopting the ground selector in preference to some permanent grounding scheme were as follows:

1. The ground selector was cheaper and simpler to install, since existing spare feeder switches could be used for the purpose and since the ground current could be adjusted to any desired value to suit relay requirements.

A grounding scheme would have involved either a costly grounding transformer of very low impedance to ensure sufficient current to actuate the ordinary overload protection, or a less costly grounding transformer of higher impedance, in which case however the ground current would have been so limited that a special ground protection would have been required which in itself would have offered serious difficulties and expense if the same selectiveness and effectiveness was to be obtained for grounds as for shorts circuit.

2. The ground selector promised, for reason of the time lag, of closing the ground switch, to allow self clearing grounds to clear without disturbance to the system.

It was originally intended to provide dead artificial grounds, so as to obtain identical current conditions for grounds and short circuits to assure effective relay action under all conditions. However, it was finally decided to introduce a very low ground resistance in form of a large water rheostat which hardly affected the magnitude of the fault current under most limited current conditions in case of distant grounds, but which offered the great advantage of keeping up the bus-bar voltage for grounds close to the terminal station, thus helping to keep the synchronous load in step.

The ground selector was put in service in 1918 and has proved very effective and beneficial as following operating records will indicate:

Table II shows an operating record of the ground selector, showing the number and kind of troubles it had to clear.

Table III shows a summary of troubles experienced on the 12,000-volt distribution. This table indicates particularly the large number of self-clearing grounds without disturbance to the system, which shows a decided advantage over a permanently grounded system.

It may be pointed out that since the installation of the ground selector not a single secondary breakdown, causing a cross short circuit to some other point of the system, has been experienced.

TABLE II

OPERATING RECORD OF GROUND SELECTOR OF 12,000-VOLT TORONTO DISTRIBUTION

	Sept. to Dec. 1918	1919	1920	1921
(A) Grounded feeders effectively cleared by action of ground selector and respective feeder protection.				
1. Permanent troubles.				
Cable troubles		4	1	1
Bad insulators		4	4	1
Bad apparatus (current transformers, Power transformers, oil switches, etc.)		5	6	3
Operator's fault		1		1
Miscellaneous		2		
Total permanent troubles		16	11	6
2. Transient troubles.				
Lightning		3	4	3
Unknown	6	15	15	19
Miscellaneous	1			4
Total transient troubles	7	18	19	26
Total of grounded feeders cleared	7	34	30	32
(B) Momentary self-clearing grounds, starting ground selector without developing into short circuit.				
1. Ground switch closing and opening momentarily	4	10	10	26
2. Momentary ground relay energization of too short duration to start ground selector switch	2	7	8	13
Total self-clearing grounds	6	17	18	39
(C) Testing faulty feeders		5	15	3

TABLE III

SUMMARY OF TROUBLES OF 12,000-VOLT TORONTO DISTRIBUTION

	1919		1920		1921	
	N	Per cent	N	Per cent	N	Per cent
1. Grounds cleared by ground selector	34	30	30	31	32	23
2. Grounds, self-clearing, without disturbance to voltage and without partial interruptions	17	15	18	18	39	29
3. Short circuits	63	55	50	51	66	48
Grand total	114	100	98	100	137	100

In all cases the trouble and partial interruption remained confined to the faulty feeder.

With the exception of one case the nature of the ground was always of sufficiently low resistance to produce sufficient ground current to effectively trip the relays. In the case which did not clear the feeder the ground current was sufficient to be distinctly noticed as overload on the respective line ammeter, enabling the operator to clear the feeder by hand without serious disturbance to the system.

In practically all cases the grounded feeders were cleared without serious disturbance to the load and without dropping other load than the one of the feeder in trouble.

In two cases 30 per cent to 50 per cent of the whole system load was lost, but in both cases the reason was that the arc produced by the ground current at the point of fault spread into the other phases and thus caused a three-phase short circuit before the feeder had time to clear.

The ground selector has proves fully up to expectation in eliminating cross short circuits and offering a means to effectively clear grounded feeders without interruption to other parts of the system.

It has, moreover, simplified operation of the Toronto distribution since it permits parallel operation of the whole distribution at all times, whereas before, the system had to be sectionalized whenever a ground occurred in an endeavor to sectionalize and localize the ground.

50,000-VOLT 30 CYCLES DISTRIBUTION SYSTEM OF THE SHAWINIGAN WATER AND POWER COMPANY

Fig. 3 shows a single wire diagram of this system. It is of a completely different nature to the Toronto system.

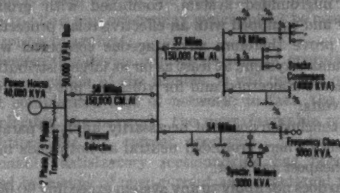

FIG. 3

The lines were of the first ones built at this voltage. The lines are of wooden pole construction with wooden cross-arms and wooden pins. Each pole is grounded by a ground wire running down the pole and being wound around the butt of the pole. The insulator pins, however, are not grounded. No overhead ground wire is provided. The ground at each pole is therefore entirely dependent on local ground conditions. The 50,000-volt system is fed from a two-phase power house by means of Scott-connected step-up transformers.

Grounds were causing serious tie-ups due to general weakening of the old insulator.

Grounding of the system was then seriously considered to eliminate this serious state of conditions, but in going further into the matter preference was finally given to the ground selector.

It would have been comparatively easy to tap the neutral point of the Scott-connected transformers and thus provide a grounded neutral at the power house end. The double line protection, however, basing on a principle requiring the back feed from synchronous machinery of the receiving end, would have become ineffective for grounds with only the generating station neutral being grounded. To make the double line protection effective also for grounds would have necessitated change to star-connected transformers with neutral grounds at all stations with synchronous machines. This was not considered desirable since it would have eliminated the possibility of running the transformer bank open delta in case of trouble in one of the single-phase units. The ground selector on the other hand assured proper double line protection without need of any change in the existing layout.

A further important point in favor of the ground selector was the fact that it produced 1.73 times greater voltage across the point of fault compared with a grounded neutral system.

It was felt that this increased voltage would materially aid in completing any insulation breakdown to a state of low-resistance ground and would assist in increasing the ground current, thus assisting effective relay action. This was considered particularly important in view of the uncertain ground conditions and in view of the very long lines with excessive reactance drop which tended to limit the fault current.

The ground selector was installed in September, 1919, and the following gives an operating record of the device:

TABLE IV

OPERATING RECORDS OF GROUND SELECTOR OF 50,000 VOLT SHAWINIGAN SYSTEM

	Sept. to Dec. 1919	1920	1921
(A) Grounded lines effectively cleared by action of ground selector and respective line protection.			
1. Permanent troubles		7	8
2. Transient troubles		12	13
Total of grounded lines cleared	8	19	21
(B) Momentary self-clearing grounds. Starting ground selector signal only, without short, or closing ground switch causing a momentary self-clearing short circuit		10	9
(C) Testing faulty feeders	5	7	1
(D) Grounds, where ground selector unable to clear line either because of high-resistance ground or because of ground selector not locking when closing		1	2
(E) Grounds developing into cross short circuits due to harmless flashover		26	11

Table IV gives a summary of the actions of the ground selector.

Table V shows the number of troubles cleared by the ground selector in relation to the number of other troubles.

It will be noticed that the results do not look as satisfactory as those shown in Tables II and III.

Considering however the character of the system, it must be realized that the results are rather better than expected.

The momentary self-clearing grounds are of a smaller percentage than in case of Table III of the 12,000-volt system, so that the advantage compared with a permanently grounded system is not so pronounced.

High-resistance grounds, where the ground selector was unable to produce sufficient current to operate relays are also somewhat more numerous than in case of the Toronto system. There were however only three cases of this nature compared with forty-eight cases where sufficient ground current was developed to actuate relays, which means only 6 per cent ineffectiveness. In view of the uncertain ground condition along the line this must be considered an unexpectedly good result.

TABLE V

SUMMARY OF TROUBLES OF 50,000-VOLT SHAWINIGAN SYSTEM

	1920		1921	
	N	Per cent	N	Per cent
Feeders cleared by ground selector	19	14	21	22
Momentary self-clearing grounds	10	7	9	9
Grounds where ground selector not effective	1	1	2	2
Grounds developing into cross short circuits but without permanently disabling line	26	19	11	12
Short circuits	81	59	53	55
Grand total	137	100	96	100

The number of secondary breakdowns, causing a cross short circuit to some other part of the system is, as seen from Table V, still undesirably large. It is to the credit of the ground selector, however, that only one of these cross short circuits actually caused a permanent insulation damage. All other cross short circuits were harmless flashovers traceable to asbestos mining districts caused by insulators whose flashover had been reduced by a combination of broken skirts from blasting and asbestos dust coating having become moist from fog or slight rain. Considerable trouble from flashovers in this district is experienced under normal voltage conditions and it is well known that sometimes insulators may be on the system unobserved which may barely be able to resist the normal operating voltage; so that the least voltage disturbance will cause the insulators to flashover.

The large number of cross short circuits is therefore less an indication of a weakness of the ground selector in that respect than a weakness in the insulation of the system at these points and has to be remedied by increasing the local safety factor of flashover even under conditions of broken skirts.

The operating results of this system indicate, similar to the Toronto system, the ability of the ground selector to avoid secondary breakdowns, as long as the system has only fair insulation, and to provide sufficient current to effectively clear faulty lines.

CONCLUSION

The operating results show conclusively that the ground selector will improve the ungrounded system to the same state of perfection as claimed for the grounded system without having to resort to an artificial grounding method.

The operating results show that the ground selector prevents permanent secondary breakdowns and permits clearance of grounded lines as effectively as the grounded system. Besides this, the ungrounded system retains the advantage of less numerous voltage disturbances and partial interruptions and the flexibility of permissible transformer connections.

The ungrounded system, combined with ground selector and equipped with an effective relay protection should prove therefore advantageous compared with the grounded system, at least as far as large distributing systems are concerned and for voltages, not exceeding 60,000-volts.

This conclusion does not necessarily apply to parallel trunk lines where grounded neutral at both ends may prove cheapest and best.

It also does not apply to any system exceeding 60,000-volts since the complication and cost may become excessive.

Each individual case, of course, will have to be studied on its own merits to arrive at a conclusion of the most desirable arrangement.

Discussion

For discussion of this paper see page 532.

The Distance Relay for Automatically Sectionalizing Electrical Net Works

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Review of the Subject.*—This paper describes a new relay for automatically sectionalizing transmission lines, similar to existing relays in many respects, but equipped with "brains" so that it can determine the location of the trouble and govern its operation accordingly. For dead short circuit its time of operation is proportional to the distance from the short circuit so that when it is used the circuit breaker nearest to the trouble always trips out first. When applied to any system it is adjusted to fit the particular length of line which it controls and no change in this setting need ever be made due to operating conditions on the remainder of the system.*

Its discrimination is obtained by a time limit which varies directly as the voltage and inversely as the current existing during the trouble. It has the incidental advantage that since the circuit breaker nearest to the trouble is opened first, the final breaker to open is the one having the longer section of line in series with it to limit the current and therefore decrease the duty required of the circuit breaker.

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A RELAY WHICH DETERMINES ITS OWN TIME OF OPERATION

THIS is a new device, which when trouble occurs, makes a calculation to determine its location. If the trouble is close to the switching point at which the relay is situated, the relay will operate quickly to disconnect the disabled section. If, however, the trouble is a long way off, the relay will operate slowly in order to allow any other relay which might be nearer to the trouble to act first.

Because this device adjusts its time of operation to suit the location of the trouble, it will operate on the most complicated sort of network; in fact, the more complicated the network, the more favorable are the conditions for its correct operation. In the past, reasonably satisfactory sectionalizing has been obtained by means of over-current and directional relays equipped with time limit features. The general scheme of operation has been to adjust the relays farthest from the power house with a short time limit and to increase this time limit on those relays which are located nearer to the power station. On a complicated system this is rather difficult and in some cases impossible, for instance, in the case of tie lines between two generating stations.

THE BASIS OF CALCULATION

In determining the distance, the relay makes use of Ohm's law and bases its calculation upon the value of the current and voltage existing at the time of the short circuit. A simple form of distance relay is shown in Fig. 1 wherein a current coil is shown arranged to close the relay contacts while a voltage coil is so arranged that, as long as voltage is applied, it will tend to overpower the current coil and prevent the contacts from being closed. These two coils are proportionate so that on the particular section of line to which they are applied the current coil will just overpower the voltage coil when a short circuit occurs at the far end of the section. If a short circuit should occur in the next section, the voltage across the potential coil of the relay will be higher and will thus prevent it from operating, whereas, if the trouble is within that section of line and close to the relay, the voltage will be lower and the relay's action, therefore, more positive. With trouble immediately at the relay, the voltage will of course be zero and, therefore, the voltage coil will offer no opposition to the operation of the current coil. Another way of looking at this simple relay is to assume that the two coils are balanced against each other and that the circuit is divided into two branches, one branch going through the voltage coil having a fixed resistance and, therefore, taking a current which is proportional to the voltage, and the other branch consisting of the current coil in series with the section of line. This latter branch is of variable resistance depending upon the location of the short circuit and the coils are so designed that the current coil will just overbalance the voltage coil when

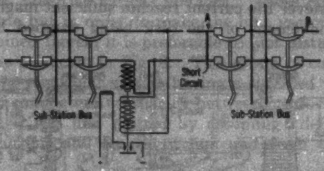


FIG. 1—PART OF A NETWORK SHOWING A SINGLE SECTION OF LINE AND AN ELEMENTARY DISTANCE RELAY

One defect is that it cannot distinguish between a short circuit at *A* and one at *B*.

Presented at the Spring Convention of the A. I. E. E., Pittsburgh, Pa., April 24-26, 1923.

the resistance is its maximum. Therefore, when the short circuit is beyond that section of line, *i. e.*, when the resistance of that particular branch becomes greater than that required to balance the relay, the relay will not operate. On the other hand, when the resistance is less than that of the full section of line, *i. e.*, when the short circuit is closer to the relay, the unbalance will be still more pronounced and the relay action more positive.

The simple relay mentioned in the preceding paragraph has two obvious defects. First, it will not work properly if the short circuit should in itself have some resistance, which resistance, being an unknown factor, would be difficult to take into account. Second, it is unable to properly discriminate in case of trouble occurring close to the substation at the other end of the section, *i. e.*, the relay cannot tell whether the trouble occurs at *A* or *B*. This simple relay would furthermore be unable to clear trouble in a distant section which for some reason had not been cleared by the proper circuit breakers on that section.

The remedy for all these defects is simple; design the relay so that when the trouble is close, it will operate instantaneously, but when the trouble is some distance away it will operate slowly so as to allow any relay which is nearer to the trouble to act.

MECHANICAL PRINCIPLE

The principle of the time-limit distance relay is shown schematically in Fig. 2. This relay is of the induction type, the disk being rotated by the current coil whenever the current reaches sufficient magnitude, —the movement of the disk being damped by the permanent magnet so that its speed is roughly proportional to the current. The movement of this disk does not directly close the contacts but instead it tightens a spiral spring. On the other end of this spring is fastened a lever which is pivoted at the center and which has the contacts on one end and the core of the voltage coil on the other. This voltage coil is so arranged that it opposes the closing of the contacts. When in the neutral position all parts of the relay are nicely balanced so that if there is no voltage on the voltage coil a very slight movement of the disk will close the contacts. Similarly, if voltage is applied to the restraining coil, its pull will prevent the contacts from being closed until the spring has been tightened up sufficiently to overcome it and since the motion of the disk is damped, this will require some time.

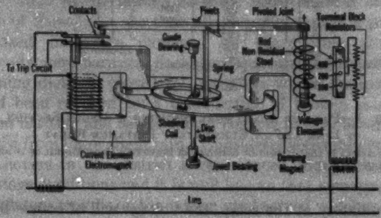

Contacts
Pivots
Pivoted Joint
Terminal Block
Resistors
Guide Bearing
Rod
Non Residual Steel
To Trip Circuit
Spring
400
200
100
Shading Coil
Disc Shaft
Voltage Element
Current Element Electromagnet
Jewel Bearing
Damping Magnet
Line

FIG. 2—SCHEMATIC DIAGRAM OF *C Z* DISTANCE RELAY

MATHEMATICAL PRINCIPLE

It is evident that for a given voltage on the voltage coil the spring must be tightened to a certain definite point before the pull of the voltage coil can be overcome and, therefore, if the voltage is maintained constant the time required for the relay to close its contacts will vary inversely as the current, or stated mathematically:

$$T \text{ varies as } 1/I$$

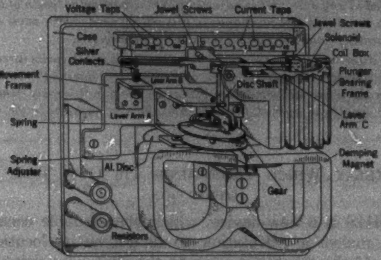

Voltage Taps
Jewel Screws
Current Taps
Jewel Screws
Solenoid
Coil Box
Case
Silver Contacts
Movement Frame
Lever Arm B
Disc Shaft
Plunger Bearing Frame
Lever Arm A
Spring
Lever Arm C
Spring Adjuster
Al. Disc
Damping Magnet
Gear
Resistors

FIG. 3—DISTANCE RELAY TYPE *C Z*

Similarly if the current is maintained constant with a resulting constant speed of the disk, the time required for the contacts to close will vary directly as the voltage, or stated mathematically:

$$T \text{ varies as } E$$

Combining these two we have

$$T = E/I = Z = \text{distance}$$

IMPORTANT DETAILS OF DESIGN

The principle which has just been described is exceedingly simple but the actual design of the relay was quite difficult, principally because of the requirements that, during time of trouble, it must discriminate between different voltages of very small value, whereas, during normal operation, the voltage coil must be able to withstand full voltage continuously. Part of the method of overcoming this trouble has been to cast cooling ribs on the case containing the potential coil, as shown in the accompanying photograph and the schematic view, Fig. 3. This loss is only about 8 watts but it is generated in a coil no larger than an ordinary spool of thread. As a result of this design, the voltage coil will discriminate quite positively between zero voltage and 5 per cent of normal voltage. At first, difficulty was encountered due to the effect of residual magnetism when suddenly reducing the voltage from normal to a small value, but this has been overcome by using a core of a special non-residual steel developed by Dr. T. D. Yensen of the Westinghouse Research Laboratory.

The inherent characteristics of the induction disk are such that within a moderate range its speed varies directly as the current so that no particular difficulty was encountered in designing the current element. But the voltage coil presented difficulties which required painstaking work to overcome. Finally the design shown in Fig. 5 was worked out to provide a device having a straight line characteristic, of rugged construction, free from the effects of friction, quick in its response, and accurate. The straight line characteristic, which was quite satisfactorily attained as shown in Fig. 6, is due to the use of a core of small cross section which saturates at a low voltage so that above this value the pull varies directly as the voltage. It should be observed that at normal voltage the pull on the core is almost 0.5 lb. Of course it is proportionally less at lower voltages but it is far from being the delicate device that might be imagined. This heavy force is required in order to insure adequate pressure on the contacts as well as to minimize the effects of friction. Friction is still further reduced and the effects of wear practically eliminated by careful jewelling of the important bearings.

FIG. 4A

FIG. 4B

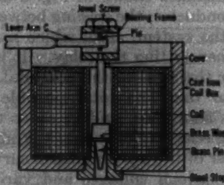


FIG. 5—TYPE *C Z* DISTANCE RELAY DETAILS OF POTENTIAL COIL

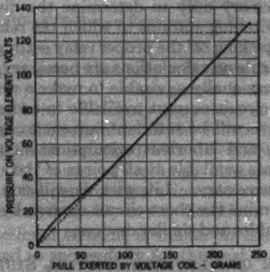


FIG. 6—TYPE *C Z* DISTANCE RELAY VOLTAGE-PULL CURVE

CHARACTERISTICS

It has been customary with the older types of relays to plot current—time curves and for this reason it will be interesting to observe in Fig. 7 a family of current-time curves taken at various voltages. These curves are complete but in order to make the best use of the data contained therein it is preferable to plot distance-time curves as shown in Fig. 8. This group of curves refers to one particular system having a specified voltage, size of conductor and spacing between conductors. The curve *B* was first plotted by assuming a definite primary current and length of line and from these figures calculating the percentage drop in voltage. The time corresponding to this voltage was then taken from Fig. 7 and the position of the straight line *B* therefore determined.

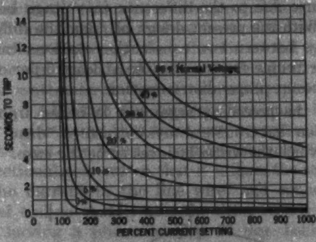


FIG. 7—TYPE *C Z* DISTANCE RELAY TIME-CURRENT CURVES AT VARIOUS VOLTAGES

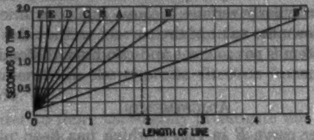


FIG. 8—TIME-DISTANCE CURVES TYPE *C Z* DISTANCE RELAY

The curves in Fig. 8 are not run up very high in time because we are not much interested in the long time characteristics of the relay. Its object is to clear trouble as quickly as possible and all adjustments should be made with that end in view. If, for instance, in Fig.

9, trouble should occur at *X* the relay at *A* should operate in, say, 0.1 second. The circuit breaker will require about 0.25 second to open, the total time required to clear the short circuit being 0.35 second. Now the relay at *B* should have sufficient time delay so that it will not close its contacts before the breaker at *A* has had a chance to open (0.35 sec.). If we double this time in order to allow an ample margin of safety, we will then have a reasonable setting for the relay at *B*. Therefore, we have drawn the dash line across Fig. 8 to indicate the maximum time which should be required to operate any relay. This is the extreme case; the average short circuit will be cleared in considerably less time. This difference in time between adjacent stations is of course due to the difference in voltage between them. With the present design of relay the difference in voltage between two consecutive stations carrying the same trouble current must be at least 5 per cent in order to secure proper discrimination.

All of the preceding argument is based upon the assumption that trouble always consists of "dead" short circuits which the writer believes to be the case. If we assume that a high-resistance short circuit exists, it will increase the time of operation of the relay but this is not particularly serious because this resistance will also cut down the value of the current so that the shock to the system is not so great and can be withstood for a longer time.

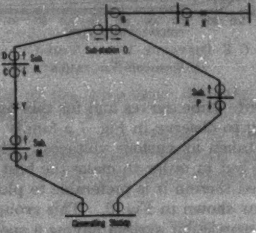


Fig. 9

ADJUSTMENTS

Consider curve *B* in Fig. 8 which has been plotted from the relay characteristics given in Fig. 7, and assume that the ampere turns on the current coil are reduced to one half. Therefore, with our assumed current, the disk will rotate at half speed which will allow the voltage to be reduced to half its value in order that the relay may operate in the same length of time. In other words by reducing the ampere turns on the current coil to one half we can reduce the distance by an equal amount, thus giving a curve *D*. Similarly by again halving the number of turns, curve *E* can be obtained. Similarly the other curves in the group are obtained by means of taps on the current coil.

Suppose the section of line to which we wish to apply this relay is so long that the adjustment shown on curve *B* is unsatisfactory. We leave the current adjustment as it is and place sufficient resistance in series with the potential coil so that its current is cut to half its original value. Therefore, in order that the pull on the coil shall be the same with a consequent equal time of operation, it will be necessary to double the voltage, which can only be done by doubling the length of the section of line. Thus, we get curve *B′* and similarily by again doubling the resistance of the potential circuit we double the distance and obtain *B″*. The other curves can be similarly extended so that adjustments can be made which are suitable to intermediate length of line section. It is thus seen that by two very simple adjustments the relay can be set to work properly on sections of lines whose length vary as much as 40 to 1.

METHOD OF SETTING RELAY

The unit of distance shown in Fig. 8 depends upon the size of the conductor, the spacing between them and the voltage of the system, all of which must be taken into account when setting the relay. It is also necessary to consider the ratio of the potential and current transformers, so that the total possible number of combinations is enormous, and in order to make the task of adjusting relays an easy one, it is important that the manufacturer of the device furnish curves or tables for systems of various voltages and various sizes of conductor showing the distances corresponding to the different relay settings. When the data are arranged in this way the problem of setting the relays is an easy one, it being necessary only to determine the distance between the switching stations to which the relay is to be applied and picking off the proper relay setting from the table. As a rule no calculations are necessary, although it may be occasionally advisable to make a rough check of the short-circuit current available in order to make sure that there will be sufficient current to operate the relay. Long and laborious calculations of the short-circuit currents available at all points of the system are certainly unnecessary. Likewise, it will not be necessary to determine the short-circuit current by means of calculating tables and similar devices.

ITS USE ON GROUNDED NEUTRAL SYSTEMS

When a short circuit occurs between two wires on a 3-phase system, the potential coil of the relay must, of course, be supplied with delta potential so that it can make its proper determination of the distance. On the other hand, it should be supplied with star voltage when a short circuit occurs between one wire and ground on a system having a dead grounded neutral. It, therefore, appears that two sets of relays should be used on such a system, one to clear short circuits between wires and the other to clear grounds. On a system having its neutral grounded through a resistance, other relay schemes must be used to clear grounded conductors.

THE DIRECTIONAL ELEMENT

In addition to the distance determining element which has been described this relay will usually require a directional element in order to definitely locate the direction of the trouble as well as its distance away. For example in Fig. 9, which represents a rather simple distribution system, it is quite evident that if trouble should occur at *Y*, both relay *C* and relay *D* at substation *N* would have the same current and voltage actuating them so that they would both try to open their circuit breakers. In order to prevent such improper operation, each relay should be equipped with a directional element which has its contacts in series with the distance contacts and acts as a sort of "check valve" to prevent the relay from operating when power is flowing toward the bus bars. Therefore, in the figure relays *B* and *D* are each shown equipped with directional element so connected that they will allow their respective relays to operate only when power is flowing away from the bus bars. Similarly the relays at substation *M* and in fact the relays on all the substations around the loop should be equipped with directional elements as shown. Of course directional elements are not necessary at the generating station and at other points where power can flow only in one direction.

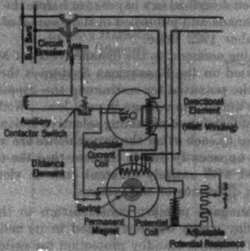


FIG. 10—*C Z* RELAY ELEMENTARY DIAGRAM

This directional element is similar to the one which has been used so successfully for the past 8 or 10 years and consists of a very sensitive watt-indicating device without mechanical restraint of any nature and so sensitive that it will operate under the most extreme possible conditions of low voltages on the system. This directional element will easily discriminate on 1 per cent of normal voltage if the short-circuit current through the relay is, say, 5 or 6 times normal. This condition is easily obtained during short circuit; in fact it is unusual to have the voltage drop to as low a value as 1 per cent of normal.

In Fig. 10 is shown a complete diagram of internal connection of the distance relay including the directional element and the adjusting taps. It should be noted that during normal conditions on the system a large number of the directional elements will have their contacts closed, but this is a matter of no importance because the cooperation of the distance element is necessary before a circuit breaker can be tripped. Therefore, the question of normal direction of power flow has nothing whatever to do with the setting or adjustment of the relay.

METHOD OF CONNECTING RELAY IN THE CIRCUIT

The experimental work which has finally resulted in the production of this relay was begun quite a number of years ago and one of the earliest fruits of the work was the startling discovery that the directional element cannot be connected in the circuit as though it were an ordinary wattmeter intended to measure power consumption. Although the problem was quite baffling at first, it was found that there were several easy solutions, the best one being to connect the current coil of the directional element so that it utilizes the star current (the current in the line wire) and connect the potential coil so that it uses the delta voltage of the system having a phase relation 30 deg. behind the current. This scheme has been successfully used for a number of years on the present type of directional overcurrent time-limit relay and it is usually assumed that this lag in the voltage circuit is for the purpose of enabling the watt element to work under a little better power factor condition when the current lags due to the low power factor of a highly inductive short circuit. While this is, of course, an advantage, the principal reason for using the connection is that it prevents improper operation of the relays under badly unbalanced conditions such as will always occur when only two wires of a 3-phase circuit are short-circuited. For example if the two coils of the relay should make use of the star current and star voltage of the circuit, it would be theoretically possible to have short circuits of such a nature that the current in the relay would, in effect, lag 180 deg. behind the voltage so that the relay would operate in the opposite direction from what it should do.

In order to more thoroughly investigate this subject and check some of the theories, an elaborate 3-phase artificial transmission line was constructed. This part of the work is described in a paper on the artificial transmission line presented before the A. I. E. E., by Mr. George H. Gray, in 1917. The conditions which exist at numerous points of a large transmission system, during almost every conceivable kind of ground and short circuit were studied and classified in a number of diagrams and charts which makes the subject easily understood. It is, therefore, unnecessary to discuss the subject in this paper.

Advantage of the Distance Relay

All dead short circuits will be cleared within one second and the majority of them in considerably less time.

Trouble will be cleared with equal rapidity from all parts of the system.

The breaker furthest from the trouble opens last, thus insuring the insertion of considerable impedance in the path of the short circuit before finally interrupting it. This reduces the strain on the breaker, and, in effect, converts every breaker on the system into a "resistance type" breaker.

It does not require an excessive amount of mathematical calculations when applying it to the distribution systems.

Conclusion

Although the distance relay as described in this paper is a new idea, it can hardly be said to be entirely untried because its use does not involve a large number of new mechanical devices or new electrical connections. The induction type of protective relay has been thoroughly tried out and found to give excellent service. The directional element has given the same good service and likewise the method of connecting it into the circuit has been thoroughly approved by years of experience. In effect, the well known directional time-element overcurrent relay has been taken and been given sufficient "brains" to enable it to determine what time limits it should have in its particular location.

Discussion

SELECTIVE RELAY SYSTEM OF THE 66,000-VOLT RING OF THE DUQUESNE LIGHT COMPANY

(Sleeper);

GROUND SELECTOR FOR UNGROUNDED THREE-PHASE DISTRIBUTION SYSTEMS

(Ackerman);

THE DISTANCE RELAY FOR AUTOMATICALLY SECTIONALIZING ELECTRICAL NET WORKS

(Crichton);

Pittsburgh, Pa., April 25, 1923.

R. N. Conwell: Mr. Sleeper's paper gives an example of one of the methods of determining whether relays are correctly set for selective operation. There is another method, that of keeping accurate records of relay operations in service, which will give equally good results but these are not obtained as promptly as by applying actual short-circuit tests. The expense, however, is much less and it is the only method applicable to those systems in which the transmission cannot be taken out of service for tests. Both methods must be preceded and followed by the same careful analytical study which Mr. Sleeper has given to his problem.

I regret that Mr. Ackerman has brought into the subject of relay protection the item of interference to signal systems. The two are separate and distinct problems and I feel sure that any system of relay protection installed on a power supply line as a remedial measure in a case of inductive interference will be an entirely different relay system from that installed solely to isolate defective apparatus or transmission lines. The scale of fundamental limiting factors of time, current and voltage in the two problems are in the ratio of 10 or 100 to 1. Furthermore, Mr. Ackerman's paper brings up the old question of grounded versus ungrounded systems from both the system protection and interference points of view. From a purely power system point of view, I believe there is little doubt as to the desirability or even necessity of grounding. Inasmuch as Mr. Ackerman's device may be considered as a Creighton arc suppressor with the high-tension phases crossed, the reduction in interference claimed for the device is not apparent, for the occurrence of a single-phase fault to ground in an ungrounded system results in the device operating to ground another phase and permitting the current required to operate the relays to flow through the ground between the fault and the ground established by the device. The condition therefore, is not unlike that found in a grounded neutral system when considered from the interference point of view.

The relay which Mr. Creighton has shown will fill a long felt need. The continual increase in the number of substations to be supplied from a transmission network has forced the use of long time settings, if arrangements could not be made for parallel line protection. The troubles and hazards incident to the use of long time settings in such cases will now be obviated by the use of the "Distance Relay."

P. Ackerman: I have been interested to hear the paper by Mr. Crichton, chiefly because I have been working on a similar principle, such scheme being in operation on one of the Shawinigan Water & Power Co's. systems for about three years. My scheme has been described in a paper read before the Engineering Institute of Canada and published in the *Journal* of the E. I. C. in the December 1922 issue.

The only thing common to the distance relay of Mr. Crichton and the one used on the Shawinigan System is the underlying idea of using the potential for restraining the tripping tendency of the current unless the voltage is very low. With respect to the detail consideration of obtaining selective action of switches located in series to each other the two schemes are widely divergent. This is apparently due to the fact that the two schemes were developed with entirely different ends in view, probably because of the difference of the systems.

Mr. Crichton gave primary consideration to the desire of obtaining a relay which was not limited in its active distance range, so as to obtain a standby protection for switches located in series to each other. He thus conceived of the distance-time relay wherein the tripping time was an approximate function of the distance to the short-circuit point. He obtained this feature by cleverly combining the voltage restraining element with a time-limit overload relay.

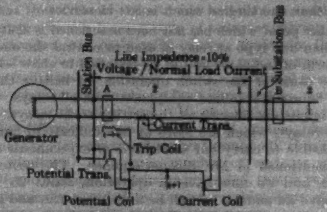
I, on my part, was anxious to obtain primarily instantaneous protection over the largest possible portion of a line so as to reduce the damage to the line and the disturbance to the system, the latter consisting largely of synchronous motor load. I was also desirous of obtaining a protection which could be successfully applied to the generators. In view of the above, I found it desirable to keep the functions of the a-c. relay and the selective time-limit feature separate.

As shown in Fig. 1 I used a balanced beam with a current solenoid at one end and a potential solenoid at the other end. The balance between current and potential solenoids can be adjusted for any desired active distance range. Thus a relay can be adjusted to be effective to the end of a line, but not beyond. Such relay will not have to be selective with any other switch and in consequence can be made instantaneous. A second set of relays is adjusted to have an active distance range beyond the substation, thus obtaining a protection for substation shorts. If this latter set of relays was made instantaneous it would become non-selective with the protection of the feeder ahead. In order to assure time selective action, therefore, this latter set of relays is arranged to trip the oil switch through a definite time relay instead of directly.

Fig. 2 illustrates the difference in time curves obtained by the two types of protection. (a) represents the time curve of Mr. Crichton's relay and (b) the time curve for the principle used on the Shawinigan system.

A distinct drawback of Mr. Crichton's relay is the fact that it cannot be applied to any case where the voltage difference between switches, located in series to each other, is less than 5 per cent. It therefore cannot be used for generator protection to act selectively with feeder switches emanating from the generator bus bar since the voltage condition of the two relays will be the same. In the principle used on the Shawinigan system such a case is taken care of by time selective difference of the two switches, as shown in Fig. 2.

A further drawback of Mr. Crichton's relay is the fact that the relay is not a strictly distance time relay but the time is also partly dependent on the actual current magnitude as will be seen from the curves in Fig. 7 of Mr. Crichton's paper. In consequence its adjustment is dependent on the connected generator capacity and it is essential that extreme operating conditions be taken into account when determining the setting.



Condition	Relative Magnetic Pull on Solenoids		Action of Relay
	Potential Coil	Current Coil	
Light Load	┃	▪	Potential Coil Overpowers Current Coil and Keeps Trip Circuit Blocked
Full Load	┃	|	Potential Coil Overpowers Current Coil and Keeps Trip Circuit Blocked
Short at end of Line at Point. 1	|	|	Potential and Current Coils just keeping balance. Relay adjusted for this balance.
Short near Power House at Point. 2.	▪	┃	Current Coil Overpowers Potential Coil and Trips Switch.
Short beyond Line at point. 3.	┃	|	Potential Coil Overpowers Current Coil and keeps Trip Circuit Blocked.

FIG. 1—SCHEMATIC DIAGRAM OF CURRENT POTENTIAL OVERBALANCE PRINCIPLE FOR A SINGLE-PHASE CIRCUIT WITH TABLE SHOWING FUNCTIONING UNDER DIFFERENT CONDITIONS.

This feature will necessitate an increase in time settings in order to assure proper time selective action for all possible operating conditions. In the principle used on the Shawinigan system the time feature is absolutely independent of operating conditions.

Another condition which will increase the time setting on Mr. Crichton's relay is the one of two substations of different distances being located in series to each other as is illustrated in Fig. 3. Substation (*C*) is assumed closer to (*B*) than substation (*B*) to (*A*). Assume (*b*) being the time curve for switch (*B*) for short circuits along the line (*B-C*). Curve (*a*) represents the time curve for switch (*A*) which is necessary to assure time selective action with switch (*B*) for short circuits beyond (*B*) but close to (*B*). The time difference (*X*) is considered the minimum necessary time distance for selective action. For short circuits at the end of line (*B-C*) the time curve (*a*) is not time selective with curve (*b*). In order to make it time selective for all conditions the time curve for switch (*A*) must be raised to time curve (a_1) which means an undesirable increase of the tripping time.

These few points mentioned must not be considered as a criticism of Mr. Crichton's relay. His scheme forms an advance in the art of relay protection in the right direction, but it has its distinct limitations as any type of relay has. As a result it can only be applied intelligently if its characteristics are fully taken into consideration. The points discussed were made to bring out more distinctly some of those limiting features which must be known in the practical application of the relay.

The fact remains that an effective protection cannot be developed without full knowledge of the system and its general operating conditions, as well as the full knowledge of the intricate characteristics of the relays to be used. Only a proper harmonizing of operating needs and relay characteristics can bring the desired results.

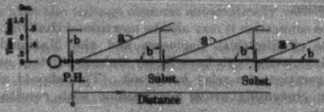


FIG. 2

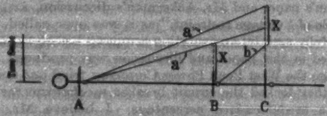


FIG. 3

W. V. Lovell: On the first page of Mr. Ackerman's paper appears the statement that the general opinion is overwhelmingly in favor of the grounded neutral, and there is little hope or desire to controvert that after hearing the papers presented on grounding at this session, but in his table, under Item 13 he states that short circuits with ground return will create heavy electromagnetic interference to telephone lines. The deduction is that the sentiment is overwhelmingly in favor of heavy interference. I don't think that is a conclusion that anyone would draw who has followed the efforts of the operating engineers to reach a solution of the problem, and surely it is not the sentiment of the Bell engineers.

I think the chief difficulty is with the broad claim in Item 13 which was probably made without very thorough analysis. I do not find in the paper data to substantiate that and similar conclusions as expressed in references No. 14 and 15. In Item 15 the question as to whether or not all of the current in the case of shorts between the two phases of the grounded system and ground will pass back through the ground and neutral, does not appear to be developed fully. There is something more to investigate on that point, particularly in view of the fact that the conclusion is drawn that the advantageous scheme is the system using the ground selector. In the case of simultaneous shorts to ground, the neutralizing action would only be secured in case both breakdowns occurred at the same point.

I would not necessarily accept the conclusions as outlined here—that the ground selector system is a most advantageous one from the standpoint of inductive coordination—without having the theory developed and supported by something in the way of operating data.

E. A. Hester: Mr. Sleeper says that in order to get credit for effective relay operation it is necessary to get it when the relays are installed and before they are called upon to operate. That is to say, in effect, that the relays thereafter are usually blamed for any failure of lines or apparatus to clear, or for faulty sequence of operation. Unfortunately, that is true in a great many companies even at the present time, although there is a tendency toward a more thorough investigation of trouble, which in a great many cases absolves the relays from blame and attaches it to faulty application, or to the failure of other apparatus. In any case, it is the duty of the protection or relay engineer to draw the attention of his organization to successful relay operations. Attention is automatically drawn to unsuccessful operations, which quite often gives executives and others not so closely in touch with the actual operation a wrong impression. It is very interesting to note that a great many companies are establishing a bureau which takes care of protection alone. This is a very encouraging sign and it is to be hoped that these bureaus, by making careful analysis of relay operations and of interruption reports, will give the protective relays a reputation, which is rightfully theirs, of apparatus, which when correctly applied, is almost unequaled in precision of operation.

In this connection it may also be mentioned that the American Institute of Electrical Engineers and the National Electric Light Association both have Relay Subcommittees engaged in the preparation of a relay handbook. This should also do its share in preventing wrong applications with resultant faulty operation.

I should like to bring out one point with reference to Mr. Crichton's paper and Mr. Ackerman's discussion, *i.e.*, that this principle of "voltage restraint," as it was once called, has been successfully used by an operating company before the apparatus described in this paper and discussion was produced. Several years ago the West Penn Power Company tried out a scheme in which they locked the circuit breaker mechanism for voltages above a predetermined percentage. I believe Mr. Crichton was somewhat interested in this application. The scheme worked out very well for a while on this system, which had very long lines and relatively few stations scattered over a large territory. The scheme and its operation have been described in other papers presented before the Institute, both at Lake Placid, New York, in 1919 and in Niagara Falls, Ontario, in 1922.

H. A. P. Langstaff: Mr. Hester has referred to the voltage restraining relay on the West Penn System. This installation was made several years ago when the West Penn System was comprised mostly of radial feeders, and under this condition, operation was quite reliable. As the system grew and additional loops as well as generating stations were added, it was found advisable to install a more up-to-date type of relay, and as this installation was being made on part of the system, operation seemed to drift toward other sections of the system where the voltage restraining relay was still in service.

This feature brought out a very important point, which was brought quite forcibly to our attention, in connection with all switch operation analyses; *i.e.*, the re-equipping of a system as large as the West Penn with new relays. The work should be complete as soon as possible. In our particular case we found that the new relays, we might say, dammed up the operations in the new relayed section and diverted them to the remainder of the system.

In connection with the new impedance relay, we are wondering whether or not Mr. Crichton is going to allow us a royalty on the original idea.

We found the that time element of the West Penn voltage restraining relay was entirely too speedy, and we found that the addition of a time delay relay materially improved the selective operation. Twelve stations were equipped with this time element relay within one month's time, thereby enabling us to materially rearrange the operating scheme of dividing the system into three parts previous to storms, and remaining in parallel, which greatly reduced the number of interruptions.

There is one point which I would like to mention in connection with balance protection, which needs to be guarded against, and that is when one tie-line is out of service for repairs or in case of trouble, the remaining lines should have a sufficiently high setting to prevent an incorrect operation due to trouble on some foreign section. It is quite possible that a radial feeder out of a substation might have a considerably higher setting than the tie-line which is left in service.

Another point which has not been mentioned is that wherever more than one type of relay is used to control the one breaker, some form of indicator should be installed to show which relay was the cause of the switch operation. I believe this feature should be made standard equipment with all relays. This has a very important bearing on the setting of the relays, especially ground relays, which are more or less difficult to figure unless it is possible to carry out a schedule of experiments similar to those mentioned in Mr. Sleeper's paper. The writer has developed, and we now have in operation, about 25 indicators which give an audible as well as a visual alarm, and also has a Veeder counter which actually automatically records the total number of relay operations. This acts as a guide to the station attendant, and gives him a check on his switching equipment.

There is also another point which has not been mentioned in any of the papers or discussions, and that is the advisability of testing all relays in the laboratory. This question has been under consideration by us for some time, and we have not yet found this to be advisable. In the last two years we have installed approximately 600 relays and not more than 10 of these have been in our laboratory. All shipments are made direct from the factory to the job, installed on panels and given more or less of a laboratory test, and the settings made directly with the relays in position. Our percentage of failures has been exceptionally low.

Before starting on our relay program, the question of source of tripping current was very thoroughly investigated, and it was finally decided to use 12-volt storage batteries and vibrating rectifiers, adjusted to give a continuous trickling charge. We now have 15 of these installations in operation. Of course, all breakers are equipped with 12-volt, 5-ampere trip coils and the plunger in this is set approximately one-half its distance up in the coil in such a way that the coil will actually operate on six volts.

During the last two years I know of no failure which can be attributed to this low-voltage service. We have had one case of complete failure, due to the blowing of a main line d-c. fuse, making all switches in this one station non-automatic, but this condition would have applied, regardless of the d-c. voltage.

Chas. McL. Moss: The difficulty of any system of balance relay protection is inherently the number of cross connections required between the current transformers of the two lines. I think you probably have not used them. Even in the diagrams we have had shown us, the connections are somewhat complicated and Mr. Sleeper admits that he left out some of them. The complications are further increased if there are more than two lines.

He has pointed out the necessity of operating lines in pairs, but has neglected to state that the Duquesne Light Company ran into some difficulties at some points where they had four lines, and if you try to pair four lines, and use all possible combinations the necessary complications cause it to fall down by its own weight, so they wisely abandoned it.

The meat of Mr. Crichton's paper was in his first few words, in which he stated that the distance relay was essentially a "self-setting" relay. We all know that we can set up any predetermined line conditions and can pack relays on that line and give the proper time setting, directional settings, current settings, to take care of any difficult conditions we can impose. Somebody comes along and puts on additional stations, involving more time settings. Instead of having normally multiple lines, we have single line operation and then in order to maintain our full protection we must reset the relays or limp along on a partially automatic protective scheme.

Mr. Ackerman, in pointing out the disadvantages of the grounded systems, brings out in a great many places in his tabulation the disadvantages of a grounded system due to transient grounds always interrupting the service. It appears to me that that difficulty should not be given so much importance because in any large system of today supplying any important service, we practically always maintain that over duplicate feeds because outside of normal maintenance we may get other faults besides grounds. With a grounded system with adequate relaying, we can clear a fault immediately, maintaining the service over the other lines. He also brings up in his tabulation the difficulty of grounded neutral system, due to the requirement of star-star transformers throughout, thus requiring tertiary windings. In a large number of systems where we have both the high and low-tension grounded, that is very simply and economically avoided by proper combinations of Y-delta transformers, making the tertiary unnecessary.

O. C. Traver: Mr. Sleeper makes a specification that single line operation must be possible, preferably without additional relays over the number required for parallel line operation. That is very laudable, but at the same time, there is one difficulty, in that without additional relays, it would take the same time to clear a short whether you have parallel line operation or single line operation. The single line must have enough time to allow others to clear. Therefore, the 1.2 seconds time mentioned by the author would be necessary to clear a fault in any case.

You have noticed in the moving pictures just shown that the arc developed first very small, and very rapidly swung across the screen out of the range of the picture. That is the sort of thing that makes trouble many times. It is the sort of thing I believe it is wise to steer clear of. Accordingly I think two sets of relays in most cases are more than justified, if you can prevent an arc from swinging into a phase-to-phase or three-phase fault. To secure the desired result you would use relays of the same general type as Mr. Sleeper has described, but two sets of them, the first set would be as nearly instantaneous as possible; the second coming into play automatically only after one line has gone. There is no material complication and the scheme works out very nicely.

I have seen a test similar to the one shown here on the screen, an arc started by a fuse, across a 110,000-volt circuit, with the full power of a large system back of it, and I was very much disappointed and greatly surprised because I expected to see some real honest-to-goodness fireworks and didn't get them. There was a ball of fire about the middle of the fuse and that ball never moved out of its tracks simply because the breaker was protected by an instantaneous relay. Although the wind was blowing at a good rate the arc had no time to move over and involve the other phases. For that reason, I am very much in favor of using as quick a time as can be had for parallel operation, and then automatically introducing a longer time for the remaining single-line operation. You usually secure an additional advantage through time grading making it possible to care for more than one line out of service at the same moment. It may not happen very often that you need to care for a condition like that, but it does happen sometimes, and this arrangement will give you entire control under all conditions.

Passing now to Mr. Ackerman's paper: He states at the start that his particular circumstances were such that grounding, either by changing to star-connected transformers or by providing an artificial grounding scheme, was not readily possible. He, therefore, hit upon a very clever scheme of taking care of the situation which was apparently troublesome. He put on something and that something worked. I cannot quite agree, however, with his conclusions as applied to the general case.

Mr. Crichton has demonstrated a mighty interesting relay. The pioneer work of the West Penn Company and Mr. Ackerman's installation have already been referred to. The earliest reference to this principle, of which I am aware, is contained in a German patent issued about 16 years ago. Since this time the difficulty, as it appeared to some of us, has been that potential connections are required, and a double set at that, to care for both phase-to-phase faults and faults to ground. The usefulness of the scheme when properly applied cannot be doubted although any universal use is seriously questioned.

Some time ago the General Electric Company developed what was called an "overload-undervoltage relay" but due to the above mentioned difficulties the time was not considered ripe for exploitation. The characteristics are about the same as for the relay described by Mr. Crichton though the method of securing them was very different. This relay also consisted of two elements but instead of a direct mechanical connection, the two elements worked independently. One of these was a voltage element and the other was an induction over-current relay of standard construction. The net action of the combination was that the voltage element automatically varied the time setting of the over-current relay.

L. P. Ferris: Mr. Ackerman has referred in several places to the matter of inductive interference, as have also two of the previous speakers in discussing his paper. In a comparison of the grounded neutral system, and the isolated system, it is well to differentiate sharply between the situation under steady state or normal operating conditions and the situation under abnormal conditions as of short circuit.

In Mr. Ackerman's Table 1, reference No. 4, operating conditions normal, he brings out a recognized difference between the grounded neutral system and the system operated without ground connection. In the first we have a source of residual current and voltage which is not present in the isolated system, so I believe he is correct in stating that the isolated system has the advantage from that standpoint.

In reference 13, condition of a ground on one wire, Mr. Ackerman draws no conclusions as to which system is the more advantageous. He states in reference to the ungrounded system that the short-circuit will have the same serious effect as in the case of the grounded system. That, I think, should be qualified because with Mr. Ackerman's scheme he grounds a phase which is not normally grounded, producing a phase-to-phase short circuit through the ground. Now, if the two grounded points and line between them embrace the parallel, you have a phase-to-phase short circuit completed through the ground which may give a more severe condition than the grounded neutral system, but that will depend to some extent upon the amount of resistance which Mr. Ackerman uses. If that resistance is small I should expect the grounded neutral system to have the advantage.

In reference No. 14 I believe Mr. Ackerman's conclusion is that the two systems stand on a par, and this is substantially correct.

In reference No. 15, I think the statement that the ungrounded system gives a metallic short circuit and neutralizes interference should be made conditional on the grounds on the two phases occurring on the same side of the parallel. If the grounds are on opposite sides of the parallel, you will have the condition of a phase-to-phase short circuit embracing the parallel and, of course, you will get induced voltages.

In reference No. 19: Mr. Ackerman claims as an advantage of the ungrounded system, greater current to actuate his relays. That same greater current would, of course, give greater induction.

Mr. Ackerman points out that with his system on the 12,000-volt network, there is a certain percentage of cases of temporary surges which clear without a short circuit, and, of course, this is an advantage to the extent to which it can be accomplished because otherwise those conditions would produce a short circuit on a grounded neutral system. Apparently, this holds for the moderate or low-voltage systems where the arcing ground is not a serious factor. Of course, where you have an arcing ground on the power system, that is a factor which is disadvantageous from the interference standpoint as well as from the power standpoint.

E. P. Peck: Mr. Ackerman evidently had a particular problem which was quite serious, and he met that problem with an excellent solution for the particular conditions. However, if we examine carefully the tables on pages 314 and 315, the application of his solution to his problem leads to conclusions which do not apply equally well to other systems and to other problems. In comparing the *B*'s and the *A*'s, we found a number of *B*'s which indicate his system is best, one clear *A* and a few *A*'s "apparently." Looking over the table in the light of experience on other systems, and re-marking it, I found a good many question marks under the *B*'s, a good many "apparently's" scratched out under the *A*'s, and a good many *A*'s filling in the blank spaces. In other words, the conclusions in the paper should be taken as applying to the particular set of conditions encountered, and not to systems in general.

One remark on the next page rather makes me think there is something special in that particular system. "Regarding the danger of high-frequency surges created by an arcing ground at the point of fault, it may be pointed out that in the writer's opinion the existence of the arcing ground should be extremely rare." This is true. Arcing grounds should be extremely rare, but they do occur on isolated systems, and when they occur the results are often quite serious, and frequently secondary troubles are produced at quite a distance from the original arcing grounds.

F. M. Billhimer: Mr. Sleeper's description of the selective relay system on the Duquesne Light Company was very interesting and instructive. I say this because of the fact that he not only described the relay system of the Duquesne Light Company, but in addition, he told us why they used this system, how they went about it to test the system after it was installed and the results of the tests after everything had been put in working order.

It was my privilege to be present at this series of tests, and the results were quite gratifying. With grounds on the 66,000-volt system, the proper breakers opened up the faulty section of the system, isolating that fault without dropping any load.

Oscillograph records obtained from this system during the test were obtained from the secondaries of 60,000 to 100,000-volt potential transformers, thereby getting the voltage direct. These oscillograms showed that the voltage unbalance due to the current of 900 amperes flowing over that insulator was not more than about ten per cent. You can readily see that synchronous apparatus will continue to operate satisfactorily under those conditions.

Yesterday, everybody seemed to agree that a resistance in the neutral was advisable, but we were not quite decided as to whether it should be high or low. After a few more carefully planned relay systems have been put into operation, equipped with ground relays, which will protect the system and service by opening the faulty section before a short circuit has occurred, we believe that the ohmic value required of neutral grounding resistors will be fairly well established. Although all systems do not present the same problems as those of the Duquesne Light Company system, I believe that you will all agree that a careful study of the conditions to be met, whether it is parallel operation of lines or some other condition, followed by working out and testing of the scheme after installation will insure excellent system protection and reliability of power service which the public utility is required to give.

H. P. Sleeper: Replying to Mr. R. N. Conwell, I would state that the primary reason for making service tests on our system was the fact that the ground relays which we are using are an entirely new development which had never had service; and in addition, we were very desirous of determining the value of ground currents which would be available on ground faults. Both of these points were completely investigated by the tests made.

Mr. Hester's point is well taken that relay performance on the system should be thoroughly investigated and broadcasted, especially the successful operations which frequently in service are overwhelmed by the undue prominence given a few incorrect operations. We will all agree, however, that it is comforting to know that important relays have operated successfully under service conditions.

In regard to the use of back-up relays for single-line operation on our balanced system, I would state that we have recognized the desirability of this additional protection and that we are at this time taking steps to install it at various points.

Mr. Moss brings out the point that any scheme of balanced relay protection is complicated. As far as this point is concerned, no system of relaying multiple lines can be said to be extremely simple. The complications of a relay system, however, can hardly be considered a serious limitation if its correct performance can be assured.

Mr. Traver's description of the 110,000-volt arc tests are very interesting, particularly relative to the small amount of destruction resulting from the instantaneous operation of the relays. The pictures which you have seen would apparently compare unfavorably with the operations which Mr. Traver has described, but I would point out the fact that on all but one of the arcs which we made, the relay action was delayed far beyond normal. This delay was caused by the fact that the ground relays were not allowed to operate and that the short-circuit relays only were in circuit, thus resulting in a long time operation and the arc achieved considerable magnitude as you have seen. However, in the one operation where the ground relays were in circuit, we were much pleased to note that the arc barely got well established before the breakers opened and the resulting damage was negligible.

As a result of the tests which we made we do not wish to give the impression that we recommend this procedure as a method of periodically testing relays. However, as a method of exhausting possibilities of new equipment, we believe that such a procedure is well justified and we feel amply repaid for the results obtained.

P. Ackerman: The criticism offered to my paper on the ground selector is not unexpected. General opinion has, in the past few years, expressed in favor of the grounded system for well known reasons. It is quite natural, therefore, that any scheme of such a radical departure such as the ground selector, claiming the same advantages as the grounded neutral system, should find its critics. Yet, I do not see the need of retracting any of my statements made in the Table I, and the General Conclusions of my paper.

One of the chief criticisms seems to center in the inductive interference problem, as expressed in items (4), (13), (14), and (15), of Table I.

As a whole there should be little difference in the inductive interference problem between the two systems except for the benefit derived from the ungrounded system as expressed in items (4) and (15).

Regarding item (15) it is to be clearly understood that this item meant to cover the case of the two phases short-circuiting through ground at the same point of the line, such as is frequently the case with lightning flashovers. Any cross short circuit of one phase at one point to another phase at another point of the system is identical with the case covered in item (13).

Regarding the fear of harm being done by arcing grounds as expressed by Mr. Peck and Mr. Ferris, I believe that the operating results are sufficient proof that no such fear need be entertained.

We do not need to argue as to whether the secondary breakdown on ungrounded systems is caused by the dynamic overvoltage breaking down depreciated insulation or by the high-frequency surges set up by the arcing ground. The fact remains that the ground selector relieves such an abnormal condition within such a brief period that no harm is done to the rest of the system. The best proof of this can be found in the operating record of the 12,000-volt Toronto distribution, which system is under my observation for the past ten years, of which period the system was operating for five years without and for five years with ground selector. There has not been a single case of a permanent secondary breakdown since the installation of the ground selector whereas previous to that secondary breakdowns were a common occurrence.

The chief advantage of the ground selector, doubtlessly, lies in its application to medium capacity and medium voltage systems with large overhead distribution. Such systems are largely exposed to temporary grounds by birds, etc., due to the small height of the insulators. With a limited capacity current of the system the chance of self-extinction of the arc, after the grounding object has fallen off, is quite pronounced. In any such case a partial interruption to the load is avoided. This advantage is plainly noticeable from Table III.

On large underground systems where most of the troubles are of a permanent nature, or where the capacity current is of such magnitude that it loses its self-extinguishing property the benefit of saving the interruption is lost, and as a result the ungrounded system combined with ground selector may be considered on par compared with the grounded system as far as selective clearance of faulty lines is concerned. Other factors will govern in such case the advisability of using one or the other system.

In closing I once more wish to emphasize my firm belief in the general opinions expressed in the conclusions of my paper.

L. N. Crichton: The allusion made by Mr. Conwell to the lack of available time settings at some points in complicated systems emphasizes one of the principle limitations on the older method of discrimination by means of definite or inverse time limits between switching stations occuring in series. It is not desirable to allow trouble, at least serious trouble, to remain on the system for more than two seconds because of the disturbance which it makes to the load. The distance relay will rid the system of most of its trouble in considerably less than one second no matter how many switching stations may be in series.

Mr. Ackerman mentioned the limitation of the distance relay, that it will not discriminate properly if the voltage difference between substations, under short-circuit conditions, is less than 5 per cent. Since the paper was written the relay has been considerably improved and as now manufactured it will operate on a difference of 3 per cent between stations, which should be sufficient to satisfy any reasonable demand. The fact that the relay does not strictly follow the mathematical law and that its time of operation does vary slightly with the current flowing is not serious. For example, if the relay should be carelessly applied and the resulting error in timing between two stations should be as great as 33 per cent which is almost an impossible condition, this would amount to only ¼ second which would neither interfere with the proper discrimination between relays nor would it materially increase the shock to the system.

The fact that the time of operation of the distance relays must be adjusted so that it is not entirely dependent upon the length of the section of line which it protects, but also depends in a small measure on the length of the next section beyond it, does not materially affect the problem. Usually this effect can be neglected and in most practical cases it will not seriously affect the period of time required to clear trouble. In this connection, it is worthwhile noting that no matter what the relay setting may be, any dead short circuit which occurs close to the bus bars will be cleared almost instantaneously. In other words, the most destructive trouble is sure to be cleared quickly.

The essential principles of Mr. Ackerman's relay scheme are similar to that of the distance relay described in this paper and the successful results which Mr. Ackerman has obtained are a vindication of the correctness of these principles.

The low-voltage scheme used by the West Penn Power Company gave a good account of itself in confining trouble to a small section of the system, but was deficient in that it did not contain any directional element. Probably this is because at that time there was no reliable directional relay available. Mr. Langstaff brings out the important point that when relays are installed on a system which has not previously had them, it is desirable to finish the job as completely as possible and trust to the relays rather than to equip part of the system and then be unable to obtain satisfactory operation on either the automatic or the hand sectionalized parts.

As mentioned by several speakers, the idea of the distance relay is not particularly new. However, I believe that most of the refinements, which have been necessary to make the old idea workable on present day systems, are original. Maybe it would be better to consider this device as a new problem in design rather than an invention. It should also be borne in mind that further improvement will probably be made from time to time, as the requirements of power systems change, but in the meantime, we have available a device which will do its part towards providing uninterrupted service to the consumer and thus advancing the electrical industry.

Lightning Disturbances on Distribution Circuits

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Review of the Subject.—A general survey is presented of the nature and distribution of failures and interruptions due to lightning upon the circuits of a large city supply system. Recommendations are given for improving conditions without specifying particular makes of arresters, in recognition of the fact that a considerable number of different types is proving adequate to meet the conditions imposed upon them on the moderate voltage systems under consideration. The subject is treated under three principal divisions: 1st. Troubles on 115-230 volt distribution circuits. 2nd. The 2300 volt circuits, and 3rd. The 13,200 volt circuits. It is shown that line failures on the low-voltage circuits are so infrequent as to be practically negligible, and that the blowing of fuses and injuries to other parts of the equipment on customer's premises are infrequent except in the case of house meters. A considerable number of meter burn outs are recorded, and it is shown that trouble from this source increases with the length of the secondary circuit. To guard against future trouble, especially as secondary networks are developed, it is recommended that a simple spark gap or other equivalent type of arrester be installed at the customer's premises as a part of the equipment in his service box. For the 2300 volt circuits, it is found that lines which run over high ground and open country are more exposed to transformer burn outs and the blowing of transformer fuses than lines in the built-up sections where buildings screen the circuit. Also, that the amount of damage to the exposed lines decreases as the number of arresters increases and that a transformer directly protected by an arrester is in a very large measure guaranteed against complete breakdown and to a less extent against the blowing of its fuses from lightning. Also, that the effectiveness of this protection is decreased with high resistance in the ground connection. Finally, for the 13,200-volt aerial circuits, it is shown that on lines at a considerable distance from generating points most of the damage from lightning is in insulator breakages and without interruption to service, but where line reactance is not sufficient to prevent flow of heavy dynamic current at the time of discharge, the conductor is often fused allowing the line to fall. To guard against these troubles, arresters should be distributed along the line at not more than two mile intervals and near generating stations the three arresters of a group should be mounted on successive poles and provided with independent ground connections.

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THE investigations which form the basis for this paper were made upon the Philadelphia Electric Company's system during the summers of 1921 and 1922. As lightning disturbances show so many eccentricities and as each storm has its own individuality both as to its path over the system and as to the intensity of its activity, it is necessary, in a study of this nature, to consider a great mass of data and at the same time analyze many individual cases, before any generalizations can be made.

The work was classified under three main divisions:

I. The low-tension distribution system, mainly 115-230 volt, three-wire single-phase with neutral grounded.

II. The main primary distribution system, 2300 volts, three-wire two-phase ungrounded.

III. The high-tension system, 13,200 volts, three-phase with neutral grounded at the generating stations only.

I. FAILURES ON THE LOW TENSION SYSTEM

It will probably be rather generally conceded by distribution engineers that lightning troubles on the low-tension system are of comparatively infrequent occurrence and of a much less serious character than those on other parts of the system. It was thought best, however, in these investigations, to obtain some quantitative results from lightning failures on these low-voltage circuits; both for the lines themselves and for the apparatus upon the customer's premises. Considering first-line failures, records were obtained for the storms occurring upon June 3rd and 10th, and July 1st and 12th, 1922, which were found to be sufficiently widespread and severe to be fairly typical. The linemens' reports upon circuit failures during these storms were, therefore, investigated, and it was found that they reported no insulator failures upon the low-voltage circuits and only three cases of low-voltage lines being down; one of these being due to a tree falling upon the circuit. On a system as extensive as the one under consideration this amount of injury could readily be passed over without further investigation.

At first sight the record of failures upon customers' premises did not offer a very encouraging field for analysis. The three classes of failures which might in some cases have been caused by lightning were the blowing of fuses, injuries to house fixtures and to meters. Monthly totals for each of these were recorded. If any considerable number of house fuses had been blown by lightning disturbances, this fact was completely masked by the large number blown by other causes. The totals ranged from 4000 to 5500 and were in some cases greater for winter months than during the lightning season. The number of injuries to house fixtures never reached 150 per month and they were quite irregularly distributed. There was, however, distinct evidence that a majority of the meter burn outs might have been due to lightning as there was a large increase in the number of such failures during the summer months. More specific information regarding these failures was therefore sought by obtaining a record of all meter repair orders issued on the days of the four storms referred to above, and upon the day following each storm. By this method a few meters might have been included which had not failed from lightning, but there could not have been enough of them to appreciably effect the results. Also, it was not possible to locate all of those which were

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injured by lightning for it was necessary to rely upon the customer for reporting the trouble and if the house was closed at the time, or if the failure did not interrupt the service, the injury would not be detected until it was too late to determine with certainty the cause of the failure.

Consideration should first be given to the geographical distribution of these meter failures, especially in comparison with the disturbances which occurred on the primary circuits during the same storms, as shown from the record of the blowing of transformer fuses. Both of these classes of failures were located on a pin map, but the comparison shows up more distinctly in Fig. 1, which gives the number of failures in parallel strips of territory 2500 feet wide running across the city from east to west. The few failures which occurred in West Philadelphia are not included as they were too scattered to have any significance and by their omission a truer picture can be obtained of conditions in the central and southern portion of the city where underground distribution is largely employed.

The most striking feature of this diagram is the comparative immunity from lightning troubles that is shown for the secondary circuits in the northern sections of the system which comprise Tacony, Germantown, Chestnut Hill and the outlying territory. This is the more remarkable when it is known that in previous years the primary circuits in these sections had suffered so severely that it had been necessary to provide much more liberal lightning arrester protection there than in any other parts of the system. No such protection had been found necessary for the secondary circuits. It is evident, therefore, that mere geographical location within the area of maximum lightning disturbances is not the determining factor in causing failures in these low-tension circuits. A hint for the solution of this problem may be obtained from wireless practise where the design of the aerial controls so largely the ability of the station to pick up distant signals. Upon this basis the explanation for the absence of meter failures in the northern territory was sought by comparing the relative lengths of the secondaries. An inspection of the secondary circuit diagrams showed that throughout this territory a majority of the customers are served from separate transformers having short secondary taps, and even where several services are run from one transformer the secondary circuits are shorter than those which prevail in the more congested districts. In order to check this theory further the meters which failed during the storms mentioned above were identified with the circuits which served them, and, except for two questionable cases where the addresses were not clearly given, all of the 105 failures were on circuits of approximately 1000 feet or more in length.

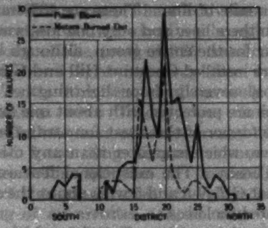

FIG. 1

This investigation also brought out the fact that in only two cases were meters which failed connected to transformers on which the fuses were blown during the same storm, and in no cases were such meters connected to transformers which were burned out by lightning. Thus all the evidence points to the fact that the lightning disturbances in the secondary circuits are not transmitted from the primaries through the transformers, but originate in the secondaries themselves and are controlled largely by the length of these circuits.

The losses occurred indiscriminately upon meters connected to ungrounded circuits or circuits grounded outside the premises or directly at the meter. No true perspective upon the protective value of these methods of connection was obtained, however, as the relative number of meters using each type of connection was not known.

On a system containing over 240,000 meters a record of 105 failures during four of the most severe storms of the season does not appear very serious and yet, in view of the efforts that are constantly being made to minimize interruptions to service, it does seem as though these conditions might be improved. Possibly the correct solution is to provide heavier insulation for the meters, but it is questionable whether, if the meters cease to function as lightning arresters, more serious damage and even loss of life may not result from lightning reaching the house circuits. Furthermore, as the load density increases, the tendency is toward the use of longer secondaries which should ultimately lead to the development of an interconnected system similar to the Edison direct-current network. In that event, these records indicate that much more serious lightning trouble may be expected.

It is known that on other systems a good deal of trouble is being experienced from meter burn outs, but the writer has been unable to obtain any data upon the extent of such damage nor whether measures are being taken to correct it. On the large city systems, at least, it would seem desirable to introduce some form of lightning protection on the secondary circuits which could be gradually extended over the system. A spark gap, or one of the other types of low-voltage arresters, should prove adequate if properly distributed. The best place to locate the arrester is on the customer's premises and it could properly be included as part of the necessary equipment of his service box. A suitable ground connection is available at this point and very little additional expense would be entitled.

II. The 2300 Volt System

The consideration of lightning troubles upon these circuits was confined to a study of transformer failures and the blowing of transformer fuses. A general survey of transformer burn outs was undertaken during the summer of 1921. It appeared necessary to cover as large a number of cases as practicable, in order to eliminate the effect of the irregular distribution of individual storms, so that the study was based upon the records of failures from January 1st, 1920 to August 1st, 1921. As these records did not specifically indicate the cause of the failure, it was assumed that all transformers removed from the line on the day of a storm or on the following day had failed from lightning. Eighty-six transformers were so removed from the aerial lines during 1920 and forty transformers up to August 1st in 1921. This represents about one third of the total number of transformers removed from these circuits for all causes during this period, and, while it may include a few transformers which were not injured by lightning, the number cannot be sufficient to alter the conclusions reached, although it may account for some of the eccentricities which appear in the records. A further check upon this method of selection was obtained for transformers which failed from lightning from August 1st, 1921 to August 1st, 1922, by having the linemen report such cases immediately after each electric storm. In this way questionable cases should be eliminated, but the results shown for failures during the later period are entirely in harmony with the conclusions reached from a study of the earlier records.

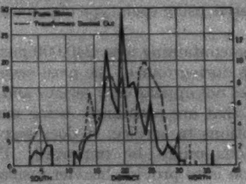

Fig. 2

The one hundred twenty-six transformers which failed from lightning from January 1st, 1920 to August 1st, 1921, were located geographically and their distribution is shown in Fig. 2. The transformers were also identified with the circuits to which they were attached and it was found whether or not they were protected with lightning arresters. In order to obtain a proper perspective of this phase of the subject, the length of all the aerial circuits of the system was recorded, together with the number of arresters and transformers connected and the number of transformer failures. This information is given below.

In some respects the records appear erratic, as should be expected in a study of lightning disturbances, though it is believed, as already noted, that some irregularities are due to errors in the records and some possibly to not having covered a sufficiently long period in the investigation. Nevertheless, the following facts are clearly brought out:

1. Circuits which have the greatest amount of lightning protection suffer less than circuits in the same district with less protection.

2. During 1920 and 1921, the Tacony District suffered more severely than any other section of the system, and the greatest losses occurred on the circuits which had the least protection.

3. Large sections in the central part of the city, which had very little arrester protection, experienced no tranformer losses.

4. A large number of circuits in different parts of the system should have additional protection.

The following information is submitted in confirmation of the above statements:

1. The circuits which have the greatest number of arresters per unit length of circuit in order of their protection are *C H*-1, *C H*-3, *T A C* 1 & 4, *F I L*-2, *G T N*-4, *F I L*-10 and *G T N*-1. These have from three to four arresters per mile of line. There are 437 transformers connected to these lines and only one loss was reported for each year for the entire group, although before the arresters were installed Chestnut Hill and Germantown suffered most severely from lightning. *C H*-4 and *G T N*-2 each are protected with about one arrester per mile and the former of these lost four transformers and the latter five, which is greater than any other circuit outside of Tacony. The exceptional feature in these records is that *G T N*-3 and 5 with about the same protection suffered no losses.

2. At the other extreme *T A C*-2, which is a very long line feeding all the territory north and west of Tacony, is very much exposed to lightning and is protected with only one arrester per seven miles of line, which means that long sections have no protection. There are 222 transformers on this circuit and 13 of these failed in 1920 and 14 in 1921. *T A C*-6 has no arrester protection, but in places is less exposed than *T A C*-2, and this has lost a total of seven transformers for the two years out of a total of 60 connected. *T A C*-3, with about the same arrester protection as *T A C*-2, but with much greater natural protection, lost three transformers out of 62. The other Tacony circuits have more arrester protection and pass through thickly settled sections. None of these have lost more than one transformer.

3. For the last year and a half no failures due to lightning have been recorded for any of the circuits out of Ludlow or Columbia substations and only two from Marshall. One of these was a 37½ kv-a. transformer which had been in service only a month and was replaced by a 50-kv-a. suggesting that the failure had been due to overload rather than lightning. There are twelve circuits from Susquehanna, eight from Ontario

and four from Hunting Park, for which no losses are recorded for this period. It is too much to expect that this perfect record will be maintained indefinitely, and it may be that a storm of unusual severity passing over this section may disclose points where additional protection should be provided, but the records cover a sufficiently long period to prove that very little lightning trouble need be expected in this territory.

4. Other circuits pretty generally distributed over the system show losses of from one to three transformers per circuit which is not sufficient to serve as a guide in locating additional arresters, but does suggest that such protection should be provided. Here the danger of basing a judgment upon inadequate data is well illustrated by the record of failures in West Philadelphia and South Philadelphia. In 1920 it seemed as though the former was much better protected than the latter, but in a single storm on the night of July 15th, 1921, eight transformers were lost in West Philadelphia and only one has been reported for South Philadelphia for this year. This failure appears to have been due to a direct stroke as the pole which carried the transformer was also injured.

A further indication of the effectiveness of arresters in the protection of these circuits is brought out by the fact that out of the total of 126 transformers which had failed from lightning, only five were directly protected with arresters, and in only one other case, a 4-kv-a. near Tacony, was there clear evidence of a failure near an arrester and this line was unprotected for a long distance on one side of the transformer. The question is frequently raised as to the part the ground resistance plays in the proper functioning of an arrester and, in order to determine whether faulty ground connections might not have been responsible for the failure of these arresters, resistance measurements were made at each of the arresters and at a number of others on the same circuits in their immediate neighborhood. In one case the ground resistance of the faulty transformer was 102 ohms and on four others near it, it ranged from 47 to 120 ohms. In another case the ground resistance was 227 ohms and on three others in the neighborhood it ranged from 70 to 167 ohms. In the third case the ground resistance was 253 ohms, while on five others in the vicinity it ranged from 27 to 77 ohms. In the last two of the five failures, it measured 364 ohms and 477 ohms respectively, and other ground resistance measurements in the same vicinity showed about the same values. It will be seen that in all but one of the cases of failures the resistance was over 200 ohms, and there the spring clip connection to the ground wire was so loose that there is no assurance that it had not fallen entirely out of place at the time of the storm. The resistance of this connection on the day the tests were made was 44 ohms. Measurements on other clips showed no appreciable resistance. It should be noted in this connection that it is difficult to obtain low-resistance grounds in much of the territory north of the city on account of the high, rocky nature of the soil.

Summarizing the above results, it appears that the arrester protection which has been provided on the most exposed circuits in Chestnut Hill and Germantown are proving successful in minimizing lightning troubles, though not completely eliminating them from these circuits. Also, it is seen that in the few cases where transformers which had arrester protection are burned out, the ground resistances were abnormally high on account of the rocky nature of this region. It is further shown that other circuits, especially in the Tacony district, needed additional protection. As noted below, the arresters which were installed in 1922 on these circuits have proved effective in reducing the losses in this territory.

It may be interesting to note that while the experience in Philadelphia as to the effectiveness of arresters in protecting transformers is in full harmony with the results obtained by Mr. D. W. Roper in Chicago (A. I. E. E. PROCEEDINGS, Vol. 35, page 655), there are other respects in which the lightning appears to affect the circuits differently in the two cities.

Transformer Fuses Blown.—Comparative figures are given below for the total number of transformer fuses blown on the 2300-volt aerial circuits by months:

	1921	1922
January	15	55
February	30	33
March	38	47
April	33	40
May	50	65
June	251	178
July	446	208
August	84	211

The increase in failures during the summer leads to the presumption that the great majority of these fuse failures are due to lightning and the improvement in the records for 1922 may be ascribed to the fact that during this year about 500 additional lightning arresters have been installed on the more exposed circuits in the northern part of the system. The balance of the failures, due to overloads on the transformer, falling of trees on the circuits, or the falling of the circuits themselves on account of ice load, wind, etc., could probably have been reduced in number if the transformers had been more heavily fused. The present practise is to fuse on the basis of one ampere of fuse capacity per kv-a. of transformer rating, five ampere fuses being the smallest in use. As the principal function of the fuse is to clear the line in case of a transformer failure rather than to protect the transformer, it should be entirely feasible to make a general increase in fuse capacities. This would be in line with the practise of many of the large operating companies. There is nothing in the records to give a clue to the probable decrease in the number of fuses blown if this practise is adopted. The really important phase of the problem is to consider how to minimize the blowing of fuses during electric storms.

As already indicated, this trouble cannot be due, except in rare instances, to lightning disturbances in the secondary circuits, for direct evidence shows that lightning surges on the secondaries, as indicated from the burning out of house meters, is not associated with primary failures, either with the blowing of fuses or the burning out of the transformers. The cause of the fuse failure must therefore be localized at the transformer itself. It is apparently due to a flashover across the transformer surfaces or in the primary coils, the insulation being reestablished, at least in part, after the blowing of the fuse has extinguished the arc.

Further information upon these fuse failures was obtained by definitely localizing with their circuits the 112 failures which occurred during the four storms in June and July, 1922, to which reference has already been made. For comparison with transformer burn outs the distribution of these failures is shown in Fig. 2. It is seen that they follow much the same course as the transformer burn outs as should be expected. The agreement would probably have been closer if the same lightning period had been considered in the two cases. The divergence which is shown for the northern territory for instance, may be explained by the large increase in the number of lightning arresters installed in this district between the periods covered by the two sets of records.

The records for these four days showed that in 17 cases fuses blew on the same transformer in two or more successive storms, and in several cases later records indicated that ultimately the transformer broke down permanently. It is evident, therefore, that the blowing of transformer fuses is serious, not only for the interruptions which they cause to the service, but also for the deterioration in transformer insulation which they indicate.

As the lightning disturbances which causes a fuse to blow comes in over the primary, the arrester protection which may be justified purely in saving the transformer from burn out, should also prove effective in reducing the trouble from the blowing of transformer fuses. Unfortunately, this investigation does not indicate that the same measure of protection may be expected in the two cases. The records collected during 1922 showed that in only five cases were transformers burned out which were protected by arresters and in one of these a very old type of arrester was employed which was probably not operative at the time. On the other hand, during the same period, 30 cases were recorded where arrester protection was not effective in preventing the transformer fuses from blowing. Apparently in all but very exceptional cases the arrester carries off a sufficient amount of the energy of the lightning surge to keep the transformer from complete failure, but in some cases it allows enough energy to pass to temporarily break down the insulation until the blowing of the fuse clears the circuit. This points to the importance of obtaining low ground resistance for the arrester in order that it may deflect as much of the lightning discharge as possible. As already noted, it is very difficult to obtain this in certain of the outlying sections of the system. It is believed, however, that this investigation shows conclusively that with the present installation, transformer failures from lightning are being kept within remarkably low limits and the blowing of transformer fuses in being minimized.

III. THE 13,200-VOLT SYSTEM

The 13,200-volt circuits of this system are, in the majority of cases, underground, and serve as feeders from generating stations to substations. There are, however, a number of aerial circuits operating at this voltage which feed substations in the outlying districts, and others which supply large power customers. The latter are increasing in number and importance each year.

A group system of numbering has been adopted to indicate in general the geographical location of the circuit. Circuits No. 704 to 706 run on the same pole line from Tacony westward, eventually separating to feed railway substations in the northern territory. Circuit No. 708 follows the river from Tacony to Bristol. Circuit No. 1126 runs from Hunting Park substation to Manayunk. Circuits No. 1301 to 1305 interconnect the above mentioned railway substations and tie in with the Chestnut Hill substation. Circuits No. 2101 and 2102 run practically from the Schuylkill Generating Station to the extreme southern portions of the city. Circuit No. 2118 is located in Frankfort which is nearer the Delaware Generating Station than Tacony. Circuit No. 2301 serves a group of manufacturing plants north of Tacony. This completes the list of circuits in operation during 1921, which was the period covered by the investigation. The general practise has been to protect these circuits at the substations by electrolytic arresters and at customers' taps by spark gap arresters.

The company's records give definite information upon the number of interruptions of each of these circuits during electric storms, and in every case where it has been possible to locate the cause of the interruption, this has been given, together with the exact location of the trouble and the extent of the damage done. There is also very clear indication that lightning was responsible for many injuries to insulators, pins and cross arms, which did not interrupt the service. These injuries were detected by the linemen who patrol the circuits after each storm and at stated intervals throughout the year. It is unfortunately impossible from the nature of these reports to differentiate between failures from lightning and from other causes. It is found, however, that there are fully three times as many insulator failures on these circuits during the lightning season as in the winter months. As most of the insulator breakages in winter are probably due to ice or high winds and failures from such causes must be less in summer, it appears conservative to assume that at least two thirds of the insulator breakages during the year are due to lightning. The reports give the number of repair jobs at each pole rather than the number of insulators replaced, so that the actual number of insulator failures will be greater than the figures given below. From the mass of data contained in these reports, the following information has been selected as it is believed that it is adequate to give a fairly clear representation of the character and extent of the damage done to these circuits by lightning during 1921:

Circuit	Length in miles	Interruptions during storms	Number of Repair Jobs: On insulators	Number of Repair Jobs: On cross arms and Pins	Interruptions per mile	Insulator repairs per mile
704	8	3	28	2	0.37	3.5
705	6	1	6	1	0.13	1.0
706	7	3	21	6	0.43	3.0
708	9	1 intermittent	3	0	0.11	0.3
1126	4	1	1	0	0.25	0.25
1301	7	2	27	6	0.29	3.9
1302	3½	1	18	3	0.29	5.1
1303	5	0	9	5	0	1.8
1304	6	0	6	1	0	1.0
1305	4	0	10	8	0	2.5
2101	9	6	12	3	0.66	1.4
2102	8½	8	14	1	0.94	1.6
2118	1½	4	0	0	2.6	0
2301	1¾	0	1	0	0	0.6

The records of failures on these circuits as given above show the customary freakishness of lightning and undoubtedly do not cover a sufficiently long period to bring out many of the peculiarities of the individual circuits. There are, however, certain features of the records which should be noted. In the first place, it appears that the circuits most distant from generating points, such as No. 1301-5, had a great many insulator failures but were remarkably free from interruptions during electric storms. Circuits somewhat nearer the center of the system such as No. 704 and 706 had much the same record of insulator failures, but experienced a greater number of interruptions. The circuits in South Philadelphia Nos. 2101-2102, which are still nearer large generators, had fewer insulator failures, but most of those which occurred during storms were accompanied with serious line trouble. It should furthermore be noted that the records of lightning trouble on 2300-volt circuits, which are much more complete than for 13,200-volt circuits, show that all through the territory north of the city, the injury from lightning to apparatus unprotected by arresters was much greater than in South Philadelphia. It is reasonable to suppose, therefore, that these 13,200-volt circuits in the Northern District had had many lightning surges induced in them which discharged themselves over insulators without completely breaking these down, the high line reactance in such cases limiting the dynamic current to comparatively low values. However, when such surges occur on circuits where the line reactance is low as on No. 2101 and 2102, the dynamic current which accompanies a discharge generally destroys the insulation and the arc which follows burns off the conductor, allowing the circuit to fall. Short circuit conditions on No. 2118 are about as severe as on No. 2101 and 2102, though the latter has much better natural protection. Three of the four interruptions recorded for this circuit were accompanied with falling of the conductors. In these cases spark gap arresters near the points of trouble failed to protect. There were only three cases recorded in 1921 in which the circuits north of this point fell during electric storms. In one case there was a direct stroke on No. 705 outside the Oxford Substation, in another both No. 704 and 706 were down at the same pole, and in the third case, one span of No. 1301 fell near the Chestnut Hill Substation. It should be noted that in the two latter cases the failures occurred near the feeding end of the circuit where the dynamic current, which might develop on short circuit, would be a maximum.

Attention should be called to the relatively good records of Nos. 705 and 708. No explanation was found for this condition on No. 705 for it is just as much exposed as No. 704 and N0. 706 being on the same pole line for most of the distance, and the circuits are transposed so that there should be no advantage of position. Circuit No. 708 had formerly been subjected to so many interruptions that it was completely re-insulated, 22,000-volt insulators being used in place of the lower voltage type employed on the other circuits and this undoubtedly accounts for the improved performance of this circuit.

Two general methods suggest themselves for reducing lightning troubles on these 13,200-volt circuits. Either to reconstruct the circuits with 22,000-volt insulators, as has been done with good results on circuit No. 708; or provide some form of lightning arrester protection on the lines themselves in addition to the arresters already in use at the terminals. The first method would involve considerable expense and while it might be employed advantageously at certain of the more exposed portions of the system, the more direct and satisfactory method would be to relieve insulation stresses by providing suitable discharge paths to ground through lightning arresters. This would in general require that the arrester should be suitable for pole mounting. On the circuits in the outlying districts at a considerable distance from the generating points there should be no difficulty in providing at reasonable expense, suitable protection by the use of spark gap or other compact form of arrester, distributed at about two mile intervals on the average.

The general opinion among operating engineers appears to be that the electrolytic or oxide film arresters

DATA UPON 2300 VOLT CIRCUITS

Circuit	Length of wire in 1000 ft.	Lightning Arresters		Transformers			
						Lost by Lightning	
		Total	Per 10,000 ft.	Protected	Total	1920	1921
Tacony							
1 & 4	18	15	8.3	8	8		
2	583	14	0.2	5	222	13	14
3	134	3	0.2	0	62	2	1
6	193	0		0	60	4	3
7	73	3	0.4	0	39	1	
8	60	6	1.0	1	38	1	
9	65	3	0.4	0	33		1
10	90	3	0.3	2	53		
11	99	4	0.4	2	42		
12 & 13	72	6	0.8	0	22		
Chestnut Hill							
1	230	216	9.0	80	110	1	
2	157	86	5.5	40	43	2	
3	169	148	8.7	69	73		
4	178	38	2.1	11	47	3	1
Germantown							
1	45	28	6.2	14	22		
2	235	47	2.0	16	56	3	2
3	105	23	2.2	8	48		
4	65	58	7.4	27	41		
5	70	11	1.6	7	10		
Filbert							
1	39	14	3.6	1	12		
2	152	125	8.2	50	59		1
3	111	13	1.2	2	42		1
4	77	4	0.5	2	40		1
5	111	23	2.1	4	63	2	
6	63	17	2.7	8	22		1
7	80	20	2.5	9	48	2	
8	65	7	1.1	2	31	1	
9	69	9	1.4	3	60		
10	288	195	6.8	91	124		
11	92	9	0.1	3	31		
12	84	15	1.8	3	17		
13	66	2	0.3	1	21		
14	52	3	1.4	3	60		
15	63	7	1.1	2	24		1
16	72	19	2.6	11	27		
Paschall							
1	93	16	1.7	4	43	1	
2	69	2	0.3	1	14		1
3	82	2	0.2	2	31	1	3
4	84	8	0.9	3	59	1	
5	54	4	0.7	2	30		
6	54	7	1.3	3	27		
7	60	2	0.3	1	19	2	
Hunting Park							
5	140	20	1.4	12	50		
6	140	19	1.3	10	75	3	1
7	60	9	1.7	3	18		
8	90	8	0.9	5	51	1	
9	102	3	0.3	6	55	1	
10	108	9	0.8	3	35	1	
11	132	4	0.3	2	58	2	
12	210	0		0	45	1	1
13	136	2	0.1	2	48	1	
14	90	3	0.3	2	40	1	1
15	27	3	1.1	2	8		
16	24	0		0	9		
17	66	0		0	35	1	1
18	27	0		0	11		
19	102	2	0.2	1	40		1
20	81	5	0.6	2	48	2	
Ontario							
1	36	0		0	18	2	
2	76	3	0.4	2	29	1	1
3	117	0		0	37		
4	27	0		0	9		
6	98	0		0	24	1	
7	51	4	0.8	2	38		
8	90	0		0	63		
9	63	0		0	37		
10	16	0		0	3		
11	50	0		0	12		

DATA UPON 2300 VOLT CIRCUITS (*Continued*)

Circuit	Length of wire in 1000 ft.	Lightning Arresters		Transformers			
						Lost by Lightning	
		Total	Per 10,000 ft.	Protected	Total	1920	1921
12	84	0		0	38	1	
13	84	0		0	56	1	
14	36	0		0	14	1	
15	48	0		0	27		1
16	12	3	2.5	4	6		
Susquehanna							
1	58	7	1.2	3	40		
2	46	6	1.4	2	4		
3	36	6	1.9	2	11		
4	26	6	2.3	14	14		
5	26	0		0	12		
6	90	3	0.3	2	38	3	
7	65	6	0.9	2	37		
8	20	0		0	9		
9	83	6	0.7	4	37	1	
10	30	0		0	16		
11	57	7	1.2	4	32	1	
12	63	0		0	36		
13	19	0		0	6		
15	57	3	0.5	2	44	2	
16	60	0		0	28		
17	36	5	1.4	4	32		
Marshall							
1	18	6	3.3	0	20*		
2	33	6	1.8	7	33	1	
4	38	6	1.5	0	38	1	
5	27	5	1.9	4	16		
6	55	18	3.2	10	56		
7	45	6	1.3	5	20		
8	36	3	0.9	0	14		
9	12	6	5.0	0	0*		
10	9	8	9.9	3	5		
11	30	6	2.0	10	35		
12	21	0		0	28		
13	45	0		0	39*		
Columbia							
1	underground						
2	24	6	2.5	8	21		
3	60	5	0.9	2	39		
4	42	3	0.7	0	17		
5	27	6	3.1	3	17		
6	45	7	1.7	3	32		
7	42	3	0.7	0	20		
8	30	6	2.0	4	8		
9	underground						
10	27	0		0	21*		
Ludlow							
1	42	3	0.7	0	34		
2	13	0		0	11		
3	21	0		0	11*		
4	60	7	1.1	4	34*		
5	underground						
6	21	0		0	25*		
7	39	3	0.8	1	37		
8	underground						
Carpenter							
1	117	5	0.4	3	40	1	
2	105	6	0.5	1	21	2	
3	underground						
4	65	0		0	33		
5	48	0		0	19	1	
6	underground						
7	92	3	0.3	1	53	3	1
8	96	6	0.6	0	61		
9	104	0		0	55	1	
10	40	0		0	6		
11	30	0		0	6		
12	120	6	0.5	8	40		
13	111	0		0	39	1	
14	reserve						
15	112	0		6	51	2	
16	106	0		0	61		
17							
18	16	0		0	6		

*Subway type omitted.

are the only types so far developed which can satisfactorily resist the tremendous dynamic discharge which may develop near large generating units.[1] The high first cost of such arresters and the difficulty in properly distributing them along the right of way, presents a serious problem in providing suitable protection for these high-voltage circuits leading out from generating points. An inspection of some of the spark gap arresters which had failed suggested that possibly this trouble might be caused by a short circuit between phases due to the simultaneous discharge of two or more of the arresters of the group. In was therefore proposed by Mr. H. S. Phelps of the Philadelphia Electric Company, to mount the three arresters of one group upon successive poles and provide independent grounds, thus introducing twice the ground resistance between phases at the time of discharge. Three groups of arresters arranged on this principle were installed on July 1st of this year on the most exposed portions of circuits No. 2101 and 2102. The day after their installation a storm of unusual severity passed over these circuits. Lightning struck an oil tank within a few rods of the circuits, fuses were blown on a transformer connected to a 2300-volt circuit on the same pole line, and other damage was done in the neighborhood, but the arresters functioned satisfactorily and no damage was done to these circuits. The record for the rest of the summer has been equally satisfactory. The question still remains whether longer experience will justify the hope that this method will provide adequate protection for such circuits and there is the further question as to what is the minimum reactance which must be provided between such arresters and the generating units in order to insure proper performance.

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[1] N. E. L. A. *Proceedings*, 1921, p. 700.

Discussion

For discussion of this paper see page 566.

Operating Experience with Current-Limiting Reactors

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***Review of the subject.**—This paper describes the troubles experienced with different types of current-limiting reactors installed on the Public Service Electric Company's system. Many of these failures are attributed to poor mechanical construction.*

The author states that the experience gained has resulted in much better designs. The conclusion is reached that if the present day reactors are properly chosen with regard to thermal capacity they are very reliable.

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THE purpose of this paper is to give an account of the operating experience with current-limiting reactors installed on the system of the Public Service Electric Company of New Jersey.

DESCRIPTION OF SYSTEM

The above-mentioned system is composed of two large sections: The Southern Division, which includes the territory between Camden and Trenton; and the Northern Division, which comprises the northeastern part of New Jersey. Since current-limiting reactors at the present time are used only in the Northern Division, merely this part of the system will be described.

The 13,200-volt, 25-cycle system is comparatively small, as it includes only 35,200 kw. generating capacity. Since this system has reactors only on the generator leads in the Marion Station, and because all the apparatus will be replaced by 60-cycle machines within the next few years, no further mention will be made of it with the exception of a brief description of the reactors later.

The bulk of the 60-cycle current is generated at Marion and Essex Stations located about three and one-half miles apart and connected together by means of five 13,200-volt tie lines. The Marion capacity is 60,500 kv-a. and Essex 85,000 kv-a. In addition there are five other smaller stations with a combined capacity of 44,500 kv-a. feeding into the system at different points, giving this Division a total capacity of 190,000 kv-a.

The 60-cycle transmission system in the Northern

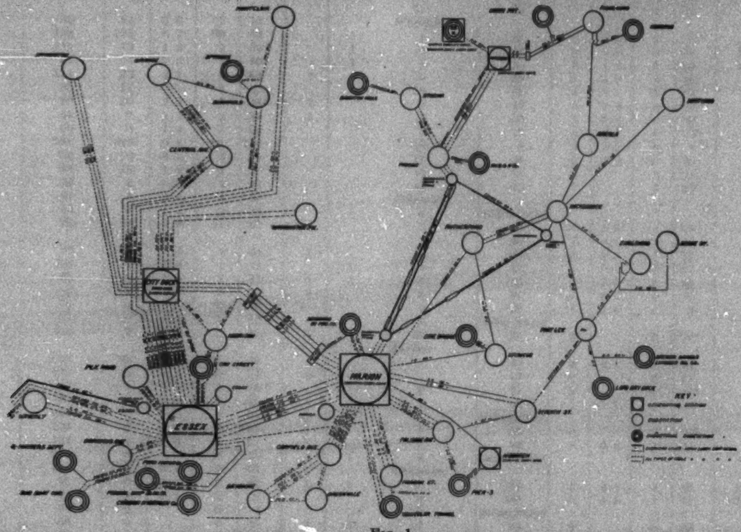

FIG. 1

Presented at the Spring Convention of the A. I. E. E., Pittsburgh, Pa., April 24-26, 1923.

Division consists of 580 miles of 13,200-volt lines and 121 miles of 26,400-volt lines. Of the 13,200-volt lines, 322 miles are overhead and 258 miles underground; of the 26,400-volt lines, 97 miles are overhead and 24 miles underground. Fig. 1 is a one-line diagram of the 60-cycle system in the north section of the Northern Division, and Fig. 2 shows the south section of this division.

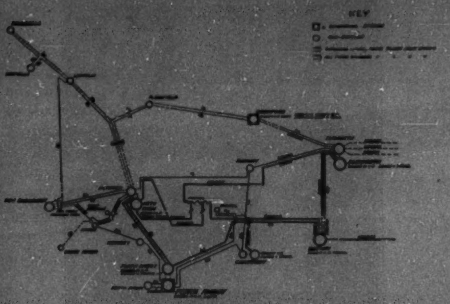
FIG. 2

These two sections are tied together with a 26,400-volt line between Essex and Bayway, and with several 13,200-volt lines between Essex and Elizabeth.

NEED FOR REACTORS

The need for current-limiting reactors was first brought to our attention in 1912, when the oil circuit breakers commenced to fail on the 25-cycle system in the Marion Station. The reactance of these turbo-generators was only 3 per cent, so in order to relieve the excessive strains on the equipment, 3 per cent reactance coils were inserted in the generator leads of each machine. (Fig. 5 shows one of these reactors.) This relieved the 25-cycle system to a great extent from oil circuit breaker and other troubles.

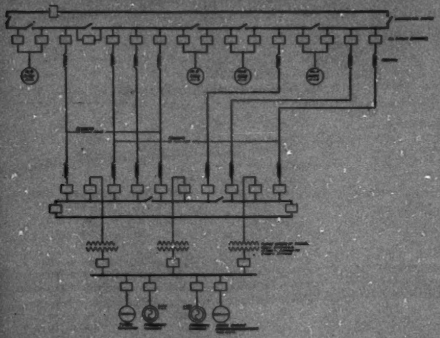
FIG. 3

In 1914 the 60-cycle system had grown to such an extent that the oil circuit breakers and other equipment were failing frequently. As a result of these failures we decided to install 5 per cent reactors on all radial 13,200-volt feeders, 2½ per cent reactors on all tie feeders, and aluminum cell arresters on the feeder side of all reactors whether the feeder was overhead or underground. This required a total of 29 sets of reactors, all of which were purchased for 5 per cent reactance with a 2½ per cent tap so that they could be used on either type of feeder. Essex Station was not built at this time. The seven tie feeders between Marion and City Dock Stations were equipped with 2½ per cent reactors at each end. It was necessary to install reactors at both ends in order to allow selective action of the balanced relays which were installed on these feeders. Before purchasing the reactors we sent a man to the factory to witness tests, which included both high

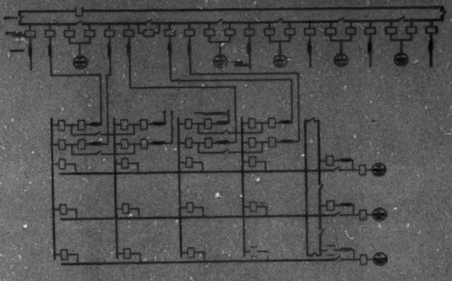
FIG. 4

FIG. 5

voltage and high frequency, and we were assured by the manufacturer's engineers that these reactors were so well designed that they would stand up under any operating condition. The factory tests seemed to prove their statements.

Fig. 6

Fig. 3 shows the Marion and City Dock busses and tie lines at the time these 60-cycle reactors were installed. Fig. 6 illustrates a typical installation at Marion.

Contrary to expectation, we experienced considerable trouble with this first installation of 60-cycle reactors; therefore we decided to install an entirely different type in 1915 in the new Essex Station. These reactors are shown in Figs. 7 and 8. The first sixteen sets (Fig. 7) proved very satisfactory, but the second installment of ten sets (Fig. 8) has been a constant source of trouble.

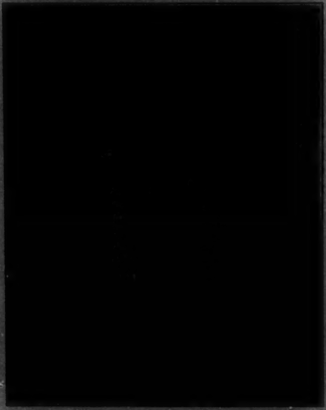

Fig. 7

Fig. 4 shows the Marion and Essex busses and tie lines after these reactors were installed.

From 1917 to the present time we have installed 18 sets of reactors of the types shown in Figs. 9, 10 and 11, and with two exceptions these have given very satisfactory service.

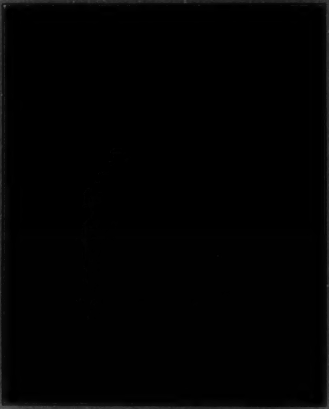

Fig. 8

Table I gives the rating, type and number of reactors installed to date at Marion, Essex, and City Dock, including the twelve 25-cycle generator reactors at Marion.

TABLE I.
REACTOR INSTALLATIONS

Group	Illustration	Number of Reactors (All 1-phase)	No. of Sets	Amperes	Per cent	Kv-a.	Year Installed
A	Fig. 5	6	2	220	3	50.	1913
		6	2	395	3	90.	1913
B	Fig. 6	72	24	175	5	66.8	1914
		12	4	250	5	95.3	1914
		3	1	350	5	133.3	1914
C	Fig. 7	6	2	150	5	57.1	1915
		9	3	175	5	66.8	1915
		12	4	250	5	95.3	1915
		21	7	350	5	133.3	1915
D	Fig. 8	6	2	550	2	83.9	1917
		9	3	250	5	95.3	1917
		15	5	350	5	133.3	1917
E	Figs. 9, 10, 11	6	2	545	1.5	63.	1917
		6	2	175	5	66.8	1919-20
		3	1	765	1.5	88.	1917
		9	3	250	5	95.3	1919-20
		18	6	350	5	133.3	1919-21
		12	4	300	8	183.	1918

Table II gives a list of the different kinds of failures and the reactor-years per failure.

TABLE II
REACTOR FAILURES

Group	Illustration	Nature of Failure				Total Number of Failures	Reactor-Years		
		Turns Short Circuited	Arced Over	Miscellaneous	Complete Failure		Total	Per Failure	Per Complete Failure
A	Fig. 5	2	..	..	..	2	120	60	..
B	Fig. 6	2	6	5	11	24	676	28.1	61.4
C	Fig. 7	2	..	..	..	2	408	204	..
D	Fig. 8	5	4	8	8	25	165	6.6	20.6
E	Figs. 9, 10, and 11	..	..	3	..	3	213	71	..

COMPARISON OF DIFFERENT TYPES OF REACTORS

For the purpose of comparing the different coils from an operating viewpoint, they have been divided into groups A, B, C, D and E, each group representing a definite type of make and a certain date of installation.

Group A, Fig. 5. Four sets of coils of this type were installed on the 25-cycle generator leads in 1913 and have given very satisfactory service with only two cases of trouble during the 10 years of operation. The first failure was caused by water getting in the reactor room and grounding one coil. In the second failure the heat of the copper caused the wooden strips supporting the coils to shrink, thus allowing so much slack in the windings that some of the turns became short-circuited. The trouble was remedied by overhauling the coils and drawing the conductors tightly over the wooden strips.

Group B, Fig. 6. This group represents a total of 29 sets of 5 per cent coils with a 2½ per cent tap, installed at Marion and City Dock in 1914. As can be seen from Fig. 6, these coils are made of stranded conductors wound in parallel layers with notched strips of specially prepared moulded non-inflammable material between the layers. Non-magnetic castings are placed at the top and bottom of the coils, and the different sections are held together by brass tie rods. All the terminals are located at the top of the reactor.

The numerous cases of short-circuited and completely wrecked coils indicated that they were not sufficiently braced, and did not have the proper clearance between layers and between coils and metal tie rods to withstand the severe operating conditions. Failures in most cases occurred between the outer turn of either the top or bottom coil and one of the brass tie rods.

In this group there have been twenty-four failures listed as follows:

2 cases of short-circuiting of turns.
6 cases of arcing from top to bottom.
3 cases of lugs burning off.
2 cases of winding burning open.
11 cases of complete failure.

After these reactors proved faulty, the manufacturer discontinued building this type and designed a new coil with the following improvements: Greater spacing between layers; wood tie rods instead of brass; heavier supports; insulators with a higher flashover; terminals brought out one on top and one on bottom.

From certain operating experience reported by various companies it would seem that the above improvements have eliminated most, if not all, of the previous faults of this make of reactor.

Group C, Fig. 7. Sixteen sets of reactors of this type were installed at Essex in 1915, nine of which were placed on tie feeders and the remaining seven sets on radial feeders.

These coils are 5 per cent with a 2½ per cent tap and are series-wound similar to Group B except that insulated cable is used. Their axis is vertical and the coils are partially enclosed by porcelain bricks with soapstone slabs at the top and bottom, held together by brass tie rods.

In this group there have been only two failures: The first occurred after two years of service when one of the coils arced from top to bottom; the second occurred when several turns short-circuited in another coil after three years of service. Both coils were repaired and have not given any further trouble.

Group D, Fig. 8. This group, which differs in design from Group C, consists of 10 sets of coils installed at Essex in 1917. These coils have two sections connected in series, each section being made up of six pancake coils in parallel. One section has a diameter approximately three inches less than the other. The coils have a horizontal axis and are partially enclosed with porcelain bricks. They are built for 5 per cent reactance and have a 2½ per cent tap.

Within three months after the installation of these reactors, three total failures occurred. In each case the coil caught fire from the arcing between a number of turns, and before the fire could be extinguished the coils were badly burned, loosened, distorted, and partially wrecked. Immediately following these cases of failure the coils were returned to the manufacturer for rebuilding and reinforcing. After the third case of trouble it was decided to return the entire ten sets for rebuilding, as it had been fully demonstrated that the coils were mechanically weak. The rebuilding and reinforcing consisted in substituting brass holding rods for the wooden rods and a general strengthening of the coils.

After the coils had been rebuilt and re-installed, they continued to give unsatisfactory service in that some of them operated very noisily at normal load, and at times of cable failures or insulator flashovers the brick-work construction of the coils was often loosened and the insulators broken. On account of the frequency of these failures it was decided to return the entire ten sets of coils to the factory for a second rebuilding. We recommended to the manufacturer that a filler more durable than "petrite" be used and also that special porcelain brick be used instead of wood for bracing that part of the coil having the smaller diameter. After it was found that this second rebuilding produced very little, if any, improvement, we decided to rebuild six of these coils ourselves. Instead of porcelain brick we used cast concrete corners so as to obtain a tight fit against both sets of pancakes. To secure ample insulation a 1/16-in. layer of sheet mica was placed between the coils and the concrete.

Fig. 9

At first it seemed that this method of rebuilding was a success, but after a few months' service several of these coils became noisy again. On account of these reactors having windings of two different diameters, it is very difficult to make them strong enough mechanically to stand up under short-circuit conditions.

Fig. 10

In addition to seventeen electrical failures, there have been several cases where the coils became noisy and had to be removed from service for mechanical repairs. At present, whenever a coil commences to become noisy, it is taken out of service and scrapped. It is needless to say that this design of reactor is not being perpetuated.

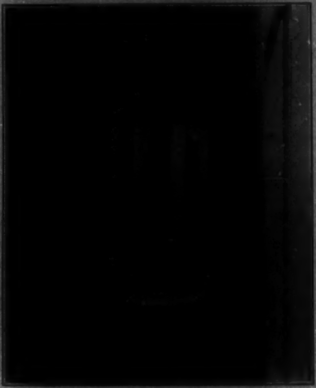

Fig. 11

Group E, Figs. 9, 10, and 11. This group represents a total of 18 sets of reactors, part of which were built for 5 per cent with a 2½ per cent tap, and the remainder were 1½ per cent coils for use on the generator tie bus. They were installed at different times between 1917 and 1920.

Fig. 12

About one year after the first installation there were two instances of trouble: The first occurred on a tie feeder and resulted in the short-circuiting of two reactors; the second occurred on another tie feeder and caused the concrete to crack on one of the coils. These demonstrations were sufficient to prove that the concrete supports were not close enough to prevent the conductors from pulling together and causing short-circuiting of the turns during times of trouble on the transmission lines.

After an investigation by the manufacturer it was decided to return part of these reactors to the factory for reinforcing, the decision applying only to those coils which were to be used on the 2½ per cent tap. Fig. 10 illustrates one of the coils after it had been reinforced with additional concrete supports half way between the old ones.

The latest type of reactor which we have installed is shown in Fig. 11. This coil is supposed to have all of the necessary characteristics and none of the bad qualities of any of the types previously described. The few of this newer design that were installed have not as yet been in service long enough to demonstrate whether they will stand up under all conditions, but we hope that they will live up to our expectations.

Fig. 12 illustrates a generator tie bus reactor rated at 5 per cent, 545 amperes, 208 kv-a. We have purchased a number of these coils for future installation.

CONCLUSIONS

It has been the opinion among a number of central station engineers that it was unnecessary to install aluminum cell arresters on the feeder side of reactors when the feeders consisted of underground cable. However, we have always felt that the arresters at times of system disturbances acted as relief valves, and since they have almost invariably discharged in cases of disturbances it would appear that they did relieve the system of at least a part of the strain.

As a word of caution it is well to point out the importance of adequate thermal capacity in reactors. From all indications a number of our failures were due to lack of thermal capacity. Since a reactor is essentially a protective device, it should be designed with a copper cross-section in excess of that of any part of the system which it protects, so that an extended period of severe duty will not cause it to fail because of lack of thermal capacity.

In spite of the troubles which we have experienced with reactors, the number of instances in which they functioned properly has far exceeded the cases of failure. Furthermore, the design of present-day reactors has been perfected to such an extent that, if properly chosen with regard to thermal capacity, the modern reactor may be considered as a very reliable piece of apparatus.

Discussion

For discussion of this paper see page 566.

Short-Circuit Forces on Reactor Supports—I

BY R. E. DOHERTY and F. H. KIERSTEAD
Associate, A. I. E. E. Associate, A. I. E. E.
Both of the General Electric Company

***Review of the Subject.**—The mechanical stress in the supporting members of any structure or apparatus under steady state is determined by the dimensions of the member and the magnitude of the resultant applied force. Under accelerated motion, however, an additional factor enters, namely, the reaction of the mass; and if the supporting members in this case are resilient, that is, spring-like, then this becomes still another factor which enters the problem of determining the mechanical stress produced by a given impressed force.*

The determination of the stress in the holding device (bolts, etc.) of reactors under short-circuit condition is just such a problem. If any motion whatever is permitted under this condition, the factors of mass and resilience are active.

This paper gives a theoretical analysis of the problem, and shows that if any motion is permitted, thus allowing the factors of mass and resilience to become active, then the maximum stress may be significantly increased above that for no motion. Illustrative calculations show that this increase in practical cases may be of the order of 25 per cent. On the other hand, if motion of the reactors is prevented by sufficient initial bolt tension, or otherwise, then the maximum stress in the holding device obviously need be only as great as that corresponding to the maximum instantaneous peak of the electromagnetic force.

A NUMBER of papers[1] has appeared in which the problem of the electromagnetic force between electric conductors or between coils, has been analyzed. The stresses, however, which such forces create in the members on which they act, are not determined merely by the value of the electromagnetic force and the dimensions of the members, as would be the case with a steady applied force, but also depend in a large measure upon other factors. Hence the analysis of such stresses is a separate problem in which the electromagnetic force is only one of a number of factors.

Scope

The present paper investigates, in a theoretical way, the character of the force which is required to hold the reactor as a whole, against short-circuit forces. Unlike the problems covered by the foregoing papers, which were concerned with the character of the electromagnetic force itself, the present investigation assumes that this force is given, and inquires into the nature of the stresses it produces in the holding device, such as bolts, etc. In other words, the problem is of a purely mechanical nature.

As such, it is the same, in fundamental respects, as a number of other problems in engineering which involve the factors of mass, resilience and a periodic impressed force.

For instance: Fly wheels for synchronous machines mechanically connected to reciprocating apparatus[2]; torsional strain of a turbo-generator shaft during generator short circuit[3]; the behavior of bus bars carrying short-circuit current[4]; the behavior of certain types of engine governors, etc. This class of phenomena is best illustrated by the simple case of the behavior of a weight, suspended by a spring, and acted upon by an alternating-current electromagnet.

Mathematically, the problem is identical with a much wider range of phenomena; that is, in addition to the above cases it involves the same form of differential equation[5] as a number of familiar problems in electrical engineering. For instance: The electric circuit containing resistance, inductance and capacity in series[6]; the series circuit involving an exciter and alternator field.[7]

The present investigation, therefore, does not deal with a new phenomenon; it merely starts with certain assumptions, which appear reasonable, and which written down mathematically, give a well known differential equation, and then applies the solution to the physical conditions of the problem. This makes it possible, within specified limits, to answer a number of questions regarding the character of the force on the holding device.

In a later paper the authors expect to present some experimental data regarding these forces.

Equations

The equations are set up under the following assumptions:

Supporting floors rigid.
Single-phase short circuit.
Current wave completely off-set.
No transients; that is, damping neglected.
Reactor unit, including concrete, etc. rigid body.
Initial tension on holding bolts same on all bolts.

---

[1] Some of which are listed in the bibliography.

[2] "Design of Fly Wheels for Reciprocating Machinery Connected to Synchronous Generators or Motors," by R. E. Doherty and R. F. Franklin, A. S. M. E. *Transactions*, 1920, V. 42, pp. 523-567.

[3] "Mechanical Effects of Electrical Short Circuits," by S. H. Weaver, *G. E. Review*, November 1915, V. 18, pp. 1066-1074.

Presented at the Spring Convention of the A. I. E. E., Pittsburgh Pa., April 24-26, 1923.

[4] "Über die Mechanischen Wirkungen des Plötzlichen Kurzschluss-stromes von Synchronmaschinen" by J. Biermanns, *Archiv für Electrotecknik*, IX, 1920, pp. 326-340.

[5] Second order, linear.

[6] "Theory and Calculation of Transient Electric Phenomena and Oscillations," by C. P. Steinmetz, 3rd Ed., p. 48.

[7] "Exciter Instability," by R. E. Doherty, JOURNAL A. I. E. E., Oct. 1922, pp. 731-744.

Reactors connected to give attraction.

Center lines of coils coincide, as in Fig. 1.

Displacement x sufficient to permit free movement of the reactor B, that is, no interference by supporting floor.

Magnetic force proportional to square of the current, other factors being constant.

Let

M = mass of reactor
i = current in reactor coils (amperes)
x = displacement of reactor B above floor (feet)
f_s = force due to bolts (lb.)
f_e = force due to currents (lb.)
W = weight of reactor = $g\,M$ (lb.)
g = gravity = 32.2 feet/sec.^{2}
k = lb. force required to elongate bolts one foot
f_0 = total force due to initial tension in bolts at $x = 0$
e = modulus of elasticity of bolt material = 30×10^6 for steel
a = total cross section of bolts in sq. inches
l = free length (inches) of bolts under tension
t = time in seconds.

The sum of all forces acting on the mass must equal zero. That is,

$$\Sigma \text{ forces} = 0 \tag{1}$$

Thus

$$f_s - M\frac{d^2 x}{d\,t^2} + f_e - W = 0 \tag{2}$$

But

$$f_s = -(f_o + k\,x) \tag{3}$$

Where

$$k = 12\,e\,a/l \tag{3a}$$

Therefore

$$f_e - M\frac{d^2 x}{d\,t^2} - f_o - k\,x - W = 0$$

or

$$\frac{d^2 x}{d\,t^2} + k\,x/M = f_e/M - \frac{(f_o + W)}{M} \tag{4}$$

The force f_e is proportional to the square of the current. Thus, let[8]

$$f_e = K\,i^2 \tag{5}$$

The current i is of the form,

$$i = I_0\,(1 - \cos\,\omega\,t) \tag{6}$$

as shown in Fig. 2.

where I_o = maximum value of a. c. component of the current i

$\omega = 2\,\pi\,f$

f = electrical frequency.

Substituting (6) in (5).

$$f_e = K\,I_0^2\,(1.5 - 2\cos\,\omega\,t + 0.5\cos 2\,\omega\,t) \tag{7}$$

Substituting this in (4),

$$\frac{d^2 x}{d\,t^2} + k\,x/M = -\,2\,\frac{K\,I_o^2}{M}\,(\cos\,\omega\,t - 0.25\cos 2\,\omega\,t) + 1.5\,\frac{K\,I_o^2}{M} - \frac{f_o + W}{M}$$

Let

$$\frac{K\,I_o^2}{M} = D \tag{8}$$

$$k/M = \alpha^2 \tag{9}$$

and

$$1.5\,D - \frac{f_o + W}{M} = E \tag{10}$$

Then the differential equation becomes,

$$\frac{d^2 x}{d\,t^2} + \alpha^2\,x = -\,2\,D\cos\,\omega\,t + 0.5\,D\cos 2\,\omega\,t + E \tag{11}$$

The solution of this equation will be in four parts as follows:

$$x = x_1 + x_2 + x_3 + x_4 \tag{12}$$

where, x_1 is given by the complementary function

$$\frac{d^2 x_1}{d\,t^2} + \alpha^2\,x_1 = 0 \tag{13}$$

x_2 is given by the particular solution,

$$\frac{d^2 x_2}{d\,t^2} + \alpha^2\,x_2 = -\,2\,D\cos\,\omega\,t \tag{14}$$

x_3, by the particular solution,

$$\frac{d^2 x_3}{d\,t^2} + \alpha^2\,x_3 = 0.5\,D\cos 2\,\omega\,t \tag{15}$$

and x_4, by

$$\frac{d^2 x_4}{d\,t^2} + \alpha^2\,x_4 = E \tag{16}$$

Solution of (13) is

$$x_1 = C_1\cos\,\alpha\,t + C_2\sin\,\alpha\,t \tag{17}$$

Solution of (14) and (15) is obtained by temporarily using vectors and substituting

$$\left.\begin{aligned} \frac{d}{d\,t}\,x &= p\,x = j\,\omega\,x \\ \frac{d^2}{d\,t^2}\,x &= p^2\,x = -\,\omega^2\,x \end{aligned}\right\} \tag{18}$$

since a cosine function only is involved. Thus, in (14)

$$x_2\,(-\,\omega^2 + \alpha^2) = -\,2\,D$$

and

$$x_2 = \frac{2\,D}{\omega^2 - \alpha^2}$$

or

$$x_2 = \frac{2\,D\cos\,\omega\,t}{\omega^2 - \alpha^2} \tag{19}$$

and similarly

$$x_3 = -\,\frac{0.5\,D\cos 2\,\omega\,t}{4\,\omega^2 - \alpha^2} \tag{20}$$

Solution of (16) is $x_4 = E/\alpha^2$ (21)

Therefore by (12)

$$x = C_1\cos\,\alpha\,t + C_2\sin\,\alpha\,t + \frac{2\,D\cos\,\omega\,t}{\omega^2 - \alpha^2} - \frac{0.5\,D\cos 2\,\omega\,t}{4\,\omega^2 - \alpha^2} + E/\alpha^2 \tag{22}$$

---

[8] The assumption is made that no deflection will be large enough to alter the value of K, that is, K = constant.

Thus the solution involves two constants, C_1 and C_2, which must be determined by the boundary conditions, such as the value of x and of $\frac{dx}{dt}$ at some particular value of t. In other words, with the constant value of displacement represented by the last term of (**22**) and with the definite functions of time represented by the 3rd and 4th terms, the constants C_1 and C_2 must be of such values that the sum of all terms must give the known boundary conditions. Or further, for instance, if $x = 0$ at $t = 0$, then the sum of the first two terms must be equal to minus the sum of the last three terms. It follows that if the boundary conditions happen to be such that they are given by the last three terms then C_1 and C_2 must be zero—that is, there would be no oscillation at the natural frequency

$$n = \frac{\alpha}{2\pi}$$

as discussed later.

To illustrate the application of equation (**22**) the following Cases will be considered.

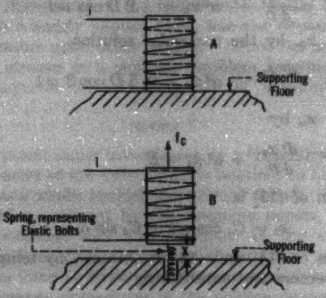

Fig. 1

Case 1. Physical location of reactors as shown in Fig. 1. Hypothetical case in which the interference of the floor to the free movement of the reactor is neglected.

Case 2. Same as *Case* 1 except that the effect of the floor is considered.

Case 3. Same as *Case* 1 except that the frequency of the electrical force is doubled.

Case 4. Same as *Case* 2 except that the frequency of the electrical force is doubled.

Case 5. Reactors rigidly fastened to I-beams instead of being bolted to a rigid floor as in the foregoing Cases.

DATA

The following data are used in all Cases excepting *Case* 5;

$$W = 1000 \text{ lb.}$$

$$M = W/g = 1000/32.2 = 31$$

Average value of magnetic force is

$$1.5\,K\,I_e^2 = 10 \times W = 10{,}000 \text{ lb.}$$

Thus, the peak value of the magnetic force, by equation 7,

$$= 4\,K\,I_e^2 = 26{,}640 \text{ lb.}$$

and,

$$\frac{K\,I_e^2}{W} = \frac{K\,I_e^2}{g\,M} = 10/1.5 = 6.66$$

$$\frac{K\,I_e^2}{M} = 6.66\,g = 214$$

Four steel bolts, each one square inch in cross-section and 10 inches free length

$$e = 30 \times 10^6 \text{ lb./sq. in. for steel}$$

Thus, by (**3a**)

$$k = 144 \times 10^6 \text{ lb./ft. elongation of bolts.}$$

By (**25**)

$$\alpha = \sqrt{\frac{144 \times 10^6}{31}} = 2160 \cong 2260 \text{ rad./sec.}^{9}$$

$$n \cong \frac{2260}{2\pi} = 360 \text{ cycles/sec.}$$

The above data may not be representative but they will serve for illustration of the application of the equations.

CASE 1

The physical relation of the reactors is shown in Fig. 1. The integration constants C_1 and C_2 are determined under the following hypothetical assumption: At $t = 0$, that is, at the moment the short circuit occurs, a shim between the reactor and the floor is suddenly withdrawn, thus permitting free motion of the reactor under the influence of all the forces assumed in the equation. This, of course, is far from a practical case, but it is of interest, and perhaps may be of importance to some readers, in helping to form an idea of the general character of the phenomenon, and in leading up to the second case, in which the effect of the floor is taken into account.

The integration constants are determined, therefore, from the boundary conditions that at

$$t = 0,\ x = 0 \text{ and } \frac{dx}{dt} = 0$$

Thus, from (**22**)

$$C_1 + \frac{2D}{\omega^2 - \alpha^2} - \frac{0.5D}{4\omega^2 - \alpha^2} + E/\alpha^2 = 0$$

and

$$C_2\,\alpha = 0$$

Therefore

$$\left.\begin{aligned} C_1 &= -\frac{2D}{\omega^2 - \alpha^2} + \frac{0.5D}{4\omega^2 - \alpha^2} - E/\alpha^2 \\ C_2 &= 0 \end{aligned}\right\} \quad (\mathbf{23})$$

---

9. In order to have $n = 3f$, thus facilitating computations.

Substituting in (22)

$$x = \left[-\frac{2\,D}{\omega^2 - \alpha^2} + \frac{0.5\,D}{4\,\omega^2 - \alpha^2} - E/\alpha^2 \right] \cos \alpha\, t$$

$$+ \frac{2\,D}{\omega^2 - \alpha^2} \cos \omega\, t - \frac{0.5\,D}{4\,\omega^2 - \alpha^2} \cos 2\,\omega\, t + E/\alpha^2 \qquad (24)$$

The angular velocity of the natural oscillations is, by (9) and (24)

$$\alpha = \sqrt{k/M} = 2\,\pi\, n \qquad (25)$$

where n = oscillations per second

Also $\omega = 2\,\pi\, f$

Substituting these relations and also (8) (9) and (10) in (24), the final equation for x becomes,

$$x = \left[2\frac{K\,I_o^2}{M\,\omega^2} \left\{ -\frac{1}{1-(n/f)^2} + \frac{0.0625}{1-\left(\frac{n}{2f}\right)^2} \right\} - \frac{1.5\,K\,I_o^2 - (f_o + W)}{k} \right] \cos \alpha\, t$$

$$+ 2\frac{K\,I_o^2}{m\,\omega^2} \left\{ \frac{\cos \omega\, t}{1-(n/f)^2} - \frac{0.0625 \cos 2\,\omega\, t}{1-\left(\frac{n}{2f}\right)^2} \right\}$$

$$+ \frac{1.5\,K\,I_o^2 - (f_o + W)}{k} \qquad (26)$$

Equation (26) is made up of two parts:

x_a = complex oscillation, given by the trigonometric terms, the average of which, over complete cycles, is zero.

and x_b = a constant deflection, determined by the difference between a constant force[10] 1.5 $K\,I_o^2$ and the sum of the weight W and initial bolt tension f_o.

The magnitudes of the different components of the complex oscillation are dependent upon the ratio of the frequency n of natural oscillations to the frequency f of the electrical system, and the second harmonic $2f$.

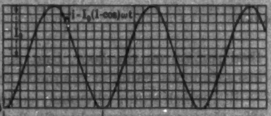

FIG. 2

The constant component x_b of the total deflection x is obviously

$$x_b = \frac{1.5\,K\,I_o^2 - (f_o + W)}{k} \qquad (27)$$

This will evidently be zero when

$$f_o + W = 1.5\,K\,I_o^2$$

or, when

$$f_o = 1.5\,K\,I_o^2 - W \qquad (28)$$

That is, x_b will be zero if the initial bolt tension f_o is the difference between the magnetic force $1.5\,K\,I_o^2$ and the weight W.

Assuming the condition given by (28), then equation (26) becomes

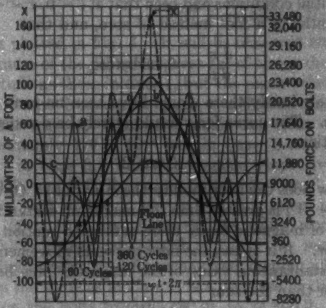

FIG. 3—CASE I. HYPOTHETICAL CASE. ASSUMES OSCILLATION ABOUT FLOOR LINE AS AVERAGE POSITION

$x = 10^{-6}$ (61.5 cos 2260 t − 85.5 cos 377 t + 24 cos 754 t)
4 $K\,I_o^2$ = 26,640 lb. a = 1st term, b = 2nd term, c = 3rd term
$d = b + c$, $x = a + b + c$

$$x = 2\frac{K\,I_o^2}{M\,\omega^2} \left[\left\{ -\frac{1}{1-(n/f)^2} + \frac{0.0625}{1-\left(\frac{n}{2f}\right)^2} \right\} \cos \alpha\, t + \left\{ \frac{\cos \omega\, t}{1-(n/f)^2} - \frac{0.0625 \cos 2\,\omega\, t}{1-\left(\frac{n}{2f}\right)^2} \right\} \right] \qquad (29)$$

The electrical frequency is taken as 60 cycles. Thus,

$$\omega = 2\,\pi\, f = 377 \text{ rad./sec.}$$

Other data are given under "DATA" above.

Thus $n/f \cong 6.0$

and $\dfrac{n}{2f} \cong 3.0$

These values substituted in (29) give,

$$x = 2\frac{6660}{31 \times 377^2} \left[\left\{ -\frac{1}{1-(360/60)^2} + \frac{0.0625}{1-(360/120)^2} \right\} \cos 2260\, t + \left\{ \frac{\cos \omega\, t}{1-(360/60)^2} - \frac{0.0625 \cos 2\,\omega\, t}{1-(360/120)^2} \right\} \right]$$

Thus $x = (61.5 \cos 2260\, t - 85.5 \cos 377\, t + 24 \cos 754\, t)\, 10^{-6}$

This is plotted as *Case* 1 in Fig. 3.

---

[10] It is interesting to consider the value of current i, on which this force depends. By equation (5), force is $K\,i^2$. But here $i_1^2 = 1.5\,I_o^2$, that is $i_1 = \sqrt{1.5}\,I_o$. This is the root-mean-square of the total current during the first cycle. That is, the steady, or average, deflection is determined by the root-mean-square current.

It will be noted that the maximum force on the bolts is 33,480 lb., whereas the maximum magnetic force is 26,640. That is, permitting free motion *increases* the stress on the bolts in this particular case. It must be remembered that curve x is the displacement under the hypothetical assumption that the average force is just balanced by the weight and initial bolt tension, and that the floor does not interfere with the free movement of the reactor. It will be observed that the amplitude of a is just equal, and opposite, to d at t = zero. If the initial conditions had been that at $t = 0$, $x = -61.5$ millionths of a foot instead of zero, and that

$$\frac{dx}{dt} = 0, \text{ as before,}$$

then, obviously there would have been no term of 360 cycles. The curve for x would merely be d under those conditions.

CASE 2

How does the floor modify the situation? Obviously if $(f_0 + W)$ is greater than the maximum instantaneous value of magnetic force, there can be no motion of the reactor. If it is less, the floor prevents movement until the instantaneous value of the magnetic force becomes greater than the sum

$$f_0 + W$$

Thus to get the actual movement of the reactor, new integration constants, C_1 and C_2, must be determined for the following known conditions:

$$f_c = f_o + W$$
$$x = O$$
$$\frac{dx}{dt} = O$$

Since $$f_o + W = 1.5\,K\,I_o^2$$

and $f_c = K\,I_o^2\,(1.5 - 2\cos\omega t + 0.5\cos 2\,\omega t)$ by equation (7)

it follows that

$$f_c = f_o + W \text{ at } \omega t = 103°$$

Referring to equation (22) the constant term

$$E/\alpha^2$$

is zero as in *Case* 1. The 3rd and 4th terms are naturally also the same. Thus by (22) and (29) and the given data, the new equation is

$$x = (C_1 \cos 2260\,t + C_2 \sin 2260\,t - 85.5 \cos 377\,t + 24 \cos 754\,t)\,10^{-6}$$

and $$\frac{dx}{dt} = (-2260\,C_1 \sin 2260\,t + 2260\,C_2 \cos 2260\,t + 32200 \sin 377\,t - 18100 \sin 754\,t)\,10^{-6}$$

When
$\omega t = 103$ degrees
$\alpha t = 6 \times 103 = 618$ degrees
$2\,\omega t = 206$ degrees
$\sin 618° = -0.978$
$\cos 618° = -0.208$
$\cos 103° = -0.225$
$\sin 103° = +0.974$
$\cos 206° = -0.899$
$\sin 206° = -0.438$

$$x = 0 = -0.208\,C_1 - 0.978\,C_2 - 85.5\,(-0.225) + 24.0\,(-0.899) = -0.208\,C_1 - 0.978\,C_2 - 2.32$$

and $$\frac{dx}{dt} = 0 = -2260\,C_1\,(-0.978) + 2260\,C_2\,(-0.208) + 32200\,(0.974) - 18100\,(-0.438) = 2210\,C_1 - 470\,C_2 + 39280$$

Solving these equations simultaneously,

$$C_1 = -17.5$$
$$C_2 = 1.3 = \text{negligible}$$

Therefore the new equation for the actual motion of the reactor is

$$x = (-17.5 \cos 2260\,t - 85.5 \cos 377\,t + 24 \cos 754\,t)\,10^{-6}$$

This is plotted as *Case* 2, in Fig. 4. Thus the reactor would leave the floor at about 0.3 cycle of ωt after the short circuit occurs, reach a maximum displacement of 106×10^{-6} feet, and return to the floor at about 0.7 cycle, thus being clear of the floor for about 0.4 cycle.

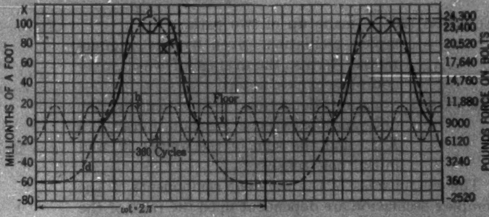

FIG. 4—CASE 2. REACTOR SUPPORTED BY FLOOR

Curve shows actual motion of reactor, assuming same initial tension on bolts as in Case 1.

$x = 10^{-6}\,(-17.5 \cos 2260\,t - 85.5 \cos 377\,t + 24 \cos 754\,t)$

$x = d + h$ for values of $d >$ zero; d = same as in Fig. 2

$4\,K\,I_0^2 = 26{,}640$ lb.

It would then remain on the floor, that is, at $x = 0$, until the instantaneous value of magnetic force again equals $(f_o + W)$, which would obviously occur at about 0.6 cycle after the reactor returned to the floor. There would therefore be a series of "jumps", one each cycle, as shown by the heavy curve x Fig. 4. Actually, of course, these jumps would be of progressively smaller amplitude, decreasing to zero as the short circuit transient dies out.

It is interesting to compare the stress thus created in the bolts with that required, as initial tension, to prevent any motion.

The force on the bolts at $x = 0$ is, by (28) and the data,

$$f_o = 1.5\,K\,I_o^2 - W = 10\,W - W = 9\,W = 9000 \text{ lb.}$$

By equation (3) the total force on the bolts is

$$f_b = -(f_o + k\,x)$$

$$k = 144 \times 10^6$$
$$x = 106 \times 10^{-6} \text{ ft.}$$

Thus

$$f_s = -(9000 + 144 \times 10^6 \times 106 \times 10^{-6}) = -24{,}300 \text{ lb.}$$

That is, the maximum force on the bolts is 24,300 lb.

The peak force is by (7)

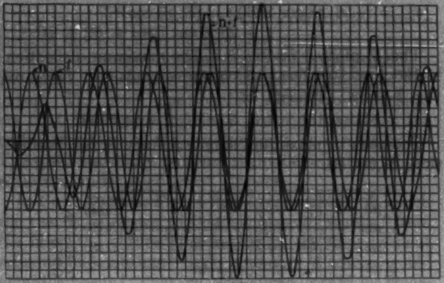

Fig. 5

$$f_m = 4\,K\,I_0^2$$

But, in the particular problem, by assumption

$$K\,I_0^2 = 6.66\,W = 6660 \text{ lb.}$$

Thus

$$f_m = 4 \times 6660 = 26{,}640 \text{ lb.}$$

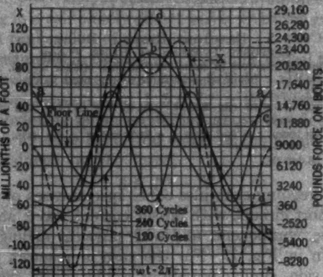

Fig. 6—Case 3. Hypothetical Case. Same Conditions as in Case 1, Except that the Electrical Frequency is Assumed to be Doubled

$x = 10^{-6}\,(56.2 \cos 2260\,t - 93.7 \cos 754\,t + 37.5 \cos 1508\,t)$

a = 1st term, b = 2nd term, c = 3rd term, $d = b + c$, $x = a + b + c$

$4\,K\,I_0^2 = 26{,}640$ lb.

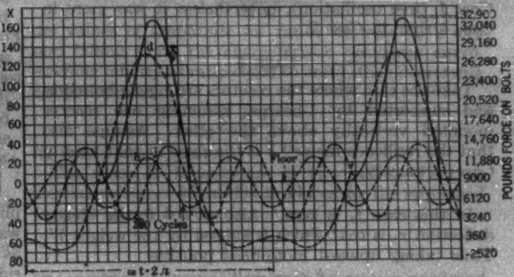

Fig. 7—Case 4. Curve Similar to Fig. 4, Case 2

All conditions being the same except that the electrical frequency is assumed to be doubled.

$x = 10^{-6}\,(-24.3 \cos 2260\,t - 35.9 \sin 2260\,t - 93.7 \cos 754\,t + 37.5 \cos 1508\,t)$

$x = d + h + j$ for values of $d >$ zero. d = same as in Fig. 6

$4\,K\,I_0^2 = 26{,}640$ lb.

Thus the maximum force on the bolts is slightly less than the maximum magnetic force.

Case 3

To study the effect of electrical frequency, assume all other data as in *Case* 1 except the electrical frequency, taking the latter as double, that is,

$$f = 120 \text{ cycles.}$$

Applying equation (29),

$$x = 2\,\frac{6660}{31 \times 754^2}\left[\left\{-\frac{1}{1-(360/120)^2} + \frac{0.0625}{1-(360/240)^2}\right\}\cos 2260\,t + \left\{\frac{\cos\omega t}{1-(360/120)^2} - \frac{0.0625\cos 2\,\omega t}{1-(360/240)^2}\right\}\right]$$

Thus

$$x = (56.2 \cos 2260\,t - 93.7 \cos 754\,t + 37.5 \cos 1508\,t)\,10^{-6}$$

This is plotted in Fig. 6. It will be observed that the amplitudes of the different waves are of about the same magnitude as those in *Case* 1, but the phase relation of the different frequencies gives a different shape of resultant wave x. Thus the maximum value of x is approximately 170×10^{-6} feet in *Case* 1, and -120×10^{-6} in *Case* 3. These correspond respectively to 33,480 lb. and 8300 lb. on the bolts. In *Case* 3, if the positive peak of 106×10^{-6} in Fig. 6 is taken instead of -120×10^{-6}, the force on the bolts would be 24,300 lb. instead of 8300 lb. Thus the result of doubling the electrical frequency and keeping all other factors the same as in *Case* 1, is a decrease in the peak force on the bolts from 33,480 lb. to 24,300 lb.

Case 4

Find the effect of the floor, other conditions being the same as in *Case* 3. Following the same procedure as in *Case* 2, new integration constants C_1 and C_2 are determined from the conditions that, at

$$\omega t = 103°$$
$$x = 0$$
$$\frac{dx}{dt} = 0$$

That is, the reactor begins to leave the floor at the moment when the magnetic force becomes equal to $(f_0 + W)$ which occurs at $\omega t = 103°$.

The forced oscillations, *i. e.* the ωt and $2\,\omega t$ terms, will be the same as in *Case* 3. Thus:

$$x = (C_1 \cos 2260\,t + C_2 \sin 2260\,t - 93.7 \cos 754\,t + 37.5 \cos 1508\,t)\,10^{-6} = 0$$

and
$$\frac{d\,x}{d\,t} = (-\,2260\,C_1 \sin 2260\,t + 2260\,C_2 \cos 2260\,t + 70{,}650 \sin 754\,t - 56{,}600 \sin 1508\,t)\,10^{-6} = 0$$

Therefore

$$C_1 = -\,24.3$$
$$C_2 = -\,35.9$$

and
$$x = (-\,24.3 \cos 2260\,t - 35.9 \sin 2260\,t - 93.7 \cos 754\,t + 37.5 \cos 1508\,t)\,10^{-6}$$

This is plotted in Fig. 7. The maximum deflection is $x = 166 \times 10^{-6}$ feet, which corresponds to a force on the bolts of 32,900 lbs. The peak magnetic force is, as before, 26,640 lb. It is very interesting to compare *Cases* 1 and 2 with *Cases* 3 and 4, the conditions in the latter two cases being the same as in the former two except that the electrical frequency is doubled. In *Case* 2, the effect of interposing the floor was to decrease the maximum deflection below that in *Case* 1, where the reactor was free to oscillate. In *Case* 4, however, the effect of the floor is to *increase* the deflection above that in *Case* 3, where the reactor was free to oscillate. Moreover, although the maximum deflection in *Case* 3 (free oscillation) was less than that in *Case* 1 (free oscillation), nevertheless it was more in *Case* 4 than in *Case* 2.

CASE 5

Take the case where the reactors are *rigidly* bolted to and supported by I-beams, assuming one reactor mounted vertically above the other. Assume also that the resilience factor[11] k is the same as that used in *Case* 1, namely,

$$k = 144 \times 10^6 \text{ lb./ft.}$$

In this case f_0 is zero. Also at $t = 0$, the steady deflection x_1 is that which is caused by the weight W. Thus taking upward motion as positive

$$k\,x_1 = -\,W$$

and

$$x_1 = -\,W/k$$

Thus equation (**26**), modified for these boundary conditions, becomes

$$x = \left[\,2\,\frac{K\,I_0^2}{M\,\omega^2}\left\{-\,\frac{1}{1-(n/f)^2} + \frac{0.0625}{1-\left(\dfrac{n}{2f}\right)^2}\right\} - 1.5\,\frac{K\,I_0^2}{k}\,\right]\cos\alpha\,t$$
$$+\,2\,\frac{K\,I_0^2}{M\,\omega^2}\left\{\frac{\cos\omega\,t}{1-(n/f)^2} - \frac{0.0625\cos 2\,\omega\,t}{1-\left(\dfrac{n}{2f}\right)^2}\right\} + \frac{1.5\,K\,I_0^2 - W}{k} \qquad \textbf{(30)}$$

Substituting,

$K\,I_0^2 = 6660 \quad M = 31.0 \quad \omega = 377 \quad f = 60$

$n = 360 \quad \alpha = 2260 \quad W = 1000 \quad k = 144 \times 10^6$

$$x = (-\,6.48 \cos 2260\,t - 86.45 \cos 377\,t + 23.6 \cos 754\,t + 62.4)\,10^{-6}$$

This is plotted in Fig. 8. The maximum deflection is 166.5×10^{-6} feet, which corresponds to a force on the I-beam supports of

$$144 \times 10^6 \times 166.5 \times 10^{-6} = 24{,}000 \text{ lb.}$$

The peak or maximum magnetic force is, by assumption, 26,640 lb.

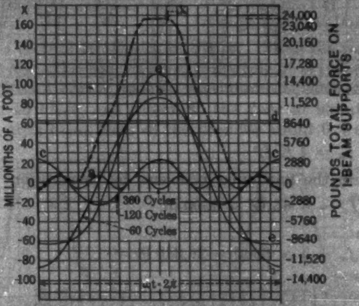

FIG. 8—CASE 5. REACTORS RIGIDLY BOLTED TO I-BEAM SUPPORTS INSTEAD OF RIGID FLOOR

Center lines of reactors in same vertical line, similar to Fig. 1. Resilience constant of *I*-Beam support same as for bolts in Case 1 (*i. e.* $k = 144 \times 10^6$)

$x = 10^{-6}\,(-\,6.48 \cos 2260\,t - 86.45 \cos 377\,t + 23.6 \cos 754\,t + 62.4)$

a = 1st term, b = 2nd term, c = 3rd term, d = 4th term, $e = b + c$

$x = a + b + c + d$

$4\,K\,I_0^2 = 26.640$ lb.

The foregoing examples cover special cases of single-phase short circuit, with off-set current wave, and sine wave a-c. component, and with physical arrangement as shown in Fig. 1. For the case of reactors placed side by side, similar equations would apply, the main difference being that moments, instead of forces, would be involved provided the reactors were held by holding bolts in the floor, and no horizontal braces between them.

The case of three-phase remains. In this, as in the foregoing cases, the various factors enter in the same way, the difference being only in the character of the magnetic force. But whatever may be its character, it may be expressed as a function of time, and treated as in the case of single-phase. Thus the magnetic force on one of three reactors would be

$$f_{e1} = K_{12}\,i_1\,i_2 + K_{13}\,i_1\,i_3 \qquad \textbf{(31)}$$

where K_{12} and K_{13} are constants.

---

[11] Ratio of force exerted on the I-beam support to deflection it produces.

The instantaneous values of short-circuit currents i_1, i_2 and i_3 in the three phases can all be expressed as functions of time, and these expressions substituted in the differential equation (**4**). Thus the problem involves longer equations with more terms, but they are of exactly the same character, and solved in the same way.

DISCUSSION OF RESULTS

The value of the results from a mathematical treatment of this sort depends upon how sound and comprehensive the assumptions are on which the equations are based. These are given at the beginning of the paper. The most questionable assumptions perhaps are that the supporting floors are rigid and that the entire structure of the reactor unit, is a rigid body. There will naturally be some slight deformation of the structure under such great stresses.

In *Cases* 2 and 4, Figs. 4 and 7 respectively, in which the reactor is shown to take successive "jumps" from the floor, the curves show return to the floor without rebound or bouncing. Such bouncing of course would actually occur. That is, the curve would be more accurate if it showed a decaying series of initially small rebounds, following the return to the floor, as now shown on the curve.

Damping is neglected. This assumption makes negligible difference excepting cases of near resonance as discussed later.

The peak magnetic force depends upon the point of the voltage wave at which the short circuit occurs. But the assumption that it occurs at the zero voltage point, resulting in a completely off-set wave, gives the greatest possible peak value of magnetic force.

The examples in *Cases* 2 and 4 assume initial tension on the holding bolts equal to $(1.5\,K\,I_e^2 - W)$. With zero initial tension, and $1.5\,K\,I_e^2 > W$, then motion would obviously be an oscillation above the floor, and about an average deflection equal to the strain of the bolts produced by the force,

$$1.5\,K\,I_e^2 - W$$

It is very important to observe that in the equation for deflection, for instance equation (**26**), the motion x of the reactor is a function of the ratio n/f. Consider what this means. The factor

$$2\,\frac{K\,I_e^2}{M\,\omega^2}$$

in equation (**26**) is the amplitude of the oscillation which would be produced in the mass M by the alternating force,[12]

$$2\,K\,I_e^2 \cos \omega\, t$$

provided the mass were disconnected from resilient supports—*i. e.* if, say, it were suspended as a pendulum, and this alternating force were applied in horizontal line, then

$$2\,\frac{K\,I_e^2}{M\,\omega^2}$$

would be the amplitude of oscillation in the horizontal line.

Now if one end of a resilient, or spring-like, horizontal support is attached to the mass, and the other end fixed, and the same alternating force applied, the amplitude of oscillation will be changed by the factor

$$\frac{1}{1-(n/f)^2}$$

It would thus be

$$2\,\frac{K\,I_e^2}{M\,\omega^2}\,\frac{\cos \omega\, t}{1-(n/f)^2}$$

This will be recognized as the term of fundamental frequency in equation (**26**).

Similarly, the second harmonic term in equation (**26**) is

$$0.5\,\frac{K\,I_e^2}{M\,(2\,\omega)^2}\,\frac{\cos 2\,\omega\, t}{1-\left(\frac{n}{2f}\right)^2}$$

the magnitude of the 2nd harmonic force[13] being one-quarter of that of the fundamental.

For illustration consider the fundamental term. If

$$n/f > \sqrt{2}$$

then obviously the amplitude of oscillation will be less than

$$2\,\frac{K\,I_e^2}{M\,\omega^2}$$

if, however,

$$\frac{n}{f} < \sqrt{2}$$

the amplitude will be greater[14] than

$$2\,\frac{K\,I_e^2}{M\,\omega^2}$$

the values of amplitude for ratios between $\sqrt{2}$ and zero falling along the characteristic resonance curve, reaching infinity at ratio equal to unity, then decreasing again, approaching

$$2\,\frac{K\,I_e^2}{M\,\omega^2}$$

for low values of ratio.

The mass M thus enters in two ways. First as inertial resistance to motion produced by the impressed force as shown by the factor

$$2\,\frac{K\,I_e^2}{M\,\omega^2}$$

and, second, as inertial resistance to motion produced by the force of the strained resilient support. Its effect here is accounted for in the factor n; since by equation (**25**)

$$n = \frac{1}{2\,\pi}\,\sqrt{k/M}$$

Thus the deflection is dependent upon the ratio n/f as well as the mass M, and it does not necessarily follow that, with the same impressed force, increasing the mass will decrease the amplitude, because the change in the frequency ratio might be such as to increase the amplitude.

---

[12] This is the fundamental term in equation (**7**).

[13] As given by equation (**7**).

[14] If $n = 2f$, this would, of course, be resonance for the $2\,\omega\, t$ term. These particular numerical values for the fundamental term alone are given merely for illustration of how the frequency ratio affects an oscillation.

To illustrate: if

$$n/f < 1.0$$

increasing the mass will decrease the amplitude,[15] whereas if, for instance,

$$n/f = 2.0$$

then doubling the mass would obviously decrease the factor

$$2\,\frac{K\,I_0^2}{M\,\omega^2}$$

by 50 per cent, but it would increase the factor

$$\frac{1}{1-(n/f)^2}$$

from 0.333 to 1.0. Thus the net result would be an *increase* from 1.0 to 1.5, that is, a 50 per cent increase.

However, the full significance of the dependence of the amplitude upon the frequency ratio is not clear without consideration of the effect of the transients. There are two of them, namely, the transient of the short-circuit current, with consequent decrease in the magnetic force; and the transient of the free oscillation due to friction or damping. Both are neglected in the equations. The justification for this is two-fold: the natural frequency n as explained later, is usually enough greater than f to give the peak deflection in the first cycle of the electrical frequency, as in the *Cases* 1 to 5 inclusive, in which, neither transient, if they had been included in the equation, would have changed the oscillations significantly. Also, even if significant transients do exist, the calculated result is on the safe side, giving calculations slightly too high.

As resonance is approached, however, the transients are very important. For instance, by equation **(26)**[16] it is evident that if $n = 0.9\,f$, the amplitude of the $\omega\,t$ term will be approximately

$$10\,\frac{K\,I_0^2}{M\,\omega^2}$$

while that of the $2\,\omega\,t$ term will be only

$$0.156\,\frac{K\,I_0^2}{M\,\omega^2}$$

thus, practically negligible. But while the $\omega\,t$ term is large, so is the $\alpha\,t$ term equally large and opposite. Hence, as shown in Fig. 5, the two waves, starting out in phase opposition, give practically zero resultant at first, which gradually builds up by successive phase shift due to slightly different frequencies, until ultimately, at 4.5 cycles of n and 5 cycles of f, the waves add in phase, giving a maximum equal to twice the amplitude of either wave. Then, for the same reason, it decreases again.

All this, provided there are no transients. But of course, there are transients. If the electrical damping were zero, then the free oscillation alone would die out due to mechanical damping, leaving only the forced oscillation, *i. e.* the $\omega\,t$ and $2\,\omega\,t$ terms. Thus, as the former dies out, it leaves the latter, which it initially opposed and canceled. Hence if the transient of the free oscillation were rapid enough, the resultant oscillation would reach a maximum earlier than if that transient did not exist, but the maximum would be only half as great—namely, the forced oscillation alone. While there are very few data on the mechanical damping, it is probable that both transients—that is, the mechanical transient and the transient of the $\omega\,t$ term—are of about equal duration, say one-quarter second. In such a case, the maximum deflection would be the result of a race between the increase in magnitude due to phase shift of waves of different frequencies, on the one hand, and to a decrease due to transients, on the other. It is clear, therefore, that at near resonance the transient cannot be neglected.

To illustrate in a very rough quantitative way the effect of transients at near resonance, take *Case* 5, *I*-beam supports, assuming,

$n = 0.9\,f$
$f = 60$ cycles
neglect $2\,\omega\,t$ term
neglect W
duration of transient of $\omega\,t$ term same as of mechanical transient $= 1/4$ second.
transients of form $\epsilon^{-\gamma t}$

By assumption[17] $\gamma \cong 20$

Applying equation (30)

$$x \cong \epsilon^{-20t}\left[-\left\{\frac{10\,K\,I_0^2}{M\,\omega^2} + 1.5\,\frac{K\,I_0^2}{k}\right\}\cos\alpha\,t + \frac{10\,K\,I_0^2}{M\,\omega^2}\cos\omega\,t + 1.5\,\frac{K\,I_0^2}{k}\right]$$

Since $n = 0.9\,f$

$\alpha = 0.9\,\omega = \sqrt{k/M}$

Hence $M\,\omega^2 \cong 1.25\,k$

and $$x \cong \epsilon^{-20t}\left[-\left\{\frac{8\,K\,I_0^2}{k} + \frac{1.5\,K\,I^2_0}{k}\right\}\cos\alpha\,t + \frac{8\,K\,I_0^2}{k}\cos\omega\,t + \frac{1.5\,K\,I_0^2}{k}\right]$$

Or $$x \cong \epsilon^{-20t}\,\frac{K\,I_0^2}{k}\left[-9.5\cos\alpha\,t + 8\cos\omega\,t + 1.5\right]$$

This is plotted in Fig. 9. The maximum value of x occurs at about two cycles, and is

$$x_m = 6.5\,\frac{K\,I_0^2}{k}$$

---

[15] Because an increase in M decreases both of the factors $2\,\dfrac{K\,I_0^2}{M\,\omega^2}$ and $\dfrac{1}{1-(n/f)^2}$, the latter being decreased since $n = \dfrac{1}{2\,\pi}\sqrt{K/M}$.

[16] Remembering that the amplitude of the free oscillation is just equal and opposite to the amplitude of the forced oscillation at t = zero. And hence if the frequencies are about equal, it will take considerable time for the two waves to gradually shift out of phase, thus building up the resultant wave. At resonance it would theoretically take an infinite time.

[17] The transient term is assumed to apply to all terms in the equation. Strictly it applies to the $\alpha\,t$ and $\omega\,t$ terms, and to only a component of the constant term. But since the latter is small, the assumption, for purpose of illustration, is justified

This corresponds, by equation (3), to a force on the support

$$f_s = k\,x_m = 6.5\,K\,I_e^2$$

That is, about 60 per cent greater than

$$4\,K\,I_e^2$$

which, by equation (7), is the maximum instantaneous magnetic force neglecting the transient.

If n is large compared to f the shape of the resultant deflection wave is that produced by the superposition of a high-frequency wave on one of lower frequency as shown in Fig. 3. Thus, the high-frequency wave a superposed on the lower frequency wave d produces the resultant curve x. If the ratio of the high-frequency n to the low-frequency f is an integer then the resultant wave repeats every cycle of the frequency f and therefore the maximum will occur in the first cycle of $\omega\,t$. If the ratio is not an integer the resultant will not repeat and will, without transients, require a few cycles of f for the relative phase of the high-frequency wave to shift to the right position to give the maximum possible resultant. However, transients probably reduce the magnitude more rapidly than the phase shift builds it up. It thus appears reasonable to assume in this case also that the maximum occurs in the first cycle of $\omega\,t$.

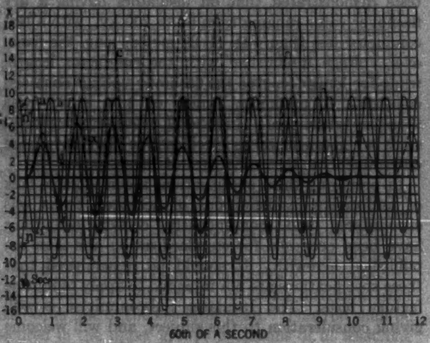

Fig. 9

$x = \varepsilon^{-20t}\,K\,I_0^2/k\,[-9.5\cos\alpha\,t + 8\cos\omega\,t + 1.5]$

a = 1st term, b = 2nd term + 3rd term, $c = a + b$, d = 3rd term

From the foregoing discussion of results, it appears that in cases where n is large compared to f the results are accurate enough to certainly indicate the general character of the phenomenon and magnitudes involved, and probably also close enough for guide in design calculations. In such cases, neglecting the transients would not involve serious error. However, it is clear from the numerical illustration just given, that where n approaches f, the transients can not be neglected. That is, the equations do not apply in such cases. But usually in problems of actual practise n is large compared with f.

Conclusions

(1) If the reactor is not held rigidly against all motion, the maximum force on the holding device (bolts, etc.) will depend in a decisive way upon the ratio of the frequency n of natural oscillation, to the electrical frequency f.

(2) If, however, the reactor is held rigidly, then the maximum force on the holding device will be independent of the frequency ratios, and need be only slightly in excess of the peak magnetic force. Or, conversely, if the initial tension in the holding device (plus weight of reactor) is equal to, or slightly greater than, the peak magnetic force, then no motion of the reactor can take place. It appears that this condition should always be sought.

(3) If the reactor is not held rigidly and n is large compared to f, say

$$n/f > 3$$

as is usually the case, then if motion is permitted, the maximum force on the holding device will occur in the first cycle of $\omega\,t$, and will usually be as great or greater than the maximum magnetic force.

(4) If $n/f < 3$ and the reactor is not held rigidly, the maximum deflection may or may not occur in the first cycle of $\omega\,t$. If it does, then the above conclusion applies. If it does not, then the transient cannot be neglected, and the equations do not apply. It is probable however, that in the latter, as in the former case, the maximum force on the holding device will be greater, than the maximum instantaneous magnetic force.

(5) The equations, which are based on definite assumptions given at the beginning of the paper, and discussed in foregoing pages, give results which indicate the general character of the phenomenon and magnitudes involved; and within limits specified,[18] could probably be used as a guide in design calculations.

The authors wish to acknowledge the valuable suggestions of Mr. W. O. Dwyer and the assistance of Messrs. D. S. Snell and R. F. Franklin.

Bibliography

"Uber die Mechanischen Wirkungen des Plötzlichen Kurzschluss-stromes von Synchronmaschen" by J. Biermanns, *Archiv für Electrotechnik*, IX, 1920, pp. 326-340.

"Repulsion and Mutual Inductance of Reactors" by H. B. Dwight, *Electrical World*, June 16, 1917, V. 69, pp. 1148-1150.

"Design of Flywheels for Reciprocating Machinery Connected to Synchronous Generators or Motors," by R. E. Doherty and R. F. Franklin, A. S. M. E. *Transactions*, 1920, V. 42, pp. 523-567.

"Mechanical Effects of Electrical Short Circuits" by S. H. Weaver, *G. E. Review*, November 1915, V. 18, pp. 1066-1074.

"Mechanical Stresses between Electrical Conductors" by K. C. Randall, *The Electric Journal*, July 1917, V. 14, pp. 283-287.

"Repulsion between Bus Bars" by S. G. Leonard and C. R Riker, *The Electric Journal*, December 1917, V. 14, 491-493.

"Some New Formulas for Reactance Coils," by H. B. Dwight, Trans. A. I. E. E., 1919 p. 1681.

"Repulsion and Mutual Inductance of Reactance Coils with the Same Axis," by H. B. Dwight, *The Electric Journal*, May 1918, p. 166.

Discussion

For discussion of this paper see page 566.

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[18] Cases where $n > 3f$.

Standardized Insulator Tests

BY THE INSULATOR SUBCOMMITTEE OF THE A. I. E. E. STANDARDS COMMITTEE

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Standardized Insulator Tests. (600 w.)
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Wet Flashover Test (Design Test). (200 w.)
Puncture Test (Design Test). (125 w.)
Corona Formation Voltage (Design Test). (60 w.)
Ultimate Mechanical Strength Test (Design Test). (140 w.)
Suspension Insulator, Proposed Test Specifications. Definitions. (30 w.)
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Puncture Test. (60 w.)
Report of 1922-23 Subcommittee (1863 w.)

AT the Pacific Coast Convention of the American Institute of Electrical Engineers, held in Portland, Oregon, July 1920, the following resolutions were passed:

> WHEREAS, there exists at present a decided lack of uniformity in methods of testing and ratings adopted by the manufacturers of pin type insulators, affecting recommended line voltages and rated dry and wet flashover values; and
>
> WHEREAS, the United States Bureau of Standards in the National Electric Code for overhead line construction has specified minimum values of flashover voltages on insulators used for transmission and distribution lines, but has not indicated the method of determining these values; therefore
>
> BE IT RESOLVED: That the members of the A. I. E. E. in session of the Pacific Coast Convention recommend that steps be taken in the near future by the Standardization Committee of the Institute to properly standardize methods of testing and rating pin type insulators, in order to obtain uniform values of flashover voltages for similar types.
>
> BE IT FURTHER RESOLVED: That a copy of this resolution be sent to the United States Bureau of Standards, Washington, D. C.

As a result of this action, the Standards Committee of the American Institute of Electrical Engineers in October 1920, appointed a subcommittee* to take up the work as outlined and advised it as follows:

> It will be observed that the resolution of the Pacific Coast Convention refers more particularly to pin type insulators, but it is desirable for this subcommittee to give attention to the question of all types of insulators for transmission lines.

As a result of the work of this subcommittee certain specifications covering the usual design tests have been drawn up and recommended to the Standards Committee. It is considered desirable, however, to obtain as much criticism as may be possible from all insulator users, before any final standardization is authorized, and it is with the desire of stimulating such criticism that the proposed specifications are presented herewith.

In undertaking to establish specified methods of conducting these tests, it was necessary to carry out investigations into the effect of test conditions. Such investigations were made at one of the insulator factories. It was found for instance that in the case of the test conductor for pin type insulators, laboratory tests demonstrated that on insulators having a dry flashover value ranging from 60 to 200 kv., with a test conductor less than 4 ft. (1.22 meter) long, the flashover voltage diminished with the conductor length, while for longer lengths the flashover voltage remained constant. A minimum length of 4 ft. (1.22) was therefore established. In general, where similar dimensions are given in these specifications they have been determined by the same sort of investigation.

In preparing these specifications a considerable lack of uniformity in definitions of accepted terms was encountered and, consequently, certain definitions have been proposed for standardization and these are published herewith. It should be remembered that these specifications are not intended to apply to routine factory tests. The Insulator Subcommittee is still giving consideration to this subject, and will welcome criticisms and suggestions leading to improvement in the form or subject matter of the specifications.

Pin Insulator Design Test Specifications as Tentatively Proposed

DEFINITIONS

The following terms are in common use and it is believed merit an exact definition or specification. The subcommittee therefore suggests the following:

A pin insulator is a complete insulator, consisting of one insulating member or an assembly of such members without tie wires, clamps, thimbles or other accessories, the whole being of such construction that when mounted on an insulator pin, it will afford insulation and mechanical support to a conductor.

A shell is a single insulating member without cement or other connecting devices.

Dry flashover voltage is the voltage at which the air surrounding a clean dry insulator or shell breaks down between electrodes, with the formation of a sustained

---

*This subcommittee now includes the following members:

R. E. Argersinger	Electrical Engr., Stone & Webster, Inc.
A. O. Austin	The Ohio Insulator Co.
A. F. Bang	Testing Engr., Pennsylvania Water & Pr. Co.
H. A. Barre	Executive Engr., Southern Calif. Ed. Co.
S. L. Case	Westinghouse Elec. & Mfg. Co.
G. I. Gilchrest	Westinghouse Elec. & Mfg. Co.
K. A. Hawley	Chief Engr., Locke Insulator Mfg. Co.
R. P. Jackson	Westinghouse Elec. & Mfg. Co.
F. Junkersfeld (Chairman)	McClellan & Junkersfeld, Inc.
A. H. Lawton	Consumers Power Co.
J. C. Martin	McGraw-Hill Co., Inc.
R. H. Marvin	The R. Thomas & Sons Co.
A. H. Moore	General Electric Co.
W. D. A. Peaslee	Chief Engr., Belden Mfg. Co.
F. W. Peek, Jr.	General Electric Co.
H. J. Ryan	Leland Stanford, Jr., University.
A. E. Silver	Electric Bond & Share Co.

Presented at the Spring Convention of the A. I. E. E., Pittsburgh, Pa., April 24-26, 1923.

arc, the test being made as described under "Dry Flashover Test."

Wet flashover voltage is the voltage at which the air surrounding a clean wet insulator or shell breaks down between electrodes with the formation of a sustained arc, the test being made as described under "Wet Flashover Test."

Puncture voltage is the voltage at which an insulator or shell is electrically punctured when subjected to a gradually increasing voltage, the test being made as described under "Puncture Voltage Test."

The ultimate mechanical strength of a pin insulator is the load in pounds at which the insulator fails either electrically or mechanically, voltage and mechanical stress being applied simultaneously as described under "Ultimate Mechanical Strength Test."

DRY FLASHOVER TEST (DESIGN TEST)

Dry flashover test shall be performed with the pin insulator mounted in a vertical position on a steel pin of circular section 1 in. (2.54 cm.) in diameter mounted on a cross-arm, and of such length that the ratio of the shortest distance from the edge of the head around the insulator to the cross-arm, to the shortest distance from the edge of the head around the insulator to the pin, shall be 1.25. The cross-arm shall be of iron pipe, preferably grounded, not less than 3 in. (7.63 cm.) and not more than 5 in. (12.7 cm.) in diameter and shall extend at least 3 ft. (0.914 m.) on either side of the center line of the insulator pin. The head of the insulator shall be fitted with a straight, smooth metallic rod or tube not less than ½ in. (1.27 cm.) in diameter extending in a direction at right angles to the cross-arm and at least 2 ft. (0.609 m.) in either direction from the center line of the insulator head. This rod shall be secured in the upper groove by means of at least one turn of wire not smaller than No. 8 A. w. g. placed in the side tie wire groove.

The character of the testing equipment and method of measuring voltage shall conform to the Standardization Rules of the A. I. E. E.

The test shall be performed by applying voltage between the rod fastened to the head and the steel pin, and raising it at a uniform rate of approximately five thousand (5000) volts per second to a value at which dry flashover occurs.

Records shall be made of barometric pressure, air temperature and humidity.

WET FLASHOVER TEST (DESIGN TEST)

The testing arrangement shall be the same as in the dry flashover test with the addition of equipment to provide a finely divided uniform spray at an angle of 45 deg. from the vertical and at a rate of 0.2 in. (5.07 mm.) per minute. The water shall have a resistance of from 3000 to 6000 ohms per in. cube (7620 to 15,250 ohms per cm. cube) and shall be delivered to the spray nozzle at a pressure of not less than 35 and not more than 50 pounds per sq. in. (2.46 to 3.51 kg. per sq. cm.) measured at the nozzle. The vertical and horizontal dimensions of the vertical area sprayed shall be measured in a plane through the vertical axis of the insulator and shall be 1.75 times the corresponding over-all projected dimensions of the insulator. The precipitation shall be determined by measurements taken, with the insulator removed, at the location of the top, center and bottom of the vertical axis of the insulator when in its test position. Individual measurements shall show a variation of not more than 25 per cent from the mean of the three measurements.

Methods of applying and measuring voltage and of recording data, shall be the same as for the "Dry Flashover Test."

PUNCTURE TEST (DESIGN TEST)

In making this test the insulator shall be immersed in oil with the pin hole, line and tie wire grooves filled with conducting material, voltage being applied between the pin hole and tie wire.

Voltage shall be applied at not more than three-fourths of the dry flashover voltage and raised at a uniform rate of approximately 5000 volts per second for insulators having an average puncture of 200,000 volts or less, and at a uniform rate of 10,000 volts per second for insulators having an average puncture voltage of more than 200,000 volts, voltage being raised until puncture occurs.

Methods of applying and measuring voltage and of recording data shall be the same as in the "Dry Flashover Test."

CORONA FORMATION VOLTAGE (DESIGN TEST)

The testing arrangement shall be the same as for the dry flashover test, using a darkened room. A voltage sufficient to cause streamers shall be applied and slowly lowered until all brush discharges disappear. The point of disappearance shall be the corona voltage.

Methods applying and measuring voltage and of recording data shall be the same as in the "Dry Flashover Test."

ULTIMATE MECHANICAL STRENGTH TEST (DESIGN TEST)

This test shall be performed in a suitable insulated testing machine where mechanical loads up to the ultimate rupture point can be applied simultaneously with a potential 15 per cent below the dry flashover voltage at a frequency not greater than 60 cycles. During the test the mechanical load shall be increased at the rate of 2000 lb. (907.2 kg.) per minute until puncture takes place. For pin type insulators, the mechanical load shall be applied at right angles to the central axis, with a suitable metal supporting pin which will not materially deflect or bend under the maximum stress imposed. Attachment shall be made to the tie wire groove by means of a steel cable loop or bridle.

Methods of applying and measuring voltage and of recording data shall be the same as in the "Dry Flashover Test."

Suspension Insulator Proposed Test Specification

DEFINITIONS

The following definitions are proposed:

A suspension insulator unit is a shell assembled with its necessary attaching members.

A string consists of two or more units connected in series.

DRY FLASHOVER TEST

Dry flashover test shall be performed with the insulator, unit or string, suspended vertically by the makers' standard cross-arm suspension hardware carried at the end of a grounded wire or suitable conductor suspended so that the vertical distance from the uppermost point of the insulator hardware to the supporting structure shall not be less than 3 ft. No other grounded structure shall be nearer than 3 ft. to any part of the unit or string. The insulator pin, or corresponding fitting, shall carry an inverted pipe tee made of ¾-in. pipe, the head of the tee being not less than 6 ft. long, the stem of the tee being coupled at the middle point of the head and having such a length that the distance from the upper surface of the horizontal head to the lowest edge of the porcelain shall not exceed 0.7 of the diameter of the lowest unit.

Potential shall be applied between the stem of the pipe tee and the grounded suspension by raising the voltage at the rate of approximately 5000 volts per second to a value at which dry flashover occurs. Records shall be made of barometric pressure, air temperature and humidity.

The character of the testing equipment and method of measuring voltage shall conform to the Standards of the A. I. E. E.

WET FLASHOVER TEST

The wet flashover test shall be performed using the same general arrangement as in the dry flashover test with the addition of equipment to provide a finely divided and reasonably uniform spray at an angle of 45 deg. from the vertical and at the rate of 2/10 in. (5.07 mm.) per minute. The water shall have a resistance of from 3000 to 6000 ohms per inch cube (7620 to 15,250 ohms per cm. cube) and shall be delivered to the spray nozzle at a pressure of not less than 35 and not more than 50 lb. per square inch (2.46 to 3.51 kg. per sq. cm.) measured at the nozzle. The vertical and horizontal dimensions of the vertical area sprayed shall be measured in a plane through the vertical axis of the unit or string and shall be 1.75 times the corresponding over-all projected dimensions of the unit or string. Precipitation shall be determined by measurements, taken with the unit or string removed, at the location of the top, center and bottom of the vertical axis of the unit or string when in its test position. Individual measurements shall show a variation of not more than 25 per cent from the mean of the three measurements.

Methods of applying and measuring voltage and of recording data shall be the same as for the dry flashover test.

PUNCTURE TEST

Puncture test shall be performed with the insulator or unit immersed in oil, voltage being applied between the cap and stud or corresponding metal fittings. Methods of applying voltage and recording data shall be the same as in the dry flashover test except that voltage application shall be begun at not more than ¾ of the dry flashover voltage.

Report of Insulator Subcommittee of the Standard Committee, A. I. E. E. for 1922-1923

The Insulator Subcommittee, Working Committee No. 3, has held four meetings, as follows:

December 8, 1922	March 9, 1923
February 2, 1923	April 6, 1923

all at Institute Headquarters, New York, N. Y.

The subcommittee has continued the study, begun last year, of the factor of safety to be recommended between line voltage and dry flashover of the insulator.

It has tabulated the data received from 27 operating companies in reply to questionnaire sent out.

The subcommittee recommends that for the present this data be used in selecting pin insulators.

The subcommittee suggests that with steel supports or grounded pins it is desirable to use the higher factors of safety while with ungrounded pins on wood poles the lower factors of safety may be justified. At present the subcommittee is not prepared to recommend what this variation should be. A discussion of the entire situation is invited.

For convenience the following table gives flashover voltages corresponding to various line voltages, as taken from this data:

Line Kv.	Flashover Kv.		
	Minimum	Average	Maximum
11	68	88	110
13.2	..	91	..
22	90	110	121
33	100	124	142
44	163	172	180
60	186	192	222
66	179	185	198

The discrepancy between the tests for 60 kv. and 66 kv. is probably due to lack of sufficient data.

In this connection, the subcommittee wishes to submit as representing practise with suspension insulators the curve published by Mr. Peek in the *General Electric Review* of February, 1922. Attention is called to the difference in factor of safety between pin and suspension insulators, particularly at operating voltages in the neighborhood of 60 kv.

ROUTINE TEST SPECIFICATIONS

The Subcommittee has given consideration to specifications covering routine factory tests on both pin and suspension units and suggests the following forms.

The Subcommittee does not take the position that these are necessarily the only tests which should be considered or, on the other hand, that all of the tests outlined must be made, but leaves this to the judgment of the Purchaser.

The Subcommittee believes, however, that if the tests are called for, they should be made as specified.

TESTS ON PIN TYPE INSULATORS

Preliminary Test. Before assembly, all shells shall be subjected to vigorous dry flashover potential at normal frequency 25 to 60 cycles for 3 minutes. If more than 5 per cent fail, the lot[1] may be retested. If on retest more than 3 per cent fail, the lot[1] shall be rejected.

Final Test. After assembly, the insulators shall be subjected to dry flashover test for two minutes. Voltages shall be such that insulators shall flashover occasionally. Insulators failing under this test shall be rejected.

TESTS ON CEMENTED TYPE OF SUSPENSION INSULATORS

Preliminary Tests. Before assembly, all shells shall be subjected to vigorous dry flashover potential at normal frequency 25 to 60 cycles for 5 minutes.

If any shell fails during the fourth or fifth minute of the test, the test shall be continued until no shell fails during the last two minutes of test. The excess time is based on the testing of quantities up to 100 at one time. For quantities greater than 100, the excess time after the last failure may be less than two minutes by agreement between manufacturer and Purchaser. If more than 5 per cent fails the lot[2] may be retested. If on retesting more than 3 per cent fails the lot[2] shall be rejected.

After assembly all units shall be subjected to flashover test at normal frequency 25 to 60 cycles for three minutes. Voltages shall be such that insulators shall flashover occasionally. All units failing under this test shall be rejected.

Mechanical Test. Not more than seven days after cementing all units shall withstand for three seconds without sign of distress a mechanical pull in line with the axis of the insulator amounting to approximately 40 per cent of the rated ultimate strength. This test shall be given before the final electrical test.

Puncture Test (For Pin and Suspension Units). Puncture voltage test shall be made on units which have passed the final routine flashover test. Rejection of units for puncture shall be based on the "Average Variation" in puncture voltage determined, as follows:

DEFINITION OF "AVERAGE VARIATION" IN PUNCTURE UNDER OIL TESTS

For this test purchaser will select from units offered for final inspection not more than ½ of 1 per cent of the total quantity and not less than 3 units.

$V_1, V_2, V_3, \ldots V_n$ = individual puncture values.

V = average puncture voltage.

$V = (V_1 + V_2 + V_3 \ldots + V_n)/n$

Let

$a_1 = V - V_1 \quad a_2 = V - V_2 \quad a_3 = V - V_3 \quad a_n = V - V_n$

Consider all the values of a as positive, that is, neglect the signs.

Let a = average variation

A = average variation, per cent.

Then

$a = (a_1 + a_2 + a_3 \ldots + a_n)/n$

$A = 100\, a/V$

Example:

Five insulators punctured at; 150, 135, 145, 138, 142 kv. respectively.

$V = (150 + 135 + 145 + 138 + 142)/5 = 142$

$a = (8 + 7 + 3 + 4 + 0)/5 = 4.5$

$A = 100 \times 4.5/142 = 3.17$ per cent

If "Average Variation," A, exceeds 10 per cent, the entire quantity shall be rejected, or at the manufacturer's option and expense an additional 2 per cent may be tested. If the "Average Variation," A, obtained from this second test only exceeds 10 per cent, the entire quantity shall be rejected.

Ultimate Mechanical Strength (Suspension units only). Not more than 7 days after cementing, ten units shall be selected for this test from each 1,000 ordered. If any fail at less than 85 per cent of the manufacturer's rated ultimate strength, an additional 20 units shall be tested at the manufacturer's expense. If none fail at 85 per cent of manufacturer's rating, the lot shall be accepted and if any fail, rejected.

Expense of Material for Tests. As far as practicable, tests shall be made upon units which have some defect and are not salable but are otherwise acceptable to the inspector for the test in question. In addition, Purchasers shall be allowed to test commercial insulators to destruction up to ½ of 1 per cent of the total number ordered. If additional units are desired for such tests, they shall be paid for by the purchaser.

We believe that the above completes the work assigned to the subcommittee. In the course of its discussions, two questions came up which it appear should be subjected to further laboratory study and the subcommittee suggests these for your consideration.

1st. The relative effect of 25- and 60-cycle tests.

2nd. It is believed that the mechanical strength of a string of insulators decreases with time. The maount of this should be ascertained.

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[1] Note: In all cases where the term "lot" is used, it should be defined as including only the number of units on the testing pan at one time.

[2] Note: In all cases where the term "lot" is used, it should be defined as including only the number of units on the testing pan at one time.

The subcommittee believes that the material included in the above report should be treated in the same way as the design test specifications previously reported on, that is, it should be published and subjected to criticism, so that it may be revised before any definite action is taken. It would be highly desirable, if possible, to have this material presented at the April meeting of the Institute in Pittsburgh, at which time the design test specifications are scheduled for discussion.

The subcommittee would like the permission of the Standards Committee to arrange for this, if possible.

Respectfully submitted,
P. Junkersfeld
Chairman, Insulator Subcommittee
of the Standards Committee, A. I. E. E.

Discussion

LIGHTNING DISTURBANCES ON DISTRIBUTION CIRCUITS (MacLaren); OPERATING EXPERIENCE WITH CURRENT-LIMITING REACTORS (Pollard); SHORT-CIRCUIT FORCES ON REACTOR SUPPORTS (Doherty and Kierstead); STANDARDIZED INSULATOR TESTS (Insulator Subcommittee of Standards Committee);

Pittsburgh, Pa., April 26, 1923.

E. C. Stone: As I understand this lightning situation, the lightning arrester must have large charging capacities. The lightning arrester must choke the dynamic current that may follow from the system. Those two conditions are almost diametrically opposite. However, in very low voltages, by which I mean under four or five thousand, the dynamic problem is comparatively simple, and you can hold down the dynamic current following with a low resistance which is not high enough to seriously interfere with your discharge but as you go to the high voltages, the problem of holding back the dynamic current without checking the surge discharge becomes more and more complicated and involves more and more complicated apparatus. At voltages of 150 to 220,000, you have to put so much insulation on to hold the line voltage, that the lightning arrester is not important. I will say that our problem for 234,000 volts is practically solved today by distributing the proper arresters. In that way we can get a high percentage of protection from four thousand volts up to some unknown quantity of 66 or 88 or 110.

I don't see the answer at the present time. Either the equipment is so expensive—I am talking about system protection rather than individual apparatus—if you have big transformers you can afford to pay a little bit for the lightning protection, but in general system protection costs so much that you can't afford to do it from perhaps 119 volts up, until you get to a voltage where your insulation is protection.

Karl B. McEachron: Attention is called in Prof. MacLaren's paper to the desirability of protecting the secondary distribution circuits, especially if those circuits are long and exposed. This point could well receive more consideration than has been given to it in the past. It is well known that the telephone companies have found it necessary to protect their subscribers' stations by the use of arresters, even when the exposed lines were not long. That such protection is necessary has been shown by the burnouts occurring where the circuits were unprotected.

Prof. MacLaren has shown that arresters having high resistance in the ground circuits fail to protect. A simple calculation will show how important this point is, being probably more important with distribution arresters than with the high-voltage equipments.

Assume that a 2300-volt arrester has an effective resistance of 25 ohms which is representative of arresters of this rating. With zero ground resistance, the discharge rate at double rated potential is 4600/25 = 184 amperes. With a ground resistance of 200 ohms, the discharge rate is only 20 amperes. Thus, the introduction of this ground resistance has reduced the discharge rate to one-sixth of its original value.

Designers of lightning arresters endeavor to get as high a discharge rate as possible, but if one succeeds in reducing the resistance from 25 to 20 ohms or less, his efforts will be of little avail if the ground resistance is not kept low.

This paper agrees with the experience of others in that the best protection is secured where the density of lightning arresters is the greatest. This is, of course, to be expected.

W. B. Kirke: I should like to bring out a few points with regard to Messrs. Doherty and Kierstead's paper that have been particularly interesting to me in a similar study that I have made on magnetic forces in reactors at The New York Edison Co. If we refer particularly to Equation 7, we will find an expression for the magnetic forces due to the currents in the reactor windings.

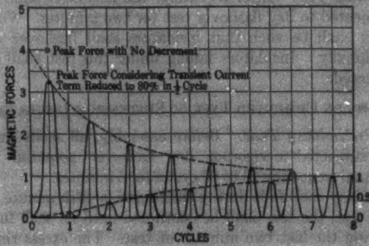

Fig. 1

If we analyze this equation for one complete cycle, we will find that the average force for that period is proportional to the 1.5 term of Equation 7. If we add up all numerical coefficients of that Equation, 1.5 + 2 + 0.5 without regard to sign, we will have a Fig. 4 which is proportional to the maximum value of the magnetic force during the first cycle. Equation 7 was derived by squaring Equation 6. If we had neglected the transient term which occurs in Equation 6 and is expressed by the figure of unity, we would have a force equation which gave over a complete cycle an average force proportional to the figure of 0.5, and a peak force proportional to the figure of 1 thus representing conditions after the transient has disappeared.

During the first cycle, however, with a totally offset current wave we found that the peak magnetic force was proportional to the figure of 4. This peak force, therefore, varies from a figure of 4 down to the figure of unity during the period of the transient, as shown in the accompanying figure.

In calculating forces in reactors under short-circuit conditions we are, therefore, likely to be greatly in error if the transient currents are not considered. As an illustration, we may take a 300-ampere circuit having a 3 per cent feeder reactor. For a short circuit at the terminals of the reactor, the effective short-circuit current, with sustained bus voltage, would be approximately 10,000 amperes and if this figure were used, it would give us magnetic forces corresponding to the figure of 0.5.

The paper of Messrs. Doherty and Kierstead brings out conclusively that it is not the 10,000 amperes that we should use nor is it the peak value of 14,000 amperes, but it is something like 28,000 amperes or 2.8 times the value of the effective sustained short-circuit current. In other words, we have a maximum force of practically 8 times that which would be derived using the effective sustained short-circuit current. If the decrement is taken into account, the ratio of 8 may be more nearly a ratio of 6.5.

J. F. Peters: Mr. Pollard's paper brings out the point very forcibly that in going into an entirely new field it is very difficult for the designer to design apparatus that is the final answer. As Mr. Pollard stated regarding these pioneer reactors, the tests at the factory seemed to demonstrate the sufficiency of the design; however, in service, weaknesses developed. The fact that the first designs were not the final answer is clearly brought out by the fact that all reactor builders have modified the construction of their coils since the days when these coils were installed. Service records have demonstrated that the modifications which have taken place since the building of these pioneer reactors have completely overcome all of the weaknesses.

With reference to the third paper by Mr. Doherty and Mr. Kierstead, I feel that the authors should be congratulated for their very interesting and valuable paper. It is a very interesting subject and it is particularly interesting to me because a few years ago I had occasion to investigate a very similar problem—that of forces between bus bars. The same differential equations, of course, apply to the two problems. I had the pleasure of witnessing some short-circuit tests on bus bars. In these tests the bus bars were installed temporarily and the bars forming one lead had considerable separation, with the supports at a considerable distance apart. The short circuit lasted some three or four seconds and during that time we could observe the oscillations of the bars increasing until adjacent bars would collide and destroy the oscillation, after which they would start to build up again.

In investigating bus bar stresses I included one of the electric damping factors, that controlling the decay of the assymmetry at the start of the short, which involves R/L as an exponent. We found that the maximum forces produced by this factor amounted to 2.8 times symmetrical force, instead of four times.

I feel that the most interesting and valuable point brought out in the paper is in supplying us with methods of determining the natural period of the coils. We can determine that value and then avoid it in the structure, then design the coil to withstand the maximum electrical forces.

H. O. Stephens: Several years ago there were some points in the design of current-limiting reactors that had not been completely substantiated by actual tests. There was some question as to whether the movement of the conductors due to the forces between them would respond to the instantaneous force resulting from the current at the peak of first half cycle of the displaced current wave. There was also some uncertainty as to the mechanical strength of the cables as they were supported in the reactors.

A large generator, built for short-circuit testing, was provided at Schenectady and a number of reactors were tested to destruction. The result of these tests demonstrated, that for all practical purposes, the peak of the displaced current wave at the first half cycle should be used in calculating the forces between turns. The actual strength of the conductors was also accurately determined. It was then possible to predict with a reasonable degree of accuracy the current that would cause the turns to be displaced in any given design of reactor and apply suitable factors of safety to obtain a reactor of sufficient mechanical strength.

Of particular interest in connection with Mr. Pollard's paper is the fact that after these tests were completed, the strength of a large number of reactors installed was calculated and it was found that some of the reactors of the type shown in Figs. 9, 10 and 11, installed by the Public Service Electric Co. of New Jersey, did not have sufficient mechanical strength and plans had already been started for adding suitable reinforcements before the failures referred to actually occurred.

The point in regard to thermal capacity, brought out in the conclusions to Mr. Pollard's paper, should not be passed over lightly. The reactor should be the last thing to give way in case of a long continued short circuit and a number of operators are specifying that the cross section of the reactor conductor should be at least equal to the cross section of the connected cable. A reactor may easily meet the usual heating guarantees and still have materially smaller cross section but the saving in losses may fully compensate the operator for the increased cost of the reactor with larger conductors.

The need for taking care of the forces between adjacent reactors has been recognized and suitable braces and supports have been provided where the calculated forces indicated their necessity. I do not believe, however, that it has been customary in general to consider the materially increased forces that may result when the natural mechanical frequency is equal to or nearly equal to the electrical frequency. The paper by Mr. Doherty and Mr. Kierstead is a real contribution to the art as it clearly demonstrates certain factors that should be avoided. Fortunately the natural mechanical frequency usually differs materially from the electrical frequency. No failures due to the force between adjacent reactors have come to my attention except one in which, through an error in installation, the braces furnished and intended to take care of this force were omitted. This case however shows that the forces are real and should be recognized in the installation unless adjacent reactors are so far apart that calculations show that they may be neglected.

C. L. Fortescue: The insulator bears to the electric industry a similar relation as the steel rail to the railway industry. It is absolutely necessary to have good insulators in order to be able to operate economically. Now. we have to develop power to meet the ever increasing demand, this means we have to develop high-voltage transmission lines and to do this we must have good insulators.

One step in order to get a good insulator is to have some standards by which to measure the quality of these insulators. These standards should be uniform for everybody and this Subcommittee of the Standardization Committee, has attempted to formulate rules by which tests should be made to determine the quality of insulators and they have produced a commendable piece of work in this specification.

The problem of the insulator manufacturer is not an easy one. He is dealing with material which is quite tricky and it goes through a large number of rather troublesome manipulations.

I think in order to help the manufacturer of insulators he ought to have sympathetic cooperation on the part of insulator users. He has quite difficult problems and he needs help; he must know under what conditions insulators fail, etc. One way in which the insulator user can help him a great deal is by understanding the manufacturing processes through which an insulator goes. So all insulator users should take an opportunity such as the one that is being presented at this convention to visit an insulator plant and see for themselves the manufacturing processes. The processes in insulator plants are very much alike, and they will not only have an interesting time in seeing these processes, but I think they will learn a lot about insulators.

H. B. Dwight: Messrs. Doherty and Kierstead give an interesting description of the increase, due to motion of the parts, which should be added to the calculated force between reactance coils.

Among the references for calculation of force they give an article by the writer, on "Repulsion and Mutual Inductance of

Reactors," published in the *Electrical World* in 1917. This article gave the first approximate formula which was worked out for the repulsion of coils with parallel axes. While its results are very close to the true values, a more accurate and convenient formula is given in the paper by the writer on "Some New Formulas for Reactance Coils," TRANS. A. I. E. E., 1919, p. 1681, formula (4). Curves for the force on coils with parallel axes are given in Fig. 3 of the A. I. E. E. paper.

Formulas and curves for the force exerted between coils with the same axis are given in the paper by the writer on "Repulsion and Mutual Inductance of Reactance Coils with the Same Axis" *The Electric Journal*, May, 1918, p. 166.

E. E. Berger: Referring to Mr. MacLaren's paper, it would seem that the problem in most cases of protecting low-tension circuits, such as 115 and 230 volts, was not very serious and that adequate protection could be supplied where needed at small cost. For 2300 and 13,200-volt service, past experience has proven that adequate protection in the form of lightning arresters is available and so the problem is simply a matter of placing arresters at the proper points of a system and in sufficient numbers. However, much valuable work can still be done in reducing the cost of such protection, or possibly increasing slightly the degree of protection.

D. W. Roper's paper, presented before the Institute in November, 1920, which I believe was the beginning of this class of investigation, has proven very valuable in determining what performance should be expected from lightning arresters designed for distribution circuits. I can say that the General Electric Company has been able to check in the laboratory to a large degree the results obtained by Mr. Roper on the Commonwealth Edison System.

My opinion is that more valuable information can still be obtained from operating lines, especially in regard to the magnitude of lightning disturbances. I feel that a very simple apparatus can be developed to measure and record the magnitude of lightning disturbances. From a study of such data taken over several lightning seasons, much light would be thrown on the whole lightning protection problem.

F. H. Kierstead: After Mr. Pollard candidly discusses his experience with current-limiting reactors of different designs he concludes that, while reactors have in the past been subject to failures of various natures, the modern reactor is a reliable piece of apparatus. Another way of putting Mr. Pollard's conclusions is that as a result of such operating experience modern reactors have been made reliable. The defects in the design and operation of early reactors which did not become apparent until the reactors had been given the test of service, have been corrected.

Failures of reactors may result from the following causes: 1—Over-voltages; 2—Insufficient thermal capacity; 3—Mechanical weakness; 4—Foreign conducting materials.

Since reactors are points of reflection, there was some apprehension as to the magnitude of the voltages that might be built up at these points, and in the early days of the design of current-limiting reactors careful consideration was given to the voltage that they might have to withstand. Experience has shown that as a result of this consideration only a very small number of failures can be attributed to over voltage and it is probable that some of these may have been due to foreign material as will be explained later.

It was always recognized that reactors should have liberal thermal capacity and it soon became the practise to construct them with materials not injured by high temperatures and I know of no failures of reactors due to high tempeature that were constructed with heat resisting materials.

While it has always been recognized that the magnetic forces in reactors were large still, as pointed out by Mr. Stephens in his discussion, their exact magnitude and destructiveness was not at first fully appreciated. Now that they are understood we should not expect a duplication of this trouble.

Experience has shown that the danger of failure due to foreign conducting materials falling or being drawn into a reactor by its magnetic field has not been fully appreciated by operators. Usually an examination reveals the cause of such a failure but sometimes the evidence is destroyed by the resulting arc. In one such case a reactor that failed due to no apparent cause was placed out of doors for some time. At a later examination positive evidence of iron rust was found on the conductors at a point where failure had occurred, clearly indicating that the failure had been caused by the presence of an iron body at this point. This is a source of trouble that can be eliminated by proper education of the operating force.

L. F. Woodruff: In 1920 Mr. O. R. Schurig and the writer undertook a study of the stresses in a-c. busbars and busbar supports due to short-circuit electromagnetic forces. A large number of oscillographic tests were made with transient-pressure-recording devices. A mathematical analysis of the motion of the bar was made on the assumption of "equivalent mass" and "equivalent stiffness" for the bar, and we obtained equations identical with those given by Messrs. Doherty and Kierstead. The oscillographic records furnished excellent checks with the theory. The results of our investigation were given in a general engineering laboratory report having a circulation only inside the General Electric Company.

It is obvious that the important principle brought forth is that conditions of electro-mechanical resonance are to be avoided. It is therefore desirable to calculate accurately the natural frequency of mechanical vibration of systems in which the equivalent mass and equivalent stiffness differ materially from the actual total mass and stiffness. The busbar is the most important illustration of such a case.

A busbar is a continuous beam, and if the assumption be made that each span is of the same length, all spans, except possibly those near the ends of the bar, will move in unison, and each span has therefore the end conditions of a beam fixed at both ends. We will set up the differential equation of motion of such a beam.

In Fig. 2 the slope of the neutral axis at any point equal to $\frac{\partial y}{\partial x}$. If the slope is assumed to be small, so that $\left(\frac{\partial y}{\partial x}\right)^2$ is negligible in comparison with $\frac{\partial y}{\partial x}$, the curvature is approximately equal to $\frac{\partial^2 y}{\partial x^2}$ and the bending moment to $E\,I\,\frac{\partial^2 y}{\partial x^2}$, where E is the modulus of elasticity of the material and I is the moment of inertia of the cross-section about the intersection of the neutral plane and the plane of the cross-section. The shear is $\frac{\partial}{\partial x}$ (moment), or $E\,I\,\frac{\partial^3 y}{\partial x^3}$.

The net vertical force acting on the section $A\,B\,C\,D$ is the difference of the shears at $A\,C$ and $B\,D$, and is equal to the rate of change of shear along the X-axis times the distance $d\,x$, or

$$E\,I\,\frac{\partial^4 y}{\partial x^4}\,d\,x.$$

If there is no external force applied, this force must be used in producing vertical acceleration. If m is the mass per unit length of beam, the mass of the section $A\,B\,C\,D$ is $m\,d\,x$, and multiplying this mass by the expression for its acceleration, and equating this to the available force, we have

$$m\,d\,x\;\partial^2 y/\partial t^2 = -\,E\,I\,(\partial^4 y/\partial x^4)\,d\,x,$$

or

$$\partial^2 y/\partial t^2 + (E\,I/m)\,(\partial^4 y/\partial x^4) = 0 \qquad (1)$$

Placing $E\,I/m = p^2$, we have

$$p^2\,\partial^4 y/\partial x^4 + \partial^2 y/\partial t^2 = 0 \tag{2}$$

On the assumption that the motion of the beam is everywhere sinusoidal with respect to time, we can write

$$y = f(x)\sin(\omega t + \phi), \tag{3}$$

which will satisfy equation (*2*) if we can find a solution for

$$p^2\,\partial^4 y/\partial x^4 - f(x)\,\omega^2 \sin(\omega t + \phi) = 0 \tag{4}$$

Differentiating (*3*) four times with respect to x, and putting $u = f(x)$,

$$\partial^4 y/\partial x^4 = (\partial^4 u/\partial x^4)\sin(\omega t + \phi) \tag{5}$$

Putting $\omega^2/p^2 = f^4$, and combining equations (4) and (5), we have

$$\partial^4 u/\partial x^4 = f^4 u \tag{6}$$

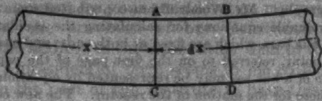

FIG. 2

By inspection, this equation is satisfied if

$u = \sin f x,$
$u = \cos f x,$
$u = \sinh f x,$
$u = \cosh f x.$

The general solution is therefore of the form

$$u = M \sin f x + N \cos f x + P \sinh f x + Q \cosh f x, \tag{7}$$

the values of M, N, P and Q being determined by the boundary conditions.

In the case of a beam fixed at both ends, the boundary conditions are that u and $\partial u/\partial x$ must vanish for $x = 0$ and $x = L$, the length of the beam.

The two conditions for $x = 0$ give

$$N + Q = 0$$
$$M + P = 0$$

The two conditions for $x = L$ give

$$M \sin f L + N \cos f L + P \sinh f L + Q \cosh f L = 0$$
$$M \cos f L - N \sin f L + P \cosh f L + Q \sinh f L = 0$$

Let $f L = \theta$. Then

$$M(\sin\theta - \sinh\theta) + N(\cos\theta - \cosh\theta) = 0 \tag{8}$$

$$M(\cos\theta - \cosh\theta) - N(\sin\theta + \sinh\theta) = 0 \tag{9}$$

From (8),

$$M = -N\,\frac{\cos\theta - \cosh\theta}{\sin\theta - \sinh\theta} \tag{10}$$

Substituting this in (9) and dividing by N,

$$\frac{(\cos\theta - \cosh\theta)^2}{\sin\theta - \sinh\theta} = -(\sin\theta + \sinh\theta),$$

or $\cos^2\theta - 2\cos\theta\cosh\theta + \cosh^2\theta = -\sin^2\theta + \sinh^2\theta,$

or $\cos\theta\cosh\theta = 1$ (11)

Values of θ which will satisfy this equation have been worked out by Strutt (*Theory of Sound*, Vol. 1), and are:

$\theta =$ 4.7300408,
7.8532046,
10.9956078,
14.1371655,
17.2787596, etc.

The smallest value of θ represents the lowest frequency or fundamental vibration, and the other values the various harmonics. In the fundamental mode of vibration only is the whole span moving in the same direction at any given instant, and resonance of any of the harmonic frequencies with the force frequency is of little importance.

We will now work back and determine the actual frequencies of the various vibrations.

For the fundamental, $\theta = 4.73$.

$$\theta = f L. \qquad f = \sqrt{\omega/p}. \qquad p = \sqrt{E\,I/m}.$$

Therefore

$$f = 4.73/L = \sqrt{\omega/p}.$$

$$\omega = \frac{22.37\,p}{L^2} = \frac{22.37}{L^2}\sqrt{\frac{E\,I}{m}}. \tag{11a}$$

For a beam of rectangular cross-section $I = B\,D^3/12$, where B is the breadth and D the depth of the beam. The mass per unit length, m, is $B\,D \times$ density. Putting in these values,

$$\omega = \frac{22.37\sqrt{\dfrac{E\dfrac{B\,D^3}{12}}{B\,D \times d}}}{L^2}$$

$$= 6.45\,D/L^2\,\sqrt{E/d} \tag{12}$$

For hard-drawn copper an average value for E is 15,000,000 pounds per square inch, and the density d = 555 lb. per cu. ft. ÷ (1728 × 32.2 × 12) = 0.000832. Putting these values in equation (12), we get

$$\omega = 880{,}000\,D/L^2$$

The frequency of the fundamental vibration, $\omega/2\pi$, is

$$140{,}000\,D/L^2 \tag{13}$$

Curves between length of span and natural frequency, which are straight lines if plotted on logarithmic cross-section paper, are shown in Fig. 3.

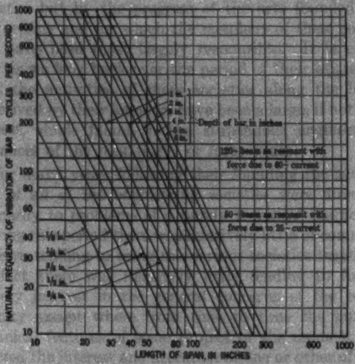

FIG. 3—CURVES SHOWING NATURAL FREQUENCIES OF VIBRATION OF COPPER BARS OF RECTANGULAR CROSS-SECTION, FIXED AT BOTH ENDS

The natural frequency of vibration of a copper bar fixed at both ends has been calculated from the partial differential equation as follows. The dimensions D (depth) and L (length) are given in inches. (E = 15,000,000 lb./sq. in.)

Fundamental vibration, 140,000 D/L^2; Second harmonic, 385,000 D/L^2; Third harmonic, 757,000 D/L^2; Fourth harmonic, 1,254,000 D/L^2. For aluminum bars, Fundamental vibration, 197,000 D/L^2.

The natural frequency of vibration of round wires has been calculated, as there is a possiblity of resonance in such apparatus as power-limiting reactors, etc. For round wires of hard-drawn copper, the natural frequency is:

Fundamental vibration, 121,000 D/L^2. For round aluminum wires: Fundamental vibration, 171 000 D/L^2.

M. Mac Laren: Mr. McEachron's illustration of the effect of high ground resistance upon the discharge rate of an arrester I think rather overstates the case for Ohm's Law cannot be applied directly here. As A. L. Atherton has so clearly shown (Paper presented at Midwinter Convention of the A. I. E. E., February 1923) the surge impedance of the circuit will generally be much greater than the resistance of the arrester and its ground connection and has, therefore, a more controlling influence in the problem. At the same time, it is important to obtain low ground resistance and where this cannot be accomplished without considerable expense the arrester density should be increased.

Mr. McEachron's statement that the best protection is secured where the density of arresters is greatest needs qualification, for the data submitted in the paper show that this only applies to the exposed circuits. There are many circuits in the city which have no arrester protection for which no injury from lightning has been recorded. One of the most important matters to determine for the exposed circuits is where the saturation point occurs beyond which it becomes economically undesirable to increase the arrester density.

Mr. Berger refers to the low cost of arrester protection for 115 and 230-volt circuits, and while this is true for an individual case, a large sum would be needed for an operating company to apply such protection at all generally to the system. For these reasons it was suggested in the paper that this type of protection might be included as part of the customer's service equipment.

R. E. Doherty: Mr. Dwight has kindly called attention to additional references to contributions which give methods of calculating the electromagnetic force. These will be added to the bibliography. Mr. Woodruff has contributed some interesting and valuable equations for calculating the oscillating frequency of bus bars, which are fixed at both ends; and has emphasized the point, referred to on the first page of the paper, that the motion of a bus bar may be represented by equations identical with those for the motion of the reactor. The new equations by Mr. Woodruff make it possible to calculate the natural oscillating frequency, using constants of the material directly, instead of an "equivalent mass" and "equivalent stiffness."

The National Engineering Societies

Their Problems of the Past, Present and Future

President's Address

BY F. B. JEWETT

In selecting a subject for his annual address to the American Institute of Electrical Engineers the retiring President has such a wide latitude of choice as to make the proper selection one of considerable difficulty. If I followed my natural inclination I should doubtless choose as a topic one of the numerous outstanding engineering problems of the day and attempt to present for your consideration my views on it. However, the provision of our Constitution which assigns the address to the close rather than to the beginning of a presidential term seems to me to indicate a desire on the part of the Institute to have this address incorporate in some measure the ideas of the retiring President on matters of general concern to the Institute rather than his ideas on some specific engineering topic which could of course be expounded in a paper or discussion at one of the technical meetings. Whether or not this was the conscious intention of the Institute at the time the Constitution was adopted, I have elected thus to interpret the situation and in what follows I shall attempt to set before you for what they are worth my present ideas on some of the problems which confront this Institute, and to a large extent the other so-called Founder Engineering Societies. If these ideas are valid and if they serve in any measure to assist our Institute to grow and develop in a way which is healthy and for the best interests of its members, for the profession as a whole and for the good of the industry and the community which we represent, I shall feel that the remarks have been worth while.

In viewing the varied activities of our national engineering societies and the problems which now confront them one cannot help but be struck by the fact that the tremendous changes of the past decade, which have so altered our whole economic, social and political life, have brought a multitude of new problems to the engineering societies. That it will require the utmost of clear thinking on the part of our members and officers, if these problems are to be solved in a manner befitting the profession of which we are a part, is a necessary corollary. That the conditions which now confront us are intricate and perplexing is not at all strange in view of the increased perplexities of life in general. In fact it would be strange indeed if a professional society such as ours could have continued uninterruptedly along the lines marked out by its founders.

In common with the other great national engineering societies, this Institute was the direct result of a feeling on the part of a group of men working in a particular field of engineering, that they and their art would profit from the establishment of a place of common meeting. It was the thought of the founders and has ever since been a guiding principle of the Institute that by thus providing a forum where engineers could meet as engineers and not as representatives of some particular business activity, and where they could discuss freely all matters of common technical interest, the art would be advanced most rapidly, the community would be best served and the members individually would receive the greatest personal benefit. The wisdom and foresight of the founders of the great national engineering societies and of the members of these societies who followed in their footsteps is evidenced not alone by the growth in size and vigor of the engineering societies but by the progress of the arts founded on engineering and for which the members of these societies have been most largely responsible. It is clear now as we look back over the period of their existence that the activities of these societies along the lines conceived by their founders have been a large, if not the largest, single factor in the stupendous engineering advances of the past thirty or forty years.

For many years following their founding the activities of all of the engineering societies were directed primarily to problems of a purely technical character. During this period there was relatively little of common concern tending to bring the various societies together. In fact there was relatively little tending to foster society interest in the general concerns of the community except where from time to time the purely technical features of some project of the moment attracted the interest and attention of one or other of the engineering bodies. But just as the growth and expansion of the fundamental sciences on which all engineering is based gradually wiped out the lines of demarcation which once separated physics and chemistry, for example, into two readily defined and wholly distinct sciences, so too the more general application of fundamental scientific knowledge and the growth of arts based on engineering broke across the barriers which once so definitely separated the civil, mechanical, mining and electrical engineering fields. Even before the outlines of these boundaries which once separated the great engineering fields began to grow indistinct, however, there was a pronounced indication of impending change within the several fields themselves.

So rapid and so extensive was the growth of the

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

industrial application of scientific knowledge that in every field of engineering, possibly more so in the field of electrical engineering than elsewhere, there sprang up great new industries, which, although they had their roots in a common field of technical knowledge, were nevertheless in many respects essentially distinct arts. While retaining undiminished their interest in those fundamental aspects of their profession which were of common technical concern, the groups of engineers who had specialized along these several arteries of development soon found themselves faced with a myriad of problems of interest to their fellow specialists but of relatively little concern to the specialists in other fields. The machinery existing in the engineering societies of the day was inadequate to satisfy the needs of all these diverse groups. This, as we are all aware, led inevitably to the formation of rather highly specialized engineering societies, such for example as the Institute of Radio Engineers or the Institute of Iron and Steel Electrical Engineers in the electrical field, and the Society of Automotive Engineers in the mechanical engineering field.

Frequently these newer societies were not purely technical in their aims but had to a greater or less extent a commercial background for part at least of their activities. In the main the active members of these specialized engineering societies were members of the founder societies and in general there was no conscious thought of any attempt to usurp the functions which had become recognized as the functions of the four national societies. The avowed purpose and desire of the founders of these societies was to handle those technical problems which were of special rather than of general interest, and to do such other things as were of special concern to the engineers who constituted their membership.

Once founded, however, it was but natural that the growth of these societies should sooner or later give rise to activities similar in whole or in part to some of the activities of the founder societies. As we are aware, this overlapping of activities between what might be termed the "parent" and the "offshoot" societies has from time to time resulted in confusion, just as the gradually overlapping fields of interest of the national societies have resulted in a confusion tending to bring about some form of joint action. On the whole, however, it seems clear that the formation of these specialized engineering societies has resulted in great benefit, not alone to the art but in fact to the founder societies as well, and the gradual clarification in overlapping fields and the spirit of cooperation and of joint action, of which there is growing evidence, bids fair to minimize the deleterious effect of such confusion as may arise.

Coincident with the expansion to the point of overlapping which has taken place in the fields of activity of the great national engineering societies and in the creation and growth of specialized engineering societies, whether wholly or partially technical in function, other factors have developed which have tended to modify the simple setting which originally surrounded the national engineering societies.

The same forces which resulted in the formation of specialized engineering societies later brought into existence associations, of which the National Electric Light Association is an outstanding example, in which the purely technical aspects of engineering, such as concern the national societies or their offshoots, were initially of relatively minor concern. These associations, which embraced in a common membership groups of men with widely diverse interests, were essentially commercial in their origin. Such technical activities as they were originally interested in were concerned more with the commercial than the scientific aspects of engineering. Gradually, however, as they grew and their problems developed, there arose within these associations activities which tended to parallel or overlap similar activities in the founder societies or their more purely technical offshoots. As in the case of these offshoot engineering societies, the overlapping activities were very largely in the hands of engineers who were also members of the national engineering societies. So long as the overlapping affected merely the internal working of an industry the results were not essentially serious. When, however, it began to evidence itself in a conflict of findings which bore upon public policy or State regulation the results began to react adversely to the industry and to the engineering profession.

While all of the foregoing factors were at work there was another and even more important influence in process of development tending to modify the original concept of the national engineering society and to change the form and scope of some of its activities. In the early days of what we might term the engineering epoch of civilization—the fifty or so years immediately following Watt's work with the steam engine—the applications of science to industry were considered largely like any other commerical business of the preceding time, namely, as matters of private concern in which the State as a whole had little or no interest. Gradually, however, with the expansion of these newer technical industries it became clear that many of them were of interest and concern to the public in ways quite distinct from the older questions of their financial solidity and their legal or illegal acts viewed solely from the standpoint of private businesses. First, with the common carriers, later with the other so-called public utilities and still later with many businesses closely akin in function to age-long commercial organizations, there grew up gradually the idea of a general State control in the interests of the public.

Initially it was logical that this control should be entrusted to those designated officials to whom in the past had been entrusted the conduct of the State's relations to matters of common public concern. In the main these officials were essentially ignorant of technical matters, so that with the best of intentions many schemes of regulation resulted disastrously through failure on the part of the supervisory officials to recognize fundamental engineering considerations.

Gradually with the increase in the magnitude and complexity of the things to be regulated by the State, or in some instances to be done by the State in the common cause, and with the increasing complexity of our modern application of a growing science to industry, there is developing an appreciation of the fact that we are more and more coming to live in what, for want of a better word, we may term a mechanical age. Further we are coming to realize that the proper direction of our affairs must to a large extent be based on the unbiased opinion of those who by training and experience are in position to know the fundamental factors which underlie satisfactory and proper physical operation; in a word, engineers. This does not mean necessarily the growth of a tendency to turn over to engineers the untrammeled control of undertakings which have a large public interest but which are primarily of an engineering character. There are too many other factors of far-reaching importance which enter into the State's relation to public utilities to make such a step either probable or desirable. What it does mean, however, is that there is a gradual and I think a rather rapidly awakening appreciation of the fact that in many directions the interests of the community make imperative the employment of men of engineering training and experience on the administrative boards charged with safeguarding the public interest. Further, there is indubitable evidence of a growing tendency in matters of great public concern involving engineering factors to seek the opinion of engineers as a basis for legislative and executive action—this on the assumption that engineers are by training and experience that group of the citizenry most competent to have a valid opinion on certain features of numerous problems affecting the State as a whole.

Growth in this direction is having a far-reaching effect, not alone on the relations of the engineering societies with each other but in their common relation to the public and to the State which represents the public. It is also having a determining effect on the proper scheme of engineering education, on the qualifications of men entitled to be classed as engineers and on the general ethics of the profession and of the relation which should exist between the State and those qualified to be classed as engineers.

In addition to the foregoing, which appears to me to be the deep underlying forces at work tending to modify our original concept of the national engineering society and to create our major problems for the future, there are certain more specific or more remote influences all tending to force us in the general direction of a closer and more intimate relationship between the national engineering societies and the development of cooperative organizations which can function for us effectively in all matters where common joint action is required.

Among these more specific influences are to be cited the Engineering Foundation and the National Research Council; the one the nucleus of what I believe is destined to be a great foundation entrusted to the guidance of the founder societies with funds contributed by those who have the faith to believe that the income from these funds will be expended wisely by the societies in fostering the growth of engineering and of the things with which engineers are vitally concerned. The other is a great national institution intended to coalesce in a common organization all of those activities of the nation interested in scientific research. As engineering and the engineering arts are the greatest consumers and beneficiaries of the results of scientific research, it is natural that the engineering societies should have a vital interest in the affairs of an organization like the National Research Council and in its success. With the establishment of the Engineering Division of the National Research Council there has been brought into being a mechanism which if properly directed will serve to tie the research activities of the engineering societies into the larger common undertaking.

Of the factors which bid fair to influence our course at a later date, but which for the moment have a less immediate bearing on our affairs, are questions of international relationships between engineering societies, whether for the better dissemination of knowledge and the advancement of the art or in the more practical affairs where benefit would be derived from a common standardization for the simplification of international trade.

In the foregoing I have attempted to state the factors which have been and are at work tending to modify the original concept of the national engineering society. I have not attempted even to enumerate all of the directions in which these forces have worked to change the form or character of our activities, nor have I attempted to discuss the relations of those regional engineering societies which have in the main the same relations to their constituent members as the national societies to their more widely distributed members. In a sense these regional engineering societies might be looked upon as sections of the national organizations established in order the better to carry out the aims of the larger organizations.

Having now surveyed the situation historically, let us make a direct comparison between the national engineering societies in the period prior to ten or fifteen years ago, with those same societies as we find them today. This should assist us in determining just how far their situation has been altered and the direction in which they should endeavor to progress if they are to achieve their initial purpose of rendering the greatest benefit to their members, to the art which is the creation of these members, and to the community of which the members are a part.

As has been indicated, these societies were initially, and for many years subsequent to their formation, associations of engineers having a common interest in some one of the fields of civil, mechanical, mining or electrical engineering. These men were banded together in institute or society for the avowed purpose of bettering their professional condition through the presentation and discussion of papers on technical subjects, which papers and discussions would at the same time tend to advance the art and benefit the public. They were interested in matters pertaining to technical education primarily from the standpoint of their own particular field and only slightly less so in education as it related to engineering in general. The societies of those earlier days had few matters of common interest and few continuing relations as societies with either the State or the public of which they were a part. They grew up with different backgrounds of experience of their members, as is evidenced by the great diversity in their qualifications for membership and in the difference in their concepts of how the qualifications for membership could best be employed to attain what was apparently a common objective.

That the results obtained by the societies in those earlier years were eminently successful is evidenced by the stupendous growth of engineering and the engineering arts which took place during the period in question.

Contrast now this relatively simple situation with the conditions of today and the problems with which the engineering societies are now faced. The national societies have grown and prospered enormously and in addition there have come into being innumerable societies which cover specialized fields of engineering interest and which in some directions are paralleling the work of the founder societies. Further, there are associations primarily commercial in their origin, which have developed engineering interests somewhat akin to those undertaken by the national societies. The national societies are finding each day an increasing number of problems in which they have a mutual interest or an interest with one or other of the different organizations just enumerated. As a result there has come into being an almost inextricable mass of relationships and interests which is each day increasing in complexity.

Further than this, there has been a material change in the concept of the relationship which should exist between the national society and some at least of its members. No longer is the national society, through its national or sectional meetings, looked upon merely as the instrumentality through which the members could advance themselves and their art professionally. Today it is considered by large sections of our membership that the national society has a duty to its members in the direction of advancing their physical and financial wellbeing.

In addition to this there has come about a decided change in the concept of the relation which should exist between the national societies acting for engineers and the State which represents the community. Evidence of this is clearly shown in the various attempts which have recently been made to coordinate the societies for certain common purposes in a federation which can speak for all on matters of common concern. That there have been and are difficulties in the carrying out of this idea has to do with the mechanism and with the diversities which exist in the constituent bodies themselves rather than in any fundamental divergence of opinion as to the necessity for some form of cooperative action in this direction.

So complex has the situation become and so insidious and alluring are many of the arguments which tend to a further complexity that it seems to me, in common with many other engineers, imperative that we take a rigid account of stock, determine anew what we believe should be our guiding principles under present conditions and then attempt the untangling of the skein in consonance with these principles.

As I view the matter, the original concept of the national engineering societies, as a mechanism for advancing the engineering art and the professional status of the members, and of exercising a controlling influence on the form and character of engineering education, is still the primary function of these societies. The means by which the several societies accomplish this end, whether through national conventions or sectional meetings is of secondary importance, nor is it necessary, as I view it, that there should be any attempt to monopolize the field to the exclusion of those societies having a membership primarily interested in some particular region of one of the major engineering divisions.

In the matter of influencing the form and character of engineering education there was never greater need of the sane and powerful influence which can be exerted only through the medium of great national professional societies. In recent years there have grown up in some of our institutions courses of instruction which purport to give young men an education destined to facilitate their rise in the engineering field or in the industries based on engineering. In the view of many engineers these courses, which directly or inferentially hold out the lure of some royal road to success, are detrimental rather than beneficial to the progress of engineering and to the arts depending upon it. To many these courses are detrimental not because of the subjects which they include but because they fail to take into account the fact that our engineering problems and our engineering industries are becoming each day more and more complex and more and more dependent on a strict and accurate knowledge of those fundamental sciences on which all engineering is based and without a knowledge of which no man can hope to build a solid future for himself or his industry. If this picture is correct, and if, as I fear, there is a tendency in some quarters for young men to consider that sound grounding in science and engineering principles is of secondary importance, it will require the utmost of attention on the part of the national engineering societies to bring our whole educational machinery into proper alignment.

I believe very firmly that the chief executives of the complex technical industries of the future are destined to be drawn more and more largely from the ranks of those men whose course of study in college, university and technical school, and whose experience, has given them a broad and accurate knowledge of fundamental scientific and engineering principles. It is obvious of course that the successful administrator must be much more than a mere skilled scientist or technical engineer. He must in addition have an ability to lead and direct men and that facility which we term, for want of a better word, "business acumen." These characteristics seem to be rather a matter of inheritance than of formal training, and from the vast number of able young men who embark on engineering and science as a life work it would seem to me reasonable to expect that there should be found an adequate number possessing the characteristics required of a chief executive. On the other hand, in industries based on science or the engineering arts so much of success or failure depends, in the last analysis, on a clear and accurate understanding of fundamental underlying factors that it is difficult in many cases to conceive of these industries being administered successfully except by men who, while possessing the necessary administrative qualifications, are at the same time, through their own training and experience, in position to have an independent judgment on the technical conclusions which form the basis for administrative decision.

If this picture is correct the national engineering societies can perform a great service to the nation through the assistance which they and their members are able to render in helping our colleges, universities and technical schools properly to plan and carry out their educational programs.

On matters which relate to standardization and to the formulation of principles and rules for guidance to good practise, there is of course much with which the national engineering societies need not concern themsleves. On the other hand, the rapidly growing interest of the State, not alone in the results of technical and commercial application of science through industry to the needs of the population but also to a greater and greater extent in the supervision of the industries which make these applications, is increasing the necessity for having this work done thoroughly and in unbiased fashion and put forth by bodies against whom can rest no suspicion of self-interest.

Our national engineering societies are in a unique position in this respect. The recognition that they are bodies which combine in a common membership, and solely through the bond of a common professional interest, the trained men of the various engineering fields, gives to their rules and their standards a force and standing not to be obtained by the rules and standards of bodies known to be organized more or less for commercial reasons. The peculiarity of this position offers both a great opporunity and a great obligation for service from the national engineering societies. The realization of this opportunity and the discharge of this obligation does not mean necessarily that the national societies, individually or collectively, must of themselves do all of the detailed work which underlies the formulation of those things which go forth under the seal of their sanction. In many cases where the general interest of the industries, of the societies and of the public at large makes desirable the sponsorship of rules and standards by the national societies or their created agencies, the collecting and sifting of data and the formulation of the proper conclusions to be drawn from this work may best be done by groups or organizations outside of the engineering societies, in fact in many cases brought into being purely by the exigencies of business. In such cases it will suffice if the engineering societies, who are looked to to sponsor accurate and unbiased rules for any who may wish to use them, review the work of others merely from the standpoint of ascertaining that it is unbiased and represents the just conslusions to be drawn from all ascertainable facts.

In those matters of concern where the questions are broader than the interests of any particular group of engineers, where our own interests as enginerers call for common action, and where the public at large and our legislators and executives have a right to expect an opinion from us as engineers, it seems clear that the efforts of the past few years in this direction must be carried forward as expeditiously as possible and a satisfactory mechanism found. Differences of opinion as to method, and difficulties which arise out of our various schemes of organization and qualifications for membership in the engineering societies, must not be allowed to stand in the way of our early ultimate success. So much publicity has been given to the organization of engineers along this line and so much has been said of the benefits to be derived from this organization that any substantial failure to achieve the results which the public has been led to expect would be forthcoming would I think lead to a state of affairs so disastrous to the engineering profession and to the public welfare that we dare not contemplate it.

Whatever, therefore, may be our individual opinions with regard to the composition or conduct of the Federated American Engineering Societies, which is our existing mechanism in this field, I am certain equally that a method must and can be found for making this organization, by evolutionary rather than revolutionary processes, all that we believe such an organization should be. It needs but a common will to succeed and a willingness to brush aside immaterial obstacles which stand in the way of achieving the ultimate objective.

As to those other interests, already mentioned, of the national societies, in the National Research Council and in the proper conduct of the trust which we have undertaken in the creation of Engineering Foundation, there is little further which need here be said. Our difficulties of the moment arise more from not having a simple, concise, thoroughly understood and agreed to picture of aim and objective than from any fundamental diversity of opinion. Until this objective is achieved and the case is set forward in a way which cannot be misunderstood, until we have organized the administration of our interests satisfactorily in conformity with the objective sought, and until we have established that close association between this administrative machinery and the governing bodies of the founder societies, without which lasting success and healthy development is impossible, it is futile and worse than futile to consider the details of intricate organization or plans for great expansion. The problem is ours and can be solved if we but set ourselves the task. It requires a critical clear-sighted analysis of the objective desired and of the methods needed to reach this objective, combined with a courage to put our conclusions into effect. When we succeed in doing this we will have built upon the groundwork already laid the foundation of a structure which will last through the ages, which will grow with time and which will be of immeasurable benefit, not alone to us of the engineering societies, but to all who are in any way affected by our works.

In conclusion it seems to me that there never was a time in the history of our engineering societies, and particularly never a time in the history of our own Institute, when the necessity for these organizations was greater, nor when their benefit both to the individual and to the community was so thoroughly worth while. The considerations which brought our national engineering societies into being are still and ever will be the primary and chief functions of institutions like our own. But to these basic objects have, with the passing of time, been added other functions the satisfactory handling of which will enhance the value of the national societies, not alone to their members but to the industries with which these members are connected and through them to the public.

Our problem now and for the future, in addition to conducting successfully those things which are clearly ours to perform, is largely one of determining what of the new things continually arising we of all other groups are best qualified to perform, setting our hands to the doing of those things in a manner befitting our profession, and of refraining courageously from undertaking anything which will detract from the doing of those things which are peculiarly ours or which others can do as well or better.

Cable Charge and Discharge

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***Review of the Subject.**—Conductivity determinations of insulating materials, and therefore the determinations of the leakage resistance of a cable, condenser or similar structure, have always been more or less unsatisfactory, due to a phenomenon often called "soaking in of the charges."*

In general, if a constant direct voltage is impressed upon a circuit, in the first moment large transient currents flow, representing the energy storage and adjustment in the magnetic and dielectric fields of the circuit. These transient currents however vanish very quickly, usually in a very small fraction of a second, and all the currents in the circuit or circuits then become constant. If a constant direct voltage is impressed upon a cable or similar structure, large transient currents also flow momentarily; but after these currents of energy adjustment in the electromagnetic field of the system have vanished, usually after a small fraction of a second, the remaining current is not constant, as in the usual electric circuit, but continues to decrease slowly, for minutes or even hours. If then the cable is discharged by short-circuiting it, after the large initial transient discharge current has passed and the voltage on the cable has become zero—in a small fraction of a second—the current coming out of the cable does not entirely vanish, but a small discharge current continues to flow for many minutes or even hours. Or, if the cable has been discharged by short-circuiting it for a short time, until its terminal voltage has become zero, and the short circuit is taken off, a terminal voltage and an electrostatic charge gradually build up again at the cable, reach a maximum after some minutes, and then gradually decrease again.

Various explanations have been proposed of this phenomenon as a hysteresis effect of the dielectric, etc.

It is shown in the following, that this phenomenon of the "soaking in of the charges" or the "electrification of the cable" is a true electric circuit transient, has nothing to do with hysteresis effects, but is the result of, and explained by the energy adjustment of a system with constant values of resistance, inductance and capacity, that is, is in no essential different from the usual rapid starting or stopping transient of the ordinary circuit, except that it is many thousands of times slower. This great slowness is due to the fact that the resistance in the transient circuit is the leakage resistance of the cable dielectric, and therefore extremely high, measured not in ohms but in hundreds of megohms.

If the dielectric of the cable consists of two or more materials, having different resistivities, or different specific capacities, or both, then the distribution of voltage through the dielectric gradually changes. At the moment of voltage application, the voltage distributes between the component dielectrics in proportion to their specific capacities; gradually however this voltage distribution changes to a distribution proportional to the resistivities of the component dielectrics, and electrostatic charges build up in the interior of the dielectric, at the boundaries between the component dielectrics. The electric quantity has to be conducted through the dielectric, and due to the very high resistivity of the dielectric, this energy readjustment within the dielectric occurs with extreme slowness, giving a transient of a duration of many minutes, but a true electric transient nevertheless, like the usual transients of a duration of milliseconds.

In the discharge of the cable, the reverse occurs, and the internal charges gradually disappear by conduction through the dielectric.

This phenomenon is discussed in its various aspects, and the equations of the slow transient derived, in the following:

It is shown that this slow transient always is impulsive or exponential, and consists of (n—1) terms, if the insulation is composed of n materials.

A number of conclusions are derived, some of which may be pertinent with regard to the mechanism of breakdown of insulation.

By this slow transient, the energy stored in the cable as condenser increases. Its apparent capacity increases. The conduction current, and therefore the $i^2 r$ losses in the dielectric, are much larger with alternating than with direct voltage. The distribution of dielectric stress between the component dielectrics changes and this may give a different dielectric strength for alternating voltage—at which the initial condition of stress distributions is permanent—and for direct voltage. Also, a time lag of insulation breakdown may result, which is not a temperature or deterioration effect. Under certain conditions, there may occur a negative time lag, that is, a condition where the insulation stands higher voltages when slowly and continuously applied than when rapidly applied for a short time.

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I. Physical Consideration

1. CABLE CHARGE

If an ideal condenser, that is, a condenser having constant capacity, $C = \frac{1}{k}$, and no losses, is connected to a source of alternating voltage, e_0, over a circuit of constant resistance, r_0, and constant inductance, L_0, it takes a charging current, which leads the terminal voltage by 90 degrees. This charging current does not pass through or into the dielectric of the condenser, but is limited to its terminals. It is preceded by a transient immediately after closing the circuit. This charging transient is oscillatory, if the circuit resistance r_0 is below, and is impulsive, if the circuit resistance is above the critical value $2\sqrt{\frac{L_0}{C}}$. This transient charging current usually is much larger than

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*Deceased.

the permanent alternating charging current, and usually vanishes in a small fraction of a second.

If the impressed voltage is continuous, the same starting transient occurs, but with its disappearance, after a fraction of a second, the current at the condenser terminals vanishes and the condenser acts as an open circuit on the continuous supply voltage, if the resistance of the dielectric of the condenser is really infinite.

If however the dielectric of the condenser has a finite, though very high, resistance r, at continuous impressed voltage e_0 the starting transient dies down not to zero, but to a finite limiting current $i_0 = \frac{e_0}{r}$. With an alternating impressed voltage, the charging current then does not lead the terminal voltage by exactly 90 degrees, but slightly less, and therefore contains an energy component of current i_0'. This energy current, though small compared with the total charging current, usually is larger than the conduction current i_0 which passes through the dielectric with a continuous voltage, as it represents not only the conduction through the dielectric, but also the losses in the dielectric due to the alternating field, of the nature of a dielectric hysteresis, etc.

This applies to an ideal condenser, that is, a condenser with perfectly uniform dielectric.

Tests show that in a real condenser, such as a cable, with a continuous impressed voltage e_0, the current remaining after the passage of the starting transient that is, after a fraction of a second, usually is not the conduction current $i_0 = \frac{e_0}{r}$, and is not constant, but is larger than i_0, often many times, and gradually but slowly decreases, for minutes and even hours, approaching the conduction current i_0. There is thus a passage of current through and into the dielectric, for a considerable time. This often is expressed as the "soaking in" of current or of charge.

The resistance of the cable insulation therefore cannot be measured by impressing a continuous voltage and reading the current, but constant voltage has to be kept on the cable until the current has become constant. Usually this is expressed by saying that the cable has first to be electrified.

2. Cable Discharge

A similar difference occurs between the ideal condenser and the cable in the discharge.

If an ideal condenser, charged to terminal voltage e_0, is discharged by short-circuiting its terminals, the discharge transient passes, usually within a fraction of a second, current and terminal voltage drop to zero and the condenser is dead. In a real condenser, as a cable however, after the passage of the discharge transient a small discharge current continues to flow, gradually decreasing, for minutes and even hours. Furthermore, if after the external discharge transient has passed and the terminal voltage dropped to zero, that is, after a fraction of a second (and usually also still after several seconds) the circuit of the cable is opened, the terminal voltage does not remain zero, but a terminal voltage reappears and gradually rises, for minutes, and corresponding charges build up on the condenser terminals, until a maximum "residual voltage" and "residual charge" has accumulated; then voltage and charge again decrease and finally vanish, often after many minutes or even hours. This residual charge may be quite considerable, sufficient to give dangerous shocks.

This phenomenon in the charge of a condenser, of the soaking in of the current and the formation of residual charges and residual voltages after discharge, does not seem to occur when the dielectric of the condenser is perfectly homogeneous, but is very pronounced when the dielectric is a composite of different materials of different electric characteristics (resistivity and specific capacity), especially if the structure of the dielectric is laminated, as in a cable.

This phenomenon is not always fully understood, and various attempts have been made to explain it as an effect of dielectric time lag, or as the result of a change of the specific capacity with time and voltage, as a hysteresis effect, etc. This is not so however, but it is a true circuit transient, as will be seen in the following, that is, a readjustment of the stored electric energy to changed circuit conditions, at constant values of circuit constants, r and C. It has nothing to do with hysteresis or time lag. The great interest of this slow transient consists in its very long duration, sometimes reaching into hours. This makes it many thousand times slower than the external condenser transient.

In a condenser with composite dielectric, like the laminated dielectric of the cable, energy storage occurs —and therefore results in transients at a change of circuit conditions—in the dielectric, at the boundaries between different dielectrics. This energy must be conducted over the resistance of the dielectric, and as this resistance is extremely high, the duration of this "internal transient," that is, "transient of internal energy readjustment," must be very great, compared with the duration of the "external transient," in which latter, the energy readjustment occurs over the resistance of the external or supply circuit; the latter resistance is measured in ohms; that of the internal transient in hundreds of megohms.

This great difference in duration, between the external transient of readjustment of terminal voltage to changed circuit conditions, and the internal transient of readjustment of internal charges within the dielectric, makes it possible to separate the two types of transient and treat them separately.

The external transient has passed and vanished, before the internal transient has appreciably started. Therefore, in dealing with the internal transient, the external transient may be considered as non-existent, but the final circuit conditions brought about by the external transient (constant terminal voltage e_0) considered as the initial circuit condition for the internal transient. Inversely, when dealing with the external transient, the internal transient may be considered as non-existing, that is, its initial circuit condition considered as permanent.

Approximate magnitudes of the quantities are, for a 12,000-volt, 2-mile, 60-cycle, 3-phase cable at 20 kv. continuous impressed voltage:

External transient: 25 amperes, 240 cycles.

Duration of external transient: 0.005 sec.

60 cycles charging current: 5 ampere maximum.

Energy component thereof: 0.2 amperes.

Initial value of slow transient: 0.37 to 2.4 milliamperes, depending on age and temperature.

Final value of slow transient: 0.11 to 1.28 milliamperes, depending on age and temperature.

Duration of slow transient: 4 min.

Residual charge: 8 kv.

3. FORMATION OF INTERNAL CHARGES

Let the dielectric of a condenser be composite, consisting of layers of two materials—as impregnated paper and petrolatum in the cable—of different specific capacities and resistivities.

Let now the circuit be closed and a continuous voltage be impressed upon the condenser. By a rapid external transient, in a small fraction of a second, the terminal voltage of the condenser then becomes constant at the value e given by the supply voltage.

An electrostatic field K thus appears between the condenser terminals giving an electrostatic flux density D in the dielectric between the condenser terminals, and an electrostatic charge Q on these terminals.

At the first moment of terminal voltage e, there can be no electrostatic charge inside of the dielectric, since such charge would have to pass through a part of the dielectric, and due to the very high resistance of the dielectric, it takes a much longer time to conduct such internal charge through the dielectric, than the time required by the external transient to produce the terminal voltage e by supplying the terminal charges Q. Thus in the first moment of terminal voltage e no internal charge can yet exist.

A line of electrostatic flux must either terminate in an electrostatic charge, or continue into infinity or into itself. Since in the first moment of terminal voltage e, or at time $t = 0$, there can be no electrostatic charge inside of the dielectric, no line of electrostatic flux can terminate in the dielectric, and all lines of electrostatic flux therefore must pass through from terminal to terminal, in the first moment of terminal voltage e. That is, the flux density D between the cable terminals must in the first moment be uniform, and the initial voltage distribution between the component dielectrics thus must be such as to give the same flux density. That is, the voltage must distribute between the different layers of the dielectric in proportion to their elastances, that is, the reciprocals of their capacities. That is, the voltage gradient in the two dielectrics must be proportional to the elastivities, or inversely proportional to the specific capacities of these two dielectrics. They would remain so, if there were no conductivity, that is, if the resistance of the dielectric were infinitely high. The transient thus were ended, and no further change occurs.

Due to the finite, (though extremely low) conductivity of the dielectrics, a current is conducted, or "leaks" through the dielectrics, and the current density of this current in the two dielectrics, is proportional to the electrical conductivities of these dielectrics and proportional to the voltage gradients in them. The initial voltage gradients however are proportional to the elastivities of the two dielectrics. The elastivities have no direct relation to the conductivities, and high resistivity may concur just as well with low elastivity, as in sulphur, as it may concur with high elastivity, in air, and inversely. It therefore is obvious that in general the current densities in the two dielectrics would not be equal. They would be equal only if it happened that the conductivities of the two dielectrics are proportional to their elastivities.

In general we therefore find, in the first moment of constant terminal voltage on the cable as condenser with composite dielectric:

All electrostatic charges are on the terminals.

Uniform and equal electrostatic flux density in both dielectrics.

No internal charges in the dielectrics.

Different current densities in the two dielectrics.

Voltage distribution between the two dielectrics by their respective capacities, that is,

Voltage gradients proportional to elastivities.

In general thus, in the initial moment of constant terminal voltage on a cable, the current densities in the two component dielectrics are different. This means that at every boundary between the two dielectrics a discontinuity or change of current density occurs, and more current flows towards the boundary from the one side, than leaves it on the other side. Consequently, an electrostatic charge builds up at the boundary between the two dielectrics, due to the difference between the two current densities on the two sides of the boundary. This electrostatic charge lowers the voltage gradient and thus the current density in the dielectric of higher current density, and raises it in the dielectric of lower current density, and thereby reduces the difference between the current densities in the two dielectrics and with it reduces the rate of building up of internal electrostatic charges on the boundaries between the two component dielectrics, until finally the current densities in both dielectrics become equal and any further building up of internal charge ceases.

The final condition thus is that of uniform current density in both component dielectrics. That means, the voltage gradients in the two dielectrics are proportional to their respective resistivities, and therefore independent of the capacities or elastivities of the two dielectrics. The electrostatic flux densities are different in the two component dielectrics. Lines of electrostatic flux terminate within the dielectric, at the boundaries between the two component dielectrics, and they terminate in electrostatic charges located at these boundaries.

4. The Slow Cable Transient

Thus there occurs, at constant terminal voltage, a gradual change in the dielectric of a cable as condenser with laminated composite dielectric, from an initial condition immediately after the appearance of the constant terminal voltage at the condenser, until final condition is reached, by a slow transient lasting many minutes and even hours. This consists of:

A change from the initial voltage distribution by capacity, to the final voltage distribution by the resistance of the component dielectrics.

A change from uniform electrostatic flux density and ununiform current density, to uniform current density and ununiform flux density.

The formation of internal charges at the boundaries between the two component dielectrics.

A current passes through the dielectric. This starts at a large value (though small compared with the external transient current) and first rapidly and then more slowly decreases, to a final minimum value of conduction current.

It thus consists of a permanent component, which is the conduction current sent by the terminal voltage through the resistance of the dielectric, and a transient component, by which the terminal voltage supplies the internal charges over a part of the resistance of the dielectric.

The energy of the permanent current is dissipated as heat in the resistance of the dielectric.

Half of the energy of the transient current is stored as the electrostatic energy of the internal charges at the boundaries between the two dielectrics, the other half is dissipated as heat in the resistance of the dielectric, over which the internal charges have to be conducted.

The energy electrostatically stored in the condenser therefore increases during the slow transient of internal charge, that is, of "soaking in of the charges," by half the energy of the transient component of the slow transient, and the apparent or effective capacity of the condenser correspondingly increases.

That is, the energy stored in the condenser, and the capacity of the condenser, reach a maximum at the end of the slow charge, when the conduction current has become a minimum. The stored energy and the capacity are a minimum at the beginning of the internal charge, that is, in the first moment of constant terminal voltage at the condenser. The latter also is the case with an alternating supply voltage, since even at low machine frequencies the duration of a cycle is short compared with the duration of the slow transient, and the internal conditions of the condenser (such as voltage distribution by capacity and not by resistance) with alternating terminal voltage therefore correspond to the initial conditions of constant terminal voltage.

A condenser with composite dielectric thus does not have a constant capacity, but has one value of capacity —the minimum—for instantaneous voltage application or alternating terminal voltage, and another value—the maximum—for continuous voltage application.

A condenser with composite dielectric thus differs from a condenser with uniform dielectric in that in the latter energy is stored as electrostatic charge at the external terminals only, while in the former energy is stored as electrostatic charge in the dielectric also, and this latter energy can be supplied—or withdrawn, in the discharge—very slowly only, as it has to flow over the higher resistance of the dielectric, and thereby gives rise to the slow terminal transient, which often is misunderstood as a hysteresis or lag phenomenon, but is not such, but is a true resistance phenomenon, and as such independent of the frequency, but calculated from the ratio of resistances and capacities of the two component dielectrics.

5. Effect on Puncture Voltage

As seen, at constant terminal voltage, the voltage distribution between the two component dielectrics changes between the initial and the final condition. That is, in the one dielectric the voltage gradient is higher in the initial conditions (and thus with alternating terminal voltage) than in the final conditions, and the reverse is the case in the other dielectric. Suppose now the first dielectric is operated closer to the limit of its dielectric strength. If then the terminal voltage is brought up rapidly, so that the initial condition of internal voltage distribution persists, at a certain terminal voltage the first dielectric will reach its disruptive gradient, and puncture of the insulation occurs. Suppose however, the voltage is brought up very slowly, so that throughout the entire voltage rise the final voltage distribution between the two dielectrics persists. At this final voltage distribution, the voltage gradient in the first dielectric is lower, for the same terminal voltage, and the disruptive gradient in the first dielectric is therefore reached only at a higher value of terminal voltage. That is, in this case a higher value of puncture voltage is required when the voltage is very slowly applied, than when the voltage is rapidly increased. This is the reverse of the usual experience—in general, the more rapidly the voltage is increased, the higher voltage the insulation will stand. However, this phenomenon of a lower voltage being sufficient for breakdown when rapidly applied than when slowly applied, has been repeatedly observed with composite insulation. In this case, a constant voltage, applied at once, may break down the insulation, while even a higher voltage when very slowly applied, may not damage it—in the former case, the dielectric passes through the initial stage of voltage distribution, where it is overstressed, while in the latter case it has passed this stage at lower terminal voltage, while still below breakdown voltage, and at the higher terminal voltages had relieved itself by the readjustment of voltage throwing a higher gradient on the second and stronger dielectric.

Inversely, if the weaker dielectric is the second one, in which the voltage gradient in the final condition is higher than in the initial condition, then with a rapidly increasing terminal voltage the insulation may show a much higher puncture voltage than with a very slowly increasing voltage, since in the latter case the same terminal voltage gives a higher gradient and thus higher stress across the second dielectric, than the same terminal voltage so rapidly applied that the initial condition persists. In this case, the insulation therefore would show a marked time lag of disruptive strength, while in the above case we may speak of a "negative time lag," that is, slow voltage application increases the disruptive strength.

It is interesting to note that here, in the second case, we get a time lag of breakdown, which is not a temperature effect, but a straight resistance phenomenon, and get it under the assumption that the relation between breakdown and breakdown gradient is instantaneous, that is, that the breakdown occurs as soon as the breakdown gradient is reached.

6. A-C./D-C. Ratio of Puncture Voltage

With alternating terminal voltage the internal conditions of the insulation always are the initial conditions. In the second case, when the voltage gradient increases with the time in the dielectrically weaker material, an alternating terminal voltage would give the same voltage gradient as exists in the initial conditions of a constant voltage equal to the maximum of the alternating voltage, and lower than the final voltage gradient of the constant voltage. Thus, if the breakdown is strictly mechanical, due to voltage stress, and no additional causes of breakdown occur with alternating voltage, then puncture would occur with an alternating voltage of a maximum equal to the constant voltage rapidly applied, but a very slowly applied constant voltage would cause a breakdown at a lower value of voltage than the maximum of alternating puncture voltage. That is, we would get a d-c./a-c. ratio of less than one. While this is against the usual experience it has been observed in a number of instances.

Dominating in the insulation puncture comes in the temperature effect, that is, local temperature rises which materially lower the breakdown voltage. As with alternating voltage energy losses occur, such as dielectric hysteresis, etc., which do not exist with constant voltage, local heating would be greater and the puncture voltage therefore lower with alternating voltage.

Thus there are three different conditions met in a composite dielectric, which have no constant direct numerical relation with each other:

1. Very slowly applied constant voltage, giving an internal voltage distribution by resistance.
2. Fairly rapid constant voltage application, giving an internal voltage distribution by capacity.
3. Alternating current voltage supply, giving the same voltage distribution as 2, but with additional energy losses and heating, the latter depending on the rate of voltage application.

The ratio of puncture voltage, d-c. and maximum a-c. in the three cases, may have any value between less than one and two to three or even more, depending on the relative constants of the two component dielectrics, on time and on temperature, etc., and research in this direction should throw a great deal of light on the mechanism of breakdown.

7. A-C. and D-C. Resistance of Dielectric

With alternating terminal voltage the internal conditions of the composite dielectric are the initial conditions of constant voltage electrification. It follows, that with alternating terminal voltage the conduction current which passes into the dielectric is not the final or permanent value of the conduction current at constant terminal voltage, but is the—usually much larger—initial or transient value of conduction current, that is, the initial value of the slow charging transient, and the $i^2 r$ loss in the dielectric therefore is much larger under alternating terminal voltage, than under continuously applied constant terminal voltage.

Thus, assuming that there were no other losses but conduction losses in the cable dielectric under alternating voltage, that is, hysteresis, ionization, etc., were absent, then the energy component of the alternating cable charging current would be—usually much—larger than the conduction current with constant terminal voltage, and the resistance of the dielectric, from energy current and alternating terminal voltage, would be much lower than the resistance measured with direct current. Nevertheless, the alternating energy current and the resistance calculated from it are a true conduction current and true ohmic resistance effect, and are independent of the frequency, though they are often misunderstood and misinterpreted as time lag or hysteresis effects. As seen, they are ohmic resistance effects of the circuit between the internal charges and the external circuit. In addition thereto occur probably true hysteresis effects, ionization effects in air by ozonization, similar ionization effects in other materials, etc., which are still little understood. Also the effects of moisture and its movement under the dielectric field.

CABLE DISCHARGES

Suppose a cable as laminated composite condenser of two dielectrics, is charged to a constant terminal voltage, and stationary conditions of voltage distribution and internal charges reached.

We now short-circuit the cable.

By the rapid external transient it discharges, within a small fraction of a second, and terminal voltage and terminal charge drop to zero. In the first moment, the internal charges, at the boundaries of the two dielectrics, remain the same. These internal charges now discharge their energy—which was half the energy of the slow charging transient, as seen above—towards the cable terminals, through the very high resistance of the dielectric, by a slow transient, a current which starts with a maximum (though very small compared with the external discharge transient) and gradually decreases to zero.

Or we just disconnect the cable from the supply voltage, leaving it open-circuited, but charged. The terminal charges and the internal charges then supply the conduction current, which passes through the dielectric, and current, charges, terminal voltage and internal voltages decrease and finally die out gradually and proportionally to each other, on small exponential transients.

The most interesting case is to discharge the cable terminals by short-circuiting for a short time (a fraction of a second) until the external cable discharge transient has passed and terminal volts and terminal charges dropped to zero, but before any appreciable discharge of the internal charges has occurred, and then to open the circuit again. This leaves the cable with zero terminal voltage and zero terminal charge, but with internal charges at the boundaries between the two component dielectrics, and corresponding internal voltages. That is, by the withdrawal of the terminal charges the internal charges as bound charges are set free and their corresponding voltages appear. These voltage gradients produce currents through the resistance of the dielectrics. These slow transient currents decrease the internal charges and build up external or terminal charges, which appear as terminal voltage. Thus a transient terminal voltage gradually builds up, increases to a maximum and then decreases again and gradually dies down to zero. Of the energy of the internal charges, half is stored as terminal charge giving residual terminal voltage:—the other half is dissipated in the resistance of the dielectric in conducting the charge through the dielectric. Assuming that the energy stored by the internal charge was half the energy of the terminal charges of the condenser. When building up a residual terminal charge, this then would contain half the energy of the internal charges, or one-quarter the energy and therefore half the voltage of the initial terminal charges. Thus a very considerable residual voltage, capable of giving dangerous shocks, may build up on a composite condenser like a cable.

8. CONCLUSIONS

In the preceding we have described the simplest case, of a laminated condenser consisting of two different dielectrics, and thus giving rise to a slow transient consisting of one exponential term.

If the condenser comprises n different dielectrics, the slow transient consists of $(n - 1)$ exponential terms, thus is more complex, but otherwise not essentially different. Observation of such slow transients on three-phase cables seems to show that they are not well represented by a single exponential slow transient, but are fairly well represented by the combination of two slow transients, the one being somewhat faster than the other. Such a case is shown in Fig. 1, with the observed currents marked by crosses. It gives three curves of the same cable, for three successive tests at different periods of the year and therefore different temperatures. This would correspond to a composite condenser having three different kinds of dielectric. In a three-phase cable we have impregnated paper as main insulation, petrolatum as impregnating material, and impregnated hemp in the triangular corner pieces between conductors and armor.

Structurally, we have considered a laminated shape, first, because it is mathematically by far the simplest case, and second, because most of the industrially important insulations are laminated; cable, condenser, high-voltage machine insulation, etc. In general, similar relations may be expected to exist in an irregularly unhomogeneous dielectric.

II. Mathematical Investigation

1. CABLE CHARGE

In the charge, or discharge, of a leaky laminated condenser, as a cable, two transients occur, an external transient giving the charge or discharge of the cable as a true condenser of constant capacity, over an external circuit containing resistance and inductance, and an internal transient, giving a readjustment of the internal charges and voltages over the resistances of the laminated dielectric.

The internal transient is so very much slower, that the external transient has completely died out, before the internal transient has appreciably started. As the result thereof, in investigating the internal transient, we may assume that the external transient has passed completely at the moment from which we began to count time for the internal transient.

The external transient is the usual transient of a condenser charge or discharge through an inductive circuit, thus of no further interest here.

The terminal conditions of the internal transient are: constant voltage impressed upon the dielectric:

$$e_1 + e_2 = e \qquad (1)$$

where e_1 is the total voltage across all the layers of the first dielectric, e_2 the total voltage across all the layers of the second dielectric:

r_1 = resistance, $k_1 = \frac{1}{C_1}$ = elastance and i_1 = current in the first dielectric, and

r_2 = resistance, $k_2 = \frac{1}{C_2}$ = elastance and i_2 = current in the second dielectric, and

i = external current.

The voltage e_1 across the first dielectric produces the current i_1 in the resistance r_1 of the first dielectric, and this gives

the conductance equations

$$\left.\begin{aligned} e_1 &= r_1\, i_1 \\ e_2 &= r_2\, i_2 \end{aligned}\right\} \tag{2}$$

With the external current i, and the current i_1 in the first dielectric, the current $(i - i_1)$ stops at the boundary of the first dielectric and there produces an electric charge and thereby a voltage change $\frac{d\,e_1}{d\,t}$. This gives:

the charging equations

$$\left.\begin{aligned} \frac{d\,e_1}{d\,t} &= k_1\,(i - i_1) \\ \frac{d\,e_2}{d\,t} &= k_2\,(i - i_2) \end{aligned}\right\} \tag{3}$$

Substituting (2) into (3) gives:

$$\left.\begin{aligned} \frac{de_1}{d\,t} &= k_1\, i - \frac{k_1}{r_1}\, e_1 \\ \frac{d\,e_2}{d\,t} &= k_2\, i - \frac{k_2}{r_2}\, e_2 \end{aligned}\right\} \tag{4}$$

(1) differentiated gives:

$$\frac{d\,e_1}{d\,t} + \frac{d\,e_2}{d\,t} = 0 \tag{5}$$

and, substituting herein (4), gives

$$k\,i = \frac{k_1}{r_1}\, e_1 + \frac{k_2}{r_2}\, e_2 \tag{6}$$

where

$$k = k_1 + k_2 \tag{7}$$

Differentiating (6) and substituting (3) therein, gives:

$$- k\,\frac{d\,i}{d\,t} + \left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2}\right) i = \left(\frac{k_1}{r_1}\right)^2 e_1 + \left(\frac{k_2}{r_2}\right)^2 e_2 \tag{8}$$

(1), (6) and (8) give us three equations in the three variables i, e_1 and e_2. Eliminating e_1 and e_2 from these three equations, leaves one equation in i, as the differential equation of the internal transient.

Multiplying (1) with $\frac{k_1\,k_2}{r_1\,r_2}$;

(6) with $-\left(\frac{k_1}{r_1} + \frac{k_2}{r_2}\right)$, and adding to (8) gives:

$$k\,\frac{d\,i}{d\,t} + k_1\,k_2\left(\frac{1}{r_1} + \frac{1}{r_2}\right) i - e\,\frac{k_1\,k_2}{r_1\,r_2} = 0 \tag{9}$$

as the differential equation in i. This is integrated by:

$$i = A + B\,\epsilon^{-ct} \tag{10}$$

(10) substituted into (9) gives:

$$B\,\epsilon^{-ct}\left\{c\,k - k_1\,k_2\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right\} - A\,k_1\,k_2\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + e\,\frac{k_1\,k_2}{r_1\,r_2} = 0$$

thus:

$$A = \frac{e}{r_1 + r_2} = \frac{e}{r} \tag{11}$$

where:

$$r = r_1 + r_2$$

and

$$\left.\begin{aligned} c &= \frac{k_1\,k_2}{k}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \\ &= \frac{\frac{1}{r_1} + \frac{1}{r_2}}{\frac{1}{k_1} + \frac{1}{k_2}} \\ &= \frac{g_1 + g_2}{C_1 + C_2} \end{aligned}\right\} \tag{13}$$

where g_1 and g_2 are the conductances and C_1 and C_2 the capacities of the two dielectrics.

From equations (1) and (6) follows:

$$\left.\begin{aligned} e_1 &= \frac{\frac{k_2}{r_2}\, e - k\,i}{\frac{k_2}{r_2} - \frac{k_1}{r_1}} \\ e_2 &= \frac{\frac{k_1}{r_1}\, e - k\,i}{\frac{k_1}{r_1} - \frac{k_2}{r_2}} \end{aligned}\right\} \tag{14}$$

The coefficient B in (10) follows from the terminal conditions, that at $t = 0$, the voltage distribution in the dielectric is that due to capacity only. That is: $t = 0$:

$$\begin{aligned} e_1 + e_2 &= k_1 + k_2 \\ i &= A + B \end{aligned} \qquad (15)$$

Substituting this into (14), and (14) into (15) gives:

$$A + B = \frac{e}{k^2}\left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2}\right) \qquad (16)$$

and since

$$A = \frac{e}{r} \qquad (11)$$

it is:

$$B = \frac{e}{k^2}\left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2} - \frac{k^2}{r}\right) \qquad (17)$$

thus:

$$i = \frac{e}{r} + \frac{e}{k^2}\left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2} - \frac{k^2}{r}\right)\epsilon^{-\frac{g_1+g_2}{C_1+C_2}t}$$

$$\left.\begin{aligned} e_1 &= \left\{\frac{r_1}{r} - \left(\frac{r_1}{r} - \frac{k_1}{k}\right)\epsilon^{-\frac{g_1+g_2}{C_1+C_2}}\right\} e \\ e_2 &= \left\{\frac{r_2}{r} - \left(\frac{r_2}{r} - \frac{k_2}{k}\right)\epsilon^{\frac{g_1+g_2}{C_1+C_2}t}\right\} e \end{aligned}\right\} \qquad (18)$$

This gives the *initial conditions* of current and fractional voltages: $t = 0$:

$$\left.\begin{aligned} i^0 &= \frac{e}{k^2}\left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2}\right) \\ e_1^0 &= \frac{k_1}{k} e \\ e_2^0 &= \frac{k_2}{k} e \end{aligned}\right\} \qquad (19)$$

and the *final conditions* of current and fractional voltages: $t = \infty$:

$$\left.\begin{aligned} i' &= \frac{e}{r} \\ e_1' &= \frac{r_1}{r} e \\ e_2' &= \frac{r_2}{r} e \end{aligned}\right\} \qquad (20)$$

thus:

$$e_1' + e_2' = r_1 + r_2$$

2. ENERGY STORAGE AND APPARENT CAPACITY

The energy stored by the external transient on the terminals of the cable as perfect condenser of capacity $C = \frac{1}{k}$ is:

$$\left.\begin{aligned} W_0 &= \frac{e^2 C}{2} \\ &= \frac{e^2}{2k} \end{aligned}\right\} \qquad (21)$$

The energy stored by the two component dielectrics of the cable as composite condenser, is in the *initial conditions*, for $t = 0$:

$$\left.\begin{aligned} W_1^0 &= \frac{e_1^{02}}{2k_1} \\ &= \frac{e^2 k_1}{2k^2}\,. \\ W_2^0 &= \frac{e^2 k_2}{2k^2} \end{aligned}\right\} \qquad (22)$$

and the total stored energy, in the initial conditions therefore is:

$$\left.\begin{aligned} W^0 &= W_1^0 + W_2^0 \\ &= \frac{e^2}{2k} \\ &= W_0 \end{aligned}\right\} \qquad (23)$$

that is, the same as in an ideal condenser, as was to be expected.

The energy stored by the two component dielectrics of the cable in the *final conditions, for* $t = \infty$, is:

$$\left.\begin{aligned} W_1' &= \frac{e_1'^2}{2k_1} \\ &= \frac{e^2 r_1^2}{2k_1 r^2} \\ W_2' &= \frac{e^2 r_2^2}{2k_2 r_2} \end{aligned}\right\} \qquad (24)$$

and the total energy stored in the composite condenser in its final conditions, is

$$\left.\begin{aligned} W' &= W_1' + W_2' \\ &= \frac{e^2}{2r^2}\left(\frac{r_1^2}{k_1} + \frac{r_2^2}{k_2}\right) \end{aligned}\right\} \qquad (25)$$

The ratio of the final to the initial joules of stored energy, and therefore of the effective capacity in the final conditions, to the capacity in the initial conditions (which latter is the capacity as ideal condenser, and the capacity under an alternating terminal voltage) thus is:

$$\left.\begin{aligned} \frac{C'}{C} &= \frac{W'}{W^0} \\ &= \frac{\frac{r_1^2}{k_1} + \frac{r_2^2}{k_2}}{\frac{r^2}{k}} \end{aligned}\right\} \qquad (26)$$

The difference of the stored energy in the final conditions, over that in the initial conditions, that is, the increase of stored energy from initial to final conditions, then is:

$$\left.\begin{aligned} W_i &= W' - W^0 \\ &= \frac{e^2 (r_1 k_2 - r_2 k_1)^2}{2k r^2 k_1 k_2} \end{aligned}\right\} \qquad (27)$$

The energy supplied by the internal transient is:

$$\left.\begin{aligned} W_i &= \int_0^\infty e\,B\,\epsilon^{-ct}\,d\,t \\ &= \frac{e\,B}{c} \\ &= \frac{e^2\,(r_1\,k_2 - r_2\,k_1)^2}{k\,r^2\,k_1\,k_2} \\ &= 2\,W_i \end{aligned}\right\} \quad (28)$$

That is, of the energy supplied by the internal transient W_i, half is stored in the dielectric as the electrostatic energy of the internal charge, W_i and the other half is dissipated as heat in the dielectric, in the $i^2\,r$ of the current which conducts these charges through the dielectric.

The efficiency of formation of the internal charges thus is 50 per cent.

3. CABLE CHARGE BY ALTERNATING VOLTAGE

At first we may neglect the losses by dielectric hysteresis, by ionization in air and other materials, by the mechanical motion of moisture and all the other losses occurring in an alternating field, which do not exist in a constant field; that is, consider the ohmic resistances of the two component dielectrics as the only source of loss.

As we are interested only in the permanent condition of alternating cable discharge, we may also omit the initial or external transient, since this is the same as with constant supply voltage and is the same as the rapid charging transient of the ideal condenser. That is, assume that the external transient has passed and permanent conditions of the alternating terminal voltage been established.

The cable charging equations then are the same as those with constant impressed voltage, except that the terminal voltage e is not constant, but is alternating, and may be expressed by:

$$e = E \sin q\,t \quad (29)$$

where:

$$\begin{aligned} q &= 2\,\pi\,f \quad (30) \\ &= 377 \text{ with } f = 60 \text{ cycles} \end{aligned}$$

The equations then are:

the *external* equation (1):

$$E \sin q\,t = e_1 + e_2 \quad (31)$$

the *internal* equations (2):

$$\left.\begin{aligned} \frac{d\,e_1}{d\,t} &= k_1\,i - \frac{k_1}{r_1}\,e_1 \\ \frac{d\,e_2}{d\,t} &= k_2\,i - \frac{k_2}{r_2}\,e_2 \end{aligned}\right\} \quad (32)$$

Differentiating (31) and substituting (32) therein, and then repeating the same operation gives the two equations corresponding to (6) and (8):

$$q\,E \cos q\,t + \left(\frac{k_1}{r_1}\,e_1 + \frac{k_2}{r_2}\,e_2\right) = k\,i \quad (33)$$

$$q^2\,E \sin q\,t + \left(\frac{k_1^2}{r_1^2}\,e_1 + \frac{k_2^2}{r_2^2}\,e_2\right) = \left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2}\right) i - k\,\frac{d\,i}{d\,t} \quad (34)$$

Multiplying (31) with $\frac{k_1\,k_2}{r_1\,r_2}$; (33) with $-\left(\frac{k_1}{r_1} + \frac{k_2}{r_2}\right)$, and adding to (34), eliminates e_1 and e_2, and leaves the differential equation for i:

$$k\,\frac{d\,i}{d\,t} + k_1\,k_2\left(\frac{1}{r_1} + \frac{1}{r_2}\right) i = E\left\{\left(\frac{k_1\,k_2}{r_1\,r_2} - q^2\right)\sin q\,t + q\left(\frac{k_1}{r_1} + \frac{k_2}{r_2}\right)\cos q\,t\right\} \quad (35)$$

This is integrated by:

$$i = C_1 \cos q\,t + C_2 \sin q\,t \quad (36)$$

Substituting (36) into (35) and equating the coefficients of $\sin q\,t$ and $\cos q\,t$ to zero, gives:

$$\left.\begin{aligned} C_1 &= q\,E\,\frac{k\,q^2 + k_1\,k_2\left(\frac{k_1}{r_1^2} + \frac{k_2}{r_2^2}\right)}{k^2\,q^2 + k_1^2\,k_2^2\left(\frac{1}{r_1} + \frac{1}{r_2}\right)^2} \\ C_2 &= E\,\frac{q^2\left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2}\right) + \frac{k_1^2\,k_2^2}{r_1\,r_2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}{k^2\,q^2 + k_1^2\,k_2^2\left(\frac{1}{r_1} + \frac{1}{r_2}\right)^2} \end{aligned}\right\} \quad (37)$$

In the expressions (37), the r, as resistances of the dielectric, are very large quantities, and in first approximation, the terms containing r in the denominator, can therefore be neglected, giving:

$$\begin{aligned} C_1 &= \frac{q\,E}{k} \\ &= 2\,\pi\,f\,C\,E \quad (38) \end{aligned}$$

which is the standard expressions of the alternating reactive charging current of a condenser of capacity C, and:

$$C_2 = \frac{E}{k^2}\left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2}\right) \quad (39)$$

(39) is $A + B$ of equation (16), that is, it is the initial value of the slow transient charging current of constant voltage E. That is:

The energy component of the alternating charging current of a conductor with composite dielectric, equals the initial charging current of the condenser at the same continuous impressed voltage, that is, the initial value of the slow constant voltage charging transient, and therefore is larger, often many times, than the true or final conduction current A, which passes through the condenser at constant voltage.

Correspondingly higher also are the $i^2 r$ losses, so that in a cable, the $i^2 r$ losses due to the true ohmic resistance of the dielectric, are higher with alternating than with continuous voltage.

The alternating charging current thus is:

$$\left.\begin{aligned} i &= \frac{q E}{k} \cos q t + \frac{E}{k^2} \left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2} \right) \sin q t \\ &= 2 \pi f C E \cos q t + (A + B) \sin q t \end{aligned}\right\} \quad (40)$$

Substituting (40) into (31) and (33) gives:

$$\left.\begin{aligned} e_1 &= \frac{k_1}{k} E \sin q t \\ e_2 &= \frac{k_2}{k} E \sin q t \end{aligned}\right\} \quad (41)$$

4. CABLE DISCHARGE

When a leaky laminated condenser, such as a cable, is charged at a continuous voltage e and kept at this voltage until stationary conditions are reached, that is, the adjustment of internal charges and voltages is completed—which may take hours—and the impressed voltage e then withdrawn and the cable discharged, the discharge transient again in general consists of two components, the external transient of short duration and the internal transient of long duration, and the two transients may be dealt with separately, as the duration, of the internal transient usually is so many thousand times longer than that of the external transient, that the external transient has entirely passed, before the internal transient has appreciably begun.

The external transient thus can be treated as the discharge of a typical condenser through an external circuit, by the standard equations, thus is of no further interest here. The investigation of the internal transient, just as in the condenser charge, can be greatly simplified by separating it from the external transient, that is, counting the time from the moment when the external transient has passed, and thus using as terminal conditions for $t = 0$ the circuit conditions left after the vanishing of the external transient, but before the beginning of the internal transient.

A leaky laminated condenser, such as cable, may be discharged

1. by short-circuiting it.
2. by open-circuiting it.
3. by closing the external circuit by a resistance r_0 (or a resistance r_0 and an inductance L).

In either case, 2 or 3.

(a) The cable may first be short-circuited for a moment, until the external transient has passed and the terminal voltage dropped to zero for $t = 0$, or

(b) The cable may at once be open-circuited, in case 2, at terminal voltage e, so that no external transient occurs, but the external charge leaks through the resistances of the dielectric, or, in case 3.

The cable may at once be closed by the resistance r_0 (and inductance L), so that the external charge due to the previously impressed voltage e passes partly through the external circuit, partly through the resistance of the dielectric.

This case 3 is of interest only, if the external resistance r_0 is of the same magnitude as the internal resistances r_1 and r_2 of the dielectric, that is, extremely high, many megohms. With the usual resistance of external circuits, the external circuit is essentially a short circuit for the internal transient, and case 3 thus becomes case 1. As the result hereof, the general case of an external circuit of resistance and inductance is of little importance, since at the very slow rate of change of the internal transient, the inductances which may usually be met in external circuits, are negligible.

5. SHORT-CIRCUIT DISCHARGE

As the rapid external discharge transient is of no special interest—being the same as in an ideal condenser—we may assume that the external transient has passed and count the time for the internal transient from the moment when terminal voltage and terminal charges have dropped to zero, but before the internal charges have begun to discharge, thus leaving the cable with zero terminal voltage and zero charge at the terminals, but with internal charges and corresponding voltages in the two component dielectrics. That is:

$$t = 0 \quad e = e_1 + e_2 = 0$$

The equations then are the same as those of the slow or internal condenser charge, (2) to (9), and are integrated by:

$$i = B \epsilon^{-ct} \quad (42)$$

and

$$\left.\begin{aligned} c &= \frac{\dfrac{1}{r_1} + \dfrac{1}{r_2}}{\dfrac{1}{k_1} + \dfrac{1}{k_2}} \\ &= \frac{g_1 + g_2}{C_1 + C_2} \end{aligned}\right\} \quad (13)$$

The coefficient B is determined by the terminal conditions $t = 0$.

At $t = 0$, *before* the external transient starts, it is:

$$e_1' + e_2' = e$$

and:

$$e_1' \div e_2' = r_1 \div r_2$$

that is, the voltages are distributed by the resistances, if we assume that stationary conditions of cable charge had been reached.

This gives:

$$\left.\begin{aligned} e_1' &= \frac{r_1}{r}\, e \\ e_2' &= \frac{r_2}{r}\, e \end{aligned}\right\}$$

By the passing of the external transient, the terminal voltage drops from e to 0, that is, this transient impresses a voltage $-e$ upon the condenser, and as during the short time of the external transient, no readjustment of internal charges can occur, the transient voltage $-e$ distributes by the capacities, that is, the two components:

$$e_1'' \div e_2'' = k_1 \div k_2$$

and as:

$$e_1'' + e_2'' = -e$$

it follows:

$$\left.\begin{aligned} e_1'' &= -\frac{k_1}{k}\, e \\ e_2'' &= -\frac{k_2}{k}\, e \end{aligned}\right\}$$

The initial voltages e_1 and e_2 thus are made up of the two components, that existing before the external transient and that impressed by the external transient: It thus is:

$$\left.\begin{aligned} t = 0:\quad e_1 &= e_1' + e_1'' = \left(\frac{r_1}{r} - \frac{k_1}{k}\right) e \\ e_2 &= e_2' + e_2'' = \left(\frac{r_2}{r} - \frac{k_2}{k}\right) e \end{aligned}\right\} \tag{43}$$

Substituting (43) into (6), gives:

$$k\, i = \left\{ \frac{k_1}{r_1}\left(\frac{r_1}{r} - \frac{k_1}{k}\right) + \frac{k_2}{r_2}\left(\frac{r_2}{r} - \frac{k_2}{k}\right) \right\} e \tag{44}$$

and, substituting (42) into (44) gives:

$$B = -\frac{(k_2\, r_1 - k_1\, r_2)^2}{r\, r_1\, r_2\, k^2}\, e$$

Thus:

$$i = -\frac{e}{r}\, \frac{(k_2\, r_1 - k_1\, r_2)^2}{r_1\, r_2\, k^2}\, \epsilon^{-\frac{g_1 + g_2}{C_1 + C_2} t} \tag{45}$$

As seen, the cable discharge current is the same as the cable charging current, except that the former does not contain the constant term.

6. DISCHARGE BY EXTERNAL OPEN CIRCUIT

Assuming that a leaky laminated condenser, such as a cable, is charged to a voltage e, and then open-circuited and allowed to discharge. Or that, before open-circuiting, the condenser is short-circuited for a short time, long enough to discharge the external transient, but short enough not to affect the internal transient.

The conductance equations are (2):

$$\left.\begin{aligned} e_1 &= r_1\, i_1 \\ e_2 &= r_2\, i_2 \end{aligned}\right\} \tag{46}$$

The capacity equations are (3), since $i = 0$:

$$\left.\begin{aligned} \frac{d\, e_1}{d\, t} &= -k_1\, i_1 \\ \frac{d\, e_2}{d\, t} &= -k_2\, i_2 \end{aligned}\right\} \tag{47}$$

Thus, substituting (46) into (47), gives:

$$\left.\begin{aligned} \frac{d\, e_1}{d\, t} + \frac{k_1}{r_1}\, e_1 &= 0 \\ \frac{d\, e_2}{d\, t} + \frac{k_2}{r_2}\, e_2 &= 0 \end{aligned}\right\} \tag{48}$$

These two differential equations are integrated by:

$$\left.\begin{aligned} e_1 &= B_1\, \epsilon^{-c_1 t} \\ e_2 &= B_2\, \epsilon^{-c_2 t} \end{aligned}\right\} \tag{49}$$

where:

$$\left.\begin{aligned} c_1 &= \frac{k_1}{r_1} \\ c_2 &= \frac{k_2}{r_2} \end{aligned}\right\} \tag{50}$$

and B_1 and B_2 are determined by the terminal conditions.

The total voltage across the condenser terminals then is:

$$e' = e_1 + e_2 = B_1\, \epsilon^{-c_1 t} + B_2\, \epsilon^{-c_2 t} \tag{51}$$

a. If the cable has been charged up to the constant voltage e, and this voltage e kept on sufficiently long to get permanent condition, then the internal charges and internal voltages are distributed by the resistances. If now the circuit has opened, without, by a momentary short circuit, discharging the external transient, then it is, at

$$t = 0: e' = e;\; e_1 \div e_2 = r_1 \div r_2 \tag{52}$$

thus:

$$\left.\begin{aligned} e_1 &= \frac{r_1}{r}\, e = B_1 \\ e_2 &= \frac{r_2}{r}\, e = B_2 \end{aligned}\right\} \tag{53}$$

thus:

$$\left.\begin{aligned} e_1 &= \frac{r_1}{r}\, \epsilon^{-\frac{k_1}{r_1} t} \\ e_2 &= \frac{r_2}{r}\, e\, \epsilon^{-\frac{k_2}{r_2} t} \\ e' &= e \left\{ \frac{r_1}{r}\, \epsilon^{-\frac{k_1}{r_1} t} + \frac{r_2}{r}\, \epsilon^{-\frac{k_2}{r_2} t} \right\} \end{aligned}\right\} \tag{54}$$

b. If the condenser is momentarily short-circuited to pass the external transient, it is for $t = 0$: $e' = 0$.

The change from $e' = e$ to $e' = 0$ occurs by the external transient, in such short time that no readjustment of the charge can occur, but upon the voltage distribution by resistance $e_1 \div e_2 = r_1 \div r_2$ is superimposed a voltage distribution of $-e$ by capacity, that is, by $e_1 \div e_2 = k_1 \div k_2$. Thus it is, at: $t = 0$: $e = 0$.

$$\left.\begin{aligned} e_1 &= \left(\frac{r_1}{r} - \frac{k_1}{k}\right) e = B_1 \\ e_2 &= \left(\frac{r_2}{r} - \frac{k_2}{k}\right) e = B_2 \end{aligned}\right\} \tag{55}$$

thus:

$$\left.\begin{aligned} e_1 &= e\left(\frac{r_1}{r} - \frac{k_1}{k}\right) \epsilon^{-\frac{k_1}{r_1}t} \\ e_2 &= e\left(\frac{r_2}{r} - \frac{k_2}{k}\right) \epsilon^{-\frac{k_2}{r_2}t} \\ &\text{and:} \\ e' &= e\left\{\left(\frac{r_1}{r} - \frac{k_1}{k}\right) \epsilon^{-\frac{k_1}{r_1}t} + \left(\frac{r_2}{r} - \frac{k_2}{k}\right) \epsilon^{-\frac{k_2}{r_2}t}\right\} \\ &= e\left(\frac{r_1}{r} - \frac{k_1}{k}\right)\left(\epsilon^{-\frac{k_1}{r_1}t} - \epsilon^{-\frac{k_2}{r_2}t}\right) \\ &= \frac{e}{2}\left\{\frac{r_1 - r_2}{r} - \frac{k_1 - k_2}{k}\right\}\left(\epsilon^{-\frac{k_1}{r_1}t} - \epsilon^{-\frac{k_2}{r_2}t}\right) \end{aligned}\right| \tag{56}$$

The maximum value of e' is given by:

$$\frac{d e'}{d t} = 0 = \frac{k_1}{r_1} \epsilon^{-\frac{k_1}{r_1}t} - \frac{k_2}{r_2} \epsilon^{-\frac{k_2}{r_2}0}$$

at the time:

$$t_0 = \frac{lg \frac{k_1}{r_1} - lg \frac{k_2}{r_2}}{\left(\frac{k_1}{r_1} - \frac{k_2}{r_2}\right) lg\, \epsilon} \tag{57}$$

7. General Transient of Condenser

Let a condenser of capacity

$$C = \frac{1}{k}$$

be connected to a source of power supply of voltage e_0, over a supply circuit of resistance r_0 and inductance L_0, where e_0 in general is a function of time, but may contain a constant term. That is, e_0 may be a constant voltage, or an alternating voltage, an oscillation, impulse or other transient, etc.

Let the dielectric of the condenser be composite, consisting of n materials, and let

e_s = total voltage across all the layers of material s,
i_s = current in material s,
r_s = resistance of all the layers of material s,

C_s = capacity and thus $k_s = \dfrac{1}{C_s}$ = elastance of all the layers of s, where: $s = 1, 2 \ldots n$. The external charging equation of the condenser then is:

$$e_0 = \sum_1^n{}_s\, e_s + L_0 \frac{d i}{d t} + r_0 i \tag{58}$$

where:
i = external current.

The conduction equation of material s then is:

$$e_s = r_s i_s \tag{59}$$

and the capacity equation of material s is:

$$\frac{d e_s}{d t} = k_s (i - i_s) \tag{60}$$

Eliminating i_s from (59) and (60) gives:

$$\frac{d e_s}{d t} = k_s i - \frac{k_s}{r_s} e_s \tag{61}$$

Equation (58) is the external, and equation (61) the internal condenser equation. These equations contain the current i and the n voltages e_s.

Differentiating (58) and substituting (61) therein, gives a second equation in i and e_s. Again differentiating and substituting (61) gives a third equation in i and e_s, and so by successively differentiating and substituting (61), we get a series of $n + 1$ successive equations in i and e_s, of the form:

$$\frac{d^m e_0}{d t^m} = \sum_1^n{}_s \frac{k_s^m e_s}{r_s^m} + L_0 \frac{d^{m+1} i}{d t^{m+1}} + r_0 \frac{d^m i}{d t^m} + \sum_1^n{}_p S_p \frac{d^{m-p} i}{d t^{m-p}} \tag{62}$$

$$S_p = (-1)^{p-1} \sum_1^n{}_s \frac{k_s^p}{r_s^{p-1}} \tag{63}$$

and:

$$m = 0, 1, 2 \ldots n \tag{64}$$

(where: $\dfrac{d^0 u}{d t^0} = u$).

From the $(n + 1)$ equation (62), the n quantities e_s can be eliminated, leaving one equation in i, of $(n + 1)^{st}$ order. Denoting:

$$\left.\begin{aligned} Q_m &= \sum \left\{ \Pi\,(m) \frac{k_s}{r_s} \right\} \\ \text{thus}\quad Q_0 &= 1;\quad Q_n = \prod_1^n \frac{k_s}{r_s} \end{aligned}\right| \tag{65}$$

That is: denoting by Q_m the sum of all the products of $\frac{k_s}{r_s}$ in groups of m, that is:

$$Q_0 = 1$$

$$Q_1 = \frac{k_1}{r_1} + \frac{k_2}{r_2} + \dots + \frac{k_n}{r_n}$$

$$Q_2 = \frac{k_1\, k_2}{r_1\, r_2} + \frac{k_1\, k_3}{r_1\, r_3} + \dots + \frac{k_2\, k_3}{r_2\, r_3} + \dots$$

$$Q_n = \frac{k_1\, k_2\, k_3 \dots k_n}{r_1\, r_2\, r_3\, r_n}$$

Multiplying (62) by Q_{n-m} and adding the $(n+1)$ equations, the terms with e_s cancel, and we get the final differential equation in i.

$$\sum_0^n{}_m Q_{n-m} \frac{d^m e_0}{d t^m} = L_0 \frac{d^{n+1} i}{d t^{n+1}} + \Big(r_0 + L_0 Q_1\Big) \frac{d^n i}{d t^n} + \sum_0^{n-1}{}_m \left\{ L_0 Q_{n-m+1} + r_0 Q_{n-m} + \sum_1^{n-m}{}_p S_p\, Q_{n-m-p} \right\} \frac{d^m i}{d t^m} \tag{66}$$

This is a differential equation of $(n+1)^{st}$ degree, showing that the general cable transient contains $(n+1)$ terms.

With the exception of S_1 and Q_0, all the S and Q in equation (66) contain r_s in the denominator. r_s however, as the resistance of a part of the dielectric, is a very large quantity, and the terms S and Q, which have r_s in the denominator, therefore are very small quantities, hence can in first approximation, be neglected in comparison with the terms L_0, r_0 and $S_1\, Q_0$.

Doing so, equation (66) simplifies to:

$$\frac{d^n e_0}{d t^n} = L_0 \frac{d^{n+1} i}{d t^{n+1}} + r_0 \frac{d^n i}{d t^n} + S_1 \frac{d^{n-1}}{d t^{n-1}} \tag{67}$$

and by (63)

$$\left. \begin{aligned} S_1 &= \sum_1^n{}_s k_s \\ &= k = \frac{1}{C} \end{aligned} \right\} \tag{68}$$

Separating the permanent term from equation (67) leaves as the transient:

$$L_0 \frac{d^{n+1} i}{d t^{n+1}} + r_0 \frac{d^n i}{d t^n} + \frac{1}{C} \frac{d^{n-1} i}{d t^{n-1}} = 0 \tag{69}$$

This equation is integrated by the exponential:

$$i = B\, \epsilon^{-ct} \tag{70}$$

and, (70) substituted into (69) gives:

$$L_0\, c^{n+1} - r_0\, c^n + \frac{1}{C}\, c^{n-1} = 0 \tag{71}$$

Equation (71) gives:

$$(n-1) \text{ roots: } \quad c = 0 \tag{72}$$

and 2 roots:

$$\left. \begin{aligned} c &= \frac{r_0 \pm j\, q}{2\, L_0} \\ \text{where:} & \\ q &= \sqrt{\frac{4\, L_0}{C} - r_0^2} \end{aligned} \right\} \tag{73}$$

or:

$$\left. \begin{aligned} c &= \frac{r_0 \pm s}{2\, L_0} \\ \text{where:} & \\ s &= \sqrt{r_0^2 - \frac{4\, L_0}{C}} \end{aligned} \right\} \tag{74}$$

Thus:

$$i = \epsilon^{-\frac{r_0}{2 L_0} t} \; (C_1 \sin q\, t + C_2 \cos q\, t) \tag{75}$$

or:

$$i = B_1\, \epsilon^{-\frac{r_0 - s}{2 L_0} t} + B_2\, \epsilon^{-\frac{r_0 + s}{L_0} t} \tag{76}$$

(75) respectively (76) is the external charging transient of the condenser, and C_1 and C_2 respectively B_1 and B_2 are the terminal conditions, determined by the terminal condition of the external circuit, in the usual manner:

$$t = 0: \qquad i = 0; \qquad e_s = 0 \tag{77}$$

The $(n-1)$ roots $c = 0$ (72), mean, that under the assumption made in (67), the internal transients are permanents, that is, are so slow that they do not come into consideration during the life time of the transient (75) (76).

Substituting (75) (76) into the general equation (66), the terms with L_0, r_0 and $\frac{1}{C}$ vanish. Separating again the permanent term, leaves as the transient term for the determination of the remaining $(n-1)$ roots, which give the internal transient, the expression:

$$\sum_0^{n-1}{}_m \sum_1^{n-m}{}_p S_p\, Q_{n-m-p} \frac{d^m i}{d t^m} = 0 \tag{78}$$

This is integrated by:

$$i = B\, \epsilon^{-ct} \tag{79}$$

and, (79) substituted into (78) gives:

$$\sum_0^{n-1}{}_m \sum_1^{n-m}{}_p S_p\, Q_{n-m-p} (-c)^m = 0 \tag{80}$$

As easily seen, in (78) all the coefficients are positive, and in (79), the successive coefficients alternately have positive and negative signs. Herefrom it follows, that all the $(n-1)$ roots of (80) are positive. That is:

The slow or internal transient of a composite condenser, containing n different materials in its dielectric, consists of $(n - 1)$ separate and real exponential transients:

$$i = \sum_{1}^{n-1} {}_m B_m \epsilon^{-c_m t} \qquad (81)$$

where:

c_m are the $(n - 1)$ roots of equation (80), and B_m coefficients derived from the terminal conditions.

As all the roots of (80) are positive, they are easily derived by approximation.

The terminal condition of the internal transient is:

$$t = 0: \quad e_s = e \frac{k_s}{k} \qquad (82)$$

and, substituting (82) into (62), and substituting (81) therein, gives the constants B_m.

Applying this to the internal transient for $n = 3$, gives:

$$(80) \qquad \sum_{0}^{2} {}_m \sum_{1}^{3-m} {}_p S_p Q_{3-m-p} (-c)^m = 0$$

$$\left.\begin{aligned} c &= \frac{m \pm s}{2k} \\ s &= \sqrt{m^2 - 4kn} \end{aligned}\right\} \qquad (83)$$

where:

$$\left.\begin{aligned} m &= \frac{k_1 (k_2 + k_3)}{r_1} + \frac{k_2 (k_3 + k_1)}{r_2} \\ &\quad + \frac{k_3 (k_1 + k_2)}{r_3} \\ n &= k_1 k_2 k_3 \left(\frac{1}{r_1 r_2} + \frac{1}{r_2 r_3} + \frac{1}{r_3 r_1} \right) \\ k &= k_1 + k_2 + k_3 \end{aligned}\right\} \qquad (84)$$

and:

$$(58) \quad e_1 + e_2 + e_3 = e$$

$$\frac{d e_1}{d t} + \frac{d e_2}{d t} + \frac{d e_3}{d t} = 0$$

$$(61) \quad \frac{d e_s}{d t} = k_s i - \frac{k_s}{r_s} e_s$$

$$(62) \quad k i = \frac{k_1}{r_1} e_1 + \frac{k_2}{r_2} e_2 + \frac{k_3}{r_3} e_3$$

$$(62) \quad -k \frac{d i}{d t} + \left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2} + \frac{k_3^2}{r_3} \right) i = \frac{k_1^2}{r_1^2} e_1 + \frac{k_2^2}{r_2^2} e_2 + \frac{k_3^2}{r_3^2} e_3$$

$$(81) \quad i = B_1 \epsilon^{-c_1 t} + B_2 \epsilon^{-c_2 t} \qquad (81)$$

$$\left.\begin{aligned} &t = 0: \quad e_s = \frac{k_s}{r_s} e \\ &k (B_1 + B_2) = e \left\{ \frac{k_1^2}{r_1^2} + \frac{k_2^2}{r_2^2} + \frac{k_3^2}{r_3^2} \right\} \\ &k (c_1 B_1 + c_2 B_2) + \left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2} + \frac{k_3^2}{r_3} \right) (B_1 + B_2) = e \left\{ \frac{k_1^3}{r_1^3} + \frac{k_2^3}{r_2^3} + \frac{k_3^3}{r_3^3} \right\} \end{aligned}\right| \qquad (85)$$

III. Numerical Calculations

1. Single Transient

The slow cable transient is measured by impressing a constant voltage upon the cable terminals and then reading volts and amperes, for a number of minutes, until the current has become practically constant. For this purpose, the voltage is brought up rapidly to the desired value, but great care must be taken not to over reach, since any reduction of the voltage gives wrong results. The voltage should be very constant, as the current fluctuates greatly with slight voltage variation; a small voltage increase causing a large increase of current, while a small voltage decrease may even cause a reversal of current. Therefore it is best to plot the observed values of current with the time as abscissas, smooth this curve and from it then take numerical values.

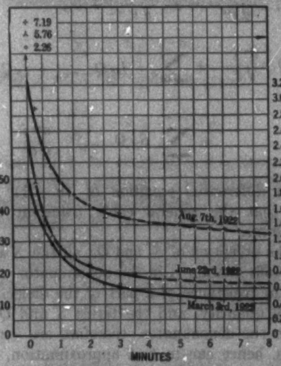

Fig. 1—D-C. Charging Current, in Milliamperes at 20 Kv., of Cable No. 245, Philadelphia, 13,000 ft.

One conductor against other two and lead sheath. Avg. of 6 tests (by kenotron).

$+\; i = 1.20 + 0.681\, \epsilon^{-0.00120 t} + 1.39\, \epsilon^{-0.0268 t}$

$⅄\; i = 0.63 + 0.497\, \epsilon^{-0.00592 t} + 1.48\, \epsilon^{-0.0353 t}$

$\bullet\; i = 0.108 + 0.206\, \epsilon^{-0.00828 t} + 0.190\, \epsilon^{-0.0317 t}$

Assuming that the curve is to be represented by the single exponential transient, as the simplest form:

$$i = A + B\,\epsilon^{-ct}$$

The simplest way of calculating A, B and c is to take from the curve three values of current at equal time intervals:

$$t_1; \qquad t_1 + t_0; \qquad t_1 + 2\,t_0$$

choosing the three times so as to cover the whole reliable part of the curve, that is, t_1 and $t_1 + 2\,t_0$ are near the ends of the curve just sufficiently away from the ends, not to be affected by the greater error of the curve ends. For instance, if readings have been taken over 8 minutes, 1, 4 and 7 minutes are good numerical values.

It is then:

$$\left.\begin{aligned} i_1 &= A + B\,\epsilon^{-ct_1} \\ i_2 &= A + B\,\epsilon^{-c(t_1+t_0)} \\ i_3 &= A + B\,\epsilon^{-c(t_1+2t_0)} \end{aligned}\right\}$$

thus:

$$\frac{i_1 - i_2}{i_2 - i_3} = \epsilon^{ct_0}$$

$$\frac{(i_1 - A)\,(i_3 - A)}{(i_2 - A)^2} = 1$$

Hence:

$$\left.\begin{aligned} A &= \frac{i_1\,i_3 - i_2^2}{i_1 + i_3 - 2\,i_2} \\ c &= \frac{\log\,(i_1 - i_2) - \log\,(i_2 - i_3)}{t_0 \log \epsilon} \end{aligned}\right\}$$

$$B = (i_1 - A)\,\epsilon^{+ct_1}$$

Given are:

The impressed voltage : e

and the capacity of the cable: $C = \dfrac{1}{k}$

From the observation of the slow transient follow: A, B and c

It is then, by equation (11):

$$A = \frac{e}{r}$$

Hence:

$$r = \frac{e}{A}$$

It is by the equations:

$$(7) \qquad k_1 + k_2 = k$$

$$(12) \qquad r_1 + r_2 = r$$

$$(13) \qquad c = \frac{\dfrac{1}{r_1} + \dfrac{1}{r_2}}{\dfrac{1}{k_1} + \dfrac{1}{k_2}}$$

thus:

$$\frac{k_1\,k_2}{r_1\,r_2} = \frac{k}{r}\,c = a$$

$$(16) \qquad A + B = \frac{e}{k^2}\left(\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2}\right)$$

thus:

$$\frac{k_1^2}{r_1} + \frac{k_2^2}{r_2} = \frac{k^2}{e}\,(A + B) = b$$

From these 4 equations follows:

$$(r_2 - r_1)\sqrt{1 - \frac{4\,a^2}{b^2}\,r_1\,r_2} = \frac{4\,a}{b}\,r_1\,r_2 + \left(r - \frac{2\,k^2}{b}\right)$$

From this equation, and (12), r_1 and r_2 are calculated. This is most conveniently done by approximation. From (7) and (13) then are calculated k_1 and k_2.

2. NUMERICAL INSTANCE

An instance is shown, in Fig. 2, the complete charging and discharging transient of a 12-kv. three-phase cable of 13,000 ft. length and the capacity, from one conductor to the other two conductors and the armor, of $C = 0.74\ \mu f$, supplied with a constant voltage of 20 kv. over a circuit of resistance $r_0 = 200$ and inductance $L_0 = 0.5$ h.

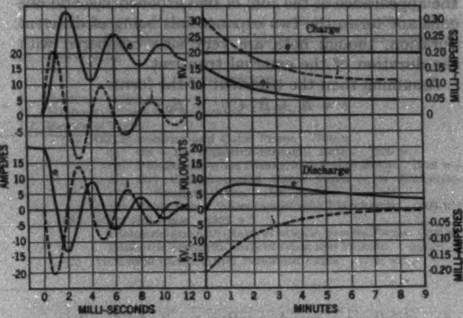

FIG. 2

CHARGE

The rapid external charging transient is:

$$e = 20{,}000\,\{1 - \epsilon^{-200t}\,(\cos 1570\,t + 0.125 \sin 1570\,t)\}\ \text{volts}$$

$$i = 25\,\epsilon^{-200t} \sin 1570\,t\ \text{amps.}$$

Duration: 0.005 sec.

The slow internal charging transient is:

$$i = 0.108 + 0.206\,\epsilon^{-.008t}\ \text{milliamperes.}$$

Duration: 125 sec.

The fractional voltages are

$$e_1 = 4200 + 11{,}200\,\epsilon^{-0.008t}\ \text{volts}$$

$$e_2 = 15{,}800 - 11{,}200\,\epsilon^{-0.008t}\ \text{volts}$$

Initial energy of charge: $W_0 = 148$ J.

Final energy of charge: $W' = 406$ J.

Energy of internal charges: $W_i = 258$ J.

DISCHARGE

The rapid external short-circuit discharge transient is:

$e = 20{,}000\, \epsilon^{-200t} (\cos 1570\, t + 0.125 \sin 1570\, t)$ volts.

$i = -25\, \epsilon^{-200t} \sin 1570\, t$ amperes.

The slow internal transient of discharge current by short circuit is:

$i = -0.206\, \epsilon^{-0.008t}$ milliamperes.

The slow internal transient of building up of residual voltage at open circuit following short-circuit discharge of terminal charges, is:

$e' = 11{,}100\, (\epsilon^{-0.0022t} - \epsilon^{-0.027t})$

This residual voltage starts in 37 sec. reaches a maximum value of 8160 volts in 100 sec. and has a duration of 455 sec.

3. DOUBLE TRANSIENT

When analyzing charging current measurements of three-phase cables, made by d-c. voltage, as shown in Fig. 1, a single exponential often does not satisfy the data, but the decrease of current during the first few minutes is far more rapid than later on, giving the curve the appearance of the sum of two exponentials and one constant term, the two exponentials being of different duration, though both are very slow compared with the duration of the external transient.

Assuming then the transient of the form:

$$i = A + B_1\, \epsilon^{-c_1 t} + B_2\, \epsilon^{-c_2 t}$$

Suppose now t_1 is the time at which the shorter transient $B_2\, \epsilon^{-c_2 t}$ has died out. Then for the part of the curve: $t \geqq t_1$,

$$i = A + B_1\, \epsilon^{-c_1 t}$$

and for this part of the curve, it is:

$$A = \frac{i_1\, i_3 - i_2^2}{i_1 + i_3 - 2\, i_2}$$

Calculating thus A for various values of t_1, $t_1 + t_0$, $t_1 + 2\, t_0$, such as: 2-4-6; 3-5-7; 4-6-8; 2-5-8; 1-2.5-4 minutes, we find a series of values for A, and for that range of time, in which approximately the same values of A are given, we can assume that the single transient equation applies, and the more rapid transient has already died out.

From this range of time we then calculate B_1 and c_1, either by the $\Sigma\ \Delta$ method, or as in the preceding (III. 1) and so get the slower transient. Subtracting this from the total transient, then gives the faster transient $B_2\, \epsilon^{-c_2 t}$, and from this the values B_2 and c_2.

This is done in Fig. 1, where the marked points give the observed numerical values, and the three curves the double transients calculated therefrom.

Discussion

For discussion of this paper see page 615.

Dielectric Strength Ratio Between Alternating and Direct Voltages

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***Review of the Subject.**—High-Voltage insulation testing has been and is usually still done by alternating voltages.*

High direct voltage was made available for testing purposes by the development of the kenotron tube. When used for testing insulation direct voltage has several advantages over alternating voltage. (1) the power necessary is often much less with direct voltage than with alternating voltage. In apparatus of high electrostatic capacity, such as long high-voltage cables, the size of the alternating-voltage testing transformer becomes excessive, thousands of kilovolt-amperes being necessary. Direct voltages are therefore preferable as they necessitate only a few kilowatts. (2) Excess direct voltage is less likely to permanently damage the insulation than excess alternating voltage. (3) If direct voltage is used conductivity tests can be made and the action of the material on the application of the voltage more thoroughly studied.

As the use of high direct voltage for testing purposes is found to be increasing, it is important to determine the relation between the insulation stress produced by direct and that produced by alternating voltages.

Little is definitely known of what is called the "dielectric strength ratio of insulation" which is the ratio of the direct disruptive voltage to the crest value of the alternating disruptive voltage. In general, this ratio might be expected to be unity. While such is the case with air some engineers have claimed, however, that some solid insulations stand a higher direct than alternating voltage.

Therefore, a very extensive set of investigations was made, with direct and with alternating voltages, on liquid and solid insulations of homogeneous and non-homogeneous structure, over a range of temperature, thickness and rate of voltage application. Their dielectric strength ratios were determined and are given and discussed in the paper.

It was found that the dielectric strength ratio may be greater than unity, and sometimes very much so, that is, that the material may stand higher and sometimes very much higher direct voltages than alternating voltages, but also that the ratio with other materials may be less than unity, that is, the material may stand higher alternating than direct voltages.

Ratios less than unity were given by oils, petrolatum, powdered glass, etc., that is, they stood higher alternating than direct voltage, though the difference rarely exceeded 10 per cent.

Ratios above unity were given by paper, cloth, solid glass and mica, etc., indicating a greater strength for direct than for alternating voltages.

The dielectric strength ratio of some materials, such as laminated paper, was found to vary with the condition and in general increase with decreasing temperature, decreasing thickness and increasing rapidity of voltage application.

Some materials, such as petrolatum impregnated cable paper, gave a very high ratio, some times exceeding two, while the component materials did not differ much from unity, petrolatum being a little below and air-dry paper a little above unity.

It is believed that the observation of the dielectric strength ratio and its changes with the condition of test, will give us a powerful tool for the investigation of insulation, and assist in solving the problem of understanding the mechanism of the breakdown of insulation in an electric field.

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I. General. (1000 w.)
II. Methods of Tests and Apparatus. (375 w.)
III. Results of Tests.
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        (a) Oil — Its Dielectric-Strength Ratio. (250 w.)
        (b) Petrolatum — Its Dielectric-Strength Ratio. (250 w.)
        (c) Dry Unimpregnated Cable Paper — Its Dielectric-Strength Ratio. (500 w.)
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        Dielectric-Strength-Ratio of Mica in Several Forms. (200 w)
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    (C) Varnished Cambric and Paraffined Paper. (750 w.)

AS the operation of electrical apparatus and circuits depends on their insulation, the maintenance and test of insulation is of foremost importance.

Insulation testing, even of direct-current apparatus, is usually done by alternating voltage, since high alternating voltages are easily obtained by the alternating-current transformer.

Two serious disadvantages arose in the use of alternating high potential testing: First, in apparatus of considerable capacitance such as underground cables, the charging current at the high test voltage is excessive, requiring uneconomically large and expensive transformers. Second, in other apparatus, corona and other dielectric losses incident to the abnormally high alternating test voltage (of three and a half and more times the normal operating voltage), may permanently damage the insulation.

With the development of the kenotron vacuum tube as rectifier, high direct voltages became available. As there is no permanent charging current with direct voltage, a kenotron rectifier of a few kw. capacity could replace a testing transformer of many hundred kw. In the absence of the intensive corona and the high dielectric losses incident to an alternating field, damage of apparatus by the high testing voltage was less to be feared with direct voltage testing.

At first it was expected that the striking distance with direct voltage would be equal to that of the maximum value of the alternating voltage, and tests made with air as dielectric corroborated this. However, engineers familiar with high-voltage, direct-current transmission claimed that apparatus could stand materially higher direct voltages than alternating voltages. When high direct voltages became more available, tests made with them showed that some solid insulation, such as that of cables, stand a higher direct voltage than alternating voltage, and it was hoped then that a constant ratio between the disruptive strength

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of direct and of alternating voltage could be found, by which the one could be expressed in terms of equality with the other. A series of tests made abroad on cables gave 2.5 as the average ratio between the direct voltage and the (effective or root-mean-square) alternating voltage which has the same disruptive effect.

Further tests made by various engineers here and abroad gave inconsistent results and different ratios between the disruptive effect of direct and alternating voltage, so that now no fixed ratio between direct and alternating voltage can be universally applied.

As the result of several years experimental investigation we have come to the conclusion, and expect to show in the following, that:

1. The disruptive effect on insulation of a direct voltage in general is different from that of an alternating voltage of a peak value equal to the direct voltage.

2. The puncture or disruptive effect of the alternating voltage (peak value) may be greater, and sometimes very much greater, than that of the direct voltage of equal value, but it may also be less. That is, the ratio: "Direct voltage divided by the peak value of the alternating voltage which gives the same disruptive effect", which we may call "dielectric-strength ratio", varies from values less than unity, when the direct voltage stress is more severe, to values much above unity, when the alternating voltage stress is more severe.

3. In air, the dielectric-strength-ratio is probably unity.

4. In solid insulation, the dielectric-strength ratio depends on the mechanical, physical, and thermal conditions of the material, and in general, seems to tend towards unity, the more homogeneous the material is.

5. In one and the same material, the dielectric-strength-ratio may vary considerably with temperature, thickness, rate of voltage application, etc.

6. In general, it seems that the mechanism of failure of insulation under high alternating voltage stress is materially different in some features from that under high direct voltage stress, and no universal and constant dielectric-strength-ratio can therefore be expected, but dependent on the feature which dominates in the failure different values must result.

The dielectric-strength ratio has been defined in the previous literature in two ways, either as the ratio: Direct voltage divided by effective or root-mean-square value of alternating voltage (in which case air would have the ratio $\sqrt{2} = 1.41$) or otherwise the ratio: Direct voltage divided by the peak value of alternating voltage, which gives to air the convenient ratio 1. We use herein, and recommend for general acceptance, the latter definition, as more rational. It gives the value 1 to air, and in general the values tend toward 1, and it gives the value 1, if the nature of the alternating voltage puncture is the same as that of the direct voltage puncture, since a direct voltage and an alternating voltage with the same peak value, should be equivalent.

Although the values given in the following tables are the averages and abstracted from thousands of individual tests, made under the greatest possible precautions, so that the experimental errors are small, they are not so consistent as to draw final conclusions from single recorded values (though these values usually are averages of 10 or 25 tests), and the conclusions are drawn from the general trend of groups of individual values. The reason for variations in results is the inherently erratic nature of disruptive tests. Dielectric tests made with air can be duplicated within two to three per cent, but in liquids like oil erratic variations occur between successive tests made with all precautions, amounting to 20 per cent to 30 per cent and more.[1] In solid insulation, the phenomenon of dielectric disruption apparently is still more complex, and the individual test results therefore are still more erratic, so that acceptable conclusions can be drawn only from the comparison of the averages of very numerous tests.

The results of this investigation seem inevitably to lead to the conclusion that the dielectric rupture under high-voltage stress is a far more complex phenomenon than is usually assumed. Puncture is not due to a mere effect of electrostatic stress, or a mere heating effect, or any specific deterioration effect, etc., but it results from a number of different effects combined in different degrees. While it is somewhat disappointing no universal "dielectric strength ratio" can be determined, which is applicable to all conditions and all apparatus, we believe that dielectric-strength-ratios can be derived for definite classes of insulation under definite operating conditions, and that the determination and study of the dielectric strength-ratio will give us an additional and powerful tool in the study of insulation failure and its causes.

Methods of Tests and Apparatus

The principal source of high, direct voltage used in the tests was the kenotron. This is a two-element vacuum tube containing a filament cathode supported within a cylindrical plate as anode. The filament is kept incandescent by means of either a transformer or a storage battery. In operation the kenotron acts simply as a unidirectional conductor, passing through only the half waves of one polarity. Fig. 1 shows some of the principal circuits used for kenotron rectification. (a) is the simplest connection. In this the kenotron is in series with the supply transformer and a condenser storing the rectified voltage. This gives a direct current, every second half cycle the vacuum becomes conducting. The principal advantage of such a connection is its simplicity, and if the load is small compared with the condenser capacity, the voltage is quite steady. Diagram (b) shows the bridge type of connection. This has the advantage of passing through both half waves in such a way that they produce the same polarity on the receiving circuit. Diagram (c) shows the full wave or double half wave type which has the principal advantage of requiring an alternating voltage source of only approximately one-third of the direct voltage. The purpose of the condenser is to smooth out the pulsations of the direct voltage. It is readily seen that the half wave connection (a) requires more capacitance than a full wave connection either (b) or (c). The capacitance required for satisfactory smoothing is dependent on the load. In testing long cables no capacitance additional to that of the cable is required. But laboratory tests on short lengths of cable or other test pieces of low electrostatic capacity necessitate the use of the condensers shown above. On account of the small direct current conducted by any type of insulation the voltage fluctuation of such a rectifier can be reduced below 2 per cent without the necessary condensers becoming large.

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[1] "Three Thousand Tests on the Dielectric Strength of Oil," by Hayden and Eddy, presented before Convention of A. I. E. E. at Niagara Falls, June 26-30, 1922.

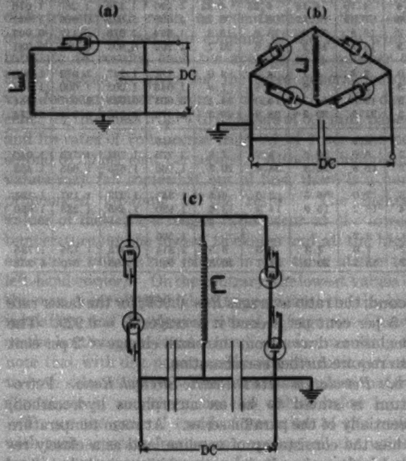

Fig. 1—Kenotron Rectifier Connections

Both the alternating and the direct-voltage data were taken on the same transformer. The direct voltage was obtained by means of the full-wave kenotron connection (c) Fig. 1 described above, using sufficient capacitance in parallel with the sample to reduce the ripples in the voltage wave to less than 2 per cent under the conditions of tests. In taking all spark voltage values special attention was given to the rate of increasing the voltage, keeping this rate as nearly the same as possible at the value noted in the data.

The various types of insulation were tested in various forms and shapes and with various electrodes but care was always taken to have no sharp corners or edges on the electrodes. All voltage readings were taken on the low side of the supply transformer and the high-tension voltage, whether alternating or direct, was obtained from a calibration curve which had been made against a sphere gap in parallel with the sample or an equivalent load. All direct-voltage data below 3000 volts were taken on a special direct-current generator with field control, the voltage being read by an indicating voltmeter in series with multiplying resistance. Whenever possible both alternating and direct voltage punctures were taken on the same sample of insulation. For instance, if the sheet of insulation under test was large enough for 10 puncture tests, 5 would be taken on alternating and 5 on direct voltage. At no time were more than 5 punctures taken on alternating voltage without then taking an equal number on direct voltage or vice versa so as to give the closest possible comparison. Each value, given in the tabulations below, usually is the average of 10 to 25 tests. Practically all of the tests were repeated at different times by different operators and under different experimental conditions.

Results of Tests

A. Oil, Petrolatum and Cable Paper

A series of tests were made on the materials entering the insulation of high-potential cables, such as petrolatum and cable paper, and the results of these tests are given in Table I. In the same table are also given the results with transil oil No. 6, as of similar character to petrolatum but far more fluid. Oil and petrolatum were tested between 2.54 cm. spheres, but cable paper was tested between 5 cm. plates. The alternating voltage values are the maximum of the voltage wave.

The table gives the tested materials: Transil oil No. 6 alone, petrolatum alone, manila paper of 0.2 mm. thickness in air dry condition alone, and the same paper impregnated with petrolatum. The table also gives the length of the gap between the spheres for oil and petrolatum, and the number of layers of cable paper used. Then it gives the approximate rate of voltage increase in per-cent-per-second of the puncture voltage. Then follow the values of direct voltage in kilovolts, and of alternating voltage, peak values for the four temperatures 25 deg., 50 deg., 75 deg. and 100 deg. cent., and finally in the four last columns the values of the "dielectric-strength ratio" derived by dividing the direct voltage by the alternating voltage. As seen, values are given for three rates of increase in terms of the puncture voltage as follows: 20 per cent, 5 per cent and 0.1 per cent rise of this voltage per second. Each of the numerical values given in Table I is the average of a minimum of 10 to 25 observations, but in many cases as many as a hundred repetitions were made.

a. Oil—Its Dielectric-Strength Ratio. The table shows incidentally that the dielectric strength of oil, with direct voltage as well as with alternating voltage, decreases somewhat with increasing temperature, but not to any great extent.

TABLE I

OIL, PETROLATUM AND CABLE PAPER

Material	Electrodes	Length of gap mm.	Rate of voltage rise per cent per sec.	Direct voltage kv				Alternating voltage kv.				Dielectric-strength-ratio			
				25 deg.	50 deg.	75 deg.	100 deg.	25 deg.	50 deg.	75 deg.	100 deg	25 deg.	50 deg.	75 deg.	100 deg.
Transil oil No. 6	2.54 cm. spheres	2	5	30.5	28.8	27.6	19.9	32.9	27.2	27.0	25.3	0.927	1.059	1.022	(0.786)
		4	5	45.8	37.5	36.2	34.0	53.0	45.0	40.4	37.7	0.864	0.833	0.896	0.902
		2	2	20.7		20.5		25.4		19.5		0.815		1.051	
		4	2	30.7		37.0		40.0		42.9		(.755)		0.863	
Petrolatum	2.54 cm. spheres	2	5	15.1	15.6	17.8	15.1	15.9	23.7	18.4	15.9	0.950	(.667)	0.967	0.950
		4	5	27.6		34.8	28.2	28.9		35.9	29.6	0.955		0.969	0.953
		No. of layers													
Cable Paper 0.2 mm. thick air dry	5 cm. plates	4	20	4.3	3.4	3.6	3.2	2.4	2.5	2.3	2.2	1.790	1.360	1.565	1.445
		8	20	9.0	8.4	7.7	7.0	7.0	6.6	5.5	5.5	1.286	1.273	1.400	1.273
		16	20	17.3	12.8	12.7	13.1	16.1	12.0	11.6	9.7	1.075	1.067	1.095	(1.340)
		32	20	33.1	26.6	24.5	26.5	30.5	28.0	28.0	28.0	1.085	0.950	0.875	0.946
		4	5	5.4	4.84	5.47	4.4	4.19	4.15	4.25	4.33	1.290	1.176	1.287	1.016
		8	5	7.6	7.5	8.7	9.0	7.39	7.32	8.97	8.72	1.028	1.025	0.969	1.032
		16	5	14.0	13.9	15.2	15.2	12.8	12.9	13.6	15.3	1.094	1.078	1.117	0.993
		32	5	22.5	13.1	15.3	20.3	18.7	13.2	15.6	20.7	1.203	0.992	0.981	0.981
		4	0.1	5.3	4.7	4.5	4.0	4.5	4.1	4.2	4.0	1.178	1.147	1.072	1.000
		8	0.1	9.2	9.4	7.8	8.3	8.8	7.8	7.8	7.8	1.045	1.205	1.000	1.064
		16	0.1	15.5	13.6	12.7	13.5	16.3	14.7	14.0	14.3	0.951	0.925	0.907	0.944
		32	0.1	27.5	25.5	24.3	22.5	31.5	29.7	28.9	28.3	0.873	0.858	0.841	0.774
Cable Paper 0.2 mm. thick, impregnated with petrolatum	5 cm. plates	1	20	7.4				3.0				2.470			
		2	20	18.3	19.0	15.0	15.1	6.6	10.6	8.7	7.8	2.775	1.793	1.725	1.940
		4	20	31.0	41.5	39.3	29.5	15.8	25.3	24.6	20.3	1.960	1.635	1.605	1.453
		1	5	9.4	8.0	6.7	7.5	7.0	6.5	5.6	6.5	1.343	1.232	1.197	1.154
		2	5	36.0	22.8	17.7	22.4	18.2	15.9	14.3	13.6	1.980	1.435	1.238	1.650
		1	0.1	5.0				3.2				1.565			
		2	0.1	16.2	14.4	13.8	14.7	7.0	7.3	8.0	8.5	2.314	1.975	1.725	1.735
		4	0.1	29.5	36.5	34.0	25.7	15.5	20.3	21.7	20.5	1.900	1.795	1.570	1.253

As would be expected from the erratic behavior of oil, discussed in a previous paper, the values of the dielectric-strength ratio of the oil, given in the last four columns, differ from each other more than any possible error of observation. However, in general the data show fairly conclusively a dielectric-strength ratio somewhat below unity,—specifically a general average of $R = 0.923$. In other words, this ratio of 92 per cent indicates that *oil has a greater dielectric strength under alternating than under direct voltage,* by about 8 per cent.

A Comparison of Ratios at Different Temperatures. The combined averages of all the ratios at 25 deg. and 50 deg. cent. is $R = 0.900$; but the combined averages of all the ratios at 75 deg. and 100 deg. cent. is $R = 0.947$. This change shows that the dielectric-strength ratio increases with increasing temperature or, as we may say, becomes more normal at higher temperatures by approaching closer to the value of unity.

Ratios at Different Gap-Lengths. The average ratio for the short oil gap is $R = 0.975$; the average ratio for the long gap is 0.871. This difference shows conclusively that the dielectric-strength ratio of oil seems to become more abnormal, that is, differs more from unity, with increasing length of oil gap. This conclusion, however, requires further corroboration.

Ratios as Affected by Rate of Increase of Voltage. For the slower rate increase of voltage, 2 per cent per second, the ratio averages $R = 0.909$; for the faster rate of 5 per cent per second it averages $R = 0.929$. The conclusions drawn from this small change of 2 per cent also require further corroboration.

b. Petrolatum—Its Dielectric-Strength Ratio. Petrolatum is stated to be an amorphous hydrocarbon, essentially of the paraffin series. At room temperature it has the consistency of vaseline, and as a closely related hydrocarbon it shows the same general characteristics as transil oil.

As seen from the table, the dielectric strength of petrolatum, under direct or alternating voltage, does not appreciably change with the temperature, between 25 deg. and 100 deg. cent., nor is there appreciable change in the dielectric-strength ratio.

The averages of the dielectric-strength ratio, taken from the table:

At:	25 deg.	75 deg.	100 deg. cent.
$R =$	0.952	0.968	0.952

Thus the ratio is essentially constant.

Again as regards variation in gap-length:

At:	2mm.	4mm.	length of gap between spheres:
$R =$	0.956	0.959	respectively, which is also

essentially constant over this small range of gap.

The average of all values of dielectric-strength ratio for petrolatum is: $R = 0.957$. Therefrom it is evident petrolatum shares with oil the characteristic that its dielectric strength is less for direct voltage than for alternating voltage, but the difference is not so great.

If the abnormal behavior of oil were due to differences in the mechanical motion produced in the ingredients of its unhomogeneous structure by the continuous dielectric field, the resultant concentration of the weaker ingredients might account for its lesser strength under the continuous direct voltage than under the alternating voltage stress, and the far more viscous petrolatum would show still less the effect of any movement and concentration—as is, in fact, the case.

c. Dry Unimpregnated Cable Paper—Its Dielectric Strength Ratio. Incidentally tests with direct and alternating voltage show no definite difference in effect on the dielectric strength of cable paper, within the range of temperature, and rate of voltage rise used in the tests. But there is a very pronounced effect on the dielectric-strength ratio, as seen from the three sets of values in the last four columns of the table which tabulate 48 results, each the average of a number of tests. Ratios are given for the three general factors viz., for temperatures from 25 deg. cent. to 100 deg., cent., for thickness from 4 to 32 layers of 0.2 mm. paper, and for rates of voltage rise from 0.1 of 1 per cent per second to 20 per cent per second. Although individual values may fall somewhat out of line, there is a very pronounced grouping of the results. The highest values of dielectric-strength ratio occur at the lowest temperature, at the lowest thickness and at the highest rate of voltage rise (shown in the table at the top left-hand corner). On the contrary, the lowest values of the ratio occur at the highest temperature, the thickest insulation, and the slowest rate of voltage rise (shown at the bottom right-hand corner). It is interesting to note that with dry paper, unlike oil, the ratio extends to both sides of unity. There are some values materially above 1, and there are also values below 1.

To get the general trend of variation of the ratio, with each separate feature, we average all the values for the same temperature, or for the same thickness, or for the same rate of voltage rise, and thus get the effect of one variable, segregated from the effect of the others. This grouping of data gives the results recorded below.

Temperature, Thickness, and Rate Data for Dry Unimpregnated Cable Paper.

Temperature:	25 deg.	50 deg.	75 deg.	100 deg.
Average ratio: R =	1.158	1.088	1.092	1.057

There is a consistent decrease of ratio with increasing temperature.

Thickness:	4	8	16	32 layers
Average ratio: R =	1.277	1.133	1.027	0.947

There is a consistent decrease of ratio with increasing thickness, from values considerably above 1 to values below 1.

Rate of voltage rise:	20 per cent	5 per cent	0.1 of 1 per cent per sec.
Average ratio: R =	1.231	1.079	0.987

There is a consistent decrease of ratio with decreasing rate of voltage application, down to values below unity.

The total average of all the values of the dielectric-strength ratio of cable paper is slightly above unity: $R = 1.100$. In other words, dry cable paper shows a slightly lesser dielectric strength for alternating than for direct voltage. Conversely stated, unimpregnated paper requires, on an average, a 10 per cent higher direct voltage than the peak alternating voltage, to puncture the same thickness of paper under otherwise identical conditions.

Since a laminated structure, consisting of a number of layers of unimpregnated cable paper, thus shows also a small deviation from the normal dielectric-strength ratio but opposite to that of either oil or petrolatum, it is of interest to consider the combination of both—that is, cable paper impregnated with the insulating hydrocarbon.

d. Cable Paper Impregnated with Petrolatum. If untreated cable paper (with a dielectric-strength ratio slightly above unity, viz: 1.100) is impregnated with petrolatum (which alone has a ratio slightly below unity—0.957) we might expect as a result of the combination of the two a ratio close to unity. On the contrary, the tests of impregnated paper show consistently very high ratios,—much higher indeed than dry paper,—and give an average ratio $R = 1.773$.

This result is very startling and its significance on insulation strength and failure is still far from being understood. The phenomenon of extra high ratio has, however, been checked and corroborated with data on several other materials of similar character.

Significant also is another factor—the enormous increase of dielectric strength, due simply to impregnation of the paper.

To get the general trend of the variation of the dielectric-strength ratio with the three controllable variables, namely temperature, thickness, and rate of voltage application, the tabulated data are again grouped for the purpose of comparison.

Temperature, Thickness, and Rate Data for Cable Paper Impregnated with Petrolatum.

Temperature:	25 deg.	50 deg.	75 deg.	100 deg.
Average ratio: R =	2.038	1.644	1.510	1.531

Thus the ratio consistently decreases with increasing temperature.

Thickness:	2	4 layers
Average ratio: R =	1.857	1.646

Thus the ratio decreases with increasing thickness

Rate of voltage rise:	20 per cent	5 per cent	0.1 of 1 per cent per sec.
Average ratio: R =	1.861	1.576	1.783

Thus the general trend is apparently that the ratio decreases with increasing slowness of voltage application, although the slowest rate shows a partial recovery.

This last group of data illustrates the difficulty of the investigation of the action of solid insulating material and emphasizes the complexity and limited knowledge of the phenomena occurring in insulating materials under electric stress. The values averaged in the middle figure (1.576 at 5 per cent rate) were taken some weeks before the other two sets of tests, and while apparently the same materials were used and treated in the same manner and the general trend of variation is the same, a considerable difference occurs in the numerical values.

It is interesting to note that the ratio of direct voltage to peak alternating voltage of petrolatum-impregnated cable-paper, $R = 1.773$, in the comparison of direct voltage with the root-mean-square value results in the ratio—$\sqrt{2}\,(R = 1.773) = 2.501$. This value is the same as has been proposed as the results of extensive tests made abroad on cables.

B. Mica and Glass

TABLE II

MICA, GLASS AND PARAFFIN

Material	Electrodes	Gap length cm.	No. of layers	Direct	Alternating	Dielec. str'gth ratio
				kv. per mm.		
Clear Mica, 0.12 to 0.18 mm. thick.....				100.5	98.0	1.025
Pasted Mica, 0.32 to 0.35 mm. thick.....				60.5	48.6	1.245
				kv.	kv.	
Mica Tape on Brass Tube............			2	14.05	8.55	1.64
			4	34.9	21.0	1.59
			8	70.7	45.1	1.57
Mica Tape without sticking compound..			2	12.7	7.3	1.74
			4	21.5	12.7	1.69
			8	66.4	27.5	(2.42)
Mica Tape with sticking compound......			2	24.5	9.6	2.55
			4	48.1	20.2	2.38
	diam.			kv. per mm.		
Glass Tubing 0.7 to 0.85 mm. thick.....	2.54 cm.			68.0	46.3	1.469
				kv.	kv.	
Powdered Glass......	2.54 cm. spheres	1.90		37	44.5	0.832
		2.54		51.9	53.8	0.965
Powdered Glass made into paste with No. 6 transil oil..........	2.54 cm. spheres	0.63		56.8	61.6	0.922
		0.95		71.2	73.8	0.965
				kv. per mm.		
Cast Paraffin 0.9 to 2.2 mm. thick......				17.4	16.8	1.036

Table II gives data on some inorganic insulation, such as mica and its compositions, and also glass. The table gives the name of the material, the data on sizes of electrodes, and either the gap length or number of layers; there are also the voltages of the tests using either direct or alternating voltage, and their ratio—that is to say, the dielectric-strength ratio of these materials. Here, as in the preceding table, each numerical value is the average of a number of tests—usually 10 or 25. For some of the materials, such as clear and pasted mica, glass, etc., the dielectric strength is given in kilovolts per mm. so as to compare the averages of the different tests made with slightly different thicknesses of material.

The Dielectric-Strength Ratio of Mica in Several Forms. Clear mica gives a dielectric-strength ratio very little above unity, that is, in pure mica the dielectric strength is practically the same for alternating as for direct voltage. Possibly this low value of ratio indicates very low dielectric losses. Built-up or "pasted" mica however already shows a materially lesser strength for alternating than for direct voltage, a ratio of 1.245. Mica tape shows still much higher ratios, and mica tape put together with some organic sticker shows very high values. A comparison is made in the following averages:

Clear Mica..................	a ratio of	1.025
Pasted Mica...............	" " "	1.245
Mica Tape..................	" " "	1.646
Mica Tape held by a sticker.	" " "	2.46

Therefore the combination of two different materials in a laminated structure, here as in the preceding section A of impregnated paper, seems to raise the ratio. The ratio increases with the increase in the difference between the materials.

Apparently there is also a slight decrease of ratio with increasing thickness of the insulation, such as observed in the preceding studies.

The Dielectric-Strength-Ratio of Glass and Paraffin. Glass was tested, in the form of thin walled glass tubes, with mercury as inner electrode and tinfoil as outer electrode. Somewhat against expectation, glass, as the average of a number of tests, gave a high ratio — 1.469; that is to say, glass is dielectrically much stronger against direct than against alternating voltage. Whether or not this high ratio indicates a lack of homogeneity of the structure of the glass — as a colloidal solution — we cannot yet judge.

The glass of the tubes was powdered and the powdered glass in air was tested between 2.54 cm. spheres. Next, the powdered glass was mixed with No. 6 transil oil to form a paste, and this paste tested between 2.54 cm. spheres. Both the glass powder with the spaces between the particles filled with air, and that with the spaces filled with oil, show a dielectric-strength ratio slightly below unity (total average 0.921). That is, glass in solid form has a much greater dielectric strength for direct than for alternating voltage; as powder, however, its dielectric strength is less for direct than for alternating voltage. The explanation of this difference is still unknown.

In the same table have been added the averages of a number of tests made on thin cast disks of paraffin. They show a ratio close to unity, slightly above 1. — 1.036. This result seems reasonable.

TABLE III
VARNISHED CAMBRIC AND PARAFFINED PAPER

Materials	Electrodes	No. of layers	Rate of voltage rise per cent per sec.	Direct voltage kv.				Alternating voltage kv.				Dielectric-strength-ratio			
				25 deg.	50 deg.	75 deg.	100 deg.	25 deg.	50 deg.	75 deg.	100 deg.	25 deg.	50 deg.	75 deg.	100 deg.
Black varnished cambric 0.3 mm. (asphaltum base)	5 cm. plates	1 2	5 5	21.8 40.2	18.1	16.0	14.5	17.4 29.4	15.9	15.6	14.7	1.253 1.367	1.138	1.026	1.014
Yellow varnished cam. 0.2 mm. (linseed oil base)		1 2 4	5 5 5	11.9 19.3 31.3	14.2 23.8 43.1	13.2 21.0 39.4	12.0 19.2 30.8	11.1 17.3 26.6	12.6 22.4 41.1	11.5 22.2 32.4	11.7 19.4 28.6	1.072 1.116 1.177	1.127 1.062 1.049	1.148 1.054 1.215	1.026 0.990 1.077
		1 2 4	5 5 5	19.4 35.7	20.1 36.0	14.0 27.6 43.5	9.5 21.8	14.4 27.6	15.0 27.1	9.2 22.7 42.2	10.2 22.0	1.347 1.293	1.340 1.329	1.525 1.216 1.031	0.932 0.990
		1 2	2 2	21.2 38.8	18.0 31.7	11.5 22.5		14.4 24.2	13.9 23.5	9.8 22.4		1.472 1.600	1.295 1.380	1.173 1.005	
Cable Paper 0.2 mm. impregnated with paraffin		1 2	5 5	20.1 32.3				14.0 22.4				1.437 1.442			

C. VARNISHED CAMBRIC AND PARAFFINED PAPER

Table III gives data on two kinds of insulating cloth — black and yellow varnished cambric, and cable paper impregnated with paraffin. Cable paper impregnated with paraffin gives in general the same characteristic as cable paper impregnated with petrolatum; as might be expected — that is, a high value of the ratio.

The data on both kinds of varnished cloth seem to show a decrease of dielectric strength with increasing temperature, and also a decrease of the dielectric-strength ratio with increasing temperature and possibly also with increasing thickness. Thus, grouping the average values as was done in previous cases, the results are as follows:

Temperature and Thickness Data for Yellow Varnished Cloth.

Temperature;	25 deg.	50 deg.	75 deg.	100 deg. cent.
Average Ratio: R =	1.297	1.222	1.171	1.003
Thickness;	1	2	3	layers
Average Ratio: R =	1.231	1.193	1.080	

The total average is $R = 1.186$.

The similarity of the change of the ratio in varnished cloth, with that in impregnated paper, raises the question whether it is not a general characteristic of compound laminated structures, to have a high dielectric-strength ratio at low temperatures, low thicknesses, and high rates of voltage rise, and conversely to have this ratio decrease with increasing temperature, increasing thickness and increasing slowness of voltage application.

Discussion

For discussion of this paper see page 615.

Cable Geometry and the Calculation of Current-Carrying Capacity

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Review of the Subject.—*The main purpose of this article is to express the calculation of current-carrying capacity in simple formulas. The allowable current for underground cables is usually limited by the maximum permissible temperature of the insulation. The temperature rise is of course a function of the ability of the cable system to dissipate the heat generated. The chief difficulty in the calculation of current-carrying capacity is the determination of the thermal resistances of the path through which the heat must flow.*

The main part of this paper deals with the errors in the standard formulas for calculating the thermal resistance and geometric properties between the conductors and the sheath. A graphical method of correcting the errors is obtained in terms of what is called the "geometric factor," the results are tabulated for 2, 3 and 4-conductor cables throughout the range of practical sizes and an empirical formula is given. The check between the results of the graphical correction method and the published experimental data on this subject is very satisfactory, and emphasizes the errors in the standard formulas.

The thermal resistance between the sheath and the duct is mentioned briefly, and an approximate method of finding the resistance between the duct and the region at base temperature is outlined.

The previous work is then combined into a simple formula giving the allowable current for n-conductor cables, there being any number of similar cables in the duct bank. The formula is also enlarged to cover the case of cables in the metric and square inch systems, and cables buried directly in the ground. The method of including the effect of induced sheath currents in single-conductor cables and of dielectric losses is shown. Finally, the procedure to use in case the cables in the duct bank are not all of the same type is outlined.

In Appendix A the geometric factor for three-conductor cables under three-phase voltage is discussed, Russell's formula for this geometric factor being compared with the experimental determinations and an empirical formula for it is given. A formula is also given for the calculation of dielectric losses in three-conductor cables. The geometric factors for three-conductor cables in all other connections (i. e., the geometric factor for one conductor against the other two and sheath, or between any two conductors, etc.) are then derived in terms of the two geometric factors already obtained.

In Appendix B are given examples of the calculation of current carrying capacity under various conditions, and of dielectric loss.

In Appendix C an example is given which shows the error introduced by using an approximate formula for the calculation of the thermal resistivity of the insulation of a three-conductor cable based upon experimental measurements, the case taken up being a table in the Research on the Heating of Buried Cables.

INTRODUCTION

THE limit in the current-carrying capacity of underground cables is the allowable temperature of the insulation at the hottest point, usually the conductor surface, (unless economic or other considerations are determining factors). For a given allowable temperature at the conductor surface and a known temperature of the earth, the allowable temperature *rise* is obtained. The allowable current is of course that current which is just sufficient to produce the allowable rise, due to the heat generated in the conductors as $I^2 r$ loss, neglecting dielectric losses for the present.

The total temperature rise of a cable is usually divided into three distinct parts: T_A, the rise of the conductors above the lead sheath; T_B, the rise of the sheath above the duct; T_C, the rise of the duct above the base temperature of the earth. The thermal path may be considered as analogous to an electrical circuit with three resistances in series. In such an electric circuit the difference of potential across each of the elements would be equal to the product of its electrical resistance by the current flowing. In this thermal path, the difference of temperature across each of the elements will be equal to the flow of heat in watts, multiplied by the thermal resistance of the element in thermal ohms.

In the present paper, the T_A rise will be considered primarily, though the other ones will be mentioned briefly, and it will be attempted to compare the various divergent methods of calculating T_A, pointing out the known errors in the various formulas, and a new method will be developed for graphically correcting them.

THE GEOMETRIC FACTOR

The temperature rise between the conductors and the lead sheath, as stated above, is equal to the product of the total watts generated in the conductors by the thermal resistance between the conductors and the lead sheath, and our problem is to determine this thermal resistance.

For single-conductor cables the thermal resistance may be obtained in thermal ohms per unit length of cable by the standard formula

$$R_{tA} = \frac{\rho \log_e (R/r)}{2 \pi} \text{ thermal ohms per unit length} \qquad (1)$$

in which ρ is the thermal resistivity of the insulating material, R is the radius over the insulation, and r is the conductor radius. This formula is rigid and exact for homogeneous insulation.

For three-conductor cables several different formulas have been given, all in the form:

$$R_{tA} = \frac{\rho \log_e F_1}{6 \pi} \qquad (2)$$

in which F_1 is a function of the dimensions of the cable,

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and is given in quite different form by different writers. The following are the various values of $\log_e F_1$:

$$\log_e F_1 = \log_e \frac{1 - \alpha\beta + \sqrt{(1-\alpha^2)(1-\beta^2)}}{\alpha - \beta}$$

Mie's formula (3)

$$\text{where } \alpha = \left(\frac{r+a}{R}\right)^2 \text{ and } \beta = \left(\frac{a - 3r}{a + 3r}\right) \cdot \alpha$$

$$\log_e F_1 = \log_e \frac{R^6 - a^6}{3 R^3 a^2 r}$$

Russell's formula No. 1 (4)

$$\log_e F_1 = \log_e \frac{R^3 - a^3}{a^3 - (a - r)^3}$$

Russell's formula No. 2 (5)

In all the above formulas R is the radius of the cable under the lead sheath, a is the radius of the centers of conductors, and r is the conductor radius. It might be mentioned that Mie's formula has been also derived by Matsumoto. Reference to the source of these formulas is given in footnote 1.

It will be noted in formula (2) that the dimensions of the cable occur only in the logarithmic function, and we shall therefore call $\log_e F_1$ the *geometric factor of the cable*, and use the symbol G_1 for it.

ERRORS IN THE FORMULAS

The available formulas for G_1 are based on certain stated assumptions. Unfortunately the values of G_1 calculated by the various formulas are quite different for cables of ordinary makeup. This indicates that the assumptions on which the formulas are based are not valid for the usual cables.

Mie states that his formula is correct for very thick insulation with small conductors and also for very thin insulation. Russell says that his formulas are correct for the case of very small conductors very far apart. It has been noted that these qualifications have not in general been heeded, and in many cases the technical press shows that the formulas have been used regardless of the size of conductor. It is desired to show that the differences in these various formulas are very great in the range of practical cables.

Formulas (3), (4) and (5) contain three variables, but since the function in each case is a ratio, it can be expressed in terms of two parameters. These will be chosen as (α) the ratio of total insulation thickness between conductors and lead to conductor diameter or $\frac{T+t}{d}$, and (β) the ratio of belt insulation thickness to conductor insulation thickness, or t/T, in which T equals the conductor insulation thickness, t equals the belt insulation thickness, and d equals the conductor diameter.

We may show the geometric factor for all cables of equal belt and conductor insulation thickness ($t/T = 1$) if we plot the geometric factor as a function of $\frac{T+t}{d}$; in other words, the thermal resistance of all cables of a given kind of insulation having the same value of this function will be the same regardless of their absolute size. Careful examination has been made of the usual forms of cable, and also the unusual cases, and it has been found that practically all present day cables fall within the limits of about 0.2 to 2.0 for the ratio $\frac{T+t}{d}$. The geometric factors in this range are plotted and shown in Fig. 1. In addition to the geometric factors from (3), (4) and (5), the geometric factor obtained from Atkinson's experimental work is also plotted, concerning which more will be said later. The differences are obviously great.

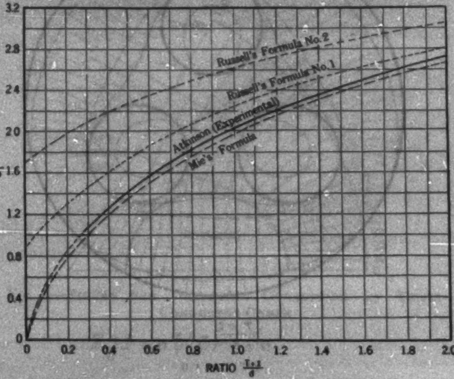

FIG. 1—GEOMETRIC FACTOR OF THREE-CONDUCTOR CABLES
Comparison of calculated values by different formulas. $t/T = 1.0$.

Before proceeding further, it is of interest to examine Fig. 1 and attempt to determine the errors for extreme cases. A ratio $\frac{T+t}{d}$ equal to zero would mean a cable (with finite conductors) of no insulation whatsoever. For this case, the thermal resistance and therefore the geometric factor must be zero, and thus both of Russell's formulas are seen to be in large error for small values of the ratio. For large values of the ratio, all the formulas approach the same correct value. For instance for a ratio of $\frac{T+t}{d}$ equal to 10, which

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1. Mie, G., *Elektrotechnische Zeitschrift*, 1905, p. 137.
Russell, A., Alternating Currents, 1914, p. 187.
Matsumoto, H., Electrotechnical Laboratory, Tokyo, Third Section, 1916, Report No. 24.

is of course an improbable practical case, G_1 has the following values for the formulas (3), (4) and (5) respectively; 4.35, 4.37 and 4.39. The most striking fact is that the curves for Russell's formulas do not pass through the origin as they should.

The errors in the various formulas are explained by their originators and are easily understood. If the twisting of the conductors is neglected, the problem is one of plane geometry, the figure to be solved being that of a cable cross-section, consisting of three circles in the vertices of an equilateral triangle, which represent the conductors, surrounded by a fourth circle representing the sheath. The approximate formulas which have been obtained have not used this actual figure, due to mathematical difficulties. Mie substitutes a figure for the three conductors, which coincides with the circles representing the three conductors at the regions nearest the sheath. His substituted figure consists of three ovals if the conductors are relatively small, and is a continuous curve dipping in between conductors if the conductors are relatively large. He uses the actual sheath. Knowing the equations of his substituted figure and of the sheath, Mie obtains a rigid expression for the thermal resistance between them. The error is of course due to the fact that the substituted figure is used in place of the three circles representing the conductors. For large values of the ratio $\frac{T+t}{d}$ the substituted figure very closely approaches the three conductors.

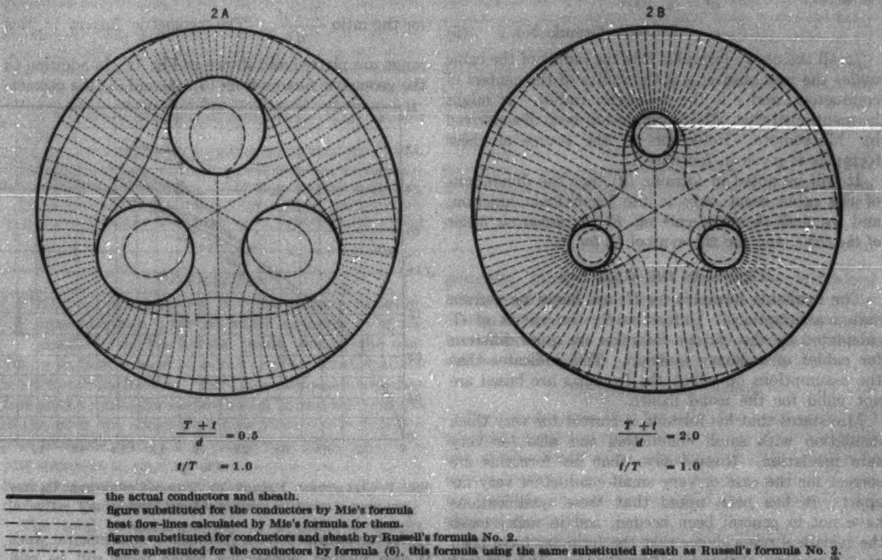
2 A
2 B
$\frac{T+t}{d} = 0.5$
$t/T = 1.0$
$\frac{T+t}{d} = 2.0$
$t/T = 1.0$
the actual conductors and sheath.
figure substituted for the conductors by Mie's formula
heat flow-lines calculated by Mie's formula for them.
figures substituted for conductors and sheath by Russell's formula No. 2.
figure substituted for the conductors by formula (6), this formula using the same substituted sheath as Russell's formula No. 2.

FIG. 2—THE SUBSTITUTED FIGURES FOR THREE-CONDUCTOR CABLES
Three conductors against the sheath.

Russell substitutes a different figure for the three conductors in his formula No. 2, his substituted figure being an oval inside the circle representing each conductor, and tangent to the conductor surface at the point nearest the cable center. For the sheath, he uses a figure tangent to the actual sheath at the points nearest the conductors and bending inward toward the cable center at the points between. Russell's formula No. 1 is derived by a different approximate method, the method of images, and no way has been devised here for graphically showing the errors in this formula.

The large error in Russell's formula No. 2 is easily explainable. The location of the greatest amount of flow of heat is undoubtedly between the conductors and the sheath through the shortest path between them and this is especially true for thin insulation. If substituted figures are to be used, the substituted sheath and substituted figures for the three conductors should be correct at this point to minimize the errors. A revision of Russell's formula No. 2 has therefore been made with the condition that the figures representing the conductors shall be tangent to the conductors at the points nearest the sheath rather than nearest the center of the cable, the substituted sheath being the same. This auxiliary formula has been derived for later purposes of checking, and has the following form:

$$G_1 = \log_e \frac{R^3 - a^3}{(a + r)^3 - a^3} \qquad (6)$$

The substituted figures are immediately understandable from Fig. 2 in which the actual conductors and sheath are shown as well as the substituted figures used in three of the formulas, Fig. 2A being a case of a relatively small value of the ratio, while Fig. 2B is for a large value of the ratio. The dotted lines are the flow lines of heat, which are defined as being lines normal to the isothermals, and such that the spaces between lines all have equal thermal resistances. The equations for these lines were obtained from Mie's formula for this purpose.

GRAPHICAL CORRECTION FOR THE ERRORS

The method devised for graphically correcting the geometric factor will now be explained. Mie's formula when applied to a cable contains errors, but it gives a *rigid solution* for the thermal conductance and geometric factor between his substituted figure and the sheath. Now the conductance between the three conductors or the substituted figure and the sheath is inversely proportional to the geometric factor. If therefore, we can by any means obtain the ratio of the conductance from Mie's substituted figure to the sheath to that from the three actual conductors for any particular cable, the product of this ratio by the geometric factor by Mie's formula will be the correct geometric factor of that cable. The same remark would of course apply to any other of the substituted figures, but since Mie's is the closest to the actual conductors, his formula and substituted figure have been chosen for this purpose.

The problem of correction is of course to graphically determine the ratio of the conductance from Mie's substituted figure to the true conductance. Due to the symmetry of a cable, it is sufficient to consider only one-half conductor. Fig. 3 shows 60 angular degrees, or one-half conductor, of the three-conductor cable shown in full in Fig. 2A. The flow lines shown were calculated by Mie's formula for them and are strictly the flow lines from Mie's figure to the sheath. Mie's equation for flow lines contains an auxiliary angle η; the line for η equal to zero is the shortest line between conductor and sheath, while the line for η equal to 180 deg. is the one midway between conductors. In Fig. 3, the lines have been plotted for η from 0 deg. to 180 deg. by steps of 10 deg.

By the definition of flow lines therefore, the conductance between the substituted figure and the sheath is the same along all of the paths between flow lines. If we take the conductance along any of these paths as unity, then the total conductance between Mie's figure and the sheath for this section would be 18 in Fig. 3.

The conductance from the conductor to the sheath must now be determined in the same units. Mie's figure coincides with the conductor for the first eight or nine paths, and each of these paths therefore has also a conductance of unity. For the subsequent paths, the conductance from the conductor to the sheath is through a longer path than from Mie's figure. Since conductance is inversely proportional to length of path, the conductance to the sheath for any particular path may be found by multiplying the conductance from Mie's figure (or unity) by the ratio of the average length of path from Mie's figure to its average length from the conductor. The conductance from the conductor to the sheath may therefore be determined path by path, the total conductance being the sum of the conductances of the 18 paths. This sum will be a number smaller than 18. If we now calculate the geometric factor for this particular cable by Mie's formula, and multiply it by the ratio of conductances, the correct geometric factor will be obtained.

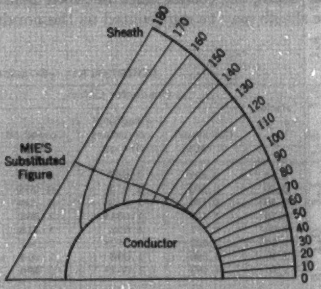

FIG. 3—FIGURE FOR GRAPHICAL CORRECTION OF THREE-CONDUCTOR CABLE

$$\frac{T + t}{d} = 0.5 \quad t/T = 1.0$$

The above method is described because of its clarity. The actual method used is based on the same principle though applied differently. The difficulty with the actual application of the method described is that a sufficient number of flow lines must be taken. It was found that the correction factor obtained by this method using 36 paths of flow instead of 18 as in Fig. 3 was slightly different, and that if 72 paths were used a still different geometric factor would be obtained, the answer obviously approaching a limit for an infinite number of flow lines. The calculation of flow lines is very laborious, since they must be plotted point by point, each point being obtained by solving an equation of degree equal to twice the number of conductors in the cable, and over forty separate figures of this kind were required.

The method actually used is the following, which is equivalent to using an infinite number of flow lines. A large figure was drawn for each case similar to Fig. 3 such that the radius of the sheath was equal to 10 inches, and the conductor, Mie's figure, and the flow lines for each 10 deg. of η were plotted on it. The length of each line from the conductor to the sheath and from Mie's figure to the sheath was carefully measured by a map measurer, and for each line the ratio of the length from Mie's figure to its length from the conductor was obtained, this being the true relative conductance along the line. The conductance from Mie's figure along each line was taken as unity.

The value of conductance between Mie's figure and the sheath was plotted against η as abscissa, the curve being of course a straight line parallel to the base. In other words, the conductance between Mie's figure and the sheath may be represented as the conductance between opposite sides of a rectangle. The relative true conductances were then plotted on the same figure against η, the true total conductance of conductors to sheath being represented by the summation of all the ordinates of this figure. The area of the figure corresponding to Mie's conductance was calculated and the area corresponding to the true conductance was measured by a planimeter. The ratio of these two areas is the ratio of conductances from the two figures, which, multiplied by Mie's geometric factor for the cable, will give its true geometric factor. The ratio of the two areas will be called the graphical correction factor.

This whole process was carried out graphically on a large scale for various proportions of $\frac{T+t}{d}$ for equal belt and conductor insulation thickness ($t/T = 1$), for half belt ($t/T = 0.5$), and for cables having no belt ($t/T = 0$). In each case the geometric factor was calculated by Mie's general formula for an n-conductor cable, the graphical correction factor was obtained as described, and multiplying these two together a corrected geometric factor was obtained. This identical process was carried out for two (round duplex), three and four-conductor cables; the results are tabulated in Table I. For practical use, the data of Table I should be expressed in curve form, G_1* being plotted against $\frac{T+t}{d}$. A separate curve should be drawn for each value of t/T. In the remainder of the article, when it is stated that the value of G_1 was obtained from Table I, it will be understood to mean from curves based on Table I.

TABLE I

CORRECTED GEOMETRIC FACTOR BY GRAPHICAL METHOD

Belt ratio t/T	Ratio $\frac{T+t}{d}$	2-Conductor Cables			3-Conductor Cables			4-Conductor Cables		
		Calculated G_1 (Mie)	Graphical correction factor	Corrected G_1	Calculated G_1 (Mie)	Graphical correction factor	Corrected G_1	Calculated G_1 (Mie)	Graphical correction factor	Corrected G_1
1.0	0.15	0.602	1.057	0.636	0.668	1.089	0.727	0.728	1.094	0.796
	0.30	0.889	1.055	0.938	1.015	1.078	1.094	1.121	1.087	1.219
	0.50	1.179	1.042	1.229	1.369	1.063	1.455	1.524	1.072	1.634
	0.75	1.456	1.031*	1.501	1.710	1.052*	1.799	1.909	1.061*	2.025
	1.00	1.681	1.022	1.718	1.978	1.045	2.067	2.210	1.053	2.327
	1.50	2.023	1.012*	2.047	2.383	1.020*	2.431	2.661	1.035*	2.754
	2.00	2.280	1.006	2.294	2.681	1.014	2.719	2.989	1.021	3.052
	2.50	2.486	1.004*	2.496	2.916	1.009*	2.942	3.243	1.015*	3.292
0.5	0.15	0.604	1.057*	0.638	0.668	1.086*	0.725	0.725	1.094*	0.793
	0.30	0.891	1.055	0.940	1.009	1.073	1.083	1.107	1.088	1.204
	0.50	1.177	1.038	1.222	1.350	1.059	1.430	1.488	1.074	1.598
	0.75	1.451	1.025*	1.487	1.672	1.045*	1.747	1.845	1.057*	1.950
	1.00	1.669	1.017	1.697	1.921	1.035	1.988	2.120	1.048	2.222
	1.50	1.995	1.007*	2.009	2.297	1.014*	2.329	2.520	1.026*	2.595
	2.00	2.242	1.004	2.251	2.572	1.009	2.595	2.817	1.011	2.848
	2.50	2.440	1.002*	2.445	2.789	1.005*	2.80	3.050	1.008*	3.074
0.0	0.15	0.613	1.055*	0.647	0.668	1.082	0.723	0.721	1.094*	0.789
	0.30	0.896	1.042	0.934	1.000	1.069	1.069	1.085	1.088	1.180
	0.50	1.178	1.028	1.211	1.321	1.053	1.391	1.435	1.068	1.533
	0.75	1.454	1.015*	1.476	1.618	1.033*	1.671	1.753	1.045*	1.832
	1.00	1.651	1.009	1.666	1.846	1.020	1.883	1.995	1.031	2.057
	1.50	1.967	1.004*	1.975	2.187	1.008*	2.204	2.352	1.015*	2.387
	2.00	2.205	1.002	2.209	2.439	1.003	2.446	2.612	1.006	2.628
	2.50	2.396	1.001*	2.398	2.638	1.002*	2.643	2.820	1.003*	2.830

*Interpolated value.

The sources of error in the method will now be examined. The main difficulty is the fact that the flow lines are strictly the flow lines from Mie's figure and not the flow lines from the actual conductors. The more closely the substituted figure coincides with the three actual conductors, the smaller will this error be, and

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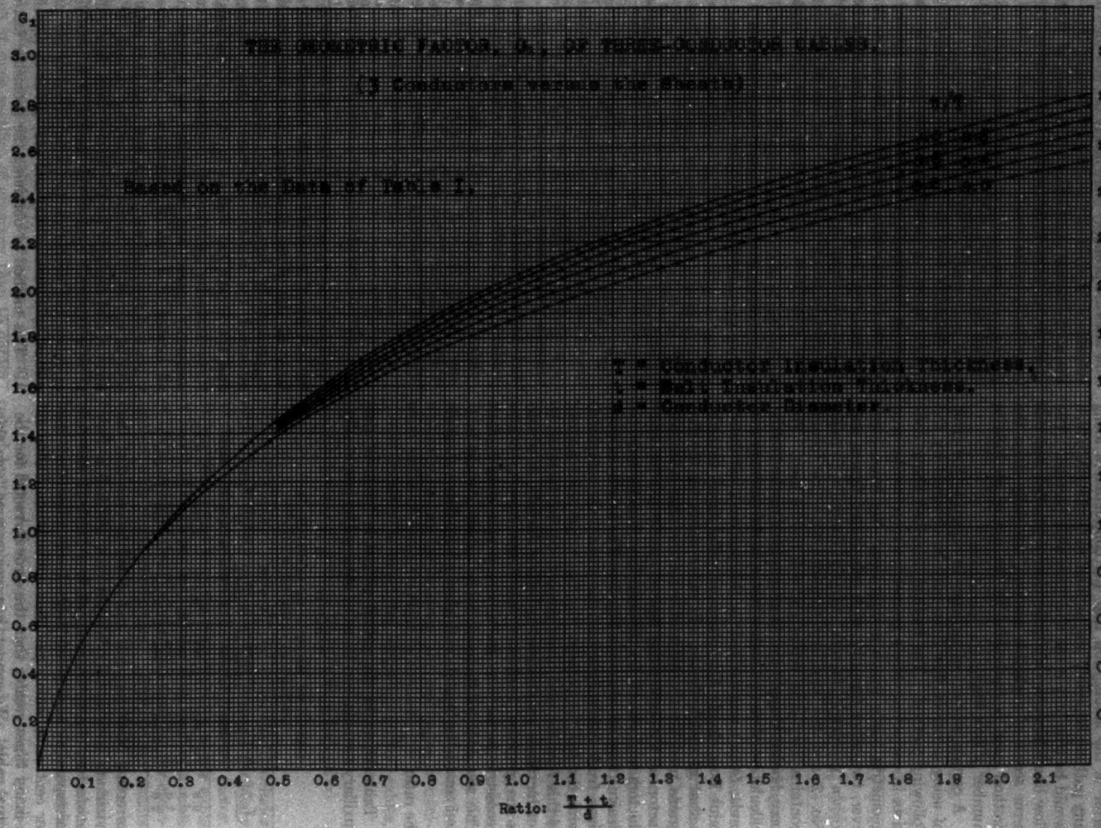
*G_1 is shown in curve form for three-conductor cables in this reprint only.


THE GEOMETRIC FACTOR, G₁, OF THREE-CONDUCTOR CABLES.
(3 Conductors versus the Sheath)
Based on the Data of Table I.
T/t
T = Conductor Insulation Thickness,
t = Belt Insulation Thickness.
d = Conductor Diameter.
G₁
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
Ratio: (T + t)/d

it was for this reason that it was decided to perform the graphical correction from Mie's formula. Another error is the assumption that the conductances along any path are inversely proportional to the length, which would be true only if the flow lines were parallel. The flow lines, however, are quite closely parallel, and this error seems of negligible amount, especially since the method used is equivalent to the use of an infinite number of flow lines.

A certain amount of evidence in regard to the error of using incorrect flow lines may be obtained by the following method. The method described above consists in correcting Mie's geometric factor to the true geometric factor which is unknown. We have, however, two other formulas with substituted figures and *known* geometric factors, namely Russell's formula No. 2 and the auxiliary formula (6). The former contains very large errors and will not be used. We may, however, apply this same graphical method to the calculation of the geometric factor of the auxiliary formula, (which was derived for this purpose) and determine how closely this corrected value agrees with its actual geometric factor. In other words, on the diagrams used for the calculation of the true geometric factor of the cable, the figures substituted for conductors and sheath by the auxiliary formula may be drawn (they are shown in Fig. 2). The lengths of flow lines from Mie's figure to the sheath are compared with the lengths between the two substituted figures of this formula, and areas proportional to the relative conductances are obtained as before, the ratio of which is the graphical correction factor transforming Mie's geometric factor into the auxiliary geometric factor. This was done for three-conductor cables of equal belt and conductor insulation thicknesses for four values of the ratio $\frac{T + t}{d}$.

Table II shows the actual geometric factor of the auxiliary formula, and its value obtained by graphically correcting Mie's formula, and it will be seen that the maximum error is only 0.8 per cent which is a very satisfactory check. A comparison of Table II with the corresponding section of Table I will show that the per cent error which has been corrected in Table II is many times greater than the error corrected for the actual cable. We cannot, however, assume from this that the corrected geometric factors of Table I are accurate to considerably more than 0.8 per cent, since an inspection of the figures would indicate that Mie's flow lines are probably more nearly the flow lines of the substituted figures of the auxiliary formula than they are of the actual conductors. The check of Table II, however, at least indicates that the error in Table I is probably quite small.

TABLE II
CHECK OF METHOD OF CORRECTION
3-Conductor Cables, $t/T = 1.0$

$\frac{T+t}{d}$	G_1 (Mie)	Graphical correction factor	G_1 of (6) by graphical correction	G_1 calculated by formula (6)	Error
0.3	1.015	0.764	0.775	0.769	+ 0.8%
0.5	1.369	0.824	1.128	1.125	+ 0.3
1.0	1.978	0.880	1.741	1.748	− 0.4
2.0	2.681	0.923	2.475	2.479	− 0.2

COMPARISON WITH EXPERIMENTAL DETERMINATIONS OF GEOMETRIC FACTOR

Fortunately another method of obtaining the geometric factor is available. The geometric factor shown in formula (2) is exactly the same for the calculation of not only thermal resistance but also of electrical resistance or capacity, the only difference in the formulas being in the constants used. If, therefore, it should be possible to obtain any one of certain electrical quantities experimentally, this determination would make it possible to find the geometric factor and thus all the other quantities. This has been done experimentally by Atkinson[2] and Sacchetto[3], who measured the resistance between three conductors and the sheath (as well as in other connections) of three-conductor cable models, composed of three tubes uniformly spaced and surrounded by a fourth tube, all located in a tub of electrolyte. Various sizes of tubes and spacings were used, and both have obtained sets of curves, in terms of charging current for the former, and capacity for the latter; from these values, three conductors against the sheath, it is possible to calculate the geometric factor in terms of our parameters by formulas (21) and (19) given in Appendix A. This method of obtaining the geometric factor is

TABLE III
COMPARISON OF GEOMETRIC FACTORS FOR THREE-CONDUCTOR CABLES

Belt ratio t/T	Ratio $\frac{T+t}{d}$	Calculated by Mie's formula	G_1 graphically corrected from Mie's formula	From Atkinson's experimental values	From Sacchetto's experimental values	Per cent deviation of (4) from (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.0	0.30	1.015	1.09	1.09	..	0.0%
	0.50	1.369	1.45	1.43	..	+ 1.4
	0.75	1.710	1.80	1.78	1.78	+ 1.2
	1.00	1.978	2.07	2.03	2.00	+ 2.0
	1.50	2.388	2.43	2.43	2.31	0.0
	2.00	2.681	2.72	..	2.54	..
	2.50	2.916	2.94	..	2.74	..
	3.18	3.168	..	..	3.00	..
	4.55	3.543	..	..	3.42	..
0.5	0.30	..	1.08	1.09	..	− 0.9
	0.50	..	1.43	1.44	..	− 0.7
	0.75	..	1.75	1.77	..	− 1.1
	1.00	..	1.99	1.98	..	+ 0.5
	1.50	..	2.33	2.33	..	0.0
0.0	0.30	..	1.07	1.07	..	0.0
	0.50	..	1.39	1.40	..	− 0.7
	0.75	..	1.67	1.68	..	− 0.6
	1.00	..	1.88	1.89	..	− 0.5
	1.50	..	2.20	2.24	..	− 1.8

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[2] Atkinson, TRANSACTIONS A. I. E. E., 1919, page 971.

[3] Sacchetto, l'Elettrotecnica, 1921, page 533.

theoretically correct, the only possible error being the experimental errors such as the location of the electrodes, which undoubtedly were reduced to a minimum. It is therefore possible to obtain values of the geometric factor by calculation from the experimental data of these two, and some of Atkinson's values have already been shown in Fig. 1.

Table III has been drawn up comparing the various experimental determinations of the geometric factor with the values obtained herein by correction from Mie's formula, shown in column (4). Column (5) shows the geometric factor obtained from Atkinson's experimental data, and column (6) that from Sacchetto's. The last two points shown in this column are for the actual values of the ratio used by Sacchetto while the others were obtained by drawing a curve through his points and reading off the values corresponding to our values of the ratio.

Column (7) shows the per cent deviation of our values from Atkinson's experimental values. The check seems quite good, especially when compared with the errors in the standard formulas as shown in Fig. 1. As to this small residual error, it is believed that in general for the small values of the ratio $\frac{T+t}{d}$, where the graphical correction is large, Atkinson's value is undoubtedly more nearly correct, while for the larger values of the ratio where the graphical correction is small, the new values are probably more nearly correct.

Referring to Sacchetto's values, it will be noted that the first one is quite close, while after that the values are low. Mie's value of the geometric factor is definitely always too small. The last five values from Sacchetto's data are 3 to 5 per cent lower than Mie's values and these values must therefore be definitely in error by at least that amount.

EMPIRICAL FORMULA FOR GEOMETRIC FACTOR

Since we have obtained the values of the geometric factor, it is of course immediately possible to derive empirical formulas which will coincide with the data of Table I. For three-conductor cables, the following formula (7) applies:

$$G_1 = (0.85 + 0.2\,\beta)\log_e[(8.3 - 2.2\,\beta)\,\alpha + 1] \qquad (7)$$

$$\text{where}\quad \alpha = \frac{T+t}{d} \quad \text{and} \quad \beta = t/T$$

Between $\alpha = 0.3$ and 2.5, this formula will give a value of the geometric factor agreeing to within 1.2 per cent with the values of Table I. Throughout most of the range, the check is within 0.5 per cent.

For an n-conductor cable it was thought that it would probably be more valuable to express a general formula at some sacrifice of accuracy than to obtain a very complicated expression or to have a different formula for each number of conductors. Formula (8) has therefore been derived:

$$G_1 = \log_e\left[\frac{n\,(8\,T + t)\,(T + t)}{4\,d\,T} + 1\right] \qquad (8)$$

where n = number of conductors in the cable.

TABLE IV
PER CENT DEVIATION OF FORMULA (8) FROM GEOMETRIC FACTOR

α	2-Conductor			3-Conductor			4-Conductor		
	β			β			β		
	1.0	0.5	0.0	1.0	0.5	0.0	1.0	0.5	0.0
0.3	− 8.8	−12.4	−15.2	+ 1.1	− 1.3	− 3.9	+ 7.2	− 0.3	+ 3.6
0.5	− 4.1	− 6.9	− 9.3	+ 1.5	+ 1.2	− 0.4	+ 4.2	+ 3.9	+ 5.0
1.0	− 0.9	− 2.3	− 3.3	− 0.9	+ 1.6	+ 3.3	− 1.3	+ 1.3	+ 6.8
1.5	0.0	− 0.5	− 1.7	− 0.7	+ 1.3	+ 4.3	− 2.9	+ 0.9	+ 8.8
2.0	+ 0.2	0.0	− 0.5	− 1.5	+ 1.0	+ 4.8	− 3.4	+ 1.5	+ 7.8

Table IV shows the error of this formula based upon the geometric factor as derived in this article. It will be noted by comparison with the correction factors shown in Table I that in most cases the error of formula (8) is much less than that of even Mie's formula, except for certain cases of non-belted cables. Cables without belts which are not in the Type *H* form are not of great practical importance, however, and it is believed that the simplicity and ease of application of this formula would make it convenient to use in cases where greater accuracy is not necessary. The simplicity is such that G_1 may be readily calculated on the slide rule, while it is dangerous and laborious to do so with Mie's or Russell's formula No. 1, since they are sixth degree expressions for three-conductor cables. It is believed that such a formula as this will be valuable when properties of cables are investigated mathematically, where for instance, the maximum or minimum value of some function is desired.

The formula is exact for single-conductor cables, since when $n = 1$ and $t = 0$, formula (8) reduces to:

$$G_1 = \log_e \frac{d + 2\,T}{d} = \log_e (D/d) = \log_e (R/r)$$

SECTOR-SHAPED CONDUCTORS

The expression "sector-shaped conductor" includes such a large variety of shapes that it seems necessary to obtain an approximate value of the geometric factor. It has been noted that the capacity of a sector cable is about 10 to 15 per cent greater than that of a cable of the same insulation thickness and same copper cross-section with round conductors. Approximate results can therefore be obtained if the geometric factor is obtained by reading a value from Table I on the basis of round conductors of the same area and the same insulation thickness, and then *decreasing* this geometric factor by 10 per cent.

It was hoped that a geometric factor might be obtained by the graphical method used for the case of round conductors. It was found, however, that Mie's flow lines were so far from being the correct flow lines for the sector conductors that no consistent results could be obtained.

Russell has given an approximate formula for three-conductor cables with sector conductors, which also applies only to the case of very small conductors with very thick insulation. This formula is obtained by a transformation of his formula No. 2, and the geometric factor is given in the following form:

$$G_1 = \log_e \frac{2\,R^2 - (c^2 + b^2)}{c^2 - b^2} \qquad (9)$$

in which R = the cable radius under the lead sheath
c = the maximum distance from the cable center to the conductor surface
b = the minimum distance from the cable center to the conductor surface.

It is difficult to state the exact error in formula (**9**), since our data are not definite enough for sector cables. However, it is quite interesting to note that although this formula is based on two approximations, the first being the approximation leading to Russell's formula No. 2 which introduces a large error, and the second being an approximation transforming his formula No. 2 into formula (**9**), these two apparently neutralize each other, with the result that formula (**9**) gives results for three-conductor cables with *round* conductors which are much more accurate than those obtained by either of his other formulas.

THERMAL RESISTANCE AND TEMPERATURE RISE BETWEEN CONDUCTORS AND SHEATH

Having obtained the values of the corrected geometric factor, it is immediately possible to calculate the temperature rise, T_A, of cables having from one to four conductors, and of course a similar process could be used to determine the geometric factor and temperature rise for cables with more conductors if desired. The formulas follow:

$$\text{Thermal resistance} = \frac{0.00522\,\rho\,G_1}{n}\ \text{thermal ohms per foot of cable} \qquad (10)$$

(Between the n-conductors and the sheath).

Temperature rise, conductors above sheath,

$$T_A = \frac{0.00522\ W\ \rho\ G_1}{n}\ \text{degrees}\ C \qquad (11)$$

where G_1 is obtained from Table I or from formulas (**7**) or (**8**)
n = number of conductors per cable
ρ = thermal resistivity of the insulation in watt-centimeter-centigrade units
$W = I^2\,r$ loss in watts per foot of cable.

$$0.00522 = \frac{1}{2\,\pi \times 12 \times 2.54}$$

The thermal resistivity may of course be obtained by inverting formula (**11**), if the temperature rise of the conductors above the sheath is known for a given loss in the conductors. It is especially important to use the correct value of the geometric factor for the determination of the thermal resistivity of cables, and it was thought that it might be interesting to illustrate this by a particular case, namely the Research on the Heating of Buried Cables. See Appendix C.

CURRENT CARRYING CAPACITY

The allowable current for underground cables is usually determined from temperature considerations except for very low-voltage cables where voltage drop may be the controlling feature. In certain cases, the laws of economy may determine the size of conductor, especially if the load factor is high. In what follows, however, only the temperature problem will be considered, but it should be borne in mind that in certain cases, the results should be tested for line drop and by Kelvin's Law.

The fundamental law on which the calculation of current carrying capacity as limited by temperature rise is based has been called "Ohm's Law for Heat." This may be expressed as follows: Temperature difference (centigrade) equals watts per foot times thermal resistance in thermal ohms per foot. This applies to a steady, continuous load, which is the only condition which will be covered in this paper.

So far the thermal resistance between the conductors and sheath is the only one that has been considered. Before giving a formula for the allowable current, we must determine the temperature rises, T_B and T_C, between the sheath and the duct, and the duct and base respectively. To do this, we must solve for the thermal resistances corresponding to these rises.

What may be roughly called the thermal resistance between the lead sheath and the duct, in other words a quantity which multiplied by the watts dissipated is equal to the temperature rise, (although it of course is not truly a thermal resistance) is primarily a function of the outside diameter of the cable and the thermal emissivity of the lead sheath. This thermal resistance may be expressed by the following formula:

$$r_{tB} = \frac{0.00411\,E}{D}\ \text{thermal ohms per foot} \qquad (12)$$

in which E is an approximate constant which might be called the surface resistivity of the sheath, almost independent of D, and expressed in degrees centigrade per watt per square centimeter, and D equals the outside diameter of the cable in inches. It might be thought that the size of the duct should be included in the formula for this thermal resistance. If the heat were transferred between the sheath and the duct by conduction, the size of the duct would enter in a marked degree. The transfer of heat, however, is by conduction in part only, and mostly by radiation and convection. The fact that T_B is almost independent of the size of duct can be illustrated by saying that for the same loss the sheath rise in a small duct is not very different from the sheath rise of a cable suspended in free air.

The duct rise, T_c, is the most difficult to determine, and in the present state of knowledge complicated mathematical calculations would probably not be justified. We will let H be the thermal resistance per foot between the duct and the region at base temperature or "sink." For a relatively small cylinder buried in the earth, the thermal resistance per foot is proportional to $\rho' \log 4\, l/D$, ρ' being the earth's resistivity, D the outside diameter of the cable, and l the distance between the cable axis and the surface of the earth. This formula due to Kennelly is practically standard in the calculation of the temperature rise of the sheath of a cable buried directly in the ground according to European practise. The temperature rise between a duct bank and the base is a similar problem, and an equivalent cylinder could be substituted for the duct. The ratio of the diameter of this cylinder to the depth below the surface would, however, be much greater than for the case of a cable buried in the ground, and errors in the formula would be greater. In fact, such a procedure is quite similar to using Russell's formula for the geometric factor for cables with large conductors and thin insulation. However, this is probably the best that can be done at present, and the errors and variations in the thermal resistivity of the soil are much greater than the errors which would be introduced by the formula. It is for this reason that the condensed rather than the full form of Kennelly's formula will be used.

We will therefore offer the following formula for H:

$$H = 0.00522 \times \rho' \times \log_\epsilon \frac{4\,l}{L} \text{ thermal ohms per foot of duct} \qquad (13)$$

in which ρ' = the thermal resistivity of the soil
l = the distance between the earth's surface and the center of the duct structure
L = the length of one side of a square duct bank.

If the duct structure is not square in cross-section, the geometric average of the two dimensions should be used for L.

Formula (13) is merely a first approximation, and may be used with the data of Shanklin[4] for the thermal resistivity of the soil. Its mathematical error has been investigated by calculating the temperature rise based on (13) and comparing it with the temperature rise calculated on the basis that the temperature rise of each duct is the rise due to the heat which it dissipates plus the rises due to the adjacent ducts, assuming that the thermal gradient as derived from (13) is true for short distances. It was also assumed that the duct structure was homogeneous with the soil in thermal characteristics. For ducts buried from 2 to 8 feet below the surface of the earth and for structures containing up to 16 ducts, the maximum difference between the two methods was only 8 per cent. This method incidentally also makes it possible to approximate the thermal gradient within the duct structure.

Atkinson[5] states that H may vary between 0.6 and 1.6 depending on the size of duct structure and upon the nature of the surrounding soil, and that 1.0 seems to be a reasonably safe average figure for conduits containing up to about 16 ducts. This is borne out by formula (13), using average values of soil resistivity. For practical numerical calculation, where the constants of the soil are unknown, 1.0 may be taken as an average value for H, and this has been done in the numerical examples in Appendix B.

The *total* thermal resistance, conductors to base for cables in ducts is of course equal to the sum of the three thermal resistances, H being multiplied by the number of similar cables in the duct to express the thermal resistance in terms of the watts per foot of *cable*, since *all* the cables are effective in heating the duct. The total thermal resistance therefore is:

$$R_{th} = \frac{0.00522\,\rho\,G_1}{n} + \frac{0.00411\,E}{D} + N\,H \text{ thermal ohms per foot} \qquad (14)$$

If we substitute 850[6] for ρ, 1200 for E, and 1.0 for H, (14) reduces to the following simple form:

$$R_{th} = \frac{4.44\,G_1}{n} + \frac{4.93}{D} + N \qquad (14')$$

thermal ohms per foot.

For armored cables buried directly in the ground, the total thermal resistance is given by a somewhat different expression. The thermal resistance between conductors and sheath is of course the same as for cables in ducts. Cables in the ground, however, usually have a protective covering of armor and jute wrappings around the lead sheath, whose thermal resistance is given by an expression similar to that for the insulation of a single-conductor cable. The thermal resistance between the outer surface of the cable and the region at base temperature is given by (13), using the outside diameter of the cable for L. If there are several cables buried fairly close together, there will in addition be an apparent increase of thermal resistance of one cable due to the presence of the others, and another term must be added represented by the summation sign,[7] which is intended to indicate that the process must be performed for all neighboring cables, excepting the one under consideration, all the cables being supposed identical in dimensions.

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[4] Shanklin, TRANSACTIONS of A. I. E. E., 1922, page 92.

[5] Atkinson, JOURNAL of A. I. E. E., 1920, page 831.

[6] This would apply to impregnated paper or varnished cambric insulation, and 650 may be used for rubber.

[7] This term is derived on the assumption that the temperature gradient is the same as that between two concentric cylinders of diameters $4\,l$ and D, [an assumption mathematically justified by formula 13] which is undoubtedly closely true if the cables are not too far apart.

The total thermal resistance, conductors to base, for cables buried directly in the ground is therefore the following:

$$R_{th} = 0.00522 \left[\frac{\rho\, G_1}{n} + \rho'' \log_e \frac{D'}{D} + \rho' \log_e \frac{4\, l}{D'} + \rho' \, \Sigma \log_e \frac{4\, l_n}{2\, x_n} \right]$$

thermal ohms per foot (15)

The following symbols apply to (**14**), (**14'**) and (**15**):

G_1 = the geometric factor from Table I, or formulas (7) or (8).

ρ = the thermal resistivity of the insulation in degrees C/watt/cm.^{3}

ρ' = the thermal resistivity of the soil in degrees C/watt/cm.^{3}

ρ'' = the thermal resistivity of protective coverings for armored cable in degrees C/watt/cm.^{3}

n = the number of conductors in each cable.

N = the number of similar, loaded cables.

D = the diameter over lead of the cable in inches.[8]

D' = the outside diameter of an armored cable in inches.

E = the surface resistivity of the sheath in degrees C/watt/cm.^{3}

H = the thermal resistance per foot of the duct from (13).

l = the distance in inches between the earth's surface and the center of the cable buried in the ground.

l_n = l of any other nearby cable.

x_n = the distance in inches between the axis of the cable under consideration and that of any other cable.

The total temperature rise for a given current per cable, there being N similar cables in the duct bank, (or in the ground), can be found by multiplying R_{th} by the $I^2 r$ loss per foot of cable, or the T_A, T_B and T_C rises may be determined separately by multiplying each of the three terms respectively of (**14**) by the watts.

The more usual problem is to find the allowable current for a given group of cables. The allowable temperature rise can be determined by subtracting the ground temperature from the allowable temperature of the insulation as determined by the A. I. E. E. rule or otherwise; if this rise is divided by the total thermal resistance, the allowable $I^2 r$ watts per cable are obtained. From the known conductor resistance, it is therefore possible to calculate the allowable current. The entire process may be expressed in the following formula giving the allowable current for n-conductor cables, there being N similar cables in the duct:

$$I = 0.281 \times \sqrt{\frac{A\,(T_e - T_0)}{n\, R_{th}}} \qquad (16)$$

where I = allowable current in amperes per conductor.

T_e = allowable temperature of the insulation according to A. I. E. E. Standardization Rule in degrees cent.

T_0 = base temperature of the ground in degrees cent.

A = cross-section of conductor in circular mils.

and R_{th} is obtained from (**14**) or (**14'**) for underground cables in ducts, from (**15**) for cables buried directly in the ground, and from (**10**) for submarine cables, T_A being the only rise in that case.

The constant in front of the radical is calculated in connection with the circular mils, A, so that the resistance is taken at 65 deg. cent. (an average operating value) including an increase of 2 per cent for stranding and 2 per cent for cabling. If the "skin" or "proximity" effects are appreciable, the constant should be divided by the square root of the ratio of a-c. to d-c. resistance. If A is expressed in square millimeters, the constant in front of the radical becomes 12.47 and if it is expressed in square inches, the constant becomes 317.0. The constant 0.281 is actually equal to $0.40/\sqrt{\rho_0}$ where ρ_0 is the resistivity of the copper in microhms per cm. cube (2.03 at 65 deg.), and from one point of view, the value of the resistivity at the operating temperature should be used. However, the variation of $\sqrt{\rho_0}$ between 55 and 85 deg. cent. is only 5 per cent, corresponding to working pressures of 30,000 and about 110 volts. Furthermore, it has been stated[5] that this variation is in part at least compensated for by changes in the surface thermal resistivity of the sheath. The use of a constant 0.281 at an average temperature thus seems allowable.

Formula (**16**) is believed to present the advantage of applying to the general case of N cables, each of n-conductors, and if formula (**8**) is used for the geometric factor, the allowable current is expressed directly in terms of the cable dimensions. Since formula (**14'**) is based on the constants used by Atkinson, and since the geometric factor of this paper agrees so closely with his geometric factor, this formula will give allowable currents for single-conductor, or three-conductor cables which will agree with those obtained by means of his tables[5].

DIELECTRIC LOSS

The effect of dielectric loss may be included by assuming (1) that the power factor throughout the entire insulation is the same as that at the hottest point, and (2) that the loss is concentrated in the conductors and must flow through the entire thermal path. The second

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[8] The outside diameter of a 3-conductor cable with round conductors is of course the following:

$$D = 2.155\,(d + 2\,T) + 2\,t + 2 \times \text{lead thickness.}$$

For sector cables, the outside diameter is equal to that of a round conductor cable of the same make-up minus from 0.3 to 0.4 times d, depending upon the shape of the sector.

assumption is of course correct for the sheath and duct rises. These two assumptions tend to give the maximum effect of dielectric loss, especially for low loss cables, though in certain cases, the effect may be greater. The assumptions do not lead to much error, since the whole effect normally is relatively small.

Since the allowable temperature of the conductor is known for a given voltage, the dielectric loss can be calculated in watts per foot by (22) based on the power factor of the insulation at that temperature. The temperature rise due to dielectric loss will then equal the loss in watts per foot multiplied by the total thermal resistance per foot from (14). The effect of the dielectric loss upon carrying capacity is obviously to diminish the temperature rise allowable for the $I^2 r$ loss. Mathematically it can be conveniently included in the calculation by finding the temperature rise due to dielectric loss as explained and subtracting this rise from T_e in equation (16).

For the case of single-conductor cables, the first term of formula (14), should be divided by 2 for this purpose, since it has been shown[9] that the temperature rise of the conductor above the sheath of a single-conductor cable is *half* as great for a given dielectric loss as for the same loss in the conductor.

INDUCED SHEATH CURRENTS

In the case of single-conductor cables bonded at both ends, the heat due to the flow of induced currents in the sheaths may have considerable effect upon the current-carrying capacity. The additional heat is generated in the sheath and therefore does not affect T_A, but does increase the T_B and T_C rises directly. If the total effective resistance due to sheath currents be calculated by a standard formula[10] and the conductor resistance at the desired temperature be known, the allowable current may be calculated conveniently by increasing the second and third terms in (14) by the ratio of the total effective resistance to the conductor resistance. (See Example 4 in Appendix B).

DIFFERENT TYPES OF CABLE IN THE SAME DUCT BANK

Formula (16) refers specifically to the case where all the cables in a duct line are exactly the same. In practise, a duct bank often contains cables of different sizes and voltages, and this formula does not apply. For the case of two different types of cable in the duct bank, a formula giving the allowable current for one type in terms of the dimensions of the two types has been derived, but it is rather formidable looking, and the following method may be easier of application. Find the allowable rise for each type of cable by subtracting the temperature of the earth from the allowable temperature of the insulation according to the A. I. E. E. rule. Express the total rise symbolically in terms of the unknown allowable watts per foot loss and the known thermal resistances. In each case, equate the total temperature rise to the allowable rise and obtain two equations for the two unknown watts per foot loss. These two equations are easily solved and the allowable watts for each type will be obtained. Note that the total rise for one type of cable is equal to the $I^2 r$ watts of that type times the total thermal resistance from (14), plus the duct rise due to the other type of cable which will equal $N H$ times the watts per foot of cable of the other type. Both watts are of course equal to $n I^2 r$. r, the conductor resistance per foot being known, the only unknowns will be the currents which may be solved from the watts. (For illustration see Example 2). The method will of course apply to any number of types of cable.

The effect of dielectric loss may be included by subtracting the temperature rise due to dielectric loss in each type from the allowable temperature for that type. The effect of induced sheath currents may be included by multiplying the terms in equation (14) corresponding to T_B and T_C by the ratio of total effective resistance to conductor resistance.

CONCLUSION

Inasmuch as the values of the geometric factor derived herewith are checked so closely by Atkinson's experimental values, it is believed that it may be stated confidently that the geometric factor, three conductors against the sheath, is known to an accuracy of 1 or 2 per cent, which seems quite sufficient since multi-conductor cables are actually not homogeneous due to the fillers. If it is desired to use a formula rather than to read the value from curves, formula (7) is by far the most accurate for three-conductor cables and (8) is fairly accurate for most practical n-conductor cables, up to $n = 4$ at least. Both are quite simple and can be easily calculated by slide rule. Of the older more standard formulas, Mie's is much the most accurate, especially in its condensed form, and this has been recognized in this country by Powell[11] in his work on current-carrying capacity.

The data of the present article have been expressed in terms of a "geometric factor." Atkinson's data are given in terms of charging current and Sacchetto's in terms of capacity. The data might equally well be given in terms of insulation resistance, dielectric loss, or even thermal resistance. In whatever form the data are given, formulas may be written with equal ease by which all the other quantities can be determined. The difficulty is, however, that such formulas, while mathematically correct, tend to give a false physical conception, such for instance as a formula for temperature rise in terms of charging current. The geometric factor has been introduced in order that all the formulas may be written directly in terms of the fundamental specific qualities of the insulation and a factor which is purely dimensional.

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[9] Schering, *Archiv für Electrotechnik*, 1922, Volume 11, page 68.

[10] See: Fisher, A. I. E. E., 1909, page 747.
Clark and Shanklin, A. I. E. E., 1919, page 917.
Capdeville, Revue Générale de l'Electricité, 1920, vol. 8, page 177.
Sacchetto, l'Elettrotecnica, 1922, page 667.

[11] Powell, A. I. E. E., 1916, page 1017.

The writer would like to emphasize here that no possible criticism is offered of Mie's or Russell's formulas, not only because both writers have carefully stated the limitations of their formulas, but also because he is an admirer and constant user of the latter's works in general and of this particular article of the former. His desire has been to show the magnitude of the errors which will occur if the formulas are used outside of the range for which they were intended, and to determine an alternative formula which will apply to the case of the more usual cables.

Appendix A

GEOMETRIC FACTORS OF THREE-CONDUCTOR CABLES FOR OTHER CONNECTIONS

In the body of this article the geometric factor three conductors against the sheath is the only one which has been considered, inasmuch as this is the only one of interest in connection with heat flow. When calculating dielectric losses and capacity, however, it is necessary to know the geometric factor under three-phase voltage, as well as under other connections, and while this has no direct bearing on current-carrying capacity, (except for the calculation of dielectric losses) it is so related to the geometric factor itself that this phase of the matter seems to have a proper place in this appendix. The geometric factor under three-phase voltage will first be discussed and then formulas will be given to obtain the geometric factor for the other connections in terms of the two geometric factors already obtained, all for three-conductor cables only.

GEOMETRIC FACTOR UNDER THREE-PHASE VOLTAGE

The usual formula for the geometric factor under three-phase voltage is given by Russell in a form which may be expressed as follows:

$$\log_e F_2 = \log_e \left[\frac{3\,a^2}{r^2} \cdot \frac{(R^2 - a^2)^3}{R^6 - a^6} \right] \qquad (17)$$

In order to simplify the constants in the formulas to be given, we will define the *geometric factor under three-phase voltage* as (3/2) $\log_e F_2$, symbolized as G_2. As in the case of Russell's formula No. 1 for the geometric factor three conductors against the sheath, formula (17) was obtained by the method of images, and it has not been possible to graphically correct the errors. This formula is theoretically correct only for the case of small conductors far apart, and it seems worth while to investigate the magnitude of the errors for the case of the usual practical cables.

While no graphical correction has been devised, it is possible to calculate this geometric factor also from the experimental work of Atkinson and Sacchetto. In Fig. 4 are plotted the geometric factors for cables with equal belt, belt equal to half the conductor insulation thickness, and no belt, as calculated by Russell's formula, and as obtained from Atkinson's data. The three points obtained by Sacchetto within the range of practical cables are also shown by points surrounded by circles.

Fortunately Russell's formula for the geometric factor under three-phase voltage gives values much closer to the experimental values than his formula for the geometric factor three conductors against the sheath, though the per cent error becomes fairly large for small values of the ratio $\frac{T+t}{d}$. As in the case of his formula for the geometric factor three conductors against the sheath, Russell's curves do not pass through the origin, but they come much closer in this case than in the former. It might be mentioned that one of Atkinson's points seems to be in error, namely his curve for 14 milliamperes, and since a continuous curve could not be drawn including data from this curve, this one value has not been included in the curve shown.

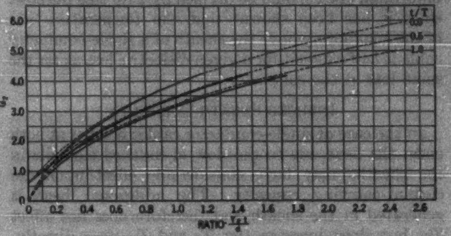

FIG. 4—THE GEOMETRIC FACTOR OF THREE-CONDUCTOR CABLES, UNDER 3-PHASE VOLTAGE

The full lines are from Atkinson's Experimental Data.
The broken lines are calculated by Russell's Formula.
The three points surrounded by circles are from Sacchetto's Experimental Data.
The dot-dash lines are an interpolation between zero and Atkinson's lowest point.

Since Atkinson's experimental data for the geometric factor between three conductors and the sheath has been so closely checked by the geometric method shown in the body of this article, and since his curves approach Russell's curves in the region where they should, it is believed that his data may safely be taken as correct for this geometric factor. Further evidence of Atkinson's correctness is that two of the three points obtained by Sacchetto shown in Fig. 4 fall exactly on his curve, the other point being not far off. Sacchetto's other two points (not shown) are for values of $\frac{T+t}{d}$ much larger than ever found in practical cables and where Russell should be accurate; they do not seem to be as close to Russell's values as would be expected.

The best determination of this geometric factor can be obtained by continuing Atkinson's curves to the right and making them asymptotic to Russell's values. If such a series of curves be drawn, it will be found that the geometric factor may be expressed by the following empirical formula:

$$G_2 = 3/2 \log_e F_2 = 2.475 \log_e [(0.52\,\delta^2 - 1.70\,\delta + 3.84)\,\alpha + 1] \qquad (18)$$

in which $\alpha = \dfrac{T + t}{d}$

and $\delta = t/T$

For α from 0.3 to 2.5, the maximum error is − 3 per cent (for $\alpha = 0.3$ and $\delta = 1.0$). For most cases the error is less than 2 per cent.

GEOMETRIC FACTORS FOR THREE-CONDUCTOR CABLES IN OTHER CONNECTIONS

Knowing G_1 and G_2, we can express the geometric factors for all other connections in terms of them, using Russell's relationships:

let $G_1 = \log_e F_1$ = geometric factor, 3 conductors against sheath (from Table I) and

let $G_2 = \dfrac{3 \log_e F_2}{2}$ = geometric factor under three-phase voltage (from Fig. 4). Then

G_3 = geometric factor, No. 1 vs. No. 2 $= 2\,G_2$

G_4 = geometric factor, No. 1 vs. No. 2 and No. 3 $= 1.5\,G_2$

G_5 = geometric factor, No. 1 vs. sheath (No. 2 and No. 3 insulated)

$$= \frac{3\,G_1 + 2\,G_2}{3}$$

G_6 = geometric factor, No. 1 vs. No. 2 and sheath (No. 3 insulated)

$$= \frac{G_2\,(6\,G_1 + G_2)}{3\,G_1 + 2\,G_2}$$

G_7 = geometric factor, No. 1 vs. No. 2, No. 3 and sheath

$$= \frac{9\,G_1\,G_2}{6\,G_1 + G_2}$$

G_8 = geometric factor, No. 1 and No. 2 vs. sheath (No. 3 insulated)

$$= \frac{6\,G_1 + G_2}{6}$$

G_9 = geometric factor, No. 1 and No. 2 vs. No. 3 and sheath $= \dfrac{4.5\,G_1\,G_2}{3\,G_1 + 2\,G_2}$

where No. 1, No. 2 and No. 3 represent the three conductors respectively

The following formulas *for three-conductor cables* based on the standard formulas, can be written in the terms of the geometric factors.

$$\text{Capacity} = \frac{0.0508\,K}{G} \text{ microfarads per 1000 feet} \qquad (19)$$

$$\text{Insulation resistance} = 0.330\,\rho_1 \times G \times 10^{-6} \text{ megohms per mile} \qquad (20)$$

$$\text{Charging current} = \frac{0.319\,E \times f \times K}{G} \text{ milliamperes per 1000 feet} \qquad (21)$$

$$\text{Dielectric loss} = \frac{0.000958\,E^2 \times f \times K \times \cos\varphi}{G_2} \text{ watts per foot} \qquad (22)$$

where G = the geometric factor for the connection used.

K = specific capacity or permittivity of the insulation.

ρ_1 = resistivity of the insulation in megohms per centimeter cube.

E = voltage in kilovolts (voltage *to neutral* if G_2 is used).

f = frequency.

$\cos\varphi$ = power factor of the insulation (for a given voltage, frequency and temperature).

Formulas (**19**), (**20**) and (**21**) can be used for any connection. If for instance, the charging current between one conductor against the other two conductors and the sheath is desired, the value will be obtained by substituting G_7 in equation (**21**). If the constants under three-phase voltage are sought, G_2 will of course be used, and the capacity and insulation resistance will be to neutral. Ordinarily the dielectric loss is of interest only under three-phase voltage, and this has been indicated by the symbol G_2 in formula (**22**). The same formula, however, may be used to determine the dielectric loss in other connections, if the constant in formula (**22**) is divided by 3, and the proper G is used.

Formulas (**19**) to (**22**) may be used for single-conductor cables also, by using $\log_e R/r$ for G or G_2, and dividing the constants in the first three formulas by 3, and by 9 in the case of equation (**22**). In this case, E must be the voltage between conductor and sheath, and the dielectric loss will be obtained in watts per foot of cable.

Appendix B

Example 1. What is the allowable current for four three-conductor cables, 4/0 A. w. g. round, insulated with 9/32 inches plus 4/32 inches impregnated paper, 1/8 inch lead for 25,000 volts working pressure?

$T = 0.281$ inches, $t = 0.125$ inches,
$d = 0.528$ inches, $D = 2.85$ inches.

$$\therefore t/T = 0.45, \text{ and } \frac{T + t}{d} = 0.77.$$

From Table I, $G_1 = 1.76$ [or 1.77 by formulas (7) or (8)]. Now $T_0 = 85 - 25 = 60$ deg. cent. Assume $T_G = 20$ deg. cent. $A = 211{,}600$ circular mils, $n = 3$, and $N = 4$. Therefore from formula (14′),

$$R_{th} = \frac{4.44 \times 1.76}{3} + \frac{4.93}{2.85} + 4 = 8.34$$

thermal ohms per foot

By (16)

$$I = 0.281 \sqrt{\frac{211{,}600\,(60\text{-}20)}{3 \times 8.34}} = 164$$

amperes per conductor

Example 2. A duct bank contains four three-conductor cables, 1/0 A. w. g. round, insulated with 9/32 inches plus 9/32 inches impregnated paper, 9/64 inches lead, for 25,000 volts working pressure, and five two-conductor cables (round duplex) 500,000 circular mils round, insulated with 3/32 inches plus 3/32 inches impregnated paper, 9/64 inches lead, for 4000 volts working pressure. What are the allowable currents?

We will call the first cables Type No. 1, and the latter Type No. 2. The earth temperature will be taken as 20 deg. cent. The following are the constants, T_0 being equal to 85 minus the working pressure:

	No. 1	No. 2
$\frac{T+t}{d}$	1.51	0.23
t/T	1.00	1.00
G_1	2.43	0.81
n	3	2
N	4	5
D	2.86	2.48
T_0	60	81
A	105,500	500 000

The thermal resistances for the two types separately are 9.33 and 8.79 thermal ohms per foot by (14′).

Let W_1 and W_2 be the allowable watt losses per foot, and I_1 and I_2 the allowable currents.

Then $\begin{cases} 9.33\,W_1 + 5\,W_2 = 60 - 20 = 40 \\ 4\,W_1 + 8.79\,W_2 = 81 - 20 = 61 \end{cases}$

Solving $\begin{cases} W_1 = 0.75 \\ W_2 = 6.60 \end{cases}$

But $\begin{cases} W_1 = 3\,I_1^2\,r_1 \\ W_2 = 2\,I_2^2\,r_2 \end{cases}$

the numerical coefficients at the end being necessary to include the number of conductors in the cable, if r is the resistance per foot of conductor. r_1 and r_2 may be calculated from resistance tables and are 0.000118 and 0.0000267 ohms per foot respectively at the operating temperatures, including increases of 2 per cent for stranding and 2 per cent for cabling. Therefore

$I_1 = 46.1$, and $I_2 = 352$ amperes per conductor.

Example 3. What is the dielectric loss in a three-conductor cable, 350,000 circular mils sector, insulated with 19/64 inches plus 7/64 inches paper, 9/64 lead for 33,000 volts working pressure, 60 cycles, assuming a permittivity of 3.3 and a power factor at operating temperature of 0.015?

$T = 0.297$ inches, $t = 0.109$, $d = 0.681$, $t/T = 0.37$, $(T + t)/d = 0.597$. From Fig. 4, $G_2 = 2.63$ [or 2.68 by formula (18)]. Diminishing the latter by 10 per cent, because the conductors are sector-shaped $G_2 = 2.37$. By formula (22),

Dielectric loss

$$= \frac{0.000958 \times (33/\sqrt{3})^2 \times 60 \times 3.3 \times 0.015}{2.37}$$

$= 0.43$ watts per foot.

Example 4. As a final example, the case of a current-carrying problem including dielectric loss and sheath loss will be given. Find the kv-a. carrying capacity of a line composed of four single-conductor cables (one being a spare) to operate at 110,000 volts, three-phase with the neutral grounded, the frequency being 60 cycles. The conductors are 400,000 circular mils, insulated with 40/32 inches impregnated paper, 5/32 inches lead. The cables are arranged in the vertices of an equilateral prism, the distance between centers of cables being 6 inches, and they are bonded at both ends at least. It will be assumed that $k = 3.3$ and that the power factor of the insulation is 0.015 at the operating temperature. $T = 1.25$ inches, $d = 0.728$ inches, and the outside diameter of the cable, $D = 3.54$ inches. The geometric factor, $\log_e R/r$, therefore equals 1.49. There are three loaded cables in the duct, and therefore $N = 3$, and of course $n = 1$. The allowable temperature will be taken as 50 deg. cent., and the earth temperature as 20.

By formula (22), the dielectric loss is 0.86 watts per foot of cable, remembering that for single-conductor cables, the constant of that formula must be divided by 9. The total thermal resistance for *dielectric loss*, conductor to base, by (14′) equals 7.7 thermal ohms per foot, the first term in (14′) being divided by 2, as stated in the text, for single-conductor cables. The temperature rise due to dielectric loss is therefore $7.7 \times 0.86 = 6.6$ degrees.

The additional effective resistance due to induced sheath currents is (by Clark and Shanklin's formula) equal to 0.0108 ohms per 1000 feet, figuring the lead resistance at 30 deg.; the conductor resistance itself calculated at 50 deg. and increased 2 per cent for stranding is equal to 0.0295 ohms per 1000 feet. The ratio of total effective resistance to conductor resistance is therefore 1.37.

Formula (14′) may now be applied as follows: $R_{th} = 4.44 \times 1.49 + 1.37 \times 4.93/3.54 + 1.37 \times 3 = 12.6$ thermal ohms per foot. Therefore from (16),

$$I = 0.281 \sqrt{\frac{400{,}000\,(50\text{-}20\text{-}6.6)}{1 \times 12.6}} = 242$$

amperes per cable.

The kv-a. carrying capacity of the group is therefore 46,100 kv-a.

Appendix C

In the text, it was mentioned that formula (11) may be used for calculation of the thermal resistivity of a cable, if the temperature rise between the conductors and sheath be measured for a known $I^2 r$ loss in the conductors, and it was stated that it was especially important to use the correct geometric factor in such a determination. An illustration of the errors introduced by the use of the incorrect geometric factor in such a measurement may be of interest.

In Table III of the Research on the Heating of Buried Cables[12] is given a tabulation of the calculated values of the thermal resistivity of paper insulated cables, single-conductor, concentric multiconductor, and cabled multi-conductor in type. The average values of thermal resistivity obtained were 1000, 950 and 600 respectively for the three types of cable. The first two were calculated by accurate formulas. For the case of the cabled multiconductor cables, the resistivity was calculated by means of Russell's formula No. 1 for round conductors, and by his formula (9) for the sector. The difference between the results is striking, and of course part of the discrepancy can be explained as due to the use of Russell's formulas as will be shown, though there will still remain a considerable difference.

Due to the limits of space, we will not show all the details of our calculations, but it may suffice to say that the correct geometric factors were obtained by the data in this article, decreasing the value by 10 per cent for sector cables as stated in the text, and using the full graphical correction for the case of the two six-conductor cables. A corrected value of thermal resistivity was then obtained by multiplying the given values by the ratio of the geometric factor by Russell's formula to its correct value.

The average value of thermal resistivity of the multi-conductor cables as published in the research was 600. If the values are corrected, the average is 720 or 20 per cent higher than the average published value. It might be added that the minimum error was − 6 per cent, and that in the case of cables No. 11 and No. 27, the errors were − 28 per cent and − 26 per cent respectively.

Discussion

CABLE CHARGE AND DISCHARGE (Steinmetz)
DIELECTRIC STRENGTH RATIO BETWEEN ALTERNATING AND DIRECT VOLTAGES (Hayden & Eddy)
CABLE GEOMETRY AND THE CALCULATION OF CURRENT CARRYING CAPACITY (Simons),

Swampscott, Mass., June 26, 1923

J. B. Whitehead: The approximate exponential shape of the charge and discharge curves of composite dielectrics has long been recognized, and many efforts have been made to explain the departure of these curves from the pure exponential form, by various assumptions as to the structure of the dielectric in various arrangements of high resistance paths in conjunction with capacities. Several of these suggestions are in approximate conformity to the experimental observations of the charge and discharge curves as observed with continuous potentials. One of the simplest of all is that used by Dr. Steinmetz in which assumption is made of a very high resistance in series with a capacity, a number of such combinations being assumed with different relative values of the resistances and capacities. Another recent analysis of the problem has been given by K. W. Wagner who, with an entirely different method of approach, through a modification of Maxwell's suggestion of a mixture of two or more dielectrics, reaches the same conclusion as does Dr. Steinmetz, namely that the charge and discharge curves have one principal exponential term accompanied by one or more additional terms of relatively much smaller magnitudes.

The behavior of dielectrics under alternating stress is very obscure and presents one of the most important problems now confronting both the physicist and the electrical engineer. None of the theories of dielectric composition as offered by the physicists is sufficient to explain the diverse behavior of ordinary insulation under alternating stress. The general case was investigated by Rowland many years ago and since by a number of others. The results of these studies will in many cases conform qualitatively to some of the main lines of experimental observation. Quantitatively, however, they are quite insufficient to explain the facts, and the anomalies of observation are so numerous as to clearly indicate that our present knowledge is quite insufficient either to explain the facts of observation, or to predict beforehand the behavior of even the simplest types of insulation. Dr. Steinmetz makes a notable contribution to the alternating-current case in reaching the conclusion that the energy component of the charging current corresponds to the initial value of the absorption current under continuous voltage. In doing this he has made certain assumptions which would appear to demand experimental confirmation. It is highly important, however, in that it appears to link up the dielectric phenomena of the continuous and alternating voltage fields.

The real problem of insulation is a knowledge of the ultimate nature of the processes which go on in the structure of dielectrics. What for example is the nature of the conductivity of the high resistance path assumed by Dr. Steinmetz, and how does the capacity in series with this resistance arise? Do they exist independently, or are both properties inherent in the structure of single molecules or atoms? We have gone so far in our studies of the wide variety of commercial insulation as to reach the conviction that it is not possible to design insulation with any certainty that its various properties can be safely predicted. Further attacks must be made through simplified problems directed toward the ultimate processes involved. The Committee on Insulation, Division of Engineering, National Research Council, working in conjunction with the Research Committee of the A. I. E. E., exists primarily for this purpose. The statement of The Problem of Insulation as prepared by these Committees is published in the June number of the Journal A. I. E. E. Considerable attention is given in this published report to the phenomenon of absorption. The Committees bespeak the interest and cooperation of all members of the Institute in attacking these problems by the methods set forth in the report.

J. Slepian: Dr. Steinmetz believes that dielectric absorption of charge is to be explained entirely by the inhomogeneity of the dielectric.

However, since conduction in a dielectric is not as simple as metallic conduction, charges may arise within an entirely homogeneous substance due solely to the passage of current. I can give some very simple examples of this.

Undoubtedly the most homogeneous dielectric imaginable is a perfect vacuum, and thanks to the general interest in radio most of us know something about conduction in a vacuum. We all know that if two electrodes in a high vacuum are sufficiently hot, current may be carried across the space between them.

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[12] *Journal* of I. E. E., 1921, Vol. 59, p. 181.

Thus the vacuum dielectric begins to leak when the electrodes are hot enough. It is now well known that the electric gradient across the space is not uniform under these conditions, and that a space charge is produced. Space charge is a very common term now. This space charge is nothing but a charge soaked into the vacuum dielectric. The transient in establishing this space charge is very rapid due to the high velcoity with which electrons move in vacuum, but it still is an absorption transient, such as this paper describes for cables. Nevertheless the vacuum is homogeneous.

Passing to the case where we actually have matter, the next case in the order of difficulty would be conduction through a gaseous dielectric. We can make a gaseous space between electrodes conducting by means of an ionizing agent. This might be a source of X-rays acting uniformly on the gaseous space or ultra-violet radiation acting on the electrodes. The distribution of gradient and charge for this kind of conduction is fairly well known. I think Mie was one of the first analysts who worked it out, and you will find his work described in J. J. Thompson's "Conduction in Gases." When current flows across a uniformly ionized air space, the gradient again is not uniform, but again space charges are produced. These space charges may be called soaked in, or absorbed charges, if you like. Again the transient in establishing such charges is of very short duration because the ions in gases also move very rapidly, but the transient exists nevertheless in spite of the homogeneity of the gaseous space.

In solid and liquid insulations the carriers of the current probably move with very slow velocity so that the time in establishing the stored up charges will be relatively very long. I do not happen to know definitely that you do get absorbed charges in strictly homogeneous liquid or solid dielectrics, but I strongly suspect that that is the case.

I do recall that in some respects the conductivity in some homogeneous dielectrics is similar to the conductivity in gases. For example, with the same gradient larger leakage may be obtained with the electrodes far apart than when close together. This shows that dielectrics do not have a simple ohmic resistance, and it seems to me that even in a homogeneous medium space charges or soaked-in charges may be produced, resulting from the nature of conduction in the dielectric.

D. E. Howes: The labor involved in obtaining reliable dielectric strength values for such a large number of conditions is enormous—a fact, which will be concurred in by all who are familiar with insulating materials.

The results in the Hayden and Eddy paper bear out the fact, which the practical engineer should consider an axiom,—that the most satisfactory test which can be given a commercial machine or material is that which corresponds most nearly to the conditions under which it is expected to be used. Short cuts should be carefully investigated before being adopted, as was wisely done by the writers of this paper. I can take no exception to their experimental procedure or results obtained. Further, I do not intend to attempt to fully explain the reasons for some of their observations. It would be, perhaps, impossible and at any rate hardly worth while. What I do wish to point out is that, from theoretical considerations alone, it is fundamentally wrong to expect any very simple or reliable dielectric strength ratio as defined by the authors of this paper, at least for practical insulating materials.

Let us borrow again Maxwell's conception (*El. & Mag.*, Vol. 1, page 453) of a composite dielectric, a conception which has proven so valuable and so actually real.

Consider two adjacent elements or strata of dielectric in series, located any place you please in a composite dielectric, whose specific inductance capacities, specific resistances are respectively denoted by K_1, K_2 and ρ_1, ρ_2.

Suppose a direct voltage be applied to the outer surfaces. It will divide between the two dielectrics so that the potential gradients will be in the ratio of ρ_1 and ρ_2. If, however, the voltage is changing rapidly, either pulsating or alternating, the voltage gradients in medium 1 and 2 will be in the ratio of K_2/K_1.

Suppose $\rho_1/\rho_2 = 1$, $K_1/K_2 = 2/4$. Then for direct voltages the gradients in the two media would be equal. For a rapidly changing voltage, the gradients in 1 and 2 would be in the ratio of 2/1 and we would expect medium *1* to fail prematurely, so that in this case the maximum value of the alternating voltage breakdown might be less than for direct voltage, unless of course, it had a much greater dielectric strength. Thus, the distribution of stress may be widely different, even though the total voltage applied be the same. With a different stress distribution throughout the insulating material, we would expect the dielectric strength to be different. Only the most homogeneous material could be expected to have a dielectric strength ratio of unity. It would be reasonable to expect that it would not vary widely for a uniform grade of material subjected to a uniform test. For this reason, direct voltage may have a limited application where its significance, relative to alternating voltage test values, has been predetermined.

It is evident that ρ_1 and ρ_2 will not vary with temperature according to the same law. Continuing, with reference to the above conception and its extension by Maxwell to account for absorption or residual charge, it is obvious that if the rate of change of voltage be very slow in comparison with commercial frequencies the relative potential gradients in different component parts of the dielectric may be determined partly by the specific resistances of those components. In other words, let us ask this question,—What is the rate of change of potential that determines whether the relative gradients shall be according to K_2/K_1, or as ρ_1/ρ_2? Is there a sharp dividing line?

In view of the well known phenomenon that a condenser exhibiting absorption does not become fully charged for quite an appreciable time, it seems reasonable to expect that there is no sharp dividing line, but that both factors contribute, and the relative importance of each will depend upon the rate of change of voltage. When the rate is very slow, the specific resistances of the component media are dominant in effect while, when the rate is rapid, the specific inductive capacities control. Thus, it is evident that the dielectric strength ratio has little meaning unless we specify very closely the rate of application of voltage.

When we consider the fact that nearly all practical solid insulating materials are composite, and analyze how they should behave under different types of fields, we must realize how little real additional information concerning dielectric behavior this so-called dielectric strength ratio actually tells us. It is realized that, on the whole, the phenomena are not as simple as explained, but I believe that in general the theoretical considerations are very well born out by the experimental data given in the authors' paper. For obvious reasons, then, I do not think that direct voltage should be employed for testing insulating materials which in service are to be subjected to alternating potentials, where the dielectric strength is to be determined. Direct voltages lower than this, however, should be useful in routine cable testing to indicate any gradual changes in their quality.

I am of the opinion that we should not draw any new conclusions as to the ultimate nature of a dielectric breakdown from the consideration of this paper. Aside from known effects of temperature etc. I believe insulation failure is dependent upon the maximum voltage attained for an appreciable length of time, regardless of its form. The ultimate cause of breakdown is doubtless, cumulative ionization, although this itself is influenced by numerous other factors.

In conclusion, I wish to express the opinion that the real nature of the mechanism of dielectric failure will not be found by any commercial or practical engineering tests that we may devise but by indirect methods. By this I mean that we should study it from a purely scientific viewpoint to determine the location and behavior of an electron in it.

D. M. Simons: I have just come across a recently published article which bears so directly upon the papers by Dr. Steinmetz and Messrs. Hayden and Eddy that I thought it might be interesting to mention it briefly, particularly one test that was made. I am referring to an article by Grünewald entitled "The Breakdown Strength of Solid, Laminate Insulating Materials with Various Types of Electrical Stress," (*Archiv fur Elektrotechnik*, 1923, Vol. XII, p. 79). Grünewald performed breakdown tests on insulation not only with alternating and continuous voltages but with various types of impulse. He did not show the effects of different rates of applying the voltage, or of different temperatures, but his work is of interest on account of the variety of impulses impressed on the insulation. His tests were mostly upon mica, and unfortunately no tests were made upon impregnated paper. Grünewald includes also a mathematical section, and while he does not obtain a solution for n-layers, he obtains an equation for the voltage distribution across two layers of insulation as a function of time, and it is interesting to note that this equation (13) can be transformed to agree with Steinmetz's equation (18).

Before proceeding to describe what I believe is the most interesting test made by Grünewald, I would like to ask if any breakdown tests have been made upon a dielectric which already contains residual charges. We might consider the case of a dielectric which has widely different breakdowns for slowly applied and rapidly applied voltages. Dr. Steinmetz has very clearly explained that the difference in breakdown strength is due to the fact that for the rapidly applied voltage the voltage distribution is determined by the relative permittivities of the layers, while for the slowly applied voltage the resistivity is the determining factor. My particular thought is what would happen if a continuous voltage close to but below the breakdown voltage was impressed on this dielectric until the residual charges had completely soaked in, and a rapidly applied voltage was then impressed on the dielectric. It seems to me that in this case the voltage distribution would be determined by the internal charges and would be according to resistivity, and the ratio of d-c. to a-c. breakdown would be reduced to or toward unity.

Grünewald did not perform this test, but he did the opposite. That is, he performed breakdown tests on condensers which had previously been charged in a direction opposite to the final test voltage, and he found that this had a very great effect in lowering the breakdown strength. As far as I can determine he broke down the previously charged condensers with single steep-front impulses only, and found that in general for this type of surge, the breakdown value was about *halved* if the condenser was previously charged in the opposite direction.

It seems that breakdown tests on dielectrics which have previously been charged in either direction may throw additional light on the fundamentals of the behavior of solid dielectrics.

Herman Halperin: In his opening remarks, Dr. Steinmetz made some comments about the relative slow progress that has been made in the art of insulation. I think these papers are a definite step in the progress that seems to be needed. The first two papers deal with stresses and the last with heat conduction through complicated structures of conductors and insulations such as multi-conductor cables.

In Dr. Steinmetz' paper he refers to the decrease in the internal transient current and gives an example which indicates a ratio of about 20 to 1. In some tests made in Chicago on long underground lines, we find that the final current was as small as 1/150th or 1/200th of its initial value. If we tried to find the equation for this decrease of current against time by using an equation with one transient, we would obtain a curve which would hold only for the data taken for one minute or so; and it was during the first minute of the test that the decrease in current was very rapid. Perhaps it would be necessary to use another transient term, as Dr. Steinmetz points out, in order to equate the entire curve.

In connection with the d-c. a-c. ratio, after we do find just what is exactly the ratio for one kind of electrical apparatus, for instance, underground cable, it would still seem necessary to discover it for joints connected with the cable. This further complicates the situation because we might secure a dielectric strength ratio of 1.78 for one cable and probably something different for the joints; and in making a test the whole line has to be subjected to the voltage at one time.

In Mr. Simons' paper, he has, I think, done a great deal in showing how accurately the thermal resistivity of a complicated arrangement of conductors and insulation can be determined. In the appendix, he shows that his more correct method would indicate an error of about 20 per cent in some results which were published by a research committee in England; so these accurate methods greatly help correlate the various researches.

Mr. Simons deals mostly with the determination of the geometric factor. But, of course, for the installed cable in this country, there are other very important factors in heat dissipation such as H, the thermal resistance of the earth per foot of duct, and the resistance of duct structure itself. While there may be variations for the thermal resistivity of insulations of as much as about 50 per cent, the heat dissipating ability of the duct structure and surrounding soil may vary several hundred per cent. A large percentage of the total drop from copper to the base soil temperature is through the soil and duct structure and there is a great deal of work to be done in this field.

H. B. Dwight: Mr. Simons has made a distinct advance in the calculation of the geometric factor of multi-conductor cables and has produced by his graphical method more accurate results than were available before. The publication of this graphical method, however, brings up at once the question whether a direct and accurate mathematical calculation cannot be made. This would be especially desirable in the present case, for the graphical method, besides having the errors of measurement inherent in any graphical method, has the theoretical approximation in that the flow lines are not those pertaining to true circles, a fact which is pointed out in the paper.

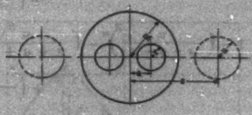

FIG. 1

A graphical method was justified because the formulas which had been published for multi-conductor cables were all approximate, since they assumed that the electric charge on each conductor was equivalent to a charge concentrated at a point of the section of the cable. This, however, gives rise to equipotential surfaces which are not circular, and so cannot coincide with the surfaces of the round wires or cables.

It appears that practically all formulas for capacitance have been based on charges concentrated at points, or uniformly distributed along lines or over surfaces. This is true, at any rate, of the published formulas for multi-conductor cables. Clark Maxwell states in Chapter 7 of his book on Electricity and Magnetism published in 1873 that every electrical problem of which we know the solution has been constructed by the inverse process of finding the shape of the conductors from the potential due to assumed charges of electricity. He also states that the only method by which one can expect to solve a new problem is by reducing it to one of the cases in which a similar problem has been constructed by the inverse process. Maxwell's statement has been quoted in fairly recent articles, and the limitation which he gives that the inverse process must always be used and that one cannot start with an assumed shape of conductor seems to hold in the most recent calculations of capacitance. A general method which would correctly use the direct process of starting with given shapes of conductors would therefore be very useful.

I wish to present an accurate method using the direct process of calculation of capacitance and geometric factor of multi-conductor cables. This method consists in first assuming a uniform charge on the surfaces of the round conductors and their images. From this the resulting charge at any point of the conductors can be calculated. This may be called the first additional charge, and it is in the form of a Fourier series, that is, a series involving cos A, cos 2 A, cos 3 A, etc. The cosines often disappear when integrated around the circle. The second additional charge can now be calculated, resulting from the first additional charge, and so on, until the terms become small.

This method has been used to determine the capacitance of a single-phase overhead line, and the result for a given example agreed with the result by the standard hyperbolic cosine formula to six significant figures. The formula for a two-conductor cable is given herewith. That for a three-conductor cable is longer.

Capacitance of two-conductor cable

$$= \frac{1}{2 \operatorname{logh} F_c}$$

$$2 \operatorname{logh} F_c$$

$$= \operatorname{logh} \frac{(s-a-u)\,(s-a-r)\,(s+a-u)\,(s+a+r)}{r\,u\,(2\,a+r)\,(2\,s-u)}$$

$$+ \sum_{n=1}^{n=\infty} \frac{L_n}{n} \left[1 - \left(\frac{r}{s-a-u} \right)^n - \left(\frac{-r}{s+a-u} \right)^n + \left(\frac{-r}{2\,a+r} \right)^n \right]$$

$$- \sum_{n=1}^{n=\infty} \frac{M_n}{n} \left[1 - \left(\frac{u}{s-a-r} \right)^n - \left(\frac{u}{s+a+r} \right)^n + \left(\frac{u}{2\,s-u} \right)^n \right]$$

$$u = \frac{R^2\,r}{a^2-r^2} \qquad s = \frac{R^2\,a}{a^2-r^2}$$

$$A_n = \left(\frac{r}{s-a} \right)^n - \left(\frac{-r}{2\,a} \right)^n + \left(\frac{-r}{s+a} \right)^n$$

$$F_n = -\left(\frac{u}{s-a} \right)^n + \left(\frac{u}{2\,s} \right)^n - \left(\frac{u}{s+a} \right)^n$$

$$B_n = -\left(\frac{r}{s-a} \right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{u}{s-a} \right)^k F_k$$

$$- \left(\frac{-r}{2\,a} \right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{-r}{2\,a} \right)^k A_k$$

$$- \left(\frac{-r}{s+a} \right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{u}{s+a} \right)^k F_k$$

$$G_n = -\left(\frac{u}{s-a} \right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{r}{s-a} \right)^k A_k$$

$$- \left(\frac{u}{2\,s} \right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{u}{2\,s} \right)^k F_k$$

$$- \left(\frac{u}{s+a} \right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{-r}{s+a} \right)^k A_k$$

For C_n and H_n use the same formulas as for B_n and G_n except change A to B and F to G.

For D_n and I_n change B to C and G to H, etc.

$$L_n = A_n + B_n + C_n + \ldots$$
$$M_n = F_n + G_n + H_n + \ldots$$

It is to be noted that the terms involve only the dimensions of the cable. The terms can be calculated with a slide rule. As there is a considerable number of these terms, the method cannot be said to be a short one.

The formula for a two-conductor cable has been used to check one of the values of geometric factor in Mr. Simons' paper within less than one per cent. For $t/T = 1$ and $\frac{T+t}{d} = 1$ the formula gives 1.730 to compare with 1.718 given by the graphical method.

R. Notvest: In connection with the Hayden-Eddy paper, it was very interesting to note the behavior of composite dielectrics under high direct as well as alternating-current potential stresses. The authors investigated practically all important commercial insulation materials from mica, oil, glass to paper and I regret that they have not extended the scope of their investigations upon slate, for slate, a composite dielectric of great commercial importance, is the classical example of the variation of the dielectric strength ratio of a given material between direct and alternating-current potentials. Slate has a rather high insulation value for direct current, but may show considerable leakage under alternating current of such low potentials as from 1500 volts up.

I have made a number of tests which plainly showed that as far as slate is concerned, leakage under high a-c. potentials cannot be ascribed to ionization of the dielectric, but is distinctly an impedance or condenser effect. This is due to the fact that slate contains among other ingredients also around 4 per cent of metallic oxides such as ferric and ferrous oxide as well as pyrites which to all practical purposes are distributed uniformly in almost molecular form throughout the otherwise electrically inert substances. Each such particle of metallic oxide is capable of assuming a definite capacity charge under alternating potentials and is interconnected with the others through an ohmic resistance of relatively high value, the 0.2 to 0.5 per cent of carbonaceous matter which slate contains. Impedance leakages actually measured on a number of various samples which were afterwards chemically analyzed, showed that variations of the metallic oxide content, that is, the capacity effect or the ohmic resistance, that is, the content of carbonaceous substances in graphitic form, corresponded very closely to estimated results based on Dr. Steinmetz formulas of a condition of a circuit having capacities and ohmic resistances in series. I am convinced that additional research on slate would produce valuable data and would give an insight of the stress distribution in composite dielectrics and the very pertinent problem of insulation in general.

Mr. Davis: Mr. Simons' paper and the formulas contained therein will undoubtedly be of inestimable value to engineers in calculating current capacities. His errors of twenty per cent of the approximate formula (I think he said twenty) and the errors published in the JOURNAL of the Institute of Electrical Engineers can hardly be considered serious when we think of the possible errors we have in the thermal resistance of this material.

I merely wanted to call attention to the fact that thermal resistivity as published by the Institute showed variations of approximately from 350 to 1250 watts per centimeter cube, per degree centigrade. Tests that we have made but not completed showed variations running from 300 watts per centimeter cube per degree centigrade to 1800. In other words, we have a variation there of about six hundred per cent. Now, the application of any of these formulas for calculation of temperature rises depends absolutely upon the use of some of those constants which really have wide variance. Consequently it gives an error of twenty per cent. That is small when the amount of thermal resistance varied about six hundred per cent.

C. F. Hanson: Referring to Table I of the paper by Hayden and Eddy the data given for impregnated cable paper are very interesting, particularly the data given for a rate of voltage rise of 5 per cent per second. The dielectric strength ratio of one paper at 25 deg. cent. is 1.343 when the rate of voltage rise is 5 per cent per second whereas the ratio of one paper at the same temperature is 2.470 when the rate of voltage rise is 20 per cent per second. One ratio is twice the other. This difference indicates that a cable of good dielectric qualities might be damaged when it is tested with direct voltage if the voltage is applied too slowly.

These data may also be interpreted in another way. The alternating breakdown voltage at 25 deg. cent. for 2 papers is 18.2 kv. when the rate of voltage rise is 5 per cent per second whereas the breakdown voltage is only 6.6 kv. when the rate of voltage rise is 20 per cent per second. The difference in breakdown voltage is about 300 per cent which is more than can be attributed to the difference in the rate of application of voltage judging by other data presented. The paper used in the former case, had a higher dielectric strength than that used in the latter case as determined from alternating voltage tests. It is then interesting to note that the dielectric strength ratio of the high dielectric strength impregnated paper is only one-half of that of low dielectric strength impregnated paper. In other words, if two cables were tested at the factory with alternating voltage and one was found to have 50 per cent higher dielectric strength than the other, and if these two cables were subsequently tested with direct voltage after installation in the field, the cable of lower dielectric strength would pass whereas the one of higher dielectric strength might fail. As cables are operated on alternating voltage, it would seem that the direct-voltage test had passed a poor cable and had eliminated a good one.

The authors state that they obtained an average value of 1.773 for the dielectric strength ratio which corresponds to 2.501 as the ratio between direct voltage and the root-mean-square alternating voltage. It must be borne in mind that this value was obtained on a comparatively thin insulation. The thickest insulation used was four layers each 0.2 mm. thick or a total thickness of about 2/64ths inch. In high-voltage cables a thickness of 40/64ths inch is used. The dielectric strength ratio would be considerably less than 1.773 for a thickness of 40/64ths inch as indicated by the author's data. Furthermore, the dielectric strength ratio obtained here is for momentary application of voltage. It may be different for a prolonged application of voltage. In testing cables the voltage is generally applied for 5 minutes. It would, therefore, be of practical interest to know what the ratio is when the time of application of voltage is 5 minutes.

Harvey L. Curtis: The paper by Dr. Steinmetz can for convenience be considered in two parts, first, the mathematical, and second, the experimental. It seems unfortunate to me that Dr. Steinmetz does not refer to the earlier mathematical work on this subject of a stratified dielectric.[1] His discussion of the energy storage in a stratified dielectric is, I think, new. However, I am not as yet willing to accept the definition of "apparent" or "effective" capacitance which is brought out as a corollary to this discussion.

It seems to me that Dr. Steinmetz overlooked at least one interesting conclusion that can be drawn from his equations. From equation (36) it is evident that the tangent of the phase difference (90 deg. minus the phase angle) is C_2/C_1. Substituting the values given in equations (38) and (39), it follows that as a first approximation, the phase difference is inversely proportional to the frequency.

In the paper above referred to, I derived an equivalent equation and called attention to the fact that this conclusion did not agree with the experimental data then available on condensers. I have hoped to see this test applied to a composite structure, such as a cable, as I believe that this will tell whether or not the theory of a stratified dielectric is of practical importance.

In the experimental part of his paper, Dr. Steinmetz measures the current which flows into a cable when a constant voltage is impressed on the cable terminals. The current which flows after the first charging current he calls "slow transient" or "internal transient." He then computes the constants in an exponential equation of two terms so as to make the observed and computed curve agree at five points. When he has done this, he finds that the theoretical and experimental curves agree within experimental error. Since the theory of a stratified dielectric shows that under certain conditions such an equation will result, he concludes that the observed phenomena are explained by this theory. However, he fails to connect the constants in this exponential equation with the dielectric constant and resistivity of the materials of the cable, which the mathematical part of the paper has shown should be possible.

Everyone who has investigated the subject agrees that there is an "internal transient" due to the stratification of a dielectric. However, there is considerable question whether, in any electrical structure, the "internal transient" which is caused by the stratification of the dielectric is an appreciable part of that which is observed. I regret that Dr. Steinmetz has not shown some experimental method by which this question can be settled.

R. J. Wiseman: The paper by Messrs. Hayden and Eddy is of great value to all who have occasion to test electrical insulation on direct and alternating voltage. With the introduction of high potential direct voltage testing the question naturally arose as to what value of voltage should be used. Should we take a value equal to the root-mean-square of the alternating voltage or the maximum value of the alternating voltage? We know that when testing with alternating voltages, insulation is subject to the maximum voltage and therefore if it is able to withstand this voltage, the insulation should be capable of withstanding a direct voltage of equal value. Instead of taking a value of direct voltage equal to the maximum value of alternating voltage, we find that a value of 2.5 times the root-mean-square alternating voltage which is 1.773 times the maximum value, has been taken as an arbitrary one in view of tests made abroad on cables. After giving due credit to the work done abroad as to what value of direct voltage in proportion to alternating voltage is necessary to cause breakdown of insulation, the question arises, do we want to make time tests on insulation using direct voltages 2.5 times r. m. s. alternating voltage or a dielectric strength ratio of 1.773.

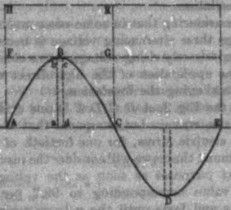

FIG. 2

I do not believe we do. The paper by Messrs. Hayden and Eddy brings out some very valuable information as to the comparison of direct and alternating voltage breakdowns of insulation under varying conditions. Their Table I shows interesting data which should cause us to defer deciding definitely on the factors to use in determining the value of direct voltages in proportion to alternating voltages. Different insulations give different values of dielectric strength ratio and for the same insulation this ratio changes with temperature, thickness and rate of voltage rise; three important quantities when testing. In all cases but impregnated paper the dielectric strength ratio is much less than 1.773 and in many cases even less than one, which means puncturing voltage on direct potentials is less than on alternating potentials. It would be unfair to test such a material at a direct voltage of 1.773 times maximum value of alternating voltage.

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[1] Maxwell: *Electricity & Magnetism*, par. 328.
Rowland: *American Journal Science* (4) 4, p. 29; 1897.
Curtis: *Bull. Bureau of Standards* 6; p. 468, 1910. S. P. 137.
Grover: *Bull. Bureau of Standards* 7; p. 519; 1911. S. P. 161

Consider the influence of thickness on the ratio. Take dry cable paper, four layers, 20 per cent per second voltage rise, the ratio is 1.790 and for 32 layers 1.085, a drop of 39.4 per cent in terms of 1.790 for a change in wall thickness from 0.8 mm. (31.4 mils) 6.4 mm. (252 mils). As we test thicknesses greater than 6.4 mm. (252 mils) it appears we may get ratios less than one for the greater thicknesses and again 1.773 is unfair.

Referring to the rate of voltage rise, we note that a rate of increase of 0.1 per cent per second gives the lowest values for the dielectric strength ratio. This rate is equivalent to taking 16.7 minutes to bring the voltage up to cause breakdown. This is a slow rise and is almost comparable to a constant voltage test. Some of the values of the ratio for dry cable paper are slightly above but the average of all values shown give a value below 1.

When we study the results obtained for impregnated cable paper we find values of ratio above and below 1.773 not consistent, the range being from 2.775 to 1.238. In view of the wide range in value of ratio it doesn't seem possible we have a right to average all values and take this average as representing the ratio for impregnated cable paper because it is the average of several variables for which the ratio itself is quite a variable.

Tables II and III also show that the ratio is different for each kind of insulation and departs considerably from 1.773.

As the paper shows that in the data reported the ratio of direct voltage breakdown to maximum value of alternating voltage breakdown is different for each kind of insulation, we should not establish a general value of direct voltage in terms of alternating voltage for insulation testing but give a value of each type of insulation.

Aside from considering that in some cases much greater value of direct voltage than alternating voltage is necessary to cause breakdown on constantly rising voltage, let us consider the effect of the long time application of the high direct voltage on the possibility of weakening the insulation.

Referring to the Fig. 2, $A\ B\ C\ D\ E$ is one cycle of an alternating voltage sine wave. Let us assume that the insulation is under effective electric stress, for one fortieth of the time each side of the maximum, that is we will consider the insulation as being stressed for test purposes as soon as the voltage reaches the instantaneous value corresponding to 99.7 per cent of the maximum value and lasts until the voltage again reaches this value when decreasing. This is shown by the curve $b\ B\ c$. The line $F\ B\ G$ represents a direct voltage equal to the maximum value of alternating voltage. The line $H\ K$ represents a direct voltage 1.773 times as great as the maximum value of direct voltage. The area $a\ b\ c\ d$ represents the product of alternating voltage and time of application and may be taken as a measure of the stress under alternating voltage. Likewise the areas $A\,F\,G\,C$ and $A\ H\ K\ C$ represent the product of direct voltage and time of application for the respective voltages.

These areas are a measure of the relative stresses which the insulation is subject to. Although we cannot say, two equal areas, one having twice the voltage of the other, give the same effective stress, still on a time basis for the sake of discussion we can say the net effect is the same. Taking this viewpoint and calling the area $a\ b\ c\ d$ unity, then the area $A\ F\ G\ C$ is 12.73 and the area $A\ H\ K\ C$ is 22.6. This means the voltage time stress is 12.73 times greater for a direct voltage equal to the maximum value of alternating voltage and 22.6 times as great for a direct voltage 1.773 times the maximum value of alternating voltage. Referring to the results which have been obtained by Messrs. Hayden and Eddy this may be a serious problem for some kinds of insulation and result in weakening of the insulation.

Another source of possible weakening of the insulation is the value of dielectric stress resulting from the high direct voltage test. The stress equivalent to the maximum value of alternating voltage is not great enough to permanently weaken the insulation without chance of recovery. It may so happen that the stress equivalent to the direct voltage will be sufficiently great to permanently damage the insulation, yet not cause it to breakdown and the usefulness of the insulation greatly lowered as a result. If a lower value of voltage had been used, possible sources of weakness would have been found, and yet the insulation have a high factor of safety.

I am a believer in direct voltage testing of insulation and look forward to the time when manufacturers and users of insulating materials will consider it as the routine method of test. However, today this method of test is purely in the formative stage and all the above questions should be seriously considered before we attempt to establish standards for the testing of insulation with direct voltages.

W. N. Eddy: Mr. Hanson brings out two points on which we would like to comment. He thinks that the increase of ratio caused in one case by increased rate of voltage application, indicates that a cable of good dielectric qualities might be damaged by the application of direct voltage at too slow a rate. We feel that such a change of the ratio rather indicates that the ratio of an insulation should not be specified without including the rate of voltage application. In fact, one of the principal points we tried to bring out in the paper was that a d-c.—a-c. ratio is meaningless unless accompanied by such factors as the rate of voltage application, the temperature, the type and shape of the insulation, etc.

He also attempts to draw definite conclusions from a comparison of two breakdown voltages and two ratios. In view of the well known difficulty of obtaining consistent breakdown data on insulation, we believe that such a practise should be avoided, and that in analyzing a collection of breakdown tests, the various values should be considered together, weighed against each other, and only general tendencies noted.

D. M. Simons: I am particularly gratified if I have been in any way instrumental in inducing Mr. Dwight to develop his formulas for the geometric factors of multi-conductor cables. When his formula for the geometric factor of three-conductor cables is available, this phase of cable engineering will apparently have received its final answer.

Several very interesting papers on the subject of the carrying capacity of cables have been published abroad at about the same time as the present paper. The Second Report on the Research on the Heating of Buried Cables[1] is an exceedingly important contribution to our knowledge of most phases of this subject. It is noted incidentally that the errors in the calculation of thermal resistivities in the first Report mentioned in Appendix C have been corrected. It is also very interesting to see that the geometric relationships of three-conductor cables were investigated both experimentally and by a graphical method. The graphical method, which is entirely different from the present one, checks my values about as closely as can be determined from the large scale graph on which the English data is plotted.

Another interesting contribution is "The Heating of Paper-Insulated Power Cables" by Sacchetto[2]. I believe that there should be some word to express the form factor or geometric modulus to take care of the dimensions of the cable in numerical calculations. I have suggested the term "geometric factor" for this purpose. I was not entirely sure that this was the best term, although it seemed sufficiently appropriate and was certainly more concise than some expressions that have been used, such as "number equal to the capacity if the S. I. C. were unity," etc. Sacchetto confirms my choice by calling a similar term a *fattore geometrico*. His geometric factor is $2/n$ times the writer's. Sacchetto also mentions the fact that Picou[3] called a corresponding factor in cable formulas the *facteur geometrique*.

I am very glad that Mr. Halperin has emphasized the possibility of errors due to the duct rise in calculating the allowable current. There is undoubtedly a great deal to be done on this section of the thermal path before we can accurately calculate in advance the temperature rise of cables.

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1. Permissible Current Loading of British Standard Impregnated Paper-Insulated Electric Cables. Second Report on the Research on the Heating of Buried Cables. JOURNAL of the Institution of Electrical Engineers, 1923, vol. 61, p. 517.

2. l'Elettrotecnica, 1923, p. 181.

3. Revue Générale de l'Electricité, 1917, vol. 1, p. 410.

Some Engineering Features of the Weymouth Station of The Edison Electric Illuminating Co. of Boston

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and JOSEPH POPE
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Review of the Subject.—*The new power station of The Edison Electric Illuminating Company of Boston, now under construction on the Weymouth Fore River is expected to have an ultimate capacity of about 300,000 kw. The present construction covers the installation of two 32,000 kw. generating units with three 1974-h. p. boilers, operating at 375 lb. steam pressure and 700 deg. fahr. In addition, a single boiler to carry steam at pressures up to 1200 lb. is to be installed. This boiler will have about the same heating surface as the normal pressure boilers. The steam generated by it will pass through a pressure reducing turbine developing about 2000 kw., and will be exhausted at 375 lb. pressure. After being reheated to the original temperature of 700 deg. the steam will be piped to the main header and used in the large turbines. If satisfactory results are obtained from the higher-pressure boiler-turbine units more of them will be installed. Three will be required to furnish sufficient steam to operate one of the 32,000-kw. units.*

The maximum steam temperature was fixed at 700 deg. in consideration of the properties of materials at the higher temperatures. While the theoretical gain from higher pressure increases up to the maximum for which any data are available, the full benefit of the thermodynamic possibilities are not at present obtainable in practise without reheating the steam at some point intermediate between the throttle and the condenser. Without reheating, the most advantageous steam pressure, both practically and economically considered, seems to be about 375 lb. gage. With reheating, it appears that about 1200 lb. is a practicable initial pressure, and that 375 lb. is an entirely satisfactory pressure at which to reheat. Accordingly a combination of these two pressures with intermediate reheating has been adopted; the higher pressure with a view to developing its possibilities, the lower pressure with the feeling that it represents the best practise in the single expansion cycle and that operation solely at that pressure will give highly economical results. The character of the future development of the plant will depend upon the relative performance of the two classes of equipment.

Feed water is to be heated by two-stage bleeding of the main units and by economizers. All normally running auxiliary equipment is to be driven by alternating-current motors. Power for driving the essential auxiliaries is to be supplied from a 2300-volt alternator of 2500 kv-a. capacity connected directly to each 30,000-kw. main generator shaft, and thus driven by the main turbine, which will have sufficient capacity for driving both generators at full load. This arrangement is expected to give a combination of greater economy and security than any heretofore used.

The switch house will be a separate structure, four stories and basement. The upper floor will carry the switchboard and the switch operating mechanism. The three lower stories will house the bus structures, oil circuit breakers and reactors, each phase being isolated to a single floor. The basement will serve as a cable vault and all incoming and outgoing circuits will pass through it. There will be two main ring busses and a transfer bus. The ring busses will be divided into sections, each section fed by two generators and with current-limiting reactors between sections. All circuits will be connected to the busses through two oil circuit breakers in series.

No prediction is made of the expected operating performance of the station, but it is calculated that under ideal conditions, acting solely as a 375-lb. single-expansion nonreheating plant, it could produce a net kw-hr. for 15,100 B. t. u. in the fuel and as a 1200 lb. compound reheating plant, it could produce a net kw-hr. for 13,600 B. t. u. These figures show the advance in heat economy that modern developments in steam engineering have made possible and the advantage which the very high-pressure reheat cycle possesses over the more conventional design. The actual plant performance will depend among other things upon the character of the load which it is found possible to put on Weymouth Station and the relative proportion of high and normal pressure equipment that may be operated.

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General Features. (450 w.)
Selection of Steam Conditions. (1180 w.)
High Pressure Installation. (550 w.)
Boiler Plant and Feed Water Heating. (750 w.)
Auxiliary Drive. (570 w.)
Switching Arrangement. (1000 w.)

IN common with many of the other public service companies serving our larger communities, The Edison Electric Illuminating Company of Boston has found its existing power stations and sites developed to their economical limits. Faced with a rapidly growing demand for service and with a broad vision into the future, it is constructing, on an entirely new site, the initial development of a station expected to have an ultimate capacity of approximately 300,000 kw.

The scope of this paper is limited to a brief description of this station and its equipment, with an interspersed discussion of such engineering features as are thought to be of more timely interest.

GENERAL FEATURES

The selected site on the Fore River in Weymouth contains about 63 acres. It is on deep water, readily accessible by large ocean going coal carriers, and has all other advantages necessary to give entire freedom in the design of a large modern station.

Fig. 1 shows the general cross-section of the station. The turbine room is to be located adjacent to the water front so that the condensing water tunnels will be direct and short. These tunnels are designed to give the same ease of flow as though they were to serve hydraulic turbines. The boiler room adjoins the turbine room on the land side and lies between the latter and the isolated building for the busses and electrical control.

The initial main generating equipment will consist of two 32,000-kw. turbines each driving a main 30,000-

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Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

kw. generator and a 2000-kw. auxiliary generator direct-connected to the shaft of the main generator. These units will be supplied with steam at a nominal pressure of 375 lb. and a total temperature of 700 deg. fahr. There will also be one 2500-kw. high back pressure turbo-generator taking steam up to 1200 lb. pressure and exhausting into the 375-lb. system. All station auxiliaries are to be driven by alternating-current motors.

The boiler installation will include three boilers each containing 19,743 sq. ft. of heating surface, with additional superheater and economizer surface, designed for 375-lb. working pressure. There will also be one boiler, of approximately the same heating surface, which will be designed to operate at a maximum of 1200 lb. It will likewise be provided with a superheater and an economizer, and also a steam reheater to restore the temperature of the exhaust steam from the high back pressure turbine to 700 deg. before it passes into the 375-lb. system. All boilers will be fired with underfeed stokers.

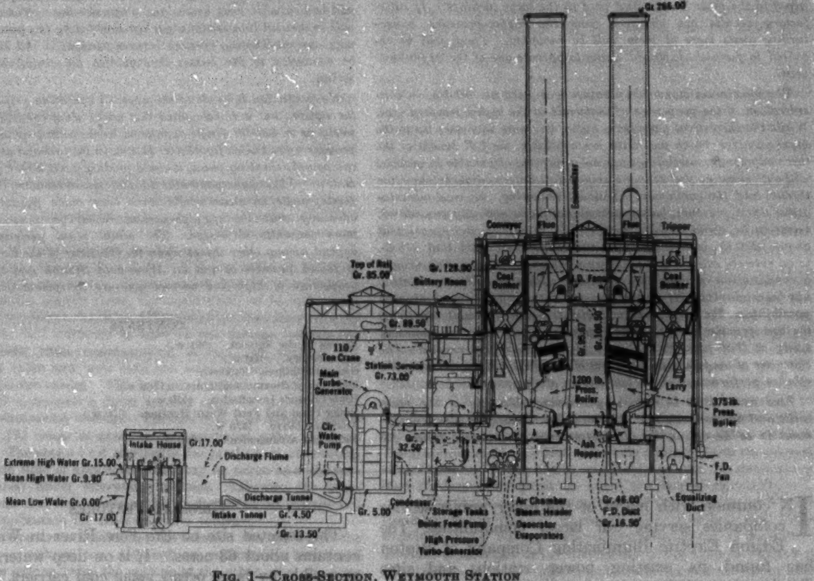

Gr. 266.00
Economizer
Conveyor
Flue
Flue
Tripper
Top of Rail
Gr. 85.00
Gr. 128.00
Battery Room
Coal Bunker
I.D. Fans
Coal Bunker
Gr. 89.50
Gr. 95.67
Gr. 108.50
110 Ton Crane
Station Service
Gr. 73.00'
Main Turbo-Generator
1200 lb. Press. Boiler
Larry
375 lb. Press. Boiler
Intake House
Gr. 17.00'
Discharge Flume
Cir. Water Pump
Gr. 32.50'
Ash Hopper
Extreme High Water Gr. 15.00
Mean High Water Gr. 9.80'
F.D. Fan
Mean Low Water Gr. 0.00'
Discharge Tunnel
Gr. 17.00'
Intake Tunnel
Gr. 4.50'
Gr. 13.50'
Gr. 5.00
Condenser
Storage Tanks
Boiler Feed Pump
High Pressure Turbo-Generator
Air Chamber
Steam Header
Deaerator
Evaporators
Gr. 46.00'
F.D. Duct
Gr. 16.50'
Equalizing Duct

FIG. 1—CROSS-SECTION, WEYMOUTH STATION

The coal handling equipment will consist of two electric unloading towers, a traveling bridge spanning the storage pile, and a system of belt conveyors for transporting the coal to or from the main storage or directly from the unloading towers to the station bunkers. The conveyor for delivering the coal to the station will be about 700 ft. in length. The traveling bridge is to carry a man trolley with bucket for reclaiming from the storage pile. A Bradford breaker will be used for crushing the coal on its way to the station.

Current will be generated at 14,000 volts, three phase, 60 cycles. It will be distributed from the station through underground circuits partly at generator voltage and partly at 25,000 volts through an adjacent outdoor transformer substation. Future transmission by overhead lines at 115,000 volts will also be provided for. The bus and switching arrangement is described later in this paper.

SELECTION OF STEAM CONDITIONS

The important bearing which the operating steam pressure and temperature have upon the possible maximum fuel economy of a station naturally called for an extended study of these features before reaching a decision in regard to them. When suitable economizers are employed the influence of steam pressure and temperature upon the efficiency of steam generation is negligible, but the influence of these factors upon the efficiency with which the heat in the steam can be utilized is in any case of definite importance.

The selection of the temperature at which the steam should be supplied to the turbine is almost entirely a problem of materials, as the beneficial effect of superheat, although largely indirect, continues in worthwhile proportion up to the maximum temperature which our engineering materials will properly withstand. In some isolated instances, European engineers have employed temperatures as high as 750 deg. to 800 deg. fahr. and in America temperatures as high as 725 deg. fahr. have been chosen by some designers. In considering this matter for the Weymouth Station it seemed best to place the nominal limit at 700 deg. fahr. It however appeared entirely practicable to employ this limiting temperature regardless of the pressure adopted.

Decision as to the most advantageous steam pressure could not be reached without a much more involved study, especially in view of the general lack of experience with the higher pressures and a decided variation in prophecy as to the net advantage to be derived through their use.

It was apparent that this study, to be complete, would have to cover the following considerations:

1. The improvement in economy of heat utilization made theoretically possible by increased steam pressure.

2. The extent to which available equipment could be expected to take advantage of the theoretical possibilities.

3. The probable effect of increased pressure upon reliability of operation and its flexibility in meeting anticipated load conditions with high over-all economy.

4. Freedom in taking advantage of future changes in the art.

5. The economic balance of carrying charges and operating costs over a long period.

The development of one part of the study to determine the effect of the higher pressures upon improved economy is illustrated by the curves in Fig. 2. All of these curves have absolute steam pressure as their abscissae.

Curve No. 1 shows the total heat in one pound of steam at a uniform temperature of 700 deg. fahr. above 79 deg. fahr., which is the temperature corresponding to one inch absolute pressure. It will be noted that the total heat shows a decrease with increasing pressure.

Curve No. 2 shows the heat remaining in the steam after perfect adiabatic expansion from the stated initial conditions to a pressure of 1 in. absolute. The vertical distance between this curve and curve No. 1 accordingly represents the B. t. u. per pound of steam theoretically available for doing work.

Curve No. 3 is a plotting of the available heat as a percentage of the total initial heat shown by curve No. 1 and represents the efficiency of the Rankine cycle at varying pressure. Its upwardly convex curvature indicates how the rate of increase in theoretical efficiency diminishes with increasing pressure.

Curve No. 4 shows the best efficiency at present to be expected of turbo-generators in converting the available heat, as shown by Curve No. 3, into useful electrical energy. In determining this curve the unit is credited with all heat recovered in the condensate by bleeding at two stages.

Curve No. 5 is the product of Curves No. 3 and No. 4 and indicates, for the different pressures, the percentage of the total initial heat in the steam which would be actually converted into electrical energy or returned to the boiler in the condensate. It will be noted that this curve takes the shape of a dome with its highest point corresponding to a steam pressure of about 600 lb. absolute.

If steam turbines could be constructed which would be equally efficient in transforming the available heat energy into useful work under all of initial steam pressure conditions, Curve No. 3 would indicate that the over-all thermal efficiency would increase with the pressure throughout the entire range considered. The most important factors which act to decrease the turbine efficiency at higher steam pressure are the increased gland and interstage leakage losses and, more particularly, the increased steam friction occasioned by the entrained water after the dew point has been reached. This lowered efficiency is particularly marked where the maximum permissible total temperature causes the higher pressures to be accompanied by diminished super heat, thus advancing the dew point to an earlier stage. The recognized method of meeting this difficulty and thus permitting the superior possibilities of higher steam pressures to be realized, is to interrupt the expansion of the steam at some intermediate pressure and restore its temperature by reheating before the expansion is continued. This may be accomplished either by employing two independent turbines, one exhausting to the other through the reheater, or by returning the reheated steam to the lower stages of the same machine from which it was extracted. The design of the Weymouth Station makes provision for employing the first of these two methods using a maximum pressure of 1200 lb.

In order that the reheater may be required to add superheat only to the steam, it is necessary that the exhaust from the high-pressure turbine be dry. The amount of energy that may be extracted from a given weight of steam between an initial condition of about 1000 lb. at 700 deg. and the dry saturated condition is only a fraction of that which can be utilized by its further expansion to vacuum after reheating. This condition permits the use of large standard design turbine units for the second expansion stage of the cycle. Thermodynamically it makes little difference in the cycle efficiency at what pressure the steam is reheated, or, indeed, if it is reheated at all. In practical operation, there is a certain pressure at reheat which gives the best combined efficiency of the two turbines working in series, but the efficiency at this pressure does not appear to be a great deal higher than at other pressures rather widely different from it. Of greater importance are the mechanical problems connected with rehandling the steam and the higher the pressure, within limits, the easier these become. For the particular situation under discussion, it was desirable to have the lower pressure portion of the plant able to function as a complete and highly economical station, so that the same considerations which control the selection of an initial pressure for the single expansion cycle were allowed to govern the choice of a reheat pressure in this case. Although it appears from Curve 5 of Fig. 2, that about 600 lb. is the most efficient pressure for a single expansion unit, cost influences make a somewhat lower pressure more economical to use. For Weymouth Station the pressure selected is 375 lb.

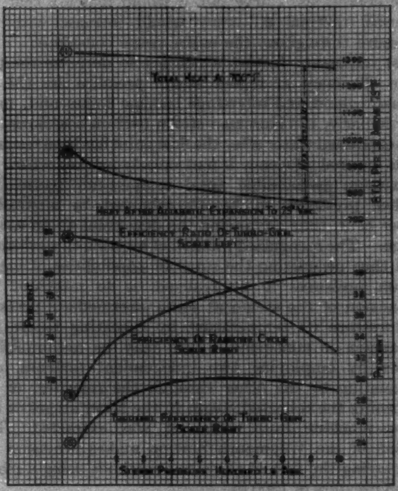


Fig. 2—Effect of Steam Pressure on Turbine Thermal Efficiency

High Pressure Installation

The high-pressure boilers will normally deliver steam at approximately 1000 lb. pressure and 700 deg. total temperature, corresponding to about 153 deg. of superheat. The entire steam output of each boiler, which under the usual operating conditions will be about 110,000 lb. per hour, will pass through a simple three-stage turbo-generator where it will be expanded down to a pressure of 375 lb. The steam will then be reheated to 700 deg. in a reheater installed in the same boiler setting, and finally be discharged into the 350-lb. header serving the main turbines. It may be noted from the quantity of steam passing through each of the high-pressure turbines that three such boiler-turbine units will be required to supply one of the main 32,000-kw. turbines at the higher loads. Operating under the conditions outlined each of the high-pressure turbines will generate about 2000 kw. Thus, a total of 6000 kw. will be generated in reducing the pressure of the steam required by one of the main units.

The high-pressure boilers will have tubes 2 inches outside diameter and 15 feet long. The tubes will be arranged in two banks with sufficient space between for the superheater and reheater. The drum will be a hollow steel forging 48 inches inside diameter with walls 4 inches thick. In order to maintain suitable drum strength the tubes and nipples, which commonly enter the drum in lines parallel with its axis are turned in pairs through an angle of 90 deg. so that the two enter the drum on a common circumference. Fig. 3 shows a cross-section of this boiler.

The generator of the pressure reducing units rated at 3000 kv-a. is to be of the induction type, if a suitable design can be worked out. This would avoid the necessity of separate excitation and careful synchronizing when being placed in service.

There are many new problems involved in the design, construction and operation of this high-pressure and reheating installation, but their nature is such that their solution may be confidently expected. It is recognized, however, that a considerable amount of development work will have to be done. It has therefore been decided to employ proven types of equipment for the larger part of the initial installation, confining the use of equipment which is as yet lacking commercial demonstration to a comparatively small supplemental installation of one high-pressure boiler with its turbine unit. This installation will afford an opportunity to demonstrate the general practicability of using very high-pressure steam in a re-heat cycle.

The ratio of high-pressure equipment and normal pressure equipment to be installed in the future will depend upon the results obtained from this initial installation and upon the experiences of others who are working along similar lines, or who are employing other methods for the utilization of high-pressure steam and the re-heat cycle.

Any estimate of the improvement in heat utilization to be expected from the use of very high steam pressure, is necessarily based on assumed properties of steam obtained by extrapolation of existing steam tables and diagrams, which may be subject to some correction. The calculated savings are so large, however, that it is felt that any probable error in these assumptions as to the properties of steam will have a relatively unimportant bearing.

Boiler Plant and Feed Water Heating

The normal pressure boilers, which are to deliver steam at 375 lb. and a final temperature of 700 deg. will be of the cross drum design, 48 tubes wide and 17 tubes high, with the superheaters between the tube banks. The exit gases will be further cooled by straight steel tube economizers which, with the induced draft fans, will be located above the boilers. The furnace arrange-

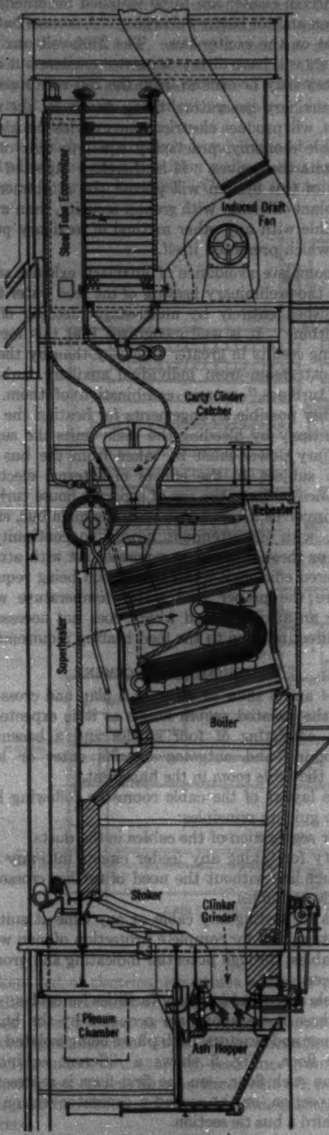


Fig. 3—Section of 1200-lb. Boiler

ment provides for firing from the low end of the boilers. The stokers are to be of the underfeed type, with clinker grinders, there being 16 retorts per boiler. The boilers will be set with the bottom of the uptake headers 25 feet above the firing floor. They will be arranged in parallel rows with the uptake ends facing a common center aisle from which all firing operations will be controlled. The stokers, which will thus be in the side aisles, will be supplied with coal from overhead longitudinal bunkers. This general arrangement of the boiler room is the same as that of the newer portion of the Edison Company's "L Street" Station, where it has proved highly satisfactory. It provides well lighted, clean and cool working quarters for boiler room operators and locates the stokers where abundant side light and ventilation are readily obtainable.

Except for the necessary modification in details, the boiler designed for operation at 1000 lb. pressure, and previously described, will be installed in accordance with the same general plan employed for the lower pressure boilers. The stoker and furnace installation will be practically the same and no attempt is to be made to isolate this installation.

Make-up water is to be provided by evaporators and the entire supply will be deaerated.

Economizers with a heating surface equivalent to about 55 per cent of the boiler surface were chosen. This selection was preceded by a study of the following influential factors:

1. The temperature at which the feed water should leave the bleeder heaters and enter the economizers.

2. The most advantageous division of the total heating surface between the boiler and the economizer.

The capacity of the feed water to absorb heat is theoretically limited by the range between two fixed temperatures,—that of the condensate and that of the saturated steam in the boiler. The practicable range of heating is, of course, somewhat narrower. The heat consumption of the main turbine, when credited with all heat returned to the system, is unquestionably the lowest when all of the heating is done by extracted steam. On the other hand, the efficiency of the boiler plant is highest when all of the heating is done in the economizer. The best over-all plant efficiency must therefore lie in some compromise between these opposing considerations. The calculations made for the Weymouth Station indicated that the most advantageous temperature to change the method of heating was between 210 deg. and 220 deg. fahr. Practically considered this is a most desirable temperature, also, for it avoids the use of superheated steam in the extraction heaters, diminishes the possibility of external corrosion of economizers and permits adequate deaeration of the feed water.

Considering the boiler and economizer in common, it is evident that for heating the water through a given total temperature range, and with a given outlet gas temperature, there will be less total heating surface required as the proportion arranged as counterflow economizer surface is increased. The relative cost of the two kinds of surface, however, exerts an opposing influence and necessitates a compromise. As to the boiler, its width is largely determined by the number of stoker retorts required, so its heating surface varies with its height. For the Weymouth Station the various considerations indicated that a boiler 17 tubes high and with 55 per cent additional economizer surface would provide the most satisfactory proportion, and that the resulting economy would justify the installation of this total amount of surface.

Decision to fire the boilers with stokers was reached only after a very careful study of the relative merits of using pulverized coal. The conditions surrounding this problem for the Weymouth Station were such that the probable savings in fuel through the use of powdered coal represented a relatively low percentage of the total fuel cost. This estimated saving was further reduced by the additional carrying charges to a margin which was too narrow to clearly call for the use of the powdered coal system. It was accordingly decided to adhere to the present practise of the Edison Company and employ stokers, at least for the initial boiler installation.

AUXILIARY DRIVE

All of the normally operating station auxiliaries are to be electrically driven, using alternating-current motors throughout. Motors of 25 h. p. and less will, in general, be served at 550 volts, and those of greater capacity at 2300 volts. Any necessary speed regulation is to be obtained through the use of brush shifting and slip-ring motors.

Power for all auxiliaries upon which the continuity of operation of the main units depends, will be supplied from auxiliary generators driven by the main turbines through direct connection to the shafts of the main generators. These auxiliary alternators will have a rating of 2500 kv-a. and will generate at 2300 volts, 3 phase. They will be mechanically in phase with the main generators so that although they will normally operate independently, they may be connected to the main bus through suitable transformers when desired. In starting up a main unit all of the auxiliaries will be started from the main bus but after the unit is in operation the important auxiliaries will be transferred to the auxiliary bus of that unit and thus be supplied from the auxiliary generator independently of the main circuits. The less important auxiliaries will be regularly operated from a 2300-volt bus fed through transformers from the main bus. A separate steam driven auxiliary turbo-generator will be provided to permit starting the station from cold in emergency.

Each generator will be excited by a 175-kw. motor generator set, consisting of a 2300-volt induction motor and a 250-volt direct-current generator. This set will be operated from the direct-connected auxiliary generator. A spare motor generator exciter and also a spare steam driven exciter are to be installed for emergency excitation and for battery charging. A storage battery will float on the exciter bus. The 2300-volt auxiliary generators will have direct-connected exciters but in an emergency may be excited from the 250-volt busses.

The auxiliary generators, being driven by the main turbines, will produce electrical energy with the highest obtainable economy, practically equal to that of the main units themselves. It is therefore expected that the use of this system will permit the attainment of higher plant efficiency with greater security, than would be possible with any other method of auxiliary power supply which presented itself.

The complete avoidance of auxiliary exhaust steam permits the preliminary heating of the feed water to be accomplished entirely by multi-stage bleeding of the main turbine. It is well recognized that this method of heating results in greater efficiency than by the use of exhaust steam from individual auxiliary turbines, "house turbines," or any combination of them. In most other possible arrangements for heating the feed water entirely by bleeding the main units the supply of auxiliary power must be taken from the bus and thus be subject to the effects of external electrical disturbances. Even the use of separate house turbines usually involves a connection with the main bus, either directly, with its attendant hazard to continuity of service, or through a motor generator set with attendant lowered efficiency,—this connection being required to secure the proper feed water temperature while carrying an auxiliary load which does not necessarily bear a direct relation to the feed heating requirements.

SWITCHING ARRANGEMENT

Figs. 4 and 5 show in diagram a plan and cross-section of the isolated switch house as it is expected to develop, consisting of four stories and a basement. All incoming and outgoing circuits enter or leave through the cable room in the basement.

In the layout of the cable room the following have been the guiding principles:

Proper segregation of the cables in the ducts.

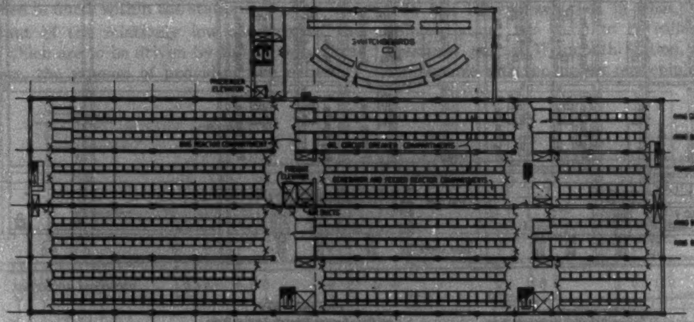
Facility for taking any feeder circuit into any outgoing duct line without the need of further crossovers in outside manholes.

Means for testing any cable at any time at suitable high voltage, and for complete protection of men working on cables, through potential indicating and grounding devices.

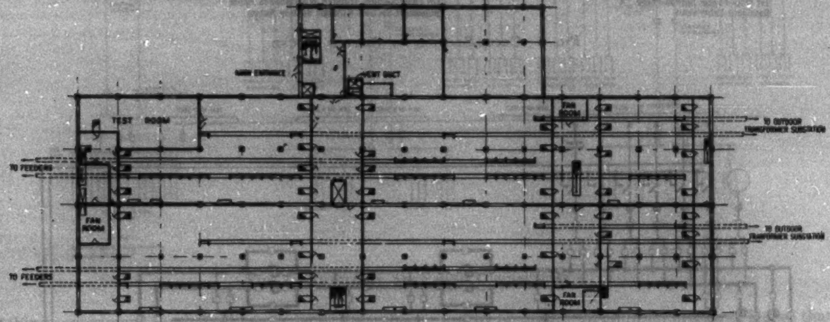
Suitable space and handling facilities for cables.

The three floors above are occupied by the busses, switch gear and reactors, each phase being isolated on a separate floor. Fig. 5 shows a different equipment section on each floor,—on the first floor is represented a feeder section, on the second a generator section and on the third a bus tie section.

On the fourth floor are located the oil circuit breaker and disconnecting switch operating mechanisms, which are interlocked to prevent the wrong sequence of operation. The switchboard room adjoins the mechanism room on the same floor, making a compact and convenient operating arrangement. On the third floor directly under the switchboard there is a conduit room to permit flexibility in adapting control circuits to future growth.


SWITCHBOARDS
PASSENGER ELEVATOR
BUS REACTOR COMPARTMENT
OIL CIRCUIT BREAKER COMPARTMENTS
FREIGHT ELEVATOR
GENERATOR AND FEEDER REACTOR COMPARTMENTS
AIR DUCTS
RING BUS Nº 1
RING BUS Nº 2
TRANSFER BUS
RING BUS Nº 2
RING BUS Nº 1

TYPICAL FLOOR PLAN PHASES A, B & C OF SWITCH HOUSE AND FOURTH FLOOR OF CONTROL BAY


MAIN ENTRANCE
VENT DUCT
TEST ROOM
FAN ROOM
FAN ROOM
FAN ROOM
TO FEEDERS
TO FEEDERS
TO OUTDOOR TRANSFORMER SUBSTATION
TO OUTDOOR TRANSFORMER SUBSTATION

CABLE VAULT OF SWITCH HOUSE AND BASEMENT OF CONTROL BAY

Fig. 4—Plan of Switch House

The switch house structure is subdivided by suitable longitudinal and cross partition walls so that each bus section is in a room by itself. Conduit is run in floors and kept out of electrical structures wherever possible. Metallic reinforcement of concrete electrical structures has also been avoided as far as practicable and where reinforcing is essential it is either discontinuous or solidly grounded, so that a high-resistance ground passing insufficient current to open the circuit will not cause heating of the reinforcement and disintegration of the concrete.

The contemplated arrangement of the 14,000-volt main power circuits may be seen from the diagram in Fig. 6. A double-ring bus is planned, in conjunction with a single transfer bus. The two-ring busses will be sectionalized with current-limiting reactors between sections. Each section will be fed by two generators. It will be possible by use of the transfer bus to interconnect any two of the sections, thus permitting complete flexibility of operation. For the initial development only one bus section will be necessary, which will be operated simply as a double bus.

The connection of all generator and feeder circuits to the bus is to be through two oil circuit breakers in series so that there may be double assurance of ability to open the circuit under any emergency condition. This object may also be attained by the use of group busses if the capacity of the individual circuits is insufficient to warrant the expense of the extra breakers that would otherwise be required. The oil circuit breakers for use on feeders and for bus sectionalizing are guaranteed to rupture 34,500 amperes at 15,000 volts. Those on generators and on large capacity

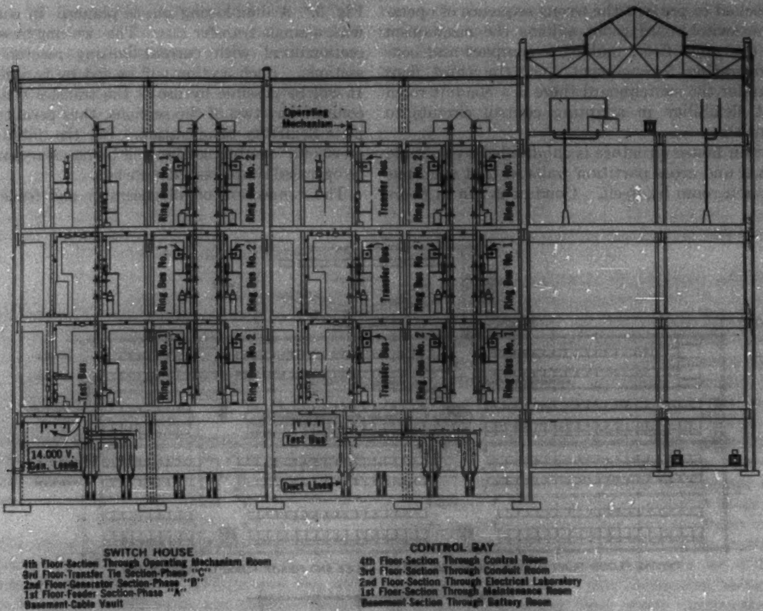

Operating Mechanism
Ring Bus No. 1
Ring Bus No. 2
Transfer Bus
Test Bus
14,000 V. Gen. Leads
Test Bus
Duct Lines
SWITCH HOUSE
4th Floor-Section Through Operating Mechanism Room
3rd Floor-Transfer Tie Section-Phase "C"
2nd Floor-Generator Section-Phase "B"
1st Floor-Feeder Section-Phase "A"
Basement-Cable Vault
CONTROL BAY
4th Floor-Section Through Control Room
3rd Floor-Section Through Conduit Room
2nd Floor-Section Through Electrical Laboratory
1st Floor-Section Through Maintenance Room
Basement-Section Through Battery Room

FIG. 5—CROSS-SECTION OF SWITCH HOUSE

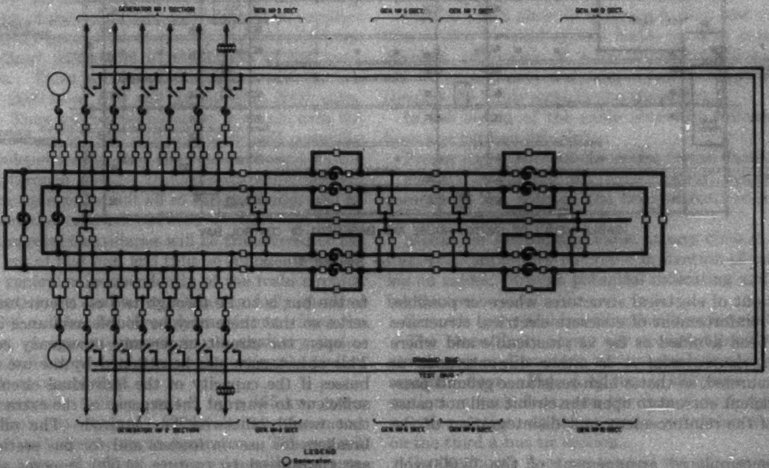

GENERATOR Nº 1 SECTION
GEN Nº 3 SECT.
GEN Nº 5 SECT.
GEN Nº 7 SECT.
GEN Nº 9 SECT.
GROUND BUS
TEST BUS
GENERATOR Nº 2 SECTION
GEN Nº 4 SECT.
GEN Nº 6 SECT.
GEN Nº 8 SECT.
GEN Nº 10 SECT.
LEGEND
Generator

FIG. 6—MAIN SWITCHING CONNECTIONS

transformer circuits which may connect with other power systems are guaranteed to rupture 58,000 amperes at this same potential.

The generators and their cable connections to the busses will be protected by balanced relays arranged to automatically disconnect any unit from the bus in the event of a ground or short circuit on the machine side of the circuit breaker. The generator leads are to be single-conductor cables carried on insulators from the machines to the switch house, through closed corridors. No cables carrying a higher potential than 2300 volts will be carried in ducts within the station.

On account of the relatively low capacity of the generators which are to be driven by the 1200-lb. pressure turbines, the expense of providing the necessary switching equipment and building space, for their direct connection to the main bus is not warranted. It is planned, therefore, to connect each group of three of these units to a separate bus through low-capacity oil circuit breakers. The group bus, will in turn, be connected to the main bus in about the same manner as a feeder.

A wiring diagram of the system for driving the station auxiliaries is given in Fig. 7. The auxiliary generators will be controlled from the main switchboard and the operator there will be responsible for keeping the 2300-volt busses constantly energized. The 2300-volt busses will be located in separate substations, one for each two units, to be provided in the auxiliary bay between the boiler room and turbine room. Truck type switch gear will be used.

Plant Efficiency

In what has gone before the discussion has centered for the most part around the efficiency of particular apparatus, conditions or methods. Of even greater interest to engineers in general is the efficiency of the whole plant as being indicative of the care and skill with which the various components have been proportioned and assembled, and the necessary compromises made between conflicting requirements.

It has been shown that in what, for purposes of differentiation, has come to be called the "Normal Pressure" part of Weymouth Station, steam pressures comparable with the highest now employed in America are to be used. In addition pressures higher than any that have been used heretofore for similar purposes are to be thoroughly tried out in conjunction with the reheat cycle.

Feed water heating is to be accomplished both by steam extraction from the main turbines and by the use of economizers. To obtain the maximum of benefit from the steam extraction method of heating motor driven auxiliaries only are to be used, supplied with power from a most efficient direct-connected auxiliary alternator. The auxiliary apparatus itself has been selected with a view to obtain the highest efficiency in both the driving and driven members.

Fig. 8 shows, in the upper diagram, an analysis of

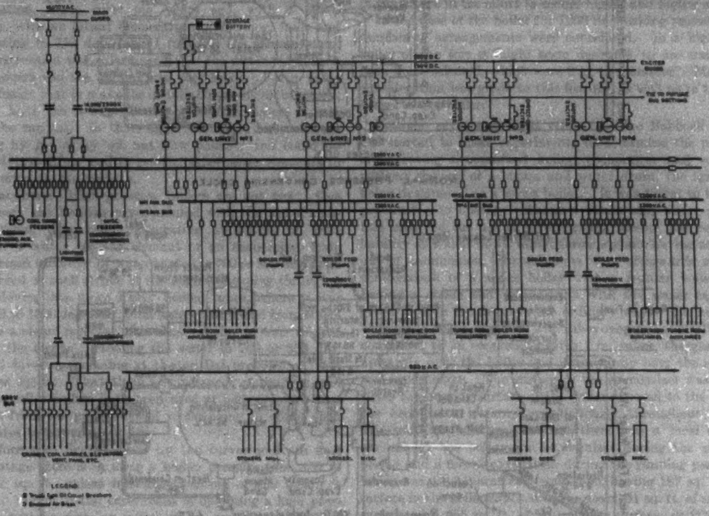
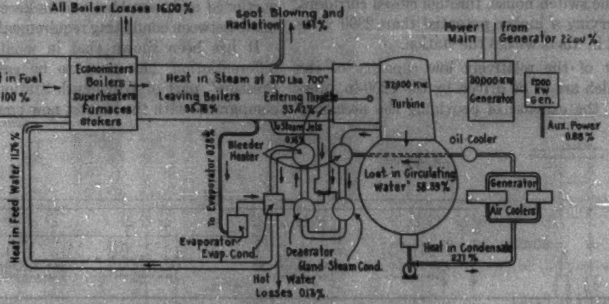
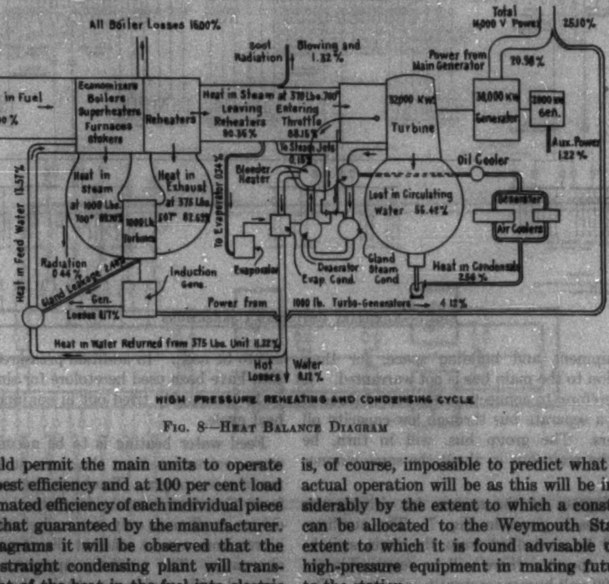

STORAGE BATTERY
EXCITER BUSSES
TIE TO FUTURE BUS SECTIONS
GEN. UNIT NO 1
GEN. UNIT NO 2
GEN. UNIT NO 3
GEN. UNIT NO 4
2300 V.A.C.
2300 V.A.C.
LIGHTING FEEDERS
BOILER FEED PUMPS
BOILER FEED PUMPS
BOILER FEED PUMPS
BOILER FEED PUMPS
TURBINE ROOM AUXILIARIES
BOILER ROOM AUXILIARIES
BOILER ROOM AUXILIARIES
TURBINE ROOM AUXILIARIES
TURBINE ROOM AUXILIARIES
BOILER ROOM AUXILIARIES
BOILER ROOM AUXILIARIES
TURBINE ROOM AUXILIARIES
CRANES, COAL LARRIES, ELEVATORS
VENT. FANS, ETC.
STOKERS
MISC.
STOKERS
MISC.
STOKERS
MISC.
STOKERS
MISC.
LEGEND

Fig. 7—Auxiliary Switching Connections

the performance which could theoretically be realized in the operation of the normal pressure (375 lb.) portion of the station of the design as described. The lower diagram shows a similar analysis for such a station operating exclusively on the reheat cycle, with the entire steam supply generated at 1000 lb. Both diagrams are based upon ideal operating conditions, *i. e.*—a constant station load of such amount and character as would permit the main units to operate at their point of best efficiency and at 100 per cent load factor. The estimated efficiency of each individual piece of equipment is that guaranteed by the manufacturer.


All Boiler Losses 16.00 %
Soot Blowing and Radiation 1.61 %
Power Main
from Generator 22.60 %
Heat in Fuel 100 %
Economizers Boilers Superheaters Furnaces Stokers
Heat in Steam at 370 Lbs 700°
Leaving Boilers 95.18 %
Entering Throttle 93.42 %
32,000 Kw Turbine
30,000 Kw Generator
2000 Kw Gen.
Aux. Power 0.88 %
To Steam Jets 0.15 %
Oil Cooler
Bleeder Heater
To Evaporator 0.78 %
Heat in Feed Water 11.76 %
Lost in Circulating Water 58.88 %
Generator Air Coolers
Evaporator
Evap. Cond.
Deaerator
Gland Steam Cond.
Heat in Condensate 2.71 %
Hot Water Losses 0.13 %

NORMAL PRESSURE CONDENSING CYCLE


Total 14,000 V Power 25.10 %
All Boiler Losses 16.00 %
Soot Blowing and Radiation 1.32 %
Power from Main Generator 20.98 %
Heat in Fuel 100 %
Economizers Boilers Superheaters Furnaces Stokers
Reheaters
Heat in Steam at 370 Lbs. 700°
Leaving Reheaters 90.36 %
Entering Throttle 88.15 %
32,000 Kw Turbine
30,000 Kw Generator
2000 Kw Gen.
Aux. Power 1.22 %
To Steam Jets 0.15 %
Oil Cooler
Heat in Steam at 1000 Lbs. 700°
1000 Lb. Turbines
Heat in Exhaust at 375 Lbs. 507° 82.63 %
To Evaporator 0.34 %
Bleeder Heater
Heat in Feed Water 13.57 %
Lost in Circulating Water 56.48 %
Generator Air Coolers
Radiation 0.44 %
Gland Leakage 2.44 %
Gen. Losses 0.17 %
Induction Gens.
Evaporator
Deaerator
Evap. Cond.
Gland Steam Cond
Heat in Condensate 2.56 %
Power from 1000 lb. Turbo-Generators 4.12 %
Heat in Water Returned from 375 Lbs. Unit 11.22 %
Hot Water Losses 0.12 %

HIGH PRESSURE REHEATING AND CONDENSING CYCLE

Fig. 8—Heat Balance Diagram

From these diagrams it will be observed that the normal pressure straight condensing plant will transform 22.6 per cent of the heat in the fuel into electric energy, which is equivalent to 15,100 B. t. u. per kw-hr. net generated, or approximately 1.05 lb. of coal of a heat value of 14,400 B. t. u. per lb.—while the corresponding figures for the high-pressure reheat plant are 25.1 per cent, 13,600 B. t. u. per kw-hr. and approximately 0.945 lb. of coal.

They above calculated results represent only the ultimate possibilities under ideal load conditions. It is, of course, impossible to predict what the results of actual operation will be as this will be influenced considerably by the extent to which a constant base load can be allocated to the Weymouth Station and the extent to which it is found advisable to employ the high-pressure equipment in making future extensions to the station.

Discussion

S. Z. Ferranti: It seems to me that when once the commercial practicability of such high pressures as it is proposed to use have been successfully demonstrated that you can then take full advantage of this provided you can go in for a very complete system of progressive feed heating (bleeding at a large number of points). As you already know, great economy results from carrying out this process very thoroughly up to the fullest practical limits. On the other hand, it is not possible to do this unless you heat the water from cold up to nearly the temperature of evaporation under working pressure, by bleeding. This in its turn necessitates a boiler which will work efficiently with very hot feed, and this can only be done by air heating to a sufficiently high temperature. I am sure you have considered this all very fully, and I should greatly appreciate your further views on the subject.

There is another point with regard to the details of your installation, and this is the necessity of taking steam backwards and forwards between the turbines and the superheaters. In my installation, made over ten years ago, with a 3000-kw. turbine, with an initial temperature of 750 deg. fahr. and re-superheating to 750 deg. fahr. I followed the same system, but the construction was so arranged that the distance between the superheaters and the turbine was very small. I then came to the conclusion that to make the installation really satisfactory, one should have the re-superheater or re-superheaters closely adjoining the turbine. In those days this would have necessitated firing by gas or by oil, but now possibly you could do this by coal-dust firing, and so get the superheater elements in close proximity to the turbine.

D. S. Jacobus: High-pressure boilers are not new. One hundred years ago Perkins experimented with the use of high-pressure steam. His steam generating apparatus of 1824 consisted of a water heating element or an economizer from which a certain quantity of water heated to 700 or 800 deg. fahr. was discharged at each stroke of the engine, escaping through a throttling device into more highly heated tubes, where it was superheated to 1000 deg. fahr. total temperature, and from which it passed to a steam receiver maintained at the pressure desired for driving the engine. Heating the water to 700 or 800 deg. fahr. would involve operating at the critical pressure of, say, 3000 lb. per sq. in. That there are instances of the sort and that some smaller boilers have been used at high pressure does not detract from the magnitude of undertaking to adapt such high pressures to power plant use.

The performance figures given by the curves indicate how little advantage there is in using a higher pressure than 350 to 400 lb. per sq. in. unless interstage heating of the steam is resorted to. Again, if the experiment of operating a high pressure steam turbine with an initial steam pressure of 1000 or 1200 lb. per sq. in. proves to be successful, 350 lb. pressure or thereabout appears to be about as high as could be employed to advantage by the main steam turbine. Possibly 1000 or 1200 lb. may not result in a plant as successful from a commercial viewpoint as one operated at, say, 600 or 700 lb. absolute pressure with a high pressure steam turbine exhausting into the main steam turbine, in which case the pressure at the main steam turbine for securing the best results would be lower than 350 lb.

Interstage heating necessarily involves additional complication and the added expense and possible difficulties in operation due to the additional complication have an important bearing on the design of the plant. Were there some more ready means of reheating the steam than passing it back to the boiler room through large pipe connections, which for a large size unit must be 20 or 24 in. in diameter for the pressures at which steam is returned to the reheaters, and which would have to be of an even larger diameter should the steam be reheated at some lower pressure, the problem would not be so difficult. Even in the case of a turbine operated at 350 lb. pressure with an initial temperature of 700 deg. fahr. there is a considerable amount of condensation in the lower stages and if there were some practical means of eliminating this condensation it would result in a higher efficiency. All these features should bring out a most interesting discussion.

Another interesting point brought out in the paper is the heating of the feed water by means of steam extracted from the turbine stages, which in this particular case leads to the best over-all plant efficiency through heating the feed water to between 210 and 220 deg. fahr. In the computations the consequent falling off in the heat absorption of the economizer and the resultant excess in the temperature of the gases is taken into account. The addition of an air heater would offset any increase in temperature of the flue gases due to the heating of the feed water, so that with an air heater the most economical temperature for heating the feed water would be higher than without an air heater, for that matter, the entire economizer might be omitted and a large air heater used instead, as is done abroad at the North Tees Station. There are so many angles to the problem that it is hard to say what plan will be adopted in the future in the effort to secure higher and higher efficiencies.

In the case of the boiler for 1200 lb. working pressure a great number of arrangements were considered. In a high pressure boiler of the sort it might seem desirable to use smaller tubes than those selected, which were 2 in. in diameter. Most careful consideration was given to this feature, as well as to the general design of the boiler.

Between seven and eight years ago The Babcock & Wilcox Co. started to make experiments to determine the possibilities of operating a combined boiler, economizer and superheater without the use of a steam and water drum, the arrangement being designated as a series boiler. It was found to be entirely practicable to operate without a steam and water drum, but this involved the use of a feed pump having 100 per cent reliability as a shut down would throw a plant having such boilers out of commission. Experiments were afterwards made on a combination of a series boiler with a regular boiler element having a steam and water drum with the water carried at a given level in the drum in the ordinary way. The experiments were made on a boiler with 2 in. tubes operated at 600 lb. pressure connected to an economizer with 1 in. tubes, the gases flowing upwardly over the boiler and downwardly over the economizer and superheater. The surface of the economizer was about 3.7 times the heating surface of the boiler exposed to the gases, and the economizer steamed continuously throughout the tests. The unit was arranged so that there was a local circulation throughout the boiler and economizer during the starting up period and a forced circulation during the running period. The boiler was a comparatively small one, having 187 sq. ft. of tube surface in the boiler exposed to the gases, 51 sq. ft. of superheater surface, and 685 sq. ft. of economizer surface, or 923 sq. ft. of heating surface in all, and was fired with fuel oil with mechanical atomizing burners. Tests were made with the boiler generating as high as 424 horse power as an average, the unit being run at times at over 500 per cent of rating, based on the total surface of the boiler and economizer and all parts were found to operate in an entirely satisfactory way. The percentage of rating at which the boiler part of the apparatus was driven in the high capacity tests was about 1500 per cent.

In developing the boiler for 1200 lb. working pressure we had before us the results secured with the use of 1 in. tubes in a great number of our marine boilers built for torpedo boat service and the results of the experiments above referred to. We further had before us the results secured with a drumless B & W boiler that we built for test purposes a number of years ago where there was water storage, the drum being replaced by a number of horizontal tubes connected to headers. A great number of different arrangements were laid out and considered and of all of these the one finally adopted was that described in the paper, approached very closely to a standard B & W boiler.

A power plant boiler under the present conditions of service must be arranged so that it can be thoroughly cleaned both inside and out. The use of 2 in. tubes, such as will be supplied for the boiler gives a convenient size for internal cleaning, whereas it is difficult, if not impossible to properly clean tubes, say, 1 in. in diameter and smaller. Were it possible to maintain at all times absolutely tight surface condensers, it would then be unnecessary to clean the interior of the boiler tubes and the art of boiler design would be considerably simplified and smaller diameter tubes could be used provided they were arranged so that they could be thoroughly cleaned on the outside. For years The Babcock & Wilcox Co. believed that it was ready to put out a B & W boiler for any desired pressure for power plant service through using the Marine B & W boiler, in fact, the test boiler of drumless construction that has been referred to was built with 2 in. tubes for a pressure of 750 lb. per sq. in. The difficulties that were experienced at some plants in maintaining the outer surface of boiler tubes clean from slag and soot when run at high ratings convinced operators of power plants that it would be a mistake to install the 2 in. tubes in groups of four as in our marine boilers, because with the tubes so arranged there are no diagonal lanes available between the tubes for cleaning the outer surface of the tubes. A new forged steel header was therefore designed and constructed where the tubes are staggered and arranged with horizontal and diagonal lanes between all of the tubes, thereby making it possible to clean the outer surfaces of the tubes through the diagonal lanes from above and below the tubes. All B & W boilers for the higher pressures are now fitted with this new form of header. The case is different from marine boilers, which have a relatively narrow width and where all of the tubes may be cleaned on the outside by opening the side casing doors, and brushing the tubes through the horizontal lanes between the rows of tubes, which would be an impractical method for the wider boilers and settings used for power plant work.

Attention is called in the paper to the use of the forged steel drum of seamless construction for the 1200 lb. boiler and to the bending of the ends of the horizontal circulating tubes which enter the drum so that they enter at circumferential lines which are twice as far apart as the distance between the horizontal circulating tubes where they enter the headers, thereby greatly increasing the efficiency of the ligaments between the tube holes. There are two circulating tubes running from the top of each uptake header to the steam and water drum and the ends of the tubes which enter the headers are bent in such a way that the tubes enter all of the headers at the same height, thereby making the circulation the same in each header. The boiler is built of A. S. M. E. Code material throughout to meet all of the requirements of the Code.

The boilers for 375 lb. pressure first considered were 14-high and 48-tubes wide with the two lowermost rows of tubes exposed for their full length to the furnace, with a horizontal baffle above the second row of tubes exposed from the downtake headers to the bottom of the inclined baffle between the first and second passes, the superheater being placed in the first pass above six rows of tubes. A change was made to a boiler 17-tubes high and 48-tubes wide with the six lowermost rows of tubes exposed to the furnace, with the superheater in the same position. This was done in order to secure the following operating advantages:

1st: The furnace temperature will be somewhat lower with the six lowermost rows of tubes exposed than with only two rows exposed for their full length to the heat of the furnace and with a high furnace of the sort that will be used this would tend to reduce the cost for brickwork maintenance. 2nd: In case a coal is burned which will cause an undue amount of slagging on the outside of the tubes exposing six rows of tubes for their full length will lead to less slagging difficulties than exposing the two rows for their full length. 3rd: In case of tube troubles due to impure feed water the tube loss would be less for a given amount of steam evaporated per hour from the boiler with six rows of tubes exposed both on account of causing the hot gases on striking the boiler tubes to be diffused over a longer length of tube surface and through providing a greater amount of boiler surface for a given amount of steam generated.

The flue gas temperatures for a given weight of steam evaporated per hour by the 17-high boilers with the six lowermost rows of tubes exposed will be but slightly lower than would be secured with the 14-high boilers with only the two lowermost rows of tubes exposed. A 16-high boiler with the six lowermost rows of tubes exposed would give about the same flue gas temperatures for a given weight of steam evaporated per boiler as a 14-high boiler with only the two lowermost rows of tubes exposed. The choice of the 17-high boilers with the six lowermost rows of tubes exposed was not therefore on the basis of the flue gas temperatures or the efficiencies, but on the broader basis of selecting a boiler which would give the best service under the conditions for use in connection with the economizers. If the 17-high boilers were fitted with a system of baffling that was considered for the 14-high boilers with the two lowermost rows of tubes exposed, the efficiency of the boilers for the same steam output under the average service conditions would be slightly more than one per cent higher than with the six lowermost rows of tubes exposed, and should no economizers be employed, this difference of efficiency would offset a considerable increase in cost of maintenance of brickwork and difficulties of operation.

P. Junkersfeld: Five considerations are stated on the third page of the paper. The third, "The probable effect of increased pressure upon reliability of operation and its flexibility in meeting anticipated load conditions with high over-all economy," and the fifth, "The economic balance of carrying charges and operating costs over a long period," are more or less interrelated. Under reliability of operation I assume that the authors have included maintenance. Apparatus of this sort naturally involves more maintenance or replacement than apparatus that has been tried out for a longer period of service. With the particular apparatus operating at 1200 lbs. steam pressure some troubles must be expected to develop during the early years, which means outages and units available less hours of the year.

Such outages do not reduce the total carrying charges per year, but will increase the carrying charges per kilowatt hour because if the turbine unit or boiler unit is out, for example, twice as many hours as a more seasoned piece of apparatus, it would double the carrying charges per kilowatt hour due to that particular apparatus. On the other side of the balance, if such turbine or boiler unit is running only half as many hours as a more seasoned unit in a year, the savings expected from the better fuel economy are much reduced. I take it, however, that these matters have been considered under considerations three and five, and the foregoing is submitted as a matter of emphasis on these two very important points in the paper.

Curve 1 Fig. 2, shows a very interesting and not usually appreciated fact, namely, that the total heat at 700 deg. fahr. decreases with increasing pressure. That is contrary to our ordinary expectations. Of course, after the careful analysis that the authors have made, we can see why it is so. However, considering Curve 1 and Curve 2 together, we see that after all, the heat available does increase with the pressure.

Curve 5 indicates that six hundred pounds should give the highest thermal efficiency. It is not clear to me whether this represents the expected performance from all types of turbines or only the particular type of turbine used in Weymouth Station.

On the third page it is stated that 1200 lb. was selected for the one new extra high pressure boiler. It would be interesting to know why 1200 lb. instead of say, 1000 or 1500 lb. was selected. There must have been some reason that led to the adoption of 1200 instead of something a little higher or a little lower.

There is frequently a tendency for too much standardization in power plants. It is a fact that the elements that go into a power plant like the elements that go into a suit of clothes,—the cloth, the thread and the buttons, should be thoroughly standardized, but you can never hope in my judgment to so far standardize power plants that they will fit every condition any more than you can hope to make a suit of clothes to fit every size of man. There would be a lot of misfits. There are a great many things that will be a good solution for one set of power plant conditions and one location, that might be a poor solution for another set.

The two most outstanding conditions that affect the proper design of plant for a given set of conditions are price of fuel and load factor. When I say price of fuel, I mean per thousand or million B. t. u., or some other equivalent that shows steaming value. Differential in price of fuel is usually a fairly definite thing. There is a substantial difference between the price of coal say in Boston, and price near the coal mines, over a long period of years. The price of coal on comparable basis may be two or three times as much in one locality as in another, and the same plant will not properly fit both localities.

Load factor, however, is a very elusive thing. It depends upon the general character of the business in the community. If every station were operated at exactly the natural consumption of power within its proper area, it would be one thing, but we are no longer operating stations,—we are operating systems, and that means one, two, three, four or even more stations in parallel. These stations on the same system are in competition with one another. The better load factor is allocated to the station with the lowest operating cost, and to an extent depending on how much lower that station operates than the other stations. Operating cost, not fixed charges, govern this allocation, because fixed charges are established when you build the station. I am assuming now that such transmission and distribution handicaps as might exist between the various stations are all taken into account in allocating load between stations.

During the life of the equipment and just as any new station is usually lower in operating cost than older stations or units on the system, so may another new station five years later be still lower. Therefore future load factor on a station or unit is a very difficult question to decide, requiring judgment and experience and the very best attention and study.

Two other points in the paper would be interesting if amplified. First,—how is it proposed to protect the reheater from overheating should the turbine load be suddenly discontinued? Second,—In view of the fact that we can so effectively heat feed water by bleeding turbines, is there much to be gained with economizers?

The final results from the Weymouth Station are very promising, indeed, as given in the paper under ideal conditions, which are the only conditions under which they can be given at this time.

L. L. Elden: The Weymouth Station now under construction will be operated in conjunction with the rest of the system substantially as follows: The area served by the Boston Edison Company resembles a square looking at it from the water side, with L Street Station located substantially at the centre of the base line. The new Weymouth Station is located at the southern terminus of the base line about eleven miles from L Street. The present area served by the company is supplied from some 85 substations.

The ultimate growth of the system may be estimated broadly from the fact that the connected load is increasing yearly at the rate of about 100,000 kw. With this growth in view, it is conceived that eventually in the interest of economy another large generating station will be required north of the city, assuming, of course, that a suitable site can be found for the purpose.

To provide a suitable link between the new station and the remote points of the present system, it is proposed to construct a 110,000-volt ring around the system, starting from Weymouth and terminating at some desirable location on the north. Lines from L Street are serving all intermediate territory and will in the end be interconnected with lines from the Weymouth Station as business requirements may dictate.

Another project now being seriously considered is the introduction of hydro power in large quantities from the St. Lawrence district. The projected point of contact is at Framingham on the westerly side of our territory where it will intersect the 110,000-volt ring previously referred to. The route of this proposed ring will be exceptionally favorable for connection on the southerly side with the proposed superpower system developed by Mr. Murray and his associates during the war period.

We also find the location of the new station favorable to the supply of service to the New Haven and Boston and Albany Railroads in the event that the electrification of these roads becomes a reality in the future. With favorable conditions as regards coal, water and distribution facilities, we feel that it is conceivable that the entire proposed development of 300,000 or more kilowatts will be easily absorbed in the near future if all of the projects now in view actually eventuate.

Operating methods to which Col. Junkersfeld referred have not as yet been fully worked out, it being evident that to determine the best method of relaying and handling the two generators comprising each unit, when connected to the main system, will require considerable study. This has not yet been completed hence it is impossible at this time to indicate the reliability factor which will be developed in actual service. It would appear from studies made up to this time that the small 1000 pound unit may prove the only unreliable elment in the whole development since all other equipment is standard and already in extensive use. When, however, it is realized that the capacity of the high-pressure unit is only of the order of 2500 kw., its importance becomes negligible in a large system like ours, where many interconnections and emergency arrangements provide means for instant absorption of loads of far greater magnitude than would be involved in the event of failure of the unit in question. Under these conditions the question of fixed charges on spare capacity at this station to meet emergencies is not important since the reserves for the system as a whole will not be carried at any one location.

H. P. Liversidge: The details of the Weymouth plant which have been presented before this Convention, indicate the very practical consideration which has been given to its design in order that the generated power may be produced at lowest over-all cost.

In the consideration of new plant layouts, there is prone to be a tendency toward maximum operating efficiency with the possible sacrifice of other factors which have a decided bearing on investment costs and over-all operating costs. It is quite evident that these various factors have all been given primary consideration in this station layout.

I believe that while this plant incorporates some radical departures from standard design, it would seem to be a very safe change. It is evident that the high-pressure section, which embodies very special features, has been so arranged in its relation to the rest of the plant that if an unforseen complication should arise, no financial penalty will be imposed even if it becomes necessary to radically change or eliminate this special part of the equipment. This, in my opinion, is a most important consideration, and in the development of large central stations, I believe we should place ourselves in the position of doing not what we might like to do from the standpoint of highest efficiency alone, but what from a business standpoint is the safe thing to do,—investment and earnings being considered.

In designing a plant, another question to be considered is reliability—the plant must operate. We are in the position to-day where departures which involve radical changes from standard design must be so planned as not to penalize the high order of service which must be rendered to-day.

I believe that this plant design presents a combination of equipment which marks a step in advance in the application of high steam pressures, and, at the same time, eliminates the possibility of complications which may have their effect in either the lowering of reliability of service or increasing maintenance costs.

G. L. Knight: There is one point I think which has not been touched on and that applies particularly to those of us who were compelled to make our decisions, especially on steam conditions, somewhat before these later developments in boiler and steam turbine designs were made available. We are not going to suffer altogether thereby, I think, but can take advantage of some of these developments by using high-pressure turbines in stations where we have adopted lower steam pressures, installing some of our new boilers at 1000 or 1200 lb. pressure and exhausting the high-pressure turbines into our present steam lines at the line pressure.

H. W. Eales: Attention was called by Mr. Liversidge to the feature of the design at this station which permitted of the elimination of the high-pressure portion and of the substitution of the more standardized methods of design in the event of any difficulties with the design. Being connected with a power station development which is utilizing pulverized fuel, I may say that the same factors have been carried out there. The design of the boiler house as laid out will permit of the substitution of any other form or method of burning coal which experience or advance in the art may direct.

Reference was made in the paper to the calculations of the Weymouth Station with reference to the choice of fuel combustion. It seems to the speaker that much of the discussion on the relative merits of pulverized fuel or stoker firing of coal in these super stations is still of the conjectural type. It is my opinion, therefore, that very little is to be gained by further discussion of the matter at this time. I would personally much prefer to delay comments until after experience with our particular station has demonstrated the results obtained.

The paper, on the sixth page pointed out two methods of securing power supply for station auxiliaries. It referred to the securing of power from the main station bus with attendant disaster to the auxiliaries in the event of general system trouble and the means of preventing that by using motor-generators floating between the house turbine bus and the station bus.

I wish to mention a third method, namely, that of using a saturated core reactor between the house turbine bus and the main station bus.

As a matter of purely experimental interest the speaker is greatly interested in the development of the Weymouth Station which uses an auxiliary alternator on the same shaft as the main generator. It may be pertinent to inquire if it is possible to go a step further and use a double-current generator, thereby deriving the excitation in auxiliary power by the same means.

F. Samuelson: Your plans are very bold and should give some really valuable information with regard to economy and reliability of extra high-pressure steam plant. It is very fortunate that it is relatively easy to ascertain the efficiency of the high-pressure end of the plant as the steam will be superheated on leaving the high-pressure turbine. With known inlet and outlet pressure and temperature of the steam it is readily found how much of the energy is converted into work. It is of course essential that the temperature measurements should be very carefully taken. There may be a slight error owing to want of exact knowledge of the specific heat of superheated steam at such high pressure of 1000 to 1200 lb. per sq. in., but this error will be small.

A very important matter in connection with the construction of steam plant at extra high pressures and temperatures, is the fact that the true elastic limit of most steels is lowered seriously as the temperature is increased. At a temperature of about 750 deg. fahr. the elastic limit may drop to half of that obtained at ordinary room temperature, and it is therfore very important that this loss of strength is understood when the factor of safety is settled upon.

As the strength of steel is so much affected by heat, I am strongly of the opinion that the temperature of the superheated steam should be controlled in some way. I have come across cases where the steam temperature has been above 800 deg. fahr. at the turbine stop valve, and where it has been necessary to lower the temperature by removing some of the superheater tubes, entailing increased drop of pressure across the superheater. I would advocate that the superheater be located in a chamber or compartment, preferable within the boiler itself, and provided with water cooled dampers, or other means which will control the flow of the hot gases. Where a reheater element is used this should be put in the same compartment as the steam passes through both these in series, and they ought therefore to be subjected to the same control. The temperature control should be automatic, regulated by some simple servo motor and pilot valve, which in turn is actuated by the temperature of the steam leaving the superheaters.

The arrangement you have adopted for providing power for the auxiliaries in the station is very economical and commendable. A somewhat similar arrangement is in use in the Birmingham Corporation power station on some 18,750-kw. sets, but in this case the auxiliaries are driven by direct current. A double unit d-c. generator set is driven direct from the main turbine, one unit provides excitation, the other supplies power for the auxiliaries.

With regard to the high-pressure turbine and its separate generator, I am not sure that this is the best ultimate arrangement. It would seem simpler to run both turbines at the same speed, putting them in tandem and connecting the two mechanically by a shaft having the usual claw coupling element at both ends. There need be no fear from out of alinement with this arrangement, and it does away with the generator, its losses, and some switching gear. The high-pressure turbine would in this case have more wheels than is indicated in the paper, but I think this would be in favor of economy, as with very high steam densities low steam speeds are desirable.

The design of the high-pressure turbine casing will no doubt present some difficulties with 1200 lb. initial and 375 lb. exhaust pressure, the mean internal pressure being in the neighborhood of 750 lb. It may be impracticable to split such a casing horizontally. The turbine set may preferably be placed at right angles from that shown on your plan with the high-pressure end nearest to the boiler house to shorten the steam pipes.

I see no reference in your paper to pre-heating the combustion air by the combustion gases on their way to the chimney. I believe this practise will be more and more taken up as it affords a means of cooling the combustion gases before they escape to the atmosphere, and also at the same time enables feed water to be heated to a higher temperature by bleeding before going to the economizer section of the boiler. With the higher feed water temperature the size of the economizer can be considerably reduced—part of this relatively expensive element is substituted for a cheaper air heating element.

There are some plants in this country in which air heating has been tried, but full information is still lacking. I think, however, that the sytem is right, and will eventually be successful.

I have not had an opportunity to check the curves giving the effect of steam pressure on the turbine thermal efficiency, but in general, I would say that they agree closely with similar investigations I have made, though not at such high pressures. I would, however, refer to curve No 4. which gives the efficiency ratio of a turbo-generator under different steam pressures, and believe that the slope of this curve is somewhat greater than it ought to be. In other words, that the efficiency of the turbine will be relatively greater at the higher pressures than indicated by this curve, which is to the good.

In conclusion, I would emphasize the great importance of proper control of the superheat in large stations such as yours at Weymouth. If such control is established the steam temperature may be raised some 50 deg. fahr. without any more risk than with a fluctuating temperature aiming at a lower temperature.

Joseph Pope: Answering the questions which Colonel Junkersfeld has asked, Curve No. 4 and consequently Curve No. 5, were prepared from data relating to the type of turbine to be installed in Weymouth Station. The manufacturers of the turbines, however, are not responsible for the high pressure end of these curves. Mr. Samuelson has also referred to Curve No. 4 suggesting that the slope is somewhat greater than he would anticipate. It is interesting to investigate the effect upon the problem in hand if the high pressure end of Curve No. 4 is raised as much as 5 per cent of its present indicated value, that is, from 72.5 per cent efficiency ratio at 1000 lb. abs. pressure to 76.0 per cent and if instead of being convex upward the curve is made a straight line of uniform slope between 300 lb. abs. and 1000 lb. abs. A table shows the result of this change most conveniently:

Abs. Press	Eff. Rankine Cycle Curve 3	Eff. Ratio Curve 4	Thermal Eff. Curve 5	Eff. Ratio as Modified	Thermal Eff. as Modified
100	30.75	83.30	25.61	83.30	25.61
200	33.75	82.80	27.92	82.80	27.92
300	35.30	82.10	28.99	82.10	28.99
365	36.20	81.49	29.48	81.50	29.49
400	36.60	81.15	29.68	81.22	29.73
500	37.50	80.00	30.00	80.35	30.12
600	38.27	78.80	30.12	70.48	30.40
700	38.83	77.45	30.07	78.61	30.50
800	39.40	75.90	29.90	77.74	30.60
900	39.77	74.28	29.52	76.87	30.52
1000	40.00	72.51	29.60	76.00	30.40

These are all slide rule figures, but sufficiently accurate for the purpose. Under the new conditions, the peak of efficiency comes at 800 lb. absolute, but this maximum is only 3.8 per cent higher than the efficiency at 365 lb. absolute, the throttle pressure actually selected, as against 2.1 per cent difference between the selected pressure and the maximum efficiency shown by Curve 5 in the paper, an amount clearly insufficient to justify the added expense. Although Curve 4, as stated, is based on data for the Weymouth machines, similar information was obtained concerning other turbine types, but the conclusions to be drawn from them were in nowise different.

The second question is "Why was 1200 lb. selected for the high pressure equipment?" It was principally because of the equipment available, and because the curve of cycle efficiency is rapidly approaching the horizontal at that pressure. In actual operation the boiler pressure will vary with the steam output and consequently with the load on the pressure reducing turbine. It is probable that the normal operating steam output will be such that the boiler pressure will be about 1000 lb. although the safety valves are to be set at 1200 lb.

The third question "How is it proposed to protect the reheater from overheating should the turbine load be suddenly discontinued?" At the time this paper was prepared there had been no opportunity to completely investigate matters of this nature and the cut of the high-pressure boiler as shown is practically as it was first offered by the manufacturers. After suitable performance data were available, more thorough study of the interaction of the boiler, superheater, turbine and resuperheater at varying loads indicated some difficulty in controlling the reheat. With an initial temperature of about 700 deg. the exhaust temperature from the turbine at light loads is also very high. In consequence the temperature leaving the reheater would be considerably above what is commonly held to be safe. A change was therefore made in the design and location of the reheater in order to give the necessary control of the reheating, but as the regulation of the initial superheat is quite satisfactory the superheater was left substantially unchanged. It is purposed now to locate the reheater above the first two passes of the tube bank with the tubes running across the boiler. The reheater tubes are in two groups separated by a vertically sliding damper forming in effect an extension of the front boiler gas baffle. By varying the vertical position of the damper the combustion gases may be made to pass over more or less of the reheater surface, and the outlet steam thereby maintained at the desired temperature. It will be understood that this same arrangement effectually takes care of the situation which may arise from sudden loss of load on the high-pressure turbine.

The fourth question "Is there much to be gained by the use of economizers, since feed water can be effectively heated by bleeding turbines?" If it be granted, and we presume that it will be, that large commercial air heaters are still decidedly experimental and without practical demonstration of economic justification sufficient to warrant their general use, a combination of feed heating by extracted steam and by economizers seems to be the best arrangement. It is realized that economizers cost money, but surface type water heaters are by no means inexpensive. The heat reclaimed from flue gases is practically net gain, whereas that saved in bled steam is partially offset by the extra live steam that has to be supplied at the turbine throttle on account of the extraction.

Mr. Eales inquired if it is possible to use a double-current generator on the same shaft as the main unit and thereby derive the excitation and auxiliary power by the same means. Mr. Samuelson calls attention to a case in Birmingham where a double unit d-c. generator set is used in this manner. The use of a direct-connected exciter in conjunction with the auxiliary alternator was considered for Weymouth and while feasible enough was abandoned in favor of the present arrangement.

The Cooling of Electric Machines

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Review of the Subject.—*The elevation in temperature of the windings of any electric machine due to the internal heat losses is usually the dominating factor in limiting the rating of the machine. This limitation found in the heating, is primarily due to the effects of high temperatures on the various types of insulation used in such machines. The American Institute of Electrical Engineers Standards specify the maximum permissible operating temperatures for the various grades of insulation. The temperature of the windings in the machines then, must be kept below this maximum permissible temperature.*

The heat losses produced in a machine are the results of electrical and mechanical losses. These losses must be transferred from the source of generation to the cooling medium. The first step of the heat flow path is by conduction through the copper, iron or insulation to the ventilating or cooling surfaces. The second step in the transfer of the heat may be called the liberation or dissipation of the heat from these surfaces to the cooling medium. This portion of the heat flow may be divided into two or more stages by the use of an intermediate cooling medium, such as oil, in an oil-insulated transformer.

This paper deals with some of the more common types of ventilating and cooling systems used in the electric machines and particularly with the laws governing the flow of heat from the cooling surfaces.

Considerable data have been published in the past giving the rate at which heat can be liberated from a surface by radiation and natural air convection currents. This is always a factor in the cooling of a machine and may be the sole means for cooling some small types of electric apparatus. Some new experimental data as well as a summary of the old data are given regarding this heat loss by natural ventilation and radiation. Tests are given for several types of surfaces, such as plane, cylindrical and corrugated surfaces.

A survey of the technical press will reveal very little accurate data regarding the rate of heat dissipation from the surfaces of electric machines by forced air convection currents. Thus, an experimental investigation was initiated for the purpose of determining some of the principal factors governing the rate at which heat can be dissipated from various types of surfaces by forced ventilation.

The results of many tests on the dissipation of heat from axial ducts are given. These tests show the influence of the air velocity, shape and surface of the air duct upon the rate of cooling. The rate of heat transfer is shown to be approximately proportional to the average air velocity, and not to the square root of the velocity as has been previously published. Another unexpected result was that the unit rate of cooling of the small circular ducts tested was practically the same as that found for the larger ducts over a considerable air velocity range. The effectiveness of the radial ducts in cooling a machine is shown by tests on ducts of several widths. The rate of heat dissipation does not differ materially from that found for axial ducts, also there is no great difference between the heat liberated from the narrow or from the relatively wide ducts.

Some interesting results were found in the cooling of a rotor. The heat loss with self-ventilation, due to its own rotation, is not proportional to the peripheral velocity. The heat dissipation curves for the rotor during rotation with separate ventilation along the periphery or air gap are very complicated being influenced by both the peripheral velocities and the forced air velocities.

The great value of proper ventilation of the end windings in electric machines is shown by the rate at which heat can be dissipated from the surfaces with forced air ventilation. Tests were made on a rotating and a stationary model with various ventilating spaces between the coil sides.

The rate of heat dissipation from cylindrical surfaces is also given with forced ventilation. The tests were made with the air flow at right angles to the cylinder or tube and the results show a much higher rate of heat flow than was found from axial or radial duct surfaces.

A summary of some miscellaneous tests is given for the heat dissipation constants found with water and oil cooling. These constants given for liquids are several fold greater than can be obtained with gases. Water, particularly, is a cooling medium with a very high rate of heat transfer coefficient.

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THE disposal of the heat resulting from the electrical and mechanical losses in any electric machine is always an important item to the designer as well as to the operator of the machine. These losses appearing all in the form of heat must be transferred from their source with a temperature gradient not to exceed the maximum safe operating temperatures. These maximum temperatures are limited by the insulation necessary to insulate the windings.

In the early days of machine design the generators, being principally of the engine type, were slow in speed and hence large in size. Due to the large surfaces, it was comparatively easy to liberate the heat with a low temperature rise. In fact with these old machines it was usually unnecessary to provide for additional ventilation other than that due to natural ventilation and windage of the rotating element. The ratings on many of these earlier machines were limited not by the temperature rise but by the performance such as commutation or regulation.

The development of the high-speed turbo-generator and the hydroelectric generator resulted in a machine of greater rating, hence greater losses, with a physically smaller machine than was used with the direct engine drive. This concentration of losses in a smaller space was possible only by greatly improving the ventilation.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

Today the commutation and regulation limitations of a machine have been practically removed so that the temperature limitation is usually the dominating factor in setting the maximum rating of electric machines. Thus to design the most economical machine, full consideration must be given to the temperature obtained at its rating. A machine with temperatures above the maximum permissible temperatures will not have a long life; on the other hand, a machine with temperatures too low mean an economic waste of material.

DISPOSAL OF THE HEAT LOSSES

The heat resulting from the internal losses of an electric machine must eventually be transferred from the machine to the air, water or the earth. This transfer of heat may be accomplished by many means. A direct system of cooling can be used such as the circulation of air or water over the heated surfaces with the heated air or water discharged at a point remote from the intake supply. An indirect system is often used for cooling a machine, such a system utilizes a fluid such as air, oil or water to "ferry" the heat from the surfaces of the machine to the outside supply of air or water. This transfer of heat may be made in several stages. Thus in an oil-insulated water-cooled transformer the heat "picked up" by the oil is transferred to the water in the cooling coils. This water then carries away the heat and distributes it either in the earth or in a body of water.

The most common system of cooling used with electrical apparatus is the direct air system. This system is especially adapted to rotating machines due to the mobility of the air. For high-voltage apparatus, such as transformers, oil is used for insulation purposes, so that the indirect system of cooling must be used. The oil merely acts as a "heat ferry" and carries the heat to the water-cooling coils in the case of a water-cooled transformer or to the ventilating surfaces in the case of a self-ventilated transformer.

Hence, one of the principal factors in the cooling of electrical apparatus is the determination of the temperature differences necessary to transfer the heat from the surfaces of the machine to the cooling fluids and from one fluid to another. The purpose of this paper is to give a summary of the various rates of heat transfer necessary for the predetermination of the machines operating temperature. Since air is the most common cooling agent it is given the greater consideration. The data given, unless noted to the contrary, are the results of experimental tests under the writer's direction and much of the data have been hitherto unpublished.

COEFFICIENT OF HEAT TRANSFER

The rate at which heat is transferred between a surface and a gas or liquid is called the coefficient of heat transfer or the heat dissipation constant. Throughout this paper the coefficient is defined in terms of watts per square inch of surface per deg. cent. difference between the surface and the fluid. This heat transfer constant is the same for a heat flow from a surface to the fluid or from the fluid to the surface, the direction of flow being dependent only upon the temperature gradient.

This coefficient is often expressed in other units; the relation of some of these constants is approximately:

1 B.t.u. / sq. ft. / deg. fahr. / hr. = 0.00366 Watts/sq. in./deg. cent.
1 Calorie / sq. cm. / deg. cent. / sec. = 27.0 Watts/sq. in./deg. cent.
1 Kilo-calorie/sq. meter/deg. cent./hr. = 0.00075 Watts/sq. in./deg. cent.
1 Watt / sq. cm. / deg. cent. = 6.45 Watts/sq. in./deg. cent.

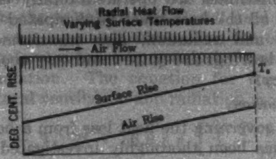

FIG. 1

$$T_s = \frac{W}{S\,K}\left(1 + \frac{1.765\,S\,K}{V}\right)$$

where T_s = Max. temperature rise of ventilating surface, deg. cent.
W = Total watts loss
S = Total ventilating surface, sq. in.
K = Coefficient of heat transfer in watts/sq. in./deg. cent.
V = Air Volume in cu. ft. per min. at 25 deg. cent.

The application of this heat dissipation constant to the cooling of electrical machines is illustrated in Figs. 1 and 2 for two conditions of heat flow.

The first condition is similar to that found in the cooling of a stator core with axial air ducts. In this case, Fig. 1, the loss is assumed to be uniformly generated along the axial ducts and the heat flow to be radial towards the ventilating surfaces. This is practically the condition found with the axial system of ventilation for laminated cores, since the resistance to the flow of heat is very much less in the radial than in an axial direction.

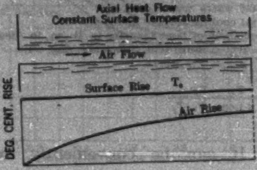

FIG. 2

$$T_s = \frac{1.765\,W}{V\left(1 - \epsilon^{\frac{1.765\,S\,K}{V}}\right)}$$

With a radial duct system of ventilation the ventilating surfaces of the ducts can be taken as having approximately a constant temperature; in this case the iron temperature rise will take a different form as shown on Fig. 2.

When the losses must be transferred from one liquid or gas to another, such as is found in the water cooling of oil-insulated transformers, it can be shown[1] that the relation given in Fig. 3 will hold regardless of the directions of fluid or heat flow.

DISSIPATION OF HEAT BY NATURAL VENTILATION AND RADIATION

The liberation of heat from electrical apparatus by natural ventilation and radiation is always a factor in limiting the operating temperatures. Many small industrial machines are operated totally enclosed, in which case all the heat must be dissipated by natural ventilation and radiation, also transformers, wires, cables and bearings are quite commonly cooled by this same agency.

A. *Radiation.*

The law governing the heat loss from a hot body by radiation has been thoroughly investigated by many authorities. It can be conveniently expressed as follows:

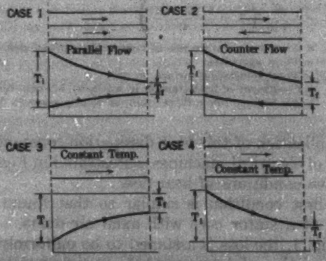

FIG. 3—HEAT TRANSFER BETWEEN FLUIDS OR BETWEEN A FLUID AND A SURFACE

T_i is initial temp. differences, deg. cent.
T_f = Final temp. difference, deg. cent.
Let W = Total watts transferred
S = Total cooling surface in sq. in.
K = Coefficient of heat transfer in W/sq. in./deg. cent.
T_m = Integrated mean tem. difference, deg. cent.
Hence $W = S\,K\,T_m$ where

$$T_m = \frac{T_i - T_f}{2.3 \operatorname{Log}_{10} T_i / T_f}$$

$$W = 36.9\, e \left[\left(\frac{T_1 + 273}{1000} \right)^4 - \left(\frac{T_0 + 273}{1000} \right)^4 \right]$$

Where W = Watts radiated per square inch of surface.
e = Relative emissivity coefficient.
T_1 = Deg. cent. temperature of surface radiating heat.
T_0 = Deg. cent. temperature of surrounding objects or air.

The emissivity coefficient (e) depends upon the color of the radiating surface and is unity for a perfect radiator, that is a dull black surface. The values of (e) for other surfaces are approximately:

Surface	Value of (e)
Lamp black	1.00
*Oils, varnishes, insulating materials	0.80 to 0.90
*Copper oxidized	0.74
*Cast iron freshly machined	0.25
*Cast iron oxidized	0.65
*Silver polished	0.03
Tinned surface	0.50
Steel surface oxidized	0.75

*By Langmuir[2].

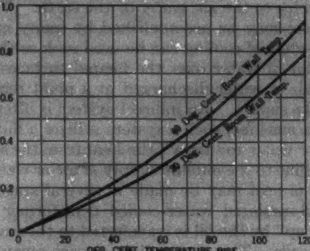

FIG. 4—HEAT LOSS BY RADIATION FROM A BLACK SURFACE

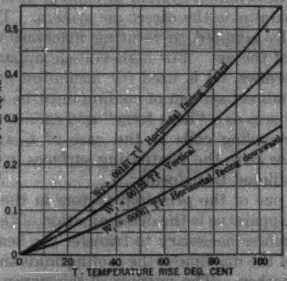

FIG. 5 HEAT LOSS BY NATURAL CONVECTION IN STILL AIR FROM A STEEL OR ALUMINUM PLATE 3 FT. SQUARE

The values of the heat loss radiated at various temperatures are shown plotted on Fig. 4. These curves show a considerable difference in the unit heat loss depending upon the room wall temperatures, also the rate of heat radiation increases considerably with increase in temperature rise.

B. *Natural Convection.*

The liberation of heat from a hot surface by natural convection is probably both by conduction and convection. That is the heat for a short distance from the surface is conducted through a thin layer of dead or stagnant air, the outer surface of this air film is set into motion by the expansion of the heated air particles and from here on the heat is ferried away by these convection currents.

Tests made by the English National Physical Laboratory[3] on the convection losses from a plane surface (3 ft. square) with different positions of the heated surface are shown in Fig. 5. As shown the heat loss by natural convection is proportional to the 5/4 power of the temperature rise above the ambient temperatures. These tests were made on both smooth steel and aluminum plates with the same results as shown.

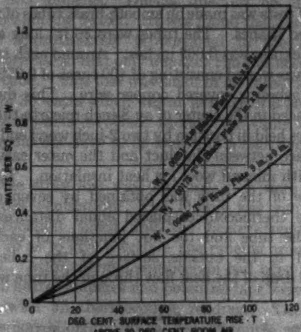

Fig. 6—Heat Loss from Surface of Vertical Plates by Natural Convection and Radiation

C. *Radiation and Convection.*

1. *Plane Surfaces.* The heat loss by radiation and convection from a plane vertical surface is given on Fig. 6. This surface was 3 feet square and was painted a dull black color. The heat loss from a 9-in. square heater[4] is also shown on Fig. 6 for a black plate and a brass plate.

The heat loss from a plane surface by natural ventilation and radiation can be put in the approximate form of

$$W = a\,T + b\,T^2$$

Where a and b are constants

T = Deg. cent. temperature rise.

W = Watts loss per square inch.

For approximate calculations with average temperatures common to electrical apparatus the natural heat loss can be taken as below:

Surface	Watts/sq. in./deg. cent. rise
Smooth plane	0.006
Cast iron	0.008
Enveloping machine	0.010 to 0.015

This enveloping surface of a motor or generator is calculated by figuring the surface of a cylinder whose general outline dimensions are approximately the same as the machine in question.

2. *Corrugated Surfaces:* The use of corrugated surfaces is quite common for oil-insulated transformer tanks. It is thus possible to increase the actual surface several fold over that obtained with the plane tank surface. The increased heat dissipating capacity due to the corrugations is principally due to the action of natural convection currents. The heat actually radiated will not be appreciably increased by the corrugations. The effective radiating surface is the minimum enveloping surface and not the actual surface. Thus the so called "radiators" used for heating a room or of a type used for cooling the oil in a self-cooled transformer function mainly by convection and not by radiation. The equation for the unit heat loss by natural ventilation and radiation, is:

$$W = a\,(T_1 - T_0)^{1.25} + 36.9\,r\,e\left[\left(\frac{T_1 + 273}{1000}\right)^4 - \left(\frac{T_0 + 273}{1000}\right)^4\right]$$

Where W = watts loss per square inch of actual surface.

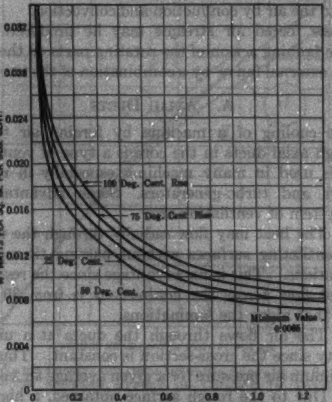

Fig. 7—Heat Dissipation from Wires or Cylindrical Surfaces by Natural Ventilation and Radiation in Still Air Based on Room Temperature of 20 Deg. Cent.

a = convection constant.

e = emissivity or radiation constant.

r = ratio of the enveloping surface to the actual surface. (Note — $r = 1$ for a plane surface).

T_1 = deg. cent. temp. of hot surface.

T_0 = deg. cent. ambient temperature.

3. *Cylindrical Surfaces:* The dissipation of heat from a cylindrical surface* by natural ventilation and

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* The total surface is taken as $\pi\,D\,L$, where (D) is the diameter and (L) the length in inches.

radiation is a function of the diameter of the cylinder or wire.[5] This is shown on Fig. 7. These tests were made with the wires or cylinders suspended horizontally in free air in a large room and no attempt was made to reduce the natural air convection currents found. Thus the smaller diameter wire can liberate the heat at a greater rate per unit surface than the larger wire. This is due to the fact that the relative thermal resistance to the flow of heat from the surface of the small wire through the encircling dead air film is less than that resistance found with the large wire. The effect of high surface temperature rise is to increase the coefficient of heat loss as shown.

DISSIPATION OF HEAT BY FORCED AIR STREAMS

In order to improve the rating of electric machines, it has long been the practise to use forced air ventilation. Radiation plays very little part in dissipating the heat from the internal motor surfaces. Most of these surfaces are at the same temperatures, hence the heat interchanged by radiation will be small if any. Thus the cooling action obtained with forced air convection is due almost entirely to the transfer of heat to the air by conduction and convection.

These forced air streams may be produced either by the fanning action of the rotating parts on the rotor, by an external fan or by a combination of the two.

A. AXIAL DUCTS

The cooling of a machine by forcing air to flow through axial ducts in the core is a type of ventilation that is used in many machines especially in railway motors and turbo-generators. Some advantages of this system of ventilation are:

1. The heat may flow radially through the iron to the ventilating surfaces without passing through the laminations. This is an advantage since the resistance to heat flow across the laminations is from 30 to 50 times that along the laminations.
2. The air flows through the ducts at a uniform velocity since the cross-section is constant. This gives a minimum air pressure for a given average air velocity.
3. Due to the rough surface obtained, a high rate of heat transfer is secured.
4. The core length of a machine will be a minimum for a given magnetic flux.

Some disadvantages of the axial system of ventilation are:

1. A fan or blower is necessary to force the air through the ducts.
2. The axial ducts must be placed in the core where the loss per unit volume is a minimum. This will give a fairly long heat flow path from the tooth zone where the maximum losses are found to the ventilating surfaces of the ducts.
3. Due to the space required by the ducts, the over-all diameter of the core will be increased.
4. The maximum iron temperature will usually be found adjacent the hottest air, that is near the end of the ducts.

The heat dissipated from the ventilating surfaces of axial ducts is influenced by the air velocity, size and shape of ducts, and the roughness of the duct surface.

DESCRIPTION OF EXPERIMENTAL APPARATUS

A diagrammatic sketch of the experimental model used in determining the heat transfer (K) from the surfaces of axial ducts is shown on Fig. 8. The first series of tests was made on smooth brass tubes of various sizes and shapes. The tubes were 36 in. long and the wall thickness was 0.063 inches. Around each tube the heater was wound consisting of 400 turns of 0.072 inches square asbestos covered copper wire. This duct was then placed in the center of an 8-inch micarta tube with the tubes held in position by two 2-inch wood end plates. The space between the duct and the outer casing was filled with mineral wool for heat insulation. The temperature of the tube was obtained by seven copper-advance thermocouples placed equal distances apart along the tube and the hot junctions of each were soldered to the outside surface of the tube. The thermal drop of the heat flow through the tube wall was very small, being a maximum of 1/5 deg. cent., so that for all practical purposes the thermocouples gave the true temperature of the ventilating surface.

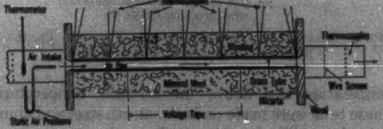

FIG. 8—APPARATUS FOR DETERMINING THE HEAT DISSIPATED FROM AXIAL DUCTS

The ventilating air was supplied by a blower driven by a variable speed d-c. motor. A maximum of 4½ in. of water, static pressure, could be obtained in the intake pipe. To obtain the total average air temperature rise five thermocouples were connected in series opposition. That is, the five hot junctions were placed in the outlet chamber with the five cold junctions in the inlet chamber and so connected that the voltage reading obtained was proportional to five times the average air temperature rise. It was necessary to thoroughly mix the air in the outlet pipe in order to obtain the average temperature. This was done by allowing the air to expand and forcing it to pass a brass gauze screen before coming in contact with the distributed thermocouple junctions.

The heater, which had terminals at each end of the duct, was supplied with current from a separate motor generator set. Potential taps were provided at each end of the heater, also two taps were taken off of a central portion of the heater. All the constants obtained were based upon the heat loss in this central portion since the temperatures there were affected very little by the stray heat loss from the ends of the tube.

METHOD OF TEST AND THEORY

The model was first calibrated for stray loss through the casing and end plates. This was done by circulating a given current and reading the final average heater temperature rise as given by the change in resistance of the heater with the corresponding heater input in volts and amperes. During this test no air was forced through the heater. It was found that the stray loss was approximately proportional to the heater temperature rise so that only one such run was necessary for each tube. This loss was small in all cases ranging from 15 to 30 watts which was an average of 5 per cent of the total watts involved, so that the value of the stray loss did not have to be known to a high degree of accuracy.

After the stray loss was obtained, runs were made with forced ventilation. The blower was run at a certain constant speed and the heater input was adjusted so as to give a surface rise ranging around 50 deg. cent. After thermal equilibrium had been established the heater amperes and potential taps were read, together with all thermocouple readings. The temperatures corresponding to the thermocouple readings were taken from a calibration curve for the thermocouple wire used.

The total cubic feet of air per minute was obtained by knowing the watts input to the air and the resulting average air rise. The watts absorbed by the air equals the total watts input minus the stray loss as given by the calibration curve corresponding to that particular heater temperature. The average temperature rise of the air was obtained from five thermocouples with the junctions in the outlet and intake pipes, hence:

$$V = \frac{1.765\, W}{T_a}$$

Where V = cubic feet of air per minute (25 deg. cent. temperature)

W = watts absorbed by air.

T_a = resulting air rise deg. cent.

The constant (1.765) is the degrees centigrade one cubic foot of air will rise with the storage of 1-watt-minute of energy. This is based upon air as weighing 0.074 lb. per cubic foot and a specific heat of 0.2418, all at 25 deg. cent. and standard atmospheric pressure (29.92 inches of mercury).

The average air velocity (v) through the duct then, is:

$$v = \frac{254\ W}{A\ T_a}$$

Where v = average air velocity in feet per minute.

A = cross-sectional area of duct in square inches.

The surface dissipation constant then is:

$$K = \frac{W}{S\ T_s}$$

Where K = dissipation constant in watts/square inch/deg. cent.

W = watts input to the air from the middle section of the heater.

S = ventilating surface of the middle duct section in square in.

T_s = average surface temperature of the middle duct section minus the average air temperature.

For each particular duct tested, five or more tests were made at various air velocities and heater inputs so as to obtain sufficient points for plotting the curves.

RESULTS AND DISCUSSION

(a) Circular ducts of various diameters, smooth surface.

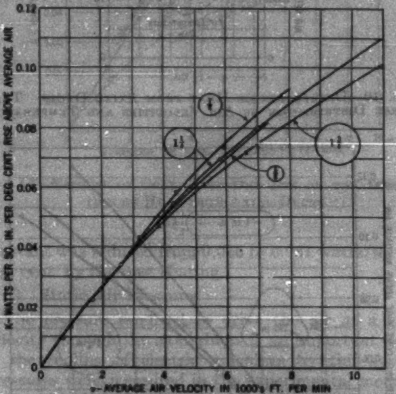

FIG. 9—WATTS DISSIPATED FROM INTERIOR SURFACE OF SMOOTH AXIAL DUCTS SHOWING EFFECT OF CHANGE IN DIAMETER

The heat transfer constant (K) plotted against air velocity for smooth brass tubes, diameters 5/8 to 1 1¾ inches, is shown on Fig. 9. There is little difference in this value of (K) for the various diameters, except for the higher velocities. The 7/8 inch duct gives the highest values and the 1¾ inch duct the lowest.

Fig. 10 gives the distribution of the air velocities across the 1¼ inch diameter duct also the temperature distribution for the same air conditions. An empirical equation for (K) for the 1¼ inch diameter duct is:

$$K = \frac{0.0154\ v}{1 + 0.048\ v}$$ where v = average air velocity in 1000's feet per minute.

The curves show practically a linear relation between (K) and (v) for air velocities up to 4000 feet per minute.

Tests made by the U. S. Bureau of Standards[6] on aeroplane radiators with ¼ and 11/32 inch diameter air cells show a heat transfer of,

$$K = \frac{0.0139\ v}{1 + 0.0135\ v}$$

This equation checks Fig. 9 rather closely up to an air velocity of 4000 feet per minute.

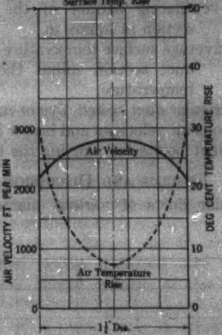

Fig. 10—Dissipation of Heat from Axial Ducts. Transverse Distribution of Air Velocities and Temperatures

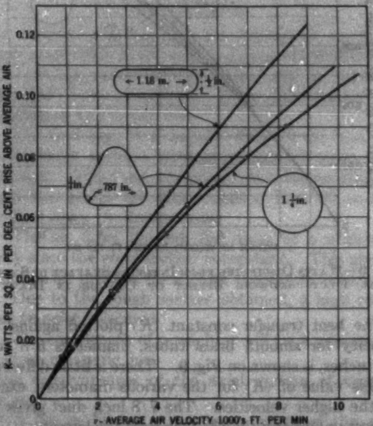

Fig. 11—Watts Dissipated from Interior Surface of a Smooth Axial Duct, Same Periphery

Tests made by H. P. Jordan[7] give the following results:

Diameter of copper tube

0.506 inches.......................$K = 0.0125\ v.$

1.236 inches.......................$K = 0.0108\ v.$

1.968 inches.......................$K = 0.0102\ v.$

The results were obtained by subtracting the stray loss at zero velocity from the losses given. The velocities tested ranged from 0 to 20,000 feet per minute. His tests show that the unit heat transfer (K) increases slightly with the temperature of the surface and air, but tests by the writer show that for all practical purposes the unit heat transfer is independent of the temperature over the range of 20 to 100 deg. cent.

(b) Effect of change in shape of duct cross-section. Fig. 11 gives the value of the heat transfer constant for three different shapes of ducts. All three have the same periphery. These curves show that the oblong duct has a better heat transfer than the triangular or circular duct also the watts dissipated are more nearly proportional to the air velocity.

(c) Effect of change in air flow through duct.

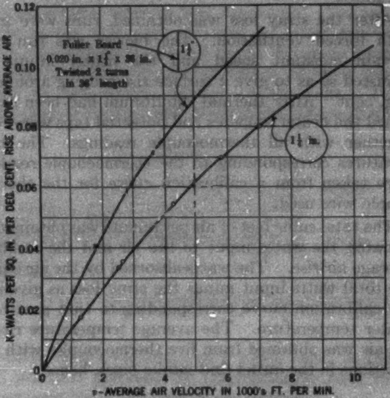

Fig. 12—Watts Dissipated from Interior Surface of Smooth Axial Duct, showing Effect of Change of Air Path

A piece of 0.020-inch fullerboard, 1¼ inch wide and 36 inches long, was placed in the 1¼ inch diameter duct and given a twist of two turns in its full length. This twist forced the air to take a helical path through the duct. Fig. 12 gives the results of this upon the heat liberated. It shows that the air was forced to mix more thoroughly and to scour the tube surface so as to give a greatly increased heat transfer over that obtained with the smooth tube.

(d) Effect of change in nature of surface.

Axial ducts in machines are formed by stacking the laminations in which holes are punched so as to form a continuous system of ducts through the core. The surface of such ducts will not be smooth, due to slight variations in the punchings and in their stacking. To study the effects of this upon the heat dissipated, a series of tests was made with a laminated duct.

Punchings of 0.017-in. iron in the form of washers 1¾ inches outside diameter and 1¼ in. inside diameter were forced in the 1¾-in. tube forming a duct 1¼ inches in diameter and 36 inches long. Thermocouples were placed at equal intervals along the duct between the laminations so as to obtain the surface temperatures.

Fig. 13 gives the values of the heat transfer constant for the smooth and laminated 1¼-in. diameter ducts. The value of (*K*) for the laminated duct is appreciably higher than that for the smooth surface duct and is almost proportional to the air velocity. This shows that a rough surface sets up eddies in the air stream, disturbing the uniform air path found in the case of the ducts with smooth surfaces. This tends to prevent a flow of relatively cool air through the center of the duct.

The surface of such ducts in actual machines may be considerably rougher (due to greater irregularities in the stacking) than the surface in this model duct. Hence, in practise, the heat dissipation constant may exceed the values shown.

(e) Effect of dust deposits upon the dissipation of heat.

Ventilated machines operating in dirty air will in time have a layer of dust deposited upon the ventilating surfaces. To show the detrimental effects of this dust upon the heating of the machine, the 1¼-in. diameter laminated duct was given a 0.030-in. coating of fine coal dust. To obtain a permanent coating of this dust in a short time, it was necessary to put a thin coating of varnish upon the surface and then to blow the coal dust through before the varnish dried. The coat of dust deposit, Fig. 14, shows a very great reduction in the heat dissipated especially at high air velocities.

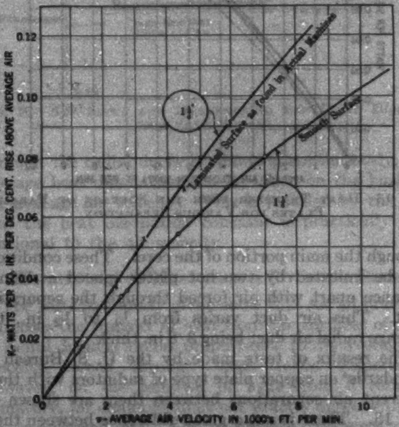

Fig. 13—Watts Dissipated from Interior Surface of Axial Duct, showing Effect of Surface

The dust deposit introduced an additional temperature drop which is independent of the air velocity, depending only upon the watts per square inch which must be conducted through this deposit. The dust coating also reduced the velocity of air passing through the duct for a given static air pressure. Thus for a static air pressure of 1½ inches the unit dissipation constant was reduced from 0.066 to 0.031 watts/sq. in./deg. cent. by the dust coating.

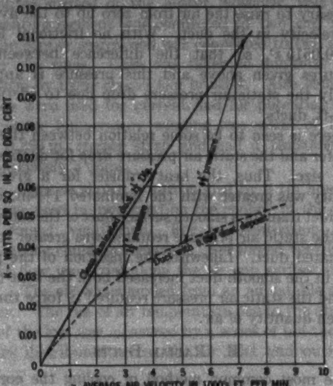

Fig. 14—Effects of Coal Dust Deposits on the Dissipation of Heat from Axial Ducts

(f) Static Air Pressures

The pressure required to drive the air at a given velocity through the axial ducts is an important factor in the cooling of electric machines by this system. Pressures were measured in the experimental model as shown in the sketch. Due to the relatively low velocity of the air in the intake and exit pipes attached to the duct, the static pressure measured is the resultant of the intake, exit and frictional losses.

The values of these pressures obtained were:

Type of duct		$p = Cv^2$
Smooth circular	1¼ in. and 1¾ in. Dia.	$= 0.086\ v^2$
" "	⅞ in. Dia.	$= 0.132\ v^2$
" "	⅝ in. Dia.	$= 0.227\ v^2$
" "	1¼ in. Dia. with paper helix	$= 0.177\ v^2$
Laminated "	1¼ in Dia.	$= 0.128\ v^2$
" "	1 ¼in. Dia. with dust deposit	$= 0.171\ v^2$
Smooth triangular	3.93 in. periphery	$= 0.097\ v^2$
" oblong	3.93 in. "	$= 0.140\ v^2$

Where p = the total static pressure drop in inches of water with an air temperature of 25 deg. cent.

v = average air velocity through the dust in 1000's ft. per minute.

Where p = the total static pressure drop in inches of water with an air temperature of 25 deg. cent.

v = average air velocity through the duct in 1000's ft. per minute.

The intake opening to the ducts was designed so as to give practically no loss at intake so that the above pressures are composed of the static pressure due to friction losses along the duct and the static pressure necessary to raise the air from zero up to the average air velocity in the duct. With no frictional losses, $p = 0.0615\, v^2$, so that the difference between the pressures given above and this pressure is approximately the static air pressure drop due to friction in the 3-ft. ducts.

There is seen to be some relation between the heat transfer and the static air pressure drop for ducts of the same size. Thus, the heat transfer for a given air velocity was greater with the laminated 1¼-in. diameter duct than for a smooth surface duct of the same size, also the static air pressure required was greater for the laminated duct. Likewise, the insertion of the paper helix in the smooth duct increased both the heat transfer and the static air pressure required to force through a given quantity of air.

B. RADIAL DUCTS

The most common method of cooling the cores of rotating machines is with the use of radial air ducts. Some advantages of the radial duct system are:

1. Radial ducts on the rotating element act as blowers, hence are self-ventilating.

2. Maximum air velocities are found where the losses are most concentrated; that is, in the tooth zone.

3. The air reaches the above tooth zone before it has become appreciably heated and the maximum air temperature is found where the iron losses per unit volume are a minimum.

4. Parallel ventilation of the end windings and the radial ducts is very simply accomplished.

Some of the disadvantages of the radial air duct system are:

1. On rotors having low peripheral speed the fanning action of the radial ducts is small, since the volume of air forced through is directly proportional to the peripheral speed for any given machine.

2. The majority of the heat due to the iron losses in the interior of the iron packet between two ventilating ducts must be conducted to these ducts through the intervening laminations. Since the resistance to the flow of heat across the laminations is 30 to 50 times the resistance to the heat flow along the laminations, it is thus necessary to limit the length of the heat flow path by close spacing of the radial ducts.

3. On high-speed machines especially induction motors, the windage noise may be excessive for certain applications.

4. On machines using small widths of teeth, the duct opening may be too small for effective ventilation and may become closed in service by duct and lint deposits.

5. On small diameter machines the air intake to the radial ducts in the rotor spider is apt to be restricted.

6. The radial ducts increase the gross core length and hence the distance between bearings.

DISSIPATION OF HEAT FROM RADIAL DUCTS

The experimental determination of the heat transfer constants from the surfaces of radial ducts is rather difficult due to the erratic disturbance set up in the air stream by the irregular separating spacers or fingers and to the conductors traversing the air flow path. Also the cross-section of the air path is constantly changing due to the changing diameter.

On machines of large diameter and shallow core depth the air velocity does not change much in passing through the main portion of the core. These conditions can be imitated by two hot plates spaced a definite distance apart with air forced through the separating duct. This air duct varies from ¼ to ½ in., the average radial air duct being 3/8 in. thick.

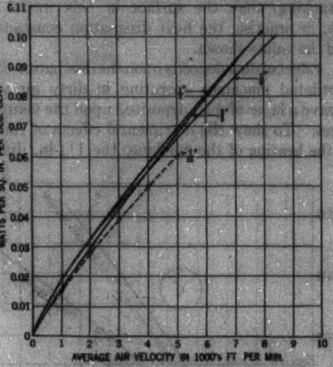

FIG. 15—HEAT TRANSFER FROM THE SURFACE OF PARALLEL PLATES FOR VARIOUS SEPARATION

The results of tests made by the U. S. Bureau of Standards[6] on copper plate type of radiators with three different air separations between them are shown on Fig. 15. There is not a great difference between these three curves and it is probable that the differences shown for the low velocities are within the range of experimental error.

Some preliminary tests[4] made on two smooth parallel hot iron plates separated a distance of 7/16 in. are also shown on Fig. 15. These values are lower than those found above. It should be noted that in all these tests the heat dissipation constant is almost directly proportional to the average air velocity. Comparison of the figures given for the 7/16-in. parallel duct with those given in Fig. 9 for the circular axial ducts will show almost the same values.

In the above experimental tests the air flow through the ducts was more uniform than that found in actual radial ducts. The tests given on Fig. 12 show a large increase in the heat transfer by a disturbance or mixing of the air flow in the ducts. Thus it is probable that the heat transfer constants given for the model radial ducts should be increased from 25 to 50 per cent for the actual values obtained in a machine due to the presence of the conductors and the ventilating fingers.

The fingers or ventilating spacers used in the radial ducts are effective not only in mixing up the air flow streams but they also increase the ventilating surfaces. Calculations will show that on the average machine the average temperature of the ventilating spacers is at least 90 per cent of the main duct surface temperatures. This additional ventilating surface on some machines will increase the effective duct surface as much as 30 per cent.

On machines of small diameter where the air velocity through the radial ducts is changing continuously, the average heat dissipation can be taken on the basis of the average air velocity. This is justified by the fact that the heat dissipation constant is practically proportional to the air velocity.

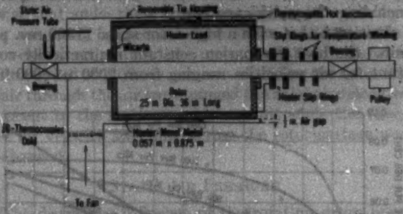

FIG. 16—APPARATUS FOR DETERMINING THE HEAT DISSIPATION FROM ROTORS

C. DISSIPATION OF HEAT FROM A ROTATING ELEMENT

The tests previously given show that for all practical purposes the transfer of heat from the surfaces of axial or radial ducts is almost proportional to the average air velocity forced through these ducts. Experience, however, on the removal of heat from a rotating armature, for example, does not exactly agree with the above observed law. Also no definite data were available regarding the effects of forced air convection currents along the surface of a rotating element. For these reasons this phase of the cooling problem was experimentally investigated on a model rotor.

DESCRIPTION OF EXPERIMENTAL APPARATUS

A sketch of the experimental rotor is shown on Fig. 16. The rotor proper was made of micarta tubing and plate, supported on a 4-in. shaft which in turn was supported between two pedestal bearings. The heating element (0.057 in. × 0.875 in. monel metal strap) was wound on the outside of this drum. The temperature of this winding was obtained by the change in resistance of a copper search coil wound in between this monel metal heater and the micarta drum. The heater straps were insulated from each other and from the copper temperature coil by a 0.005 in. paper sleeve. The outer surface of the heater, however, was bare. The heating element was connected to two slip rings also the copper search coil was connected to two other copper slip rings. Care was taken to minimize contact drop by using copper leaf brushes. Also separate brushes were used for current and for potential taps. The copper temperature winding was made of 0.005 in. × 0.250 in. copper strap and had a resistance of approximately 3 ohms.

The drum as shown is surrounded by a tin housing. This was used in investigating the cooling of the rotor with forced ventilation. For self-ventilation tests this housing could be removed. For forced ventilation the air flow was axial along the air gap surface of the rotor. The air was supplied from an adjustable speed blower. The rise of air temperature due to the absorption of the loss was measured by a series of ten thermocouples distributed across the air inlet and outlet.

PROCEDURE OF TESTS

The first series of tests was made with self ventilation, with the tin housing removed, allowing the rotor to rotate in free air. With the drum rotating at a fixed speed the current was thrown on so as to give a temperature rise ranging from 30 to 75 deg. cent. The current and rotor speed were held constant until thermal equilibrium was reached after which all readings were taken.

The power was supplied by a d-c. generator. The watts dissipated were found from the product of the current and the voltage drop as measured by two pairs of copper leaf brushes on the slip rings.

The temperature of the surface was found by the resistance reading of the copper temperature winding. This method of getting the temperature was accurate since there was no flow of heat in towards the center of the rotor after steady condition had been reached, and the temperature difference between the two sides of the monel strap did not exceed ½ deg. cent.

With forced ventilation a series of tests was made with the drum stationary. During each run the blower speed as well as the heater input were held constant. With the drum rotating at a definite speed a series of runs was taken with various blower speeds and heater inputs. This way 5 series of runs were made with rotor speeds of 60, 200, 600, 1000 and 1300 rev. per min. Under these different conditions the static air pressures were measured.

Discussion of Results

Fig. 17 gives the rate at which heat is dissipated for self-ventilation and forced ventilation. Under self-ventilation the ordinate is obtained by dividing the total watts dissipated by the total surface (2470 sq. in.) and by the rise in deg. cent. of the surface above room air temperature. With forced ventilation the deg. cent. rise used is the rise of the heater strap above the average air temperature. This is equivalent to the temperature of the surface minus the adjacent air temperature in the air gap.

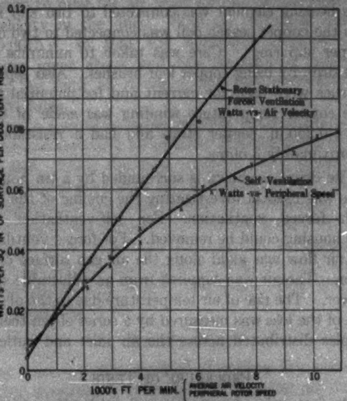

Fig. 17—Dissipation of Heat from the Surface of a Rotor, 25 in. Dia., 36 in. Long

For self-ventilation the abscissa is expressed as the peripheral speed of the rotor in 1000's ft./min. For the curve with forced ventilation the abscissa represents the average air velocity through the air gap with the drum stationary.

The cubic feet of air per minute was obtained from the watts absorbed and the corresponding rise in air temperature.

hence $V = \dfrac{1.765\ W}{T}$

where V = cu. ft. of air per minute.

where W = watts absorbed by air.

T = rise in air temperature deg. cent.

The curve with self-ventilation bends over with high rotor speeds. This indicates that some of the hot air is carried around with the rotor. With forced ventilation, however, the air is forced to pass through the duct. The dissipation of heat is proportional to the number of air molecules passing a given surface, and to the temperature difference between the surface and those air molecules. This is shown by the dissipation constant being roughly proportional to the air velocity with forced ventilation. The curve gives 0.004 watts/sq. in./deg. cent. with zero air velocity. This is due to the fact that there was some air leakage caused by convection currents. With self-ventilation about 0.008 watts/sq. in./deg. cent. is dissipated with the drum stationary due to convection and radiation.

The curve obtained with forced ventilation and drum stationary agrees very closely with the curve shown on Fig. 13 which gives the corresponding values for an axial duct through punchings.

By extending the curve shown for natural ventilation it is estimated that with a peripheral velocity such as is obtained in turbo-generators of 20,000 to 25,000 feet per minute the dissipation constant will have reached its maximum value of about 0.10 to 0.12 watts per sq. in. per deg. cent. rise. With forced ventilation under the conditions shown, there seems to be no limit to the amount of heat which can be dissipated. In practise, however, there are several factors which limit to a certain extent the maximum air velocity, such as friction and windage or blower input.

Curves given on Fig. 18 were all obtained with forced ventilation and the drum rotating at various speeds including the stationary run.

It is seen that with a high rotor speed (1300 rev. per min.) the heat dissipation constant is practically constant for air velocities ranging from 2000 to 6000 ft./min. Conversely it is seen that with the rotor stationary this dissipation constant is practically proportional to the air velocity through the air gap. With a rotor speed of 600 rev. per min. an intermediate condition is shown.

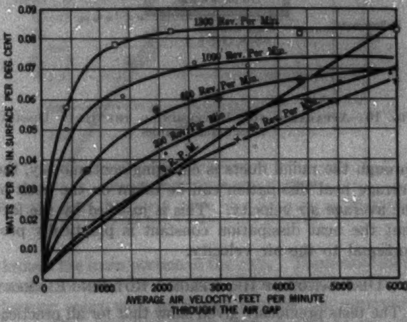

Fig. 18—Watts Dissipated from the Surface of a Rotating Rotor (25 in. Diam., 36 in. Long), with Forced Axial Ventilation through a ½-in. Single Air Gap.

It has been explained before that the curve on Fig. 17 found with natural ventilation indicates that particles of hot air are carried around the rotor when under rotation which causes the curve to bend over with high rotor speeds. This same condition happens with the drum rotating in a stator and with forced ventilation through the air gap. In addition to this the rotating surface of the drum at high speeds with its film of rotating air particles offers a very high resistance to the axial flow of air through the air gap.

Although these results show that with respect to the rotor the heat dissipation is by no means proportional to the average air-gap velocity yet with respect to the stator the conditions are entirely different and a heat dissipation roughly proportional to the air velocity is to be expected. With the high rotor speeds a high dissipation constant is obtained with low average axial air velocities. This indicates thorough mixing of the air and good scouring action of the rotor surface. However, under these conditions with a long core the temperature of air would rise to an excessive value which would also mean an excessive iron temperature unless other means of ventilation of the core were used.

Thus it is seen that under certain conditions more heat can be dissipated from the rotor stationary than when rotating with the same temperature rise and with the same quantity of air through the air gap per minute.

The air resistance along the stator is comparatively low so that under these conditions, high axial air velocities are found adjacent to the stator while the axial air velocity adjacent to the rotor is small. Thus with a rotor speed of 1300 rev. per min. doubling the quantity of air through the air gap changed the axial air velocities adjacent to the stationary air conduit while the effective air velocity sweeping across the rotor surface was unchanged.

STATIC AIR PRESSURE

The disturbing influence of the rotating drum upon the flow of air through the air gap is shown by the static air pressure curves, Fig. 19. Thus an average air velocity of 3000 ft./min. through the air gap requires a total static pressure of 0.9 in. of water with the drum stationary. With the drum rotation at 1300 rev. per min. or a peripheral speed of 8550 ft./min. requires 1.97 in. of water static pressure to force through the same amount of air. Also for an average air velocity of 5000 ft. min. through the gap requires a pressure of 2.5 in. with the rotor stationary and 4.34 in., with a rotor speed of 1300 rev. per min.

The empirical equation for these pressure-velocity curves is:

$$p = (0.0152\, v^{3/2} + 0.1\, V)\, V$$

where p = total static air pressure drop.
v = rotor peripheral velocity in 1000's ft./min.
V = average air velocity through air gap in 1000's ft./min.

From the equation it is seen that the factor $(0.0152\, V\, v^{3/2})$ represents the additional static air pressure required to overcome the disturbing influence of the rotor. The factor $0.1\, V^2$ represents the pressure required with the rotor stationary.

With a rotor peripheral speed of 20,000 ft./min., such as found in turbo-generators, and an average air-gap velocity of 5000 ft./min. the total static air pressure which would be required for the apparatus tested amounts to 9.3 inches of water.

In machines using small air gaps such as induction motors it is seen that very little, if any effective ventilation or cooling can be obtained by trying to force air through the air gaps. The resistance of the air path is too high for the static air pressures usually available.

With the drum rotating at 1000 rev. per min. and with no forced ventilation the watts dissipated due to the natural fanning action of the rotor amounted to only 0.008 watts/sq./in./deg. cent. rise of rotor windings, which is a negligible quantity.

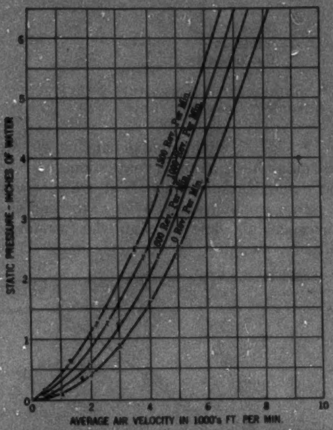

FIG. 19—TOTAL STATIC AIR PRESSURE REQUIRED TO FORCE AIR THROUGH A ½-IN. SINGLE AIR GAP, 36 IN. LONG WITH ROTOR AT VARIOUS SPEEDS. ROTOR DIAM. 25-IN.

D. DISSIPATION OF HEAT FROM END WINDINGS

The designer of rotating electric machines tries to reduce the length of the heat flow path in the windings to a minimum. Thus the end windings of these machines are usually ventilated so that the copper loss can be liberated after having been conducted through the insulating material. Due to the great surface available in these ventilated end windings their temperatures are usually lower than the imbedded windings, in fact many short core machines can be cooled almost entirely by these ventilated windings[8]. The tests described below were made for the purpose of investigating the rate of heat transfer from these windings as affected by air velocities and coil arrangement.

(a) *Description of Experimental Apparatus*: For investigating the cooling of a rotating end winding due to its own fanning action an experimental rotor was made as shown on Fig. 20. This was a 600-volt, Class A insulated winding in which the core length was reduced to a minimum. Thus the armature coil can be considered as two end windings connected together. The ventilating space between the coils was approximately 1/8 in. and a minimum of banding wire was used so as to obtain the maximum ventilation.

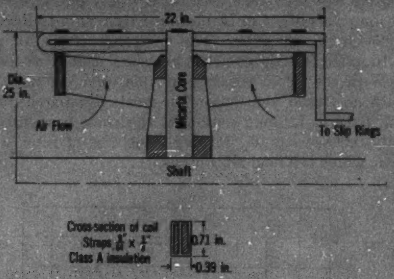

Fig. 20—Experimental Model for Determining the Dissipation of Heat from Rotating End Windings

The determination of the heat transfer constants from such a winding was subject to difficulties due to the irregular air flow through the windings. The irregularity was due to the unevenness of the coil separations also part of the coils, on the diamond portion, crossed each other forming a checker-board effect, while the remainder of the coil sides were parallel to each other. For these reasons tests were made on this model with self-ventilation only.

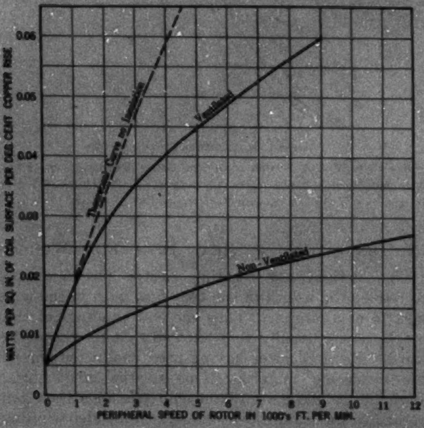

Fig. 21—Dissipation of Heat from Surfaces of Rotating End Windings

The windings were all connected in series and direct current was circulated for heating the windings. The average temperature of the copper was obtained by the change in resistance method.

Tests and Discussion

The results of the tests obtained are shown on Fig. 21. The ventilation surface of the end coil was taken at its mean length of turn times the outside periphery. Tests were also made with the winding non-ventilated, that is, with the intake air cut off. The great differences shown by the two curves bring out clearly the importance of ventilating the winding. These two curves given for the insulated windings show a bending over of the curves with increasing speed due to the thermal drop through the insulation.

The similarity of the curves given for the ventilated winding to that given on Fig. 17 for the heat transfer from a rotating cylinder in free air should be noted.

The curve shown in dotted lines is for the surface heat transfer and gives the values with no drop through the insulation, or with the insulation removed.

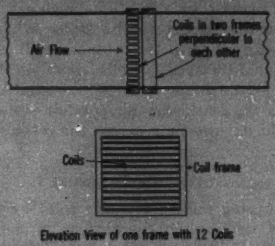
Air Flow
Coils in two frames perpendicular to each other
Coils
Coil frame
Elevation View of one frame with 12 Coils

Fig. 22—Experimental Model for Testing the Heat Dissipation from End Windings

(b) *Test with Separate Ventilation.*[4] In order to investigate more fully the ventilation of end windings, an experimental model was made as shown in Fig. 22. This model consisted of a wood frame holding the coils to be tested with the proper clearance between them. The coils were 1 in. by 2¼ in. by 12 in. long. They were wound with a special wire heater and insulated with cotton tape, treated as in practise. The average surface temperatures of the coils were obtained by the change in resistance of a fine copper wire distributed over the surface of the coils. Two frames containing the sample coils were used. These could be arranged with the coils parallel and in line with each other, or by rotating one frame 90 deg. the coils in the two separate frames would cross each other, forming the checker-board effect such as is formed on the diamond portion of the end windings.

The air volume passing through the ventilating space of these coils was measured by the electric method previously described. The average air velocities were obtained by dividing the volume by the cross-sectional area of the ventilating spaces.

TESTS AND DISCUSSION

The results of these tests are shown on Fig. 23. The values given for the 1/8-in. and ¼-in. air spaces are a mean value for all the curves. This one curve is not only common for both of these ventilating spaces but is also common for both coil arrangements.

That is, the heat transfer coefficient is the same with the coil axes parallel and at right angles to each other. The values given for the 3/8-in. and ½-in. air spaces with the coils parallel lie below this mean curve while the values for these spacing with the coils at right angles are above the mean curve. It should be noted that the rate of increase of the heat transfer per 1000 foot air velocity is practically the same as that given for the laminated axial ducts, Fig. 13. The curves for the end windings do not pass through zero for zero air velocity. This ordinate represents the heat dissipated by natural convection and radiation.

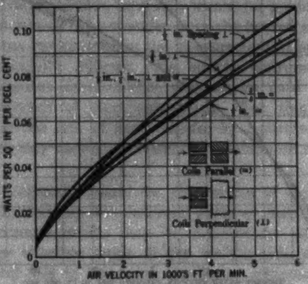

FIG. 23—DISSIPATION OF HEAT FROM END WINDINGS BY FORCED VENTILATION

E. DISSIPATION OF HEAT FROM CYLINDRICAL SURFACES

(*a*) *Plain Cylindrical Surfaces.* The current capacity of wires or cables either bare or insulated placed in a given air velocity stream will depend upon the rate of heat liberation from the ventilating surface. Tubular type of air coolers are also used for cooling the ventilating air in the closed circuit system of ventilation. Another application where heat is transferred from cylindrical surfaces is in the tubular type of air-oil coolers.

The transfer of heat from cylindrical surfaces with forced air convection at right angles to the cylinder axis is much greater than the coefficient previously given. For example, Kennelly and Wright,[9] found the following equation to hold for wires placed at right angles to an air stream:

$$K = \left(\frac{1.21}{d} + 595\right)\frac{\sqrt{1 + 20.3\,v}}{10^4}$$

where K = heat transfer in watts/sq. in./deg. cent.
d = diameter of wire in inches.
v = air velocity in 1000's ft./min.

These tests were made on small wires of 0.0045 in. to 0.0272 in. diameter. The above equation does not hold for wires larger than these. With a 0.0201-in. diameter wire (No. 24 B & S gage), the heat transfer on the basis of a 1000 foot per minute air velocity will be 0.303 watts/sq. in./deg. cent.

Hughes[10] tests on the heat transfer from the outside surfaces of copper tubes by forced air convections currents at right angles to the tubes are plotted on Fig. 24. These tests show a great variation of the constants with the tube diameter.

(*b*) *Cylindrical Surfaces with Fins.* In the tubular type of cooler or heat interchanger where a liquid is forced through the tube bore and the air flow is external at right angles to the tube, it will be found that the limiting resistance to the heat flow will be on the air side, due to the high rate of heat transfer on the liquid side. Hence it is usually advisable to increase the effective air-ventilating surface by fastening metal fins on the outside of the tube. In this way the effective ventilating surfaces can be increased several fold.

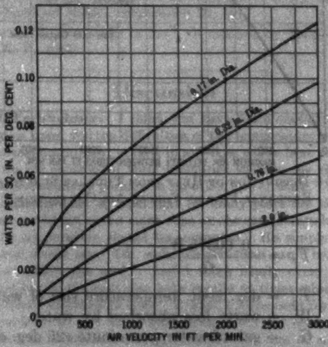

FIG. 24—DISSIPATION OF HEAT FROM THE OUTSIDE SURFACE OF COPPER TUBES BY FORCED AIR CONVECTION AT RIGHT ANGLES TO TUBE.

Tests on such a cooler with 5/8 in. outside diameter copper tubes, equipped with 5 fins per inch of tube, are shown on Fig. 25. The heat transfer given is figured on the basis of the total ventilating surfaces composed of the outside tube surface and the total fin surfaces. The fin surface was six times the outside tubular surface yet the average heat transfer coefficient is much higher than that found with a uniform air flow through axial ducts.

Dissipation of Heat by Liquids

A. Water. Some special motors and generators are cooled by forcing water through suitable pipes or ducts. Many oil-insulated transformers are also cooled by the same means. Water is especially adapted as an efficient cooling agent due to its high thermal capacity per unit volume and to the ease at which heat can be interchanged between it and the surrounding surfaces. The equation giving the relation between the water temperature rise, loss and water flow is approximately

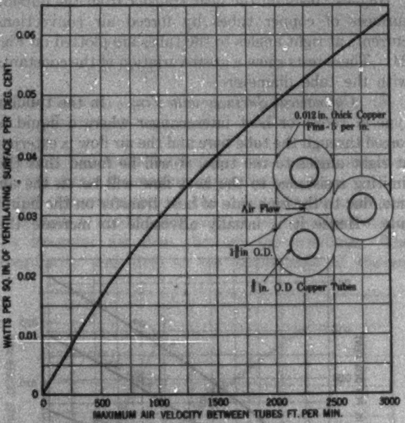

Fig. 25—Dissipation of Heat from Tubular Surfaces with External Fins by Forced Air Convection

$$T_w = \frac{0.00375\,W}{G}$$

where T_w = deg. cent. temperature rise of water.

W = watts loss absorbed by water.

G = gal. of water per minute (25 deg. cent.).

The heat transfer between a moving water column in a tube and the tube surface as found by Richter,[11] McAdams and Frost[12] is shown on Fig. 26. This rate of heat transfer is enormous compared to that obtained with air.

The values shown for zero water velocity are due to natural convection currents, these particular values vary considerably and in practise may be increased several times.

B. Oil. The use of oil as a "heat ferry" is quite common in transformers and bearings. The viscosity of oil is higher than that of water also its specific heat and coefficient of thermal conductivity are lower than the corresponding values for water. All of these factors have the effect of giving oil a lower rate of heat transfer than is obtained with water.

Data calculated from results obtained by Derby[13] on the rate of heat transfer with oil are shown on Fig. 27.

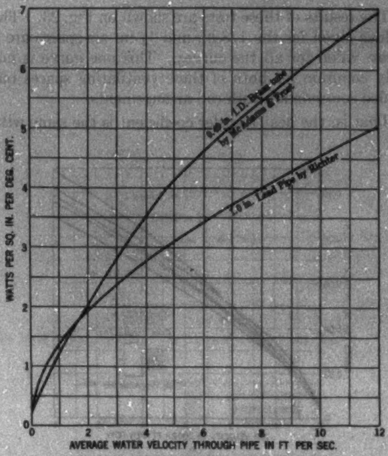

Fig. 26—Coefficient of Heat Transfer between a Metallic Surface and a Forced Water Stream

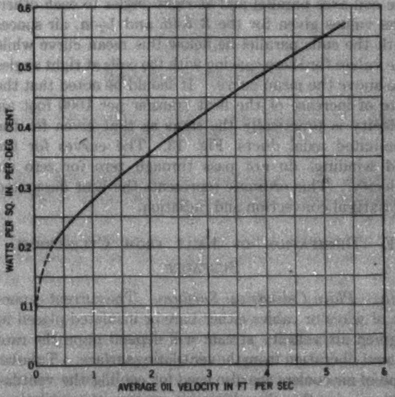

Fig. 27—Coefficient of Heat Transfer between a Metallic Surface and a Forced Oil Stream. Viscosity of Oil—50 Saybolt Sec.

The oil tested had a Saybolt viscosity of 50 seconds which is equivalent to common transformer oil at about 50 deg. cent. The heat transfer is shown to be considerably lower than those values given for water.

The equation connecting the oil temperature rise, rate of flow and watts loss absorbed for transformer oil at 50 deg. cent. is approximately

$$T_0 = \frac{0.0095\,W}{G}$$

where T_0 = deg. cent. oil temperature rise.
W = watts absorbed by the oil.
G = oil flow in gal. per min.

The above data as given are by no means complete. The writer has attempted to give a summary of some of the principal factors to be considered in the cooling of electric apparatus, especially means for predetermining the rate of heat transfer from the heated surfaces. Problems encountered in the measurement of heat flow are especially difficult. No simple instruments of measurement are available such as are used for the measurement of electrical energy. For this reason tests on the rate of heat transfer by separate experimenters may vary considerably due to different methods used.

The constants found for the cooling of a machine vary appreciably for different parts of the machine. Approximate results may be obtained by taking an average effective value of the cooling constant[14] and applying this to the total ventilating surfaces.

The writer wishes to thank Messrs. C. E. Wilson, T. S. Taylor, and C. J. Fechheimer for their valuable assistance in the investigation of this problem, and the Westinghouse Elec. & Mfg. Co. for providing means for conducting these series of tests.

Bibliography

Note: The references are numbered as they appear in the paper.

1. Engineering Thermodynamics by C. E. Lucke. Published by McGraw Hill Co.

2. Laws of Heat Transmission in Electrical Machinery by Irving Langmuir. A. I. E. E. 1913.

3. The Transmission of Heat by Radiation and Convection by Ezer Griffiths and A. H. Davis. Special Report No. 9. Department of Scientific and Industrial Research, London, England. Published by His Majesty's Stationery Office.

4. Heat Dissipation. Reports of Westinghouse Elec. & Mfg. Co. Research Laboratory by T. S. Taylor.

5. Current Capacity of Wires and Cables by George E. Luke. *Electric Journal*, April 1923.

6. Results of Test on Radiators for Air Craft Engines. Report No. 63 of National Advisory Committee for Aeronautics.

7. On the Rate of Heat Transmission between Fluids and Metal Surfaces by H. P. Jordan. I. M. E. 1909.

8. Longitudinal and Transverse Heat Flow in Slot-Wound Armature Coils by C. J. Fechheimer. A. I. E. E. 1921.

9. The Convection of Heat from Small Copper Wires by A. E. Kennelly, C. A. Wright and J. S. Van Bylevelt. A. I. E. E. 1909.

10. On the Cooling of Cylinders in a Stream of Air by J. A. Hughes and E. Coy. *Philosophical Magazine*, 1916.

11. Double Pipe Heat Interchangers by G. A. Richter, *Chem. & Met. Magazine*, March 24, 1920.

12. Heat Transfer by W. H. McAdams and T. H. Frost, *Journal of Ind. & Eng. Chem.* Jan. 1922.

13. Recooling of Lubricating Oil from Turbine Bearings and Gears by S. E. Derby. *Power Mag.* August 2, 1921.

14. Heating of Railway Motors in Service and on Test Floor Runs by G. E. Luke. A. I. E. E. 1922.

MISCELLANEOUS REFERENCES

Heat Loss from Radiators by F. P. Anderson. *H. & V. Eng. Journal.* December 1921.

Heat Dissipation from Self-cooled Oil Filled Transformers by J. J. Frank & H. O. Stephens. A. I. E. E. 1911.

Effect of Barometric Pressure on Temperature Rise by V. M. Montsinger. A. I. E. E. 1916.

Radiation of Heat by W. W. Coblentz. U. S. Bureau of Standards Paper No. 357.

The Dissipation of Heat from Various Surfaces and The Flow of Air through Small Brass Tubes by T. S. Taylor. A. S. M. E. 1920.

The Heat Paths in Electrical Machinery by H. D. Symons and M. Walker. I. E. E. 1911.

Temperature Distributions in Electrical Machinery by B. G. Lamme. A. I. E. E. 1916.

Heat Transmission by W. E. Dalby. I. M. E. 1909.

Heat Transmission by A. J. Wood and E. F. Grundhofer, 1920 Penna State College, Bulletin No. 30.

Investigation of Water-Air Radiators for Cooling Generators and Motors by G. H. Reist and E. F. Freiburghouse. *G. E. Review.* February 1920.

Heat Dissipation, a Problem in High Rated Units by G. E. Luke, *Electrical World.* September 22, 1917.

Internal Temperatures of A-C. Generators by Ralph Kelly. A. I. E. E. 1917.

Discussion

C. J. Fechheimer: I think you have heard enough already to impress upon you the difficulties of the temperature problem, in electrical machinery; in fact, in any case where heat problems arise, we are confronted with difficulties such as the electrical engineers encounters perhaps in no other line of work. In fact, these difficulties are so great that for many years the electrical engineer would not spend much time on the consideration of how best the temperature should be computed. He simply passed it on and would say, "Oh, well, temperatures can't be figured anyhow; what's the use?" And I think nearly every electric designer has in the past met with results at times which came as complete surprises to him. It wasn't infrequent for him to find machines in test which heated, say, fifty per cent more or less than he estimated, and if in any other line of work the electric engineer couldn't calculate better than fifty per cent he would lose his job quickly.

These two papers are an index of the fact that we are going ahead; we are getting away from rule-of-thumb methods on the question of heating of electric machinery, getting them on a more scientific basis. It is high time that we are doing more; it is high time that a few devote their undivided attention to this very important problem.

I think perhaps one reason why the electrical engineer did not advance more rapidly in his study of the heating problem was because he tried to consider the subject as a whole. He took his final results from tests and endeavored to analyze them and found it was quite impossible. He got so many inconsistencies that he didn't know where he was. Now, the only way, after all, that we can make progress is by attacking the *elements* of a complicated problem. That is what the authors are doing. I know that in Mr. Luke's paper he has given the results of investigation on particular elements that go into the heating problem.

Especially in connection with the ventilation of machinery we are going ahead quite rapidly. We are realizing that there is a distinct connection, for instance, between the character of flow of water and that of air, and since we use air almost exclusively for cooling, we can take the results of the work of hydraulic engineers, or apply them directly to some of our problems in connection with the flow of air.

For the most part the engineer who has worked with air problems has considered air to be highly compressible whereas water is not, and therefore air could not be considered in the same manner as water. Now, as a matter of fact, the compression to which air is subjected in electric machinery is ordinarily of the order of one or two per cent or even less of the total pressure. Atmospheric pressure corresponds to approximately 407 inches of water, and it is seldom we run more than ten or twelve inches, which is less than three per cent.

Now, considering that very fact means that we can take the work of the hydraulic engineers and use their formulas for computation of such things as pressure drops in our machines, when they can be applied. We can use their method of attack in many cases and build up models of parts of machines and cause air to flow through them (or water, if it can be more readily used), and determine our constants, so we are better able to compute the air pressure drops as we can compute the electric pressure drops in our external networks, although, of course, the computation is far more difficult in the case of the air circuit than in the electric circuit. Then if we determine experimentally the characteristics of the generator of pressure, that is the fan, we have solved a very large part of the ventilating problem, if the air circuit is in an enclosed chamber such as in the high-speed turbine generators. Once we have determined how the air flows through the machine, and the volume and pressure, we have gone a long ways toward solving this very complex problem.

One more thing in connection with Mr. Rice's paper; electrical engineers have seldom used the method of dimensions for solving their problems. It is a method which is quite foreign to most of their methods of attack. I might suggest that they look into that a little further, but in using that method of dimensions great caution must be observed because some item might be overlooked and the solution will in consequence be entirely in error.

V. M. Montsinger: In Fig. 6 Mr. Luke gives some curves for the dissipation of total loss from large vertical plates. According to these curves and from 0 to 120 deg. cent. rise the loss varies approximately as the temperature rises raised to the powers of 1.3 and 1.36. For temperature rises used in practise I have found that the exponent is somewhat less than these values.

Suppose we consider rises from 20 to 70 deg. cent. and calculate the exponent considering both radiation and convection. If we differentiate the radiation formula we can calculate the exponential values for different values of rises for any given room temperature as follows:

$$k\,(T_2^4 - T_1^4) = (T_2 - T_1)^n\,k$$

$$\log\,(T_2^4 - T_1^4) + \log k = n \log\,(T_2 - T_1) + \log k$$

Differentiating, we get

$$n = \frac{4\,T_2^3}{(T_2 + T_1)\,(T_2^2 + T_1^2)}$$

The following gives the values of n for a room temperature of 20 deg. cent. (T_1 = 293 deg.)

$(T_2 - T_1)$	n
10	1.051
20	1.104
30	1.150
40	1.200
50	1.245
60	1.292
70	1.339
Average	1.197

Since approximately one-half the loss from a plain black surface is dissipated by convection and the other half by radiation and since the exponent is 1.25 for convection the general average exponential value should be about 1.225. This I find checks fairly closely with test results within this range of rises.

In second column on the fourth page, Mr. Luke gives a formula for the total loss from an irregular surface. For corrugated surfaces I have found that it is necessary to introduce a correction factor, say R, in the convection part of the formula to take care of restricted air circulation in deep corrugations. For instance, if we should have a corrugation 12 in. deep and with an air space 1 in. wide the value of R is about 0.5. In other words the convection is only about one-half as effective within this corrugation as for a plain surface. This reduction is a function of the width divided by the depth of the air space in the corrugation.

I hope to publish in the near future an article dealing with dissipation of loss by radiation and convection from plain and irregular surfaces and in which will be given a formula that will hold for any reasonable shape of corrugation, etc.

G. E. Luke: Mr. Montsinger has given some data indicating that the dissipation of heat from large vertical plates by radiation and convection should vary approximately as the temperature rise raised to the 1.225 power or $W = a\,T^{1.22}$ where (W) is watts loss per square inch of surface and (a) is a constant.

This equation is approximate for a given temperature range. The values of 1.30 to 1.36 given in the paper were averages for a temperature rise of 20 to 120 deg. cent. The exponents of (T) based on temperature rises of 20 to 70 deg. cent. would be reduced and would not differ materially from the above value given by Mr. Montsinger.

The exact value of this exponent is not well established, for example, Frank and Stephens[1] give data which result in a value of approximately 1.31 for this exponent based on a temperature rise up to 70 deg. cent. To determine the exact value of this equation will require exceedingly accurate tests and will probably be dependent upon the conditions of test such as size and position of plate and upon the natural air convection currents found in the room used for the tests.

There is no doubt but that the heat loss from a unit surface of a radiator or corrugated surface by natural convection will be less than that found from a plane surface so that a factor (R) as suggested by Mr. Montsinger would be desirable in the general heat loss equation from such surfaces. This constant will not only vary with the depth of the corrugations but will depend upon the height of the vertical surfaces and also upon the degree of freedom the air has in flowing through the radiator.

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[1] "Heat Dissipation from Self Cooled Oil Filled Transformers," by J. J. Frank and H. O. Stephens, A. I. E. E., 1911.

Free and Forced Convection of Heat in Gases and Liquids

BY CHESTER W. RICE
Associate, A. I. E. E.
Research Laboratory, General Electric Co.

Review of the Subject.—*The general problem of heat transfer requires a knowledge of the laws of conduction, radiation and convection. In 1822, Fourier gave us the first thoroughly scientific definition of conductivity and reduced the problem of heat conduction to an exact science, with a power and completeness which left little room for extension or improvement even to the present day. The law of radiation was first suggested by Stefan in 1879 as a result of an analysis of some experiments made by Tyndall. In 1884 Boltzman deduced the law theoretically from the principles of thermodynamics and electromagnetics. Thus the laws of conduction and radiation have been accurately known for a long time, while the problem of convection has received relatively little study. This fact is surprising when we consider the important part played by convection in almost all cases of heat transfer. A complete mathematical solution of a convection problem would require a knowledge of the hydrodynamic laws of viscous fluids for stream line and turbulent motion, combined with the Fouier equations of heat conduction in a moving medium. At present our lack of the hydrodynamic laws for turbulent motion renders a rigorous solution impossible. Therefore in most of the theoretical work so far attempted the simplifying assumption of an inviscid fluid has been found necessary. The theoretical results obtained when viscosity is neglected are in general far from the experimental facts. Langmuir's study of the problem showed that the viscosity is a factor of first importance which cannot be neglected. He therefore adopted a film theory as an approximation. The reason for the existence of a film around a hot body may be seen as follows: Consider a horizontal wire maintained at a given temperature in a fluid, the fluid adjacent to the wire will become heated and rise while the cooler fluid of greater density will flow into its place. Thus a convection current is set up by the difference in density between the hot and cold fluid. This condition is usually referred to as free convection. At the surface of the wire the fluid is stationary due to viscosity. As we proceed from the surface of the wire the velocity of the convection currents increase until a distance is reached at which the critical velocity conditions in the fluid are exceeded and the stream line flow bursts into turbulent motion. The discontinuity between the stream line and turbulent motion constitutes the outer boundary of the film. At the inner boundary the fluid has the temperature of the hot surface and at the outer boundary the temperature of the ambient fluid. The actual configuration of the outer boundary is unknown. As an approximation we might assume that it was an eccentric ellipse or cylinder, etc., and determine the size and eccentricity so as to best fit the experimental results. For ease of calculation Langmuir adopted the simplest approximation and assumed that the outer boundary of the relatively stagnant film was a cylinder concentric with the wire. He thus reduced the hopelessly complex problem of convection to one of conduction in the steady state.*

The question of how the film thickness varies with the size and shape of the body and properties of the ambient fluid is determined by the method of dimensions plus experiments. Raleigh has frequently pointed out the great power of the **method of dimensions** *in obtaining the solution of physical problems. He says—"It often happens that simple reasoning founded upon this principle tells us nearly all that is to be learned from even a successful mathematical investigation, and in numerous cases where such a mathematical investigation is beyond our powers, the principle gives us information of the utmost importance." As already stated the problem of convection is at present beyond our reach and therefore we have to be content with the information which we can obtain from the* **method of dimensions.**

Recently Davis has attacked the problem by dimensional methods starting from certain general hydrodynamic considerations which do not take into consideration the existence of a film. In the present papers on Free and Forced Convection Langmuir's film theory has been extended by dimensional analysis and it is felt that the results obtained give a clearer insight into the mechanism of convection, as well as more useful equations. An attempt has also been made to bring together the available dcta and correlate them as far as possible. A striking example of the practical results which may be expected from a thorough understanding of the laws of conduction, radiation and convection is found in Langmuir's gas filled lamps. His study and application of these laws allowed him to more than double the efficiency of the incandescent lamp.

FREE CONVECTION OF HEAT

I. Introduction

A RIGOROUS mathematical solution of a problem in free convection would require an exact knowledge of the hydrodynamic laws of viscous fluids for stream line and turbulent motion, combined with the Fourier equations of heat conduction in a moving medium. The problem of turbulent fluid motion has been attacked by Stokes, Kelvin, Rayleigh, Reynolds and others without as yet having reached a satisfactory solution. A summary and discussion of their work may be found in Lamb's Hydrodynamics.[1]

In most of the theoretical work so far attempted on convection the simplifying assumption of an inviscid fluid has been found necessary. A summary of this theoretical work on heat transfer, with numerous references, may be found in Russell's[2] valuable paper. Langmuir[3] preferred to retain the idea of viscosity as a factor of first importance and therefore adopted a film

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[1] Horace Lamb, Hydrodynamics Camb. Univ. Press 3rd Edition, p. 591.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

2. Alex. Russell, *Phil. Mag.* Vol. 20, pp. 591-610, Oct. 1910.

3. a. Irving Langmuir *Phy. Rev.*, Vol. XXXIV, No. 6, p. 401, 1912

b. Irving Langmuir, A. I. E. E., Vol. XXXI, Part 1, p. 1229, 1912.

c. Irving Langmuir, A. I. E. E. Vol. XXXII, Part 1, p. 301, 1913.

d. Irving Langmuir, *Trans. Am. Elec. Chem. Soc.*, Vol. XXIII, p. 299, 1913.

theory as his approximation. More recently Davis[4] has attacked the problem of convection by the method of dimensions starting from certain hydrodynamic considerations.

II. Langmuir Film Theory of Convection

When a horizontal wire is maintained at a given temperature in a fluid, the fluid adjacent to the wire becomes heated and rises while the cooler fluid of greater density flows into its place. Thus a convection current is set up by the difference in density between the hot and the cold fluid. At the surface of the wire the fluid is stationary since the viscosity requires that there is no slip at the boundary between the wire and the fluid. As we proceed from the surface of the wire the velocity of the convection currents increase until a distance is reached at which the critical velocity conditions in the fluid are exceeded and the stream line flow bursts into turbulent motion. The discontinuity between the stream line and turbulent motion constitutes the outer boundary of the film. At the inner boundary of the film the fluid has the temperature of the hot surface and at the outer boundary the temperature of the ambient fluid. In order to simplify this picture for purposes of calculation Langmuir assumed that the outer boundary of the relatively stagnant film of fluid was a cylinder concentric with the wire, and of a definite diameter. He thus reduced the complex problem of free convection to a problem of conduction in the steady state.

The general equation for the conduction of heat after the steady state has been reached is

$$W = (A/L)\, k\, \Delta t \qquad \text{watts} \qquad (1)$$

where W = watts of heat flow
A = Area of cross-section of heat flow path in square cm.
L = Length of heat path in cm.
k = Heat conductivity of medium expressed in watts per cm. per deg. cent.
Δt = Temperature difference causing the heat flow in deg. cent.

For the case of free convection from a wire, we substituted the problem of conduction between concentric cylinders as illustrated in Fig. 1, the inner cylinder being at the temperature of the wire and the outer cylinder having the temperature of the ambient fluid. Applying equation (1) to the element $d\,r$ we obtain for the total heat flow for a length of L cm.

$$W = -\frac{2\,\pi\, r\, L}{d\, r}\, k\, d\, t \qquad \text{watts} \qquad (2)$$

This can be written

$$\frac{W}{2\,\pi\, L}\,\frac{d\, r}{r} = -\,k\, d\, t \qquad (3)$$

Integrating between limits $r = D/2$ and $r = b/2$ and $t = T_2$ and $t = T_1$

$$\frac{W}{2\,\pi\, L}\int_{r=D/2}^{r=b/2}\frac{d\, r}{r} = -\int_{T_2}^{T_1} k\, d\, t \qquad (4)$$

If the heat conductivity k is independent of temperature we obtain

$$\frac{W}{2\,\pi\, L}\,\log_e b/D = (T_2 - T_1)\, k \qquad (5)$$

or

$$W = \frac{2\,\pi\, L\, k\, \Delta\, t}{\log_e b/D} \qquad \text{watts} \qquad (6)$$

Now if k is a function of the temperature as is the case in all real fluids we go back to equation (4) and obtain after integrating the left hand side and interchanging the limits of integration for T

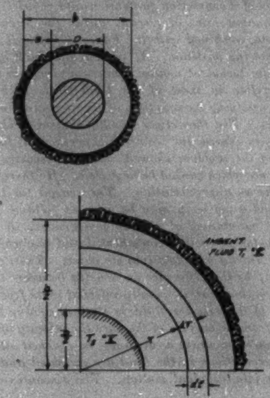

Fig. 1—Free Convection in Gases and Liquids

$$W = \frac{2\,\pi\, L}{\log_e b/D}\cdot\int_{T_1}^{T_2} k\, d\, t \qquad \text{watts} \qquad (7)$$

or

$$W = \frac{2\,\pi\, L}{\log_e b/D}\left[\int_0^{T_2} k\, d\, t - \int_0^{T_1} k\, d\, t\right] \qquad (8)$$

Langmuir has used the following abbreviation for the thermal conduction

$$\varphi = \int_0^T k\, d\, t \qquad \text{watts per cm.} \qquad (9)$$

where
T = Absolute temperature of fluid in deg. K.
k = Heat conductivity in watts per cm. per deg. cent.

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[4] a. A. H. Davis, *Phil. Mag.*, Vol. 40, p. 692, 1920. (Paper dealing with Free and Forced Convection).

b. A. H. Davis, *Phil. Mag.*, Vol. 41, p. 899, 1921. (Paper dealing with Forced Convection for Cylinders and a relation to the mechanical resistance to flow).

c. A. H. Davis, *Phil. Mag.*, Vol. 43, p. 329, 1922. (Paper Free Convection from Cylinders).

d. A. H. Davis, *Phil. Mag.*, Vol. 44, p. 920 and p. 940, 1922. (Papers on Free and Forced Convection).

If the heat conductivity k is given in calories per second per cm. per deg. cent. then we have

$$\varphi = 4.19 \int_0^T k\,d\,t \qquad \text{watts per cm.} \qquad (10)$$

Langmuir has also put

$$S = \frac{2\pi L}{\log_e b/D} \qquad \text{cm.} \qquad (11)$$

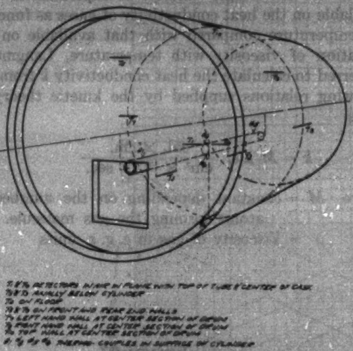
T1 & T2 DETECTORS IN AIR IN PLANE WITH TOP OF TUBE & CENTER OF CASK
T3 & T4 AXIALLY BELOW CYLINDER
T5 ON FLOOR
T6 & T7 ON FRONT AND REAR END WALLS
T8 LEFT HAND WALL AT CENTER SECTION OF DRUM
T9 RIGHT HAND WALL AT CENTER SECTION OF DRUM
T10 TOP WALL AT CENTER SECTION OF DRUM
θ1 θ2 θ3 θ4 THERMO-COUPLES IN SURFACE OF CYLINDER

FIG. 2—FREE CONVECTION TESTS IN CASK

FIG. 3

and called it the "Shape Factor." The factor S might also be termed the "Shape Conductance" since it measures the ease with which heat is conducted through a body of given form without reference to the intervening medium.

By the definitions of equations (11) and (9) we may write equation (8) in the following abbreviated form

$$W = S\,(\varphi_2 - \varphi_1) \qquad (12)$$

or putting $\varphi_2 - \varphi_1 = \Delta\varphi$

$$W = S\,\Delta\varphi \qquad \text{watts} \qquad (13)$$

CALCULATION OF "SHAPE CONDUCTANCE"

Langmuir has pointed out that all problems in steady heat flow can theoretically be reduced to the form given by equation (13).

This "shape conductance" is the same as the "shape capacity" in electrostatics or "shape permeance" in the magnetic circuit, provided Heaviside units are used, or as the "shape conductance" in the electric circuit. In brief, the shape conductance has the same value for that large class of problems which are governed by the divergence theorem and the Fourier-Ohm law. The combination of these two laws results in Laplace's equation and therefore to determine the shape conductance we require the solution of Laplace's equation so as to fulfill the boundary conditions imposed by the problem. It is not in general possible by known mathematical methods to obtain the required solution except for a relatively few simple geometrical forms. These will be found in works on Electricity and Magnetism, Hydrodynamics, Heat and Sound. Some further useful shapes have been calculated by methods of approximation in a paper by Langmuir,[4] Adams and Meikle. Reference may also be made to a paper[5] by the present writer in which an experimental method of obtaining the solution of any flow problem is discussed.

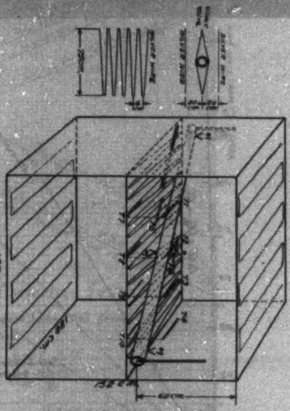

FIG. 4—FREE CONVECTION ON TESTS IN OVEN

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[4] Irving Langmuir, E. Q. Adams and G. S. Meikle, Flow of Heat Through Furnace Walls, *Trans. Amer. Electro Chem. Soc.*, Vol. XXIV, p. 53, Sept. 1912.

[5] Chester W. Rice, A. I. E. E., Vol. XXXVI, p. 905, 1917.

For the purpose of easy reference the shape conductances for the three simplest geometrical forms are given below.

I. Parallel plane surfaces

$$S = A/t \qquad \text{cm.} \qquad (14)$$

Fig. 5

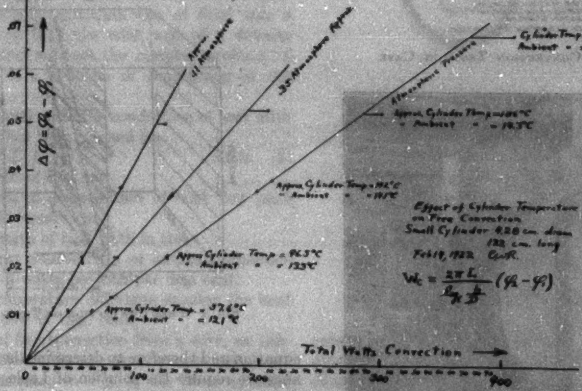

Fig. 6

where A = Area of surface in sq. cm.
t = thickness of material between surfaces in cm.

II. Concentric cylinders

$$S = \frac{2\pi L}{\log_e b/D} \qquad \text{cm.} \qquad (15)$$

where L = Length of cylinder in cm.
b = Diameter of outer cylinder in cm.
D = Diameter of inner cylinder in cm.

III. Concentric Spheres

$$S = \frac{2\pi}{1/D - 1/b} \qquad \text{cm.} \qquad (16)$$

where D = Diameter of inner sphere in cm.
b = Diameter of outer sphere in cm.

CALCULATION OF THE THERMAL CONDUCTION φ

To calculate the function φ for different absolute temperatures it is necessary to express the heat conductivity of the gas or liquid as a function of the absolute temperature.

Due to the meager amount of experimental data available on the heat conductivity of gases as function of temperature compared with that available on the variation of viscosity with temperature, Langmuir[3a] preferred to calculate the heat conductivity k from the following relations supplied by the kinetic theory of gases

$$k = M\,\mu\,c_v \quad \frac{\text{cal.} \times \text{cm.}}{\text{cm}^2 \times {}^\circ\text{C} \times \text{sec.}} \qquad (17)$$

where M = constant depending on the number of atoms forming the gas molecule.
μ = Viscosity of gas in c. g. s. units
c_v = Specific heat per gram at constant volume.

For the variation of viscosity with temperature he used Sutherland's equation which is

$$\mu = \frac{K\,T^{1/2}}{1 + C/T} \quad \frac{\text{dynes} \times \text{cm}}{\text{cm per sec.} \times \text{cm}^2} \qquad (18)$$

Where K and C are constants which depend on the nature of the gas

T = Absolute temperature of gas in deg. K

For the variation of the specific heat with temperature he used the values given by Pier[6] of the form

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[6] M. Pier, *Z. f. Electrochem* 15, p. 536. 1909 and 16, p. 899, 1910.

$$c_v = F\,(1 + \alpha\,T)$$

cal. per gram per deg. cent. (19)

Where F and α are constants of the gas

T = Absolute temp. deg. K

For air $F = 0.1614$; $\alpha = 0.0002$

H_2 $F = 2.21$; $\alpha = 0.0002$

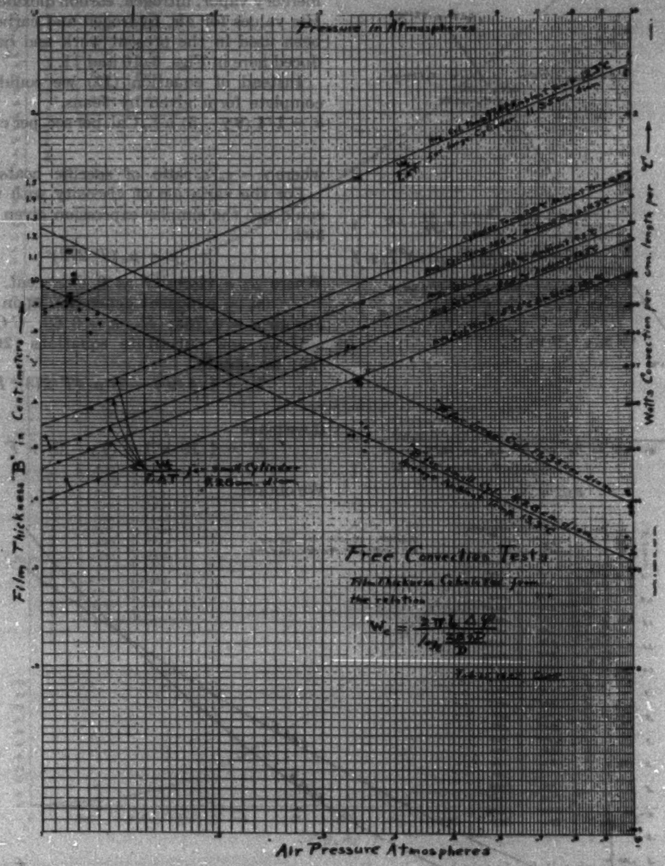

Fig. 7

CO_2 $F = 0.124$; $\alpha = 0.00095$ (Below 1100 deg. K.)

Substituting (18) and (19) in (17) he obtained for the heat conductivity of the gas as function of temperature.

$$k = M\,F\,(1 + \alpha\,T)\,\frac{K\,T^{1/2}}{1 + C/T}$$

Cal. per sec. per cm per deg. cent. (20)

Substituting $A = 4.19\,M\,F\,K$ we obtain

$$k = A\,(1 + \alpha\,T)\,\frac{T^{1/2}}{1 + C/T}$$

Watts per cm. per deg. cent. (21)

which is the relation in the form used by Langmuir in calculating the values of the function φ. We have from equations (9) and (21)

$$\varphi = \int_0^T k\,d\,t = A \int_0^T \left[\,(1 + \alpha\,T)\,\frac{T^{1/2}}{1 + C/T}\,\right] d\,T \qquad (22)$$

and since it is only for large values of T that the temperature coefficient α is important it can be shown that

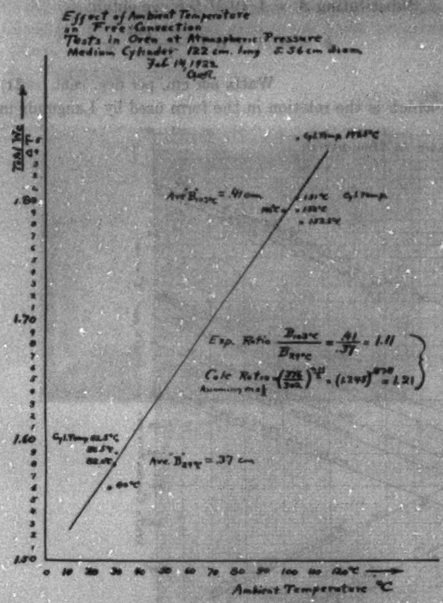

FIG. 8

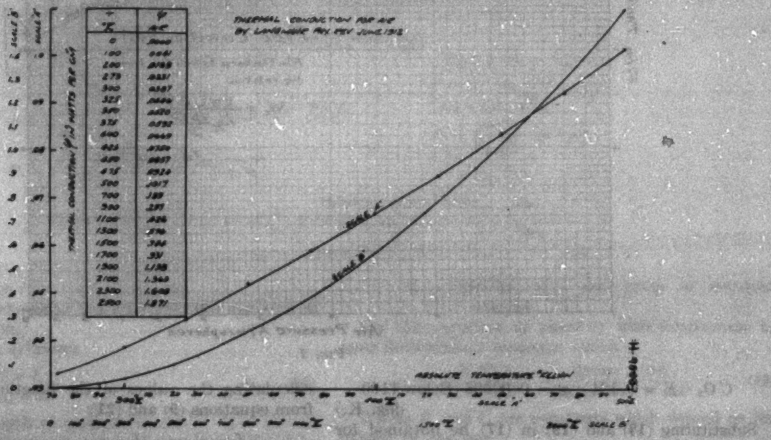

FIG. 9

(22) may be written with sufficient approximation as

$$\varphi = A\,(1 + 0.6\,\alpha\,T) \int_0^T \frac{T^{1/2}\,d\,T}{1 + C/T} \qquad (23)$$

From which we obtain

$$\varphi = A\,(1 + 0.6\,\alpha\,T)\,(2/3\,T^{3/2} - 2\,C\,T^{1/2} + 2\,C^{3/2}\tan^{-1}\sqrt{T/C}) \qquad (24)$$

The values of the constants taken and the corresponding values of φ have been tabulated for air, hydrogen, mercury vapor, nitrogen, carbon dioxide by Langmuir.[3d] His values for air, hydrogen and carbon dioxide have been used in the present work and have been reproduced here in Figs. 9, 10 and 11.

Instead of equation (17) we could use the more complete form given by Jeans.[7]

$$k = 1/4\,(9\,\gamma - 5)\,\mu\,c_v \text{ Cal. per sec. per cm. per deg. cent} \qquad (25)$$

where $\gamma = c_p/c_v$ ratio of specific heats.

For the variation of viscosity with temperature we could use the simpler expression given by Jeans[7] page 302.

$$\mu = \mu_0\,(T/273)^n \qquad (26)$$

Where μ_0 = viscosity at 0 deg. cent.

n = exponent depending upon the gas.

For air $n = 0.754$; H_2, $n = 0.69$; CO_2, $n = 0.98$.

If we now substitute equations (26) and (19) in (25) we obtain.

$$k = 1/4\,(9\,\gamma - 5)\,\mu_0\,(T/273)^n\,F\,(1 + \alpha\,T) \qquad (27)$$

If we place

$$N = 4.19 \times 1/4\,(9\,\gamma - 5) \times F \times (1/273)^n \times \mu_0$$

We obtain

$$k = N\,(1 + \alpha\,T)\,T^n \text{ watts per cm. per deg. cent.} \qquad (28)$$

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[7] Jeans. Dynamical Theory of Gases, Camb. Univ. Press, Second Edition, p. 318, 1916.

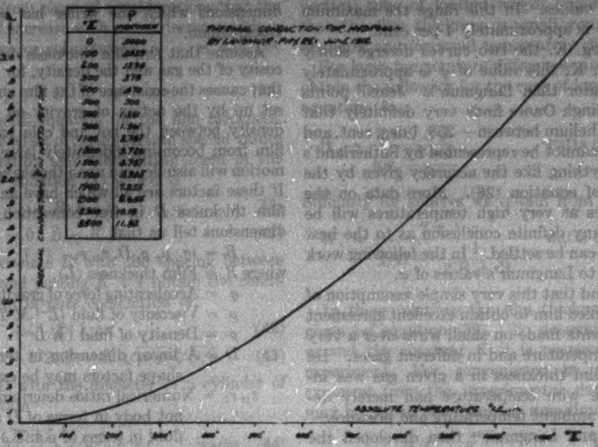

FIG. 10

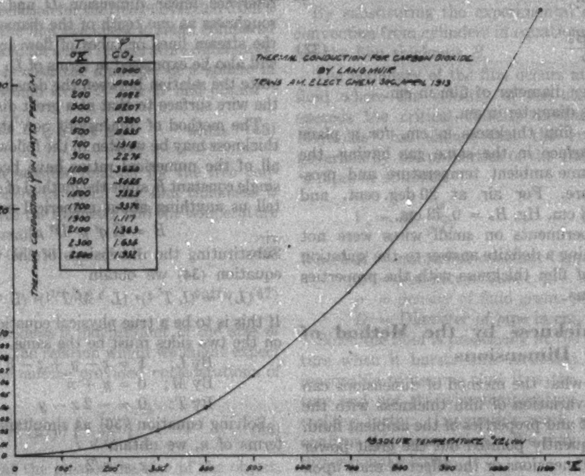

FIGURE 11

From which

$$\varphi = \int_0^T k\,d\,T = \frac{N}{n+1}\left(1 + \frac{n+1}{n+2}\,\alpha\,T\right) T^{n+1} \qquad (29)$$

For the purposes of approximation we may assume $n = 1$ in the parenthesis and obtain

$$\varphi = \frac{N}{n+1}\,(1 + 2/3\,\alpha\,T)\,T^{n+1} \qquad (30)$$

For air we have the following values

$\mu_0 = 0.0001705$ c. g. s. units
$F = 0.1614$
$\gamma = 1.40$
$n = 0.754$
$\alpha = 0.0002$

Substitute these values in eq. (29) we obtain for air

$$\varphi_{air} = 1.825 \times 10^{-6}\,(1 + .0001275\,T)\,T^{1.754} \qquad (31)$$

This curve is parallel with Langmuir's between 300 deg. *K.* and 1000 deg. K. and gives approximately 5 per cent higher values. In this range the maximum variation in $\Delta \varphi$ is approximately 4 per cent.

Above 1000 deg. K. the two curves diverge slowly and at 2500 deg. K. this value of φ is approximately 18 per cent greater than Langmuir's. Jeans[7] points out that Kamerlingh Onnes finds very definitely that the viscosity of helium between − 258.1 deg. cent. and 183.7 deg. cent. cannot be represented by Sutherland's formula with anything like the accuracy given by the simple relation of equation (**26**). More data on the viscosity of gases at very high temperatures will be required before any definite conclusion as to the best form of equation can be settled. In the following work we have adhered to Langmuir's values of φ.

Langmuir[3] found that this very simple assumption of a film theory enabled him to obtain excellent agreement with his experiments made on small wires over a very wide range in temperature and in different gases. He found that the film thickness in a given gas was independent of the wire temperature and merely depended upon the ambient temperature and pressure.

E. Q. Adams and Langmuir[3a] then developed the following relation to connect the film diameter with the wire diameter:

$$\log_e b/D = \frac{2\,B_o}{b} \qquad (\mathbf{32})$$

where b = outer diameter of film in cm.
D = wire diameter in cm.
B_o = The film thickness in cm. for a plane surface in the same gas having the same ambient temperature and pressure. For air at 20 deg. cent. and 76 cm. Hg, $B_o = 0.43$ cm.

Langmuir's experiments on small wires were not suitable for obtaining a definite answer to the question of the variation of film thickness with the properties of the ambient gas.

III. Film Thickness by the Method of Dimensions

Let us now see what the method of dimensions can tell us about the variation of film thickness with the shape of the object and properties of the ambient fluid. Rayleigh[8] has frequently pointed out the great power of the method of dimensions or the effect of scale upon physical phenomena, in obtaining the solution of physical problems. He says—"It often happens that simple reasoning founded upon this principle tells us nearly all that is to be learned from even a successful mathematical investigation, and in numerous cases where such a mathematical investigation is beyond our powers, the principle gives us information of the utmost importance."

The most complete discussions of the principle of dimensions which the writer has seen are those of Buckingham.[9]

Assume that the film thickness depends on the viscosity of the gas and the density, for it is the viscosity that causes the existence of the film and it is the motion set up by the action of gravity on the difference of density between the hot and cold gas that keeps the film from becoming indefinitely large. Obviously the motion will also depend upon the shape of the hot body. If these factors are all which have an influence on the film thickness B in free convection, the method of dimensions tell us that

$$B = (g, \mu, \rho, D, r_1, r_2 \ldots) \qquad (\mathbf{33})$$

where B = Film thickness (L)
g = Accelerating force of gravity ($L\,T^{-2}$)
μ = Viscosity of fluid ($L^{-1}\,M\,T^{-1}$)
ρ = Density of fluid ($M\,L^{-3}$)
D = A linear dimension in terms of which all shape factors may be expressed (L).
r_1, r_2 = Numerical ratios describing the form of the hot body in terms of D and condition of fluid in terms of μ and ρ.

In the case of a wire we might select the diameter as reference linear dimension D and then express the roughness as one tenth of the diameter. The form of the stream lines or tubes of flow around the cylinder will also be expressed in terms of D. Other ratios will state the relation between the density or viscosity near the wire surface to that at a great distance.

The method of dimensions now states that the film thickness may be written in the following form in which all of the numerical ratios have been lumped in the single constant K since the method of dimensions cannot tell us anything about numerical multipliers

$$B = K\,g^x\,\mu^y\,D^z\,\rho^n \qquad (\mathbf{34})$$

Substituting the dimensions of the various factors in equation (**34**) we obtain

$$(L)^1 = (L\,T^{-2})^x\,(L^{-1}\,M\,T^{-1})^y\,(L)^z\,(M\,L^{-3})^n \qquad (\mathbf{35})$$

If this is to be a true physical equation the dimensions on the two sides must be the same. Therefore

$$\left.\begin{array}{ll} \text{By } L; & 1 = x - y + z - 3\,n \\ \text{By } M; & 0 = y + n \\ \text{By } T; & 0 = -\,2\,x - y \end{array}\right\} \qquad (\mathbf{36})$$

Solving equation (**36**) as simultaneous equations in terms of n, we obtain

$$\left.\begin{array}{l} x = n/2 \\ y = -\,n \\ z = \dfrac{2 + 3\,n}{2} \end{array}\right\} \qquad (\mathbf{37})$$

Substitute these values of the exponents in (**34**) and obtain

$$B = K\,g^{n/2}\,\mu^{-n}\,D^{\frac{2+3n}{2}}\,\rho^n \qquad (\mathbf{38})$$

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[8] Nature, Vol. XXXII, p. 314, 1885 or *Sci. Papers*, Vol. II, p. 424, 1881-1887.

[9] E. Buckingham, *Journ. Wash. Acad. Sci.*, Vol. IV, No. 13, p. 347, July 1914.
Buckingham, *Phy. Rev. N. S.*, Vol. IV, No. 4, p. 345, Oct. 1914.
Buckingham, *Phil. Mag.* Vol. 42, No. 251, p. 696, Nov. 1921.

Since we know from physical considerations that the film thickness will increase with μ and decrease with ρ, n must be negative or substitute $n = -m$ and get

$$B = K\,(\mu/\rho)^m\,D^{\frac{2-3m}{2}}/g^{m/2}\ \text{cm.} \tag{39}$$

This is the general equation for the film thickness. Let us now see the effect of trying certain special values of m in this equation and substituting them in the general heat flow equation (1) which is

$$W = (A/B)\,k\,\Delta t \tag{40}$$

If $m = 0 \quad B = K\,D$ (41)

Here we may associate D with the spacing between parallel surfaces and putting $K = 1$ obtain the condition for a problem in pure conduction or,

$$W = (A/D)\,k\,\Delta t \tag{42}$$

If $m = 1/2, B = K\,\sqrt{\mu/\rho}\,\sqrt[4]{D/g}$ (43)

If we associate D with the diameter of a cylinder of length L we obtain

$$W = \frac{\pi\,g^{1/4}\,D^{3/4}\,L}{K}\sqrt{\rho/\mu}\,k\,\Delta t \quad \text{watts} \tag{44}$$

which is a possible relation for the free convection from a cylinder in which the film thickness is small compared with the diameter and for small temperature differences.

If we associate D with the diameter of a sphere in equation (43) we would write $A = \pi\,D^2$ and obtain

$$W = \frac{\pi\,g^{1/4}\,D^{7/4}}{K}\sqrt{\rho/\mu}\,k\,\Delta t \quad \text{watts} \tag{45}$$

Here we have a possible relation for the free convection from a sphere in which the film thickness is small compared with the diameter and the temperature differences are small.

If $m = 2/3, \; B = (K/g^{1/3})\,(\mu/\rho)^{2/3}$ (46)

$$W = \frac{g^{1/3}\,A}{K}\,(\rho/\mu)^{2/3}\,k\,\Delta t \quad \text{watts} \tag{47}$$

Here the film thickness is independent of the size of the body. This is the relation which we might expect to hold for a plane surface provided considerations of stability permit.

If $m = 1 \quad B = K\,(\mu/\rho)\left(\frac{1}{g\,D}\right)^{1/2}$ (48)

Here we see that the greater the size of the object, the thinner the film. This looks more like a "draught effect" than ordinary free convection. We might associate D with the diameter of a short vertical tube whose length is a constant C times the diameter. We would then have

$$A = \pi\,D \times C\,D \tag{49}$$

and obtain

$$W = \frac{\pi\,C\,g^{1/2}\,D^{5/2}}{K}\,(\rho/\mu)\,k\,\Delta t \tag{50}$$

This is a possible relation for the convection in a short vertical tube of limited length under the conditions that the film thickness is small compared with the diameter and for small temperature differences.

For large temperature differences and thick films we obtain the following for the case of long cylinders from equation (8)

$$W_c = \frac{2\,\pi\,L\,\Delta\,\varphi}{\log_e \frac{2B+D}{D}} \quad \text{watts convection} \tag{51}$$

Similarly for spheres we obtain from equation (16)

$$W_c = \frac{2\,\pi\,\Delta\,\varphi}{1/D - \frac{1}{2\,B + D}} \quad \text{watts convection} \tag{52}$$

In these relations the film thickness has the form given by equation (39). To determine the exponent m and the constant K it is necessary to appeal to experiment. For convenience in determining the exponent from the experimental data equation (39) may be put in the following non-dimensional form

$$B/D = K\left(\frac{\mu}{\rho\,g^{1/2}\,D^{3/2}}\right)^m \tag{53}$$

By substituting the experimental data for the free convection from cylinders in equation (51) we can solve for the film thickness B.

The boundary of the film occurs at the point in the fluid where the velocity of the stream line motion exceeds the critical velocity in the fluid. Osborne Reynolds[10] has shown that the critical velocity is governed by the properties of the fluid. For the case of pipes he obtained

$$V_c = M\,\frac{\mu}{\rho\,D} \quad \text{cm. per sec.} \tag{54}$$

Where M = numerical constant (approximately 2500).
μ = viscosity of fluid c. g. s. units
ρ = density of fluid grams per cu. cm.
D = Diameter of pipe in cm.

Since the fluid is practically at the ambient temperature when it bursts into turbulent motion, it seems reasonable to assume that it is the ambient temperature and not the wire temperature which is the controlling factor. Langmuir's work appears to amply confirm this assumption. We may now plot the non-dimensional relation

$$B/D \text{ vs. } \frac{\mu}{\rho\,g^{1/2}\,D^{3/2}} \tag{55}$$

on log log paper for all of the available data and should find that the points fall on a single curve. The exponent m for a certain range is the slope of the curve in the range considered. Having determined the exponent m the value of the constant K is determined from equation (53).

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[10] Osborne Reynolds, *Phil. Trans. Roy. Soc.*, Part III, p. 935, 1883 also *Phil. Trans. Roy. Soc. A* part I, p. 123, 1895.

IV. Experimental

FREE CONVECTION FROM LARGE HORIZONTAL CYLINDERS IN AIR

The following tests on the free convection from large horizontal cylinders were undertaken to extend the available data into the region where the film thickness is small compared with the cylinder diameter. Under this condition the law of variation of the film thickness can be most accurately tested. The cylinders used consisted of heavy wall, polished, silver plated copper tubes provided with internal electric heating units. The units were constructed by first winding two layers of mica paper tape on a brass tube of proper size and then winding it with nicrome ribbon, after which, two more layers of mica paper tape were applied. This brought the outside diameter up to a point where it was a snug fit into the copper tube. The spacing between the turns of the nicrome ribbon was approximately 1½ mm. Four nicrome-ideal thermocouples were provided to measure the cylinder temperature. The couples were silver soldered into small copper pipe plugs which were later screwed into holes which had been tapped in the copper tube. Three of the couples were situated along the top of the tube, one in the middle and one about 10 cm. from each end, the fourth was at the bottom of the cylinder in the middle. The dimensions of the cylinders were as follows:

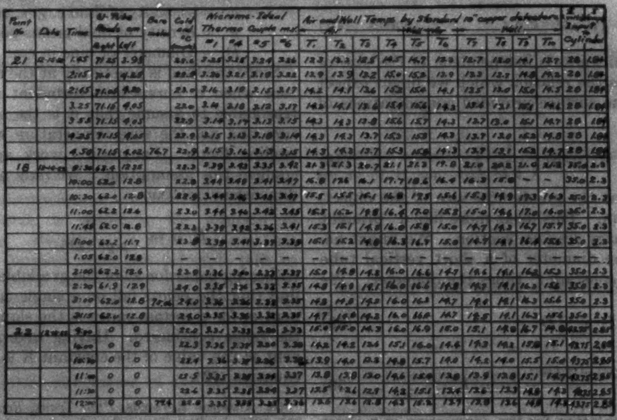
1. Large cylinder, 11.35 cm. outside diameter, with a wall thickness of 0.635 cm. and 152.5 cm. long.
2. Medium cylinder 5.56 cm. outside diameter with a wall thickness of 0.476 cm. and 122 cm. long.
3. Small cylinder, 4.28 cm. outside diameter with a wall thickness of 0.635 cm. and 122 cm. long.

FREE CONVECTION TESTS IN CASK USING AIR

The effect of reduced air pressure was studied by placing one of the cylinders a little below the axis of a large wooden cask arranged for evacuation. The cask was built of paraffin-treated wood and was 1.8 meters inside diameter by 2.1 meters long. The inside of the cask was lined on all sides, except the ends with water pipes for the purpose of controlling the ambient temperature. A heavy pressboard lining was placed inside of the cooling coils. The ambient temperature was measured by specially calibrated copper resistance detectors of the three lead type. The normal resistance of the detectors was approximately 10 ohms. A differential meter was used in connection with the detectors which read the temperature directly in deg. cent. The location of the detectors and cylinder in the cask is shown in Fig. 2. Fig. 3 is a photograph of the set-up. Power was supplied to the heater from a direct-current source through small bare copper wires (approx. 0.075 cm. diam.). A similar pair of small wires was provided for measuring the voltage drop across the heater. The ammeters and voltmeters used in the tests were carefully calibrated. The potential differences developed by the nicrome-ideal thermocouples on the cylinder were measured by a portable potentiometer. The pressure difference between the air in the cask and the outside was measured with a mercury *U*-tube. A standard barometer on the same floor of the building was used to reduce the pressures in the cask to absolute values.

Tests were made on the large and small cylinders in the cask. Table I is a sample data sheet from which an idea of the duration of the tests, variations in tem-

TABLE I
SAMPLE DATA SHEET
FREE CONVECTION TESTS IN CASK USING AIR
SMALL SILVER PLATED CYLINDER (4.28 CM. DIA. BY 122 CM. LONG)



Point No.	Date	Time	U-Tube Reads cm		Barometer	Cold end °C	Nicrome-Ideal Thermo Couple mv.				Air and Wall Temps by Standard 10Ω copper detectors										E	I
			Right	Left			#1	#4	#5	#6	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	Input to Cylinder	
21	12-15-[illegible]	1.45	71.25	3.95		22.6	3.25	3.25	3.24	3.26	12.3	12.3	12.5	14.5	14.7	12.3	12.7	12.0	14.1	12.7	28	1.04
		2:15	71.0	4.25		22.9	3.20	3.21	3.19	3.22	12.9	13.9	13.2	15.0	15.2	13.9	13.3	12.7	14.8	14.2	28	1.04
		2:45	71.05	4.20		23.0	3.16	3.10	3.15	3.17	14.2	14.1	13.6	15.2	15.4	14.1	13.5	13.0	15.0	14.5	28	1.04
		3.25	71.15	4.05		22.0	3.14	3.10	3.12	3.17	14.2	14.1	13.6	15.4	15.6	14.3	13.6	13.1	15.1	14.6	28	1.04
		3.55	71.15	4.05		22.9	3.14	3.14	3.13	3.15	14.3	14.3	13.8	15.6	15.7	14.3	12.7	13.0	15.1	14.7	28	1.04
		4.25	71.15	4.05		22.9	3.15	3.13	3.18	3.14	14.3	14.3	13.7	15.3	15.3	14.3	13.7	13.0	15.2	14.8	28	1.04
		4.56	71.15	4.02	76.7	22.9	3.15	3.16	3.13	3.15	14.3	14.3	13.7	15.3	15.6	14.3	13.7	13.1	15.2	14.7	28	1.04
18	12-14-[illegible]	9:30	63.4	12.35		22.3	3.39	3.42	3.35	3.42	21.3	21.3	20.7	21.1	21.3	19.8	21.0	20.2	21.0	21.2	35.0	2.3
		10:00	62.0	12.8		22.8	3.44	3.49	3.41	3.47	16.8	17.6	16.1	17.7	18.6	16.4	16.3	15.8	–	–	35.0	2.3
		10:30	62.0	12.8		22.9	3.44	3.46	3.48	3.47	15.8	15.5	16.1	16.8	17.5	15.6	15.3	14.9	17.3	16.3	35.0	2.3
		11.00	62.2	12.6		23.0	3.44	3.46	3.42	3.45	16.5	16.2	14.8	16.4	17.0	15.2	15.0	14.6	17.0	16.0	35.0	2.3
		11:45	62.0	12.6		23.3	3.39	3.42	3.36	3.41	15.3	15.1	14.5	16.0	15.8	15.0	14.7	14.3	16.7	15.7	35.0	2.3
		1:00	63.2	11.7		23.8	3.39	3.41	3.37	3.39	16.1	15.2	14.8	16.3	16.7	15.0	14.9	14.1	16.4	15.6	35.0	2.3
		1.05	62.0	12.8		–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
		2:00	62.2	12.6		23.9	3.36	3.40	3.37	3.37	15.0	14.8	14.8	16.0	16.6	14.7	14.6	14.1	16.2	15.3	35.0	2.3
		2:30	61.9	12.9		24.0	3.35	3.36	3.32	3.35	14.8	14.8	14.1	16.0	16.6	14.8	14.7	14.1	16.3	15.6	35.0	2.3
		3:00	62.0	12.8	75.06	24.0	3.36	3.36	3.32	3.35	14.8	14.8	14.2	16.0	16.3	14.7	14.6	14.1	16.3	15.6	35.0	2.3
		3:15	62.0	12.8		24.0	3.35	3.36	3.32	3.35	14.7	14.8	14.2	16.0	16.6	14.7	14.5	14.1	16.3	15.6	35.0	2.3
22	12-[illegible]	9:30	0	0		22.0	3.31	3.33	3.30	3.33	15.4	15.0	14.3	16.0	16.9	16.0	15.1	14.8	16.7	14.8	43.75	2.85
		10:00	0	0		22.3	3.36	3.37	3.30	3.30	14.2	14.2	13.6	15.1	16.0	14.6	14.3	14.2	15.8	15.1	43.75	2.85
		10:30	0	0		22.4	3.36	3.37	3.26	3.30	13.9	14.0	12.8	14.8	15.7	14.0	14.2	14.0	15.5	15.0	43.75	2.85
		11:00	0	0		22.5	3.35	3.28	3.24	3.27	13.8	13.8	13.0	14.6	15.8	13.8	13.9	12.8	15.1	14.7	43.75	2.85
		11:30	0	0		22.6	3.35	3.38	3.24	3.27	12.5	12.6	12.9	14.2	15.1	12.6	12.6	12.3	14.8	14.3	43.75	2.85
		12:00	0	0	77.4	22.8	3.35	3.38	3.25	3.36	12.6	12.6	12.8	14.3	15.2	12.7	12.8	12.6	14.8	14.4	43.75	2.85

perature, etc. can be judged. Table II is a summary of the points obtained with the large cylinder in the cask. The average temperatures are here recorded. The radiation correction was calculated from the Stefan-Boltzman law which may be written[30]

$$W_r = 5.7 A\, E\, [\,(T/1000)^4 - (T_0/1000)^4\,] \text{ watts} \quad (56)$$

Where 5.7 = Black body radiation constant
A = Area of radiating surface in sq. cm.
E = Relative emissivity of surface
T = Absolute temperature of hot body deg. K
T_0 = Absolute temperature of surroundings deg. K.

An emissivity of three per cent was assumed for the polished silver surface of the cylinders. This is the mean value given by Langmuir[31] between 325 deg. K and 900 deg. K.

Table III is a summary of the data obtained with the small cylinder in the cask.

TABLE II

SUMMARY OF TESTS ON LARGE CYLINDER (11.42 CM DIA BY 152.5 CM LONG) IN CASK USING AIR

Point No	Atm. Pres. cm Hg Mercury	Atm. Pres. in Atmos's	Cylinder Temp °C	Amb Temp °C	Wall Temp °C	ΔT of Cyl °C above Amb	W watts total input	W_r watts radiation estimate	W_c watts convect. $W-W_r$	$\frac{W_c}{\Delta T}$	$\frac{W_c}{\Delta T L}$ L length °152.5 cm
8	8.5	.112	65.3	12.2	11.8	53.1	87.3	6.06	81.2	1.53	.01000
9	8.65	.1136	50.7	12.6	12.6	38.1	55.8	4.0	51.8	1.36	.00892
5	8.5	.112	98.0	12.00	13.6	85.0	130	12.0	118	1.39	.00912
10	8.65	.1136	91.0	12.0	12.3	79.0	131	10.0	121	1.53	0.1002
11	26.27	.346	96.5	13.3	14.6	83.2	202.5	11.1	191.4	2.30	.01508
12	26.5	.348	98.4	13.0	14.5	85.4	209.5	11.5	198.0	2.32	.0151
1	74.8	.985	100.4	13.7	15.7	86.7	330	12.0	318	3.67	.02405
2	74.4	.978	99.2	9.94	11.84	89.26	330	12.0	318	3.56	.0233
4	75.6	.995	95.2	11.2	13.2	84.0	334	11.0	323	3.89	.0255
13	75.7	.995	100.0	11.8	13.8	88.2	347	12.0	335	3.80	.0249

TABLE III

SUMMARY OF TESTS ON SMALL CYLINDER (4.26 CM DIA. BY 122 CM. LONG.) IN CASK USING AIR

Point No (Test)	Atm. Pres. cm Hg Mercury	Atm. Pres. in Atmos's	Temp Cylinder °C	Amb Temp °C	Wall Temp °C	ΔT above amb.	W total watts input	W_r watts radiation estimate	W_c watts conv. $W-W_r$	$\frac{W_c}{\Delta T}$	$\frac{W_c}{\Delta T L}$ L = length = 122 cm
39	75.84	.998	231	12.8	14.3	218.1	423	16.2	406.8	1.86	.01522
38	9.3	.122	182.2	15.3	15.7	166.9	129	10.1	118.0	.713	.00583
37	26.9	.354	188.3	13.3	14.5	175.0	205	10.5	194.5	1.11	.00908
36	76.4	1.005	187.5	14.2	15.5	173.3	310	10.50	299.5	1.73	.01416
avgs			186.0	14.26							
35	7.69	.101	142.0	14.1	14.1	127.9	87.7	6.49	81.21	.635	.0052
34	8.84	.116	144.2	15.1	15.0	129.1	87.69	6.49	81.2	.629	.00515
33	27.05	.356	139.2	13.8	14.5	125.4	131.37	6.27	125.1	1.00	.00819
32	27.15	.357	139.2	14.9	15.4	124.3	131.37	6.27	125.1	1.006	.00823
31	75.9	1.0	140.3	13.4	14.4	126.9	202.68	6.27	196.4	1.55	.0127
30	75.8	1.0	148.0	13.3	14.5	134.7	215.76	6.92	208.84	1.55	.0127
avgs			142.15	14.1							
19	8.15	.107	97.5	13.2	13.2	84.3	51.52	3.3	48.20	.572	.00469
21	9.57	.126	92.7	13.5	12.6	79.2	51.52	3.16	48.36	.610	.00499
20	9.1	.12	93.7	13.3	13.3	80.4	51.52	3.16	48.36	.602	.00493
17	25.2	.332	97.2	13.4	13.9	83.8	80.5	3.3	77.2	.925	.00757
18	25.86	.340	97.9	13.3	14.4	84.6	80.5	3.3	77.2	.920	.00753
22	77.4	1.017	96.8	12.6	14.4	84.2	124.5	3.3	121.2	1.44	.0118
16	74.15	.975	97.9	11.04	11.9	86.86	124.5	3.4	121.1	1.395	.01143
14	75.04	.988	98.3	15.3	16.2	83.0	124.5	3.4	121.1	1.46	.01196
avgs			96.5	13.26							
29	8.35	.11	59.4	12.4	12.0	47.0	26.4	1.5	24.9	.53	.00433
28	9.53	.1252	53.8	11.2	10.0	42.6	23.94	1.31	22.63	.532	.00436
26	26.6	.35	59.2	14.0	13.8	45.2	38.16	1.48	36.7	.812	.00665
25	27.35	.36	57.1	12.95	13.3	44.15	38.4	1.48	36.92	.837	.00685
23	77.2	1.016	57.6	11.50	12.1	46.1	60.0	1.48	58.52	1.27	.0104
15	74.08	.975	58.3	10.7	11.1	47.6	60.0	1.5	58.50	1.23	.01007
avgs			57.57	12.125							

FREE CONVECTION TESTS IN OVEN USING AIR

The effect of ambient temperature on free convection was studied by placing the medium cylinder in a large oven which was provided with heating units at the top and bottom. The units could be operated together or separately. Fig. 4 is a skeleton view of the oven set up and Fig. 5 is a photograph of one of the ovens. Preliminary tests showed that large convection currents were set up in the oven which vitiated the results, when both heating units were operated. Subsequently all tests were made using the top unit only. Another point observed in the preliminary tests was that the radiation from the heating unit seriously affected the readings of the copper wire temperature detectors. The writer therefore, resorted to a network of 0.010 in. diam. (0.0254 cm.) bare polished silver wire for the purpose of determining the ambient air temperature. One net work was placed about 20 cm. below, and another 20 cm. above the cylinder. A third on the level with the cylinder passed alternately over and under the cylinder. The silver wire was supported at the ends and in the middle on asbestos strings. Table IV is a sample record sheet. The resistance of the silver wire detectors was measured by a portable (Leeds and Northrup) bridge which was checked against a standard. The resistance of the detectors at 0 deg. cent. was determined from the relation

$$R_0 = \frac{R_1}{1 + \alpha\, t_1} \text{ ohms} \quad (57)$$

where R_1 = observed resistance of detector at temperature t_1 deg. cent.
α = 0.00344 the temperature coefficient of a sample of the silver wire determined experimentally.

In determining the R_0 the oven was left closed over night without heat and the ambient temperature t_1 determined as the average of the ten copper resistance detectors. The temperature for another value of resistance was then obtained from the relation

$$t = \frac{R_t - R_0}{\alpha\, R_0} \text{ deg. cent.} \quad (58)$$

Table V is a summary of the R_0 determinations. Table VI is a summary of the data obtained in the oven.

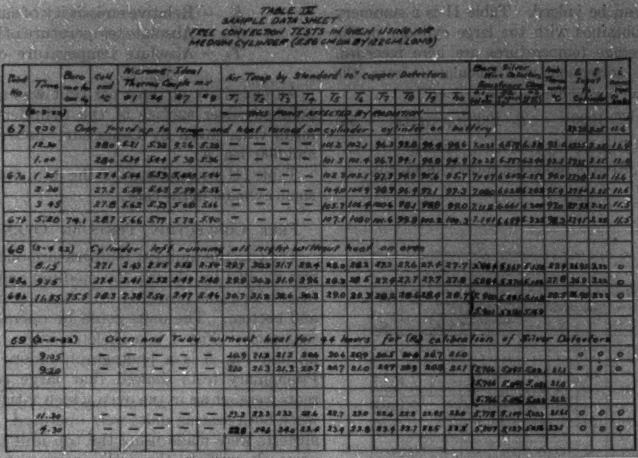


TABLE III
SAMPLE DATA SHEET
FREE CONVECTION TESTS IN OVEN USING AIR
MEDIUM CYLINDER (2.56 CM DIA BY 122 CM LONG)

Point No.	Time	Barometer cm Hg	Cold end °C	Nichrome-Ideal Thermo Couple mv #1	#4	#7	#9	Air Temp by Standard 10" Copper Detectors T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	Bare Silver Wire Detectors Resistance Ohms #1	#2	#3	Amb. Thermometer °C	E Input to Cylinder	I	i
	(8-3-32)							— THIS POINT AFFECTED BY RADIATION —																
67	9.30	Oven forced up to temp. and heat turned on cylinder. cylinder on battery																				27.30	3.25	11.6
	12.30		28.0	5.21	5.32	5.26	5.28	—	—	—	—	101.2	102.1	96.3	93.8	94.9	94.6	7.033	6.57[illegible]	6.28[illegible]	93.6	27.25	3.25	11.6
	1.00		28.0	5.34	5.44	5.38	5.36	—	—	—	—	101.5	102.4	96.7	94.1	94.8	94.9	7.035	6.557	6.246	93.3	27.25	3.25	11.6
67a	1.35		27.4	5.44	5.53	5.485	5.44	—	—	—	—	102.3	103.1	97.3	94.9	95.6	95.7	7.047	6.602	6.252	94.0	27.30	3.25	11.6
	2.30		27.2	5.58	5.65	5.59	5.58	—	—	—	—	104.0	104.9	98.9	96.4	97.1	97.3	7.080	6.626	6.263	95.4	27.44	3.25	11.6
	3.45		27.8	5.62	5.73	5.68	5.66	—	—	—	—	105.7	106.4	100.6	98.1	98.8	99.0	7.112	6.661	6.300	97.0	27.35	3.25	11.5
67b	5.20	74.1	28.7	5.66	5.77	5.78	5.90	—	—	—	—	107.1	108.0	101.6	99.8	100.2	100.3	7.141	6.689	6.328	98.3	27.45	3.25	11.5
68	(8-4-32)	Cylinder left running all night without heat on oven																						
	8.15		27.1	2.43	2.55	2.52	2.50	29.7	30.3	31.7	29.4	28.0	28.2	27.3	27.6	27.4	27.7	5.88[illegible]	5.267	5.15[illegible]	27.9	26.95	3.22	0
68a	9.45		27.4	2.41	2.52	2.49	2.48	29.9	30.3	31.9	29.6	28.3	28.5	27.4	27.7	27.7	27.8	5.884	5.270	5.15[illegible]	27.8	26.9	3.22	0
68b	16.55	75.5	28.3	2.38	2.54	2.47	2.46	30.7	31.2	32.6	30.3	29.0	29.3	28.2	28.6	28.4	28.7	5.900	5.285	5.168	28.5	26.95	3.22	0
																		5.901	5.286	5.169				
69	(8-6-32)	Oven and Tube without heat for 44 hours for (R) calibration of Silver Detectors																						
	9.05		—	—	—	—	—	20.9	21.3	21.2	20.6	20.6	20.9	20.5	20.6	20.7	21.0					0	0	0
	9.20		—	—	—	—	—	21.0	21.3	21.3	20.7	20.7	21.0	20.7	20.9	20.8	21.1	5.766	5.095	5.0[illegible]	21.1	0	0	0
																		5.766	5.0[illegible]	5.0[illegible]	21.4			
																		5.766	5.0[illegible]	5.0[illegible]	21.2			
	11.30		—	—	—	—	—	23.3	23.3	23.3	22.6	22.7	23.0	22.4	22.9	22.95	23.0	5.778	5.104	5.033	21.65	0	0	0
	4.30		—	—	—	—	—	22.8	24.6	24.0	23.4	23.4	23.8	23.4	23.7	23.6	23.8	5.807	5.127	5.056	23.5	0	0	0

V. Analysis of Experiments

Effect of Cylinder Temperature

In Fig. 6 the watts convection for the small cylinder have been plotted against the corresponding values of $\Delta\varphi$. Here the ambient temperatures are approximately constant and therefore if the film thickness is independent of cylinder temperature a linear relation between W_c and $\Delta\varphi$ is to be expected from equation (**51**). Fig. 6 shows this to be the case for the three low temperature points. The high temperature points show too large values of W_c for the values of $\Delta\varphi$. Fearing trouble from oxidation all the low temperature points were taken first and the high temperature points last. After the test was completed and the cylinder inspected, it was found to be badly tarnished. This undoubtedly greatly increased the radiation which accounts for the higher points being considerably off the curves.

Effect of Air Density

To determine the relation between air density and free convection the watts convection per cm. of length per deg. cent. temperature difference have been plotted against the air pressure on log log paper in Fig. 7. If equation (**44**) applied strictly the convection loss would be proportional to the square root of the air density, since from the kinetic theory of gases, we know that both the heat conductivity and viscosity are independent of density. For small temperature differences the appropriate value of heat conductivity may be taken as the average value of the heat conductivity for the fluid at the cylinder temperature and the ambient temperature, since for small temperature differences we have

$$k_{avg}\,\Delta t = \Delta\varphi \qquad (\mathbf{59})$$

Fig. 7 shows that equation (**44**) is approximately fulfilled. To see the variation of the film thickness with the air density the small cylinder points have been calculated from equation (**51**) assuming the film thickness determined by the air density for the ambient temperature. The straight lines in Fig. 7 drawn to represent the mean values of B as function of air density are square root lines. These lines are not the best average lines, but do not depart more than about 10 per cent from the worst points.

Effect of Ambient Temperature

In Fig. 8 the watts convection per deg. cent. for the high and low temperature points obtained from the oven tests have been plotted against the ambient air temperature. We see from this that the convection increases with the ambient temperature. The equation for the film thickness on the assumption that $m = 1/2$ is

$$B = K\sqrt{\mu/\rho}\,\sqrt[4]{D/g} \qquad (\mathbf{60})$$

The density of a gas as function of the absolute temperature is

$$\rho = \rho_0\,(273/T) \quad \text{grams per cu. cm.} \qquad (\mathbf{61})$$

where ρ_0 = Density of gas at 0 deg. cent. and 76 cm. of mercury in grams per cu. cm.

T = Absolute temperature deg. K.

The viscosity is given by equation (**26**) with sufficient accuracy as

$$\mu = \mu_0\,(T/273)^n \quad \text{c. g. s. units} \qquad (\mathbf{62})$$

where μ_0 = viscosity of gas at 0 deg. cent. in c. g. s. units

T = Absolute temperature of gas deg. K.

n = Exponent depending on the gas.

From the above relations we obtain

$$\sqrt{\mu/\rho} = \sqrt{\mu_0/\rho_0}\,(T/273)^{\frac{n+1}{2}} \qquad (\mathbf{63})$$

Using the following values for air,

$\mu_0 = 0.0001705$ c. g. s.

$\rho_0 = 0.00129$ grams per cu. cm. at 76 cm. Hg.

$n = 0.754$

we obtain *for air*

$$\sqrt{\mu/\rho} = 0.364\ (T/273)^{0.878} \qquad (64)$$

From the oven tests we have for the average ambient temperatures for the high and low points the following:

High temperature, $T = 103.2 + 273 = 376.2$ deg. K.

Low temperature, $T = 28.8 + 273 = 301.8$ deg. K.

By equation (**60**) and (**64**) the ratio of the film thickness as determined by the ambient temperature will be

Calc. $B_{103 deg. cent.}/B_{29 deg. cent.}$

$$= (376.2/301.8)^{0.878} = 1.21 \qquad (65)$$

We may now compare this with the values of B determined from the experiments as calculated from equation (**51**).

For the low temperature point we have, from Fig. 9.

Cylinder 82.3 deg. cent. = 355.3 deg. K. $\varphi_2 = 0.053$

Ambient 28.8 deg. cent. = 301.8 deg. K. $\varphi_1 = 0.039$

$W c = 83.7$ watts $\qquad \Delta\varphi = 0.014$

$$\log. \frac{2B + D}{D} = \frac{2\pi L \Delta\varphi}{W c}$$

$$= \frac{2\pi \times 122 \times 0.014}{83.7} = 0.128$$

The anti-log then gives

$$\frac{2B + D}{D} = 1.137$$

and $\qquad B_{29 deg. cent.} = 0.37 \qquad (66)$

For the high temperature points we have

Cylinder 150.4 deg. cent. = 423.4 deg. K. $\varphi_2 = 0.0747$

Ambient 103.2 deg. cent. = 376.2 deg. K. $\varphi_1 = 0.0152$

$W_c = 85.0 \qquad \Delta\varphi = 0.0152$

from which we obtain

$$B_{103 deg. cent.} = 0.41 \text{ cm.} \qquad (67)$$

This gives as the experimental ratio

Exp. $\qquad B_{103 deg. cent.}/B_{29 deg. cent.} = 1.11 \qquad (68)$

This value shows the right trend but is 9 per cent too low. The check, however, is considered satisfactory, considering the accuracy of the experiments.

VI. Summary of Available Data on Free Convection from Cylinders

Gases

The data used have been converted into watts convection per cm. length of cylinder. A radiation correction has been made where necessary, using the values of emissivity given by Langmuir[3a]. The watts convection have then been plotted against $\Delta\varphi$ as in Fig. 6. By this means we obtain an average of all the points for a given wire size having the same ambient conditions. The values of watts convection for a given value of $\Delta\varphi$ read off the curves are those given in Table VII. Langmuir's values of φ as given in Figs. 9, 10 and 11 have been used.

TABLE V

THREE DETERMINATIONS OF R_0

POINT NO	BARE SILVER WIRE DETECTOR OHMS CORRECTED FOR LEADS			AVG. OF DETECTOR READINGS CORRECTED °C	CALCULATED VALUE OF R_0 OHMS, α = .00386		
	#1 20 CM BELOW	#2 20 CM ABOVE	#3 LEVEL OF CYL.		#1 20 CM BELOW	#2 20 CM ABOVE	#3 LEVEL OF CYL.
49	5.609	4.976	4.902	20.16	5.301	4.652	4.584
70	5.776	5.069	4.995	20.13	5.299	4.651	4.583
71	5.717	5.013	4.943	22.86	5.300	4.648	4.583
AVG					5.300	4.651	4.583

The data by Langmuir[3a] were taken in practically open air, using small platinum wires. Wire temperatures up to 1600 deg. cent. were used with an ambient temperature of 27 deg. cent.

Ayrton and Kilgour[11] worked with small platinum wires in air at temperatures up to 300 deg. cent. with an ambient temperature in the vicinity of 10 deg. cent. In these experiments the wire was placed in the axis of a water jacketed cylinder having an internal diameter of 5.08 cm. diameter.

Petavel[12] worked with a platinum wire 0.1106 cm. diameter in the axis of a water cooled cylinder 2.06 cm. diameter. Wire temperatures up to 1100 deg. cent. were used with an ambient temperature of 16 deg. cent. His tests ranged from a pressure of 1/10 to 100 atmospheres in air, oxygen, hydrogen, carbon dioxide, nitrous oxide and some preliminary tests in liquid carbon dioxide and nitrous oxide.

Kennelly[13] worked with small copper wires in a large tank with air pressures ranging from approximately ½ to 2¼ atmospheres. The wire temperatures were around 180 deg. cent. with an ambient of approximately 20 deg. cent. His data as studied and corrected for radiation by Langmuir[3a] have been taken.

Only a portion of the data by the various investigators has been worked out, but we believe that we have obtained a representative selection.

The collected data have been plotted in the non-dimensional form of equation (**55**) in Fig. 12, and are seen to lie on a single curve on log log paper within the probable experimental errors. The single straight line which seems best to fit the points has a slope of $m = 0.54$. The value of K found by averaging the K's for all of the points as calculated from equation (**53**) assuming $m = 0.54$ is $K = 4.4$. The line of slope $m = 1/2$ has also been drawn in such a manner

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[11] Ayrton & Kilgour, *Phil. Trans. Roy. Soc. A*, CIXXXIII, p. 371, 1892.

[12] Petavel, *Phil. Trans. Roy. Soc. A.*, Vol. 197, p. 229, 1901.

[13] A. E. Kennelly, TRANS. A. I. E. E., XXVIII, (1), p. 363, 1909.

TABLE III

SUMMARY OF TESTS IN OVEN USING AIR
MEDIUM CYLINDER (5.56 CM DIA BY 122 CM LONG)

Point No. (Test)	Ab Pressure cm Hg. Mercury	Ab Pressure in Atmos	Cylinder Temp °C	Ambient Temp by Silver Wire Detector #3 in Level of Tube °C	Ambient Temp by Silver Wire Detector #2 (6" above) Tube °C	Ambient Temp by Silver Wire Detector #1 (6" below) Tube °C	Res in Ohms of Silver Wire Detector #3 Corrected for Leads	Res in Ohms #2	Res in Ohms #1	Floor Temp °C by Couple	Wall Temp °C by Couple	Ceiling Temp °C by Couple	Heater Wire Temp °C by Couple	ΔT °C of Cylinder above Temp at Level of Cylinder	W Total Watts Input to Cylinder	W_R Watts Radiated calculated	W_c Watts Convection $W-W_R$	$\frac{W_c}{\Delta T}$	$\frac{W_c}{\Delta T L}$ (L in cm)
60	76.9	1.013	80.0	26.85	29.2	24.6	5.006	5.119	5.731					53.15	86.0	2.9	83.1	1.56	.0128
63	77.14	1.014	82.5	28.9	32.4	28.0	5.034	5.169	5.810					53.6	86.6	2.7	83.9	1.565	.01282
68a	75.5	.994	82.0	28.8	31.1	26.75	5.026	5.149	5.787					53.2	86.6	2.8	83.8	1.58	.01295
68b	75.5	.994	82.5	28.6	32.1	27.7	5.049	5.165	5.806					53.9	86.8	2.7	84.1	1.59	.01303
Avg			82.3	28.8													83.7	1.58	
64	75.95	.999	96.0	43.7	51.6	38.7	5.272	5.475	6.005					52.3	88.6	3.3	85.3	1.61	.0132
65	73.9	.973	97.0	44.3	52.3	39.9	5.280	5.488	6.026					52.7	88.6	3.3	85.3	1.62	.01328
Avg			96.5	44.0													85.3	1.62	
61	76.7	1.010	149.5	104.0	118.2	94.0	6.281	6.593	7.021					45.5	88.0	4.0	84.0	1.85	.01517
67a	74.1	.975	146.0	98.5	114.4	90.7	6.133	6.482	6.950					47.5	89.0	4.0	85.0	1.79	.01467
67b	74.1	.975	151.0	103.6	117.7	95.8	6.218	6.565	7.044					47.4	89.3	4.0	85.3	1.80	.01475
72a	76.3	1.003	153.5	105.6	120.0	98.1	6.250	6.606	7.085	76.0	86.5	1080	3350	47.9	89.3	4.0	85.3	1.78	.0146
72b	76.3	1.003	152.0	104.3	120.5	96.6	6.226	6.576	7.058					47.7	89.5	4.0	85.5	1.79	.01468
Avg			152.8	104.8													85.0	1.80	

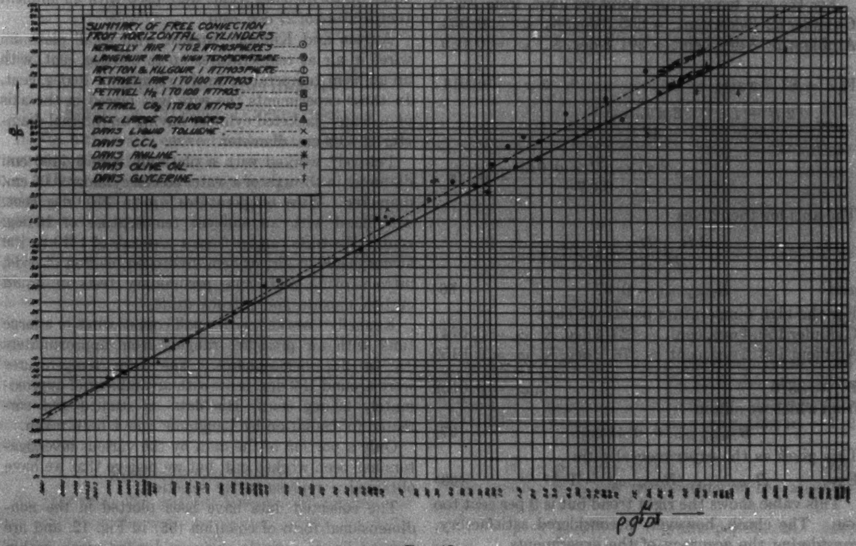

FIG. 12

as to give most weight to the first 13 points because the accuracy of determining B/D is greatest for the small values, assuming equal accuracy in determining the watts convection.

To see the error involved in using the simple relation $m = 1/2$, we have for Langmuir's highest point

$$\text{Obs.}\quad B/D = 27.4;\ \frac{\mu}{\rho\, g^{1/2}\, D^{3/2}} = 19.9 \tag{69}$$

$$\text{Obs.}\quad W_c = 0.84;\quad \Delta\varphi = 0.537;\ D = 0.00404 \text{ cm.} \tag{70}$$

For the same ambient condition on the square root curve, we have

$$B/D = 15.0;\ \frac{\mu}{\rho\, g^{1/2}\, D^{3/2}} = 19.9 \tag{71}$$

$$\Delta\varphi = 0.537 \quad D = 0.00404 \text{ cm.}$$

$$W_c = \frac{2\,\pi\, L\,\Delta\varphi}{\log_e \dfrac{2\,B + D}{D}} = \frac{2\,\pi \times 1 \times 0.537}{\log_e 31.0}$$

$$= 0.985 \text{ watts} \tag{72}$$

Thus, the watts convection which would be obtained by using the simple square root relation is only 17 per cent higher than Langmuir's observed value.

If we now take Petavel's point for air at 100 atmospheres pressure which will be seen to lie about as far off the curve as any in the low range, we have

$$\text{Obs.}\quad B/D = 0.142;\ \frac{\mu}{\rho\, g^{1/2}\, D^{3/2}} = 0.0013 \qquad (73)$$

$$\text{Obs.}\quad W_c = 11.3;\ \Delta\varphi = 0.450;\ D = 0.1106\ \text{cm.} \qquad (74)$$

And for the same ambient condition on the square root curve

$$B/D = 0.120;\ \frac{\mu}{\rho\, g^{1/2}\, D^{3/2}} = 0.0013 \qquad (75)$$

$$\Delta\varphi = 0.450;\ D = 0.1106\ \text{cm.}$$

$$W_c = \frac{2\pi L \Delta\varphi}{\log_e \dfrac{2B + D}{D}} = \frac{2\pi \times 1 \times .45}{\log_e 1.24}$$

$$= 13.15\ \text{watts} \qquad (76)$$

For this point the calculated value of watts convection is 16 per cent higher than Petavel's observation which is probably not far from the experimental error.

The value of the constant K, assuming $m = 1/2$, has been taken as $K = 3.4$ which is the average value for the first 13 points.

We may conclude that the simple square root relation can be used over the entire range of available data without exceeding the probable experimental errors, and is therefore preferable to a fractional exponent until such is definitely required by more precise data.

LIQUIDS

A few tests were made, in a light petroleum oil (No. 12 Transil Oil), on the free convection from a horizontal grid of 0.010 in. diam. (0.0254 cm.) silver wires. The spacing between the wires forming the grid was approximately 2.5 cm. and the total length of wire used was 1560 cm. The grid was submerged to a depth of 15 cm. in a large rectangular oil bath leaving 15 cm. of oil between the grid and the bottom of the

TABLE VII

SUMMARY OF COLLECTED DATA ON FREE CONVECTION FROM HORIZONTAL CYLINDERS

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
SAMPLE AND GAS	CYL. DIAM. CM.	ABS. GAS PRESS. ATM.	AMB. TEMP. °C	CYL. TEMP. °C	CONV. WATTS PER CM. LENGTH	Δφ WATTS PER CM.	b CM.	B CM.	$\frac{B}{D}$	$\frac{\mu}{\rho}$ C.G.S.	$\frac{\mu}{(\rho \sqrt{g} D)}$ C.G.S.	[illegible] X m=½	[illegible] X m=[illegible]
KENNELLY	.0114	.987	20°	185°	.096	.050	.279	.1338	11.70	.165	4.08	5.80	5.48
AIR	.0114	2.22	20°	185°	.112	.050	.188	.088	7.73	.069	1.048	5.73	5.60
	.0631	.987	20°	185°	.184	.050	.374	.1425	2.21	.165	.272	4.20	4.46
●	.0631	2.22	20°	185°	.233	.050	.258	.0865	1.37	.069	.121	3.94	4.28
LANGMUIR	.00049	1.0	27°	1027	.89	.537	.226	.1105	224	.160	12.90	6.15	5.48
AIR	.00691	1.0	27°	1027	.97	.537	.224	.1085	15.7	.160	8.31	5.27	4.82
	.0251	1.0	27°	1027	1.30	.537	.336	.156	6.22	.160	1.89	5.08	5.44
●	.051	1.0	27°	1027	1.71	.537	.368	.159	3.12	.160	.443	4.68	4.85
AYRTON AND KILGOUR	.0031	1.0	10.5	298	.164	.096	.1134	.0551	17.75	.163	26.2	3.47	3.06
	.0051	1.0	13.0	301	.192	.096	.110	.0530	10.40	.165	12.9	2.90	2.66
AIR	.0162	1.0	12.0	300	.220	.096	.220	.1044	6.48	.166	2.4	4.43	4.23
	.0206	1.0	12.3	300	.266	.096	.194	.0865	4.20	.166	1.8	3.39	3.36
◐	.0236	1.0	15.9	306	.290	.096	.250	.1132	4.80	.168	1.3	4.27	4.22
	.0456	1.0	14.5	302	.307	.096	.244	.1042	2.33	.167	.63	3.54	3.56
PETAVEL	.1106	100	16.	916	11.30	.450	.142	.0157	.142	.00169	.0013	3.96	5.16
AIR	.1106	10	16.	916	4.34	.450	.212	.0507	.458	.0169	.0130	4.02	4.76
■	.1106	1	16.	916	2.03	.450	.446	.1077	1.015	.169	.130	4.20	4.58
PETAVEL	.1106	100	16.	916	22.4	2.8	.201	.0452	.409	.011	.00357	4.17	5.02
HYDROGEN	.1106	10	16.	916	12.5	2.8	.452	.1707	1.56	.11	.0357	5.00	5.63
■	.1106	1	16.	916	7.7	2.8	1.09	.490	4.44	1.1	.357	4.53	4.55
PETAVEL	.1106	34.8	16.	616	5.1	.202	.1615	.0165	.161	.00226	.00196	3.16	4.05
CO_2	.1106	10.2	16.	616	2.7	.202	.177	.0332	.301	.0077	.0067	3.68	4.52
⊟	.1106	.99	16.	616	1.3	.202	.236	.0917	.83	.0783	.069	3.16	3.52
RICE	4.28	1.0	16.0	142	1.66	.037	4.92	.32	.075	.140	.00053	3.27	4.42
AIR	4.28	.35	14.0	142	1.08	.037	5.32	.52	.122	.417	.0016	3.13	4.07
▲	4.28	.11	14.0	142	.672	.037	6.05	.886	.207	1.33	.0048	3.00	3.68
	5.56	1.0	22.6	81.6	.667	.016	6.32	.38	.068	.162	.000535	3.46	4.46
	5.56	1.0	102.8	152.4	.697	.015	6.37	.405	.073	.233	.000500	3.06	4.05
	11.35	1.0	12.3	97.4	2.00	.022	12.15	.40	.035	.164	.000181	3.21	4.59
	11.35	.35	12.3	97.4	1.29	.022	12.46	.66	.057	.418	.000465	3.08	4.26
	11.35	.11	12.3	97.4	.81	.022	13.45	1.05	.093	1.3	.00110	2.80	3.64
	AVG. LAST 13 POINTS—3.4 AVG——————4.4												

TABLE VIII

SUMMARY OF FREE CONVECTION DATA IN #12 TRANSIL OIL
SILVER WIRE .010" (.0254 CM) DIAMETER AND 1560 CM LONG

POINT NUMBER	AMB TEMP °C	WIRE TEMP °C	Δt °C	WC CONV WATTS	$\frac{WC}{L}$
11	24.9	37.8	12.9	878	.0563
12	25.2	42.8	17.6	1396	.0894
13	25.8	57.7	31.9	2840	.182
14	26.5	50.6	24.1	2126	.137
15	27.4	75.2	47.8	4990	.320
16	21.4	43.4	22.0	1944	.125
17	22.9	96.1	73.2	7900	.506
18	24.6	124.0	99.4	11670	.748
19	23.5	83.3	59.8	6200	.397
20	25.0	109.0	84.0	9550	.612
21	27.0	133.0	106.0	13120	.842
36	22.0	45.1	23.1	1944	.125
37	22.4	62.5	40.1	3870	.248
38	23.7	83.2	59.5	6220	.400
39	26.0	111.5	85.5	9596	.615
40	28.5	143.6	115.1	14160	.908
AVG	24.8				

TABLE IX

CALCULATION OF HEAT CONDUCTIVITY OF #12 TRANSIL OIL
FROM FREE CONVECTION TESTS
SILVER WIRE .0254 CM DIAMETER

AMB TEMP °C	WIRE TEMP °C	$t - t_a$ °C	$\frac{W_c}{L}$	$\varphi - \varphi_a$ $.35 \frac{W_c}{L}$	Δφ WATTS PER CM	Δt °C	k_{avg} $\frac{\Delta\varphi}{\Delta t}$	t_{avg} °C
24.8	35.	10	.045	.0158				
24.8	45	20	.105	.0368	.0210	10	.0021	40
24.8	55	30	.175	.0613	.0245	10	.0025	50
24.8	65	40	.250	.0876	.0263	10	.0026	60
24.8	75	50	.325	.114	.0260	10	.0026	70
24.8	85	60	.405	.1420	.0284	10	.0028	80
24.8	95	70	.485	.170	.0280	10	.0028	90
24.8	105	80	.570	.200	.030	10	.0030	100
24.8	115	90	.665	.233	.033	10	.0033	110
24.8	125	100	.760	.266	.033	10	.0033	120
24.8	135	110	.865	.303	.037	10	.0037	130
24.8	145	120	.970	.340	.037	10	.0037	140

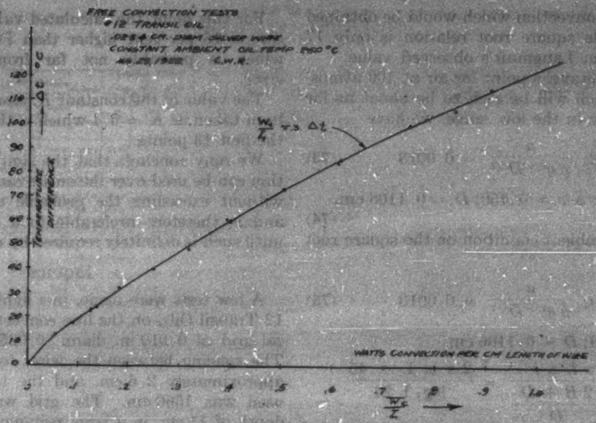

FIG. 13

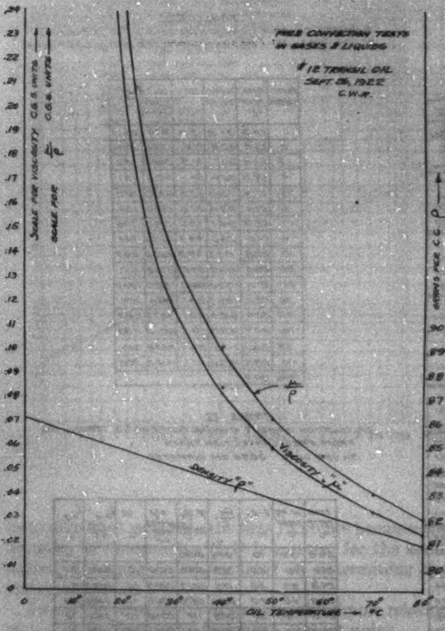

FIG. 14

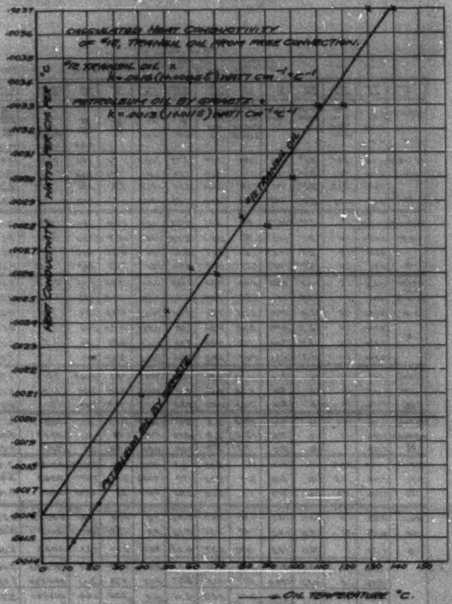

FIG. 15

tank. The ambient oil temperature was measured by mercury thermometers and the temperature of the wire determined by the resistance method, using equations (**57**) and (**58**). Table VIII is a summary of the data obtained.

In Fig. 13 the watts convection per cm. length of wire has been plotted against the temperature difference between the wire and the ambient fluid. Since heat conductivity data are not available the procedure adopted was to calculate the heat conductivity from the free convection data and compare the order of magnitude of the result with that obtained by Graetz[14] for the heat conductivity of a certain petroleum oil which he used.

In these calculations we have assumed the relations

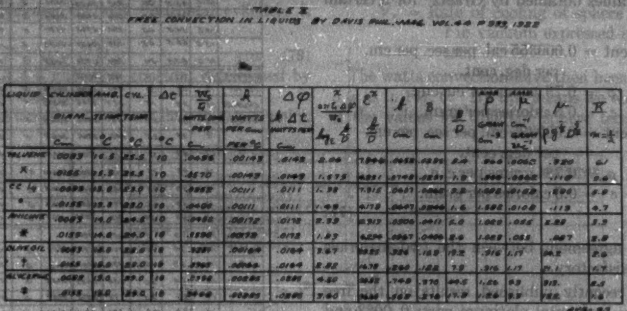

TABLE X
FREE CONVECTION IN LIQUIDS BY DAVIS PHIL. MAG. VOL. 44 P 533, 1922

LIQUID	CYLINDER DIAM. cm	AMB. TEMP °C	CYL. TEMP °C	Δt °C	$\frac{W_c}{L}$ WATTS PER cm	k WATTS PER cm PER °C	$\Delta\varphi = k\,\Delta t$ WATTS PER cm	$x = \frac{2\pi L\,\Delta\varphi}{W_c} = \log_e \frac{b}{D}$	$e^x = \frac{b}{D}$	b cm	B cm	$\frac{B}{D}$	AMB ρ GRAM cm^{-3}	AMB μ GRAM cm^{-1} sec^{-1}	$\frac{\mu}{\rho g^{\frac{1}{2}} D^{\frac{3}{2}}}$	K $m = \frac{1}{2}$
TOLUENE	.0083	16.5	25.5	10	.0436	.00143	.0143	2.06	7.846	.0652	.0285	3.4	.866	.0062	.320	6.1
×	.0155	15.5	25.5	10	.0570	.00143	.0143	1.575	4.831	.0748	.0231	1.9	.866	.0062	.110	5.6
CCl$_4$	.0083	13.8	23.0	10	.0552	.00111	.0111	1.38	7.315	.0607	.0262	3.2	1.602	.0102	.290	5.0
•	.0155	13.3	23.0	10	.0466	.00111	.0111	1.49	4.170	.0647	.0266	1.6	1.582	.0108	.113	4.7
ANILINE	.0083	14.0	24.6	10	.0462	.00172	.0172	2.39	8.313	.0690	.0411	5.0	1.023	.056	2.29	3.3
*	.0155	14.6	24.0	10	.1596	.00172	.0172	1.83	6.254	.0967	.0406	2.6	1.023	.055	.867	2.8
OLIVE OIL	.0083	12.0	22.6	10	.3281	.00164	.0164	3.67	39.25	.326	.163	19.2	.916	1.17	94.2	2.6
†	.0155	15.0	25.0	10	.3963	.00164	.0164	2.82	16.78	.260	.122	7.9	.916	1.17	31.1	1.7
GLYCERINE	.0083	13.0	23.0	10	.2398	.00285	.0285	4.50	90.02	.748	.370	44.5	1.26	9.3	913.	2.5
‡	.0155	13.0	23.0	10	.3406	.00285	.0285	3.60	36.60	.568	.278	17.9	1.26	9.3	182.	1.6

AVG. = 3.7

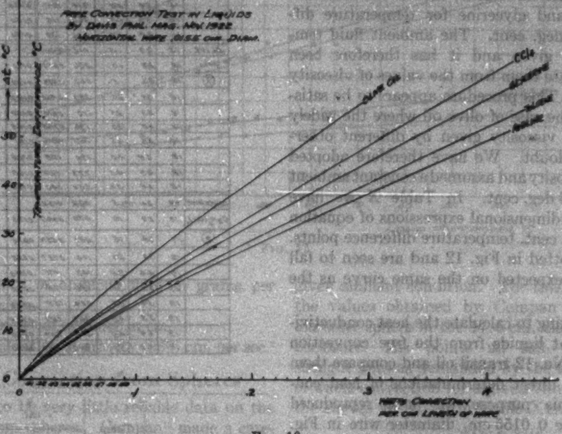

Fig. 16

which we have found to hold for gases as given by equations (59), (51) and (39) in which $m = 1/2$ and $k = 3.4$. We have also assumed that the film thickness merely depends upon the ambient properties of the liquid. Fig. 14 gives the density and viscosity taken from some available data on No. 12 transil oil as a function of temperature. The viscosity data were translated from second Saybolt to c. g. s. units using a chart given by N. MacCoull.[15]

The heat conductivity calculations are given in Table IX. The values of watts convection given in column five were read from the smooth curve of Fig. 13 at equal temperature intervals. In the fifth column and first line we have

$$\varphi_1 - \varphi_a = \int_0^{T_1} k\,d\,T - \int_0^{T_a} k\,d\,T$$

and from the second line

$$\varphi_2 - \varphi_a = \int_0^{T_2} k\,d\,T - \int_0^{T_a} k\,d\,T$$

Taking the difference we have the value given in column six or

$$\Delta\varphi = \varphi_2 - \varphi_1 = \int_0^{T_2} k\,d\,T - \int_0^{T_1} k\,d\,T = \int_{T_1}^{T_2} k\,d\,T$$

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[14] Graetz Ann. der Physik, Vol. 25, p. 353, 1885.

[15] N. Mac Coull Lubrication (Texas Co.), May, 1921.

For small values of $T_2 - T_1$ we may take k at the average of the two temperatures and write

$$\varphi_2 - \varphi_1 = \Delta\varphi = k_{avg}\,(T_2 - T_1)$$

or

$$k_{avg.} = \frac{\Delta\varphi}{\Delta t}$$

as given in column eight. In Fig. 15 the calculated heat conductivity of No. 12 transil oil is compared with the following values obtained by Graetz[14] for a certain petroleum oil

$$\left.\begin{aligned} k \text{ at } 12.7 \text{ deg. cent} &= 0.000355 \text{ cal. per sec. per cm. per deg. cent.} \\ \rho &= 0.79 \text{ grams per cu. cm.} \\ c &= 0.5 \text{ calories per gram} \\ k \text{ at } 22.7 \text{ deg. cent} &= 0.000395 \text{ cal. per sec. per cm. per deg. cent.} \\ \rho &= 0.784 \text{ grams per c. c.} \\ c &= 0.5 \text{ calories per gram.} \end{aligned}\right\} \quad (77)$$

The general agreement between the two materials is considered satisfactory.

Davis[4d] has recently published results on the free convection from horizontal platinum wires 0.0083 and 0.0155 cm. diameter in toluene, carbon tetrachloride, aniline, olive oil and glycerine for temperature differences up to 50 deg. cent. The ambient fluid temperatures are not given and it has therefore been necessary to estimate them from the values of viscosity which he assigns. This procedure appears to be satisfactory except in the case of olive oil where the widely different values of viscosity given by different observers leaves us in doubt. We have therefore adopted Davis' value of viscosity and assumed a constant ambient temperature of 18 deg. cent. In Table X we have calculated the non-dimensional expressions of equation (55) for the 10 deg. cent. temperature difference points. The results are plotted in Fig. 12 and are seen to fall as well as can be expected on the same curve as the gases.

It is now interesting to calculate the heat conductivities of the different liquids from the free convection test as we did for No. 12 transil oil and compare them with the available direct determinations of heat conductivity. For this purpose we have reproduced Davis' data for the 0.0155 cm. diameter wire in Fig. 16 and obtained from the smooth curves the points used for the calculations given in Table XI. The calculated heat conductivities are plotted in Fig. 17. In Table XII we have compared the calculated values with the experimental values by H. F. Weber (Landolt-Börnstein Tabellen).

The variation is not greater than 5 per cent and is therefore probably within the experimental errors. Thus free convection tests can be used as a convenient method of determining the heat conductivity of fluids when the viscosity and density are known.

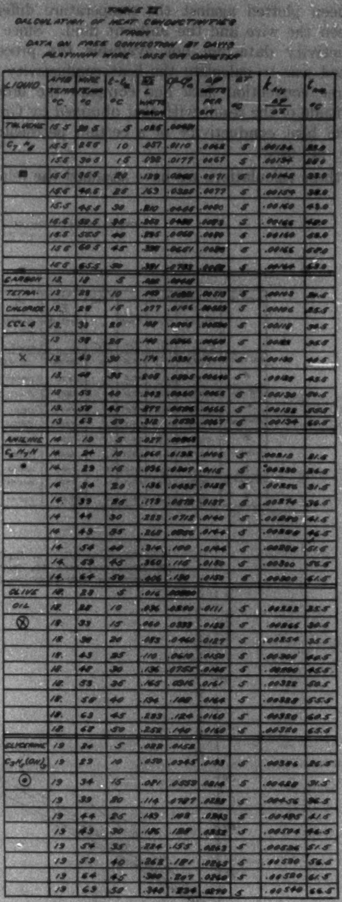

TABLE XI
CALCULATION OF HEAT CONDUCTIVITIES
FROM
DATA ON FREE CONVECTION BY DAVIS
PLATINUM WIRE .0155 CM DIAMETER

LIQUID	AMB TEMP °C	WIRE TEMP °C	$t - t_a$ °C	$\frac{EI}{L}$ WATTS PER CM	$\varphi - \varphi_a$	$\Delta\varphi$ WATTS PER CM	ΔT °C	k_{avg} $\frac{\Delta\varphi}{\Delta T}$	t_{avg} °C
TOLUENE	15.5	20.5	5	.026	.0048				
C_7H_8	15.5	25.5	10	.057	.0110	.0062	5	.00124	23.0
	15.5	30.5	15	.092	.0177	.0067	5	.00134	28.0
■	15.5	35.5	20	.129	.0248	.0071	5	.00142	33.0
	15.5	40.5	25	.163	.0325	.0077	5	.00154	38.0
	15.5	45.5	30	.210	.0405	.0080	5	.00160	43.0
	15.5	50.5	35	.262	.0488	.0083	5	.00166	48.0
	15.5	55.5	40	.295	.0568	.0080	5	.00160	53.0
	15.5	60.5	45	.330	.0651	.0083	5	.00166	58.0
	15.5	65.5	50	.391	.0733	.0082	5	.00164	63.0
CARBON	13.	18	5	.022	.0044				
TETRA-	13	23	10	.049	.0096	.00515	5	.00103	20.5
CHLORIDE	13.	28	15	.077	.0146	.0053	5	.00106	25.5
CCL4	13.	33	20	.108	.0205	.0059	5	.00118	30.5
	13	38	25	.140	.0266	.0061	5	.00122	35.5
×	13	43	30	.174	.0331	.0065	5	.00130	40.5
	13.	48	35	.208	.0395	.0064	5	.00128	45.5
	13	53	40	.243	.0460	.0065	5	.00130	50.5
	13.	58	45	.277	.0526	.0066	5	.00132	55.5
	13	63	50	.312	.0593	.0067	5	.00134	60.5
ANILINE	14	19	5	.027	.0086				
C_6H_7N	14	24	10	.060	.0192	.0106	5	.00212	21.5
●	14	29	15	.096	.0307	.0115	5	.00230	26.5
	14	34	20	.136	.0435	.0128	5	.00256	31.5
	14.	39	25	.179	.0572	.0137	5	.00274	36.5
	14	44	30	.223	.0712	.0140	5	.00280	41.5
	14	49	35	.268	.0856	.0144	5	.00288	46.5
	14	54	40	.314	.100	.0144	5	.00288	51.5
	14.	59	45	.360	.115	.0150	5	.00300	56.5
	14.	64	50	.406	.130	.0150	5	.00300	61.5
OLIVE	18.	23	5	.016	[illegible]				
OIL	18.	28	10	.036	.0200	.0111	5	.00222	25.5
⊗	18.	33	15	.060	.0333	.0133	5	.00266	30.5
	18	38	20	.083	.0460	.0127	5	.00254	35.5
	18.	43	25	.110	.0610	.0150	5	.00300	40.5
	18.	48	30	.136	.0755	.0145	5	.00290	45.5
	18.	53	35	.165	.0916	.0161	5	.00322	50.5
	18.	58	40	.194	.108	.0164	5	.00328	55.5
	18.	63	45	.223	.124	.0160	5	.00320	60.5
	18.	68	50	.252	.140	.0160	5	.00320	65.5
GLYCERINE	19	24	5	.022	.0152				
$C_3H_5(OH)_3$	19	29	10	.050	.0345	.0193	5	.00386	26.5
⊙	19	34	15	.081	.0559	.0214	5	.00428	31.5
	19	39	20	.114	.0787	.0228	5	.00456	36.5
	19	44	25	.149	.103	.0243	5	.00485	41.5
	19	49	30	.186	.128	.0252	5	.00504	46.5
	19	54	35	.224	.155	.0263	5	.00526	51.5
	19	59	40	.262	.181	.0265	5	.00530	56.5
	19	64	45	.300	.207	.0260	5	.00520	61.5
	19	69	50	.340	.234	.0270	5	.00540	66.5

TABLE XII

Liquid	Temp. deg. cent.	k cal. cm. sec. deg. cent.	Ratio	Source
Toluene............	9 to 15	0.000307	1.05	Weber
C_7H_8	12	0.000293		Calculated
Carbon tetra-chloride.	9 to 15	0.000252	1.05	Weber
CCl_4............	12	0.000240		Calculated
Aniline............	9 to 15	0.000408	0.86	Weber
C_6H_7N............	12	0.000477		Calculated
Olive Oil............	6.6	0.000392	0.87	Weber
	6.6	0.000450		Calculated
Glycerine............	9 to 15	0.000670	0.87	Weber
$C_3H_8O_3$............	12	0.000768		Calculated

For long cylinders we have obtained

$$W_c = \frac{2\pi L \Delta\varphi}{\log_e \dfrac{2B + D}{D}}$$

watts convection (78)

where L = Length of cylinder in cm.

$\Delta\varphi$ = Thermal conduction in watts per cm.

D = Cylinder diameter in cm.

B = Film thickness in cm.

The film thickness for long horizontal cylinders is given by

$$B = K\sqrt{\mu/\rho}\,\sqrt[4]{D/g} \quad \text{cm.} \quad (79)$$

where K = 3.4 a numerical constant determined by experiment.

μ = Viscosity of ambient fluid in c. g. s. units.

ρ = Density of ambient fluid in grams per cu. cm.

D = Diameter of cylinder in cm.

g = Acceleration of gravity (980) cm. per sec.²

VII. Free Convection from Spheres

There appears to be very little reliable data on the free convection from spheres. Compan[16] made a careful study of convection from a lamp black covered copper sphere 2.0 cm. diameter, in spherical enclosures of 16 cm. 14.5 cm. and 8.3 cm. diameter. The range of pressures covered was from approximately 6 atmospheres to 1/1000 of a mm. Compan gives the rate of cooling in deg. cent. per second for different sphere temperatures. The radiation correction is obtained as the rate of cooling under the best vacuum conditions which he was able to obtain. We have calculated the watts convection from his data by the following relation

$$W_c = 4.19\, C\, M\, (V_t - V_v) \text{ watts convection} \quad (80)$$

where C = 0.0952 = specific heat of copper at the sphere temperature in calories per gram per deg. cent.

M = 38.54 = Mass of sphere in grams (value given by Compan)

V_t = Rate of cooling of sphere at temperature t in deg. cent. per second

V_v = Rate of cooling of sphere at temperature t in vacuum expressed in deg. cent. per sec.

The watts convection have then been plotted against $\Delta\varphi$ as in Fig. 6. By this means we obtain an average of the values for different sphere temperatures for a given ambient condition. For this work we have used the values obtained by Compan with the 2.00 cm. diameter sphere in the 14.5 cm. diameter metal sphere enclosure. His points all fall very accurately on straight lines as demanded by the theory. The values of watts convection for a given value of $\Delta\varphi$ as read off the curves are given in Table XIII.

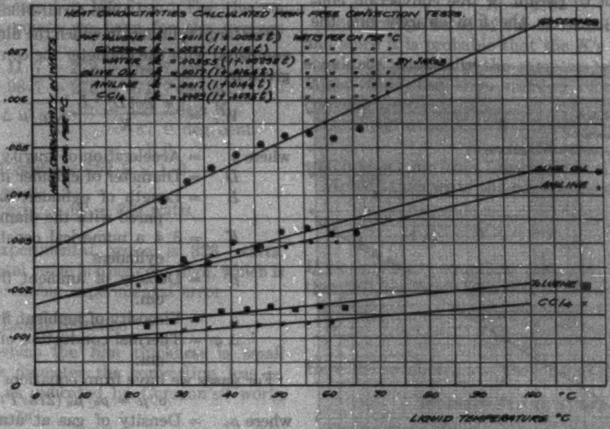

FIG. 17

TABLE XIII.

FREE CONVECTION FROM SPHERE 2.00 CM. DIAMETER AT 98.56 DEG. CENT.

ENCLOSURE 14.5 CM. DIAMETER SPHERE AT 0 DEG. CENT.

Air Pressure atmos.	Total Watts Convection W_c	$\Delta\varphi$ watts per cm.	B/D	μ / $\rho\, g^{1/2} D^{3/2}$ C.G.S units	Assuming $m = 2.3$ K
0.0737	0.143	0.0251	—	0.0202	—
0.530	0.424	0.0251	1.445	0.00281	72.3
1.02	0.571	0.0251	0.615	0.00146	47.4
2.03	0.805	0.0251	0.322	0.000735	39.3
4.13	1.142	0.0251	0.1905	0.000361	37.2
5.90	1.335	0.0251	0.1545	0.000253	38.7

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[16] Paul Compan, *Ann. de Chemie et de Physique*, VII, Series, Vol. 26, p. 488-574, 1902.

For spheres equation (**52**) gives

$$W_c = \frac{2\pi\,\Delta\varphi}{1/D - \dfrac{1}{2B+D}} \quad \text{watts} \qquad (81)$$

From which

$$B/D = 1/2\left(\frac{2\pi\,\Delta\varphi\,D}{W_c - 2\pi\,\Delta\varphi\,D}\right) \qquad (82)$$

In Fig. 18 we have plotted the values of B/D against $\dfrac{\mu}{\rho\, g^{1/2} D^{3/2}}$ on log log paper. The three high-pressure points are seen to lie on a straight line of slope $m = 2/3$. For larger values of B/D the enclosure is probably too small to permit of free convection conditions. At low pressures the film entirely fills the enclosure and the problem is then one of pure conduction. This fact is useful for eliminating convection in determinations of the heat conductivity of gases.

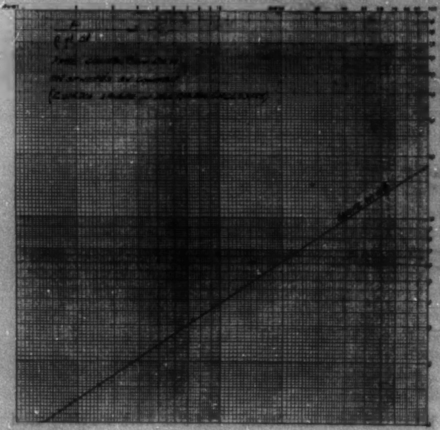

Fig. 18

If we assume that Compan's three high-pressure points accurately represent the conditions of free convection for spheres we have

$$B = (K/g^{1/3})\,(\mu/\rho)^{2/3} \quad \text{cm.} \qquad (83)$$

In the last column of the table are the values of K assuming this relation. The average value for the three high-pressure points gives $K = 38.4$. Here the film thickness is independent of the size of the body. More tests are needed before this conclusion can be definitely held.

VIII. Free Convection from Large Surfaces

For large surfaces the film thickness will usually be small compared with the dimensions of the body and therefore the "shape conductance" will have the simple form of equation (**14**) and the convection loss may be written

$$W_c = (A/B)\,\Delta\varphi \quad \text{watts} \qquad (84)$$

The variation of the film thickness B with the size of the body and ambient fluid properties can be found by determining the values of m and K in the general equation (**39**) for the film thickness, from experiment on similar bodies of various sizes or, by variations of the ambient fluid properties.

Where such experiments are not available a good approximation can usually be obtained by substituting an equivalent cylinder or sphere, etc., for the desired object.

LARGE CYLINDERS

For large cylinders in which the film thickness is small compared with the diameter the convection loss will be given by

$$W_c = \frac{\pi\, g^{1/4} D^{3/4} L}{K}\sqrt{\rho/\mu}\,\Delta\varphi \quad \text{watts} \qquad (85)$$

where g = Acceleration of gravity (980) cm. per sec.^{2}
D = Diameter of cylinder in cm.
L = Length of cylinder, assumed long compared with the diameter, in cm.
K = 3.4 a numerical constant for horizontal cylinders
ρ = Density of ambient fluid grams per cu. cm.
μ = Viscosity of ambient fluid in c. g. s. units
$\Delta\varphi$ = Thermal conduction of fluid in watts per cm.

For gases we have from equations (**61**) and (**62**)

$$\rho/\mu = \rho_0/\mu_0\,(273/T)^{n+1} \qquad (86)$$

where ρ_0 = Density of gas at atmospheric pressure and 0 deg. cent. in grams per cu. cm.
μ_0 = Viscosity of gas at 0 deg. cent. in c. g. s. units
T = Absolute temperature of the gas in deg. K.
n = Exponent depending on the nature of the gas; values are given by Jeans.[7]

For air at atmospheric pressure (76 cm. Hg) and 0 deg. cent. we have

ρ_0 = 0.00129 grams per cu. cm.
μ_0 = 0.0001705 c. g. s. units
n = 0.754

Substituting these values in equation (**86**) we obtain for air at atmospheric pressure

$$\rho/\mu = 7.56\,(273/T)^{1.754} \qquad (87)$$

$$\text{and } \sqrt{\rho/\mu} = 2.75\,(273/T)^{0.877} \qquad (88)$$

For small temperature differences of the order of 100 deg. cent. for air, we may take as a good approximation

$$\Delta\varphi = k_{avg.\ temp.} \times \Delta t \qquad (89)$$

If we now take the value of the specific heat of air at 300 deg. K. and assume it constant, we may write equation (28) *for air*

$$k = 3.4 \times 10^{-6}\, T^{0.754}$$

watts per cm. per deg. cent. (90)

If we now substitute equations (88), (89) and (90) in equation (85), we obtain the following *for air.*

$$W_c = 0.0067\, D^{3/4} L p^{1/2} \frac{T^{0.754}_{avg}}{T^{0.877}_{amb}} \Delta t \text{ watts} \qquad (91)$$

where D = Cylinder diameter in cm.
L = Cylinder length in cm.
p = Absolute air pressure in atmospheres
T_{avg} = Average of cyl. and amb. temp. deg. K.
T_{amb} = Ambient air temp. deg. K.
Δt = Temperature difference deg. cent.

Let us now apply this relation to the large silver plated cylinder $D = 11.35$ cm. $L = 152.5$ cm. For the atmospheric pressure points we have for the average values

$$\begin{aligned} T_{amb} &= 11.7 + 273 = 284.7 \text{ deg. K.} \\ \text{Cyl. Temp. } T &= 98.7 + 273 = 371.7 \text{ deg. K.} \\ T_{avg} &= 328.2 \text{ deg. K.} \\ \Delta t &= 87.0 \text{ deg. cent.} \\ p &= 1.0 \end{aligned}$$

$$W_c = 0.0067 \times (11.35)^{3/4} \times 152.5 \times 1.0 \times \frac{(328.2^0)^{.754}}{(284.7)^{0.877}} \times 87 = 304 \text{ watts}$$

The observed experimental value is 323 watts. Thus, the experimental value is 6 per cent high, which is not far from the probable experimental error.

LARGE SPHERES

For spheres in which the film thickness is small compared with the diameter the convection loss is, assuming $m = 2/3$ as indicated by Compan's[16] work,

$$W_c = \frac{g^{1/3}\, \pi\, D^2}{K} (\rho/\mu)^{2/3} \Delta \varphi \qquad (92)$$

where g = Acceleration of gravity (980) cm. per sec.^{2}
D = Diameter of sphere in cm.
K = 38.4 a numerical constant for spheres
ρ = Density of ambient fluid in grams per cu. cm.
μ = Viscosity of ambient fluid in c. g. s. units
$\Delta \varphi$ = Thermal conduction of fluid in watts per cm.

From equation (87) we have *for air*

$$(\rho/\mu)^{2/3} = 3.85\, (273/T)^{1.17} \qquad (93)$$

If we now substitute equations (89), (90) and (93) in (92), we obtain for moderate temperature differences *in air*

$$W_c = 0.0024\, p^{2/3} \frac{T^{.754}_{avg}}{T^{1.17}_{amb}} \pi D^2 \Delta t \qquad (94)$$

Here the significance of the symbols is the same as in equation (91).

CIRCULAR DISKS

For a circular disk 19.1 cm. in diameter, Langmuir[34] obtained, assuming equation (84), the following values for the film thickness in air at atmospheric pressure and a room temperature of 300 deg. K.

Horizontal, upper surface exposed $B = 0.41$ cm.
Vertical position $B = 0.45$ cm.
Horizontal, surface exposed downward $B = 0.92$ cm.

There are no data available to guide us in selecting the exponent m for this case. Tentatively we will assume $m = 1/2$ in equation (39) and associate the linear dimension with the diameter D of the disk. We then have

$$B = K \sqrt{\mu/\rho}\, \sqrt[4]{D/g} \quad \text{cm.} \qquad (95)$$

The diameter of the disk was 19.1 cm. and taking $B = 0.45$ cm., we obtain

$$K = 3.05$$

The average value of the K's for cylinders was 3.4 and since it is probable that the K's for the two cases should be practically the same we will use the K obtained from the large number of cylinder tests for convenience.

By substituting $K = 3.4$ in equation (95), we obtain

$$B = 0.485 \text{ cm.}$$

which is not far from Langmuir's values.

We will then write the convection as

$$W_c = \frac{\pi\, g^{1/4} D^{7/4}}{4\, K} \sqrt{\rho/\mu}\, \Delta \varphi \quad \text{watts} \qquad (96)$$

If we substitute equations (88), (89) and (90) in equation (96), we obtain for a disk *in air*, assuming $K = 3.4$,

$$W_c = 0.0017\, D^{7/4} p^{1/2} \frac{T^{0.754}_{avg}}{T^{0.877}_{amb}} \Delta t \text{ watts} \qquad (97)$$

Here the symbols have their usual significance.

INFINITE PLANE SURFACES

The problem of free convection from large or infinite plane surfaces involves many uncertainties. A horizontal surface exposed upwards involves questions of instability with the possibility of cyclonic action. Extended vertical surfaces will introduce "draught effects" which may require separate consideration. A horizontal plane surface exposed downward cannot produce convection. In spite of these objections it is interesting to study the case in which the free convection per unit area of surface is independent of the size of the body and apply it as a tentative approximation for the convection from large surfaces. To obtain this condition from our general equation for the film thickness we put $m = 2/3$ in equation (39), and obtain

$$B = (K/g^{1/3})\, (\mu/\rho)^{2/3} \quad \text{cm.} \qquad (98)$$

From which

$$W_c = (g^{1/3}/K)\, (\rho/\mu)^{2/3} A\, \Delta \varphi \quad \text{watts} \qquad (99)$$

For air at atmospheric pressure we have

$$(\rho/\mu)^{2/3} = 3.85\, (273/T)^{1.17} \qquad (100)$$

Since no tests are available on large plane surfaces, we will have to assume a value of K from the data on other surfaces. For spheres Compan's work indicates $m = 2/3$ and $K = 38.4$. In view of the small amount of data and rather limited size of enclosure used by Compan, it seems best to take our K from the cylinder and disk tests. For convenience we will assume a film thickness

$B = 0.485$ which corresponds to $K = 3.4$ in equation (**95**). Substituting this value in equation (**98**) gives

$$K = 16.5$$

Substituting equation (**89**), (**90**) and (**100**) in (**99**), we obtain the following *for air*

$$W_c = 0.0056\, p^{2/3} \frac{T_{avg.}{}^{0.754}}{T_{amb}{}^{1.17}} A\, \Delta t$$

watts convection (**101**)

From all of these relations we see that for a constant ambient air temperature the watts convection per deg. cent. temperature difference increases approximately as the 3/4 power of the average of the absolute temperatures of the surface and the ambient air. If we plot W_c v. s. Δt from these relations on log log paper, we find that for a constant ambient temperature the watts convection for temperature differences from 100 deg. cent. to 500 deg. cent. lie practically on a straight line of slope 1.21. This means that in this range the convection loss increases as the 1.21 power of the temperature difference. The equations of Dulong and Petit[17] and Lorenz[18] express the free convection loss as proportional to the 1.23 and 1.25 power respectively of the temperature difference.

The use of any of these expressions as approximations for the convection from large surfaces gives the same variation of the convection with the average absolute temperature of the surface and ambient air. The effect of ambient temperature and the air pressure depend on the assumed value of m. Further tests on large surfaces are needed to indicate the most suitable approximation for general work.

IX. Summary and Conclusions

1. The present work gives additional evidence in favor of Langmuir's film theory of convection which states that the loss of heat by free convection from a body takes place as if there were a film of relatively stationary gas around the body through which the heat is carried entirely by conduction and therefore if the film thickness is known the free convection loss can be calculated by the ordinary laws of heat conduction in the steady state.

2. The heat conduction between two surfaces in a fluid may be written

$$W = S\,(\varphi_2 - \varphi_1) \qquad \text{watts} \quad (\mathbf{102})$$

Where S = "Shape conductance" of the heat flow path measured in cm.

$$\varphi_2 - \varphi_1 = \Delta\varphi = \int_{T_1}^{T_2} k\, d\, T$$

in watts per cm. (**103**)

Where k = Heat conductivity of the fluid in watts per cm. per deg. cent.

T_2 = Absolute temperature of hot body in deg. K.

T_1 = Absolute temperature of cold body in deg. K.

3. The method of dimensions gives the following general relation for the film thickness

$$B = K\,(\mu/\rho)^m \frac{D^{\frac{2-3m}{2}}}{g^{m/2}} \text{ cm.} \qquad (\mathbf{104})$$

where K = Numerical constant to be determined by experiment

m = Numerical exponent to be determined by experiment

μ = Viscosity of ambient fluid in c. g. s. units

ρ = Density of ambient fluid in grams per cu. cm.

g = Acceleration of gravity (980) cm. per sec.^{2}

D = Linear dimension describing the body under consideration measured in cm.

4. In applying relation (**102**) to the film theory of convection the inner boundary of S is the body from which the convection is desired and the outer boundary is obtained by adding the film thickness to the body. Having thus determined the configuration of the corresponding conduction problem the value of S may be calculated or determined experimentally. Calculations of S are only possible in the simplest cases but fortunately the experimental method is simple and can always be applied.

5. For the case of free convection from long horizontal cylinders the available experiments include cylinders from 0.004 cm. to 11.35 cm. in diameter with temperatures up to 1600 deg. cent. and pressure ranging from 1/10 to 100 atmospheres. Tests in air, hydrogen, carbondioxide and several liquids show that the following simple relations hold within the probable experimental errors.

$$W_c = \frac{2\pi L\,\Delta\varphi}{\log_e \dfrac{2B + D}{D}}$$

watts convection (**105**)

where L = Length of cylinder in cm.

$\Delta\varphi$ = Thermal conduction in watts per cm.

D = Diameter of cylinder in cm.

$$B = 3.4\,\sqrt{\mu/\rho}\,\sqrt[4]{D/g} \qquad \text{cm.} \quad (\mathbf{106})$$

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[17] **Dulong and Petit**, *Ann. de Chim. et de Phys.* 7 (1817). See discussion by Langmuir, (3d).

[18] **L. Lorenz**, *Ann. Phys.* 13, p. 582, 1881. See discussion by Langmuir, (3d) p. 319 and 320.

where μ = Viscosity of ambient fluid in c. g. s. units
ρ = Density of ambient fluid in grams. per cu. cm.
D = Cylinder diameter in cm.
g = Acceleration of gravity (980) cm. per sec.^{2}

6. For gases the ambient properties as a function of temperature are given by the following relations with sufficient accuracy.

$$\rho = \rho_0 (273/T) \quad \text{grams per cu. cm.} \qquad (107)$$

$$\mu = \mu_0 (T/273)^n \quad \text{viscosity c. g. s.} \qquad (108)$$

From which

$$\rho/\mu = \rho_0/\mu_0 (273/T)^{n+1} \quad \text{c. g. s.} \qquad (109)$$

where ρ_0 = Density of gas in grams per cu. cm. at 0 deg. cent. and 76 cm. Hg.
μ_0 = Viscosity of gas in c. g. s. units at 0 deg. cent.
T = Absolute temperature of gas deg. K.

Jeans[7] gives a list of values of n from which the following have been taken

Air, $n = 0.754$; H_2, $n = 0.69$; CO_2, $n = 0.98$

For air at atmospheric pressure and 0 deg. cent. we have used the following values

ρ_0 = 0.00129 grams per cu. cm.
μ_0 = 0.0001705 in c. g. s. units (independent of pressure)
n = 0.754

From this we obtain *for air* at 76 cm. Hg.

$$\rho/\mu = 7.56 (273/T)^{1.754} \qquad (110)$$

7. From these relations we may write the film thickness for long horizontal cylinders *in air* as a function of temperature and pressure as follows

$$B = 0.00162\, p^{1/2} D^{1/4} T^{0.878} \quad \text{cm.} \qquad (111)$$

where p = Absolute air pressure in atmospheres
D = Diameter of cylinder in cm.
T = Absolute temperature of ambient air deg. K.

8. For moderate temperature differences we may write

$$\Delta \varphi = k_{avg.} \Delta t \quad \text{watts per cm.} \qquad (112)$$

where $k_{avg.}$ = Heat conductivity of fluid in watts per cm. per deg. cent. for the average of the surface and ambient temperatures.
Δt = Temperature difference deg. cent.

9. For air at moderate temperatures (up to 1000 deg. K.) we may write with fair accuracy

$$k = 3.4 \times 10^{-6}\, T^{0.754} \text{ watts per cm. per deg. cent. (independent of pressure)} \qquad (113)$$

where T = Absolute temperature of air deg. K.

10. The free convection from spheres is given by the following relations

$$W_c = \frac{2 \pi \Delta \varphi}{1/D - \dfrac{1}{2B + D}} \quad \text{watts convection} \qquad (114)$$

where $\Delta \varphi$ = Thermal conduction of fluid in watts per cm.
D = Diameter of sphere in cm.
B = Film thickness in cm.

The scanty data indicate the following form for the film thickness

$$B = (K/g^{1/3}) (\mu/\rho)^{2/3} \quad \text{cm.} \qquad (115)$$

where K = 38.4 a numerical constant
g = Acceleration of gravity (980) cm. per sec.^{2}
μ = Viscosity of ambient fluid in c. g. s. units
ρ = Density of ambient fluid in grams per cu. cm.

11. For large surfaces the film thickness will usually be small compared with the dimensions of the body and therefore the "shape conductance" will have the simple form of equation (14) and the free convection loss may be written

$$W_c = (A/B) \Delta \varphi \quad \text{watts} \qquad (116)$$

where A = Area of surface sq. cm.
$\Delta \varphi$ = Thermal conductivity of fluid in watts per cm.
B = Film thickness in cm. depending on the form and size of the body as expressed by the general equation (104).

12. For large cylinders in which the film thickness is small compared with the diameter, the convection loss is given by

$$W_c = \frac{\pi g^{1/4} D^{3/4} L}{K} \sqrt{\rho/\mu}\, \Delta \varphi \quad \text{watts} \qquad (117)$$

where g = Acceleration of gravity (980) cm. per sec.^{2}
D = Diameter of cylinder in cm.
L = Length of cylinder in cm.
K = 3.4 a numerical constant for horizontal cylinders
ρ = Density of ambient fluid grams per cu. cm.
μ = Viscosity of ambient fluid c. g. s. units
$\Delta \varphi$ = Thermal conduction of fluid in watts per cm.

For moderate temperature differences *in air* we may write equation (117) as follows

$$W_c = 0.0067\, D^{3/4} L\, p^{1/2} \frac{T_{avg.}^{0.754}}{T_{amb.}^{0.877}} \Delta t \text{ watts} \qquad (118)$$

here p = Absolute air pressure in atmospheres
$T_{avg.}$ = Average of cyl. and amb. temperature deg. K.
T_{amb} = Ambient air temperature deg. K.
Δt = Temperature difference deg. cent.

13. For large spheres in which the film thickness is small compared with its diameter the free convection will be given by

$$W_c = (A/B) \Delta \varphi \quad \text{watts convection} \qquad (119)$$

where A = πD^2 = Area of sphere in sq. cm.
$\Delta \varphi$ = Thermal conductance watts per cm.
B = Film thickness in cm.

The scanty data available indicate that the film thickness will be given by equation (**115**) from which the convection is

$$W_c = \frac{g^{1/3}}{k}\left(\frac{\rho}{\mu}\right)^{2/3} \pi D^2 \Delta \varphi \qquad (120)$$

where K = 38.4 a numerical constant

The other symbols have their usual significance.

For moderate temperature differences *in air*, we may write equation (**120**) as follows

$$W_c = 0.0024\, p^{2/3} \frac{T_{avg.}^{0.75}}{T_{amb}^{1.17}} \pi D^2 \Delta t \quad \text{watts} \qquad (121)$$

Here the symbols have their usual significance.

14. For circular disks in which the film thickness is small compared with the diameter, we will assume that $m = 1/2$ in the general equation (**104**) for the film thickness and write the free convection as

$$W_c = \frac{\pi\, g^{1/4} D^{7/4}}{4 K} \sqrt{\rho/\mu}\, \Delta \varphi \quad \text{watts} \qquad (122)$$

here D = Diameter of disk in cm.

The other symbols have the same significance as in equation (**117**). For horizontal and vertical disks we may assume $K = 3.4$ with fair accuracy.

For moderate temperature differences *in air* we may write equation (**122**) as follows:

$$W_c = 0.0017\, D^{7/4} p^{1/2} \frac{T_{avg.}^{\ 0.754}}{T_{amb.}^{\ 0.677}} \Delta t \quad \text{watts} \qquad (123)$$

Here the symbols have the same significance as in equation (**118**).

15. If the free convection per unit area is to be independent of the size of the body, the method of dimensions tells us to put $m = 2/3$ in the general equation (**104**) for the film thickness and obtain

$$B = (K/g^{1/3})\,(\mu/\rho)^{2/3} \quad \text{cm.} \qquad (124)$$

If we apply this relation to convection from large surfaces in which the film thickness is small compared with the dimensions of the body we obtain

$$W_c = \frac{g^{1/3} A}{K} (\rho/\mu)^{2/3} \Delta \varphi \quad \text{watts} \qquad (125)$$

where K = Numerical constant to be determined from experiment

A = Area of surface in sq. cm.

the other symbols having their usual meaning.

The value of K has been calculated on certain assumptions from which we obtain

K = 16.5

We may apply equation (**125**) as a tentative approximation for the convection from large surfaces. *For air* we then obtain for moderate temperature differences

$$W_c = 0.0056\, p^{2/3} \frac{T_{avg.}^{\ 0.754}}{T_{amb.}^{\ 1.17}} A \Delta t \quad \text{watts} \qquad (126)$$

16. The above equations for the free convection *in air* from large bodies show that for moderate temperature differences with constant ambient air temperature the watts convection per deg. cent. temperature difference increases approximately as the 3/4 power of the average of the absolute temperature of the surface and the ambient air.

17. Free convection tests constitute a convenient method of obtaining the heat conductivity of liquids when the viscosity and density are known. The following results (excepting water) have been obtained by this method.

	Water	$k = 0.00555\ (1 + .00298\ t)$	watts	cm^{-1}	deg. cent.$^{-1}$
No. 12	Transil Oil	$k = 0.0016\ (1 + .0095\ t)$	watts	cm^{-1}	deg. cent.$^{-1}$
	Toluene	$k = 0.0011\ (1 + .0095\ t)$	"	"	"
	Glycerine	$k = 0.0027\ (1 + .016\ t)$	"	"	"
	Olive oil	$k = 0.0017\ (1 + .016\ t)$	"	"	"
	Aniline	$k = 0.0017\ (1 + .014\ t)$	"	"	"
	CCl_4	$k = 0.0009\ (1 + .0095\ t)$	"	"	"

18. We have seen that the physical assumption of Langmuir's film theory combined with dimensional reasoning gives equations from which the free convection, from bodies of various shapes may be calculated in any gas or pure (non-plastic) liquid. It therefore seems certain that when a successful mathematical solution of the actual convection problem is obtained it will show the existence of the relatively stationary film with turbulent boundary as postulated by Langmuir's film theory.

The writer wishes to express his indebtedness to Mr. R. E. Doherty for the use of his equipment in connection with the tests on the large cylinders in the cask and to Mr. C. W. Cutler for his careful work in connection with these tests.

FORCED CONVECTION OF HEAT

I. Introduction

In a companion paper[1], Langmuir's film theory was applied to the calculation of the free convection from various bodies. The present work shows how it may be used to determine the forced convection.

The heat loss from any surface is the sum of two parts, the radiation and the convection. Thus the total heat loss is

$$W = W_R + W_c \quad \text{watts} \qquad (1)$$

The heat loss by radiation, W_R, may be calculated from the Stefan-Boltzman equation

$$W_R = 5.7\, A\, E\, [\,(T/1000)^4 - (T_0/1000)^4\,] \quad \text{watts} \qquad (2)$$

where 5.7 = black body radiation constant (most recent value)

A = area of radiating surface in sq. cm.

E = relative emissivity of surface

T = absolute temperature of hot body deg. K.

T_0 = absolute temperature of surroundings deg. K.

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[1] Chester W. Rice, Free Convection of Heat in Gases and Liquids.

The quantity E is unity for the ideal black body and less than one for other surfaces. E varies to some extent with temperature, but for most practical purposes may be considered a constant for any one substance. Values of E are given by Langmuir[2].

II. Langmuir's Film Theory of Convection

Langmuir's film theory of convection assumes that the heat loss by convection, W_c, consists essentially of heat conducted away from the body through a film of relatively stagnant gas at the surface. He assumes that the temperature of the outside surface of this film is the same as that of the ambient fluid; in other words, the whole temperature drop occurs across the film. If the ambient fluid is quiet we have the case of free convection previously treated.[1] If a rapid relative motion exists between the body and the ambient fluid we have the case of forced convection. In this case the impressed wind velocity is large compared with the velocity of the convection currents set up by the action of gravity on the difference in density between the hot and cold fluid. At a velocity of 100 cm. per sec. in air the forced convection loss is already many times the loss in still air.

To see the physical reason for the existence of a relatively stagnant film, consider the case of a fluid flowing in a pipe. At low velocities the motion of the fluid is in stream lines. If the velocity is raised above the critical velocity, v_c, where

$$v_c = 2500 \frac{\mu}{\rho D} \text{ cm. per sec.}^3 \qquad (3)$$

μ = viscosity of fluid c. g. s. units
ρ = density of fluid in grams per cc.
D = diameter of pipe in cm.

eddies burst into existence. Near the surface of the tube the motion will be in stream lines, due to the action of viscosity, and at a certain distance from the surface a transition will occur from the stream line condition to the turbulent motion which fills the center of the tube. The discontinuity between stream line and turbulent motion constitutes the inner boundary of the film. The outer boundary is the inside wall of the tube. The fluid in contact with the surface will have the temperature of the tube and the inner boundary will be approximately at the temperature of the turbulent fluid. Thus, we assume that the whole temperature drop occurs across the relatively stagnant film at the surface of the tube.

A similar phenomenon exists when a body of any form is bathed in a stream of fluid. The assumption of a film theory, therefore, reduces the complex problem of convection to a case of heat conduction in the steady state from the surface of the body to the surface of discontinuity which constitutes the outer boundary of the film.

As an example, consider the case of a cylinder bathed in a stream of fluid flowing at right angles to the axis. Here the true surface of discontinuity is extremely complicated and unknown. As an approximation we might assume an eccentric ellipse or an eccentric cylinder, etc., and determine the size and eccentricity so as to best fit the experimental results. From the point of view of calculations the simplest assumption of all is that the film boundary is a concentric cylinder. As a first approximation we will therefore adopt the simplest assumption and substitute the problem of conduction between concentric cylinders.

The general equation for the conduction of heat after the steady state has been reached is

$$W = (A/L)\, k\, \Delta t \qquad \text{heat flow in watts} \qquad (4)$$

where A = Area of heat flow path in sq. cm.
L = Length of heat path in cm.
k = Heat conductivity of medium expressed in watts per cm. per deg. cent.
Δt = Temperature difference causing the heat flow deg. cent.

Applying this to an element of volume in the case of the flow of heat between concentric cylinders and assuming the heat conductivity independent of temperature, we obtain.[1]

$$W = \frac{2\pi\, L\, k\, \Delta t}{\log_e b/D} \qquad \text{heat flow watts} \qquad (5)$$

where L = Length of cylinder, assumed long compared with the diameter, in cm.
k = Heat conductivity of fluid assumed independent of temperature in watts per cm. per deg. cent.
Δt = Temperature difference deg. cent.
b = Diameter of outer cylinder in cm.
D = Diameter of inner cylinder in cm.

If k is a function of temperature, as is the case in all real fluids, Langmuir[3] has pointed out that the appropriate form is

$$W = \frac{2\pi\, L\, \Delta\varphi}{\log_e b/D} \qquad \text{watts} \qquad (6)$$

$$\text{where } \Delta\varphi = \int_{T_1}^{T_2} k\, d\,T \qquad \text{watts per cm.} \qquad (7)$$

T_2 = Absolute temperature of inner cylinder deg K.
T_1 = Absolute temperature of outer cylinder deg. K.

Langmuir has used the following abbreviations

$$S = \frac{2\,\pi\, L}{\log_e b/D} \qquad \text{cm.} \qquad (8)$$

and called it the "shape factor." The factor S might also be called the "shape conductance" of the heat flow path. Calculation of S is only possible in the simplest cases, but fortunately it can be determined experimentally for any shape of flow path. A further discussion of this subject will be found in the paper on Free Convection[1]. We may now express any heat conduction problem in the following form

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[2] Irving Langmuir, *Trans. Am. Electrochem. Soc.*, Vol. XXIII, p. 299, 1913.

[3] Irving Langmuir, *Phys. Rev.*, Vol. XXXIV, No. 6, p. 401, 1912.

$$W = S \, \Delta \varphi \qquad \text{watts} \quad (9)$$

In applying relation (9) to the film theory of convection, the inner boundary of S is the body from which the forced convection is desired and the outer boundary is obtained by adding the film thickness to the body.

III. Film Thickness by the Method of Dimensions

We will assume from physical considerations that the film thickness may be represented by an equation of the following form:

$$B = K \rho^x \mu^y D^z v^n \qquad (10)$$

where K = A numerical constant

ρ = Density of fluid in grams per cc.

μ = Viscosity of fluid in c. g. s. units

D = Linear dimension, describing the form of body, in cm.

v = Velocity of fluid stream in cm. per sec.

Substituting the dimensions of the symbols in equation (10), we obtain

$$(L)^1 = (M \, L^{-3})^x \quad (L^{-1} M \, T^{-1})^y \quad (L)^z \, (L \, T^{-1})^n \qquad (11)$$

In order to have this represent a true physical equation we must have the dimensions of the various quantities equal on both sides of the equation. We therefore write the following simultaneous equations:

$$\left.\begin{array}{ll} \text{By } L, & 1 = -3x - y + z + n \\ \text{By } M, & 0 = x + y \\ \text{By } T, & 0 = -y - n \end{array}\right\} \qquad (12)$$

Solving in terms of n we obtain

$$\left.\begin{array}{l} x = n \\ y = -n \\ z = n + 1 \end{array}\right\} \qquad (13)$$

Substituting the values of the exponents in (10), we have

$$B = K \rho^n \mu^{-n} D^{n+1} v^n \qquad (14)$$

Since it is evident that the film thickness will decrease with increasing velocity, we should change the sign of n or substitute

$$n = -m \qquad (15)$$

and obtain for the general expression for the film thickness in forced convection

$$B = K D \left(\frac{\mu}{\rho D v} \right)^m \qquad \text{cm.} \quad (16)$$

For large surfaces in which the film thickness is small compared with the dimensions of the body, the shape conductance may be written

$$S = A/B \qquad \text{cm.} \quad (17)$$

and the watts convection will then be given by the following expression

$$W_c = (A/B) \, \Delta \varphi \qquad \text{watts} \quad (18)$$

If we now substitute equation (16) in (18), we have

$$W_c = \frac{A}{K D} \left(\frac{\rho D v}{\mu} \right)^m \Delta \varphi \qquad \text{watts} \quad (19)$$

This is our general expression for the forced convection from large surfaces in which the film thickness is small compared with the size of the body. For small temperature differences we may write

$$\Delta \varphi = k_{avg.\ temp.} \, \Delta t \text{ watts per cm.} \qquad (20)$$

and obtain

$$W_c = \frac{A}{K D} \left(\frac{\rho D v}{\mu} \right)^m k_c \, \Delta t \qquad \text{watts} \quad (21)$$

For the case of cylinders or pipes we would associate D with the diameter and write

$$A = \pi D \times L \qquad \text{sq. cm.}$$

Substituting this value in (21) we obtain

$$W_c = \frac{\pi L}{K} \left(\frac{\rho D v}{\mu} \right)^m k_c \, \Delta t \qquad \text{watts} \quad (22)$$

This is the form of relation obtained by Davis[4].

Davis has also shown that the relations derived by Boissinesq[5] and Rayleigh[6] may be thrown into this form. According to the present film theory this is the form of relation which should hold only for large cylinders or tubes in which the film thickness is small compared with the diameter and for small temperature differences.

From the kinetic theory of gases we have for the heat conductivity

$$k = M \mu c_v \qquad (23)$$

where M = numerical constant depending on nature of gas

μ = Viscosity of gas in c. g. s. units

c_v = Specific heat of gas per gram at constant volume

We also have, for the ratio of the specific heats

$$c_p / c_v = \gamma \qquad (24)$$

We may write

$C = \rho \, c_p$ specific heat per cc. at constant pressure (25)

If we now substitute μ from equation (23) in equation (21) we obtain with the use of equation (24) and (25) the following

$$\frac{W_c}{A \, k \, \Delta t} = \frac{K}{D^{1-m}} \left(\frac{C_v}{k} \right)^m \qquad (26)$$

This is the relation derived from dimensional considerations by Nusselt[7] for forced convection in tubes.

As a matter of interest we may note that if we assume $m = 1$ in equation (26) and substitute

$$Q = A_v \qquad \text{flow in cc. per sec.} \quad (27)$$

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[4] A. H. Davis, *Phil. Mag.*, Vol. 40, p. 692, 1920.
A. H. Davis, *Phil. Mag.*, Vol. 41, p. 899, 1921.
A. H. Davis, *Phil. Mag.*, Vol. 43, p. 329, 1922.
A. H. Davis, *Phil. Mag.*, Vol. 44, p. 940, 1922.

[5] Boussinesq, Comptes Rendus, CXXXII, p. 1382 (1901), Free Convention.
Boussinesq, Comptes Rendus, CXXXIII, p. 257 (1901), Forced Convection.

[6] Rayleigh, Nature XCV, p. 66, 1915 or *Sci. Papers*, Vol. VI, p. 300, 1911-1919.

[7] Wilhelm Nusselt, *Zeit. des. Verh. Deut. Ing.*, Band 53, Nr. 43, p. 1750, 1909. Band 53, Nr. 44, p. 1808, 1909. Band 54, Nr. 28, 9 Juli, 1910.

we obtain

$$W_c = K\,Q\,C\,\Delta t \qquad \text{watts} \quad (28)$$

In this case the heat transfer is proportional to the rate of flow of material in cc. per sec., the specific heat per unit volume, and the temperature difference, which is a commonly used type of expression for forced convection.

IV. Forced Heat Convection from Cylinders

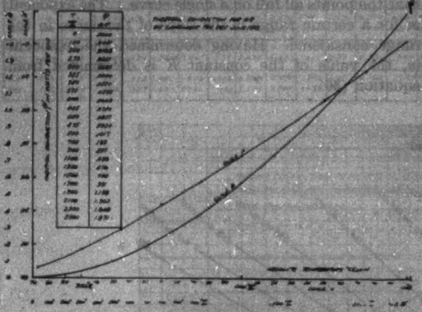

FIG. 1

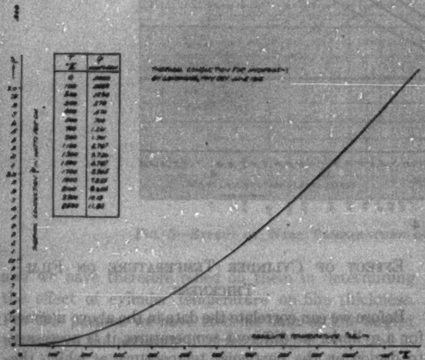

FIG. 2

A representative portion of the data by the various investigators has been converted into watts convection per cm. length of cylinder. For small wires the radiation is negligible. In Hughes' data on large cylinders his radiation corrections have been applied. Langmuir's[2] values of φ as given in Figs. 1, 2, and 3 have been used.

The data by King[8] were taken in the open air, and the wire under test was mounted in a fork at the end of a rotating arm. He worked with platinum wires of 0.0153 cm. to 0.00278 cm. in diameter. The velocity was varied from about 17 cm. to 800 cm. per second. Wire temperatures from 227 deg. cent. to 1004 deg. cent. were used. The ambient air temperature was approximately 17 deg. cent.

Kennelly[9] investigated the forced convection from copper wires in the open air. As in King's work, a fork carrying the wire was mounted at the end of a whirling arm. Kennelly's data as summarized by Langmuir[10] has been used. In this investigation the copper wires had diameters of 0.0101 cm., 0.0159 cm., and 0.0204 cm. Wind velocities from standstill to approximately 1800 cm. per second were studied. Wire temperatures up to approximately 300 deg. cent. were used. The ambient temperature was in the vicinity of 20 deg. cent. More recently Kennelly[11] has studied the effect of air pressure on forced convection from a platinum wire 0.0114 cm. diameter. The rotating arm device was installed in a large steel tank in which the air pressure could be controlled. Wind velocities from standstill to 1900 cm. per second were used and the pressure varied from approximately 0.4 of an atmosphere to 4 atmospheres. Wire temperatures of approximately 410 deg. cent. and 558 deg. cent. were employed. The ambient air temperature was in the vicinity of 18 deg. cent.

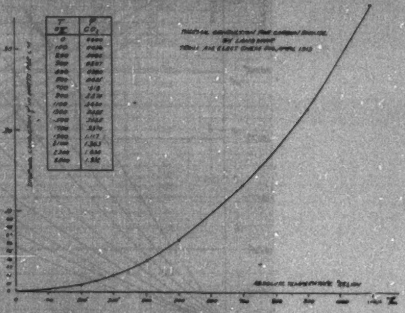

FIG. 3

Hughes[12] studied the forced convection from copper tubes in a wind tunnel. The tubes were heated by steam at atmospheric pressure and the temperature of the tube surface was assumed to be 100 deg. cent. The ambient air temperature has been assumed to be 15 deg. cent. Wind velocities from zero to 1500 cm. per second were studied. The cylinders had diameters of 0.43 cm., 0.81 cm., 1.93 cm., 5.06 cm., and 15.5 cm. The first four cylinders were approximately 100 cm. long. The 15.5 cm. diameter one was only 20 cm. long, and, from the experimental arrangement, it would appear to be subject to large end effects. The data on this cylinder have therefore not been considered.

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[8] L. V. King, *Phil. Trans. Roy. Soc. A.*, Vol. 214, p. 373, 1914.

[9] A. E. Kennelly, C. A. Wright, and J. S. Van Bylevelt, A. I. E. E., vol. XXVIII, Part I, p. 363, 1909.

[10] Irving Langmuir, A. I. E. E., Vol. XXXI, Part I, p. 1237, 1912.

[11] A. E. Kennelly and H. S. Sanborn, *Proc. Amer. Phil. Soc.*, Vol. LIII, p. 55, 1914.

[12] J. A. Hughes, *Phil. Mag.*, Vol. 31, p. 118, 1916.

Worthington[13] and Malone studied the forces convection from a platinum wire 0.0256 cm. in diameter by rotating it at the end of an arm in a large tank of water. The experiments covered velocities from 0 to 16 cm. per second. Wire temperatures in the vicinity of 50 deg. cent. were used. The ambient water temperature is not given but has been assumed to be 20 deg. cent.

From the point of view of the film theory of forced convection we have obtained

μ = Viscosity of fluid in c. g. s. units

ρ = Density of fluid in grams per cc.

v = Velocity of fluid stream in cm. per sec.

m = Numerical exponent to be determined by experiment.

To determine the value of the exponent we plot the non-dimensional relation

$$B/D \text{ vs. } \frac{\mu}{\rho D v} \qquad (31)$$

on log: log paper for the available data, and should find that the points all fall on a single curve. The exponent m for a certain range is the slope of the curve in the range considered. Having determined the exponent m, the value of the constant K is determined from equation (30).

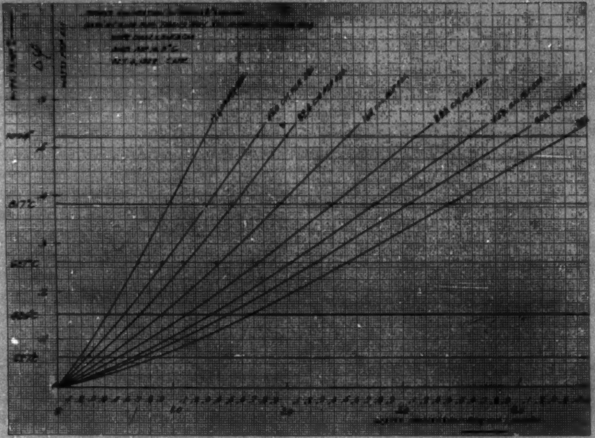

FIG. 4

$$W_c = \frac{2\pi L \Delta\varphi}{\log_e b/D} \text{ watts convection} \qquad (29)$$

where L = Length of cylinder, assumed long compared with the diameter, in cm.

$\Delta\varphi$ = Thermal conduction in watts per cm.

D = Diameter of cylinder in cm.

b = $2B + D$ = Outside diameter of film in cm.

For the film thickness we obtained

$$B = K D \left(\frac{\mu}{\rho D v} \right)^m = \text{Film thickness in cm.} \qquad (30)$$

K = Numerical constant to be determined by experiment

EFFECT OF CYLINDER TEMPERATURE ON FILM THICKNESS

Before we can correlate the data in the above manner for a cylinder at different temperatures it is necessary to decide on the particular temperature at which the viscosity and density are to be taken. In the case of free convection, Langmuir[3] showed that the film thickness was determined solely by the viscosity and density of the ambient fluid irrespective of the wire temperature.

This was shown to be the case by plotting the watts convection against the thermal conduction $\Delta\varphi$ for different cylinder temperatures with constant ambient temperature conditions. The curves obtained were straight lines as required by equation (29), if b is to be independent of the cylinder temperature. King's[8] data cover the widest variation in wire temperature,

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[13] Worthington and Malone, *Journ. Franklin Inst.*, CLXXXIV, p. 115, 1917.

TABLE I

FORCED CONVECTION FROM CYLINDERS IN AIR
DATA BY KING PHIL. TRANS. ROY. SOC. A. VOL. 214, 1914
WIRE DIAMETER = .0153 CM.
AMBIENT AIR TEMPERATURE = 16.8 °C.
BAROMETER ASSUMED = 76 CM H_g

AIR VEL.	AMB. TEMP.	WIRE TEMP. = 500 °K GEOM. MEAN = 381 °K			WIRE TEMP. = 653 °K GEOM. MEAN = 451 °K			WIRE TEMP. = 900 °K GEOM. MEAN = 510 °K			WIRE TEMP. = 1090 °K GEOM. MEAN = 562 °K			WIRE TEMP. = 1277 °K GEOM. MEAN = 610 °K		
v	$\frac{\mu}{\rho D v}$	W_c PER CM LENGTH	$\frac{a}{D}$	$\frac{B}{D}$	W_c PER CM LENGTH	$\frac{a}{D}$	$\frac{B}{D}$	W_c PER CM LENGTH	$\frac{a}{D}$	$\frac{B}{D}$	W_c PER CM LENGTH	$\frac{a}{D}$	$\frac{B}{D}$	W_c PER CM LENGTH	$\frac{a}{D}$	$\frac{B}{D}$
17.4	.55	.138	7.324	3.46	.435	9.116	4.06	.703	10.024	4.52	1.00	11.134	6.40	1.405	10.381	4.70
[illegible]	.212	.263	4.753	1.09	.572	5.366	2.18	.908	5.960	2.49	1.295	6.521	2.75	1.745	6.586	2.81
82.4	.116	.331	3.456	1.23	.721	3.000	1.41	1.041	4.477	1.74	1.521	4.874	1.94	2.015	6.129	2.07
165	.058	.424	2.630	.910	.905	2.890	.947	1.375	3.204	1.13	1.925	3.430	1.24	2.56	3.651	1.32
282	.034	.549	2.103	.556	1.127	2.351	.680	1.790	2.540	.773	2.370	2.753	.882	3.14	2.845	.326
452	.021	.667	1.850	.425	1.328	2.065	.537	2.030	2.223	.615	2.77	2.387	.700	3.60	2.497	.752
648	.016	.712	1.779	.392	1.480	1.916	.469	2.260	2.090	.603	3.07	2.130	.534	3.97	2.249	.647
882	.012	.825	1.644	.327	1.660	1.786	.392	2.525	1.900	.452	3.42	2.024	.616	4.41	2.106	.655

We see that column XIII yields the most nearly constant value of K. We will therefore assume that the film thickness in forced convection is determined by the viscosity and density of the air at the geometric mean of the wire and ambient absolute temperatures. Data on larger wires at high temperatures are, however, needed before the question can be definitely settled.

TABLE II

FORCED CONVECTION FROM CYLINDERS
EFFECT OF CYL. TEMP. ON FILM THICKNESS
WIRE DIAM = .0153 CM
AIR VELOCITY = 28.7 CM PER SEC.

I	II	III	IV	V	VI AMB TEMP.		VII AVG. TEMP.		VIII GEOM. MEAN	
$\frac{B}{D}$	WIRE TEMP. °K	AMB TEMP. °K	AVG. TEMP. °K	GEOM. MEAN TEMP. °K	$\frac{\mu}{\rho D v}$	m = .62 K	$\frac{\mu}{\rho D v}$	m = .62 K	$\frac{\mu}{\rho D v}$	m = .62 K
1.17	500	290	395	381	.10	4.89	.170	3.5	.160	3.66
1.36	653	290	435	451	.10	5.68	.235	3.16	.219	3.53
1.56	900	290	595	510	.10	6.59	.350	3.01	.265	3.59
1.79	1090	290	690	562	.10	7.47	.455	2.91	.315	3.65
2.06	1277	290	784	610	.10	8.55	.378	2.98	.365	3.63

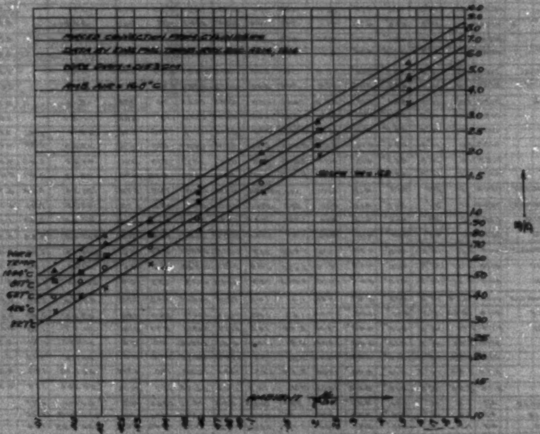

FIG. 5—EFFECT OF WIRE TEMPERATURE ON FILM THICKNESS IN FORCED CONVECTION

and we have therefore relied on them in determining the effect of cylinder temperature on film thickness. For this purpose we have used his data on the largest wire, since the accuracy in determining the film thickness is best with large wires and high wind velocities. These data are given in Table I, and in Fig. 4 we have plotted the forced convection per cm. length of wire against $\Delta \varphi$ for each velocity. The result shows a decided curvature which indicates that the film thickness increases with the wire temperature, while the ambient conditions remain constant. To bring the fact out in another manner, we have plotted on log: log paper the values of B/D vs. $\mu/\rho D v$ for different wire temperatures with constant ambient temperature conditions. From Fig. 5 a mean slope of $m = 0.62$ has been selected. We now wish to choose the temperature condition for the viscosity and density which will make the curves of Fig. 5 fall on a single line; that is, have a single value of K in equation (30). We have therefore recorded in Table II the value of K obtained from equation (30), assuming $m = 0.62$ for different assumed values of the viscosity and density. In column I are the values of B/D as read from the curves of Fig. 5. In columns II, III, and IV are given the corresponding values of wire temperature, ambient temperature, and average of wire and ambient temperature. Column V is the geometric mean of the wire and ambient absolute temperature, or

$$T_{geom.\ mean} = \sqrt{T_{wire} \times T_{amb.}} \quad \text{deg. } K \qquad (32)$$

For air we have with sufficient accuracy[1]

$$\mu/\rho = 0.132\ (T/273)^{1.754} \quad \text{c. g. s. units} \qquad (33)$$

where T = Absolute air temperature deg. K.

We see that column VIII yields the most nearly constant value of K. We will therefore assume that the film thickness in forced convection is determined by the viscosity and density of the fluid at the geometric mean of the wire and ambient fluid temperatures. Data on larger wires at high temperature are, however, needed before the question can be definitely settled.

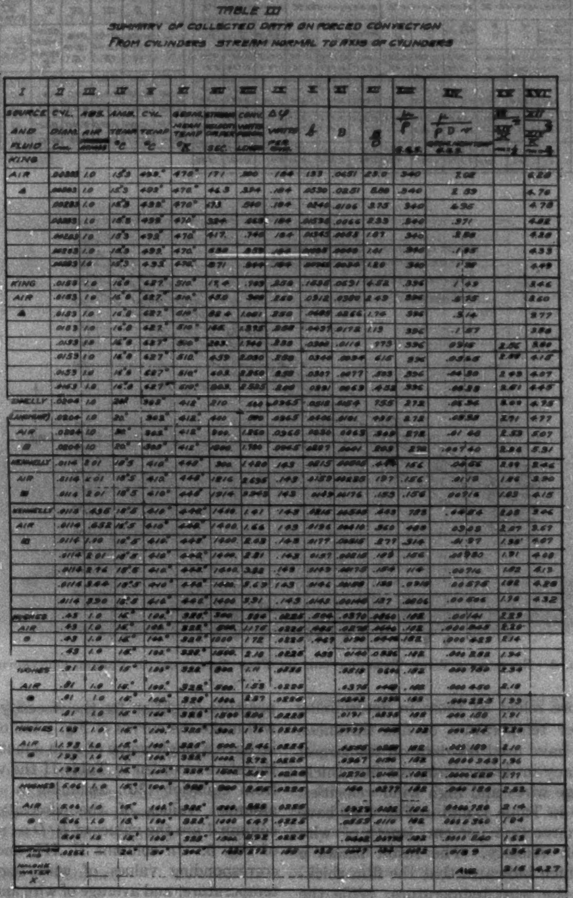

TABLE III
SUMMARY OF COLLECTED DATA ON FORCED CONVECTION
FROM CYLINDERS STREAM NORMAL TO AXIS OF CYLINDERS

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI
SOURCE AND FLUID	CYL. DIAM. cm.	ABS. AIR ATMOS	AMB. TEMP °C	CYL. TEMP °C	GEOM. MEAN TEMP °K	STREAM VELOCITY CM PER SEC.	CONV. WATTS PER CM LENGTH	$\Delta\varphi$ WATTS PER CM.	b	B	$\frac{B}{D}$	$\frac{\mu}{\rho}$ C.G.S.	$\frac{\mu}{\rho D v}$ GEOM. MEAN TEMP C.G.S.	[illegible]	[illegible]
KING															
AIR	.00283	1.0	15.3	493°	470°	17.1	.300	.164	.133	.0651	23.0	.340	7.02		6.28
▲	.00283	1.0	15.3	493°	470°	46.3	.394	.164	.0530	.0251	8.88	.340	2.59		4.70
	.00283	1.0	15.3	493°	470°	173	.540	.164	.0240	.0106	3.75	.340	.696		4.78
	.00283	1.0	15.3	493°	470°	324	.663	.164	.01536	.0066	2.33	.340	.371		4.82
	.00283	1.0	15.3	493°	470°	417.	.740	.164	.01345	.0053	1.87	.340	.288		4.20
	.00283	1.0	15.3	493°	470°	630	.855	.164	[illegible]	[illegible]	1.41	.340	.195		4.33
	.00283	1.0	15.3	493°	470°	871	.944	.164	[illegible]	.0034	1.20	.340	.138		4.49
KING	.0153	1.0	16.0	627°	510°	17.4	.703	.250	.1636	.0631	4.52	.396	1.49		3.46
AIR	.0153	1.0	16.0	627°	510°	45.0	.988	.250	.0912	.0380	2.49	.396	.575		3.60
▲	.0153	1.0	16.0	627°	510°	82.4	1.061	.250	.0685	.0266	1.74	.396	.314		3.77
	.0153	1.0	16.0	627°	510°	165	1.395	.250	.0437	.0172	1.13	.396	.157		3.88
	.0153	1.0	16.0	627°	510°	283.	1.740	.250	.0300	.0116	.773	.396	.0915	2.56	3.90
	.0153	1.0	16.0	627°	510°	459	2.030	.250	.0340	.0094	.616	.396	.0566	2.58	4.16
	.0153	1.0	16.0	627°	510°	603.	2.260	.250	.0307	.0077	.503	.396	.0430	2.43	4.07
	.0153	1.0	16.0	627°	510°	883.	2.525	.250	.0281	.0063	.412	.396	.0293	2.61	4.45
SHELLY	.0204	1.0	20°	303°	412°	210	.660	.0965	.0312	.0154	.755	.272	.0634	3.00	4.75
(LANGMUIR)	.0204	1.0	20°	303°	412°	460	.890	.0965	.0406	.0101	.495	.272	.0289	2.71	4.77
AIR	.0204	1.0	20°	303°	412°	900.	1.260	.0965	.0330	.0063	.309	.272	.0148	2.53	5.07
■	.0204	1.0	20°	303°	412°	1800.	1.780	.0965	.0207	.0041	.203	.272	.00740	2.84	5.31
KENNELLY	.0114	2.01	18.5	410°	448°	300.	1.420	.143	.0215	.00505	.443	.156	.0455	2.09	3.46
AIR	.0114	2.01	18.5	410°	448°	1216	2.635	.143	.0159	.00225	.197	.156	.0113	1.86	3.90
■	.0114	2.01	18.5	410°	448°	1914	3.343	.143	.0143	.00176	.155	.156	.00716	1.83	4.15
KENNELLY	.0114	.436	18.5	410°	448°	1400.	1.41	.143	.0215	.00505	.443	.720	.0454	2.08	3.46
AIR	.0114	.652	18.5	410°	448°	1400.	1.66	.143	.0196	.00410	.360	.480	.0302	2.07	3.67
■	.0114	1.00	18.5	410°	448°	1400.	2.03	.143	.0177	.00316	.277	.314	.0197	1.98	4.07
	.0114	2.01	18.5	410°	448°	1400.	2.31	.143	.0157	.00216	.193	.156	.00980	1.91	4.08
	.0114	2.76	18.5	410°	448°	1400.	3.33	.143	.0149	.00175	.154	.114	.00716	1.82	4.13
	.0114	3.44	18.5	410°	448°	1400.	3.63	.143	.0146	.00158	.138	.0916	.00575	1.82	4.28
	.0114	3.90	18.5	410°	448°	1400	3.91	.143	.0143	.00146	.127	.0806	.00506	1.78	4.32
HUGHES	.43	1.0	15°	100°	328°	300	.884	.0225	.504	.0370	.0860	.182	.00141	2.29	
AIR	.43	1.0	15°	100°	328°	600.	1.176	.0225	.485	.0275	.0640	.182	.000705	2.20	
●	.43	1.0	15°	100°	328°	1000	1.72	.0225	.467	.0190	.0442	.182	.000423	2.14	
	.43	1.0	15°	100°	328°	1500.	2.10	.0225	.438	.0140	.0326	.182	.000282	1.94	
HUGHES	.81	1.0	15°	100°	328°	300	1.11	.0225		.0513	.0634	.182	.000750	2.34	
AIR	.81	1.0	15°	100°	328°	500.	1.53	.0225		.0376	.0464	.182	.000450	2.18	
●	.81	1.0	15°	100°	328°	1000	2.27	.0225		.0248	.0306	.182	.000225	1.93	
	.81	1.0	15°	100°	328°	1500	2.86	.0225		.0191	.0236	.182	.000150	1.91	
HUGHES	1.93	1.0	15°	100°	328°	300	1.76	.0225		.0777	.0403	.182	.000314	2.20	
AIR	1.93	1.0	15°	100°	328°	500.	2.46	.0225		.0555	.0287	.182	.000189	2.10	
●	1.93	1.0	15°	100°	328°	1000	3.72	.0225		.0367	.0190	.182	.0000943	1.96	
	1.93	1.0	15°	100°	328°	1500	5.05	.0225		.0270	.0140	.182	.0000629	1.77	
HUGHES	5.06	1.0	15°	100°	328°	300	2.65	.0225		.140	.0277	.182	.000120	2.52	
AIR	5.06	1.0	15°	100°	328°	500.	3.50	.0225		.0927	.0182	.182	.0000720	2.14	
●	5.06	1.0	15°	100°	328°	1000	6.47	.0225		.0563	.0110	.182	.0000360	1.84	
	5.06	1.0	15°	100°	328°	1500	8.92	.0225		.0442	.00875	.182	.0000240	1.63	
[illegible] AND MALONE WATER X	.0256	—	24°	80°	302°	1420	2.72	.122	.132	.0047	.184	.0082	.0189	1.34	2.40
													AVE.	2.16	4.27

Summary of Collected Data on Forced Convection from Cylinders

In Table III we have given a summary of the collected data on the forced convection from cylinders in a stream of fluid normal to the axis of the cylinder. For the water point we have used the recent data on heat conductivity by Jakob[14]. Since the temperature difference is small we have written

$$\Delta \varphi = k_{\text{average temp.}} \Delta t \quad \text{watts per cm.} \qquad (34)$$

where $k_{\text{average temp.}}$ = Heat conductivity of water at the average of the wire and ambient temperature in watts per cm. per deg. cent.

Δt = Temperature difference between the wire and ambient water deg. cent.

In Fig. 6 the data have been plotted on log: log paper. From this we see that for small values of B/D

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[14] M. Jakob, *Ann. der Physik*, Vol. 63, p. 537, 1920.

and $\mu/\rho D v$ the exponent in equation (30) is approximately represented by $m = 1/2$. For larger values of B/D and $\mu/\rho D v$ an average slope of $M = 2/3$ may be employed.

We may now summarize our conclusions on the forced convection from cylinders in a stream of fluid flowing at right angles to the axis, as follows:

$$W_c = \frac{2 \pi L \Delta \varphi}{\log_e b/D} \text{ watts convection} \quad (35)$$

where L = Length of cylinder assumed long compared with the diameter measured in cm.

$\Delta \varphi$ = Thermal conduction in watts per cm.
D = Diameter of cylinder in cm.
b = $2 B + D$ = Outside diameter of film in air.

For the film thickness we have

$$B = K D \left(\frac{\mu}{\rho D v} \right)^m = \text{Film thickness in cm.} \quad (36)$$

where μ = Viscosity of fluid for the geometrical mean of the wire and ambient temperature measured in c. g. s. units.
ρ = Density of fluid for the geometrical mean of the wire and ambient temperature measured in grams per cc.
D = Diameter of cylinder in cm.
v = Velocity of fluid in cm. per sec.

When $\frac{\mu}{\rho D v}$ is less than 0.013 and B/D is less than 0.24, we have $m = 1/2$ and $K = 2.15$. When $\frac{\mu}{\rho D v}$ is greater than 0.013 and B/D is greater than 0.24, we have $m = 2/3$ and $K = 4.27$.

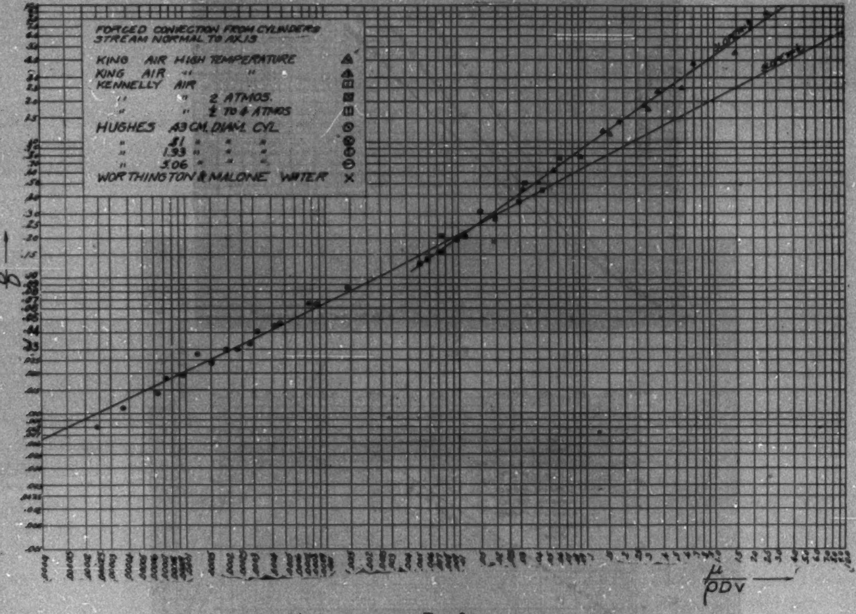

FIG. 6

V. Forced Convection from Spheres

For the case of forced convection from spheres we have only the meager data given by Compan[15] for a 2-cm. diameter lamp black-covered copper sphere. The tests were made in a wind tunnel. Compan gives the rate of cooling in deg. cent. per second for different sphere temperatures. The radiation correction is obtained as the rate of cooling in the best vacuum which he was able to obtain (less than 1/1000 mm.). The method of reducing his data to watts convection is given in the companion paper on free convection[1]. In Fig. 7 we have plotted the watts convection against $\Delta\varphi$ for the different velocities. A decided curvature is apparent, showing that here also the film thickness is affected by the sphere temperature with constant ambient conditions. As in the case of cylinders we have assumed that the appropriate temperature at which the viscosity and density should be taken to give the film thickness is the geometric mean of the sphere and ambient temperatures. It has been shown[1] that the convection from spheres on the film theory is given by

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[15] P. Compan, *Annales de Chemie et de Physique*, VIII, Series 26, p. 488, 1902.

$$W_c = \frac{2\pi\Delta\varphi}{1/D - 1/b} \quad \text{watts} \quad (37)$$

where $\Delta\varphi$ = Thermal conduction in watts per cm.
D = Diameter of sphere in cm.
b = $2B + D$ = Outer diameter of film in cm.

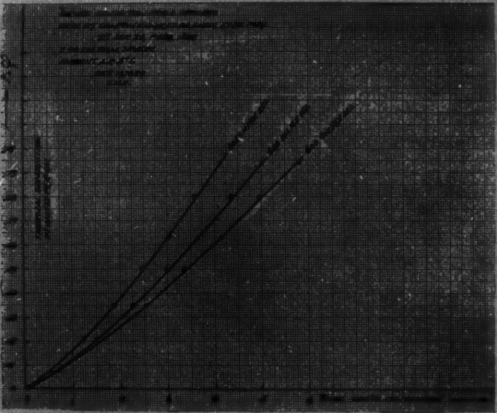

Fig. 7

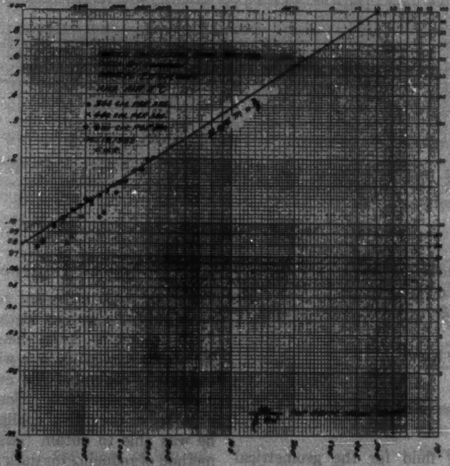

Fig. 8

From the above we obtain

$$B/D = 1/2 \left(\frac{2\pi \Delta \varphi D}{W_c - 2\pi \Delta \varphi D} \right) \qquad (38)$$

In the general equation (16) for the film thickness we associate D with the diameter of the sphere and determine K and m from the experiments. To this end we have calculated B/D and $\mu/\rho D v$ from the experimental data in Table IV and plotted the results on log: log paper in Fig. 8. From Fig. 8 we observe that the majority of points fall on a single curve having a slope $m = 2/3$. The three lowest temperature points show systematic departures which are probably due to experimental errors. The average value of the constant K is 35, assuming $m = 2/3$.

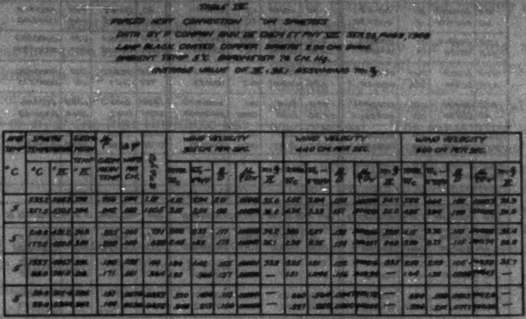

TABLE IV

FORCED HEAT CONVECTION ON SPHERES

DATA BY P. COMPAN ANN. DE CHEM ET PHY. XXVI SER. [illegible]

LAMP BLACK COATED COPPER SPHERE 5.00 CM. DIAM.

AMBIENT TEMP 5°C. BAROMETER 76 CM. Hg.

AVERAGE VALUE OF K = 35, ASSUMING m = 2/3

Amb. Temp.	Sphere Temperature		Geom. Mean Temp.	μ/ρ Geom. Mean Temp.	Δφ Watts per cm.	[illegible]	Wind Velocity [illegible] cm. per sec.					Wind Velocity 440 cm. per sec.					Wind Velocity [illegible] cm. per sec.				
°C	°C	°K	°K				Total W_c	$W_c -$ [illegible]	B/D	$\mu/\rho D v$	$m=2/3$ K	Total W_c	$W_c -$ [illegible]	B/D	$\mu/\rho D v$	$m=2/3$ K	Total W_c	$W_c -$ [illegible]	B/D	$\mu/\rho D v$	$m=2/3$ K
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.17	4.12	[illegible]	[illegible]	[illegible]	35.6	[illegible]	[illegible]	[illegible]	[illegible]	34.7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	257.5	530.5	[illegible]	[illegible]	[illegible]	1.005	[illegible]	[illegible]	[illegible]	[illegible]	36.0	[illegible]	3.33	[illegible]	[illegible]	35.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.731	[illegible]	2.33	.177	[illegible]	34.2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.15	[illegible]	[illegible]	[illegible]	35.4
	172.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.45	[illegible]	.173	[illegible]	36.1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.116	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.163	[illegible]	33.8	2.25	[illegible]	[illegible]	[illegible]	33.5	[illegible]	[illegible]	.106	[illegible]	35.7
	[illegible]	[illegible]	[illegible]	.171	[illegible]	[illegible]	[illegible]	[illegible]	.137	[illegible]	—	1.51	[illegible]	[illegible]	[illegible]	—	1.64	1.30	[illegible]	[illegible]	—
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.530	[illegible]	.110	[illegible]	—	[illegible]	[illegible]	[illegible]	[illegible]	—	[illegible]	[illegible]	[illegible]	[illegible]	—
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	—	[illegible]	[illegible]	[illegible]	[illegible]	—	[illegible]	[illegible]	.0771	[illegible]	—

We may summarize our data on spheres for the available range by noting that the *forced* convection is given by equation (37), in which

$$B \;\; = 35.\, D \left(\frac{\mu}{\rho D v} \right)^{2/3} \qquad \text{cm.} \qquad (39)$$

where D = Sphere diameter in cm.

μ/ρ = Viscosity divided by density of the fluid, for the geometrical mean of the sphere and ambient temperatures, measured in c. g. s. units.

v = Velocity of fluid in cm. per sec.

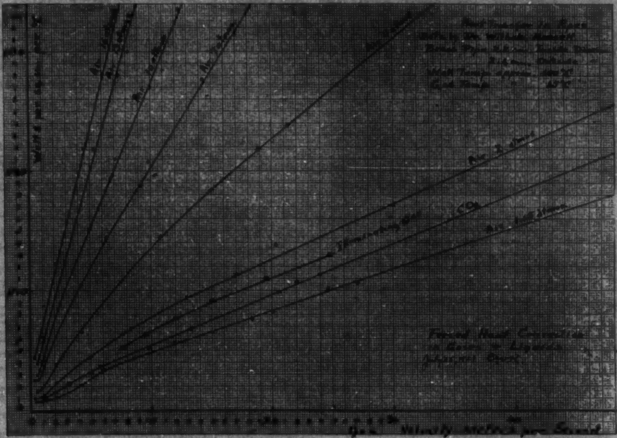

FIG. 9

VI. Forced Heat Convection in Pipes

For this case we have the valuable contributions of Nusselt[7]. His data were given in the form of curves, the units being kilogram calories per hour per square

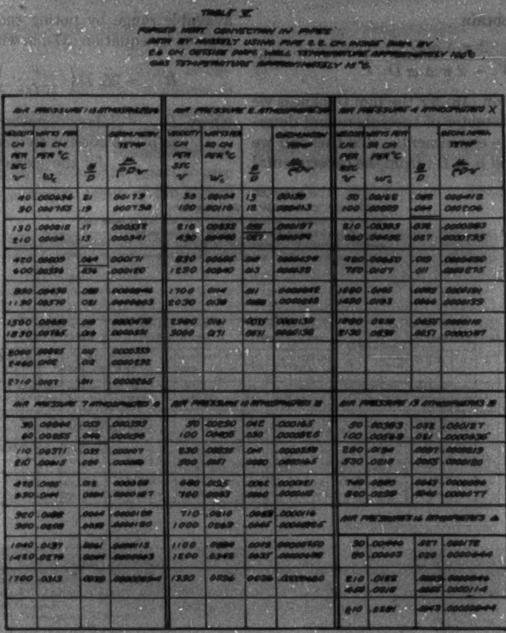

TABLE X

FORCED HEAT CONVECTION IN PIPES
DATA BY NUSSELT USING PIPE 2.2 CM. INSIDE DIAM. BY
2.6 CM OUTSIDE DIAM. WALL TEMPERATURE APPROXIMATELY 100°C.
GAS TEMPERATURE APPROXIMATELY 15°C.

AIR PRESSURE 1.15 ATMOSPHERES ●

VELOCITY CM PER SEC v	WATTS PER 50 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
40	.000696	.21	.00173
90	.000765	.19	.000738
130	.000812	.17	.000332
210	.00104	.13	.000341
420	.00209	.064	.000171
600	.00396	.036	.000120
850	.00430	.029	.0000846
1130	.00570	.021	.0000603
1500	.00650	.019	.0000478
1830	.00765	.016	.0000391
2090	.00825	.015	.0000339
2460	.0112	.012	.0000292
2710	.0107	.011	.0000265

AIR PRESSURE 2 ATMOSPHERES ●

VELOCITY CM PER SEC v	WATTS PER 50 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
50	.00104	.13	[illegible]
100	.00116	.12	.000413
210	.00232	.055	.000197
430	.00440	.027	[illegible]
830	.00685	.018	.0000436
1290	.00940	.013	.0000338
1700	.0114	.011	.0000242
2030	.0138	[illegible]	.0000218
2980	.0161	.0075	.0000139
3000	.0171	.0071	.0000136

AIR PRESSURE 4 ATMOSPHERES X

VELOCITY CM PER SEC v	WATTS PER 50 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
50	.00168	.082	.000418
100	.00283	.064	.000206
210	.00383	.038	.0000983
380	.00492	.027	.0000735
480	.00650	.019	.0000480
750	.0107	.011	.0000275
1080	.0145	.0080	.0000191
1490	.0193	.0066	.0000139
1980	.0218	.0055	.0000110
2130	.0238	.0051	.0000097

AIR PRESSURE 7 ATMOSPHERES ●

VELOCITY CM PER SEC v	WATTS PER 50 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
30	.00844	.059	.000393
60	.00855	.046	.000196
110	.00371	.035	.000107
210	.00615	.020	[illegible]
420	.0105	.012	.000069
630	.0141	.0084	.0000187
920	.0188	.0064	.0000128
980	.0208	.0059	.0000120
1040	.0137	.0061	.0000113
1420	.0270	.0044	.0000083
1700	.0313	.0030	[illegible]

AIR PRESSURE 10 ATMOSPHERES ▣

VELOCITY CM PER SEC v	WATTS PER 50 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
50	.00290	.042	.000165
100	.00406	.030	.0000826
230	.00836	.014	.0000357
500	.0151	.0080	.0000165
680	.0125	.0062	.0000121
700	.0183	.0060	.0000118
710	.0210	.0063	.0000116
1000	.0269	.0045	.0000826
1100	.0284	.0045	.00000750
1200	.0342	.0035	.00000688
1330	.0336	.0036	.00000620

AIR PRESSURE 13 ATMOSPHERES ▨

VELOCITY CM PER SEC v	WATTS PER 50 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
50	.00383	.032	.000127
100	.00569	.021	.0000635
280	.0134	.0097	.0000213
530	.0218	.0055	.0000120
740	.0250	.0043	.0000086
860	.0259	.0040	.0000077

AIR PRESSURE 16 ATMOSPHERES △

VELOCITY CM PER SEC v	WATTS PER 50 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
30	.00440	.037	.000178
60	.00603	.026	.0000844
210	.0182	.0063	.0000246
460	.0218	.0055	.0000114
610	.0261	.0043	.00000844

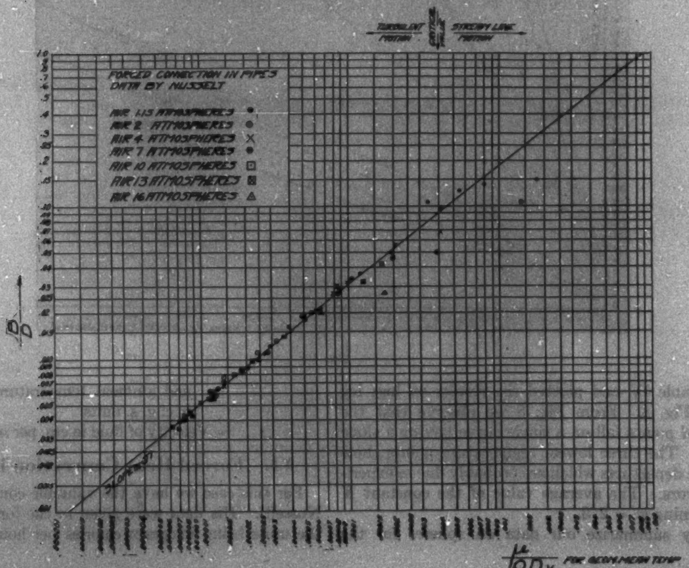

Fig. 10

meter of surface per degree centigrade temperature difference against mean gas velocity in meters per second. We have reproduced his curves as well as the scale would permit in Fig. 9, the units being changed to watts per square cm. per deg. cent. against meters per second velocity. In Table V we have calculated B/D and $\mu/\rho D v$ for air at different densities. For small values of B/D the film thickness has been calculated from the relation.

$$W_c = (A/B)\,\Delta\varphi \qquad \text{watts convection} \qquad (40)$$

where A = Area of heat flow path in sq. cm.

B = Film thickness in cm.

$\Delta\varphi$ = Thermal conduction watts per cm.

For the larger values of B/D the film thickness has been calculated from the relation

$$W_c = \frac{2\pi L\,\Delta\varphi}{\log_e D/b} \qquad \text{watts convection} \qquad (41)$$

where L = Length of pipe, assumed long compared with the diameter, measured in cm.

$\Delta\varphi$ = Thermal conduction in watts per cm.

D = Diameter of pipe in cm.

b = $D - 2B$ = Inside diameter of film in cm.

In Fig. 10 the values of B/D have been plotted against $\mu/\rho D v$ for the geometrical mean temperature on log : log paper. The points lie on a single curve as required by the theory. The mean slope is $m = 0.77$ and the constant $K = 37$. Without producing an appreciable error we may force the slope to $m = 3/4$ and use $K = 30$. Obviously we should not attempt to apply the film theory of forced convection below the critical velocity, that is, for values of $\mu/\rho D v$ greater than 0.0004 where stream line motion exists.

To summarize our results on the heat transfer in pipes, we use equation (40) when the film thickness is small compared with the pipe diameter and equation (41) when the film is more than 5 per cent of the pipe diameter. The film thickness is given by

$$B = K D \left(\frac{\mu}{\rho D v} \right)^m \qquad \text{cm.} \qquad (42)$$

D = Pipe diameter in cm.

μ/ρ = Viscosity divided by density of fluid at the geometric mean temperature of the pipe and contained fluid measured in c. g. s. units

v = Mean velocity of flow in cm per sec.

The best values of the exponent and corresponding value of the constant are

$m = 0.77$ and $K = 37$.

Without producing an appreciable error we may use the simpler values

$m = 3/4$ and $K = 30$.

VII. Experiments on Forced Heat Convection in Gases and Liquids

The present work was undertaken at the suggestion of Dr. Whitney to determine the relative forced heat convection in various gases. It is included here as a supplement to the data already discussed.

Fig. 11 will give a general idea of the apparatus used. Fig. 12 is a cross-section of the constricted section showing the torpedo-shaped heating unit with the principal dimensions.

The procedure for making a run at atmospheric pressure consisted in scavanging out the apparatus by running the centrifugal blower and passing gas into and out of the system until a state of sufficient purity was reached. The apparatus was quite tight but in order to insure pure gas during the tests a small constant inflow and outflow was maintained.

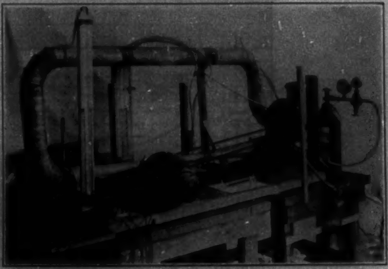

Fig. 11—Forced Heat Convection in Gases and Liquids

Upon leaving the centrifugal fan in a four-inch diameter pipe, the gas entered a reducing cone which was connected to the two-inch constriction. The velocity was here measured in the center of the pipe with a pitot tube, and is therefore approximately the maximum velocity, not the mean velocity. The pressure and velocity orifices of the pitot tube were connected to a large diameter U-tube (3/4 in. inside diameter) which was filled at first with water and later with a mineral oil of density 0.88 grams per cc. at 25 deg. cent. The oil and large diameter U-tube were used to reduce the meniscus errors. The velocity was calculated from the usual equation for pitot tubes, the constant C being assumed unity.

$$v = C\sqrt{2 g h \rho_1/\rho_2} \qquad \text{cm. per second} \qquad (43)$$

where g = 981 cm. per sec.^2

h = difference of levels in U-tube in cm.

ρ_1 = density of liquid in U-tube grams per cc.

ρ_2 = density of gas being measured, grams per cc.

The velocity of the gas was again increased by entering the constricted annular space around the torpedo-shaped heater. The velocity in this annular space was calculated from the observed central or maximum velocity in the two-inch pipe by multiplying by the ratio of areas (1.37) and is the value plotted in the curves. After leaving the two-inch pipe the gas entered an expansion cone which brought it again to a four-inch pipe. The gas next passed through a cooler consisting of a helical copper tube with water circulation which was located in the upper part of the four-inch diameter pipe. The gas then returned to the intake of the centrifugal fan. A check on the velocity of flow in the different gases was obtained from the fan revolutions. The velocity was found to be proportional to the fan revolutions for all gases within the experimental error. The temperature of the gas approaching the heater was measured at the beginning of the reducing cone and the temperature of the gas leaving the heater was measured at the end of the expansion cone. The gas entered the heater in the vicinity of 25 deg. cent. and left the heater from 1 deg. to 3 deg. cent. higher. The temperature of the heater was determined from the known temperature coefficient of copper according to the following relations

$$R_0 = \frac{R_t}{1 + 0.0042\,t} \quad \text{ohms} \quad (44)$$

$$t = \frac{R_t - R_0}{0.0042\,R_0} \quad \text{deg. cent.} \quad (45)$$

During a run on a particular gas the temperature of the heater was maintained approximately constant by adjustment of a series rheostat in order to maintain the viscosity, density, and heat conductivity of the gas approximately constant. The resistance of the heater was determined by voltmeter and ammeter readings with reasonable accuracy. The variation of heater temperature for a given run might be 80 deg. ± 5 deg. cent.

For the tests in water a similar heater was placed in one side of a 20-inch by 2-inch diameter glass U-tube. The water was passed into the U-tube by an expanding cone from a 3/4-inch water pipe. After leaving the heater the water passed out of the other arm of the U-tube into the waste pipe. The velocity in this case was calculated from the quantity of flow as measured with stop watch and pail. The velocities are therefore mean velocities and are not strictly comparable with the velocities obtained by the pitot tube for the gas tests. The velocities plotted in the curves for water are these mean velocities. Correction to a common velocity basis was not considered necessary as the tests on water were quite rough and merely desired to check the order of magnitude.

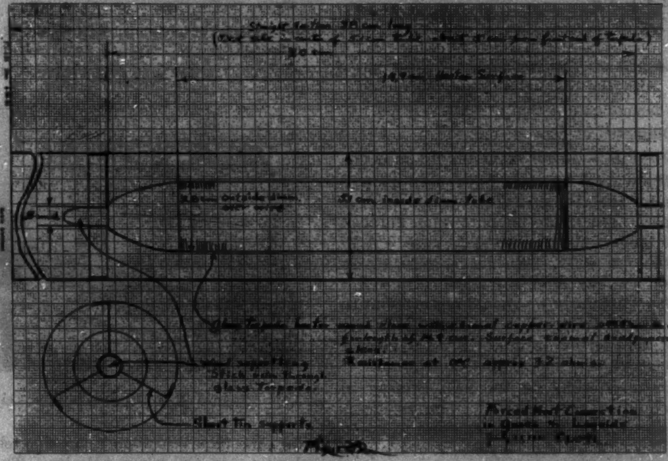

FIG. 12

During the tests on water with day light at a proper angle on the glass U-tube, a line of demarcation could be observed between the water very close to the heater and that at some distance. At high velocities this film appeared very thin, less than a millimeter, while at the low velocities it became thicker and wavy or spouting in appearance. A study of the surface condition of a heated body in a liquid in this manner would undoubtedly be of considerable interest in adding to our knowledge of heat transfer and the intimately related problem of fluid friction.

The radiation and conduction losses have been neglected. With this type of apparatus the conduction paths are lead losses, straight conduction through the gas and conduction from the wire on the cylindrical part of the glass torpedo to the sheet tin supports. These losses are all small compared with the forced heat convection.

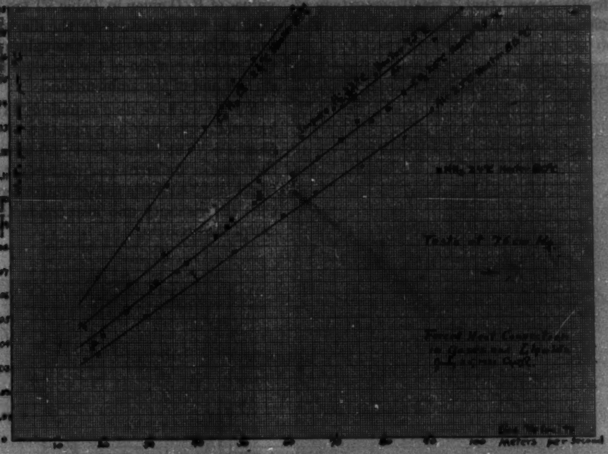

Fig. 13

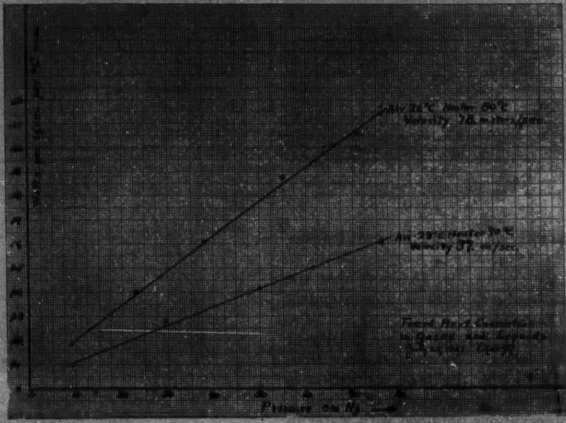

Fig. 14

Fig. 13 summarizes the results for different gases at atmospheric pressure as obtained with the above apparatus. The ordinates give the watts dissipated per sq. cm. of surface per deg. cent. temperature difference. In figuring the temperature difference the mean of inflowing and outflowing gas temperature was used and the average temperature of the heater obtained from the resistance. The average condition for each gas is tabulated on the curve Fig. 13. The heater temperature for the individual points varying by as much as ± 10 deg. cent. in some cases.

The abscissas give the velocity measured by the pitot tube situated in the middle of the 2-in. pipe about 4 inches in front of the torpedo multiplied by the ratio of the two areas, *i. e.* multiplied by 1.37.

We have not as yet obtained a run with pure hydrogen. The difficulty here is that a small percentage of one of the heavier gases produces a marked effect on the density and consequently on velocity and heat transfer determinations.

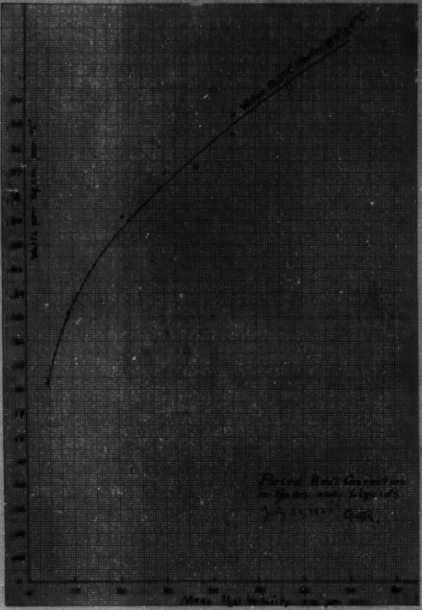

Fig. 15

Examination of the curves Fig. 15 shows that the heat convection at high velocities is approximately proportional to the gas velocity.

In Fig. 14 the effect of varying the air density at constant velocity is shown. We may observe from this that for high velocities the heat convection is approximately proportional to the density of the gas.

In Fig. 15 the observations obtained with water in the same type and dimensions of apparatus are given. The ordinates are the watts per square centimeter of heater surface per degree centigrade temperature difference. The abscissas give the mean velocity over the heater surface in the constricted annular space.

In Fig. 16 the heat convection is given for constant mean velocity as a function of the heater temperature.

In these experiments the film thickness is small compared with the dimensions of test body and therefore we may treat the problem as the flow of heat between parallel plane surfaces. We have therefore calculated the film thickness from equation (18). The values of B/D given in Table VI are obtained by dividing the film thickness by the diameter D of the heater unit which was 2.8 cm. The values of $\mu/\rho\, D\, v$ are for the geometric mean of the absolute temperature of the surface and of the ambient fluid.

For air we have used equation (33) to determine the kinemetic viscosity as a function of the absolute temperature T at atmospheric pressure. In considering the effect of different gas pressures it should be recalled that both the kinetic theory and experiment have shown that the viscosity and heat conductivity are independent of the pressure.

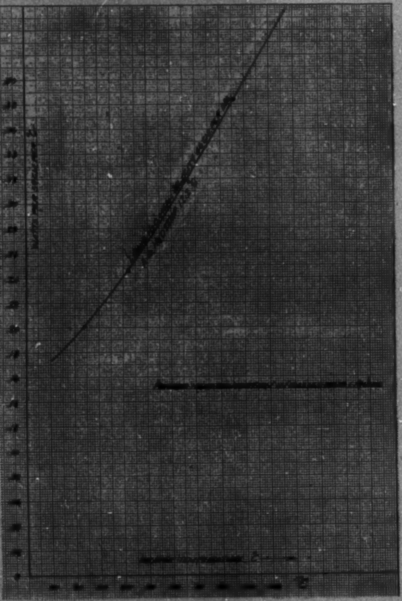

Fig. 16

Langmuir's values of φ as given in Fig. 1 have been used for air. For carbon dioxide at atmospheric pressure and 322 deg. K. we have taken the following values.

ρ = 0.001675 grams per cc.

μ = 0.0001625 cm.$^{-1}$ grams sec.$^{-1}$.

The heat conductivity has been calculated from the following relation given by Jeans[16]

$$k = 1/4\,(9\gamma - 5)\,\mu\,c_v \text{ cal. sec.}^{-1}\text{ cm.}^{-1}\text{ deg. cent}^{-1} \qquad (46)$$

For CO_2 we have assumed

$\gamma = C_p/c_v = 1.3$ Ratio of specific heats (47)

$c_v = 0.165$ cal. grams^{-1} at const. vol.

For the average temperature of the surface and ambient CO_2, of 323 deg. K. we have

$k = 0.000045$ cal sec.$^{-1}$ cm.$^{-1}$ deg. cent.$^{-1}$

Miltiplying by 4.19 we have

$k = 0.0001885$ watts cm.$^{-1}$ deg. cent.$^{-1}$

For small temperature differences we may write

$$\Delta\varphi = k_{avg.\ temp.}\,\Delta t \text{ watts per cm.} \qquad (48)$$

For our temperature range in CO_2 we thus obtain

$\Delta\varphi = 0.00943$ watts per cm.

For ammonia by a similar process we have obtained for 324 deg. K. the following

$\rho = 0.00065$ grams per c.c.

$\mu = 0.000126$ cm.$^{-1}$ grams sec.$^{-1}$

For the heat conductivity of NH_3 at 325 deg. K.

$k = 0.00036$ watts per cm. per deg. cent.

H-9511c

Assuming $\gamma = 1.31$; $c_v = 0.39$ cal gram^{-1} at const. val.

For ethyl chloride we have taken the following approximate values to represent the condition at 319 deg. K.

$\rho = 0.00244$ grams per c.c.

$\mu = 0.000106$ cm.$^{-1}$ grams sec.$^{-1}$

$k = 0.000139$ watts per cm. per deg. cent.

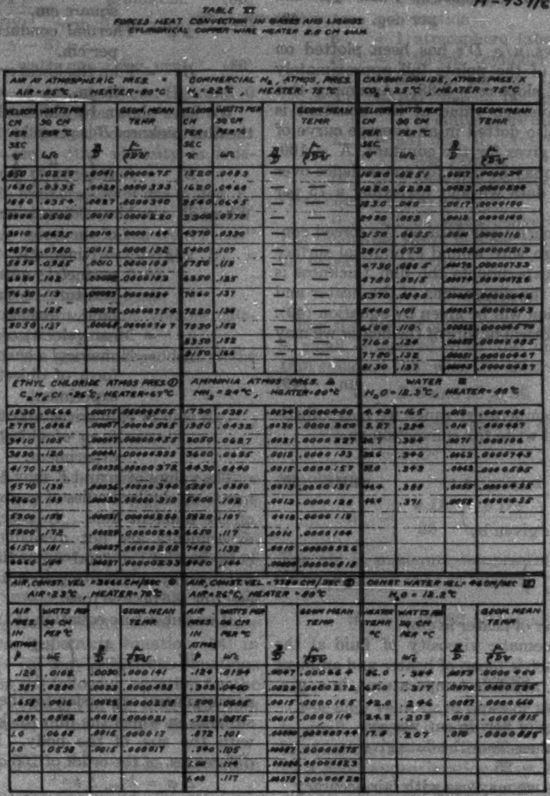

TABLE XI

FORCED HEAT CONVECTION IN GASES AND LIQUIDS

CYLINDRICAL COPPER WIRE HEATER 2.8 CM DIAM

AIR AT ATMOSPHERIC PRES. • AIR = 25°C, HEATER = 80°C				COMMERCIAL H_2, ATMOS. PRES. H_2 = 22°C, HEATER = 75°C				CARBON DIOXIDE, ATMOS. PRES. X CO_2 = 25°C, HEATER = 75°C			
VELOCITY CM PER SEC v	WATTS PER 30 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$	VELOCITY CM PER SEC v	WATTS PER 30 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$	VELOCITY CM PER SEC v	WATTS PER 30 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
850	.0229	.0041	.0000475	1520	.0483	—	—	1020	.0251	.0027	.000034
1630	.0335	.0029	.0000393	1620	.0466	—	—	1220	.0282	.0023	.0000284
1880	.0354	.0027	.0000340	2540	.0645	—	—	1830	.048	.0017	.0000190
[illegible]	.0500	.0018	.0000220	3300	.0770	—	—	2490	.053	.0013	.0000140
3010	.0635	.0014	.0000164	4370	.0930	—	—	3150	.0655	.0011	.0000110
4870	.0780	.0012	.0000132	5400	.107	—	—	3810	.073	.00085	.00000813
5850	.0925	.0010	.0000103	5750	.113	—	—	4730	.0865	.00076	.00000733
6880	.102	[illegible]	.0000103	6350	.125	—	—	4780	.0915	.00074	.00000726
7630	.113	.00083	.00000834	7060	.137	—	—	5370	.0940	.00066	.00000646
8500	.125	.00075	.00000754	7220	.138	—	—	5460	.101	.00067	.00000643
9050	.137	.00068	.00000707	7930	.152	—	—	6100	.110	.00062	.00000570
				8350	.152	—	—	7160	.124	.00055	.00000485
				9150	.166	—	—	7780	.132	.00051	.00000447
								8130	.137	.00049	.00000427

ETHYL CHLORIDE ATMOS PRES. ① C_2H_5Cl = 26°C, HEATER = 67°C				AMMONIA ATMOS PRES. △ NH_3 = 24°C, HEATER = 80°C				WATER ■ H_2O = 12.3°C, HEATER = 80°C			
1930	.0666	.00075	.00000805	1730	.0381	.0034	.0000400	4.43	.165	.013	.000486
2750	.0868	.00057	.00000565	1980	.0432	.0030	.0000350	5.27	.234	.014	.000457
3410	.105	.00047	.00000455	3050	.0627	.0021	.0000227	26.7	.354	.0071	.000106
3830	.120	.00041	.00000333	3600	.0695	.0019	.0000193	[illegible]	.340	.0063	.0000743
4170	.123	.00038	.00000372	4430	.0840	.0015	.0000157	[illegible]	.343	.0062	.0000585
4570	.138	.00036	.00000340	5280	.0980	.0013	.0000131	[illegible]	.355	.0055	.0000435
4860	.143	.00033	.00000310	5400	.102	.0013	.0000126	[illegible]	.371	.0052	.0000436
5300	.158	.00031	.00000288	5820	.107	.0012	.0000118				
5900	.172	.00029	.00000263	6650	.117	.0011	.0000104				
6150	.181	.00027	.00000252	7480	.132	.0010	.00000926				
6660	.184	.00027	.00000233	9480	.144	[illegible]	.00000818				

AIR, CONST. VEL. = 3666 CM/SEC ● AIR = 23°C, HEATER = 70°C				AIR, CONST. VEL. = 7780 CM/SEC ⊕ AIR = 26°C, HEATER = 80°C				CONST. WATER VEL. = 46 CM/SEC ⊠ H_2O = 12.2°C			
AIR PRES. IN ATMOS p	WATTS PER 30 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$	AIR PRES. IN ATMOS p	WATTS PER 30 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$	HEATER TEMP °C	WATTS PER 30 CM PER °C W_c	$\frac{B}{D}$	GEOM. MEAN TEMP $\frac{\mu}{\rho D v}$
.120	.0102	.0030	.000141	.124	.0194	.0047	.0000664	36.0	.384	.0053	.0000460
.387	.0280	.0022	.0000488	.303	.0460	.0028	.0000272	65.0	.317	.0070	.0000586
.658	.0416	.0022	.0000250	.500	.0605	.0015	.0000165	42.0	.246	.0087	.0000660
.907	.0562	.0018	.0000021	.723	.0875	.0010	.0000114	24.2	.203	.010	.0000815
1.0	.0608	.0016	.0000017	.872	.101	[illegible]	.00000944	17.0	.207	.010	.0000885
1.0	.0538	.0015	.0000017	.940	.105	.00087	.00000875				
				1.00	.114	[illegible]	.00000823				
				1.00	.117	.00076	.00000828				

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[16] J. H. Jeans Dynamical Theory of Gases, *Camb. Univ. Press*, p. 318, 1916. (Second Edition).

Assuming $r = 1.13$; $c_v = 0.243$ cal gram.$^{-1}$ at const. volume.

For water the values of viscosity and density given in Fig. 17, were taken from Kaye and Laby (Physical and Chemical Constants).

The heat conductivity of H_2O is from Jakob's[14] recent work, which may be summarized in the range from $t = 0$ deg. cent. to $t = 100$ deg. cent. by

$$k = 0.00555 \quad (1 + 0.00298\, t) \quad \text{watts per cm. per deg. cent.} \qquad (49)$$

In Fig. 18, B/D vs. $\mu/\rho\ D\,v$ has been plotted on log : log paper. All of the points fall approximately on a single curve of slope $m = 0.88$. Assuming this value of slope the average value of the constant is $K = 25$. We have also dotted in an average curve of slope $m = 1.0$ which gives for the constant, $K = 100$. This simpler relation may be used for convenience over the present range of data without exceeding a 15 per cent error.

We may now summarize our data on the forced convection from the rough wire wound cylindrical surface as follows: In these experiments the film thickness is small compared with the dimensions of the heater and therefore the convection will be given by equation (**40**).

In view of the cylindrical nature of the heater we may write

$$A = \pi D L \qquad \text{cm.}^2 \qquad (49)$$

Also for the small temperature range we may substitute equation (**48**) and write the convection as

$$W_c = \frac{\pi D L}{B}\, k_{avg\ temp.}\, \Delta t \qquad \text{watts} \qquad (50)$$

The film thickness is given by

$$B = K D \left(\frac{\mu}{\rho D v} \right)^m \qquad \text{cm.} \qquad (51)$$

Where D = diameter of heater in cm.

$\mu/\rho = \nu$ = kinematic viscosity of fluid at the geometric mean of heater and ambient fluid, absolute temperatures, in c. g. s. units.

v = Velocity of flow in cm. per sec.

The best average values from the data give

$m = 0.88$; $K = 25$

By forcing the slope we may use with fair accuracy

$m = 1.0$; $K = 100$.

A comparison of these data with Nusselt's data on the heat transfer in smooth pipes shows that for the same value of $\mu/\rho\ D\,v$ Nusselt's value of B/D is approximately five times the value which we obtained for the rough copper wire heater. This means that our rate of heat transfer is approximately five times as great as Nusselt's. A portion of this difference is probably due to the increased area of the rough surface; the remainder being due to the difference in the flow conditions.

VIII. Forced Convection from Large Surfaces

For high velocities and large bodies the film thickness will usually be small compared with the dimensions of the body and therefore the "shape conductance" will have the simple form given by equation (**17**) and the forced convection will be

$$W_c = (A/B)\, \Delta \varphi \qquad \text{watts convection} \qquad (52)$$

Where A = Area of cross-section of heat flow path in square cm.

$\Delta \varphi$ = Thermal conductance of fluid in watts per cm.

B = Film thickness or length of heat flow path in cm.

We have shown by the method of dimensions that the film thickness B is given by

$$B = K D \left(\frac{\mu}{\rho D v} \right)^m \qquad \text{cm.} \qquad (53)$$

Where D = Linear dimension characterizing the form of the body

$\mu/\rho = \nu$ = Kinemetic viscosity of fluid for the geometric mean of the absolute temperature of the surface and ambient fluid, measured in c. g. s. units.

v = Mean velocity of fluid in cm. per sec.

K = Numerical constant, depending on the form of object, to be determined by experiment

m = Numerical exponent, depending on the form of object, to be determined by experiment.

The values of m and K may be determined by experiments on similar bodies of various sizes or by variations of the ambient fluid. In practical work, where such experiments are not available for the body under consideration, a good approximation can usually be obtained by substituting an equivalent cylinder, sphere, etc., for the desired form.

If we substitute equation (**53**) in (**52**) we obtain

$$W_c = \frac{A}{K\ D} \left(\frac{\rho D v}{\mu} \right)^m \Delta \varphi \qquad \text{watts} \qquad (54)$$

as our general expression for the forced convection from surfaces in which the film thickness is small compared with the size of the body. For small temperature differences, of the order of 100 deg. cent., we may write

$$\Delta \varphi = k_{avg.}\, \Delta t \qquad \text{watts per cm.} \qquad (54)$$

where $k_{avg.}$ = Heat conductivity of fluid for the average of the surface and ambient fluid temperatures measured in watts per cm. per deg. cent.

Δt = Temperature difference between surface and ambient fluid in deg. cent.

Cylinders

For cylinders in a stream of fluid at right angles to the axis we have for the case of thin films, namely; B/D less than 0.05

$$m = 1/2 \text{ and } K = 2.2 \qquad (55)$$

If we substitute these values in equation (54) writing for the area A of length L.

$$A = \pi D L \qquad \text{sq. cm.} \qquad (56)$$

We obtain

$$W_c = \frac{\pi L}{2.2}\left(\frac{\rho D v}{\mu}\right)^{1/2} \Delta\varphi \quad \text{watts} \qquad (57)$$

For gases we have the following relations

$$\rho = \rho_0\,(273/T) \qquad \text{grams cm.}^{-3} \qquad (58)$$

$$\mu = \mu_0\,(T/273)^n \qquad \text{cm.}^{-1}\ \text{gram sec.}^{-1} \qquad (59)$$

$$k = 1.05\,(9\,\gamma - 5)\,\mu\,c_v \qquad \text{watts cm.}^{-1}\ \text{deg. cent}^{-1} \qquad (60)$$

where ρ_0 = Density of gas at 76 cm. Hg. and 0 deg. cent. in grams cm.$^{-3}$

μ_0 = Viscosity of gas at 0 deg. cent. in cm.$^{-1}$ gram sec.$^{-1}$ (the viscosity is independent of pressure).

T = Absolute temperature of gas in deg. K.

n = Exponent depending on gas. The following values are taken from Jeans[14] page 302, Air, $n = 0.754$; H_2, $n = 0.69$; CO_2, $n = 0.98$

$\gamma = c_p/c_v$ Ratio of specific heats

c_v = specific heat at constant volume calories gram^{-1} deg. cent.$^{-1}$

The heat conductivity of a gas is independent of the pressure.

For Air we have

$\rho_0 = 0.00129$ grams cm.$^{-3}$ at 76 cm. Hg.

$\mu_0 = 0.0001705$ cm.$^{-1}$ gram sec.$^{-1}$

$n = 0.754$

From which we have for air at atmospheric pressure

$$\rho/\mu = 7.56\,(273/T)^{1.754} \qquad \text{cm.}^{-2}\ \text{sec.} \qquad (61)$$

$$\sqrt{\rho/\mu} = 2.75\,(273/T)^{0.877} \qquad (62)$$

We have shown[1] that the heat conductivity of air at moderate temperature may be written

$$k = 3.4 \times 10^{-6}\, T^{0.754} \ \text{watts cm.}^{-1}\ \text{deg. cent.}^{-1} \qquad (63)$$

For moderate temperature differences of the order of 100 deg. cent. we may substitute equation (54) in equation (57). For *air* we may substitute equations (62) and (63) and obtain the following equation for the forced convection from cylinders in *air* where the film thickness is small compared to the cylinder diameter.

$$W_c = 0.0018\, L \sqrt{p\, D\, v}\, \frac{\Delta t}{T_{avg}^{\ .123}} \ \text{watts convection} \qquad (46)$$

where L = Length of cylinder in cm.

p = Absolute air pressure in atmospheres

D = Cylinder diameter in cm.

v = Air velocity in cm. per sec.

Δt = Temperature difference between surface of cylinder and ambient air in deg. cent.

$T_{avg.}$ = Average absolute temperature of surface and ambient air deg. K.

In deriving equation (64) we have used the arithmetic mean temperature in determining the kinematic viscosity since for moderate temperature differences the geometric and arithmetic means are approximately the same. As a check on the relation let us apply it to calculating the convection given by Hughes for the 1.93 cm. diameter cylinder given in Table III. We have

L = 1 cm. (Length of cylinder under consideration)

p = 1 atmosphere (absolute air pressure)

D = 1.93 cm. (cylinder diameter)

v = 500 cm. per sec. (air velocity)

Δt = 100 deg. − 15 deg. = 85 deg. cent. (Temperature difference)

$T_{avg.}$ = 331 deg. K. (Average of Cyl. and Amb. Air Temp.)

W_c = 2.46 watts convection observed

From equation (64) we have

W_c = 2.34 watts convection calculated

The calculated result is 6 per cent low, which is within the experimental error.

SPHERES

For the case of spheres in which the film thickness is small compared with the diameter, we associate D in equation (53) with the sphere diameter and take

$$m = 2/3 \text{ and } K = 35 \qquad (65)$$

If we substitute these values in equation (54) writing

$$A = \pi D^2 \qquad \text{sq. cm.} \qquad (66)$$

we obtain

$$W_c = \frac{\pi D}{35}\left(\frac{\rho D v}{\mu}\right)^{2/3} \Delta\varphi \qquad (67)$$

For *air* at moderate temperature differences we may write equation (67) as follows

$$W_c = 0.00082\, D^{5/3}\,(p\,v)^{2/3}\, \frac{\Delta t}{T_{avg.}^{0.416}} \qquad \text{watts} \qquad (68)$$

PIPES

For high velocity in pipes or where the film thickness is small compared with the diameter we associate D in equation (53) with the diameter of the pipe and take as a good approximation

$$m = 3/4 \text{ and } K = 30$$

Substituting these values in equation (54) we obtain

$$W_c = \frac{\pi L}{30}\left(\frac{\rho D v}{\mu}\right)^{3/4} \Delta\varphi \quad \text{watts} \qquad (69)$$

For *air* at moderate temperature differences we may write equation (69) as

$$W_c = 0.002\, L\,(p\, D\, v)^{3/4}\, \frac{\Delta t}{T_{avg.}^{0.561}} \qquad \text{watts} \qquad (70)$$

PLANE SURFACES

For plane surfaces the film thickness should be independent of any linear dimension. This condition requires us to put $m = 1$ in equation (53), from which

$$B = K \frac{\mu}{\rho\, v} \quad \text{cm.} \tag{71}$$

Substituting this relation in equation (52) we obtain for infinite plane surfaces

$$W_c = \frac{A\, \rho\, v}{K\, \mu} \Delta \varphi \quad \text{watts convection} \tag{72}$$

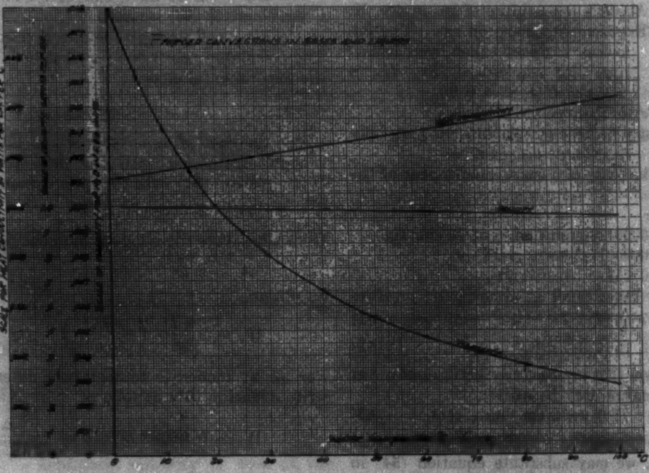

FIG. 17

There are no data available on the forced convection from large plane surfaces but we may obtain an estimate of the value of K from Nusselt's data on heat transfer in pipes, and our data obtained with a cylindrical heater in the middle of a tube. Another way of viewing the matter is to observe that the conditions pertaining in an infinite pipe should be similar to an infinite plane surface. From Table V we have for air at 10 atmosphere pressures

A = 1 sq. cm. (Area of surface)
p = 10 atmospheres (Air pressure)
v = 1330 cm. per sec. (Mean Air Velocity)
Δt = 100 deg. cent. − 15 deg. cent. = 85 deg. cent.
W_c = 0.0336 × 85 = 2.86 watts per sq. cm.
$\Delta \varphi = \varphi_2 - \varphi_1 = 0.0585 - 0.036 = 0.0225$ watts per cm.
$\rho/\mu = 7.56\,(273/327)^{1.734} = 5.5$ at Geom. mean temp. and 76 cm. Hg.

Substituting these values in equation (72) and solving for K we have

$$K = \frac{1 \times 10 \times 5.5 \times 1330 \times 0.0225}{2.86} = 575. \tag{73}$$

From Table VI we write the following point for air at atmospheric pressure

A = 1 sq. cm. (Area of surface)
p = 1 atmosphere
v = 8500 cm. per sec.
Δt = 80 deg. cent. − 25 deg. cent. = 55 deg. cent.
W_c = 0.125 × 55 deg. cent. = 6.88 watts per sq. cm.
$\Delta \varphi$ = 0.0144 watts per cm.
ρ/μ = 5.6 at Geom. mean temp.

Substituting these values in equation (72) and solving for K we obtain

$$K = 100 \tag{74}$$

The value of K estimated from Nusselt's data is seen to be approximately six times as large as the value obtained from our data. In our tests the high-velocity gas was constricted in its flow over the wire wound heater by the surrounding tube. In Nusselt's tests the gas flowed through a smooth brass pipe. Thus a portion of the difference in the values of K is to be expected from the difference in test conditions and the remainder due to the greater roughness of our wire wound heater. In a subsequent paper, connecting the film thickness with the mechanical resistance to flow, it will be shown that the heat transfer from a rough surface, equivalent to coarse sand, is approximately twice that obtained from a smooth surface.

Data on the heat transfer at high velocities from smooth and rough surfaces are needed to select the best value of K for engineering calculations. For the present we will take an average of the two values obtained above or

$$K = 340 \qquad (75)$$

For moderate temperature differences we may substitute equation (54) in (72) and obtain for large plane surfaces at high velocities

$$W_c = 0.003 \frac{A \rho v}{\mu} k_{avg.} \Delta t \quad \text{watts} \qquad (76)$$

Where

A = Area of surface in sq. cm.

ρ = Density of fluid in grams per cc. for the average of surface and ambient temperatures

μ = Viscosity of the fluid in c. g. s. units for the average of surface and ambient temperatures

v = Mean velocity of fluid over surface in cm. per sec.

$k_{avg.}$ = Heat conductivity of fluid, for the average of surface and ambient temperatures, in watts per cm. per deg. cent.

Δt = Temperature difference deg. cent.

The arithmetic mean has been used instead of the geometric mean temperature in determining the kinematic viscosity of the fluid since, for the moderate temperature differences here under consideration, the two means are practically equal.

For *air* we may substitute equations (61) and (63) in (76) and obtain

$$W_c = 0.0014\, A\, p\, v \frac{\Delta t}{T_{avg.}} \quad \text{watts convection} \qquad (77)$$

Where A = Area of surface in sq. cm.

p = Absolute air pressure in atmosphere

v = Air velocity in cm. per sec.

Δt = Temperature difference deg. cent.

$T_{avg.}$ = Average of absolute temperature of surface and ambient air in deg. K.

From this we see that for the same temperature difference the forced convection decreases as the ambient temperature is raised.

A rough check on equation (77) may be had by applying it to calculate the heat dissipated from the rotor of a large turbo-alternator for the following assumed conditions:

Rotor diameter . . . . D = 114 cm.
Rotor length . . . . L = 376 cm.
Rotor area . . . . $A = \pi D L$ = 135,000 sq. cm.
Air pressure . . . . p = 1 atmosphere
Velocity of air in air gap . . . . v = 4570 cm. per sec.
Mean air temperature . . . . = 30 deg. cent.
Mean rotor surface temperatures . . = 65 deg. cent.
Surface film drop . . . . Δt = 35 deg. cent.
Average of surface and ambient temperature . . . . = 321 deg. K.

$W_c = 0.0014 \times 135{,}000 \times 1 \times 4570 \times 35/321 =$ 94 kw.

The above assumed conditions represent the approximate values obtaining in practise with 100-kw. rotor excitation.

IX Forced Heat Convection in Mixtures

To determine the free or forced heat convection in mixtures we require a knowledge of k, ρ and μ for the mixture under consideration.

The viscosity of gaseous mixtures have been investigated experimentally and theoretically at some length by Graham[17], Maxwell[18], Puluj[19], Sutherland[20], Thiesen[21] Schmitt[22]. Chapman[23] has recently studied the problem from the theoretical side in three classical papers, where the experimental and theoretical material are fully discussed.

The heat conductivity of mixtures of gases has not received much experimental study. Data on the heat conductivity of Helium and Argon is given by Wachsmuth[24] and we have data on mixtures of oxygen and hydrogen by Wassilewja[25].

Chapman[23] discusses the problem from the theoretical point of view from mon-atomic gases. His equations are probably also approximately correct for calculating the heat conductivity of poly-atomic mixtures though this question has not been checked.

In Fig. 19 we have taken the data on heat conductivity for mixtures of Helium and Argon by Wachsmuth[24] as recorded by Chapman[23] first reference. The viscosity data is by Schmitt[22] as given by Chapman[23], first reference.

In Fig. 20 mixtures of oxygen and hydrogen are similarly treated. The data on viscosity are by Schmitt[22] and the heat conductivity by Wassilewja[25]. In both cases the densities have been taken from Kaye and Laby, Physical and Chemical Constants.

We have seen from equation (76) that the forced convection at high velocities from large plane surface for moderate temperature differences may be written as

$$W_c = \frac{A v}{K} \left(\frac{k \rho}{\mu} \right)_{avg.} \Delta t \quad \text{watts} \qquad (78)$$

From this we see that the heat transfer per square

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[17] Graham, *Phil. Trans. Roy. Soc.*, 1846.

[18] Maxwell, *Sci. Papers*, Vol. II, p. 72.

[19] Puluj, Carl's Repertorium XV, p. 590. See also Meyer "Kinetic Theory." p. 200.

[20] Sutherland, *Phil. Mag.*, Vol. 40, p. 421, 1895.

[21] Theisen, *Verh. D. Deutsch. Phys. Ges.*, 4, p. 238, 1902.

[22] Schmitt, *Ann. d. Phys.* 30, p. 393, 1909.

[23] Chapman, *Phil. Trans. Roy. Soc. A.*, Vol. 211, pp. 433-483, 1912.
Phil. Trans. Roy. Soc. A., Vol. 216, pp. 279-348, 1915.
Phil. Trans. Roy. Soc., A., Vol. 217, pp. 115-195, 1917.

[24] Wachsmuth "Halle Diss," 1907; "Phys. Zeitschrift," 7, p. 235, 1908.

[25] Wassilewja, *Phys. Zeit.* 5, p. 737, 1904.

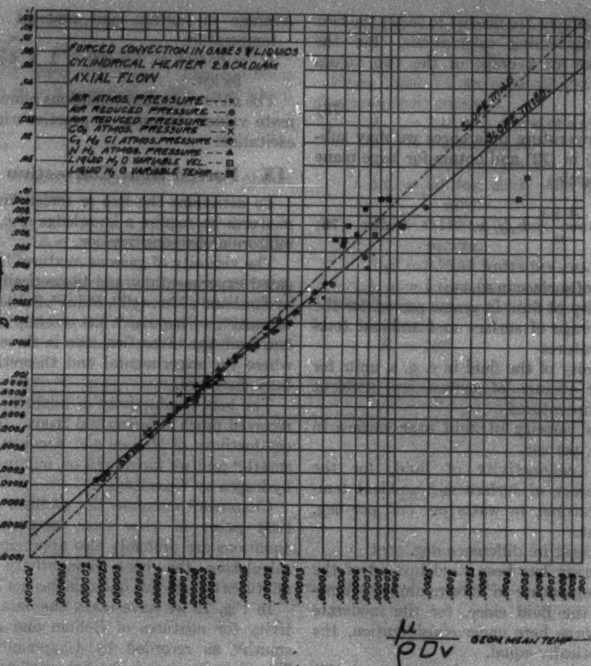
FORCED CONVECTION IN GASES & LIQUIDS
CYLINDRICAL HEATER 2.8 CM. DIAM.
AXIAL FLOW
AIR ATMOS. PRESSURE
AIR REDUCED PRESSURE
AIR REDUCED PRESSURE
CO2 ATMOS. PRESSURE
C2 H5 Cl ATMOS. PRESSURE
N H3 ATMOS. PRESSURE
LIQUID H2 O VARIABLE VEL.
LIQUID H2 O VARIABLE TEMP.
SLOPE 77/40
SLOPE 77/100
GEOM. MEAN TEMP.

Fig. 18

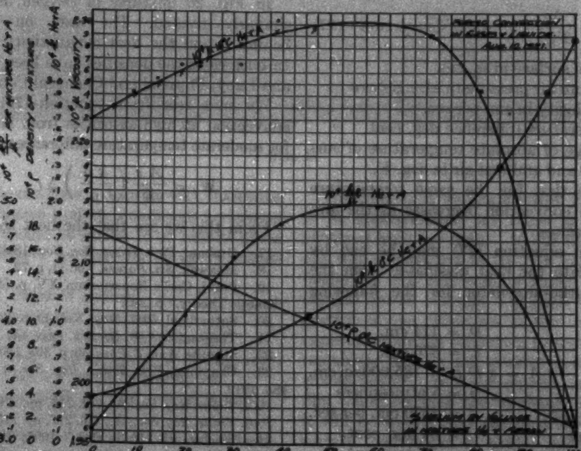
Forced Convection in Gases & Liquids Aug. 10 1921.
10⁴ ρ Density of Mixture
10⁴ μ Viscosity
10⁴ k/ρ He + A
10⁴ k/ρ for mixture He + A
10⁴ k/c He + A
% Helium by Volume in Mixture He + Argon
100% He

Fig. 19

centimeter per deg. cent. temperature difference for a given velocity is proportional to $\left(\frac{k\rho}{\mu}\right)_{mix.}$ for the fluid under consideration. For brevity we may therefore call this quantity the forced heat convection coefficient of the fluid. In Fig. 19 and 20 we have calculated the forced heat convection coefficients for the two available mixtures. The heat conductivity in the curves is given in calories per second per cm. per deg. cent. and therefore must be multiplied by 4.19 before substituting in equation (76). From these curves it will be observed that for mixtures of a light with a relatively heavy gas, the heat transfer coefficient has a maximum value for a mixture in the neighborhood of 60 per cent by volume of the lighter gas. For the 60 per cent helium and argon mixture the forced heat convection coefficient is about 1.6 times the value for pure helium or pure argon. For 60 per cent H_2 by volume in O_2 the convection coefficient is about 1.8 times as large as for either gas alone. In our forced heat convection tests using H_2 from the laboratory mains we found for the ratio of the forced heat convection in hydrogen to that in air the approximate value 1.3. On the basis of the heat transfer coefficient for the pure gases we should have obtained approximately 1.0 instead of 1.3 as the ratio. From Fig. 20 we see that this discrepancy would correspond to a mixture of 95 per cent H_2 and 5 per cent O_2. An analysis of the H_2 from the mains showed it to be about 95 per cent pure, the principal impurity being nitrogen.

For the case of viscosity Graham[17] first observed the curious fact that the addition of a moderate amount of light gas to a much more viscous and heavy gas may actually increase the viscosity of the latter. The same phenomena was commented on by Maxwell (Bakerian Lecture 1866).

Mixtures of hydrogen or helium with heavy gases therefore constitute a means of increasing the forced heat transfer over that obtainable for either gas alone.

The question of mixtures of liquids may be treated in the same manner.

X. Forced Convection in Hydrogen

For pure H_2 we will take the following values:

$$\rho = 0.00008987\ (273/T) \text{ grams per cc.} \qquad (79)$$

$$\mu = 0.000085\ (T/273)^{0.69} \text{ cm.}^{-1} \text{ gram sec.}^{-1} \qquad (80)$$

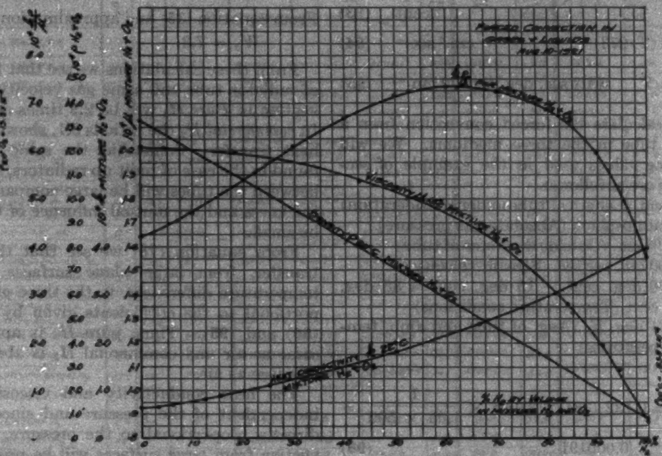

Fig. 20

$$c_v = 2.21\ (1 + .0002\ t) \text{ Cal. gram}^{-1} \text{deg. cent.}^{-1} \qquad (81)$$

$$\gamma = 1.419 \qquad (82)$$

Assuming the specific heat constant at its value for 300 deg. *K*., we have from equation (60)

$$k = 33.8 \times 10^{-6}\ T^{0.69} \text{ watts per cm. per deg. cent.} \qquad (83)$$

$$\mu/\rho = 0.95\ (T/273)^{1.69} \qquad \text{c. g. s. units} \qquad (84)$$

$$\frac{k\rho}{\mu} = 0.464/T \qquad \text{watt sec. deg. cent.}^{-1} \text{ cm.}^{-3} \qquad (85)$$

For commercial H_2 we will assume a mixture of 95 per cent H_2 and 5 per cent N_2 by volume. As we have no data on the properties of this mixture, we will assume that it is similar to the H_2 and O_2 mixture of Fig. 20 and modify the constants assumed above for pure H_2 in the same ratio as observed for a 95 per cent H_2 and

5 per cent O_2 mixture. We thus assume the following values to represent *commercial* H_2

$$\rho \quad = 0.000148\ (273/T) \text{ grams per cc.} \qquad (86)$$

$$\mu \quad = 0.000102\ (T/273)^{0.69} \text{ cm.}^{-1} \text{ gram sec.}^{-1} \qquad (87)$$

$$k \quad = 31.9 \times 10^{-6}\ T^{0.69} \text{ watts per cm. per deg. cent.} \qquad (88)$$

$$\mu/\rho = 0.69\ (T/273)^{1.69} \qquad \text{cm.}^2 \text{ sec.}^{-1} \qquad (89)$$

$$\frac{k\,\rho}{\mu} = 0.602/T \qquad \text{watt sec. deg. cent.}^{-1} \text{ cm.}^{-3} \qquad (90)$$

For air we have obtained the following values

$$\rho \quad = 0.00129\ (273/T) \text{ grams per cc.} \qquad (91)$$

$$\mu \quad = 0.0001705\ (T/273)^{0.754} \text{ cm.}^{-1} \text{ gram sec.}^{-1} \qquad (92)$$

$$k \quad = 3.4 \times 10^{-6}\ T^{0.754} \text{ watts per cm. per deg. cent.} \qquad (93)$$

$$\mu/\rho = 0.132\ (T/273)^{1.754} \qquad \text{cm.}^2 \text{ sec.}^{-1} \qquad (94)$$

$$\frac{k\,\rho}{\mu} = 0.475/T \qquad \text{watt sec. deg. cent.}^{-1} \text{ cm.}^{-3} \qquad (95)$$

Let us now calculate the forced convection from a single radiator tube in the three mediums. We will assume the same values used in the example of section VIII which are as follows:

L = 1 cm. (Length of tube under consideration)
p = 1 atmosphere (Absolute gas pressure)
D = 1.93 cm. (Outside diameter of tube)
v = 500 cm. per sec. (Stream velocity)
Δt = 100 deg. cent. – 15 deg. cent = 85 deg. cent (Temperature conditions)
$T_{avg.}$ = 331 deg. K. (Avg. of tube and amb. temperatures)

For air

$$\frac{\mu}{\rho\,D\,v} = 0.132\ (331/273)^{1.754} \times \frac{1}{1.93 \times 500} = 0.000191 \qquad (96)$$

This value is less than 0.013 and therefore we use $m = 1/2$ and $K = 2.15$ in equation (**36**) and obtain

$$B/D = 2.15\ (0.000191)^{1/2} = 0.0297 \qquad (97)$$

From this we see that we are dealing with a thin film and may therefore employ equation (**64**) in calculating the forced convection.

$$W_c = 0.0018\ \sqrt{1.93 \times 500} \times 85/331^{0.123} = 2.34 \text{ watts} \qquad (98)$$

For Pure H_2

$$\frac{\mu}{\rho\,D\,v} = 0.00136 \qquad (99)$$

We therefore employ $m = 1/2$ and $K = 2.15$ and obtain

$$B/D = 0.0794 \qquad (100)$$

Here the film thickness is getting rather thick, and it is best to calculate the forced convection from equation (**35**). We may substitute the approximation of equation (**54**) and obtain

$$W_c = \frac{2\,\pi\,L\,k_{avg.}\,\Delta t}{\log_e \dfrac{2\,B + D}{D}} \qquad \text{watts} \qquad (101)$$

From equation (**100**)

$$B \quad = 0.153 \text{ cm.} \qquad (102)$$

$$W_c = \frac{2\,\pi \times 33.8 \times 10^{-6} \times 54.5 \times 85}{0.144} = 6.85 \text{ watts} \qquad (103)$$

For commercial H_2

$$\frac{\mu}{\rho\,D\,v} = 0.00099 \qquad (104)$$

From equation (**36**) or from figure 6 we have

$$B/D = 0.0675 \qquad (105)$$

From equation (**35**) and approximation of equation (**54**)

$$W_c = 7.3 \qquad \text{watts} \qquad (106)$$

From these calculations we see that for the same size of radiator tube and same gas velocity the forced convection in *Pure* H_2 will be 2.9 times as great as in air. In *commercial* H_2 the ratio is 3.1, showing a still greater heat transfer. Somewhat the same ratio should be found in the case of built up radiators, since the spacing between the tubes will be large compared with the film thickness and the mutual influence of the tubes should be small.

From equation (**78**) we see that the rates of heat transfer, from large plane surfaces with moderate temperature differences, in the three gases will be proportional to the coefficients given by equations (**85**), (**90**) and (**95**). Thus, pure H_2 is approximately the same as air and commercial H_2 is about 1.3 times as effective as air.

The heat conductivity and viscosity of a gas is independent of the pressure and since the density is directly proportional to the pressure, the forced convection from large surfaces will be proportional to the gas pressure. For cylinders the effect is less pronounced. Equation (**57**) shows that for moderate temperature differences and thin films, the rate of heat transfer will be proportional to the square root of the absolute pressure of the gas. The necessity for using large areas in steam condensers to obtain a good vacuum is thus apparent. We may also observe that other things being equal a heavy vapor cycle has an advantage over steam.

From equation (**57**) we may observe that the forced convection per sq. cm. of tube surface is inversely proportional to the square root of the tube diameter. Thus, generally speaking, small radiator or condenser tubes are more efficient than large ones.

In Table VII the properties of a few gases and liquids are given for purposes of comparison. The values given for the liquids are chiefly from Landolt Börnstein Tabellen. The heat conductivities of the gases have been calculated from equation (46) with the exception of "Impure H_2." For mixtures of a light with a heavy gas equation (46) does not hold and the values given are those previously estimated from Wassilewja's[33] experimental data on mixtures of H_2 and O_2.

The specific heats of the mon-atomic gases Neon, Krypton and Xenon are not available but fortunately the kinetic theory of gases provides us with a method of calculating their values. For mon-atomic gases we assume that any possible internal atomic energy does not contribute to the specific heat and we therefore have from Jeans[16] (page 198, equation 512)

TABLE VII

FORCED CONNECTION IN GASES AND LIQUIDS.

NAME OF FLUID	TEMP °C	$\frac{C_p}{C_v}$ γ	SPEC HEAT AT CONST VOL C_v Cal gram⁻¹ C⁻¹	VISCOSITY μ cm⁻¹ gram sec⁻¹	DENSITY ρ gram cm⁻³	HEAT COND k watt cm⁻¹ °C⁻¹	ν $\frac{\mu}{\rho}$	$\frac{k\rho}{\mu}$	RELATIVE TO AIR AT 0°C	$k\sqrt{\rho/\mu}$	RELATIVE TO AIR 0°C
– GASES –											
HYDROGEN H_2	0°	1.42	2.33	.0000850	.0000899	.00161	.946	.00170	.98	.00166	2.62
HYDROGEN H_2	100°	1.42	2.38	.000105	.0000658	.00200	1.60	.00125	.718	.00158	2.49
IMPURE H_2	0	—	—	.000102	.000148	.00152	.690	.0022	1.27	.00183	2.89
(95% H_2 – 5% N_2)	100°	—	—	.000127	.000108	.00188	1.18	.0016	.92	.00172	2.72
AIR	0°	1.40	.170	.000171	.00129	.000231	.133	.00174	1.00	.000634	1.00
AIR	100°	1.40	.173	.000216	.000945	.000292	.229	.00128	.735	.000610	.963
HELIUM He	0°	1.63	.766	.000189	.000178	.00147	1.06	.00139	.800	.00143	2.26
HELIUM	100°	1.63	.766	.000236	.000130	.00183	1.82	.00101	.58	.00136	2.14
NEON Ne	0°	1.64	.147	.000302	.000900	.000452	.336	.00135	.776	.00078	1.23
ARGON A	0°	1.67	.0146	.000210	.00178	.000164	.118	.00139	.800	.000477	.75
KRYPTON Kr	0°	1.68	.0358	.000235	.00371	.0000898	.0635	.00141	.810	.000356	.561
XENON Xe	0°	1.67	.0228	.000211	.00585	.0000503	.0361	.00139	.800	.000265	.418
NITROGEN N_2	0°	1.41	.175	.000166	.00125	.000234	.133	.00176	1.01	.000642	1.01
CARBON DIOXIDE CO_2	0°	1.30	.165	.000139	.00197	.000161	.0706	.00228	1.31	.000605	.955
CARBON MONOXIDE CO	0°	1.40	.173	.000163	.00125	.000224	.131	.00171	1.00	.00062	.978
AMMONIA NH_3	0°	1.31	.39	.000096	.000762	.000266	.126	.00211	1.21	.00075	1.185
METHANE CH_4	0°	1.32	.448	.000104	.000717	.000336	.145	.00232	1.33	.000885	1.40
ETHYLENE C_2H_4	0°	1.26	.321	.000097	.00127	.000206	.0764	.00270	1.55	.000745	1.18
ETHYL CHLORIDE [illegible]	12.5°	1.13	.243	.000094	.00273	.000123	.0344	.00358	2.06	.000665	1.05
STEAM H_2O	100°	1.33	.340	.000132	.000581	.000326	.227	.00144	.828	.000684	1.08
MERCURY (VAPOR) Hg	357°	1.67	.0149	.000625	.00383	.0000075	.161	.000605	.348	.000243	.383
— LIQUIDS —											
WATER H_2O	0°	–	—	.0178	1.00	.00556	.0178	.310	178	.0415	655
WATER H_2O	100°	–	—	.00284	.958	.00721	.00296	2.44	1400.	.134	211
MERCURY Hg	0	–	—	.0169	13.60	.0620	.00124	50.0	28800.	1.76	2780.
MERCURY Hg	50°	–	—	.0141	13.47	.0782	.00105	75.5	43300	2.44	3850.
ALCOHOL C_2H_5OH	20°	–	—	.0119	.794	.00177	.0119	.149	857	.0163	25.7
GLYCERINE $C_3H_5(OH)_3$	20°	–	—	8.50	1.26	.00365	6.75	.000540	.31	.00140	2.21
[illegible] CCl_4	20°	–	—	.00969	1.594	.00106	.0061	.175	100.	.0136	21.5
FIE TRANSIL OIL				.21	.85	.0019	.247	.0077	4.43	.00383	6.05

$$c_v = 3/2 \frac{R}{J\ m} \text{ Cal gram}^{-1} \text{ deg. cent.}^{-1} \text{ at const. vol.} \quad (107)$$

Where R = Gas constant = 1.35×10^{-16}
J = Mech. equiv. of heat = 4.19×10^7 ergs
m = Atomic mass grams

We also have

$$m = N_0/\rho_0 \quad \text{grams} \quad (108)$$

where N_0 = Number of atoms in 1 cc. of gas at 0 deg. cent. and 76 cm. Hg = 2.75×10^{19}
ρ_0 = Density of gas at 0 deg. cent. and 76 cm. Hg. in grams per cc.

Substituting (108) in (107) we have

$$c_v = 3/2 \frac{R\,N_0}{J\,\rho_0} \text{ Cal. gram}^{-1} \text{ deg. cent.}^{-1} \quad (109),$$

and substituting the numerical values we have for the specific heat of a mon-atomic gas

$$c_v = .000133/\rho_0 \text{ Cal. gram}^{-1} \text{ deg. cent.}^{-1} \text{ at const. volume} \quad (110)$$

IX. Summary and Conclusions

1. The present investigation shows that Langmuir's film theory of convection can be successfully applied to the calculation of forced convection.

2. The method of dimensions gives the following general expression for the film thickness B.

$$B = K D \left(\frac{\mu}{\rho D v} \right)^m \quad \text{cm.} \quad (111)$$

where K = Numerical constant to be determined by experiment
m = Numerical exponent to be determined experimentally

$\mu/\rho = \nu$ = Kinematic viscosity of the ambient fluid for the geometric mean of the ambient and surface temperatures in c. g. s. units.

D = Linear dimension describing the body under consideration measured in cm.

v = Ambient fluid velocity, cm. per sec.

3. The forced convection from long cylinders in which the fluid flows normal to the axis has been tested for a wide range of conditions These tests include cylinders from 0.0028 cm. to 5.06 cm. diameter at temperatures from room to 1000 deg. cent. and air pressures ranging from 1/2 atmosphere to 4 atmospheres. One point is also available for water. For this wide range the forced convection is given by the following relation:

$$W_c = \frac{2\pi L \Delta\varphi}{\log_e b/D} \quad \text{watts convection} \quad (\mathbf{112})$$

Where L = Length of cylinder in cm.

$\Delta\varphi$ = Thermal conduction of fluid in watts per cm.

D = Diameter of cylinder in cm.

b = $2B + D$ = Outside diameter of film in cm.

The film thickness B is given by equation (**111**) in which we associate D with the diameter of the cylinder and use the following values for the experimental constants:

When $\frac{\mu}{\rho D v}$ is less than 0.013 and B/D is less than 0.24, we have $m = 1/2$ and $K = 2.15$

When $\frac{\mu}{\rho D v}$ is greater than 0.013 and B/D is greater than 0.24 we have $m = 2/3$ and $K = 4.27$

4. The forced convection from spheres is given by

$$W_c = \frac{2\pi \Delta\varphi}{1/D - 1/b} \quad \text{watts convection} \quad (\mathbf{113})$$

Where $\Delta\varphi$ = Thermal conduction in watts per cm.

D = Diameter of sphere in cm.

b = $2B + D$ = Outer diameter of film in cm.

The film thickness B is given by equation (**111**) in which we associate D with the diameter of the sphere. From the meager data we obtain $m = 2/3$ and $K = 35$.

5. The forced convection in pipes is given by the following relation:

$$W_c \quad \frac{2\pi L \Delta\varphi}{\log_e D/b} \quad \text{watts convection} \quad (\mathbf{114})$$

Where L = Length of pipe under consideration, cm.

$\Delta\varphi$ = Thermal conduction in watts per cm.

D = Internal diameter of pipe cm.

b = $D - 2B$ = Internal diameter of film in cm.

The value of B is given by equation (**111**) in which $m = 0.77$ and $K = 37$ or without appreciable error we may force the slope to $m = 3/4$ and use $K = 30$.

6. The tests using the copper wire heater included air, CO_2, NH_3, C_2H_5Cl, Impure H_2 and water. The tests were made at high velocities and the conditions were such as to give thin films. Moderate temperature differences we used. We may therefore summarize the data as follows:

$$W_c = \frac{\pi D L k_{\text{avg. temp.}}}{B} \quad \Delta t \quad \text{watts} \quad (\mathbf{115})$$

The film thickness B will be given by equation (**111**) using $m = 0.88$ and $K = 25$ or by forcing the slope to $m = 1$, we may use $K = 100$ with reasonable accuracy.

7. For large surfaces the film thickness will usually be small compared with the dimensions of the body and we may, therefore, write the forced convection

$$W_c = A/B \, \Delta\varphi \quad \text{watts} \quad (\mathbf{116})$$

Where A = Area of surface in square cm.

$\Delta\varphi$ = Thermal conduction in watts per cm.

B = Film thickness in cm.

If we substitute the general expression for the film thickness as given by equation (**111**) in equation (**116**) we obtain as our general expression for the convection from large bodies having thin films.

$$W_c = \frac{A}{K D}\left(\frac{\rho D v}{\mu}\right)^m \Delta\varphi \quad \text{watts} \quad (\mathbf{117})$$

Here the slope m and constant K have to be determined for a given geometrical form by experiments on similar bodies of various sizes or from a single body under different ambient fluid conditions.

8. For moderate temperature differences we may write with sufficient approximation.

$$\Delta\varphi = k_{\text{avg. temp.}} \Delta t \quad \text{watts per cm.} \quad (\mathbf{118})$$

where $k_{\text{avg. temp.}}$ = Heat conductivity of fluid at the average of the surface and ambient fluid temperatures in watts per cm. per deg. cent.

Δt = Temperature difference between surface and ambient fluid in deg. cent.

9. For cylinders in which the film thickness is small compared with the diameter, namely B/D less than 0.05, we have obtained $m = 1/2$ and $K = 2.2$. Therefore, we obtain for moderate temperature differences

$$W_c = \frac{\pi L}{2.2}\left(\frac{\rho D v}{\mu}\right)^{1/2} k_{\text{avg.}} \Delta t \quad \text{watts} \quad (\mathbf{119})$$

10. For gases we have the following relations

$$\rho = \rho_0 (273/T) \quad \text{grams per cc.} \quad (\mathbf{120})$$

$$\mu = \mu_0 (T/273)^n \quad \text{cm.}^{-1} \text{ gram sec.}^{-1} \quad (\mathbf{121})$$

$$k = 1.05\,(9\gamma - 5)\,\mu c_v \quad \text{watts per cm. per deg. cent.} \quad (\mathbf{122})$$

where ρ_0 = Density of gas at 76 cm. Hg. and 0 deg. cent. in grams per cc.

μ_0 = Viscosity of gas at 0 deg. cent in c. g. s. units

(The viscosity is independent of pressure)

T = Absolute temperature of gas in deg. K.
n = Exponent depending on gas. The following values are taken from Jeans[14], page 302; Air, $n = 0.754$; H_2, $n = 0.69$; CO_2, $n = 0.98$
$\gamma = c_p/c_v$ = Ratio of specific heats
c_v = Specific heat at constant volume in cal per gram per deg. cent.
k = Heat conductivity of gas and is independent of the pressure.

For Air we have

ρ_0 = 0.00129 grams per cc.
μ_0 = 0.0001705 c. g. s. units
n = 0.754

From which we have *for air* at atmospheric pressure

$$\rho/\mu = 7.56\,(273/T)^{1.754} \qquad \text{cm.}^{-2}\ \text{sec.} \qquad (123)$$

In the paper on Free Convection[1] it was shown that the heat conductivity of air at moderate temperatures may be written

$$k = 3.4 \times 10^{-6}\, T^{0.754} \quad \text{watts per cm. per deg. cent.} \qquad (124)$$

11. Application of these relations to equation (119) gives the following expression for the forced convection from cylinders in *air* under the conditions of thin films and moderate temperature differences.

$$W_c = 0.0018\, L \sqrt{p\, D\, v}\, \frac{\Delta t}{T_{avg.}^{0.123}} \quad \text{watts convection} \qquad (125)$$

where L = Length of cylinder under consideration in cm.
p = Absolute air pressure in atmospheres
D = Diameter of cylinder in cm.
v = Air velocity in cm. per sec.
Δt = Temperature difference between surface and ambient fluid deg. cent.
$T_{avg.}$ = Average of absolute temperature of surface and ambient fluid deg. K.

12. For spheres in which the film thickness is small compared with the diameter we obtain

$$W_c = \frac{\pi D}{35} \left(\frac{\rho D v}{\mu} \right)^{2/3} \Delta \varphi \quad \text{watts} \qquad (126)$$

For *air* at moderate temperatures we may write it

$$W_c = 0.00082\, D^{5/3}\, (p\, v)^{2/3}\, \frac{\Delta t}{T_{avg.}^{.416}} \quad \text{watts} \qquad (127)$$

where D = Diameter of sphere in cm.
The other symbols have the significance given in equation (125).

13. For pipes in which the film thickness is small compared with the diameter we obtain

$$W_c = \frac{\pi L}{30} \left(\frac{\rho D v}{\mu} \right)^{3/4} \Delta \varphi \quad \text{watts} \qquad (128)$$

For *air* at moderate temperature differences

$$W_c = 0.002\, L\, (p\, D\, v)^{3/4}\, \frac{\Delta t}{T_{avg.}^{.361}} \quad \text{watts} \qquad (129)$$

Here D is the inside diameter of the pipe and the other symbols are as defined in equation (125).

14. For plane surfaces we have taken the following relation to represent average conditions:

$$W_c = 0.003\, \frac{A\, \rho\, v}{\mu}\, \Delta \varphi \quad \text{watts} \qquad (130)$$

For *air* at moderate temperature differences, we may write it

$$W_c = 0.0014\, A\, p\, v\, \frac{\Delta t}{T_{avg.}} \quad \text{watts} \qquad (131)$$

15. We observe from these relations that in gases the forced convection for the same temperature difference decreases as we increase the ambient temperature. In liquids the opposite effect takes place in general.

16. The forced convection in fluid mixtures requires a knowledge of k, ρ and μ for the mixture. It has been shown that for a certain mixture of a light with a heavy gas the forced convection from plane surfaces may be considerably greater than for either gas alone. This effect is shown in our tests on the forced convection using impure hydrogen. Here the observed forced convection was approximately 1.3 times as great as it would be in pure hydrogen or in air.

17. For pure hydrogen we have the following relations:

$$\rho = 0.00008987\,(273/T) \quad \text{grams per cc.} \qquad (132)$$

$$\mu = 0.000085\,(T/273)^{0.69} \quad \text{c. g. s. units} \qquad (133)$$

For moderate temperatures we may use

$$k = 33.8 \times 10^{-6}\, T^{0.69} \quad \text{watts per cm. per deg. cent.} \qquad (134)$$

18. For commercial hydrogen we have assumed the following values:

$$\rho = 0.000148\,(273/T) \quad \text{grams per cc.} \qquad (135)$$

$$\mu = 0.000102\,(T/273)^{0.69} \quad \text{c. g. s. units} \qquad (136)$$

For moderate temperatures

$$k = 31.9 \times 10^{-6}\, T^{0.69} \quad \text{watts per cm. per deg. cent.} \qquad (137)$$

19. The forced convection from a smooth cylinder under the following conditions

L = 1 cm. (Length of tube under consideration
p = 1 atmosphere (Absolute gas pressure)
D = 1.93 cm. (Outside diameter of tube)
v = 500 cm. per sec. (Steam velocity)
Δt = (100 deg. cent. − 15 deg. cent. = 85 deg. cent. (Temperature conditions)
$T_{avg.}$ = 331 deg. K. (Avg. of tube and ambient absolute temperatures) gives for *air*

$$W_c = 2.34 \text{ watts} \qquad (138)$$

For *pure hydrogen*

$$W_c = 6.85 \text{ watts} \qquad (139)$$

For *commercial hydrogen*

$$W_e = 7.3 \text{ watts} \qquad (140)$$

Thus under the assumed conditions the forced convection from the radiator tube in *pure* H_2 is 2.9 times as great as in *air*. In *commercial* H_2 the ratio is 3.1, showing a still greater heat transfer. In actual built up radiators the same general effect should be observed, since the spacing between the tubes will be large compared with the film thickness and the mutual influence of the tubes should not be great.

20. The heat conductivity and viscosity of gases are independent of the pressure, and, since the density is directly proportional to the pressure, the forced convection from large plane surfaces at high velocities will be directly proportional to the gas pressure. For cylinders the effect of pressure is less pronounced, since the forced convection is only proportional to the square root of the pressure.

21. The forced convection per sq. cm. of tube surface, for thin films, is inversely proportional to the square root of the cylinder diameter. Thus, generally speaking, small radiator tubes are more efficient than large ones.

22. From Table VII we see that the forced convection from a large plane surface in water is approximately 178 times as great as for air. It is also evident that transil oil is not as effective as water. This presents the problem of using dielectrics having better heat transfer properties than oil for certain engineering applications.

23. We have seen that the physical assumption of Langmuir's film theory combined with dimensional reasoning gives equations from which the forced convection from bodies of various shapes may be accurately calculated in any gas or liquid. In the paper on Free Convection[1], similar assumptions were found to give simple solutions for free convection problems. It, therefore, seems certain that when a successful mathematical solution of the actual convection problems is obtained, it will show the existence of a relatively stationary film with a turbulent boundary as postulated by Langmuir's film theory.

Acknowledgment is made to Dr. W. R. Whitney for suggesting the investigation and for the inspiration which his interest in the subject largely supplied. The writer is also indebted to Mr. E. W. Kellogg for the many valuable suggestions which he contributed during the investigation, and to Mr. E. P. Lawsing, for his valuable assistance in making the tests.

Discussion

V. M. Montsinger: I think I am safe in making the statement that there is no other problem which offers any more difficulties than does that of heating and cooling. This is especially so in investigating free convection. In fact there are usually so many factors to be taken into consideration that it is practically impossible to arrive at any simple rule or formula that will hold for all the conditions met with in practise. I think one of the most important points to be considered in deriving a formula for free convection is that of the size of the area, that is, whether the heated surface is concentrated in a small area or whether it covers a large area several feet in height. Mr. Rice has considered this within a certain range of sizes. Another important point is the range in temperature used.

As most of the data used by Mr. Rice were obtained either from relatively small areas or at fairly high temperatures, that is, above 100 deg. cent. rise and as a contributing part to this compendium, I would like therefore to give in brief some of the results of my experience covering a period of several years in cooling mostly by free convection, of large areas at relatively low temperature rises, less than 100 deg. cent. rise. This lower region of temperature rise is a very important one from a practical standpoint for the reason that the limiting temperature of all class A insulating materials used so extensively in electrical apparatus comes within this range.

First, in reference to the question of the film of inactive air next to the heated surface I think perhaps it may be of interest if I give in a very general way the results of a recently conducted experimental observation which confirms very conclusively the existence of a film of apparently stationary gas on the heated surface.

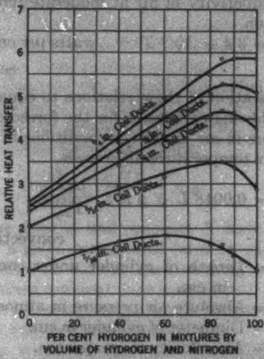

Fig. 1—Curves showing Effect of Size of Coil Ducts on Cooling of Vertical Coils by Free Convection in Gases at 200 Lb. (als.) Pressure

The experiment consisted in cooling by free convection a series of vertical transformer coils separated by various ducts of different sizes in, (1), a light gas of hydrogen, (2), in a heavy gas of nitrogen and (3), in a mixture of the two gases.

For the larger sizes of coil ducts used around ½ in. to ¾ in., the relative heat transfer or convected losses for a given temperature rise for the two gases checked fairly closely the values calculated by Mr. Rice's formula in which the loss varies directly with the product of heat conductively and the square root of the gas density and inversely with the square root of the viscosity.

But for the smallest coil duct used around $^3/_{16}$ in., it was found that there was practically no difference in the cooling by either of the gases when under 200 pounds absolute pressure. What appeared at first to be somewhat puzzling occurred when mixtures of these gases were used varying from 100 per cent nitrogen to 100 per cent hydrogen by volume. For instance, with a 50-50 mixture, the loss dissipated for a given rise was approximately 50 per cent greater than for either of the pure gases, and this difference in cooling with the pure gases and their mixtures gradually decreased as the mixture approached either one of the pure gases.

This hump in the efficiency curve gradually disappeared (as the coils with larger ducts were used) by moving over in the direction of the ¾ in. ducts and, of course, as the ducts became larger the cooling became better and better for the hydrogen gas, as compared with nitrogen. This is illustrated in Fig. 1.

The important point however is this: what was the cause of it? According to Mr. Rice's formula, nitrogen has a thin film of inactive gas on the surface and hydrogen gas has a relatively thick film—about 2.6 times greater than that of nitrogen at 50 deg. cent. and at 200 pounds pressure. Naturally then for small constricted paths the thick hydrogen film was clogging the circulation whereas the thin film of nitrogen allowed a much more free circulation and it so happened for this particular size of duct that the cooling was about equal, whereas for unrestricted circulation the cooling of hydrogen ranges from 2 to 3 times as great as that of the nitrogen.

Beginning with hydrogen as the cooling medium and as nitrogen was gradually added, thus forming a mixture, the gain due to decreasing the film thickness was up to a certain point more than the loss due to the addition of the poor cooling gas (and consequently loss of equal amount of hydrogen) but finally at approximately 50-50 mixture the conditions of gain and loss reversed themselves and the cooling again decreased. This phenomena proves conclusively the existence of a gas film upon which Mr. Rice's formulas are based.

Second, in reference to the question of the variation of free convection loss with temperature rise as given by Mr. Rice's equation (126), I wish to make the following comments: if we choose some definite ambient temperature, say 30 deg. cent., the equation expresses the loss as a function of the average of the absolute temperatures of the heated surface and ambient raised to the 0.754 power multiplied by the temperature difference in degrees cent. If we calculate the loss by this equation and plot loss as ordinates and temperature rise as abscissa on double-log paper we find that the loss varies up to 100 deg. almost exactly as the temperature railed to the 1.06 power. But as Mr. Rice states in the paper from 100 to 500 deg. the average exponential value is around 1.25. For example the following tabulation gives the values of the exponent between each 100 deg. rise up to 500 deg. cent. as calculated from his formula.

For temperature rise Between limits of:	Loss varies approx. as temperature rise raised to power of:
0–100 deg. cent.	1.06
100–200 deg. cent.	1.14
200–300 deg. cent.	1.24
300–400 deg. cent.	1.265
400–500 deg. cent.	1.37

Average from 100 to 500 deg. cent. = 1.254.

However, I have found as I pointed out in my article in June 1916 proceedings that the exponent of 1.25 holds from 0 to 100 deg. and I am quite certain that loss by convection for large areas does not vary even approximately as the first power of the temperature rise as given by equation (126).

I have been using for over ten years this value of the exponent, namely 1.25, in connection with the cooling of large selfcooled transformers having tanks with irregular surface where the greater part of the heat is dissipated by convection and I find that it always checks very closely with the test results. The difference between the loss calculated on the basis of using an exponent of 1.06 and one of 1.25 is quite an appreciable amount. For instance, if the constant is so chosen that the lines cross at 10 deg. rise the difference in loss at 100 deg. rise is in the ratio of about 1 to 1.5.

To obtain laboratory accuracy on the loss of heat by free convection from rather tall vertical surfaces I recently made a series of tests on a cast iron plate about 3.15 in long by 13.1 in. wide by $1^1/_{16}$ in. in thickness. In the plate were imbedded sheath wire units of equal resistance about 2 in. apart so as to obtain a uniform temperature throughout the whole area including both sides of the plate. This eliminated the necessity of making corrections for stray losses in case one side of the plate had been blanketed, as is sometimes done in investigations of this kind.

The plate was suspended in a vertical position in the air in an open room having a constant temperature. Fifteen thermocouples were soldered in holes in the surface on one side and five on the other side of the plate, the five being used merely as a check to see if both sides were at the same temperature. Direct current was used to supply the loss. All readings of volts and amperes and thermocouples were taken with a potentiometer. For convection, the air was used as the ambient. For radiation the temperature of the walls of the room about 10 feet distance was used for the ambient although the air and wall temperatures were usually the same.

Three series of runs were made with both sides of the hot plate under the same conditions, namely:

(1) painted a lamp black; (2) nickel plated and polished, and (3) with the surfaces partly oxidized and set permanently at about 325 deg. cent. before making any test.

For the first condition the paint began to scale off when the temperature reached approximately 100 deg. cent. or about 72 deg. rise and the test had to be discontinued. Up to this point the loss by convection was taken as the difference between the total loss and the loss calculated by the standard radiation law and assuming that the emissivity factor was 0.9 that of a perfect black body. The radiation constant used was 3.68×10^{-11}

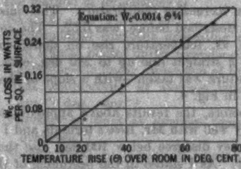

FIG. 2—HEAT LOSS BY FREE CONVECTION FROM PLANE 31.5" HEIGHT AND 13.1" WIDTH, PAINTED LAMP BLACK

The convection loss points fell on a straight line on double-log paper the equation of the line being

$$W_c = 1.40 \times 10^{-3} \theta^{1.266} \qquad (1)$$

in which W_c is the watts per square inch of surface and θ is the temperature rise in deg. cent. The loss is plotted against temperature rise raised to the 5/4 power in Fig. 2.

For the other two conditions of the plate surface the emissivity factor for radiation was assumed to be that obtained by subtracting from the total loss for a given rise around 50 or 60 deg. cent. the convection loss according to equation (1). The resulting value of emissivity was used throughout the range of test temperatures in determining the convection loss points.

According to the above method of procedure the emissivity factor for the nickel plated and polished surface was 0.07.

Up to about 150 deg. cent. rise the convection loss points fell in a straight line on double-log paper, the slope of the line being about 1.26. From 150 to 300 rise the points gradually drew away from the straight line and at 300 deg. rise the loss was about 25 per cent higher than the straight line. Fig. 3 shows the loss plotted against temperature rise raised to 5/4 power.

At first it was thought that this departure from a straight line on double-log paper was due to a change in the law but (as will be seen later where tests were made with the *surfaces* partly oxidized did not show this departure), it was apparently due to a gradual oxidation of the surface which at first was not discernable to the eye but which gradually increased the emissivity factor. At about 300 deg. rise the surface became so tarnished that the tests were discontinued.

For the last set of tests the surfaces were painted black and then subjected to a temperature of 325 deg. cent., for about a day, so as to let it get "set" before making any test. Some of the paint came off and what remained turned a brownish color.

Tests were taken first with the temperature decreasing and second with an increasing temperature. The emissivity factor, determined as described before, was 0.52. The convection loss where plotted vs. temperature rise on double-log paper fell practically on a straight line up to 280 deg. cent. rise—as far as the temperature was taken—with a slope of the line of 1.25. This is shown in Fig. 4.

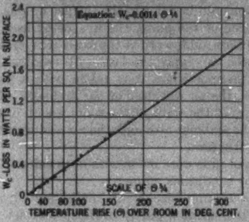

FIG. 3—HEAT LOSS BY FREE CONVECTION FROM VERTICAL PLANE 31.5 IN. HEIGHT AND 13.1 IN. WIDTH. NICKEL PLATED AND POLISHED

As stated before, the results above approximately 100 deg. rise agree fairly well with Mr. Rice's formula and I do not think that because they disagree below 100 deg. rise they contradict the film theory. As I stated in the beginning of the discussion the conditions of cooling a large plate are so radically different from that of cooling a small area where the heat *is* concentrated and especially at low temperature differences that possibly the same equations would not be expected to hold for both conditions.

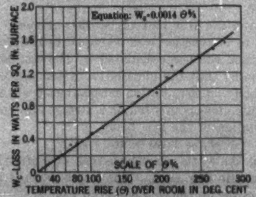

FIG. 4—HEAT LOSS BY FREE CONVECTION FROM VERTICAL PLANE 31.5 IN. HEIGHT AND 13.1 IN. WIDTH. SURFACES PARTLY OXIDIZED BEFORE TESTS

The point which I wish to emphasize is that for a large vertical surface and for temperature rises used in most electrical apparatus the loss can be expressed close enough for practical purposes by a simple exponential equation in which the loss varies as the 5/4 power of the temperature rise. This agrees with the results found by Messrs. Ezer, Griffiths and A. H. Davis conducted under the auspices of the Department of Scientific and Industrial Research, and shown in their Special Report No. 9 issued last year on "The Transmission of Heat by Radiation and Convection." This was for temperature rises up to 100 deg. cent. and for vertical planes up to approximately 35 in. in height. For heights of 69 and 104 in. the values of the exponent were 1.3 and 1.34 respectively. The constant in their formula was the same (1.28×10^3) for all heights above 12 in. (30 cm.). For smaller heights than 12 in. they found that the constant increased. The constant I found from the tests on the hot plate was 1.4×10^3 or about 9 per cent higher than Griffiths and Davis' value, for corresponding heights.

Chester W. Rice: (by letter Oct. 18, 1923.) In the above discussion Mr. Montsinger called my attention to the fact that the free convection from vertical plane surfaces varies as the 5/4 power of the temperature difference whereas my equations make the convection vary approximately as the first power of the temperature rise for small temperature differences. In view of this lack of agreement between the theory and experiment for low temperature differences, it seemed desirable to reexamine the available data on the basis of a more general expression for the film thickness so as to obtain, if possible, a single expression which would be universally applicable for both large and small temperature differences. In a subsequent paper it will be shown that the desired universal expression can be readily found and that it leads to the following conclusions.

(1) The more general expression for the effective film thickness, here developed, is seen to be superior to the simpler expression of the previous paper since the resulting equations account accurately for the convection from large and small bodies at both high and low temperature differences.

(2) The general expression for the effective film thickness obtained by the method of dimensions for free convection is

$$B = K\,D\,(k/\mu\,c_p)^p\,[\nu/(\alpha\,g\,\Delta\,t)^{1/2}\,D^{3/2}]^n \qquad \text{cm.} \quad (1)$$

where K = Experimental constant depending on the system of similar bodies under consideration
D = Characteristic linear dimension of body in cm.
k = Heat conductivity at average temperature watt cm.$^{-1}$ deg. cent.$^{-1}$
μ = Viscosity of fluid at average temperature cm.$^{-1}$ gram sec.$^{-1}$
c_p = Specific heat at constant pressure for average temperature Joule gram^{-1} deg. cent.$^{-1}$
p = Experimental exponent
ν = μ/ρ = Kinematic viscosity of fluid for average temperature cm.^{2} sec.$^{-1}$
α = Coef. of density change per deg. cent. for average temperature in deg. cent.$^{-1}$, for ideal gases $\alpha = 1/273$
g = Acceleration of gravity = 980 cm. sec.^{2}
$\Delta\,t$ = Difference between surface and ambient temperature deg. cent.
n = Experimental exponent for the system of similar bodies under consideration.

(3) When dealing with *ideal gases*, the first factor in equation (1) can be neglected since it does not vary greatly from one gas to another. Under these conditions we write the effective film thickness

$$B = K_1\,D\,[\nu/(\alpha\,g\,\Delta\,t)^{1/2}\,D^{3/2}]^n \qquad \text{cm.} \quad (2)$$

(4) The free convection from *long horizontal cylinders* in *gases* and *liquids* is given by

$$W_c = 2\,\pi\,L\,\Delta\,\varphi/\log_e[(2\,B + D)/D] \qquad \text{watt} \quad (3)$$

where L = Length of cylinder in cm.
$\Delta\,\varphi$ = Thermal conduction in watts per cm.
D = Diameter of cylinder in cm.
B = Film thickness in cm. from Eq. (1).

A sufficiently close approximation is usually obtained by taking $n = 1/2$; $K = 2.12$ and $p = 1/4$ in equation (1).

(5) The free convection from *long vertical cylinders* in *gases* is given by equation (3) and the film thickness by equation (2). The meager available data give $n = 2/3$ and $K_1 = 8.65$.

(6) The free convection from *spheres* in *gases* is given by

$$W_c = 2\pi \Delta \varphi/[1/D - 1/(2B + D)] \quad \text{watts} \quad (4)$$

and the film thickness by equation (2) in which $n = 1/2$ and $K_1 = 2.02$.

(7) For *large surfaces* the film thickness will usually be small compared with the film thickness and the shape conductance will then take the simple form $S = A/B$. For moderate temperature differences we may also take $\Delta \varphi = k_{avg.} \Delta t$. Under these conditions, our general expression for free convection becomes

$$W_c = (A\, k\, \Delta t/K\, D)\, (\mu\, c_p/k)^p\, [\,(\alpha\, g\, \Delta t)^{1/2}\, D^{3/2}/\nu]^n \quad \text{watts} \quad (5)$$

For *ideal gases* we may omit the first factor without great error and obtain

$$W_c = (A\, k\, \Delta t/K_1\, D)\, [\,(\alpha\, g\, \Delta t)^{1/2}\, D^{3/2}/\nu]^n \quad \text{watt} \quad (6)$$

(8) For *large long horizontal cylinders* in *gases* and *liquids* we take $K = 2.12$; $n = 1/2$ and $p = 1/4$ in equation (5). For the convection in *air* we obtain

$$W_c = 0.000785\, A\, (1/D)^{1/4}\, p^{1/2}\, (1/T_{avg.})^{.125}\, \Delta t^{5/4} \quad \text{watt} \quad (7)$$

where A = $\pi D L$ = Area in square cm.
D = Cyl. diam. in cm.
L = Length of long cylinder in cm.
p = Air pressure in atmosphere
$T_{avg.}$ = Average of cyl. and amb. absolute temperature deg. K
Δt = Temperature difference deg. cent.

(9) For *large long vertical cylinders* in *gases*, we take $n = 2/3$ and $K_1 = 8.65$ in equation (6). For *air* we obtain the following approximation

$$W_c = 0.00166\, A\, p^{2/3}\, (1/T_{avg.})^{.415}\, \Delta t^{4/3} \quad \text{watt} \quad (8)$$

(10) For *large spheres* in *gases* we take $n = 1/2$ and $K_1 = 2.0$ in equation (6). For *air* we then obtain the following approximation.

$$W_c = 0.00088\, A\, (1/D)^{1/4}\, p^{1/2}\, (1/T_{avg.})^{.125}\, \Delta t^{5/4} \quad \text{watt} \quad (9)$$

(11) For a *long thin vertical plane* surface (ribbon surface) in *gases* we may take as an approximation $n = 1/2$ and $K_1 = 1.46$ in equation (6). We also take D equal to the vertical height H in cm. For *air* we then obtain

$$W_c = 0.00121\, A\, (1/H)^{1/4}\, p^{1/2}\, (1/T_{avg.})^{.125}\, \Delta t^{5/4} \quad \text{watt} \quad (10)$$

(12) The more general theory of the present paper shows that *free convection tests can not be used as a primary method of obtaining heat conductivities.* The method, however, is still of interest where an approximation value of the heat conductivity already exists. Below, the heat conductivities have been recalculated by this method.

No. 12 Transil Oil = $0.0017\,(1 - 0.0026\,t)$ watt cm.$^{-1}$ deg. cent.$^{-1}$
Toluene = $0.0015\,(1 - 0.0029\,t)$ " " "
Glycerine = $0.0033\,(1 - 0.0038\,t)$ " " "
Aniline = $0.0020\,(1 - 0.0018\,t)$ " " "
CCl_4 = $0.0012\,(1 - 0.0038\,t)$ " " "
Olive Oil = $0.0016\,(1 + 0.0055\,t)$ " " "

The value for olive oil should not be given much weight due to the large uncertainty concerning the viscosity. The present method gives, excepting olive oil, negative temperature coefficients whereas, the method of the previous paper gave positive values.

An oversight was made in Section V under *Effect* of *Ambient Temperature.* Here the experiments were tested by calculating the film thickness for the high and low ambient temperatures from the observed convection. These calculations showed that the film thickness increased with increasing ambient temperature as was required by the theory and therefore the agreement was considered satisfactory. Now the heat conductivity increases with the temperature, but not quite as fast as the film thickness increases and therefore the convection per degree cent. should decrease with increasing ambient temperature, whereas the experiments show an increased convection per deg. cent. for the high ambient temperature. Thus the *oven tests* are contrary to the theory of the paper as well as the more general theory outlined above. It is probable that convection currents were the cause of the trouble.

In Section VII of the paper, we used Compan's data on free convection from spheres to determine the constants in the convection laws. In the course of the recent work, I have reexamined Compan's data and compared it with the work of Peclet (*Traité de la Chaleur*, Third Edition, Vol. 111, pp. 418-481) and M'Farlane (*Proc.* Royal Soc. London, Vol. 20, p. 90, 1871 and 1872) with the result that Compan's experiments do not agree at all with the others. It seems probable that the trouble with Compan's work is due to the use of a small sphere covered with a thick coat of lamp black. In the above summary of the recent work Compan's data have therefore been neglected. His data on forced convection have likewise been neglected in the following discussion on forced convection.

A recent study of a fund of valuable data on the heat transfer to liquids flowing in pipes, which has come to the writer's attention, showed that here also a more general expression for the film thickness is necessary to properly correlate the heat transfer in liquids with that in gases. In a subsequent paper the new data will be presented and the following conclusions reached.

(1) The more general expression for the effective film thickness, here developed, is superior to the expression of the previous paper in that it allows us to correlate the heat transfer in gases and liquids.

(2) The general expression for the effective film thickness obtained by the method of dimensions is

$$B = K_0\, D\, (k/\mu\, c_p)^p\, (\mu/\rho\, D\, v)^x \quad \text{cm.} \quad (11)$$

where K_0 = Experimental constant depending on the system of similar bodies under consideration
D = Characteristic linear dimension of body in cm.
k = Heat conductivity at average temperature watt cm.$^{-1}$ deg. cent.$^{-1}$
μ = Viscosity of fluid at average temperature cm.$^{-1}$ gram sec.$^{-1}$
c_p = Specific heat at constant pressure for average temperature Joule gram^{-1} deg. cent^{-1}
p = Experimental exponent
ρ = Density of fluid at average temperature gram cm.$^{-3}$
v = Mean velocity of fluid relative to the surface cm. sec.$^{-1}$
x = Experimental exponent

(3) When dealing with *ideal gases* the first factor in equation (11) can be neglected since it does not vary greatly from one gas to another. Under these conditions we obtain

$$B = K\, D\, (\mu/\rho\, D\, v)^x \quad \text{cm.} \quad (12)$$

which is the expression assumed for both gases and liquids in the previous paper.

(4) For thin films and moderate temperature differences we obtain for *gases* and *liquids.*

$$W_c = (A\, k\, \Delta t/K_0\, D)\, (\mu\, c_p/k)^p\, (\rho\, D\, v/\mu)^x \quad \text{watt} \quad (13)$$

where A = Area of surface cm.^{2}
Δt = Temperature difference deg. cent.

(5) For *ideal gases* the factor $(\mu\, c_p/k)^p$ may be neglected with sufficient approximation so that

$$W_c = (A\, k\, \Delta t/K\, D)\, (\rho\, D\, v/\mu)^x \quad \text{watt} \quad (14)$$

This relation was tested over a wide range of conditions in different gases in the previous paper with good agreement.

(6) For *smooth pipes* well above the critical velocity, the available data in *gases* and *liquids* is approximately satisfied by taking $p = 1/2$, $x = 5/6$ and $K_0 = 60$ in equation (13).

(7) The film theory naturally suggests a relation between the mechanical resistance to flow and the heat transfer. A study of the heat transfer and mechanical resistance for pipes leads to the following relation:

$$W_c = (A\, k\, \Delta t\, R/v\, \mu)\, (\mu\, c_p/k)^p \quad \text{watt} \quad (15)$$

where R = mechanical resistance dynes cm. $^{-2}$

(8) We may write equation (15) in the following form

$$W_c = (A\ c_p\ \Delta\ t\ R/v)\ (k/\mu\ c_p)^{1-p} \qquad \text{watt} \quad (16)$$

which is seen to conform to Reynolds Law as given by Stanton.

(9) If we substitute Lees equation for the resistance to flow in *smooth pipes* in equation (16), and assume $p = 1/2$ we obtain for *gases* and *liquids*.

$$W_c = (k/\mu\ c_p)^{1/3}\ A\ \rho\ c_p\ v\ \Delta\ t\ [0.0009 + 0.0763/(v\ D\ \rho/\mu)\}^{.35} \qquad \text{watt} \quad (17)$$

This equation takes into account the variation of the resistance with the degree of turbulence and therefore should be applicable over the entire range above the critical velocity.

For *ideal gases* we may take with sufficient approximation

$$W_c = 1.16\ A\ \rho\ c_p\ v\ \Delta\ t\ [0.0009 + 0.0763/(v\ D\ \rho/\mu)^{.35}] \qquad \text{watt} \quad (18)$$

For *air* equation (18) becomes approximately

$$W_c = (0.417\ A\ p\ v\ \Delta\ t/T_{avg.})\ [0.0009 + 0.00012\ T_{avg.}^{\ .615}/(v\ D\ p)^{.35}] \quad \text{watt} \quad (19)$$

where p = absolute air pressure in atmospheres
$T_{avg.}$ = Average of surface and ambient absolute temperature

(10) For *rough pipes* (fine sand paper) we will assume $x = 1$, $p = 1/2$ and $K_s = 136$ in equation (13).

For *ideal gases* we then obtain

$$\overline{W}_c = 0.00635\ A\ \rho\ v\ k\ \Delta\ t/\mu \qquad \text{watt} \quad (20)$$

For *air* we have

$$W_c = 0.0031\ A\ p\ v\ \Delta\ t/T_{avg.} \qquad \text{watt} \quad (21)$$

(11) For cylinders in *ideal gases* the values obtained in the previous paper still hold. We therefore take $x = 1/2$ and $K = 2.2$ in equation (14) and obtain for thin films

$$W_c = (A\ k\ \Delta\ t/2.2\ D)\ (\rho\ D\ v/\mu)^{1/2} \qquad \text{watt} \quad (22)$$

where D = Diameter of cylinder in cm.

For *air* we obtain

$$W_c = 0.0018\ L\ \sqrt{p\ D\ v}\ \Delta\ t/T_{avg.}^{\ .123} \qquad \text{watt} \quad (23)$$

where L = Length of cylinder in cm.

(12) The heat transfer in gaseous mixtures are covered by the general equations which apply to liquids and gases.

In conclusion the writer wishes to express his thanks to Mr. Montsinger for his valuable criticism of the paper which brought out the limitations of the previous theory and stimulated the writer to work out the present more general relations.

The Electrical Plant of Transocean Radio Telegraphy

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***Review of the Subject.**—A description of the expansion of the Transocean Communication System of the Radio Corporation of America from a few isolated plants to a unified group of electrical plants all controlled for communication purposes from a central traffic in New York City, with a summary of the technical conditions covering the design of the Radio Central Station and of the technical conditions to be met in operating efficiently a modern radio communication system.*

AT the beginning of 1920 the United States Government removed the war restriction on commercial radio service, and the Navy Department restored to the Radio Corporation of America those stations which were built and equipped in 1914 by the Marconi Wireless Telegraph Company of America for transocean service.

In addition to the agreements previously entered into with countries in Europe for transocean radio service, the Corporation faced the situation arising out of the Great War, in which practically every European country demanded direct radio commiuncation with the United States.

The need for the provision of modern facilities for carrying on radio communication with those countries with which agreements had already been made, was imperative, and hardly less imperative was the need for the expansion of our facilities to meet the new situation.

The radio equipment in all of the installations restored to the Corporation was of obsolete type and based on the use of damped waves, except in the case of the New Brunswick station. At that station the Navy Department had instructed the General Electric Company to install high-frequency alternator equipment and to modify the antenna circuit to meet the requirements of their system. Accordingly, an alternator equipment was installed which was able to supply to the antenna circuit 200 kilowatts at the high frequency to which the antenna circuit is tuned. The antenna at this station had been erected as an inverted L, approximately a mile long and 550 feet wide. This was changed to the multiple-tuned type by adding five tuned down leads, equally spaced along the length of the antenna, and connecting them through a balanced distribution system to the ground and counterpoise wires. This installation has been described by technical papers read in 1920 and 1921.

Operation of the system of the Radio Corporation started with two transmitting stations — at New Brunswick, New Jersey, and at Marion, Massachusetts. Each of these transmitting stations had its corresponding receiving station at Belmar, New Jersey, and at Chatham, Massachusetts, respectively. New Brunswick was used for communication with England, and Marion for communication with Norway. The telegraphic operation of the English circuit was centered in Belmar, and the operation of the Norwegian circuit was centered in Chatham. Messages to England or Norway were telegraphed to Belmar and Chatham respectively, where they were copied and transmitted over the radio circuit via New Brunswick and Marion. Similarly, messages from England and Norway were received in Belmar or Chatham, were copied by hand, and re-telegraphed to New York. This process involved several relays of telegraph operators with the consequent high expense and possible delays and errors.

With the present system of operation, the Radio Corporation has six transmitters on the Atlantic coast, two in Tuckerton, one in New Brunswick, one in Marion, and two in the Radio Central station on Long Island. All these transmitters are controlled directly from the traffic office in New York City.

Only one receiving station is needed for all incoming messages. This receiving station is located at Riverhead, Long Island. It has a single antenna of a new and special type, which will be described later. This antenna intercepts the waves from all European transmitting stations. The receiving apparatus, also of a new type, separates this conglomeration of ether waves which come in over the receiving antenna, into separate messages which are automatically relayed over telephone wires so that all messages are received and copied in the same traffic office in New York.

The transmitting station on Long Island — known as "Radio Central" — and the receiving station at Riverhead, Long Island, represent the modern system of the Radio Corporation. The stations at New Brunswick, Marion, and Tuckerton, are adaptations of the modern transmitting apparatus developed by the General Electric Company and antennas built before the war. The characteristic features of the transmitting system are: The high-frequency alternator, the multiple-tuned antenna, the speed or wave-length regulator, and the magnetic amplifier.

In the Riverhead receiving station the method of centralization has been carried to its logical conclusion by concentration of all radio apparatus in the one station, and concentration of all reception in New York. The advantage of such concentration is obvious. New receiving circuits for communication with any new station in Europe can be added at a negligible cost by installing a new set of receiving apparatus on some of the shelves provided for that purpose in the Riverhead receiving station.

The Radio Central transmitting station has been

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planned in such a way that the cost of additional transmitting units will be a minimum. The choice of the site of the Radio Central transmitting station was carefully considered, looking forward to a growth of international radio communication which would require as much as twelve transmitters in this new station. Two of these twelve transmitters are already completed.

The principal considerations in selecting the site for the Radio Central station were:

1. The site must be within a reasonable distance from New York — the center of traffic.

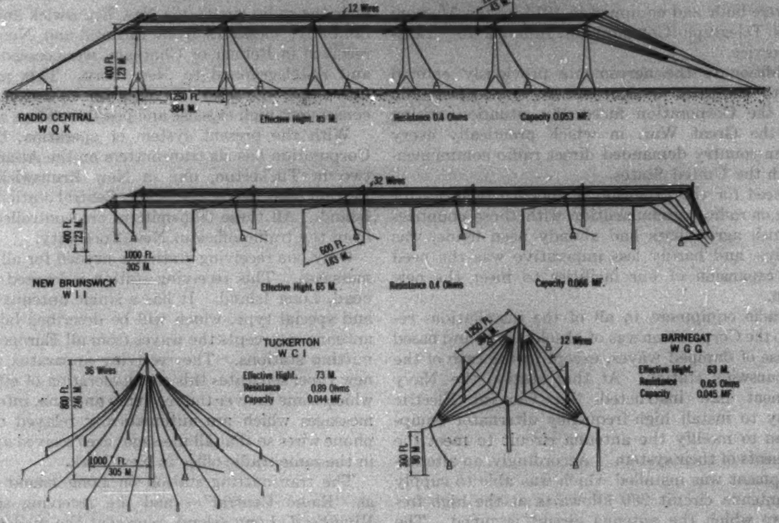
12 Wires
150 Ft.
43 M.
400 Ft.
123 M.
1250 Ft.
384 M.
RADIO CENTRAL
W Q K
Effective Hight. 85 M.
Resistance 0.4 Ohms
Capacity 0.053 MF.
32 Wires
400 Ft.
123 M.
1000 Ft.
305 M.
600 Ft.
183 M.
NEW BRUNSWICK
W I I
Effective Hight. 65 M.
Resistance 0.4 Ohms
Capacity 0.066 MF.
TUCKERTON
W C I
Effective Hight. 73 M.
Resistance 0.89 Ohms
Capacity 0.044 MF.
36 Wires
800 Ft.
246 M.
1000 Ft.
305 M.
1250 Ft.
384 M.
12 Wires
BARNEGAT
W G G
Effective Hight. 63 M.
Resistance 0.65 Ohms
Capacity 0.045 MF.
300 Ft.
90 M.

Fig. 1

2. A large tract of land of a desirable nature must be available, at a moderate cost.

3. A good power supply must be within easy reach.

4. There must be direct and reliable wire line communication with New York City.

The site selected on Long Island fulfilled these requirements in an ideal way, but another desideratum which, in the past, had been the deciding factor in selecting sites for transmitting stations, was not fulfilled in the Long Island location — a natural low ground resistance. The Long Island ground consists of quartz sand of extraordinarily high resistance. The decision, therefore, regarding the selection of this site was a grave responsibility for the engineers of the Radio Corporation. It meant a radical departure from the generally accepted theories. It implied that practical operation rather than technical considerations was to be the controlling factor. The engineers thus undertook to remedy by new developments in the technique what nature had failed to provide — a good ground. Much progress had already been made to reduce ground resistance by multiple tuning and ground equalizers, but this experience had been gained in stations like New Brunswick, Marion, and Tuckerton, where the natural ground resistance was low. However, we had sufficient faith in the further possibilities of development of improved grounding methods to take the responsibility for starting the construction of the new station while investigation was going on to find a solution for the grounding problem. The development work of the new ground system required as much time as the completion of the rest of the station, but by the time the station was ready to go into service the ground system was also ready and proved to be succesful beyond the most sanguine expectations.

The Radio Central transmitting station of the Radio Corporation of America is the first of our stations that has been planned and designed from the beginning to meet modern requirements, the other stations having been made to conform to modern practise by modification of equipment installed in earlier times. The Radio Central type of station is being duplicated in Poland and Sweden. This station has been frequently described and while its 400 ft. steel towers with 150 ft. cross arms are quite well known, little has been published regarding the technical performance of the plant.

RADIATION

The transmission value of the transmitting station is expressed by the product of the effective height — usually given in meters — and the charging current of the antenna circuit — given in amperes. In deciding upon the value of meter amperes that would be required at our Long Island station, we took advantage of the experience gained from work done with the signals transmitted from the Nauen station in Germany and the Carnarvon station in England. As a result of the preliminary work in this connection, a figure of 50,000 meter amperes was decided upon and the antenna circuit was designed to give this value with full power on one transmitting unit.

As this figure of 50,000 meter amperes is made up of two factors, effective height of and current in the antenna circuit, the values assigned to each of these factors must be so chosen that the cost of the antenna, cost of power equipment, and cost of operation and maintenance, will result in the most economical investment. In order to determine the most economical height of antenna, it was necessary to check carefully, the varying effects of capacity, effective height, wave length, voltage and current. The result of these investigations showed that if the first cost of the transmitting station be plotted against the height of the towers for a given value of meter amperes at a given wave length, a curve is obtained showing a distinct minimum. This minimum is not sharp but shows that there is a minimum cost of station for the given meter ampere value over a small range in the height of the towers.

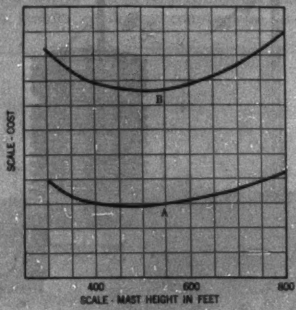

FIG. 2

Design Data: Radio central type antenna; wave length 16,000 M.; Voltage maximum 120,000 M.; meter ampere value: Curve A-50,000, Curve B-100,000.

Fig. 1 gives for comparison the principal dimensions, effective height, and resistance of the four types of antenna used in the Atlantic shore stations of the Radio Corporation. The effective heights are determined by measurements of radiation.

Fig. 2 shows the calculated cost for antenna structures at different heights for two typical stations of 50,000 and 100,000 meter amperes radiation.

The antenna voltage limitations which had been experienced at our older stations necessitated an investigation of the insulators that should be used in connection with these antennas. This work has been described recently in a paper read by Mr. W. W. Brown on March 7th, last, before the Institute of Radio Engineers, and this shows that by careful design and arrangement of parts, we have been able to raise the working voltage of our antennas from around 60,000 to 150,000. In a recent test of the insulators actually installed and operating at our Radio Central station at a voltage of approximately 120,000, it was found that the voltage distribution over the double insulator unit, by means of which the wires are suspended from the bridge arm of the towers, is roughly 45 per cent and 55 per cent, the insulator nearer the tower having the smaller proportion of the voltage.

POWER

The power to operate the station is generated in the Long Island Lighting Company's plant at Northport, L. I., and carried by a three-phase network at 22,000 volts, a distance of 30 miles to the radio station. At the radio station, the power is transformed to 2300, two phase, to drive the induction type motors connected through step-up gears to the high-frequency alternators.

ANTENNA

The suspension of the antenna wires followed current transmission line practise. The wires run the full length of the antenna; standard transmission line clamps are fastened to the wire at each tower suspension point. These are shackled to the insulators suspended from the tower bridge arm. As the working voltage at which this antenna would operate, was higher than that used at our other transocean stations, the design of this circuit was carefully considered with respect to corona losses. The operation of this antenna at 135,000 volts showed that the corona limit was not reached on any portion of the circuit, although there is not a very great reserve where the inner wires unshielded by the suspension insulators, pass across the face of the steel tower.

The antenna consists of 12 parallel wires 5/16 in. diameter 7500 feet long and spaced on an average about 14 feet apart forming an approximately horizontal plane about 150 feet wide. The wires are stretched from dead end structures close to the building to the first tower cross arm then from cross arm to cross arm in a straight line to the sixth tower, then again to a dead end structure at the ground level at the far end.

The self supporting type of tower was selected for use with this antenna. It is equipped with a bridge arm, its length 150 ft. over-all—fixed to the top of the tower. The insulators carrying the antenna wires are suspended from the lower face of this bridge.

Many reasons entered into the decision to use this type of tower, three of which may be mentioned here. One consideration was, the average height of the antenna wires. With a group of similar antenna wires, equally loaded, suspended on a springstay between two towers, the height above ground of the point of suspension of a wire decreases as the distance between this point and the nearest tower is increased. With a similar group of wires suspended from the bridge arm of a tower, there is no similar variation.

Another engineering consideration was the variation in antenna constants caused by high winds. The suspension of the group of antenna wires from a spring-stay slung between the tops of two masts or towers, has been used at our New Brunswick, Marion, and similar stations. It has been found that whenever there is a high wind blowing across the antenna wires, the spring stay assumes a new position varying with the strength and direction of the wind. With gusty winds of high velocity, this change of position is continuously occurring. There is, in addition, the variation in the position of the antenna wires due to the cross wind on the wire span between the spring stays. The result of these changes in position of the wires is that the constants of the antenna circuit change, and detune the antenna from the alternator which is operated at an accurately regulated wave length. The resulting fluctuations in radiation have been so great at times as to seriously impair the commercial effectiveness of this station. Now, with a fixed point of suspension, such as the tower bridge arm, the only variations in position of the wires, are those due to the wind on the wire span between the towers. Those due to the variation in the position of the spring stay are not present.

The antenna circuits at all of our stations are equipped with variometers to correct for these changes and our experience is that the variations are less severe with Radio Central type of antenna than with that of New Brunswick.

As Long Island is well within that zone of the United States in which sleet and glare formation must be expected on all structures exposed to the weather during the winter months, provision has been made to melt such ice as may form around the antenna wires. The heating current for sleet melting, is supplied from the power house, at 60-cycles, through special transformers and reactances. The antenna wires are connected together at the far end of the circuit. By opening switches at the power house end of this circuit, the wires can be disconnected from the radio frequency feeder circuit and the 60 cycle power circuit can be connected. If the several downleads were connected directly to the antenna wires throughout their length the path of the heating current would be short-circuited. Two satisfactory methods have been used to avoid such short circuit. One method consists in dividing the wires into four groups and connecting only the wires belonging to one group at each of the four intermediate points. At both ends, all wires are connected. The other method consists of making the connection of each wire through a specially designed condenser.

The inductors used at each downlead of the multiple tuned antennas are installed without any protection from the weather. This type of installation has proved satisfactory except at some locations close to the sea where the spray from the sea water deposits salt on the insulators.

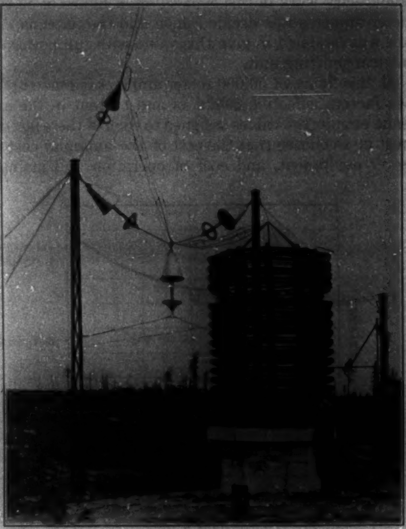

Fig. 3

The standard outdoor coil is shown in Fig. 3. Fig. 4 shows coils housed in frame structures lined with copper.

At stations where more than one antenna circuit is installed, attention must be given to the disposition of the several antennas and of their individual feed circuits in order to minimize their mutual interaction. In enlarging or remodelling an existing station, it is not always expedient to attempt to bring all antenna circuits to the close proximity of the power house. This is particularly true of a station where the original antenna circuit is of umbrella design and where a second antenna circuit is to be installed, which can be operated simultaneously with the first and on a long wave length differing from that of the first by only a few per cent. Such a situation confronted us at our Tuckerton station. The space immediately surrounding the power house was occupied by the umbrella antenna, which was in continuous commercial use. The new antenna could be erected on some vacant land just beyond the boundary of the space occupied by the umbrella antenna provided this antenna circuit could be fed with power at radio frequency from the power plant. The study of a transmission line that would be suitable for supplying to this antenna from the power plant, 200 kw., at frequencies of around 18,000 per sec. with little loss on the line, disclosed that this was quite practical. The antenna has been erected, this high-frequency line has been installed and the circuit has been operated very satisfactorily now for over a year. The power delivered to the antenna circuit is 92 per cent of the power supplied to the line.

The success of this type of antenna feed circuit will have a profound effect upon the design of stations operating two or more antenna circuits simultaneously.

GROUNDING SYSTEM

The first decision to be made in the development of the ground system was whether it should be of the buried wire type, or the type known as "counterpoise" or "earth screen." The New Brunswick station has a ground system combining counterpoise and buried wires. Experience had shown that while the counterpoise type might be ideal, from a theoretical point of view, it would be undesirable from the point of view of practical maintenance.

Fig. 4

A counterpoise consists of a network of wires mounted on poles. These wires carry fairly high potential and the failure of any one wire will cause interruption of service until the fault is located and repaired. The overhead system of wires is also undesirable because it is an obstruction, making the maintenance of the overhead antenna wiring difficult and expensive. Theoretical considerations indicated that a buried wire system would be as effective as an insulated counterpoise provided that its dimensions and design were carefully planned with reference to the character of the soil.

To determine the basic factors for the design of a buried ground system, measurements were made of wave propagation on wires of different lengths buried in the Long Island soil. As a result it was found that the velocity of wave propagation on a wire in this soil is about one-tenth of the velocity of wires suspended in the air. It was found, furthermore, that the resistance of the wire is a function of the wavelength. With increasing length of the wire, the conductivity increases as a linear function up to a length of one-quarter wavelength, where it reaches a maximum, after which it becomes a periodic function of the wavelength and the length of the wire. The results of these measurements showed that the maximum length of wire which could be used effectively must be something less than one-quarter wavelength of the wave propagation in the buried wire.

Measurements of wave propagation in the buried wires indicated that while lengths as great as 1200 feet could be used economically in the Long Island soil, it was furthermore determined, through calculations of the electric field distribution around the antenna, that 76 per cent of the electric lines of force radiating from the antenna would be collected by these ground wires if they were made 1000 feet long. One thousand feet on each side of the center line of the antenna was therefore considered sufficient; the result is that the Long Island antenna, in effect, stands on a plate of copper 2000 feet wide and 3 miles long, and therefore the functioning of this antenna is made independent of the resistance of the soil.

The combined antenna and ground system offers a total equivalent resistance to the antenna currents of only 40 hundredths of an ohm, made up as follows:

Radiation resistance.......	0.05 ohms at 16,500 meters
Ground resistance..........	0.10 ohms
Tuning coil resistance......	0.15 ohms
Conductor resistance........	0.05 ohms
Insulator and other losses...	0.05 ohms
Total....	0.40 ohms

The unit is operated with 200 kw. in the antenna, and the antenna current is 700 amperes, resulting in a radiation of 60,000 meter amperes.

A special plow was constructed by which the wires could be laid cheaply. The plow carried a coil of wire. It had a blade which introduced the wire in the ground at a depth of twenty inches. The plow was drawn by two Ford tractors.

The ground network consists of wires each 2000 feet long buried in the ground a depth of 15 to 20 inches in lines at right angles to the line of the antenna with the center point of the ground wire under the center line of the antenna. The ground wires are spaced 10 feet apart and as the antenna is 7500 feet long there are therefore 750 such wires making the total length of buried wire approximately 1,500,000 feet. The ground wires are connected to a heavy underground bus which runs in the ground under the center line of the antenna. There is also an aerial bus feeder which is connected to the buried bus through inductive reactances in such a manner as to make all paths to ground of equal reactance, resulting in equal distribution of the antenna current to all sections of the ground system.

CONSTANCY OF WAVELENGTHS

A factor of great importance is that of maintaining the frequency or wavelength radiated absolutely constant for reasons that will be referred to later. In radio stations using high-frequency generators of the alternator type the speed of the alternator determines the frequency of the waves radiated. In many other forms of transmitters the frequency is affected by the antenna constants if not actually controlled by the antenna, with the result that as the antenna wires are blown about by wind; and ground and insulators are affected by dry, wet or frosty weather; changes in frequency will constantly occur. In the case of the alternator the problem resolves itself into maintaining the driving motor at constant speed regardless of voltage or frequency fluctuations in the power supply or the telegraph load fluctuations to which it is subjected by the alternator. This is accomplished by a system of relays operated in synchronism with the telegraph key by means of which the voltage applied to the motor terminals and the resistance in series with the wound rotor is varied so that the motor torque is always just equal to the load to which it is applied. Tendency to change speed on account of the telegraph load is thereby eliminated. Speed fluctuations due to changes in the power supply are not so easily disposed of however. A portion of the generator output is utilized to energize a tuned circuit of low resistance adjusted to have a natural period slightly different from the frequency at which the generator is maintained so that if the alternator frequency varies only a few hundreths of one per cent in one direction, there will be a large increase in the current in this resonant circuit, or if the variation is in the other direction, there will be a correspondingly large decrease. A portion of the current in this resonant circuit is rectified and we are thus provided with a direct current which varies up or down practically instantaneously with the slightest change in the alternator frequency. This direct current is made to control the voltage supplied to the motor terminals reducing the voltage to counterbalance a tendency towards increase in speed and vice versa. In order that there may be a visual indication of what is going on, a recording ammeter is inserted in the rectified current circuit; a fine straight line on the ammeter chart indicates a constant frequency, a thick line indicates small and continuous variations of frequency and so forth. Under usual conditions of operation, irregularities of the ammeter chart line can be included within two parallel lines 1/8 in. apart representing maximum frequency variations not exceeding one in 5000 or 4 cycles per second, or 3 meters when operating at 20,000 cycles and 15,000 meters.

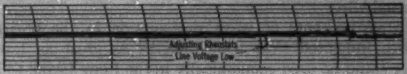

Fig. 5
New Brunswick, N. J.

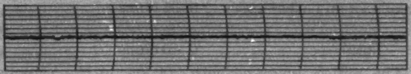

Fig. 6
Radio Central, Long Island.

Fig. 5 is a section of speed control ammeter chart from the New Brunswick station; the irregularities in this chart are due to various adjustments being made while in operation.

Fig. 6 is a section from a Speed Control Ammeter chart for one of the transmitters at the Radio Central Station.

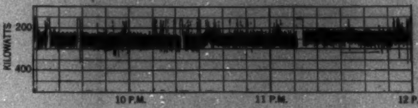

Fig. 7

Fig. 7 is the corresponding section of the wattmeter chart of the same transmitter.

RECEIVING SYSTEM

The centralized receiving system is located at Riverhead, Long Island. The antenna is of a new type which gives uni-directional reception. This system is so oriented as to receive signals from the over-ocean transmitter and annul signals from all other directions, including the powerful home transmitter nearby.

The antenna consists of two copper wires strung on ordinary poles like a telephone line, and extending over a distance of nine miles, (15,000 meters). This antenna feeds a number of separate receiving circuits of different wave lengths without the slightest mutual interference or weakening of the signals.

Important as it is, from the point of view of centralization, to be able to receive an indefinite number of signals from the same antenna, the greatest importance in the use of this new receiving system is its remarkable properties of suppressing atmospheric disturbances or the so-called "static" which hitherto has been the bane of radio communication. The attainment of these results is not an accident; it is the result of development work covering a number of years. The "wave antenna" as now used in Riverhead, is the practical answer to the receiving problem of today. The principle of directive reception has almost unlimited possibilities, and, by economic laws, the receiving system should be developed along these lines until its cost begins to equal the transmitting system. Then will the total cost of a complete circuit, transmitter and receiver, reach its ultimate minimum. However, this economic balance is far from reached as yet. The principles of reception by long antennas were laid down in two papers presented to the A. I. E. E. in 1919, one by Weagant describing a system of balanced long loops, and the other by Alexanderson describing a system of open wires balanced against each other.

In this development the controlling idea is a mental picture which we now have of the nature of the disturbance which we wish to suppress. We call it "static" because it was assumed, in the past, that it was of the nature of static electricity. The hypothesis which is the basis of our modern work is, however, different. We imagine the ether as a disturbed ocean with waves of every length rolling in from all directions. These waves are of the same nature as the signal waves. Those disturbing waves which are of different wave length from our signals, can be shut out by the same means as we use for shutting out other signals; that is, by tuning. But the disturbing waves which have the same wave length as our signal and are in all respects of the same nature, pass through our tuning system like the signal. We must therefore find some basis for discrimination other than wave length.

If we can construct a receiver which is sensitive only to waves coming from one direction, then we can shut out waves from all other directions, even if they have the same wave length. This idea started us on the work of directive reception. Theoretically, there is no limit to the improvement attainable in this direction. We might build a receiving antenna focussed on one transmitting station in Europe, but such receiving antenna would cover a very large area.

A complete theoretical analysis of the wave antenna has been given in a recent paper read this year before the A. I. E. E. by Messrs. Beverage, Rice and Kellogg. For those who wish only to understand the characteristics of our modern receiving system, in order to make use of it, the following popular explanation may be of some guidance.

Imagine the antenna to be a long, narrow lake, and that the wind is the incoming signal, and further that a cork, floating on the waves of water that beat against the shore is the detector. If the observer stands at one end of the lake, he will observe waves beating against his shore only when the wind blows lengthwise to the lake and from the end opposite to this location. When, on the other hand, the wind blows from his end of the lake, the beating waves appear at the opposite end, while his shore is calm. This, at least, would be the case if the lake has smooth sand beaches on which the waves could spend their energy without reflection. But, if the lake ends have steep rocky shores, the water waves will be reflected back and forth and thereby make the surface of the whole lake rough. The waves, which indicate the "signal wind," would thus appear at both ends of the lake, regardless of the longitudinal direction of the wind. This reflection must be avoided. The wave antenna is therefore made with ends corresponding to the sandy beach. The antenna terminates in a resistance which is carefully adjusted to absorb all wave energy and reflect none. The practical advantages of the use of the wave antenna are the elimination of about 90 per cent of the extraneous disturbances known as static.

A valuable practical feature of the form in which the wave antenna has been developed is the method of reflecting the signal so that the "surge resistance" which absorbs the static can be located in the receiving building. This is accomplished by erecting a two-wire line and making the same two wires function both as an antenna and as a transmission line for radio frequency waves. The two wires in parallel act as the antenna. At the far end of the line they are connected together through the primary winding of a special transformer. One end of the secondary of this transformer is connected to the middle point of the primary winding; the other end in connected to ground. The secondary winding feeds the current back into the two wires in series as a transmission line and a second transformer at the front end of the line couples the transmission line to the receiving set. The midpoint of the transformer winding connected to the lines is grounded through the "surge resistance." By this connection, the windings on the two halves of the transformer are opposed for currents flowing over the two wires in parallel, that is, for the antenna effect, and produce no effect upon the receiver.

The resultant reception characteristic curve shows that reception residuals of static of a few per cent may occur in certain directions in the back area of the diagram. The residuals are practically negligible in most cases, but when there is very strong interference or strong sharply directional static in their general direction, an appreciable improvement may be obtained by balancing the residuals to absolute zero for some particular direction in the back area. This is illustrated by Fig. 8.

The final balancing of static and interference is accomplished by the use of an artificial line. This line is fed by currents coming from only the same direction as the undesirable residuals. The phase of these currents may be made anything desired with respect to the phase of the residuals in the secondary of the transformer to which the surge resistance is connected. By making the intensity of the voltage on the artificial line the same as the residual voltage intensity, and by making the phase displacement 180 degrees, the residual currents are readily balanced for any particular direction in the back area.

With this antenna system, extremely satisfactory multiplex reception is being carried out at Riverhead.

Six sets of receiving equipment are normally coupled to this one antenna system, and the signals on six transoceanic circuits are separated by tuning, and copied simultaneously, each independent of the electrical operation of the other sets.

For this purpose, the antenna output transformer is built with several secondaries and the artificial lines are made up to accommodate a number of receiver sets. Many precautions are necessary in the design and arrangement of the receiving equipment to eliminate cross talk and "beat note" interference between the different sets. With this end in view, the equipment for reception of long waves has been completely remodelled.

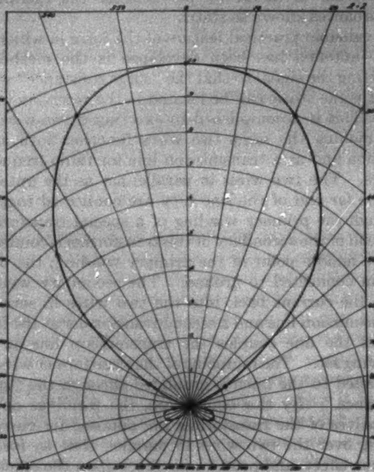

Fig. 8—Directive Curve of Wave Antenna

In the first place, all of the different elements in each receiving set must be thoroughly shielded. The tuning inductances are all balanced pairs of coils placed in an inner shielding of copper to eliminate the losses in the iron casing of the outer shield. In spite of the shielding, cross talk and beat note interference occurred until suitable chokes and filters were placed in both the positive filament and positive plate leads of all coupling tubes, amplifiers, detectors, and oscillators.

The receiving apparatus is arranged in line, with the antenna input panel at one end and the audio frequency output panel at the other end. The intervening units are placed in correct sequence so that the signal currents pass in progressive order along the line through all the various units from input to output panel without looping back over this same path.

The elements of a set are mounted on a sub-panel which is placed in an iron box, the front door of which may be opened. All adjustments of tuning and filament control which are likely to be made frequently on a set tuned to a fixed wave length can readily be made without opening the front door of the iron boxes because such control handles are mounted on the outer doors in such a manner as to engage with the controls on the sub-panel when the iron door is closed.

These receivers are set up on racks holding three sets per rack. Each set is arranged as a complete unit on a shelf and the shelves are arranged in three tiers on the racks. The Riverhead station is equipped with three racks making space for the accommodation of nine receiving sets.

Fig. 9 gives the general view of the receiving equipment.

The signals received from the wave antenna are strong; so usually a total of four stages of amplification is sufficient to bring the intensity of normal European signals up to a strength that is rather uncomfortable to the ear.

Since all the local long wave stations, except Marion, are either behind or in the case of Rocky Point, at right angles, to the direction from which the European signals come, directive reception alone lowers the intensity of the local stations so much that tuning can easily eliminate their interference. Interference as strong for instance, as that from Marion, can be eliminated when the wavelength differs by not less than 3 per cent. For interference of considerably less intensity than that from Marion, as for instance, that from stations in Europe, or from a local station, reduced by directive reception, a 2 per cent difference in wavelength is sufficient.

Fig. 9

For wavelength difference of 2 per cent and less the constancy of frequency of the transmitting station becomes of very great importance. Extremely good frequency regulation at the transmitting station will allow the use of filter circuits by means of which interference on wavelengths differing less than 2 per cent from that of the desired signal can be eliminated.

The receiving station at Riverhead, L. I., is about 70 miles east of New York and the next phase of the problem was the automatic transfer of the radio signals to the central control office in New York City in order to eliminate the double handling of traffic, the slowing up of the circuit, and the other delays inseparable from the older system. The requirements of this circuit were studied and then the American Tel. & Tel. Co. was requested to provide a suitable tone circuit from Riverhead to our Broad Street office, New York City. For a period of several months experiments were conducted over this temporary line, during which it was demonstrated that it was feasible to send these tone signals over a 70-mile circuit without detriment to the readability of the signals. Continuous commercial operation over a single tone circuit was started about July 1st, 1921. Subsequently additional tone circuits were built for the commercial operation and control of Riverhead station in this manner.

Central Operating Room

The operating room at the city offices is the place where the written message is converted to the dot and dash of the Morse code. The continental code is used in radio as in all other international telegraphic communication. During the last few years a great change has taken place in the transmission of the message. Whereas formerly the manually operated telegraph key was used almost universally for speeds of transmission of 40 words per minute or less, this has been entirely displaced by the machine transmitter. The advantages of machine transmission over hand transmission are (1) that the operator is required to work a typewriter keyboard only and need not necessarily be a skilled telegraphist, (2) that one operator can transmit messages in this manner at rates up to 100 words per minute, whereas the best that can be done by hand is 35 or 40 words per minute, (3) that all characters are perfectly formed and do not vary with the different operators, and (4) the machine is tireless and has no lost time. The telegraphic manipulation is actually accomplished by first punching the message on a paper tape and subsequently passing the punched paper tape through the mechanical transmitter which is an automatically operated telegraph key.

The transmitter sends telegraph impulses over the control wires between the city office and the transmitting station and operates the relay system at that station.

In order that a check can be kept on the performance of the automatic transmitter, the control wires, and the relay system of the transmitting station, a radio receiving set is provided at the city office which makes audible or visible to the operator the actual signal being transmitted into the ether. This receiver is a very simple piece of apparatus, since the reception of the signal from the nearby high power transmitter is not at all difficult, although of course, as there are so many transmitters operating in one locality with only small wavelength separation, very efficient tuning equipment must be provided.

The reception of a message at the city office requires a reversal of the above process. The signal as received at the receiving station is in the form of audio frequency current, the frequency of which is variable as desired, these signal currents are transferred to the city office by telephone wires. At the city office it is necessary to further amplify the currents before they are introduced into the telephone or the recorder. It is possible to use aural reception at speeds up to 35 or 40 words per minute. Better speeds can be secured at times by a combination of aural and recorder reception. At speeds over 40 words per minute tape reception must be used exclusively. It is possible for some tape readers to copy as fast as 60 words per minute but generally for speeds over 40 words per minute; the work is divided up among an increased number of operators; 40 to 70 words per minute two operators; 70 to 100 words per minute three operators, and so forth. The development of the tape recorder used for transoceanic radio reception was ably described in a paper presented to the Inst. of Radio Engineers by J. Weinberger in 1921.

The electrical equipment of the operating room of a city office, handling a large number of circuits, requires careful planning. In the city office of the Radio Corporation of America at 64 Broad Street, New York City, there are at present in continuous operation,

6 transoceanic receivers
6 local Monitor receivers
6 automatic transmitters

and over 30 land wires. To these will soon be added a number of new services.

Power supplies of different types are provided for the various electrical and mechanical devices and measures have been taken to prevent inductive interference effects between instruments.

Wavelength Distribution

The economical wavelength for communication over a certain distance can be selected by the practical rule that the economic range of a station for reliable communication is about 500 to 1000 times the wavelength. If too short a wave is selected the signals will be weak in daytime and strong but variable at night. This variation is most noticeable during the period when darkness exists over the area between the communicating stations. In some parts of the world it is possible to use short waves to advantage because the absorption is comparatively lower than on long waves and variations are unimportant but generally speaking for distances over 3000 miles the reliability of wavelengths of over 11,000 meters is so much greater than that of shorter waves. Long waves have therefore been universally adopted for long distance communication.

It can now be readily seen that since the ability to receive distinct signals depends on the separation of different frequencies there is a definite limit to the number of "channels" of communication between stations that can be set up.

If the wavelengths between 11,000 and 22,000 meters are divided into 2 per cent bands there are 35 "channels." If into 1 per cent bands, there are 70 "channels." Except to such extent as directional reception will permit the number of one way channels open for such, long distance communication is limited to the number of these bands.

If we suppose our plans to be based on the use of 1 per cent bands, it is evidently necessary first that each transmitter shall cause no radiation outside of the 1 per cent band allotted to it and furthermore shall maintain its actual radiation frequency exactly on the center of such band; and second that each receiver shall be capable of separation of currents from those differing only 1 per cent in frequency. The above requirements imposed upon the transmitter have already been proved practicable. But the realization of the full possibilities of radio communication requires that all transmitters of antiquated type which take undue space in the ether be replaced.

There are, however, other difficulties that cannot be so easily overcome. For instance while it is quite possible for the receiving station to separate currents of frequencies differing 1 per cent if the voltages induced at the station at the different frequencies are equal, it is not an easy matter to separate the currents when the voltage induced in one case is 1000 times the voltage induced in the other. This is the situation where in the case of a transatlantic circuit the receiving station in America receives from Europe on 15,000 meters and the transmitting station in America sends to Europe at the same time on 15,150 meters. In such cases, as described above, it is necessary to increase the separation between frequencies to 3 per cent and in order that such large separation may not be too numerous a rule has been established by precedence and informal agreement, that all the transmitters in one locality shall transmit on wavelengths close together. We have such a case in the concentration of American transmitters between 16,000 and 17,500 meters. In this band of wavelengths there are operating at present the following stations:

15,900 Meters	Tuckerton No. 1 Transmitter
16,300 Meters	Kahuku No. 1 Transmitter
16,465 Meters	Radio Central No. 1 Transmitter
16,700 Meters	Tuckerton No. 2 Transmitter
16,975 Meters	Kahuku No. 2 Transmitter
16,975 Meters	Annapolis Compensating Wave Arc
17,145 Meters	Annapolis Signalling Wave Arc
17,500 Meters	Radio Central No. 2 Transmitter

It is planned to operate transmitters in Sweden, Poland and Argentine in the near future on wavelengths 18,000 meters to 19,000.

The French Government station at Lyon operates at 15,500 meters and there are a number of additional European and American Transmitters operating between that wavelength and 11,000 meters, while other Government and Commercial stations in France are at present operating at wavelengths from 19,000 to 22,000 meters.

The congestion of the ether is therefore not a mere matter of looking into the future, but a real present day problem. The necessity for traffic regulation is at least enough to prevent reckless driving so to speak, is just as apparent as the undesirability of hidebound regulations until such time as the limit of possible improvements in technique have been more definitely determined.

Such is the present situation in the long distance radio ether. The congestion is due to the necessity for the use of the longer waves for long distance work and the fact that all high-power stations are broadcast stations; much improvement is possible in existing practise but radically new methods of operation must also be considered, such for example as directional radiation on shorter waves. With the realization of such possibilities the situation will take on a new aspect.

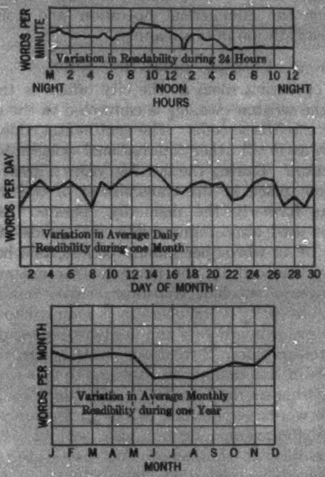

Fig. 10

Project of New Communications

Sufficient statistics are now available by means of which the technical and financial possibilities of new circuits of communication can be accurately predetermined.

Fig. 10 shows the daily, monthly and yearly reception curves for a typical transatlantic circuit. The ordinates of these curves show the capacity of the circuit at the different times of the day and year respectively. By the capacity of the circuit we mean the practically possible speed of reception in five-letter code words per minute. The capacity of the circuit is a function of the strength of the signal and the intensity of the disturbance. The intensity of the signal is measured in absolute units of microvolts per meter.

Experience has shown that under any given condition of atmospheric disturbance, there is a direct proportionality between the strength of the signal measured in microvolts per meter and the traffic capacity of the circuit measured in words per minute. The proportionality defined above is almost exact between the limits of oral reception ranging from 5 to 40 words per minute and it can be considered as substantially correct up to the highest speeds that are used. This simple relation between strength of signal and words per minute has given us a practical method of measuring the intensity of atmospheric disturbances.

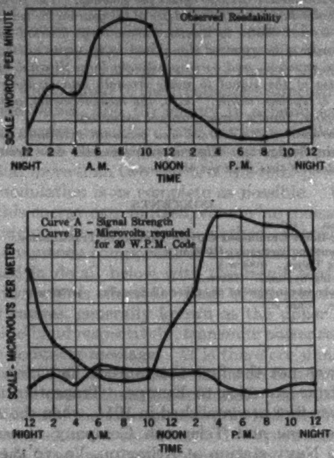

Fig. 11—Typical Daily Variation of Signal and Disturbances on Vertical Antenna During Summer Months at Belmar, N. J.

As an actual standard method of measurement an artificial signal is introduced into the receiving system and regulated so that the capacity of the receiver is 20 words per minute. The number of microvolts per meter which must be introduced to permit reception at 20 words per minute is thus a direct measure of the intensity of disturbance.

Fig. 11 shows a typical daily curve of variation of signal strength and disturbance, measured on a simple vertical antenna.

If a transmitting station is to be designed for a new geographic location, measurements of disturbances are taken in that location. The results of these measurements, which may be taken over a large part of a year, show what strength of signal will be needed during the different months of the year to carry a desired traffic. Fig. 12 shows a typical chart of this kind. Comparison between this chart and the known typical yearly chart for a transatlantic circuit gives a direct indication of the capacity of the new circuit in terms of the circuit in operation. The chart for the projected circuit shows the capacity of a 50,000-meter ampere and of a 100,000-meter ampere transmitting station.

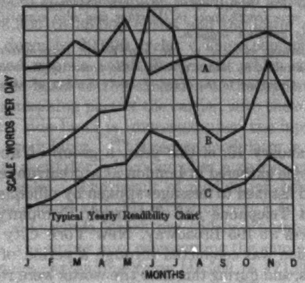

Fig. 12

Curve A — Typical East-West circuit.
Curve B — Typical North-South circuit. Calculated for 100,000 M.A.
Curve C — Typical North-South circuit. Calculated for 50,000 M. A.

Thus it can be stated that guess work has been eliminated from the development of radio communication, and that sound foundations, both technically and financially, can be laid for all future expansions of our system.

Transatlantic Radio Telephony

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Review of the Subject.*—The first transmission of the human voice across the Atlantic was accomplished by means of radio in 1915. Since that time substantial progress has been made in the art of radio telephony and in January of this year another important step was taken in the accomplishment of transoceanic voice communication. At a prearranged time telephonic messages were received in London from New York clearly and with uniform intensity over a period of about two hours.*

These recent talking tests were part of a series of experiments on transatlantic telephony which are now under way, the results of which to date are reported in the paper.

A new method of transmission radiating only a single side-band is being employed for the first time. As compared with the ordinary method of transmission, this system possesses the following important advantages:

The effectiveness of transmission is greatly increased because all of the energy radiated is effective in conveying the message; whereas in the ordinary method, most of the energy is not thus effective.

The stability of transmission is improved.

The frequency band required for transmission is reduced, thus conserving wave length space in the ether and also simplifying the transmitting antenna problem.

An important element of the high-power transmitter is the water-cooled tubes, by means of which the power of the transmitted currents is amplified to the order of 100 kilowatts or more. The direct-current power for these tubes is supplied from a 60-cycle, a-c. source through water-cooled rectifier tubes.

A highly selective and stable type of receiving circuit is employed. Methods and apparatus have been developed for measuring the strength of the electromagnetic field which is delivered to the receiving point and for measuring the interference produced by static.

The transmission tests so far have been conducted on a wave length of 5260 meters (57,000 cycles per second). The results of the measurements during the first quarter of the year on the transmission from the United States to England show large diurnal variations in the strength of the received signal and in the radio noise strength, as is to be expected, and correspondingly large diurnal variations in the ratio of the signal to noise strength and in the resulting reception of spoken words. Also, the measurements, although as yet incomplete, show a large seasonal variation.

The character of the diurnal and seasonal variations is clearly indicated in the figures. The curves present the most accurate and complete data of this kind yet obtained.

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ON January 15, of this year, a group of about 60 people gathered in London at a prearranged time and listened to messages spoken by officials of the American Telephone and Telegraph Company from their offices at 195 Broadway, New York City. The transmission was conducted through a period of about two hours, and during this time the words were received in London with as much clearness and uniformity as they would be received over an ordinary wire telephone circuit. During a part of the time a loud speaker was used in connection with the receiving set, instead of head receivers. The reporters present easily made a transcription of all the remarks, both with the head sets and with the loud speaker.

These tests were made possible by cooperation between the engineers of the American Telephone and Telegraph Company and the Western Electric Company, and the engineers of the Radio Corporation of America and its associated companies. The sending apparatus was installed in the station of the Radio Corporation of America, at Rocky Point, L. I., in order to make use of that company's very efficient multiple-tuned antenna. The receiving apparatus was installed in the buildings of the Western Electric Company, Ltd., at New Southgate, England.

This was not the first time speech had been transmitted from America to Europe. Transatlantic telephony was first accomplished in 1915, when the American Telephone and Telegraph Company transmitted from the Navy station at Arlington, Va. to the Eiffel Tower, Paris. In these earlier tests, however, speech was received in Paris only at occasional moments when transmission conditions were exceptionally favorable. The success of the present tests indicates the large amount of development which has been carried out since this first date.

The recent talking tests were carried out as part of an investigation of transatlantic radio telephony. This investigation is directed at determining (1) the effectiveness of new methods and apparatus which have been developed for telephonically modulating and transmitting the large amounts of power necessary for transoceanic operation, (2) the efficacy of improved methods for the reception of this transmission and for so selecting it as to give an extremely sharp differentiation between the range of frequencies transmitted and all the frequencies outside of this range; and (3) determining the transmission characteristics for transatlantic distances and the variation of the characteristics with the time of day and the season of the year, including the measurement of the amount of static interference.

The tests are being continued, particularly as regards the study of transmission efficiency.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

SINGLE SIDE-BAND ELIMINATED CARRIER METHOD OF TRANSMISSION

The method of transmission used in these experiments is what we know as the single side-band eliminated carrier method[1]. With this method,[1] the narrowest possible band of wave lengths in the ether is used, and all of the energy radiated has maximum effectiveness in transmitting the message.

As has been pointed out in other papers[2], when a carrier is modulated by telephone waves, the power given out is distributed over a frequency range, and may be conveniently considered in three parts: (1) energy at the carrier frequency itself, (2) energy distributed in a frequency band extending from the carrier upward, and having a width equal to the frequencies appearing in the telephone waves, and (3) energy in a band extending from the carrier downward, and having a similar width. The power at the carrier frequency itself makes up somewhat more than two-thirds of the total power, even when modulation is as complete as possible. Furthermore, this energy can, in itself, convey no message, as is self evident. In the present method, therefore, the carrier-frequency component is eliminated, by methods explained in detail below with the result that a large saving in power is effected. Each of the remaining frequency ranges, generally known as the upper and the lower side-band respectively, transmits power representing the complete message. It is therefore unnecessary to transmit both of these side-bands, so that in the present method one of them is eliminated. In this way the transmission of the message uses only half the frequency band required in the usual method of operation. Similarly the frequency-band accepted by the receiving set is narrowed to conform to a single side-band as compared with the usual double side-band reception, and as a result the ratio of signal to interference is improved. Certain other advantages of this method will be brought out in the further discussion.

While these advantages of the single side-band eliminated carrier method hold good for radio telephone transmission generally, they become of the utmost importance in transoceanic work, because of the necessity of conserving power in a system where the transmitting powers are large, and also because the very limited frequency range available for long distance transmission makes it imperative that each part of the range shall be utilized with the greatest of care. Before discussing the method further, the circuits and apparatus which are actually used in the tests will be described.

THE TRANSMITTING SYSTEM

The transmitting system is shown in simplified circuit form in Fig. 1. It is illustrated as grouped into three parts: The low-power modulating and amplifying stages, shown below in light lines; the high-power amplifiers, shown in heavy lines above and to the right; and the rectifier which supplies the power amplifier with high-tension direct current, shown in the upper left-hand portion of the diagram.

Referring first to the low-power portion of the system, it will be seen that the voice currents (from either a telephone line or a local microphone) are fed into a balanced type of modulator circuit and are modulated with a carrier current of a frequency of about 33,000 cycles. The operation of the balanced type of modulator in suppressing the unmodulated carrier component is explained in the Colpitts and Blackwell carrier current paper referred to above. The result of this modulating action is to produce in the output circuit of modulator No. 1, modulated current representing the two side-bands, for example, the upper one extending from 33,300 to 36,000 cycles and the lower one from 32,700 down to 30,000 cycles. These components are impressed upon a band filter circuit which selects the lower side-band to the exclusion of the upper one and of any remaining part of the carrier, with the result that only one side-band is impressed upon the input of the second modulator. This second modulator is provided with an oscillator which supplies a carrier current of 88,500 cycles. The result of modulation between the single side-band and this carrier current is to produce a pair of side-bands which are widely separated in frequency, the upper one, representing the sum of the two frequencies, extending from 118,500 to 121,200 cycles and the lower one, representing the difference between the two frequencies, extending from 58,500 down to 55,800 cycles. In this second stage of modulation there is a relatively wide separation between the two-side bands which facilitates the selection at these higher frequencies of one side-band to the exclusion of the other. Another important advantage is that it allows a range of adjustment of the transmitted frequency without changing filters. This is accomplished by varying the frequency of the oscillator in the second step. In the present case, the frequency desired for transmission is that corresponding to the lower side-band of the second modulator. The lower side-band of from 58,500 to 55,800 is therefore selected by means of the filter indicated. This filter excludes not only the other side-band but also any small residual of 90,000-cycle unmodulated carrier current which may get through the second modulator circuit if it is imperfectly balanced.

Having prepared at low power the side-band currents of desired frequency it is necessary to amplify them to the required magnitude for application to the transmitting antenna. This amplification is carried out in three stages. The first stage increases the power to about 750 watts, and is shown in Fig. 1 together with the modulating circuits. This amplifier employes in its last stage three glass vacuum tubes rated at 250 watts each and operating at 1500 volts.

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[1] For a more complete exposition of this method see U. S. patent No. 1449382 issued to John R. Carson to whom belongs the credit for having first suggested it. Also see Carson patents Nos. 1,343,306 and 1,343,307.

[2] "Carrier Current Telephony and Telegraphy" by Colpitts and Blackwell. JOURNAL A. I. E. E., April, 1921.

"Application to Radio of Wire Transmission Engineering" by Lloyd Espenschied. *Proc. Inst. Radio Engrs.*, Oct. 1922.

"Relations of Carrier and Side-bands in Radio Transmission" by R. V. L. Hartley. *Proc. Inst. Radio Engrs.*, Feb. 1923.

The output of the 750 watt amplifier is applied to the input of the larger-power amplifying system beginning with the 15 kw-amplifier of Fig. 1. This consists of two water-cooled tubes in parallel, operating at approximately 10,000 volts. The output of this amplifier is applied by means of a transformer to the input of the 150-kw. amplifier which consists of two units of ten water-cooled tubes each, all operating in parallel at about 10,000 volts.

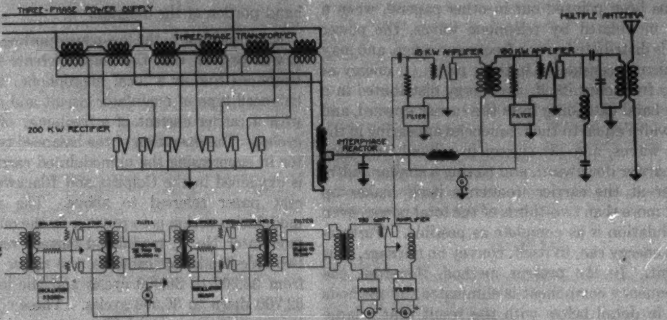

FIG. 1—SINGLE SIDE-BAND ELIMINATED CARRIER TRANSMITTER

The high-voltage, d-c. supply is furnished by a large vacuum tube rectifier unit rated at 200 kw. It employes water-cooled tubes similar to those used in the power amplifiers except that they are of the two-electrode type. The rectifier operates from a 60-cycle, three-phase supply circuit and utilizes both halves of each wave. The two sets of rectified waves are combined by means of an inter-phase reactor which serves to smooth out the resultant current and by distributing the load between tubes of adjacent phases increases the effective load capacity of the rectifier. The ripple is further reduced by the filtering retardation coil and condensers shown.

FIG. 2

Reproductions of the apparatus comprising the transmitter system as described above are given in Figs. 2, 3, 4 and 5.

Fig. 2 shows the apparatus comprising the low-power stage of the transmitting system. The right-hand rack contains the two weak-power modulating units and the two single-side-band selecting filters. The left hand rack is the 750-watt amplifier unit. The three radiation-cooled tubes of 250 watt capacity each will be seen near the top. Below are the smaller amplifying stages. The power supply board is shown in the center of the photograph.

FIG. 3

Fig. 3 is a side view of the 15-kw. amplifier unit. The face of the panel from which the control handles protrude is on the left. Mounted in the cage behind the front panel are two water-jackets for accommodating the water-cooled tubes, also a coiled hose for increasing the electrical resistance of the water supply circuit (the water-cooled anodes of the tubes being operated above ground potential).

The final amplifier of 150 kw. capacity is shown in Fig. 4. It comprises two units each of 75 kw. Each unit contains 10 water-cooled tubes which can be seen mounted in their water jackets. To the right of these units is located the 200-kw. rectifier unit shown in Fig. 5. The unit contains actually 12 tubes, there being two tubes for each of the six half waves. The pancake coils on the top of the rack are protective choke coils to guard the transformer secondary winding against steep wave fronts in case of tube failure.

Fig. 4

From the above description it will be understood that the transmitting system is one in which the useful side-band is first developed by modulation and filtration at low powers and then its power is built up to a large value in a succession of powerful amplifiers. It will be appreciated, therefore, that the large power amplifiers and in particular the water-cooled tubes which are their essential elements represent one of the major problems of the development.

Fig. 5

High-Power Tubes

The development of the high-power tubes is described quite fully in another paper[3]. The present discussion is, therefore, limited to a few of the outstanding features.

In the design of high-power tubes for use in this system the main problem is to insure the ready disposal of the large amounts of heat generated at the anodes. For the conditions of use in the present type of system where the tube is employed as an amplifier, the power which must be disposed of as heat at the anode is of the same order of magnitude as the power which the tube will deliver to the antenna. In the case of the present equipment, therefore, the tube must be so designed as to operate continuously with a heat dissipation at the anode of more than 10 kw. It is obviously difficult to secure so large a dissipation in a tube enclosed with glass walls, and a tube was therefore designed in which the anode forms a part of the wall of the containing vessel and the heat generated in it is removed by circulating water. The tube used is shown in Fig. 6. The lower cylindrical portion is the anode which is drawn from a sheet of copper. The upper portion is of glass and serves both to support and insulate the grid and filament elements.

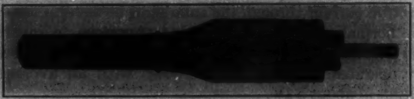

Fig. 6

The three principal difficulties met in the construction of these tubes are the making of a vacuum-tight seal between the copper and the glass, the provision of adequate means for conducting through the glass wall the large currents necessary to heat the filament, and the obtaining of the necessary vacuum for high-power operation.

The first of these problems was solved by the development of a new metal to glass seal. In making this seal the glass and metal parts are brought into contact while hot, the temperature being high enough for the glass to wet the metal. The part of the metal in contact with the glass is made so thin that the stresses which are set up when the seal cools are not great enough to fracture the glass or to break it away from the metal at the surface of contact. Seals made in this way are sufficiently rugged to stand repeated heating and cooling from the temperature of liquid air to that of molten glass without deterioration.

A seal employing the same principle but differens in form is also used at the point where the leadt carrying the filament current pass through the glass walls of the tube. The lead is made of copper 0.064 in. in diameter and passes through the center of a copper disk, 0.010 in. thick, the joint between the lead and the disk being made vacuum-tight by the use of a high-melting-point solder. The disk is sealed to the end of a glass tube which is in turn sealed into the glass wall of the vacuum tube.

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[3] Bell System Technical Journal, July 1922.

In exhausting the tubes it has been found necessary to subject all the metal parts to a preliminary heat treatment in a vacuum furnace during which the great bulk of the occluded gasses is removed. By this method the time of exhaust can be considerably reduced but the vacuum conditions to be met are so stringent that the final processes of evacuation must be carefully controlled and often occupy as much as twelve hours.

The tubes are operated at a plate voltage of 10,000 volts and are capable of delivering 10 kw. at this voltage in a suitable oscillatory circuit. For this performance an average electron current of 1.35 amperes is required. The total electron current that the filament must be capable of supplying to insure steady operation is about 6 amperes.

When the tubes are used to amplify modulated currents with large peak values such as are characteristic of telephone signals it is essential that the maximum electron current through the tube shall be several times the normal operating current and therefore to insure the necessary high quality of transmission these tubes are operated for telephone purposes with an average output of about 5 kw.

THE RECEIVING SYSTEM

In the method of transmission ordinarily employed in radio telephony by which the carrier and both side-bands are sent out from the transmitting station and received at the distant end, detection is readily accomplished merely by permitting all of these components to pass through the detector tube. The detecting action whereby the voice-frequency currents are derived, is accomplished by a remodulation of the carrier with each sideband.

With the present eliminated carrier method of transmission the side-band is unaccompanied by any carrier with which to remodulate in the receiving detector. It is necessary, therefore, to supply the detector with current of the carrier frequency obtained from a local source. Thus, in the present experiments, if a current of the original carrier frequency, 55,500 cycles, is supplied to the detector it will remodulate or "beat" with the received side-band of, say 55,800 to 58,500 cycles and a difference-frequency band of 300 to 3000 cycles, *i. e.*, the voice frequency band will result.

The arrangement actually used, however, is not quite so simple as this. It is shown schematically in Fig. 7. Reception is carried out in two steps, the received side-band being stepped down to a lower frequency before it is detected. The stepping down action is accomplished by combining in the first detector the incoming band of 55,800 to 58,500 cycles with a locally generated current of about 90,000 cycles. In the output circuit of the detector the difference-frequency band of 34,200 to 31,500 cycles is selected by a band filter and passed through amplifiers and thence to the second detector. This detector is supplied with a carrier of 34,500 cycles which, upon "beating" with the selected band, gives in the output of the detector the original voice-frequency band.

The object of thus stepping down the received frequency is to secure the combination of a high degree of selectivity with flexibility in tuning. The high selectivity is obtained by the use of a band filter. It is further improved by applying the filter after the frequency is stepped down rather than before. To illustrate this improvement assume that there is present an interfering signal at 60,000 cycles, 1500 cycles off from the edge of the received telephone band. This is a frequency difference of about 2½ per cent; but after each of these frequencies is subtracted from 90,000 cycles, the difference of 1500 cycles becomes almost 5 per cent. This enables the filter to effect a sharper discrimination against the interfering signal. Furthermore, the filter is not required to be of variable frequency as would be the case were it employed directly at the received frequency since by adjusting the frequency of the beating down oscillator the filter is in effect readily applied anywhere in a wide range of received frequencies. The receiving method, therefore, enables the filter circuit, and indeed also the inter-

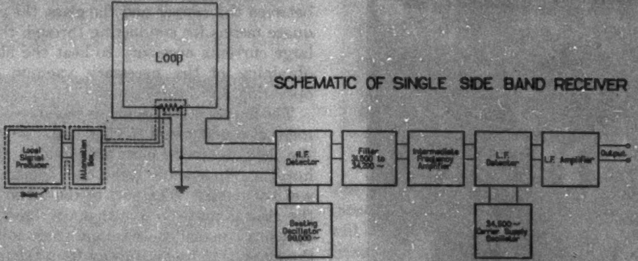
SCHEMATIC OF SINGLE SIDE BAND RECEIVER
Loop
Local Signal Producer
Attenuation Box
Shield
H.F. Detector
Filter 31,500 to 34,200 ~
Intermediate Frequency Amplifier
L.F. Detector
L.F. Amplifier
Output
Beating Oscillator 90,000 ~
34,500 ~ Carrier Supply Oscillator

FIG. 7

mediate frequency amplifiers, to be designed for maximum efficiency at fixed frequency values without sacrificing the frequency flexibility of the receiving set.

A photograph of the receiving set used in the transatlantic measurements is reproduced in Fig. 8. The signals are received on a square loop six feet on a side and wound with 46 turns of stranded wire. The first box contains the beating oscillator and high-frequency detector, the second box the filter and amplifying apparatus for the intermediate frequency and the third box the final detector and amplifier. The shielded box at the left of the picture, which is connected to the loop by means of leads in the copper tube, is the apparatus for introducing the comparison signal of known strength into the loop for measuring purposes. This receiving and measuring set is described more in detail in a paper by Bown, Englund, and Friis in the *Proceedings* of the Institute of Radio Engineers for April 1923.

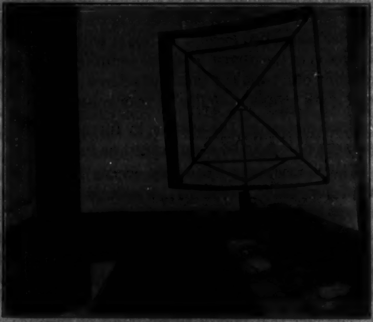

FIG. 8

Although it was this very selective and reliable method of intermediate-frequency reception which was used in London, it is quite possible to receive the single-side-band transmission by means of a regular heterodyne receiving set. Even a self-regenerative set will suffice under some conditions. It is necessary, however, to adjust the frequency of the oscillator very carefully to that of the transmitting carrier frequency, otherwise serious distortion of the received speech will result. Also it is, of course, necessary that the tuning be not too sharp if ordinary tuned circuits and not filter circuits are employed. One might expect that some difficulty would be experienced in maintaining the frequency at the receiving end in sufficiently close agreement with the sending frequency. In the tests no particular difficulty was experienced, the oscillators at the two ends being so stable that only an occasional slight readjustment of the receiving oscillator was required. With the development of more stable oscillators, doubtless, the frequency with which readjustments are required, will be further reduced. If serious distortion of the received speech is to be avoided the two frequencies must be within about 50 cycles, an accuracy of 0.1 per cent at 50,000 cycles.

TRANSMISSION ADVANTAGES OF THE SYSTEM

Since the present experiment represents the first use of the single-side-band eliminated carrier type of system some further discussion of the characteristics and advantages of the system is appropriate.

The importance of the system in conserving frequency range will be appreciated when it is realized that the total frequency range available for transatlantic telephony is distinctly limited. Just what the most suitable range is has not been accurately determined but it seems limited to below 60,000 cycles (5000 meters) because of the large attenuation suffered during the daylight hours by frequencies higher than this. On the lower end of the frequency scale, transatlantic telegraphy at present pretty well preempts frequencies below 30,000 cycles (10,000 meters). Therefore, for the present at least transatlantic telephony is limited to a range of some 30,000 cycles. Now transmission of speech requires as a minimum for good quality a single-side-band 3000 cycles wide. Allowing for variations and clearances between channels it is doubtful if the channels could be made to average closer than one every 4000 cycles for single-side-band transmission and one every 7000 cycles for the ordinary double side-band transmission. This means that even were the whole range from 30,000 to 60,000 cycles devoted to telephony to the exclusion of telegraphy, only about four channels could be obtained by the older methods and some seven by the present one.

It is a rather interesting commentary to note that a somewhat similar situation as to limitation in frequency range exists in the case of carrier-current transmission over wires. The transmission efficiency falls off with increase in frequency and limits the range of frequencies which can be economically used, in much the same way as it is limited in long distance radio transmission. It is because of this limitation in the case of wires and the value which attaches to conserving the frequency range consumed per message that single side-band transmission was first developed for wire carrier current systems. Its development in wire transmission has been of considerable aid in adapting the method to the present purpose of transatlantic operation.

The second of the outstanding characteristics of the present system resides in the large power economy which it permits. Transatlantic telephony requires hundreds of kilowatts of high-frequency power. Since it is difficult and expensive to produce this power it is important that every effort be made to increase its efficiency or effectiveness in transmitting the voice. To illustrate how the present system effects economies in power, consider the case of a carrier wave completely modulated by a single frequency tone. In such a completely modulated wave, only 1/3 of the total power contains the message, the remaining 2/3 conveying only the carrier frequency which can as well be supplied from an oscillator of small power at the receiving station. It is obvious, therefore, that by eliminating the carrier only 1/3 as much power need be used as would be required were all the elements of the completely modulated wave transmitted. To realize the maximum advantage of this mode of operation, the system eliminates the carrier at low power and, thereby, the high-power apparatus is devoted exclusively to the amplification of the essential part of the signal.

If, after having suppressed the carrier, both side-bands were transmitted, their reception would require perfect synchronism between the carrier resupplied at the receiving end and that eliminated at the sending end, a condition which is practically impossible to meet without transmitting some form of synchronizing channel, which is, indeed, much the same as transmitting the carrier itself. If the receiving carrier is not synchronized, the two side bands will interfere with each other upon being detected. By eliminating one side-band, this interference is prevented and reception may be carried on, using a locally supplied frequency which is only approximately equal to that of the suppressed carrier. The two may differ by as much as 50 cycles before the quality of the received speech is greatly impaired. The importance to the carrier suppression method of eliminating one side-band will, therefore, be appreciated. The present system eliminates one side-band while still in the low-power stage. While it would be possible to do this selecting after they have both been raised to the full transmitting power, this would require the use of a filter of high-power carrying capacity, which would make the filter very costly and also render the system inflexible to change of wave length. The present system overcomes both of these difficulties by filtering out one side-band at low power levels and by the use of the double modulation method.

Another very important reason for the transmission of a frequency band as narrow as is possible lies in the difficulty of constructing an antenna to transmit more or less uniformly at these long waves a band of frequencies which is an appreciable fraction of the main carrier frequencies. For example, in the ordinary method of transmission an antenna which was intended to transmit a 30,000-cycle carrier and its two speech side-bands would need to be designed to transmit all the frequencies from 27,000 cycles to 33,000 cycles, a band which is equal to 20 per cent of the carrier frequency. This band is considerably wider than that given by the resonance curve of a highly efficient long wave antenna. To accommodate both side-bands would require flattening out the resonance curve either by damping, which means sacrifice in power efficiency, or by special design of the antenna, possibly throwing it into a series of interacting networks and causing it to become a rather elaborate wave filter. The importance, from the antenna standpoint, of narrowing the frequency band required to be transmitted is, therefore, evident.

It is extremely important that the received signal be affected as little as possible by changes in the transmission efficiency of the medium. The voice frequency currents produced at the receiving end, after detection, are proportional to the product of the carrier wave and the side-band. If the carrier as well as the side-band is transmitted through the medium, then a given variation in the transmission efficiency of the medium will affect both components and will change the received speech in proportion to the square of the variation, as compared to the first power if only the side-band is transmitted and the carrier is supplied locally. Thus it will be seen that the omission of the carrier from the sending end and the resupplying of it from the constant source at the receiving end gives greater stability of transmission.

Without discussing the system in further detail the advantages of it may be summarized as follows:

1. It conserves the frequency (wave length) band required for radio telephony, which is particularly important at long wave lengths.

2. It conserves power, in that all of the power transmitted is useful signal-producing power. This is particularly important also in long distance transmission which requires the use of large powers.

3. The fact that only a single band of frequencies is transmitted simplifies the antenna problem at long wave lengths, where the resonance band becomes too narrow to transmit both side-bands.

4. As compared with a system which eliminates the carrier but transmits both side-bands the simple side-band system has the important advantage of not requiring an extreme accuracy of frequency in the carrier which is resupplied at the receiver. Were both side-bands transmitted very perfect synchronism would be required for good quality.

5. It improves the transmission stability of the radio circuit since variations in the ether attenuation affect only one (the side-band) of the two components effective in carrying out the detecting action in the receiver.

6. The receiving part of the overall system has two advantages:

a. It need accept only half of the frequency band which would be required in double side band transmission, thereby accepting only half of the "static" interfering energy.

b. By stepping down the frequency of the received currents and filtering and amplifying at the low-frequency stage a very sharp cutoff is obtained for frequencies outside of the desired band and a very stable and easily maintained amplifying system is obtained.

STUDY OF TRANSATLANTIC TRANSMISSION

We come now to a consideration of the second major part of the investigation, namely, that having to do with the transmission of the waves across the Atlantic. It will be evident, from what has been said earlier, that the transmission question is essentially one of how best to deliver, through the variable conditions of the ether to the receiving station, speech-carrying waves sufficiently free from interference to be readily interpretable in the receiving telephone. The transmission efficiency of the medium varies with time of day and year, and is different for different wave lengths. The interference conditions are also influenced by these same factors.

Now we can study this transmission medium in much the same way we would a physical telephone circuit, by putting into it, at the sending end, electromagnetic waves of a known amount of power and measuring the power delivered at the receiving end. The interference at the receiving station likewise may be measured and the ratio of the strength of the signal waves to the interfering waves may be taken as a measure of freedom from interference; this in turn being directly related to the readiness with which the messages are understood. Accordingly, there has been included as an integral part of the investigation of transatlantic radio telephony, the development of suitable methods and apparatus for measuring the strength of the signal waves and of the interfering waves, as they arrive at the receiving station. The apparatus[4] employed in measuring the field strength of the received signals has been outlined above under Receiving System and need not be gone into further. However, a word of explanation about the method which is employed in making the measurement may be helpful. It will be recalled that the specially designed receiving set is provided with a local source of high frequency from which can be originated signals of predetermined strength. A measurement of the field strength of a signal received from the distant transmitter is made by listening first to the distant signal and then to the locally produced signal, shifting back and forth between these signals and adjusting the strength of the local signal until the two are substantially of the same strength. Then, knowing the power delivered by the local source, the power received from the distant station is likewise known. The relation between the power in the input of the radio receiving circuit to the field strength required to deliver that power is known through the geometry of the receiving antenna (in this case a loop) and, therefore, the measured power of the signal can be translated directly into the field strength of the received waves.

The measurement tone signal is transmitted from the Rocky Point sending station by substituting for the microphone telephone transmitter a source of weak alternating current of about 1/100 watt at a frequency of approximately 1500 cycles. This tone modulates the radio telephone transmitter in the same way that voice currents would and is radiated from the antenna as a single-frequency wave of 5260 meters (57,000 cycles per second). It, therefore, constitutes a means of sending out a single-frequency continuous wave for measurement purposes. At the receiving end this continuous wave is demodulated to the same tone frequency which it originally had.

For measuring the strength of the received noise, *i. e.*, the radio frequency currents arising from static or other station interference, the method is quite similar. In this case, however, the noise received is so different from that which can be set up artificially in any simple manner that no attempt is made to compare it directly with a local noise standard. Instead the volume of the interfering noise is expressed in terms of its effect in interfering with the audibility of a local tone signal by measuring the local signal which can just be definitely discerned through it. This is a threshold type of measurement which is necessarily difficult to carry out with accuracy. In order to increase the sharpness of definition of the local signal and to make it correspond more closely to speech reception the signal tone is subjected to a continuous frequency fluctuation. The comparison signal has therefore a warbling tone which occupies a frequency band not unlike that of the voice. This method of measuring the interference is discussed in more detail also in the measurement paper referred to above.

Procedure in Making Transmission Measurements. The three quantities which are included in the transmission measurements, namely, the signal strength, the noise strength, and the percentage of words received correctly, are observed one after another in what might be termed a unit test period. Although the duration of this test period and the order of making the measurements has been changed somewhat during the course of the experiments, the following program is representative of the conditions under which the data presented below were taken.

A 25-minute test period divided as follows:

5 minutes of tone telegraph identification signals (for receiving adjustment purposes).

10 minutes of disconnected spoken words.

10 minutes of a succession of five-second tone dashes separated by five-second intervals, (for measurement of the received field strength, the intervals between the dashes being used for throwing on the local receiving source and adjusting its strength to equal that of the received signals by alternately listening to one and then the other).

Immediately following this test period the London observers measured the noise level.

This unit test period was repeated every hour over a period which varied from several hours to as long as two days' duration. Most of the test periods ran for about 28 hours, starting about eleven o'clock Sunday morning and continuing until about three o'clock Monday morning, London time. During this time the telegraph load through the Rocky Point station of the Radio Corporation was sufficiently light to enable one of the two antennas to be devoted to these experiments. The measurements were started January 1, 1923 and are still in progress.

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[4] It is described in detail in the paper entitled, "Radio Transmission Measurements" by Bown, Englund, and Friis, *Proc. Institute of Radio Engrs.*, April 1923.

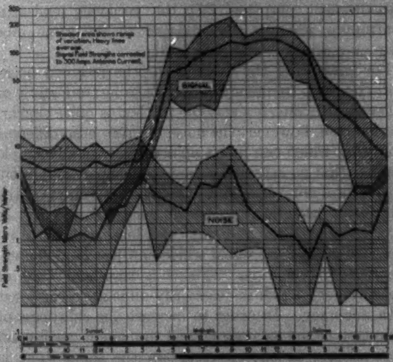

Fig. 9—Transatlantic Radio Transmission Measurements. Diurnal Signal and Noise Variation, January 1,—February 23, 1923.

At the present time (April) the results for the first three months of the tests are available. These results are not yet sufficiently complete nor do they cover a sufficient number of variables in terms of time, wave length, etc., to enable any very definite conclusions to be drawn from them. They do illustrate, however, the usefulness of the methods employed, and even in their incomplete state show some factors of considerable interest.

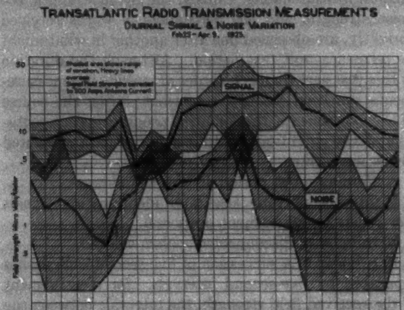

Fig. 10

The results of the measurements of received signal strength and received noise are given in Figs. 9 and 10. The data have been divided and plotted in these two sets of curves because the transmission conditions across the North Atlantic appeared to suffer a rather rapid change about February 23rd. Fig. 9 therefore covers the winter period from January 1 (when the test started) to February 23; and Fig. 10 covers the next period from February 25 to April 9.

The curves are plotted between time of day as abscissas and field strength in microvolts per meter as ordinates. The time during which darkness prevailed at Rocky Point and at London is indicated by the block-fills on the time scales. The overlap of these block-fills indicates the time during which darkness extended over the entire transatlantic path. For Fig. 9 the darkness-belt is as of February 1 and for Fig. 10 as of March 21. The curves show the mean of the results and also the boundaries of the maximum and minimum values observed.

Received Signal Strength. The outstanding factors to be noted concerning the signal strength curves are:

1. The diurnal variations are plainly in evidence. During the first test period covered by Fig. 9, for example, the field strength varied in the ratio of the order of 15 to 1 between day and night conditions, running about 100 microvolts per meter during the night and averaging about 6 microvolts per meter during the day. The diurnal variation is also to be seen in Fig. 10 although the variations between night and day transmission are less marked.

The measured daylight values lend support to the Austin-Cohen absorption coefficient. The average of the observed daylight values for the period of these tests is between 7 and 8 microvolts per meter, while the calculated value is 9.5. Concerning the high field strengths obtaining at night, it should be noted that the maximum observed value of 237 microvolts per meter does not exceed the value of some 340 microvolts which it is estimated should obtain at London were no absorption present in the intervening medium, *i. e.*, were the waves attenuated in accordance with the simple inverse-with-distance law. While no definite conclusions can yet be drawn from these results as to the cause of the diurnal variations, this indication that the upper limit of the variation is the no-absorption condition suggests that the diurnal fluctuations are controlled by the absorption conditions of the medium rather than by reflection or refraction effects.

2. An indication of the seasonal variation which apparently occurs in developing from winter to early spring is found in a comparison of the signal strength curves of Fig. 9 and 10. On the whole the signal strength received in the second test period is considerably less than that received for the first period. This drop in the average of the 24 hours is caused by a large decrease in the night-time transmission efficiency. The daylight transmission does not change much, but

what little change there is lies in the direction of an increase as the season advances.

3. A decrease in the transmission efficiency is observed between the time of sundown in London and sundown in New York, that is, during the period when the sunset condition intervenes in the transmission path. This dip is particularly noticeable in the signal strength curve of Fig. 10. It is not noticeable in Fig. 9, except for the fact that the rise in signal strength corresponding to night conditions in London is delayed until the major part of the transmission path is in darkness.

Strength of Received Noise. The variation in the strength of received noise is shown by the noise curves of Figs. 9 and 10.

1. The diurnal variation of that portion of the noise which is due to atmospheric or "static" disturbances is somewhat obscured by the presence of artificial noise, *i. e.*, noise caused by interference from other stations. The rise in the noise curve at 12 noon is known to be due to artificial interference. In general, however, the large noise values shown to prevail throughout the night in London between about 6 p. m. and 4 a. m. are known to be due to atmospherics. This diurnal variation shows up quite prominently in both figures.

The maximum noise is reached at 2 a. m. London time. Up to this time the night belt extends over London and a sector of the earth considerably to the east and including Europe, Africa and Asia. The noise begins to drop off shortly thereafter and reaches its minimum at sunrise in London. This could be accounted for on the assumption that the major source of the noise lies considerably to the east of London and that transmission of the stray electric waves to London is gradually diminished in efficiency as daylight overtakes the path of transmission.

2. The seasonal variation, as shown by a comparison of the noise curve of Fig. 9 with that of Fig. 10, is not so great as is the case with the transmission efficiency of the signal. However, the noise level is noticeably higher during the second period of the tests,[5] as shown by the average curve of Fig. 10, particularly during the night when the maximum noise obtains.

This indicates that the noise is largely of continental origin lying to the east or south east of London which is in agreement with rough observations made by means of a loop and suggests that the employment of directional antennas would be of considerable advantage. It is expected to include such antennas in the further measurement work.

In connection with these noise curves it should be noted that what they represent is in reality the strength of a local warbling tone-signal, expressed in terms of equivalent field strength in microvolts, which is just definitely audible through the noise. The actual value of the noise currents, were they measured by an integrating device such as a thermocouple, for example, would be a number of times larger than indicated.

Ratio of Signal to Noise Strength; Words Received. The noise curve of Fig. 9 and that of Fig. 10 can, therefore, be read as "The strength of the signal tone which can just be heard through the noise." It can, therefore, be directly compared with the signal curve itself and the difference between the two curves is a measure of the level of the actual signal strength above that which would just permit of the signals being heard. Actually, the difference between the two curves, as shown in the figures, is proportional to the *ratio* of the signal to the noise strength, because the curves are plotted to a logarithmic scale.

This signal to noise ratio is plotted in Fig. 11 for the test period which corresponds to Fig. 9, and Fig. 12 for the test period which corresponds to Fig. 10. These ratio curves are derived by going back to the original data and taking the ratio for each unit measurement period and spotting it upon the chart as shown by the black points. An average is taken of the points for each hour of the 24-hour period as shown by the circle points. The dash line curves of Figs. 11 and 12, therefore, trace the average diurnal variation of signal to noise ratio.

These curves show:

1. That the signal-to-noise ratio reaches its minimum during the time when the sunset period intervenes between London and New York.

2. During the night in London the ratio increases more or less continuously and reaches a maximum around the time of sunrise in London.

3. During the course of the daylight period in London the ratio starts out high and drops rather rapidly during the forenoon and assumes a more or less constant intermediate value during the afternoon until sundown. It is during this afternoon period in London that the business hours of the day in London and New York coincide, so that this is the most important period from a telephone communication standpoint.

The drop in the very low ratios obtaining in London in the early evening is due to the fact that an increase in noise occurring at this time is accompanied by a decrease in transmission efficiency from America. This may readily be seen by referring to Fig. 10. The noise increases as the night belt, proceeding westward, envelopes England and improves the transmission of atmospherics, which arise possibly in continental Europe, Asia and Africa. As the shadow wall, proceeding westward, intervenes between England and America, the transmission efficiency of the desired signals from America drops and it is not until the night belt extends as far west as America that the transmission efficiency improves sufficiently to overcome the disadvantage in London of the large noise values which night there had brought on. Conversely, the high signal to noise ratio, obtaining at about sunrise in London, appears to be due to the fact that as the termination of the night belt, moving westward, intervenes between England and the source of atmospherics to the east, the noise level drops rapidly and has reached low values by the time sunrise arrives in London. At this time, however, darkness still extends to the west and the transmission efficiency from America is at its maximum. It is, therefore, due to this interplay between these two factors, signal strength and noise strength, controlled very largely by the transition periods between day and night, that the signal to static ratio varies diurnally in the manner pictured in Figs. 11 and 12.

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[5] The results obtained more recently than time has permitted to include in the curves show a continual rise in the level of the noise during April.

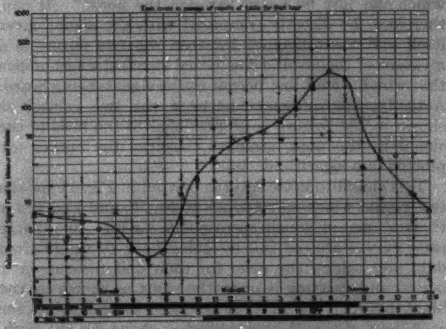

Fig. 11—Transatlantic Radio Transmission Measurements. Diurnal Variations of Signal to Noise Ratio, January 1—February 23, 1923.

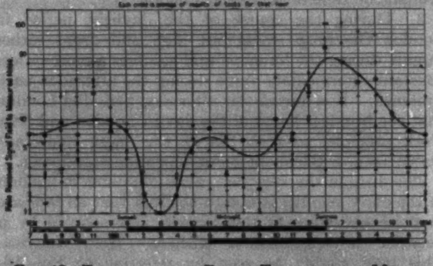

Fig. 12—Transatlantic Radio Transmission Measurements. Diurnal Variations of Signal to Noise Ratio. February 23—April 9, 1923.

Concerning seasonal variation, shown by a comparison of Figs. 11 and 12, the following can be said: The diminution in signal-to-noise ratio in the second test period as compared with the first is caused by the fact that the signal strength has decreased and at the same time the noise has somewhat increased. There is just one other point and that concerns the dip in the ratio occurring at night in London between 12 midnight and 3 a. m. This dip is due to an increase in the noise which occurs around 2 a. m. (A further reduction during this time, and one which extends the time of minimum ratio from sundown on through the night until 2 a. m. is shown by the April measurements which time has not permitted including in the curves).

During each test period lists of disconnected words were spoken over the systems. As an approximate and easily applied method of indicating the talking efficiency of the circuit, note was made of the percentage of the words which were correctly received.

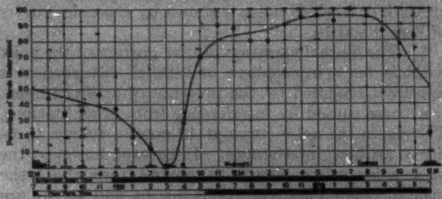

Fig. 13—Transatlantic Radio Transmission Measurements. Diurnal Variation of Words Understood, January 1—February 23, 1923.

Each circle is average of all tests for that hour including triangular points. The latter are known to be cases in which low percentage is due to unnatural causes.

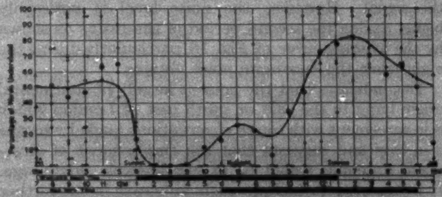

Fig. 14—Transatlantic Radio Transmission Measurements. Diurnal Variation of Words Understood, February 25—April 9, 1923.

Each circle is average of all tests for that hour including triangular points. The latter are known to be cases in which low percentage is due to unnatural causes.

The curves of Figs. 13 and 14 show the manner in which the percentage of the words which were correctly received varies through the 24 hours. Each point corresponds to the percentage of words correctly received during one unit test period. In many of these tests the interference was noted to be caused by radio telegraph stations, and the data in which the interference is of this character, in so far as identified, are indicated by the triangular dots. It will be seen that most of the poor receptions were due to this cause. Especially is this true of tests at 12 noon at which time severe interference from sources local in London was experienced. The circle points are the average of results for each hour's tests. The dash line curve is a smoothing out curve of these points.

It is interesting to note that these curves of actual word count conform very well in general shape with those of Figs. 11 and 12 which also really measure receptiveness although in a less direct manner. Reception is best during the late night and early morning, drops off during the day, reaching a minimum during the evening. Furthermore, the night reception is shown to be considerably better for the January-February period than for the February-March period. The curve of Fig. 14 corresponds quite closely with that of Fig. 12. The curve of Fig. 13 does not show as much of a peak as does that of Fig. 11 which is, of course, due to the fact that above a certain ratio the percentage of words understood is high and cannot rise above 100 per cent.

CONCLUSION

As has been indicated this is a report of work which is still in progress. To date:

A new type of radio telephone system affording important advantages for transatlantic telephony has been developed and put into successful experimental operation across the Atlantic.

Sustained one-way telephonic transmission has been obtained across the Atlantic for the first time by means of this system.

The advantages of this system which had been anticipated, particularly, in respect to economies of power and wave lengths, have been realized. Furthermore, it has been demonstrated that the high-power water-cooled vacuum tubes which have seen their first prolonged operation in this installation are admirably adapted for use in high-power radio installations and particularly for use as high-power amplifiers, in the type of system we have described. Also, the method of reception has proved itself to be eminently satisfactory for use with the single side-band type of transmission and to possess important advantages for radio telephony in respect to selectivity and amplification.

Methods have been developed for measuring the strength of the received signals and the strength of the received interfering noise and these methods have been successfully applied in the initiation of a study of the variations to which transatlantic transmission is subject.

The results of the transmission measurements show that, at 5000 meters, the diurnal variations are large, as was to be expected, and give evidences of a large seasonal variation which was, indeed, also to be expected. The results indicate that it will probably be desirable to use a wave length longer than 5000 meters. The measurements are now being made to include the longer wave lengths.

Discussion

W. V. Lovell: There are just one or two comments I would like to make on the paper on Transatlantic Radio Telephony, which bring out the need for the development of nomenclature keeping pace with the development of new methods. If we do not keep pace, we are going to have quite a bit of confusion. I don't think that the enterprising people who go ahead and do things ought to hold back until they can get approved language to use.

In this paper we find reference to "radio noise strength" for instance. The whole subject of acoustics is one that must be developed a little bit more for both radio and line telephony. And along with the study of acoustics we want to pay some attention to nomenclature.

In previous papers dealing with long distance radio I have found reference to static level, and that appears to be the same thing as what is meant here by radio noise strength. I don't think that either term is really descriptive. "Static level," like a great many other things that have been called static doesn't remain stationary. At the same time, we could find difficulty with the use of the term "radio noise strength." Apparently the dimensions of the antenna and characteristics of the receiving set are involved as well as the condition in nature which is to be defined or evaluated.

While people are going ahead with the development, the committee on standards should give attention to the terms used as well.

Frequency Measurement in Electrical Communication

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Review of the Subject.—*The need for increased accuracy in the measurement of any frequency between one and several million cycles per second is first considered.*

A brief discussion of general types of frequency standards leads to the reasons for choosing a continuously operating generator of alternating current. The generator developed is, in effect, a regenerative system comprising a 100-cycle tuning fork, of low decrement, maintained in vibration by a vacuum tube amplifier.

The absolute value of the frequency of the alternating current delivered by this generator is determined by counting the number of cycles executed in an accurately known time. A small synchronous motor is used for making this count. Recording apparatus for comparing the rate of the fork with various time indicators is described.

Data are given showing the effect upon the frequency of such external conditions as temperature, potential of power supply, circuit constants of the amplifier and vacuum tubes. This information is obtained either by comparisons with a constant frequency or by determinations of the absolute value of the frequency. The ratio of the rate of the fork to the rate of a carefully maintained clock has been found to be constant to within six parts in 1,000,000 over considerable periods of time.

For measuring the frequency of any alternating current used in electrical communication in terms of the known frequency there has been developed a decade arrangement of harmonic producers and a special modulator-rectifier circuit for combining known harmonics of the base and comparing other frequencies with them. These circuits are described in detail.

THE art of electrical communication has at its disposal such a wide variety of methods for the transmission of intelligence that it now employs alternating currents having frequencies which cover the entire range between a few cycles per second and several million. Refinements in these methods have reached a point where it is imperative that determinations of the frequency of any of these alternating currents may be made with an accuracy considerably higher than has been possible hitherto.

For example, in the field of radio broadcasting, it has already become necessary to establish a carefully planned assignment of wave-lengths for the many stations now operating throughout the country. Great care must be exercised in maintaining these stations at their allotted frequency in order that the general scheme may be effective in preventing undue interference.

Accuracy with respect to frequency is also of much interest to the electro-physicist. In certain of his indirect measurements it is considered one of the known variables and many determinations, such as those of the absolute value of inductance or of capacity, would be greatly facilitated if more significant figures could be written into the frequency term on the right-hand side of the equation.

A survey of existing practise discloses much that is worthy of consideration. The work of Prof. D. C. Miller is, of course, known throughout the country. For many years he has been recognized as one of the leading authorities on the standardization of tuning forks. Much has also been accomplished by the Bureau of Standards, both in the calibration of tuning forks and in the construction of electrically resonant circuits from inductances and capacities of known value. A discussion of these or of other noteworthy efforts to obtain increased accuracy in the measurement of frequency cannot, however, be entered upon here. It is sufficient to say that, with the particular requirements of electrical communication in mind, it was felt desirable to undertake the development of a suitable standard of frequency and of means for measuring a wide range of frequencies in terms of this standard.

Recently introduced methods for multiplex telephony and telegraphy by means of carrier currents over wires have placed exceedingly rigorous limits on the frequency adjustment of certain types of apparatus. It is sometimes necessary to hold such circuits as oscillators or filters to within 0.1 per cent of a given value under commercial operating conditions. It is, therefore, apparent that the calibrating devices used in the manufacture and maintenance of such circuits must be reliable to 0.01 per cent and that the primary standard should be good to 0.001 per cent. The goal which we have set for ourselves, then, is the development of a primary standard of frequency, the absolute value of which is known to one part in 100,000, and of means for comparing the frequency of any alternating current having a frequency between one and several million cycles per second with this standard.

SOURCE OF CURRENT OF CONSTANT FREQUENCY

In selecting the form of the primary standard, a number of general types were considered. After examining methods for determining the absolute value of the standard and for comparing it with other frequencies it was thought that a continuously operating generator of alternating current would be preferable to any indicator having a known relation between response and frequency. It was further felt that the frequency of the alternating current should be fixed by some inherent property of the generator rather than that it should be controlled or regulated by any reaction consequent upon a departure from a given value. Inasmuch as the determination of the absolute value of the frequency of the alternating current involves counting the number of cycles which it executes in some

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Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

finite time it is necessary that the rate of the generator be sufficiently constant to permit us to use the average value over this interval as the instantaneous value at any time.

The most promising method of meeting the above conditions appears to be one in which some highly resonant system is maintained in continuous vibration by the application of energy from an external source. In general such arrangements operate through some form of trigger action, controlled by the vibration of the resonant system, which periodically releases energy from the driving source. This will be found to be equally true for a clock, for an organ pipe or for any other continuously maintained vibrating system. Any such device constitutes a regenerative system.

The period of vibration of the resonant element, to be acceptable in our standard, must be little affected by changes in external conditions, including those associated with the source of energy maintaining the vibration. This in turn requires that the trigger action be extremely sensitive and at the same time capable of accurately controlling the driving energy.

Since resonant elements of small decrement are more effective than those of higher decrement in controlling the frequency of such regenerative systems, it is generally true that mechanically vibrating bodies are superior to electrically resonant circuits.

The reasoning so far has led us to a device having the general characteristics of a good clock. In fact, if we think of a clock as a standard of frequency it will be found that we have little more to ask in the matter of accuracy. There remains, however, the problem of comparing the frequency of the mechanically vibrating element—the pendulum—with the frequency of any alternating current. It is a simple matter to obtain from a clock, by means of some form of contact, an alternating current having a frequency in the neighborhood of one cycle per second. Such an alternating current is not entirely satisfactory for our purpose because of the high frequencies to which our comparisons must be carried. Any departures from a fixed frequency on the part of the fundamental are multiplied in absolute value as the frequency is stepped up to its higher harmonics. Because of this effect we should be seriously troubled by any minute variation in period of successive cycles, due to irregularities in the contact or in the mechanism, as we multiply such a fundamental frequency sufficiently to use it in the calibration of radio apparatus.

Difficulties of this nature may be reduced in two ways: First by applying the driving force as uniformly as possible and second by making the frequency as high as the demands of the work will permit, thus reducing the multiplication necessary to reach the higher frequencies.

The requirements thus far laid down for the energy supply portion of our regenerative system are admirably met by thermionic amplifiers. By using high amplification the controlling elements may be made extremely sensitive and consequently may be coupled so loosely to the vibrating element as to be practically without effect upon it. Further, the driving force is not disturbed by any irregularities such as are present in the usual clock mechanisms. This removes one of the major causes of variation in the length of successive periods.

For the particular problem in mind it is not essential to measure accurately frequencies differing by less than 100 cycles per second. This led to the choice of a 100-cycle tuning fork as the resonant member of our system. The decrement of such a device is, perhaps, not as small as that of a good pendulum but its frequency is more suitable for our present needs. In a more complete solution of the general problem it would be desirable to investigate the possibilities of a pendulum maintained in vibration by a thermionic amplifier. Such an arrangement could be adjusted to generate a frequency of one cycle per second which is, in some respects, the logical frequency for a master standard.

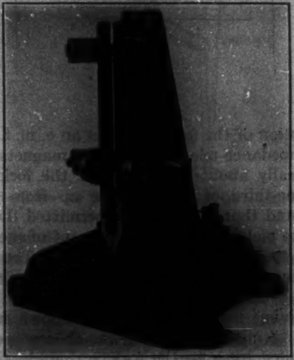

FIG. 1

With this brief general discussion of the factors involved in the development of a frequency standard for alternating-current measurements, we may proceed to a detailed description of the apparatus which has been set up.

The fork used in the major portion of this work was made by the Standard Scientific Company. It was especially designed to have a low decrement. In order that there should be small energy loss in the mounting and no variation in energy absorption due to change in location, the fork is mounted on a heavy cast iron base, as shown in Fig. 1. It is keyed into its socket and is clamped firmly by a nut on the underside of the base. The photograph also shows the electromagnets used for coupling the fork to the vacuum tube amplifier. Energy picked up by one electromagnetic system is amplified in the vacuum tube circuit. The output of this amplifier, in turn, drives the fork through the other electromagnetic system. By this method we see that the only resonant member is the fork itself, and thus the frequency is determined by the fork alone. Also, the coupling may be made very small so that the decrement of the fork is little affected by the driving apparatus. This method of maintaining a tuning fork in vibration was suggested in 1919 by Dr. Eccles.[1]

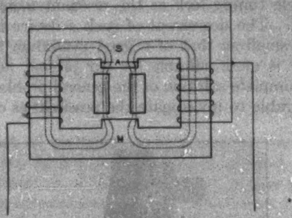

FIG. 2

The motion of the fork produces an e. m. f. in a pair of high-impedance telephone receiver magnets arranged symmetrically about the prongs of the fork approximately one-third of the distance up from the base. It was found that this position permitted the receiver magnets to pick up a sufficient amount of energy without introducing noticeable damping. The distance between the prongs of the fork and the magnet poles is a little over 0.025 centimeters.

The driving magnet is made as shown in Fig. 2. This type of magnetic circuit was chosen because the forces applied by it to the fork are nearly sinusoidal. It is a modification of the magnetic circuit used by Eckhardt, Karcher and Keiser[2] in a somewhat different system for driving a tuning fork by means of vacuum tubes. The proper dimension for the pole width (a) was determined empirically, the value chosen being that for which a certain small amplitude could be obtained with a minimum input current. The ohmic resistance of the two windings in series is 1600 ohms and the reactance at 100 cycles is between 5000 and 6000 ohms.

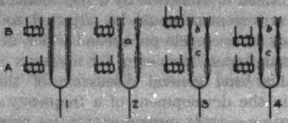

FIG. 3

The location of the driving magnet along the length of the prongs is of interest. It was found by experiment that for a given system the maximum amplitude could be obtained by placing the driving magnet near the center of impact of the prong, other conditions being equal. The location chosen, however, is determined from other considerations. As is well known, a tuning fork has several modes of vibration, as shown in Fig. 3. Any one of the simpler modes may be sustained by the electrical system provided the magnets are properly located and the windings are connected in the proper sense. However, the energy necessary to drive the fork in successive modes increases with the order of the mode so that a system designed to drive a fork at its fundamental frequency is not likely to have sufficient amplification to drive it at a frequency higher than that of its third mode. Diagram 1 of Fig. 3 shows the normal mode and the arrangement of the control and driving magnets, respectively, at *A* and *B*. By proper connection, the electrical input to the fork at *B* may be made to be in phase with the energy picked up at *A*, thereby maintaining the fork in vibration. It will be observed from diagram 2 that a reversal in the connection of the coils would cause the fork to be vibrated in its second mode. Again, from diagram 3, it will be observed that if the driving magnet is above the node *b* the amplifier must introduce no phase change if the fork is to be driven in its third mode, whereas if the magnet is between the nodes *b* and *c*, the phase must be reversed. If, then, we place the driving magnet between the node of the second mode and the upper node of the third mode, we find no tendency for the fork to vibrate in these modes when the phase is correct for the fundamental. The nodes were found by means of dust figures and agreed very closely with the theoretical values as given by Lord Rayleigh.[3] In setting up the fork and driving magnets care was taken to make the air-gaps symmetrical, within narrow limits, so that there should be no unsymmetrical lateral force on the fork which might tend to alter its frequency.

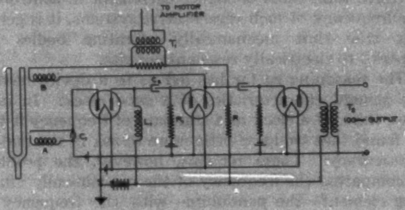

FIG. 4

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[1] *Phys. Soc. Proc.* 31, August, 1919, p. 269.
Phys. Soc. Proc. 32, August, 1920, pp. 345-360.

[2] *Phys. Rev.* 17, April 1921, pp. 535-536.
Jour. Opt. Soc. Amer. 6, November 1922, pp. 949-957.

[3] Theory of Sound—Vol. I.

A schematic of the amplifier circuit is shown in Fig. 4. *A* and *B* are respectively the control and driving magnets of the fork. The retard coil, L_1, the condenser, C_2, and the resistance, R_1, constitute the usual direct coupling between the two tubes. C_1 is a small fixed condenser which is found to be necessary for most efficient operation. This condenser has about one tenth the capacity which would resonate the control coil at 100 cycles per second. Its effect upon the frequency of the system will be considered later. In this circuit the use of transformers was avoided because of the possibility of introducing a phase change in the amplifier. The amplifier delivers sufficient energy to drive the fork at any desirable amplitude. In the system actually set up, only part of the total output energy is used. This is accomplished by putting a high series resistance, *R*, in the circuit between the second stage of the amplifier and the driving magnet. This has a stabilizing effect on the regenerative system by making the load impedance of the amplifier more nearly constant.

The system thus far described provides solely for maintaining the fork in vibration. Since we are interested in obtaining an alternating current for use in various measurements, provision must be made for supplying energy to external circuits in such a manner that there can be no reaction upon the regenerative system which might cause a variation in its frequency. This is accomplished by means of the third tube shown in the figure. Any change in the load impedance of this tube produces an entirely negligible change in its shunting effect on the regenerative system. The transformer, T_1, supplies a second load circuit, the purposes of which will be discussed later.

Fig. 5

In the photograph of Fig. 5 the fork drive amplifier is shown on a panel together with associated apparatus. This panel is the one containing five tubes. Although the schematic shows but two tubes in the regenerative amplifier, there are actually provided two pairs of tubes, the grids and plates of one pair being in parallel with the corresponding grids and plates of the other. The filaments of the two pairs of tubes may be lighted independently. The fork may, therefore, be driven by either pair of tubes, one pair being placed in operation before the other is turned off, thus maintaining the fork with no discontinuity in the output of the 100-cycle current from the transformer, T_1. This is done so that tubes or batteries may be replaced at any time without interrupting the operation of the system.

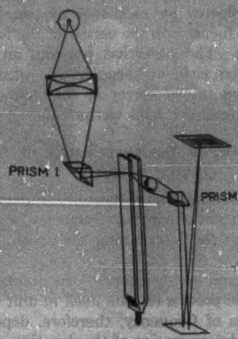

Fig. 6—Path of Rays Through Optical System

In order to study such external conditions as may affect the rate of the form certain additional apparatus has been provided. The fork and driving magnets are placed in a cast iron tank provided with a glass cover, ground to fit. This tank is immersed in a temperature controlled bath. Means are also provided for exhausting the tank so that the effect of pressure variations may be studied.

In order that observations of the amplitude of the fork may be easily and quickly made, an optical system has been set up by means of which a band of light, the width of which varies linearly with the amplitude of the fork, is projected on a scale. This scheme is based on the fact that the corner of a fork prong is not a sharp angle but is a quadrant of a cylinder of very small radius. If this cylinder is illuminated by a bright source of light one element will appear to an observer as a very bright narrow line parallel with the prong of the fork. As the fork vibrates this illuminated element will apparently broaden into a band the breadth of which indicates the amplitude of vibration. An enlarged image of this band may be projected upon a scale by a suitable objective. The lens used is a Bausch and Lomb triple aplanat which has a large aperture and a fairly flat field. It is effectively about 20 inches from the scale and 0.5 inch from the illuminated band, thus giving a magnification of about 40.

In the actual system it is necessary to use mirrors in order that the light path might be included within the restricted dimensions of the tank. Total reflection prisms are used for 90 deg. reflection and a silvered mirror for 180 deg. reflection. The path of the rays may be followed from the sketch shown in Fig. 6. Light from the lamp and condensing lens enters from above through the water and the glass cover of the tank. It is then reflected by means of the total reflection prism, 1, and focused on the corner of the fork nearest the observer. The objective projects an image of the illuminated element through the total reflection prism, 2, down to the mirror at the base of the fork which in turn reflects it back to the scale.

DETERMINATION OF ABSOLUTE VALUE OF FREQUENCY OF SOURCE

Having described the generator for developing an alternating current of constant frequency, we shall next consider means for determining the absolute value of this frequency.

We understand by "frequency" the number of times a certain cyclic process repeats itself in unit time. All determinations of frequency, therefore, depend essentially upon a determination of the duration of the time interval during which the cyclic process repeats itself a known number of times.

It is evident that any determination of the duration of an interval of time can be effected only by comparison with another interval of time which is taken as standard. The basic part of our problem then, is the choice of a satisfactory standard of time. From the point of view of fundamental physics the most desirable standard of time would, perhaps, be that interval required for light to travel unit distance. As the velocity of light is invarient the unit of time is thus determined by the choice of a unit of length, which is fundamental to any system of units. Failing such a standard, however, we may content ourselves with the mean solar day as determined astronomically. This interval of time is constant and known to an accuracy considerably in excess of our present needs. Taking the mean solar day as the standard of time the problem then becomes one of comparing the time interval between two successive cycles of the standard alternating current with the mean solar day. Obviously one of the best methods is to count the number of cycles executed by the alternating current during one day. If the number of these cycles is the same on successive days it is fair to ascribe a certain degree of constancy to the frequency of the current. However, the fact that the current makes the same number of cycles each day does not insure its constancy throughout the day. In order to obtain more complete information on this point the standard of time must be subdivided into smaller units and the number of cycles occurring in each of these small units determined. To accurately subdivide the day it is necessary to use some device such as a clock (unless observations are made on the transit of various stars across the meridian) and this involves the same difficulties which occur in comparing the frequency of the alternating current directly with the mean solar day. About the best that can be done then, is to design the system in such a way that its frequency is little affected by variations in external conditions and to reduce the magnitude of such variations as far as possible. Special attention must be paid to breaking up all conditions which are of a cyclic nature, such as diurnal variations. If this is done and comparisons are then made with a good clock which is similarly cared for—such as the clock of the United States Naval Observatory—a fixed ratio between the rates of the two may be taken as evidence that each is constant to the precision observed.

In considering means for comparing the rate of the fork with the rate of a clock we were embarrassed by the wide variety of methods available. In particular, certain optical schemes offered alluring possibilities. However, in view of the desirability of making a continuous count of the number of cycles occurring throughout an entire day, it was decided to employ some such device as the impulse wheel. This impulse wheel, or phonic wheel, is, in effect, an electric motor operated in absolute synchronism with the fork.

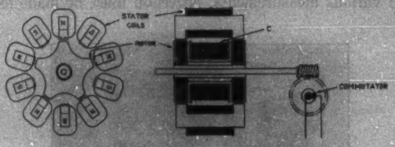

FIG. 7

The first apparatus set up employed the usual type of impulse wheel. This was similar to one used by Dadurian[4] in his study of contact driven forks. The contacts for controlling the current to this wheel were carried on a secondary fork, which acted as a relay.

This arrangement was not well suited to continuous operation and has been replaced by a synchronous motor having much higher efficiency and so arranged that it may be operated directly on the alternating-current output from a small vacuum tube. The diagram shown in Fig. 7 will make clear the electrical and magnetic circuits. The rotor consists of two laminated silicon steel disks machined with five projections as shown. These disks are mounted on the ends of a soft iron core which is bored for the shaft and slotted radially to reduce eddy current losses. The rotor is mounted within a stationary coil, *C*, in such a way that it revolves freely. The stator consists of ten laminated units, each provided with a coil as shown. These stator coils are all connected in series in such a manner that, considering either side of the motor, any two adjacent poles are of opposite polarity. The operation of the motor is as follows:

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[4] *Phys. Rev.* 13, May 1919, p. 337.

A small direct current is passed through the stator winding, polarizing the units as indicated. Alternating current is passed through the rotor coil, making all the poles on either end of the rotor alternately north and south. If, now, the rotor is turned up to synchronous speed—that is, one revolution for every 5 cycles of the fork—it will pull into step provided the electrical input is sufficient to overcome electrical and frictional losses. The friction is made small by using ball bearings and by filling in the spaces between the rotor poles so as to eliminate windage.

The vacuum tube amplifier furnishing the alternating current to the motor is controlled by the regenerative system through the transformer, T_1, in the feed-back circuit of the amplifier shown in Fig. 4. This motor amplifier is shown at the top of the panel in Fig. 5. Like the fork amplifier it is provided with duplicate tubes to insure continuous operation.

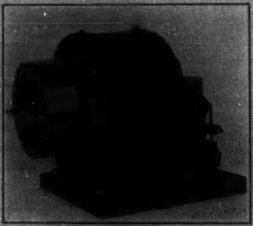
FIG. 8

As in most synchronous motors, there is a slight tendency toward hunting. This may be overcome by the use of a small mercury-filled fly-wheel provided with damping vanes which absorb practically all energy other than that of uniform rotation.

The question has been raised as to whether or not such a synchronous motor could slip a cycle or half a cycle and again pull into step, thus registering incorrectly the number of cycles over any given time. A number of tests have shown that this is not possible with the present motor. The most convincing was to observe a disk, containing a single white segment, mounted on the motor shaft and illuminated by a mirror vibrating subsynchronously at a frequency one-fifth that of the current driving the motor. By this means it is possible to tell whether the motor advances or lags behind normal synchronous speed since, at synchronous speed, the marking segment always appears at a fixed angle when illuminated by the mirror. As a further check, the motor was made to slip a cycle by aggravating the hunting or by loading momentarily, but in no case did it again pull into step after falling behind more than a small fraction of one cycle.

The motor is geared to a commutator by means of a worm and pinion of such ratio that the commutator revolves once in every 100 cycles of the driving current. The motor, with the fly-wheel and commutator, is shown in Fig. 8.

A current interrupted by this commutator may be made to operate any suitable recording device. Various arrangements have been used for registering the impulses from the commutator. For continuous operation the cycles are counted by an electrical clock, driven by impulses from the commutator. Should the fork be adjusted accurately to 100 cycles per second, the clock would keep correct time. This clock, together with the laboratory standard clock, is shown in the photograph of Fig. 9. The dial at the right registers the impulses from the commutator. Its mechanism is provided with means for transmitting every sixtieth impulse to the dial at the left, which indicates the minutes and hours in the usual manner. The clock in the center is the laboratory standard which was used in most of the preliminary work. With this arrangement it is easy to compute, from the readings on the dials, the average frequency of the fork over long intervals of time.

FIG. 9

FIG. 10

In order that measurements may be made with great accuracy in a shorter time, a two-pen Wheatstone recorder was arranged so as to mark simultaneously, on a single tape, second impulses, from the laboratory standard clock, and commutator impulses as controlled by the fork. Care was taken to have the tape moving at a sufficiently uniform speed to permit measurements to be made with an accuracy better than 0.01 of a second. In this connection it is interesting to examine the accuracy with which the commutator, driven by the synchronous motor, indicates 100-cycle intervals. This was tested as follows:

A small amount of the 100-cycle current from the output of the regenerative circuit was passed through a distortion-producing amplifier giving a sharply peaked wave. The current from this amplifier was interrupted by the commutator and recorded by a Dudell oscillograph. The peak of the wave is used to indicate a definite portion of the cycle. The instant at which the commutator closes the circuit is indicated by a shift of the zero axis on the record, due to the introduction of a small battery into the circuit. A series of records show that the commutator makes successive contacts at the same part of the cycle to well within one-tenth of a cycle. These tests were made without the mercury-filled fly-wheel and, therefore, include errors due to the hunting of the motor. A number of these oscillograms is shown in Fig. 10.

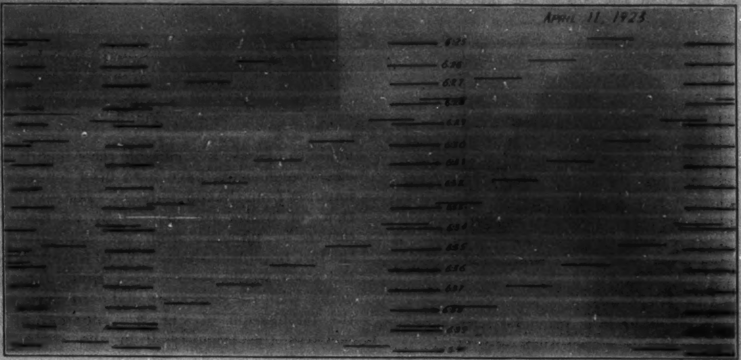

FIG. 11

In Fig. 11 a series of tape records taken at one minute intervals is shown. The upper dashes represent second intervals as marked by the clock; the lower dashes indicate 100-cycle intervals as marked by the commutator. It will be noticed that a dash marked by the fork lags behind the corresponding dash from the clock about 0.2 of a fork interval more than did the fork dash taken a minute previous. This means that while the clock records sixty complete clock intervals, the fork records about 59.8 fork intervals. In this manner, by making a measurement at the beginning and at the end of any suitable period of time we may find the number of fork intervals in any given whole number of clock intervals. Since our fork intervals correspond to 100 cycles, the total number of cycles executed in any known time may be easily computed and the rate of the fork thus determined. The accuracy of the method is fixed partly by the precision with which measurements of length can be made on the tape and partly by the regularity in operation of intermediate relays and of the recorder. It has been found that measurements may be depended upon to within 0.01 second, which gives an accuracy better than one part in 6000 over a period of one minute or than one part in 360,000 over a period of one hour. Many sets of tape records have been made to check the rate of the fork and the stability of the recording apparatus. In Table I are given the results of observations made on the number of cycles occurring in successive five minute intervals, as indicated by the laboratory standard clock, for a period of two hours. It will be observed that the average difference between any single observation and the average rate for the entire time is about 6 parts in 1,000,000. This indicates either that the fork and the laboratory clock are both constant, or that they both vary together, within this limit. This last supposition is hardly conceivable.

In order to observe sudden changes in the frequency of the fork, such as might be occasioned by mechanical jarring or by some slight change in external conditions, it has been found advantageous to employ the well known method of beats. By comparing a high harmonic of the fork frequency with the frequency of the current from a suitable vacuum tube oscillator a very small change in the frequency of the fork becomes readily detectable. It has been found possible by passing current from the fork system through a special harmonic producer, (to be described later) to obtain

TABLE 1

Time: As indicated by the Laboratory Clock. Observations made every 300 seconds.

Lag: The fraction of a fork interval by which a fork dash lags behind the corresponding clock dash.

Difference: The amount by which the lag decreases during 300 seconds. This is measured on every 299th fork impulse. Indicating this decrease by D the number of cycles made in 300 seconds is then 100 (299 + D).

Frequency: $\frac{100\,(299 + D)}{300} = 99.6667 + 1/3\,D.$

Deviation: The difference between the frequency observed for a given interval and the mean frequency throughout the run.

Time	Lag	Difference	Frequency	Deviation
3:00	0.439			
3:05	0.400	0.039	99.6797	0.0006
3:10	0.366	0.034	99.6780	0.0011
3:15	0.332	0.034	99.6780	0.0011
3:20	0.294	0.038	99.6794	0.0003
3:25	0.253	0.041	99.6804	0.0013
3:30	0.216	0.037	99.6790	0.0001
3:35	0.178	0.038	99.6794	0.0003
3:40	0.141	0.037	99.6790	0.0001
3:45	0.105	0.036	99.6787	0.0004
3:50	0.068	0.037	99.6790	0.0001
3:55	0.033	0.035	99.6784	0.0007
4:00	0.995	0.038	99.6794	0.0003
4:05	0.961	0.034	99.6780	0.0011
4:10	0.924	0.037	99.6790	0.0001
4:15	0.884	0.040	99.6800	0.0009
4:20	0.849	0.035	99.6784	0.0007
4:25	0.815	0.034	99.6780	0.0011
4:30	0.772	0.043	99.6810	0.0019
4:35	0.738	0.034	99.6780	0.0011
4:40	0.700	0.038	99.6794	0.0003
4:45	0.664	0.036	99.6787	0.0004
4:50	0.625	0.039	99.6797	0.0006
4:55	0.585	0.040	99.6800	0.0009
5:00	0.547	0.038	99.6794	0.0003
Mean Value		0.0371	99.6791	0.0006

a sufficient amount of the 500th harmonic to give an easily perceptible beat with a current of 50,000 cycles supplied by a vacuum tube oscillator. Over small intervals of time the frequency of the oscillator will remain fixed, therefore any slight changes in the frequency of the fork, magnified 500-fold in absolute value by the harmonic producer, will result in a considerable change in the rate of these beats. In this way it is possible to determine quickly and accurately the effect of certain variables on the frequency of the fork. It is possible, for example, to observe the result of such transients as occur when the amplitude of the fork is suddenly increased or decreased. The precision of this method is evidently high. In fact, since the frequencies between which beats are observed are of the order of 50,000 cycles, and since it is easy to detect differences in beat frequency of one cycle in 10 seconds, a change in frequency of one part in 500,000 is readily observed. It should be understood, of course, that although this method is very useful for determining the order of magnitude of various effects it should not be relied upon for absolute quantitative data.

In order that an accurate check might be kept on the rate of the laboratory standard clock, provision has been made for recording United States Naval Observatory time signals, as sent out by Arlington, on the tape of the Wheatstone recorder simultaneously with second impulses from the clock. It is thus possible to check the average rate of the laboratory clock to approximately 0.01 second per day. The electrical equipment for recording the Arlington signals consists of a radio receiver and of a vacuum tube amplifier containing a resonant circuit tuned to the audio frequency. This audio signal is rectified and the resulting direct current used for operating one pen of the recorder. The jack panel shown at the bottom of the rack in the photograph of Fig. 5 furnishes means for connecting any source of electrical impulses to the recorder so that comparisons may be made between the rates of any two such sources.

CHARACTERISTICS OF STANDARD SOURCE

Observations of the rate of the fork have been made over a considerable period of time to determine its frequency and the extent to which it is affected by variations in external conditions. The data of Table I is an example of the constancy of the ratio of the rates of the fork and of the laboratory clock. At present we are unable to say which has the greater absolute constancy.

It is probable that the temperature of the fork is one of the most important of the factors affecting its frequency. A large amount of data has been taken showing the rate of the fork at various room temperatures and also at the temperature of melting ice. The average change in frequency between 0 deg. and 22 deg. cent. is 0.0109 per cent per 1 deg. cent.

In order that the frequency of the fork may be relied upon to one part in 100,000 it is apparent that its temperature must be known to be better than 0.1 deg. cent. While this is not particularly difficult, under average laboratory conditions, it is highly desirable that some other material be found from which to build the fork. A steel having such coefficients of expansion and of elasticity that the resultant coefficient of frequency is small would be desirable. It is, of course, necessary that the properties of the material be such that the fork has as low a decrement as possible. The use of steel has advantages since it lends itself so readily to electromagnetic driving. Other materials, notably fused quartz, have such commendable properties that it would undoubtedly be profitable to consider means for coupling the vacuum tube amplifier to a fork built of such material.

The relation between the amplitude at which the fork is driven and its frequency is also of much interest. The problem, however, concerns the properties of the amplifier more than it does those of the fork. It is very difficult to readjust the system so that the fork may be maintained at various amplitudes without at the same time introducing changes which have a far greater influence upon the frequency than does the change in amplitude, *per se*. It is certain that anything which alters the phase relation between the currents in the windings of the control and driving magnets is of vital importance in regard to both the frequency and the amplitude. The present regenerative system has been so designed that it is in a stable oscillating condition with the fork vibrating at a double amplitude of approximately 0.25 mm. In general a small amplitude is to be desired because it is accompanied with small frictional and hysteresis losses. With large amplitudes these losses would introduce appreciable damping into the resonant system.

In studying the influence of the amplifier circuit upon the frequency of the fork tests have been made in which the tubes, the plate and filament batteries and the condenser across the control magnet were varied.

As has been previously mentioned, the amplifier contains two pairs of tubes, either of which may be used to drive the fork. Due to slight differences in the connections to these tubes the frequency of the fork when driven by one pair differs by one part in 200,000 from its frequency when driven by the other. When both pairs are operating simultaneously the frequency differs by about one part in 100,000 from the frequency obtained with the amplifier normally used.

In order to show what happens when changes, such as aging, occur in the tubes themselves a series of tests was made in which five pairs of tubes were used, successively, in one pair of sockets. During these tests there was no observable variation in the frequency. This means that any change must have been less than one part in 500,000.

The batteries supplying the plate potential and the filament current are, as many of us have observed, the least constant of the elements of any vacuum tube circuit. Tests have been made, therefore, to determine what change in frequency may be expected as a result of moderate variations in these batteries. On the curve of Fig. 12 the departure of the frequency of the standard sources from the value which it has at normal plate potential is plotted against the plate potential. From this curve it is seen that the battery may drop nearly 5 volts from its usual operating potential without introducing a change of more than one part in 100,000. Since this battery can be maintained to much better than 5 volts over long periods of time errors due to the plate battery will not be large enough to give trouble.

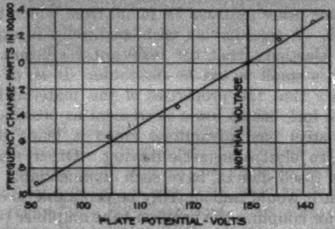

Fig. 12

In fact, in bringing the frequency of the fork to exactly 100 cycles per second, it is planned to make the final adjustment by means of the plate potential.

Observations made on the frequency of the fork as a function of the filament current showed no detectable change until the current was reduced to 80 per cent of its normal value. With storage batteries of adequate capacity, therefore, the variation of potential during discharge will be quite without disturbing effect upon the operation of the standard.

Reference has already been made to the condenser which is placed across the coils of the control magnets. The effect upon the frequency of such changes as might occur in this condenser was investigated by varying its capacity over the range between 0.01 μ f. and 0.1 μ f. The decrease in frequency with increase in capacity is plotted on the curve of Fig. 13. It has been found that a capacity of 0.05 μ f. gives satisfactory results with respect to both efficiency and stability. The slope of the curve at this point is such as to indicate a change of frequency of three parts in 1,000,000 for each 1 per cent change in capacity. The temperature coefficient of this condenser is known to be about 0.003 per cent per 1 deg. cent. It is thus evident that the resultant effect upon the frequency is in the neighborhood of one part in 100,000,000 for each 1 deg. cent. change in the temperature of the condenser.

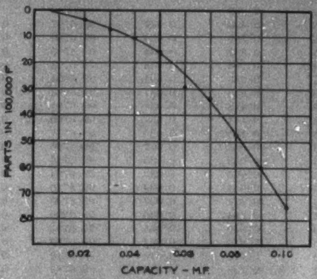

Fig. 13

As a result of the tests described above it appears that, by suitably maintaining the fork and its associated equipment, the frequency of the alternating current produced may be relied upon, both as to constancy and as to absolute value, to one part in 100,000.

MEASUREMENT OF FREQUENCY IN TERMS OF THE STANDARD

The third major portion of our problem concerns means for comparing the frequency of any alternating current which may be used in electrical communication with the frequency of the alternating current delivered by our standard. This is accomplished by a system of harmonic generation. In order to compare the higher radio frequencies with the 100-cycle standard it is, of course, necessary to go to very high harmonics. To accomplish this without the chance of error a decade arrangement of harmonic producers is employed.

The circuit shown in Fig. 14 constitutes the first stage of this system. The design of the harmonic generator, shown at the left, is based on the principle that if the energy of any periodic wave is liberated during a small fraction of the cycle the wave will contain a large number of harmonics. This fact can readily be demonstrated theoretically. The operation of the circuit is as follows:

The plate potential for the tube V_1 is obtained from a condenser, C, which is charged to the potential of the battery E_b through a resistance R. The potential E_c, of the battery in the grid circuit is more than sufficient to prevent the plate from drawing current from the condenser. If, now, we impress a sine wave upon the terminals of the transformer T_1 there will be set up in the grid circuit of the tube a sine wave e. m. f. which will act in series with the potential of the battery E_c. Let us assume that this sine wave has a sufficient amplitude so that at some time during the cycle the negative potential on the grid is reduced to a point which will permit the plate to draw current. It will be observed that as current flows from the condenser an e. m. f. is set up across the transformer T_2. This transformer is connected in such a manner that this e. m. f. tends to further reduce the negative potential on the grid. The action, therefore, builds itself up so that the space current increases very rapidly, thus reducing the charge on the condenser more rapidly than the battery can replace it through the resistance. This finally results in a decrease in the space current, due to the decrease in the potential across the condenser, which in turn immediately reacts to increase the negative grid potential, thereby reducing the plate current still further. As soon as the current to the plate becomes less than the current at which the battery charges the condenser through the resistance the potential across the condenser, and, therefore, the potential acting in the plate circuit, begins to increase. By this time, however, the grid is sufficiently negative to prevent the further flow of current to the plate. As the impressed sine wave passes through its cycle it again reaches a value at which current may flow, and, the condenser having acquired another charge, the process is repeated. In this manner the sine wave serves to release the charge on the condenser through the transformer T_2 at periodic intervals and in such a manner that the greater part of the energy is expended in a very small fraction of the cycle. In practise it has been possible, as previously mentioned, to adjust a circuit of this type so that it will give 500 or more harmonics of the 100 cycle fork.

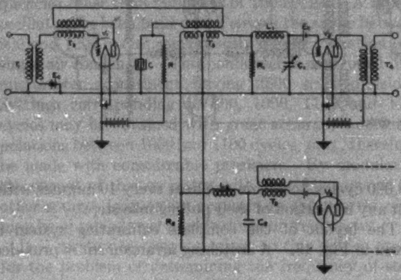

FIG. 14

The output side of the transformer T_2 of the harmonic producer is divided into two portions connected, respectively, across the resistances R_1 and R_2. Shunted across R_1 is a series circuit made up of the inductance L_1 and the variable capacity C_1. By suitably adjusting C_1 any of the harmonics generated in the preceding vacuum tube may be selected and impressed upon the amplifier tube V_2. The constants of this resonant circuit are so chosen that the first nine multiples may be readily selected. A sine wave alternating current having any one of these frequencies may, therefore, be obtained in the output of the transformer T_4.

The resistance R_2 is also shunted by a series tuned circuit. This circuit however, is permanently adjusted for the tenth harmonic of the frequency impressed upon the preceding harmonic producer, that is, for 1000 cycles. The potential across the condenser C_2 is introduced into the grid circuit of a second harmonic producer from which may be obtained any multiple of 1000 cycles. The output of this second harmonic producer supplies two tuned circuits in the same manner as does the first. This process can be repeated as often as necessary in order to obtain sufficiently high frequencies, or until the system becomes unstable.

In order to obtain alternating currents having frequencies other than those furnished directly by any of the harmonic producers a modulating circuit is employed. This is shown schematically in Fig. 15. This circuit also provides for comparing any two nearly equal frequencies. The balanced modulator, shown on the left, is similar to one described in a paper on carrier current transmission by Colpitts and Blackwell.[5] With respect to a voltage impressed upon the transformer T_1 this circuit has the properties of any vacuum tube amplifier, the current flowing in the output circuit reproducing the wave form of the potential impressed upon the transformer. Any voltages impressed upon the transformer T_2, however, set up equal and opposite currents in the two halves of the output transformer and do not cause a current to appear in the output circuit. Voltages impressed upon the transformer T_2, however, do react with voltages impressed upon the transformer T_1, by virtue of the non-linear amplification characteristic of the tubes. Under these con-

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5. Jour. A. I. E. E. 40, April, 1921, pp. 301-305.

ditions there appear in the output circuit alternating currents having frequencies corresponding to the sum and to the difference of the input frequencies. Thus, if we impress a voltage having a frequency of 1000 cycles upon the transformer T_1 and a voltage having a frequency of 10,000 cycles upon the transformer T_2, there will appear in the output of the transformer T_3, in addition to the amplified 1000-cycle current, alternating currents having frequencies of 9000 and 11,000 cycles. Any component of the output current may be selected by means of the tuned circuit and amplified in the single vacuum tube shown. The two tubes at the right of the schematic constitute a balanced rectifier. These tubes have on the grid a constant potential of such value that when there is no voltage across the input transformer there will be no plate current in either tube. Should voltage be impressed upon the transformer T_4 the potentials impressed upon the grids of the two tubes will be such that they will alternately draw current from the plate battery. This will set up across the outer ends of resistances R_1 and R_2 an alternating potential having a frequency equal to that impressed upon the transformer. Since this voltage is symmetrical with respect to the meter there will be no motion of the needle, unless the frequency is extremely low. Voltages impressed upon the transformer T_4 cause both tubes to draw current from the plate battery during that portion of the wave which makes the total grid potential less negative than that of the grid battery. Under these conditions there will be no difference of potential across the outer ends of the resistances R_1 and R_2. If, however, two voltages of nearly the same frequency are impressed upon the two input transformers, they will not effect the two tubes equally. At the instant when they are in such phase relation that their sum will be effective on one tube their difference will be effective on the other. Under this condition the bridge circuit, composed of the two resistances and the two tubes, will be unbalanced and the needle of the meter will be deflected. As the phase relation changes so that the two voltages are 90 deg. apart, the effects upon the two tubes become symmetrical and the meter will stand at its zero position. As the phase continues to change, the resultant voltages will unbalance the bridge in the opposite direction and the deflection of the meter will be reversed. It is thus seen that the deflection of the meter indicates the phase difference between the two voltages and may be used in determining the frequency difference between them, so long as this difference is not too great to be followed by the needle.

Because of the balanced arrangement of the input circuit of this rectifier the frequencies of currents from two vacuum tube oscillators or similar devices may be compared without any tendency on the part of one to influence the operation of the other. In fact it has been found possible to adjust two oscillators, operating at 10,000 cycles, to within one beat every 10 minutes without any indication of their pulling into step.

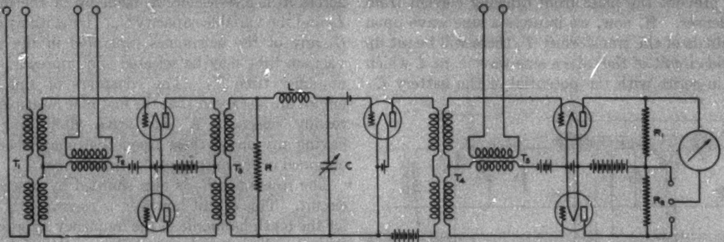

Fig. 15

The layout of the complete calibrating system is given in Fig. 16. A switching arrangement is provided so that any of the inputs to the modulator-rectifier circuit may be connected to the output of any of the amplifiers associated with the several harmonic producers, or to any of a number of vacuum tube oscillators.

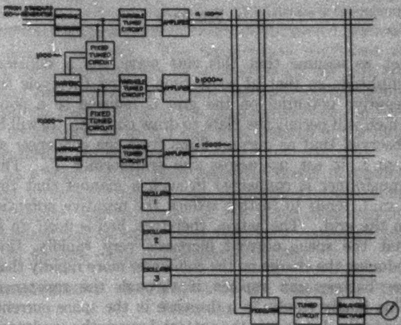

Fig. 16

It is thus possible to obtain, in the output of the balanced modulator, a current component having a frequency equal to the sum or to the difference of any two of the frequencies available in the outputs of the harmonic producers. Such a component may be selected by the tuned circuit, amplified, and so impressed upon the rectifier. One of the oscillators may then be adjusted to deliver a current of this same frequency by connecting it to the rectifier and setting for a zero beat on the meter.

In general, any of the oscillators may be thus adjusted to have a frequency equal to the sum or to the difference of the frequencies of any two of the other circuits. By repetition of this process it is possible to build up frequencies corresponding to any multiple of 100 cycles which is less than 1,000,000.

For frequencies in excess of 1,000,000 cycles we are not interested in such close subdivision and it is found more expedient to start with a base of 1000 or of 10,000 cycles.

Means have also been provided for obtaining frequencies other than multiples of 100 cycles. A special oscillator has been built which covers the range between 900 and 1200 cycles continuously by the variation of a single air condenser. This condenser is controlled by a micrometer adjustment having 2500 scale divisions. Settings corresponding to 900, 1000, 1100 and 1200 cycles may be obtained with great accuracy and interpolations between 1000 and 1100 cycles, may, therefore, be made with considerable precision. By modulating the output of this oscillator with that from one of the other sources frequencies accurate to the nearest cycle may be obtained over a wide range of frequencies.

It is rarely necessary to resort to this device except for the problem of determining the frequency of some particular current. For calibrating oscillators or wavemeters the multiples of 100 cycles give sufficiently close steps.

This general method of producing a particular frequency is preferable to one in which all the harmonics are developed in a single circuit. In the latter case it is difficult to distinguish between harmonics, particularly at the higher frequencies, and there is always a possibility of selecting the wrong one. Moreover, it is practically impossible, with an electrically resonant circuit, to select a given harmonic entirely and exclude adjacent harmonics. For a large part of our calibrating work it is sufficient if we have even multiples of 1000, 10,000 or 100,000 cycles. These are given directly by the harmonic producers in a manner which makes it very convenient to select the particular one required and to compare other frequencies with it.

For the adjustment of apparatus at a distance from the source of current of known frequency secondary standards, of the form best adapted to the needs of the particular problem, may be employed. In this connection we find that frequency standards possess a very interesting property which they share with no other standard, namely, their absolute portability. By the use of telephone lines or of radio a current having the same frequency as that from our standard source, or from the harmonic producer system, may be obtained at any distance from the standard. As a matter of fact, currents obtained from harmonics of the present fork have already been transmitted to a number of points, throughout the United States and Canada, for checking the calibration of such secondary standards as were being used in the adjustment of communication equipment. During the installation of the Havana—Key West cable the calibrations of oscillators in Cuba were frequently checked directly against the frequency of the fork in New York.

Discussion

J. W. Horton: A comment in connection with the frequency measuring work of the Bureau of Standards may be of interest. From time to time they have transmitted standard frequencies from the radio station at the Bureau. On several occasions these have been compared with our standard as follows: The transmitted wave was received in the usual manner, amplified in a radio frequency amplifier and impressed upon a detector circuit which was loosely coupled to a variable frequency oscillator. This oscillator was adjusted, in accordance with our base frequency, to the frequency scheduled to be transmitted by the Bureau. Any difference between the two sources, one in Washington and the other in New York, appeared as a low frequency in the output of the detector. Comparisons made on frequencies of from 500,000 to 1,000,000 cycles per second showed in no case a difference of more than 2500 cycles. On one occasion, at a frequency of 1,000,000 cycles per second, the observed difference was about 25 cycles, being below the audible range.

Telephone Equipment for Long Cable Circuits

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Review of the Subject.—*Some of the important developments contemplated in the apparatus and equipment for long toll cable circuits are described. The large number of equipment units per station in the cable plant and the greater number of stations in a given length of cable than in an open-wire system, have made the economic importance of the equipment design such that a comprehensive program of development, affecting many types of equipment, has been undertaken. The outstanding features of some of the more important of these, including the telephone repeater equipment, test board equipment, and signaling equipment, are described. The necessity for compactness in the dimensions of equipment units, uniformity in assembly arrangements, and simplicity in design, together with the need of careful correlation of the electrical and mechanical requirements, are emphasized. The methods proposed for meeting these requirements generally, are described.*

* * * * * *

INTRODUCTION

THE use of lead-covered cables in place of bare copper wires for long distance telephone lines has been an important development and much interesting information on this subject has already been presented to the Institute. The engineering and construction features involved in a cable system of this sort were described by Mr. Pilliod in his article on the Philadelphia-Pittsburgh Section of the New York-Chicago cable, while the transmission characteristics of such a system were brought out in the recent paper by Mr. Clark. It is the purpose of the present paper to deal with some of the important developments in apparatus and equipment which are contemplated for the cable plant.

A cable system requires repeater stations at more frequent intervals throughout its length than an open-wire line, because of the much smaller gage conductors which it employs. Consequently, in such a system, a greater proportion of the plant investment is represented by the equipment within the offices than is the case with open-wire construction. Furthermore, the number of equipment units per station in a cable system is ordinarily much larger than in an open-wire office, due to the fact that the chief advantages in the use of long cable circuits, in place of open-wire construction, have occured on routes carrying heavy traffic where many circuits are needed. Thus, the requirements of the cable plant have been such as to emphasize the economic importance of the equipment design.

To meet these requirements it has been necessary to undertake a comprehensive plan of development affecting many types of equipment. This has involved careful consideration of both the electrical arrangements and the mechanical design, which are being closely coordinated with the purpose that both should contribute to the highest degree of efficiency in the functioning of the system.

It is not possible in this paper to give many details concerning these developments, but it is desired to present some of the principal features as applied to typical cases. Among the more important of these are the telephone repeater equipment, the testboard equipment and the signaling equipment. These are closely associated with each other in their operation, as well as in their physical location, and it has been necessary, in the design of all units to have due regard to the system as a whole.

TELEPHONE REPEATER EQUIPMENT

The function of the telephone repeater as an amplifier in long distance lines is well known. The telephone repeater in its present form has been the chief factor in making long distance cable telephony practicable, and it is probable that the developments in connection with telephone repeaters have been among the most rapid and comprehensive of any in the toll equipment. It will, therefore, be very interesting to note, in the illustrations which follow, the principal steps which have been taken in working out the form of the equipment to the degree of efficiency now required for the cable plant.

FIG. 1—BOX TYPE REPEATER INSTALLATION AT SALT LAKE CITY, UTAH

In Fig. 1 is shown one of the original forms of repeaters, a number of which were installed as early as 1914. This particular illustration was taken from an installation at Salt Lake City, Utah. In this case the repeater apparatus was assembled in boxes designed to mount on the wall, each box containing a one-way amplifier.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 25-29, 1923.

Two such one-way units were required to form what is now known as a two-way, two-element repeater. The balancing networks, associated coils, etc., were mounted on separate racks while the batteries were mounted in an adjacent enclosed cabinet. The floor space required for a two-way repeater employing this type of apparatus was in the neighborhood of 15 square feet per unit, including the usual allowances for aisle space and associated apparatus.

Fig. 2 shows the first type of vacuum tube telephone repeater designed for commercial manufacture. The particular unit shown is a single "through line" set, that is, one which remains connected to a through circuit at all times instead of being used at a switching point to establish built-up connections at the will of the operator, as a "cord circuit" repeater. The "cord circuit" repeater of the same type was similar in arrangement and dimensions to the "through line" repeater shown. In this type of repeater, all of the apparatus for a two-way circuit was mounted together on one rack. The testing equipment and signaling apparatus were duplicated in each repeater set. This apparatus, as well as the balancing networks and other miscellaneous apparatus, was mounted on the same rack as the repeater. A unit of this type required about 10 square feet of floor space.

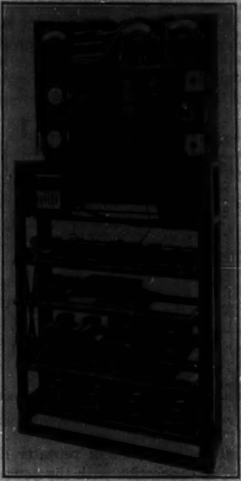
Fig. 2—Wide Rack Through Line Repeater

Fig. 3 shows the type of "through line" set which was standardized in 1917 for use on open-wire lines. Fig. 4 shows a group of "cord circuit" sets of the same type of design, together with the testing unit. This form of set was a great improvement over the earlier types. It employed, however, many of the same large types of individual pieces of apparatus as were used in the former sets, and the testing equipment, which was mounted on the middle rack of the group, was required to be duplicated for each group of 6 repeaters. This type of set has been used in many of the smaller installations, but it has not met the requirements for large cable installations. The average floor space area required was about 6 square feet per set.

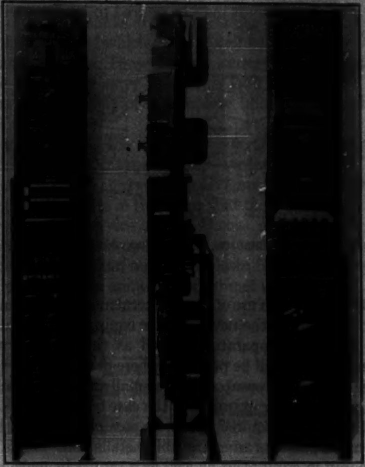
Fig. 3—Floor-Mounted Through Line Repeater

Fig. 4—Group of Six Cord Circuit Repeaters and Associated Testing Unit

Fig. 5 shows the proposed assembly of the type of telephone repeater which is now being developed for all classes of installations employing the two-way, two-element circuit. Fig. 6 shows the general arrangement proposed for a group of sets of this type in a large installation, as in a cable office. This set is expected to have many advantages adapting it particularly to cable installations. When mounted as shown in Fig. 6 it will occupy but 1.5 square feet of floor space per unit. Fig. 7 shows how this set may be arranged in small installations where it may be desired to be mounted on a low rack.

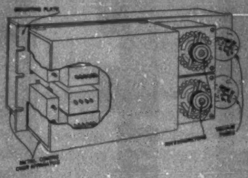

FIG. 5—TYPICAL ASSEMBLY OF PANEL MOUNTED REPEATER SET

By the uniform use of these general mounting arrangements for all of the new repeater equipment, including the accessory apparatus as well as the repeater sets themselves, it will be possible, where desired, to serve a large number of repeaters with a small amount of testing equipment. For example, in the case of the voltmeters and ammeters required, it will be possible to employ but one meter panel for an entire cable installation. Thus, an economy in equipment as well as a saving in space will be effected.

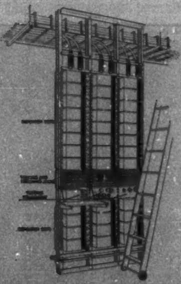

FIG. 6—GROUP OF PANEL MOUNTED REPEATERS AS ARRANGED IN A LARGE INSTALLATION

Fig. 8 shows how some of the principal features of this proposed type of set, which distinguish it from the earlier types of repeaters, are related to the circuit arrangement, as well as to the mechanical design. Previously, the apparatus which it is now proposed to mount in distinct groups on separate panels, as indicated in this diagram, was assembled together in one repeater unit. Several types of sets were accordingly necessary to meet the various field conditions. This is to be avoided in the new design by separating from the basic repeater unit such apparatus as may be required to be different under different conditions of use. For example, the basic repeater unit in the new repeater is to be the same for both "through line" and "cord circuit" use, and for large installations as well as for small ones. The signaling apparatus which may have different features in different types of offices, and which may not be needed with "through line" repeaters in some cases, will be furnished as a separate panel from the basic repeater unit. The apparatus which will permit the repeater to be used for cord circuit operation is also to be furnished as a separate unit and may be used in place of the through signaling unit, without changing the basic repeater. The filter which will determine the cut-off frequency of the repeater will also be furnished as a separate piece of apparatus mounted on the repeater set, so that the repeater may be suited to any desired type of line by providing the proper filter.

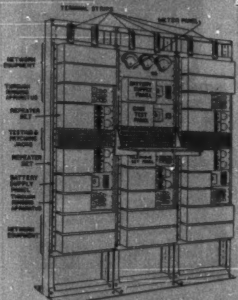

FIG. 7—PANEL MOUNTED REPEATERS AS ARRANGED IN SMALL INSTALLATION

Another interesting phase of repeater development has been that in connection with the power supply for the vacuum tubes. This includes (1) a source of filament current (2) a source of plate potential and (3) a source of grid potential. In meeting the requirements of cable installations the principal improvements desired have included the use of batteries common to as many repeaters as possible in place of individual batteries, closer regulation of potentials and the elimination of dry cells, where practicable.

In some of the earliest installations a separate six-volt storage battery was used to supply the filament current for the vacuum tubes of each repeater set. Later, the filament current supply was taken from an 11-cell central office storage battery through a rheostat. As the potential of the 11-cell central office battery, normally 24 volts, varied from 20 to 28 volts during the operation of a charge and discharge routine, it was necessary to adjust the rheostat at frequent intervals to maintain constant current in the vacuum tube filaments. With the greatly increased number of repeaters per station which has occurred in cable systems, the maintenance involved in readjusting the filament currents would have become prohibitive on this basis. Accordingly, for the larger installations, duplicate 11-cell batteries normally floated from generators and provided with emergency cells to maintain voltage during an emergency discharge are proposed. By this improved arrangement it is expected to be possible to maintain the filament voltage within one volt up or down from its normal value, even during an emergency discharge, until the batteries are almost completely discharged. This improved regulation will entirely eliminate adjustments of the individual repeaters during operation to secure proper values of filament current.

For the plate voltage supply dry cells have sometimes been used in small installations. These are now being displaced, to a large extent, by small storage cells, two groups being used so that one group may be charged while the other is in service. In the large cable installations, the current drain on the 130-volt plate batteries has sometimes reached values as great as four or five amperes, so that it is now planned to float these batteries also, instead of operating on the charge and discharge basis. It is expected by this means to obtain regulation of the plate voltage within plus or minus five volts from the normal value of 130, at all times.

Consideration is being given to another possible improvement in the power arrangements for large repeater installations. This involves the proposal to use a storage battery common to all of the repeaters for supplying the grid potential. If it is found that this arrangement is practical, it will permit the elimination of the individual dry cell batteries which have been employed, with a consequent saving in maintenance which might be appreciable. It seems likely that a very small storage battery would serve for this purpose since the current drain is negligible.

The amount of power required to operate the vacuum tubes in a large telephone repeater installation is considerable. Each filament requires a current in the neighborhood of one ampere and, while the filaments are connected in series in such a way as to utilize as efficiently as practicable the full potential of the central office battery, the load on this battery sometimes amounts to several hundred amperes.

Fig. 9 gives some idea as to the size of the storage batteries for a typical office of this kind. The large cells in wooden tanks are those making up the filament batteries, each of which is an 11-cell battery of 24-volt nominal rating. These two batteries in parallel are large enough to carry the office load for at least 12 hours in the event of a complete failure of charging equipment.

Fig. 10 shows the charging generators and power switchboard for a typical cable repeater office. The gas engine drives emergency generators to float filament and plate batteries in the event that the regular electric power supply service fails. Alarms are provided to indicate any abnormal condition such as a voltage higher or lower than normal, a blown fuse, etc. These are grouped in an annunciator cabinet on the wall.

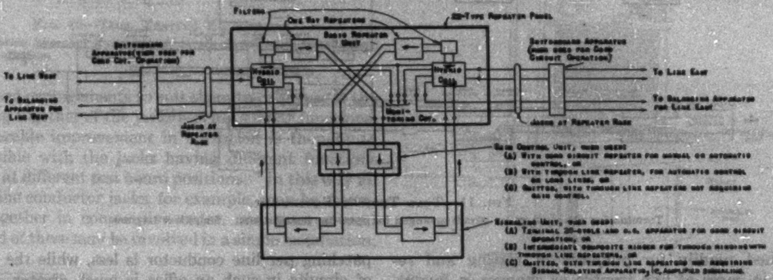
Filters
One Way Repeaters
22-Type Repeater Panel
Basic Repeater Unit
Switchboard Apparatus (when used for Cord Cct. Operation)
To Line West
To Balancing Apparatus for Line West
Hybrid Coil
Jacks at Repeater Rack
Monitoring Cct.
Hybrid Coil
Switchboard Apparatus (when used for Cord Circuit Operation)
To Line East
To Balancing Apparatus for Line East
Jacks at Repeater Rack
Gain Control Unit, when used:
(A) With cord circuit repeater for manual or automatic control, or
(B) With through line repeater, for automatic control on long lines, or
(C) Omitted, with through line repeaters not requiring gain control.
Signaling Unit, when used:
(A) Terminal 20-cycle and D.C. apparatus for cord circuit operation, or
(B) Intermediate composite ringer for through ringing with through line repeaters, or
(C) Omitted, with through line repeaters not requiring signal-relaying apparatus, ie, amplified signaling.

FIG. 8—SCHEMATIC CIRCUIT DIAGRAM SHOWING TWO-WAY TWO-ELEMENT REPEATER ARRANGED TO EMPLOY ONE TYPE OF BASIC REPEATER UNIT FOR ALL CLASSES OF SERVICE

FIG. 9—TYPICAL STORAGE BATTERY ROOM FOR CABLE REPEATER STATION

FIG. 10—TYPICAL POWER ROOM FOR CABLE REPEATER STATION

Test Board Equipment

The "test board" forms an important part of the cable plant equipment, since it is the one point in the office where all of the lines and the equipment as a whole may be reached readily for purposes of testing and rerouting, in cases of line trouble or changes in layout. The form and arrangement of the test board equipment consequently have an important bearing on the effectiveness with which the facilities are handled by the maintenance forces. This is particularly true in the cable plant where the number of circuits involved is large.

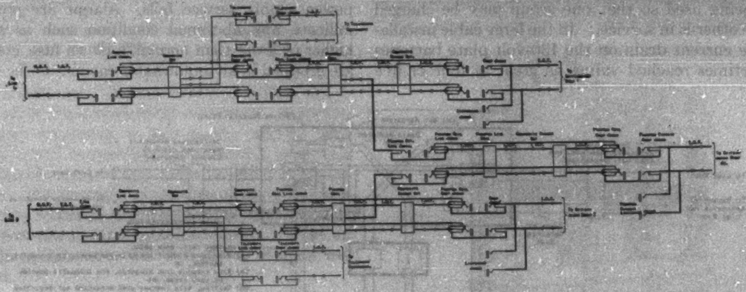

FIG. 11—TOLL TESTING EQUIPMENT

Terminating phantom group circuit arranged for open-line installations. Simplified diagram.

In general, all toll line conductors are brought into a central office, from the outside, through a cable, and first appear at a rack called a "distributing frame" where they are soldered to exposed terminal lugs so that they may be reached for the purpose of connecting them to apparatus within the office. This arrangement is well suited to the permanent connections but is not intended to permit frequent changes or the ready removal of the apparatus for line testing.

The necessity for rearranging the connections between the apparatus and the lines in cases of line trouble make it desirable at times to be able to make such changes quickly, and the means which have been provided for this purpose are located at the "test board". Here both the line conductors and the apparatus units are wired to "jacks" which are arranged to permit the transfer of the normal connections by the insertion of plugs wired to flexible cords. A temporary connection made in this manner, through a conducting cord wired to two plugs, is called a "patch." The apparatus for determining the location of line troubles is also located at the test board and is wired to cords and plugs so that it may be connected to any line in the office readily, upon the occurrence of line trouble, without necessitating changes in soldered connections.

The test boards used for the open-wire plant have been designed to take care of 40 to 80 line conductors in one position, that is, in a board three feet long. The amount of testing and patching work required on open-wire lines has been such as to make this a convenient number of circuits to handle within this space. In cable installations, however, the amount of testing and patching per line conductor is less, while the number of circuits in such an office is much greater. Consequently, in cable offices it is possible to concentrate a larger number of wires within a given test board space. This is desirable from the standpoint of economy in space as well as from that of convenience in operation.

One of the first steps considered in the development of efficient testboard equipment for cable installations has been the reduction of the number of jacks per circuit. In open-wire installations it has been the practise to equip each line circuit and each equipment unit, such as a composite set or phantom coil, with a full complement of jacks suited to provide the maximum degree of flexibility in "patching", thus permitting the ready interchanging of individual equipment units, lines and drop circuits. In cable installations, where the circuits are more likely to be uniformly equipped with the same types of associated apparatus and where the line troubles are less frequent, it is expected to be possible to eliminate certain of the jacks, such as those associated with the composite sets and phantom sets, thus simplifying the equipment for terminating the toll lines. Fig. 11 shows the typical open-wire arrangement for a terminating phantom group circuit in which the maximum number of jacks is furnished. This requires a total of 48 jacks. Fig. 12 shows the arrangement of a terminating phantom group circuit as planned for a cable installation. In this case a total of but 30 jacks is required.

Another important development expected in the test board arrangements to suit them to cable use is the grouping together of the jacks serving similar functions. Considerable improvement in operation is thought to be possible with the jacks having different functions located at different test board positions. In this way all of the line conductor jacks, for example, may be assembled together in consecutive order, and since several hundred of these may be involved in a single installation, this should greatly facilitate the identification of the desired circuits by the attendant in the process of patching and testing. This grouping of the jacks should also effect a saving in testing equipment, since it will eliminate the need of the line testing apparatus, such as the Wheatstone bridge, at positions where the line conductors will not appear.

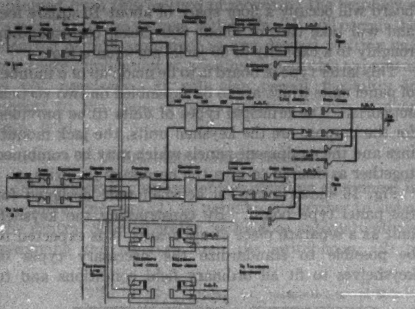

FIG. 12—TOLL TESTING EQUIPMENT

Terminating phantom group circuit arranged for cable installations. Simplified diagram.

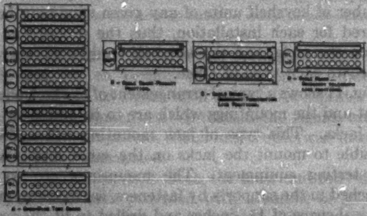

FIG. 13—TOLL TESTING EQUIPMENT

Typical assembly arrangement for jacks at toll test board.

Fig. 13 illustrates both the open-wire and the proposed cable methods of grouping the jacks. In the arrangement for open-wire circuits the jacks associated with both the lines and equipment are terminated adjacent to each other in the same test board panel. In cable installations the jacks having similar functions are to be grouped together, the groups having different functions being mounted in different panels which may be located in different test board positions. These various groups, in the latter case, are planned as follows:

1. Primary line testing position for testing and "patching" toll lines only. This position to be used for locating faults in the cable circuits and equipped with a Wheatstone bridge and voltmeter, to permit the necessary electrical measurements for this purpose. This position is also to be used for making temporary changes in the assignments between the lines and the equipment as a whole, but is not to be arranged to permit changes in individual equipment units, such as composite sets, phantom sets, etc., The jacks to be located at this board are to include those designated as "line jacks" and "equipment jacks", in Fig. 12.

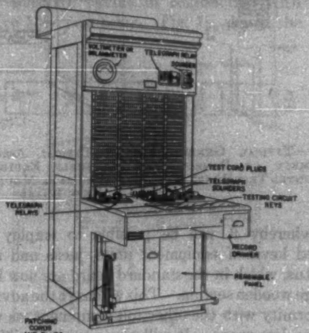

FIG. 14—TYPICAL ASSEMBLY OF FLOOR-MOUNTED TEST BOARD. ONE POSITION

2. Secondary terminating line positions for testing and "patching" the lines between the "drop" side of the equipment and the toll switchboard circuit. This position is to be used for determining the general nature of a trouble and its general location, *i.e.*, whether it is in the direction of the line or in the direction of the "drop", and for clearing troubles not requiring line tests. This position is not to be equipped with Wheatstone bridge testing apparatus as at the primary board. The jacks to be located at this position are to include those designated as "secondary line jacks", "drop jacks" and "listening jacks", in Fig. 12.

3. Secondary telegraph line positions for testing and "patching" the telegraph line circuits. This position is to be used solely for interchanging telegraph lines and telegraph equipment, in cases of temporary changes in

assignment and is not to be equipped with line testing apparatus. This position will permit changes to be made in the telegraph assignments without interfering with the telephone circuits. The jacks to be located at this position are to include those designated in Fig. 12 as "telegraph line jacks" and "telegraph drop jacks".

A further and more extensive improvement in test board design is anticipated as a result of development work whereby it will be possible to employ panel mounted keyshelf equipment units, jacks and testing apparatus, which in the standard board are now housed in a large wooden section. This will have the advantage of uniformity with the other toll equipment, as well as requiring less space. It will also permit flexibility in the use and installation of the various combinations of keyshelf equipment, jack equipment and other testing apparatus which may be required to suit each case. While this arrangement will have its chief advantages when applied to large cable installations, it will also be well suited to open-wire use and small installations, since its design will permit the highest degree of flexibility with respect to both the amount and type of equipment.

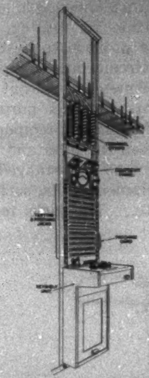

Fig. 15—Typical General Assembly of Panel-Mounted Test Board

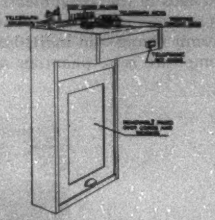

Fig. 16—Typical Assembly of Keyshelf Equipment Unit for Panel-Mounted Test Board

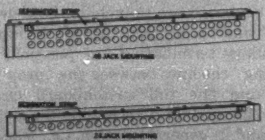

Fig. 17—Typical Assembly of Jack Mountings for Panel-Mounted Test Board

These points may be illustrated by comparing the general features of the two types of boards. Fig. 14 shows the assembly of a one-position section of the present type of board employing a wooden framework. This board, with the necessary allowances for aisle space, requires a floor area amounting to about 24 square feet, while it houses a maximum of about 1000 jacks corresponding roughly to about 40 jacks per square foot. Fig. 15 shows a typical position employing the proposed panel mounting method. Such a board will occupy a floor space of about 10 square feet and will take care of about 600 jacks, corresponding roughly to a capacity of 60 jacks per square foot.

This latter type of board is to be made up of a number of panel units which are to be assembled on two vertical supports. The principal types of units to be provided for the purpose are the keyshelf units, the jack mountings and the equipment panels which may be combined together as desired to give the necessary facilities.

Fig. 16 shows a typical keyshelf unit designed for the panel type board. By constructing the keyshelf unit as a separate piece of apparatus, it is expected to be possible to standardize the necessary types of keyshelves to fit all ordinary field conditions and to specify the desired type of keyshelf to go with any particular arrangement or number of jacks. The number of keyshelf units of any given type may be as desired for each installation, thus the proportion between the jacks and the keyshelf equipment may be suited to each type of office.

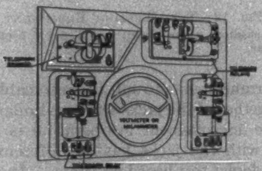

Fig. 18—Typical Assembly of Apparatus Panel for Panel-Mounted Test Board

Fig. 17 shows a typical arrangement of the jack equipment and the mountings which are to be employed for the jacks. This type of jack mounting will make it possible to mount the jacks on the same supports as the testing equipment. The mountings are to be attached to the supports by fasteners, each occupying a vertical space of 1¾ inches and drilled to fit the usual drillings in the supports. This will permit the close association of the jacks with the desired testing apparatus. It will also be possible, by this means, to use for the jacks only such of the available vertical space as may be desired, the remainder being used for other equipment, thereby effecting economy in the use of the space. This arrangement is thus expected to be advantageous both in large installations, where the various groups of jacks are desired to be arranged at separate primary and secondary positions, and in small installations in which but a few jacks may be required for all purposes.

Fig. 18 shows the proposed assembly of a typical panel equipped with voltmeter, telegraph relay and sounder, such as are usually mounted above the jack field. The advantages of using the panel mounting for this equipment is expected to be the same from the standpoint of convenience and economy as that obtained from this method of mounting the keyshelves in relation to the various jack fields. It will permit the location of this equipment at any desired position and eliminate duplication at positions where this apparatus is not necessary. It will also permit flexibility in regard to the type of panel associated with a given jack field.

SIGNALING EQUIPMENT

The principal purposes of the signaling equipment in telephone lines are (1) to permit a subscriber to signal the central office operator, as he does automatically in removing the receiver from the switchhook, (2) to permit the central office operator to ring the subscriber's bell and (3) to permit the operators at different central offices to signal each other. At repeater stations this equipment serves to pass the signals around the telephone repeaters which might otherwise interfere with their transmission.

The switchboards in both the local and toll central offices have been provided with a source of 20-cycle current for signaling, as current of this frequency is suited to operate the subscriber's bell directly. This frequency has also been satisfactory for operating the signaling apparatus in the various local trunk circuits and in short toll lines without superposed telegraph. The introduction of superposed telegraph and telephone repeaters, however, has prevented the use of 20-cycle signaling on many toll circuits. It has accordingly been expedient to use a higher frequency for the majority of the toll circuits, and the one which has been most generally used for this purpose has been 135 cycles. It is desired, therefore, to bring out here some of the more important features of the 135-cycle signaling arrangements which are being developed for cable systems.

The signaling equipment for cable systems has been required to meet more severe conditions than those ordinarily encountered on open-wire lines. In order not to interfere with the direct-current telegraph system used on the cables, it has been necessary to limit the signaling current to a few milliamperes. Furthermore, the characteristics of the cable apparatus have been such as to attenuate the signaling currents to a greater extent than in open-wire systems. It has accordingly been necessary to undertake the design of signaling equipment of greater sensitivity, as well as to provide a source of supply of signaling current possessing a high degree of freedom from harmonics and capable of being closely regulated.

The desired increase in sensitivity of the signaling system is expected to be obtained through the use of a highly selective circuit in conjunction with a very sensitive 135-cycle relay. Fig. 19 shows the general scheme of the circuit which is being developed for the purpose. It is seen that this includes a filter which may be inserted in the line circuit between the signal-receiving apparatus and the switchboard "drop" to give the desired terminal impedance at the signaling frequency, thus preventing the low impedance shunt on the signaling relay which would otherwise be caused by the "drop" circuit at a frequency as low as 135 cycles. The filter is arranged so that it need be inserted in the circuit only in such cases as may require the increased signaling range obtained in this manner, thus, it may be omitted on the shorter circuits where it is often unnecessary. The general circuit arrangement shown in Fig. 19 will permit the ready interchange of different types of signaling apparatus. With this arrangement of the apparatus, any "ringer" of any desired frequency combination such as 20-135 cycles, 20-20 cycles, 135-135 cycles, etc., may be connected temporarily to the system when desired, by means of "patching" cords, without requiring any changes in the permanent wiring.

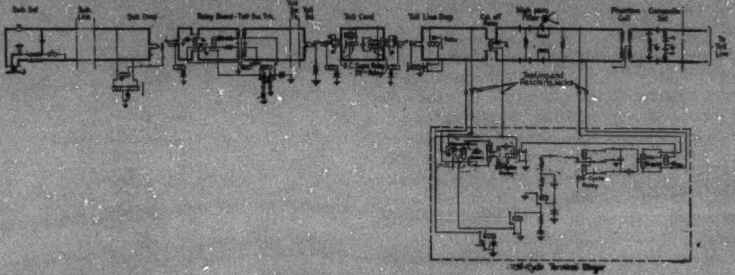

FIG. 19—TOLL SIGNALING EQUIPMENT

135-cycle ringdown toll signalling system, using terminal ringer employing make-contact 135-cycle relay. Simplified diagram.

Much of the sensitivity and selectivity which may be obtained with this signaling system are due to the design of the 135-cycle relay. As shown in Fig. 20, the relay is designed to make it capable of close and accurate adjustment. Both the magnetic air gap and the contact spacing may be adjusted, about 0.0015 inch or 0.038 millimeter being used ordinarily for the latter. The relay is mechanically tuned, the natural period of the reed with its adjustable weight corresponding closely to the frequency of the signaling current. A stop pin is provided which prevents undue vibration of the reed due to transient impulses or excessive currents. Also, the relay circuit is electrically tuned by a shunt capacity and a series inductance and capacity. It is thus very selective and is relatively free from the ordinary sources of interference such as those caused by mechanical vibration, telegraph signals, switchhook impulses, voice currents, etc. The sensitivity of the relay is such that it will operate on as little power as 30 microwatts, corresponding to a current in the neighborhood of 0.25 milliampere.

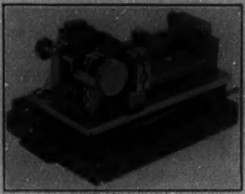

FIG. 20—GENERAL STRUCTURE OF 135-CYCLE RELAY

This relay is to be mounted so that it may be inserted in the circuit or removed from it in the manner of a plug and jack, without requiring changes in the permanent wiring and without affecting the circuit operation, except to interrupt the signal-receiving system while the relay is removed. Thus it will be very convenient to make the necessary adjustments of the relay separate from the circuit with which it may be associated in service. The relay will be well protected from mechanical interference, such as building vibration, by padding in the mounting which prevents rigid mechanical connection between the relay and its external support.

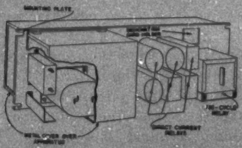

FIG. 21—TYPICAL ASSEMBLY OF 135-CYCLE COMPOSITE RINGER SET

The assembly arrangements proposed for all of the new signaling apparatus are such as will fit in closely with the panel mounting methods designed for the remainder of the toll equipment in cable installations. Fig. 21 shows the assembly proposed for the new composite ringer set. This method of mounting the composite ringer set is a very desirable one since it permits a complete composite ringer set to be manufactured as a unit and installed as such.

Fig. 22 shows the manner in which a number of these composite ringer panels are planned to be mounted together with the associated testing equipment in a large installation. This close association of the composite ringers with the jacks and testing equipment is expected to be of considerable advantage in facilitating the maintenance of the apparatus.

The need in cable systems of a particularly well-regulated source of 135-cycle signaling current, with sufficient capacity for a large number of lines, has made it desirable to undertake the development of both a special type of interrupter and a motor-generator for this purpose. Close frequency regulation is also very desirable in signaling over cable circuits, in view of the increased sensitivity in receiving which may be obtained by the use of very selective receiving apparatus.

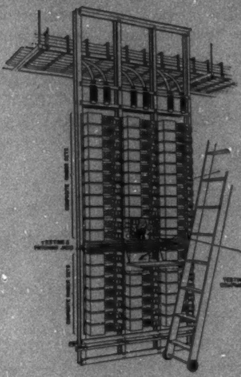

FIG. 22—TYPICAL ASSEMBLY OF GROUP OF COMPOSITE RINGER SETS

Fig. 23 shows the circuit arrangement of the interrupter which is expected to be provided for this purpose. This includes a vibrating reed actuated by an electromagnet when direct current is applied. The contacts on the reed being in series with the battery circuit, intermittent operation is secured in the manner of an ordinary buzzer, the speed of operation for a given applied voltage being determined by the natural period of the reed. The actuating circuit of the vibrator also includes the primary side of a transformer, the secondary side of which is connected to a filter for suppressing harmonics in the output. The maximum output capacity of the interrupter is about three-fourths of a watt.

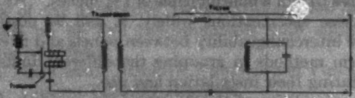

Fig. 23—135-Cycle Interrupter
Simplified circuit diagram.

Fig. 24—General Structure of Vibrator for 135-Cycle Interrupter

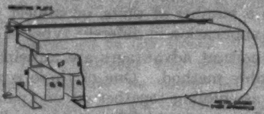

Fig. 25—Assembly of 135-Cycle Interrupter

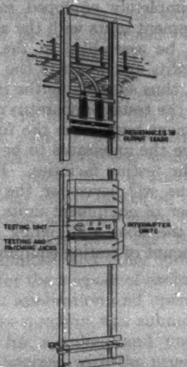

Fig. 26—Typical Assembly of Group of 135-Cycle Interrupter

Fig. 24 shows the vibrating element to be employed in this interrupter. This is equipped with an adjustable weight on the reed so that the frequency of the output may be regulated. For an input voltage variation of 20 to 28 volts, the output frequency will vary about 4 cycles.

Fig. 25 shows the assembly proposed for the complete interrupter unit. This includes the vibrator, filter, transformer, etc., mounted on a panel under metal covers. Fig. 26 shows a typical arrangement of a group of interrupters with the associated testing equipment. These assembly arrangements are uniform with those previously described for other types of equipment.

The 135-cycle motor-generator developed for signaling purposes is shown in Fig. 27. This outfit includes two motor-generators, one for regular service and one for reserve, on one panel, the control equipment being mounted on a second panel. The wave form of the output of the motor-generator, which is provided with a filter, is practically free from harmonics. Its voltage range is from 30 to 40, while its frequency range is from 134 to 136 cycles, including full-load to no-load conditions. The output capacity of this machine is approximately 10 watts.

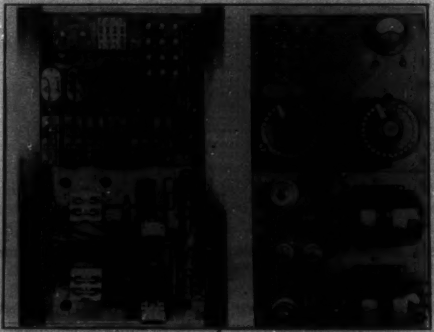

Fig. 27—Assembly of 10-Watt, 135-Cycle Motor-Generator Set and Associated Apparatus

General Assembly Arrangements

In the preceding descriptions some mention has been made of the panel assembly method which is expected to be used with much of the cable equipment. It might be well to speak briefly here of some of the features of this mounting method which are expected to have general application.

The large number of equipment units per station in the cable plant has been one of the principal factors in determining the requirements for efficiency in the design of the equipment. These requirements have been mainly (1) compactness in dimensions, (2) uniformity in assembly arrangements and (3) simplicity in design.

To make possible the housing of the equipment for a cable installation within a building of reasonable dimensions, it has been necessary to economize carefully in space. Fig. 28 shows in a general way how the panel assembly method may be employed to accomplish this. As seen from this view, the equipment panels are assembled on vertical supports consisting of I-beams or channels which extend from the floor to the ceiling, rolling ladders being employed to reach the apparatus above when necessary. As has been shown in the preceding views of individual equipment units, all of the adjustable apparatus is mounted on the front of each equipment panel and the material not requiring adjustment when in operation is mounted on the rear, since by this means the rear aisles can be made narrower than the front aisles. Thus, all available space is utilized to the extent that the floor area required per equipment unit in a cable installation will be as small as the maintenance, manufacturing and installation requirements for the apparatus will permit.

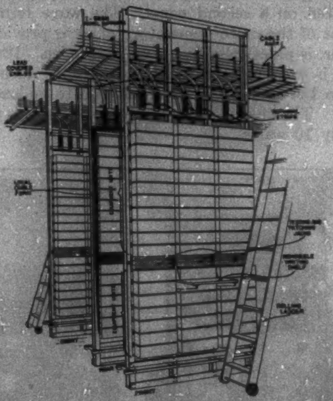

FIG. 28—GENERAL VIEW SHOWING TYPICAL GROUP OF EQUIPMENT UNITS EMPLOYING PANEL MOUNTING METHODS

In view of the many different types of equipment units having a variety of special functions which are required to make up a complete cable installation, a high degree of uniformity in design is necessary to permit efficiency in their use. It is expected that this will be accomplished effectively by applying the panel assembly method in a uniform manner to practically all types of equipment units. All panels are to be of a uniform length, designed to mount on vertical supports spaced 19½ inches between centers. The height of the different panels will vary, according to the amount of apparatus in each unit, but this vertical dimension is in all cases to be a whole multiple of 1¾ inches. By applying these specifications widely, it will be possible to secure interchangeability between panels and to employ uniform methods in grouping the different units, thus facilitating their installation and use.

The simplicity of the design of the equipment units comprising the various panels is also of great importance. This has necessitated careful attention to the forms of the individual pieces of apparatus, in order that these might fulfill their specific functions efficiently while at the same time fitting in well with the general equipment arrangements. To this end, new types of apparatus, such as repeating coils, retardation coils, etc., are being developed especially for cable use. Much will also be accomplished toward the simplification of the panels by carefully avoiding duplication in the accessories to the different types of equipment and by dissociating from the individual units of all types any pieces of apparatus capable of being made common to a number of units, or subject to different methods of application in different types of offices.

Other important advantages are anticipated in the panel assembly method. One of these is that it will permit the assembling together on one panel of all of the pieces of apparatus of different types which may be desired to form a distinct equipment unit. In large installations, completely equipped racks including a number of equipment units with the associated testing apparatus may be assembled in the factory on the supporting uprights and wired to the terminals at the top of the rack, thus simplifying the installation work. The location of the testing apparatus on the same rack with the equipment panels has the further advantage that it will place the apparatus to be adjusted within easy reach of the testing facilities. Furthermore, the testing apparatus will serve for the maintenance of a greater number of equipment units when all of the space between the floor and ceiling is utilized, thus reducing the amount of testing equipment required.

Technical Committee Annual Reports 1922-1923

REPORT OF PROTECTIVE DEVICES COMMITTEE

To the Board of Directors:

Following the policy adopted in previous years, the work of the Protective Devices Committee has been carried on by five subcommittees with subjects assigned as follows:

1. Oil Circuit Breakers, Switches and Fuses.
2. Current-Limiting Reactors.
3. Lightning Arresters.
4. Grounding of Systems.
5. Relays.

The work of these various subcommittees has been reflected in a number of papers presented under the sponsorship of the Protective Devices Committee at various meetings during the year. Further activities are given in the individual reports appended.

During the year it was suggested that a study be made of the subject of automatic substations but on account of the fact that so much apparatus is involved that it could not properly be assigned to any of the existing subcommittees there was some question as to whether the study of this subject is a function of the Protective Devices Committee. The question was, therefore, submitted to the Board of Directors for a decision. The Chairman was advised that such a study could properly be made by the Protective Devices Committee in view of the fact that automatic substations consist essentially of an assembly of apparatus controlled by certain protective devices. It was stipulated, however, that other technical committees should be kept informed of such findings as concern apparatus in which they are interested. The subject is to be handled in the way in which the Protective Devices Committee deems most advisable. On account of the fact that this decision was reached so late in the year, it was decided that this year's committee would not take any action but that recommendation be made that a succeeding committee appoint a special subcommittee on this subject.

Some attention has been given to the question of establishing relations with the British Oil Switch Research Committee and other foreign committees of a like nature for the purpose of exchanging information on the design, application and operation of oil circuit breakers. The question was submitted to the Board of Directors for a ruling and your Chairman was instructed to proceed with the negotiations and make such arrangements as would be satisfactory to the Protective Devices Committee. Accordingly, Mr. B. G. Jamieson was appointed a committee of one and instructed to look into the possibilities of establishing some method whereby interchange of oil circuit breaker data may be obtained. Mr. Jamieson is now in Europe and is giving this matter his attention. It is expected that the succeeding committee will be able to report considerable progress.

Subcommittee on Circuit Breakers, Switches and Fuses

B. G. Jamieson, *Chairman*

The subcommittee feels gratified in being able to report that the following matters essential to a revision of oil breaker rating which were specifically mentioned in the last annual report and since submitted have been finally approved by your committee. This action establishes the following as essentials in the determination of the interrupting capacity of breakers.

1. That the operating duty of the breaker, formerly known as the "duty cycle," shall be considered as beginning and ending with the breaker in the open position.

2. That the mechanical and electrical condition of the breaker shall be substantially the same at the end as at the beginning of its operating duty though its interrupting capacity may be materially reduced.

3. The breaker shall not emit flame during its cycle of operating duty.

It is believed that the adoption of the terms "unit operating cycle" and "operating duty" removes some elements of ambiguity which characterized the former definition of intended equivalence. The term "unit operating cycle," now understood to mean one closing and one opening operation, and the term "operating duty," embracing any number of unit cycles of operation, are uniformly applicable to breakers with operating duty ratings consisting of any number of cycles.

The establishment of the open position of the breaker as the origin of all operating duty makes it possible to fix breaker ratings for one cycle of operation. The subcommittee believes it unwise that the single cycle rating should be made the commercial standard but it also believes that the fixing of this rating is a protection against the exigency, although remote, of initially closing of heavy duty breakers on short circuits.

The subcommittee further desires to draw attention to the fact that concomitantly with the deliberations of the committee during the year, oil breaker design has undergone an evolution contemplating the elimination of oil throwing. On this account the committee omitted reference to this feature in its proposal for rating revisions.

It is further suggested that since the re-rating of existing types of breakers, which is a proceeding of considerable magnitude, has not been completed at this time, that when this action shall have been finished the data in condensed form be made the subject of a special report of the incoming subcommittee as a supplement to this report. This special report should include a statement of rating of interrupting capacity of the old and new forms of breakers of the same type which will differentiate between the effect of the factors fixed by this committee and others, such as design modifications of the breakers for the purpose of elimination of oil throwing.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1924.

The subcommittee also desires to again emphasize the necessity of further research and test to determine the possible effect or weight of other factors varying with the system involved, now the subject of research in other countries, such as for example, power factor and voltage reestablishment characteristics. It is hoped that early system tests will bring out any facts of importance in this connection, particularly if they be of a character which might influence the interrupting capacity ratings.

The subcommittee would also draw attention to the valuable work of similar import done in connection with fuses by some of the operating companies. The data obtained are a valuable contribution to the general phenomena of electrical energy interruption.

Subcommittee on Current-Limiting Reactors

N. L. POLLARD, *Chairman*

FAILURES

During the past year the Committee prepared and sent out a questionnaire requesting information on operating experience with current-limiting reactors. This questionnaire was sent to twenty-five of the larger operating companies using reactors.

The answers were not received in time to permit tabulating the information, but in general they indicate fewer failures than in previous years. Practically all the failures reported were for reactors that had been installed for several years. The latest designs are much better mechanically and electrically and will undoubtedly stand the strains that they will be subjected to in the rapidly growing systems of today.

HIGH-VOLTAGE REACTORS

The manufacturers are prepared to build reactors for any voltage in operation today for the protection of high-voltage tie lines. Reactors of this type are constructed like transformers without the iron core and are oil immersed in steel tanks. The highest voltage these have been built for to date is 66,000.

USE OF SHUNTING RESISTORS WITH REACTORS

One manufacturer reported further indications of the value of resistors shunting reactors having been obtained in recent tests made in disconnecting transformers from a large generating system. The tests showed that a resistor shunting a reactor reduces the over-voltages which arise due to the inductive kick.

THERMAL CAPACITY

Operating companies are beginning to realize the importance of installing reactors having a greater thermal capacity than formerly. This change in opinion has resulted from a number of causes:

1. During the past few years there has been a number of instances where short circuits have remained on systems for comparatively long periods of time and as a result it seemed very desirable to install reactors having a greater thermal capacity.
2. The low price of copper and the high price of coal has made it economical to purchase reactors of lower losses than in the past.

UNUSUAL INSTALLATIONS

During the past year there has been a number of unusual installations of reactors, as follows:

One manufacturer reported receiving an order of 72-60-cycle, 96-kv-a., 478-volts, 200-ampere reactors for installing on a 27,600-volt system. This is the first system operating at as high voltage as 27,600 where such a large number of current-limiting reactors will be used. The same company received another order for 36 outdoor feeder reactors rated at 60 cycles, 120 kv-a., 381 volts, 214 amperes, to be installed outdoors on 22,000-volt circuits.

STANDARD SPECIFICATIONS

The subcommittee is continuing work on a standard specification which they hope to be able to complete before the end of the year.

PAPERS

A paper covering operating experience with different types of reactors was presented at the Spring Convention in Pittsburgh. Several other papers that will deal with the design features of reactors are practically ready for presentation whenever the Meetings and Papers Comittee can arrange to use them.

Subcommittee on Lightning Arresters

F. L. HUNT, *Chairman*

During the past year the Lightning Arrester Subcommittee has been active in discussion and further study of the classification of lightning arresters, and has made some progress toward the preparation of a specification for lightning arresters. This work has been greatly facilitated by the cooperation of two of the large manufacturing companies, both of which have built up in their testing laboratories equipment which may be called lightning generators, and which are useful in making comparative tests on the performance of various classes of lightning arresters.

During the year the Committee visited the testing laboratory of the Westinghouse Electric & Manufacturing Company, at East Pittsburgh, where a demonstration of their lightning generator was given, and comparative tests made on several different classes of arresters. The Committee has also visited the testing laboratory at the General Electric Company's plant at Schenectady, where a very complete and instructive series of tests were made, bringing out the important features in the testing of lightning arresters and comparisons between some of the important characteristics of different types of arresters.

Definite progress can be reported on the development of a technical definition and specification of lightning arresters, although the work is not sufficiently completed to warrant the presentation at this time of any final results.

New types of arresters are being developed by some of the manufacturers as the result of experimental work which has been urged by this Committee, and as a result of the papers that have formerly been presented by this Committee. During the year the Committee arranged for a paper by Mr. A. L. Atherton on the development of the autovalve lightning arrester, which was read at the Midwinter Convention, and a paper by Mr. Malcolm MacLaren on lightning disturbances on distributed circuits, which was read at the Spring Convention at Pittsburgh.

Subcommittee on Grounding of Systems

E. C. STONE, *Chairman*

This is the second year of the existence of the Subcommittee on Grounding of Systems. The subcommittee having been created by action of the general committee on October 14th, 1921.

The principal activities of the subcommittee this year consisted in analyzing and editing the vast fund of information received in reply to the inquiry sent out last year and in arranging the program for the symposium on Grounding of Systems which was a very important part of the Spring Convention of the American Institute of Electrical Engineers held in Pittsburgh, April 24-27, 1923.

The data compiled from the inquiry above referred to included extensive information on thirty six of the principal operating companies in the United States with a total generating capacity of 6,371,850 kv-a. and a total transmission line mileage of 31,408 miles.

This report has now been printed and was presented at the 1923 Spring Convention as the first technical report of this subcommittee under the title "Present Day Practises in Grounding of Transmission Systems." This report was in two sections; the first section covering Systems Transmitting at Generated Voltage, being prepared by Mr. W. W. Woodruff of The Philadelphia Electric Company and the second section covering Systems Transmitting at Higher Than Generated Voltage by the Chairman of the Subcommittee.

As an added special assignment, data were also presented on the protection of underground cables operating at over 30,000 volts.

With the cooperation of the Chairman and members of the Protective Devices Committee, the Chairman of the Subcommittee was fortunate in obtaining, for the Spring Convention, a group of very interesting and valuable papers, summarizing the present state of the art as regards Grounding of Transmission Systems. These papers included the following:

General Considerations in Grounding the Neutral of Power Systems, by H. H. Dewey.

The Neutral Grounding Reactor by W. W. Lewis.

Operating Performance of a Petersen Earth Coil, by J. M. Oliver.

Third Class Conductors and Mechanism of Arcing Ground, by C. P. Steinmetz.

Voltages Induced by Arcing Grounds, by J. Slepian and J. F. Peters.

All of these papers were presented at the 1923 Spring Convention in conjunction with the Technical Report of this subcommittee and were actively discussed.

Acting under instructions given by the General Committee at the meeting held in Chicago, September 22nd, 1922, the Subcommittee this year sent out three inquiries on the following subjects:

Simultaneous Cable Failures
Protection of High-Voltage Cables
Trouble Report Forms

Copies of these inquiries are appended.

A considerable number of very interesting and complete reports have been received in reply to these inquiries, especially covering the subject of Simultaneous Cable Failures. There is still a number of important companies to be heard from however, and the subcommittee expects, during the summer, to obtain additional data from these companies.

It is hoped to submit an analysis and report of all data received in reply to these inquiries at the Fall meeting of the Protective Devices Committee.

RECOMMENDATION

The investigations of the subcommittee have developed the fact that:

There is still wide divergence of opinion as to the proper manner in which to ground a given transmission system;

That systems of different characteristics require different treatment; and

That special problems arise from time to time in connection with the operation of the grounded and ungrounded systems.

It is therefore our recommendation that the Subcommittee on Grounding of Systems be continued next year with instructions to follow the general situation as to practise in grounding of transmission systems and to make such special investigations along similar lines as the General Committee may see fit to assign to it.

APPENDIX—REQUESTS FOR INFORMATION

Simultaneous Cable Failures

The last general meeting of the Committee brought out the fact that a number of companies were suffering from several cables failing simultaneously at widely separated points.

This subject was assigned to the Grounding of Systems Committee. Accordingly the following information is requested:

1. System characteristic
   (a) Voltage
   (b) Frequency

(c) Type of system—3-phase 3-wire, 3-phase, 4-wire or what.

2. Is neutral grounded or ungrounded?

3. If neutral is grounded please give electrical characteristic of ground, such as dead ground, ohms, resistance, or reactance, etc.

4. How many miles of cable are in operation at this voltage?

5. Please give number of cases by years where two or more cables have failed simultaneously or closely in succession. For each case, please furnish year in which failure occurred, number of cables failing, type of failure as to ground or short, location of failures with reference to each other and to the system, cause and any other pertinent descriptive matter.

Please furnish a brief description for systems at each voltage at which you operate.

Protection of High-Voltage Cables

The protection of high-voltage cables having been assigned to the Subcommittee on Grounding of Systems, the following information pertaining to same is requested:

1. Please indicate number of miles of cable operated at each voltage which you use above 20,000.

2. For each voltage, please furnish the following information:

(a) How many conductors in one sheath.

(b) Are step-up and step-down transformers provided for each individual run of cable or are a number of cables operated from a common bus?

(c) Is the neutral grounded or ungrounded?

(d) If grounded, is resistance, reactor, or dead ground used? If former, how many ohms?

(e) Is switching performed on these cables by automatic circuit breakers at the cable voltage, or are the breakers installed on the low-voltage side of the step-up and step-down transformers?

(f) If lightning arresters are used, please give type, location and other pertinent data.

(g) What other protective devices if any are used?

Trouble Report Forms

Please furnish if possible copies of all report forms used by your company in reporting and recording system troubles.

Subcommittee on Relays

E. A. Hester, *Chairman*

During this year the activities of the Relay Subcommittee have been centered in the preparation of the Relay Handbook, which is being compiled jointly by this subcommittee and the Relay Subcommittee of the N. E. L. A. Electrical Apparatus Committee. It is estimated that at the present time the work on this handbook is something more than fifty per cent complete and it is hoped that it will be published some time before the end of the present calendar year. Leaflets have been prepared giving complete table of contents, purpose, physical description, and price of the handbook, together with a list of the contributors. The United States and Canada and some other foreign countries have been divided into districts and representatives appointed to handle the sales and distribution of the handbook in each district.

In order to finance the publication of this handbook by the National Electric Light Association, it is necessary that a certain minimum number of advance orders be obtained. An active sales campaign is now being carried on and more than a sufficient number of advance orders are in prospect to insure its publication.

This handbook will contain in convenient form the work of the various relay committees of the two societies for the past few years. All the relays and relay schemes which have been approved of merit by actual experience are fully described. All of the accompanying details of relay application are given in such a manner as to present the subject from every angle, thus obviating the necessity of referring to other publications and searching among widely scattered reports for desired data. It will represent the combined experience of the most progressive relay engineers in the country and will compare with the best electrical and mechanical handbooks of recent issue.

This relay handbook will be taken as a basis for all future committee work and the annual reports of the relay committees will be in the nature of additions to or revisions of the material contained therein. Reissues of the handbook will be printed as often as warranted by the demand.

Three important relay papers have been presented under the auspices of this subcommittee during the year. The first of these was a complete description of the protective scheme used on the high-tension transmission ring of the Duquesne Light Company around the city of Pittsburgh, together with a description of the actual service tests made in the field to determine its effectiveness. The second was a paper describing a means of obtaining the advantages of a grounded system on a delta-connected transmission and distribution system where a ground could not be conveniently obtained. The third was a description of a new relay which has recently been developed to a commercial stage. This relay takes advantage of the voltage drop between the point of short circuit and the point of application and is, therefore, automatic in its selection of faulty sections of line and does not require elaborate calculations for purposes of making settings.

It is felt that all three of these papers have added distinctly to the data made available through the work of the Protective Devices Committee.

REPORT OF THE ELECTROPHYSICS COMMITTEE

To the Board of Directors:

ADVANCES IN ELECTROPHYSICS 1922-1923

It is difficult to point out any outstanding advance in electrophysics for the past year. The literature, however indicates a steady progress. Greatest interest appears to be concentrated in researches on the constitution of matter and the structure of the atom. Apparently considerable work is being done with reference to the possibility of breaking down atomic nucleus or disintegrating the elements. There is possibility of a great quantity of future work on this subject, first brought forward by Rutherford in 1920.

LECTURES AND PAPERS

The Electrophysics papers published during the year have covered a great variety of subjects such as colloid chemistry, mathematical papers on networks and transmission lines, radiation from transmission lines, electromagnetics, arc characteristics, gaseous conduction, vacuum tubes, heat conduction and convection and dielectrics, lightning, etc. In addition to these there were many papers bordering on the practical that could have been classed as electrophysics.

There have been three Electrophysics sessions during the year, one at the Midwinter Convention and two at the present Annual Convention in conjunction with the Research Committee. It is significant that two past presidents presented electrophysics papers at the Midwinter Convention and four took part in the discussion. An electrophysics paper is being presented by a past president at the Convention. The discussions at these sessions have always been interesting and exceedingly well attended.

These papers represent pioneer work, fundamental work. The Institute should continue to encourage this work and to preserve its records as it has done in the past.

F. W. PEEK, JR., *Chairman.*

MARINE COMMITTEE

To the Board of Directors:

The art of electricity on shipboard is steadily going forward.

During the past year the activities of your Committee have been somewhat divided as a great number of its members were appointed to represent various interests on the Sectional Committee of the American Engineering Standards Committee for "Electrical Installation on Shipboard," "Marine Rules" referred to in last year's report and matters pertaining to that Committee being of first importance have been given preference.

The industry with which your Committee is associated, while not in a healthy condition, is gradually returning to normal and use of the "Marine Rules" is evident. There has been evidence during the past year that the ship owners and operators and Naval Architects are beginning to realize not only the saving in fuel, but the saving in maintenance, the increased speed and precision which can be obtained by the use of electrical auxiliaries. Further, there appears to be considerable interest developing in Diesel electric drive for small vessels.

Five meetings were held. The first, September 15th, at which time the Committee was organized, subcommittees appointed and the following work left from the previous year was undertaken.

A. To write specification for standard appliances.

B. To detail power apparatus for auxiliary machinery.

C. Propulsion.

D. Historical.

E. Radio.

F. Wires and Cables.

G. To have Steamboat Inspection Service include some requirement for electrical knowledge of licensed engineers and a separate electrical license for motorships.

H. Publicity.

I. Interior Communication Apparatus.

J. Editing.

Standard Appliances, Subcommittee (A), had done considerable work in standardizing fixtures and appliances; the major portion of their work has been reported direct to the American Engineering Standards Sectional Committee to avoid delay.

Power Apparatus, Subcommittee (B), the work of this committee has largely been modification of existing rules to suit changing conditions and experience gained from recent electrical installations and have also prepared specifications for Electric Elevators on shipboard.

Propulsion, Subcommittee (C), a committee was appointed as usual, although the conditions of electric propulsion are such at the present time as to require very little activity on the part of the committee. The subcommittee of last year prepared some rules for the care and maintenance of electrical machinery and these have been sent to those vitally concerned. No attempt will be made at present to take up the standardization of electrical propulsion apparatus as it is now in a formative state and it is desired to obtain some actual operating results from the large number of electrically propelled ships in service before standardization is attempted.

Historical, Subcommittee (D). The outline for this committee was somewhat modified from that given in last year's report as the Chairman of subcommittee, Mr. H. L. Hibbard, Marine Engineer, Cutler-Hammer Manufacturing Company, was elected Chairman of the Sectional Committee, "Electrical Installation on Shipboard" "Marine Rules" of the American Engineering Standards Committee and his duties were proportionally increased. It is gratifying to report that after four years of review of the many historical articles and books written on the subject, the Committee has completed a fifty page chronology of the electrical applications in the marine field from the beginning up to the present time. The report deals with electricity on merchant and naval vessels, domestic and foreign, treats of the many changes through which the art has passed and makes reference to the more important papers previously published. Your Committee considers the report a valuable asset and it is the hope that some means may be provided for publishing same. The committee's work in the future will consist of keeping the report up to date.

Radio, Subcommittee (E), this committee has had little to do this year and it is probable it will be discontinued in the near future as the Radio Committee of the Institute may take over all radio work.

Wires and Cables, Subcommittee (F), the work of this committee has been in collecting data on the wire and cable now in use with a view to making such modifications as appear warranted.

License and Education of Engineers, Subcommittee (G), as predicted in last year's report, the duties of this committee have been most arduous as it has encountered obstinate opposition where it expected assistance. The reason for this condition is that those in authority least of all of those concerned fail to realize the necessity for some action and this attitude is no doubt due to lack of knowledge of those in authority of the nature and extent of electrical apparatus being installed on ships today and for the past five years. Your Committee has had considerable correspondence with the Board of Supervising Inspectors, Steamboat Inspection Service and one personal interview with the Board. The results are not what was expected or what they should be. In order to give the matter a public hearing a paper entitled, "The Electrical Engineer on Shipboard," was read by your Chairman before the New York Section of the American Institute Electrical Engineers, November 1922; the meeting was well attended, discussions numerous and interesting and emphasized in detail perhaps more than the paper the abuse to which electrical plants on shipboard are subjected today.

Your Committee plans to have a somewhat similar paper presented at one of the sessions of the annual meeting of the Naval Architects and Marine Engineers in November this year. Mr. A. E. Waller Designing Engineer of the Ward Leonard Company, Chairman of this Committee, is deserving of great credit for the persistance and methodical manner in which this subject has been kept before the authorities and in the public press and the Committee feels quite confident that the desired results will be realized during the coming year.

Publicity, Subcommittee (H), recently appointed, has prepared several news items of interest in connection with marine electrical work and more will follow.

Interior Communication, Subcommittee (I), specifications for Telephones, Revolution Indicators and Electrical Thermometers have been prepared by the subcommittee, approved by the Marine Committee and are about to be forwarded to the Sectional Committee of the American Engineering Standards Committee.

Editing, Subcommittee (J), has completed all work accepted by the Marine Committee.

The Committee as a unit has worked harmoniously and the entire membership is to be congratulated for the attendance and enthusiasm. The coming year will require considerable effort on the part of the Subcommittee for License and Education of Engineers and also no doubt will be called upon to support the Sectional Committee of American Engineering Standards Committee to consummate its purposes. The question of varnished cambric and telephone cable and specifications for tank gages should receive the attention of next year's Committee.

G. A. Pierce, *Chairman.*

REPORT OF THE INDUSTRIAL AND DOMESTIC POWER COMMITTEE

To the Board of Directors:

The work of this committee of the Institute is rather difficult to carry on because it covers such a wide field of electric power application. In many cases the electric drive is noteworthy from the aspect of the mechanical design of the drive and because of the very ingenious system of control which is used. Only in comparably few cases do these applications represent any marked departure in apparatus design or power consumption. The elements that interest the central station man, such as efficiency, power factor and apparatus design, are often lacking. Applications of this kind have their greatest economical interest, not in the power saved, but in the increased output of the operators. In other words, the interest in many cases is one of general economy, rather than that of purely engineering proclivity.

We are ever striving to appropriate for personal use more and more of this world's goods. At the same time we are striving to reduce our hours of productive labor in order, as we claim, that we may have more time for leisure and intellectual cultivation. At any one time the goods which can be obtained are directly proportional to the number of persons participating in the distribution. An increase in wages does not affect this distribution as a whole, but an unbalanced wage scale may cause an unequal distribution of goods giving certain classes of persons more than their share and operating as a hardship to other classes.

The great problem which leaders of industry are facing today is increased production per person in order that more goods may be produced with less hours of productive labor per person. This output has been very greatly increased during the last twenty-five years by the substitution of the electric drive for the steam engine drive. The advantage of the electric drive is not so much one of economy in power consumption as of increased flexibility. Improvements in the electric motor have made many of these applications possible and also have materially reduced the personal hazard to the operator. By far the largest factor contributing to the success of the electric drive has been the improvement in control apparatus. The electric motor is restricted in its range of characteristics. The additional functions required for a particular drive must be supplied by the control equipment.

In viewing this situation broadly, we must not overlook the contribution made by the mechanical engineer in developing new and improved methods of drive and his able cooperation with the electrical engineer in adapting the electric drive to special machinery. In many cases the machine itself has been re-designed in order to take full advantage of the electric motor and the automatic features of its control.

In order to be of interest to the Institute membership as a whole, this committee should arrange for presentations illustrating the broad tendencies now existing, and also the lines along which development is taking place, describing the particular application selected for purposes of illustration.

This work can perhaps best be carried on by cooperating with Institute sections to arrange programs on industrial application subjects in addition to papers presented at Institute meetings. Many of the typical applications are of considerable local interest; for example, the textile industry in New England and the south, the lumber industry on the Pacific coast, the automobile industry in and about Detroit, etc. By this cooperation, it may be possible to hold several meetings on the subjects of this committee during the year and present material which will be not only of interest to the particular industry selected, but which will also illustrate in a broader way the trend of power applications.

Electric power can be broadly divided into two classes: (a) its generation and distribution, and (b) its utilization. All power generated must be sold at an average net profit, otherwise the industry would not exist. The companies are vitally interested in the utilization of their output and many papers have been written describing the load factor, power factor and other general matters of interest. In the main, however, the engineers of these companies have devoted their energy toward solving the immediate problems connected with the generation and distribution of this power. It is only recently that they have realized the advantages and economies of the electric drive for the auxiliary equipment in their power plants. The large per cent of connected load is made up of constant speed, squirrel-cage induction motors which are usually started and stopped by manual control. Both the motor and its controller are so far standardized that they can be obtained as a stock article from all manufacturers of this apparatus. The application of these motors consists largely in determining the size of the motor necessary and devising the mechanical means for connecting the motor to the load. This work can usually be done by one of the power company's salesmen without the assistance of an expert. The more difficult applications represent a relatively small per cent of the connected load, but they are very important because in many cases they determine whether central station power can be economically used in a particular industry. In addition to these special problems, a study of production in an industry often shows that a machine that was previously driven at constant speed from the line shaft, and later on was connected to its own constant speed induction motor, can be improved and the output of its operator materially increased by adapting it to a motor and control which can quickly start and stop the machine, and, perhaps, operate it at different speeds.

POWER FACTOR

This is of great importance to the distributing companies. Careful investigations of the design of the induction motor indicates that very little improvement can be made in the power factor of this machine, but considerable may be done by more carefully applying the motor to operate closer to its normal load. The power factor can be improved mainly by means of additional devices such as static condensers, phase advancers or synchronous condensers. Many applications permit the use of synchronous motors which also can be used for power factor correction. Care should be taken in recommending the use of synchronous motors as it is only in relatively large sizes, in proportion to the rest of the load, that power factor correction can be economically obtained in this way. Where there are a large number of relatively small motors, 50 h. p. or less, it will often prove more economical, both as to first cost and to operation, to use induction motors and provide one synchronous condenser for the whole installation.

MAGNET CLUTCHES

The application of the synchronous motor has been materially widened by the use of the magnetic clutch which permits the motor to accelerate under no load and then gradually connect the load to the motor after synchronous speed is reached. Recently in flour mill electrification this factor has been taken advantage of. The group drive in this class of work necessitating fairly large horse power motors, also in cement mills where individual drive of large horse power is required. In rubber mill line drive the use of the clutch permits the use of synchronous motors and an additional advantage has been gained for quick stopping as the load can be disconnected from the motor very quickly and the brake which becomes a part of the clutch in such drives can be used for the quick stopping of the mills.

THE SYNCHRONOUS MOTOR

The design of the synchronous motor has been improved so that considerable starting torque can be obtained and the motor can be directly coupled to the load in many cases without the use of clutches. This is particularly true where the starting torque is less than full-load torque. It has also be found that a synchronous motor can be quickly stopped by overexciting the field and short-circuiting the primary windings through a resistance. These improvements in the design of the motor have enabled engineers to extend its range of application. Where synchronous motors are applied, automatic control is more satisfactory and is being generally used.

SQUIRREL-CAGE INDUCTION MOTORS

The design of these motors has not been materially changed for a number of years, but improved forms of automatic starting devices are being placed on the market in increasing numbers and the tendency seems to be toward the "push button method" of starting for many applications.

Remote control is desirable for all types of motors and for most applications. It does not depend on the judgment of the operator and it is much safer respecting the personal hazard aspect because it removes the switching means from the immediate locality of the operator. In many cases it is more economical than manual starting; it permits the starting means to be located near the motor and reduces wiring expense. It is often more convenient, as it enables the motor to be started and stopped from several places. Push buttons can be located wherever most convenient to the work. If the application involves a personal hazard it is very desirable to have ready means of stopping the machine from all positions of the operator. All of these considerations are tending to increase the number of automatic starters in proportion to the total number of starters used.

VOLTAGE CONTROL

This method of controlling a direct-current motor has come rapidly to the front in the last year or two. It consists in providing an individual generator for each motor, the motor being controlled by changing the value and direction of the generator field. This system was first advocated by Mr. H. Ward Leonard about 25 years ago, but its development was retarded by certain inherent difficulties experienced in its application. It is the scheme which is recommended for mine hoists where large d-c. motors are used, and it is also the recognized method of controlling the larger motors used for reversible steel mill drive.

Recently this system of control has been successfully applied to electric elevators, electric shovels and it is now being used to some extent on reversing planer equipment. It has some very desirable characteristics and undoubtedly will be applied to skip hoists, which are very similar to elevators.

PAPER MACHINE

The sectional drive for paper machine has come to the front in the last few years. The improvements which have been completed during the past year indicate that it is not only practical, but very desirable to operate paper machines by individual motors applied to each section. This has proven a difficult problem and has been worked on at various times by engineers during a considerable period of time. Its successful solution is a distinct advance in the application of the electric drive for industrial use.

POWER STATION AUXILIARIES

The electric drive has practically superseded the small steam turbine for the operation of power house auxiliaries. The attempt is being made to use a-c. motors for all applications on account of the greater convenience in securing a-c. power and the greater reliability of this source of power as it can be obtained from static transformers rather than rotating machinery. Considerable success has attended the use of a-c. motors and some stations are being entirely equipped with this form of auxiliary drive. The electrical characteristics of d-c. motors for certain applications are so much better that they are in general the ones that are used for these particular applications, and where a power station has a reliable source of d-c. power it is an advantage to operate these auxiliaries with d-c. motors.

OIL WELL APPLICATIONS

The electric drive has superseded the gas and steam engine for oil well pumping, and also the steam engine for oil well drilling on many large properties. The pumping equipment has been universally satisfactory and much more effective than the gas engine. Experiments are still being conducted on the electric drive for drilling purposes.

LUMBER INDUSTRY

The electric drive has been used as a source of power for most of the operations connected with the lumber industry, particularly for all those operations in and about a mill. More recently the electric motor has been successfully applied to logging donkeys and experiments are being continued which engineers believe will result in a further substitution of the electric motor for the steam engine.

WOOD WORKING MACHINERY

Many of the operations in wood working are most economically performed at a high speed. Recently an adjustable frequency system of control has been used, some a-c. induction motors being coupled to an induction generator from which different frequencies can be obtained. This is particularly interesting as one of the few applications so far employing a variable frequency method of control. Undoubtedly, other applications will be made of this method.

PAPER CALENDERS

Calenders require a low speed for "making up" which consists of threading the paper back and forth between the rolls. After the machine is "made up" it is gradually brought up to speed. A small auxiliary motor has generally been used for driving the calendar at the very low speed required for threading the paper between the rolls. This requires a separate motor for each machine, together with the necessary gearing and clutch for disconnecting this motor when the machine is being operated at normal speeds. Recently the low speed has been obtained by supplying power at a low frequency. This power can be obtained from a single machine and can be wired to all of the calenders. Calenders are usually made up at different times so that the capacity of this low-frequency machine can be much less than the total requirements for all of the calenders. The horse power required for driving at the low speed is very small compared with the high-speed drive all of which tends to make this method of low-speed control very economical and convenient. At the same time it simplifies the gearing for each individual machine.

ELECTRIC STEAM GENERATORS

The development of the electric steam generator has made possible the complete electrification of mills utilizing process steam, and although its use cannot be justified in all cases it has a field of application and is destined to be used to a large extent in locations where cheap water power is available and the cost of coal is high.

APPARATUS

The tendency is toward using larger units for electric drive and controlling these units automatically. This has resulted in the developing of larger sizes of magnetic contactors and improvement in the operation of all contactors. The small, as well as the large sizes of magnetic contactors, have been carefully studied and new designs placed on the market. Some of the changes made may not prove permanent improvements, but in general, the performance of the magnetic contactor has been materially improved so that today it is a very reliable piece of apparatus and represents a high grade product.

There has been a very decided tendency toward the enclosing of electrical apparatus. Such enclosures afford protection to both persons and apparatus, and in many cases, it decreases the fire risk. This has resulted in motors being provided with enclosing covers, and in control equipment, of mounting them in cabinets.

DOMESTIC APPLIANCES

The small electric motor has come into very general use in the home, office, store and on the farm. Below is a typical list of these appliances.

Home

1. Washing Machines
2. Ironing Machines
3. Vacuum Cleaners
4. Domestic Pumps
5. Dish Washers
6. Refrigerators
7. Buffing and Grinding Motors
8. Oil Burning Equipment
9. Small Ventilating Equipment—Ventura Fans
10. Sewing Machines

Office and Store

1. Tabulating and Adding Machines
2. Envelope Openers
3. Meat Slicers
4. Meat Grinders and Choppers
5. Coffee Grinders and Peanut Grinders
6. Popcorn Machine

Farm

1. Milking Machines
2. Cream Separators
3. All items included in items 1 to 11 under home
4. Bone Grinders

There has been a decided increase in the demand for small motor-driven appliances on account of the general efficiency of such devices, and also, the fact that they remove much of the drudgery from work which formerly was done by hand. The difficulty in obtaining domestic servants and office help has made many of these appliances absolute necessities in every-day life.

The modern device has been materially improved so that it is good from the mechanical standpoint. It is easy to operate, it has a better appearance and a longer life. These devices are all of the "fool-proof" type.

SAFETY TO PERSONS

One of the most gratifying developments of the past few years has been the increased attention given to the personal safety of operators and other persons employed in industry. The economical loss due to accidents is well known and fully recognized, but an improvement in working conditions from this viewpoint could not be made so soon. It required time to study in detail the source of accidents and to develop appliances to meet this demand.

The electric drive has contributed very largely to the improvements made in working conditions on account of its flexibility, its remote control feature and the possibility of automatic control. Great improvements have also been made in the design of industrial machines. The elimination of line shafting and belting was a great indirect step in advance. The enclosure of the motor and control protects the operator from injury and the automatic features have rendered this device safe to start and stop. It, also, has permitted the introduction of emergency stop devices where necessary. Among the automatic features, might be mentioned the locking of the disconnecting switch which prevents the accidental starting of the machine while an attendant is working on it. Also, low-voltage protection which prevents the accidental starting of the machine after failure of power.

CONCLUSION

Our committee hopes that section meetings can be arranged for the discussion of electric application to the industries and that we can work out a plan of cooperation with the local sections to further this work. We shall welcome any suggestions from members calling our attention to subjects of importance or any other matters connected with our committee work.

H. D. JAMES, *Chairman.*

REPORT OF THE INSTRUMENTS AND MEASUREMENTS COMMITTEE

To the Board of Directors:

The Instruments and Measurements Committee submits the following annual report, giving briefly a summary of its activities during the past year, together with mention of the progress and developments of the electrical art included in the field of this Committee.

At the Midwinter Convention, held at New York, a session on Friday afternoon, February 17th, was assigned to this Committee. A most interesting and valuable group of papers was provided and presented. The discussion was very active and was limited only by the time available. The papers were as follows:

Application and Limitation of Thermocouples for Measuring Temperature, by I. B. Smith, Leeds & Northrup Co., Philadelphia, Pa.

The increasing use of thermocouples in electrical measurements makes this paper most timely. The applications and limitations are clearly set forth.

Measurement of Power in Polyphase Circuits, by C. Fortescue, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

The author believes the present method of measuring power in polyphase circuits is not an equitable one and suggests a better.

Volt-Ampere Meters, by R. C. Fryer, Union Gas & Elec. Co., Cincinnati, O.

There is a decided tendency to change rate systems from kilowatt demand primary charge to a kilovolt-ampere basis. The author describes the various methods available for measuring kilovolt-amperes. This is the first paper on this subject presented before the Institute.

Expansion of Oscillography by Portable Instrument, by J. W. Legg, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Latest developments have made the oscillograph a really portable device and it is now coming into its own as an instrument for assisting the operating man in solving many heretofore unknown problems. Mr. Legg had one of the devices at the meeting and after the session adjourned spent considerable time in demonstrating it to the members.

Measurement of Transients, by F. E. Terman, Leland-Stanford University.

The subject of transients is always an interesting one. The author presents one method of obtaining information on this subject.

Balance Methods in A-C. Measurement, by P. A. Borden, Hydro-Electric Power Commission, Toronto, Ontario.

The author describes a method of obtaining more accurate measurements of alternating-current quantities.

At the annual convention in June, the Committee has prepared and there will be presented one paper as follows:

The Standardization of Electrical Measuring Instruments, by H. B. Brooks, Bureau of Standards, Washington, D. C.

Your Committee in previous reports has mentioned some preliminary investigations made on the subject of Instrument Standardization, but it was not until this year that it was decided to go ahead with the actual work of preparing what is hoped will develop into an American Standard. A Subcommittee, consisting of Messrs. H. B. Brooks, Chairman, F. P. Cox, Paul MacGahan and G. L. Crosby, was appointed and has held several meetings. The first portion of this work has been completed this year and is presented in Mr. Brooks' paper. It deals with definitions, terminology, etc. The Committee plans to continue this work and to issue from time to time additional sections until we have a complete American Standard. In its work the Committee has studied the English, French and German specifications. In order to bring the first part of the specifications more definitely before the members, the Committee has arranged for a paper to be presented in June, as mentioned above. It is hoped that this paper will receive the closest attention and that the discussion on this part of the report will lead to a satsifactory agreement among the members.

DEVELOPMENTS

The developments during the past year in watthour meters has been chiefly in that portion dealing with demand measurements for alternating-current meters. Three of the manufacturers—General Electric, Sangamo and Westinghouse—have placed on the market a register which may be applied to a standard watthour meter and which will give the integrated kilowatt demand in any specified time interval in addition to the usual kilowatt hours. This development has simplified demand measurements in the alternating-current systems.

The Warren synchronous motor (General Electric Company) has been mentioned in the 1919 and 1920 reports. The Committee can now add that two other manufacturers (Sangamo and Westinghouse) have brought out synchronous motors. The tendency of the art, therefore, seems to be to get away from spring-driven clocks for the measurement of time in various electrical measuring devices and use instead a synchronous motor. It is expected that the future will see continued developments along these lines.

Meters for the measurement of volt-amperes are appearing on the market and have been described in Mr. Fryer's paper, presented at the Midwinter Convention. Your Committee anticipates that the next few years will see continued development along these lines.

In the instrument field for commercial service, the chief tendency shown in the past year has been toward the more general use of smaller instruments. This has been brought about largely by the necessity to conserve space on large switchboards. This demand is being met by practically all of the manufacturers of instruments.

In the 1921 report, a method of testing current transformers developed by Dr. Silsbee of the United States Bureau of Standards is mentioned. This is being offered to the trade by the Leeds & Northrup Company, Philadelphia, Pa. The Committee has been advised that this year another transformer testing device has been placed on the market by the Esterline-Angus Company of Indianapolis.

Several other devices for Laboratory use have been reported to your Committee but these are more in the nature of improvements to existing apparatus and, therefore, have not been described in detail.

In order that the activities abroad in the instrument field may be brought before our members in convenient form, the Committee this year arranged to review and abstract articles appearing in the foreign technical press. This important work has been under the direction of Mr. Ivring B. Smith, committee of one, and his report which has been approved by the Main Committee is attached. It is suggested that future Committees continue this work, so that this part of the annual report will constitute in time a valuable historical record of the work in foreign fields.

Subcommittee on Instrument Development in Europe During the Past Year

IRVING B. SMITH

CURRENT AND VOLTAGE MEASUREMENT

Moving Iron Instruments. A comprehensive discussion of the various types of moving iron ammeters, voltmeters, frequency meters, ohm meters, capacity meters and wattmeters. There is considered the relative performance of the different types on direct and on alternating current, the compensation for hysteresis, for frequency variation, temperature variation, influence of external magnetic fields and form of scales.

Dutilh (*Soc. Franc. Elect.* Bull. 1 pp. 387-409, Nov., 1921).

New High-Pressure Voltmeter. A gold leaf electroscope arranged to indicate voltage. The leaf is enclosed in and electrically connected to a metal case supported by an insulator. A beam of light permits measuring the deflection of the leaf. An earthed plate is placed at a greater or less distance according to the voltage to be measured, and acts on the leaf through an adjustable window. The law relating to the angular deflection (X) of the leaf to the potential (Y) was found to be of the form $Y = a\,x^2 + b\,x + c$. The theory of the apparatus is given. Its chief advantages are ease of operation and safety in use.

L. Amaduzzi (*Elettrotecnica* 9. pp. 52-56, Jan. 25, 1922).

Shunted Dynamometer Ammeters. This paper describes a dynamometer ammeter, suitable for a-c. or d-c., reading up to 300 amperes, made by the firm of Carpentier, the instrument possessing several novel features. The fixed coil and the moving coil are put in parallel across a shunt which carries the main current and has a potential drop up to 0.5 volt, the circuits of the two coils having time-constants of about the same order. In order to increase the couple in ammeters intended for less precise work, the coils may contain a laminated iron core. Each coil is in series with a suitable manganin resistance, and in order to reduce the temperature coefficient of the instrument a nickel shunt is connected in parallel with the moving coil and a portion of its manganin series resistance. The instrument is found to be very effectively compensated for changes of temperature, and frequency errors are negligible. To render the ammeter unaffected by external magnetic fields when used to measure direct currents, it is enclosed in a sheet-iron case. The current range is varied by changing the shunt. A description is also given of dynamometer testing sets arranged to measure voltages and currents on alternating current with a precision comparable with that obtained with moving-coil instruments on direct current. A universal dynamometer to measure voltage, current and power is also described. The paper is illustrated by photographs of the instruments, and diagrams of connections are given.

L. Joly (*Soc. Franc. Elect.*, Bull. 2 pp. 291-301, June, 1922).

Modification of Campbell's Arrangement for Measuring Currents of Telephonic Frequency. For measuring currents of telephonic frequency, the simplest arrangement is that devised by A. Campbell. This arrangement is, however, not suitable for measuring lower frequencies in most cases. It is shown how the original arrangement can be modified so as to be suitable for the measurement of frequency over a wide range. The generalized frequency bridge is thus obtained and its principle explained. Some special cases of this generalized bridge are found very simple and satisfactory for the purpose.

S. Chiba (Inst. El. Eng. Japan, J. No. 405, pp. 294-300, April, 1922).

CAPACITY MEASUREMENT

Valve Method for Determining Capacity. A small transmitting valve or a receiving valve has its grid and filament connected together. The filament current is measured by an ammeter in circuit. The condenser, whose capacity is to be determined is included in the anode circuit, and a static voltmeter (a Wulf quadrant electrometer) is joined in parallel with it. A sensitive mirror galvanometer serves to measure the anode current. An anode voltage sufficient to give the saturation current in the valve is used; this voltage depends on the filament current; in the experiments here described anode voltages of 300 volts were used, the saturation voltage being about 30 to 50 volts.

The condenser is first short-circuited. On removing the short-circuiting connection the condenser accumulates a charge proportional to the time. The charging current will remain constant so long as the difference between the anode potential and the back e. m. f. of the condenser exceeds the saturation voltage. Measurement of the time t required for the anode current I_a to charge the condenser (of capacity C) to the measured potential V, allows C to be determined at once from the relation $C = I_a\, t/V$. The method can be used for widely different capacities. In the experiments here described the capacities varied from 0.01 to 1.0 microfarad.

Karolus and (Prince) Reuss. (*Phys. Zeits.* 22, pp. 362-363, June 15, 1921).

Development of Maxwell's Capacity Bridge. It is shown that in a general network of electric conductors, in which there is a galvanometer, together with one or more sources of e. m. f. and a condenser having one set of plates permanently connected to a junction, while the other set is alternately connected n times per sec. to one or other of a pair of junctions, when the variables are adjusted so that there is no galvanometer deflection, the permanently-connected plates of the condenser may be connected to any point in the network without disturbing the zero galvanometer deflection. The theorem is extended to include inductances in the network, and is illustrated by its application to Maxwell's bridge method for determining the capacity of a condenser in electromagnetic units.

V. A. Bailey (*Phil. Mag.* 43, pp. 1107-1112, June, 1922).

CARBON DIOXIDE MEASUREMENT

Carbon-Dioxide Meter. A small motor pumps the flue gas into a measuring chamber and rotates it rapidly. Within the chamber a measuring wheel is taken along by friction. At the same time air is pumped into an identical chamber rotating a similar wheel in the opposite direction. The difference in the two torques can be calibrated in terms of $C\,O_2$.

W. Waldhansen, A. E. G. Mitteilungen, Aug., 1922.

Carbon-Dioxide Recorder. Two platinum spirals in two separate cells form 2 arms of a Wheatstone's Bridge. One cell saturated air. Galvanometer of moving coil type. Can be secured either as a recorder or an indicator.

Cambridge & Paul Instrument Co., London, Eng., Nov. 10, 1922.

DIELECTRIC CONSTANT MEASUREMENT

Valve Method of Finding Dielectric Coefficient of a Liquid. Two oscillatory circuits are loosely coupled and their tuning is adjusted until suitable beats are obtained in the anode circuit of one of them. These beats are detected by utilizing condensers, a potentiometer, and an aperiodic galvanometer suitably connected. By altering the dielectric of the condenser across the oscillatory circuit the beat rate changes. The known condenser variation required to restore the original beat rate enables the dielectric coefficient to be computed.

R. Whiddington (*Cambridge Phil. Soc. Proc.*, 20, pp. 445-446, Nov., 1921).

FLOW MEASUREMENT

Recording Flow Meter. A device of H. Tinsley & Co. for recording electrically and at a distance, the rate of flow of water over a weir. Also arranged to integrate the total flow. The method depends upon the depth of immersion of electrodes placed in the water above the weir. The higher the water level the deeper the immersion, and the less the electrical resistance. By specially shaping the one electrode the conductivity may be made a measure of the flow of water. A second similar set of electrodes with constant depth of immersion, excited by the same d-c. voltage, forms part of the "control" circuit. The recording instrument is a "thread" recorder operated by an ohm meter arrangement of circuits connected to the two electrode systems. The deflection is proportional to the rate of flow of water. A simple type of intermittent integrating mechanism is added. The apparatus can be placed at a very considerable distance from the weir, as the currents are very small and the line resistance may be made negligible.

J. of Scientific Instruments, May, 1922.

FREQUENCY MEASUREMENT

Deflection Frequency Meter. The frequency meter described has a very open and long uniform scale. The principle employed is that of a circuit made resonant for a particular frequency, say 50 cycles per sec. Any departure from this frequency causes a deflection of the moving element, which is fitted with soft-iron vanes in the shape of two quadrants set at 90 deg. to one another and subjected to the influence of coils in two parallel circuits (the l circuit and the r circuit), the current in the former of which lags by 90 deg. behind that in the latter. The instrument is now made by Messrs. Nalder Bros. and Thompson, Ltd.

C. L. Lipman (*Electrician*, 87, pp. 458-459, Oct. 7, 1921).

GALVANOMETERS

The Moll Galvanometer is a moving coil instrument with a compact electromagnetic field instead of the customary permanent magnet. It is quick in action and suitable for thermopiles.

J. Scientific Instruments, May, 1922.

High-Sensitivity Galvanometer. Sullivan Type of increased sensitivity made by H. W. Sullivan, London, Eng. 10,000 m/m per micro-ampere for a period of 24 sec. and a resistance of 8000 ohms.

Moving-Coil Galvanometer of High Sensitivity. The theory of the moving-coil galvanometer is considered in this paper, and it is shown how to make a reflecting type of instrument as sensitive as possible. The conditions for maximum sensitivity are $K/K_0 = r/r_0 = H/H_0$, where K = total moment of inertia, K_0 = moment of inertia of coil alone, r = total resistance, r_0 = resistance of coil, H = intensity of magnetic field, H_0 = smallest value of H for which motion is aperiodic. By using a quartz fiber suspension and extremely thin electrolytically-deposited silver strips, hanging loosely on either side of the fiber to produce the resisting torque, a sensitivity of 21 mm. per microvolt can be obtained.

F. Zernike (*K Akad. Amsterdam*, Proc. 24, 6 and 7, pp. 239-245, 1922).

New High-Frequency Vibration Galvanometer. By means of a mechanical device, the vibrations of the reed of a Brown telephone receiver are optically magnified to such an extent that 1000-cycle alternating currents of a few microamperes may be easily measured. In combination with vacuum amplifiers, currents several thousands times smaller may also be measured.

L. V. King (Roy. Soc. Canada, *Trans.* 15 Sect. 3. pp. 143, 1921).

New Form of Thermo-galvanometer. A simplified form of thermo-galvanometer in which the magnetic system of an ordinary Deprez d'Arsonval galvanometer is employed. The moving part consists of two loops in series, one consisting of two thick wires of different metals, the second of two very fine wires of the same metal. The two loops are joined in series so that when the current passes through them, the heat produced in the fine-wire loop, owing to its ohmic resistance, produces a difference of temperature between the two ends of the thick-wire loop. This causes a thermoelectric current to flow round the loop, superimposed on the main current, and thus a deflection is caused. A sensitivity of 1 mm. per milliampere is attainable with single loops, but the same principle can be applied when multiple loops are used. If one of the metals used in the thermoelectric loop be iron, it is necessary to compensate for the inequality produced in the magnetic field.

A. Competti (*N. Cimento*, 22, pp. 321-322, Nov.-Dec., 1921).

Quick-Action Registering String Galvanometer. A detailed description of the instrument, including reproductions of the records given by it. An investigation was made of the effect of temperature, and of secular change of the tuning-fork used for regulating the film speed. It was found that the greatest variation in the number of vibrations per sec. was less than 1 in a thousand. It was found also that the absolute error of a single exposure point was less than 0.001 sec.

E. V. Angerer (*Zeits. Instrumentenk.* 42 pp. 1-6, Jan., 1922).

HUMIDITY MEASUREMENT

New Electric Hydrometer. The instrument, which is on the market, consists of a number of thermocouples in series, with alternate junctions in juxtaposition as in a thermopile, one set being covered with muslin which is kept moist with distilled water while the other set is exposed to the air. This arrangement is connected with a galvanometer and acts as a wet-and-dry-bulb hygrometer, the psychrometric difference being indicated by the deflection of the galvanometer. In addition the air temperature is measured by an electric resistance thermometer. The psychrometric difference and the air temperature are actually recorded by the movement of two pointers across a dial on which is engraved a monogram from which the relative humidity is easily read. The recording portion may be at a distance from the psychrometer, and indeed may serve several, being connected with each at will. For use in cases where the air temperature is not required the instrument has been adapted to read relative humidity directly off a single scale, while an autographic recording device has also been devised.

J. Cartus (*Zeits. Vereines deutsch.* Ing. 65, pp. 767-768 July 16, 1921).

INDUCTANCE MEASUREMENT

Direct-Reading Inductance Meter. The method consists in measuring the potential difference at the terminals of the inductance, alternating current being supplied by a magneto, and the potential difference being rectified before being applied to a direct-reading voltmeter. The inductance to be measured is in series with a permanent inductance, which is relatively large. Under these conditions it is shown that the reading of the voltmeter is very approximately in proportion to the inductance under test. The effect of the resistance of the inductance on the result is considered.

R. Barthelemy (*Rev. Gen. d'El.* 11, pp. 419-424, March 25, 1922).

MAGNETIC MEASUREMENTS

Flux Density in Air. Essentially an analytical balance for balancing the force deflecting a short conductor in a magnetic field. Read by means of beam of light and mirror. Claim remarkable sensitivity and accuracy being able to detect a change of 1 line per sq. cm.

W. P. Canby, *Jour.* of the Inst. of Elec. Eng. (British) Jan., 1923.

Magnetic Testing of Permanent Magnets. The paper describes a method of calibrating Hartmann and Braun's apparatus—used originally for measuring apparent remanences—so that it could be employed for determining the part of the hysteresis curve lying mostly in the upper left-hand quadrant. The true remanence, coercive force, and reversible permeability of a number of short cylindrical bar magnets of different qualities are measured by using the method.

J. Wurschmidt (*Zeits. f. Physik.* 10. 2. pp. 91-110, 1922).

LIGHT MEASUREMENT

A Universal Photometer without Diffusing Screen. The principle of the apparatus, the construction of which is illustrated and described in detail, is as follows. A small real image of the source to be measured, placed at a suitable distance, is projected on the eye of the observer, who thus sees a lens uniformly illuminated and forming one of the photometric surfaces. The second photometric surface is formed similarly by the light from a small comparison lamp. The two surfaces are brought into juxtaposition by means of a Lummer-Brodhun cube. Equality of brightness is obtained by interposing a wedge of absorbing material or suitable screens. The apparatus is contained in a small box easily transported. The method has the advantage that any one of a number of sources (such as individual lamps in a room or a street) can be studied separately. In addition it eliminates the great loss in brightness inseparable from the use of diffusing screens. The apparatus can be applied to measure the brightness of a surface, and it has various special applications, notably the determination of the loss of light in optical instruments.

C. Fabry & H. Buisson (*Rev. d'Optique* 1, pp. 1-12, Jan., 1922).

POWER FACTOR MEASUREMENT

Power Factor Meter. The combined electromagnetic effect of the current and pressure windings is such that for any power factor the system takes up a position of neutral equilibrium. The reading is independent of load. Made by Nalder Bros. and Thompson, Ltd.

London Electrician, Jan. 26, 1923.

POWER MEASUREMENT

Remote Power Indicator. The Cambridge & Paul Scientific Instrument Co. produce the Fawsett Indicator used for automatically indicating the power in a three-phase circuit at a distance of several miles. The actual operating circuit is a thermo-electric one making use of a uni-pivot d-c. galvanometer which may be scaled in thousands of kilowatts. Pilot wires laid with the transmission cables may be made use of to carry the current.

London Electrician 88, pp. 71, Jan. 20, 1922.

Differential Electrodynamometer. Method used at the Laboratoire Central d'Electricité for standardization of a-c. wattmeters and for power measurements consists essentially in balancing the power to be determined (or a known fraction) against another power which can easily be measured, for example the power dissipated in a non-inductive circuit.

The method is reduced to the measurement of pressure and a knowledge of the value of several resistances. It is a null method not requiring a knowledge of the frequency.

P. de la Gorce (*Camptes Rendus* 174, pp. 607-609, Feb. 27, 1922).

Electrostatic Wattmeter. After a historical review of the quadrant electrometer and its application to the measurement of alternating currents and dielectric losses, the author describes recent improvements by means of which he obtains a sensitiveness of over 400 mm. at 2 meters with 100 volts on the needle and 0.1 volt between the quadrants. This instrument has a good control of the electrical and mechanical zeros. The use of the instrument at 0.1 per cent power factor is considered, and it is shown that an accuracy up to 5 per cent is attainable, using a good wave form and avoiding errors due to surface leakages from the leads and operating switches. It gives results of value on capacities as low as 1/1000 microfarad, working on a 100-volt circuit at 50 periods. The mathematical theory of the instrument is given, and its application to modern precision measurements (as, for example, the measurement of induction) is described.

G. L. Addenbrooke (*Electrician* 88, pp. 466-469, April 21, 1922).

POTENTIOMETER

A-C. Potentiometer. Consists of two potentiometers supplied with alternating current in quadrature with each other. In series with the first potentiometer is a reflecting dynamometer. This allows comparison between the effective values of alternating and direct current. Arrangement is provided for securing perfect quadrature.

H. Tinsley & Co., London, England.

A-C. Potentiometer. Apparatus for measuring the ratio of transformation and phase angle of transformers. Diagrams of theoretical and practical connections are given.

Elettrotecnica 9, pp. 344-346, May 25, 1922.

A Portable Deflection Potentiometer. The author describes a potentiometer in which a millivoltmeter is used instead of a zero galvanometer. The only adjustment of the potentiometer resistance required once the instrument is set, is a rough one to bring the reading conveniently on the scale of the millivoltmeter. Two potential points are taken off the potentiometer resistance, and the derived circuit from them contains the standard cell, or alternatively the e. m. f. to be measured, and the millivoltmeter. In the first setting, the two zero potential points, between which the resistance is r, are used and the series rheostat is adjusted so that the millivoltmeter reads zero in the ordinary way. The e. m. f. to be measured then replaces the standard cell and the deflection of the millivoltmeter is noted after altering the resistance between the potential points to r' is necessary. The voltage to be measured is then $r' e/r + u + i r^1 (R - r)/R$ where e is the e. m. f. of the standard cell, u is the reading of the voltmeter, i the current in the derived circuit, and R the resistance of the main circuit). It is shown that the last term may be neglected without appreciable error, and $r' e/r$ is a difference of potential proportional to r' the potentiometer dials may be graduated directly in these terms, so that the value of the e. m. f. to be measured reduces to the sum of the readings of the millivoltmeter and the potentiometer dials. The whole of the dial and other switches, resistances, etc., are arranged in a compact case, which is described and illustrated in the paper. A special form of voltage multiplier enables voltages up to 900 volts to be measured and the resistance that this should have for minimum error is discussed. The exact procedure in use is detailed both for voltage measurement and for that of currents by means of shunts. An accuracy of 1 in 1000 is obtainable in all cases, and in favorable cases a much higher accuracy is obtainable.

A. Iliovici (*Soc. Franc. Elect.*, Bull. 2. pp. 117-136, March, 1922)

TEMPERATURE MEASUREMENT

Optical Pyrometer. A disappearing type of optical pyrometer produced by H. Tinsley & Co. in cooperation with the National Physical Laboratory. A Wheatstone's Bridge arrangement of connections permits of offsetting the scale zero to such an extent as to permit the full scale length to cover any desired range of temperature. Direct reading in temperature with one or more ranges.

J. Scientific Instruments, May, 1922

Total Radiation Pyrometer. Produced by the Foster Instrument Co. and arranged to indicate record or control high temperatures. It is proposed to so construct the control device as to permit holding to any desired time-temperature program.

J. Scientific Instruments, May, 1922.

Total Radiation Pyrometer. Describes a new form of radiation pyrometer in which an image of the radiator is formed on a blackened plate to which are attached two fine thermo-junction wires. The observer has an eyepiece by means of which he ensures that the plate is more than covered by the image of the radiator, and a galvanometer attached to the thermo-element then gives a measure of the heating effect of the image on the plate, *i. e.*, of the temperature of the radiator. Full deflection is generally reached in about 6 to 8 seconds from exposure to the radiation. The instrument is termed an "ardometer."

G. Keinath (*Elekt. Zeits*, 42, pp. 1384-1387, Dec. 1., 1921).

RESISTANCE MEASUREMENT

New Ohm Meter, the "Meg." Produced by Evershed and Vignoles, London, England. It does not replace the "Megger" entirely but has some advantages in portability and cost. Tests on 500 volts.

Direct-Reading Ohmmeter. A high resistance is placed across the line with a sliding contact near the middle. Connecting a sensitive voltmeter between slider and ground, a point is found giving full scale deflection on the voltmeter. A fixed resistance is then shunted across the voltmeter resulting in the voltmeter reading directly the resistance in ohms to ground.

C. Dufrene (*Revue Generale de l'Electricite,* April 1, 1922).

Direct-Reading A-C. Ohmmeter. The author previously described the use of a magneto in conjunction with a synchronous commutator and moving coil voltmeter for the direct reading of inductance. He applies the same apparatus in the direct measurement of capacity and low resistance by alternating current. It may be used to measure earth resistance with electrolytic capacity and subject to parasitic currents. The measurement is independent of the speed of the magneto. The method has been used for capacities from several microfarads down to two or three thousandths of a microfarad.

R. Barthelemy (*Rev. Gen. d'el.* pp. 891-894, June 17, 1922).

Leakance Bridge. Wagner's method of measuring leakance is only applicable when the cable sheath is insulated from earth. The present article describes (1) a form of bridge in which the effective capacity and leakance under working conditions can be accurately determined when the cable is still on the drum (*i. e.* with the sheath partially earthed), and (2) a double bridge for similar measurements on cables after laying. The article contains a useful discussion of the action of the earth-capacities of various parts of the bridges and of the generator circuit and shows how their effect is eliminated.

F. Fischer (*Teleg. u. Fernspr. Techn.* 10, pp. 137-140, Oct., 1921).

A-C. Bridge for Telephone Cables. In measuring the a-c. resistance of telephone cables difficulties arise because of the capacities against ground. An improved Wien bridge is suggested wherein an auxiliary resistance which duplicates the bridge branches, makes the arrangement symmetrical against ground. Great accuracy is claimed.

K. Kupfmuller & P. Thomas (*Elektrotechnische Zeits.*, April 6, 1922).

Thomson Bridge. In the usual connections for the Thomson Bridge errors may arise due to yoke resistance and to variable contacts in ratio coils. The author shows how a second measurement may be made and all of the above errors eliminated.

R. Hiecke (*Elek. u. Maschinenban* 40, pp. 158-161, April 2, 1922).

SPEED MEASUREMENT

Stroboscope. The stroboscope method of calibrating tachometers and regulating governors is in use and has been found to give high accuracy with little trouble.

Oscilloscope. While this is not essentially a speed measuring device, it is for use in the examination of objects when under rapid motion. It depends for its action upon the principle of the stroboscope. Instead of a shutter an "Osglim" lamp and induction coil is used.

A double set of gears permits driving the interrupter from the device whose motion is being studied. In this manner the stroboscope is either maintained in synchronism with the machine being tested or else by means of another set of gears. A slow motion effect may be produced enabling one to study the full cycle of operation of any moving part. It is put out by Herbert Kennedy & Co., Ltd., London, England.

INSTRUMENT TRANSFORMERS

Current Transformer. Everett Edgecumbe Co., England, have produced a portable current transformer having a large number of ranges. It is combined with a small testing set.

London Electrician, Jan. 19, 1923.

Short-Circuit Proof Current Transformer. Paper describes the latest types of "bar transformers" for currents of 300 amperes and up, suitable in secondary energy supply to operate a wattmeter, a relay and a meter. For currents between 10 and 300 amperes a similar type but with a loop as primary is described. This latter type is overload proof. A small spark gap built into the transformer gives protection against sudden overvoltages. Some of the 300-ampere, single-bar transformers have withstood a dead short circuit from a 60,000 kv-a. generator in which a crest value of 114,000 ampere was reached before the circuit breaker opened.

(*Siemens Zeits.*, May and June, 1922), W. Skirl.

Transformers for Measuring Apparatus. Current and potential-transformers for ammeters, voltmeters, wattmeters, and relays must satisfy conditions quite different from those of ordinary transformers. These conditions are separately analyzed, and it is shown that they are satisfied in the transformers of the Societe de la Metallurgique Electrique. Special attention is given to transformers for relays in order to obtain a given time-lag.

V. Candie (*Rev. Gen. d'El.* 11, pp. 193-201, Feb. 11, 1922).

High-Voltage Electric Meter with Fault Indicator. To give warning when one of the fuses on the potential transformer burns out or fails to make circuit, auxiliary windings are placed over the usual potential coils. These serve to light two miniature lamps to unlike intensities if a fuse is out. It is claimed that without such indication a meter may be registering as much as 70 per cent low for long periods.

A. Kaeppele (*Elektrotechnische Zeits.*, March 9, 1922).

TESTING TRANSFORMERS

High-Voltage Testing Transformer. Describes a 400,000-volt testing transformer set. Based upon the Dessauer principle, one single-phase transformer of 200,000 volts and two single-phase transformers of 100,000 volts each, are arranged in series with their low-voltage windings excited from two insulating transformers, resulting in a total potential against ground of 400 kv. This simplifies the securing of the insulation of the individual transformer windings against core and ground and guarantees at least a partial testing voltage in case of a breakdown of one of the transformers.

D. Vogelsang (*Elektrotechnik und Masch.*, May 28, 1922).

A 350,000-Volt Testing Transformer. The transformer described is designed to give currents up to one ampere at 350,000 volts, 50 cycles, in view of the appreciable capacity currents required in cable testing at these high voltages. The oil-immersed type was chosen in preference to the open type as affording greater security from the effects of damp, etc., although the latter would have been cheaper and in some ways more convenient. One terminal of the high-tension winding is permanently earthed to the case to which one end of the primary winding is also connected. The primary is supplied at 5000 volts through an auxiliary transformer, on the low-tension side of which all voltage regulation is effected. The secondary is divided into sections so that voltages of 87,500, 175,000 or the full 350,000 can be obtained. During the tests a spark-gap method was used to measure the secondary voltage. Special care was taken in the arrangement of the insulation, which is partly of oil and partly by screens of solid insulating material, as well as the coverings on the conductors composing the coils. The disposition of the coils and insulation is such that the positions of the equipotential surfaces can be calculated easily, and the screens and surfaces where there is a change of insulating material are arranged as far as possible to lie on equipotential surfaces. The magnetic circuit consists of a central core and two return paths. Concentric windings of circular form are employed, with the low-tension windings inside the high-tension windings, and much heavier insulation is provided for the turns near the high-tension terminal than for the rest of the winding; these are altogether of much more substantial construction. The different layers and sections of the windings are separated by carefully placed screens. A special feature of the apparatus is the large, single, high-tension exit terminal, which is of the condenser type, with five concentric metal sleeves separated by a special insulating material. The projecting portion is enclosed in a sort of cylindrical chimney filled up with oil, which projects about 5 feet above the top of the main case. Excluding this chimney, the case which is cylindrical is about 9 ft. high and about 8 ft. in diameter, and the total weight of the apparatus is 15 tons.

R. Massot & P. Bunet (*Rev. Gen. d'El.* 11, pp. 752-757, May 20, 1922).

Timing Apparatus

Electric Timing Device. For short intervals of time. An inductive resistance in a Wheatstone's Bridge employed for calibrating an electrical chronometer.

J. of the Inst. of Elec. Eng. (British), Dec., 1922.

Vibration Measurement

Vibrograph for Turbo-Generator Shafts. Describes the construction and method of use of a vibrograph designed for use on large turbo-generators. The description given is fully illustrated by diagrams, and examples of the results obtained, showing the effect of loading at various speeds, are reproduced in the paper.

J. Geiger (*Zeits. Vereines deutsch. Ing.* 66, pp. 437-330, May 6, 1922).

Measurement of Vibrations of Structures. Describes apparatus designed for studying the vibrations in a building, etc., from the point of view of acceleration. The principle of the method consists in suspending a heavy mass M in such a manner that it makes a very light contact with a surface. The force of contact is adjusted to such a value that the vibration forces are just insufficient to produce separation of the mass from the surface. The additional force f required to produce this separation under static conditions is determined by means of a subsidiary experiment, so that the acceleration y, which is due to the vibration, is given by $f = M y$. Instruments are described for measuring vibrations in the horizontal and vertical directions, and the method of determining the instant of separation is discussed.

P. Prache (*Soc. d'Encouragement,* Bull 134, pp. 177-186, Mar., 1922).

G. A. Sawin, *Chairman.*

REPORT OF THE COMMITTEE ON TRANSMISSION AND DISTRIBUTION

To the Board of Directors:

The Report of the Committee on Transmission and Distribution for the year 1922-1923 is submitted under the following headings:

1. Report of the Cable Research Committee.
2. Review of Construction Problems in Overhead Transmission and Distribution.
3. Review of Construction Problems and Tendencies in Underground Transmission and Distribution Practise.
4. Testing of Underground Transmission and Distribution Cables.
5. Review of Inductive Coordination.
6. Review of Papers Submitted during the Year.

As has been the custom in previous years, the report of your committee summarizes under the various headings the advances made in the transmission and distribution field during the past year together with future improvements to be expected but not, as yet, fully developed.

Report of the Cable Research Committee

Maximum Permissible Temperature for Impregnated Paper Insulation. The investigation on the maximum permissible temperature for impregnated paper insulation, which is being conducted by the Massachusetts Institute of Technology for the Research Committee is progressing favorably, and the preliminary investigations have been practically completed. During the course of these investigations it was discovered that none of the existing machines or devices for testing paper were sufficiently accurate to serve as a measure of the depreciation of the paper with heat. Accordingly, the college staff in charge of these investigations devised a new machine for testing the folding endurance of strips of paper, which appears to give very consistent results.

A number of the cable manufacturers have placed orders for similar machines. It is expected that the heat runs on the samples that are available will start very shortly, and a portion of the results should be available during the course of the year. In this connection it is of interest to note that a report recently presented to the Institution of Electrical Engineers, (English) has resulted in raising their maximum permissible temperature from 50 deg. cent. to 65 deg. cent. Private information from some English engineers indicates that a number of the leading engineers in England consider that a further increase is warranted, particularly for low-voltage cables.

Available Supply of Manila Hemp Rope Fiber Paper. All recent American specifications for high-voltage cable include a requirement that the paper shall consist solely of manila hemp rope fiber stock. Some of the cable manufacturers have found difficulty in getting as much paper as they desired, and appararently the difficulty is due primarily to the shortage of second-hand manila hemp rope fiber ship's hawsers, which is the material from which the cable paper is made. The shortage is due in part to the gradual change from manila hemp rope fiber hawsers to steel hawsers, and partly also to the considerable reduction in international shipping. These causes may result in a gradually diminishing supply of raw material over a period of years, and with the increasing use of high-tension cable, it may be desirable to investigate the use of other fibers for making cable paper. Some foreign companies are using wood pulp paper, or a mixture of wood pulp and manila hemp rope fiber.

Previous reports have related the improvements in the quality of paper insulation that have been secured in the past few years, and the reductions in the thickness of insulation that had been found to be satisfactory with the modern grade of impregnated paper insulation. One company has reduced the thickness of insulation on cable for all voltages by about 50 per cent and the experience in service with the cables having this reduced thickness of insulation indicates that the cable is just as satisfactory in all respects as the older cable with thicker insulation. It is suggested that the purchasers of cable could do much to relieve any shortage in paper that might occur by making a similar reduction in the thickness of the insulation which they specify, as this would reduce the amount of paper required for cable insulation by about one-half.

Dielectric Strength. The leading cable manufacturers have made some marked improvements in the dielectric strength of their insulation, and the present indications are that they will ultimately be able to offer to customers high-voltage cables with just as low a dielectric power factor as any heretofore furnished and at the same time a materially higher dielectric strength. Improvements of this kind should very shortly result in the manufacture of cables for materially higher voltages than any that have heretofore been considered practicable.

One large company is installing this year several miles of single-conductor cable for operation at 44 kv. Another company is installing some single-conductor cable for operation at 66 kv., and a third group of companies is making plans for the trial operation of a short length of single-conductor cable at a much higher voltage. Several of the leading cable manufacturers will cooperate in this experiment. One company that has now a number of miles of 33-kv. cable in service will make a considerable addition to this class of cable during the year, and as the operating results to date do not indicate that the cable is operating at its maximum voltage limit, it is proposed to operate one or two lines about 14 miles in length at 44 kv. The transformers for this trial operation will be so wound that they will permit keeping full load on the cables at either 33 kv. or 44 kv.

In England one company has on order some 3-conductor cable with round conductors, intended for operation at 44 kv., and guaranteed for this pressure. Arrangements are being made, however, for the trial operation of this cable at 66 kv., and in case this experiment is not successful, a trial will be made at 55 kv. Apparently this company, like one of the large American operating companies, is of the opinion that the upper voltage limit of a cable can be definitely determined only by experimental operation.

Investigation of Dielectric Loss and Ionization. An arrangement has been made by the Research Committee with the Harvard Engineering School to make an investigation into the subject of ionization loss and ionization in high-voltage cables, both single-conductor and three-conductor. This subject is also receiving considerable attention from the leading cable manufacturers, and apparently the investigation is very necessary, for until the nature of the so-called ionization loss can be definitely determined, and this loss materially reduced or eliminated, this particular property of high-voltage cables may be the limiting feature in raising the operating voltage, and particularly so in those portions of the country where the ground temperature during the winter months is below the freezing point.

High-Voltage Tie Lines between Generating Stations. Two of the larger operating companies with stations on both sides of the cities which they serve are seriously considering plans for high-voltage tie lines between their large generating stations, so that these tie lines may make available, in case of an emergency, the spare generating capacity in any one of the several generating stations. In this way spare capacity in one station would be made available at any portion of the system, and the calculations indicate that the reduction in generating capacity would more than offset the cost of such a tie line. The preliminary plans for these tie lines contemplate the line being carried through the outlying districts of the city, in which portions the line when first installed may be in part overhead, but it is assumed that as the cities expand, due to their normal growth and development, these overhead portions may have to be placed underground.

High-Voltage Cable Joints. In impregnated paper insulation, the dielectric strength at right angles to the paper is very much higher than the dielectric strength along the surface of the paper. It is for this reason that the lead sheath of the cable must be trimmed back a considerable distance from the exposed ends of the copper in making the joints. As the voltage increases the length of the joint will have to increase proportionately, and in the case of some of the higher voltages now being considered, the joint may be four or five feet long. Even this length would be materially shorter than that used in England for cables which are armored and buried directly in the ground.

This indicates the desirability of some research in connection with high-tension cable joints for the purpose of determining if changes in the present methods of joint construction can be made which will materially reduce the length of the joint now indicated as necessary for the higher voltages. If such a reduction in length is not possible, then it may be necessary to devise new types of conduit and manhole construction for the installation of the very high-voltage cables.

Dielectric Loss Guarantees. In this country, the leading cable manufacturers have recently been making high-voltage cable with dielectric losses that compare very favorably with the best results obtained abroad. Private advices from foreign engineers, however, indicate that the American manufacturers are maintaining a somewhat wider margin between their average dielectric loss results, and their guarantees, than is customary abroad. It is assumed, however, that as the American manufacturers become more familiar with the properties of the impregnating compound and the paper which affect the dielectric losses, they will be able to make closer dielectric loss guarantees.

REVIEW OF CONSTRUCTION PROBLEMS IN OVERHEAD TRANSMISSION AND DISTRIBUTION

A continued tendency toward standardization of materials and methods of construction is noted. Not only have new records been made in extra high-voltage transmission, but there is obvious a continued trend toward higher voltages in nearly all classes of lines.

Supporting Structures. With the increasing scarcity of certain timbers heretofore widely utilized for poles, there is evident a further increase in the employment of steel structures. Some new and economical designs not only for suburban but for city service, are in evidence and a demand for standardization of steel structures is being manifested.

Steel towers remain the ultimate standard for trunk transmission lines, some new designs for extra high-tension voltage transmission having recently appeared. The modification of existing lines to secure the clearances required for higher voltages, has been accomplished in some recent instances by raising structures without the discontinuance of service.

With continued higher prices of wood poles, it is natural that close attention is being given to improved methods of preservation. Several new processes for the treatment of entire poles have been advocated in order to avoid insect depredations above the ground line. A problem as yet unsolved is that of rendering such poles satisfactory for climbing. The open tank treatment continues to be the general standard and is rapidly succeeding the brush treatment formerly in such wide use. The puncture treatment as a means of securing a deeper penetration of the preservative has many advocates. In some sections of the country the all-important problem is not the protection of poles from fungus rot, but rather from the ravages of termites. Various systems of gassing or otherwise poisoning these insects in situ, as a method of saving infected poles, have been tried with some indications of future success.

New and improved specifications for the purchase of poles are now being prepared by various commercial organizations, it having been found in many cases that conditions in the pole market have so changed as to necessitate the preparation of new specifications.

Insulators. There have been in the past year no new developments of outstanding importance. Manufacturers have continued to improve materials, processes and designs with most satisfactory results to operating companies. A standardization in design, particularly in pin-type insulators utilized for lower voltages, is becoming evident. Suspension insulators having higher mechanical strengths than the older types are beginning to appear. In order to secure satisfactory insulator operation there must be close cooperation between insulator manufacturers and users in order that the units selected may be suited to the particular conditions of the transmission line upon which they are to be used. Too often in the past there have been failures resulting from the selection of units based entirely upon catalog ratings, rather than on the physical and electrical characteristics of the units.

In the attachment of conductors much care must be utilized, whether the line be of the pin or suspension type. In either case, too rigid a form of attachment is apt to result in breakage of the conductor and in the case of pin-type insulators, some of the mechanical clamps which have been advocated have caused a high depreciation in insulator heads.

The line of demarkation between suspension and pin-type units continues to be in the neighborhood of 66,000 volts. While many companies prefer suspension insulators for lines of this voltage others continue to use pin-type with entire satisfaction.

Additional forms of pins are appearing on the market, particularly those for higher voltage service especially calculated to reduce stresses in the insulator heads.

Conductors. The selection of the various metals which have been employed for conductors, such as copper, copper-clad steel, aluminum, steel-cored aluminum and steel is largely dependent upon economic conditions and the relative prices of the metals employed. For some extra long spans bronze cables are being experimented with. New types of weather-proof insulation have recently appeared and have some beneficial qualities.

Line Hardware. There is noted a continued tendency toward standardization of minor details in line hardware. The general trend today in distribution practise is to utilize longer and heavier cross-arms than those employed but a few years back, this being a natural requirement of heavier lines and higher voltages, with consequently greater clearances. There is also a definite tendency toward the use of angle cross-arm braces, particularly for the heavier arms. Rolled thread bolts are gradually replacing cut thread for nearly all standard purposes. In eye bolts and anchor rods, a tendency toward the use of larger eyes is noted. Careful attention to galvanizing continues to be observed. The three-bolt guy clamp so long considered standard for nearly all guying, has been found inadequate in holding qualities and new and improved designs are appearing on the market. There has recently been made available a larger series of insulators for guys.

Line Sectionalizing and Protective Devices. During the past year several new types of line switches have appeared on the market and many satisfactory types are now available. There is noted a tendency toward simplification and an increase in the cross-section of current-carrying parts. Considerable attention is being given to methods of avoiding corrosion in the contact-making surfaces of air break switches.

Fuses for higher voltage service are still unsatisfactory in many respects, as has recently been proved by some extensive tests undertaken by operating companies.

Street Lighting Construction. New and improved devices for series street lighting circuits are coming on the market. In overhead work there is noted a tendency toward the abandonment of span construction in favor of long mast arms. The use of insulating transformers for special services, as for instance, where branch loops are carried through trees, or in other hazardous locations, is noted. There is also observed a tendency in some instances toward the employment of pole-type, constant-current transformers of relatively small size, fed from constant potential circuits utilized only for street lighting purposes. This system is said to result in a considerable saving in substation space and pole space and also in fewer outages.

4000-Volt Distribution. While higher voltage distribution continues to increase, nevertheless, 2300-volt and 4000-volt systems remain the general standard for urban distribution. Although 4000-volt, 4-wire distribution has been practised in this country for 25 years or more by some companies, there has recently been noted a tendency toward changing over many 2300-volt, 3-phase, 3-wire or 2-phase, 4-wire to 4000-volt systems, particularly in some of the larger cities which have heretofore preferred the older systems. This movement will probably result in more uniform practises in 4000-volt distribution.

Rural Lines. The rural line problem continues to be one in which economic considerations are paramount. Various means have been suggested whereby this much desired service can be rendered at reduced cost, but always it is necessary to give the closest attention to the maintenance of safe construction. The possibilities of long span construction, having regard to the necessities of climatic conditions, are being recognized, and some cheap but satisfactory lines have been constructed.

REVIEW OF CONSTRUCTION PROBLEMS AND TENDENCIES IN UNDERGROUND TRANSMISSION AND DISTRIBUTION PRACTISE

It has been apparent during the last year that there is an increasing interest in problems connected with the construction and operation of underground systems. This has been brought about by the fact that such systems are becoming larger and the voltages being used are becoming higher.

Some of the causes of trouble on underground cables have been studied and definite remedies have been found. For example, expansion and contraction of cables under various load conditions have been determined and it has been found and quite generally recognized that the most effective way of overcoming troubles from this source is to construct the duct entrances into manholes in such a way as to avoid undue wear and exposure of cables. The use of concrete duct end bells, permitting cables to be fireproofed well into the ducts and providing broad bearing surfaces for the cables have been found particularly advantageous.

The heating of duct runs is another subject which has been given much attention and it is now generally recognized that duct runs should be so constructed that no cable will be required to dissipate its heat solely by conduction through other ducts. This has had a material effect upon shape of conduit runs, and it is now standard practise for some companies to build their duct runs 2 wide and 4 high, and in some cases build them in the form of a *U* so that each duct can radiate heat directly to the surrounding earth.

The use of the kenotron for routine testing of high-voltage cables with direct-current promises to result in the ability to detect incipient trouble in cables so that weak spots may be removed and the troubles due to cable failures in service largely eliminated. During the year 1922 one company was able by means of such testing to almost eliminate failures in service on a number of 6600-volt rosin oil cables. The method used by this company consists in applying periodically a moderately high-voltage direct current and measuring the variation of current with time. Normally the current decreases very rapidly and approaches a constant value but should it first decrease and then begin to increase it is taken as an indication of incipient trouble and the cable is broken down and replaced at some convenient time. If further investigation proves this to be a general characteristic of all paper-insulated cables, this method of testing should be very important and an extremely valuable one, since the indications are positive, while the application of high voltage which either does or does not break down the cable is undoubtedly a less useful test.

In recent years, but particularly during the last few months, a great deal has been done to improve the reliability of service on a-c. underground distribution systems through the adoption and use of various schemes for connecting distributing mains in multiple. This idea has been developed to the point where the reliability of service from a-c. networks is equivalent to that of generating stations and some companies have even gone so far as to recommend the abandonment of d-c. distribution systems. Recent progress has been such that a number of companies have definitely decided to curtail the growth of their d-c. systems.

TESTING OF UNDERGROUND TRANSMISSION AND DISTRIBUTION CABLES

There have been no innovations or marked changes in the standard practise for testing underground cables since the last report of this Committee, and the general practise continues substantially as described in that report.

The manufacturers are giving more attention to the development of tests for the control of raw products. The rapid increase in operating voltages emphasizes the importance of absolute knowledge of the quality of the materials going into cables and therefore the necessity for careful control of the materials and manufacturing processes.

It is understood that the principal research work of the manufacturers during the past year has been devoted very largely to investigations of those factors which cause variations in the quality of cables and in test results, particularly on high-voltage cables. It is obvious that the advancement of the art must include not only improved average quality but greater uniformity in quality for as operating voltages increase, a lower "factor of assurance" may have to be accepted. A lower "factor of assurance" may be permissible if it is accompanied by a smaller variation in the quality as shown by greater uniformity in the results of the various tests which are made. But much research work is still necessary before it will be possible to secure a thorough understanding and accurate control of the obscure and apparently very sensitive causes of variations which are now experienced.

There has been no change in the standard acceptance testing methods. Most cable, particularly that for high-voltage service, is being purchased under the N. E. L. A. specifications for paper-insulated underground cable or specifications very similar to them, the acceptance tests being carried out as prescribed in those specifications.

There is, however, a feeling on the part of some engineers that in puncture tests of samples graduated time tests should be made. It is well known that the puncture voltage obtained when raising the voltage by 30-second steps, as prescribed by the existing specifications, will be considerably higher than the maximum voltage which the cable will withstand indefinitely. While the determination of the latter value is only practicable in connection with large and important installations, the 30-second period might properly be increased considerably.

The opinion has also been expressed by some engineers that specifications should include a test for thermal resistivity similar to that provided in certain English specifications.

Review of the Inductive Coordination

Probably the most outstanding result of the year in inductive coordination was the publication on December 9th, 1922 of the third report of the Engineering Subcommittee, Joint General Committee of the Bell Telephone System and the National Electric Light Association. This report contains preferred detailed practises covering the physical relations between electrical supply and signal systems which, taken with the second report of the Joint Engineering Committee containing detailed principles, forms a basic code for guidance in the cooperative settlement of specific cases. Due to the complexity of the subject and, in many instances, lack of supporting technical data, these reports necessarily treat the subject in a qualitative manner.

The joint work has now entered upon a third stage of development; namely, the determination of limits or tolerances for inclusion in a subsequent addition of the practises. A Joint Subcommittee on Development and Research, working under the supervision of the Joint Engineering Committee has been organized and is actively working on the determination of mutually satisfactory values for quantitative limits or tolerances in the practises. This Subcommittee is also cooperating for the improvement and extension of remedial measures in general.

Review of Papers Submitted During the Year

In the June 1922 issue of the Journal an article on "Rating of Cables in Relation to Voltage" was presented by the Subcommittee on Wires and Cables of the Standards Committee.

After a brief discussion of the need of a comprehensive research on fundamental cable problems, this paper presents summaries of various papers and articles dealing with geometric relations which affect dielectric stress; dielectric failure of air; ionization of gas in solid insulation; dielectric failure of transformer oil; electric properties of petrolatum; residual charge, power factor and associated effects; grading of insulation; and miscellaneous data.

A paper on "Potential Gradient in Cables" was submitted by Messrs. W. I. Middleton, C. L. Dawes, and E. W. Davis in the August, 1922 issue of the Journal.

This paper deals with a discussion of the logarithmic formula as applied to the variation of stress in single-conductor cables, its modifications and the effect of internal heat on the potential gradient. Considerable experimental data and curves, accumulated from many breakdown tests, are given. Results of tests on cables with large ratios of dielectric diameter to conductor diameter are included with the modification of the logarithmic formula as applied to them. Results of breakdown tests on three-conductor cables are also shown and the calculated rupturing stresses are compared with those for single-conductor cables.

In the same issue of the Journal, Mr. Donald M. Simons has contributed an article entitled, "Rating of Cables in Relation to Voltage."

This paper is a bibliography on dielectrics and includes references to articles in periodical literature dealing with the physics of dielectrics from the view-point of energy loss and electric strength.

In the June 1922 issue of the Journal, Mr. D. W. Roper presented a paper on "Dielectric Losses and Stresses in Relation to Cable Failures."

A compilation of the cable failure records covering a period of 18 years on the transmission lines of a large operating company was used by the author in the preparation of this paper. All failures caused by external injury to the lead sheath, lightning, failure on test and all joint troubles are eliminated. The author discusses the remaining failures from the viewpoint of dielectric loss and curves showing the relation of current per conductor, temperature, and watts loss per foot are given. Curves showing the interrelation between critical current, critical temperature and dielectric loss are shown for several sizes of cable. The relation between insulation thickness and dielectric stress is discussed and the present practise of American and foreign cable manufacturers in the matter is pointed out. The conclusion of the writer is that the cable failures considered have been largely dielectric loss failures. It is also pointed out in conclusion that high-voltage cables having a low dielectric loss can be safely operated at temperatures materially higher than are possible with high-loss cables.

A paper entitled "A New Method for the Routine Testing of Alternating-Current High-Voltage Paper Insulated Cable" was presented by Messrs. Howard S. Philips and E. Dean Tanzer in the March, 1923, issue of the JOURNAL.

In this paper the authors present a method developed for the routine testing of cables. The need of a satisfactory routine test is discussed and the development of this method of testing is described. The method of applying this test is explained and the highly successful results obtained by a large operating company in indicating incipient trouble on rosin oil cables prior to failure of the cable is pointed out. Various charts showing results obtained on cables in good condition and those having incipient trouble are given. In conclusion the advantage of this test and the need for further investigation is brought out.

In the June 1922 issue of the JOURNAL, Mr. Donald M. Simons submitted an article entitled "On the Minimum Stress Theory of Cable Breakdowns."

This paper presents a discussion and further investigation of Fernie's theory and data that cable breakdowns are governed by the dielectric stress at the sheath, or in other words the minimum stress. Assuming that the inner layers of insulation may be over-stressed without complete rupture of the cable due to the stable equilibrium of the remainder of the insulation and also that insulating materials have a critical breakdown gradient, the author shows that the direct result of these two hypotheses is that the minimum stress at breakdown is a constant though it is not in itself the criterion.

In the February 1922 issue of the JOURNAL, Mr. Robert J. Wiseman presented a paper entitled "The Apparent Dielectric Strength of Cables."

After a brief discussion of theories previously advanced in connection with the dielectric strength of cables, the author presents the assumption that that which has been considered the dielectric strength of the dielectrics is not the "true dielectric strength" but an "apparent dielectric strength" which is a function of the radii of the conductor and insulation. Reasons supporting this assumption are pointed out and discussed. Comparison is made of the dielectric strength of cables computed from the same data from four different viewpoints, namely the minimum stress theory, the average stress theory, the increase in conductor diameter theory when $R/r > 2.72$ and the ionization theory. The results are shown and discussed and it is pointed out that the ionization theory appears to apply most satisfactorily.

In the June 1922 issue of the JOURNAL, an article on "Effects of the Composite Structure of Impregnated Paper Insulation on its Electric Properties" was submitted by Messrs. Wm. A. Del Mar and C. F. Hanson.

Electrical networks representing the conditions in cable dielectrics are presented and discussed and a quantitative expression is obtained of the power factor and dielectric loss in terms of the resistivities and specific capacities of the elements of the insulation. The electrical function of the paper in impregnated paper insulation is also discussed and experiments are cited which indicate that the failure of such insulation is due to ionic motion of the oil. The conclusion is drawn that in designing and using high-voltage cable particular care should be taken to impose every possible restraint upon ionic motion in the oil and practical means of accomplishing this are given.

The October 1922 issue of the JOURNAL contained an article entitled "Electrical Characteristics of Transmission Systems" by Mr. Herbert Bristol Dwight.

The author describes and explains a method of making electrical calculations for long constant-voltage transmission lines, taking into account the effect of step-up and step-down transformers and making a direct calculation for the complete system without any trial-and-error procedure. The author uses as a specific example a 220,000-volt, 60-cycle line 200 miles in length with assumed average values of transformer core and condenser losses and gives the solution of the line characteristics. A circle diagram of the line, based on this method and giving results for all loads, is also shown.

"A Graphic Method for the Exact Solution of Transmission Lines" is the title of an article contributed in the November 1922 issue of the JOURNAL by Mr. Collis H. Holladay.

This paper presents a quick method of constructing the vector diagram of a power line, from which it is possible to read directly the voltage, current, power and power factor at either end of the line. The method of constructing the diagram is described and explained. To illustrate the procedure by this method, the calculation for a three-phase line is presented.

Mr. H. Goodwin, Jr., presented a paper on "Qualitative Analysis of Transmission Lines" in the January 1923 issue of the JOURNAL.

The process proposed in this paper is called "Qualitative Analysis of Transmission Lines" because of its similarity to chemical qualitative analysis. Based upon a previous presentation by Mr. Percy H. Thomas, the "critical load" for transmission lines is discussed and the most important parts of Mr. Thomas's presentation are abstracted. The detail development of the formula for the critical load is given and the effect of the various line constants upon it are discussed.

Working from the critical load, a vector interpretation of the action of a long high-voltage transmission line is given. From this vector interpretation it is possible to predict certain essential operating characteristics. Practical examples of transmission lines operating at critical loads, which show the possibility of predicting desired conditions of operation, are given.

In the July 1922 issue of the JOURNAL, Mr. R. J. C. Wood presented an article on "220-kv. Transmission."

This paper deals with the writer's experience in connection with the change-over of the Southern California Edison Company's System from 150-kv. to 220-kv. Problems of insulation, corona losses, charging currents, tower design and protective devices are considered and discussed. Wiring diagrams of the Southern California Edison Company's system are given and the change of transformers, oil circuit breakers, insulators, towers, etc. for successful 220-kv. operation are described. In conclusion the author points out that 220-kv. transmission appears, the further investigation is carried, to differ only in degree from transmission at lower voltages; that the unequal voltage distribution over long insulator strings can be eliminated by shield rings; that standard suspension insulators can be used; that suitable transformers and switches have been developed; and that corona constants and charging currents can be sufficiently well determined to permit of intelligent design of lines.

In the April 1922 issue of the JOURNAL, Mr. Henry Flood, Jr. presented an article on "The Superpower System."

After a brief review of the essential elements of the superpower plan and discussion on the effect of a cheaper and more adequate power supply upon industrial activity, this paper deals with the steam electric power production facilities for the proposed superpower zone. It is brought out that the existing facilities form an important element and are to be used to a very large extent as the nucleus of the system. The effect of amortizing less efficient steam-electric plants will be to materially increase the resulting economy of the balance of the existing plants, which, with the proposed new water power developments, will be used to carry the peak load, permitting the base-load steam-electric plants to operate at very high capacity factors with resulting low production costs. The location of base-load steam-electric plants in the bituminous coal region is found to be uneconomical at the present time, but the location of large stations in the anthracite region is found to be very attractive. The use of process fuel is not recommended under present conditions. In conclusion the importance of locating base-load steam electric plants with reference to coal delivery routes is pointed out.

In the same issue of the JOURNAL was another article on "The Superpower System" by Mr. L. E. Imlay.

The proposed hydroelectric developments and transmission systems in connection with the superpower system are dealt with in this article. After a brief discussion of the factors governing hydroelectric development, the author describes the three principal types of hydroelectric development, namely, plants depending on uniform stream flow, plants on rivers having variable stream flow, and plants on streams regulated by storage reservoirs. The important rivers of the superpower system zone are classified as to the most suitable type of development.

The transmission and distribution systems are then considered and it is brought out that the transmission lines should operate at 220-kv. and the distribution lines at 110-kv. The requirements and difficulties to be met in the successful operation of an interconnected system are discussed together with the benefits to be derived from such a system.

In conclusion it is pointed out that a proper attitude of federal and state regulatory bodies is essential to the success of the superpower project.

The November 1922 issue of the JOURNAL contained an article on "Failure of Disk Insulators on High-Tension Transmission Lines" contributed by Mr. Harrison D. Panton.

This article presents the actual operating performance of disk insulators over a period of nine years. The failures considered are those located on the annual insulator test, failures due to lightning not being included. The records indicate that the longest life is to be expected from disks nearest the conductor and the shortest from those at the tower or grounded end of the assembly. It is pointed out that these results are contradictory of conclusions deduced from recent research work in connection with disk insulator assemblies and, if borne out by further investigation and experience of operating companies, indicate the necessity of devising some means of increasing the life of the disks in the top third of the assembly, especially the disk attached to the tower.

Mr. G. W. Lapp presented a paper in the July 1922 issue of the JOURNAL on "An Overpotential Test for Insulators."

The author discusses the characteristics of routine electrical tests and describes a new overpotential test with its application and effects. It is brought out that this test is superior to the flashover, high-frequency and impact tests in that the punctures produced by this over potential applied in a smooth sine wave of low frequency are of marked suddeness and violence, indicating that this test eliminates with fairness material that would be likely to puncture in service on account of the low dielectric strength of the total path of puncture. It is also pointed out that in the overpotential test, involving as it does a higher intensity of applied pressure, the insulators passing the test have not been weakened and a part of their useful life sacrificed.

In the October 1922 Issue of the JOURNAL a paper on "Test and Investigations on Extra High-Tension Insulators" was submitted by Messrs. C. Coleridge Farr and Henry E. R. Philpott.

In this paper, the viewpoint of the purchasing engineer is taken with special reference to methods of test for acceptance and tests for porosity and deterioration of insulators. The author's experience with insulators on a 66,000-volt line in New Zealand is described and tests to overcome the many failures due to inherent faults that are either overlooked or "get by" the factory tests are discussed. The methods of test suggested are a measurement of insulation resistance with a 1000-volt megger, a bare flashover pressure at 50 cycles for 15 seconds, a bare flashover at high frequency for 15 seconds, and a porosity test consisting of a hydrostatic pressure of 2000 pounds per square inch applied to the insulator through a strong aqueous solution of fuchsin for approximately a week. A general discussion of these tests is given with the results of their application to several types and makes of insulators including those of British, American, New Zealand, Australian and German manufacture. The conclusions drawn are that insulators for extra high-tension work, before being put into service, should be subjected, individually in the case of suspension units and on each shell in the case of pin insulators, to flashover pressure for a definite period at both high and low frequency; that a percentage of each shipment of insulators should be subjected to a hydrostatic pressure test for porosity; and that insulators can and are being made which will not be overstressed by such tests.

In the December 1922 issue of the JOURNAL, Mr. Karl Willy Wagner presented a paper on "Physical Nature of the Electrical Breakdown of Solid Dielectrics."

This paper deals with the electrical rupture of solid insulating materials and advances the theory that the electrical rupture of solid dielectrics is a phenomenon of overheating by current. After pointing out the general outline of the theory, the author describes the various experiments he has conducted to disclose the nature of the rupture phenomena. The results of these tests are shown and their support of the author's theory discussed.

Mr. Malcolm MacLaren contributed an article on "Lightning Disturbances on Distribution Circuits" in the March 1923 issue of the JOURNAL.

A general survey of the nature and distribution of failures and interruptions due to lightning upon the circuits of a large city supply system is presented in this paper. The subject is treated under three principal divisions, namely, troubles on the 115 to 230-volt circuits, the 2300-volt circuits and the 13,200-volt circuits. It is shown that line failures on the low-voltage circuits are so infrequent as to be practically negligible and that the blowing of fuses and injuries to equipment on the customer's premises are infrequent with the exception of the house meters. Trouble from meter burn-outs increases with the length of the secondary circuits. It is recommended that a simple spark gap or other equivalent type of arrester be installed on the customer's premises as a part of the equipment in the service box.

For the 2300-volt circuits it is shown that lines running over high ground and open country are more subject to transformer burn-outs and blowing of fuses. It is also shown that damage to exposed lines decreases as the number of arresters increases and that the effectiveness of the protection is decreased with high resistance in the ground connection.

For 13,200-volt aerial circuits it is shown that on lines at a considerable distance from generating points most of the damage from lightning is in insulator breakages but where the line reactance is not sufficient to prevent flow of heavy dynamic current, at the time of discharge, the conductor is often fused, allowing the line to fall. It is recommended that arresters be distributed along the line at not more than two mile intervals and that near generating stations the three arresters of a group be mounted on successive poles and provided with independent ground connections.

"Corona in Air Spaces in a Dielectric" is the title of a paper presented in the September 1922 issue of the JOURNAL by Mr. J. E. Shrader.

This paper deals with the investigation of the formation of corona in air spaces in various dielectric materials, with air spaces excluded as much as possible and also with air spaces of definite thicknesses, extent and location at atmospheric pressure. The author shows the results of this investigation by curves and the variation of power factor with potential gradient. The conclusion drawn is that the potential gradient to produce corona depends on the thickness of the air space and also upon its extent.

In the same issue of the JOURNAL, was an article on "The Action and Effect of Moisture in a Dielectric Field" by Mr. Delafield Du Bois.

In order to get a picture of the action of moisture in a dielectric field the author has assumed a simple hypothetical case and attempts to follow it to its logical conclusion. A pure dielectric of a homogeneous and plastic nature between parallel electrodes and subject to electric stress is assumed. A very small globule of conducting moisture is then assumed to be placed in the dielectric and its action analyzed under constant potential stress and alternating stress.

Following up the analysis the author finds that such a dielectric containing particles of moisture would show absorption and residual charge and many other characteristics of actual insulation. It is also found that certain unusual test data, obtained while testing cables, could be explained in accordance with his analysis.

E. B. Meyer, *Chairman.*

REPORT OF THE LIGHTING AND ILLUMINATION COMMITTEE

To the Board of Directors:

Following the established practise, the first part of the Committee's report reviews the year's activities of the Committee, while the latter part is devoted to the progress of the art.

Committee Activities

In the organization meeting held October 19th, 1922, plans for the year were outlined and the work of the Committee assigned to individual members.

In the second meeting held on February 14th, 1923, the progress of the work was approved, and the possibility of starting new activities discussed. Prof. Scott pointed out the desirability of encouraging further publicity along the line represented by Illumination Items. At the request of the Committee, he is looking into the question with the view of recommending activity on the part of the Committee.

The leading established activities of the Committee are the arrangement for convention papers and the department of Illumination Items, contributed to the Journal of the American Institute of Electrical Engineers.

Convention Papers. Dr. Shackelford, with the assistance of other members, has taken charge of the Committee's work in supplying suitable papers for conventions. The interest of the Association membership in lighting is such that it has seemed desirable to provide papers of broad general interest rather than those appealing particularly to specialists. Likewise, it has seemed preferable to present a small number of papers at each convention rather than grouping a larger number less frequently, and such a plan has been followed by the Committee.

At the Pittsburgh convention, a paper on Theatre Lighting by Messrs. Mott and Jones was presented. A paper on Incandescent Lamp Quality by Messrs. J. W. Howell and H. Schroeder is arranged for the June convention.

Negotiations are well under way with a number of authors for papers ranging from street lighting to railway signal lighting. These will be available for next year's committee, and are likely to provide for all the convention time available to this phase of the Association's activities.

Illumination Items. Last year's Committee initiated the plan of contributing a section to the Journal made up of short articles on timely topics, presented in a form to interest engineers who do not specialize on lighting. This department has been continued during the past year and the inquiries received seem to indicate an increasing interest on the part of the readers of the Journal. Mr. W. M. Skiff has taken charge of collecting the material for this purpose, and he has been assisted in this connection by various members of the Committee. In the twelve numbers of the Journal, including that of May 1923, fifty columns of articles have appeared. Ample material has been maintained on hand at the General Office. In addition, many shorter articles have been placed at the disposal of the Editor for use as fillers.

Lighting Progress

There has been a considerable revival of interest in lighting growing more and more apparent since the first of the calendar year. Many new installations are being made and old ones are being revamped, these changes being accompanied with a tendency toward higher standards of illumination.

Perhaps the best numerical indication of lighting activity is the relative sale of incandescent lamps. The year 1921 recorded a slump of about 20 per cent in the number of so-called "large" incandescent lamps sold, as compared with 1920.

The figures for 1922 show a full recovery. While the number of lamps sold, viz.: 203,000,000 is slightly less than in 1920, the total wattage and lumen capacity is higher.

The relative quantities of large lamps sold for the last three years are as follows:

	Year		
	1920	1921	1922
Relative number of incandescent lamps	100	78.5	97.5
Relative total lumens or candle-power	100	85.6	118.
Relative total watts	100	81.	102.5

In addition to the large lamps, about 85,000,000 miniature lamps were sold in 1922.

Illuminants. There was considerable development in manufacturing methods and ingenious machinery for incandescent lamps, which, among other things, make for simplicity and flexibility in meeting changes in demand. Along with these improvements, it has been found more practicable to make tipless lamps and some types are being changed to this form.

The specifications for incandescent lamps have been changed to average lumens throughout life. Since lamp quality is independent of rating, this provides a measure of excellence, which is universally applicable, and therefore, is a strong influence toward the production of better lamps.

Slight changes have been made in the larger sizes of sign lamps to increase the end-on candle power.

There has also been some development in the application of tints and colors to lamps by the coating with refractory material forming a diffusing surface.

A number of 30,000-watt incandescent lamps were made and used for experimental purposes. These demonstrated the practicability of building much higher power lamps than at present, but in the absence of a sufficient demand, they are not being commercialized at the present time.

Arc lamps are still being used to quite an extent in street lighting, especially for the "white way" lighting and some improvement has been made in electrodes and in light controlling equipment, resulting in increased effectiveness.

There has been considerable experimentation along various lines of so-called gaseous conductor illuminants, although no predominating practicable development has been reported.

Lamp Equipment. The term "luminaire" as a broad designation for a lighting fixture or lighting unit was mentioned in last year's report. It has been approved by a number of electrical groups and is receiving considerable use. It is still too new to tell whether or not it will ultimately predominate in this country. Reports from Europe indicate that it is looked upon with favor there, and so has the possibility of world-wide adoption.

A considerable number of new equipments have been developed and put on the market. Few of these embody radical changes or new lighting principles.

A new type of industrial unit consists of a combination of diffusing globe with a steel reflector. This unit gives excellent diffusion, minimizes glare and has a relatively high utilization of light. The enclosing globe not only permits the use of clear bulb lamps, but also minimizes dust accumulation and facilitates cleaning. While its higher cost may prevent its application to as broad a field as the *R L M* reflector, its merit seems likely to assure wide use in the better classes of installations. This luminaire is being furnished by several manufacturers under the trade-name of "Glassteel."

Several new forms of prismatic reflectors have been developed.

The tendency toward the use of squat globes having a large diameter compared to the depth, seems to be extending rapidly, a number of new makes having been brought out.

The luminaires having imitation candles or otherwise employing diffusing round bulb lamps, are still being shown prominently for home lighting. Quite a variety of supplementary glassware is being presented to further diffuse the light of such equipment. Where this glassware is applied, it is quite practicable to use clear bulb regular lamps and thus save the additional cost of the somewhat special round bulbs.

The portable floor and table lamps are finding considerable use in home lighting and, to some extent, are being applied in other classes of artistic lighting. The indirect and semi-indirect types of portables are being utilized in rapidly increasing numbers.

Some of the recent developments in glassware and shades indicate a growing interest in tinted light.

Street lighting practise indicates the growing popularity of the ornamental types having single light pole top lanterns. Where formerly the use of such equipment was more or less limited to so-called "white-way" districts, they are today being used throughout the cities and notably in residence sections. The four and five light ornamental standards which were so popular a few years ago, are in many instances being changed over to the single light arrangement, some manufacturers having developed parts to facilitate the transformation. For highway lighting, in addition to the unit reported last year, at least two new equipments have since been developed. A rapidly growing demand has been reported.

To facilitate the selection of street lighting units, several manufacturers have made extensive installations of various ornamental equipments from which an excellent idea of appearance may be obtained, either lighted at night or cold in the daytime.

A number of developments are being made to increase the flexibility of control of electric lighting circuits, these include mercury switches for distant operation and carrier-current control.

Practise. The practise in lighting is still advancing rapidly toward higher illumination levels and better diffusion, with the result that effectiveness of illumination is being more harmoniously combined with artistic effects. Each advance suggests greater possibilities of the future.

School lighting is still one of the active fields. While most of the new installations represent acceptable practise, there is still a need of revamping of most of the older installations in the interest of conserving the eyesight of pupils, especially during evening sessions.

More interest in improved home lighting has been shown during the past year than ever before.

Store lighting is always a leader in practise, due to its close relation to profitable merchandising. During the year, attention has largely centered around the show window. Investigations reported have shown, by actual statistics, the increased drawing power of brightly lighted windows and of colored light. It has also been shown that higher intensities than previously used are effective in overcoming the distraction of external reflections in window glass, both by day and by night. The new practises are making liberal use of color and spotlight effects, used much as they are applied in theatrical stage lighting.

The year has witnessed considerable activity in theatre lighting and art gallery lighting, both of which have been subjects of papers and reports. Rather novel lighting effects were employed by a German Opera Company which toured the eastern part of the country during the past season. Most of the equipment employed is not applicable to the conditions existing in the United States, although certain features are of considerable interest.

Surveys of industrial lighting practise have been reported, including food industries, laundries, textile mills, etc.

The United States Public Health Service has conducted an investigation of lighting conditions in the post offices. Results have been reported in the technical press and elsewhere. They indicate that the standards now in use are considerably below those necessary for welfare and economy of operation.

The sign lighting field has been stimulated quite remarkably during the past year and a number of stupendous signs, involving rather novel features, have been erected. The power requirements of these are especially heavy in the brightly lighted metropolitan areas, due to the rather large lamps necessary to produce a conspicuous effect. Some of the signs represent lighting loads equivalent to those of towns of one or two thousand population.

Due to data presented on the relation of wall finishes to lighting effects, a number of the paint manufacturers have actively taken up the problem of painting for light and are promoting recent improvements.

Street lighting is still receiving considerable attention. The mounting number of traffic accidents is evidence of the need of better illumination, while the value of improved appearance is better appreciated. Highway lighting has been extended with remarkable rapidity. It is making night driving on interurban highways safer and more convenient. Where travel is heavy, headlights alone are not entirely adequate.

Many special applications have shown increased activity, among these may be mentioned railway signaling and aerial navigation.

For quite a number of years, horticulturists have evinced considerable interest in the application of artificial light to the control of plant growth. As the result of these investigations, the art appears to be approaching a practical state. A very comprehensive and elaborate project is nearing completion. An experimental laboratory has been so designed that factors, such as temperature, moisture and carbon dioxide contents, as well as the artificial lighting, can be accurately controlled. In this instance, it will be possible to obtain higher intensities of artificial light over relatively large areas, than ever before used.

Codes and Standards. Last year mention was made of the adoption of the Illuminating Engineering Society's Industrial Lighting Code as an American Engineering Standard. Since then the Motor Vehicle Lighting Code and the System of Nomenclature and Photometric Standards have been similarly approved.

The State of Massachusetts determined to set about enforcing the motor vehicle rule that tail lights must be such as to render registration plate numerals visible at 60 feet. An investigation revealed that practically none of the equipment in use was complying. At the request of state officials, a joint committee of the Illuminating Engineering Society and the Society of Automotive Engineers formulated specifications. The enforcement in Massachusetts started January 1, 1923, and it is understood that similar action is being undertaken in each of the states represented in the conference of motor vehicle administrators.

Committees of the Illuminating Engineering Society are revising the School Lighting Code put out in 1918 by that organization. A sectional committee is being organized under a joint sponsorship of the American Institute of Architects and the Illuminating Engineering Society to consider establishing the School Lighting Code as an American Engineering Standard.

Another lighting project which is active in the American Engineering Standards Committee is that of colored signal lights for traffic and other purposes. A conference of all interested groups, which was held last summer, showed a strong demand for such action.

The world standardization of incandescent lamp bases was the subject of extensive discussion at the recent meeting of the Electrotechnical Commission held at Geneva. The tendency seems to be toward the adoption of the Edison base, which has been known abroad as the American base. The changes involved are principally with regard to details of screw threads of both lamp bases and sockets.

The meeting of the International Commission on Illumination is scheduled to be held in the United States in 1924. Preliminary plans have been made, although arrangements have been somewhat delayed by the unsettled conditions abroad.

Education. Lighting education is playing an important part in the "Electrify" movement of the Joint Committee of the electrical industry, to inform the American Public regarding the advantages of electric service. In connection with this movement, a rather comprehensive book on incandescent lighting practise has been issued.

The Lighting Sales Bureau of the National Electric Light Association has felt the need of more trained illuminating engineers in the employ of central station companies. A committee has been organized to encourage the employment of such engineers and provide, in cooperation with the Illuminating Engineering Society, for a suitable training course.

The Chicago Section of the Illuminating Engineering Society, in cooperation with the Armour Institute and the architectural societies, conducted a course on lighting for architects in Chicago and the Illuminating Engineering Society has organized a general committee to encourage and guide similar activities elsewhere. This committee has outlined courses for architects and contractors.

To educate school children in the correct use of the eyes, field representatives of the Eyesight Conservation Council of America have been giving illustrated talks in the grade and other schools in various sections of the country. The relation of lighting to eye protection is being stressed.

Evidence is plentiful of the expanding public interest in better lighting. Not the least of this is the insistent demands upon the few lighting service departments for demonstrations and lectures. These requirements are taxing the capacity of the limited number of illuminating engineers available. The demonstration method of creating interest and giving instruction has proved its value.

One manufacturers' organization has been very successful in impressing the principles of good lighting by means of dialogues depicting their application to particular situations.

All in all, the year may be characterized as one of healthily increasing progress.

G. H. Stickney, *Chairman.*

REPORT OF THE POWER STATIONS COMMITTEE

To the Board of Directors:

Scope of the Committee

"The duties of the technical committees" having been defined by the Secretary of the Institute as "the promotion and coordination of Institute activity in their respective field," the scope of the Power Stations Committee has comprehended four main activities:

First. Routine analysis and recommendation of papers submitted by the Meetings and Papers Committee.

Second. Securing of papers on subjects related to the committee's jurisdiction for publication in the A. I. E. E. Journal.

Third. Special research investigation with reports and papers resulting therefrom.

Fourth. Resumé of the year's progress on power station work.

Accomplishment

During the past year the first activity has been carried out as occasions presented. As to the second, various papers have been secured on the initiative of the committee. As to special research work, of the various suggestions made it has been found that only a few could be favorably considered at the present time, although it is believed that this work of the committee will in future years bring substantial results to the Institute and its membership.

Emphasizing the point raised in last year's report of this committee, that A. I. E. E. committees should deal rather with the scientific and technical aspects of problems, while leaving to other national societies, like the National Electric Light Association, American Electric Railway Association and the Association of Edison Illuminating Companies, new developments and their application, it is believed that much greater benefit to the Institute and its membership may be secured through the agency of this committee if it may receive from the membership at large suggestions covering (a) desirable papers for presentation to the Institute, (b) special lines of technical research such as might be carried out most readily in specially equipped laboratories, (c) test results or (d) information best procurable from operating power stations. Should such suggestions through inadvertance cover subjects more properly handled through other societies' committees, there is provided through the Committee on Coordination a medium for the proper allocation of the work, at the same time affording to all Institute membership an opportunity to secure assistance possibly unavailable without such freedom of suggestion. In like manner the related committees of the other societies have arranged to refer to the Power Stations Committee problems requiring special technical or scientific treatment.

Resume of the Year's Progress in Power Station Work

It is scarcely possible to attempt to review power station progress without trenching upon ground covered by the 1923 reports of the Electrical Apparatus and Prime Movers Committees of the N. E. L. A. and the Power Generation Committee of the A. E. R. A. to which particular reference is made for a wealth of information and detail. Certain points, however, will be emphasized here.

Temperature Indicators

Temperature indicators installed for many years back have been of the resistance coil or thermocouple type, arranged for occasional use or at periodic intervals. Later switchboard type instruments for indicating temperatures were used in many stations, but recently an insistent demand for an accurate curve drawing instrument has been met and many are now in operation both on steam and water wheel driven units. Troubles of various sorts with the detecting coils or couples appear to have been largely overcome by more careful and substantial construction and stronger armor particularly at lead exits.

Temperature Standards

Papers and discussions presented at last year's A. I. E. E. Convention as a result of cooperative tests by manufacturers and operators on various large generators presage an early revision of existing standards so that the present maximum temperature of 150 deg. cent. for class B insulation will be reduced to 125 deg. cent. and a single type detector standardized consisting of a twenty inch resistance coil instead of the present choice between coil and couple. In case of turbine generators this seems likely to be accompanied by a change in design employing radial ventilation.

Turbine Generator Units

For the past two years very complete analyses of the performance of the seventy odd largest steam turbines units have been made by which both users and manufacturers of turbines and generators, alike, have benefited through elimination of defects shown to be inherent, and through improvement in design for future construction.

The advance in the size of generators is notable in those being produced for the Hudson Street Station of the Brooklyn Edison Company and the Crawford Avenue Station of the Commonwealth Edison Company having a capacity of 62,500 kv-a. at 80 per cent power factor. While not immediately affecting generator design or characteristics, it is not improbable that the present tendency toward higher steam pressures and superheats, and suggestions looking toward furnaces in which refractory material is substantially replaced by water tube surfaces, may bring about a unit type of power station from the boiler to the bus which will have material influence on generator design. For several new stations main turbines are under construction for operating pressures of 550 pounds per square inch and total temperatures of 725 deg. fahr., and smaller trial turbines are being built for pressures of 1000 to 1200 pounds.

The lubrication of steam turbine generator units has long been a source of anxiety to many of the most careful operators. Until recently the subject has not been approached in the spirit of scientific analysis which its importance warranted. Within the past year through the agency of the Prime Movers Committee of the N. E. L. A. arrangements have been perfected in cooperation with the representatives of the Bureau of Standards, manufacturers of lubricating oil, and the turbine builders whereby a tentative set of specifications previously drafted will be tested by oils taken from many of the most important units in the country to determine whether these specifications impose all the necessary and sufficient conditions for purchase. It is believed that this step marks a distinct advance in the state of the art and will produce information of permanent scientific value for power stations.

WATER WHEEL GENERATORS

The race for size is close between the water wheel and the turbine driven type; the year sees the construction of both types in the gigantic capacity of 65,000 kv-a. and voltages around 12,000. In the case of the Niagara Falls Power Company's machines guarantees for generator efficiencies have reached the remarkable figure of over 98 per cent so that in combination with their transformers an efficiency of 97 per cent is obtained. This installation is noteworthy, among many other features, in suspending with the added weight of the water wheel from a single Kingsbury bearing 69 inches in diameter a total weight of over 1,100,000 pounds; there is further interesting provision in the case of each main unit for mounting an auxiliary a-c. generator on the shaft to supply power to an auxiliary bus from which through a motor generator set excitation to the main unit is secured.

FLY WHEEL EFFECT IN WATER WHEEL DRIVEN A-C. GENERATORS

In the application of the a-c. generators for water wheel drive, the flywheel effect ($W R^2$) required for speed regulation or other reasons is an important factor in the first cost of the generating equipment. As it is important to keep the initial investment in a hydroelectric installation to a minimum, a more general appreciation of the effect of $W R^2$ on the initial cost will be beneficial to the industry, particularly in the case of moderate and small capacity low head propositions.

Standard designs provide a value of $W R^2$ which general experience has proved satisfactory. By various means, involving either the addition of weight to the generator rotor, the use of flywheels or possibly a combination of both practically any requirement within reason can be met, but the investment is correspondingly increased, not only due to the increased cost of materials entering into the unit but also due to the special design involved.

The value of flywheel effect required in a water wheel generator installation is primarily a question of application and not one of generator design. The generator design, however, is affected. A study of the application involves a knowledge of the hydraulic setting, the electrical system layout, number and size of units, ratio of actual load fluctuations to size of system and the service expected of the generator. With these factors in mind the value of flywheel effect needed or the effectiveness of the value recommended by either the generator manufacturer or the water wheel builder can be determined. Such an analysis is particularly pertinent to the moderate capacity open flume setting where it is primarily a question of speed regulation, and the matter of stresses in penstocks or wheel casings are not controlling, as in such cases it is often found that the operating condition of from 10 to 30 per cent load change rather than the 100 per cent load change, which common practise takes as the basis for guarantees on speed regulation, a difference of from 1 to 1½ per cent in speed regulation will often show from 5 to 15 per cent saving in cost in the generating unit.

It is recommended that this matter receive more generally the consideration that experience warrants should be given to it, as by so doing it will clarify what is often a more or less obscure factor, but one that is always involved in hydroelectric applications.

SMALL ALTERNATING-CURRENT VERTICAL WATER WHEEL GENERATORS

In the past few years the smaller isolated, automatic, low head hydroelectric station has become of increasing importance, as it offers an economic and cheap means of developing the country's water power in many parts of the country. Also in some locations the larger developments have been made, but there still remains a large number of small developments awaiting suitable equipment for an economical utilization of the power. Many of these smaller developments are located close to load centers and, therefore, tend to more economical development than the larger water powers located at distant points with their attendant high cost of transmission.

The vertical unit has established itself as the most practical arrangement for the small and moderate-capacity low-head hydroelectric plant. Very definite requirements are placed on the generator for these installations. It must be economical in first cost and performance, of design proportions and arrangements to fit the modern hydraulic setting, and it must have the same factor of safety in design and reliability in operation required of the larger units.

Complete lines of such small vertical water wheel generators have been designed.

Fire Fighting

The occurrence of frequent generator fires in past years led many companies to install apparatus for fighting fires, either by water or by steam, with provision or more recent machines whereby automatic differential relay apparatus as installed for the purpose of protection against internal grounds or short circuits by opening armature field circuit breakers could also be made to shut off any outside supply of air and if desired trip the turbine throttle. Within the year by concurrence between the Fire Underwriters Electrical Bureau and representatives of the N. E. L. A. the first steps have been taken toward the establishment of any definite requirements. The present general recommendations are included in the 1923 report of the N. E. L. A. Electric Apparatus Committee.

Closed Ventilation Systems for Generators

During the past year several companies, notably those at Hartford and St. Louis, have conducted extensive tests on the injection of inert gas into a closed circuit system for smothering incipient fires, the gases used being carbon dioxide and nitrogen, with preference at present lying in favor of the former. The earlier closed systems used air coolers of the spray type and so satisfactory has been the performance that one important company has decided in future installations to abandon the use of louvres or doors by which the closed system could at will be converted into one of straight-through ventilation.

The outstanding progress of the year in this field has been the successful operation for several months in the Amsterdam Station of the Adirondack Power and Light Company, as well as several instances abroad for even a longer time, of the so-called radiator- or surface-condenser type of air cooler, where cooling water passing through what is practically a surface condenser absorbs the heat of the generator air; multiple metal disks or fins of about ½-in. radial depth to the tube and spaced from 5 to 11 to the inch afford a heat transfer sufficiently great to prevent the cost of the apparatus becoming prohibitive. A further advantage is found in the hanging of the condenser units directly below the generators so as to reduce to a minimum the volume of enclosed air with the consequent diminution of the risk of entire combustion of windings in case of fire, and concomitant ease and economy of fighting with inert gas the fire when once started. Many other installations are either under course of construction or consideration.

Generator Voltage Regulator

When a station is on hand operation, the field is manipulated to keep a constant alternating-current voltage, and the exciter is kept at a constant voltage. Therefore, if a regulator operated this rheostat through the medium of a motor, the station would function the same as if it were on hand operation.

Heretofore, the difficulty with a regulator that operated the field rheostat directly has been sluggishness and the inability to prevent hunting when the rheostat was operated at a high speed due to the inertia of the moving parts. These difficulties have been overcome and rheostatic type of regulator has been in service for over twelve months in a large steam power station.

It is often desirable to operate a synchronous condenser used for voltage control at the end of a transmission line, over a wide range of lagging and leading kv-a. In order that the condenser may take a heavy lagging current from the line, it is necessary that its field current be reduced to very low values. This low current in turn requires that the voltage across the collector rings of the condenser be reduced to, in some cases, as low as 3 or 4 volts.

The standard broad range vibrating regulator (alternating-current control type) is able to control the exciter voltage down to a point approximately the residual voltage of the machine. If an attempt is made to lower the exciter voltage still farther, the regulator contact remain open permanently, thus inserting the entire rheostat in the exciter shunt field, but the exciter voltage, of course, cannot drop below that corresponding to residual. This residual voltage is, in general, many times that required across the condenser collector rings under maximum lagging conditions.

A special broad range regulator has been developed which is capable of working over the wide range required, consisting of a combination of the standard vibrating regulator, which operates on the exciter rheostat, and a motor operated rheostatic regulator which operates on the main field circuit of the condenser.

In the Hell Gate Station in New York, all main generating units are designed with direct-connected exciters. In order that stable exciter operation can be assured or at least the possibility of unstable operation reduced to a minimum, the exciter is operated throughout its stable voltage range and resistance inserted in the generator field for low voltages. This is accomplished by a unique combination of exciter and generator field rheostats. The rheostat face plate is divided into three sections. The first thirty-eight steps operating in the exciter field rheostat bring the exciter voltage to a predetermined minimum stable operating condition. From this point or over the middle section, the generator rheostat is in operation, and as soon as the generator rheostat is cut out, the exciter voltage is raised by 180 steps in the exciter. The combination rheostat is designed for motor operation.

CENTRAL CONTROLLING AND DISPATCHING SYSTEMS

Interesting equipments of centralized control apparatus have been built for the central control of automatic substations and hydroelectric generating stations. The entire control, under the direction of the load dispatcher and the answer back signals for each station or substation are carried over one or in some cases two pairs of telephone wires leased from the telephone company. These telephone wires may, in addition, be used for communication purposes. The selector equipment used in this method of control is composed entirely of relays and has no notching or rotary selector switches.

In order that the load dispatcher may know the totalized load on the various stations or substation, the meter readings are transmitted by means of a variable frequency to the load dispatcher's office where they are again transformed into a continuous meter indication.

POWER STATION AUXILIARIES

In the design of recent power stations, there is a definite trend toward the use of electrically-driven auxiliaries. This has been due chiefly to the following reasons:

1. High condensation losses and the complication of steam piping for small steam units.
2. Difficulty in designing small auxiliary turbines for the high pressures and superheat used in modern practise.
3. A wider choice of methods of obtaining heat balance is available.
4. Flexibility and ease of control.

The economies resulting from the increasing use of steam bled from the main turbine for the heating of feed water is accentuating this tendency toward the maximum possible use of electric auxiliaries, resulting in a variety of schemes for maximum possible economy of heat balance, and necessitating the utmost safeguarding of electric auxiliaries for reliability of operation and continuity of air supply.

The supply of auxiliary power has been taken either from the main bus or from house alternators or in installations made during the past year from a separate a-c. or d-c. generator mounted on the shaft of the main generator. While advocates of all three systems are found, perhaps the greatest weight of engineering opinion is today in favor of the last source of supply and in order to provide a greater degree of protection than is securable by relay protection on a direct connection between main and auxiliary bus, certain plants like the Colfax Station of the Duquesne Light Company have provided special form of motor generator set for heat balance.

TRANSFORMERS

The year has seen notable advance in size, voltage, efficiency, and general perfection of large transformers. Many installations on the Pacific Coast have been made for a voltage of 220,000, and the Tokio Company is installing four 150,000 volt service transformers to have a banked rating of 63,000 kv-a. The Kansas Electric Company is installing three-phase, 15,000 kv., 60-cycle self-cooled transformers, and in the water cooled type single-phase units with a capacity of 20,000 kv-a. have been furnished the Southern California Edison Company. In the 25-cycle class single-phase units rated at 22,000 kv-a. are being installed with the Niagara Falls Power generators.

The year has brought out a new system for self-cooling large transformers by a bank of radiators either on the transformer or separately located so arranged that, where needed, radiation by means of oil convection currents, may be increased by blowing air against the radiator surfaces.

The use of the oil conservator for transformers has been so favored that it is considered by many standard practise for the highest voltages and largest sizes.

A new development during the year is a transformer with tank nitrogen filled above the oil, the nitrogen being secured from the air by inspiration over a deoxidizing substance.

Pressure relief diaphragms have been improved by resorting to glass instead of metal, the former being shown by test to rupture at more uniform pressures and more quickly so that there is less liability of tank rupture on account of the inertial of the material, before relief can be afforded by the breaking of the diaphragm.

The latest power development of the Southern California Edison Company going into operation in 1923, and having an ultimate capacity of 150,000 kv-a. in six units. The initial installation consists of three generating units rated as 25,000 kw., 89.4 per cent power factor, 11,000 volts, 50 cycles, 428 rev. per min. The layout of the station is on the unit basis; two generating units stepping up to 220,000 volts through a single bank of transformers rated at 55,500 kv-a. and connecting to a double high-tension bus.

One of the factors fixing this arrangement is that the charging kv-a. of 280 miles of circuit at 220,000 volts requires the capacity of two generators to prevent self excitation. Other unique features in the station are: totally enclosing of the generator units to permit use of a humidifier; use of graded insulation on the transformers with the high-tension neutral solidly grounded; use of relay in grounded neutral to operate generator field rheostats as an arc suppressor; installation of sufficient number of high-tension breakers to sectionalize the transmission circuits at this point and in accordance with accepted practise in solidly grounded neutral systems above 88,000 volts, the high-tension breakers will be rated at 187,000 volts with 373,000 volt rain test. The station will also operate at 60 cycles with increased speed and generator voltage.

With the rapid growth of this system to its present generating capacity, an economic survey has justified a change in voltage to 220,000 volts, in order to handel the increased power from Big Creek to the Los Angeles districts. The only equipment necessary to make this change has been additional insulator units with strings equipped with static shields and auto-transformers to step up from 150,000 to 220,000 volts. The transformers will have graded insulation and will be considered as part of the transmission system, there being no breakers on the high-tension side.

SYNCHRONOUS CONDENSERS

In connection with large distribution systems, the importance of good regulation through control of power factor is well recognized. For this purpose, synchronous condensers have come into general use on long transmission lines; well over 150,000 kv-a. in units of from 10,000 to 30,000 kv-a. capacity, have been installed in the immediate past by three Pacific Coast companies alone; the use of all sizes, however, down to as low as 50 kv-a. is rapidly extending.

All synchronous condensers of modern design are arranged for a-c. self starting. On units of 5000 kv-a. and above, considerable saving can be realized by introducing oil pressure into the bearings before starting so as to lift the rotor in the journals. In addition to the saving in size of starting auto transformers the power demand upon the system is materially lessened which may be very desirable with limited generating capacity.

AUTOMATIC EQUIPMENT

The installation of automatic hydroelectric power stations has been signalized the past year by the operation of the Searsburg, Vt. station of the New England Power Company with a generator of 5000 kv-a., the largest unit in such service. This station is completely automatic including operation of hydraulic governors, and is more completely described in articles included in the bibliography.

Further tendencies in this direction are shown by the equipping by the Southern Power Company of some of their larger generators, normally manually operated, with automatic attachments so that the moving of a single switch by the operator serves for a complete shut down of the unit and its auxiliaries. Further possibilities for the power station have been developed during the year in the installation by the Alabama Power Company of automatic switch closing devices by which the circuit breaker is closed several times in predetermined cyclic succession after the initial opening of the circuit for whatever cause.

SWITCHES AND CIRCUIT BREAKERS

The year has seen developed for both manufacturers and operators decided preference for the "dead" type of circuit breaker which is now standardized for the newer and heavier duty apparatus. Previously inaccuracy was the bane of the bushing-type current transformer, but the New Hudson Avenue Station of the Brooklyn Edison Company is installing on its circuit breakers bushing-type transformers designed to be accurate for both operating and metering purposes.

The truck type of circuit breaker for both power house and substation has seen notable progress during the year in power house installation, as well as the allied type—also to be installed in the new Brooklyn Station—by which the tank, cover and auxiliary mechanism are first lowered so as to disconnect from the bus or feeder, rendering the parts innocuous for inspection or repairs and obviating the use of disconnecting switches.

BUS STRUCTURE

The outstanding development in recent bus and cell structure construction has been isolated phases which were brought about a year ago in the Hell Gate and Calumet Stations with segregations effected in each by horizontal arrangement of the three phases, whereas during the year installations have been made with vertical phase separation in the Cahokia Station of the St. Louis Company, the Hudson Avenue Station of the Brooklyn Edison Company, and in additions to the switching equipment at the Westport Station in Baltimore, O Street Station of the Boston Elevated, and the South Street Station of the Narragansett Electric Lighting Co. in Providence.

REACTORS

No outstanding developments have taken place other than a realization by manufacturers and operators alike that further increases in staunchness and arcing distances must be provided to secure necessary reliability of service; the more recent designs incorporate such improvements.

DISCONNECTING SWITCHES

Aside from designs improved for more staunchness and greater margin of safety for current-carrying capacity as a result of tests which demonstrated a year or more ago the inadequacy of the existing designs, the chief advance has been the development of improved types of mechanical interlocks of disconnecting switches and their associated circuit breakers.

BENCHBOARDS

For the first time benchboards have been constructed of metal instead of slate or stone previously used; with the tendency in large stations for only low voltage control on the board itself, metal construction affords obvious advantages.

TERMINAL ROOMS

While used in some stations for many years, there has only very recently developed a general tendency to provide a terminal room for all control cable and wires reaching to the switchboard so that utmost flexibility, ease and dispatch may be afforded for inspection and repair or replacement of any necessary parts. This appears to be a development of real importance for future power houses of major size.

LIGHTNING ARRESTERS

For power station use as well as substation a new type of arrester is offered called the Autovalve Type consisting of a gap backed by columns of disklike conducting material with intervening strips of mica and functioning by ready discharge of current at over-voltage but arresting a subsequent dynamic flow.

VENTILATION

The subject of ventilation has received during the year special treatment in the reports of the Prime Movers and Electric Apparatus Committee of the N. E. L. A. to which reference is made for considerable data emphasizing the importance of the subject for future power house design, especially where it is desirable to utilize all air flow in the furtherance of better economy in station heat balance. It would appear that this subject has received in the past very inadequate attention and largely by reason of the many inherent difficulties. Much research work appears to be in order.

ILLUMINATION OF POWER STATIONS

During the year a movement has been started through the agency of the Electric Apparatus Committee of the N. E. L. A. and the Society of Illuminating Engineers for a careful and much needed study of the subject, and reports of value may be expected in the near future.

RADIO

The increasing use of transmission of intelligence by radio by power companies for inter-communication between power stations and out districts, and for general purposes of load-dispatching has led to the opinion among many engineers that future system inter-communication will be entirely accomplished by this means, particularly in view of the progress that has been made through carrier service on transmission lines. At the same time the United States Government regulations at present existing impose hindrances and possible embarrassments that merit attention on the part of power stations contemplating the use of such service.

COMMITTEE ON COORDINATION

Last year's report referred to the organization of the Committee on Coordination which has functioned this year through two meetings and requisite correspondence. The chief usefulness of such a committee consists in its ability to function upon occasion on the initiative of some of its represented member societies and it will probably not originate activities, at least, as long as its status remains unofficial.

The personnel of the Committee on Coordination until otherwise determined consists of the chairman of the respective national societies' committees represented and not of certain individuals. As a very helpful step toward justifying its *raison d'etre* committee members exchange minutes of their respective national committees with any necessary correspondence. The Chairman of the Power Stations Committee of the A. I. E. E. is *ex-officio* chairman of the Committee on Coordination.

During the year it became evident that the work of the committee would be greatly furthered if there should be a representative on it from the American Society of Mechanical Engineers, and by the authority of the President of the A. I. E. E., the Chairman of the Power Stations Committee was instructed to present this matter to the officials of the A. S. M. E., who have appointed as their representative the Chairman of their Professional Division.

The present membership of the Committee on Coordination is made up of the following Committee Chairmen:

American Society of Mechanical Engineers, Professional Division

Association of Edison Illuminating Companies, Turbines and Generators

American Electric Railway Association, Power Generation

National Electric Light Association, Electrical Apparatus

National Electric Light Association, Prime Movers

American Institute of Electrical Engineers, Electrical Machinery

American Institute of Electrical Engineers, Protective Devices

American Institute of Electrical Engineers, Power Stations (Chairman)

The Committee is endeavoring at present to arrive at agreement by the represented societies on a definition, suitable for all occasions, of the term "Net Generator Output."

The Committee has also used its good offices directly and through the represented memberships to have taken under further review the tentative regulations proposed by a Subcommittee of the National Board of Fire Underwriters for the Installation or Use of Pulverized Coal Systems, to the end that the state of the art, particularly as affecting power stations, might be promoted rather than retarded by the premature adoption of regulations unnecessarily drastic in the light of scientific knowledge, technical experience, and operating requirements.

Appendix

Important Articles on Power Station Design and operation that have Appeared in American and Foreign Technical Journals.

A—STATION DESIGN AND OPERATION

"The Quality of Hydroelectric Service, Yesterday and Today" by Alex. E. Bauhan. *General Electric Review*, June 1922. This article relates to the quality of service that is to be expected from a properly equipped and ably operated hydro-plant and shows why such service is comparable with and in many cases is better than that rendered by the metropolitan steam plant.

"Hazards in Hydroelectric Plants" by Alex. E. Bauhan. *General Electric Review*, September 1922. Some of the electrical and mechanical hazards present in a low head hydroelectric plant are described as well as effective precautions taken against these hazards.

"Utilization of Surplus Flood Water to Suppress Backwater upon Water Power Developments" by Floyd A. Nagler. *General Electric Review*, October 1922. The author discusses the use of surplus head water to suppress the backwater by the production of a more or less imperfect form of hydraulic jump.

"Operating Pilot Board and Load Dispatching System of the New York Edison Company" by W. H. Lawrence. *General Electric Review*, March 1923. A rather detailed discussion of a very large and complicated load dispatching board system.

"Indicating the Total Power from Several Generating Stations" by H. P. Sparkes. *Electric Journal*, April 1922. The article describes a system whereby the load dispatcher may be instantaneously informed of the entire system load.

"The Hell Gate Station of the United Electric Light & Power Company" by H. W. Leitch. *Electric Journal*, May 1922, Power, May 2, 9 and 16, 1922. Description of an ultimate 300,000 kw. steam station having a novel station layout in all respects.

"Switching Equipment for the Hell Gate Station" by C. M. Moss and J. B. MacNeill. *Electric Journal*, May 1922. A complete description of the isolated study of Massachusetts stations emphasizing the advantages of electrical equipment from the upkeep standpoint.

"European Practises Tend Toward Greater Economy" by A. Dyckerhoff. *Electrical World*, March 4, 1922. Standardization of equipment, waste heat boilers, gas turbines, large mercury rectifiers.

"Developments in Power Station Design". *Engineering*, January 13, February 24, May 12, June 2, July 30, 1922. A series of articles dealing with the latest foreign developments.

"First Large Plant Using Pulverized Fuel Exclusively." *Electrical World*, April 15, 1922, Power, April 18, 1922. Description of the Lakeside Station of the Milwaukee Electric Railway & Light Company.

"The Blackburn (England) Power Station" by L. C. Kemp. *Electric Journal* (England) January 1922. A complete description of the plant is given in this article.

"Five Systems with an Output of over 1,000,000,000 kw-hr. *Electrical World*, April 29, 1922. Data on output and peak load of the largest generating and distributing systems in the United States and Canada, having yearly outputs in excess of 100,000,000,000 kw-hr.

"Operating Interconnected Hydro Plants for Best System Economy" by J. W. Andre. *Electrical World*, May 13, 1922. A description of the methods in use on the Southern California Edison Company's system, showing how the use of steam standby plants is minimized.

"Calumet to Reinforce Industrial Section of Chicago System". *Electrical World*, June 3, 1922, *Power*, May 30, 1922. A description of the latest Commonwealth Edison Company's station to be put into service.

"Semi-outdoor Hydroelectric Plant with Backwater Suppressor" by John A. Sirnit. The author describes the use of the Thurlow backwater suppressor in the installation of the Alabama Power Company at the Mitchell Dam.

"Gennevilliers Steam Power Station" by G. Mercier. *Revue Industrielle*. Phase layout in this station is given.

"The Queenston-Chippewa Hydroelectric Development" by F. Q. Gaby. *Electric Journal*, August 1922, *Power*, June 27, 1922. The author describes the general details of the entire plant.

"The 55,000-h. p. Turbines for the Queenston Power Plant" by F. H. Rogers. *Electric Journal*, August, 1922. The article describes very completely the most powerful hydraulic turbines in the world.

"The 45,000-kv-a. Waterwheel Generators for the Queenston Station" by H. U. Hart. *Electric Journal*, August 1922. A detailed description is given of the largest waterwheel generators in the world.

"Some Elements of Air Flow in Electrical Machinery" by C. J. Fechheimer. *Electric Journal*, August, September, November, December, 1922, February, March, May, 1923. A rather detailed description of problems of air flow are given.

"Steam Engineering Practise in Modern Central Stations" by T. E. Keating. *Electric Journal*, December 1922. This article discusses a few of the thermal and operating problems that are today under consideration by central station engineers as well as their influence on the economic generation of power in modern steam stations.

"The Tangential Impulse Waterwheel" by Ely C. Hutchinson. *Electric Journal*, February, March 1923. The author describes the modern impulses waterwheel and its adjuncts.

"The Commercial Economy of High Pressure and High Superheat in the Central Stations" by George A. Orrok. A. S. M. E. *Transactions*, December 4, 1922. The effect of regeneration as well as the effect of increased temperatures and pressures is covered.

"The Choice of Steam Conditions in Modern Power Stations" by L. E. Kemp. *Electrician*, June 30, 1922. This article gives an analysis of the problem from the English point of view.

"Operating Expenses of Six Plants" *Electrical World*, January 21, 1922. A March and April, 1922. A very complete description of the large plant now supplying base load power to Paris. This plant represents the best European practise as pertaining to "Superpower" stations.

"Heat Balance and Electrical Equipment in the Gennevilliers Station" by R. H. Andrews. *Power*, August 15, 1922. Wiring diagram and description of switch houses. Description of closed system method used for cooling air to generators. Station heat balance compared with those of other large stations such as Colfax and Delaware.

"The New Superpower Plant near Paris Approaching Completion." *Electrical World*, August 5, 1922. A review of the above article in English.

"Steam Power Plant at Comines" by J. Reyval. *Revue General de l'Electricité*, July 15 and 22, 1922. A description of the second largest steam power station in France.

Caribou 165,000-Volt Development" by J. A. Koontz. *Electrical World*. September 23, 1922. A description of a 1010 foot head hydroelectric plant with 15,000 h. p. impulse wheels and transmission line of 165,000 volts.

"Innovations in New Brooklyn Power House". *Electrical World*. November 11, 1922. Description of the 400 000 kw. Hudson Avenue Station of the Brooklyn Edison Company.

"Design Features of Cahokra Station". *Electrical World*, December 31, 1922. Description of the large central station burning pulverized coal to supply St. Louis with electrical energy.

"The Pit River Power Developments" by Frank G. Baum. *Electrical World*, January 27, February 3, 1923. Details of a very large 220,000-volt hydroelectric development on the Pacific coast.

"Eight Systems with Output of 1,000,000,000 kw. in 1922." *Electrical World*, April 7, 1923. Reports of over 92 companies in the United States and Canada with outputs of over 100,000,000 kw-hr. in 1922.

"Full Automatic Hydroelectric Station" by E. B. Collins, May 19, 1923. 5000 kv-a. automatic generating station in inaccessible site tied in with other hydroelectric stations. Functioning under different conditions explained.

"The Mechells Power Station of the Birmingham Corporation" by W. Mobile Twelvetrees. *Engineering* January 26, February 2, 16, 23, March 9, 23, 30, April 13, 1923. General description of the Mechells Power Station of the City of Birmingham, England.

"The Improvement of Power Factor" by Dr. Fisbert Kapp. *Engineering* November 24, December 1 and 8, 1922. Discussion of the economical trend of improvement, methods of securing improvement with special reference to the use of synchronous motors. Gives data as to changes made for service based on power factor improvement by customers, by British Power Company.

"The Lancashire Electric Undertaking" *Engineering*, December 22, 1922. General description of extension of plant of the Lancanshire Electric Power Co. at Radcliffe, England.

B—Prime Movers and Generator Design and Operation

"Maintaining the Efficiency of Turbine Lubricating Oil" by C. H. Hapgood and F. R. C. Boyd. *General Electric Review*, September 1922. The improvements made in modern centrifugal purifiers and their application to power plant problems are discussed.

"Cooling of Turbo Generators" by A. R. Smith. December 1922, February 1923, May 1923, *General Electric Review*. The article discusses the various methods of cooling ventilating air when recirculated through turbo generators, the heat transfer, and the physical considerations and design of surface air coolers.

"Closed Air Circuit System of Ventilation" by John G. Monson. *General Electric Review*, December 1922. The author discusses the general principles of generator cooling by means of a closed air circuit.

"Modern Jet Condensers in Small Lighting Plants" by R. E. Hellmes. *General Electric Review*, January 1923. A variety of novel jet condenser installations are described.

"The 40,000 kv-a. Shawinigan Falls Waterwheel Generator" by J. Ralph Johnson. *General Electric Review*, May 1923. The article describes the design and construction of a very large modern water wheel generator.

"Hydraulic Turbine Governors" by W. R. Kepler. *Electric Journal*, February 1922. A general discussion of the subject is treated.

"Systematic Inspection Scheme for Preventing Breakdown" by Alex E. Bauhan. *Electrical World*, May 13, 1922. The article describes a very thorough plant of systematic inspection in a low head hydroelectric plant.

"Cracking in Condenser Tubes" H. Moore and S. Beckinsale. *Engineering*, March 17, 1922. The results of investigations of low temperature annealing applied to condenser tubes is given.

"The Size Factor in Steam Turbines". *Engineering*, March 31, 1922. A study of the factors affecting the size of rotor in steam turbines.

"Extinguishing Generator Fires with $C\,O_2$" by J. B. Wheeler. *Electrical World*, July 22, 1922. Arrangement of fire-extinguishing equipment at the Ashley St. Plant of the Union Electric Light and Power Company is described and results of tests given.

C—Boiler Plant Design and Operation

"The Control of Blower Motors" by Henry G. Issertell. *General Electric Review*, May 1922. The author has devoted his article to showing how to select the proper motor and form of control for various types of blowers.

"Preparations, Transportation and Combustion of Powdered Coal" by John Blizard, *Bulletin* 217, Bureau of Mines. A very complete manual on powdered coal.

"The Elasticity of Pipe Bends" by Sabin Crocker and S. S. Sanford. A. S. M. E. *Transactions*, December 7, 1922. This article covers a number of experiments on pipe bends conducted by the Detroit Edison Company.

"Boiler Room Performance and Practise at the Colfax Station of the Duquesne Light Company," by C. W. E. Clarke, Mechanical Engineering, May 1922. The operation and operating details of this superpower plant are quite fully described.

"400-lb. Pressure Adopted." *Electrical World*, September 16, 1922. A description of the boilers to be installed at the Waukegan, Ill. station of the Public Service Company of Northern Illinois.

"Fuel Economy in Steam Power Plants" J. B. C. Kershaw. *Beama*, October 1922. A series of articles covering very completely European practises in this line.

"Ash Handling for Central Stations" by John Hunter and Alfred Cotton. A. S. M. E. *Transactions*, December 4, 1922 A very general descriptive article of the various methods employed in large and small central stations.

"Natural Gas as Fuel for Central Stations" E. A. Quinn. *Electrical World*, March 10, 1923. The author describes a new type of burner for gas as fuel.

"Present Day Boiler Room Operation" by I. E. Moultrop and R. E. Dillon. A. I. E. E. Journal, February 24, 1922. A description of how combustion efficiency and economical loading of boilers and turbines are attained in a large central station.

"A New System of Feed Water Deaeration". *Electrician*, January 19, 1923. Diagram and description of "closed feed" system whereby condensate from main turbine may be passed through condenser of house turbine, thus improving economy, make up water being deaerated in the process.

"A New Development in High-Pressure Steam Generation." *Power*, July 25, 1922. Description of a special high-pressure boiler and economizer developed by Thomas E. Murray and Power Specialty Company and installed at the Municipal Gas Company's plant at Albany, N. Y. Radical type superheater installed at rear of furnace. Data of tests given.

D—Fuels and Combustion Studies

"Generating Power from Waste" by H. S. Bastian. *Electrical World*, February 25, 1922. Description of the methods used in storing and burning "hogged fuel" or sawmill refuse at the plant of the Portland Railway, Light and Power Company.

"Fuel Economy and Production Expenses" by Allen M. Perry. *Electrical World*, July 15, 1922. Data given for electrical plants burning coal, oil, gas and "hogged fuel."

"Boiler Capacities Obtainable per Cubic Foot of Furnace Volume" T. B. Stillman. *Power*, Jan. 31, 1922. Results on burning oil under a boiler at 658 per cent rating.

"Pulverized Lignite or Boiler Fuel". *Engineering*, November 17, 1922. Data and results of tests of Australian Lignite Burned in Pulverized Form.

"Recovering Fuel from Ashes by Magnetic Separator". *Engineering*, January 5, 1923. Refers to a magnetic separator for removing unburnt fuel from ashes. Devised by Messrs. Fried Krupp and J. Germany.

"The Driving of Handling Machinery" H. Blyth. *Electrician*. December 29, 1922. Advantages of water drive for coal and ash handling equipment with special reference to water trough conveyors for ashes.

"Coal Handling and Electricity Supply" by George F. Ziminer. *Electrician* December 29, 1922. Methods used for handling coal at Blackburn Electric Station, England. Traveling crane unloads barges by grab bucket, or railroad cars by inverting them. General features of crane.

Nicholas Stahl, *Chairman*.

ELECTRICAL MACHINERY COMMITTEE

To the Board of Directors:

The activities of the Electrical Machinery Committee during the past year have been limited in scope and have been concerned almost entirely with the review of papers sent by the Chairman of the Meetings and Papers Committee for examination and report. Eighteen papers were thus submitted for review.

While the industry has made great strides in the development of electrical generating units of gigantic capacity, reaching 62,500 kv-a. in individual units, this work has not advanced to a stage which permits its outline and description for presentation to the members of the Institute. This will be the task of the Committee during the coming year.

B. A. BEHREND, *Chairman.*

REPORT OF TELEGRAPHY AND TELEPHONY COMMITTEE

To the Board of Directors:

The work of the Telegraphy and Telephony Committee was handicapped early in the year by the sickness and death of Mr. R. E. Chetwood, who had been appointed its chairman. The Committee prepared the following statement to express the feelings of the members of the Committee with regard to the death of Mr. Chetwood:

"The Telegraphy and Telephony Committee of the American Institute of Electrical Engineers wish to express the feeling of great loss of the members of the Institute in the death of the Committee's late chairman, Mr. R. E. Chetwood. The communication art loses in him one of its outstanding figures, not only in technical ability, but also in personality."

This statement was read by President Jewett at the opening of one of the sessions on communication at the February Convention.

In accordance with the custom of the last few years, the report of our committee is a brief summary of the advances which have been made or which have come into prominence in the communication art during the past year.

MACHINE-SWITCHING TELEPHONE CENTRAL OFFICES

The year has been marked by a considerable increase in the number of machine-switching telephone central offices.

A paper was read at the midwinter convention by Messrs. Craft, Morehouse and Charlesworth, describing the so-called "panel" or power-driven type. Offices of this kind have been cut into service in New York, Philadelphia, Seattle, Providence and Buffalo. The step-by-step machine-switching system, which is of the direct-control type, has been installed in a number of the smaller cities. Further extensions of both panel and step-by-step machine switching systems are under way.

LONG DISTANCE TELEPHONE CABLES

The extremely important developments which have been carried out in long distance telephone cables are well described in the following papers:

(1) "Philadelphia-Pittsburgh Section of the New York-Chicago Cable," by J. J. Pilliod, in the A. I. E. E. JOURNAL for July, 1922.

(2) "Telephone Transmission over Long Cable Circuits," by A. B. Clark, in the A. I. E. E. JOURNAL for January, 1923.

(3) "Telephone Equipment for Long Cable Circuits," by C. S. Demarest, to be presented at the A. I. E. E. June 1923 Convention.

SUBMARINE CABLE TELEGRAPHY

Several of the Western Union Telegraph Company's transatlantic cables are now being operated directly between New York and London. In this service, special regenerative repeaters are employed which make use of phonic wheel motors controlled by electrically operated tuning forks. Other developments which were required in connection with this system include a new form of repeater and terminal equipment for the land line extensions.

The same company is having manufactured a trial length of an improved type of ocean cable developed by the Western Electric Company. Tests of this cable will be made in the near future and if they result successfully, future transatlantic cables are expected to be of that type.

It is reported that an a-c. telegrpah system is being operated, superimposed on an ordinary d-c. channel, on a cable of $K R = 0.9$ without affecting the speed of the d-c. circuit. Details of this system are promised for publication later.

LONG DISTANCE RADIO TELEGRAPHY AND TELEPHONY

On January 14, 1923, a group of about 60 people gathered in London and listened for two hours to speeches transmitted by radio from America. This was a part of a series of tests being carried out on long distance radio telephony. A complete description of the methods employed and the results obtained will be found in a paper by Messrs. Arnold and Espenschied on "Transoceanic Radio Telephony" to be presented at the June, 1923, A. I. E. E. Convention. This work is being actively continued.

Since atmospheric disturbances or "static" constitute one of the principal limitations to long distance radio communication, a great deal of effort has been directed towards finding methods of reducing its effect. The most successful line of attack has been the employment of directive receiving systems. The greater the degree of selectivity, the smaller will be the amount of interference from static coming in directions other than that from which the signals are being received. In transoceanic radio communication, the natural conditions are fortunate in this respect, since European signals arrive from the northeast, while the preponderance of static is from the southwest. On the Pacific Coast, static comes mainly from the east and southeast, and is favorable to the reception of trans-Pacific signals. A paper presented before the Institute at the Midwinter 1923 Convention on "The Wave Antenna" by Messrs. Beveridge, Rice and Kellogg, describes and explains the most successful form of directive radio receiving system for long wave lengths which has been devised.

Considerable valuable work has been done by various investigators in determining the direction of static at different points on the earth's surface. This knowledge will be of great benefit in connection with the employment of directive receiving antennas along the lines mentioned above.

RADIO TELEPHONE BROADCASTING

The interest in radio telephone broadcasting has steadily developed, and considerable progress has been made, particularly in the telephonic quality of the material sent out by broadcasting stations.

The Department of Commerce, on March 20, 1923, called together a second National Radio Conference, in order that it might have the benefit of the advice of those qualified in the art, and interested in the industry, in the formulation of such rules and regulations particularly with regard to wave length assignments, as were consistent with the advancement of the art, and were possible under the existing laws. As a result of this conference, a much larger range of wave lengths has been made available for broadcasting, and a new scheme of assigning wave lengths has been set up, which should very much decrease interference between broadcasting stations.

INDUCTIVE COORDINATION OF COMMUNICATION AND POWER LINES

In the last two annual reports, reference has been made to the work undertaken in cooperation by the National Electric Light Association and the Bell Telephone System in setting up practises for the coordination of the lines of the utilities interested, so as to prevent interference to the communication service. The joint general committee of these two organizations has issued a third progress report, in which the principles previously agreed upon have been supplemented by a set of recommended practises. The committee has also appointed a subcommittee on development and research, to undertake a cooperative study to determine what practical measures, if any, may be developed and adopted, to lessen the effect of those characteristics of both power and communication systems which contribute to inductive interference.

The development of the art of communication at high frequencies has raised some interesting and novel questions regarding interference at these high frequencies.

STRENGTH OF POLE LINES

Of great interest in connection with the extremely important question of strength of pole lines is a paper on "Wind Shielding Between Conductors of Telegraph and Telephone Lines" by P. J. Howe, in the January, 1923, A. I. E. E. JOURNAL.

PRINTING TELEGRAPHY

In addition to increasing the number of its multiplex printing telegraph circuits, the Western Union Telegraph Company has put into service a number of "forked-series" circuits of this class. In the latter circuits, one or more of the multiplex channels are dropped at, or extended to other places from intermediate stations along the route of the main circuit. For example, a New York-San Francisco multiplex circuit has one of its channels dropped at Los Angeles, thus giving the latter city direct and separate means of communication with New York and with San Francisco on a circuit previously used only for traffic from San Francisco to New York and vice versa.

Because of the constant increase in the number of securities listed by the New York Stock Exchange and the growth in the volume of sales made through that exchange, it has become desirable to operate at higher speed than in the past, certain groups of tickers which record stock transactions. This has been satisfactorily accomplished by substituting tape transmission for the directly connected keyboard transmitters formerly used. The tape is prepared for transmission by means of keyboard perforators which can be operated at high speed by skilled operators who are no longer limited in their manipulation of the keyboard by considerations of the varying line time required for the transmission of different characters.

RAILROAD TELEGRAPHY AND TELEPHONY

The importance of adequate and reliable communication facilities on the railroads is appreciated by those in charge of railroad operation, probably more than ever before. This has led to the adoption of greater mileage of telephone despatching and telephone message circuits, of greater mileage of trunk line facilities equipped with loading coils and telephone repeaters, of interest in telegraph printers for certain long distance telegraph business, of stronger open wire construction, and of proper installation of aerial and underground cable where conditions warrant it.

The use of radio on passenger trains, principally for receiving material from broadcasting stations for the entertainment of the passengers, has been experimented with by a number of railroads during the year. In one or two cases, railroads have made use of radio in emergency conditions, for locating trains, and for ascertaining the condition of the lines in connection with damage by severe storms.

MILITARY SIGNALING

During the year the Signal Corps have brought out for the air service of the Army three types of radio transmitters for aircraft use. The smallest is a telephone set used only between planes in formation flying, the maximum range being about five miles. The others are more powerful telephone and telegraph sets with rated telephone ranges between plane and ground of 30 and 250 miles respectively. All are designed on the master oscillator-power amplifier system, to assure constant frequency with changing antenna characteristics in flight. The receivers of all the sets are identical, consisting of single circuit tuners and "superheterodyne" amplifiers of six stages.

Until this year, radio communication for infantry units smaller than a regiment had not been feasible. This year has seen the production, however, of a telegraph set for battalions employing a fixed loop of one turn with an area of less than 1 square meter for both transmission and reception over a distance of 4 miles and affording 10 working waves between 74 and 76 meters. The weight including storage batteries for 8 hours operation, is 78 lb. and is divided between two men for transportation in limited field movements. A break-in feature permits the receiving operator to break the transmitting operator at will and every operator in the net is cognizant of the readiness of all other operators in the net without the necessity for calling them. The development of this equipment serves a long felt need in a field where wire lines are ordinarily maintained in action only at tremendous sacrifice of personnel.

The Signal Corps has in operation a radio network extending over the country. This net consists of twelve primary stations, located at the administrative headquarters and other important points, supplemented by about sixty secondary stations, located at posts, camps and stations.

EDUCATION IN COMMUNICATION ENGINEERING

There has been a considerable development of interest in the technical school in communication engineering. This is due to the large demand for graduates of technical schools by telephone and telegraph operating and manufacturing companies, to the stimulating effect of the important part which communication played in the world war, and to the growth of radio telephone broadcasting.

Elective communication courses are generally offered in electrical engineering schools, and in some cases communication courses are required. Many of the schools, however, have not appreciated the value of properly arranged communication courses, in teaching the fundamentals of electromagnetic action which underlie not only communication engineering, but all electrical engineering.

Graduate work in communication engineering, and courses in physics especially adapted for communication students, are given in a number of the colleges. Signal Units of the Reserve Officers' Training Corps have been installed in eleven of the technical schools. Research work in electrical communication is being carried out in a number of technical schools. There is a considerable demand for students with extra training in mathematics and physics to enter research and development work in the larger communication companies.

The communication companies, because of the highly technical nature of the work required of their men, are spending considerable effort in setting up company schools, lecture courses, etc., for training their employees.

FINE WIRE TELEPHONE CABLE

Advances in the art are permitting the use of much smaller wires in cable, particularly for the shorter subscribers' cables. Cables in all sizes from 1200 pair down, of 24 A. w. g. gage are coming into very extensive use.

JOINT CHICAGO AND NEW YORK A. I. E. E. MEETING

An interesting demonstration was given during the Midwinter Convention of the possibilities of present-day telephony, when simultaneous meetings were held in Chicago and New York, and the two audiences were made effectively one in being connected by telephone lines and loud speakers. Addresses, papers and discussions given in part at each place were heard by all, as well as though the complete audience had been at a single point.

The above report has been prepared from material contributed by the following members of the committee: G. A. Cellar, H. P. Charlesworth, H. W. Drake, L. F. Morehouse, C. W. Rice, J. K. Roosevelt, General Edgar Russell, A. B. Smith, and C. A. Wright.

OTTO B. BLACKWELL, *Chairman.*

REPORT OF THE EDUCATIONAL COMMITTEE

To the Board of Directors:

The past year has seen an exceptional gain in interest in the problems of engineering education on the part of the profession at large. Significant movements have been inaugurated both in the colleges and in outside circles looking toward better coordination between preparatory training and professional education on one hand, and between the latter and the industrial demand on the other. Specific changes in educational practise growing out of these movements have not yet appeared, but may be anticipated.

CHANGE IN CURRICULUM

The deans of fourteen engineering colleges in the middle west have held two conferences on changes in curriculum which have culminated in the following resolutions:

"In order to meet the constantly enlarging responsibilities of the engineering profession, we favor an advance in engineering education at this time that shall provide five years of collegiate training for those engineering students whose aim is to become qualified to take positions among the creative leaders in the profession, and that such advance shall be made in substantial accordance with the following plan:

1. Include in the four-year engineering curricula a substantial proportion of fundamental and humanistic subjects, omitting, if necessary, a sufficient amount of the more advanced technical work. It is desirable that, so far as possible the curricula in the different branches of engineering shall be sufficiently uniform to permit students to defer their final choice of a specialty at least to the end of the second year.

2. Add a fifth year of advanced work, mostly or

wholly technical, and specialized to such an extent as desired.

3. The first four years of work shall lead to a bachelor's degree and the fifth year to an advance degree in engineering."

Among other considerations urged in support of these changes is the necessity of upgrading many of the applicants for admission to state institutions from certificated high schools which do not give a sufficient mathematical preparation for standard engineering courses.

This subject of the proper length and scope of engineering courses is being vigorously debated. Three parties can be identified easily: First, those who believe in the efficacy of the present standard four-year courses and who hold that whatever additional time is available should be given to graduate studies and research in the engineering field; second, those who believe that engineering training should be strictly professional in its methods and scope and that it should be preceded by two, three, or four years of work in the liberal arts or general sciences; and third, the group whose views are embodied in the resolutions quoted above.

THE COOPERATIVE PLAN

There is a slow but steady growth in the application of the cooperative plan of alternate periods of instruction and employment. The prototype of such courses has been in operation at the University of Cincinnati since 1906. Recent evolution tends to produce variants of the original plan. At Massachusetts Institute of Technology cooperation is carried on only with specific industries engaged in manufacturing, electric railway operation and electric power supply, and with programs of practical experience organized on lines as definite as the curriculum of studies. At Worcester Polytechnic Institute students in electrical engineering are offered the option of engaging in practical work for a continuous period of fifteen months between the junior and senior years.

The pedagogic merits of the cooperative plan of coordinated instruction and experience are of great interest but the possibilities presented by this plan for placing education on a plane of approximate self-support for both students and institutions are worthy of especial consideration. Demand for admission to colleges is rising rapidly and tends to outrun existing plant and teaching facilities. It is a serious question whether a great enlargement of facilities can be financed through private endowment or public support. This question bears a particularly serious relation to engineering education because of its high overhead and operating costs.

THE SUPPLY OF GRADUATES AND THE PROFESSIONAL DEMAND

Attendance at engineering colleges passed the post-war peak in 1922 and has since declined appreciably. The peak with respect to numbers graduating occurs this year. In the past fifteen years the electrical industries of America have nearly quadrupled in magnitude while the number of men graduating each year in electrical engineering has advanced only about one fourth. With the prospect of a decline from the present peak the question of the adequacy of the supply of young engineers begins to assume serious proportions. Owing to the youth of the profession accessions to the ranks of electrical engineers during the past fifteen or twenty years have been almost entirely cumulative. With the rate of loss from death, superannuation and diversion to other fields rising and with the industry ever advancing in magnitude and complexity a stationary rate of accessions is certain to leave a serious deficit in the relation of supply to demand.

MOVEMENTS FOR INVESTIGATION AND DEVELOPMENT

Within the past year the National Industrial Conference Board has established a Joint Conference Committee on Engineering Education with a like committee from the Society for the Promotion of Engineering Education. The scope of the activities projected by these joint committees is well expressed by the following excerpts from an official statement.

"It was emphasized as a primary consideration that American industry's efficient development and contribution to the national welfare is closely bound up with the character and efficiency of the professional engineers educated in the various engineering schools, and required in industry as technical experts and inventors, as managers and executives of industrial enterprises. Equally important is the development of competent men as instructors in the engineering schools.

For these reasons and also because engineering schools are, after all, a part of the industrial system, and must therefore function efficiently if the system as a whole is to be efficient, it was agreed that there must be a close and cooperative relationship between engineering schools and American industry in its various branches; and adequate methods must be developed under which each will help the other in the common cause of adequately training professional engineers.

The Committee is not concerned with details of college curricula, but with the broader aspects of the problem, such as the best way of fitting the graduate of the engineering schools into industry; whether and how best to give the student industrial training coincident with his academic studies; an adequate supply of high-grade instructors and a better understanding of their responsibilities by both industry and the engineering schools."

The Society for the Promotion of Engineering Education has further created from its own membership a Board of Investigation and Coordination whose objects are stated to be:

"1. To ascertain the facts in engineering education, such as concerns (a) teachers, their origin, training, experience and effectiveness; (b) teaching facilities; (c) curricula; (d) students and graduates, their origin, training, experience and effectiveness.

2. To ascertain present and future requirements in the fields served by engineer graduates.

3. To present the facts and requirements for their bearing on the training of the engineer to the end that he may (a) develop himself and his profession; (b) realize and fulfill his obligation to society.

4. To maintain close contact with engineering schools enabling them to participate in the investigations; and reporting to them from time to time; the end that the developments may be continuous from the initial contact between the colleges and the agencies of the Board.

5. To secure the necessary funds for these purposes."

It is proposed to place a salaried director, with adequate staff, at the head of this Board and to create a Board of Counsellors representing the several industrial fields to which engineering education is related.

A movement of such scope and aims is of profound interest to the Institute and its sister professional societies. It is important that there should not be too many agencies of investigation and promotion working in an uncoordinated manner. It is also important that due weight should be given to the views and interests of the engineering professions as well as those of the colleges and the industries.

The importance of gaining the cooperation of the major engineering societies has been recognized by the Society for the Promotion of Engineering Education in its appointment of a committee for this express purpose.

It is the sense of the Educational Committee that the Institute should indicate its readiness to participate actively in any broad movement concerned with the development of engineering education. In view of the establishment of the Joint Conference Committee referred to above and the active steps already taken toward setting up its program, it is the feeling of this Committee that the Institute should await some indication of the details and order of this program before initiating active measures on its own account and that the Institute should act as far as practicable in concert with its sister professional societies.

WM. E. WICKENDEN, *Chairman.*

REPORT OF COMMITTEE ON ELECTROCHEMISTRY AND ELECTROMETALLURGY

To the Board of Directors:

This committee has been somewhat more active during the past year than for several years previously. The situation covering past and prospective activities was discussed at the beginning by correspondence with the various members of the committee. Four or five of the seven members have shown an active interest in the work. No regular committee meetings have been held as there was no special occasion therefore. It was intended to hold a meeting at the Pittsburgh Convention but only two of the committee members were in attendance.

The particular activity of the committee for the year was the preparation for and conduct of a session on Electric Furnaces which was held under its auspices at the Spring Convention. In addition efforts have been made through committee members to obtain papers of Institute caliber along electrochemical and electrometallurgical lines and several such papers are now in course of preparation so that it is hoped the committee will be in position next year to request a session for papers devoted to such subjects.

It would appear that the proper function of this committee should be that of a technical liaison between the A. I. E. E. and the American Institute of Mining and Metallurgical Engineers and the American Electrochemical Society. We should observe the technical activities of those societies and report with the correct Electrical Engineering slant those matters in which A. I. E. E. members should be interested. Any technical subjects which are investigated jointly by one of these societies and the A. I. E. E. should be under the direction of this committee for the A. I. E. E.

At the present time it seems that the most serviceable line of action for this committee lies in stating from time to time the status of the art and advancements in the art with particular reference to the power supply and the electrical equipment applications and such features of the chemical and metallurgical processes as directly affect the power and equipment. It is hoped that the next session under the auspices of this committee will have papers on the general subject of Electrical Equipment and Power Requirements for Electro-chemical and Electro-Metallurgical Service.

Since the apparatus and applications thereof in which this committee's activities are associated are so diverse that the whole field could not be covered in a brief resumé and since there do not appear to be any outstanding advances which have not been more specifically within the sphere of some other technical committee to report, no attempt is being made at this time to report the progress of the art.

J. L. McK. YARDLEY, *Chairman.*

Electrical Loud Speakers

BY A. NYMAN
Associate, A. I. E. E.
Westinghouse Elec. and Mfg. Co.

Review of the Subject.—*An electrical loud speaker is a device applied to radiophone reproduction of speech and music.*

This paper brings out the mechanical and electrical essentials of this device and gives a short description of present-day forms and methods used in developing and testing new types.

A technical analysis of one successful form brings out the various factors involved and the means used for achieving satisfactory results. These results are illustrated by appropriate test curves.

The paper is a record of results obtained through technical analysis of this apparatus and is intended to direct attention to the necessity of similar analysis continued and extended.

THE popularity of radiophone broadcasting has created a considerable demand for an electrical loud speaker. Electrical loud speakers have been used for a number of years for such purposes as train announcing. However, this type of loud speaker was developed primarily for speech reproduction and as long as it possessed a certain degree of clearness, was considered satisfactory.

The major part of radiophone broadcasting is music and the aesthetic value of music is its vital element. Therefore, music reproduction should be so close to the original as to maintain its aesthetic value.

It has been found that music reproduction requires the presence of notes ranging in frequency from 25 cycles per second to 5000 cycles per second. The quality of reproduction is affected to a large extent by the loudness of individual frequencies; hence, the necessity of bringing in each frequency at a value proportional to the original volume. It can readily be seen that the quality of the pick-up instrument or microphone, as well as the design of the transmitting and receiving systems is of utmost importance.

Apart from the pick-up and transmission, the following essential features pertain to the loud speaker alone:

1. Uniform intensity of sound at all frequencies from 25 cycles to 5000 cycles.

2. Absence of resonance points capable of responding at a frequency different from applied or giving an excessive volume of sound when their frequency is applied.

3. The ability to reproduce a combination of frequencies with a volume of each frequency proportional to the input.

4. Absence of distorting harmonics at any individual frequency applied.

Feature 1 is particularly important in reproducing every kind of sound; for example: A weak or missing range of frequency is noticeable even to an untrained ear. However, if it is near either end of total range, *i.e.*, below 400 or above 3000, an untrained ear may sometimes fail to detect this defect. Similarly an individual missing frequency can be occasionally overlooked. A loud range distorts the quality to a considerable extent and a loud individual note has a very unpleasant blasting effect.

Feature 2, if overlooked is particularly liable to give blasting or an unnatural ring of certain notes. The fundamental may be suppressed and a harmonic of an altogether different pitch come through, possibly considerably louder than the applied note.

Feature 3, dealing with combination of frequencies, is particularly noticeable in speech reproduction. Normal vowel sounds consist of a fundamental of rather small volume and harmonics often much larger than the fundamental. Unless the proportionality is maintained the sound of the voice changes giving the impression of a changed pitch; tenor voice may sound like bass; soprano like contralto, or vice versa. The higher harmonics again determine the individual characteristics of the voice. Thus, in order to recognize a person's voice, the higher harmonics up to the 20th or 30th must be included and kept at their proportional value. What is true of voice is true of most musical instruments. The pitch, or the individuality may appear changed unless a proportionality is maintained.

Feature 4: Certain materials have qualities which give them peculiar forms of vibration. Thus, the vibrations of brass are usually different from aluminum, wood or micarta. This is generally due to a number of harmonics, each modifying the original note. In a loud speaker the pleasing quality and the naturalness of reproduction are dependent to a very great extent on the choice of materials, particularly of the material carrying a large amount of energy of sound.

STRUCTURES OF LOUD SPEAKERS

A number of structures have been used successfully as loud speakers.

Fig. 1 shows a loud speaker operating on the same principle as an ordinary telephone receiver. It has a thin iron diaphragm held at a small distance from two magnetic pole pieces, which are energized by a permanent magnet and also by two coils, one on each pole piece. The volume that can be obtained from this type of loud speaker is somewhat limited on account of the close spacing between the diaphragm and the pole pieces. Moreover, certain notes are accentuated, due to the resonance of the diaphragm.

Fig. 2 shows a moving coil type of loud speaker. A

Presented at the Annual Convention of the A. I. E. E. Swampscott, Mass., June 26-29, 1923.

circular coil is located in a round air-gap, with an iron core in the center. This air-gap is traversed by a strong magnetic field, excited by an inner coil which carries direct current while the circular coil mentioned above carries sound-producing alternating current and is attached to the center of a diaphragm. Very satisfactory results can be obtained with this type of loud speaker.

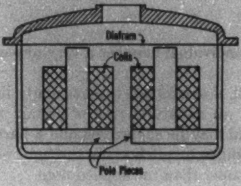

Fig. 1

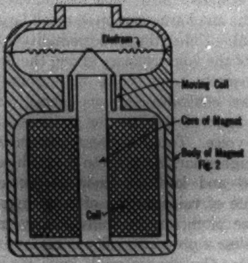

Fig. 2

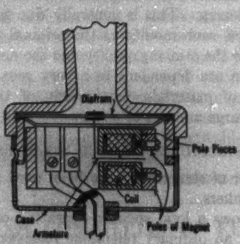

Fig. 3

Fig. 3 shows what may be termed the enclosed armature type. A small iron armature is located in the center of a coil and suspended by two thin piano wires. The coil is surrounded by two U-shaped pole pieces, forming two air-gaps. A permanent magnet produces magnetic flux in these air-gaps. The current in the coil causes diametrically opposite pole pieces to be energized simultaneously, which causes the armature to rock. This rocking is communicated through a thin connecting rod to the center of a diaphragm.

Fig. 4 shows the sound distribution for a loud speaker constructed on this principle. The curve is taken by a method described below. The loudness is fairly uniform over the range. The curve shows the frequency from 100 cycles to 10,000 cycles for abscissa and loudness for ordinate.

Fig. 5 shows the "relay type" loud speaker recently developed. Its construction is similar to that of a polarized telegraph relay. A thin iron armature is located between four pole pieces, each carrying a coil. These pole pieces are magnetized by an L-shaped magnet and the coils are connected in such a manner that diametrically opposite pole pieces exert simultaneous attraction. The armature operates through a rod on a corrugated aluminum diaphragm.

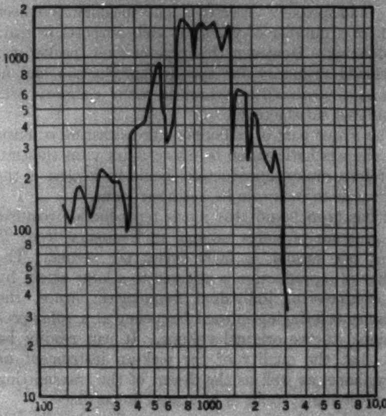

Fig. 4

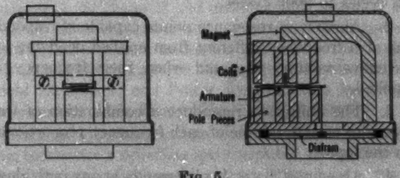

Fig. 5

Fig. 6 gives a representative curve of this loud speaker. The range is fairly wide, while no part of it is exaggerated in volume.

METHODS OF TESTING

The four essential features of the loud speaker have been investigated by different test methods, partly dynamical and partly physiological; *i.e.*, depending on aural observations.

Fig. 7 shows a diagram of an oscillator designed to cover a range from 150 to 10,000 cycles. A number of steps of condenser capacity raise the frequency about 50 to 100 per cent while for each step the movement of an inductive coil on and off an iron core gives gradual variations of frequency. Each step of condenser is calibrated for frequency at different coil settings which are indicated on a graduated scale. The coil acts as an inductance and also as a transformer. Operating the set at 20 watts, the amount of power drawn to the loud speaker is small giving good voltage and frequency regulation on load. In order to make the loud speaker circuit equivalent to a tube circuit, a resistance equal to the tube impedance is included in series with the loud speaker. Although the voltage on the oscillator remains fairly constant throughout the whole range, for quantitative measurements, the voltage can be checked at each reading.

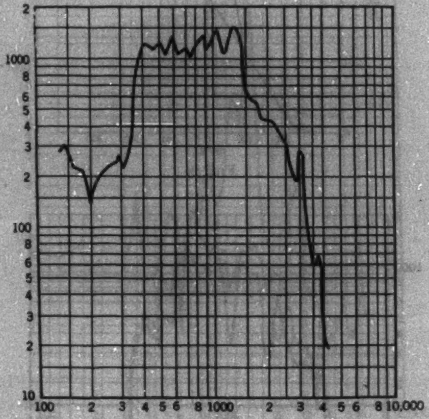

FIG. 6

Fig. 8 shows the pick-up arrangement for measuring the sound from a loud speaker. A condenser transmitter pick-up is considered very close to the ideal sound-receiving instrument and has been used by many investigators as a sound standard. The pick-up of this transmitter is amplified through a resistance amplifier precluding distortion and the resultant current measured on a milliammeter. The last stage containing a step-down transformer, is also used for checking the voltage at each frequency. Hence, any possibility of reduction of received current at low frequencies is balanced by a corresponding reduction of the measured value of voltage.

Sound volume tests were conducted as follows:

The oscillator was operated through the complete range at fairly constant voltage while measurements of sound by condenser transmitter were recorded and corrected by the value of voltage measured at each frequency. This arrangement gives a complete cycle from current to current and is evidently equivalent to the cycle from sound to sound. In addition, a point is obtained at 60 cycles to determine the loudness of very low notes.

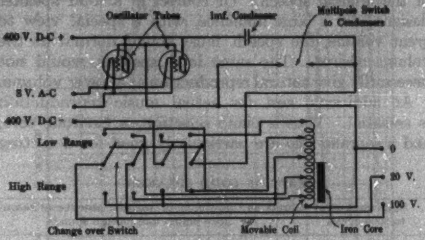

FIG. 7

The above test gives valuable data for investigation of the uniformity of sound and of the absence of resonance points. Listening to the sound, while performing this test, makes it possible to detect any foreign noise, rattle, or sound at a diffferent frequency from that applied.

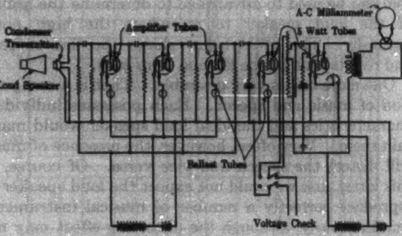

FIG. 8

The ability to reproduce accurately any kind of musical sound or speech can be tested best by actual music and speech reproduction. Again a condenser transmitter has been used for the pick-up of sound. A number of stages of amplification (resistance coupled) bring the current to the loud speaker while an audibility meter is so arranged that the volume can be cut down to any suitable loudness. Each note on the piano repeated several times is one of the best means of detecting any disturbing harmonics. Each note should come through clear and similar to the original piano note. Low notes, in particular, should be checked for the presence of the fundamental tone. Some designs of loud speakers while giving a loud note at these pitches are found to be completely devoid of the fundamental—the note is just the sum of all overtones.

Speech transmission over the same circuit gives a splendid test for quality and recognizability of reproduction. For proper speech reproduction, the volume should be adjusted to equal approximately the loudness of the original speech. Of course, in a loud speaker designed for a large audience, with a special view to great volume, the speech must sound normal at the volume desired. The same loud speaker would not necessarily give natural reproduction at a lower volume.

An additional test for actual music reproduction is essential. Thus a piano selection, a baritone solo, and a soprano solo are particularly good for detecting any faults in quality. In addition a violin or a flute solo can be used to advantage to determine the ability of the loud speaker to reproduce the high notes naturally. The table below shows the list of tests and results that can be learned from each.

TESTS OF LOUD SPEAKER

1. Measurement of volume 60 to 5000 cycles............	Uniformity of volume, absence of resonance points and foreign sounds
2. Musical scales on piano.....	Accurate reproduction of quality on each note, particularly the low note.
3. Speech..................	Clearness of articulation. Individuality of voice.
4. Piano Selection...........	Clearness and naturalness on abrupt tones
5. Baritone and soprano singing	Clearness and naturalness of sustained notes.
6. Flute or violin............	Reproduction of high notes.
7. Speech and Music........	Naturalness of superimposed sounds.

One more test is very desirable. That is a combination of music and speech. Each possesses individual characteristics and the ideal loud speaker would maintain them. Very often, however, the presence of music will distort the speech and vice versa. Of course, in this latter case we could not expect the loud speaker to reproduce correctly a number of musical instruments simultaneously, although the distorting effect may not be as noticeable as in the case of speech and music.

RESONATING SYSTEMS

Considered from a mechanical standpoint a loud speaker is invariably a complicated resonance system. Certain subdivisions of resonance, are, however, possible.

1. *Mechanism as a whole.* In most loud speakers, the diaphragm is the surface producing sound. The force of a magnetic field is in all types except the moving coil type counteracted by a strain in the diaphragm. In a loud speaker of the type shown in Fig. 1, that is, the telephone receiver type, this action is automatic. The diaphragm pulls down until its tension is equal to the pull of magnetic field. In the type shown in Figs. 3, 4 and 5, the normal position of armature is such that the magnetic pull is zero. Actually, however, it is very difficult to keep the armature in this position. Generally there is a little pull one way or another, balanced by the strain in the diaphragm.

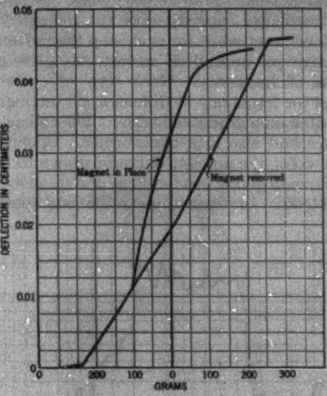

FIG. 9

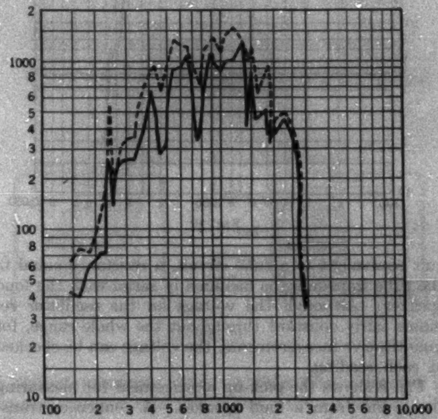

FIG. 10

For a movement of the diaphragm, the magnetic field begins to exert a force helping this movement. If the magnetism is increased by using a stronger magnet, the force of the magnet may be made so large that it pulls the diaphragm over. Normally, a balanced condition may be obtained where very little force is required to produce a certain movement. Fig. 9 illustrates this fact. The two curves show the variation of force on the diaphragm with movement of the diaphragm, and show that with the magnet the diaphragm requires distinctly smaller force for the same movement.

In this way, the strength of the magnet and the tension of the diaphragm determine the force for certain movements and, consequently, the resonant frequency of the whole mechanism. By adjusting the magnetism in a way to get a very close balance, this resonant frequency may be placed very low. As a rule, the damping at these low frequencies is high enough to conceal the resonance; however, the whole of the low range will be found raised. This is demonstrated on Fig. 10, showing two curves for one loud speaker, one with a 0.015-in. gap and the other with a 0.010-in. gap. The latter had a close magnetic balance; hence, all notes and the low notes in particular are increased. These curves were taken on relay type loud speaker. It is evident that similar adjustment is possible on all types except the moving coil type; in the latter the resonance point is determined entirely by mechanical strain and the mass of moving parts.

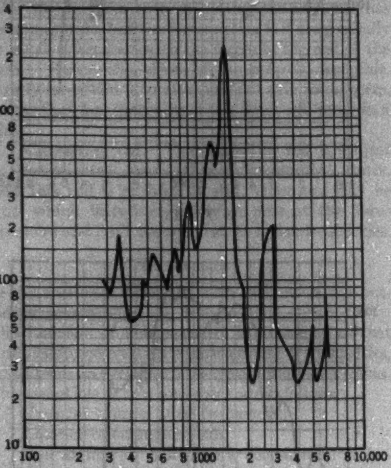

Fig. 11

2. *Diaphragm and Horn.* The nature of the diaphragm as a vibrating structure is such that it has a number of resonant notes. Fig. 11, taken on a very stiff small diaphragm, illustrates these resonance points. The lower resonant points can be attributed to the effect of the horn. 1500 cycles is the fundamental of the diaphragm with harmonics at 2900, 5000 and 6000 cycles and higher. The ratios are not exactly 1:2:3:4, but modified by the presence of horn and the nature of the diaphragm.

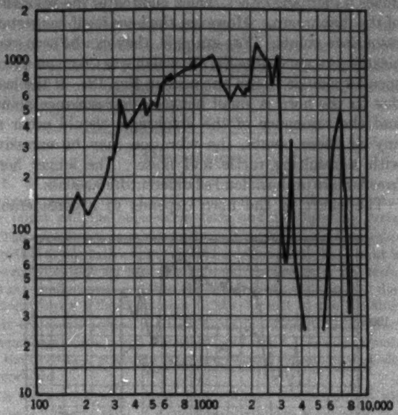

Fig. 12

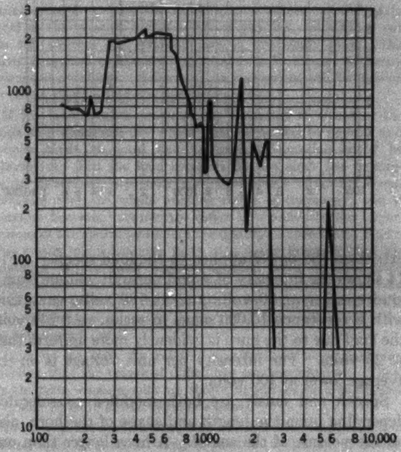

Fig. 13

Fig. 12 shows the resonance in the diaphragm corrugated to reduce the effect of harmonics.

Fig. 13 shows the same kind of curve for a flat diaphragm without corrugation. The resonance points are now more pronounced.

Both these curves were taken without a horn and show the diaphragm resonance only.

The effect of the horn is not independent of the qualities of the diaphragm, because the horn constitutes a load on the diaphragm. If this load is large, it produces a damping effect, which eliminates the resonance of the diaphragm. However, the horn itself has certain resonance points. For example, they can be seen very clearly on a curve like Fig. 10 which was taken on a loud speaker similar to the one used in Fig. 12. If the load due to the horn is small between its resonance points and the resonance of the diaphragm should occur at any of these points, the vibration may be excessive with a resulting rattle and noise. The longer horn favors more uniform load at different frequencies.

The effect of the horn depends to a considerable extent on the power applied to the loud speaker. Fig. 14 shows the intensity of sound per unit voltage from a loud speaker at 20 volts and at 100 volts, each time with a resistance 10,000 ohms in series. At 100 volts the load due to the horn is evidently more uniform than at 20 volts. The intensity per unit voltage is on the average lower at 100 volts.

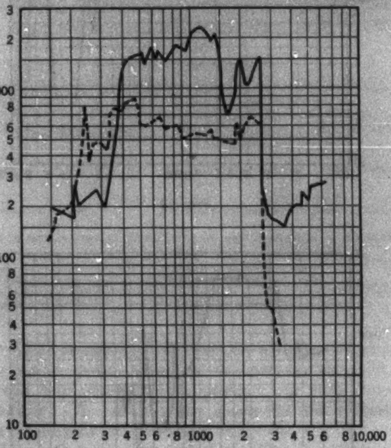

FIG. 14

Speech and music are both modified considerably depending upon the length and shape of the horn, also on the volume of the sound. A horn longer than one quarter wave length of the lowest pitch available gives the best reproduction. However, in practise the length of the horn seldom exceeds three feet, approximately one-fourth of the wave length of 90 cycles, the fundamental of the horn. If the horn is shorter than one foot (270 cycles fundamental the bass and baritone voices are likely to be distorted, since their fundamental, which is below 270 cycles would be reduced. It has been found that a loud speaker with a magnetic balance and a horn about two feet long is capable of very good reproduction of even very low frequencies.

Careful study has been made of materials to be used in diaphragm and in the horn in so far as it affects the quality of reproduction. Aluminum or micarta diaphragms apparently give the best results while a wood horn or horn made of some "dead" material like hard rubber, is least likely to introduce a strange quality.

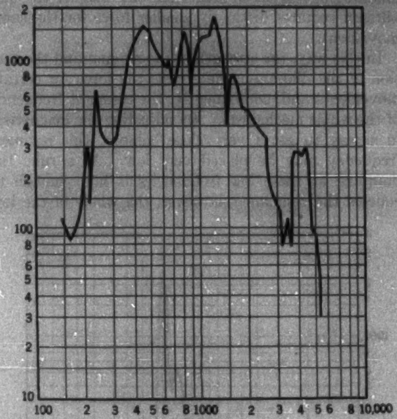

FIG. 15

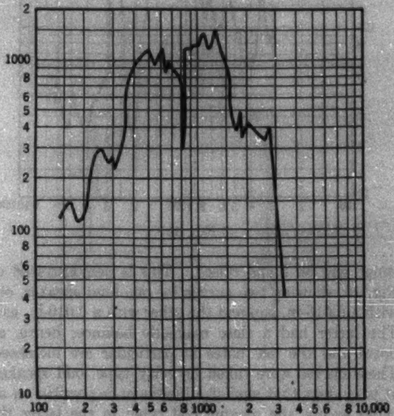

FIG. 16

III. *Armature.* The armature of a loud speaker of enclosed armature or relay type is a strip of steel very short and stiff, but nevertheless possessing a resonance point within the audible range. It has been found that frequencies above this resonance point are difficult to reproduce. Thus, Fig. 15, taken on a loud speaker with a very small stiff armature, shows a range on higher notes extending to 5000 cycles.

Another effect of resonance of armature is the introduction of foreign notes. An example is seen in the Fig. 16 showing a reduction of volume at 800 cycles. At this frequency it was observed that the note had a strange high pitched harmonic; however, damping the armature by a piece of rubber cleared this note and brought up the volume of its fundamental. The trouble was eventually overcome by using a much stiffer armature.

IV. *Strip.* The strip supporting the armature has a resonance note, but the forces acting in it are generally very small compared with the forces in the rest of the system. Hence the effect of the strip is negligible. The only exception is in attaching the strip to supports. It seems that any looseness at this point will result in a rattle.

V. *Connecting rod.* The connecting rod is subject to a complicated torsional and longitudinal strain. Unless this rod is sufficiently stiff, vibrations may be set up which introduce a foreign note at the lower frequencies and limit the sound at the higher frequencies.

CONCLUSION

In conclusion a brief summary will cover the outstanding points.

The function of loud speakers is considered as that of a device for converting electric current of frequencies, ranging from 25 cycles to 5000 cycles, into sound waves.

The essentials of this conversion are as follows:

1. Uniform volume at all frequencies.
2. Absence of strange sounds.
3. The ability to reproduce a combination of frequencies correctly.

Four fundamental types of loud speakers are discussed:

1. Receiver type.
2. Moving coil type.
3. Enclosed armature type.
4. Relay type.

Test methods are outlined for:

1. Measuring the volume of sound.
2. Testing the quality of reproduction.

The effect of various parts of a loud speaker on its operation are considered, viz:

1. The magnetic structure.
2. The diaphragm.
3. The horn.
4. The details.

The art of designing a loud speaker is extremely new. The empirical work for ascertaining the effect of various factors is only in its embryo stage. Eventually, we may expect to design a horn or a vibrating structure with the same facility as an electric motor because a loud speaker is really an electric motor, though its load is less tangible than the load of most motors.

The design of a loud speaker must be based on a scientific analysis of this load and of its reaction on the motor. This involves considerable acoustic research work, mechanical research on vibrating structures and electrical work on the effects of vibrating parts in an electromagnetic structure.

Discussion

E. W. Kellogg: Mr. Nyman has mentioned a series of tests of loud speakers which I think may well be classified into what we might call the engineering tests and the listening tests.

The engineering tests are made with pure sine wave current supplied to the instrument and are to determine, first, what is the sound output per watt input for all frequencies in the useful range; and second, whether the force which acts on the diaphragm bears a linear relation to the current in the winding.

If the sound output per watt input is different for different frequencies, wave form distortion will result, because certain of the overtones present will be exaggerated and others partially suppressed.

If the force-current relation is non linear, the instrument will cause a different type of wave form distortion, producing harmonics which were not in the original.

The test for linear relation between current and force may be carried out by supplying the instrument with a sine wave current strong enough to give the diaphragm a maximum deflection equal to the greatest it will attain in practise. A trained ear can tell whether the resulting tone is practically pure. The current supplied to the instrument should be filtered to insure absence of overtones. Otherwise the instrument will be blamed for faults which are in the source of current.

If the loud speaker produced no distortion of either of the kinds just described, the other tests—listening to speech and various kinds of vocal and instrumental music—would be superfluous. But the loud speakers do not meet the requirements mentioned, especially the requirement of uniform sensitiveness over the whole working range of frequency, and it therefore becomes necessary to judge how seriously the imperfections of the instrument affect the quality of speech and music. This means many listening tests with many listeners, and the balancing of one fault against another, with an integrated appraisal of the various instruments compared. The loud speaker will be sold on the basis of the listening tests but it will be developed and improved through the information gained by the engineering tests.

It is in the tests for relative sensitiveness at various frequencies that the principal difficulties are encountered. These are the tests whose results are shown by curves such as Figures 4, 6, and 10 to 16 of Mr. Nyman's paper.

In view of their importance and the fact that this method of testing loud speakers is being applied in a number of places, I should like to see an interchange of ideas and experiences, and if possible an agreement on test conditions and methods of presenting the results, which will make the results obtained in different laboratories directly comparable.

Without pretending to be especially in a position to lay down standardization rules in regard to tests of this kind, I think that thought on the subject may be promoted and accelerated by my attempting to do so, because it will show what difficulties or possible sources of error have occurred to me and tentative ways of avoiding them. Others who have had more experience than I have, can perhaps rule out some of these precautions as unnecessary and will probably add others that I have overlooked.

In order to get the proposition into concrete form I am going to suggest a tentative set of test conditions, the general method of testing to be as described in Mr. Nyman's paper, but the following conditions imposed:

(1) The oscillator which provides the current for testing should be provided with a filter to eliminate overtones.

(2) The amplifier between the electrostatic transmitter and the galvanometer should be provided with a tuning, or else a filter so that only the fundamental frequency will affect the galvanometer.

(3) The test room to be at least 15 feet by 15 feet, 10 feet high, with its ceiling and three of its walls heavily damped and the remaining wall hard.

(4) The loud speaker to face the middle of the hard wall, 6 feet above the floor and 6 feet from the wall to the center of the bell of the horn.

(5) The electrostatic transmitter to be located in a hole in the wall, directly in front of the loud speaker, and the floor between the loud speaker and the transmitter to be padded with damping material.

(6) The sound pressures recorded to be divided by 2 to reduce the results to equivalent free space pressure.

(7) Results to be given in dynes per square centimeter sound pressure per volt-ampere supplied to the instrument, accompanied by a statement of the instrument impedance and the maximum peak value of voltage or current which it can handle without distortion.

(8) Departure from standard test conditions to be stated.

Let me now give my reasons for some of the provisions named:

First, the oscillator which provides the current for testing should be provided with a filter to eliminate overtones. Let us take the case of very low tones. You noticed in the curves given in Mr. Nyman's paper that the sensitiveness of practically all of the instruments tested was at least 10 times as great for tones above 500 cycles as for tones below 150 cycles. Suppose we supply the instrument with an electric current of 100 cycles, and then measure the sound output by means of the electrostatic transmitter with amplifiers as indicated in Fig. 8 of the paper. If there are any overtones present in the electric current, those overtones may be reproduced so strongly that they will count very much more in the galvanometer reading than the fundamental, whereas we are only interested in the fundamental. Even if the current supplied to the loud speaker is a pure sine wave, the instrument itself may generate overtones. This will occur if the current-forec relation is non linear. The overtones thus generated may be more strongly radiated than the fundamental, and since the purpose of the test is to determine not simply how much noise is produced but how much sound of fundamental frequency is radiated, I have called for a filter or tuning between the electrostatic transmitter and the galvanometer. A vibration galvanometer is an alternative.

Next in regard to the size of the test room: The arrangement I have suggested places the loud speaker fairly close to the electrostatic transmitter, and the other walls farther away. The instrument gives out sound in all directions, and even where we have done the best we can at damping, there will be reflections of a substantial fraction of the sound that strikes the side and back walls. The reflected waves will interfere, producing at certain wave lengths subtractions and at other wave lengths additions to the sound coming directly from the loud speaker. The intensity of the interfering waves will be much reduced by placing the walls that are producing these objectionable reflections well away from the loud speaker. This means using a fairly large room, as well as damping the walls.

Why use a hard wall and locate the electrostatic transmitter in the middle of it, in a hole in the wall? I mentioned that hole in the wall construction because I am not sure that all the electrostatic transmitters that would be used are constructed so that the air pressure cannot equalize on the front and the back of the diaphragm for the longest waves. In most of them, the back is housed in pretty well, perhaps well enough for all practical purposes, so that there will not be pressure equalization. But guarding against any possibility of it, suppose we put the instrument in a hole in the wall that fits it tightly. In some testing arrangements I believe the electrostatic transmitter has been placed some distance from the wall. This means that the sound waves given out strike the wall behind the electrostatic transmitter and come back to produce interference effects. If we place the electrostatic transmitter close to the wall so that it is at a pressure loop and motion node for all wave lengths—then we will get a pressure doubling for all waves, provided the wall is perfectly reflecting.

Next is the question of the distance of the electostatic transmitter from the loud speaker. I mentioned six feet because I think that represents something like the distance that listeners would sit from a loud speaker in a home. I would place the transmitter closer to the loud speaker than six feet, except for the fact that I have some reason to believe there is a pressure variation very close to the horn of a loud speaker which does not represent energy actually radiated. There is a circulation of air which may be described as simply surging back and forth. It is represented by a reaction on the diaphragm not of energy dissipation or energy radiation, but an inertia factor, and may be compared to the magnetic field around an antenna less than a quarter of a wave length from the antenna. You would not think of using such a field as a measure of the energy actually radiated by the antenna. You would go at least a wave length or two away. I should be very glad to hear from any one who can answer the question whether there is such an effect as I have mentioned. I tried to look it up but could not find any literature on the subject, except Rayleigh's formula for the reaction of air on a diaphragm ("Theory of Sound", Vol. II., page 165).

I also suggested that results be given in dynes per square centimeter sound pressure per volt-ampere supplied to the instrument, accompanied by a statement of the instrument impedance and the maximum peak value of voltage or current which it can handle without distortion. While the quality of the loud speaker is pretty much a question of *relative* amplitudes at different frequencies, if we could give the distance and have conditions equivalent to the radiation in free space, without any surrounding walls, and give them in terms of sound pressure per volt-ampere supplied the instrument, we should also have a measure of the sensitiveness of the instrument, which is desirable for purposes of comparison.

The last condition mentioned in my list is that departures from standard test conditions are to be stated—of course that might for some time to come mean stating all conditions, but I think it highly desirable that they be stated in detail.

A. E. Kennelly: The method of testing loud speakers which is presented to us in this interesting paper is a valuable one for getting rapid empirical results. It seems to me that much more definite and scientific information can be obtained through the medium of the motional impedance circle. But in any case, whether we use one method or another, we may consider that the ideal instrument is one which produces a linear relation between the pressure of the air at the transmitter diaphragm and the pressure in free air at a defined point in front of the loud-speaking receiver, that the ratio of the two would measure the efficiency of the system; and that it should be the same all over the acoustic range considered.

It is not reasonable to expect that any such relation can be obtained in the near future, and we can only hope to arrive at it by a succession of approximations.

An ordinary diaphragm has its resonant point or a series of resonant points. The horn has another; the transmitter has another; the electrical apparatus involved connecting them, may have another or a series of others. So it is only by suitably combining a series of such resonances that we can hope to produce conditions that will produce an equalization over the whole working acoustic range as far as possible. There is so much to be done in this direction, that we cannot expect perfection in the near future.

I think we have all been struck with the enormous advances that have been made in the last few years in the direction of frequency measurements, of repeater circuits, and of Atlantic radio transmissions, that it is evident one year produces at this favored time as much development as might have been expected in a dozen years only a little while ago.

W. H. Martin: Mr. Nyman's four requirements for a good loud speaker can, I think, be adequately expressed by two requirements; first, that the efficiency be constant over the frequency range and, second, that the device used produces no frequencies except those imposed upon it.

In going over this paper, I notice that the so-called "relay type" structure shown in Fig. 5, is described as a new development. Comparing the structure of Fig. 5 with the so-called "balanced armature" type of Fig. 3, it appears to me that the relay type is really a small modification of the balanced armature type. Magnetically the two are practically the same. The relay type appears to have the disadvantage, moreover, of having a more complicated winding system.

In the matter of obtaining a loud speaker which will be distortionless for the frequency and volume range required for reproducing speech and music, it seems to me that the important problem is not so much that of devising a new magnetic structure for converting from electrical power to mechanical motion; it is rather that of properly communicating this mechanical motion to the air. Indications are that there is a larger field for improvements in the vibratory and acoustical parts of the structure than in the electro-mechanical parts.

It would help in reading this paper if the author would state what the units of loudness are in which the curves of the paper are plotted. By loudness we might interpret these units to mean sound power, sound pressure, or, as is often meant by loudness in acoustical work, sensation in the ear. Since the second and third are respectively proportional to the square root and logarithm of the first, it is essential to know what the units are in order to interpret these curves properly.

Referring to the measuring apparatus used, the paper does not indicate that the transmitter and amplifier were calibrated to determine whether or not they are as free from distortion as it is assumed. In regard to the method of testing referred to in the paper, I can say that experience with this method has shown that unless more adequate means are used than are described in this paper for eliminating harmonics from the currents sent through the loud speaker and those given out by the amplifier, the results are open to question, particularly at the lower end of the frequency range.

Paul G. Andres: Two causes contributing materially toward the delinquency of a loud speaker in the matter of faithful reproduction or quality of tone are magnetic and mechanical hysteresis.

Considering the first, the presence of iron in the voice current magnetic circuit causes the diaphragm or armature to respond somewhat differently from what it would were no iron used with the result that foreign harmonics are introduced which tend to distort the output.

Mechanical hysteresis or the mechanical lag of the various movable members contributes further distortion. Spoken words and musical sounds have wave fronts that differ decidedly from pure sine waves. They may be very steep or peaked and consequently distortion may easily be introduced due to the diaphragm, armature or other parts of the vibrating system all of which possess a certain element of inertia or have natural vibration periods of their own not being able to follow the impressed voice currents as readily as they might a pure sine wave, particularly if that be of low frequency. Then too, internal losses in the movable members add to the deviation of the acoustical output from the electrical input.

It would appear as though only an aural comparison of the output with the input can be used in passing final judgement on quality, although a test independent of the opinion of individual observers to determine faithfulness of reproduction or an indication of the distorting factors mentioned would be most desirable if such a one could be evolved.

A. Nyman: I will take up first the discussion by Mr. Kellogg. The first point Mr. Kellogg brought out was a question of engineering tests by the uniformity of volume over the frequency range. Mr. Kellogg made the statement that when the volume is uniform over the whole range of frequencies, it will not be necessary to have the other tests. I can't agree with that statement. The last speaker, I think, brought the point out very clearly. It is a question of the wave form of a combination of different frequencies. An individual frequency might have a wave form that is fairly steep, but a combination of frequencies might have a wave form which is considerably steeper than that, and a test at a single individual frequency will not give an indication of the effect of combined frequencies. That is only one of the things that affect it. Then there are numberless possibilities of combinations of sounds, such as music and singing, say, or different instruments in the orchestra, each one having a combination of different frequencies, and this combination has to be reproduced with exactness.

I fail to see how any test of individual frequency can possibly disclose the operation of the loud speaker in a complicated mixture of frequencies. So from this point of view I believe it will be always essential to have the actual acoustic tests performed on the loud speaker before it can be approved.

The second point brought out by Mr. Kellogg, on the standardization of tests, is very good. This is exactly what ought to come about. As far as the method of tests is concerned, I can't agree with Mr. Kellogg exactly. On the matter of sine waves for the oscillator and a tone filter in the receiving system, he is correct. I believe these steps should be taken, and we have attempted in our work to produce a current as close to a sine wave as possible.

On the other hand, that does not eliminate the test without the tone filter, because the tone filter will indicate the reproduction of the note that you have applied, but that does not exclude the possibility of some other frequencies appearing and distorting the sound; and you couldn't indicate these other frequencies except by making a test without the tone filter.

As far as the arrangement of the room for testing the loud speaker is concerned, Mr. Kellogg's suggestions are good, but I have one exception to offer. Mr. Kellogg suggests placing the loud speaker at a certain distance from the horn in a room especially constructed for the purpose. That is all right, provided the loud speaker were used under those conditions in actual practise. In actual practise, the loud speaker may be placed in a resonant room or in a room highly damped. And in a resonant room, it isn't so much a question of how much sound you get in a certain direction but rather it is, how much energy of sound is produced by the loud speaker.

I believe the problem is really complicated and for final standardization both tests would be required; that is, a directional loudness measurement and an energy of loudness measurement.

Methods for measuring the total energy of sound have not been developed. The only thing I can suggest would be an arrangement similar to the measurement of energy of illumination; in other words, by taking a polar curve around the loud speaker, that will indicate the energy.

With very simple tests that I have conducted, moving the pick-up in front of the horn, I have observed that on most of the frequencies we have to deal with, the pick-up of sound at any point of the horn, provided the horn is designed properly, is practically the same. It may vary 20 or 30 per cent. Of course, this variation is not appreciable.

Mr. Kellogg is right in saying that the condenser transmitter is not uni-directional. I have taken polar curves of condenser transmitters and have found that especially on the low frequencies the pick-up from the back and the sides may be as high as 50 per cent of the sound from the front. This makes it so much more important to use either the method suggested by Mr. Kellogg or placing the transmitter directly in front of the horn, where the sound from the forward direction is so much larger than any reflected sound can be.

On the point brought up by Mr. Kennelly with regard to motional impedances, as I see it the motional impedance test is an essential feature of an electrical analysis of a loud speaker. Of course, the tests described in this paper are empirical tests applied on a practical scale in the engineering development. They are not pure research work. Motional impedance should be carried out on the loud speakers and on combinations of loud speakers and horns, and I believe that will be one of the best methods for determining the characteristics of different types and the operation of the system as a whole.

Mr. Martin brought up the point of efficiency of loud speaker determining its characteristic—I believe the answer to this is the same as to Mr. Kellogg's statement, that is that the efficiency at individual frequencies will not determine alone the operation of the loud speaker. You should take the efficiency of individual frequencies in combined tones; that is, where a mixture of frequencies is applied, and determine whether the individual frequencies will come out in the values proportional to the input.

Mr. Martin is correct in saying the difference between the relay type and the enclosed armature type is very small. The difference is simply practical. In our manufacturing experience, we have found that the relay type is cheaper to manufacture and is more adaptable to changes necessary for proper operation.

The curves that have been plotted do not refer to any particular unit of sound. All of these curves are only relative curves. We start with a certain voltage and we obtain a certain voltage. It is the comparison of the efficiency of loud speaker transmitter systems that we are after. This transmitter—the condenser transmitter—is the most perfect transmitter that has been found practical, and naturally the loud speaker, for practical purposes, should be adjusted to this most perfect transmitter. But all results are purely relative. The transmitters that we have been using have all been tested and their resonance frequencies are well outside the range of the curves that have been shown.

Artificial Transmission Lines With Distributed Constants

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Review of the Subject.—*Artificial transmission lines have been very valuable in developing the theory of transmission of power. This theory has been developed in great detail for steady state conditions, and it is now important to extend it to cover transient conditions. This can best be done upon artificial transmission lines but the usual lumpy type of line introduces parasitic reflections caused by the discrete units of inductance, and therefore are of very limited use for this purpose. Attempts have been made from time to time to produce artificial lines with distributed constants, but the results were either unsatisfactory or expensive and bulky.*

A method, originally proposed by Pupin, has been developed by which small compact and cheap artificial transmission line coils can be designed and built to represent any desired configuration of circuit. These involve tinfoil wound in with the wire between the different layers, insulated from the winding by the usual form of condenser dielectric paper. One of the most difficult parts is to arrive at a coil with the proper constants. Formulas have been derived for doing this which will calculate the result given by the coil within the limits of manufacturing error, and which can be used in a straightforward manner similar to the design of other electrical apparatus. Either single or polyphase lines may be represented, and a single-phase line representing the average configuration of a 110,000-volt line approximately 1000 miles (1600 km.) long has been constructed and successfully used for test over a period of about one year. A three-phase line of similar characteristics, but 400 (640 km.) miles long, has also been constructed and is now being used in experimental investigations. The cost of such lines is low, being approximately 6 cents per mile per phase, excluding the cost of mounting.

There are certain limitations of equivalence, but these are in secondary effects which do not cause serious error. One feature contributing largely to the success of this type of line is the possibility of operating at low potentials with moderate amounts of power, and obtaining readings by means of vacuum tube, or Triode, repeaters, which operate without drawing enough power from the circuit to disturb the conditions being investigated. The lines as constructed will easily withstand 110 volts a-c. which is sufficient for most test purposes.

Introduction. (600 w.)
Development of Artificial Line with Distributed Constants. (950 w.)
The Use of Loop Mile and Wire Mile Equivalent Circuits. (570 w.)
Design of Single-Phase Line with Round Wire Coils. (670 w.)
Examples of Single-Phase Round Wire Coils. (520 w.)
Design of Artificial Line with Rectangular Wire Coils. (450 w.)
Examples of Three-Phase Rectangular Wire Coils. (890 w.)
Tests Upon the Single-Phase Smooth Artificial Line. (210 w.)
Limitation of Equivalence of the Smooth Type of Line. (1460 w.)
Conclusions. (260 w.)
Appendix I. Design Formulas for Round Wire Coils. (850 w.)
Appendix II. Design Formulas for Rectangular Wire Coils. (550 w.)
Appendix III. Calculation of Single-Phase Round Wire Coils. (200 w.)
Appendix IV. Calculation of Three-Phase Rectangular Wire Coils. (50 w.)
Appendix V. Conversion Formulas. (120 w.)
Appendix VI. The Use of Vacuum Tubes in Recording Transien's. (700 w.)
Table of Symbols. (300 w.)
Tables of Data Upon Coils. (Included in Text.) 4
Figures 13
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INTRODUCTION

THE theory of the transmission of electricity has been materially advanced through the use of artificial transmission lines in the laboratory. It is difficult to test full sized lines due to the large spaces they cover, simultaneous readings at various points involving complicated communications, while the expense of construction renders the elimination of errors imperative. As longer and longer lines are necessary their behavior must be predicted with correspondingly greater accuracy. In addition, modern large power networks introduce still further complications, and with many lines interconnected to many sources of power the behavior of the system is not only difficult but in many cases impossible of exact mathematical solution. The use of artificial lines which cost a very small fraction of the actual line may thus represent a direct and economical method of investigating the phenomena involved, and may in many cases save expense and errors in designing the actual system.

Artificial lines of various types have been in use for about thirty years, one of the first investigations being in connection with submarine signaling (Bibliography No. 1) and later Pupin (Bibliography No. 2) developed artificial equivalents in connection with his work upon loaded telephone lines. At the present time these artificial equivalents are used extensively in telephone and telegraph work for balancing circuits, or as standards of transmission distance, while a large number are used for educational purposes in the laboratories of engineering schools. In a few cases power companies have built equivalents of a portion of their system for investigations of the power distribution. (Bibliography No. 3).

The usual construction involves the use of lumped constants, or in other words, a coil having the desired values of resistance and inductance is connected in the proper manner to one or more condensers of the correct value of capacity, the group being equivalent to a section of line of definite length. These units may be connected in either star or delta combinations, and are usually referred to as a T or π section, the arrangement being shown in Fig. 1. If the equivalent section of line is short the values of the total inductance, capacity and resistance per section in the actual line and its artificial equivalent may be made identical without introducing more than a negligible error. If the equivalent section is long, however, correction factors must be used. A very complete description of the design and use of such artificial lines will be found in the special

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books upon the subject by Kennelly. (Bibliography 4 and 5).

While this type of construction is in many ways satisfactory and useful it has certain very definite limitations which prevent the experimental development of transmission theory along certain lines. At present the most important of these is the investigation of transient phenomena. The lumpy type of line loses its equivalence to the actual line when the flow of current is not a repeating function, since the constants behave as separate units towards the wave fronts set up in changing from one state of flow to another, resulting in very large reflections from the inductance lump and very large conduction by the capacity lump. If the inductance were entirely free from distributed capacity then a vertical wave front would be completely reflected. However, the inductance coil must always contain a certain amount of distributed capacity and therefore the reflection of a wave front will depend upon the ratio of the inductance to the distributed capacity that actually exists. Since this capacity is not designed but merely occurs as a result of winding wire in a coil, it is a more or less hap-hazard feature of the usual artificial line. The wave resulting from a transient impressed upon a lumpy line consists of a multiplicity of superimposed reflections which distort it very far from its correct form. (Bibliography No. 6). It is therefore necessary to make an artificial line in which the inductance and capacity are intimately associated at all points in a manner similar to that which actually exists in the real line.

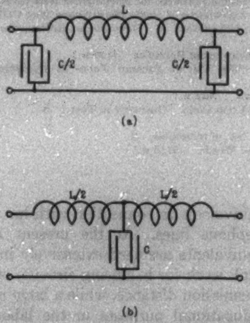

FIG. 1—(A) LOOP-MILE π SECTION OF LUMPY ARTIFICIAL TRANSMISSION LINE

If the constants are properly chosen it may also represent a wire-mile.

(B) A SIMILAR SECTION OF THE *T* TYPE, THE CONSTANTS HAVING VALUES GIVING THE SAME NOMINAL LENGTH AS (A).

DEVELOPMENT OF THE ARTIFICIAL LINE WITH DISTRIBUTED CONSTANTS

This type of line is referred to as a "smooth" line in distinction from the more usual lumpy line. From time to time attempts have been made to produce such a smooth line for the study of transient or other conditions. The Taylor and Muirhead (Bibliography No. 1) artificial cable was smooth, but included only capacity and resistance, and therefore could not support a traveling wave. The method used was to slit a sheet of resistance metal in a staggered manner to give the proper resistance, the sheet being separated from an unslitted sheet of similar dimensions by a proper dielectric to give the desired capacity. In 1898 Pupin (Bibliography No. 2) made a smooth line by winding spools of insulated wire with tinfoil interspersed between the layers. This gave the desired characteristics, but did not have sufficient dielectric strength, and failed under the impressed voltage so frequently that it was abandoned. Later Union College (Bibliography No. 7) in collaboration with Steinmetz constructed a smooth line consisting of large glass cylinders lined with tinfoil and wound with a single layer of wire. This had high dielectric strength but was a bulky design as the single layer of wire gave relatively low inductance for the material used, while the thickness of the dielectric resulted in low capacity. It also suffered severely from breakage of the glass cylinders. The amount constructed was equivalent to about 130 miles (209 km.) of single-phase line, occupied 1300 cubic feet (37 cu. m.), and weighed about 3½ tons (3200 kg.).

The Research Division of the Electrical Engineering Department of the Massachusetts Institute of Technology started an investigation of the possibilities of making a satisfactory smooth artificial line in the fall of 1920. The work has been done largely by graduate students and research assistants. Mr. L. H. Becker began the first attempts at making coils with distributed constants, and the first tests were carried out by Messrs. M. F. Gardner and E. J. Arnold, all of these being research assistants. The design of the single-phase line was carried out by the author, and the more extensive tests, which are to be described in another paper, were carried out by Messrs. Scott, Van Ness and Jackson, graduate students, under the supervision of Mr. S. M. Jones, research assistant. The design of the three-phase line was done as a graduate thesis by Messrs. P. T. Coffin and L. O. Buckner, and the test work upon this line is now under way by a group of graduate students.

The first lines of endeavor dealt with lattice-wound coils, the capacity being obtained by copper plating over the insulation, or embedding the coil in Wood's alloy. This gave unsatisfactory results due to large dielectric hysteresis and variations in the resulting capacity. After considerable experimenting it was decided to develop the Pupin type of coil. The final type of construction consists of a multilayer coil wound with bare, or enamel insulated, copper wire of rectangular or circular cross section. Between each layer of winding, and also on the inside and outside of the coil, is wound a layer of tinfoil, insulated from the conductor by impregnated condenser paper. Each layer of tinfoil projects at one end so that all the layers may be soldered together. There must also be a break in each layer along the same radial line, so that the tinfoil does not make a short-circuited secondary for the main winding. This results in a coil having the capacity uniformly distributed with respect to the conductor. The leakage is also uniformly distributed, but there is a certain periodicity of constants due to the existence of mutual inductance between turns in each coil. This does not appear to interfere with its equivalence to an actual line, as shown from tests made upon the lines. One important facility contributing largely to the success of this construction is the availability of three electrode vacuum tube amplifiers for use in making tests. The operation of the usual type of metering equipment and oscillographs requires so much power that if they are directly connected to the line the resultant equivalent load is very large and distorts the line conditions seriously. Such equipment can only be connected to the end of the line which is attached to the power supply without introducing serious errors. By means of the vacuum tube amplifier or repeater, however, oscillographs and other devices may be operated by the plate current of the vacuum tube and controlled by the charge on the grid. If the grid is properly polarized it draws negligible power from the system and therefore does not introduce any undesirable incidental loads. Thus tests may be made with small amounts of power and moderate voltages, the dielectric strength required being low, and as a result the line construction compact. With the high grade materials now available coils of this construction can be made to withstand a working potential of 100 volts per mil of condenser dielectric. This allows a safety factor of from two to three.

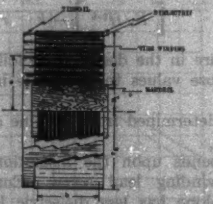

FIG. 2—CROSS-SECTION OF ROUND WIRE TYPE OF SMOOTH ARTIFICIAL LINE COIL

The arrangement with rectangular wire is the same, but usually somewhat longer compared to the diameter.

Owing to the interdependance of the various constants in the coil the design is somewhat complicated. A change in any one dimension affects all the other constants and cut-and-try methods are very laborious and wasteful. Therefore a method of design has been worked out which will give reasonably accurate results. It introduces of necessity several design constants, which have to be more closely determined by a sample coil, but usually one or two samples will suffice to predict the results within five to ten per cent, which is as close as coils of this sort can be manufactured commercially. If greater accuracy is necessary the coils must be wound slightly oversize and then unwound until they fit the required constants. Since the capacity reduces directly with the first power, and the inductance approximately proportionately to the square of the turns unwound, it is always possible to unwind an oversize coil to a point where the desired ratios are obtained. The equivalent size of conductor represented by the artificial line is roughly proportional to the size of conductor actually used in winding the coil, and varies but very slightly for a small change in the number of turns. Thus unwinding the coils changes the equivalent spacing rapidly, the equivalent length of line slowly, and the equivalent conductor size remains practically constant.

THE USE OF LOOP-MILE AND WIRE-MILE EQUIVALENT CIRCUITS

As mentioned above the transmission line may be represented by T or π sections having the values of inductance, capacity, resistance and leakance existing in the actual line. In the same way these constants may be worked into the smooth type of artificial line. In this case, however, since the constants are distributed, the actual values of the constants are used, and no correction factors applied. No attempt has been made to introduce leakance into the design, since this is rather an indefinite constant, depending, in the actual line upon the condition of the insulators and the weather and in the artificial line upon the amount of moisture present and the type of dielectric. Since it is usually very low in aerial lines no serious error is introduced by this omission provided its resultant magnitude is comparable to that observed in practise.

One point which frequently leads to confusion is the use of either wire or loop-mile equivalents. The wire-mile equivalent is a circuit equivalent to one wire of the transmission line represented, and is of no use by itself, always requiring a return circuit of similar nature. The loop-mile equivalent is a circuit representing both sides of the transmission line, that is the line going out and the line returning, and represents the complete transmission system by itself. A loop-mile circuit may be made from one or more wire-mile circuits. In the case of the representation of a single-phase line either one is satisfactory, the terminal conditions being identical, and the loop-mile equivalent usually being simpler to construct. In the representation of a three-phase line three wire-mile equivalent circuits must be used. Fig. 3 will perhaps make this clearer. Case (a) represents an artificial π line section with two groups of inductance and capacity. This can be broken up into two wire-mile equivalents as shown in (*b*), the circuit on either side of the line *m-n* being a wire-mile equivalent. It will be noted that this requires the use of larger capacity for each circuit, since the individual condensers are connected in series. Case (*c*) is a loop-mile equivalent circuit having the same terminal characteristics as either of the others, but requiring only one inductance coil and the minimum size of capacity. Case (*d*) shows the arrangement of an equivalent circuit for a three-phase line with equilateral spacing. This neglects the capacities to ground. Case (*d*) can easily be converted to (*e*), the condensers being connected in star instead of delta. It will be seen that this latter case consists of three wire-mile equivalent circuits. In some cases the constants are spoken of as inductance or capacity to neutral, which is merely a different way of stating the wire-mile values.

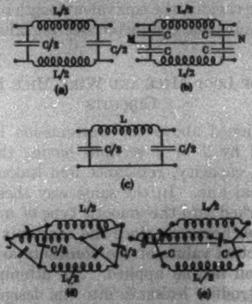

Fig. 3

(A) Circuit Equivalent of a Single-Phase Transmission Line,

(B) The Same Circuit as (A) Broken up to Show Two Wire-mile Equivalent Sections Combined to Represent the Line

(C) The Same Section Length as (B) with the Constants Lumped to Give a Loop-mile Section

(D) A Three-phase Transmission Line Represented by Lumped Constants

(E) The Same Circuit as (D) with the Capacities Connected in Star Instead of Delta

(The values of the constants in all the different sections are marked to show the values giving the same nominal length of section for each type of circuit). It will be noted that this is the same as three of the wire-mile sections used in (b).

The same arrangements can be used in the smooth line artificial coils, except that the capacity is of course distributed. The hardest thing to get in the design is sufficient capacity, particularly with round wire. Therefore, where it is possible to do so the loop-mile equivalent is desirable from the standpoint of economy and compactness. It is also the only type that can be easily obtained with round wire, and for this reason the three-phase lines are constructed with flat strip. While the flat strip is a little more difficult to obtain, usually standard sizes can be used, and it introduces greater flexibility in the design, since the resistance can be changed by altering the thickness without seriously affecting the capacity, which is dependent upon the width of the copper strip and the thickness of the dielectric. The three-phase design later described may of course be also used for single-phase design, since it is merely a method of arriving at the constants of the coil, and these may be chosen to represent any desired form of line within the possible limits of the artificial coils. It should also be borne in mind that any coil containing inductance, capacity and resistance will represent some kind of transmission line, but the relations of the constants may be such that the actual line represented is of extreme or grotesque configuration.

DESIGN OF SINGLE-PHASE LINE WITH ROUND WIRE COILS

Appendix I gives the derivation of the design formulas. The resultant useful formulas are as follows:

$$l = \frac{0.915\, l_e^{1/4}\ (\delta + k\, r)^{3/4}\ y^{1/4}\ 10^3}{(H\, K)^{3/4}\ r^{1/4}} \tag{12}$$

$$\delta = \left\{ 0.003 \left[\log_{10} \frac{2\,S}{d} \right]^{1.6} \left[\frac{y\, r}{l_e} \right]^{0.4} \left[K / H^{0.6} \right] - k \right\} r \tag{14}$$

The procedure in the design is as follows:

1st. Choose values to substitute in (14):

$\log \dfrac{2\,S}{d}$ is determined from the line configuration.

y	depends upon the insulation or setting of winding lead screw. Enamel insulation alone has less dielectric loss than fabric covered wire. With this type of insulation (y) is found to be about 1.3. Obviously it can never be less than unity.
l_e	may be arbitrarily fixed as a long line is represented by several sections. If the line is short it is made equal to the length of line to be represented. More than ten miles per section with the round wire coils makes bulky units, but the cost per mile is not greatly affected. A larger equivalent length for fixed equivalent spacing and thickness of dielectric requires an increase in the size of wire used. Thus if the wire radius is found to be very small it is advisable to increase l_e or δ.
K	depends upon the dielectric used. For the usual types of condenser paper and impregnation it varies from 2.5 to 3.0. A good average value for trial is 2.8.
H	depends upon the shape of the coil. Its maximum value is 6.06×10^{-4} and therefore a value slightly less than this should be used and the coil shape arranged to approach the maximum as closely as possible.

k should be taken as 0.215 unless considerable bedding occurs, in which case it is decreased somewhat.

δ is usually chosen for dielectric strength, and modified if necessary to alter the characteristics of the coil. It should never be less than 0.002 inches (0.005 cm.) and where possible should consist of two sheets of paper to obtain greater uniformity.

2nd. The constants having been determined and a value of l_c chosen, find r from (14) by cut-and-try or graphical solution.

3rd. Find l from (12). (Formula (13) Appendix I may also be used.)

4th. Check resistance of coil against resistance of actual line of same length (l_c) from formula:

$$R_1 = \frac{l\,\rho \times 10^{-6}}{\pi\, r^2} \quad \text{(See Appendix I)} \tag{10}$$

The resistance will vary very nearly directly with the value r^2 as shown in Fig. 4, so correct to the proper size of wire if necessary and repeat the last three steps, δ being corrected from (14).

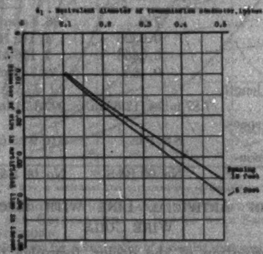

FIG. 4—CURVE SHOWING THE RELATIONS BETWEEN THE DIAMETER OF WIRE WOUND IN THE SMOOTH ARTIFICIAL LINE COIL AND THE EQUIVALENT DIAMETER OF TRANSMISSION LINE CONDUCTOR REPRESENTED

The upper curve is for a wire spacing of 18 ft. (5.5 m.) and the bottom curve is for a spacing of 6 ft. (1.83 m.). The equivalent length of section varies from 2 to 40 miles (3.2—64 km.), increasing with the diameter of wire in coil. The length of wire in the coil varies from 5000 to 50,000 cm. (164-1,640 ft.), also increasing with the diameter of wire. The design constants were taken as follows:

$K = 2.5 \qquad y = 1.5 \qquad H = 0.6 \times 10^{-3} \qquad \delta = 0.0025$ in.

5th. The values of r, l and δ having been determined, next calculate the mean winding radius, turns per layer, number of layers, etc.

H is maximum when $b/c = 1$ and $c/a = 0.06$ to 0.8 (Bibl. No. 8). The coil dimensions can be easily obtained from the following relations:

$$a = \sqrt[3]{\frac{l\,c\,2\,(y\,r)^2}{\pi\, b\,(c/a)^2}} = 1.20\,\sqrt[3]{l\,(y\,r)^2} \tag{37}$$

(if the optimum values of c/a and b/c are used.)

$$c = 0.662\,a = Q\,p = Q\,(2\,r + 2\,\delta + f) \tag{38}$$

$$b = c = 2\,s\,y\,r \quad r_1 = a - c/2 \quad r_2 = a + c/2$$

Bibl. No. 8)

Since H is not critical the nearest values giving Q, n and s whole numbers should be used.

Winding the Coil. A mandrel is made with its diameter equal to $2\,r_1$. A roll of tinfoil should be procured about one inch wider than the dimension b, and the dielectric paper should be about 1/2 in. wider than b. It is convenient to have a hot iron to melt the wax on the overlapping edges of this paper and thus fasten it in place. The winding is started by first covering the mandrel with paper, and then placing one sheet of tinfoil in place. The tinfoil must be cut to leave a gap about 1/8 in. wide at the ends to prevent short-circuited turns. All these gaps should be on the same radial line so that the ends can all be soldered together without producing short circuits from layer to layer. The tinfoil is then covered with the proper thickness of dielectric paper, and the winding started, one layer being completed. Another layer of dielectric tinfoil and dielectric is put in place, and another layer of wire wound, this process being repeated until the proper number of layers are completed. The coil is finished off by a final layer of tinfoil, and well covered with heavy paper for protection. In placing the dielectric paper it is a good plan to have the overlapping ends either carefully cut to fit in the tinfoil gap, or distributed as evenly as possible around the circumference, to avoid building up a bump upon one side of the coil, which tends to lower the capacity. The tinfoil is placed so that one side projects beyond the dielectric paper. These projecting edges are now all soldered together and the lead wire soldered on. The whole coil is then impregnated, preferably in vacuum. Ceresin wax is desirable for this as it is more nearly moisture proof than paraffin. Tests upon sample coils should always be made after they have been impregnated in exactly the same manner as the finished coils, since the capacity is changed quite materially by this treatment, usually being increased about ten per cent. Moisture affects the capacity, leakance and dielectric hysteresis of the coil, therefore the coils should be mounted in a tight box and preferably calcium chloride or other moisture absorbing compound used to take up the small amount that penetrates the walls.

EXAMPLES OF SINGLE-PHASE ROUND WIRE COILS

The single-phase line of this type in the Massachusetts Institute of Technology laboratories was built as an approximate duplicate of an aerial line with 10 ft. (3 m.) spacing. This would correspond to about average practise upon a 110-kv. line. No limitation was set upon the equivalent conductor size providing it was within reasonable limits. The results of the first four sample coils are given in Table I. The coil constants were chosen to give representation of the line by loop-mile equivalents. The final specifications are given in Table II, and the design check for the average constants in Appendix III.

TABLE I

TRIAL SINGLE-PHASE ROUND WIRE COILS

Loop Mile Equivalent

Coils designed with 0.002 in. (0.0051 cm.) paper, 0.003 in. (0.0076 cm.) tinfoil

K assumed $= 2.5$, $H = 0.6 \times 10^{-3}$, $y = \dfrac{D}{2r} = 1.60$

Coil No.	Wire size B & S	R ohms	L mh	C mf	l_e miles	d inches	S feet	y	z inches	l cms	
No. 1	No. 18	9.25	115	0.180	27.0	0.54	10.0	1.6	0.002	44,000	Design
		9.71	144	0.206	33.0	0.61	26.5	1.27	0.003	45,700	Test
No. 2	No. 21	10.10	58	0.084	13.0	0.375	10.0	1.6	0.002	24,000	Design
		11.05	68.5	0.080	13.0	0.375	31.0	1.1	0.003	24,600	Test
No. 3	No. 23	11.70	40	0.056	8.6	0.28	10.0	1.6	0.002	17,500	Design
		12.85	46.7	0.0534	9.5	0.285	24.0	1.13	0.0032	19,000	Test
No. 4	No. 23	13.3	47.6	0.060	9.2	.285	18	1.15	0.0025	20,000	Design
		12.9	46.3	0.058	9.8	.29	19	1.13	0.0025	19,300	Test

On coils No. 1, 2 and 3, y came out lower than expected which increased the inductance. The paper dielectric ran thicker than specified which tended to decrease the capacity, but at the same time the dielectric constant was higher than expected so the variation in capacity was minimized. The equivalent length of section and diameter of conductor check reasonably with the design, but the spacing is in error, due to the difference in inductance and capacity both being in the direction to increase spacing, except in the first coil. The specifications were given in terms of layers and turns per layer, which increased the length of wire in coil slightly, as the layer pitch was greater than expected.

TABLE II

SINGLE-PHASE ROUND WIRE COILS

Loop Mile Equivalent

FINAL COIL SPECIFICATIONS

Wire size	No. 23 B & S enamel covered
Diameter of winding mandrel	1.8 inches. (4.57 cm.)
Total number of turns	792
Turns per layer	36
Number of layers	22
Thickness of dielectric, one sheet	0.002 inches. (0.0051 cm.)
Thickness of tinfoil	0.003 inches. (0.0076 cm.)
Gap in tin foil	0.25 inches. (0.6 cm.)
Tolerance	$L/C = 640 \begin{cases} +3\% \\ -5\% \end{cases}$

Coil No. 4 was designed upon the data obtained from the first three, and fairly close results were obtained. Breakdown tests upon the dielectric showed erratic results. The voltage varied from 320 to 1000, 60-cycle a-c. root-mean-square value. Most of the 8 coils tested for breakdown withstood from 450 to 600 volts. Wind wire with highest practicable tension. Impregnate coil after winding. Solder all tinfoil together on one end and attach flexible lead. The first and last layers to consist of tinfoil and dielectric. Protect outside of coil with 0.005-in. (0.012 cm.) fish paper.

FINAL COIL DIMENSIONS

126 coils of this type were manufactured representing approximately 1000 miles of single-phase line. The average results are:

Weight of complete coil	1.88 pounds (0.858 kg.)
Outside diameter	3.5 inches (8.9 cm.)
Inside diameter	1.75 inches (4.45 cm.)
Winding length	$1^1/_{16}$ inches (2.7 cm.)
Coil length over insulation	1.5 inches (3.81 cm.)
Coil length over tinfoil, total	2.0 inches (5.08 cm.)
Winding depth	$^7/_8$ inches (2.22 cm.)

FINAL COIL CONSTANTS AND EQUIVALENT VALUES[1]

	Equivalent per coil	126 coils
Resistance	7.9	996.4 ohms
Inductance	0.0326	4.112 Henries
Capacity	0.0565	7.116 Micro-farads
Leakance	0.465	58.7 Micro-mhos.
Equivalent length of line	7.87 miles (12.6 km.)	992.1 Miles (1590 km.)
Equivalent spacing	8 ft. 9 in.	8 ft. 9 in. (2.67 m.)
Equivalent conductor	No. 00 B&S	No. 00 B&S. (9.2mm.)

DESIGN OF ARTIFICIAL LINE WITH RECTANGULAR WIRE COILS

The derivation of the design formulas is given in Appendix II. The resultant formulas used in the design are as follows:

Mechanical dimensions of coil:

$$a = 3.2\,(z/L)\,(w\,R\,t/\rho)^2\,10^{-10} \qquad \text{(I)}$$

$$b = 2\,r_1 = y\,w\,s \qquad \text{(II)}$$

$$c = Q\,p = n\,p/s \qquad \text{(III)}$$

$$r_1 = b/2 = y\,w\,s/2 \qquad \text{(IV)}$$

$$r_2 = r_1 + c = r_1 + n\,p/s \qquad \text{(V)}$$

Winding Dimensions of coil:

$$n = \frac{5\,\rho\,L\,10^8}{z\,w\,R\,t} \qquad \text{(VI)}$$

$$Q = n/s \qquad \text{(VII)}$$

$$P = (t + 2\,\delta + f) \qquad \text{(VIII)}$$

$$s = \frac{a \pm (a^2 - y\,w\,n\,p)^{1/2}}{y\,w} \qquad \text{(IX)}$$

NOTE: (IX) is derived from (7) and (8).

---

[1] The equivalent line values are calculated by the more accurate formulas given in Appendix V. If the approximate formulas in Appendix I are used slightly different results are obtained.

Dielectric thickness:

$$\delta = \frac{88\, w\, K\, q\, L}{N\, c\, Z} \qquad \text{(X)}$$

The procedure in the design requires the assumption of a definite value of thickness of copper strip, the width and dielectric thickness then being chosen to give the desired characteristics. The constants of the line to be represented may be determined from any desirable formulas, those given in Appendix I being satisfactory. The procedure in the design will then be as follows:[2]

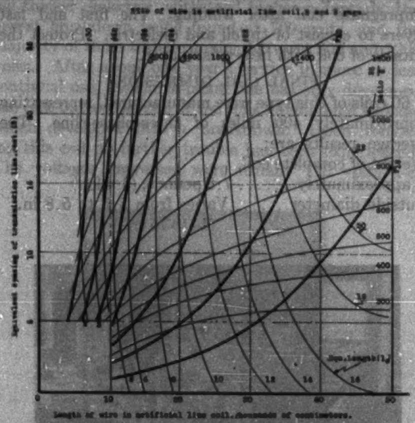

FIG. 5—CHART SHOWING THE RELATIONS BETWEEN THE VARIOUS DIMENSIONS AND EQUIVALENTS OF ROUND WIRE SMOOTH LINE LOOP-MILE SECTIONS

Values are calculated for a fixed value of dielectric thickness of 0.0025 in. (0.00635 cm.) Since they are all interdependent they can all be plotted to the same coordinates. For example, 20,000 cm. of No. 22 B & S wire will result in a section having an equivalent length of 10 miles with equivalent spacing of ten feet, and the ratio $2\,S\,/\,d = 700$, thus the equivalent conductor diameter is 0.34 in. The curves are plotted for average values of design constants and therefore give approximate results only.

1st Determine values of L, R and C desired from line configuration and length of section desired to represent.
2nd Assume values of design constants z, q, y, ρ and K.
3rd Assume several values of thickness of copper strip.
4th Solve for (w/n) from formula (X).
5th Solve for $(w\,n)$ from formula (VI).
6th Determine values of (w) and (n) from last two steps.
7th Solve for value of (a) from formula (I).
8th Solve for value of (s) from formula (IX).
9th All other dimensions may then be determined by direct substitution in the desired formulas.

It will be seen that the width of copper strip is determined from the design and that a definite value is necessary for any given thickness. It is usually desirable to try several sizes for the preliminary steps. As the formulas are based upon the fixed relation that the winding length is equal to the internal diameter, (see Appendix II) it is easy to form a mental picture of the size of the coil from the average winding radius (a) determined as in the 7th step above.

As it is impossible to avoid variations in design constants due to non-uniformity of materials and manufacturing methods, a sample coil should next be made up, from which the correct design constants can be determined for the exact conditions encountered and the final design arrived at by the same methods.

It should be remembered that it is possible to make a coil which satisfies the requirements of L, R and C but does not meet the requirement that the winding length is equal to the internal diameter. After building a sample some slight modifications may be found desirable to make it fit the imposed conditions more accurately so that the finished coil may ultimately have a winding length different from the inner diameter by a small amount.

EXAMPLES OF THREE-PHASE RECTANGULAR WIRE COILS

The line in the Mass. Inst. Tech. laboratories was designed to duplicate an average of present standard practise upon 110-kv. lines. The following characteristics were taken from data upon a number of existing lines:

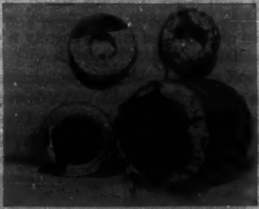

FIG. 6—SEVERAL TYPES OF SMOOTH ARTIFICIAL LINE COILS

Upper left is lattice wound embedded in Wood's Alloy. Upper right is round wire single-phase 8 loop-mile section. Lower left is experimental three-phase coil, and lower right the final three-phase coil of 20 wire miles per section.

Length of section	20 miles. (For convenience) (32 km.)
Voltage	110,000 volts
Spacing	120 in. (3.05 m.)
Conductor	No. 0000 B & S
Arrangement of conductors	Equilateral

From these data the constants per section are:

Resistance per wire (R)	5.185 ohms
Inductance per wire (L)	0.04183 henries
Capacity per wire (C)	0.2855 microfarads

---

[2] The design of this first line has been made under the limited assumption of equilateral spacing, and absence of the ground plane. The extension of the design to the more general case is discussed later on in the paper. The methods are the same, different constants being used.

Three thicknesses of copper strip were tried and No. 28 B & S seemed to give the best design. Two coils were made up, supposedly duplicates, and the results obtained are shown in Table III.

From these values it appeared that either the dielectric constant or the constant (q) in the capacity formula had been assumed too small, and that the tension used in winding had stretched the wire, increasing its resistance.

Coil No. 3 was then made up with the same specifications except that thicker dielectric was used and the results, as shown in the tabulation, check quite well except that the resistance is still a little high. Since no attempt was being made to duplicate an exact line, but merely to obtain one that would represent average conditions, this was considered close enough and the final design was based upon this information. The actual equivalence is shown in Table III.

TABLE III

THREE-PHASE RECTANGULAR WIRE COILS

Wire Mile Equivalent

Calculated Values		Test Values				
		Coil No. 1	Coil No. 2	Error	Coil No. 3	Error
L	0.0418	0.0410	0.0420	2%	0.0425	1.7%
C	0.2855	0.3470	0.3270	21%	0.2850	0.0%
R	5.185	5.862	5.879	13%	5.946	14.5%

	Calculated Values	Test Values Coil No. 3	Error
Length	20 miles (32 km.)	20.2 miles (32.4 km.)	1.0%
Spacing	120 in. (3.05 m.)	120 in. (3.05 m.)	0.0%
Size of conductor	212,000 c. m. (107 sq. mm.)	186,000 c. m. (94 sq. mm.)	12.3%

The fact that the length and spacing of coil No. 3 both check so closely is of course merely a coincidence with this particular coil.

The final coil specifications and measurements upon finished coils are given in Table IV. The calculations of the final coil are given in Appendix IV. It will be noted that the specifications are not exactly the same as the calculations, slight changes having been made to meet the commercial conditions. Thus the nearest standard size of fiber tubing for a hub had a diameter of 4.16 in. (10.6 cm.) making the value of r_1 actually 2.08 in. (5.3 cm.) instead of 1.88 in. (4.8 cm.) as designed. To compensate for the increase in diameter, the spacing of the turns was increased slightly as well, which resulted in the final coils being of substantially the same characteristics as expected.

TABLE IV.

THREE-PHASE RECTANGULAR WIRE COILS

Wire Mile Equivalent

FINAL COIL SPECIFICATIONS

Wire Size, No. 28 B & S tinned bare (Rectangular)	Width 0.104 in. (0.264 cm.) Thickness 0.013 in. (0.033 cm.)
Inside Diameter	4.16 in. (10.6 cm.)
Outside Diameter	5.50 in. (14.0 cm.)
Total number of turns	725
Turns per layer	29
Number of layers	25
Thickness of dielectric, two sheets, each	0.003 in. (0.0076 cm.) 0.006 in. total (0.015 cm.)
Width of dielectric	5 in. (12.7 cm.)
Thickness of tinfoil	0.002 in. (0.0057 cm.)
Width of tinfoil	6 in. (15.2 cm.)
Gap in tinfoil	0.375 in. (0.95 cm.)
Length of layer	4.0 in. (10.2 cm.)
Core length	5.00 in. (12.7 cm.)
Over all length	5.5 in. (14.0 cm.)
Tolerance	L/C = 160 ± 10 per cent

Wire to be wound with uniform tension. Solder all projecting ends of tinfoil, and attach flexible lead. Impregnate coil after winding. The first and last layers to consist of tinfoil and dielectric. Protect the outside of coil with heavy fish paper.

FINAL COIL DIMENSIONS

60 coils of this type were manufactured, representing approximately 400 miles of three-phase line. The average results are:

Weight of complete coil, approximately	7.5 pounds, (3.4 kg.)
Outside diameter	Varies from 5.5 to 5.8 in. (14 to 15 cm.)
Inside diameter	4.125 in. (10.5 cm.)
Over all length	Approximately 5.5 in. (14 cm)
Winding depth	Varies from 0.69 to 0.87 in. (1.75 to 2.2 cm.)

Fig. 7—The 1000 Mile Single-Phase Smooth Line Assembled in Boxes. One Box is Opened to Show Arrangement of Coils

FINAL COIL CONSTANTS AND EQUIVALENT VALUES

	Per coil	Total average, per wire
Resistance	5.51 to 6.02 ohms	114.0 ohms
Inductance	0.041 to 0.043	0.84 henries
Capacity	0.251 to 0.312	5.25 microfarads
Equivalent length of line	19.1 miles (30.5 km.)	382.75 miles (613 km.)
Equivalent spacing	144 inches (3.56 m.)	144 inches (3.56 m.)
Equivalent conductor diameter	0.426 in. (1.08 cm.)	0.426 in. (1.08 cm.)

Both sets of lines were manufactured by the General Radio Company of Cambridge, Mass. They are particularly fitted for the manufacture of this type of apparatus since it involves production methods very similar to those used in the manufacture of radio apparatus. The author would like to take this occasion to express his appreciation of their assistance in developing these lines and in carrying out some of the experimental work.

A number of the coils were tested for breakdown potential between the wire and tinfoil. The voltage required was rather erratic due to variations in dimensions and quality, the single-phase coils breaking down under an impressed a-c. voltage of from 320 to 1000 volts, root-mean-square value. The three-phase coils, having thicker dielectric, withstood higher potentials, the lowest breakdown being found at 700 volts, and the majority of those tested running between 800 and 1200 volts. After about one year's service in more or less continual use, the single-phase line has only developed one case of trouble, one of the coils having high leakage which developed into a breakdown. Upon removing this coil no further trouble has been experienced. The voltages impressed were usually 110 volts a-c. or d-c., and higher values occurred in parts of the line during some of the tests.

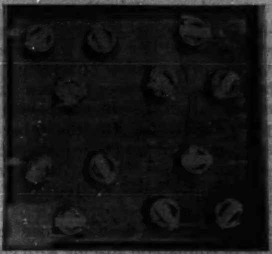

Fig. 8—The Arrangement of Four Sections of the Smooth Three phase Line in Containing Box

Coils are spaced so that mutual between them is negligible, but can be moved to obtain desired value if required. The cage in corner of box is for moisture absorbing compound.

The constants of the coils were measured in the development work by means of bridge measurements at 1000 cycles, for the inductance and capacity and leakance, the resistance being measured upon a d-c. bridge. After assembly into the final line the average constants were then determined by the free and grounded impedance method with a-c. potentiometer readings at 60 cycles. These latter readings of course give the values that should be used in computing the line, and very little difference in constants was found, being less than the individual error between different coils due to variations in manufacture.

The cost of coils is interesting. It will vary with the type of line used, the form of representation and the size of conductor. In general the larger the transmission line being represented, in terms of spacing and conductor size, the more expensive will be the artificial line coils. The single and three-phase lines being described both cost almost exactly 60 cents per mile per phase for the coils without mounting. The single-phase coils are smaller but represent approximately eight miles of line. The three-phase coils are larger and more expensive, but represent about twenty miles of line each. The three-phase line of course requires three coils for each section and therefore is about three times as expensive to build as the single-phase line, since single coils would be of about the same cost per mile. The labor cost depends to a large extent upon the number of layers per coil, since it is the placing of the tinfoil and paper that is the expensive operation.

Fig. 9—The 400 Mile Three Phase Smooth Line, as it Stands in the Laboratory, in Use for Transient Investigations

Spacing cribs are placed between boxes to eliminate mutual inductance vertically.

Tests Upon the Single-Phase Smooth Artificial Line

An extensive study of the behavior of the line under d-c. transient conditions was made by graduate students as already mentioned. The d-c. transients were chosen as a preliminary step to the investigation of a-c. transients, since the former can be calculated with reasonable facility while the theory of the latter is still extremely embryonic. Fig. 10 shows the current transient upon 992 miles (1590 km.) of the line, the far end being open with no load and the current being taken at the near end. It will be seen that there is no evidence of partial reflections and that the line appears to behave in the same manner as the actual line it represents. Theoretical checks upon the transients

Fig. 10—The Direct Current Transient at the Home End of 992 Miles of the Single-Phase Artificial Transmission Line, the Far End Being Open

confirm this. Fig. 11 shows the current at the near end of the same length of line when the far end was short-circuited. In these curves the reflections occur at the proper intervals for the travel of the wave front over the whole line. The mathematical analysis of these oscillograms shows that they check the expected results with excellent fidelity. The details will be omitted here as another paper is to be devoted to the discussion of the test details, and these are merely shown as examples indicating the absence of lumpiness in this type of artificial line.

FIG. 11—THE DIRECT-CURRENT TRANSIENT AT THE HOME END OF 992 MILES OF THE SINGLE-PHASE ARTIFICIAL TRANSMISSION LINE, THE FAR END BEING SHORT-CIRCUITED

LIMITATION OF EQUIVALENCE OF THE SMOOTH TYPE OF LINE

There are certain limitations in representing an actual system by this type of artificial line. Some of these involve the characteristics of the materials used in the coil, while others deal with the possibilities of arrangement to represent unsymmetrical configurations, conditions of unbalance and the like.

Under the first classification the most noticeable is the effect of dielectric hysteresis, which is present in the artificial lines and is absent or negligible in the actual line. This means that the apparent leakance is a function of frequency, and must be minimized by maintaining the hysteresis loss as small as possible. Its effect upon the transient is to distort the wave front and cause it to deviate from the perpendicular. The effect is not large if good materials are used and should not cause serious error. There is also no control over the actual leakance, but its magnitude appears to be of the proper order. The leakance in the artificial coils is entirely distributed, while in the actual line it is lumped at the towers. This difference does not appear to cause any discrepancy since the effect of leakance in aerial lines is usually very small.

In the actual transmission line skin effect is present, while this effect is practically absent in the artificial line coils. It is of small magnitude compared to other factors, but may be of great importance in dealing with steep wave fronts and high-frequency transients. The dielectric hysteresis of the artificial line is of somewhat similar character, and therefore exact equivalence under high-frequency conditions would appear to depend, at least in part, upon the possibility of obtaining equivalence of skin effect by dielectric hysteresis.

The ordinary lumpy type of artificial line has a definite periodicity of arrangement of discrete constants and therefore acts as a filter, with fixed values of cut-off frequency, the usual type being a low pass filter. It is important to have this cut-off value very different from any of the frequencies encountered in the test work in order to have correct representation. Strictly speaking an artificial line is only equivalent to the actual line for one frequency. If the cut-off frequency is remote from the operating frequencies, however, the discrepancies for the different frequencies are very slight. In dealing with transients, high frequencies occur in two ways. The period of reflection or oscillation of the transient may be of high frequency, and a steep wave front may be represented as a Fourier's Series containing a very large, or perhaps infinite number of frequencies. Thus it is extremely necessary that the characteristics of the line shall not alter as the frequency is increased, except in the same way that the characteristics of the actual line would alter. In the smooth type of artificial line there is a certain amount of periodicity due to a number of separate coils being used, and the mutual inductance between turns in each coil being lumped within the coil, there being no mutual between separate sections. On the other hand the capacity is entirely distributed with respect to the wire. This is to be the subject of further investigation. Probably if any cut-off value exists it is of very high frequency.

The three-phase line has been designed to represent an isolated line of equilateral spacing with currents and voltages that either are balanced or add up to zero, so that there is no residual potential between neutral and ground. The effect of the ground plane has, for the time being, been neglected. These assumptions do not coincide with the actual case, where the ground plane must be considered, but were chosen as the basis upon which to build up the experimental and theoretical investigation. Mutual inductance between coils has also been omitted, the coils being mounted far enough apart so that the mutual between them is negligible. This again does not entirely coincide with practise

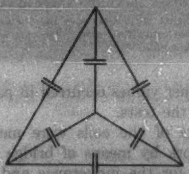

FIG. 12—THE ARRANGEMENT OF CONDENSERS USED BY GRAY IN LUMPY ARTIFICIAL LINE TO REPRESENT THE EFFECT OF THE GROUND PLANE

as mutual exists between the different circuits of a three-phase transmission line, but if the limitations of current and voltage stated above are adhered to, the mutual upon an equilaterally spaced line cancels out and does not affect the distribution.

There are several ways in which these added factors may be introduced. Gray (Bibl. No. 3) has made a careful study of the equivalence of artificial and actual lines, and constructed a lumpy type of artificial line to represent an actual transmission system. His theoretical treatment takes into account the effect of the ground plane upon the capacities between conductors, as well as mutual inductance between circuits and the effects of unbalance. The condenser circuit used for the equivalent artificial section is shown in Fig. 13. The three condensers in star represent the capacities of the lines to ground, and the three condensers in delta represent the capacities between wires. By the usual star-delta transformation formulas (Bibl. No. 4) it is possible to transform this combined circuit to either one star or one delta combination of capacities. The only difficulty is that in doing so the ground plane connection disappears, since the neutral of the condensers must represent the neutral zone of the system of transmission conductors. If the distance between conductors is small with respect to the distance to ground, then all the capacities in Gray's system, Fig. 13, can be assumed symmetrical with small error. With the configuration of the usual transmission system this will result in less error than the probable accuracy of the test work and the variations in the coils themselves, unless each one is individually adjusted by unwinding as mentioned above. In this case the capacities of all the wires to ground can be replaced by one capacity connected from the neutral of the system of conductors to the equivalent ground plane. This has not been tried, but should represent any desired configuration with reasonable accuracy. It introduces, however, discrete units of capacity which give periodicity and to a certain extent destroy the smoothness of the artificial line. This may have an undesirable effect under transient conditions, but it is improbable that it would be of very great magnitude since the capacities to ground are small compared to those between conductors. If the spacing of the conductors is not equilateral, unsymmetrical capacities are needed. Usually this would require a section consisting of two coils with one value of capacity and the third coil with a different value. This can be obtained readily with the type of smooth line described by working through two designs for the desired characteristics, using the rectangular wire type of coil, the flexibility of the design allowing changes in capacity without altering the equivalent length of section. Possibilities in this direction and the accuracy of equivalence of the three-phase line which has just been constructed are to be the subject of further investigation.

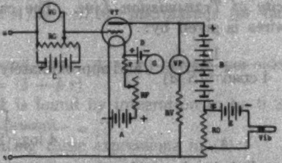

FIG. 13—VACUUM TUBE REPEATER CIRCUIT USED FOR OBTAINING OSCILLOGRAMS WITHOUT DRAWING POWER FROM THE ARTIFICIAL TRANSMISSION LINE

Mutual inductance between the artificial line coils in any one section will be required if a current or potential distribution exists which impresses a residual voltage between the neutral of the transmission system and ground. In the actual case this will result in a current flowing in the ground. Therefore, in order to calculate the value of this mutual inductance, it is necessary to know the distance of the return path through the ground from the electrical center of the system. This is variable due to the characteristics of the soil adjacent to the transmission line. Gray has assumed that the center of the return path of the ground current lies 20 feet (6.1 m.) below the surface and adds this distance to the height of the transmission system above ground in order to calculate the required mutual inductance. It is probable that this is correct for some cases and represents an average condition, although other cases might depart from it materially. Upon this basis the mutual introduced has a magnitude of about 20 per cent of the self inductance of the coils, and so can be readily taken care of. If the transmission line coils are assembled so that this value of mutual inductance exists it will then represent a different actual line from the one it would represent if no mutual inductance were present. This occurs on account of the fact that, although the mutual inductance cancels out for zero summation, or balanced currents and voltages, it nevertheless reduces the effective value of inductance in the coils, the net resultant self-inductance being equal to the total self inductance less the mutual inductance. Gray gives the formulas for computing the mutual and self inductance under any assumed conditions as:

$$L_s = 2\left(\log_e \frac{2\,S'}{d_s} + 1/4\right) \qquad \textbf{(39)}$$

abhenries per cm. per wire

$$L_m = 2\,(\log_e S'/S) \quad \text{abhenries per cm. per wire} \qquad \textbf{(40)}$$

The net resultant self inductance of the given length of artificial line will then be $(L_s - L_m)$ per wire, and this value should check the value obtained from the formulas of Appendix I where no mutual is taken into account. If the mutual inductance for a 110-kv. line of No. 0000 B & S conductors spaced 120 in.

(3.05 m.) is considered, the distance S' being taken as 50 ft. (16.3 m.), then:

$$L_m/L_s = \frac{\text{Log}_e\,(600/120)}{\text{Log}_e\,(1200/0.46)} = 0.204$$

[Omitting the term 1/4 in (39)]

Thus for this case the mutual inductance is about 20 per cent of the total self inductance. In the case of the smooth three-phase artificial line discussed it was designed to have a value of self inductance equivalent to the value of $L_s - L_m$ as found by these formulas. Therefore if the mutual inductance is introduced to correct it for equivalence under special conditions it will change the line which it represents. Since the capacity and resistance remain the same and the inductance is reduced, it will have the effect of reducing the equivalent length of section and the equivalent diameter of conductor about 10 per cent. As the total capacity per coil remains unchanged, its value per unit length of equivalent line is thus increased and this will decrease the $2\,S/d$ ratio about 20 per cent. Since the equivalent diameter of wire has been decreased a lesser amount the spacing will be slightly decreased. Thus in general the effect upon the equivalent actual line of introducing mutual between the artificial line coils is to decrease the length, conductor size and spacing by an amount smaller in proportion than the amount of mutual introduced, when expressed as a per cent of self inductance.

CONCLUSIONS

Artificial transmission lines can be successfully built with distributed constants, which will in the main duplicate the behavior of actual transmission lines, not only under steady state conditions but also under transient conditions. Certain properties of the materials used in the coils introduce effects in the artificial line which are not present in the actual line, and the change in dimensions causes some of the effects present in the actual line to be absent in the artificial line. However, as shown by the correspondence between the experimental work and the theory for relatively simple cases, the results of these divergences from exact equivalence are very small, since the discrepancies are of secondary nature, and a great deal of information can be obtained by the use of such artificial lines in studying transient and other conditions. Single-phase lines can be easily and accurately duplicated. Three-phase lines require more complexity of circuit, particularly where special configurations of line or special load conditions are to be represented, but the method of design and construction is sufficiently flexible to allow of this being accomplished. The accurate introduction of the ground plane effects and mutual inductance between wires requires further analysis and study and the possibilities of duplicating networks will be developed with further experimental work. Complete transmission systems can be represented by such coils, but in order to be fully useful the effect of terminal apparatus must be introduced by artificial equivalents as well, and this is the subject of separate investigations at the present time. From the economical standpoint this type of transmission line is cheap, compact and rugged

Appendix 1

DERIVATION OF DESIGN FORMULAS FOR ROUND WIRE COILS

Inductance of Transmission Line. From the usual theory is derived the inductance formula:

$$L_1 = 4\log_e(2\,S/d) + \mu/2 \quad \text{abhenries per cm.} \qquad (1)$$

The error in omitting the term $\mu/2$ is very small. For the configuration usually used in high-tension power lines this error is less than 2 per cent, which is smaller than the error resulting from differences in materials and dimensions of the artificial coils and can thus be neglected. Omitting the term $\mu/2$ and substituting the proper conversion factors the formula becomes:

$$L_0 = 1.482\left(\log_{10}\frac{2\,S}{d}\right)10^{-3}\,\text{henries per loop mile} \qquad (2)$$

Capacity of Transmission Line. The capacity between wires is given by:

$$C_1 = \frac{1}{4\cosh^{-1}(S/d)}\text{, and approximately,}$$

$$C_1 = \frac{1}{4\log_e(2\,S/d)} \qquad (3)$$

The result being in statfarads per cm.

The errors in the approximate formula are very small for aerial lines where the ratio $2\,S/d$ is large. For the average configuration the error cannot be detected in ordinary slide rule calculations. Therefore the approximate form is used. By substituting the proper conversion factors, (3) becomes:

$$C_0 = \frac{19.41 \times 10^{-9}}{\log_{10}(2\,S/d)} \quad \text{farads per loop mile} \qquad (4)$$

Resistance of Transmission Line. The resistance of the line is given directly by the formula:

$$R_1 = \frac{2\,l_e \times 5280\,\sigma}{d_1^2 \times 10^6}\ \text{Ohms.}$$ (Total for the length being considered). The value of σ is about 10.4 per cm. foot at 20 deg. cent. for copper. (5)

These three formulas determine the constants of the line it is desired to reproduce, the units being in the usual American system. The leakance is omitted as discussed in the body of the paper.

Inductance of the Artificial Line Coil. The formula of the Bureau of Standards (Bibl. No. 8) is simple to use and gives reasonably accurate results:

$$L_2 = [l^{4/3}/D^{2/3}]\,H \times 10^{-3}\ \text{millihenries} \qquad (6)$$

The form factor H does not vary rapidly for small changes in coil dimensions, and so may be assumed constant. (This is the factor G in the Bureau of Standards nomenclature.)

Since D is a function of the wire radius it is convenient to express: $D = 2\,y\,r$. From which:

$$L_2 = \frac{l^{5/3}}{(y\,r)^{1/3}}\,H \times 10^{-3} \text{ millihenries} \qquad (7)$$

Capacity of Artificial Line Coil. The best formula for the calculation of capacity is open to question. If the successive turns of wire were not so closely adjacent probably a logarithmic form similar to that of transmission line to ground would be more accurate. But the electrostatic field is actually so different that it does not hold. If the wire beds in the tinfoil and dielectric some form of concentric cylinder capacity formula would apply. If there is no bedding and the electrostatic field is uniform, the capacity formula for flat plates could be modified to give close results. Probably the actual case is a combination of the various circumstances enumerated, but an empirical investigation showed that the flat plate formula gives the closest results. Thus:

$$C_2 = (K\,A/36\,\pi\,x)\,10^{-5} \text{ microfarads for flat plates} \qquad (8)$$

With round wire: $A = \pi\,r$ per cm. per side of wire.
And $x = (\delta + k\,r)$
Where k is found by integration to be 0.215 if no bedding is present.

Substituting these expressions in (8):

$$C_2 = \frac{K\,r\,10^{-5}}{36\,(\delta + k\,r)} \text{ microfarads per cm. of wire per side of layer} \qquad (9)$$

Resistance of Artificial Line Coil. From the dimensions of the coil the resistance must be:

$$R_2 = \frac{l\,\rho \times 10^{-6}}{\pi\,r^2} \text{ ohms} \qquad (10)$$

The value of ρ will vary from 1.7 to 1.8 microhms per (cm.)[3]

Combination of Fundamental Formulas into Design Formulas. (2) and (4) multiplied by the equivalent length of the line in miles will give the total values of inductance and capacity. If the equivalent length in terms of these constants is solved for, there is obtained:

$$l_e = \sqrt{L_0\,C_0/28.8} \times 10^6 \text{ miles} \qquad (11)$$

Solving for the length of wire wound in the coil from (7), (9) and (11) there is obtained:

$$l = \frac{0.915\,l_e^{3/4}\,(\delta + k\,r)^{3/8}\,y^{1/8}\,10^3}{(H\,K)^{3/8}\,r^{1/8}} \qquad (12)$$

Solving for l again from (2) and (7):

$$l = \left[\frac{2.36\,\log_{10}(2\,S/d)\,l_e\,(y\,r)^{1/3}\,10^3}{H}\right]^{0.6} \qquad (13)$$

Eliminating l from (12) and (13) and solving for δ:

$$\delta = \left\{ 0.003\left[\log_{10}\frac{2\,S}{d}\right]^{1.6} [y\,r/l_e]^{0.4} \; [K/H^{0.6}] - k \right\} r \qquad (14)$$

Formula (14) determines the thickness of the dielectric that must be used in terms of known constants, or factors that can be assumed. It would be more convenient to have it in such form that the wire size would be determined as a function of δ since the dielectric is generally chosen from considerations of operating potential. Unfortunately the large number of fractional exponents make this difficult except by graphical methods. See Fig. 5.

NOTE UPON THE MINIMUM SIZE OF COIL REQUIRED

In equation (12) for a given case all the terms are constant, or nearly so, except those containing δ and r. Thus it can be written:

$$l = M\,\frac{(\delta + k\,r)^{3/8}}{r^{1/8}} \quad \text{Where } M \text{ is a constant including all other terms} \qquad (15)$$

Thus if $(\delta + k\,r)^{3/8}\ /r^{1/8}$ is made a minimum, the length of wire required in the coil will be a minimum. Substituting 0.215 for k, differentiating this expression with respect to r, and equating the result to zero for minimum, this condition becomes: $\delta/r = 0.43$.

Equation (14) may be written: $\delta = (B - k)\,r$ Where B represents the first expression within the brackets.

Thus $$B = (\delta/r) + k \qquad (16)$$

And substituting the value of δ/r found from test for minimum:

$$B = 0.645 \qquad (17)$$

From this relation it would be simple to find a definite value for r directly from the line configuration values required in (14). Unfortunately it usually results in a prohibitive value of δ. Substituting the value of B just determined in (14), the value of δ becomes:

$$\delta = 0.43\,r \qquad (18)$$

This implies the use of either extremely small wire or very heavy dielectric, both of which are objectionable for other reasons, and it would appear that the theoretical minimum length of wire cannot be taken advantage of in ordinary designs.

Appendix II

DERIVATION OF DESIGN FORMULAS FOR RECTANGULAR WIRE COILS

Inductance of Artificial Line Coil. Maxwell's fundamental inductance formula can be modified to fit in well with the design, and is therefore made use of, a correction factor (z) being included, the value to be determined from empirical data. The Bulletin of the

Bureau of Standards, Vol. VIII, p. 135 gives this formula as:

$$L = 4\,\pi\,a\,n^2 \left(\log_e \frac{8\,a}{G} - 2\right) 10^{-9} \text{ henries} \qquad (19)$$

The errors involved are due to distortion of the current sheet and the shape of the coil. Comparison of this formula with one of the more accurate ones over a range of shapes likely to be used for this purpose gives an average correction of about plus 13 per cent. The formula as it will be used in the following derivation is thus written:

$$L = 1.134\,z\,.\,4\,\pi\,a\,n^2 \left(\log_e \frac{8\,a}{G} - 2\right) 10^{-9} \text{ henries} \qquad (20)$$

Capacity of Artificial Line Coil. Due to edge effect and bedding of the conductors a very accurate capacity formula is impossible. The one for two parallel flat plates is used:

$$C = (8.85\,K\,A\,10^{-8})/(\delta) \text{ microfards}$$

If the total area of one side of the copper ribbon is assumed as one plate and the projected area upon the tinfoil as the other plate, then:

$$C = (2\,\pi\,a\,w \times 2\,n \times 0.0885 \times 10^{-6}\,K\,q)/(\delta)\ M\,F \qquad (21)$$

The correction factor (q) is inserted to adjust for variations determined empirically.

Resistance of Artificial Line Coil. The resistance of the coil will of course be given by the expression:

$$R = (2\,\pi\,a\,n\,\rho)/(w\,t) \text{ ohms} \qquad (22)$$

Leakage of Artificial Line Coil. Predetermined attempts to design for leakage are omitted as discussed in body of paper.

* * * * * * * * * *

Combination of Above Fundamental Formulas into Useful Design Formulas. Since $y = b/w\,s$ and $c = n\,p/s$. G may be expressed as

$$G = 0.2235\,(y\,w\,s + n\,p/s) \qquad (23)$$

Also it is evident that

$$a = r_1 + (n\,p/2\,s) \qquad (24)$$

Dividing (**24**) by (**23**) we obtain

$$a/G = \frac{2\,r_1\,s + n\,p}{0.447\,(y\,w\,s^2 + n\,p)} \qquad (25)$$

If this can be reduced to a numeric constant then the term $[\log_e a/G]$ in (**20**) acquires a fixed value and results in great simplification. This can be done if we make $b = 2\,r_1$, as then $r_1 = y\,w\,s/2$ and substituting this value in (**25**)

$$a/G = \frac{y\,w\,s^2 + n\,p}{0.447\,(y\,w\,s^2 + n\,p)} = 2.235 \qquad (26)$$

Substituting (**20**) in (**26**) we then get

$$L = 1.262\,z\,a\,n^2\,10^{-8} \text{ henries} \qquad (27)$$

It must always be remembered that this involves fixing the dimensions by the relation $b = 2\,r_1$.

Now dividing (**21**) by (**27**) we obtain

$$C/L = \frac{88.2\,w\,K\,q}{\delta\,n\,z} \qquad (28)$$

And multiplying this by (**4**) gives

$$\frac{R\,C}{L} = \frac{5.538\,a\,K\,\rho\,q\,10^2}{\delta\,z\,t} \qquad (29)$$

By rearranging (**27**) we can write

$$a\,n^2 = 7.93\,(L/z)\,10^7 \qquad (30)$$

And from (**28**) and (**29**)

$$\delta = \frac{88.2\,w\,K\,q\,L}{n\,z\,C} = \frac{5.538\,a\,K\,\rho\,L\,q\,10^2}{z\,R\,C\,t}$$

From which $\quad a\,n = 0.1593\,(w\,R\,t/\rho) \qquad$ (**31**)

(a) and (n) may now be obtained by simultaneous solution of (**30**) and (**31**), giving

$$n = \frac{4.975\,\rho\,L\,10^8}{z\,w\,R\,t} \qquad (32)$$

And $\quad a = 3.205\,(z/L)\,(w\,R\,t/\rho)^2\,10^{-10} \qquad$ (**33**)

By means of these and one or two self evident combinations all dimensions of the coils may now be determined.

RECAPITULATION

(all dimensions in cm.)

Mechanical Dimensions of Coil

$$a = 3.2\,(z/L)\,(w\,R\,t/\rho)^2\,10^{-10} \qquad (I)$$

$$b = 2\,r_1 = y\,w\,s \qquad (II)$$

$$c = Q\,p = n\,p/s \qquad (III)$$

$$r_1 = b/2 = y\,w\,s/2 \qquad (IV)$$

$$r_2 = r_1 + c = r_1 + n\,p/s \qquad (V)$$

Winding Dimensions of Coil

$$n = \frac{5\,\rho\,L\,10^8}{z\,w\,R\,t} \qquad (VI)$$

$$Q = n/s \qquad (VII)$$

$$p = (t + 2\,\delta + f) \qquad (VIII)$$

$$s = \frac{a \pm (a^2 - y\,w\,n\,p)^{1/2}}{y\,w} \qquad (IX)$$

Note: (IX) is derived from (**25**) and (**26**).

Dielectric Thickness.

$$\delta = \frac{88\,w\,K\,q\,L}{n\,z\,C} \qquad (X)$$

The numerical constants in (I), (VI) and (X) have been shortened to approximate figures as the accuracy is still greater than that possible in actual design due to variations in materials and methods of assembly.

Owing to the flexibility of the design in using flat strip it is possible to make various sizes of copper satisfy the same criteria of line constants by changing the dimensions of the coil. Therefore the formulas have been developed from the standpoint of using a given size of strip, since usually definite sizes are commercially available, and a few preliminary trials will quickly determine the one probably most suitable, parallel designs being carried through partway as with most electrical machine design. There are so many considerations to be taken into account in completing the design, such as desired voltage, length of section, flexibility of connections, etc., that no general rule to cover every case can be made.

Appendix III

CALCULATION OF DESIGN OF SINGLE-PHASE ROUND WIRE COILS

(Loop mile equivalent section)

The line to be represented is 110 kv., spaced ten feet (3.05 m.) with a conductor diameter of 0.30 in. (0.763 cm.). Using these values and average values of the various constants gives

S = 10 ft. (3.05 m.)
d = 0.3 in. (0.763 cm.)
$2\,S/d = 800$
$\text{Log}_{10}\,2\,S/d = 2.9031$
$y = 1.13$
l_e = 8 miles (12.8 km.)
$K = 2.8$
$H = 6 \times 10^{-4}$
δ = 0.0025 in. (0.00635 cm.)

The values required for equation (**14**) are as follows

$$[\log_{10} 2\,S/d]^{1.6} = [2.9031]^{1.6} = 5.5$$

$$\left[\frac{y\,r}{l_e}\right]^{0.4} = [1.13 \times r/8]^{0.4} = 0.455\,r^{0.4}$$

$$K/H^{0.6} = 2.8/(6 \times 10^{-4})^{0.6} = 239$$

Thus substituting in (**14**) we get

$$\delta = (0.003 \times 5.5 \times 0.455\,r^{0.4} \times 239 - 0.215)\,r = 0.00635 \text{ cm. (by assumption)}$$

Next try the value of r for No. 23 B & S wire. This is about 0.03 cm. Substituting this above we get $\delta = 0.0068$ cm.

Therefore the required insulation would be a little larger than that assumed. Referring to Fig. 5 it will be seen that a spacing of a little less than ten feet is to be expected with this combination. So make a new calculation with smaller spacing and the other constants the same, say S = 9 feet (2.75 m.), then

$$[\log_{10} 2\,S/d]^{1.6} = [\log_{10} 720]^{1.6} = [2.858]^{1.6} = 5.35$$

Using the same value of r, and all previous constants the same

$$\delta = (0.003 \times 5.35 \times 0.112 \times 239 - 0.215)\,0.03 = 0.00639 \text{ cm.}$$

This result gives a value of δ less than one per cent different from the desired value and so is satisfactory. The values for solution of (**12**) to determine l must now be derived:

$l_e^{3/4} = (8)^{3/4} = 4.75$
$(\delta + k\,r)^{3/8} = (0.00639 + 0.215 \times 0.03)^{3/8} = 0.195$
$y^{1/4} = (1.13)^{1/4} = 1.03$
$(H\,K)^{3/8} = (6 \times 2.8 \times 10^{-4})^{3/8} = 0.0913$
$r^{1/8} = (0.03)^{1/8} = 0.644$

Hence: $l = (0.915 \times 4.75 \times 0.195 \times 1.03 \times 10^{3}) / (0.0913 \times 0.644) = 14{,}800$ cm.

This value can be checked from (**13**)

$$\log_{10} 2\,S/d = 2.858$$

$$(y\,r)^{2/3} = (1.13 \times 0.03)^{2/3} = 0.1005$$

Thus: $l = (2.86 \times 2.858 \times 8 \times 1.005 \times 10^{3}) / (6 \times 10^{-4}) = 14{,}900$ cm.

This checks the previous value, and l will be taken as 15,000 cm. The resistance of the actual line will be, from (5): take $\sigma = 10.4$, then, $R_1 = (2 \times 8 \times 5280 \times 10.4)/(0.3^2 \times 10^6) = 9.4$ ohms. The resistance of the artificial line coil per section will be: from (**10**): take $\rho = 1.75$, then, $R_2 = (15{,}000 \times 1.75 \times 10^{-6}) / (\pi\,0.03^2) = 9.25$ ohms. Which checks closely with the desired values. The actual dimensions of the coils may now be obtained from (**37**) and (**38**):

$$a = 1.20 \sqrt[3]{15{,}000\,(1.13 \times 0.03)^2} = 3.08 \text{ cm. (1.21 in.)}$$

$$c = 0.662\,a = 2.04 \text{ cm. (0.80 in.)}$$

$$b = a = 3.08 \text{ cm. (1.21 in.)}$$

From a, $Q = a/p = 3.08/(0.06 + 0.0128 + 0.0076) = 38.4$, so make layers 38

From b, $s = b/2\,y\,r = 3.08/(2 \times 1.13 \times 0.03) = 44.2$, so make turns per layer 44.

* * *

These results are not exactly the same as the final coil specifications. The final coils were given the last design adjustment after a trial of several samples in order to correct for the particular manufacturing variations encountered. The wire obtained for the actual coils also ran somewhat oversize which necessitated changes. The procedure to be followed after this step in the design would be to make up several samples and then obtain corrected values for the constants under the particular conditions, from which the final coil specifications can be drawn up.

Appendix IV

CALCULATION OF FINAL DESIGN OF 3-PHASE ARTIFICIAL LINE COILS

The values of the constants as determined from the trial coils have the following values:

$C = 0.2855$ μf.
$L = 0.0418\,H$.
$R = 5.185$ ohms.
$K = 2.8$
$q = 1.40$
$z = 1.14$
t = No. 28 B & S tinned 0.01304 in.
$y = 1.25$
$\rho = 1.8 \times 10^{-6}$
$\delta = 0.00634$ in.
$f = 0.002$ in.

From formula:

(X) $w/n = 0.365 \times 10^{-3}$
(VI) $w\,n = 192.0$

Thus n = 725……Total number of turns
and w = 0.264 cm. = 0.104 in.
Width of strip

(I) a = 5.56 cm. = 2.19 in.
Average winding radius
(VIII) p = 0.0705 cm. = 0.0277 in.
Layer pitch
(IX) s = 28.2...... Turns per layer
Make s = 29 to avoid fractional layers
(VII) Then Q = 25........ Number of layers
(II) b = 9.58 cm. = 3.77 in.
Axial length of winding
(III) c = 1.76 cm. = 0.695 in.
Radial depth of winding
(IV) r_1 = 4.79 cm. = 1.88 in.
Inside radius of coil
(V) r_2 = 6.55 cm. = 2.58 in.
Outside radius of coil

Appendix V

Conversion formulas for determining the configuration of a transmission line represented by one or more artificial line coils.

$$l_e = 0.31\sqrt{19.9\,C^3 + 36 \times L\,C \times 10^4} - 1.38\,C \tag{34}$$

$$d_1 = 0.234\sqrt{l_e/R} \tag{35}$$

$$\text{Log}_{10}\,s = 38.9 \times 10^{-3}\,[l_e/C] + \log_{10} d_1 - 0.301 \tag{36}$$

l_e to be taken in miles
d_1 " " " " inches
S " " " " inches

One mile = 16.0×10^4 centimeters.

Effect of variations in coil constants upon equivalent line dimensions.

+ 10 per cent error in	R gives + 0 per cent error in l_e. L gives + 4.5 per cent error in l_e. C gives + 4.7 per cent error l_e.
+ 10 per cent error in	R gives + 4.8 per cent error in d L gives + 2.4 per cent error in d C gives + 2.5 per cent error in d
+ 10 per cent error in	R gives a − 4.4 per cent error in S L gives a − 38 per cent error in S C gives a − 23 per cent error in S.

Appendix VI

THE USE OF VACUUM TUBES IN RECORDING TRANSIENTS

In general it is most convenient to record the transients upon the smooth transmission lines by means of oscillograms. For current readings the vibrator of the oscillograph may be inserted directly in the circuit without adding sufficient resistance to cause serious error. For voltage readings, however, the power required to operate the oscillograph represents a very large equivalent load upon the transmission line due to the small scale upon which the work is performed. Therefore it can only be used directly when connected to the terminals of the source of power. The interesting cases usually involve the voltage at other points on the line, and for obtaining these use may be made of three-electrode vacuum tubes, or Triodes, of the usual type developed for communication work. Providing the grid is maintained negative, practically no current is drawn, and the change in the charge upon the grid may be caused to control sufficient current in the plate circuit to successfully operate the oscillograph.

Fig. 13 shows one of the circuits used for this purpose. It is necessary in dealing with transients to eliminate reactance in the repeater circuit to as great an extent as possible. For this reason it is not possible to use a transformer to couple the plate circuit to the oscillograph, and the impedances of the two circuits thus connected being of very different magnitudes, the combination is not very efficient. The resistance *R O*, Fig. 13, is used as a potentiometer connection, one terminal and a variable tap being connected to the oscillograph vibrator Vib. through the battery *E*. The steady flow of current through the plate circuit due to the battery *B* is thus balanced out of the vibrator circuit, and the oscillograph will be operated by the variations in current produced by changes in the charge upon the grid. The voltmeter *V P* measures the plate potential, and, if small batteries are used, must be left connected all the time if the apparatus is to be calibrated. Care must be taken that the resistance in this d-c. voltmeter does not contain inductance or distributed capacity of large enough amount to by-pass the transient to some extent. *R F* is the usual filament control resistance, and should be of low value to give close adjustment. If the circuit is to be calibrated to give quantitative readings the filament temperature is very important in order to obtain duplication of results, and usually cannot be maintained sufficiently accurately with an ammeter. Therefore the resistance *r* and single dry cell *D* are introduced so that the filament current can be adjusted by means of the galvanometer *G*. The resistance *r* should have such a value that *G* will be on zero when the proper filament current is obtained. If only proper duplication of the wave without its quantitative value is required, then this refinement can be omitted.

The battery *C*, resistance *R G* and voltmeter *V G* are used to adjust the grid potential. *M* and *N* are the input terminals. If the resistance between *M* and *N* is extremely high, the return circuit of *C* through to the filament is blocked off and the battery will not be effective. Therefore it is sometimes necessary to connect about 1/2 megohm across *M*-*N* to ensure proper control of the grid potential and to increase potential of battery *C*. This potential should be adjusted so that the current in the plate circuit lies midway on the straight portion of the triode characteristic curve when no potential is impressed upon M-N. Sometimes, if it is known that the transient voltage is to extend much further in one direction than the other, it is desirable to adjust to a different part of the curve in order to extend the range of the repeater without distortion. Three 5-watt power tubes in parallel connected at $V\ T$ were found to give sufficient current variation to operate the oscillograph successfully. About 5 volts plus or minus can be recorded without serious distortion. The repeater can be calibrated upon a d-c. circuit with fair accuracy, if care has been taken in eliminating reactance, and its faithfulness of reproduction can be observed by impressing an a-c. wave upon one vibrator direct and upon a second through the repeater, superimposing them upon the viewing device of the oscillograph. With correct adjustment the maximum deviation from duplication should not be greater than the width of the beam of light from the oscillograph mirror.

The operation of the repeater under transient conditions can also be checked either by photographing transient waves in a circuit where sufficient power can be drawn off to operate one vibrator direct without distorting the circuit. This can be visually observed if a switch is made up to alternately open and close a contact, causing transients in an experimental circuit, synchronously with the viewing device.

Other combinations can be used, but this seems to be the simplest to make up with the ordinary laboratory apparatus available. Distortion can be reduced by some form of push-pull circuit, and the use of a high inductive reactance and low capacity reactance as a transient block and pass system can be substituted for the potentiometer arrangement $R\ O$ and E. But this is likely to introduce difficulties due to distributed capacity and similar attendant troubles.

LIST OF SYMBOLS USED IN FORMULAS

A	Surface area of one plate of condenser in sq. cm.
C	Capacity in microfarads per section per wire. (Or to neutral).
C_1	Capacity between transmission wires in statfarads per cm.
C_o	Capacity between transmission wires in farads per loop mile.
C_2	Capacity of artificial line coil in microfarads.
D	Winding pitch per turn $= b/s = y\,w = 2\,y\,r$. Dimensions in cm.
G	$0.2235\ (c + b)$.
H	Form factor for inductance formula, see Bib. No. 8.
K	Dielectric constant. 2.5 to 2.9.
L	Inductance in henries per section per wire. (Or to neutral).
L_o	Inductance of transmission system in henries per loop mile.
L_1	Inductance of transmission system in abhenries per cm. per pair of wires.
L_2	Inductance of artificial line coil in millihenries.
L_s	Self inductance of one wire to ground return—abhenries per cm. per wire.
L_m	Mutual inductance between two wires having common return abhenries per cm. per wire.
Q	Total number of whole layers of winding.
R	Resistance in ohms per section per wire.
R_1	Resistance of total length of actual line, ohms per loop mile.
R_2	Resistance of artificial line coil in ohms.
S	Spacing of transmission line, in same units as d unless otherwise stated.
S^1	Distance between transmission wire and common ground return, inches.
a	Average winding radius in cm.
b	Axial length of winding in cm.
c	Radial depth of winding in cm.
d	Diameter of actual transmission line conductor. In same units as S unless otherwise stated.
d_1	Diameter of transmission line conductor in inches.
f	Thickness of tinfoil in cm.
k	A constant in capacity formula, usually equal to 0.215.
l_e	Equivalent length of transmission line in miles.
l	Length of wire wound on artificial line coil, in cm.
n	Total number of turns per coil.
p	Layer pitch or thickness of one layer, in cm.
q	Capacity correction factor, usually about 1.10 to 1.5.
r	Radius of round wire used in coils, cm.
r_1	Inside radius of winding cm.
r_2	Outside radius of winding in cm.
s	Turns per layer in winding.
t	Thickness of strip conductor in cm.
w	Width of strip conductor in cm.
x	Average distance between surface of round wire and tinfoil, cm.
y	$b/w\,s$, the ratio of total winding length to net possible winding length. 1.3 to 1.6.
z	Inductance correction factor. 1.14 to 1.21.
δ	Thickness of dielectric in cm.
ρ	Resistivity of conductor in microhms per cm.^{3} 1.72 to 1.85 for copper.
σ	Resistivity of transmission conductor in ohms per circular mil foot. 10.4 to 11.
μ	Permeability of transmission line conductor.

Bibliography

1. H. A. Taylor and Alex. Muirhead, British Patent Mo. 684 of 1875.

1A. C. F. Varley, British Patent No. 3453 of 1862.

2. M. I. Pupin, "Propagation of Long Electrical Waves," TRANS. A. I. E. E., March 1899, Vol. XVI, pp. 93-142.

3. G. H. Gray, "Design Construction and Test of an Artificial Power Transmission Line for the Telluride Power Company of Provo, Utah." TRANS. A. I. E. E., Vol. XXXVI, p. 789, 1917.

4. A. E. Kennelly, "Hyperbolic Functions Applied to Electrical Engineering." McGraw-Hill, 1916.

5. A. E. Kennelly, "Artificial Electric Lines." McGraw-Hill, 1917.

6. A. E. Kennelly and U. Nabeshima, "The Transient Process of Establishing a Steadily Alternating Current on a Long Line, from Laboratory Measurements on an Artificial Line." *Proc. Amer. Phil. Soc.*, Vol. LIX, p. 325, 1920.

7. J. H. Cunningham, TRANS. A. I. E. E., Vol. XXX, Part 1, pp. 245-256. February 1911.

8. Circular of the Bureau of Standards, No. 74, p. 293, 1918.

Discussion

J. F. Peters: I would like to comment on the paper by Professor Dellenbaugh. The performance of a simple transmission line at normal frequency, where the supply is at one end and the load at the other, can be calculated very accurately and with a small amount of labor. When the load or supply is distributed at several points, or there are a number of branches, the matter becomes considerably more complicated and an artificial line may be desirable; it may offer the easiest method of solution. However, the solution of the performance of the transmission line is not the important problem. The important problem is the solution of the transmission line including the rotating machinery; that is, it should include the reactions of the generators and the synchronous condensers that are generally used on high-voltage transmission.

The reactions in the machines are rather complicated, which would point more toward the solutions by means of artificial set-ups. But the reactions are individual; that is, vary widely for different machines, and particularly vary widely between large and small machines. So that in making a set-up of an artificial line it is necessary to obtain approximately the reactions that will be obtained in the final machine. That being the case, it is necessary to use machines in this miniature or artificial transmission line that have appreciable dimensions.

With reference to abnormal voltage conditions the ones that we are particularly interested in are those in the nature of surges or traveling waves. When an abnormal voltage or surge of this nature strikes an artificial line made up of lumped constants such as those described by Professor Dellenbaugh, although he obtains uniformly distributed capacitance, the conductance is still lumped, and a surge reaching one unit of that artificial line will first penetrate a few turns. Thus the inductance per turn will be only the inductance of that turn. As the surge penetrates further into the coil, it encounters very much higher inductance per turn, because then the inductance is the inductance of those turns plus the mutual inductance of the whole coil. The performance of a surge on a coil of this kind would pass through a wide cycle of constants periodically. It is very interesting to note the absence of these secondary reflections in the oscillograms. However, the secondary reflections in this set-up would have a reflection of approximately one-fiftieth of the length of the whole line, and it is possible that the oscillograph would not record that. It seems to me that with the facilities of artificial lines to date, the results obtained can be used only to corroborate calculations.

O. R. Schurig: An increasing number of the electrical-engineering problems met by transmission-line and operating engineers call for experimental investigations on miniature (or artificial) circuits. The problems are of two kinds:

(1) High-frequency problems, arising in connection with lightning, switching, arcing grounds, etc. The high-frequency problems ordinarily involve measurements at only a few points of a system, the currents frequently are relatively small (*i. e.* far below normal-load values), and the phenomena are of a transient character calling for a sensitive, high-frequency oscillograph. Uniformly distributed line constants are essential.

(2) Normal-frequency problems in large system networks, such as the determination of currents, voltages and their relative phases, both in normal operation and during short circuits. The normal-frequency problems often involve a large number of measuring instruments (frequently thirty, or more, in polyphase circuits); the currents are of the order of normal-load magnitudes or greater. Both steady-state and transient-state observations are called for. Indicating instruments exclusively may be used in many cases, while oscillographs are required in others. Lumped-line constants are generally permissible, though sometimes uniformly distributed line constants are preferable.

A question at once arising is whether there is one type of miniature system which will permit the solution of both kinds of problems, that is of both the high-frequency and the normal-frequency problems. Since the "lumpy" type of line is not applicable to the high-frequency problems, the question becomes, more specifically, whether the lines of the type described in the paper are practically applicable to both high-frequency and normal-frequency problems.

The practical success of a miniature system for the solution of normal-frequency network problems calls for the minimum of complications due to measuring instruments, in view of the large number of practically simultaneous observations desired. In the solution of high-frequency problems the practical objection to the use of special measuring instruments (such as those employing vacuum-tube amplifiers) is much diminished because, as a rule, only a few of the special instruments are required.

Hence it is of importance to know how large the rating of the miniature equipment must be made in order that standard instruments may be employed without serious error, for both steady-state and transient-state observations at normal frequencies. It is conceivable that an increased current rating of the miniature circuit would also permit simplifying the measurements of high-frequency transients. It is the object of the following discussion to establish the minimum current rating of the miniature system which will permit the use of standard measuring instruments for normal-frequency problems.

As the author properly points out, the errors due to instruments are of two kinds:

(1) That due to the impedance introduced by series-connected instrument elements, such as ammeter or wattmeter current coils;

(2) The error due to the current abstracted from the circuit by shunt-connected instrument elements, such as voltmeter or wattmeter potential circuits.

The error due to series-connected instruments will be seen from figures for typical ammeters and wattmeters. It will not be attempted to give impedance data for the large variety of instruments of different manufacture. The data given in Table I are representative values indicating the order of the magnitudes. It is to be understood that materially larger values of impedance than those given as well as smaller values, are met among the different types of instruments.

If, for example, 10-ampere and 5-ampere current ratings are chosen, typical values of voltage drop at maximum current are those of Table I. If the ammeter and the wattmeter current-element are connected in series, as for power-factor measurements the combined voltage drops, due to the instruments, are seen to be a negligible percentage of the generator voltage in a 100-volt miniature circuit. However, the drops exceed 5 per

TABLE I

Approximate impedance drops at maximum r. m. s. current at 60 cycles per second

	For 5-ampere rating	For 10-ampere rating
Ammeter	0.50 volt	0.25 volt
Wattmeter current coil	0.65 volt	0.30 volt
Ammeter and watt meter element in series combination		
Voltage drop	1.15 volt	0.55 volt
Per cent drop in a 100-volt miniature circuit	1.15 per cent	0.55 per cent
Per cent drop referred to 10 volts	11.5 per cent	5.5 per cent
Oscillograph*		
Voltage drop	0.3 volt	0.3 volt
Per cent drop in a 100-volt miniature circuit	0.3 per cent	0.3 per cent
Per cent drop referred to 10 volts	3 per cent	3 per cent

Note: The data given apply to one of the customary types of commercial oscillographs, for which the vibrators have 1.5 ohm resistance and a current rating of 0.2 ampere; shunts are used for current values in excess of 0.2 ampere.

cent for the 10-ampere rating, and 10 per cent for the 5-ampere rating, when expressed as a percentage of the line drop over a line having a voltage drop of 10 per cent of the generator terminal voltage. The reason for referring the per cent error, due to series-connected measuring instruments, to the *line drop*, rather than to the generator terminal voltage, is that in network problems the division of currents over parallel-connected lines depends largely on the relative impedances of the lines.

The above considerations call for a miniature-line current rating of the order of 10 amperes for a 100-volt circuit, or 5 amperes for a 200-volt circuit, so that the errors due to the series elements of typical portable ammeters and wattmeters may not approach or exceed 10 per cent. Smaller current ratings will lead to larger errors due to the series elements of the instruments for the circuit conditions under discussion, because, roughly speaking, the impedance drops of the series elements at rated currents increase inversely as the current ratings.

The second error mentioned was that due to the current abstracted by voltmeter or wattmeter potential coils. The figures for typical voltmeter and wattmeter potential coils are those of Table II. It is seen that the largest errors due to shunt elements are those of the oscillograph, which gives a 4 per cent error for a 5-ampere circuit rating. For a 200-volt system, the percentage values for current abstracted will be the same as those given in Table II, provided the 300-volt elements are employed.

TABLE II

Approximate current at 60 cycles per sec., at 100 volts

Voltmeter (150-volt element)	0.050	amp.
Wattmeter potential coil (150-volt element)	0.030	amp.
Wattmeter potential coil and voltmeter in parallel combination (150-volt elements)		
Total current	0.080	amp.
Per cent current referred to 5 amperes	1.6	per cent
Per cent current referred to 10 amperes	0.8	per cent
Oscillograph* (at full-scale deflection)		
Current	0.2	amp.
Per cent current referred to 5 amperes	4	per cent
Per cent current referred to 10 amperes	2	per cent

*See note applying to Table I

It may, therefore, be concluded that an a-c. miniature circuit-current rating of not less than 5 amperes and preferably 10 amperes is required to limit the errors, due to the customary portable instruments and oscillographs, to values safely below 10 per cent. In regard to instrument errors, the 200-volt circuit rating is preferable to the 100-volt rating. If, therefore, the type of miniature circuit having distributed circuit constants, such as that described in the paper, can be constructed for the increased current ratings mentioned, the number of different types of artificial circuit could be materially reduced.

The current ratings established apply, of course, to the lumpy type of miniature system, as well.

D. C. Jackson: From the standpoint of the practising engineer, the value of this struggle with artificial transmission lines and other artificial circuits is to aid in discovering the analysis which will enable one to preconceive the reactions that will occur on an actual line, rather than make the design of the line without a full and adequate comprehension of all the factors that will arise.

But there is another important feature, which was referred to in the President's address, that I want to emphasize, and that is the relation of these things to the educational process.

A good many years ago there was that roomfull of line that has been referred to this morning as the Steinmetz artificial line, and at that time Dr. Pender (who was one of my colleagues at the Massachusetts Institute of Technology) and I had some discussions over the best methods of teaching the subject of power transmission from the theoretical aspects, for the purpose of enabling our students to have a correct vision of the problems. We also about that same time fell into some considerable discussions of the problems of skin effect. The consequence was that there was started a series of experimental investigations of skin effect in various conductors which resulted in a number of papers. Perhaps a quarter of the references in the bibliography accompanying Mr. Dwight's paper are papers that arose as a consequence of that argument between Pender and myself, which was later also taken up and carried still farther by Dr. Kennelly when he became a colleague of ours.

At the same time Dr. Pender and I struggled with the problem of a transmission line which could be long, preferably as long as a quarter of a wave length with 60 cycles per second. Consequently, a replica of a line about 1000 miles long with conductors of two or three thousand circular mils and spacing for a couple of hundred thousand volts, was made up. It is quite an expensive affair but it does not occupy a roomful of space by any means. It was made up in that characteristic form that was shown in a paper some years back by Dr. Pender, of pancake coils placed in a toroidal arrangement so as to secure substantially distributed resistance and self-inductance without much mutual induction between coils.

That line was made up with large enough conductors so that it would carry sufficient current for enabling us to put oscillographs, ampere-meters and voltmeters in any part of the line we chose to without disturbing its performance and it works admirably for steady state conditions up to several hundred cycles, per second, but the capacity is lumpy and it does not represent distributed effect with transient conditions.

It is on that line that Professor Ricker, for instance, a good many years ago, as shown in a paper of his, found experimentally the difference in admittance for odd and even harmonics which exist between even and odd quarter wave lengths.

Later Dr. Bush took up this aspect of our work. He came to me one day and urged that our students will not be meeting the present situation in the art unless we can deal with transients experimentally in the laboratory on long lines. One of the outcomes is the artificial line equipment described in the papers by Professor's Bush and Dellenbaugh.

What I want to emphasize is this: that while the sort of work described by Dr. Bush and Professor Dellenbaugh in their papers, and referred to by Mr. Dwight in his paper is directly serviceable to the art, the fact is that that work which you find described in the papers of Bush and Dellenbaugh, and the papers that are referred to by Mr. Dwight that came from Kennelly, Laws, Pierce, and so on, was actually planned for educational purposes and was carried out for those purposes; that the philosophy of the thing is to bring the senior students and graduate students in the engineering school into contact with the most advanced philosophy of engineering structures that we have; and, if possible, carry them on into experimental and mathematical work which advances the state of knowledge to some degree as well as soundly founding them for creative work in the art.

That is not what the physicist would call fundamental research, but it is definite and effective engineering research, and when carried on by senior students and graduate students in laboratories that are associated with the under-graduate work, it affects the junior students by causing a tremendous stir of the heart and expansion of the ambition. If this ideal can be more fully adopted by the engineering schools of the United States and even greater sympathy for it be exhibited by the industries of the United States, it will during the next decade change the whole aspect of our engineering practise, because it will give us a more and more effective group of young engineers coming forth into the profession year by year.

The fact that the thing has been fruitful—it has been carried on for a dozen years at the Massachusetts Institute of Technology as a definite philosophy of engineering education and is carried on in other institutions also to a considerable degree;—that it has been fruitful is shown by the fact that in the bibliographies of the papers presented today and in the discussions related to them you will find that a considerable number of the papers referred to have been produced as a consequence of this philosophy. For instance, papers by Lyons, Kennelly, Laws, Pierce, Affel and others come within the category. Professor Bush as a graduate student came in contact with the ideal and he has been very effectively carrying it on since he joined our staff, along with Professor Dellenbaugh, Professor Kennelly and a number of our other colleagues on the staff of the department from which these papers came.

I do not wish to divert attention from the important problems of engineering practise and the philosophy underlying engineering practise in this matter of long distance transmission of power, but it seems to me that it is worth while introducing here this educational aspect of these papers on account of its great interest to us as electrical engineers since, of the various branches of the engineering profession, probably electrical engineering is more soundly based on the fundamental physics and mathematics than any of the other branches.

Harold W. Buck: There are a few unorthodox considerations that I want to speak of in connection with this general question of power transmission particularly in connection with the papers under discussion. These papers are extremely interesting and exhibit wonderful mathematical and engineering skill but why do we have so many papers on this particular subject? It looks like a confession of weakness somewhere.

One of the speakers has referred to the fact that troubles on transmission systems increase as the square of the size of the network. I think that is quite true. Our very high-voltage systems of great length of today are beginning to bring up difficulties and complications which are really very serious. The papers are the result of this situation and are an attempt to analyze the phenomena and to predict what is going to happen under various conditions.

I think a good test of engineering progress is whether simplification is resulting. If we look around and into our power transmission problems at very high voltage at the present time and apply that test, it seems to me that the result is not satisfactory.

The transmission of power from a piston for instance to a rotating wheel through a connecting rod is a very simple proposition but when the connecting rod is lengthened out to such a distance that its inertia and elasticity become factors which cannot be controlled then some other method must be found.

A transmission line is merely a connecting rod and in the very high-voltage lines of great length the inertia and elasticity are becoming difficult factors to handle and the papers under discussion prove it.

Our big transmission systems at very high voltage are not getting more simple but more complex and I think we will have to admit whether we like it or not that many new troubles are developing in connection with them.

Take for instance a 200-mile line at 220,000 volts. We have here a charging current of the order of 50,000 kv-a. This requires for satisfactory control a synchronous condenser installation approximating in size the power house itself. We accept it from force of habit and tradition but it is a grotesque situation from the standpoint of real engineering. The switches are becoming of enormous size and cost and all controlling apparatus is increasing in complexity and cost. Worst of all instability is increasing. Satisfactory adjustments may be made for a given set of conditions but if a sudden change occurs as must frequently happen things are thrown badly out of balance.

All of these things we accept as a matter of course because we have gradually been led into them but it would be well to consider under all the circumstances whether we are not really approaching the limit of transmission of power by alternating current.

There is a great deal of loose talk in the papers nowadays of transmission of power a thousand miles at a million volts and super-power bus bars running from New York to San Francisco. This is all, of course, absurd and if any such thing were attempted under the present state of the art I do not think there would be mathematicians enough in the country to figure out what had happened after it was all over.

This is all more or less heretical but it seems to me that engineers should pause occasionally in the evolution of engineering and see whether they are really headed in the right direction. After all, reduced to its lowest terms, transmission of power is merely the transfer of electrons from one point to another. If we consider all that is involved in a three-phase 220,000-volt transmission it seems almost obvious that we are carrying out this physical operation in a very clumsy way.

I have no suggestions to offer except to go slow. However, physicists are learning more and more about electrons, their characteristics and methods of controlling them. I cannot help feeling that before this great long distance super-power problem is worked out satisfactorily some radically new method of transferring electrons from one point to another will be worked out.

F. S. Dellenbaugh, Jr.: I would like to add just one thing, that we have since this paper was written built an artificial cable by this same method. The formulas employed and the constants are a little different but I don't think there is any necessity for describing that separately.

I want to offer one word of apology on the systems in measurement Dr. Kennelly said, "What a perfectly horrible lot of units you have!" I admit it. The difficulty lies in the fact that to make it practical, you must deal with transmission lines in the unit you are used to. You don't design 16,000 centimeters of transmission line—that doesn't mean much to you. You deal in miles, unfortunately, but still you do. And so we have miles in length, feet in spacing, and inches in diameter. The fundamental formulas deal with centimeters in C. G. S. or absolute units. So we must convert from the metric or absolute units of our fundamental formulas for the coils to ultimate dimensions, obtaining the line in miles, feet and inches; and as a result I have attempted to put in the duplicate units all the way through.

It is very interesting to note that the work of Mr. Boyajian on the penetration of transients fits in directly with this kind of work, even though he dealt with transformers and we are dealing with transmission lines. It is the same thing, and it fits particularly on the lumpy type of line. In the old type of lumpy line, if we applied a transient or surge, it would partially penetrate the inductance coils. If they were perfect inductances, with no distributed capacity, there would be no penetration and the surge, when it hit the inductance, would immediately be totally reflected, and therefore the lumpy type of line would be absolutely useless for transient investigation. Practically, the coils must have distributed capacity, and so a transient on a lumpy line does penetrate the coils and does give you curves which represent more or less roughly the same curves you get on smooth lines, but filled with a lot of ripples due to partial reflections. The result is that the behavior of the lumpy line under transient condition depends upon the distributed capacity of the coils, which is an entirely haphazard thing and is not taken into account in the design. If you wish to look at it in that way, all this distributed line has done is to design the distributed capacity of the coil so that it is what you want, and leave out the lumped capacities.

Both Mr. Boyajian and Mr. Peters mentioned skin effect, dielectric losses and their relations. Mr. Peters also mentioned that the penetration of the transient, even in the distributed constant lines, as it went through a single coil, had to go through an infinite number of varieties of inductance. That, of course, is perfectly true, and is one of the points that worried us when we built the lines. So our first line was built with 8-mile sections. We built about 1000 miles of it in order that the sections might be small compared to the total length, and that the total length might be enough to record easily on the oscillograph.

We were very much pleased to find that what we might call the "lumped mutual" of the distributed line has no apparent effect on the oscillograms and that they check remarkably well with the theory. Therefore, we might say that the proof of the pudding is in the eating in this case, and that those minor reflections which occur due to the penetration through the coil are of such a small magnitude that they do not show and are not recorded. That is, we don't attempt to say that we have a line which is exactly the same as an actual transmission line, because the only thing that is exactly the same is another transmission line built the same way. Even then the insulators and ground conditions would be different. But we have built a line which behaves very nearly the same way, near enough so that if we get reasonable checks we know that our theory is on the right track.

If we take fundamental physical laws and arrive at results, then take a transmission line which is somewhere near an actual line and test it and arrive at certain experimental results, then if the experimental results and the theoretical results coincide with reasonable accuracy, we can say. "Well, our theory must be pretty good." The final test is to go out and try it on a real line.

We did that last spring and found that our artificial line was very much better than the real line, because it checked the theory much closer. That of course, is a necessary and expected result because the real line is complicated with a lot of factors that can be eliminated entirely in the artificial line.

For the general study of transients and the development of theory in connection with them, we feel the artificial lines are very good, but we admit entirely all of these questions of errors: The effect of dielectric hysteresis; the fact that you still have skin effect but not as much as you have in the actual line; and the fact that there is a certain amount of lumped mutual and a certain value of cut-off due to periodic structure. They must be there, but the evil of their being there is far less than with the lumpy type of artificial line.

Mr. Schurig mentioned the question of errors due to current carrying capacity and was kind enough to state that the distributed lines can be made larger. With the distributed lines made with round wire, there is only one possible coil that will represent a certain combination of circumstances, so that you cannot make those larger; but with the distributed lines having flat wire, you can make them of any size you please because there are an infinite number of coils that will represent any particular set of circumstances, and as a result you can make the flat wire lines to carry any current you please. It is merely a question of size and expense.

The cost of the 2 lines that we have built—one a thousand miles single-phase, the other 450 miles 3-phase—was 60 cents per mile per phase for both. This covers the manufacture of the coils by an outside concern, and included their profit, I think.

For our work, we have found it much more convenient to use vacuum-tubes and meters drawing small power because that is the way our laboratory works. After you once get a vacuum-tube repeater trained and used to you, and not nervous about the particular person running it, it behaves very well. Every person that takes it up has the same troubles over again. It is apparently impossible to eliminate them by instruction. After it once goes, it behaves very well.

For instance, we have made a number of transient investigations with a 6-volt storage battery for our supply of power. That gets you away from any question of leads through conduits affecting the first surge. We measure the voltage rise on the line at different points, and the current through the line at different points, with 5 or 6 volts impressed on the near end. You can go up, of course, as far as you please, up to the limit of dielectric strength.

With regard to the errors Mr. Schurig mentioned, I would like to point out one thing: The series connection gives practically negligible errors under most conditions, particularly with transient work. The shunt errors are more serious than he pointed out, if you are dealing with transients. That is, his results were calculated for load conditions and more or less for steady work. What would calculate as a very small shunt error under his method might result in a rather large error if you dealt with transients on a line open circuited, for instance. I do not believe you can get transients on an open-circuited line with any kind of volt-meter across it; that is, even a very high resistance will pull it down greatly. We have found it necessary to go to several megohms resistance across the line in order to use the repeater without error due to the shunt resistance.

I would like to say a word on the scale of the system. The scale can be made extremely small, providing you are willing to go into the difficulties of testing with it. It is a question of a balance between the testing technique and the amount of room you want to give up to the line. Our thousand mile single-phase line is mounted in a rather loose framework. We leave plenty of room around and it stands about 4 feet high; in fact, it is very nearly 4 feet on a side—not quite, but we will say a 4-foot cube will include 1000 miles of single-phase line. The 3-phase line is very much larger on account of the way we have mounted the coils. They have been mounted so that mutual inductance can be eliminated; that means a lot of waste room. I don't remember the exact dimensions of the boxes, but they are, roughly, 4 feet square and about 5 feet high and take up, I should say, about 25 per cent more room than the single-phase line in cubic feet. Both of these are used for small current and both of them could be designed in considerably smaller space if it was necessary.

A General Consideration of T- and Pi-Type Artificial Electric Lines

In Connection with a Proposed Compensated Pi-Line

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***Review of the Subject.**—A theory of the general artificial electric line circuit, including the T-and π-lines as its special case, is here laid down. This theory is applied to the determination of a simple method for designing a compensated π-type artificial electric line whose electrical characteristics can be more closely approximated to the actual uniform line than a common T-line or π-line, when required to work, not at a single frequency, but over a definite range of frequencies.*

IT IS well known that *T*-type or π-type artificial electric lines may be made externally equivalent to a uniform line, but they represent different characteristics at other frequencies. When compared with a uniform line, the characteristics of the *T*-line and π-line are in opposition to each other if they are represented by a complex number. (Bib. 3.)

In the same way that we can construct an artificial line which has better frequency characteristics than the *T*-line or π-line, by combining these two lines properly, we may obtain a similar result by using such artificial lines as are shown in Fig. 3. This construction is intermediate between that of the *T*-line (Fig. 1) and π-line (Fig. 2), and this type of artificial line will here be called the "Compensated π-line."

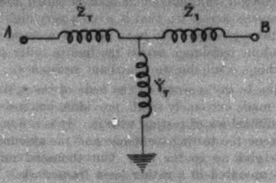

FIG. 1—T-LINE SECTION

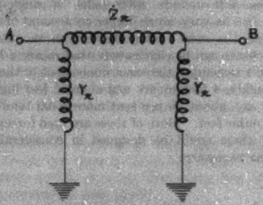

FIG. 2—PI-LINE SECTION

I. THE THEORY OF THE COMPENSATED π-LINE

In Fig. 3, $Z_{c\pi}$, Z_π and Z_c represent impedance between *A B*, *C D* and *D B* respectively. Z_c will be called the "Compensating impedance." $Y_{c\pi}$ represents the shunt admittance.

We will consider the compensated π-line which is to represent the uniform line (Fig. 4). In Fig. 4, Z_0 and θ_0 denote the surge impedance and the total hyperbolic angle (vector attenuation constant) respectively. We put

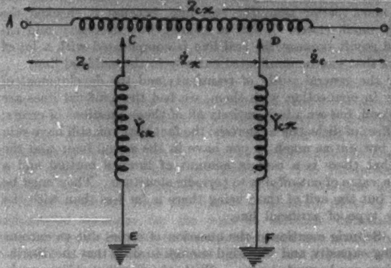

FIG. 3—COMPENSATED PI-LINE SECTION

$$\beta = \frac{2\,Z_0}{Z_{c\pi}} \qquad \text{numeric} \ \angle \qquad (1)$$

which represents the degree of compensation. In order to have the conditions such that the compensated π-line and the uniform line have the same electrical behavior at the terminals *A* and *B*, we will consider the case when the receiving end is short-circuited. The currents at the receiving end *B*, and at the sending end *A*, are represented by I_B and I_A. In Fig. 3 we get

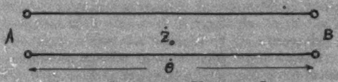

FIG. 4—CONJUGATE UNIFORM LINE

$$I_A = [1 + Y_{c\pi}\,(Z_\pi + Y_{c\pi}\,Z_c\,Z_\pi + 2\,Z_c)\,]\,I_B \qquad \text{amp.} \ \angle \qquad (2)$$

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 25-29, 1923.

In Fig. 4, we get (Bib. 1, 2)

$$I_A = I_B \cosh \theta \qquad \text{amp. } \angle \qquad (3)$$

According to the condition of equivalency, we have

$$\cosh \theta = 1 + Y_{c\pi} (Z_\pi + Y_{c\pi} Z_c Z_\pi + 2 Z_c) \qquad \text{numeric } \angle \qquad (4)$$

From the equation (1)

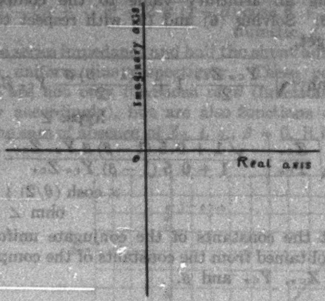

FIG. 5—β-PLANE

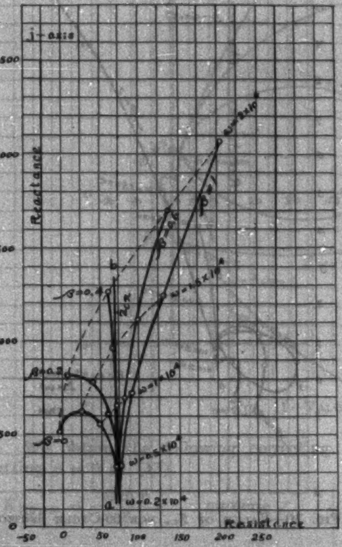

FIG. 6—VALUE OF $Z_{c\pi}$ FOR DIFFERENT VALUES OF β

$$\left.\begin{aligned} Z_c &= \frac{\beta Z_{c\pi}}{2} \\ Z_\pi &= (1-\beta) Z_{c\pi} \end{aligned}\right\} \text{ohm } \angle \qquad (5)$$

are obtained. By (4) and (5), we have

$$\cosh \theta = 1 + Y_{c\pi} Z_{c\pi} \left\{ 1 + \frac{(1-\beta)\beta}{2} Y_{c\pi} Z_{c\pi} \right\} \qquad \text{numeric } \angle \qquad (6)$$

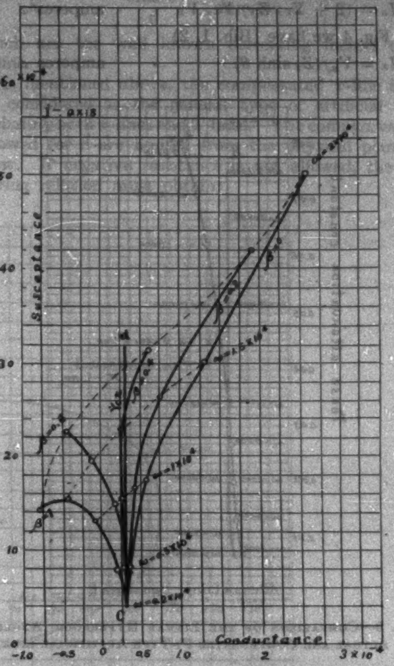

FIG. 7—VALUE OF $Y_{c\pi}$ FOR DIFFERENT VALUES OF β

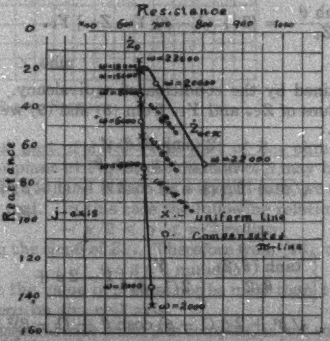

FIG. 8—COMPARISON OF THE SURGE IMPEDANCE OF ACTUAL AND COMPENSATED PI-TYPE ARTIFICIAL LINE

Next we will consider the case when the receiving end is freed. The voltage at the receiving end and the current at the sending end are represented by V_B and I_A. In Fig. 3 we obtain

$$I_A = (2 + Y_{c\pi} Z_\pi)\, Y_{c\pi} V_B \qquad \text{amperes } \angle \qquad (7)$$

In Fig. 4 we have (Bib. 1, 2)

$$I_A = V_B/(Z_0 \sinh \theta) \qquad \text{amperes } \angle \qquad (8)$$

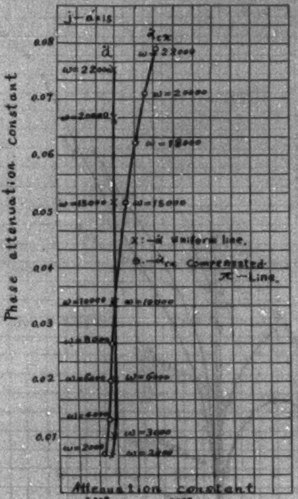

FIG. 9—COMPARISON OF THE LINEAR HYPERBOLIC ANGLE OF ACTUAL AND COMPENSATED PI-TYPE ARTIFICIAL LINE

From (5), (7) and (8)

$$\frac{\sinh \theta}{Z_0} = [2 + (1 - \beta)\, Y_{c\pi} Z_{c\pi}]\, Y_{c\pi} \qquad \text{mho } \angle \qquad (9)$$

is obtained by the condition of equivalency. Solving the values of $Z_{c\pi}$ and $Y_{c\pi}$ from (6) and (9) we get

$$Z_{c\pi} = \frac{2\,(\sinh \theta/\theta)\, Z}{\{1 - \beta + \beta \cosh \theta + \sqrt{\beta^2 + 2\beta\,(1-\beta) \cosh \theta + (1-\beta)^2}\}} \qquad \text{ohm } \angle$$

$$(10)$$

$$Y_{c\pi} = \left\{\frac{\tanh(\theta/2)}{\theta/2}\right\} \frac{Y}{2\,(1-\beta)}\, \{1 - 2\beta + \sqrt{\beta^2 + 2\beta\,(1-\beta)\cosh\theta + (1-\beta)^2}\} \qquad \text{mho } \angle$$

where Z and Y represent the total impedance and the total admittance of the conjugate uniform line. The following relations being well known

$$\left.\begin{aligned} \theta &= \sqrt{Z\,Y} \text{ hyps. } \angle \\ Z_0 &= \sqrt{Z/Y} \text{ ohm } \angle \end{aligned}\right\} \qquad (11)$$

we can design the compensated π-line by equation (10) assigning an arbitrary value to the compensating factor β. Solving (6) and (9) with respect to θ and Z_0, we get,

$$\theta = 2 \sinh^{-1} \sqrt{\frac{Y_{c\pi} Z_{c\pi}}{2}\, 1 + \frac{(1-\beta)\,\beta}{2}\, Y_{c\pi} Z_{c\pi}} \qquad \text{hyps. } \angle$$

$$Z_0 = \sqrt{\frac{Z_{c\pi}}{Y_{c\pi}}} \left\{ \frac{\sqrt{1 + 0.5\,(1-\beta)\,\beta\, Y_{c\pi} Z_{c\pi}}}{1 + 0.5\,(1-\beta)\, Y_{c\pi} Z_{c\pi}} \times \cosh(\theta/2) \right\} \qquad \text{ohm } \angle \qquad (12)$$

So that the constants of the conjugate uniform line can be obtained from the constants of the compensated π-line, $Z_{c\pi}$, $Y_{c\pi}$ and β.

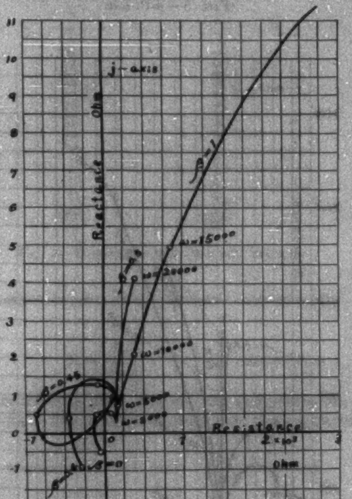

FIG. 10—VALUE OF $Z_{c\pi}$ FOR 60-CYCLE SECTION

II. THE RELATION BETWEEN THE CORRECTING FACTOR AND THE COMPENSATING FACTOR

We can understand easily by (10) that the series impedance and the shunt admittance of the compensated π-line are obtained by multiplying the correcting factors

$$K_1 = \frac{2 \sinh \theta / \theta}{\{1 - \beta + \beta \cosh \theta + \sqrt{\beta^2 + 2\beta(1 - \beta \cosh \theta + (1 - \beta)^2}\}} \text{ numeric } \angle \tag{13}$$

$$K_2 = \frac{\tanh(\theta/2) + \{(1 - \beta)(\theta/2)\}}{1 - 2\beta + \sqrt{\beta^2 + 2\beta(1 - \beta)\cosh\theta + (1 - \beta)^2}} \text{ numeric } \angle$$

into the series impedance and half the shunt admittance of the uniform line respectively. These correcting factors are not only functions of θ (functions of frequency accordingly), but are also functions of β.

In the case of absence of Z_c, *i. e.*, $\beta = 0$, it is a pure π-line. We obtain the following correcting factors by putting $\beta = 0$ in (13),

$$\left.\begin{aligned} K_{1(\beta=0)} &= \frac{\sinh \theta}{\theta} \quad \text{numeric } \angle \\ K_{2(\beta=0)} &= \frac{\tanh(\theta/2)}{\theta/2} \quad \text{numeric } \angle \end{aligned}\right\} \tag{14}$$

which are the well-known correcting factors for the π-line. On the contrary, the pure T-line is obtained in the case $Z_\pi = 0$. We get the following correcting factors by putting $\beta = 1$ in (13).

$$\left.\begin{aligned} K_{1(\beta=1)} &= \frac{\tanh(\theta/2)}{\theta/2} \text{ numeric } \angle \\ K_{2(\beta=1)} &= \frac{\sinh \theta}{\theta} \quad \text{numeric } \angle \end{aligned}\right\} \tag{15}$$

which are the well known correcting factors of a T-line.

As mentioned above, the compensated π-line is to be considered as the general type of artificial line, including both π-lines and T-lines. The complex values allowed for β are limited from the physical point of view. If negative resistance is not allowed, the following relation must obtain:

$$0 \leqq \text{real part of } \beta Z_{c\pi} \leqq \text{real part of } Z_{c\pi} \text{ ohm } \angle \tag{16}$$

In the β-plane in Fig. 5, the domain of β may be determined by (13) and (16). As a special case, we consider β to be real. In this case the following relation holds:

$$-\pi/2 < \gamma < \pi/2 \text{ radian}$$

γ being the slope of $Z_{c\pi}$. Therefore from (16), we get

$$0 \leqq \beta \leqq 1 \text{ numeric}$$

So that the value of β must not be more than 1 nor less than zero on the real axis.

The general case when β is complex, is not so simple, but in a practical case it will be convenient to study the nature of the correcting factors by trial, choosing proper

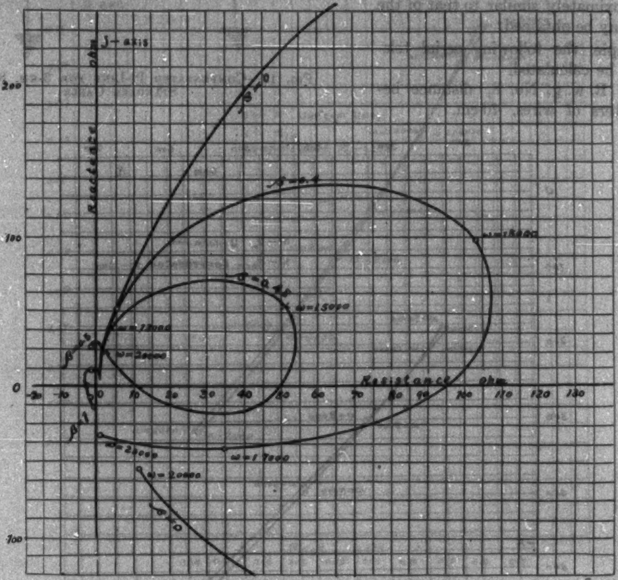

FIG. 11—VALUE OF $Y_{c\pi}$ FOR 60-KM. SECTION

values for β from a consideration of the physical construction of the artificial line.

III. HOW TO DESIGN THE COMPENSATED π-LINE

In the compensated π-line, resistances and inductances are connected in series for the impedance $Z_{c\pi}$ etc., and for the admittance $Y_{c\pi}$, condensers and resistances are connected in parallel, so that their frequency characteristics are the straight lines parallel to the imaginary axis. Not only so, but the imaginary parts are proportional to the frequencies. Hence, if we take such a value of β that the frequency characteristics of $Z_{c\pi}$ and $Y_{c\pi}$ are similar to those above mentioned, the compensated π-line whose nature is approximately similar to that of the uniform line, may be obtained.

As an example, the following aerial telephone line will be calculated.

Copper wire, B. w. g. No. 8, distance between the centers of wires, 40 cm.

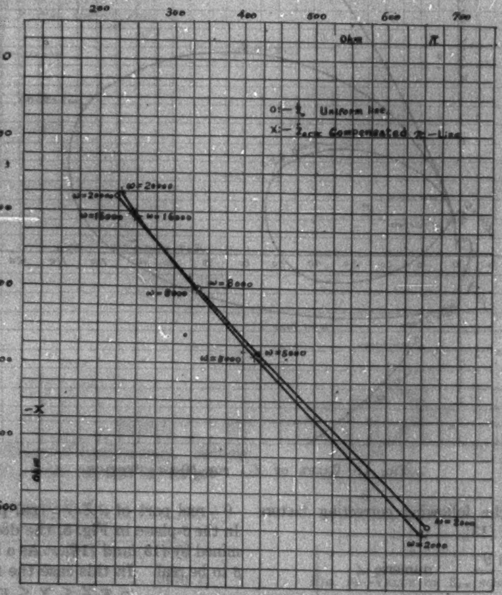

FIG. 13—COMPARISON OF THE SURGE IMPEDANCE OF ACTUAL AND COMPENSATED PI-TYPE STANDARD CABLE

Series resistance r = 2.494 ohms/km.
Series self inductance l = 0.0022 henrys/km.
Leakage conductance $g = 10^{-6}$ mhos/km.
Shunt capacitance c = 0.005285×10^{-6} farads/km.

The compensated π-line which corresponds to the 30 km. section of the above line was calculated. The total series impedance and the total shunt admittance were obtained as follows:

$$Z = 30\,[2.494 + j\,\omega \times 0.0022] \quad \text{ohms} \angle$$
$$Y = 30\,[10^{-6} + j\,\omega \times 0.005285 \times 10^{-6}]\ \text{mhos} \angle$$

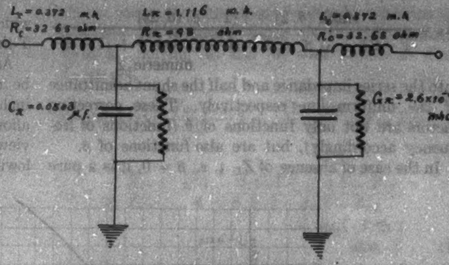

FIG. 12—COMPENSATED PI-LINE FOR 3-KM. SECTION OF STANDARD CABLE

$Z_{c\pi}$ and $Y_{c\pi}$ are calculated for the various real values of β (0, 0.2, 0.4, 0.6 and 1) in the range of frequencies between 2000 and 20,000 in angular velocity. The results are shown in Figs. 6 and 7. In the various values of $Z_{c\pi}$ and $Y_{c\pi}$, we see that those which correspond to $\beta = 0.4$, are satisfactory, because the characteristic is nearly vertical and the imaginary parts are proportional approximately to the frequency. Then two straight vertical lines, $a\,b$ (Fig. 6) and $c\,d$ (Fig. 7) are drawn as a proper approximation to the characteristics of $Z_{c\pi}$ and $Y_{c\pi}$. The constants of the compensated π-line were chosen so that the characteristics of $Z_{c\pi}$ and $Y_{c\pi}$ are $a\,b$ and $c\,d$. Thus we have

$R_{c\pi} = 70$ ohms $R_c = 0.2 \times 70$ ohms
$L_{c\pi} = 0.0645$ henries $L_c = 0.2 \times 0.0645$ henries
$G_{c\pi} = 3.2 \times 10^{-6}$ mhos $R_\pi = 0.6 \times 70$ ohms
$C_{c\pi} = 0.157 \times 10^{-6}$ farads $L_\pi = 0.6 \times 0.0645$ henries

The surge impedance $Z_{0c\pi}$ and the linear hyperbolic angle $\alpha_{c\pi}$ were calculated by (**12**). The corresponding values of the uniform line were also calculated by (II). The results of the calculation are given in Table I, Figs. 8 and 9. As shown by the table and the figures, the surge impedances and the linear hyperbolic angles of both lines coincide with each other approximately. Hence we can say that this compensated π-line agrees satisfactorily in all electrical respects with the uniform line. Moreover this type is convenient for calculation, since it is not a composite line as is the case of the combined T-π-type.

Similar calculations were made for the same telephone line, increasing the length of section to 60 km. The results are given in Figs. 10 and 11. In this case, it is difficult to choose the value of β properly. If we cannot have $Z_{c\pi}$ and $Y_{c\pi}$ with the desired vertical characteristics etc. for the various values of β, the length of one section must be decreased.

As another example, it was found that the proper value of β should be 0.4, for the 3-km. section of the

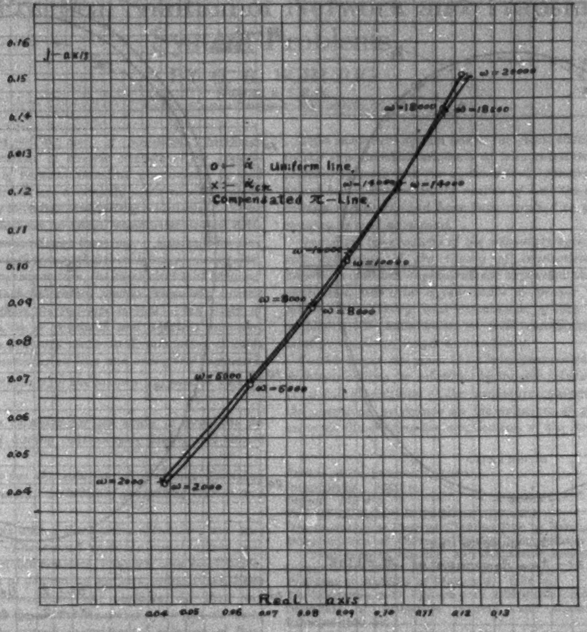

FIG. 14—COMPARISON OF THE LINEAR HYPERBOLIC ANGLE OF ACTUAL AND COMPENSATED PI-TYPE STANDARD CABLE

TABLE I.

LINEAR HYPERBOLIC ANGLE AND SURGE IMPEDANCE OF THE AERIAL LINE AND THE COMPENSATED π-TYPE ARTIFICIAL LINE

ω	$\alpha\,\angle$	$\alpha_{c\pi}\,\angle$	$Z_0\,\angle$	$Z_{0c\pi}\,\angle$
2000	$.00219 + j\,.00699$	$.00210 + j\,.00686$	$690\ \diagdown\overline{12^\circ.07}$	$685\ \diagdown\overline{11^\circ.40}$
3000	$.00223 + j\,.01035$	....	$666\ \diagdown\overline{8^\circ.55}$	....
4000	$.00224 + j\,.01374$	$.00217 + j\,.0134$	$657\ \diagdown\overline{6^\circ.56}$	$653\ \diagdown\overline{6^\circ.20}$
5000	$.00224 + j\,.01715$	....	$653\ \diagdown\overline{5^\circ.30}$	....
6000	$.00225 + j\,.02030$	$.00218 + j\,.0202$	$650\ \diagdown\overline{4^\circ.45}$	$645\ \diagdown\overline{4^\circ.25}$
7000	$.00225 + j\,.02392$	$.00220 + j\,.0235$	$649\ \diagdown\overline{3^\circ.83}$	$644\ \diagdown\overline{3^\circ.60}$
8000	$.00225 + j\,.02734$	$.00222 + j\,.0270$	$648\ \diagdown\overline{3^\circ.35}$	$641\ \diagdown\overline{3^\circ.04}$
10,000	$.00225 + j\,.03420$	$.00222 + j\,.0340$	$647\ \diagdown\overline{2^\circ.69}$	$644\ \diagdown\overline{2^\circ.57}$
15,000	$.00225 + j\,.05115$	$.00243 + j\,.0515$	$646\ \diagdown\overline{1^\circ.82}$	$646\ \diagdown\overline{1^\circ.83}$
20,000	$.00225 + j\,.0675$	$.00267 + j\,.0707$	$645\ \diagdown\overline{1^\circ.35}$	$680\ \diagdown\overline{2^\circ.26}$
22,000	$.00225 + j\,.0750$	$.00278 + j\,.0793$	$645\ \diagdown\overline{1^\circ.22}$	$801\ \diagdown\overline{5^\circ.05}$

standard cable whose linear constants are

$r = 54.7$ ohms/km.
$l = 0.622$ m. h./km.
$g = 3.11 \times 10^{-4}$ mhos/km.
$c = 0.0336$ μ f./km.

The result is shown in Fig. 12 where the values of $Z_{0c\pi}$, $\alpha_{c\pi}$, Z_0 and α are compared in Figs. 13 and 14 and Table II. They coincide remarkably.

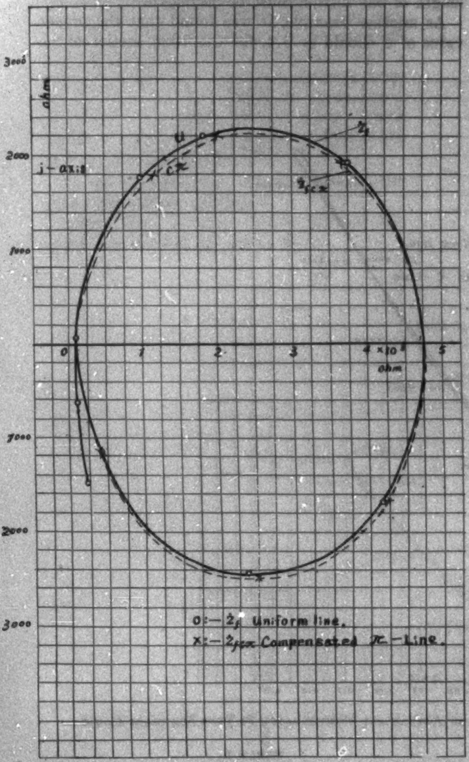

FIG. 15—OPEN-CIRCUIT IMPEDANCE OF 60-KM. ARTIFICIAL LINE

TABLE II.

LINEAR HYPERBOLIC ANGLE AND SURGE IMPEDANCE OF THE STANDARD CABLE AND THE ARTIFICIAL LINE

ω	$\alpha \angle$	$\alpha_{c\pi} \angle$	$Z_0 \angle$	$Z_{0c\pi} \angle$
2000	.04341 + j .04238	.0429 + j .0428	901.7 $\overline{\backslash 43°1'25''}$	902 $\overline{\backslash 43°.96}$
5000	.06654 + j .06914	.0659 + j .0695	571 $\overline{\backslash 42°50'23''}$	570 $\overline{\backslash 43°.23}$
8000	.0825 + j .0892	.0821 + j .0900	452 $\overline{\backslash 42°3'30''}$	452 $\overline{\backslash 42°.23}$
10,000	.0912 + j .0102	.0912 + j .104	404 $\overline{\backslash 41°.47}$	406 $\overline{\backslash 41°.62}$
12,000	.0985 + j .112	.0987 + j .112	370 $\overline{\backslash 40°.90}$	370 $\overline{\backslash 41°.06}$
14,000	.105 + j .1225	.1055 + j .122	343 $\overline{\backslash 40°.30}$	344 $\overline{\backslash 40°.44}$
16,000	.1114 + j .1325	.1104 + j .1336	321.5 $\overline{\backslash 39°.66}$	323 $\overline{\backslash 39°.92}$
18,000	.1166 + j .1422	.1166 + j .1415	303.5 $\overline{\backslash 39°.06}$	305 $\overline{\backslash 39°.04}$
20,000	.1215 + j .1515	.123 + j .149	289 $\overline{\backslash 38°.44}$	289 $\overline{\backslash 37°.78}$

IV. COMPARISON WITH OTHER TYPES

If we choose the same sectional length, the compensated π-line is superior to the T-line (or π-line) of course, but the former is more complex than the latter in its construction. Therefore it is safe to compare it with the section of the T-line and π-line of half the length. In the above two representative cases, we found that the compensated π-line could be designed to be more satisfactory than the T-line or π-line of half length. Moreover, in these particular cases, this type gave better results experimentally than the combined T-π-type. In this case, the total length of the compensated π-line was taken as 60 km. and the length of the section was taken as 30 km. For the combined T-π-line, the total length was taken as 60 km., but for the length of each section 15 km. was chosen.

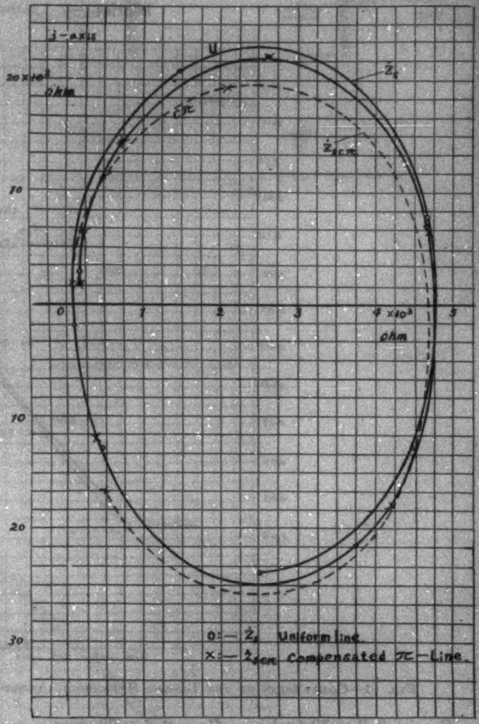

FIG. 16—SHORT-CIRCUIT IMPEDANCE OF 60-KM. ARTIFICIAL LINE

The short-circuit impedance of the compensated π-line $Z_{sc\pi}$, the open-circuit impedance of the same $Z_{fc\pi}$, the short-circuit impedance of the conjugate uniform line Z_f, and the open-circuit impedance of the same Z_f are given in Figs. 15 and 16. These are to be compared with Figs. 14 and 15 in a previous paper. (Bib. 3).

Bibliography

1. A. E. Kennelly. "The Application of Hyperbolic Functions to Electrical Engineering Problems."

2. H. Ho. Hado Sindo oyobi Hirai. P. 215.

3. H. Nukiyama and Y. Shoji. "The Combined T-π-Type Artificial Electric Line." *Denki Gakkai Zasshi*, June, 1922.

4. A. E. Kennelly. "Artificial Electric Lines."

5. A. E. Kennelly. "Tables of Complex Hyperbolic and Circular Functions."

6. A. E. Kennelly. "Chart Atlas of Complex Hyperbolic and Circular Functions.

A Miniature A-C. Transmission System

For the Practical Solution of Network and Transmission-System Problems

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Review of the Subject.—*The use of so-called "artificial lines" — i. e. miniature models of electric circuits — for experimental laboratory studies of transmission-line phenomena is well known. The miniature models are commonly made so as to represent, in true proportion, the electrical constants of a real line. Thus there are artificial telephone lines, artificial submarine cables, artificial long-distance power-transmission lines, etc. A miniature line is a true model of a real line to the extent that the miniature circuit has, for any desired degree of approximation, the same electrical behavior as the full-size circuit. Such laboratory models frequently permit — far more conveniently than the full-size circuit — the study of actual circuit phenomena in a practical and efficient manner. When their limitations are properly understood, miniature circuits may be of great value to transmission-line and operating engineers.*

Miniature electric circuits may for the present purpose be divided into two general classes:

(1) *Miniature circuits intended for the study of problems on* **long lines,** *such as lines having continuously distributed circuit constants.*

(2) *Miniature circuits for the solution of problems involving* **complete system networks,** *inclusive of generating-station and substation apparatus.*

The following paper deals with a three-phase miniature a-c. system of the network type. The circuit includes synchronous machines, transformers, adjustable resistors, reactors, and condensers, for complete representation of generating stations, substations, lines and loads. The circuit connections are variable, so that any system having not more than the available number of circuit elements may be represented for the correct experimental solution of low-frequency problems.

This miniature system has so far given about three years of service in the experimental analysis of transmission system behavior, for existing systems and for systems to be constructed. The problems solved have been within the realms of both the designing and the operating engineers, and have been applied to power systems in this country as well as abroad.

One of the by-products of the miniature system is the confirmation and extension of the theory of transmission-line phenomena. Several prominent engineers have expressed the belief, and made the prediction, that after a few years of proper use of the miniature system, the theory and calculation of the present transmission problems will have been so well established that experimental solutions will no longer be necessary.

In this paper are given (1) a brief discussion of some of the present problems calling for solutions by the miniature experimental method, (2) a full description of the miniature equipment, (3) an outline of the operating procedure in the solution of problems, and (4) an example illustrating the application of the miniature equipment.

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Problems of Stability of Machines. (175 w.)
The Miniature A-C. System with Adjustable Circuit Constants. (500 w.)
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INTRODUCTION

A TYPE of minature electric circuit which has been found to be of great practical value is one designed primarily for the experimental solution of problems arising in transmission networks. One of the problems is the determination of current division in a system network, both under normal load and during short circuit. Furthermore, the proper use of relays often requires definite knowledge as to both magnitudes and relative phases of currents and voltages at numerous points of a system under short-circuit conditions. The solution of these problems involves not only the line characteristics and the transmission network connections, but also the type and connections of transformers, the characteristics of synchronous apparatus and their location with respect to the transmission network. Likewise, the stability of rotating machinery, under changing loads or for different methods of system operation, is a highly important practical problem in connection with transmission system design and operation.

While a number of the problems of the type men-

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tioned in the preceding paragraph can be and have been solved theoretically by calculation, experimental solutions are often desirable—not only as a means of verifying the calculations and thus establishing the practical value of, and the fullest confidence in, the theoretical solution, when the latter is possible,—but also as the only available practical procedure for problems not readily subject to theoretical solution.

The requirements of problems of the above type, suggest a laboratory miniature electric system embodying the following features:

> Two or more generators capable of representing separate generating stations.
>
> Two or more banks of transformers permitting a variety of connections in accordance with central-station practise.
>
> Adjustable resistor, reactor and capacity units, for polyphase (three-phase) circuits.
>
> A sufficient number of the line units to permit representation of interconnected networks (or at least of portions thereof).
>
> Switching arrangements for facilitating the practical procedure of interconnecting the various elements of the miniature system in any manner desired.
>
> Provision for inserting measuring instruments, actual relays, current transformers, etc. at any point of the system.
>
> A sufficiently high rating of the miniature apparatus such as to render inappreciable the errors introduced by the connection of practical measuring apparatus to the system.

A miniature system designed to meet these requirements has been installed and experimented with in the General Engineering Laboratory at Schenectady. The conception of the plan is due to Mr. H. H. Dewey, and a considerable amount of preliminary work preparatory to the construction of the miniature system was done by Mr. W. W. Lewis. Acknowledgements are also due to Messrs. D. P. Savant and A. R. Miller for their work in connection with the design and installation of the miniature system. The miniature system was built in 1919 and has since then been applied to the solution of a variety of electric-circuit problems.

OBJECT

It is the object of this paper (1) to review and discuss the nature of some of the practical problems to which this miniature system is applicable, (2) to describe its essential parts, and (3) to illustrate by an example its application to practical problems.

General Discussion of Problems

Under this heading will be shown the practical nature, and the importance to transmission system design and operation, of some of the problems which called for the construction of the miniature transmission system. The results of the experiments made will not be presented here. In some cases these results were applicable to one central-station system only, in others they have been of a fundamental character and therefore applicable more generally. A number of problems of each kind considered below have been solved with the aid of the miniature system.

DETERMINATION OF CURRENT DIVISION DUE TO NORMAL LOADS

This problem, already referred to, is one of vital importance to the economical layout and efficient operation of a transmission system. For a purely radial system of transmission from generating station to substations, without interconnecting feeders between substations, calculations of current flow in the feeders are relatively simple. Experimental solution is then of no particular advantage. For complex networks, however, having numerous interconnecting tie lines, thus offering a multiplicity of paths of current flow from the generating stations (one or more), the calculations are quite lengthy and often impractical. The complications of calculations are due not only to the circuit connections but also to the use of reactors—frequently causing dissimilar impedance angles in the various conducting elements of a network—and to dissimilar power factors of the loads on the system. Consideration has been given to the possible merit of approximating the solution of load-current problems by a d-c. experimental circuit, with the result that this sort of approximation cannot in general be expected to give reliable results, on account of the last-mentioned factors.[1] The experimental solution thus calls for an a-c. miniature system. With the proper equipment it is possible to determine, by the closing or opening of a switch, in the laboratory, the effect of adding to or withdrawing from the system a feeder or a tie line.

To summarize: The predetermination of normal current flow in the branches of networks is frequently desirable for the best design of new transmission systems as well as for the practical study of system additions or changes. The experimental solution of these problems calls for a miniature a-c. system.

DETERMINATION OF CURRENTS DURING SINGLE-PHASE SHORT CIRCUITS, FOR THE PROTECTION OF CIRCUITS AND ASSOCIATED PROBLEMS

When all three phases of a three-phase system are simultaneously short-circuited, the sustained short-circuit currents are balanced three-phase currents in all the circuit elements affected by the short circuit, regardless of the number of generating stations, the transformer connections or the circuit connections. For all other types of short circuits, an unbalanced current flow results. Such short circuits are

> Short circuit between two phases,

---

[1] See discussions of the paper on "Experimental Determination of Short-circuit Currents in Electric-Power Networks," A. I. E. E. Midwinter Convention, New York, 1923.

One-phase-to-ground short circuit on grounded-neutral circuit.

Simultaneous ground short circuits occurring on two phases of a grounded-neutral circuit.

While there are a great many cases where consideration of three-phase short-circuit currents, as obtained by the short-circuit calculating table, is sufficient for the determination of relay settings, circuit-breaker ratings, electromagnetic stresses, etc., detailed studies of single-phase short circuits are often necessary, when it is considered that a large number of short circuits are of the single-phase variety.—According to L. C. Nicholson,[2] the short circuits due to lightning on an aerial system were observed to be proportioned as follows: 60 per cent two-wire short circuits, 10 per cent three-phase short circuits, and 30 per cent single-wire-to-ground flashovers.—During short circuits, causing an unbalanced current division among the phases, the voltages between the lines generally become distorted from their normal balanced condition (represented by an equilateral triangle of voltage vectors). This distortion may be very pronounced at the points at which relay operation is desired. The distortions of the voltage triangle are due to the tendency for the voltage between the points short-circuited to approach zero. If, for example, a short circuit occurs between phases A and B on a three-phase circuit, the phases of which are A, B and C, the voltage V_{AB} will approach zero at the point of short circuit, while the voltages V_{BC} and V_{CA} at the point of short circuit will tend to become equal, each being less than its normal magnitude before the short circuit. Moreover, the relative phases of currents and voltages throughout the system will be far different from those applying to normal load conditions or to three-phase short circuits. Calculation of power-directional relay torques (their magnitude and direction) for the types of short circuits under discussion is often extremely laborious in view of the above considerations. Miniature tests in the laboratory on a suitable a-c. miniature system enable practical data on the performance of power-directional relays to be obtained under short circuits simulating those in actual service. Furthermore, miniature studies of this kind permit the factors affecting voltage and current relations during short circuits and their bearings on relay performance to be analyzed. Consequently a miniature laboratory transmission system serves not only in the solution of protection problems for the selected systems tested in miniature, but should aid, above all, in the advancement of knowledge towards a fuller understanding of short-circuit phenomena.

OTHER SHORT-CIRCUIT PROBLEMS

If a station with Y-connected generators has one generator neutral grounded, the other neutrals being ungrounded, the current contributed by each of the ungrounded generators to a line-to-ground short circuit will differ from that obtained when all the generator neutrals are grounded. This problem lends itself to laboratory solution by a miniature transmission system. A similar problem is due to the use of multiple Y-connected transformers when only one transformer neutral is grounded.

For short-circuit current determinations the reactance values, under short-circuit conditions, of generators and transformers must be known, as well as the paths traversed by the short-circuit current components.

Other problems within this class are the determination of transient[3] and sustained reactances, of an alternator when short-circuited between terminals or from one or more terminals to neutral in the various possible ways.

Since relay operation commonly takes place during the short-circuit transient, the rates of short-circuit current decrement are of importance in the adjustment and behavior of relays, and in circuit-breaker application. The difficulties of short-circuit current determination increase when several generating units of dissimilar rates of current decrement feed current into a short circuit. Their combined effects on the short-circuit current transient, on relay behavior throughout a system, and on circuit-breaker requirements may be studied experimentally in miniature. To vary the time-constants of alternator windings, inductances and resistances external to the miniature alternators may be employed. Fundamental data on this kind of problem may be gathered by miniature tests with the aid of oscillograph records of current and voltage, and may be confirmed, in some cases, by direct connection of relays to the miniature circuit.

PROBLEMS ON STABILITY OF SYNCHRONOUS AND INDUCTION MACHINES

Both synchronous machines and induction motors will perform their normal functions and run in a stable manner only if operated within certain limits of load and circuit conditions, but will become unstable[4] under other operating conditions. One of the problems of growing importance, within this class, is that of the stability of operation of induction motors and synchronous motors at the receiver end of long high-voltage aerial transmission lines. For this kind of circuit the voltage, current and power conditions for stable motor operation may be materially different from those pertaining to motors operating on constant-voltage mains. Various problems of stability of both synchronous and induction machines lend themselves to solution by calculations, without tests, provided the machine characteristics and the circuit constants are known. Nevertheless, our understanding of instability problems of this type can hardly be said to have been advanced to the stage at which experimental solutions for the verification of calculations are no longer required.

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[2] Discussion, A. I. E. E. TRANSACTIONS 1911, part 1, p. 350

[3] The "transient" reactance is the reactance determining the r. m. s. a-c. component of initial short-circuit current.

[4] The word "unstable" is used here in the following sense: A synchronous motor becomes unstable when the relations of power supplied to the motor and power demanded by the motor are such as to cause it to drop out of synchronism and come to a dead stop; similarly, in induction-motor operation instability occurs at the breakdown point.

Fig. 1—Miniature Electric-Power Transmission System, View of Resistance and Reactance Units, of Switchboard Wiring, of Alternators and of Exciter

Total floor space 18 ft. by 20 ft., including space for operator and instrument table.

The Miniature A-C. System with Adjustable Circuit Constants

The miniature system consists of the following equipment (see Fig. 1):

Generators: two type *A H I*, 3-phase, 4-pole, 3.75 kv-a., 1800-rev. per min., 110-volt, 60-cycle alternators, each coupled to a 10-h. p., 1700-rev. per min., 230-volt, d-c. motor. These generators are arranged for delta and star connections, and are of the revolving-field type. Each alternator may be operated as a synchronous motor driving its d-c. machine as a generator. A three-kw., motor-driven exciter set is provided.

Transformers: six type *M*, air-cooled, single-phase, 2 kv-a. 110/220-440-volt transformers. They may be connected to form two three-phase banks, thus permitting the various customary transformer connections, including auto-transformer schemes.

Reactors: twenty-four reactors, single-phase, air-cooled, iron-core type with fixed air-gap. Each reactor has 15 taps brought to a terminal board. In this way, 128 reactance values are available, ranging from 0.2 to 30 ohms at 60 cycles. The sustained current-carrying capacity is 10 amperes. The reactance values are constant for current values up to 20 amperes. The effective resistance of these reactors is negligible for all but the smallest values of reactance.

Resistors: twenty-four single-phase resistors, of "German silver" wire, wound on asbestos tubes; resistance range from 0.2 to 30 ohms, in 74 steps. The reactance, at 60 cycles per sec., of these units is less than 2 per cent of their resistance, and hence negligible. Their sustained current-carrying capacity is 10 amperes.

Condensers: The condensers are of the oil-immersed, flat, paper type. Enough units of approximately 2 and 5 microfarads are available to permit three-phase representation of lines several hundred miles long. The condenser units are not adjustable. The capacity values desired for any problem are obtained by series and parallel connections of the fixed units available. In Fig. 1 the condensers are not shown. Provision is made for connecting them to the back of the switchboard.

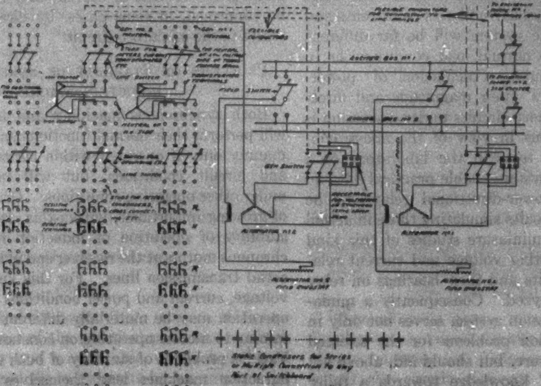
Static Condensers for Series or Multiple Connection to any Part of Switchboard

Fig. 2—Wiring Diagram and Panel Layout for Miniature Transmission System

Switchboard: A slate switchboard for interconnecting generators, transformers and line units is shown in Fig. 1 (rear view). The permanent circuit connections are shown in Fig. 2. Jumpers with copper terminals to fit the studs and wing nuts on the front of the switchboard are provided for making the particular system connections required for any problem. Brass short-circuiting strips are used for bridging across the front terminals not needed for series-connected instruments or other series equipment.

OPERATION OF MINIATURE SYSTEM AND MEASUREMENTS

The resistor, reactor and condenser units may be used to represent either one phase (to neutral) of a three-phase balanced system under conditions of balanced current flow, or a full three-phase system for problems involving unequal current division among the phases and distorted line voltages. In the former arrangement (single phase to neutral) 24 independently adjustable circuit units may be represented, while in the latter arrangement 8 three-phase separately adjustable circuit elements of a network may be reproduced in miniature. The two three-phase alternators, together with the 60-cycle laboratory mains, permit representation of three separate generating stations of a three-phase system. For problems involving loads, one or both alternators may be operated as synchronous motors to simulate separate loads, or any number of the available resistor and reactor units may be connected to represent single-phase or three-phase load impedances.

For the study of problems involving generating stations with step-up transformers and a transmission network, one method of operation is as follows (other methods are often desirable and may be employed within the range of conditions permitted by the equipment):

TABLE I

	Volts terminal voltage	Amperes full-load line current	Full-load kv-a.
Generators	110	19.7	3.75
High-tension line conditions for delta-delta transformer connections, ratio 1:4	440	4.9	3.75

Under these or similar conditions, the circuit will accommodate a variety of relays, also current transformers. Frequently, the reduction factors from actual system values to miniature system values of voltage, current and impedance may be chosen such that the insertion of these devices introduces but negligible changes of circuit conditions. In other cases, proper allowance is made in the circuit constants for the impedance of the series devices inserted.

Under the circuit conditions of Table I, the constants of the apparatus of the miniature system are those shown in Table II.

It is often convenient to observe sustained short-circuit conditions with the aid of indicating instruments. Such observations are permitted in view of (1) the overload capacity of the principal equipment and (2) the wide range of the circuit constants.

TABLE II
CIRCUIT DATA FOR MINIATURE SYSTEM

Generators (When Delta-connected)		
Three-phase rating	3.75 kv-a.	
Current	19.7 amperes	
Voltage	110 volts	
Resistance per coil	0.17 ohms	1.8%
"Transient" reactance	0.48 ohms	5 %
Synchronous reactance	4.4 ohms	45. %
Transformers (Delta-delta connection)		
Total equivalent resistance		1.1%
Total equivalent reactance		1.6%
Total equivalent impedance		1.9%
*Resistors**		0.4% to 58%
*Reactors**		0.4% to 58%

*Based on three-phase rating of 3.75 kv-a. at 440 volts.

Oscillograph measurements may be made for the determination of short-circuit transients (limited, of course, to those at fundamental frequency). The miniature system is not applicable to the solution of high-frequency problems, such as arise in connection with lightning disturbances, arcing grounds, switching, etc. Experimental data on such problems may be obtained from tests of actual systems or, in some cases, from laboratory tests of "smooth" miniature circuits (*i. e.* circuits with continuously distributed, rather than lumped, circuit constants).

The lumped circuit units of resistance, reactance and capacity in the miniature system under consideration do, however, permit correct representation of the steady-state behavior of long transmission lines at any one desired value of (low) frequency—such as 60 cycles per sec.

Example

A simple example is given below to illustrate the method of procedure in a miniature test, the magnitude of the circuit values employed, and the accuracy of the results obtained. Thus the items to be dealt with are the following:

(1) the derivation of the miniature-system circuit data employed in the tests, from the full-size data,

(2) the results of the miniature tests,

(3) the conversion of these results into full-size system values,

(4) the determination, by means of calculations, of the accuracy of the results,

(5) a comparison of the results obtained by miniature tests with those measured on the full-size system.

DETERMINATION OF LOAD-CURRENT FLOW IN A SYSTEM NETWORK

The circuit to be considered is that of Fig. 3A, which was reduced to that of Fig. 3B for purposes of simplification. It is desired to determine the normal current flow throughout the network when the following data are given: Four loads, the impedance values of the network branches and the generator bus voltage. The data appear in column 2, Table III. The problem is one of the simplest which has been encountered and can, of course, be solved by calculation. Its value should, therefore, not be judged on the basis of complexity. On the other hand, its merit lies in the fact that full-size system tests could be made for comparison with the miniature tests. Furthermore, the procedure in a more complex test differs in no essential respects from that outlined in this case.

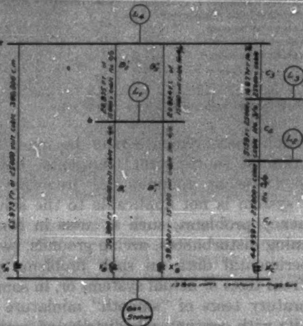

FIG. 3A—ONE-LINE CIRCUIT DIAGRAM FOR THREE-PHASE, 60-CYCLE NETWORK WITH BALANCED LOADS

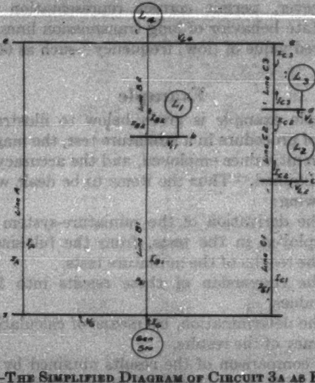

FIG. 3B—THE SIMPLIFIED DIAGRAM OF CIRCUIT 3A AS EMPLOYED IN MINIATURE TESTS

One of the most essential steps in the procedure is the *choice of suitable conversion factors* to be applied to the full-size data. Each problem to be solved generally calls for different reduction factors. Separate factors apply, in any one case, to the values of current, voltage, power, and impedance. A number of items which determine the reduction factors are: current, voltage, and kv-a. rating of full-size circuit; the range of the miniature-system circuit constants; the type of problem, that is, whether short-circuit problem or normal-current problem; the method of measurement to be employed, *i. e.* whether oscillograph or indicating-meter method; the accuracy of the results desired.

For the problem under consideration, the miniature series impedances representing the lines should be as large as possible, on account of accuracy, and the current values should likewise be sufficiently high to give reasonably precise readings on the available indicating ammeters and wattmeters of low series impedance. A current value between 5 and 10 amperes, say 6 amperes, is a convenient one for the average of the currents in the principal lines A, B_1, and C_1 (in Fig. 3B). Thus a total generator current I_o of approximately 18 amperes is obtained, representing a full-size-system total generator output of 10,400 kw., or 11,820 kv-a. at a power factor of 0.88 (see bottom of column 2, Table III.) If, then, 440 volts are used — this value being the rated voltage of the high-voltage miniature equipment — *and a line-to-neutral, single-phase representation of the (balanced three-phase) system is made,* the three-phase miniature

TABLE III
FULL-SIZE SYSTEM DATA AND CORRESPONDING MINIATURE TEST VALUES

1	2		3	4	
Symbols refer to Figs. 3A & 3B	Full-size Data for balanced $3-\phi$ system		Data Reduced to Miniature scale by conversion factors*	Miniature system data employed in test	
Line A reactor X_A'	0 $+j\,1.20$ ohms				
Line A cable Z_A''	2.42 $+j\,2.40$ ohms				
Total Line A, Z_A	2.42 $+j\,3.60$ ohms		3.70 $+j$ 5.50	3.70 $+j$ 5.54	
Reactors $X_B' = X_B''$	0 $+j\,3.65$ ohms				
Impedances of Cables B_1' & B_1''	2.04 $+j\,1.39$ ohms				
Total line B_1, Z_{B1}	1.02 $+j\,2.52$ ohms		1.56 $+j$ 3.85	1.51 $+j$ 3.87	
Impedances of Cables B_2' & B_2''	1.25 $+j\,0.855$ ohms				
Total line B_2, Z_{B2}	0.625 $+j\,0.427$ ohms		0.95 $+j$ 0.65	1.00 $+j$ 0.69	
Reactor X_C'	0 $+j\,5.12$ ohms				
Cable C 1	2.92 $+j\,1.76$ ohms				
Total line C 1, Z_{C1}	2.92 $+j\,6.88$ ohms		4.46 $+j$ 10.50	4.68 $+j$ 10.46	
Total line C 2, Z_{C2}	0.34 $+j\,0.204$ ohms		0.52 $+j$ 0.31	0.25 $+j$ 0.40	
Total line C 3, Z_{C3}	1.105 $+j\,0.668$ ohms		1.68 $+j$ 1.02	1.61 $+j$ 1.08	
	kw., $3-\phi$	p. f.	Single − phase kw.	Single −phase kw.	p. f.
Load L 1	1862	0.82	1.24	1.20	0.84
Load L 2	1060	0.695	0.707	0.670	0.696
Load L 3	200	0.50	0.133	0.129	0.495
Load L 4	not given			4.18	0.94
Total generator load	10,400	0.88	6.93	6.53	0.86
Line voltage bus a	13,800 volts		440 volts to neutral	440 volts to neutral	

*The full-size-to-miniature reduction factors are as follow: For computing miniature system values, divide the full-size values by these quantities:

500	for power or kv-a.
18.1	for voltage
27.6	for current
0.655	for resistance, reactance, or impedance

system value of kv-a. equivalent to the full-size value of 11,820 kv-a. is:

$$\frac{3 \times 440 \times 18}{1000} = 23.7 \text{ kv-a.}$$

This gives a kv-a. reduction factor of $\frac{11{,}820}{23.7} = 498$, or approximately 500, which latter value will be used. From the figures already given, the voltage reduction factor is $\frac{13{,}800}{440\sqrt{3}} = 18.1$. Consequently, the current reduction factor is $\frac{500}{18.1} = 27.6$, and the impedance reduction factor is $\frac{18.1}{27.6} = 0.655$; that is, the miniature-system resistance and reactance values are numerically larger than those of the full-size system. With these ratios, the desired miniature-circuit values of Table III, column 3, were obtained. Capacity charging currents, being practically negligible for the purpose of these tests, were left out of consideration. The effect of measuring instruments on the circuit impedances will be illustrated for the case of line B_1. From Table III, column 3, the desired miniature value of Z_{B1} is $1.56 + j\,3.85$ ohms. A 10-ampere ammeter and a 10-ampere current-coil of a wattmeter (both of type P3) introduce additional series impedances of $0.044 + j\,0.019$ ohms and $0.030 + j\,0.014$ ohms, respectively, representing a total combined impedance of $0.074 + j\,0.033$ ohms due to the instruments. For line B_1 the resulting errors are then $\frac{0.074}{1.56} \times 100 = 5$ per cent for the resistance component, $\frac{0.033}{3.85} \times 100 = 1$ per cent for reactance component, and, likewise, 2 per cent for impedance magnitude. The impedance error is negligible. In other cases, the resistance and reactance box settings were reduced in accordance with the instrument impedances, if the latter were more appreciable.

The loads L_1, L_2, L_3 and L_4 were represented as constant impedances by adjustable resistance and reactance units. It is quite feasible, when desired, to employ one or both of the 3.75 kv-a. alternators as motors in place of constant-impedance loads, as already indicated.

In column 4 of Table III, are shown the miniature-system data employed in the tests. These values differ slightly from those of column 3, *i. e.* from the desired miniature values, on account of the steps of the variable resistors and reactors. The differences in question, however, do not exceed 6 per cent for the values which determine the accuracy of the results.

From the four load values and the circuit impedances given in Table III, *the experimental miniature results and the corresponding full-size results* shown in Table IV were obtained. The results comprise the currents throughout the network and the voltages at the substation busses, for balanced generator voltages and balanced loads. A sufficient number of phase angles were determined, with the aid of indicating wattmeters, to establish definitely the phases of all currents throughout the system.

TABLE IV
RESULT OF MINIATURE TESTS FOR CIRCUIT DATA OF TABLE III AND FIG. 3

	Miniature Test Values		Data converted to full-size scale by conversion factors
	Amperes		Amperes
Current I_A	5.88	27.5° lag*	162
" I_{B1}	8.25	33.1° lag*	228
" I_{B2}	4.74		131
" I_{C1}	3.27	37.6° lag*	90.2
" I_{C2}	1.07		29.5
" I_{C3}	0.80		22.1
" I_{L1}	3.51		97.0
" I_{L2}	2.39		66.0
" I_{L3}	0.645		17.8
" I_{L4}	11.1		306
" I_G	17.4		480
	Volts to neutral		Volts line-to-line
Voltage V_G	440		13,800
" V_{L1}	407		12,770
" V_{L2}	404		12,680
" V_{L3}	403		12,640
" V_{L4}	402.5		12,620

*Lag angles referred to voltage V_G to neutral.

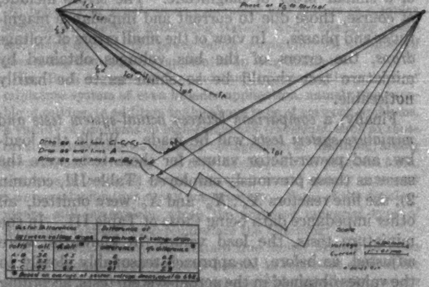

FIG. 4—VECTOR DIAGRAMS OF VOLTAGE DROPS OVER THE LINES OF FIG. 3B FOR TEST CONDITIONS OF TABLE IV

These voltages were obtained by multiplying test values of full-size currents by actual-system line impedances. Disagreement between vector voltage drops is due to test errors.

Vector calculations for the accuracy of the miniature results were made, as indicated in Fig. 4. The vector voltage drops over each of the three paths from bus *a* to bus *e* (see Fig. 3b) were obtained by complex algebra, by multiplication of test current values (converted to full scale, from Table IV last column) with actual-system impedances (from Table III, column 2). The maximum per cent mutual disagreement between the voltage *drops* so found is 7.5 per cent, while their per cent deviation from the average of the voltage drops is of the order of 4 per cent or less. It may, therefore, be concluded that the errors of the current values are of a similar order of magnitude. The errors include, of course, those due to current and impedance magnitudes and phases. In view of the small errors of voltage *drops*, the errors of the bus voltages obtained by miniature test should be so small as to be hardly noticeable.

TABLE V.
COMPARISON OF MINIATURE-CIRCUIT AND ACTUAL-SYSTEM TESTS

	Results of Miniature Tests				Results of Actual system tests		% discrepancy between miniature & full-size results; % based on full-size values
	Small-scale values		Equivalent full-scale values				
	Amperes	Power factor	Amperes	Amperes corrected for voltage*	Amperes	Power factor	For current
Current I_A	4.00	0.68	111	106	105	0.77	0.9%
" I_{B1}	9.77	0.92					
" B_1'	4.88	0.92	134	128	130	0.88	1.5%
" B_1''	4.89	0.92	135	129	135	0.92	4.5%
" I_{C1}	4.18	0.89	115	110	108	0.91	1.7%
	Single – phase kw.		3 – phase kw.		3 – phase kw.		for kilowatts
Kilowatts line A	1.20		1800		2000		10%
" " B_1	3.94						
" " B_1'	1.97		2950		2850		3.5%
" " B_1''	1.97		2950		3100		4.8%
" " C_1	1.64		2460		2450		0.4%
			Line Voltage				
	Line-to-neutral volts			corrected†	Line voltage		
Voltage V_G	440		13,800	14,400	14,400		

*Corrected by factor 138/144 due to difference of generator voltages in miniature and full-size tests.
†Correction factor 144/138 applied.

Finally, a *comparison between actual-system tests and miniature-system tests* will be made. While the load-kw. and power-factor values for these tests were the same as those previously employed (Table III, column 2), the line reactors X_B', X_B'' and X_C' were omitted, all other impedance data being those of Table III. In the miniature tests, the load values (in kilowatts) were adjusted, as before, to approach reasonably closely to the values obtained in the actual-system tests, according to the lower part of Table III. Comparative results of both tests, shown in Table V, indicate discrepancies of less than 5 per cent. with the exception of the values of power for line *A*, which differ by 10 per cent. In the consideration of these differences, the complications involved in the actual-system measurements—made at mutually remote points under varying loads, with switchboard instruments,—should be taken into account. It may, therefore, be concluded that the miniature-test data are a good equivalent of the actual-system behavior. Moreover, the preceding comparisons established a relatively good accuracy of the actual-system tests.

Conclusion

The miniature a-c. electric power transmission system described permits the experimental solution, in the laboratory, of operating and design problems for actual transmission systems. Generating stations, transformers, substations, the transmission network and loads for systems may be correctly represented in the laboratory. Conditions of normal operation as well as short-circuit conditions may be reproduced. High-frequency transients, such as traveling waves, cannot be experimented with on this type of miniature system, on account of its lumped circuit elements, but the low-frequency transients, such as those of alternating-current generators, are correctly shown in the miniature circuit. Adjustable circuit constants, and quickly variable circuit connections, are used to permit miniature representation of the majority of the customary circuit conditions and system connections.

It has been possible with the aid of this miniature system not only to solve practical problems for transmission and operating engineers, but also to confirm, in several instances, the theory underlying calculations of circuit phenomena. The experimental laboratory solution has been compared, and found to be in good agreement, with the results of tests made in the field on commercially operating circuits and with the results of calculations.

Discussion

G. M. Armbrust: Mr. Schurig describes a very practical method for the solution of the most important problems in the design and operation of a transmission system.

The tendency of design of transmission systems generally is toward radial distribution or some limited form of network. This sectionalizing has resulted from numerous and extensive operating troubles occurring on a solidly inter-connected network such as the 12,000-volt transmission system of the Commonwealth Edison Company of Chicago, to which I refer. However, the radial scheme of distribution is expensive as reserve line capacity is required for every unit of load. It is therefore desirable to have some means of quickly solving a number of problems to determine the most economical design within the limits of the capacity of apparatus and protective schemes.

These problems are numerous as every group of load units has different geographic relations and electrical constants. The rapid growth of load makes these problems continuous.

This also brings about another condition; the time required to obtain and install cable makes necessary many temporary arrangements which must operate for a considerable time before an ultimate scheme can be realized.

The miniature a-c. scheme provides means for the practical solution of such cases without too great an amount or complication of equipment. The solution of problems mentioned is perhaps of the greatest commercial value, but as Mr. Schurig points out the scheme permits the investigation of many other problems.

In applying the miniature to the cable transmission system mentioned, it is apparent that solutions can be obtained without the use of a great number of units. Most of the problems would be single phase where a larger number of units are available, while in those requiring three-phase solutions it would not be necessary to represent the system in detail; sections could be represented by single units.

In studying the scheme, I had hoped that the miniature system could be reduced to a smaller scale, perhaps comparable with the d-c. calculating table, but limitations of a-c. measuring instruments make this impractical.

I should like to hear Mr. Schurig's opinion on this, regarding the possible minimum scale of the equipment.

V. Bush: It will not be long in my opinion before every large power system will install a calculating table as a part of its engineering equipment. Because of the much greater information available these tables will probably gradually be constructed as a-c. duplications. In addition to the advantages of the a-c. form mentioned by Mr. Schurig, such tables will allow of the investigation of such matters as are involved in power factor, the proper location and size of correcting apparatus, etc.

An a-c. network may be a single-wire or three-wire representation. The former is of course less expensive and may be installed first, and later form part of a three-wire representation. Single-phase short circuits, unbalance, etc. may be studied on the complete artificial system; but care must be used in order to properly represent the inherent tendency of rotating machinery to balance phases. This is a similar matter to that of proper representation of sustained short-circuit conditions. The constants of the piece of apparatus which represents a generating station must be altered to simulate the field reaction.

Either lumped or distributed constants may be used in the representation. Where only a few types of lines are involved, the latter is probably not much more expensive than the former, and it allows also of some small study of transient effects.

The difficulty of measurements on the a-c. system, without disturbing conditions on adding the meter, may be avoided by the use of the vacuum tube repeater. We have found at the Massachusetts Institute of Technology that this does not involve any great difficulty in maintaining calibration. Current or voltage at any point in the network may be readily measured in this manner without drawing appreciable current. Hence the a-c. representation may be made as small in size as desirable.

There is one point in the artificial system that warrants considerable attention. This is the representation of the generating stations where several are simultaneously connected to the network. It is necessary that the relative phase displacements of the voltages supplied to the corresponding points in the artificial network be adjusted upon each change of load to correspond exactly to the phase displacements that will occur automatically under similar conditions in the actual system. Generating stations may be represented by a voltage supplied from a coil adjustable in a circular field, such as in a polyphase regulator; and the phase adjustment may then be made by rotating the coil manually. Similar treatment is necessary for any large synchronous apparatus in the system. It is not necessary that the coil of the regulator have internal constants corresponding to those of the station represented, for the constants can be added externally and the "generated" voltage maintained manually inside of these. The phase adjustment at each artificial station is a matter depending upon governor regulation. When full data on this point are available it is apparently possible to set up correct rules for the adjustment of phase upon change of load.

The transformers may of course be represented by equivalent networks, and all constants reduced to those corresponding to one voltage chosen as base.

A considerable amount of study is necessary before all the points in the manipulation of an artificial network of this sort are worked out. Some such means of attack is, however, imperative in order to properly lay out and operate systems of the magnitude at present projected.

O. R. Schurig: The miniature power generating and transmission system described in the paper has its chief application, as already pointed out, in the solution of *system network* problems, such as the determination of (1) normal load distribution in the lines and in generators, (2) system voltage regulation for balanced or unbalanced loads, (3) transient and sustained short-circuit currents at any point of the system for any kind of short circuit, (4) the behavior of relays during short circuits, (5) current-limiting reactor magnitudes and their best location (6) behavior of synchronous generators and motors in complex systems with several generating stations under changing loads, and short circuits, (7) power factor correction, (8) stability of generating and receiving apparatus on long lines, etc.

Dr. Bush suggests, if I understand him correctly, a type of miniature system of even broader application, namely one capable of solving, in addition to the above type of network problems, a variety of problems on high-frequency transients, such as those involving traveling waves, peak voltages due to lightning, arcing grounds, and switching. If fully developed in a practical form, the type of artificial circuit suggested would clearly represent a universal miniature circuit construction. Some of the developments, still incomplete, which the proposed universal type of miniature system would require are the following:

(1) Development of miniature line units: Miniature units of the "smooth" type would be needed to represent each of the sizes of cables as well as units to represent electrically the various types of aerial line construction. A far greater variety of artificial line units than now available would be called for in the representation of an average transmission system. Furthermore, the development of equivalent miniature generator, motor and transformer units—to meet the small scale of the proposed miniature system—would be necessary, as indicated by Dr. Bush.

(2) Measuring instruments and methods: A miniature system should permit measurements of current, voltage and power to be made with considerable facility. Several years of experience with practical miniature network tests have indi-

cated the necessity of having connected in circuit from 20 to 30 measuring instruments, on an average, for substantially simultaneous readings by the observers. That is a considerable number of instruments, covering a large table, when the customary portable type of instrument is used. In the low-current type of "smooth" miniature system proposed by Dr. Bush, special measuring methods such as those involving vacuum-tube amplifiers are needed. Though considerable progress has been made by Dr. Bush and his associates in the development of such measuring methods, the vacuum-tube amplifier scheme in its present state is probably too delicate and complicated for practical measurements when quick results and a large number of measurements are wanted.

One factor of prime importance in power system network investigations is the measurement of *power*. Experience has shown a visual direct-reading power indicator (such as an ordinary portable wattmeter) to be most essential. For the "smooth" type of miniature circuit, we need a new direct-reading wattmeter of much reduced current and voltage consumption.

The above considerations lead to the following summary:

There is a definite field for each of the two types of miniature circuit under discussion:

The network type of miniature system including synchronous machines and lumped transmission line units, as described in the paper, has already established its practical usefulness for the solution of normal-load and short-circuit problems at fundamental frequency; a large bulk of the pressing problems, both of the routine and research type, in transmission networks may be solved in a practical manner; the current rating of the generator and line equipment should be not less than 5 or 10 amperes. Thus, the equipment necessary and the methods of testing are substantially the customary and well-established ones, including power measurements by portable indicating wattmeters. The use of the miniature equipment by central-station and transmission engineers involves no radical departure from full-size system operation. It is not desirable to further broaden the use of the network type of miniature system by substituting "smooth" circuit elements for the "lumpy" type in an attempt to make it applicable also to high-frequency transient problems. The change would introduce new and unsolved problems of design. If "smooth" type of circuit elements having a current rating of an order of magnitude lower than 5 or 10 amperes were used, the convenient direct-reading type of instruments would no longer be applicable for voltage and power measurements. The vacuum-tube type of instrument has, up to the present time, not reached a state of development in which it can be considered practical for quick results when a large number of substantially simultaneous observations are wanted.

The "smooth" type of miniature line, on the other hand, is called for in high-frequency transient investigations. The current rating of the equipment is preferably below 5 amperes, and a good many of the tests may be so conducted as to require but a small number of measuring instruments, which (in accordance with the practise developed by Dr. Bush[1] and Professor Dellenbaugh[1]) are of the vacuum-tube amplifier type. The design of equipment and the manipulation of the tests call for a high degree of specialization—somewhat outside the field of central-station and transmission-line engineering but well within the field of a University laboratory. A number of the large engineering schools now have technical engineering laboratories supported by the industry. Thus, the transmission engineer's problems calling for smooth-line tests in miniature may be assigned to the proper university laboratory for solution, as has already been done in the past. The new contact thus afforded between the industry and the university seems to be beneficial not only to professors and students; but also to the industry as well.

In reply to Mr. Armbrust's question referring to the possible minimum scale of the a-c. miniature-system equipment, I will say that the size depends on the use to which the miniature system is to be put: If to be used for current measurements only, a much smaller equipment will suffice than for a miniature system intended for power and voltage measurements as well. It is assumed, of course, that indicating instruments are to be employed, at least part of the time. For tests involving *power, voltage and current observations* at several points of a system, a 10-ampere continuous rating at 440 volts (20-ampere short-time rating) has been found to be about the minimum for the line units of resistance and reactance. The figures given are those for the miniature system at Schenectady, as described in the paper. A lower current rating will introduce appreciable errors due to the change of circuit conditions brought about by the insertion of the customary portable measuring instruments, as indicated by Mr. Armbrust.

The minimum practical scale of an a-c. equipment for *current observations only* has not been definitely determined, because there has been no call for such an equipment. It is not at all unlikely that the current rating may be reduced to a value of the order of 50 milliamperes, the measurements to be made with thermocouple a-c. ammeters, some types of which have a voltage drop well below the maximum permissible value set by miniature-line requirements. The design of inductance units for the low current values, such as 50 milliamperes, should not offer great difficulties. In this connection, it will be recalled that there are a number of load division and short-circuit problems which call for current measurements, but not for power and voltage measurements. However, it has been felt in the past that the restriction of the equipment to observations of current only, was not desirable in view of the greatly increased use obtained from a device giving power and voltage data in addition to current. It should be pointed out here that the size and cost of the equipment are not to be expected to be proportional to the current rating on account of the increased number of turns required for the inductance units of lower current rating.

Another matter bearing on the size of the miniature equipment is *the number of circuit units required*. At first thought it might appear that the requisite number of miniature units must closely approach the number of network branches of an entire transmission system. As a matter of fact, there are relatively simple circuit transformations, of a mathematical or geometrical sort, that may be applied to complicated networks to reduce the total number of elements in the mesh of circuits. Transformations of the kind that I have in mind have been worked out by Kennelly[2], Fortescue[3], Evans[4] and others. It is often surprising how much labor is saved by the simplest of transformations.

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[1] See papers, by the respective authors, read at Annual Convention.

[2] *Elec. World & Eng.* Sept. 16, 1899, p. 413.

[3] *Elec. Jour.* Aug. 1919, p. 350.

[4] *Elec. Jour.* Aug. 1919, p. 345.

A Simplified Method of Analyzing Short-Circuit Problems

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***Review of the Subject.**—This paper presents the theoretical basis for a theorem by which the practical analysis and visualization of short-circuit phenomena can be greatly simplified. The theorem follows from the approximation of neglecting resistance in the application of Kirchhoff's Law to closed circuits. Thus in any problem of short circuits in which the effect of resistance is negligible in the initial moment, and this includes many of them, the theorem applies. It is: If the resistance of a closed circuit is zero, then the algebraic sum of the magnetic linkages of the circuit must remain constant.*

Illustrative examples are given, including the transformer, the single-phase and polyphase alternator, and the induction motor.

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THE practical analysis and visualization of many short-circuit problems may be greatly simplified by the *theorem of constant flux linkages.* This theorem has been mentioned in an incidental way on two previous occasions,[1] but its importance appears to warrant further emphasis. Attention was drawn to its possible greater usefulness, by the mathematics and some of the conclusions in a recent Institute paper on *Electromagnetic Forces.*[2]

SCOPE

The purpose of this paper, therefore, is to present the theoretical basis for the theorem, and illustrate its use by a few practical examples.

It is based on Kirchhoff's law of voltages, and follows from the approximation of neglecting the resistance in the circuits. It is stated as follows: *If the resistance of a closed circuit is zero, then the algebraic sum of the magnetic linkages of the circuit must remain constant.*

The theorem thus applies in any case in which the effect of resistance can be neglected. This includes most practical problems, because the effect of resistance in determining the *initial* rush of current is usually negligible. Its effect in determining the transient, or decay of current is, of course, decisive; but usually the problem is to determine the initial current rush. That is, the result of first importance is the initial condition, immediately following sudden short circuit; and this is, of course, practically the condition which would exist if there were no transient—in other words, the condition which would exist if the resistance in the circuits were zero. The application to practical problems, therefore, is very broad.

THEORETICAL BASIS

In most short-circuit problems, Kirchhoff's two laws are dealt with. The second law states that the sum of all e. m. fs. around a closed circuit must equal zero. That is,

$$\Sigma e = 0 \qquad (1)$$

Simple Case. It follows that in a simple circuit[3] as shown in Fig. 1,

$$r\,i + \frac{d}{d\,t}(L\,i) = 0 \qquad (2)$$

where
r = resistance (ohms)
L = inductance (henries)
i = current (amperes)

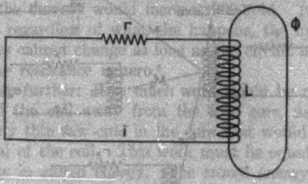

FIG. 1

Referring to equation (2), if

$$r = 0$$

then

$$\frac{d}{d\,t}(L\,i) = 0 \qquad (3)$$

That is, the product $L\,i$ is constant. But what does the product $L\,i$ represent? The inductance L is equal to the "flux-turns," or "linkages" per ampere. That is,

$$L = \frac{\phi N}{i \times 10^8}$$

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[1] Reactance of Synchronous Machines by Doherty and Shirley, TRANS. A. I. E. E. 1918, Vol. 37, Part 2, p. 1209.

Short-Circuit Current of Induction Motors and Generators, by Doherty and Williamson, TRANS. A. I. E. E., 1921, Vol. 40, p. 509.

[2] By Carl Hering, JOURNAL A. I. E. E., Feb. 1923, p. 139.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

[3] Neglecting capacitance which is practically never a significant factor in short-circuit problems.

Thus the product $L\,i$ represents $\phi\,N$, which is the magnetic linkages of the circuit. Therefore, *the magnetic flux*[4] *linked with a closed circuit of zero resistance cannot change.*

It will be shown that this is true not only for the simple circuit shown in Fig. 1, but in general, even for a circuit which is part of a network and which may, in addition, be in mutual inductive relation to other circuits. The only conditions are, that the circuit shall be closed and shall contain no resistance.

General Case. The validity of the theorem for the general case will now be established.

Consider the circuits shown in Fig. 2, which are assumed to be in relative motion.[5]

Applying equation (1) to the closed circuit A,

$$i\,r + \frac{d}{d\,t}\,(L\,i) + \frac{d}{d\,t}\,(M_1\,i_1 + M_2\,i_2 + M_3\,i_3 + \ldots) = 0 \tag{4}$$

or in general

$$i\,r + \frac{d}{d\,t}\,(L\,i + \Sigma\,M\,i_s) = 0 \tag{5}$$

where i_s = current in secondary or inductively related circuits. Neglecting resistance,

$$\frac{d}{d\,t}\,(L\,i + \Sigma\,M\,i_s) = 0 \tag{6}$$

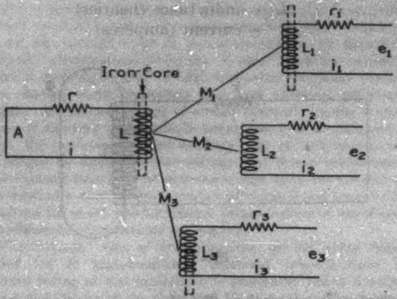

FIG. 2

or

$$L\,i + \Sigma\,M\,i_s = \text{constant} \tag{7}$$

But the expression

$$L\,i + \Sigma\,M\,i_s$$

represents the magnetic linkages of the closed circuit A. Hence, the theorem applies to a detached closed circuit as shown in Fig. 2, in mutual inductive relation to any number of secondary circuits.

It remains to be proved that it holds also for the perfectly general case, as illustrated in Fig. 3, in which, as in Fig. 2, the circuits are assumed to be in relative motion and in addition the closed circuit A is made up of branches of a network.

Thus consider *any* closed circuit, as A in Fig. 3.

(a) in relative motion to, and inductive relation with, any number of secondary circuits.

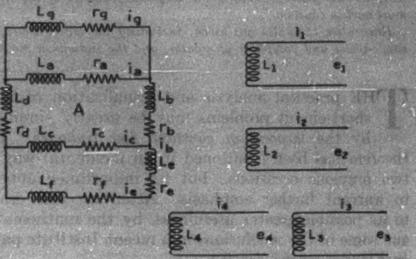

FIG. 3

(b) in multiple connection with other circuits.

Let

$i_a, i_b, i_c,$ etc. be the currents in the several individual branches of the closed circuit.

$L_a, L_b, L_c,$ etc. be respectively the inductances of these branches.

$r_a, r_b, r_c,$ etc. be the corresponding resistances.

$M_1, M_2, M_3,$ etc. be the mutual inductances between the closed circuit and the secondary circuits.

$i_1, i_2, i_3,$ etc. be the currents in the secondary circuits.

All the quantities are considered as variables. The units are: i's in amperes; r's in ohms; L's and M's in henries. By Kirchhoff's Law

$$\Sigma\,e = 0$$

Thus, summing the voltages around the circuit A,

$$(i_a\,r_a + i_b\,r_b + i_c\,r_c + i_d\,r_d) + \frac{d}{d\,t}\,(L_a\,i_a + L_b\,i_b + L_c\,i_c + L_d\,i_d + M_g\,i_g + M_e\,i_e + M_f\,i_e + M_1\,i_1 + M_2\,i_2 + M_3\,i_3 + \ldots) = 0 \tag{8}$$

That is,

$$i\,r + \frac{d}{d\,t}\,(\Sigma\,L\,i + \Sigma\,M\,i_s) = 0 \tag{9}$$

Neglecting resistance this becomes

$$\frac{d}{d\,t}\,(\Sigma\,L\,i + \Sigma\,M\,i_s) = 0 \tag{10}$$

But, as before, the quantity in the parenthesis represents the number of magnetic linkages of the circuit. The equation states that the rate of change of the number of linkages is zero; *hence the number is constant.* Therefore the theorem is general.

---

[4] More accurately, the magnetic linkages. For instance, two lines linking half the turns would obviously represent one total linkage. Thus on account of partial linkages, it is not quite accurate to say the "magnetic flux" is constant, but only the flux representing complete linkages is constant.

[5] And therefore both the M's and L's are variable: M's variable by relative motion; L's may be variable either by saturation or by proximity to moving iron of secondary circuits.

Thus to repeat: *If the resistance of a closed circuit is zero, then the algebraic sum of the magnetic linkages of the circuit must remain constant.*

It is well at this point to interpret the physical meaning of this statement. Magnetic linkages are expressed numerically by the product of the magnetic flux and the number of turns with which the flux is linked,[6] or, in general, by the product of the inductance L or M by the current. Therefore in a closed circuit without resistance if the inductance, either self or mutual, changes for *any* reason, or if the current changes in any mutually related circuit, then there must be a compensating change in current in the closed circuit to maintain constant its magnetic linkages. Do not misunderstand. The distribution of linkages among the different branches or component, series elements of the closed circuit need not remain the same, but the algebraic sum of all linkages must remain constant.

However, in special cases of symmetrical branches such as the star-connected polyphase machine, with sine wave flux distribution on open circuit, it can be shown[7] that the flux linkages in *each branch* or phase remains constant when the terminals are short-circuited.

ILLUSTRATIVE EXAMPLES

Transformer. Consider a few simple illustrations to visualize what this means. In Fig. 4A, a transformer primary is connected to a battery, the secondary being open. Of the total flux ϕ, established by and linking with the primary, part of it ϕ_m links the secondary, and the remainder ϕ_{l1} passes between the windings, that is, as leakage flux. Now close switch s_2 (Fig. 4B), and assume zero resistance in the secondary. Obviously no electrical change will follow. Open switch s_1 (Fig. 4C). What happens? We know that the secondary linkages

$$\phi_m N_2 = i_1 M$$

cannot change, and hence a secondary current i_2 must appear such as to satisfy the condition

$$i_2 L_2 = i_1 M = \text{constant} \qquad (11)$$

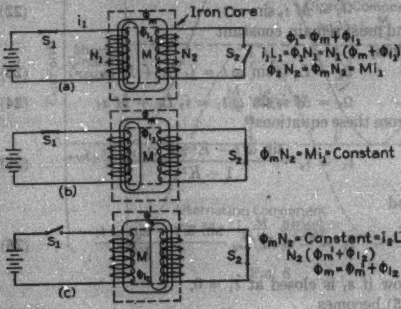
Iron Core
i_1
S_1
N_1
M
N_2
S_2
ϕ_m
ϕ_{l_1}
$\phi_1 = \phi_m + \phi_{l_1}$
$i_1 L_1 = \phi_1 N_1 = N_1(\phi_m + \phi_{l_1})$
$\phi_2 N_2 = \phi_m N_2 = M i_1$
(a)
S_1
M
S_2
$\phi_m N_2 = M i_1 = \text{Constant}$
(b)
S_1
M
ϕ_{l_2}
S_2
$\phi_m N_2 = \text{Constant} = i_2 L_2 = N_2(\phi_m' + \phi_{l_2})$
$\phi_m = \phi_m' + \phi_{l_2}$
(c)

FIG. 4

It is interesting to follow this further. Suppose the secondary winding is now taken completely out of the transformer, that is, away from any iron as indicated in Fig. 5A. The theorem still applies. The inductance L_2' is very much less, perhaps 0.005 of its former value, L_2. It does not matter. The linkages must remain constant. Hence the current i_2 must correspondingly increase so that

$$i_2 L_2 = i_2' L_2' \qquad (12)$$

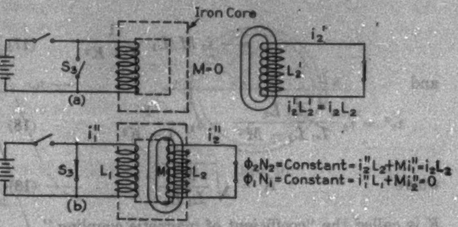
Iron Core
S_3
M=0
i_2'
L_2'
$i_2' L_2' = i_2 L_2$
(a)
i_1''
i_2''
S_3
L_1
M
L_2
$\phi_2 N_2 = \text{Constant} = i_2'' L_2 + M i_1'' = i_2 L_2$
$\phi_1 N_1 = \text{Constant} = i_1'' L_1 + M i_2'' = 0$
(b)

FIG. 5

Thus the current would increase 200 fold. In other words, regardless of what else happens, the magnetic linkages cannot change as long as the circuit is closed and the resistance is zero.[8]

A step further: How much work would be required to pull the coil away from the iron core, assuming infinitely thin saw cuts in the core that would permit removal of the coil. This work must be equal to the increase in stored energy. The stored energy is

$$1/2\, L_2\, i_2^2 \text{ before}$$

and

$$1/2\, L_2'\, i_2'^2 \text{ after}$$

Work is $$W = 1/2\,(L_2'\, i_2'^2 - L_2\, i_2^2) \qquad (13)$$

But by (12),

$$L_2\, i_2 = L_2'\, i_2'$$

Solving for i_2' and substituting in (13),

$$W = 1/2\, L_2\, i_2'\,(L_2/L_2' - 1) \qquad (14)$$

If L_2 is large compared with L_2', as here, (14) becomes,

$$W \simeq 1/2\, L_2\, i_2^2\,(L_2/L_2')$$

That is, the work is approximately equal to the initial stored energy multiplied by the ratio

$$L_2/L_2' = 200$$

or $$W = 200\,(1/2\, L_2\, i_2^2)$$

---

[6] Keeping in mind the statement previously made regarding partial linkages.

[7] *Short-Circuit Current of Induction Motors and Generators,* Doherty and Williamson, TRANS. A. I. E. E., Vol. 40, p. 537.

[8] If the circuit contains some resistance, as of course it must, the linkages must decrease at a rate just sufficient to generate the $i\,r$ drop.

Next consider what happens when the same secondary coil, now shown in Fig. 5A is again placed in the transformer as before, except that before this is done, the switch s_1 is closed as in Fig. 5B. The primary is thus closed under the condition of zero magnetic linkages, and must therefore remain in that condition. Thus the number of magnetic linkages of primary is

$$i_1'' L_1 + M i_2' = \text{constant} = 0$$

and of the secondary

$$i_2'' L_2 + M i_1'' = \text{constant} = i_2 L_2 \qquad (16)$$

From which

$$i_1'' = - i_2 \frac{M L_2}{L_1 L_2 - M^2}$$

$$= - i_2 M/L_1 \frac{1}{1 - K^2} \qquad (17)$$

and

$$i_2'' = i_2 \frac{L_1 L_2}{L_1 L_2 - M^2} = i_2 \frac{1}{1 - K^2} \qquad (18)$$

where $$K = \sqrt{\frac{M^2}{L_1 L_2}} \qquad (19)$$

K is called the "coefficient of magnetic coupling."

From the case of Fig. 5B, it is clear that the effect of the closed primary in maintaining zero flux[9] is to force the flux, which is linked with the secondary, out of the main magnetic path into the leakage paths. Thus the current i_2'' is as much greater than i_2 (which corresponds to Fig. 4C) as the reluctance of the leakage paths is greater than the reluctance of the main magnetic path. This is the ratio of the short-circuit current to the magnetizing current, and is given by equation (18), thus

$$i_2''/i_2 = \frac{1}{1 - K^2}$$

The foregoing examples involving direct current have been given for the purposes of illustrating with emphasis what would happen if the resistance were zero. Although actually with direct current the magnetic flux and the current in such cases as illustrated would gradually die out or decay, nevertheless for sudden changes or for many alternating-current conditions in which the change, or tendency to change, is rapid, the results are very closely those which would exist if the resistance were zero.

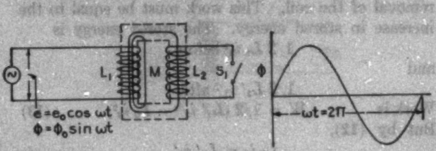

FIG. 6

Consider now the transformer as shown in Fig. 6. Assume that the primary is connected to an infinite alternating current system, that is, that the alternating voltage holds up after the switch s_1 is closed. Before short circuit, the number of magnetic linkages of the primary was

$$\Omega_1 = L_1 i_0 \sin \omega t \qquad (20)$$

and of the secondary

$$\Omega_2 = M i_0 \sin \omega t \qquad (21)$$

where

i_0 = magnetizing current
$\omega = 2 \pi f$
f = electrical frequency
t = time

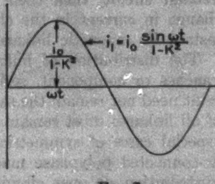

FIG. 7

After short circuit Ω_1 is still given by (20), since the voltage and therefore the alternating flux must hold up. But Ω_2 is the value of magnetic linkages existing at the moment t_1 when the secondary is short-circuited by closing switch s_1. It is,

$$\Omega_2 = M i_0 \sin \omega t_1 \qquad (22)$$

and must remain constant

Thus,

$$\Omega_1 = L_1 i_0 \sin \omega t = i_1 L_1 + M i_2 \qquad (23)$$

$$\Omega_2 = M i_0 \sin \omega t_1 = i_2 L_2 + M i_1 \qquad (24)$$

From these equations[10]

$$i_1 = i_0 \frac{\sin \omega t - K^2 \sin \omega t_1}{1 - K^2} \qquad (25)$$

and

$$i_2 = - i_0 M/L_2 \frac{\sin \omega t - \sin \omega t_1}{1 - K^2} \qquad (26)$$

Now if s_1 is closed at $t_1 = 0$,
(25) becomes

$$i_1 = i_0 \frac{\sin \omega t}{1 - K^2} \qquad (27)$$

and (26) becomes

$$i_2 = - i_0 M/L_2 \frac{\sin \omega t}{1 - K^2} = - M/L_2\, i_1 \qquad (28)$$

Equation (27) is plotted in Fig. 7.

The similarity of (27) and (18) is at once apparent, the only difference being that now the current is alternating. That is, the relation between the current before and after short circuit is the same in either case, namely,

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[9] Or linkages, to be accurate

[10] Assuming L_1, L_2 and M constant. If they are not constant, the expressions for i_1 and i_2 will be more complicated.

$$\frac{1}{1 - K^2}$$

Assume now that s_1 is closed at the instant $\omega t_1 = \pi/2$, that is, when the secondary encloses, or is linked with, maximum flux. The fact that the secondary linkages thus "caught" by the short circuit must remain constant, gives interesting consequences. Consider these consequences first in the light of the equations, and then from the physical point of view. By equation (25), for

$$\omega t_1 = \pi/2$$

$$i_1 = i_0 \frac{\sin \omega t - K^2}{1 - K^2} \quad (29)$$

for values of $t > t_1$, since the equations obviously hold only for conditions after the short circuit. And by (26)

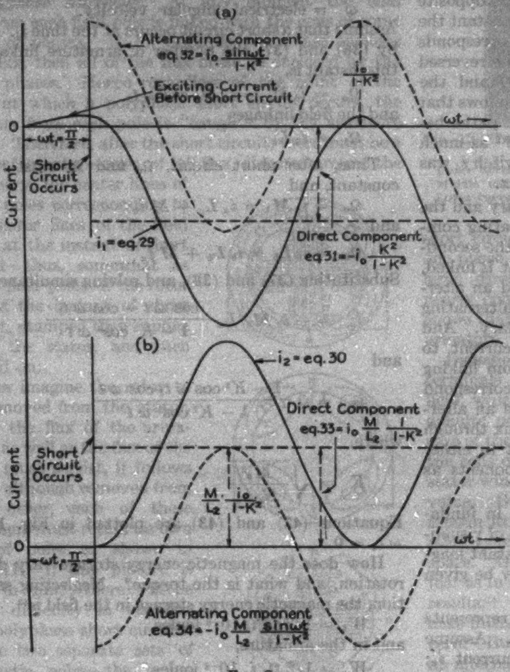

Fig. 8

$$i_2 = -i_0 M/L_2 \frac{\sin \omega t - 1}{1 - K^2} \quad (30)$$

also for values of $t > t_1$.

It will be observed that (29) and (30) each involve a constant or "direct" component of current in addition to the "alternating" component. In (29), for instance, the direct component is

$$i_1' = -i_0 \frac{K^2}{1 - K^2} \quad (31)$$

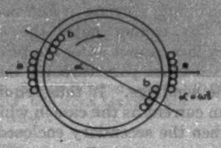

Fig. 9

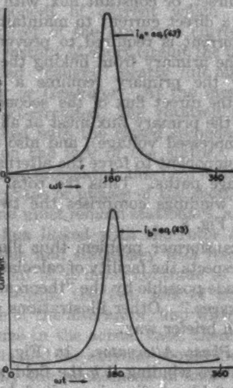

Fig. 10

and the alternating component is

$$i_1'' = i_0 \frac{\sin \omega t}{1 - K^2} \quad (32)$$

In (30), the direct component is

$$i_2' = i_0 M/L_2 \frac{1}{1 - K^2} \quad (33)$$

and the alternating component is

$$i_2'' = -i_0 M/L_2 \frac{\sin \omega t}{1 - K^2} \quad (34)$$

Equations (**29**) and (**30**) are plotted in Fig. 8, showing graphically the relation between the alternating and direct components.

Now examine the physics of the problem. Consider the case in which the switch s_1, Fig. 6, is closed when the secondary is linked with maximum flux, that is, when $\omega t = \pi/2$. This flux, or linkages rather, must remain constant. But obviously one-half cycle later, the flux in the primary is full value in the opposite direction, since the impressed voltage is maintained. That is, the primary is trying to force flux through the closed secondary, which must not only prevent this opposite flux from entering, but also must maintain constant the flux with which it is linked. This instant corresponds to the maximum current in Fig. 8. Since the reversed primary flux cannot enter the secondary, and the secondary flux cannot enter the primary, it follows that both fluxes must pass through the leakage paths between the windings. It thus requires twice[11] as much maximum current as the case in which the switch s_1 was closed when the secondary enclosed zero flux.

Thus the actual current in both the primary and the secondary comprises a direct and an alternating component of about equal maximum values. In the secondary the direct, or constant flux, with which it is linked, requires a direct current to maintain it, and an alternating current is required to prevent the alternating flux of the primary from linking the secondary. And similarly the primary requires a direct current to prevent the direct flux of the secondary from linking it (since the primary flux must at all times correspond to the impressed voltage), and also requires an alternating component to force the alternating flux through the leakage paths. Thus the total current in either of these windings comprises the two components as shown in Fig. 8.

The transformer problem thus illustrates in fundamental respects the facility of calculation and of visualization made possible by the "theorem of constant magnetic linkages." Other illustrations will now be given in a much briefer way.

Single-Phase Alternator. In Fig. 9, $a\,a$ represents the armature winding, $b\,b$ the field winding. Assume that the field winding is excited by the current i_0; that the field winding, having zero resistance, is short-circuited at the collector rings.[12] Thus as long as the permeance of the magnetic circuit is not changed, i_0 would continue to flow, by the condition that the field linkages must remain constant. But whatever happens, the field linkages cannot change. Remove the rotor from the machine. The field current will increase because the inductance has decreased. Replace it, and the current is once more i_0.

When running with open armature circuit, the number of field linkages is

$$\Omega_b = i_0 L_b \tag{35}$$

and the armature linkages,

$$\Omega_a = i_0 M \tag{36}$$

where,

$$M = M_0 \cos \omega t \tag{37}$$

M_0 = mutual inductance between armature and field windings in position $\alpha = 0$, Fig. 9.

ω = electrical angular velocity.

Assume that a short circuit occurs at the time t_1. Then by (**36**) and (**37**) the number of armature linkages at this instant is,

$$\Omega_a = i_0 M_1 = i_0 M_0 \cos \omega t_1 \tag{38}$$

and the field linkages

$$\Omega_b = i_0 L_b \tag{39}$$

Thus, after short circuit, Ω_a and Ω_b must remain constant, and

$$\Omega_a = i_0 M_1 = i_a L_a + M i_b \tag{40}$$

and

$$\Omega_b = i_0 L_b = i_b L_b + M i_a \tag{41}$$

Substituting (**37**) and (**38**), and solving simultaneously,

$$i_a = - i_0 M_0/L_a \frac{\cos \omega t - \cos \omega t_1}{1 - K^2 \cos^2 \omega t} \tag{42}$$

and

$$i_b = i_0 \frac{1 - K^2 \cos \omega t_1 \cos \omega t}{1 - K^2 \cos^2 \omega t} \tag{43}$$

where,

$$K = \sqrt{\frac{M_0^2}{L_a L_b}}$$

Equations (**42**) and (**43**) are plotted in Fig. 10 for $\omega t_1 = 0$.

How does the magnetic energy storage vary during rotation, and what is the torque? Neglecting saturation, the magnetic energy storage in the field is[13]

$$W_b = 1/2\, \Omega_b i_b 10^{-8} \text{ joules} \tag{44}$$

and in the armature,

$$W_a = 1/2\, \Omega_a i_a 10^{-8} \text{ joules} \tag{45}$$

But Ω_a and Ω_b are constant. Therefore the energy of each circuit varies directly with the current. The total energy is

$$W = W_a + W_b = 10^{-8}/2\, (\Omega_a i_a + \Omega_b i_b) \tag{46}$$

Since the electrical power supply is cut off, the change in energy must be supplied mechanically. That is, the energy supplied to the magnetic field during rotation through the angle $d\,\alpha$ is equal to the mechanical work done: Torque X angle. Hence,

$$d\,W = T\,d\,\alpha$$

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[11] Actually $(1 + K^2)$ times as great. K^2 is of the order of 0.99 in a transformer.

[12] The exciter voltage, of course, supplies only the $i\,r$ drop, and therefore corresponds to the initial rate at which the flux would actually die down if the collector rings were short-circuited on the given machine. Without resistance, it would not die down.

[13] Stored energy = $W = 1/2\, L\, i^2\, 10^{-8} = 1/2\, (L\, i)\, i\, 10^{-8}$ joules. But, $L\, i = \Omega$ Hence, $W = 1/2\, \Omega\, i \times 10^{-8}$ joules.

That is, the torque is

$$T = \frac{d\,W}{d\,\alpha} \qquad (47)$$

where $\alpha = \omega\,t$.

Differentiating (46),

$$T = \frac{d\,W}{d\,\alpha} = 10^{-8}/2\left(\Omega_a\frac{d\,i_a}{d\,\alpha} + \Omega_b\frac{d\,i_b}{d\,\alpha}\right) \qquad (48)$$

Thus the electromagnetic torque during short circuit is determined by the slope of the current curves.

Polyphase Alternator. At any moment the *total* flux from each field pole is linked with the coils of some phase or phases. If, for example, one phase encloses zero flux, then all of the flux must be enclosed by the other phases. Hence regardless of the point of the cycle at which a polyphase short circuit occurs, the full flux from each pole is "caught" by the armature coils. Therefore, after the short circuit, there must be a series of armature poles, of full flux, disposed around the periphery, the center lines of these poles corresponding to the center lines of the field poles at the instant of short circuit—thus, somewhat as if the field poles of the rotor had at the instant of short circuit, stamped their replica upon the stator, and then moved on.

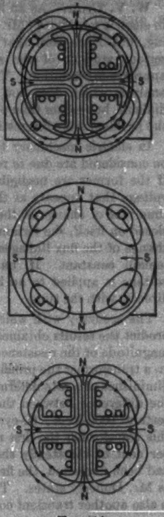

Fig. 11

Now imagine the rotor to be removed from the stator. Since the flux of the armature, as well as the flux of the field, must persist, it follows that, although removed from each other, each of these members must retain its own set of poles as shown in Fig. 11. If this picture is kept in mind, there will be little difficulty in visualizing polyphase short circuits. These two separate sets of magnetic poles—the armature and the field—both of about equal flux per pole, must persist, regardless of what happens, so long as the circuits are closed, and the resistance is zero. Thus if the rotor is again put into place and rotated, these two sets of poles, one rotating, the other stationary, pass over each other, and the currents adjust themselves so as to maintain constant flux linkages. When the poles are in the same relative position as that when the short circuit occurred, the currents will be of the value existing the instant before short circuit, namely, zero in the armature, and no load exciting current in the field. One pole pitch, *i. e.*, one half cycle, later, when the poles are opposing each other, thus forcing double flux through the leakage paths, the currents are maximum. Thus in the alternator, as in the transformer, if the armature, or secondary, "catches" any flux when the short circuit occurs, there must be a direct component of current, and also the maximum value of the total current occurs one half cycle after the moment of short circuit. Fig. 12 is an oscillogram of a three-phase short circuit of an alternator, showing the current of one phase, the field current and the line voltage. The large direct component of current which produces nearly complete offset of the armature current wave, shows that at the instant of short circuit the particular armature phase had "caught" nearly full flux.

Fig. 12—Three-Phase Short Circuit of a 10,000 Kv-a., 6600-Volt, 63-Cycle Generator

Upper curve-current in phase a.
Middle curve-field current.
Lower curve-armature volts.

Induction Motor or Generator. At the moment the terminals are short-circuited, the rotating magnetic field, which is linked with the closed secondary, has a definite position on the face of the stator. Like the case of the polyphase alternator, this field is linked with the coils of some phase or phases of the short-circuited stator winding, and must remain stationary, and at full value. But the flux linked with the rotor must also remain at full value and stationary *with respect to the rotor winding.* That is, it must continue to rotate in space. In other words, the two sets of poles exist, just as in the case of the alternator, and with the same results.[14] Of course in the actual case, there is resistance in the circuits and the currents would die down to zero. But initially the phenomena of short circuit in an alternator and in an induction motor are practically identical.

As a further illustration of this method of analysis it can be shown that the initial current rush to an induction motor[15] is the same when the line switch is closed upon the motor at standstill, as when the motor is running at synchronous speed.[16] In either case, the number of flux linkages in the closed secondary, or rotor, circuits is zero, and must remain so after the line switch of the primary circuit is closed. The primary circuits are open and of course contain zero linkages before the switch is closed. But consider the closed circuits formed by the switch, namely those each of which is made up of a motor phase and the corresponding alternator phase in series. In each of these circuits, each electrically disconnected from the other, a definite value of flux existed, or was linked with the circuit, at the moment the switch was closed, the value of flux linkages in each circuit depending, of course, upon the alternator pole position at that moment. The value of flux thus caught in each circuit must remain constant. But since, assuming constant line voltage, the flux in the alternator phase must continue to vary, through fixed values, and thus (by assumption of no leakage) can not hold the constant component of flux which must continue to exist somewhere in the circuit, it follows that it must exist in the primary phases of the induction machine. In other words there must be a direct, or stationary magnetic field around the periphery of the induction motor, just as in the previous illustrations. Also since the voltage is held up at the terminals of the motor there must also be a rotating field. That is, there exists a stationary, and a rotating set of poles around the stator periphery; and none of this flux can link with the rotor. Therefore, at standstill, the secondary must contain direct current to prevent the stationary primary flux from entering; and must contain alternating current to prevent the rotating flux from entering. And these components of current must be of the same magnitude since the fields they oppose are of the same value. On the other hand, the primary winding must contain direct current to sustain the constant or stationary component of flux in the linkage paths, and an alternating component to sustain the rotating field—both of same magnitude. Thus the total current in either the primary or secondary windings must contain a direct and an alternating component of current of the same magnitude

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[14] For further discussion of this, see "Short Circuit of Induction Motors and Generators" by Doherty and Williamson, Trans. A. I. E. E., Vol. XL, 1921, p. 509.

[15] If the motor is of sufficient size so that the resistance is practically negligible in the first moment after short circuit.

[16] To the Author's knowledge, no tests have been made to confirm this, but it follows from the theorem.

If the rotor is driven by external means at synchronous speed when the switch is closed, then, since neither the rotating nor the stationary field of the stator (both, the same as in the foregoing case) can enter the rotor, all conditions will be the same as before, excepting the rotation. Now the stationary field of the stator of course requires direct current in the stator windings, but it produces due to rotation of the rotor, alternating current in the closed rotor circuits; also the rotating field of the stator requires alternating currents in the stator windings, but being stationary with respect to the rotor, produces therein a direct current. And all currents are of about the same maximum values, and hence the same as at stand-still.

SUMMARY

Summarizing briefly the points brought out in the paper, it follows from Kirchhoff's Law of Voltage, that *if the resistance of a closed circuit is zero, then the algebraic sum of the magnetic linkages of the circuit must remain constant.* Thus if the reluctance of the magnetic path linking the closed circuit is increased, the current in the circuit must increase, and vice versa, to maintain constant the magnetic linkages. Or if a separate magnetomotive force is impressed on the magnetic circuit linking the closed circuit, tending to change the linkages of the closed circuit, then the current in the latter must adjust itself to balance the new m. m. f. and thus maintain constant linkages. The application of the foregoing theorem to the problems of short circuits of electrical apparatus accomplishes two important results: In the first place it tremendously simplifies the mathematics and calculation of the short-circuit current and related problems; also it makes it possible to visualize easily the phenomena—which, of course, is always highly desirable in any analysis.

Discussion

W. V. Lyon: As an introduction to the study of transients in electric machinery, Mr. Doherty's paper should prove interesting, but I am sure that neither Mr. Doherty nor any other serious student of the subject would be satisfied with this approximate solution. There is no doubt that a more careful analysis which does not neglect the dissipated forces is more difficult, but rather than proving a deterrent this difficulty makes the problem more interesting.

In electric machinery the only voltages that need ordinarily be considered are due to resistance and to variable flux linkages. If the former are negligible in comparison with the latter, the voltage that appears at the terminals of any circuit equals the time rate of change of the total flux linkages. If the circuit is closed on itself, the terminal voltage is zero, and the time rate of change of the flux linkages is zero; that is, the total flux linkages remain constant. If, on the other hand, a constant direct voltage is applied to an idle circuit, the flux linkages increase directly with the time.

The accuracy with which this "theorem of flux linkages" will predict the results obtained in practise depends upon the relative magnitude of the resistance and the self and mutual inductances. In a transformer the principal component of the transient current diminishes at a rate determined by the ratio of the resistance and the difference between the self and mutual inductances. In a certain 1000-kv-a., 60-cycle transformer this ratio is 56. In this transformer the first rush of current on short circuit is 163 per cent of the steady short-circuit current. If the resistance had been neglected, the first rush would have been 200 per cent, as Mr. Doherty states. The error is nearly 25 per cent. There is also another transient component, which is however but 2 or 3 per cent as large as the one just mentioned. This component diminishes at a rate determined by the ratio of the resistance and sum of the self and mutual inductances. This ratio is ordinarily 5 or 10 per cent, and might safely be assumed to be zero.

In a 75-h. p. induction motor we have tested, the ratio of the resistance to the self inductance was about 2.5 for the stator and 4 for the rotor. The two components of the transient current, instead of being constant as would be the case if the resistance were negligible, diminish at a considerable velocity. The one of higher frequency diminishes at a rate of 74 hyperbolic radians per second, and the one of small frequency at a rate of 48 hyperbolic radians per second. In this motor the first rush of current in one phase would be to a maximum value of about 3300 amperes if the resistance were negligible. Actually the first current rush has a maximum value of about 2200 amperes. In this case, neglecting the resistance leads to an error of 50 per cent, whereas if the resistance is taken into account, students have been able to compute the first current rush with an error of less than 10 per cent.

From these examples it would seem that resistance plays a larger part in determining the first current rush than we might be led to believe from a cursory study of the subject.

A. Boyajian: I think Mr. Doherty's paper is a valuable contribution to the electrical literature in that it gives a physical interpretation to the short-circuit transient, a subject which has been dealt with almost exclusively mathematically in the past, starting with differential equations and finding a complete solution that will satisfy them. It is brought out in this paper in an extremely interesting and visual manner that the short-circuit transient of flux and current is produced by the flux which is trapped in the short-circuited coil. The fundamental principle that no flux can enter or leave a short-circuited coil of zero resistance is used masterfully in analyzing the short-circuit transient of a number of electrical apparatuses.

Of course, ordinarily, short-circuited coils have a certain amount of resistance, by virtue of which flux can gradually penetrate into them or leave them, and the transient decays in a short time to a negligible value in accordance with the attenuation constant of the circuit, as is well known. But Mr. Doherty, I think, is to be commended for maintaining the presentation of the principle simply by ignoring the calculation of the rate of decay, inasmuch as the mathematics of these transients are quite old, but the physical interpretation of the phenomenon new to many.

R. E. Doherty: Professor Lynn considers the principle proposed in this paper as, at best, a poor approximation. What the paper presents is the proof of a useful theorem—at least the author has found it to be useful. Like any other theorem, if it is applied where it is not applicable, unreasonable results follow.

If any one is interested in seeing how the theorem may be *applied* for practical results in the case of induction motors, such as Prof. Lynn mentions, I would refer him to the method outlined in a previous A. I. E. E. paper.[1]

Now I am in complete accord with Prof. Lyon as to the desirability of rigorous attack of any problem of this sort; but in practical engineering I have found that I can stick to rigor up to the point where it leads to hopeless complication. Then I have to do something else—because an answer must be obtained.

The value which I feel is in this method of attacking short-circuit problems lies in the fact that it is possible to visualize what is going on, and in a large number of cases, to get actual quantitative calculations in a very simple manner. In those cases in which it is not possible to make quantitative calculations, one at least can obtain accurately the initial condition, that following the instant or the moment of the short-circuit; and if one who is working on such problems has some notion of what the effect of resistance is, he can from that, estimate close enough for practical purposes the rate of decay of the current.

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[1] Short-Circuit Current of Induction Motors and Generators, by Doherty & Williamson. TRANS. A. I. E. E., Vol. 40, 1921, p. 509.

Proximity Effect in Wires and Thin Tubes

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Review of the Subject.—*Formulas are given, with examples, for the following new problems in proximity effect resistance ratio: Thin Tube and Infinitesimal Wire; Two Thin Tubes in Return Circuit; Two Thin Tubes in Parallel; Insulated Cable Sheaths in Single-Phase Circuit; Cable Sheaths in Three-Phase Circuit, Flat Spacing; Finite Wire and Infinitesimal Wire; Two Wires in Parallel; Three-Phase Circuit, Triangular Spacing; Three-Phase Circuit, Flat Spacing.*

Reduction formulas for Bessel functions, suitable for skin effect problems, are tabulated, and a table of values of the first five orders, for use in drawing curves, is given.

* * *

THE calculation of the alternating-current resistance ratio due to skin effect or proximity effect, has been worked out for a number of shapes and combinations of conductors. The case of an isolated wire was solved by Clerk Maxwell in 1873, (Reference 1). This solution is usually expressed in terms of Bessel functions of the first kind. In 1886, Lord Rayleigh gave the calculation for skin effect in an infinite plane conductor, (Reference 2), and in 1909 Dr. A. Russell published a calculation for a concentric main, that is, a return wire inside of a tube (Reference 3).

The effect on the resistance, caused by current distortion in conductors in armature slots, was treated by A. B. Field, 1905, (Reference 4), R. E. Gilman, 1920, (Reference 5) and W. V. Lyon, 1922, (Reference 6).

In 1915, A. E. Kennelly, F. A. Laws and P. H. Pierce published some very precisely measured curves of various shapes and combinations of conductors taken up to 5000 cycles, (Reference 7). These were very valuable, for in 1915 the only calculation which was in a form to be applied to a finite conductor and checked by test was the calculation for an isolated wire, which they checked exactly. Several of the other test curves have since been checked by calculations, and the curves have been of undoubted help to those deriving the new formulas. Similar test curves up to 100,000 cycles were published in 1916 by A. E. Kennelly and H. A. Affel, (Reference 8).

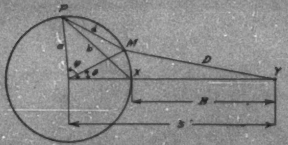

FIG. 1—THIN TUBE AND INFINITESIMAL WIRE

The proximity effect resistance ratio in a return circuit of two thin straps close together was published by the writer in 1916, (Reference 9).

In 1918, (Reference 10), calculations were published by the writer for skin effect in an isolated tube, and in an isolated thin strap, the latter at low frequency. (See also Reference 11 in which the exact skin effect ratio of a tube is given in terms of Bessel functions of the first and second kinds). In the 1918 paper, the "principle of similitude" was enunciated by the writer, that for a certain shape of cross section of circuit the resistance ratio is constant for a given value of f/R, thus correlating tests made at radio frequencies with those made for electric furnace work, etc. A proof of this principle was published by J. Slepian (Reference 12).

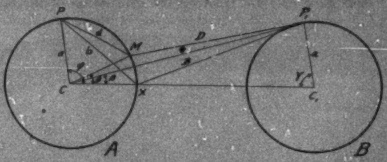

FIG. 2—TWO THIN TUBES IN RETURN CIRCUIT

The proximity effect in a return circuit of two wires was calculated for low frequencies in 1920 by H. L. Curtis (Reference 13). A complete solution was published by J. R. Carson in 1921, (Reference 14), and an alternative solution is given in this paper, (Formula VII).

The skin effect of a cable with steel armor wires and sea water return was calculated by J. R. Carson and J. J. Gilbert in 1921, (Reference 15).

The proximity effect of two thin tubes at low frequencies was given by the writer in 1922 (Reference 16). This result is checked and extended to high frequencies in this paper.

In 1922 a valuable mathematical paper on skin effect was published by Chas. Manneback (Reference 17), in which he describes a new method of attacking a skin effect problem. As with other new methods of solving skin effect problems mentioned in the above summary, this method is first used to obtain the standard formula for an isolated wire. The method is then applied to the case of the proximity effect of a finite wire and an infinitesimal wire, and an expression for the current density at any point of the section of the finite wire is obtained. No expression for the resistance ratio R'/R was obtained, and at first sight, the integration of the resistance losses over the section of the finite wire

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would appear to involve very complicated integrals in Bessel functions. The method the writer has adopted to overcome this apparent limitation of Manneback's method is to make use of a device involving Poynting's Theorem, given by J. R. Carson in Reference 14, equation (18).

The writer has used the above general method to obtain new formulas for ten cases of proximity effect resistance ratio (see Part II). The formulas are good for high frequencies as well as low. Except in the case of a return circuit of two wires, and the low-frequency range of two thin tubes, the problems are new and the values of the resistance ratio were previously not obtainable. The problems are of practical value in electric furnace work, underground cable work and radio work, as indicated in the examples. See also the examples in Reference 16.

I. THIN TUBE AND INFINITESIMAL WIRE

Let $X\,M\,P$ be an extremely thin tube of radius a and thickness t, and let Y be a very small return conductor. The calculation applies to the case in which the ratio of the thickness of the tube to its diameter is very small, and so the change in current density from the inner to the outer surface is inappreciable compared with the changes in current density in different parts of the tube due to the proximity of the return conductor. The return current is assumed concentrated at the point Y.

Let the current density at M be $i_{(\theta)}$. Then the resistance drop at M is $i_{(\theta)}\,\sigma$ where σ is the specific resistance of the conductor, absolute units being used. Similarly, if i_0 is the current density at X, the resistance drop at X is $i_0\,\sigma$.

The difference between the inductance drops at X and M due to flux caused by the current I_1 in the return conductor Y is given by the well-known inductance formula and is

$$j\,\omega\,2\,I_1 \operatorname{logh}\frac{D}{B} \text{ volts per cm.}$$

Let the current density at P be $i_{(\varphi)}$ and the element of area of section at P be $a\,t\,d\,\varphi$. The difference between the inductance drops at X and M due to flux caused by the current $i_{(\varphi)}\,a\,t\,d\,\varphi$ at P is

$$j\,\omega\,2\left(\operatorname{logh}\frac{d}{b}\right) i_{(\varphi)}\,a\,t\,d\,\varphi$$

This may be integrated around the tube and the difference between the inductance drops at X and M due to current in the tube is

$$j\,\omega\,2\int_{\varphi=0}^{\varphi=2\pi}\left(\operatorname{logh}\frac{d}{b}\right) i_{(\varphi)}\,a\,t\,d\,\varphi$$

Adding the resistance and inductance drops

$$i_{(\theta)} = i_0 + j\,\frac{2\,\omega}{\sigma}\,I_1 \operatorname{logh}\frac{D}{B}$$
$$+ j\,\frac{2\,\omega}{\sigma}\int_{\varphi=0}^{\varphi=2\pi}\left(\operatorname{logh}\frac{d}{b}\right) i_{(\varphi)}\,a\,t\,d\,\varphi \qquad (1)$$

This is the same as Manneback's equation (5) Reference 17.

Let $\quad l^2 = \dfrac{\omega\,2\,\pi\,a\,t}{\sigma}$ as in equation (9)

Reference 16.

Then $j\,\dfrac{2\,\omega}{\sigma} = j\,\dfrac{l^2}{\pi\,a\,t}$

$$B = X\,Y = s - a$$
$$d^2 = M\,P^2 = a^2 + a^2 - 2\,a^2\cos(\theta - \varphi)$$
$$b^2 = X\,P^2 = a^2 + a^2 - 2\,a^2\cos\varphi$$
$$D^2 = Y\,M^2 = s^2 + a^2 - 2\,a\,s\cos\theta$$

The series expansions given by H. L. Curtis, Note 1, Reference 13, where their derivation may be found, must now be used.

$$\operatorname{logh} d/a = -\{\cos(\varphi - \theta) + 1/2\cos 2(\varphi - \theta) + \ldots + 1/n\cos n(\varphi - \theta) + \ldots\} \qquad (2)$$

$$\operatorname{logh} b/a = -\{\cos\varphi + 1/2\cos 2\,\varphi + \ldots + 1/n\cos n\,\varphi + \ldots\} \qquad (3)$$

$$\operatorname{logh} D/s = -\left\{a/s\cos\theta + \frac{a^2}{2\,s^2}\cos 2\,\theta + \ldots + \frac{a^n}{n\,s^n}\cos n\,\theta + \ldots\right\} \qquad (4)$$

First, let the effect of the return current be neglected, and put $i_{(\varphi)} = i_0$

$$i_{(\theta)} = i_0 + \frac{j\,l^2}{\pi\,a\,t}\int_{\varphi=0}^{\varphi=2\pi}(\operatorname{logh} d/a - \operatorname{logh} b/a)\,i_0\,a\,t\,d\,\varphi$$

Using the series expansions, all the terms of the integrals are zero, and $i_{(\theta)} = i_0$ when the effect of the return current is neglected. This result, of course, is known from the conditions of the problem.

Now, for the present, neglect the effect of i_0 and find a value of $i_{(\theta)}$ which will balance the effect of the return conductor.

$$i_{(\theta)} = \frac{j\,l^2}{\pi\,a\,t}\,I_1\left(\operatorname{logh} D/s + \operatorname{logh}\frac{s}{s-a}\right)$$
$$+ \frac{j\,l^2}{\pi\,a\,t}\int_{\varphi=0}^{\varphi=2\pi}(\operatorname{logh} d/b)\,i_{(\varphi)}\,a\,t\,d\,\varphi$$

The nth term of $\operatorname{logh} D/s$ is $-\dfrac{a^n}{n\,s^n}\cos n\,\theta$

Find a solution $i_{n(\theta)}$ of the equation

$$i_{n(\theta)} = -\frac{j\,l^2}{\pi\,a\,t}\,I_1/n\,\frac{a^n}{s^n}\cos n\,\theta$$
$$+ \frac{j\,l^2}{\pi\,a\,t}\int_{\varphi=0}^{\varphi=2\pi}(\operatorname{logh} d/b)\,i_{n(\varphi)}\,a\,t\,d\,\varphi \qquad (5)$$

Let $\quad i_{n(\theta)}{}^{(1)} = -\dfrac{j\,l^2}{n}\,\dfrac{I_1}{\pi\,a\,t}\,\dfrac{a^n}{s^n}\cos n\,\theta \qquad (6)$

$$i_{n(\theta)}{}^{(2)} = i_{n(\theta)}{}^{(1)} + \frac{j\,l^2}{\pi\,a\,t}\int_{\varphi=0}^{\varphi=2\pi}(\operatorname{logh} d/a - \operatorname{logh} b/a)\,i_{n(\varphi)}{}^{(1)}\,a\,t\,d\,\varphi \qquad (7)$$

$$i_{n(\theta)}{}^{(2)} = i_{n(\theta)}{}^{(1)} + \left(\frac{j\,l^2}{\pi\,a\,t} \right)^2 I_1/n\, \frac{a^n}{s^n}$$

$$\int_{\varphi=0}^{\varphi=2\pi} \cos n\,\varphi \Big[\sum_{k=1}^{k=\infty} \frac{\cos k\,(\varphi-\theta)}{k}$$

$$- \sum_{k=1}^{k=\infty} \frac{\cos k\,\varphi}{k} \Big] a\,t\,d\,\varphi$$

Now $\cos A \cos B = 1/2 \cos (A+B) + 1/2 \cos (A-B)$ The integrals of all the terms are zero except when $k = n$.

$$i_{n(\theta)}{}^{(2)} = i_{n(\theta)}{}^{(1)} + \left(\frac{j\,l^2}{\pi\,a\,t} \right)^2 I_1/n\, \frac{a^n}{s^n}$$

$$\int_{\varphi=0}^{\varphi=2\pi} \left[\frac{\cos n\,\theta}{2\,n} - \frac{1}{2\,n} \right] a\,t\,d\,\varphi$$

$$i_{n(\theta)}{}^{(2)} = i_{n(\theta)}{}^{(1)}$$

$$+ \left(\frac{j\,l^2}{\pi\,a\,t} \right)^2 I_1/n\, \frac{a^n}{s^n} \Big(\frac{\pi \cos n\,\theta}{n}$$

$$- \pi/n \Big)\, a\,t$$

$$i_{n(\theta)}{}^{(2)} = i_{n(\theta)}{}^{(1)} - \frac{j\,l^2}{n}\, i_{n(\theta)}{}^{(1)}$$

$$- \left(\frac{j\,l^2}{n} \right)^2 \frac{I_1}{\pi\,a\,t}\, a^n/s^n \qquad (8)$$

Put $i_{n(\theta)}{}^{(3)} = i_{n(\theta)}{}^{(1)} + \frac{j\,l^2}{\pi\,a\,t} \int_{\varphi=0}^{\varphi=2\pi}$ (logh d/a

$$- \text{logh}\, b/a)\, i_{n(\varphi)}{}^{(2)}\, a\,t\,d\,\varphi$$

$$= i_{n(\theta)}{}^{(2)} - \frac{j\,l^2}{n}\, \frac{j\,l^2}{\pi\,a\,t} \int_{\varphi=0}^{\varphi=2\pi} (\text{logh}\, d/a$$

$$- \text{logh}\, b/a)\, i_{n(\varphi)}{}^{(1)}\, a\,t\,d\,\varphi$$

$$- \left(\frac{j\,l^2}{n} \right)^2 \frac{j\,l^2}{\pi\,a\,t} \int_{\varphi=0}^{\varphi=2\pi} (\text{logh}\, d/a$$

$$- \text{logh}\, b/a)\, \frac{I_1}{\pi\,a\,t}\, \frac{a^n}{s^n}\, a\,t\,d\,\varphi$$

from (7). The last integral is zero.

$$i_{n(\theta)}{}^{(3)} = i_{n(\theta)}{}^{(2)} + \left(\frac{j\,l^2}{n} \right)^2 i_{n(\theta)}{}^{(1)}$$

$$+ \left(\frac{j\,l^2}{n} \right)^3 \frac{I_1}{\pi\,a\,t}\, \frac{a^n}{s^n} \text{ as in} \qquad (8)$$

$$= i_{n(\theta)}{}^{(1)} \left[1 - \frac{j\,l^2}{n} + \left(\frac{j\,l^2}{n} \right)^2 \right]$$

$$- \left(\frac{j\,l^2}{n} \right)^2 \frac{I_1}{\pi\,a\,t}\, a^n/s^n \left[1 - \frac{j\,l^2}{n} \right] \qquad (9)$$

Similarly,

$$i_{n(\theta)}{}^{(4)} = i_{n(\theta)}{}^{(3)} - \left(\frac{j\,l^2}{n} \right)^3 i_{n(\theta)}{}^{(1)}$$

$$- \left(\frac{j\,l^2}{n} \right)^4 \frac{I_1}{\pi\,a\,t}\, a^n/s^n$$

$$= i_{n(\theta)}{}^{(1)} \Big[1 - \frac{j\,l^2}{n} + \left(\frac{j\,l^2}{n} \right)^2$$

$$- \left(\frac{j\,l^2}{n} \right)^3 \Big]$$

$$- \left(\frac{j\,l^2}{n} \right)^2 \frac{I_1}{\pi\,a\,t}\, a^n/s^n \Big[1$$

$$- \frac{j\,l^2}{n} + \left(\frac{j\,l^2}{n} \right)^2 \Big] \qquad (10)$$

If this process is continued indefinitely, the value of $i_{n(\theta)}$ is given by convergent infinite series, provided the values of l and n are such as to make the series convergent. Since n is equal to 1 or greater, the series give the value of $i_{n(\theta)}$ for l less than 1. The sum of the infinite series is

$$\frac{1}{1 + \frac{j\,l^2}{n}}$$

Thus $i_{n(\theta)} = - \frac{I_1}{\pi\,a\,t}\, \frac{a^n}{n\,s^n}\, \frac{j\,l^2}{1 + \frac{j\,l^2}{n}} \cos n\,\theta$

$$- \frac{I_1}{\pi\,a\,t}\, \frac{j\,l^2}{n}\, \frac{a^n}{n\,s^n}\, \frac{j\,l^2}{1 + \frac{j\,l^2}{n}} \qquad (11)$$

and the value of $i_{(\theta)}$ which will balance the effect of the return conductor is

$$i_{(\theta)} = \frac{j\,l^2\,I_1}{\pi\,a\,t} \sum_{n=1}^{n=\infty} \frac{a^n}{n\,s^n}$$

$$- \frac{j\,l^2\,I_1}{\pi\,a\,t} \sum_{n=1}^{n=\infty} \frac{a^n}{n\,s^n}\, \frac{j\,l^2}{(n + j\,l^2)}$$

$$- \frac{I_1}{\pi\,a\,t} \sum_{n=1}^{n=\infty} \frac{a^n}{n\,s^n}\, \frac{j\,l^2}{1 + \frac{j\,l^2}{n}} \cos n\,\theta$$

$$i_{(\theta)} = \frac{j\,l^2\,I_1}{\pi\,a\,t} \sum_{n=1}^{n=\infty} \frac{a^n}{n\,s^n}\, \frac{1}{1 + \frac{j\,l^2}{n}} (1 - \cos n\,\theta) \qquad (12)$$

To this must be added the quantity i_0 to obtain the total value of $i_{(\theta)}$

$$i_{(\theta)} = i_0 + \frac{I_1}{2\,\pi\,a\,t} \sum_{n=1}^{n=\infty} a^n/s^n\, \frac{2\,j\,l^2}{(n + j\,l^2)}$$

$$(1 - \cos n\,\theta)$$

$$\text{The total current} = I = -I_1 = \int_{\theta=0}^{\theta=2\pi} i_{(\theta)}\, a\, t\, d\,\theta$$

$$I = 2\,\pi\, a\, t\, i_0 - I \sum_{n=1}^{n=\infty} a^n/s^n \frac{2\,j\,l^2}{(n + j\,l^2)}$$

$$i_0 = \frac{I}{2\,\pi\, a\, t}\left[1 + \sum_{n=1}^{n=\infty} a^n/s^n \frac{2\,j\,l^2}{(n + j\,l^2)}\right] \qquad (13)$$

$$i_{(\theta)} = \frac{I}{2\,\pi\, a\, t}\left[1 + \sum_{n=1}^{n=\infty} a^n/s^n \frac{2\,j\,l^2}{(n + j\,l^2)} \cos n\,\theta\right] \qquad (14)$$

This value of $i_{(\theta)}$ should now be checked by writing $i_{(\varphi)}$ in accordance with it and substituting in the original equation (1).

i_0 is given by (13) and $i_{(\varphi)}$ by (14), changing θ to φ.

$$\operatorname{logh} D/B = \sum_{n=1}^{n=\infty} \frac{a^n}{n\, s^n}(1 - \cos n\,\theta)$$

$$\operatorname{logh} d/b = \sum_{k=1}^{k=\infty}\left[\frac{\cos k\,\varphi}{k} - \frac{\cos k\,(\varphi - \theta)}{k}\right]$$

Substituting all these in (1) and carrying out the integration, we obtain equation (14).

This furnishes a check on equation (14) and further shows that it applies for all frequencies, that is, for all values of l. While the expression was derived by means of series which were convergent for l less than 1, the expression (14) itself is a convergent series for all values of l, that is, for all frequencies, so long as a/s is less than 1, and it has been shown to satisfy the fundamental equations of the problem, irrespective of the value of l.

It is now desired to find an expression for R'/R.

$$R = \frac{\sigma}{2\,\pi\, a\, t}$$

$$R' = 1/I^2 \int_{\theta=0}^{\theta=2\pi} i_{(\theta)}\, a\, t\, d\,\theta\, \hat{i}_{(\theta)}\, a\, t\, d\,\theta \frac{\sigma}{a\, t\, d\,\theta}$$

where $\hat{i}_{(\theta)}$ is the conjugate of $i_{(\theta)}$, that is, it is the same except that j is replaced by $-j$. The integration of the product of the two series is very similar to that used in deriving (8), all the terms being zero except when the coefficients taken from the two series are alike.

$$R'/R = \frac{2\,\pi\, a\, t}{I^2} \cdot \frac{I^2\, a\, t}{(2\,\pi\, a\, t)^2}\left[2\,\pi + \sum_{n=1}^{n=\infty} \frac{\pi\, a^{2n}}{s^{2n}} \frac{2\,j\,l^2}{(n + j\,l^2)} \frac{(-2\,j\,l^2)}{(n - j\,l^2)}\right]$$

$$R'/R = 1 + \sum_{n=1}^{n=\infty} \frac{a^{2n}}{s^{2n}} \frac{2\,l^4}{(l^4 + n^2)}$$

as in Formula I, Part II.

II. TWO THIN TUBES IN RETURN CIRCUIT

For the case of two thin tubes, equation (3′) of Maneback's paper was not used. Instead, an element of current, $i_{(\gamma)}\, at\, d\,\gamma$ was assumed at P_1, the current density $i_{(\gamma)}$ being considered to be constant at first. The following expansions from H. L. Curtis' paper, Reference 13, are required:

$$\operatorname{logh} B/q = -\sum_{n=1}^{n=\infty} \frac{a^n}{n\, q^n} \cos n\,\beta \qquad (15)$$

$$\operatorname{logh} D/q = -\sum_{n=1}^{n=\infty} \frac{a^n}{n\, q^n} \cos n\,(\beta - \theta) \qquad (16)$$

$$\operatorname{logh} q/s = -\sum_{n=1}^{n=\infty} \frac{a^n}{n\, s^n} \cos n\,\gamma \qquad (17)$$

$$\frac{\cos n\,\beta}{q^n} = 1/s^n\left[1 + \sum_{k=1}^{k=\infty} \frac{\underline{|n + k - 1}}{\underline{|n - 1}\,\underline{|k}} \frac{a^k}{s^k} \cos k\,\gamma\right] \qquad (18)$$

$$\frac{\sin n\,\beta}{q^n} = 1/s^n \sum_{k=1}^{k=\infty} \frac{\underline{|n + k - 1}}{\underline{|n - 1}\,\underline{|k}} \frac{a^k}{s^k} \sin k\,\gamma \qquad (19)$$

The same process for finding $i_{(\theta)}$ is now gone through as for a thin tube and an infinitesimal wire, and when $i_{(\gamma)}$ is constant the same result is obtained, namely eq. (14). This is to be expected, for a round tube or wire with uniform current density has the same external effect as if the current were concentrated at its center.

Now assume that there is the same current density in tube B as has just been calculated for tube A, and the resulting current density in tube A can be found. This again can be assumed for tube B. The process gives a symmetrical sequence of expressions, as given in Formula II, Part II. The expression for R'/R is obtained by direct integration, as previously described.

III. TWO THIN TUBES IN PARALLEL

This case is calculated in the same way as the preceding, except that I is used instead of $-I$ for the second conductor, with the result given in Formula III.

IV. INSULATED CABLE SHEATHS IN SINGLE-PHASE CIRCUIT

The assumptions for this problem are given under Formula IV, Part II. The calculation is similar to that of Formula I, the total current in the sheath being zero.

V. CABLE SHEATHS IN THREE-PHASE CIRCUIT, FLAT SPACING

The current is I in cable A,

$$I e^{j\frac{2\pi}{3}} = I\left(\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right)$$

$$= I\left(-1/2 + j\frac{\sqrt{3}}{2}\right) \text{ in cable } B\text{, and}$$

$$I e^{j\frac{4\pi}{3}} = I\left(\cos\frac{4\pi}{3} + j\sin\frac{4\pi}{3}\right)$$

$$= I\left(-1/2 - j\frac{\sqrt{3}}{2}\right) \text{ in cable } C.$$

VI. FINITE WIRE AND INFINITESIMAL WIRE

The formula for $i_{(r\theta)}$ is derived in Mr. Manneback's paper, equations (14) and (19). It is

$$i_{(r\theta)} = \frac{I}{\pi a^2}\frac{j\alpha a}{2}\frac{J_0(j\alpha r)}{J_1(j\alpha a)}$$

$$+ \frac{I_1}{\pi a^2} j\alpha a \sum_{n=1}^{n=\infty} a^n/s^n \frac{J_n(j\alpha r)}{J_{n-1}(j\alpha a)}\cos n\theta \quad (20)$$

where $$\alpha^2 = \frac{j4\pi\omega}{\sigma} \quad (21)$$

In order to derive an expression for R'/R, the writer replaced $i_{(r\theta)}$ by $E/\sigma - e_{(r\theta)}/\sigma$ (see note 5 of Manneback's paper) where E is the voltage induced by flux cutting the axis of the wire.

By eq. (10) of J. R. Carson's paper, Reference 14,

$$j\mu\omega H_\theta = \frac{\partial}{\partial r} e_{(r\theta)} \quad (22)$$

where μ is the permeability, assumed = 1 for this problem, and H_θ is the tangential component of the magnetic force.

At the surface,

$$H_{(a\theta)} = \alpha/\omega \frac{I\sigma}{\pi a^2}\left[-\frac{j\alpha a}{2}\frac{J_0'(j\alpha a)}{J_1(j\alpha a)}\right.$$

$$\left.+ j\alpha a \sum_{n=1}^{n=\infty}\frac{a_n}{s_n}\frac{J'_n(j\alpha a)}{J_{n-1}(j\alpha a)}\cos n\theta\right] \quad (23)$$

By eq. (18) of Carson's paper, the true energy transferred to or from one cm. of wire through its surface, according to Poynting's theory, is

$$\hat{I} I R' = \text{real part of } \frac{a}{4\pi}\int_{\theta=0}^{\theta=2\pi} \hat{e}_{(a\theta)} H_{(a\theta)}\, d\theta \quad (24)$$

Since this involves only values at the surface of the wire, the integration is not complicated.

$$e_{(a\theta)} = E - \frac{I\sigma}{\pi a^2}\left[\frac{j\alpha a}{2}\frac{J_0(j\alpha a)}{J_1(j\alpha a)}\right.$$

$$\left.- j\alpha a \sum_{n=1}^{n=\infty} a^n/s^n \frac{J_n(j\alpha a)}{J_{n-1}(j\alpha a)}\cos n\theta\right] \quad (25)$$

The term involving E in $\hat{I} I R'$ is the real part of

$$\frac{a}{4\pi}\int_{\theta=0}^{\theta=2\pi} \hat{E} H_{(a\theta)}\, d\theta$$

By Carson's paper, (17),

$$4\pi I = \int_{\theta=0}^{\theta=2\pi} H_{(a\theta)}\, a\, d\theta \quad (26)$$

E is a voltage caused by flux generated in the axis by the current I in the finite wire and by the current $-I$ in the return wire. Current densities of the form $A\cos n\theta$ do not produce any voltage in the axis, nor any total current, when integrated around the circle. E is therefore in quadrature with I, and there is no real part of $\hat{E} I$, nor of

$$\int_{\theta=0}^{\theta=2\pi} \hat{E} H_{(a\theta)}\, d\theta$$

We have therefore, as before, the integral from 0 to 2π of the product of two Fourier series (23) and (25). The terms will be zero except where the coefficients of θ taken from the two series are alike.

$$\text{Put } j\alpha a = bj\sqrt{j} \quad (27)$$

where $$b^2 = \frac{4\pi a^2\omega}{\sigma} \quad (28)$$

so that b is a real quantity. Omit the imaginary parts of the terms of the product and formula VI is obtained.

VII. TWO WIRES IN RETURN CIRCUIT

The procedure is the same as for two thin tubes in return circuit, and the same expansions are required. In the course of the calculation it is required to find the current density $i_{(r\theta)}$ due to current density in the return wire

$$i_{(u\gamma)} = c_k J_k(j\alpha u)\cos k\gamma$$

The result, after integrating over the surface of the return wire, that is, from $\gamma = 0$ to 2π and from $u = 0$ to a, is

$$i_{(r\theta)} = \sum_{n=1}^{n=\infty} c_k a^{n+k}/s^{n+k} \quad \frac{J_n(j\alpha r)}{J_{n-1}(j\alpha a)}\cos n\theta$$

$$\frac{\underline{/n+k-1}}{\underline{/n-1}\,\underline{/k}} J_{k+1}(j\alpha a) \quad (29)$$

This is only a partial expression used in the course of the calculation. The total value of $i_{(r\theta)}$ is

$$\frac{I}{\pi a^2}\left[A_0 J_0(j\alpha r) + \sum_{n=1}^{n=\infty} N_n J_n(j\alpha r)\cos n\theta\right] \quad (30)$$

As previously stated, Formula VII is an alternative solution, the problem having been solved in J. R. Carson's paper, Reference 14.

VIII. Two Wires in Parallel

Since in two wires in parallel, the current crowds to the furthermost part of each wire, it is to be expected that their effect on each other will be less than if the current in one were concentrated at the center. So, also, in a return circuit the currents crowd toward each other and the effect is correspondingly greater. This is illustrated by the three values of R'/R given in the example. A similar result was found in connection with thin tubes, Formulas I, II and III.

IX. Three-Phase Circuit, Triangular Spacing

In this problem, the time phase angles for three-phase currents given in connection with Formula V, cable sheath in three-phase circuit, must be used, and in addition the direction angle $\pi/6$ must be added to, or subtracted from, the angles θ, γ etc. With triangular spacing, the three wires and their currents are symmetrical, and each current can be obtained from the one preceding it in the order A, B, C, by multiplying by $e^{j\frac{2\pi}{3}}$

X. Three-phase Circuit, Flat Spacing

In this problem, as in V, the direction angle is π, used with the middle conductor.

XI. Bessel Function Reduction Formulas

The first three or four orders of Bessel Functions of the required value of b are usually required. They can be worked out with less work than the remainder of the problem generally requires.

XII. Table of Bessel Functions

This table gives the values of Bessel Functions of the first five orders, and their first derivatives, for argument $x\, j \sqrt{j}$, where x is any whole number from 1 to 10. These values are useful for drawing curves of proximity effect. Interpolated values cannot be obtained from this table, but must be separately derived by means of the reduction formulas given in XI, from the values of ber x etc. given in Reference 11.

Part II

Formulas for Proximity Effect Resistance Ratio

I. ○ · Thin Tube and Infinitesimal Wire

$$l^2 = \frac{2\pi a t \omega}{\sigma}$$

a = radius of tube in cm.
t = thickness of tube in cm. (t is assumed very small compared with a).
s = axial spacing
ω = $2\pi f$
f = frequency in cycles per second
σ = specific resistivity in absolute units.

$$R'/R = 1 + \sum_{n=1}^{n=\infty} a^{2n}/s^{2n} \frac{2 l^4}{(l^4 + n^2)}$$

For infinite frequency, $R'/R = \dfrac{s^2 + a^2}{s^2 - a^2}$

The above formula applies when the two conductors form a return circuit, and also when they carry equal currents in parallel, and are remote from other conductors. Example, $s/a = 4$, $l = 1$, $R'/R = 1.0641$.

II. ○ ○ Two Thin Tubes in Return Circuit

$$A_n = \frac{2a^n}{s^n} \frac{(l^4 + j l^2 n)}{(l^4 + n^2)}$$

$$B_n = \frac{1}{2} A_n \sum_{k=1}^{k=\infty} \frac{a^k}{s^k} A_k \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}$$

$$C_n = \frac{1}{2} A_n \sum_{k=1}^{k=\infty} \frac{a^k}{s^k} B_k \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}$$

$$D_n = \frac{1}{2} A_n \sum_{k=1}^{k=\infty} \frac{a^k}{s^k} C_k \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}$$

etc.

$N_1 = A_1 + B_1 + C_1 + \ldots$
$N_2 = A_2 + B_2 + C_2 + \ldots$
$\ldots\ldots$
$N_n = A_n + B_n + C_n + \ldots$
$\ldots\ldots$

$$R'/R = 1 + 1/2\,|N_1|^2 + 1/2\,|N_2|^2 + \ldots + 1/2\,|N_n|^2 + \ldots$$

$|N_n|^2$ is the square of the absolute value of N_n. $\underline{|0} = 1$. Example, $s/a = 4$, $l = 1$, $R'/R = 1.0685$. The same value was obtained by the low frequency calculation given by the writer in Reference 16.

III. ○——○ Two Thin Tubes in Parallel

Let A_1, B_1, $\ldots A_n$, B_n, C_n etc. have the same values as for two thin tubes in return circuit.

$M_1 = A_1 - B_1 + C_1 - D_1 + \ldots$
$M_2 = A_2 - B_2 + C_2 - D_2 + \ldots$
$\ldots\ldots$
$M_n = A_n - B_n + C_n - D_n + \ldots$
$\ldots\ldots$

$$R'/R = 1 + 1/2\,|M_1|^2 + 1/2\,|M_2|^2 + \ldots + 1/2\,|M_n|^2 + \ldots$$

Example, $s/a = 4$, $l = 1$, $R'/R = 1.0600$

IV. ⊙ · Insulated Cable Sheaths in Single-Phase Circuit

Sheath replaced by a copper sheath of the same resistance, and mean radius and of thickness t, and considered as a thin tube. It is insulated from other sheaths except at one point.

Cable inside sheath replaced by a solid wire of the same resistance, and of radius a, and whose current density is considered uniform.

Return cable and its sheath represented by an infinitesimal wire.

$$l^2 = \frac{2\pi c t \omega}{\sigma}$$

c = radius of sheath

$$\frac{\text{Resis. loss in sheath}}{\text{Resis. loss in cable at zero freq.}}$$

$$= \frac{a^2}{c t} \sum_{n=1}^{n=\infty} c^{2n}/s^{2n} \frac{l^4}{(l^4 + n^2)}$$

If the sheaths are connected at both ends, the currents and losses are approximately calculated by usual reactance formulas.

V. INSULATED CABLE SHEATHS IN THREE-PHASE CIRCUIT, FLAT SPACING

⊙ · · OUTSIDE CONDUCTOR

Two cables and their sheaths represented by infinitesimal wires. Axial Spacing = s.

$$\frac{\text{Resis. loss in sheath}}{\text{Resis. loss in cable at zero freq.}}$$

$$= \frac{a^2}{c t} \sum_{n=1}^{n=\infty} (1 - 1/2^n + 1/2^{2n})\, c^{2n}/s^{2n} \frac{l^4}{(l^4 + n^2)}$$

· ⊙ · *Middle Conductor*

$$\frac{\text{Resis. loss in sheath}}{\text{Resis. loss in cable at zero freq.}}$$

$$= \frac{a^2}{c t} \sum_{n=1}^{n=\infty} (2 - \cos n\pi)\, c^{2n}/s^{2n} \frac{l^4}{(l^4 + n^2)}$$

Example. 2,000,000 c.m. single-conductor cables, 60 cycle.

$c = 2.97$ cm., $t = 0.030$ cm., $s = 10.7$ cm.

$c/s = 0.278$ $\sigma = 2100$ at 75 deg. cent.

$l^2 = 0.1005$

$$\frac{\text{Resis. loss in sheath}}{\text{Resis. loss in cable at zero freq.}}$$

= 2.8 per cent for single-phase circuit.

= 2.1 per cent for three-phase circuit, flat spacing, outside conductor

= 8.4 per cent for three-phase circuit, flat spacing, middle conductor.

VI. ● · FINITE WIRE AND INFINITESIMAL WIRE

All Bessel functions have argument $b\, j \sqrt{j}$

$$= j\sqrt{j} \sqrt{\frac{4\pi a^2 \omega}{\sigma}}$$

$$u_0 + j\, v_0 = \text{ber}\, b + j \,\text{bei}\, b = J_0(b\, j\sqrt{j})$$

$$u_0' = \frac{d\,u_0}{d\,b} = \text{ber}'\, b$$

$$u_n + j\, v_n = J_n(b\, j\sqrt{j})$$

$$u_n' = \frac{d\,u_n}{d\,b} \text{ etc.}$$

$$R'/R_0 = 1 + \frac{2(u_0'^2 + v_0'^2)}{(u_0 v_0' - u_0' v_0)} \sum_{n=1}^{n=\infty} a^{2n}/s^{2n} \frac{(u_n v_n' - u_n' v_n)}{(u_{n-1}^2 + v_{n-1}^2)}$$

The above formula applies when the two conductors form a return circuit, and also when they carry equal currents in parallel, and are remote from other conductors. R_0 = resistance of isolated conductor. As is well known,

$$R_0/R_{d.c.} = \frac{b}{2} \frac{(u_0 v_0' - u_0' v_0)}{(u_0'^2 + v_0'^2)}$$

This quantity is tabulated in Scientific Paper No. 169 of the Bureau of Standards, by E. B. Rosa and F. W. Grover, page 226. (Reference 18.)

VII. ●● TWO WIRES IN RETURN CIRCUIT

All Bessel functions have argument $b\, j\sqrt{j}$

$$= j\sqrt{j}\sqrt{\frac{4\pi a^2 \omega}{\sigma}}$$

$$j\sqrt{j} = \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

$$\underline{/0} = 1$$

$$A_0 = \frac{b\, j\sqrt{j}}{2 J_1}$$

$$A_1 = -\,a/s\, \frac{b\, j\sqrt{j}}{J_0}$$

$$A_2 = -\,a^2/s^2\, \frac{b\, j\sqrt{j}}{J_1}$$

. . . .

$$A_n = -\,a^n/s^n\, \frac{b\, j\sqrt{j}}{J_{n-1}} \qquad n \neq 0$$

$$B_n = \sum_{k=1}^{k=\infty} - A_k \frac{a^{n+k}}{s^{n+k}} \frac{\underline{/n+k-1}}{\underline{/n-1}\,\underline{/k}} \frac{J_{k+1}}{J_{n-1}}$$

$$C_n = \sum_{k=1}^{k=\infty} - B_k \frac{a^{n+k}}{s^{n+k}} \frac{\underline{/n+k-1}}{\underline{/n-1}\,\underline{/k}} \frac{J_{k+1}}{J_{n-1}}$$

. . . . . .

$N_1 = A_1 + B_1 + C_1 + \ldots\ldots$

$N_n = A_n + B_n + C_n + \ldots\ldots$

etc.

$$R'/R_0 = 1 + \frac{1}{2|A_0|^2 (u_0 v_0' - u_0' v_0)} \sum_{n=1}^{n=\infty} |N_n|^2 (u_n v_n' - u_n' v_n)$$

$$R_0/R_{dc} = \frac{2}{b} |A_0|^2 (u_0 v_0' - u_0' v_0)$$

These formulas apply very closely to cables as well as wires.

VIII. ●—● TWO WIRES IN PARALLEL

Let $A_0, A_1, B_1 \ldots A_n, B_n, C_n$ etc. have the same values as for two wires in return circuit.

$$M_1 = A_1 - B_1 + C_1 - D_1 + \ldots$$
$$M_2 = A_2 - B_2 + C_2 - D_2 + \ldots$$
$$\ldots\ldots$$
$$M_n = A_n - B_n + C_n - D_n + \ldots$$
$$\ldots\ldots$$

$$R'/R_0 = 1 + \frac{1}{2\,|A_0|^2\,(u_0\,v_0' - u_0'\,v_0)} \sum_{n=1}^{n=\infty} |M_n|^2\,(u_n\,v_n' - u_n'\,v_n)$$

Example. $s/a = 4,\ b = 10$

$R'/R_0 = 1.101$ for two wires in parallel.

$= 1.114$ for finite wire and infinitesimal wire.

$= 1.129$ for two wires in return circuit.

The third value agrees with that given by J. R. Carson, Reference 14, Fig. 7.

IX ●●● THREE-PHASE CIRCUIT, TRIANGULAR SPACING

$$A_0 = \frac{b\,j\,\sqrt{j}}{2\,J_1} = \frac{b}{2\,J_1}\left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)$$

$$A_n = a^n/s^n\,\frac{b\,j\,\sqrt{j}}{J_{n-1}}\left(\cos\frac{2\,\pi}{3} + j\sin\frac{2\,\pi}{3}\right)\ n \neq 0$$

$$F_n = a^n/s^n\,\frac{b\,j\,\sqrt{j}}{J_{n-1}}\left(\cos\frac{4\,\pi}{3} + j\sin\frac{4\,\pi}{3}\right)$$

$$B_n = \sum_{k=1}^{k=\infty} a^{n+k}/s^{n+k}\,J_{k+1}/J_{n-1}\,\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\left(A_k\cos\frac{k\,\pi}{3} + F_k\right)\left(\cos\frac{2\,\pi}{3} + j\sin\frac{2\,\pi}{3}\right)$$

$$G_n = \sum_{k=1}^{k=\infty} a^{n+k}/s^{n+k}\,J_{k+1}/J_{n-1}\,\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\left(A_k + F_k\cos\frac{k\,\pi}{3}\right)\left(\cos\frac{4\,\pi}{3} + j\sin\frac{4\,\pi}{3}\right)$$

$$C_n = \sum_{k=1}^{k=\infty} a^{n+k}/s^{n+k}\,J_{k+1}/J_{n-1}\,\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\left(B_k\cos\frac{k\,\pi}{3} + G_k\right)\left(\cos\frac{2\,\pi}{3} + j\sin\frac{2\,\pi}{3}\right)$$

$$H_n = \sum_{k=1}^{k=\infty} a^{n+k}/s^{n+k}\,J_{k+1}/J_{n-1}\,\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\left(B_k + G_k\cos\frac{k\,\pi}{3}\right)\left(\cos\frac{4\,\pi}{3} + j\sin\frac{4\,\pi}{3}\right)$$

etc.

$$M_n = A_n + B_n + C_n + \ldots$$
$$N_n = F_n + G_n + H_n + \ldots$$

$$R'/R_0 = 1 + \frac{1}{2\,\hat{A}_0\,A_0\,(u_0\,v_0' - u_0'\,v_0)} \sum_{n=1}^{n=\infty}\left\{\hat{M}_n\,M_n + \hat{N}_n\,N_n + (\hat{M}_n\,N_n + M_n\,\hat{N}_n)\cos\frac{n\,\pi}{3}\right\}(u_n\,v_n' - u_n'\,v_n)$$

$\hat{A}_0$ is the conjugate of A_0, that is, the imaginary part of $\hat{A}_0$ is (-1) times the imaginary part of A_0. Example. 500,000 c. m. three-conductor cable, 60 cycle, $s/a = 2.5,\ b = 1.4,\ R'/R_0 = 1.055$.

X ●●● THREE-PHASE CIRCUIT, FLAT SPACING

$$A_0 = \frac{b\,j\,\sqrt{j}}{2\,J_1} = \frac{b}{2\,J_1}\left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)$$

In the following, $n \neq 0$

$$A_{an} = A_{bn}\left[\cos\frac{2\,\pi}{3} + j\sin\frac{2\,\pi}{3} + 1/2^n\left(\cos\frac{4\,\pi}{3} + j\sin\frac{4\,\pi}{3}\right)\right]$$

$$A_{bn} = \frac{b\,j\,\sqrt{j}}{J_{n-1}}\,a^n/s^n = b/J_{n-1}\,a^n/s^n\left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)$$

$$A_{cn} = A_{bn}\left(\cos\frac{4\,\pi}{3} + j\sin\frac{4\,\pi}{3}\right)$$

$$A_{dn} = A_{bn}\left(1/2^n + \cos\frac{2\,\pi}{3} + j\sin\frac{2\,\pi}{3}\right)$$

$$B_{an} = \sum_{k=1}^{k=\infty} J_{k+1}/J_{n-1}\,\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\left[(A_{bk} + A_{ck}\cos k\,\pi)\,a^{n+k}/s^{n+k} + A_{dk}\,\frac{a^{n+k}}{(2\,s)^{n+k}}\right]$$

$$B_{bn} = \sum_{k=1}^{k=\infty} J_{k+1}/J_{n-1}\,\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\,A_{ak}\,a^{n+k}/s^{n+k}$$

$$B_{cn} = \sum_{k=1}^{k=\infty} J_{k+1}/J_{n-1}\,\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\,A_{dk}\,a^{n+k}/s^{n+k}$$

$$B_{dn} = \sum_{k=1}^{k=\infty} J_{k+1}/J_{n-1}\,\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\left[A_{ak}\,\frac{a^{n+k}}{(2\,s)^{n+k}} + (A_{bk}\cos k\,\pi + A_{ck})\,a^{n+k}/s^{n+k}\right]$$

C_{an}, C_{bn} etc. are obtained from the same formulas as B_{an}, B_{bn}, etc., respectively, except that A is changed to B, and similarly for D_{an}, D_{bn}, etc., change B to C. This process can be continued indefinitely.

Outside Conductors.

Let $L_n = A_{an} + B_{an} + C_{an} + \ldots$

R'/R_0 of outside wires = 1

$$+ \frac{1}{2\,|A_0|^2\,(u_0\,v_0' - u_0'\,v_0)} \sum_{n=1}^{n=\infty} |L_n|^2\,(u_n\,v_n' - u_n'\,v_n)$$

Middle Conductor.

$M_n = A_{bn} + B_{bn} + C_{bn} + \ldots$

$N_n = A_{cn} + B_{cn} + C_{cn} + \ldots$

R'/R_0 of middle conductor

$$= 1 + \frac{1}{2\,|A_0|^2\,(u_0\,v_0' - u_0'\,v_0)} \sum_{n=1}^{n=\infty} \left\{ \hat{M}_n\,M_n + \hat{N}_n\,N_n + (\hat{M}_n\,N_n + M_n\,\hat{N}_n) \cos n\,\pi \right\} (u_n v'_n - u_n' v_n)$$

Putting the B's and C's = 0 is equivalent to assuming two of the conductors to be infinitesimal. Formulas VIII, IX and X apply very closely to cables as well as wires. Example: 2,000,000-c.m. single-conductor cables, flat spacing 60 cycle. Neglect the lead sheaths. $s/a = 5$, $b = 2.7$ for 75 deg. cent., $R'/R_0 = 1.03$ for each outside cable, and 1.13 for the middle cable. Each of these figures is multiplied by $1.22 = R_0/R_{dc}$ in order to obtain R'/R_{dc}.

XI. Bessel Function Reduction Formulas

Special

$u_0 \quad = \text{ber}\,x$

$v_0 \quad = \text{bei}\,x$

$$u_1 = + \frac{1}{\sqrt{2}}\,(u_0' - v_0')$$

$$v_1 = + \frac{1}{\sqrt{2}}\,(u_0' + v_0')$$

$$u_2 = - \frac{\sqrt{2}}{x}\,(u_1 - v_1) - u_0 = \frac{2\,v_0'}{x} - u_0$$

$$v_2 = - \frac{\sqrt{2}}{x}\,(u_1 + v_1) - v_0 = - \frac{2\,u_0'}{x} - v_0$$

$$u_2' = - \frac{1}{\sqrt{2}}\,(u_1 + v_1) - \frac{2\,u_2}{x} = - u_0' - \frac{2\,u_2}{x}$$

$$v_2' = + \frac{1}{\sqrt{2}}\,(u_1 - v_1) - \frac{2\,v_2}{x} = - v_0' - \frac{2\,v_2}{x}$$

General

$$u_{n+1} = - \frac{n\,\sqrt{2}}{x}\,(u_n - v_n) - u_{n-1} \qquad n \neq 0$$

$$v_{n+1} = - \frac{n\,\sqrt{2}}{x}\,(u_n + v_n) - v_{n-1} \qquad n \neq 0$$

$$u_n' = - \frac{1}{\sqrt{2}}\,(u_{n-1} + v_{n-1}) - \frac{n\,u_n}{x} \qquad n \neq 0$$

$$v_n' = + \frac{1}{\sqrt{2}}\,(u_{n-1} - v_{n-1}) - \frac{n\,v_n}{x} \qquad n \neq 0$$

Use these formulas with the table of $u_0 = \text{ber}\,x$, $u_0' = \text{ber}'\,x$ etc. in Reference 11.

These formulas are also suitable for Bessel functions of the second kind by changing u to l and v to m. Thus $\text{ker}\,x = l_0$ and $\text{kei}\,x = m_0$, etc.

TABLE I

BESSEL FUNCTIONS, $u_n + j\,v_n = J_n\,(x\,j\,\sqrt{j})$, $v_n' = \frac{d}{d\,x}\,u_n$

x	1	2	3	4	5	6	7	8	9	10	x
u_0 = ber x	+0.984 382	+0.751 734	−0.221 38	−2.563 42	−6.230 08	− 8.858 32	− 3.632 9	+20.974 0	+73.935 7	+138.840	u_0 = ber x
v_0 = bei x	+0.249 566	+0.972 292	+1.937 59	+2.292 69	+0.116 03	− 7.334 75	−21.239 4	−35.016 7	−24.712 8	+ 56.370	v_0 = bei x
u_0' = ber′x	−0.062 446	−0.493 067	−1.569 85	−3.134 65	−3.845 34	− 0.293 08	+12.764 5	+38.311 8	+65.600 8	+ 51.195	u_0' = ber′ x
v_0' = bei′x	+0.497 397	+0.917 014	+0.880 48	−0.491 14	−4.354 14	−10.846 22	−16.041 5	− 7.660 3	+36.299 4	+135.309	v_0' = bei′ x
u_1	−0.395 868	−0.997 078	−1.732 64	−1.869 25	+0.359 78	+ 7.462 20	+20.368 9	+32.506 9	+20.719 2	− 59.478	u_1
v_1	+0.307 557	+0.299 775	−0.487 45	−2.563 82	−5.797 91	− 7.876 67	− 2.317 2	+21.673 5	+72.054 3	+131.879	v_1
u_1'	−0.476 664	−0.720 532	−0.635 99	+0.658 74	+4.251 33	+10.206 52	+14.677 5	+ 5.866 4	−37.108 0	−132.087	u_1'
v_1'	+0.212 086	−0.305 845	−1.364 13	−2.792 83	−3.327 80	+ 0.235 45	+12.780 7	+36.832 2	+61.749 0	+ 45.127	v_1'
u_2	+0.010 411	+0.165 279	+0.808 37	+2.317 85	+4.488 48	+ 5.242 91	− 0.950 4	−22.889 0	−65.869 2	−111.779	u_2
v_2	−0.124 675	−0.479 225	−0.891 02	−0.725 36	+1.422 10	+ 7.432 44	+17.592 4	+25.438 9	+10.134 8	− 66.610	v_2
u_2'	+0.041 623	+0.327 788	+1.030 93	+1.975 73	+2.049 97	− 1.454 56	−12.493 0	−32.589 1	−50.963 2	− 28.840	u_2'
v_2'	−0.248 047	−0.437 789	−0.286 47	+0.853 82	+3.785 30	+ 8.368 74	+11.015 1	+ 1.300 6	−38.551 6	−121.987	v_2'
u_3	+0.013 788	+0.065 612	+0.130 44	−0.282 63	−2.094 35	− 6.430 04	−12.876 5	−15.420 4	+ 3.166 6	+ 72.353	u_3
v_3	+0.015 629	+0.144 210	+0.565 38	+1.437 76	+2.454 41	+ 1.901 46	− 4.407 2	−22.575 0	−54.538 7	− 81.423	v_3
u_3'	+0.039 433	+0.093 575	+0.072 00	−0.914 09	−2.922 76	− 5.747 81	− 6.249 2	+ 3.979 6	+38.354 6	+104.463	u_3'
v_3'	+0.048 634	+0.239 418	+0.636 27	+1.073 55	+0.695 57	− 2.498 96	−11.222 9	−25.707 4	−35.563 4	− 7.513	v_3'
u_4	−0.002 60	−0.040 97	−0.193 27	−0.493 10	−0.628 67	+ 0.648 3	+ 6.083 5	+19.094 7	+38.667	+ 46.579	u_4
v_4	−0.000 13	−0.008 30	−0.093 02	−0.499 85	−1.727 62	− 4.230 2	− 7.116 9	− 5.288 8	+14.032	+ 70.500	v_4
u_4'	−0.010 40	−0.080 56	−0.234 32	−0.323 71	+0.248 34	+ 2.770 0	+ 8.745 2	+17.319 5	+19.140	− 12.148	u_4'
v_4'	−0.000 78	−0.024 83	−0.183 52	−0.716 65	−1.834 36	− 3.071 1	− 1.921 9	+ 7.703 5	+34.545	+ 80.465	v_4'
u_5	+0.000 20	+0.006 80	+0.058 50	+0.273 08	+0.851 04	+ 1.830 5	+ 2.209 0	− 1.821 3	−18.619	− 58.722	u_5
v_5	−0.000 18	−0.004 84	−0.025 54	−0.033 53	+0.211 43	+ 1.475 6	+ 5.342 3	+12.812 8	+21.384	+ 15.198	v_5
u_5'	+0.000 95	+0.017 84	+0.104 78	+0.360 76	+0.815 11	+ 1.007 4	− 0.847 2	− 8.623 9	−26.955	− 53.427	u_5'
v_5'	−0.000 85	−0.011 00	−0.028 32	+0.046 69	+0.565 64	+ 2.220 0	+ 5.589 6	+ 9.233 7	+ 5.504	− 24.511	v_5'

Part III. List of References

For a more complete list of articles on this subject, see the bibliography published in Reference 7.

1. J. Clerk Maxwell, Electricity and Magnetism, Vol. 2, 1873, paragraph 689.

2. Lord Rayleigh, *Phil. Mag.*, 1886, pp. 382, 469.

3. A. Russell, *Phil. Mag.* Vol. 17, 1909, p. 524.

4. A. B. Field, "Eddy Currents in Large Slot-Wound Conductors," Proc. A. I. E. E., July, 1905, p. 659.

5. R. E. Gilman, "Eddy Current Losses in Armature Conductors," Trans. A. I. E. E., 1920, p. 997.

6. W. V. Lyon, "Heat Losses in Stranded Armature Conductors," Jour. A. I. E. E., Jan., 1922, p. 37.

7. A. E. Kennelly, F. A. Laws and P. H. Pierce, "Experimental Researches on Skin Effect in Conductors," Trans. A. I. E. E., 1915, p. 1953.

8. A. E. Kennelly and H. A. Affel, "Skin Effect Resistance Measurements," *Proc.* Inst. of Radio Engineers, May, 1916.

9. H. B. Dwight, "Skin Effect of a Return Circuit of Two Adjacent Strap Conductors," *The Electric Journal*, April, 1916, p. 157.

10. H. B. Dwight, "Skin Effect in Tubular and Flat Conductors," Trans. A. I. E. E., 1918, p. 1379.

11. H. B. Dwight, "A Precise Method of Calculation of Skin Effect in Isolated Tubes," Jour. A. I. E. E., Aug. 1923, p. 827.

12. J. Slepian, Discussion on Reference 10, Trans. A. I. E. E., 1918, p. 1401.

13. H. L. Curtis, Scientific Paper No. 374 of the Bureau of Standards, Washington, D. C., 1920.

14. J. R. Carson, "Wave Propagation over Parallel Wires: The Proximity Effect," *Phil. Mag.*, April, 1921, p. 607.

15. J. R. Carson and J. J. Gilbert, "Transmission Characteristics of the Submarine Cable," *Journal* of the Franklin Institute, Dec., 1921, p. 705.

16. H. B. Dwight, "Skin Effect and Proximity Effect in Tubular Conductors," Journal A. I. E. E., Mar., 1922, p. 203.

17. Chas. Manneback, "An Integral Equation for Skin Effect in Parallel Conductors," *Journal of Math. and Physics*, April, 1922.

18. E. B. Rosa and F. W. Grover, Scientific Paper No. 169 of the Bureau of Standards, Washington, D. C., Table XXII, page 226.

Floating Neutral n-Phase Systems

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Review of the Subject.—*Although this paper treats of the general case of floating neutral polyphase systems, one of its prime objects is to find the very simplest method of solving three-phase unbalanced star circuits. The method presented below is considered much simpler than the one in which the star circuit is replaced by an equivalent delta circuit; also simpler than the method of replacing one unbalanced circuit by two balanced circuits, one being subjected to direct phase rotation and the other to opposite phase rotation. Although these methods are fairly useful for the three-phase case, they become very unwieldy for a number of phases greater than three. Not only is this method more simple than any so far proposed, but it is a direct method, based on a straight forward application of Kirchhoff's Laws. A final and very important advantage of the method is that all the results are perfectly general, applying equally as well to a single-phase system as to any polyphase star system.*

CONTENTS:

Review of the Subject. (165 w.)
Introduction. (185 w.)
(I) Description of Types of Circuits. (100 w.)
(II) Notation. (220 w.)
(III) General Expression for Current in any Branch. (225 w.)
(IV) General Expression for Voltage in Neutral for Any Branch. (60 w.)
(V) General Expression for Voltage Between Geometrical Neutral and True Neutral. (175 w.)
(VI) Discussion of Effect of Phase Rotation and Tabulation of Operators to Assist in Calculations. Sample Calculations. (1000 w.)
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INTRODUCTION

WHEN an a-c. receiving circuit is operated without grounding the neutral, the potential of the neutral in respect to the supply lines may take on a very wide range of values depending on the constants of the various branches or phases and on which way the source alternator may be rotating. Some use of this last fact has been made in determining what is called Direction of Phase Rotation.[1]

It is proposed in this paper to investigate such *floating neutral systems*, under the following sub-headings:

I. Description of Types of Circuits.
II. Notation.
III. General Expression for Current in any branch of an *n*-phase system.
IV. Voltage to neutral for any branch.
V. Voltage between geometrical and true neutral.
VI. Discussion of Effect of Phase Rotation and Tabulation of operators to assist in calculations. Sample Calculation.
VII. Discussion of Resonance. Some Test Results.
VIII. Bibliography.

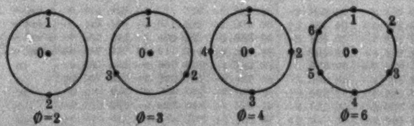

FIG. 1

I. Description of Types of Circuits. Referring to Fig. 1, ϕ stands for the number of branches or phases between lines and neutral, 0.

For example, for $\phi = 3$, there are three branches to neutral. Between 0 and 1 is located one branch which may contain resistance, inductance and capacity in any desired combination. Similarly the branches 02 and 03 may include any desired constants. In the general case there are ϕ branches all tied together at 0.

Fig. 2 shows the alternating-current sources corresponding to Fig. 1. In both Fig. 1 and Fig. 2 the branches are numbered in clockwise sequence around the circle.

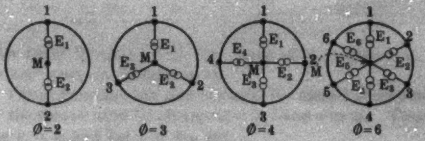

FIG. 2

II. Notation. The voltage from *M* to 1 will in all cases be taken as standard phase.

As the large majority of voltages, currents, impedances and admittances in this paper are complex numbers, ordinary type will be used for them. Script letters will be used for ordinary numbers.

Let $\alpha = \cos \dfrac{2\pi}{\phi} + j \sin \dfrac{2\pi}{\phi}$ for counter clockwise rotation.[3]

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[1] W. V. Lyon, Determining Phase Rotation, See Bibliography No. 9.
T. W. Varley, Determining Phase Rotation, See Bibliography No. 8.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 25-29, 1923.

[2] Defined in Part (V).

[3] This definition takes the point of view of the vector diagram of the voltages. If the sequence in which $\overline{01}$, $\overline{02}$, $\overline{03}$, etc. receive the impulse of voltage is considered, then the terms counter clockwise and clockwise must be interchanged. It is worth noting that the A. I. E. E. has no standard definition for Phase Rotation.

$$\alpha = \cos\frac{2\pi}{\phi} - j\sin\frac{2\pi}{\phi} \text{ for clockwise rotation}^{1}$$

where ϕ = number of phases or branches.

$E_1 = E\,\alpha^0$ = Voltage between M and 1.
$E_2 = E\,\alpha^1$ = " " M and 2.
$E_3 = E\,\alpha^2$ = " " M and 3.
$E_n = E\,\alpha^{n-1}$ = " " M and n.

where E = numerical value of impressed voltage to geometrical neutral.[2]

Z_1 = Impedance between 0 and 1.
Z_2 = " " 0 and 2.
Z_n = " " 0 and n.
Y_1 = Admittance between 0 and 1.
Y_2 = " " 0 and 2.
Y_n = " " 0 and n.
I_1 = Complex quantity expression for current in $\overline{01}$.
I_2 = " " " " " " $\overline{02}$.
I_n = " " " " " " $\overline{0n}$.
E_{on} = " " " " voltage between true and geometrical neutral.

III. General Expression for Current in any Branch. Fig. 3 represents three receiver branches and three corresponding source branches of the general case

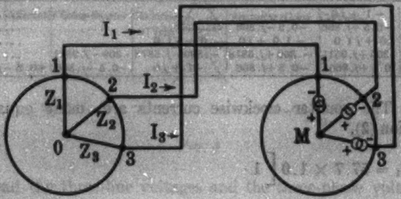

Fig. 3

where it is assumed that there are any number of branches up to n in number.

Applying Kirchhoff's Laws to this general case.

$$I_1 + I_2 + I_3 + \ldots I_n = 0$$
$$-I_1 Z_1 + E_1 - E_1\alpha + I_2 Z_2 = 0$$
$$-I_1 Z_1 + E_1 - E_1\alpha^2 + I_3 Z_3 = 0$$
etc.
$$-I_1 Z_1 + E_1 - E_1\alpha^{n-1} + I_n Z_n = 0$$

Solving the above equations,

$$I_1\,(1 + Z_1/Z_2 + Z_1/Z_3 + \ldots Z_1/Z_n)$$
$$= E_1\left(\frac{1-\alpha}{Z_2} + \frac{1-\alpha^2}{Z_3} + \ldots \frac{1-\alpha^{n-1}}{Z_n}\right)$$

$I_1 =$

$$\frac{E\,\alpha^0\left(\dfrac{1-\alpha}{Z_2} + \dfrac{1-\alpha^2}{Z_3} + \ldots \dfrac{1-\alpha^{n-1}}{Z_n}\right)}{Z_1\,(1/Z_1 + 1/Z_2 + 1/Z_3 \ldots 1/Z_n)}$$

In general then, for the pth current

$I_p =$

$$E/Z_p\left[\alpha^{p-1} - \frac{\alpha^0/Z_1 + \alpha^1/Z_2 + \ldots + \alpha^{n-1}/Z_n}{1/Z_1 + 1/Z_2 + \ldots + 1/Z_n}\right] \quad 1)$$

$$I_p = E\,Y_p\left[\alpha^{p-1} - \frac{Y_1\alpha^0 + Y_2\alpha^1 \ldots Y_n\alpha^{n-1}}{Y_1 + Y_2 + \ldots Y_n}\right] \quad (2)$$

The use of this formula is illustrated by the problem of Part VI.

IV. General Expression for Voltage to Neutral for any Branch. Equations (1) and (2) give this quantity at once as

$I_p Z_p =$

$$E\left[\alpha^{p-1} - \frac{\alpha^0/Z_1 + \alpha^1/Z_2 + \ldots \alpha^{n-1}/Z_n}{1/Z_1 + 1/Z_2 + \ldots 1/Z_n}\right] \quad (3)$$

$$I_p Z_p = E\left[\alpha^{p-1} - \frac{Y_1\alpha^0 + Y_2\alpha^1 \ldots Y_n\alpha^{n-1}}{Y_1 + Y_2 + \ldots Y_n}\right] \quad (4)$$

V. General Expression for Voltage between Geometrical Neutral and True Neutral. The *geometrical neutral* is defined as the center of the circle located by the potentials of the various lines. Thus in Fig. 1, if point 1 is taken as zero potential and if points 2, 3, 4, etc. are located on a circle a properly graduated scale will read the potential difference between any two lines.

The *true neutral* is fixed by the intersection of the various $I_1 Z_1$, $I_2 Z_2$, $I_3 Z_3$, etc. vectors and coincides with the geometrical neutral only in the case of a balanced load.

Since $I_p Z_p$ is the voltage between the pth line and the true neutral and since $E\,\alpha^{p-1}$ is the voltage between the pth line and the geometrical neutral, their difference is the voltage between the true and geometrical neutral. Hence from (4),

$$E_{on} = E\,\alpha^{n-1} - E\left[\alpha^{n-1} - \frac{Y_1\alpha^0 + Y_2\alpha^1 \ldots Y_n\alpha^{n-1}}{Y_1 + Y_2 + \ldots Y_n}\right]$$

$$E_{on} = E\,\frac{Y_1\alpha^0 + Y_2\alpha^1 + \ldots + Y_n\alpha^{n-1}}{Y_1 + Y_2 + \ldots Y_n}$$

$$= E\,\frac{\sum_1^n Y_n\,\alpha^{n-1}}{\sum_1^n Y_n} \quad (5)$$

VI. Discussion of Effect of Phase Rotation and Tabulation of α-Operators to Assist in Calculations. Sample Calculations. With the purpose of giving some physical interpretation to the results above obtained, it is desirable to follow the wanderings of the true neutral in these floating neutral polyphase systems. The travels of this neutral are almost unlimited. It can take up any position from infinite distance from the geometrical neutral at resonance down to zero distance from geometrical neutral for a balanced load. If the alternator supplying the system is stopped and brought up to speed in the opposite direction, then this E_{on} vector might in some particular case move from a 10:30 o'clock vector to a 6:30 o'clock vector of less than half its original magnitude. The locus of this true neutral when constant voltage but varying frequency is maintained on a given circuit will in some cases be a circle, in others a cusp, in others an ellipse. It has been only in recent years that attention has been called to the fact that phase rotation has such a pronounced effect on this E_{on} vector.[4]

To illustrate and to aid in the use of the formulas, the following example is solved:

Find the voltage between the true neutral and the geometrical neutral for a system where the line voltages are 100 and where the constants of the three branches are, $Y_1 = 1.0$; $Y_2 = 1.0 - j\,1.0$; $Y_3 = 1/2 + j\,1.0$; (a) when phase rotation is clockwise and (b) when phase rotation is counter clockwise.[5]

TABLE I.

CLOCKWISE OPERATORS

ϕ	α^{-5}	α^{-4}	α^{-3}	α^{-2}	α^{-1}	α^{0}	α^{1}	α^{2}	α^{3}	α^{4}	α^{5}
2	....	....	....	....	$-1 + j0$	1	$-1 + j0$	....	....	....	....
3	....	....	....	$-0.5 - j.866$	$-0.5 + j.866$	1	$-0.5 - j.866$	$-0.5 + j.866$	....	....	....
4	....	....	$0 - j1.0$	$-1.0 + j0$	$0 + j1.0$	1	$1.0 + j0$	$-1.0 + j0$	$0 + j1.0$	....	....
5	....	$.309 - j.9511$	$-.809 - j.5878$	$.809 + j.5878$	$.309 + j.9511$	1	$.309 - j.9511$	$-.809 - j.5878$	$-.809 + j.5878$	$.309 + j.9511$	....
6	$+0.5 - j.866$	$-0.5 - j.866$	$-1 + j0$	$-0.5 + j.866$	$0.5 + j.866$	1	$0.5 - j.866$	$-0.5 - j.866$	$-1 + j0$	$-0.5 + j.866$	$+0.5 + j.866$

COUNTER CLOCKWISE OPERATORS

ϕ	α^{-5}	α^{-4}	α^{-3}	α^{-2}	α^{-1}	α^{0}	α^{1}	α^{2}	α^{3}	α^{4}	α^{5}
2	....	....	....	....	$-1 + j0$	1	$-1 + j0$	....	....	....	....
3	....	....	....	$-0.5 + j.866$	$-0.5 - j.866$	1	$-0.5 + j.866$	$-0.5 - j.866$	....	....	....
4	....	....	$0 + j1.0$	$-1 + j0$	$0 - j1.0$	1	$0 + j1.0$	$-1.0 + j0$	$0 - j1.0$	....	....
5	....	$.309 + j.9511$	$-.809 + j.5878$	$-.809 - j.5878$	$.309 - j.9511$	1	$.309 + j.9511$	$-.809 + j.5878$	$-.809 - j.5878$	$.309 - j.9511$	....
6	$+0.5 + j.866$	$-0.5 + j.866$	$-1 + j0$	$-0.5 - j.866$	$+0.5 - j.866$	1	$0.5 + j.866$	$-0.5 + j.866$	$-1 + j0$	$-0.5 - j.866$	$+0.5 - j.866$

Using equation (5) and Table I:

(cw) $E_{on} =$

$$57.7\,\frac{1.0 + (1.0 - j\,1.0)\,(-\,0.5 - j\,0.866) + (0.5 + j\,1.0)\,(-\,0.5 + j\,0.866)}{1.0 + (1.0 - j\,1.0) + (0.5 + j\,1.0)}$$

(ccw) $E_{on} =$

$$57.7\,\frac{1.0 + (1.0 - j\,1.0)\,(-\,0.5 + j\,0.866) + (0.5 + j\,1.0)\,(-\,0.5 - j\,0.866)}{1.0 + (1.0 - j\,1.0) + (0.5 + j\,1.0)}$$

$$E_{on} = -\,34.25 - j\,10 \quad \text{cw.}^{6}$$
$$E_{on} = \quad 45.8 - j\,10 \quad \text{ccw.}^{7}$$

To further illustrate the use of the method of this paper, the currents are calculated.

The clockwise currents are, using equation (2).

$$I_1 = 57.7 \times 1.0 \left[1 - \frac{1.0 + (1.0 - j\,1.0)\,(-\,0.5 - j\,0.866) + (0.5 + j\,1.0)\,(-\,0.5 + j\,0.866)}{1.0 + (1.0 - j\,1.0) + (0.5 + j\,1.0)} \right]$$

$$= 57.7\,(1 + 0.5928 + j\,0.1732)$$
$$= 92.1 + j\,10.0 = 92.6 \text{ amperes}$$

$$I_2 = 57.7 \times (1.0 - j\,1.0)\,[(-\,0.5 - j\,0.866) + 0.5928 + j\,0.1732] = -\,34.65 - j\,45.3$$
$$= 57.2 \text{ amperes}$$

$$I_3 = 57.7 \times (0.5 + j\,1.0)\,[(-\,0.5 + j\,0.866) + 0.5928 + j\,0.1732] = -\,57.35 + j\,35.3$$
$$= 67.6 \text{ amperes}$$

The counter clockwise currents are, using equation (2),

$$I_1 = 57.7 \times 1.0 \left[1 - \frac{1.0 + (1.0 - j\,1.0)\,(-\,0.5 + j\,0.866) + (0.5 + j\,1.0)\,(-\,0.5 - j\,0.866)}{1.0 + (1.0 - j\,1.0) + (0.5 + j\,1.0)} \right]$$

$$= 57.7\,(1 - 0.7928 - j\,0.1732) = 11.93 - j\,10.0$$
$$= 15.6 \text{ amperes}$$

$$I_2 = 57.7 \times (1.0 - j\,1.0)\,[(-\,0.5 + j\,0.866) - .7928 - j\,0.1732] = -\,34.65 + j\,114.6$$
$$= 118.5 \text{ amperes}$$

$$I_3 = 57.7 \times (0.5 + j\,1.0)\,[(-\,0.5 - j\,0.866) - 0.7928 - j\,0.1732] = 22.7 - j\,104.8$$
$$= 107 \text{ amperes.}$$

A quicker way to solve problems of this type is to lay off to scale a triangle of the three line voltages and on this graph locate the true neutral from the calculated value of E_{on}. All phase voltages may then be scaled off as the distance from the true neutral to the corners of the triangle. A phase voltage times the phase admittance equals phase current.

VII. Resonance. Resonance in an *n*-phase circuit may be defined as the condition under which

$$Y_1 + Y_2 + Y_3 + \ldots Y_n = 0 \qquad (6)$$

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4. See Numbers 8, 9, 14 of Bibliography.
5. This problem is taken from page 377 of the 1917 Edition of Alternating Currents and Alternating-Current Machinery, by D. C. & J. P. Jackson.
6. The solution given in D. C. & J. P. Jackson's Textbook.
7. This solution is not mentioned in the above textbook.

With conditions approaching this, large currents will flow.

For the single-phase case, $\phi = 2$, resonance occurs when $Z_1 = -Z_2$. For the three-phase case, $\phi = 3$, it occurs when $Z_1 = -\frac{Z_2 Z_3}{Z_2 + Z_3}$. For $\phi = 4$, it occurs when

$$Z_1 = -\frac{Z_2 Z_3 Z_4}{Z_3 Z_4 + Z_4 Z_2 + Z_2 Z_3}$$

As a resonant circuit will best illustrate the possibilities of a floating neutral system, the following test was carried out. A three-phase, star-connected circuit was prepared with the following constants: Number one branch an air-cored inductance of 0.1745 henry and 4.6 ohms d-c. resistance, number two branch a capacity of 106.2 microfarads, and number three branch a capacity of 105.6 microfarads. In each branch an ammeter was placed. A voltmeter was arranged to read the three-line voltages and the three-phase voltages. Line voltages were maintained equal by adjusting the currents taken by an auxiliary three-phase load from the same alternator.

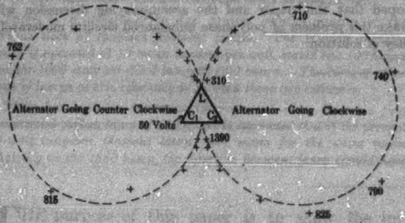

Fig. 4

Speeds were varied from 300 to 1400 rev. per min. and readings of currents and voltages were taken for each speed. The results were then reduced to a standard line voltage of fifty, as represented by the triangle of Fig. 4. On Fig. 4 is plotted the true neutral as found by swinging from the appropriate corners arcs equal to the phase voltages. The neutrals so found are shown by crosses with the corresponding alternator speeds close by. By using formula (5) and the constants above the locus of the true neutral was found to be two circles somewhat larger than the two circles indicated by the test data. The explanation of this is that the resistance during the tests, *i. e.*, with alternating current, was slightly greater than that given by a d-c. measurement. It is interesting to note that if the resistance could be reduced to zero the locus for both directions of alternator rotation would be a straight line upward from the top vertex of the triangle and downward from the center of the base of the triangle.

VIII. BIBLIOGRAPHY

1. A. E. Kennelly, Determination of Current in 3-Pointed Star Systems, *Elec. World*, Vol. 34, 1899, p. 268.

2. A. E. Kennelly, Equivalence of Deltas and 3-Pointed Stars, *Elec. World*, Vol. 34, 1899, p. 413.

3. A. Burt, Three-Phase Power Factor, Trans. A. I. E. E., XXVII, 1908, p. 801.

4. H. P. Wood, Application of Symbolic Functions, *Elec. World*, June 16, 1910, p. 1597.

5. C. H. Johnson, Solution of Problems in Star-Connected Unbalanced 3-Phase Circuits, *Elec. World*, Dec. 15, 1910.

6. L. G. Stovkis, Analysis of Unbalanced 3-Phase Systems, *Elec. World*, May 1, 1915, p. 1111.

7. Gilman and Fortescue, Single-Phase Power Service from Central Stations, Trans. A. I. E. E., XXXV, 1916, p. 1329.

8. T. W. Varley, Determining Phase Rotation, *Elec. World*, Mar. 10, 1917, p. 466.

9. W. V. Lyon, Determining Phase Rotation, *Elec. World*, May 19, 1917, p. 968.

10. C. J. Fechheimer, Phase Rotation, *The Electric Journal*, 1917, p. 111.

11. V. Karapetoff, Harmonics in M-Phase Systems, *Elec. World*, Mar 30, 1918, p. 660.

12. C. L. Fortescue, Method of Symmetrical Coordinates, Trans. A. I. E. E., Vol. 37, 1918, p. 1027.

13. L. A. Doggett, Study of Unbalanced 3-Phase Loads, *Elec. World*, Sept. 6, 1919, p. 512.

14. W. V. Lyon, Unbalanced 3-Phase Circuits, *Elec. World*, June 5, 1920, p. 1304.

15. D. C. & J. P. Jackson, Alternating Current and Alternating-Current Machinery, The Macmillan Co., 1917, Ch. VIII.

16. Magnusson, Alternating Currents, McGraw-Hill Book Co., 1921, Ch. XXV.

17. Lawrence, Principles of Alternating Currents, McGraw-Hill Book Co., 1922, Ch. XII.

Discussion

C. W. Bates: At the bottom of page 1031, Prof. Doggett states:

"Resonance in an *n*-phase circuit may be defined as the condition under which the total admittance is equal to zero." I think that is subject to correction, the correct statement being "Resonance in an *n*-phase circuit may be defined as the condition under which the reactive component of the total admittance (or the total susceptance) is equal to zero." It is rather out of the question to eliminate the power component and, besides, that does not affect the question of resonance.

Just before that Prof. Doggett says: "A quicker way to solve problems of this type is to lay off to scale a triangle of the three line voltages," etc. The vector diagram should not be used for direct calculation except in very special cases but it should be used to interpret the complex quantities as they are determined. Having calculated the complex expression for the distance between the true neutral and the geometric neutral and consequently having located the true neutral on the vector diagram, it is a simple problem to determine the various voltages from the true neutral to the various phase terminals, by using the vector diagram as a visual aid to the expression of the proper terms whose algebraic sum will give the voltage desired. For this purpose the diagram does not need to be drawn with particular accuracy and the final results are obtained with no loss of accuracy whatever.

The graphical picture is a splendid thing and I think that this paper would be improved if the illustrative problem, which Prof. Doggett has on page 1031, were drawn out graphically, both to show to the man who is not familiar with all of the details what is going on and to serve as an aid to calculation. I think that Prof. Doggett is to be congratulated on having called attention to this method of locating the floating neutral. This problem is one which occurs very often and there are very few men outside of the teaching staffs of the colleges and a very few recent graduates who can solve this problem (as well as some similar ones) without a great deal of trouble.

V. Karapetoff: Formulas equivalent to Prof. Doggett's final result (5) were deduced and published by me in 1900 in a little book entitled "Ueber Mehrphasige Stromsysteme bei Ungleichmassiger Belastung" (Enke, Stuttgart), and I have given them to my students every year at Cornell. I am not raising any question of priority, but wish to indicate that a much more general result can be arrived at in a simpler manner than Prof. Doggett's, and that his eq. (5) follows directly from the more general formula.

Let a be the vector of the voltage between the two neutral points. Then the *net* voltage acting upon the load in phase 1 is $E_1 - a$, and the current in phase 1 is equal to $Y_1 (E_1 - a)$. Thus we have

$$\left.\begin{array}{l} I_1 = Y_1 (E_1 - a) \\ I_2 = Y_2 (E_2 - a) \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ I_n = Y_n (E_n - a) \end{array}\right\} \qquad (\mathrm{I})$$

Adding these equations together and remembering that $\Sigma I = 0$, we find

$$\Sigma Y E = a \Sigma Y \dots\dots\dots\dots\dots\dots (\mathrm{II})$$

from which

$$a = (\Sigma Y E) / \Sigma Y \dots\dots\dots\dots\dots\dots (\mathrm{III})$$

Eq. (III) is the fundamental equation for any n-phase star-connected polyphase system, with unsymmetrical voltages and with an unbalanced load. Knowing a, the currents are found from eqs. (I). Doggett's eq. (5) is a special case of expression for a when the generator voltages are symmetrical.

Eqs. (I) and (III) have been found of inestimable value in many problems, especially in the study of V and T connections, because these connections may be considered as special cases of star-connected windings with unsymmetrical voltages.

L. A. Doggett: I quite agree with Mr. Bates' correction. Professor Karapetoff has in a few brief steps arrived at a very comprehensive expression for the vector voltage between the two neutrals. Its very generality, however, conceals many rather remarkable particular deductions. The particular aspect which is stressed in my paper is that of the effect of phase rotation on the location of the neutral. In reading Professor Karapetoff's 1900 monograph some seven years ago I failed to get any help on this question of phase rotation. It is easy to see that, given this general expression (III above) and Professor Karapetoff on hand to amplify and explain, many practical deductions would follow. He has given us one such in the last paragraph of his discussion.

In closing it appears to me that an old problem which has never satisfactorily been solved in the literature ordinarily available to the electrical engineer, has been resurrected. It is hoped that this paper and the accompanying discussion will make this problem of polyphase unbalanced circuits moderately easy of solution.

The Quality of Incandescent Lamps

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Review of the Subject.—*The new specifications for incandescent lamps determine their quality in terms of life to burnout at a specified mean efficiency. Formerly it was measured by the life to 80 per cent. of initial value. The new tests, therefore, conform more nearly to the actual practise in the use of tungsten filament lamps, as many of these lamps are above 80 per cent at time of burnout.*

The data on carbon lamps in the paper are based on a life exponent originally determined by Mr. Howell, one of the authors of this paper. Curves showing the relation between life, efficiency and candle power was given by him in a paper presented before the Institute on April 10, 1888. Practically the same life-candle power relation has been found to apply to the GEM, tantalum and tungsten filament lamps. Nowadays the exponent used is that applying to the life-efficiency relation and is different for each kind of filament as they have different candle power-efficiency relations. All data are based on commercial ratings and guarantees.

The quality of tungsten filament lamps has greatly improved since their commercial introduction in 1907 as is shown by the fact that the 40-watt vacuum lamp is now over eight times as good as then.

There is a difference in the present relative quality of the various sizes of tungsten filament lamps. The 10-watt vacuum lamp, if operated at 10 mean lumens per watt, would live 190 hours and the 1000 watt gas-filled lamp 35,000 hours. The lives of other sizes of lamps at this efficiency is between these two extremes.

There has been an enormous improvement since Edison's first commercial carbon lamp of 1880. It is estimated that if the present 40-watt tungsten filament lamp were made for the same mean efficiency as the 1880 lamp, the tungsten filament lamp should have a life of over a hundred and fifty thousand years. And this does not include the enormous improvement in the larger sizes of lamps due to Dr. Langmuir's invention of the gas-filled lamp. Owing to these enormous differences, the other term of quality measurement, indicating the mean efficiency for a given life, is therefore used to show the improvements since 1880. A table gives these data in chronological order.

It is estimated that in 1880 about 50 lumen-hours of light were obtained for one cent, covering the cost of current and lamp renewals. The amount now obtainable is very much greater, due to lamp improvements, reduction in their prices and reduction in rates for current. The public has utilised these by using more light. With the present 40-watt tungsten filament lamps and with current at the present approximate general average rate of 4½ cents kw-hr., 1700 lumen-hours can be had for one cent. It is impossible to show what part of this actual increase is due to lamp improvements except to indicate what would have been obtainable with one factor without the other. Thus had there been no rate reduction, 432 lumen-hours would now be had due to actual lamp improvements. Without the lamp improvements 190 lumen-hours would now be had due to the reduction in the general average rate. To indicate the gain due to Dr. Langmuir's invention of the gas-filled lamp, in which the high wattage sizes are the most efficient, 3820 lumen-hours can be had with the 1000 watt lamp at 4½ cents per kw-hr.

About half a billion dollars were spent in the United States in 1922 for current used for lighting. If the same amount of light were produced by the original 1880 bamboo carbon lamp, the cost would have been increased 3½ billion dollars requiring about fifty billion extra tons of coal, equal to about ten per cent of the total coal production in the United States.

THE purpose of this paper is to show the improvements have been made in incandescent electric lamps since they first became commercially available.

Quality is the term used by incandescent lamp engineers in connection with the relative excellence of lamps. The two main factors which determine quality are efficiency and life. With a given quality these two factors bear a reciprocal relation to each other, that is, the life of a lamp can be increased but at a sacrifice of efficiency, and vice versa.

Attention is called to the fact that Mr. Howell, one of the authors of this paper, presented before the Institute on April 10, 1888, a paper entitled "The Maximum Efficiency of Incandescent Lamps". This paper gave, for the first time, data on the relation between the life and efficiency of incandescent lamps. Occasional reference to this paper will be made.

An improvement in quality might be reflected as an increase in efficiency or in life, without a reduction of the other factor. Quality, therefore, may be measured either in terms of efficiency for a given life, or in terms of hours life for a given efficiency. In the 1888 paper the latter term was used, the quality of lamps being stated as "The life they will give when burned at a given efficiency."

The quality measure is useful to lamp engineers in determining the usefulness of suggested improvements in lamps, in comparing the performance of different types of lamps and in maintaining the product up to the highest possible standard.

STANDARD SPECIFICATIONS FOR DETERMINING LAMP QUALITY

The Bureau of Standards at Washington, D. C., have for some time established specifications on lamps for use by the various departments of the United States Government. These specifications were established by them after conference with manufacturers and engineers. The present specifications are different from those in use up to about two years ago.

The difference between the present and previous specifications can perhaps best be explained by quoting the introductory statement of Circular 13 of the Bureau of Standards:

"The most notable of these changes is the abandonment of the long-established provision that the life of test lamps shall be considered as ended when the candle power has fallen to 80 per cent of the initial value. The specification of such an end point is convenient and

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reasonable in the testing of carbon lamps, because those lamps will often burn for a long period after they have become so blackened that they should not be continued in use. In tungsten lamps, however, means have been found to prevent excessive blackening of the bulbs, so that the lamps normally burn out before their efficiency has fallen enough to justify replacing them. The new tests will, therefore, be based on the total life to the time of burnout, thus conforming more nearly to actual practise in the use of lamps."

"The performance of the lamp throughout its life will also be taken into account through — the evaluation of life-test results on the basis of average efficiency throughout life, instead of the initial efficiency."

It will be noted that the inherent quality of lamps is now measured by the number of hours life to burnout at a given average efficiency throughout life. Lamps may be operated on test at an efficiency that will make the duration of the test about 500 hours. For purposes of comparison of different lots of each size of lamp and for a permanent historical basis of record, mean efficiencies are given in the specifications for each lamp, which efficiencies were originally based on an average life of 500 hours.

Basis of Data

In Mr. Howell's 1888 paper, a curve was shown indicating the relation between the life and efficiency of carbon lamps, which at that time had untreated filaments. This curve was based on the author's calculations, obtained from the tests of many lamps during a period of over five years, that the life of lamps varied inversely as the 3.65 power of the candle power. This life-candle power exponent however was not given in the paper. In the intervening years, even with the tests of many more lamps, this exponent has remained unchanged. It has even been found that the same life-candle power exponent applies to the treated carbon, the Gem (metallized carbon) and practically to the tantalum and tungsten filament lamps.

Nowadays it is customary to make calculations on the basis of the life efficiency exponent. This exponent varies with the different lamps as the candle power-efficiency curve is different for each kind of filament. The life-efficiency exponents used in the calculations of the data given herein are as follows:

5.5 for the untreated carbon filament lamp
5.8 for the treated carbon filament lamp
5.8 for the Gem (metallized carbon) lamp
6.2 for the tantalum lamp
6.7 for the tungsten filament lamp

The exponent for tungsten filament lamps has been considered to be different for the different sizes of lamps, but it is now believed by many that the figure 6.7 applies to all sizes, so this figure is used herein.

Improvement in Quality of Tungsten Filament Lamps

Limitation of space prevents giving data showing the improvement in all the different sizes of tungsten filament lamps from their commercial introduction to date. The most important of the vacuum lamps is the 40-watt, 115-volt size. The suggested standard comparison efficiency given in the specifications for this lamp is 9.9 mean lumens per watt. The actual life obtained at this mean efficiency, based on the commercial ratings in effect at various times from the introduction of this size of lamp in 1907 to date, is as follows:

TABLE I.
QUALITY OF 40-WATT 115-VOLT VACUUM TUNGSTEN FILAMENT LAMP, 1907-1923

Date	Initial efficiency	Hours life to burnout	Spherical reduction factor	Mean Efficiency in percent of Initial	Inherent Quality Hours life to burnout at 9.9 mean L-P-W
Nov. 1907	1.25 w-p-h-c-p	800	78%	85%	63
Apr. 1910	*1.23 do	1000	78%	85%	78
Oct. 1912	1.17 do	1000	78%	86%	120
Jan. 1914	1.10 do	1000	78%	87%	196
Oct. 1914	1.03 do	1000	78%	88%	327
May 1917	1.35 w-p-s-c-p	1000	—	87%	261
July 1917	1.33 do	1000	—	88%	312
Aug. 1918	1.36 do	1000	—	87%	250
Apr. 1920	1.35 do	1000	—	87%	261
May 1921	9.9 l-p-w	1000	—	89%	458
Oct. 1921	10.1 do	1000	—	90%	564
May 1922	10.1 do	1000	—	89%	524
May 1923	10.1 do	1000	—	89%	524

*The International candle in use since April 1910 is 1.6 per cent less than the standard used prior to that date.

It will be seen that a great improvement in the quality of this lamp has been made since its introduction sixteen years ago; in fact the quality then was but 12 per cent of what it is now. Due to war conditions there were slumps, but the lamp is now about twice as good as it was in 1914. These data, derived from commercial ratings, show average figures—there are of course occasional fluctuations.

Difference in Relative Quality Between Various Sizes of Tungsten Filament Lamps

A difference in relative quality between the various sizes of lamps is indicated by the difference in their rated initial efficiency. Table II shows the present difference in relative quality between the various standard sizes of 115-volt tungsten filament lamps as determined from their commercial ratings.

It is impractical to chart the relative quality given in the above table in terms of hours life at a given efficiency. A chart, Fig. 1, has therefore been prepared showing the present relative quality in terms of the mean efficiencies (given in Table II) of the various lamps for 500 hours life.

TABLE II.

DIFFERENCE IN RELATIVE QUALITY BETWEEN VARIOUS SIZES OF 115-VOLT TUNGSTEN FILAMENT LAMPS

Lamp	Commercial Rating initial L-P-W	Mean L-P-W for 500 hours life	Relative Quality — Hours life at 10 mean L-P-W
10 watt Vacuum	8.2	8.73	190
15 watt Vacuum	8.7	9.16	275
25 watt Vacuum	9.5	9.56	375
40 watt Vacuum	10.1	9.98	485
50 watt Vacuum	10.2	9.84	440
60 watt Vacuum	10.2	9.61	370
75 watt Gas-filled	11.8	11.9	1,600
100 watt Gas-filled	12.9	13.0	2,950
150 watt Gas-filled	14.3	14.3	5,400
200 watt Gas-filled	15.3	15.4	9,100
300 watt Gas-filled	16.5	16.6	15,200
500 watt Gas-filled	18.1	18.1	26,400
750 watt Gas-filled	19.1	18.0	25,300
1000 watt Gas-filled	20.0	18.8	35,000

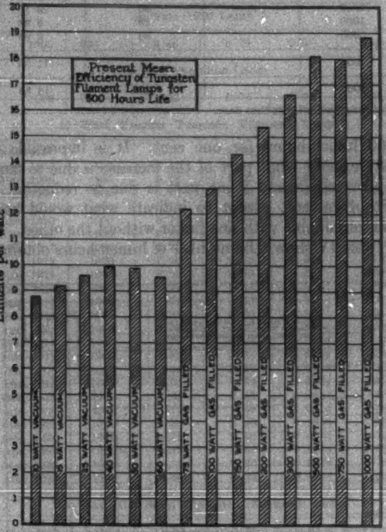

FIG. 1. PRESENT MEAN EFFICIENCIES OF TUNGSTEN FILAMENT LAMPS FOR 500 HOURS LIFE

IMPROVEMENT IN THE MOST GENERALLY USED LAMP, 1880-1923

The first commercial incandescent lamp had a carbonized paper filament. About the middle of the year 1880, carbonized bamboo was used. The term "watts" had not been adopted at that time, so the bamboo lamps were made for an efficiency of "eight lamps per horse power". The lamps were designed to give 16 candle power in a horizontal direction when new and had a rated commercial life of 600 hours to burn out. Expressing these figures in the present international candle power standard, their quality was such that it is computed that they would have given a mean efficiency of 1.12 lumens per watt during 500 hours total life. In the latter part of 1881 lamps were made at a higher efficiency for use on lighting circuits having close voltage regulation. They were made "ten lamps per horse power" rated for 600 hours life to burn out. In the years following the quality was improved by the experience gained in manufacture, the changes in the filament, etc. The most important improvements were in:

1888	Asphalted bamboo — carbon filament
1893	Treated bamboo — carbon filament
1894	Treated cellulose — carbon filament
1905	Metallized carbon filament (Gem lamp)
1906	Tantalum filament
1907	Tungsten filament
1911	Drawn tungsten wire
1912	"Getters" (chemicals used to improve the lumen maintenance)
1913	Gas-filled tungsten filament lamp.

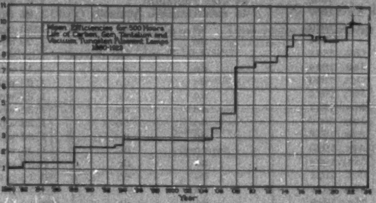

FIG. 2. IMPROVEMENT IN LAMPS, 1880-1923

The aggregate of these advances brought about by changes in type of filament and improvement in quality is staggering in amount. For example, the quality of the present 40-watt vacuum lamp is such that if it were made for the same mean efficiency as Edison's bamboo carbon lamp of 1880, it is estimated that the tungsten filament lamp would have a life of over one hundred and fifty thousand years. And this does not take into consideration the enormous improvement in the larger sizes of tungsten filament lamps due to Dr. Langmuir's brilliant invention of the gas-filled lamp.

Owing to such enormous differences, the improvements are therefore indicated in terms of their mean efficiencies for 500 hours life. Table III gives the mean efficiencies of the most popular size of vacuum lamp in use from 1880 to date. These data in chart form are shown in Fig. 2. To indicate the improvement made possible by Dr. Langmuir's invention, the present mean efficiencies of the 75 and 1000-watt gas-filled tungsten lamps are added at the bottom of Table III.

Due to these various improvements, the rated initial efficiencies were improved as will be seen from the figures in the third column.

TABLE III.

MEAN EFFICIENCIES FOR 500 HOURS LIFE OF CARBON, GEM, TANTALUM AND VACUUM TUNGSTEN FILAMENT LAMPS, 1880-1923

Date	Lamp	Rated Initial efficiency	Rated Hours life to burnout	Reduction factor	Mean efficiency (% initial)	Mean LPW for 500 hrs. life
1880	16 c.p. carbon (bamboo)	8 per h. p.	600	77%	65%	1.12
Oct. 1881	do do	10 per h. p.	600	77	65	1.41
1888	do (asphalted bamboo)	3.1 wphcp	600	77	70	2.30
1892	do (treated bamboo)	3.1 wphcp	600	77	75	2.45
1894	do (treated cellulose)	3.1 wphcp	600	82½	80	2.80
1900	do do	3.1 wphcp	600	82½	81	2.83
1904	do do	3.1 wphcp	600	82½	82	2.87
1905	50 watt GEM	2.5 wphcp	600	82½	82	3.56
1906	44 watt Tantalum	2.0 wphcp	700	78	85	4.46
Nov. 1907	40 watt Tungsten (pressed)	1.25 wphcp	800	78	85	7.29
Apr. 1910	do do	*1.23 wphcp	1000	78	85	7.62
1911	do (drawn)	1.23 wphcp	1000	78	85	7.62
Oct. 1912	do do	1.17 wphcp	1000	78	86	8.01
Jan. 1914	do do	1.10 wphcp	1000	78	87	8.66
Oct. 1914	do do	1.03 wphcp	1000	78	88	9.31
July 1916	do do	1.32 wpscp	1000	....	88	9.31
May 1917	do do	1.35 wpscp	1000	....	87	9.01
July 1917	do do	1.33 wpscp	1000	....	88	9.24
Aug. 1918	do do	1.36 wpscp	1000	....	87	8.93
Apr. 1920	do do	1.35 wpscp	1000	....	87	9.01
May 1921	do do	9.9 l-p-w	1000	....	89	9.77
Oct. 1921	do do	10.1 l-p-w	1000	....	90	10.10
May 1922	do do	10.1 l-p-w	1000	....	89	9.98
May 1923	do do	10.1 l-p-w	1000	....	89	9.98
May 1923	75 watt Tungsten (gas-filled)	11.8 l-p-w	1000	....	91	11.90
May 1923	1000 watt Tungsten (gas-filled)	20.0 l-p-w	1000	....	85	18.80

*The International candle in use since April 1910 is 1.6 per cent less than the standard used prior to that date.

VALUE OF LAMP IMPROVEMENTS

The cost of current to the public is now very much less than in 1880. The public has increased the amount of light it has used with every reduction in rate for current and every improvement in lamps. It is difficult to determine the cost of current in the first few years, but it is assumed in the following calculations to have averaged 20c. per kw-hr., from 1880 to 1883, 15c. from 1883 to 1888 and 13c. from 1888 to 1890. Data beginning with 1890 are available on the general average rate for which current was sold so these figures are used in the calculations.

The 1880 lamp had a mean efficiency of 1.09 lumens per watt during its rated life of 600 hours, consumed about 93 watts and had a list price of one dollar. At 20c per kw-hr., about 50 lumen-hours could be obtained for one cent, covering the cost of current and lamp renewals.

The present 40-watt tungsten filament lamp has a list price of 32 cents. If it were of the same quality as the 1880 lamp it would have a mean efficiency of 1.01 lumens per watt during its rated life of 1000 hours. At the present approximate general average rate of 4½c. per kw-hr. such a lamp would produce 190 lumen-hours for one cent.

If on the other hand with no reduction in rate for current, the present 40-watt lamp, which has a mean efficiency of 9 lumens per watt during its 1000 hours rated life, would at 20c. per kw-hr., produce 432 lumen-hours for one cent.

With the combination of the reduction in rate and improvement in lamps, it is now possible at the approximate general average rate of 4½c. per kw-hr., to obtain 1700 lumen-hours for one cent. It is impossible to state exactly what part of the increase is due to lamp improvements and what part is due to reduction in cost of current, except to indicate what would have been obtainable with one factor without the other.

Table IV shows the number of lumen-hours obtained from time to time for one cent with the most popular size of lamp used. It is based on the prevailing lamp quality and the prevailing approximate general average rate for current. This is shown diagramatically in Fig. 3. At the bottom of Table IV is given for comparison, the number of lumen-hours now obtainable with the 75 and 1000-watt gas-filled tungsten filament lamps.

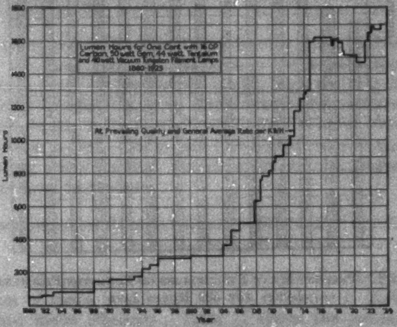

FIG. 3. LUMEN-HOURS FOR ONE CENT AT APPROXIMATE GENERAL AVERAGE RATES FOR CURRENT, 1880-1923

TABLE IV.
LUMEN-HOURS FOR ONE CENT. 1880-1923

Date	Rated life in hours	Mean lumens per watt during rated life	List price of lamp	Approximate general average rate per kw-hr.	Lumen hours for one cent with prevailing quality and prevailing rate per kw-hr.
		16 C-P Carbon Lamp			
1880	600 (a)	1.09	$1.00	$0.20	50
Oct. 1881	600 (b)	1.37	1.00	0.20	61
1883	600 (b)	1.37	0.75	0.15	83
1888	600 (b)	2.33	0.75	0.13	143
1890	600 (c)	2.28	0.75	0.12	155
1893	600 (c)	2.38	0.75	0.11	176
1894	600 (c)	2.72	0.75	0.10	220
1895	600 (c)	2.72	0.50	0.09½	246
1896	600 (c)	2.72	0.22	0.09	283
1900	600 (c)	2.75	0.20	0.08½	300
1904	600 (c)	2.79	0.20	0.07	364
		50 Watt GEM Lamp			
1905	600	3.45	0.25	0.07	440
		44 Watt Tantalum Lamp			
1906	700	4.23	0.75	0.06	501
		40 Watt Vacuum Tungsten Filament Lamp			
Nov. 1907	800	6.78	1.50	0.06	635
Oct. 1908	800	6.78	1.10	0.05½	759
Nov. 1908	800	6.78	1.00	0.05½	785
Sept. 1909	800	6.78	0.90	0.05½	815
Jan. 1910	800	6.78	0.80	0.05½	870
Apr. 1910	1000	6.78	0.80	0.05½	904
Apr. 1911	1000	6.78	0.70	0.05	967
Mar. 1912	1000	6.78	0.55	0.05	1020
Oct. 1912	1000	7.20	0.45	0.05	1175
July 1913	1000	7.20	0.35	0.05	1250
Jan. 1914	1000	7.77	0.35	0.05	1280
Apr. 1914	1000	7.77	0.30	0.05	1300
Oct. 1914	1000	8.38	0.30	0.05	1595
Apr. 1915	1000	8.38	0.27	0.04½	1620
May 1917	1000	8.10	0.27	0.04½	1570
July 1917	1000	8.32	0.27	0.04½	1610
Jan. 1918	1000	8.32	0.30	0.04½	1590
Aug. 1918	1000	8.04	0.30	0.04½	1535
Oct. 1918	1000	8.04	0.35	0.04½	1515
Apr. 1920	1000	8.10	0.40	0.04½	1470
May 1921	1000	8.80	0.40	0.04½	1600
Oct. 1921	1000	9.10	0.40	0.04½	1650
Apr. 1922	1000	9.10	0.35	0.04½	1690
May 1922	1000	9.00	0.35	0.04½	1670
May 1923	1000	9.00	0.32	0.04½	1700
		75 Watt Gas-Filled Tungsten Filament Lamp			
May 1923	1000	11.90	0.55	0.04½	2270
		1000 Watt Gas-Filled Tungsten Filament Lamp			
May 1923	1000	18.80	4.25	0.04½	3820

(a) 93.25 watts (b) 74.6 watts (c) 50 watts

The amount of light now obtainable for one cent, with the 40-watt vacuum and the 75 and 1000-watt, gas-filled tungsten filament lamps at various rates for current, is given in Table V. As in Table IV, the cost of lamps is considered as their list price. Large consumers of current obtaining low rates are usually large purchasers of lamps and hence obtain them at less than list price. Under such circumstances the amount of light obtained for a given expenditure would be somewhat greater than stated. For comparison, the amount which would have been produced by the 1880 lamp is also shown. These data are shown diagramatically in Fig. 4.

As previously mentioned, it is impossible to exactly state what part of the increase is due to lamp improvement and what part is due to the rate reduction. The improvement in lamps is indicated by the fact that the rated mean efficiency of the present 40-watt vacuum tungsten filament lamp is 8¼ times, the 75-watt gas-filled is nearly eleven times and the 1000-watt is 17¼ times that of the 1880 lamp. The rate reduction depends, of course, on the rate used.

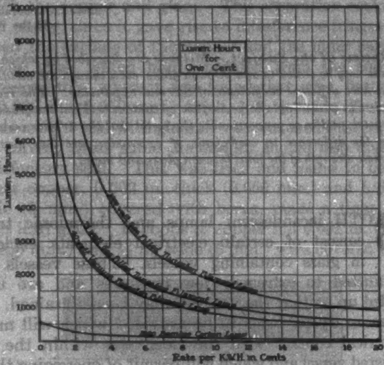

FIG. 4. LUMEN-HOURS NOW OBTAINABLE AT VARIOUS RATES FOR CURRENT

TABLE V.
LUMEN-HOURS FOR ONE CENT AT VARIOUS RATES FOR CURRENT

Rate per kw-hr.	Lumen-Hours for One Cent with			
	1880 carbon lamp	40-watt vacuum tungsten filament lamp	75-watt gas-filled tungsten filament lamp	1000-watt gas-filled tungsten filament lamp
1 c.	391	5000	6860	13200
2	288	3200	4150	7700
3	228	2370	3190	5600
4	188	1870	2510	4230
5	160	1550	2080	3460
6	140	1333	1760	2930
7	124	1150	1540	2530
8	111	1022	1360	2230
9	101	918	1220	2000
10	92	832	1110	1800
12	78	703	865	1510
14	69	608	755	1300
16	61	536	670	1140
18	55	480	603	1020
20	50	432	574	905

EFFECT OF LAMP IMPROVEMENTS

Improvements in lamps may be utilized either to get more light for the same money or to get the same amount for less money. The American public has chosen to get more light and each advance in lamp quality has resulted in increased intensity of lighting everywhere.

It is estimated that over half a billion dollars were paid in 1922 for electricity used in the United States for electric lighting. If the present day intensity of lighting were produced by using the bamboo carbon lamp of 1880, the cost of lighting in 1922 would have been increased three and one half billion dollars. This would have required about fifty-billion additional tons of coal, about ten per cent of the total coal production in the United States, to generate the amount of light actually used.

Discussion

For discussion of this paper see page 877.

The Art of Sealing Base Metals Through Glass

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Review of the Subject.—*Methods are described by which base metals may be sealed to and through glass, even though the metal and glass have different coefficients of thermal expansion. The method consists in providing a large surface of contact between the glass and the metal, and in so proportioning the metal that the stresses resulting from the difference in coefficients of expansion are less than the ultimate strength of the joint between glass and metal.*

Four different types of seals are discussed:

First, the flattened wire seal for small electrical conductors.

Second, the ribbon seal for special purposes.

Third, the disk seal for commercial manufacture of seals for carrying currents of the order of 100 amperes.

Fourth, the tube seal in which metal and glass tubing are joined together.

BOTH incandescent lamps and thermionic tubes consist of certain electrical elements enclosed in glass containing vessels. These vessels are exhausted to extremely low pressures and are then sealed up so that the vacuum may be maintained. It is necessary for leads to be provided which will make electrical contact with the electrodes within the exhausted vessel and which will permit of energizing these electrodes by means external to the vessel.

In the incandescent lamp, the electrode consists of a fine wire which is heated to a high temperature by passing current through it. For many years platinum was used as that part of the electrical conductor, passing through the walls of the glass enclosing vessel, which carried current to and from the filament. There are two reasons why platinum was used. First, it does not oxidize while the glass is being applied; consequently, the glass comes in contact with a clean surface of platinum and this was thought to be a desirable feature. Second, of the metals readily available, ten years ago, platinum was the only one which had a coefficient of expansion approximately the same as that of the lead glass in use at that time. Both lead glass and platinum have a coefficient of expansion approximately 9×10^{-6} per deg. cent. Other metals, such as gold, silver, copper, iron, nickel, etc., have coefficients of expansion appreciably greater than that of glass. Consequently when attempts are made to cover round wires of these metals with glass, while good contact may be made with the glass hot, yet when the glass and wire cool down to room temperature, the metal contracts further than the glass, and separates from the glass, leaving very small openings between the wire and the glass, through which air readily enters the vacuum container. Consequently, it became axiomatic, in the incandescent lamp manufacturing industry, that the seal wire, as that portion of the conductor coming directly in contact with the glass was called, must have the same coefficient of expansion as that of the glass through which it passed. In consequence, we find that when the price of platinum increased enormously several years ago, platinum substitutes were offered for use, which had approximately the same coefficient of expansion as the platinum which they replaced.

Alloys of iron and nickel have the property of a varying coefficient of expansion, depending upon the relative proportions of the two component parts. An alloy may be obtained having practically any coefficient of expansion from zero to 14×10^{-6} per deg. cent. One of the earliest platinum substitutes consisted of a core of a nickel iron alloy sheathed with platinum. Another substitute at present in use in the lamp manufacturing industry consists of an alloy core sheathed with copper and usually coated with dehydrated borax. In the substitution of either of these alloys for platinum, a larger diameter of the substitute wire is usually required. The alloy core is of considerably higher resistance than the platinum which it replaces. The copper sheathed wire in this respect is better than the platinum sheathed wire in that the copper is of lower resistance than the platinum. The high resistance is objectionable because, carrying the normal current, the wire heats and of course expands. If this expansion is sudden, that is, if the wire heats up before the surrounding glass, it is quite likely to split the glass from its internal wedging action. Consequently, it has been found advisable to

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use only a short length of the platinum substitute and to weld copper wire to each end of the short length. The solid copper being a better heat conductor than the alloy wire itself, serves to take heat away from the wire and thus keep the resulting expansion at a minimum. Welding the copper wire to the substitute also serves to close up any small openings which may occur between the copper sheath and the alloy core which otherwise would give rise to very small leaks, developing after several days' or weeks' use. Thus we have the practically universal use of a round cylindrical wire which has approximately the same coefficient of expansion as that of glass, passing through the glass.

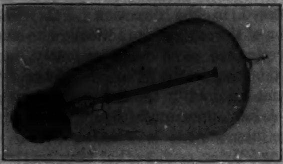

Fig. 1

Measurements made upon tungsten and molybdenum showed that they had coefficients of expansion approximately 1/3 that of the ordinary lead glass. The well-known Pyrex glass has approximately the same coefficient of expansion as that of tungsten. Thus satisfactory seals are made between tungsten and Pyrex glass through the intermediate use, however, of a second glass which seals satisfactorily to the tungsten and to which the Pyrex glass is, in turn, attached.

In the telephone plant, thousands of tiny incandescent lamps are used for indicating the condition of subscribers' lines. These lamps are approximately ¼ in. in diameter (6 mm.) and 1¾ in. long (4½ cm.). Several years ago, experiments were made to determine upon a practicable substitute if any, for the platinum used in these lamps. Trial lots of lamps were made up using various sizes of platinum and platinum substitutes at that time available. Exceedingly fine wires of copper and iron were tried, also copper wire covered with dehydrated borax was tried. As was expected, these showed slight leaks. No. 34 B & S gage copper wire was then prepared in a different way. Placed on an anvil, it was struck at right angles with the sharp edge of a hammer, thus flattening the wire locally to a section approximately 0.001 in. thick (0.025 mm.) and 0.030 in. wide (0.75 mm.). This wire was covered with dehydrated borax and the flattened portion placed within the glass. Tests showed that a seal made in this way was vacuum tight. An experimental machine was soon put together, using a cam-actuated hammer to flatten the wire, after which the wire was drawn by motor-driven rolls through a borax coating bath and then through a dehydrating oven. Repeated tests showed that wire thus made sealed with lead glass without leaks. Larger sizes of wire were tried. Fig. 1 shows a 40-watt, 110-volt incandescent lamp made with the flattened copper. With the use of larger sizes of wire it became more and more difficult to flatten the wire by a single hammer blow. It was found that several successive blows were necessary. If the copper was not flattened sufficiently, separation occurred between the copper and glass. This separation could be seen easily, since it gave rise to interference fringes between the surface of the copper and the inner surface of the glass which had separated from the copper. Consequently, when it was found that wire, flattened to 0.002 in. (0.050 mm.) thick, 1/8 in. wide (3 mm.) made a tight seal with glass, it was reasoned that copper foil having these same dimensions should also make a satisfactory seal. This was tried and found to be so. Sheet copper, 0.002 in. thick (0.050 mm.) and 1/8 in. wide (3 mm.) covered with borax, made a tight seal with lead glass. Ribbons of copper foil 0.002 in. thick (0.050 mm.) and ¼ in. wide (6 mm.) were covered with borax and found to be tight. This thin ribbon, however, was mechanically weakened by oxidation outside of the seal. Consequently, ribbons were made from 0.004 in. (0.1 mm.) and 0.008 in. (0.2 mm.) sheet copper, ¼ in. wide (6 mm.), the ribbon being flattened locally at the sealing-in point, the increased thickness outside the seal making the ribbon mechanically stronger. All of these seals were covered with dehydrated borax.

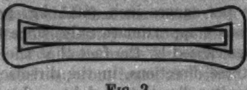

Fig. 2

Fig. 3

The question arose as to the part played by the borax in the seal manufacture. Consequently, a seal was made with 0.002 in. (0.050 mm.) copper ribbon without borax. It was found that such a seal was tight, although the ribbon outside of the seal was exceedingly weak, due to oxidation. 0.004 in. ribbon (0.1 mm.), ¼ in. wide (6 mm.) apparently sealed in satisfactorily, though showing a slight leak under test. Ribbon 0.008 in. thick (0.2 mm.) by ¼ in. wide (6 mm.) when sealed in showed visible leaks along the edges of the ribbon, but not at the center. Fig. 2 shows a cross-section, not to scale, of such a seal. As previously explained, the leak is shown by interference rings produced in the small space between the glass and metal which are separated. We found that by tapering off the two edges of the strip by filing to a knife edge, tight seals could be obtained between glass and 0.008 in. (0.2 mm.) copper ribbon, that is, the cross-section of the ribbon instead of being a rectangle, was made a parallelogram having two rather acute angles. Tight seals, using larger and larger sizes of copper were rapidly made in succession, until a ribbon 0.015 in. thick (0.38 mm.) and 1 in. wide (25.4 mm.) was sealed through glass without the use of borax, and without leaking. As before, the edges of the ribbon were filed to a knife edge. Such a seal is shown in Fig. 3 and a cross-section in Fig. 4.

The results of these preliminary experiments were so much at variance with belief and previous experience as to be open to considerable doubt as to the reliability of methods of test. Vacuum tight seals were, however, produced by different operators using material from different sources. As a result of these experiments and tests, a complete chemical and mathematical investigation was inaugurated as well as further experimental work either to prove or disprove the results of the initial experiments which have just been recorded.

In order that the explanation of the action of this seal, known as the ribbon seal, may be clear, the various steps in the manufacture will be explained somewhat in detail. A piece of 0.015 in. (0.37 mm.) sheet copper is cut approximately 1 in. wide (25.4 mm.) by 4 in. long (10.1 cm.). The edges of the sheet copper for a length of approximately 1 in. (25.4 mm.) at the center are filed away so that the angle at the edge is approximately 8 deg. or 10 deg. A glass tube is provided, into the end of which it is desired to seal the ribbon. The end is flattened to provide an opening approximately 1 1/8 in. (28 mm.) by 1/8 in. (3 mm.). The sheet copper is then heated in the glass working fires for several seconds to a good red heat. It is then so placed within the end of the glass tube that the glass will come in contact with that section provided with the sharpened edges. The glass is pressed in contact with the ribbon and melted in place at as high a temperature as can be maintained without injury to the glass or the copper. It is then annealed and finally allowed to cool off. It will readily be seen that this operation is the exact parallel to that employed in sealing in platinum or platinum substitute wires.

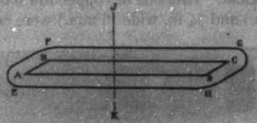

FIG. 4

In previous attempts to seal base metals through glass, in practically every instance round or nearly round wires were used, that is, wires were used in which the perimeter of the cross-section was the minimum possible for the area of cross-section. In the case of the copper ribbon seal, with a ribbon width of 1 in. (25.4 mm.) and thickness of 0.015 in. (0.37 mm.), the perimeter of the cross-section is very great compared to the perimeter of a circle of equal cross-section. In other words, for a given cross-section of metal, the ribbon affords a great deal more surface of contact between metal and glass than if a cylindrical conductor were used. Due to this great surface of contact, the stresses produced between metal and glass due to the difference in coefficients of expansion never become large enough to fracture either the joint between metal and glass or the glass itself.

In Fig. 4, A, B, C, D, is the cross-section of the copper ribbon, not to scale, and E, F, G, H is the enclosing glass. We find that the copper ribbon is actually stretched in the direction AC. The cubical contraction of the copper is practically the same as that of an unrestrained piece. Consequently, copper contracts in thickness in the direction of the line JK by that amount required by the cubical contraction of the copper. A few approximate figures may in this connection be interesting.

Assuming 525 deg. cent. as that temperature at which the glass commences to stretch the copper, room temperature of 25 deg. cent. coefficient of expansion for copper of 17×10^{-6} per deg. cent., and coefficient of 9.1×10^{-6} for glass, then the strain in a 1 in. length of copper will be approximately $(17.2 - 9.1) \times 10^{-6} \times 500 = 0.00405$ in. (0.1 mm.) Tests on soft copper show that this elongation corresponds approximately to a stress of 8700 pounds per square inch (612 kilograms per sq. cm.). The thickness of ribbon in the direction JK of Fig. 4, being approximately 0.015 in. (0.37 mm.), the force necessary to stretch a 1 in. length of the seal will be approximately 131 lb. (59.4 kg.). This stress may be assumed to be carried as shear between the glass and the copper along planes AB and CD. AB and CD are both approximately 5/32 in. (4 mm.) so that the shearing stress per square inch necessary to stretch the copper the required amount is approximately 840 pounds per square inch (59.1 kg. per sq. cm.). These figures are approximate only, since they neglect complications caused by the fact that the copper is in tension in three directions, in the directions AC and JK, and also in a direction at right angles to the cross-section of the figure. However, the figures indicate the order of magnitude of the stresses involved.

The stretching of the copper has been directly measured on a somewhat different type of seal, the disk seal, which will be described later. There can be no doubt that the glass is sucked in by the copper in the direction JK. Two parallel spots were lapped on the glass surrounding such a seal and the distance, JK, on Fig. 3, carefully measured with a micrometer. The copper was then dissolved in nitric acid, leaving only the surrounding glass shell. Subsequent measurements showed that the glass had sprung outwardly a measurable amount.

Glass may be regarded as a more or less viscous liquid. It has no definite melting point such as simple materials have. Upon heating it becomes more and more fluid until at the temperature at which it is applied to copper, it has the consistency of rather thin molasses. Glass at this temperature wets the copper, just as at room temperature, water will wet glass. Apparently as the hot glass and copper cool off, the adhesion of the glass to the copper is stronger than the cohesion within the glass, since in every case of a seal properly made, fracture will occur, not between the glass and copper, but in the glass itself, a thin film of glass being left adhering to the copper. Consequently the shearing strength of the joint between the glass and copper may be taken as equal to the shearing strength of the glass itself. Now the adhesion of the glass to the copper is entirely independent of the thickness of the copper or the thickness of the glass. The stresses, however, which the joint may be called upon to resist, are directly dependent upon the thickness of the glass or the copper or both. It is not possible to seal a heavy block of copper to a heavy block of glass since the stresses which the joint will be called upon to resist will be greater than the strength of the glass near the joint. However, it is entirely possible to seal a very thin section of either substance directly to the other; for example, a circular microscope cover glass may be melted to and will adhere to a large block of copper and a disk of sheet copper of approximately the same dimensions may be melted to a large block of glass. In both of these cases the stress which the joint between glass and copper is called upon to resist is less than the strength of the joint. Consequently, again referring to Fig. 4, we are led to the conclusion, which is amply sustained by experiment, that no matter what the metal, the angle *B A D* may always be made sufficiently acute so that the stress between the metal and glass is always less than the shearing strength of the joint; that is, it is entirely possible to seal any metal through glass, provided that the glass wets the metal when hot and further provided that the metal does not melt at the temperature necessary for it to be wetted by the glass. Seals have been made between lead glass and base metals, such as, iron, brass, and silver. Copper is peculiarly satisfactory for this service, since soft copper passes its elastic limit at a comparatively low stress per square inch. Consequently, for a given width of ribbon, thicker copper may be used than if other metals are used. This is fortunate, because copper has good electrical and heat conductivity.

Referring again to Fig. 4, attempts have been made to determine the most desirable cross-section of the copper ribbon. The various methods of considering the stresses indicate approximately the same most desirable cross-section. It has been previously stated that the copper ribbon is stretched in the direction *A C*. This occasions a compressional stress in the glass *A F C* and under compression this glass may be assumed a column, or, again, it has been stated that the glass *F C* is drawn towards the glass *E D* in the direction *J K*, by contraction of the copper ribbon in thickness. The glass *A F C* then may be considered as a beam rigidly supported at both ends and deflected downwards under the load applied by the copper. In this case the deflection of the beam at each point throughout its length would be proportional to the thickness of the copper ribbon at that point. Under such conditions bending stresses will occur in the glass beam opposite points *A* and *B* at which points there is a change in rate of deflection. Preliminary consideration thus indicates the desirability of avoiding all corners on the cross-section of the ribbon.

Mr. T. C. Fry has made a very complete mathematical analysis of the stresses and strains occurring in a ribbon seal, in order to determine the best section of the copper, the "best section" being that having the maximum cross-section for a given width of ribbon. The curve indicated by Mr. Fry for cross-section of ribbon lies between the cosine curve of the glass considered as a column in compression and the curve of deflection of a beam rigidly fixed at both ends such that maximum fiber stress is nowhere exceeded. Thus in Fig. 4, the surface *A B* should be curved and be approached by surface *A D* tangentially, and surface *A B C* should be a smooth curve without any sudden change in direction. The calculations, however, cannot be made with as great precision as might be desirable because of unknown factors in the calculation. We do not know the exact temperature at which solidification of the glass may be said to commence on cooling, nor do we know the elastic constants of glass and metals under various conditions of temperature and stress. Practically, if a ribbon seal fails, the angle at the edge is made sharper and a new seal made, rather than attempting to calculate the exact dimensions.

In the introductory paragraphs an advantage for platinum was claimed in that it did not oxidize and thus permitted closer contact between glass and metal. In the description of the seal manufacture it was stated that the copper ribbon was heated red hot before covering with glass. This heating of the copper ribbon of course oxidizes it and this oxide is not removed before being brought into contact with the glass. These two statements are at variance. As a matter of fact, perfectly good seals result when the copper has been oxidized as stated. If care is used to work in a reducing atmosphere and to use glass which does not reduce, as lead glass will, it is quite possible to make entirely satisfactory seals between glass and clean copper. Apparently, a reasonably thin coating of oxide does no harm, whatever, to the seal. When hot, this oxide is black, but as the seal cools off, the cupric oxide changes to cuprous oxide and it is this latter oxide which gives rise to the crimson color of the seal.

Fig. 5 shows a microphotograph of a section of the seal between copper and glass. The copper may be recognized from its etched surface. The black line immediately next to the copper is a thin layer of cuprous oxide approximately 0.0003 in. in thickness (0.007 mm.) Immediately next to the thin layer of oxide will be seen a section of glass. This photograph is given to show the exceedingly close adhesion of the glass to the copper.

So far as we can find from consideration of the various factors entering the copper-lead glass seal, chemical reactions play a small and unimportant part. While certain reactions do occur, these are relatively slow and minute in comparison with the physical phenomena which make the seal possible.

Several interesting features of the ribbon seal are evident upon consideration. The copper ribbon within the glass, it will be remembered, is in tension in every direction. Heating the copper will thus initially reduce stresses because the copper will be expanded in the direction of the tension forces acting upon it. In other words, such a seal will not fail with sudden passage of current through the ribbon, because such heating as may occur in the ribbon actually lessens the stresses in the glass. A ribbon seal made from 0.008 in. sheet copper (0.4 mm.) and 7/16 in. wide (11 mm.) will continuously carry 40 amperes. It will, however, easily carry several hundred amperes for a short time. In fact it will carry enough current to make the ribbon visibly hot outside the seal. Again, various electrodes for use in vacuum apparatus may be formed from that portion of the ribbon within the enclosing chamber. Such an application may be seen in Fig. 6, showing the anode for a mercury rectifier tube made in one piece with the iron ribbon which passes through the glass member of the stem. Further applications may be easily imagined.

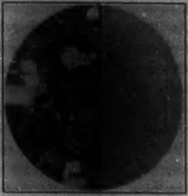
FIG. 5

FIG. 6

In the discussion of the ribbon seal it was assumed that the copper in Fig. 4 was stretched in the direction *A C* by the clamping effect of the glass upon the surfaces *A B* and *C D*. If this is the case, then it should be possible to construct a seal in which the glass comes only in contact with the copper ribbon at the edges of the ribbon and not along the center, that is, a seal such as that shown in Fig. 7 should be possible of construction. Next consider that Fig. 7 is a cross-section, not of a ribbon, but of a circular disk, taken at right angles to the plane of the disk along the diameter of the disk. Since, according to our assumption, glass does not touch the copper near the center of the disk, it makes no difference what form the glass assumes where it is not in contact with the copper; that is, it should be possible to seal a circular diaphragm into a glass tube in the manner shown in Fig. 8. It will be remembered that in the discussion of the ribbon seal, the edges of the ribbon were sharpened in order to prevent separation of the ribbon and glass. If the disk has square edges, then whatever separation occurs between glass and copper will do no damage provided separation does not extend to the inside of the tube. Further, since the separation in this case is not objectionable, we might just as well leave off the glass from the edge of the disk and apply glass only to the two faces. This is shown in Fig. 9. This then results in two pieces of glass tubing, one end of each being flared and the two flares melted to opposite faces of a single copper disk. As a matter of fact, such a seal is entirely possible. The final step in the development of the disk

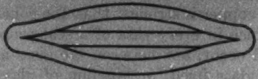
FIG. 7

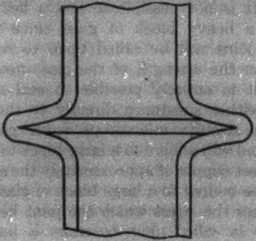
FIG. 8

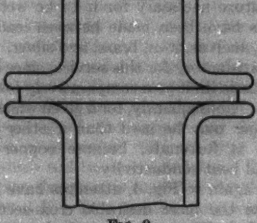
FIG. 9

seal is shown in Fig. 10. A round copper wire is passed through a centrally located hole in the disk and is soldered to the disk. Tubing on one face of the disk is almost entirely removed, leaving only a torus of glass in contact with one face. Fig. 11 shows such a disk seal. This type of seal has certain advantages over the ribbon seal. First, the electrical conductor is round and thus of a shape easily obtainable commercially. It is more rigid than a corresponding section of copper ribbon. The disk can easily be punched out on a punch press and requires no filing or machining.

In this seal the stresses are more simple than in the ribbon seal. The copper disk is stretched radially in all directions upon cooling. The glass is subjected to compression only without bending stresses such as occur in the ribbon seal. The disk contracts in thickness by an amount sufficient to compensate for the restricted radial contraction.

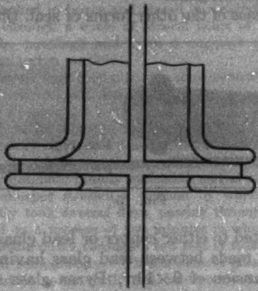

Fig. 10

Care must be taken to prevent glass on one side from running over the edge and making contact with glass on the opposite side of the disk, for if it should make contact, the seal will be broken, because the glass around the edge will not contract as far as the copper disk contracts in thickness.

Fig. 11

As in the ribbon seal, so also in this disk seal, the copper is under tension, though in two dimensions instead of in three. In consequence, when the disk is heated by passage of current through the central conductor, the copper when first expanded reduces the tension stresses so that the corresponding stress in the glass is actually lessened and passes through zero before any possible destructive stresses are applied to the glass by the copper. For example, a certain disk seal, made with a disk 1 3/16 in. diameter (3 cm.) and No. 6 B & S gage copper wire, (0.162 in. diameter — 4.1 mm.) normally carries a current of 90 amperes. Such a seal will carry any current which the copper wire is capable of conducting short of fusing the copper wire. Such a seal will carry 1200 amperes suddenly applied, though a current of 700 amperes will fuse the wire in air if applied for several minutes. As a matter of fact, the disk seal will remain tight even with the copper conductor operating at red heat. Thus this type of seal has the very desirable feature of withstanding heavy overload. It, of course, has the further desirable feature that it has the full conductivity of the given size of copper wire, there being no intermediate portion having a higher resistance.

There seems to be no reason why disk seals of any desired conductivity cannot be made if it is found necessary to make them. 3/8 in. diameter (10 mm.) of copper conductor, is as large as we have so far found it necessary to try out. For exceedingly heavy currents, still another type of seal is available, which will be referred to a little later.

The thickness of the disk is determined by the subsequent use to which the seal is to be put. For example, it is quite possible to seal a disk 1 in. in diameter (25.4 mm.) and 3/32 in. in thickness (2.5 mm.) between two flared glass tubes. However, such a seal once cooled to room temperature will not withstand subsequent heating and cooling. The explanation seems to be that even though the copper is thoroughly annealed when red hot with application of glass yet the subsequent stretching of the copper as it cools off, hardens the copper to an appreciable degree, thus raising the elastic limit and thus increasing the stress which the copper passes through the joint to the glass. Thus in proportioning a seal which is to withstand a great number of repeated cycles of heating and cooling, it is necessary to decrease the thickness of the disk until the maximum stress which the copper can pass to the glass is less than the ultimate strength of the glass. In the case of the 1 in. diameter (25.4 mm.) disk, a convenient thickness is 0.020 in. (0.5 mm.) or 0.030 in. (0.75 mm.).

Glass may be sealed to opposite faces of a copper cent and if carefully annealed, the cent may be made a great circle of a glass sphere.

Theoretical calculations of the relation between thickness and diameter of a disk indicate dimensions of the same order of magnitude as those found by experiment, although here again, as in the case of the ribbon seal, the elastic constants of the glass and copper are not definitely known, nor is the temperature at which the glass may be assumed to solidify a definite quantity.

There are thus, three methods by which copper may be sealed through glass for use as electrical conductors; first, the borax-coated, flattened copper wire; second, the ribbon seal, and third, the disk seal.

The function of the borax in the case of the flattened wire seems to be to provide material next to the copper, which by its low melting point decreases the temperature difference over which the copper is stretched upon cooling. For example, on a certain flattened seal wire with borax the thickness of the flat was 0.0015 in. (0.037 mm.) and without borax it was found necessary to decrease the thickness to 0.0009 in. (0.023 mm.). It is necessary only on the smaller seals where the copper should be as strong mechanically as possible. For ribbon seals and disk seals, borax is not used, the dimensions of the copper being sufficiently large to withstand ordinary handling.

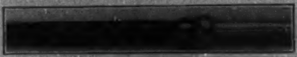
Fig. 12

With the success attending the development of the ribbon and disk seals, further experiments were made to determine the possibility of joining metal tubing to glass tubing. For example, a ½ in. diameter (13 mm.) copper tube was spun outwardly in a flange at one end and the resulting flange was sealed to glass tube in a manner similar to that employed in making disk seals. The over-all diameter of such a seal, however, is of course considerably larger than the diameter of the copper tube. Consequently the experiment shown in Fig. 12 was tried and found to be practicable. In this case a copper tube is machined at one end to provide a thin wall of copper. Glass is melted to the outside and joined to a glass tube of suitable size. Here, again, the copper is under tension as the seal cools, yet the adhesion of the glass to the copper is sufficient to stretch the copper as it cools. Further experiments showed that if the glass accidentally ran across the end of the tube, the seal failed at that point because the glass in this case impeded the contraction in thickness of the copper tube. To avoid this difficulty, the expedient used on the ribbon seal was adopted and the edge of the copper tube machined to a knife edge. Then should any glass pass over the edge, failure of the seal would not occur.

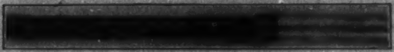
Fig. 13

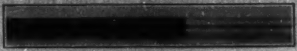
Fig. 14

Fig. 13 shows a seal of this type in which the copper tube has been reduced to a knife edge at the open end. Here the glass is applied outside of the copper and in this form is known as an external seal. Fig. 14 shows glass tubing applied to the inside of the same sort of copper tubing. This is known as an internal seal. It has been found that there is a difference in the behavior of internal and external seals made with the same dimensions of the copper tube. An internal seal will resist sudden heating much better than an external seal, while an external seal will resist sudden cooling much better than an internal seal. So far, there seems to be no limit to the size of copper tubing which can be joined to glass tubing.

Fig. 15 shows a seal between a 3½ in. diameter (9 cm.) copper tube and a 5 in. (12.8 cm.) diameter glass tube. In this seal the copper tubing is materially reduced in thickness over that portion which comes in contact with the glass.

As in the case of the other forms of seal, the tube seal is not restricted to either copper or lead glass. Fig. 16 shows a seal made between lead glass having a coefficient of expansion of 9×10^{-6}, Pyrex glass with a coefficient of expansion of 3×10^{-6}, and copper with a coefficient of expansion of 17×10^{-6}. In this case a copper tube is sealed to a Pyrex glass tube at one end and to a lead glass tube at the opposite end.

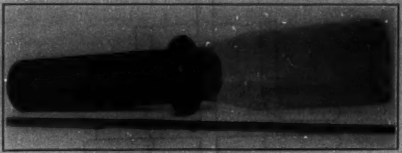
Fig. 15

If it should ever be necessary to seal exceedingly heavy copper conductors through glass, such, for example, as a copper shaft 2 in. or 3 in. in diameter, the easy way to accomplish this would be to select a copper tube slightly flared at one end, which would slide easily over the copper shaft. To the flared end of the copper tube the glass could be sealed and then after the seal was made, the copper tube could be brazed or welded to the copper shaft.

Fig. 16

In conclusion, there does not seem to be any limit to the size of the seals which can be made between metal and glass, so long as the parts are properly proportioned. The practical limit is reached in the laboratory, due to the weight of the parts to be handled and the necessity for keeping the parts in the proper relative relation. This, however, is purely a matter of design of mechanical means for so supporting the weights that they may be moved into proper relative positions.

It has thus been shown that where the parts are properly proportioned the difference in coefficients of expansion of the metal and the glass may be made of no effect upon the strength of the resulting seals.

Discussion

THE QUALITY OF INCANDESCENT LAMPS

(Howell and Schroeder) and

THE ART OF SEALING BASE METALS THROUGH GLASS

(Houskeeper), Swampscott, Mass., June 29, 1923

C. F. Scott: About fifteen years ago the tungsten lamp was evolving quite rapidly. Mr. Houskeeper and I were associated in those days in the Westinghouse Lamp factory, trying to help along the progress of the lamp.

A common method of making the tungsten lamp then before the days of the present long wire was the mixing of fine powdered tungsten with carbon and some kind of glue and then squirting it through a diamond die and taking that soft thread catching it on a piece of paper which was moved back and forth, and then cutting the pieces into almost hairpin shaped filaments. These were then packed in boxes and placed in a carbonizing furnace. They went through a cycle of several hours of carbonizing to burn out the mucilage and leave a compact combination of carbon and tungsten particles.

These hairpins were then placed in some neutral gas, a mixture made of hydrogen and nitrogen, and the current was passed through the filaments, which burned out the carbon and sifted together the little tungsten particles forming a little tungsten hairpin, not very rigid, and a half dozen of these hairpins, more or less were assembled together in a lamp, each of these hairpins as an individual, individually measured, and so on.

Mr. Houskeeper devised a method by which that process which usually took several days, passing through many hands, was reduced to a process of five minutes in which the squirted filament passing down through a few feet of tubing was carbonized by high temperature. Then a current passed through the filament and it came forth this continuous so-called wire of tungsten, which might be a thousand feet long with a uniformity which was otherwise, previously impossible. That led to a wire formed lamp, one filament wound continuously as they are wound now. I think Mr. Houskeeper's name would have been widely known in connection with that process which would have been a very important thing if the rate of progress had been less rapid. The wired lamp of the General Electric came at about the same time and was commercially developed at the same time that this was being developed, so that this stage of the tungsten filament lamp was shortlived on account of the progress.

Another important problem in this old incandescent lamp field has been the problem of getting the current from the outside to the inside of the enclosed lamp or bulb. It seems to me that many of the research processes which Houskeeper used in that earlier work have characterized this next development. There is no phenomenally new discovery. He takes the metals, the common metals, not platinum or some other particular thing, but the common metals and any kind of glass apparently, and by a little physical study of the way of combining, and the form, and the strength, and coefficient of expansion, he forms a new combination which is remarkable. The ability to take the things that we are familiar with, and get some new and almost startling outcome is to me as remarkable or possibly more remarkable than discoveries in new fields.

H. Lemp: In 1885 while connected with the Schuyler Electric Light Company of Hartford, Conn. Mr. Merle J. Wightman and myself were engaged in developing what we called the "Series Incandescent Lamp" for arc light circuits. The plan used before the advent of the series incandescent lamp was to use a number of the regular 100-volt lamps in multiple to absorb the current used in connection with arcs, and we then conceived the idea that it would be much better if a lamp could be made with a filament large enough to take the whole ten amperes of a commercial arc light circuit and be connected in series with arc lamps, instead of in groups of lamps in multiple requiring complicated compensating devices.

As all the early incandescent lamps were limited to something like a one-ampere current, to go from one ampere to ten amperes at that time required some research work as to the manner of passing these relatively big currents through the glass sealing, and if one cares to look at past records one will find an early patent to Mr. Wightman and myself for the use of a platinum ribbon in place of the platinum wire, then commercially used for leading in conductors for incandescent lamps.

We thought that if we could make the conductor in such a form that its surface of contact with the glass would be very large in proportion to its thickness, any heat developed would be more easily absorbed by the glass without cracking and the relative expansion by heat of the metal and glass would be less, and so we started out to make these platinum ribbons to conduct the current through the seal for these series incandescent lamps. Many hundreds of these lamps were commercially exploited.

When we found that we were successful in that attempt we grew bolder. We said "Now, if the platinum in the sheet form will carry those heavier currents and not crack the glass seal, why not take some other metals, iron and copper," and then we tried thin copper ribbons and sealed them in the manner shown by Mr. Houskeeper this morning, without however thinning the edge, and we were very much astonished to see that the seals did not crack; they seemed all right, but after a while the air leaked in and destroyed the vacuum.

Mr. Houskeeper has told me this morning why we failed and I want to pay my tribute to him for this lesson.

I think that the method of sealing he has shown us today is a great step ahead in the art, particularly as it will enable us to manufacture larger electron tubes for the purpose of rectifying alternating currents of magnitude.

C. H. Sharp: With reference to the paper of Messrs. Howell and Schroeder I wish to call attention to one statement which I think should be added, and that is that 500-hour test criteria apply to, I believe, the Lamp Works and not necessarily to other lamp testing organizations which may use a different basic life value for determining the efficiency at which lamps are to be operated on life test.

Mr. Howell has been connected with the manufacture of incandescent lamps longer than almost any other man in the world. He has a familiarity with the history of lamp manufacture, with the romantic history of that art which few, if any, other men possess. It would be very desirable if we could have a record of Mr. Howell's knowledge and recollection of the circumstances of the evolution of the lamp; of the troubles, of the successes which the incandescent lamp manufacturers have experienced in the course of all the years of the development of this art.

A. L. Atherton: The very important work that Mr. Houskeeper has described opens up many possibilities. In thinking over the possibilities, a question comes up which I would like to ask. Some applications for this type of seal will doubtless require materials other than copper on account of chemical limitations. In trying to bond other metals to glass there has been some experience with what seems like an electrolytic action by which gas bubbles are evolved at the contacts. This is particularly true with metals like nickel which have other advantages over copper. I would like to ask Mr. Houskeeper if this sort of thing has been encountered in the work which he has referred to with metals other than copper, and if some preventive methods have been devised.

W. G. Houskeeper: In answer to the last question as to the sealing of other metals than copper into glass, if I understood the question correctly it was a question of the evolution of gas bubbles between the metal and the glass. That can be very satisfactorily taken care of by giving the metal a heating in vacuum before the glass is applied. If sheet nickel, for example, be maintained at approximately 800 deg. cent. for ten or twenty minutes in vacuum you can apply the glass to the surface any time in several weeks without bubbles between the glass and the nickel.

Transmission Line Transients

BY V. BUSH
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***Review of the Subject.**—There is great need for a better understanding on the part of transmission engineers of the traveling waves on lines. The complete analysis of such waves is inherently complicated, but there are certain approximate relations by which the progress of waves on our ordinary systems may be readily studied. Before any theory is accepted for practical use, however, it is necessary to check it carefully by experiment. This paper gives quantitative checks of the principal aspects of the theory of the traveling waves due to suddenly applied continuous potentials, as originally given by Heaviside.*

It is shown that this theory based on fixed line constants predicts accurately the progress of waves as far as magnitudes, speed of propagation, and shape of wave except at the wave front are concerned. The effect of steepness of wave front is discussed with relation to its practical aspects. It is proposed to follow this paper shortly with similar results on alternating-current transients.

Tests on artificial lines and on an aerial line in the field are compared with computed results. Working rules for considering the progress of transients are discussed, together with certain results that may be obtained by their use. An extension of approximate analysis is given in an appendix, which is sufficiently accurate for transmission lines of the present length, and which is sometimes easier to handle than the complete theory.

Reflections at apparatus, and at line junctures, is discussed qualitatively. There are several cases in which extreme potentials may arise. The most common case is when a cable and aerial line are used in conjunction. Another case occurs upon the sudden interruption of heavy current. The superposition of transients in a polyphase circuit, as affected by the sequence of closing of the switch points appears also to involve the possibility of difficulty.

The paper is non-mathematical. It attempts to gather together existing theory in simple form, rather than to extend analysis.

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Reflection and Refraction at Junctions. (275 w.)
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Effects in Polyphase Circuits. (375 w.)
Appendix. (1800 w.)

THE purpose of this paper is to present briefly the results of an investigation on transmission line transients recently performed in the Department of Electrical Engineering, Mass. Inst. of Tech., by Messrs. S. M. Jones, J. A. Scott, B. Van Ness, and D. C. Jackson, Jr., graduate students in the department. The object of the research was to check experimentally part of the theory of transients on transmission lines and cables, and to investigate qualitatively certain phenomena of reflection and wave form.

Qualitative experimental work has often been carried on along these lines. Quantitative work is attended with considerable difficulty in manipulation.

It is however, of great importance to our progress in the extension and operation of large systems that the theory of their electrical behavior be carefully checked in not only qualitative but also quantitative manner, in order that reliance upon the theory may be justified. The development of transient theory is particularly dependent upon the introduction of assumptions and approximations. The only final test of such theory is then a careful comparison with practise.

It is not ordinarily possible, for many reasons, to carry on accurate transient measurements on the actual transmission line. The use of artificial lines is hence imperative. The theory as so far developed is an approximation when applied to either one or the other. It is necessary that it be shown to give correct results to engineering accuracy on both.

The present research was made possible by the development of a smooth artificial transmission line which is described in an accompanying paper by Prof. F. S. Dellenbaugh, Jr. Laboratory work was also supplemented by tests in the field on the Georgia-Alabama tie line, performed during the past summer by Prof. Dellenbaugh and Mr. Jones. This line was made available through the courtesy of the Georgia Ry. & Power Co. and the Alabama Power Co.

In presenting the results, some of the simpler transient theory is briefly reviewed. Certain generalizations are also drawn which it is believed will be of service to transmission engineers. Mathematics has been avoided in this paper in the hope that it may the better serve those who hesitate to attempt the careful reading of a mathematical paper. Plenty of mathematical analysis will be found in the references.

There have been several classifications of circuit transients, for example those of Steinmetz and Kennelly. For the present purpose it is convenient to divide them into two classes, which may be called traveling wave transients and machine transients. The first merges into the second. When a transmission network is, disturbed in any way, for example by a short circuit, events in the first instant are determined by the progress of traveling waves, which proceed over the circuit and are reflected and refracted at junctions and apparatus. This period is immediately followed by a second one during which the procedure is determined largely by the electrical and mechanical characteristics of the machines connected to the network. The first transient is caused by the readjustment of the energy stored in the lines, and the second by the readjustment of the

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Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

energy stored in the machines. The first is over in an interval usually so short that the second has hardly begun before it is over. Hence the two types may be considered separately without serious error. This investigation has to do only with the first type of transient, characterized by propagation of traveling waves of voltage and current.

All tests were made by suddenly applying or removing steady or continuous potentials. The results however, apply to most cases of the application of alternating potentials, for ordinarily the initial waves of voltage and current have become much attenuated before great change has occurred in an applied potential of commercial frequency. Thus if the line is 30 miles (49 km.) long, waves have time to travel over the system 10 times before an applied 60-cycle voltage has varied through one tenth cycle, and the most interesting effects are usually produced during the first one or two passages of the wave. The application of a polyphase voltage may be considered as the separate application of single-phase voltages, and the results added to obtain the total transient. It is, of course, necessary to include in the summary those waves which are induced in one wire by a wave in a parallel wire. This summation cannot be strictly carried out where the effects of iron, corona, etc. render the equations of the circuit nonlinear. Most of the polyphase transients of practise may, however, be qualitatively studied by means of the rules governing single-phase, continuous potential transients.

The transients of practise are often rendered seriously complicated by the presence of arcs, of which the resistance is not a constant. Progress even along these lines can, however, be expedited by examining the laws of simple cases.

SIMPLE WAVES AND REFLECTIONS

When a switch is closed, applying a potential E between a pair of wires of a transmission line or between one wire and ground, a wave of voltage of height E proceeds out along the line. Except for the effects of dissipation, due to resistance and leakance, this wave proceeds without attenuation or distortion, and at the speed of light, 3×10^5 km./sec. This means simply that the potential appears suddenly at points out on the line, the distance to which it has appeared at an instant t seconds after closing the switch being $v\,t$, where v is the velocity of light. Beyond this distance there is at this instant no effect whatever. This is called a rectangular wave, and may have a nearly perpendicular wave front. Such a wave is indicated as shown in Fig. 1.

A voltage wave of this sort is always accompanied by a wave of current of height $I = E\sqrt{\frac{c}{l}}$, where c and l are the capacity and inductance per unit length of the line. The current is directed out along one wire, and back along the other wire or ground, and similarly appears suddenly at distant points on the line.

On reaching the end of the line, these waves are reflected. If the end is open or short-circuited, reflection is complete. Reflection may be either with or without change of sign. The natures of the voltage and current wave reflections are always opposite; that is, if the voltage is reflected with change of sign, the current will be reflected without change of sign, and vice versa.

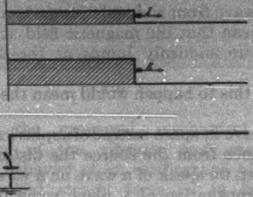

FIG. 1

The progress of the voltage and current waves over a simple line may thus be mapped out, neglecting the effects of losses. In Fig. 2 is shown the progress on a line open at the distant end when a voltage is applied suddenly at the near end. The successive pictures are drawn for fractions of a period T which is twice the time necessary for a trip of the wave down the line and back. The nature of the reflections may readily be determined from the fact that the current at the open end must always be zero.

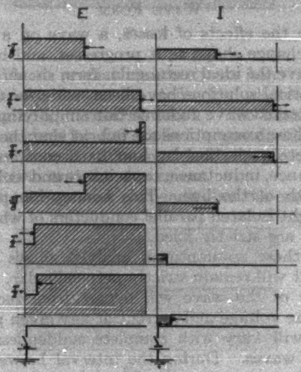

FIG. 2

From such a diagram we may readily construct the diagram showing the time variation of voltage and current for any point on the line. Thus Fig. 3 shows the variations at the midpoint of the line for the above example.

Strictly, when a switch is closed a spherical wave proceeds off into space, spreading out as do the waves from a radio antenna. The diagrams above show the effect at one part only of the field, namely at the wire. It is the effect at the wire in which we are usually interested. Such a diagram can, nevertheless, not tell the whole story of what happens, and it is well to bear this in mind. For example, the fact that the current on the wire at the wave front suddenly jumps to a value I does not at all mean that the magnetic field at a distance r from the wire suddenly jumps at the same instant to $\frac{2I}{r}$. For this to happen would mean the propagation of a plane wave in space instead of a spherical wave. At a long distance from the source the distinction disappears. When we speak of a wave on a wire, we usually mean the propagation of a plane wave in space and guided by the wire.

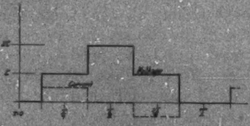

Fig. 3

Wave Form

Due to the effects of losses, a wave on a line will actually change shape as it progresses, and hence will not preserve the ideal rectangular form shown in Fig. 1. Mathematical solutions have been made for the shape of the distorted wave under certain simplifying assumptions. These assumptions are in brief that the line may be completely specified by four fixed constants $r\ l\ c\ g$, the resistance, inductance, capacitance and leakance per unit length of the line. This assumption postulates two straight isolated parallel conductors of which these constants are strictly fixed.

Under these assumptions, analysis shows that the wave front will remain strictly perpendicular, although the shape of the wave will depart from rectangular form. The voltage and current at any given point on the line will vary with complete suddenness on the arrival of waves. During the interval between wave advents the current or voltage at the point will vary slowly, due both to the attenuation of the wave, and its distortion in shape.

Heaviside first attacked this problem in a comprehensive manner. Some of the mathematical processes he used were lacking in rigor, but they are rapidly being put upon a firm foundation and extended by Carson[1] and a few others. A great deal has been written on the subject since the time of Maxwell and Hertz, but it is in general in such form as to be interpreted only with great difficulty for the cases in which the transmission engineer is interested.[2] Unfortunately the expressions which result from the analysis of traveling waves are complicated. Apparently this cannot be helped, except by the possible introduction of graphical methods, for the phenomena themselves are inherently complicated.

Experimental Checks on Theory

In Fig. 4 is shown the current entering a smooth artificial line with the distant end short-circuited, and a steady voltage suddenly applied at the home end. The oscillogram includes a 1000-cycle timing wave for convenience in measurement.

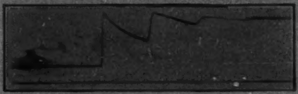

Fig. 4

The artificial line used is one of those described by Dellenbaugh, and is shown in Fig. 5. It consists of 126 coils, having the capacity uniformly distributed by means of tinfoil and paper inserted between layers of wire. It represents 990 miles (1612 km.) of two No. 00 copper conductors, spaced 8 ft. 9 in. (267 cm.). This extreme length was used to better bring out in the oscillograms the wave forms of current and voltage. It will be noted, for example, in the oscillogram that, due to the very long length of line, the final steady current is less than the initial surge.

Fig. 5

The transients were computed principally from Heaviside's[3] and Carson's formulas, with Mr. Carson's much appreciated assistance in interpretation of difficult points in connection with some of the later work.

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[1] J. R. Carson. Theory of the Transient Oscillations of Electrical Networks and Transmission Systems. Trans. A. I. E. E., Vol. XXXVIII, p. 345.

[2] Consult for example H. Poincare, L'Eclairage Electrique, Vol. XL, 1904, p. 120.

[3] Heaviside, Electromagnetic Theory, Vol. II, pp. 201, 378.

In Fig. 6 is plotted the computed curve, by the full line, for the above case. On this same diagram is plotted, dotted, a copy of the oscillogram of Fig. 4. The correspondence is seen to be excellent except immediately at the wave front. Reasons for the departure of theory from the observed form at the wave front will be discussed below. Several other oscillograms for various terminal conditions were also computed, the agreement being in general satisfactory.

These measurements may be considered a check upon theory as far as the amount and nature of reflections and the wave shape except at the wave front are concerned. With this confirmation of analysis we are justified in proceeding by the same methods to more complicated cases. We should, however, give particular consideration to the departure from expected results at the wave front.

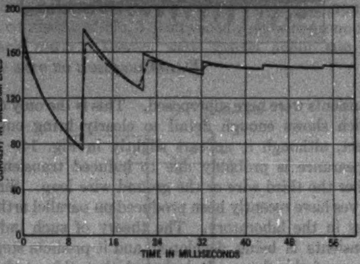

Fig. 6

Oscillograph Data

Possible oscillograph errors were studied.

We have, due to Kennelly, a treatment of the correction factors for an oscillograph vibrator of measured constants, which may be applied for oscillograms taken of steady state phenomena. These will show the errors present at each frequency of harmonic. There is no similar method available by which corrections may be applied to oscillograms of transients except for perpendicular changes. The best that can be done in other cases is to estimate the probable deviations.

Fig. 7

The overshoot and lack of perpendicularity of the oscillogram were studied for cases where a voltage was applied directly from a battery, and known to be "rectangular". This was done for various vibrators and damping fluids. A typical curve is shown in Fig. 7. This was taken at the same drum speed as used for the other oscillograms. The approximate amount of overshoot when present could then be subtracted from a perpendicular rise in the curves to be analyzed. It will be noted that the vibrator follows even a perpendicular rise with very little deviation. This deviation in the case of a perpendicular rise of current can also be calculated from the measured mechanical frequency and damping of the vibrator and shown to be small. The behavior of an oscillograph vibrator to a suddenly applied steady force is, in fact, exactly analogous to the behavior of a series resonant circuit under suddenly applied constant e. m. f. Having measured by Kennelly's methods the vibrator constants, it is possible to predict accurately its behavior under such a sudden force. Its deflection plotted against time will give a damped sinusoid. The initial rate of deflection is given by the applied torque divided by the moment of inertia. The overshoot is given by the final deflection and the decrement.

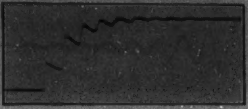

Fig. 8

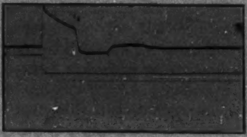

Fig. 9

Since the errors in a perpendicular rise were small, it was assumed that oscillograph errors in other parts of the curves could be neglected.

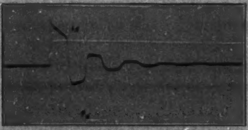

Fig. 10

Shorter Lines

Fig. 8 shows a curve taken with an artificial line 330 miles (537 km.) long. It is similar to the preceding except that reflections come at more frequent intervals, and the final steady current builds up to a higher value.

The oscillogram of Fig. 9 gives the home end entering current for a suddenly applied constant voltage when the distant end of the 990-mile (1612 km.) line is open. In this case the current reverses in sign, and flows alternately into and out of the line, subsiding to a steady value of zero current since the leakage is negligible. Fig. 10 is for a line 330 miles (537 km.) long under the same conditions.

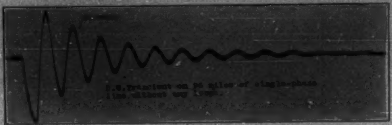

Fig. 11

Finally in Fig. 11 is shown the current for an artificial line only 96 miles (156 km.) long with the distant end open. Here the reflections follow so rapidly that the curvature of wave front soon masks the theoretically sharp cornered form.

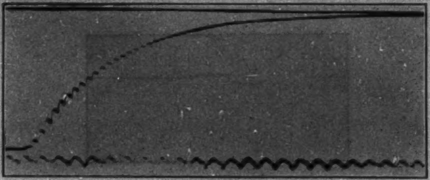

Fig. 12

Field Tests

Oscillograms for comparison taken on the Gadsden-Lindale tie line are shown in Figs. 12, 13 and 14. The first was taken with the distant end grounded, and the others with it open.

The method of taking these curves was the same as

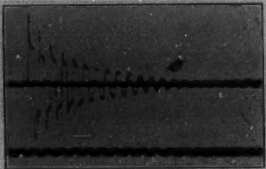

Fig. 13

that used on the artificial line. A low value of applied e.m.f. was used, and the oscillograph connected directly in the line. The voltage was applied between two wires of the three-phase power line. The line constants were determined from separate a-c. measurements.

Computations again showed these curves to agree with theory as well as could be expected in view of the experimental difficulties involved; Fig. 15 shows a computed curve superposed on the beginning of one of the enlarged oscillograms.

Several points of interest may be noted. One is the shape of wave front, which here again appears rounded and masks the rectangular appearance given by simple theory. Another point is the slight indication of stepped wave front appearing in Fig. 13. Apparently two transients were here superposed. This is the only curve which shows enough detail to clearly bring out the effect, although it appears slightly in Fig. 14. This appearance is probably due to induced transients in either the third wire or the ground wire loop. Similar curves have recently been produced on parallel artificial lines in the laboratory. The theory of such induced transients is being developed, and it predicts stepped waves of this form. It is hoped that this work will warrant later publication.

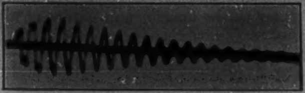

Fig. 14

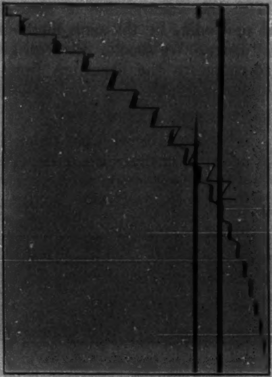

Fig. 15

In Fig. 14 the initial amplitudes are abnormally low. The experiment was repeated many times in the attempt to locate the cause of this departure from theory, and only one case was obtained in which the effect was absent, which is the curve of Fig. 13. Various expedients were tried, such as using a storage battery instead of a generator to supply the constant potential, applying a large capacity across the source of voltage and with its center point grounded, and so on. Curves of applied voltage taken simultaneously indicate that the initial low value is not due to terminal apparatus, for the drop of applied voltage on closing the switch is very slight. Different pairs of line conductors were tried, but always with the same result. There was apparently some disturbing effect in the system usually present. It may be noted that the second wave being larger than the first represents net energy supplied from the line to the battery.

The line on which these curves were taken consisted of three No. 3/0 conductors flat spaced 14 ft. (427 cm.). The part of the line used was 37 miles (60 km.) long. Located as shown in Fig. 16 were two overhead ground wires consisting of No. 3/0 stranded steel wire grounded at each tower. Towers were spaced about 600 ft. There were no transpositions.

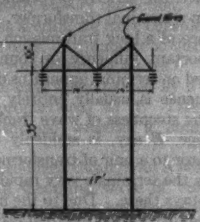

Fig. 16

Line Constants

Neither the artificial line nor the actual line of practise corresponds exactly to the ideal line which is mathematically analyzed. This ideal line is considered to have four constants which are absolutely fixed, and which are the same at every point of the line. Hence it is to be expected that there will be a departure of experimental results from those indicated by this basic theory. No case in which these simplifying assumptions are not made has ever been completely analyzed mathematically.

Our experimental results indicate, except for the peculiar initial departure noted above, that the actual progress of waves in practise follows that predicted by theory except at the wave front. Simple theory indicates wave fronts that diminish in height but remain strictly perpendicular. The wave fronts of practise are rounded off.

The physical aerial line departs from the simple case assumed in analysis in the following particulars:

1. *Skin Effect.* This is better designated as the imperfect penetration of current into the wire at the wave front. Due to this imperfect penetration the front of the wave meets a resistance greater than the continuous current resistance of the conductor. This effect undoubtedly changes the wave front slightly.

2. *Ground Effect.* This includes the effect of the ground, the ground wires, parallel lines, or other neighboring structures. In each of these there will be induced accompanying transients, due both to electrostatic and electromagnetic induction. But little is now known of the extent of the modification introduced by these factors. The effect of a short-circuited ground wire loop is probably worth considerable study. The presence of the ground offers in practise the possibility of transients between wire and ground as well as between wires. These can only occur if the terminal apparatus is grounded, or contains sufficient capacity to ground. The constants which apply for such a transient are doubtful on account of the imperfect conductivity of the ground.

3. *Radiation from the Line.* This factor has recently received considerable attention. A paper by Manneback[4] shows its effect on single waves to be negligible to engineering accuracy.

4. *Periodicity in the Line.* This is due to several causes such as the sag of the line wires, the lumping of leakance and, to a small extent, capacity, at the towers, etc. This probably gives the line a slight filter effect and tends to remove high frequencies from the wave front.

5. *Corona.* The height of voltage waves is probably greatly affected in practise by corona. This is of much importance practically, for the high-voltage line is thereby relieved of some of the grief due to traveling waves. The effect of corona on transients has not as yet been investigated. An experimental study of voltage and harmonic distribution on a line due to corona was recently made at the Institute, but has not yet been reported upon. This work was carried on by means of artificial corona leaks applied to an artificial transmission line in the steady state. These leaks were built of thermionic tubes or triodes so controlled as to pass current in a manner similar to that passed by corona on an aerial line. It is thought possible to throw some light on the effect of corona on transients by using these leaks in a transient investigation.

For transients on power cables there is an additional modifying factor:

6. *Effect of Solid Dielectric.* The loss in the dielectric appears as a leakance on the line. Assuming a constant phase angle of dielectric, the corresponding leakance is proportional to the frequency, and not a constant, This again tends to remove high frequencies from the wave front.

The artificial line used in the tests is practically free from factors 1, 2, 3 and 5. It includes factor 4, periodicity, in a somewhat different manner from, and to a greater extent than the real line. The capacity and resistance of the artificial line are strictly distributed. Its inductance is to a certain extent lumped in sections due to the mutual inductance between turns. This introduces a filter action, the amount of which is now being studied. It also undoubtedly results in a higher speed of propagation of the extreme front of the wave than would be found on the line being represented. Both effects influence principally the steepness of wave front.

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[4] Chas. Manneback. Radiation from Transmission Lines. Jour. A. I. E. E., Feb. 1923.

The artificial line also includes factor 6, and to about the correct extent to simulate cable action. The leakance of both real and artificial lines is negligible except for the effect of loss in the dielectric. This again modifies wave front.

It is hence not yet possible for the artificial line to yield complete information regarding the steepness of wave front to be found on physical lines. In other respects it gives reasonably accurate duplication, and its use will probably increase for the study of all kinds of transients upon networks which are too complicated for mathematical analysis. It is of importance to inquire how important in practise is this matter of steepness of wave front, which cannot be either experimentally or theoretically determined at present.

WAVE FRONTS AND OVERVOLTAGES

During the system transient characterized by traveling waves we are interested primarily in the abnormal voltages which may appear at various parts of the network, their duration and the number of times they are repeated. Later, during the second class of transient, when machine constants enter as a controlling factor, we are principally interested in abnormal currents, and in the mechanical stresses and the switch duty thereby imposed. During the first part of the disturbance following a switching operation, however, we are interested in waves of current only in so far as they may result in abnormal voltages.

It is this first transient that we are here studying. It is ordinarily over long before a relay or a switch can act. Mechanical stresses due to high currents which might appear in this brief interval are also of such short duration that they cannot produce mechanical breakdown with the usual amount of inertia present. The only factor capable of causing damage during the brief interval of the first transient is an overvoltage. We are interested in traveling waves only in so far as they may result in high-voltage stresses, long maintained, or often repeated.

Much has been written about steepness of wave front, and an exaggerated idea of its importance has as a result been popularly produced. We need consider the steepness of a wave only as it affects the voltages produced by the wave. Steepness of wave front even of a current wave can in itself produce no harm.

The transformer is the usual terminal apparatus on physical lines. This is a system with decidedly distributed constants, containing capacity both between turns and to ground. The impact of an incoming wave of voltage over a line upon a terminal transformer is in the first instant exactly equivalent to the sudden application to the terminal or terminals by any other means of the same voltage as that produced by the incoming wave. By the first instant, in this connection, is meant the time during which the transformer transient is determined by its capacity between coils and to ground, and before appreciable current has begun to flow in the windings. It is during this period that the maximum potentials ordinarily occur. It makes little difference in the effect produced whether the voltage thus suddenly applied grows to its maximum value in one or one hundred microseconds. The worst case that can happen, for a given maximum voltage, is to apply it directly and suddenly to the transformer terminals without an intervening line. The effect of the line on the wave front can be only in one direction, namely to decrease slightly the suddenness of application. As far as voltage waves are concerned, therefore, we have nothing to fear on account of steepness even when the strictly perpendicular wave front impinges upon a terminal transformer.

The presence of a long transmission line may indeed have an important influence in connection with the effect produced on terminal apparatus during switching, but this influence is usually entirely apart from any question of the steepness of wave front. When a continuous voltage, E say, is suddenly applied directly from a generator to a pair of transformer terminals, the voltage across the terminals very rapidly arrives at the value E and stays there. Due to the inductance of the source and the distributed electrostatic capacity of the transformer, the terminal voltage will, in fact, rise instantaneously to nearly double, and then oscillate about its final value E. The duration of this oscillation is fortunately exceedingly brief, as it is determined by the natural period of a circuit consisting substantially of the inductance of the leads in series with the distributed capacity of the transformer and of the source. Hence in switching a transformer onto a generator, we do not ordinarily have to consider, as far as insulation is concerned, that there is any overvoltage produced at all. It takes time as well as excess voltage to produce a breakdown.

If now, however, a long transmission line be introduced between the source and the terminal transformer, a new condition must be considered. Upon closing the switch a wave of voltage of height E runs down the line. Upon arriving slightly attentuated at the transformer, this wave is reflected in the first instant nearly as though the end of the line were open, for at first the transformer inductance prevents it from drawing appreciable current. The voltage wave is thus reflected and nearly doubled. This doubled voltage then persists across the transformer until the arrival of the next wave, or until the flow of current into the transformer drags it down. We thus have across the transformer terminals an overvoltage which can last for a considerable period, and hence may be capable of producing breakdown. The presence of the line, while it has been able to affect the amount of voltage produced or the suddenness of its application only in a favorable direction, has nevertheless introduced a longer duration of the overvoltage which may be serious.

The distribution of potential in a transformer during the interval after the sudden application of a terminal voltage is determined by the distribution of capacity, much as is the case with a string of suspension insulators. End coils will hence receive more than a proportional share of the overvoltage. This is the reason for wider spacing on transformer end turns. When current begins to flow in the windings a redistribution of voltage occurs, as well as a decrease in the voltage at the terminals. This matter has been treated by Blume and Boyajian.[5]

A wave does not progress into the transformer in any such manner as it progresses along the line, for the mutual inductance and capacity both between coils and to ground profoundly modify the action.

Thus far in this section we have considered incoming waves of voltage only. It remains to discuss the effect of the accompanying incoming waves of current, for a voltage wave is always accompanied by a current wave approximately proportional to it. This matter can, however, be dismissed without a detailed separate investigation.

The voltages produced throughout a piece of terminal apparatus by an incoming wave of voltage may be completely analyzed from a knowledge of the voltage only. Any failure to compute the voltage produced at any point will be due to lack of analytical ability, and not to deficiency of premises. The system is completely fixed once the constants of the apparatus and of the line, and the amount of voltage, are given. The wave of voltage is accompanied by a wave of current. Can this wave of current, penetrating the apparatus, produce disastrous effects? Obviously it can produce only those voltages which have already been found from an analysis of the effect of the voltage wave. We are interested in overvoltages. Hence if we will completely treat the effect of an incoming wave of voltage, we can forget about the accompanying wave of current, knowing that we have completely taken it into account.

It follows, therefore, that the steep wave front of current is entirely innocuous, in any case where the sudden application of the corresponding voltage does no harm. This may seem strange, for the impact of a perpendicular wave of current on a piece of highly inductive terminal apparatus looks dangerous offhand; but reflection will show that the danger is very likely to be entirely visionary.

A perpendicular rise of current in a coil entirely devoid of distributed capacity would indeed result in an infinite voltage between turns. But such a condition can never exist, first, because we cannot build an inductance which is devoid of distributed capacity, and second, because even if we could, we could not cause an instantaneous rise of current in the windings by any means whatever. It is well to remember that the current can never change with entire suddenness by any finite amount of change, however small, in any finite length of conductor, however short, or however connected.

A perpendicular wave of current traveling down a line and meeting a terminal inductance with negligible distributed capacity will be entirely reflected in the first instant and not penetrate at all. The accompanying voltage wave will be completely reflected and doubled. After the production of this double voltage, the current will begin to flow in the coil, beginning at an initial rate of $2E/L$ amperes per second, where E is the height of the incoming voltage wave. As current flows, the terminal voltage will drop.

When the capacity between turns of the coil is considerable, and the capacity to ground negligible, an incoming wave of voltage will at first be reflected with change of sign, and the terminal voltage will be instantaneously zero. The current wave will be reflected and doubled. This is because the lumped capacity across the line acts in the first instant like a short circuit. The period during which it thus acts is exceedingly brief, for it soon charges to a potential $2E$. Thereafter reflection is as though it were not present, and the same effect is produced as in the preceding paragraph.

When capacity to ground is also considerable, reflection will be incomplete and of either sign, depending upon the relation of inductance and capacity.

An incoming voltage wave may thus be reflected at terminal apparatus in such manner as to practically double. The accompanying current wave can produce no greater voltage than this doubled voltage, but it may produce it in unexpected places. If, for example, an air core choke coil is inserted in the line, with very little distributed capacity to ground, then the advent of a steep wave may throw nearly doubled voltage across the turns of the coil. The effect is nearly the same in the first instant as though the line at the far end of the coil were short-circuited. The effect of the incoming waves can be analyzed much more easily in nearly every case by considering the effect of the voltage wave only. The current wave which accompanies it can produce no other voltages than those determined by this analysis.

It is not meant by this discussion to indicate that steepness of wave front can always be entirely ignored. Sometimes the lack of entire suddenness of voltage application is important. In most cases in practise, however, it may be considered to be only a modifying factor, the influence of a line upon which is always in the advantageous direction and increasing with length of line.

We may thus conclude that the principal factors to be considered in traveling waves are the magnitude and the duration of the voltage produced, and the portions of the apparatus which receive the overvoltage. In regard to these factors, experiment both upon the artificial line and the physical line checks reasonably well with elementary theory.

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[5] Trans. A. I. E. E., Vol. XXXVIII, p. 577.

ATTENUATION

A wave of voltage arriving at the open end of a line is entirely reflected without change of sign, and hence completely doubles. The end of the line experiences momentarily a maximum voltage of twice the value of the incoming wave. This maximum voltage is altered after the first instant by attenuation and change of wave form.

This is illustrated in Fig. 17, which is an oscillogram of voltage at the distant end of an open artificial line of length 330 miles (537 km.). The voltage here does not initially rise to twice its final steady value because of attenuation in this long length of line. This oscillogram, and others of the voltage on the line, were taken by means of the thermionic tube repeater attachment described by Dellenbaugh, in order to avoid disturbing the electrical condition of the line. For a shorter line the voltage would more nearly double. The attenuation factor is

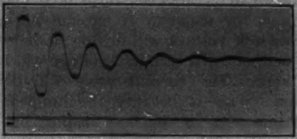

FIG. 17

$$E^{-\left(\frac{r}{2l}+\frac{g}{2c}\right)t}$$

For an aerial line g is usually negligible. In the case of the No. 00 conductor line spaced 8 ft. 9 in. which is represented by the artificial line here used, the ratio of r to l is 216, and hence the exponent α is 108. The time of propagation over 330 miles (537 km.) is approximately 0.00177 seconds. The duration of overvoltage at the receiving end is thus 0.0036 seconds. This may be checked by the timing wave on the oscillogram. The damping factor for one run over the line is

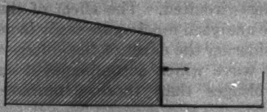

FIG. 18

$$\epsilon^{-108t} = \epsilon^{-0.191} = 0.826$$

In the oscillogram Fig. 17 we find by measurement that the voltage at the end rises instantaneously to approximately the value given by this attenuation factor, that is by $2 \times 0.826 \times E = 1.65\,E$, where E is the suddenly applied potential.

It would appear offhand that it should rise to a value much less than this because of distortion. A wave when running over a line assumes the shape shown in Fig. 18, even on the ideal line with fixed uniform constants. The decrease in height of wave front is given by the attenuation factor. In the appendix is derived an expression for this wave form which it is felt may have some utility on account of simplicity.

For a short line the attenuation factor is small. Hence on a short line the voltage at the open receiving end will reach very nearly double the applied voltage. The high voltage will of course persist for a much shorter period than on the longer line, but it will be more frequently repeated.

REFLECTION AND REFRACTION AT JUNCTIONS

When a wave meets a junction between lines of different constants it is partially reflected and partially refracted into the second line. If we write

$$\gamma = \sqrt{\frac{c}{l}}$$

and use subscripts for the two lines, the coefficients of reflection and refraction are

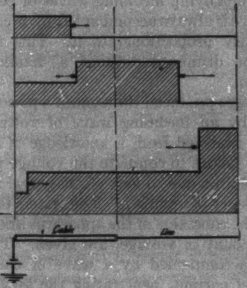

FIG. 19

$$\frac{\gamma_2}{\gamma_1 + \gamma_2} \text{ and } \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2}$$

The resistance of each line is assumed small, as will always be the case in practise. One is the coefficient of reflection and the other the coefficient of refraction. The coefficient of reflection for the voltage wave is the coefficient of refraction for the current wave, and vice versa. The voltage is always increased on passing into a line of smaller γ. These rules are sufficient to map out the progress of waves over junctions when attenuation and distortion are neglected.

When a power cable and aerial line are joined, the ratio of inductance to capacity is so widely different for the two parts that γ for the aerial line may be neglected to a first approximation compared to γ for the cable. Reflection and refraction are thus practically complete at the junction during the first instant. Neglecting attenuation and distortion the progress of waves over such junctions may hence be readily mapped. In Fig. 19 is shown the progress for an aerial line and cable in series, a voltage being suddenly applied to the cable, and the distant end of the line being open. For short lines where attenuation and distortion are small, it will be seen that several times normal voltage may be produced in this manner. Very little current penetrates into the aerial line, although of course the ratio between voltage and current waves in each line is always γ for that line.

REFLECTIONS AT APPARATUS

A wave impinging upon a lumped inductance devoid of distributed capacity will be in the first instant completely reflected.

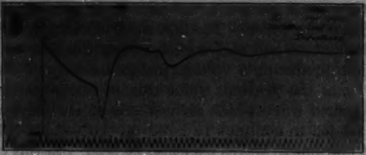

FIG. 20

The effect of the distributed capacity in the coil is to make this reflection in the first instant incomplete, the reflection coefficient being roughly that given above for junctions between lines.

When time has elapsed sufficient for the inductance to show appreciable current through its windings, it reflects less of the wave. Finally it may reflect with reversed sign if the line is long enough for this effect to have time to occur before the next wave arrives.

In the oscillogram of Fig. 20, taken on a line with an inductance load, this effect is clearly shown. The curve is of current at the home end. The initial surge of current is identical with that of Figs. 4 and 9, for until the arrival of the reflected wave back at the home end, the conditions at the distant end can exert no influence on the home end current. The first incoming reflected wave of Fig. 20 is comparable in the first instant with that of Fig. 9 where the distant end is open, but is somewhat less in height on account of distributed capacity in the terminal inductance. This wave, however rapidly reverses as the inductance begins to draw current, and before the arrival of the second wave has become of opposite sign, and nearly the same as the corresponding part of the wave of Fig. 4, differing from it also on account of the resistance of the terminal inductance. The second incoming wave repeats this effect with opposite sign and attenuated by an additional trip down the line and back. Fig. 21 shows the voltage produced across the inductance during this transient.

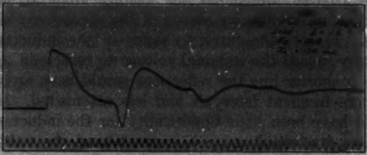

FIG. 21

Fig. 22 shows the corresponding behavior with a terminal condenser. Such a condenser acts in the first instant like a short circuit, so that the wave of current comes back without change of sign. This condition is reversed as the condenser accumulates a charge.

The relation of inductance to capacity in the terminal load may be such that no reflection at all occurs in the first instant. The same effect may be obtained by a terminal resistance equal to $1/\gamma$ of the line. This effect is shown in Fig. 23, which is an oscillogram of entering current under this condition. There is a slight remaining indication of a reflection, due probably to a capacity in the resistance used, and only approximate equality between the terminal resistance and $1/\gamma$ of the line.

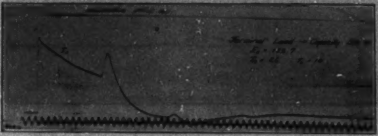

FIG. 22

OPENING OF CIRCUIT

When a line is opened, by a switch or by a wire breaking, the current is suddenly decreased at that point from the previous value I to zero. A wave of current, a depression in this case, then runs over the line and reduces the current to zero on the line as it goes. This wave of current is accompanied by a wave of voltage of height I/γ, which is superimposed upon the voltage previously existing on the line. The voltage appearing across the line just outside the break is hence in the first instant equal to $E - I/\gamma$, and may be large or small, positive or negative, according to the value of the current broken.

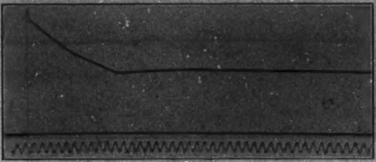

FIG. 23

This is shown in Fig. 24 and Fig. 25. In the former, taken on the full length of the line, and interrupting short-circuit current, the value of I/γ is less than E so that the voltage at the break remains positive, but nearly zero. In the latter, using one-third the length of line, the current is three times as great so that the voltage rises to nearly twice E negative. The voltage of the curves is the voltage between lines just outside the switch. It was measured by the thermionic tube attachment, for otherwise the line conditions would have been much affected by the presence of the oscillograph.

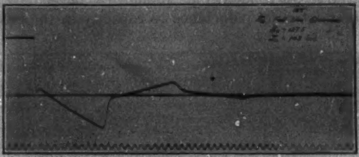

Fig. 24

The voltage across the break is different. If the break occurs in the middle of the line, as by breaking a wire, the voltage that appears is $2\,I/\gamma$. If at the end, it may be instantaneously approximately $E + I/\gamma$, for the voltage applied by the source will nearly double for an instant due to its inductance.

The voltage appearing at a break may be very large, and has nothing to do directly with the normal voltage of the circuit. Thus if the line tested above were made 33 miles (54 km.) long, the voltage appearing on opening a short circuit would be about 9 times the voltage existing before the switch was opened.

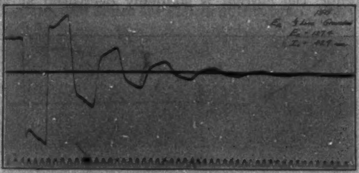

Fig. 25

In ordinary alternating-current switching the oil switch holds the arc for several cycles, and then snuffs out at the zero of current. This prevents such effects as just described occurring in normal switching. When a wire breaks, the case is different.

The maximum voltage on breaking a circuit need not appear at the point of breaking. The voltage wave running over the line may be nearly doubled at an open end or a transformer winding. There may be open ends present even when heavy currents are broken, for the line may divide into branches, one of which is open. Reflections and refractions at bifurcations may be computed by the usual reflection and refraction coefficients, using γ for the parallel combination of the two lines into which the wave passes.

A special case may occur when a line, terminating in a transformer at the distant end and carrying a heavy current, is suddenly interrupted at the near end. In this case the wave of voltage running over the line is reflected without change of sign at each end, and hence builds up continuously, and to a large extent, particularly if the line is short in length. Some of these cases have been discussed by Wagner.

An oscillogram for this last condition is shown in Fig. 26. The voltage measured is that at the distant end of the line across the terminal inductance. The switch opened at the home end during the straight portion of the curve. The first wave of voltage arrived at the distant end and approximately doubled by reflection, producing a voltage instantaneously of about five times the previous value due to resistance drop. The second wave which arrived was of the same sign and tended to continue to build up the voltage. The third and fourth waves may also be seen. In this case the line was very long, 990 miles, so that the drop in voltage due to the change of current in the terminal inductance, which occurred in the interval between wave arrivals, prevented the voltage from actually building up in value by reason of the successive reflections. In fact the current change in the inductance actually caused the terminal voltage to reverse in sign. With a shorter line the reflections would have arrived at more frequent intervals and less attenuated, there would have been little opportunity for the inductance to alter its initial value of current, and the voltage would have built up to a high value. In practise such an effect would be likely to result in a circuit breakdown, and very probably at a point distant from the initial break.

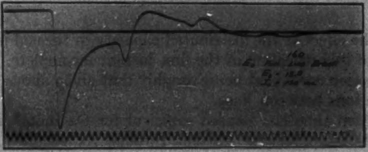

Fig. 26

Effects in Polyphase Circuits

The above discussion has been confined to single-phase circuits and with continuous applied potentials. The commercial polyphase circuit introduces two new factors for consideration.

If the line is short, then the first transient is practically over before the applied potential can have varied much at commercial frequencies. In this case continuous potential analysis suffices to give an excellent idea of the transient. On the other hand if the line, or combination of lines forming the system, was 750 miles long, a trip down the system and back would occupy one half-cycle. Thus a wave started by a potential of one sign would, on its return, become superimposed upon a voltage of opposite sign. If long cables are involved, the speed of propagation is decreased so that this effect is more likely to at least partially appear. If the dielectric constant of the cable paper is taken as 4, the speed of propagation will be approximately halved.

Also in polyphase circuits several single-phase circuits are involved and may carry transients simultaneously. A wave in one circuit will, in general, be accompanied by an induced wave in a parallel circuit. The presence of ground provides other circuits in which transients may occur. The combined effect of these transients may, for the case of assumed fixed constants, be obtained by analyzing each one separately and combining the results. Interesting cases may easily arise.

For example, take the case of closing a three-phase oil switch on a long line. The switch points do not close exactly simultaneously. When the first point closes, a transient follows between the wire thus excited and ground. When the second point closes this is followed by a second transient line to ground, and a transient between wires. The third point produces a transient wire to ground and two transients between lines. There are also induced transients. Suppose the points close in a particular sequence and at intervals corresponding to the reflection interval. The separate transients may easily superpose in such manner that voltages are produced much in excess of those produced by one transient alone.

Little has been done on polyphase transients of this type as yet. They offer an attractive field for investigation. It is quite possible that the occasional mysterious breakdowns which occur in systems upon closing a particular switch may result from especially unfortunate combinations of primary transients, and that some of these effects might be considerably modified by a slight change in an oil switch.

Appendix

The analysis of waves on lines as found in the literature usually makes certain simplifying assumptions in the mathematics, which amount to neglecting changes in wave form.[6] The result is a solution which is expressed in terms of rectangular waves, and which hence gives no information, for example, concerning the maximum voltages produced on reflections. Carson's extension of Heaviside's analysis gives exact solutions, which unfortunately involve laborious computations in order to attain numercial results. In communication circuits, which are electrically very long, it is usually necessary to adhere to exact solutions, and Carson's mechanical methods greatly reduce the labor involved. The power transmission circuit is in general of much less electrical length. It hence appears desirable to develop solutions which are approximate, but in which the degree of approximation is carried one step further than is usually the case. Such solutions will be sufficiently accurate for the transmission line of lengths now in service, and will be more easily employed than the complete solution. This note will outline how this can be done for one case of interest, namely the propagation of waves on a line open at the distant end when a steady voltage is suddenly applied at the home end.

The differential equations are

$$\left\{\begin{aligned} \frac{\partial i}{\partial x} &= -c\,\frac{\partial e}{\partial t} \\ \frac{\partial e}{\partial x} &= -R\,i - L\,\frac{\partial i}{\partial t} \end{aligned}\right. \tag{1}$$

when the line is assumed without leakance, as is warranted in the case of the aerial line. A solution of these equations for e is then

$$e = \epsilon^{-\alpha t} \sum_n (A \sin n\,t + B \cos n\,t)\,(a\,m \cos m\,x - b\,m \sin m\,x)$$

where $\alpha = \dfrac{R}{2\,L}$ (2)

and $m^2 = L\,C\,(\alpha^2 + n^2)$ (3)

Determining the constants A, B, a and b of integration for the terminal conditions above outlined, we have

$$e = E - \frac{4\,E}{\pi}\,\epsilon^{-\alpha t} \Sigma_k \frac{1}{2\,k+1} \sin m_k\,x\,(\cos n_k\,t + \alpha/n_k \sin n_k\,t) \tag{4}$$

where $m_k = \pi\,(2\,k+1)\,/2\,l$, l is the length of line and K is given the values 0, 1, 2, 3 . . . At this point it is usual to introduce the approximation of assuming α/n_k negligible in comparison to unity. With this assumption equation (4) may be written

$$e = E - \frac{4\,E}{\pi}\,\epsilon^{-\alpha t} \Sigma_k \frac{1}{2\,k+1} \sin m_k\,x \cos n_k\,t \tag{5}$$

which can be altered into the form

$$e = E - \frac{2\,E}{\pi}\,\epsilon^{-\alpha t} \Sigma_k \frac{1}{2\,k+1} \sin (m_k\,x + n_k\,t) - \frac{2\,E}{\pi}\,\epsilon^{-\alpha t} \Sigma_k \frac{1}{2\,k+1} \sin (m_k\,x - n_k\,t) \tag{6}$$

and again neglecting α^2 in expression (3) we may write

$$e = E - E/2\,\epsilon^{-\alpha t}\,4/\pi\,\Sigma_k \frac{1}{2\,k+1} \sin m_k\,(x + v\,t) - E/2\,\epsilon^{-\alpha t}\,4/\pi\,\Sigma_k \frac{1}{2\,k+1} \sin m_k\,(x - v\,t) \tag{7}$$

This solution states that the voltage on the line after closing the switch is given by a constant value E less the sum of two waves, each of height $E/2$ and wave length $4\,l$, of rectangular form, one of which moves to the left and the other to the right at the velocity of light v. At the time $t = 0$, the two waves coincide as shown in Fig. 27, and the voltage on the line is given as everywhere zero. At the end of an eighth period we have the conditions shown in Fig. 28, with a wave of voltage out to the distance $v\,t$. Further diagrams will show the course of reflections, etc. The diagram of Fig. 28 is drawn neglecting the factor $\epsilon^{-\alpha t}$. When this is included the rectangular waves will shrink in size as they proceed. A discrepancy will now be immediately noted. Redrawing the diagram for this instant and adding the waves we see that the solutions predicts a small voltage out on the line at distances beyond the front of the wave. This we know to be contrary to fact, and it is produced by the neglecting of α/n_k in the derivation. In fact to the same approximation we could have written unity in place of $\epsilon^{-\alpha t}$.

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[6] For example consult Wagner, Elektromagnetische Ausgleichsvorgange in Freileitungen und Kabeln—Pub.

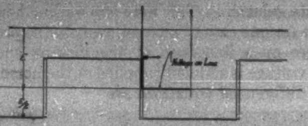

Fig. 27

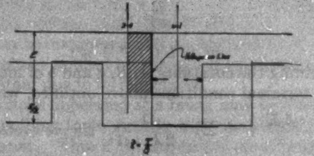

Fig. 28

The solution also gives no correct information regarding the voltage produced at the first reflection at the end of the line, and we are interested in the correct value of this voltage.

Let us therefore review the derivation and retain the terms α/n_k but neglect $(\alpha/n_k)^2$ compared to unity. For the line reviewed in the paper α/n_k has a maximum value of about 0.2 for a length of 500 km., and $(\alpha/n_k)^2$ a corresponding value of 0.04. The neglecting of the squared term is hence seen to be warranted in a typical case, but the neglecting of the first power is liable to lead to error.

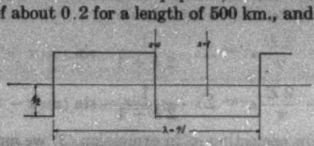

Fig. 29

We may hence continue to write (3) as

$$m_k\,v = n_k$$

The term of equation (4)

$$(\cos n_k\,t + \alpha/n_k \sin n_k\,t)$$

may be written

$$\cos\,(n_k\,t - \alpha/n_k)$$

for neglecting $(\alpha/n_k)^2$ we have

$$\cos \alpha/n_k = 1$$
$$\sin \alpha/n_k = \alpha/n_k \qquad (8)$$

The expression (4) thus becomes

$$e = E - \frac{4\,E}{\pi}\,\epsilon^{-\alpha t}\,\Sigma_k\,\frac{1}{2\,k+1}\,\sin m_k\,x\,\cos\,(n_k\,t - \alpha/n_k)$$

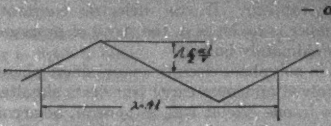

Fig. 30

and transforming as before

$$e = E - E/2\;\epsilon^{-\alpha t}\,4/\pi\;\Sigma_k\,\frac{1}{2\,k+1}\,\sin m_k\,(x + v\,t - \phi_k) - E/2\;\epsilon^{-\alpha t}\,4/\pi\;\Sigma_k\,\frac{1}{2\,k+1}\,\sin m_k\,(x - v\,t + \phi_k) \qquad (9)$$

where $\phi_k = \dfrac{\alpha}{m_k\,n_k} = \dfrac{\alpha}{m_k^2\,v}$

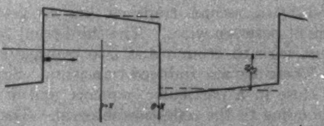

Fig. 31

We have as before two waves which run in opposite directions over the line, attenuated by a factor $\epsilon^{-\alpha t}$, but they are no longer of rectangular shape. The shape of the component waves is constant, so we shall examine it at time $t = 0$. The wave of equation (9)

$$-\,E/2\;4/\pi\;\Sigma_k\,\frac{1}{2\,k+1}\,\sin\,(m_k\,x - m_k\,\phi_k)$$

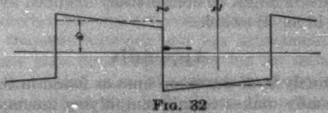

Fig. 32

may, by the use of (8), be written

$$-\,E/2\;4/\pi\;\Sigma_k\,\frac{1}{2\,k+1}\,\sin m_k\,x + E/2\;4/\pi\;\Sigma_k\,\frac{1}{2\,k+1}\,m_k\,\phi_k\,\cos m_k\,x \qquad (10)$$

The first term gives the previous rectangular wave shown in Fig. 29. The second term is a correction term to be added to take account of distortion. It may be written

$$E\,\frac{4\,\alpha\,l}{\pi^2\,v}\;\Sigma\;\frac{1}{(2\,k+1)^2}\,\cos m_k\,x \qquad (11)$$

This is plotted in Fig. 30. It consists of a fundamental term of wave length $4\,l$ and height $E\,\frac{4\,\alpha\,l}{\pi^2\,v}$, plus a 3d harmonic of 1/9 the amplitude, and so on.

The combination of Fig. 29 and 30 gives the complete wave shape shown in Fig. 31. Similarly the other wave of equation (9) is plotted in Fig. 32. The progress of the voltage on the line may now be obtained by combining these component waves as before.

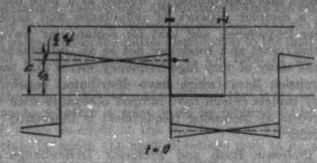

FIG. 33

The positions at $t = 0$ are shown in Fig. 33. The voltage on the line again adds up to zero everywhere as it should at this instant. Move the waves slightly in position as shown in Fig. 34, and we have the conditions one eighth-period after closing the switch. It will be noted that beyond the point to which the wave of voltage has progressed we now have very approximately zero voltage on the line.

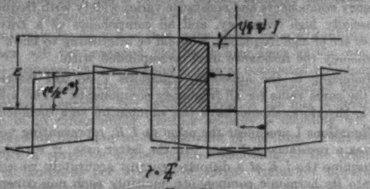

FIG. 34

A quarter-period after switch closing we have the position of Fig. 35. Summing the waves we have the resultant voltage as shown in Fig. 36. This is the instant that the voltage has just arrived at the distant end of the line. An instant later, reflection has occurred, and the combination of the waves gives Fig. 37, and so on.

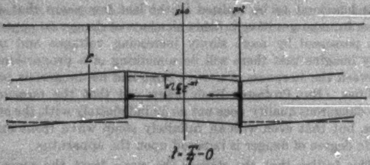

FIG. 35

The voltage produced at the distant end on the first reflection is given by adding the values of the component waves for this instant. This gives for the instant just before reflection

$$e_n' = E + E/2\,\epsilon^{-\alpha t} - \epsilon^{-\alpha t}\,E\,\frac{4\,\alpha\,l}{\pi^2\,v}\,(1 + 1/9 + 1/25 + \ldots) \qquad (12)$$

$$- E/2\,\epsilon^{-\alpha t} - \epsilon^{-\alpha t}\,E\,\frac{4\,\alpha\,l}{\pi^2\,v}\,(1 + 1/9 + 1/25 + \ldots)$$

But $(1 + 1/3^2 + 1/5^2 + \ldots) = \pi^2/8$

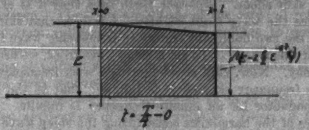

FIG. 36

and hence (12) may be rewritten

$$e_n' = E\left[1 - \epsilon^{-\alpha t}\left(1 + \frac{\alpha\,l}{v}\right)\right] \qquad (13)$$

At the instant under consideration

$$t = l/v$$

Also
$$\alpha\,t = \frac{\alpha\,l}{v} = \frac{\pi\,\alpha}{2\,n_0}$$

and $\epsilon^{-\alpha t}$ may be written to the same degree of approximation as before

$$\epsilon^{-\alpha t} = 1 - \alpha\,t$$

Inserting these in (13) we have

$$e_n' = E\,[1 - (1 - \alpha\,t)\,(1 + \alpha\,t)]$$

and thus to the same approximation

$$e_n' = 0$$

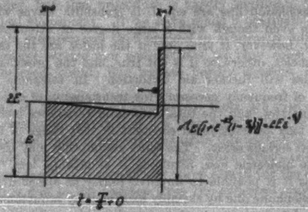

FIG. 37

which is physically evident for the instant before the wave arrives.

Similarly an instant after reflection we have

$$e_n'' = E\,[1 + (1 - \alpha\,t)^2]$$

or $e_n'' = 2\,E\,[1 - \alpha\,t]$

and as before

$$e_n'' = 2\,E\,\epsilon^{-\alpha t}$$

The voltage produced at the distant end on the first reflection is thus given by applying the attenuation factor $\epsilon^{-\alpha t}$ and doubling the resultant voltage. The expression for the current is unaltered by this second approximation.

Other cases of interest may be treated by the same method of approximation.

Discussion

A. Boyajian: I believe one of the puzzling discrepancies, viz., the nature of the wave front, mentioned in this paper is capable of a comparatively simple physical explanation.

Professor Bush states that theory shows the wave front ought to be perpendicular, regardless of the losses; that is, although the wave front will gradually diminish in amplitude due to losses, yet it should always remain perpendicular, but that contrary to theoretical considerations the oscillograms show a sloping and rounding of the wave fronts.

Of the factors that are mentioned as affecting wave front, one is skin effect. Professor Bush says that it is expected to influence the wave front slightly. I believe that skin effect is one of the two dominant factors in influencing wave front, and that for this reason. To get a steep wave front, we need infinite frequency. Now, skin effect increases with frequency, and for infinite frequency skin effect is infinite; that is, the resistance of a conductor for an infinite frequency is infinite. Since the voltages of the harmonics constituting the steep wave front are finite but the impedance of the line to them is practically infinite, therefore the infinite frequency which an absolutely steep wave front represents will be wiped out instantly. It, therefore, is impossible to maintain a rectangular wave front on a line of which the resistance increases with frequency, wiping out the higher harmonics faster then the lower harmonics and rounding out all corners, as the oscillograms show. A very good illustration of this is afforded by water waves. By throwing a pebble into a pool we can create an infinitely complex jagged wave at the very point where the stone falls into the water. What do we expect to notice five or ten feet away? An almost pure sine wave, free from all the steepness and jaggedness of the original wave. Here also the higher harmonics which are responsible for the steepness and jaggedness of the original wave are wiped out very fast, and only the lower harmonics reach any considerable distance, and the wave is then nicely rounded.

The second dominant factor in rounding steep wave fronts is the dielectric loss. According to the theory of the so-called "distortionless circuit," if the resistance and dielectric losses of a transmission line are equal, it transmits waves without distortion even though with a large degree of attenuation. This theory, however, is true only if the losses are the same for all frequencies, otherwise it is not true, and the presence of dielectric loss will further aid distortion. In telephone work the range of frequency necessary for transmission of clear speech is quite narrow, and the total conductor resistance including skin effect may be considered constant in that range of frequencies without serious error. The same assumption may also be made for the dielectric loss, assuming it as leakage loss. For these reasons it is possible to improve purity of speech-transmission on a telephone line by suitably loading it by inductances. However, when we come to the general problem of a steep front on a rectangular wave, which covers the range from direct current in the main body of the wave to practically infinite frequency at the very wave front, the increase of both resistance and dielectric losses with frequency can not be ignored, and the distortionless circuit does not exist. At the higher frequencies the loss in the dielectric is not leakage but true dielectric hysteresis loss which increases with frequency and has a distorting and rounding influence similar to skin effect. If the conductor and dielectric losses are or may be assumed equal to each other for any given frequency (although of a different value at each different frequency) the calculation of the distortion is considerably simplified.

In this connection I also wish to point out an inherent limitation of lumped artificial transmission lines. So far as operating frequency phenomena are concerned, lumped laboratory models are no doubt entirely satisfactory to duplicate the phenomena occurring on uniform lines, but when the problem is that of very high frequencies, steep wave fronts, reflections, etc., is the laboratory model with lumped constants still an exact equivalent of the transmission line with distributed constants? A paper entitled "Abnormal Voltages in Transformers" presented before the Institute a number of years ago by Blume and Boyajian, analyzed in considerable detail the behaviors of lumped coils to very high-frequency voltages and steep waves. It was shown there that coils ordinarily treated as a pure inductance react as a complex network of inductance and capacity at the higher frequencies, and that for rectangular wave fronts they initially react as a condenser not as an inductance. Evidently, the study of very high-frequency waves and transients of transmission lines (which have distributed constants) by laboratory models (which have lumped constants with varying characteristics at increasing frequencies) is inherently limited and difficult.

Both Professor Bush and Professor Dellenbaugh deserve much credit for their painstaking investigations on artificial transmission lines.

J. F. Peters: In Professor Bush's paper I was rather surprised at the disagreement between the calculated surges and those recorded by oscillograms, Figs. 6 and 15. I note that the author states the mathematical solutions are based on the assumption that the line may be completely specified by four fixed constants,—r, l, c, g,—resistance, inductance, capacitance and leakance, per unit length of line. Calculations apparently are based on one value of resistance for all frequency components. Since a rectangular wave is made up of a large number of frequency components and resistance varies widely with frequency, especially high frequency, and all components of a traveling wave are of high frequency, it would be interesting to know what frequency the resistance used corresponded with.

Referring to the appendix, equations 1 do not restrict R to the same value for all frequency components. The solution of 1, therefore, should have the summation sign included $\epsilon^{-\alpha t}$

Equations 1 are linear the same as $I\ R$, representing voltage across a resistance is linear. In the latter case, if we want to determine the $I\ R$ of a distorted current accurately we resolve I into its harmonic components, multiply each component by the value of R that corresponds to that particular frequency and then add up the component voltages. In the same way, if for each frequency component in equation 9, corresponding to the different values of k, we use the value of alpha corresponding to that frequency, much better agreement will be found between calculated and measured values and in no way will we have violated the restrictions imposed by equations 1.

Joseph Slepian: I found the discussion on page 884 on wave fronts and over-voltages very valuable although some statements contained therein I think require modification.

It has come to be realized in the last few years that steep wave front surges have in them elements of danger to apparatus not possessed by more slowly increasing voltages and many may imagine that there will be something of a proportionality between the steepness of a wave front and its dangerous potentialities. But Professor Bush points out that the measure of the dangerous quality does not go up indefinitely with the steepness, but that even with an infinitely steep wave front only a finite degree of danger is imposed upon the apparatus.

This feeling that the steeper the wave front the more disastrous the effect of the wave on insulation has been responsible for numerous arguments as to the value of high-resistance lightning arresters. It is argued although such arresters do not have enough discharge capacity to reduce the energy or the voltage in the surge in any marked degree, they do taper off the wave front and thus remove the very dangerous character of the wave. However, even with infinitely steep waves there is a limit to the measure of this dangerous character.

Whether a wave front is steep or not, is of cours a relative matter. A wave can never be infinitely steep; there must be some slope to it. To be precise the steepness should be referred to some interval of time.

In all the cases, the time interval that the steepness of the wave should be referred to is the period of some natural oscillation of the apparatus being considered. For example, in the case of a transformer, when a voltage is suddenly impressed and maintained, the voltage distribution within the transformer will differ from the final voltage distribution; and the readjustment of voltage from the initial distribution to the final distribution will ordinarily take place in the form of an internal oscillation.

It is the time of this internal oscillation that is the determining factor as to whether a wave is to be referred to as steep or tapered, when its effect on the transformer is being considered. If the time in which the applied voltage rises to its maximum value is short, compared to the natural period of the internal oscillation, the internal oscillation will have practically its maximum amplitude and the maximum abnormal concentration of voltage on the internal parts of the transformer will be produced. But if the time in which the voltage rises to its maximum value is long compared to the natural period of these oscillations, the amplitude of these oscillations will be very much reduced and the distribution of voltage in the transformer will be more nearly the normal distribution.

The same thing is true as regards the multiplication of voltage on a transmission line, due to reflection at its end. The transmission line also has a natural period, namely: the time for the wave to run down to the end and back again.

Professor Bush states that with short lines a doubling of voltage will be produced just as in long lines at the remote end—assuming the remote end is open or connected to inductive apparatus. However, since lines are usually energized through inductive apparatus, the natural period of the line is going to come in in determining whether actually a doubling will be obtained or not, due to the transient of the line. That is, to produce a surge on a line requires something of the order of several amperes per kilovolt of surge. Inductive apparatus cannot deliver this current instantly; the surge produced by switching on a generator or transformer must build up relatively gradually, because of the inductance of the apparatus. If it builds up slowly compared to the natural period of the line, a doubling of voltage at the far end over that at the near end will not be produced. There may be an abnormal rise of voltage due to a transient including machine constants, but not one due to the line alone.

In practise, there usually are transformers, inductive apparatus at both ends, at the near end the inductance of the apparatus and at the far end the capacity of the apparatus will both operate to prevent a rise in the voltage at the far end due to switching at the near end, provided the line is not too long. That is, if the natural period of the line is short, compared to the transients involving the machine constants, the line itself will introduce no new element of danger in raising voltage.

The only case that I can see where this doubling on short lines may be obtained is in the case of short-circuits. In the case of a short circuit, there is of course practically no inductance at the point where the short-circuit occurs and so waves may be produced sufficiently steep to double by reflection even on short lines. Fortunately, the polarity of the voltage induced by the short circuit is usually such as to merely reverse the voltage at the reflection point instead of doubling the normal voltage; that is, the voltage changes from plus normal voltage to minus normal voltage. Thus, the amplitude of the surge at reflection is twice normal voltage, but the actual voltage to ground is only normal. However, as regards its effect on the internal distribution of voltage within the reflecting apparatus it is still a surge of twice normal voltage.

Modern transformers are designed so as to be able to stand the sudden application of normal voltage. Hence if a surge does not greatly exceed normal voltage, we do not need to worry about its wave front provided the transformer is properly designed.

V. Bush: In regard to Mr. Boyajian's discussion, there is one point that I wish to emphasize. We have at present an analysis of waves on transmission lines only under the assumption of strictly constant R, L, C and G for the line, and we have no analysis for any other condition. If we could have an analysis which would take into account skin effect, for instance, or many other disturbing factors, it would be very valuable; but apparently such analysis is beyond the range of our present analytical ability. It is hence the purpose of this paper to show what an analysis, based on recognized limited assumptions, corresponds to in the actual case; and how far experiments on smooth artificial lines may be expected to yield reliable information in regard to the transient surges of practise.

In regard to Mr. Peters' comments: We can get a closer approximation to the observed waves if we will correct alpha, that is the attenuation factor, term by term in accordance with the frequency, but that procedure is hardly legitimate from a mathematical standpoint. If we have made a complete analysis on certain premises and then proceed to arbitrarily introduce into our answer certain new assumptions, we may come nearer to an observed result; but we are hardly rigorous in our mathematics.

In regard to Mr. Slepian's comments, there are many cases where we encounter waves that are very nearly perpendicular, and I am pleased that he has enlarged upon this particular point. For example, when an arc breaks, the wave which ensues is initially very nearly a perpendicular wave except for the correction which is due to the passage from the spherical wave to a plane one.

The Standardization of Electrical Measuring Instruments[1]

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Review of the Subject.—*This paper points out the advantages, both to the maker and the customer, of national standard specifications for manufactured articles. The history of standardization in England is outlined. British specifications for electrical instruments appeared first in 1909, and in improved and enlarged form in 1919. French specifications were adopted in 1921, German in 1922.*

The outstanding features of the three foreign specifications are compared. The British go into much detail concerning scale construction and marking, the French concerning definitions and temperature rises. The German specifications contain very good definitions of instruments and their parts, principles, etc. They prescribe severe tests for mechanical and thermal robustness, and have an elaborate scheme of symbols to indicate the grade, operating principle, kind of current, test voltage, etc.

An account is then given of the consideration which the subject of instrument standardization has received in this country. The National Meter Committees have done a related piece of work by having the Meter Code prepared and revised, and have made recommendations to the makers from time to time concerning matters of instrument standardization. The Instruments and Measurements Committee of the Institute has considered the foreign specifications, and has determined by a personal canvass that a majority of American makers are in favor of standardization. However, this canvass also showed that most of them would have felt much freer to discuss proposed standard specifications rather than the abstract question as to whether specifications should be formulated. The Instruments and Measurements Committee therefore appointed a subcommittee of four members to prepare such tentative specifications, which are given as an Appendix to the paper. The specifications omit some important topics because the Subcommittee wished to include at the start only those on which general agreement could be had without difficulty.

The paper concludes with a discussion of some of the features of the tentative specifications.

INTRODUCTION

While the United States has led the world in applying the principles of interchangeability of parts and of quantity production to its manufacturing processes, it has only recently begun to appreciate the advantages of national standard specifications. The outbreak of war brought this question forward in very unmistakable fashion, and there is good reason to believe that at no distant time the advantages of "national uniformity of buying and selling requirements" will be as much appreciated here as abroad.

The purposes to be served by such specifications may be regarded from two standpoints. The manufacturer is benefited because he can devote his energies to certain definite lines for which the demand can be confidently anticipated. With the limitation of his output largely to standard grades, ranges, and finishes, unit-costs are decreased, demands can be more closely estimated, raw materials can be bought in fewer kinds and sizes and in larger quantities, and overhead costs are reduced. If the national specifications under which the maker works have attained a good standing abroad, a further important advantage is secured, in that surplus stocks are immediately available for export instead of goods made up specially with the attendant increased cost.

From the purchaser's viewpoint, standard specifications are useful in selecting the most suitable apparatus for given requirements, and in obviating the necessity for individual study and the preparation of independent specifications. It is only when the articles bid on have been made under the same requirements as to quality and performance that an intelligent selection can be made from a number of bids. As an illustration of the condition that is produced by lack of standard requirements, we may cite current transformers. As now made, these are classified by each manufacturer in such general terms that a purchaser can readily pick out the extreme grades, especially with the relative prices as a guide. But it is not possible for him to choose the best transformer from the offers of several manufacturers, for the rating "50 volt-amperes, compensated for 25 volt-amperes," for example, does not throw much light on the matter, in the absence of uniform practise as to permissible ratio and phase-angle errors. The purchaser is compelled to fall back on his impressions of the relative standing of the manufacturers concerned.

Both manufacturer and purchaser lose in the absence of national specifications, on account of which different individual specifications necessarily arise. The diversity of requirements that may thus arise increases engineering and manufacturing costs and hence the cost to the purchaser. A case in point is that of Government specifications. The Navy Department, which very properly buys its supplies on definite specifications, is compelled to go to the expense of preparing complete specifications for electrical instruments, both for power-plant use and for radio purposes. If American standard specifications were in existence, the Navy Department could use them, with only a few added clauses to take care of the special requirements which exist only in naval work.

INSTRUMENT STANDARDIZATION ABROAD

The work of national standardization in Great Britain began in 1901 when the Council of the Institu-

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[1] Published by permission of the Director of the Bureau of Standards.

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tion of Civil Engineers appointed a committee to consider the advisability of standardizing various kinds of iron and steel sections. The cooperation of three other British societies (Mechanical Engineers, Iron and Steel Institute, Naval Architects) was at once secured, and the work of preparing specifications was begun. A year after the formation of the "Engineering Standards Committee" it was suggested that the subject of electrical machinery and apparatus be included in its scope, and with the cooperation of the Institution of Electrical Engineers this work was begun. That the results of the labors of the Standards Committee have been appreciated is shown by its growth from a single committee of seven members to a great organization, the British Engineering Standards Association, consisting of a Main Committee of 24 members. The Report of the Association dated July 1, 1919, showed that 26 sectional committees appointed by the main committee were functioning, and that they had appointed 78 subcommittees and 144 "panel committees," making a total of 249 committees with an aggregate membership of over 1100. Another indication of the importance of the work is found in the range of subjects for which specifications have been prepared or are under way. As given in reports of January 1, 1923 these included:

Civil Engineering and Building Trade	25
Mechanical	36
Electrical	76
Automotive (Aircraft and Automobile)	164
Transport	16
Ships and Their Machinery	42
Ferrous Metals	10
Non-Ferrous Metals	46
Chemical, Including Chemical Engineering	14
General	4

The first British Standard Specification for electrical instruments was No. 49 on ammeters and voltmeters, adopted in 1909, as the result of nearly two years of work by a subcommittee. Shunts, resistors, and instrument transformers were not included in this specification.

In 1919 this specification was superseded by No. 89 on indicating ammeters, voltmeters, wattmeters, frequency and power factor meters. At the same time two other related specifications were issued; No. 81 on instrument transformers and No. 90 on recording (graphic) ammeters, voltmeters, and wattmeters. Specification No. 89 was a very distinct advance over its forerunner, No. 49.

French standard specifications for electrical measuring instruments, instrument transformers, and shunts were prepared by a Technical Committee[2] with the collaboration of representatives of the makers, the Laboratoire Centrale d'Électricité, and a consulting engineer (Mr. A. Iliovici) specializing in electrical instruments. These specifications[3] were adopted by the Chambre on January 20, 1921.

German specifications for electrical instruments were prepared by a committee of the Verband Deutscher Elektrotechniker. A proposed draft was published[4] in March, 1921, and was intended to become effective on July 1, 1922. This draft evoked criticisms and suggestions which were used in preparing a considerably revised one, which appeared[5] a year later. This revised draft was adopted by the Verband at its 1922 meeting, and is to go into effect on July 1, 1923.

German specifications for instrument transformers[6] were prepared by the Verband, and went into effect on July 1, 1922.

It is of interest to compare the outstanding features of these three national specifications. In general, the British specifications are probably the most satisfactory from the American viewpoint, though they leave some important points untouched. They treat with considerable detail the important question of scale construction and marking. Three classes as regards accuracy are provided, called Sub-standard, First Grade, and Second Grade respectively. Insulation and dielectric strength are carefully specified. Limits of error are given for the three grades, and limiting values of influences of operating conditions (temperature changes, frequency changes, external magnetic fields, etc.).

The French specifications open with an elaborate set of definitions, which in the main are very good. Instruments are classified first into two groups, Laboratory and Industrial, the former including special or delicate devices used either to measure very small magnitudes or to measure ordinary magnitudes with high precision.[7] The second broad classification is denoted Industrial Instruments, and is further divided into Standard Instruments, Control[8] Instruments, Switchboard Instruments, and Ordinary Instruments, of which latter all that is said is "This group includes all measuring instruments of which the essential quality is a low price." Requirements stated for the other classes include accuracy, damping, mechanical robustness, heating, and dielectric strength. The French specifications are the only ones which give a list of recommended ampere ranges[9] for the current coils of instruments.

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[2] Of the "Chambre Syndicale des Constructeurs de gros Matériel Électrique."

[3] They were published in *Revue Générale de l'Électricité*, vol. 9, pp. 119c-134c, May 28, 1921.

[4] *Elektrotechnische Zeitschrift*, vol. 42, p. 324, March 31, 1921.

[5] *Elektrotechnische Zeitschrift*, vol. 43, p. 290, March 2, 1922. Some minor corrections and changes were published in the same journal, vol. 43, p. 1074, August 17, 1922.

[6] *Elektrotechnische Zeitschrift*, vol. 42, pp. 209, 836; Mar. 3, July 28, 1921.

[7] Actually, the French specifications do not cover these special laboratory devices, nor do either of the other national specifications.

[8] This is a literal translation; "working-standard" is perhaps a better rendering.

The French specifications go into great detail in regard to the permissible temperature rises of conducting materials and various kinds of insulating materials in instruments.

The German specifications divide instruments as follows:

Precision Instruments,	first class	
"	"	second "
Operating	"	first "
"	"	second "

They contain a very good set of definitions of instruments and their parts, operating principles, ranges, kinds of cases, etc. They introduce a concept not found in either of the other specifications, namely, "minimum creeping distance," which is defined as the shortest path along which current can flow over the surface of an insulator between metal parts which have a difference of potential between them.

In addition to prescribing limits of accuracy and for the effects of temperature change, etc., the German specifications are unique in specifying a maximum distance of the pointer from the scale, and in requiring switchboard voltmeters and ammeters to be capable of carrying continuously a voltage or current 20 per cent greater than full-scale value. When it is considered that this overload means a heat loss in the instrument 44 per cent more than that caused by rated voltage or current; that voltmeters are ordinarily installed to work on about three-fourths of full-scale value; and that even full-scale value on a switchboard ammeter usually corresponds to an overload condition of the associated apparatus, it seems that the German requirement will involve some economic waste of materials in the construction of instruments.

In addition to the stipulated accuracies of precision instruments, the German specifications permit additional errors in the following cases:

For instruments having voltage circuits exceeding 250 volts, 0.1 per cent additional;

For instruments with interchangeable series resistors, 0.1 per cent additional;

For instruments with interchangeable shunts, 0.2 per cent additional.

Another outstanding feature of the German specifications is the extent to which they carry the use of symbols to serve as concise statements of certain features. In addition to the letters *E*, *F*, *G*, *H* denoting the four classes above mentioned, symbols are provided for the following kinds of current: d-c., a-c.; two-phase, balanced three-phase, unbalanced three-phase, and four-wire three-phase. Position symbols show that instruments are for use in horizontal, vertical, or oblique position. Another set of symbols denotes the kind of mechanism (moving-iron, induction, etc.). This is a desirable feature in instruments for general laboratory work, because it assists the user to select the most suitable instruments for such cases as unusual frequencies, for example. The use of a black, brown, red, blue, or green star denotes that the instrument is for a particular operating voltage, the black denoting "not over 40 volts," and the green "901 to 1500 volts."

The French and German specifications impose tests for robustness, while the British specifications ignore this important point. The French requirement of "twice the current corresponding to full-scale deflection, applied suddenly and for a very short time" is virtually a mild test of *mechanical* robustness only, because the thermal inertia of current windings would make the resulting rise of temperature very small. The German requirement is much more severe, and may properly be called a test of mechanical and thermal robustness. It specifies that the test current shall be 10 times that corresponding to the maximum value of the range of measurement, and that it shall be applied as follows:

9 impulses of 0.5 second in intervals of 1 minute each, then

1 impulse of 5 seconds duration.

With this current, which generates heat in the windings at 100 times the normal rate, the instrument receives a severe mechanical test and a searching test for any thermally weak spots.

The question of damping is variously handled. The British specifications require the pointer to settle to a definite indication "in a reasonable time," and in an appendix the "extreme limits of reasonable time" are given as follows:

Scale length in inches Not over	Time in seconds
4.5	2.5
7	3
12	5

The German requirement is more logical in that it bases the time to come to rest on the length of the pointer, and on the class of instrument, as follows:

Precision instruments,	both classes,	$3 + L/100$	seconds,
Operating "	first class,	$3 + L/50$	"
" "	second "	$4 + L/50$	"

where L is the length of the pointer, measured in millimeters.

The French specifications characterize damping by two quantities:

1. The ratio of the first swing to the steady deflection when the appropriate current is suddenly passed through the instrument.

2. The time required for the pointer to come to rest, under the same conditions.

Incidentally, for practical purposes the pointer is assumed to come to rest when it is within 1 per cent of the steady deflection.

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[9] The same values are also recommended as the primary ranges of current transformers. For voltage transformers a list of recommended transformation ratios is given.

The British specifications require zero adjusters on (spring-controlled) sub-standard and first-grade instruments. The German specifications require them for precision apparatus of both grades, and the French make no mention of the matter.

CONSIDERATION IN THE U. S.

The Meter Committee of the Association of Edison Illuminating Companies has used its influence to encourage the standardization of watt-hour meters. With the corresponding Committee of the National Electric Light Association, it inaugurated the preparation by Electrical Testing Laboratories of the Meter Code, which has been of great service to the meter industry.

Since the checking of watt-hour meters involves the use of indicating instruments, it is only natural that the national Meter Committees should at times make recommendations concerning them. Among these the following work of the N. E. L. A. Meter Committee and Apparatus Committee may be noted:

Proposed Standardization System of Connections for Instruments and Meters used with Current and Voltage Transformers, 1913 and 1914 Reports.

Standardization of Shunts for Instruments and Meters, 1914 and 1915 Reports.

Standardization of Design and Maintenance Method for Instrument Transformers, 1916, 1917, and 1918 Reports.

Standardization of Meter and Instrument Design, 1921 and 1922 Reports.

Standardization of Instrument Transformers, 1922 Report.

The Instruments and Measurements Committee of the Institute has a keen interest in all questions relating to instrument standardization. At the request of this committee, the writer presented a paper[10] three years ago, in which the British Specifications were mentioned, with a table of required accuracies for various kinds and classes of instruments. A little later this committee took up the question of instrument terminology. A list of words relating to instruments, their operating principles, parts, functions, etc., was drawn up, and the various divergencies and inconsistencies were discussed with the hope of establishing a list of recommended terms which might eventually supersede the imperfect terminology in use. While some useful points were brought out in the discussion, the committee felt that the standardization of the terminology of instruments was a part of a larger job and the matter was therefore held in abeyance.

In 1921 the proposed German specifications for instruments and for instrument transformers were published. In order to get these specifications in convenient form for discussion, they were translated into English. Copies of the translation were sent to the members of the Instruments and Measurements Committee and the Meter Committees of the N. E. L. A. and the Edison Association. The Instruments and Measurements Committee appointed the writer as a subcommittee of one to canvass the sentiment of the instrument manufacturers of this country as to the necessity or desirability of American standard specifications covering their products.

Before this canvass was begun, the French specifications were translated, and copies were sent to the above-named committees. Copies of the French, German, and British specifications were sent out early in 1922 to the American manufacturers of electrical instruments and accessories, and soon after the personal canvass was begun. In all, twenty-one concerns were visited. Some of them make a complete line of instruments and accessories, while others make accessories (for example, instrument transformers) as adjuncts to other lines, such as watt-hour meters.

The results of the canvass showed that a majority of the makers were in favor of standard specifications, though some questioned the necessity or advisability of their preparation at the time. Two important ideas, however, came to the surface. The first was that many of the manufacturers would have felt much freer to express opinions if proposed American standard specifications had been placed before them, instead of only the abstract question as to the desirability of formulating such specifications. The second was that it would be entirely proper for the A. I. E. E. Instruments and Measurements Committee to prepare such specifications to serve as a basis for criticisms and suggestions.

Following out this suggestion, Chairman Sawin of the Instruments and Measurements Committee appointed a subcommittee for this purpose. The membership of this subcommittee included Mr. F. P. Cox of the General Electric Co., Mr. G. L. Crosby of the Roller-Smith Co., Mr. P. MacGahan of the Westinghouse Electric & Mfg. Co., and the writer, as chairman. The three foreign specifications were carefully considered, and what appeared to be the best features of each were woven into a preliminary draft. This was carefully revised by the subcommittee. The final draft, as here given, was prepared in a joint session of the subcommittee and the Instruments and Measurements Committee.

The Instruments and Measurements Committee authorized the writer to prepare a paper presenting the tentative specifications, in order to get them before the entire membership of the Institute. They are given as an Appendix herewith. All makers and users of electrical instruments are earnestly invited to send written comments, and criticisms of the scope and subject matter of the specifications to the writer, care Bureau of Standards, Washington, D. C., for the guidance of the subcommittee in the further development of these tentative specifications.

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[10] The Accuracy of Commercial Electrical Measurements, Jour. A. I. E. E., vol. 39, p. 495, 1920.

It will be noted that a number of important topics are not covered in these specifications, though they are carefully treated in the foreign ones. This is not because the subcommittee considers these topics unimportant, but rather because it is felt best to begin with things about which general agreement can easily be reached. On the matter of accuracy of calibration, for example, considerable difference of opinion exists. Probably no customer would care to buy instruments concerning which absolutely no statements or guarantees of accuracy are given by the maker. However, since undue importance is sometimes attached to high accuracy, and because the accuracy of instruments in service is so largely under the user's control, it was felt that for the present the specifications should leave it to the maker and customer to agree on the accuracy required in each case. The same statement applies to the permissible limits for the various influences of operating conditions.

Some specific comments on other features of the appended specifications will now be given.

I-2. *Kinds of Instruments Included.* Under this heading the German specifications limit current and voltage capacities to 1000 amperes and 20,000 volts, because "With larger values of current and voltage it is difficult to keep within the prescribed limits of accuracy, and the checking of the accuracy becomes difficult."

III-1. In defining the words "instrument" and "indicator" the subcommittee followed the example of the British specifications, but with an important difference in the definition of "indicator". There is no question as to the need for the distinction between instrument and indicator, and the only argument against taking this step was that early in the history of electric lighting in this country the word "indicator" was sometimes[11] used to denote a device of low accuracy. It was finally agreed that this usage was now practically obsolete, and was therefore no bar to the use of the word "indicator" in its new sense.

Attention is invited to the distinction between the terms "series resistor" and "multiplier". These terms should not be confused.

III-4. *Damping.* For the present this term is simply defined. Section 8302 of the Standards of the Institute prescribes the method by which damping shall be measured. Three quantities are to be observed, as follows: The number of swings made by the pointer in coming to rest; the time (in seconds) required for the pointer to come to rest; the overshooting, in per cent of the angular displacement due to the initial disturbance.

III-5, 6. *Torque and Weight.* For the present these sections are limited to simple definitions of these quantities and a statement of the units employed. In view of the extensive mention of the ratio of torque to weight as a criterion of the degree of excellence of an instrument as regards frictional error, it should be stated that this time-honored criterion is now known to be incorrect save when the moving elements of the instruments to be compared have equal weights. (Equality of excellence of pivots and jewels is assumed). The use of the simple ratio of torque to weight is based on the incorrect assumption that the area of pivot in contact with the jewel is independent of the weight, whereas it must necessarily increase with the weight. By the simple rule of torque to weight, certain small American instruments having very light coils would be expected to stick so badly as to be useless, whereas actually they work very well. If we use the expression, torque divided by weight to the nth power, and take for n a value greater than unity and less than 2, more concordant results will be obtained. A prominent German specialist[12] in electrical measuring instruments gives as a satisfactory criterion, based on practical experience, the quotient, torque divided by the 1.5-power of the weight.

IV-1. It is not felt opportune to make a subdivision of instruments as to "class" or "grade". A suggested classification, which it was not thought advisable to adopt at this time, was as follows:

1. As to use:

(a) Portable Instruments, Precision Grade
(b) " " Utility "
(c) Switchboard " High "
(d) " " Industrial "

It was also suggested that "laboratory-standard" instruments form a third class (in addition to portable and switchboard).

IV-2c. The adjective "electrodynamic" is advanced for consideration as a logical substitute for the lengthy term "electrodynamometer-type" and the abbreviated (and not strictly correct) term "dynamometer"[13]. The French, Germans and Italians use the forms corresponding to "electrodynamic" (électrodynamique, elektrodynamisch, elettrodinamico).

It should be emphasized that in drawing up these specifications the effort has been made to maintain a harmony of treatment and of substance with the Standards of the A. I. E. E. It should also be noted that the present specifications are not put forward as a finished product. In their present form, however, it is felt that they will be valuable for their intended use as a tangible basis for discussion and as a nucleus about which finished specifications may in due time be realized. Such specifications should be prepared with the approval of the Institute, and in accordance with the procedure of the American Engineering Standards Committee.

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[11] This was not the general practise, however, for the device by which early Edison stations measured the voltage at feeder-ends was known as the Howell indicator.

[12] Dr. G. Keinath, of the Siemens & Halske A. G. of Berlin. See his book "Die Technik der elektrischen Messgeräte," second edition, pages 20-22.

[13] The term "dynamometer" means "that which measures force." A spring balance is a dynamometer, and some forms of spring balance are called dynamometers.

Appendix

PROPOSED AMERICAN SPECIFICATIONS FOR ELECTRICAL MEASURING INSTRUMENTS

OUTLINE

I. *Introduction*
  1. Origin and Purpose
  2. Kinds of Instruments Included

II. *Standards*
  1. General
  2. Units; Legal Standards
  3. Reference Temperature
  4. Standard Ambient Temperature

III. *Fundamental Definitions*
  1. Instruments and Their Parts
  2. Rating
  3. Scales
  4. Damping
  5. Torque
  6. Weight
  7. Error and Correction
  8. Influences of Operating Conditions
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    - (b) Frequency "
    - (c) Voltage "
    - (d) External-Field "
    - (e) Power-Factor "
    - (f) Position "

IV. *Classification*
  1. As to Use:
    - (a) Portable Instruments
    - (b) Switchboard "
  2. As to Principle of Operation
    - (a) Dynamometer (Electrodynamic)
    - (b) Permanent-Magnet Moving-Coil
    - (c) Moving-Iron
    - (d) Induction
    - (e) Electrothermic
    - (f) Electrostatic
  3. As to Kind of Protection by the Case:
    - (a) Dust-Proof
    - (b) Moisture-Proof
    - (c) Rust-Resisting
    - (d) Water-Tight (submersible)

V. *Requirements*
  1. Construction
  2. Rating
  3. Insulation Resistance and Dielectric Strength
  4. Scale and Pointer
  5. External Shunts
  6. Marking

I. INTRODUCTION

1. *Origin and Purpose.* These specifications were prepared for the Instruments and Measurements Committee of the American Institute of Electrical Engineers by a subcommittee appointed for the purpose. In their present form they are intended to serve as a tangible basis for discussion on the part of makers, users, and laboratory men engaged in testing electrical measuring instruments, in order to determine whether American standard specifications of this general nature would be of advantage to all concerned, and if so, what changes in and additions to these specifications are required to make them suitable for the purpose.

The objects of these specifications are as follows:

1. To standardize the principal terms used in describing instruments and their parts, functions, operating principles, etc.

2. To promote a more complete understanding between the maker and the user concerning the essential characteristics of instruments for definite applications.

3. To eliminate diversity of purchase specifications for instruments for similar uses.

The limitations of the specifications may be stated as follows:

1. They are of necessity limited to certain broad classes of electrical instruments, and there will always be special devices not falling entirely within their scope. In many cases, however, the specifications may be used together with certain additions to cover the special features.

2. They are subject to revision from time to time as the art of instrument making advances, or as the requirements of practise change. They are not intended to restrict development or prevent improvement, but rather to guide progress along the most efficient lines.

3. They are not intended to cover the necessary legal provisions of a contract, but only the technical requirements of a purchaser in ordering or a maker in bidding.

2. *Kinds of Instruments Included.* These specifications apply for the present to the following kinds of indicating electrical instruments for direct current and for alternating current:

Ammeters,
Voltmeters,
Wattmeters,
Frequency Meters,
Power-Factor and Phase Meters,
Reactive-Factor Meters.

These specifications are not intended to apply to indicating instruments provided with arrangements for curve drawing, contact making, etc. They do not apply to the following kinds of instruments:

(a) Small instruments of types and sizes which are used where the requirements are not severe, and where low cost is essential; for example, small polarized-vane ammeters used on automobiles, battery-charging outfits, etc.

(b) Instruments constructed for very special requirements.

II. STANDARDS

1. *General.* Electrical measuring instruments shall conform to such general requirements of the Standards of the American Institute of Electrical Engineers as shall be applicable to instruments, when such requirements are not definitely covered in the following specifications.

2. *Units, Legal Standards.* The accuracies specified by maker and purchaser are to be based on the legalized international electrical units.

3. *Reference Temperature.* The standard temperature of reference for instrument characteristics shall be 20 deg. cent.

4. *Standard Ambient Temperature.* For purposes of rating instruments, the standard ambient temperature shall be 40 deg. cent. See definition of rating, below.

III. FUNDAMENTAL DEFINITIONS

1. *Instruments and their Parts.* An *instrument* is a measuring device which indicates the present value of the quantity under observation. The term "instrument" includes the indicator (as defined below) together with any accessory apparatus such as shunts, shunt leads, resistors, reactors, condensers, or instrument transformers.[14]

The *mechanism* is the arrangement for producing and controlling the motion of the pointer. It includes all the essential parts necessary to produce this result, but does not include the base, cover, scale, or any parts, such as series resistors or shunts, whose function is to make the readings agree with the scale markings.

The *moving element* includes the pointer and the parts which move with it.

The *indicator* is the mechanism and the scale, built into the case, including any accessory devices (resistors, shunts, etc.) which are built into the case or non-removably attached to it.

Examples: An instrument (ammeter) for 500 amperes direct current consists of the indicator (which may be thought of as essentially a millivoltmeter) together with a 500-ampere shunt and a pair of shunt leads. Another instrument (wattmeter) consists of the indicator (which is essentially a 5-ampere 110-volt wattmeter) together with a current transformer and a potential (voltage) transformer.

An instrument is said to be *self-contained* when all accessory apparatus necessary to cause the scale reading to correspond with the numerical value of the quantity measured is enclosed within or permanently attached to the indicator case.

The *current circuit of an indicator* is that winding (or other conducting path) of the indicator which carries the current to be measured, or a definite fraction of it, or a current dependent upon it.

The *voltage circuit of an indicator* is that winding of the indicator to which is applied the voltage to be measured, or a definite fraction of it, or a voltage dependent upon it.

The *current circuit of an instrument* is that winding (or other conducting path) of the instrument which carries the entire current to be measured.

The *voltage circuit of an instrument* is that winding of the instrument to which the entire voltage to be measured is applied.

Example: An a-c. wattmeter for 1000 amperes, 6600 volts, consists of an indicator (which is essentially a 5-ampere, 110-volt wattmeter with its scale marked to read primary power) together with a current transformer of 1000: 5 amperes and a potential (voltage) transformer of 6600:110 volts. The current circuit of the indicator is its 5-ampere winding, and its voltage circuit is its 110-volt winding. The current circuit of the instrument is the 1000-ampere winding of the current transformer, and the voltage circuit of the instrument is the 6600-volt winding of the potential (voltage) transformer.

A *series resistor* is a resistor forming part of the voltage circuit of an indicator or an instrument.

A *multiplier* is a particular type of series resistor which is used to extend the voltage range of an instrument beyond some particular value for which the instrument is already complete.

A *shunt* is a resistor connected in the circuit to be measured and in parallel with the current circuit of an indicator.

A *reactor* is a device used for the purpose of introducing reactance, and usually has a high time-constant.

Shunt leads are leads which connect the current circuit of an indicator to the shunt.

2. *Rating.* The *rating of an instrument* is a designation assigned by the manufacturer to indicate its operating limitations. The full-scale marking of an instrument does not necessarily correspond to its rating.

3. *Scales.* The *indication range* is the range within which the electrical quantity (current, voltage, power, etc.) is to be indicated without reference to accuracy.

The *measurement range* is that part of the range of indication within which the requirements for accuracy are to be complied with.

The *scale length* is the length of the arc described by the end of the pointer in moving from the zero position to the end of the scale.

4. *Damping.* This is a term applied to instrument performance to denote the manner and the rapidity with which the pointer settles to its steady reading after a change in the value of the measurement quantity.

5. *Torque.* The *torque* of an instrument is defined as the turning moment which is developed by its mechanism when holding the pointer in the position of full-scale deflection. Torque is to be expressed in millimeter-grams, and should be accompanied by a statement of the angle corresponding to full-scale deflection.

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[14] It is proposed to cover instrument transformers by separate specifications.

6. *Weight.* The *weight* of a moving system includes one-half the weight of the springs, if any. It is to be expressed in grams.

7. *Error and Correction.* The error of indication is the difference between the indication and the true value of the quantity being measured. It is the quantity which must be algebraically subtracted from the indication to get the true value. A positive error denotes that the indication of the instrument is greater than the true value.

The correction has the same numerical value as the error of indication, but the opposite sign. It is the quantity which must be algebraically added to the indication to get the true value. If T, I, E and C represent respectively the true value, the indicated value, the error, and the correction, the following equations hold:

$$E = I - T$$
$$C = T - I$$

Example: A voltmeter reads 112 volts when the voltage applied to its terminals is actually 110 volts. Then

Error = 112 − 110 = +2 volts
Correction = 110 − 112 = −2 volts.

8. *Influences of Operating Conditions.* (a) The *temperature influence* is defined as the percentage change in the indication which is caused solely by a difference in room temperature of ± 10 deg. cent. from the reference temperature (20 deg. cent.)

(b) The *frequency influence* (in other than frequency meters) is defined as the greatest percentage change in the indication which is caused solely by a change of ± 10 per cent from the rated frequency.

(c) The *voltage influence* (in other than voltmeters) is defined as the greatest percentage change in the indication which is caused solely by a change of ± 10 per cent from the rated voltage.

(d) The *external-field influence* is defined as the percentage change in the indication which is caused solely by an external field of an intensity of 5 gausses produced by a current of the same kind and frequency as that on which the instrument operates, with the most unfavorable phase and position of the external field.

(e) The *power-factor influence* (in wattmeters) is defined as the percentage change of the indication which is caused solely by the lowering of the power factor from unity to 0.50, current lagging, at the rated voltage and frequency.

(f) The *position influence*, in other than gravity-controlled instruments, is defined as the maximum displacement of the pointer which is caused solely by an inclination of 30 deg. in any direction from the normal position of use. It is to be expressed as a percentage of the scale length.

IV. CLASSIFICATION

Electrical instruments may be classified as follows:

1. *As to Use:*
    - (a) Portable instruments
    - (b) Switchboard "
2. *As to Principle of Operation:*
    - A. *Electromagnetic*
        - (a) Dynamometer (Electrodynamic)
        - (b) Permanent-Magnet Moving-Coil
        - (c) Moving-Iron
            - (1) Plunger
            - (2) Vane
            - (3) Repulsion
        - (d) Induction
    - B. Electrothermic
        - (a) Expansion
        - (b) Thermocouple
    - C. Electrostatic

NOTE: Instead of "permanent-magnet moving-coil" the following terms were also suggested: "d'Arsonval;" "fixed-field." Instead of the principal heading A, "Electromagnetic," the term "electrodynamic" was also suggested.

3. *As to Kind of Protection:*
    - (a) Dust-Proof
    - (b) Moisture-Proof
    - (c) Rust-Resisting
    - (d) Water-Tight (Submersible)

The terms "portable" and "switchboard" are self-defining. A third classification as to use is suggested, namely, "laboratory-standard."

In *dynamometer* (*electrodynamic*) *instruments* one or more coils move within the field produced by a fixed coil or coils.

In permanent-magnet moving-coil instruments a coil moves within the field of a permanent magnet.

In *moving-iron instruments* one or several pieces of soft iron are caused to move by the magnetic field of a fixed coil or coil system. Various forms of this instrument (plunger, vane, repulsion) are distinguished chiefly by mechanical features of construction.

In *induction instruments* the torque is produced by fixed coils acting upon moving conducting parts (disks, drums, etc.) in which currents are produced by electromagnetic induction.

Electrothermic instruments depend for their operation on the heating effect of a current. Two distinct types are (1) the expansion type, including the "hot-wire" and "hot-strip" instruments; (2) the thermocouple type, where one or more thermocouples which are heated directly or indirectly by the passage of a current supply a direct current which flows through the coil of a suitable direct-current mechanism, such as one of the permanent-magnet moving-coil type.

Electrostatic instruments depend for their operation on the forces of attraction and repulsion between bodies charged with electricity.

A *dust-proof instrument* is provided with a case which excludes dust from the mechanism.

A *moisture-proof instrument* is one in which moisture is excluded from the mechanism, or which is so constructed that moisture will not damage the mechanism.

A *rust-resisting instrument* is one whose case and parts are made of rust-resisting materials, or are specially finished to resist the corrosive effects of moist air.

A *water-tight (submersible) case* withstands for one hour complete immersion in a tank containing sufficient water to cover all parts to a depth of at least three feet, without any visible trace of penetration of water into the interior.

V. REQUIREMENTS

1. *Construction.* The construction of electrical instruments shall be mechanically sound, suitable to their class and purpose, and shall be such as to give assurance of permanence in the accuracy of the indications. All materials must be suitable for the purpose for which they are used.

2. *Rating.* The rating of the circuits of an instrument shall be equal to, or less than, the maximum current or voltage to which they may be continuously subjected without exceeding the permissible temperature rises. These rises are to be those specified for the various classes of insulating materials by the Standards of the A. I. E. E., and refer to the standard ambient temperature of 40 deg. cent.

If the rating of an instrument differs from the full-scale marking, the rating shall be marked on the instrument.

The limiting observable temperature of shunts shall not exceed 120 deg. cent. It shall be measured by mercury, alcohol, or resistance thermometers or by thermocouples, any one of these devices being applied to the hottest accessible part of the shunt. *Exception.* This rule shall not apply to shunts having no soldered joint and made of material which is not permanently changed in resistivity if continuously subjected to a higher temperature.

3. *Insulation Resistance and Dielectric Strength.* Tests for insulation resistance and dielectric strength shall be made on finished instruments.

The *insulation resistance* between all the electrical circuits of an indicator connected together and the case shall not be less than 20 megohms, as measured with a d.c. test voltage of 500 volts. The insulation resistance between the current circuit and the voltage circuit of an indicator (where both exist) shall not be less than 5 megohms.

When indicators are specified to have one or more of the circuits internally connected to the case, the necessary exceptions to these requirements are allowed.

Dielectric strength tests shall be made with a wave form which is nearly sinusoidal and of a frequency between the limits of 15 and 65 cycles. Tests shall be made between all the electrical circuits of the device connected together and the case. *Instruments having a voltage circuit* shall withstand for 60 seconds a test of twice rated voltage plus 1000 volts. *Instruments without a voltage circuit* shall withstand for 60 seconds a test of 1000 volts. These are effective (root-mean-square) values.

4. *Scale and Pointer.* The *preferred value of each scale division* should be either 1, 2, or 5 of the units measured, or any decimal multiple or submultiple of these numbers. In the case of multiple-range instruments exceptions to this rule may be necessary, but should be avoided where reasonably possible.

The *angle subtended by a scale division* shall preferably not be less than 0.5 degree in portable instruments, or 1 degree in switchboard instruments. When smaller angles are used, the legibility is decreased.

The *numbers marked on the scale,* except in the lower part of non-uniform scales, shall preferably be by steps of 1, 2, or 5, or a decimal multiple or submultiple of any of these numbers. The figures shall be of such shape as to minimize the risk of different figures being confused with one another, and shall be so spaced as to render individual numbers clearly distinguishable from adjacent numbers.

In determining the influences of operating conditions (a) to (e) inclusive, page 8, ammeters and voltmeters should be tested with rated current or voltage respectively. Wattmeters should be tested with rated voltage and such a current (not exceeding the rated current) as will give a suitable deflection, preferably not less than that corresponding to one-half of the maximum reading. Frequency meters (except in the test for voltage influence) should be tested at rated voltage. Power-factor meters and reactive-factor meters should be tested with rated voltage and rated current, unity power factor.

5. *External Shunts.* The *main terminals* of the shunt shall be so constructed that slight variations in the manner of connecting it in the circuit (such as might occur in an average workman-like installation) shall not alter the indication of the instrument by more than 0.25 per cent.

The *thermal electromotive force* produced by continuous operation of the shunt at rated current shall not exceed the value which would cause a change in the reading (at rated current) of 0.25 per cent. The connections to the circuit should be made so that the opportunity for the escape of heat will be the same at both terminals.

6. *Marking.* The *indicator* shall be distinctly marked with the following particulars in such a way that they will be visible from the front of the case: Name (or symbol) of manufacturer; serial number; designation of the quantity measured; the words "direct-current" or "alternating-current" or their abbreviations; the rated current, voltage or frequency (or the ranges of these quantities) or such of these as apply; the maximum

current and voltage, in the case of wattmeters, power-factor meters, and reactive-factor meters. Of these, only the scale marking itself and the designation of the quantity measured shall be conspicuous.

In the case of alternating-current instruments the following additional items are required: Ratio of the appropriate current transformer expressed thus: 1200:5 or 1200/5; ratio of the appropriate potential (voltage) transformer expressed thus: 6600:110 or 6600/110.

The indicator of an ammeter having separate shunts shall also be marked with the drop of the shunt with which it is to be used, corresponding to full-scale deflection.

Instruments having separate shunts or series resistors should be marked to indicate this fact.

Separate shunts, if not interchangeable, shall be marked as follows: Name (or symbol) of manufacturer; serial number of the indicator with which it was calibrated; the line current corresponding to full-scale deflection of the indicator; the rated current, if this is less than the preceding, and the drop at rated current. When the shunts are designed to be used with devices taking sufficient current to be an appreciable proportion of the whole, this fact shall be indicated.

Interchangeable separate shunts shall bear the above markings, except that the serial number may be omitted.

Discussion

W. B. Kouwenhoven: On the fifth page of Mr. Brooks' paper, where he lists the proposed types as portable and switchboard instruments of various grades. I think that it would be advisable to add a third grade under the portable class. This third grade might be known as laboratory instruments. It should include instruments that can be used in rough, ordinary work; something better than an indicator but not a precision instrument. I feel that a sub-classification giving the type of controlling force employed in the instrument would be of value and interest to the user. The controlling forces found in common use in deflection meters are:

1. Resisting force of a spring; 2. Torsion of a filament; 3. Attraction of gravity; 4. Attraction or repulsion of permanent magnet.

On the eighth page, item d under heading 8, where Mr. Brooks discusses the effects of the external field on the deflection of the instrument, I think the last clause of that paragraph might well be modified to read as follows: That the external field be placed in such a location as regards phase and position that it will cause the maximum error in the instrument.

One point that I think should be added to these specifications, is: What is the effect on the deflection of standing under load for one hour? A great many instruments change considerably on that test.

Another requirement that might be added is that an instrument should be insect proof.

C. M. Green: There is one point at which I have been very much perplexed in connection with the use of instruments and in this connection I would particularly refer to "Standards of the Institute of Electrical Engineers" under date of 1922, page 25, "Kinds of Current," Rule 3104—Direct Current, 3108—Pulsating Current and 3112—Continuous Current. To my mind these apparently are normally rules or definitions without any actual defining.

It seems to me that our book of standards is in a most deplorable condition in reference to the definition of the various kinds of currents, and if there is a single member here who can from these rules determine definitely where direct current leaves off and a pulsating one begins, I surely should appreciate it if he would help me out.

I have heard complaints regarding some half-wave tungar rectifiers not fulfilling their rating, the rectifier being rated at 6 amperes and the customer claimed that it gave 11, which was more than he wanted. However, with two ammeters connected in the circuit with the batteries which are being charged from the half-wave tungar rectifier, the d'Arsonval type of instrument, which gives the average value of the current which is effective in charging the battery, recorded 6 amperes, the rating of the outfit, whereas an inclined-coil instrument giving the r. m. s. value of the current, or its heating value, recorded 11 amperes. Can anyone tell from our rules 3104 and 3108 whether the current flowing through this circuit was a direct or pulsating current? For all intents and purposes the battery was being charged just as effectively at 6 amperes as though it had been from a normal d-c. source, and as such was the current flowing into the battery a direct or pulsating one as determined by the above rules?

A. E. Kennelly: I would like to call attention to one point, namely, that if this work is going to be serviceable to the American Institute of Electrical Engineers, it must be serviceable to our fellow groups of engineers in other countries than this, and therefore it must be expressed in one and the same system of units. The units employed in these specifications are in the main metric, as I think we will all agree that they should be, but there are a few of the specifications such as for example appear in the immersion problem on the ninth page, which call for immersion under three feet of water.

It seems to me that in order to make the specifications universally suitable to all possible parts of the world, one and the same system of units should be used throughout.

J. R. Craighead: On the fifth page of Mr. Brooks' paper he states that the Committee was fully agreed that the word "indicator" is obsolete as indicating an instrument of somewhat inferior accuracy. It has not seemed to me that the word is obsolete in that sense. I was glad to note that Professor Kouwenhoven, requested that there should be inserted in the table on the fifth page a place for an instrument which would come between the grades indicated. He said that it should have an accuracy "better than an indicator, but somewhat less than a precision instrument." The language would suffer a loss if we cut out the use of the word "indicator," for that purpose because its use is active at the present time.

On the seventh page the definitions of the circuits of indicators and instruments are so drawn that they do not refer to definite mechanical parts of the apparatus; rather, they refer to the circuit connections. I cannot say that this is necessarily objectionable, but it is a fundamental change from the way that such definitions have usually been made. That is, if we build a five-ampere instrument and set it on the shelf and we have a current transformer for use with that instrument, in any given case the transformer may or may not be used. If we use the instrument alone, the current circuit of the instrument is then both the indicator circuit and the instrument circuit. The instrument and the indicator are identical. If, on the other hand, we connect this circuit to the secondary of the current transformer, the circuit becomes the indicator circuit, and it is no longer correct to refer to it as the instrument circuit. The primary circuit of the current transformer then becomes the instrument circuit.

Similarly in attaching a millivoltmeter to a shunt, the instrument becomes the shunt plus the parallel circuit made by the millivoltmeter winding. The indicator circuit is the millivoltmeter winding, but if you use the millivoltmeter direct, the millivoltmeter winding becomes the instrument circuit.

I am mentioning this to call attention to the action this committee has taken and to note that it is considerably different from previous practise.

Paragraph 8 on the eighth page is divided into a, b, c and d. The word "greatest" implying a maximum error is implied in paragraphs b, c and d but omitted in paragraph a. It would seem that as this case is strictly parallel to the others the error should also be defined as a maximum.

Under paragraph d the statement regarding the location of the field is not quite complete. That is, if we form a field of five gausses and intrude an instrument into that field, the field is then more intense than five gausses at some points and less intense at other points. There should be some other closer definition as to the point at which the five gausses are to be maintained in order to get a definite thing which can be repeated at different places. That is, the resultant field would be very largely affected by the size of the coil used to maintain it, and by the exact way in which the coil was placed with relation to the instrument itself.

H. B. Brooks: I am sure that the favorable comments made by Dr. Kouwenhoven and Dr. Kennelly are appreciated by the Subcommittee which has had the task of preparing these specifications.

Dr. Kouwenhoven suggestion for a third grade of portable instrument is in line with recent developments. The General Electric, Weston, and Westinghouse companies have all produced smaller portables for general purposes, such as trouble location.

The comments of Dr. Kouwenhoven and Mr. Craighead about the specification with respect to the effect of external field are very good and the specification should be modified accordingly.

In regard to Dr. Kouwenhoven's question about the rating, it may seem puzzling that we have two standard temperatures for instruments. The explanation is that the twenty-degree requirement refers to instrument performance; that is, to accuracy and other operating characteristics, with the exception of ability to stand overloads. The forty-degree requirement refers to the rating; that is, an instrument must be capable of carrying the specified load for the specified time with the external temperature at the unfavorable point of forty degrees centigrade.

I agree with Dr. Kouwenhoven's suggestion, at least as far as portable instruments are concerned, that there should be a requirement limiting the permissible change in accuracy caused by standing in circuit. In the case of switchboard instruments it might be sufficient to specify that they should stand in circuit with a given load for say one hour before the accuracy test is begun.

Mr. Green reports the interesting circumstance that two perfectly correct ammeters connected in series in the output from a rectifier may show values as different as six and eleven amperes. It is unfortunate that there is not available in standard text books a good discussion on the subject of the measurement of pulsating currents. There are only two things in print about this, of which I am aware. The first is a discussion by Mr. P. MacGahan and was published only for the engineering department of the Westinghouse Company. The second is an article on "Selecting Ammeters for Various Current Measurements," by Mr. V. H. Todd of the Westinghouse Company, in *Electrical Review* (Chicago), October 30, 1920. It is really quite important to to choose the proper ammeter and it depends upon whether you are interested in the effective value of the current or in the average value. If you are charging a battery with a pulsating current you want an instrument which measures the average value; when operating lamps or heating devices an ammeter which measures the effective value should be used.

Dr. Kennelly is quite right in saying that if American specifications are to be most useful for American engineers, they must also be suitable for foreign engineers, and I am sure the committee will make the necessary corrections.

As to Mr. Craighead's comments about the word "indicator," I believe that word is no longer used in instrument catalogs or in technical articles, but I am glad to be corrected if such is not the case.

There is a very good reason for using the word in the sense that the committee proposes. We have tried in vain to find another English word which will express the thought and still leave the word instrument for the complete measuring equipment; that is, the indicating device with all of its accessories.

As to the suggestion of Mr. Craighead that the word "greatest" should appear on page 901 in connection with the definition of temperature influence, I would say that to the best of our knowledge temperature effects on instruments within a range of a plus or minus ten degrees centigrade are always linear. For that reason we did not feel it necessary to put in the word "greatest," but the effect of frequency, for example, is often quite different from linear, and the effect of the other quantities may also be safely specified by putting the word "greatest" in the three definitions.

Pellet Type of Oxide Film Lightning Arrester

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Review of the Subject.—*Practise has confirmed the theory that a valve type of lightning arrester is the best; that is, one which has a very large resistance or impedance at rated voltage, but which has a resistance of only a comparatively few ohms at say double rated voltage. The aluminum arrester was the first of this type and its success is well known. Somewhat later the oxide film (OF) arrester was developed to overcome the disadvantage of charging, and this type too has had remarkable success.*

A modification of this "OF" type is now announced. Briefly, pills or pellets of lead peroxide, the chemical powder used in the present "OF" type, are made and coated with an insulating film of powder. They are about the size of the familiar sugar coated pills. These are simply poured in an insulating tube, terminals put on each end in contact with the pellets, a series gap added, and the arrester unit is complete.

In action, each pellet is a miniature "OF" cell, so the arrester tube contains a number of them in series and parallel. The over-voltage surge discharges through the pellets, which have a low resistance after the powder film is punctured, and sealing occurs at the contact of the various pellets. No dynamic or normal frequency current passes.

The characteristics are similar to those of the present "OF" arrester, giving it the fundamental characteristics a successful arrester must have. The pellet construction permits of a simple, flexible and comparatively cheap design.

THE theory and principle of the oxide film lightning arrester have been pretty thoroughly taken up in previous papers[1] and its successful operation in thousands of installations has proved the results obtained in the laboratory. From the first, efforts have been directed toward applying this principle to a form of construction other than the present one, so that a less expensive unit could be made. For small capacity of power apparatus, there is a limit to the money which can be expended for protective apparatus, and it is for such conditions that the type of arrester to be described, was developed.

FIG. 1—LEAD PEROXIDE PELLETS BEFORE BEING COATED WITH AN INSULATING POWDER

The main difference between this new pellet type of oxide film (*OF*) lightning arrester and the present *"OF"* cell is based upon a relation explained by colloidal chemistry.

A small part of the subject of colloids deals with grains, and states that a small grain will adhere to a larger one and cover it. As a familiar example, sugar adheres and coats blueberries, when the two are shaken together. Also in the extreme, it is for the same reason that lead pencils write on paper, and chalk on boards, that is, the plane surface in these cases is the larger grain.

This principle has been made use of in applying the basic principle of the oxide film arrester to this new pellet type. If lead peroxide (PbO_2), the filler of the *"OF"* cell, be made into a round pill or pellet, and shaken up with a fine powder, the pellet will become coated with this powder. This then is what is done: Lead peroxide pellets are shaken up with an insulating powder and thoroughly coated. Fig. 1 shows the pellets before being coated and Fig. 2 after being coated. Each pellet is now a miniature *"OF"* cell, consisting of an outside film, which is the insulating powder, and the active internal filler, which is the original pellet. To make a lightning arrester these covered pellets are placed in an insulating tube, of say porcelain, and bare electrodes attached to each end of the tube in contact with the pellets. The electrodes act both as terminals and covers for the container. Between the terminals of the container, there are hundreds of these miniature *"OF"* cells in series and parallel. The over all length of the container depends upon the voltage rating of the arrester desired, and the chosen diameter of the tube is related, (to a certain extent), upon protective value and life of the arrester.

FIG. 2—LEAD PEROXIDE PELLETS AFTER BEING COATED WITH AN INSULATING POWDER AND BEFORE BEING USED IN A LIGHTNING ARRESTER

The film of insulating powder apparently acts as a porous spacer and not as a solid insulation. This is shown in Fig. 3, in a way, by the microphotograph of a covered pellet magnified to 25 diameters. In the photograph, due to the high magnification, only the center of the pellet is in focus. The insulating film, acting as a spacer and not as a solid, eliminates the objectionable time lag of establishing a spark, that solids have. This fact is borne out in tests — the equivalent sphere gap (E. S. G.) being low. A high-voltage impulse discharge goes through this pellet arrester in a number of

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[1] TRANSACTIONS, A.I.E.E., 1918, Vol. XXXVII, page 871.
TRANSACTIONS, A.I.E.E., 1920, Vol. XXXIX, page 1981.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

parallel paths and sealing or recovery occurs at the contact surfaces of all pellets effected by the discharge in the path of the discharge current.

While the basic principle of this pellet arrester is the same as the present used *"OF"* unit, the difference in construction of the pellet type causes some modification in the film action. The leakage current is very much less in the pellet type. It is less than one milliampere when new, and contrary to expectations, continued service does not increase it. It seems that sealing or recovery is more complete between peroxide surfaces than between a lead peroxide and a metal surface, — the condition which exists in the present *"OF"* cell. This fact is of course beneficial. The pellets become a bit irregular due to continued use and the sealing surfaces can be easily seen on each one. Fig. 4 shows a picture of a lot of pellets taken from a 2300-volt arrester which had been given repeated discharges from a lightning generator[2] while also connected to a 2300-volt 60-cycle circuit of high kw. capacity. The worn condition of the pellets can be readily seen.

FIG. 3—A LEAD PEROXIDE PELLET AFTER BEING COATED WITH AN INSULATING POWDER

Magnification is 25 diameters. Only a central ring is in focus.

FIG. 4—LEAD PEROXIDE PELLETS AFTER BEING COATED WITH AN INSULATING POWDER AND AFTER EXTENSIVE USE IN A LIGHTNING ARRESTER

It might be expected that such small contact surfaces would be quickly worn away, but such does not seem to be the case. A contact surface once punctured by the discharge, and sealing having occurred, is still available for further use; that is, the life of the contact surface between pellets is great. The pellets do become a bit irregular in shape from service, but they are amply strong to carry a heavy impulse discharge. Although dynamic does not follow the impulse, over voltage dynamic tests show the pellets also to have ample strength to carry a heavy dynamic discharge. Life tests show hardly any deterioration although an excess of the insulating covering of the pellets becomes blown or shaken off. The necessary amount of covering seems to adhere regardless of service. This fact, however, is beneficial rather than harmful as it improves the protective value.

The construction and size of the pellets for both successful and best operation is of course important. A successful arrester must (1) discharge the surge freely at some definite voltage above normal; (2) the free discharge must be sufficient to relieve the voltage strain; (3) the insulating film on the pellets must reseal, that is, prevent the dynamic from following the abnormal surge and (4) it must have endurance and life. All of these factors are related to the size of pellet, and the best size has been determined by extensive laboratory tests. Pellets, as now used, are about one eighth of an inch in diameter.

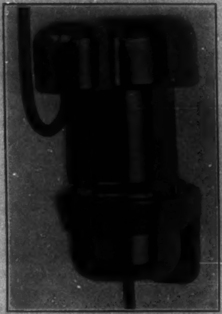

FIG. 5—PELLET TYPE OXIDE FILM LIGHTNING ARRESTER FOR 2300-VOLT SERVICE

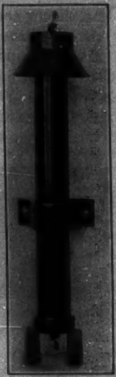

FIG. 6—PELLET TYPE OXIDE FILM LIGHTNING ARRESTER FOR 15,000-VOLT SERVICE

The electrical characteristics of this pellet arrester are very similar to those of the present *"OF"* arrester, therefore oscillograms of representative tests are not shown, as they would simply be duplicates of those published heretofore in the papers already referred to.

The future of this type of lightning arrester appears to be very bright — it has all the characteristics demanded by the basic theory of lightning arresters, and the mechanical form of pellets lends itself to a convenient, flexible and relatively cheap construction. Fig. 5 shows a 2300-volt arrester and Fig. 6 a 15,000-volt arrester.

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[2] *General Electric Review*, 1921, Vol. 24, page 946.

Discussion

K. B. McEachron: The pellet arrester represents an extension of the oxide film principle in an ingenious manner. The problem of making, commercially, small pellets of lead peroxide, and coating them with litharge has been solved in a satisfactory manner.

In design it is possible to increase the discharge rate by increasing the number of pellets in parallel and to increase the operating voltage by increasing the number of pellets in series. The arrester is simple in construction and offers some advantages over the cellular type in the matter of assembly.

A large number of tests have been made on the arrester while in the developmental stage and also after being put in production. A volt-ampere curve (expressed in r. m. s. values) taken on a group of 3000-volt arresters using the discharge from a lightning generator is shown in Fig. 1. It will be seen that the voltage across the arrester at currents below 100 amperes lies between 4 and 6 kv. This value is practically the same as obtained from other good arresters of the same voltage rating. With an instantaneous current, 1500 amperes, the voltage is approximately 8.5 kv. This voltage is considerably lower at these higher currents than any other arrester we know of.

It is probably true that under lightning conditions in service on a distribution circuit, that most of the discharges will be of low current value, but the reserve capacity is present in the pellet arrester if needed.

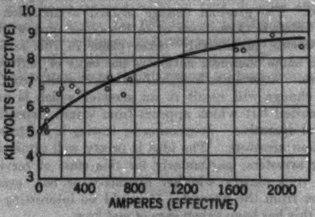

Fig. 1—Volt-Amperes Characteristic of Pellet Arresters (3000-volt)

V. E. Goodwin: It has been a source of gratification to me to note the interest and progress which has been made in recent years in the methods of test and in the development of lightning arresters. This is a line of work in which we can have no prejudice and no favorites.

It is only a few years ago that arresters were tested with a static machine and Leyden jars. A little later the lightning generator was brought out with larger condensers operated at 100 kv. and later at 250 kv. At that time we entered what might be termed the wood splitting age as blocks of wood could easily be shattered by discharges of this magnitude. Recently we have all heard of the 2,000,000-volt generator.

Great improvement has likewise been made in the methods of measurement of these impulse voltages. First these measurements were made with needle gaps and later by sphere gaps which were more capable of measuring the crest values of voltage. Recently we have been able to study the decrement as well as the crest values of voltage. This decrement in voltage is of great importance as it gives a fair idea of the value of the voltage at any time during the discharge of the arrester. This is accomplished by means of gaps having greater and known time lags, such as gaps with shunt capacitance and resistance.

A few months ago we had an excellent paper by Mr. Atherton describing the development of a new type of valve arrester. Today we have this paper by Mr. Lougee which gives a general description of pellet type oxide film arrester. This is another valve type arrester. Valve type arresters have ideal characteristics from an operating standpoint as they allow little or no dynamic current to follow the lightning discharge. Any flow of dynamic current is objectionable and should it continue for over a cycle or so a serious interruption to service is liable to follow. The chief difference between it and the standard oxide film arrester is that the cell element in the former is the individual pellet. The operating voltage per cell is much lower which allows of great flexibility in design.

The pellet design has an excellent volt-ampere characteristic, as Mr. White has just shown by the curve on the blackboard. This means that the arrester has low impedance to lightning impulses and will handle heavy discharges without excessive rise in voltage. The arrester also has a high dynamic voltage failure point. This is another important consideration since arresters are frequently called upon to function during line troubles when the line regulation may be momentarily very bad.

C. P. Steinmetz: The pellet type of oxide film lightning arrester, adequately fills a gap for which no economically suitable lightning arrester was heretofore available, namely that of primary distribution circuits of 2300 volts and over, up to 13,200 volts and more.

The first great advance in lightning arrester engineering was made by Alexander J. Wurtz, in his investigation of the non-arcing metals. This gave us the multigap lightning arrester. Its characteristic is, that the dynamic which follows the static discharge is cut off at the end of the first half wave, and therefore does not disturb the circuit. The multigap arrester remained the standard of the best type of arrester until electric transmission voltages had increased to values at which arcing grounds could exist. Then the multigap arrester failed, for these higher voltage circuits, as it is almost instantaneously destroyed by an arcing ground, and for sometime the protection of transmission lines against lightning appeared almost hopeless indeed.

The problem was solved by the development of the aluminum cell arrester. Its characteristic is, to let the static pass freely, but not to allow the dynamic to follow at all. Therefore the aluminum arrester can take care of arcing grounds for a reasonable length of time. It quickly became the standard of excellency, used wherever protection was of importance. Its only disadvantage is, that it requires attention, that is, systematic charging. This disadvantage was overcome by the development of the oxide film arrester. This also is a counter-e. m. f. or valve type of arrester and like the aluminum cell, has the same high discharge rate, the same ability to cope with arcing grounds, and the other good features of the aluminum arrester, but it requires no attention. It therefore rapidly replaced the aluminum arrester.

For low voltage primary distribution circuits, such as 2300 to 13,200 volts, and more particularly for the protection of smaller apparatus, such as lighting transformers, distributed on such circuits, the standard forms of aluminum and of oxide film arresters were not economical, in view of the relatively low cost of the individual protected apparatus. The problem therefore arose, to produce an arrester of the type of oxide film, which is economically suited for these circuits. This has been solved by Mr. Lougee in making this arrester economical for lower voltages by putting it into pill form, as we may say. But though in pill form, it is a real lightning arrester; it has the same high discharge rate, the same self-healing qualities, etc. It has lost none of the advantageous features of the standard oxide film arrester.

In its present form, the pellet arrester is a development which had been practically completed some years ago; with apparatus as lightning arresters, where the final decision of excellency after all is the practical experience, it is our fixed policy to try it out thoroughly for years, first in our laboratories, and then in industrial circuits, by the installation and operation of numerous arresters, so that, when we finally present it to you, our statements are not the expectation and hope based on limited laboratory experiment, but are the result of years of experience on hundreds of arresters in real industrial service.

A. L. Atherton: There is one interesting and very significant point which is illustrated, or rather emphasized, by the description by Mr. Lougee of this new form of the oxide film lightning arrester. That point, fundamental to the lightning arrester industry, is that the manufacturer for whom Mr. Lougee speaks, and the one for whom I am speaking, the Westinghouse Company, are both striving in lightning arrester development toward pretty much the same thing.

This is important in lightning arresters because some other manufacturers are not striving toward those same ideals. The special significance of the basic agreement of these two companies lies in the fact that both are fundamentally manufacturers of what we might call primary or working equipment, and are only suppliers of lightning arresters and other accessory types of equipment because it is realized that these are necessary if the systems for which apparatus is supplied are going to perform in such a way as to promote the growth of the electrical industry as a whole.

Lightning arresters are a minor part of our business. The engineers who develop and design lightning arresters are charged primarily with the problem of protecting apparatus and only secondarily with the provision of apparatus which will be a profitable business.

That the two organizations which approach the problem from this broad viewpoint aim at the same characteristics as a fundamental, is significant because it is a rather clear proof that those characteristics are the right ones. There are quantities of devices made and sold and used with the idea that they are protective devices, but in the design of which all of the principles that have to do with correct protective equipment are completely neglected.

This point is stressed because I believe that the cause of the use of these inferior devices is the lack of general understanding of the requirements for lightning arresters to give performance. I don't think very many people make poor investments knowingly and willingly.

In this connection I would like to make a suggestion in regard to the presentation of information on lightning arresters. Such information should be in very definite, specific, quantitative terms, which it has not very generally been in the past. When we speak about resistance of an arrester, it is better to say so many ohms than to say it is a low resistance. This sort of information is lacking in Mr. Lougee's paper, probably justifiably, but was supplied to some extent in one of the discussions.

There are one or two points in connection with the arrester itself on which I would like to comment. Every one has found it very difficult to make solid dielectrics in which the breakdown voltage is reliable, particularly when the voltages required are low. Mr. Lougee probably has a very vivid realization of this from work in development, to a reliable point, of the varnish film on the older form of oxide film arrester. In this case the voltage that was aimed at was something like three hundred volts breakdown per film.

The dimensions of the pellet arresters are not shown, but judging from the cut, Fig. 5, there is a column of pellets, something like six or seven inches long, in the 2300 volt arrester. This would mean something like sixty pellets in series, or sixty miniature oxide film cells, as they are called by Mr. Lougee, in series for 2300 volts.

This represents a normal voltage rating of forty volts or so per cell and means that the breakdown per dielectric film has to be of the order of forty or fifty volts to give a reasonable total breakdown voltage.

It certainly is an accomplishment if solid material has been provided which is reliable in breakdown at voltages of that order. Of course, the curve given in discussion has no bearing on this point since it indicates only characteristics after breakdown.

The matter of life of arresters under discharge, referred to on page 1020, is one of particular importance. More definite information about the conditions in the tests referred to would be of interest.

An arrester, even of the distribution type, has to carry several hundred amperes and sometimes even a thousand amperes of surge current. Laboratory tests which we have made indicate that the life of any kind of arrester varies very greatly with the current. Types that fail after two or three thousand discharges, at an overload condition of 5000 amperes, seem to last indefinitely with currents of the order of two hundred to five hundred amperes. The reference in the footnote describes tests in which the currents are of the order of fifty amperes. It would be interesting to know further about these tests, in order to know their significance, and to have data on life under conditions which approach service conditions in severity.

This point is of particular importance in the type of arresters described. The gradual change toward a condition of open circuit, which is taken care of in the older type with the cell testing device, presumably takes place in this arrester also, since the same chemical action is the basis of performance in both cases. After some length of service it would seem that this gradual change will result in an arrester which is practically open-circuited but which appears from visual inspection to be intact and operative. Inspection by testing is not feasible with distribution arresters. The idea is pretty well established that lightning arresters, particularly for this class of service, must indicate very clearly whether operative or not, even by an inspection from the ground. It would be interesting to know whether this has been taken care of and how.

E. R. Stauffacher: On the first page the statement is made—"To make a lightning arrester these covered pellets are placed in an insulating tube, of say, porcelain, and bare electrodes attached to each end of the tube in contact with the pellets."

I should like to inquire whether the construction of a commercial arrester is such that the wire which connects between the line and the column of pellets goes directly to the column or whether there is an air gap interspersed. Probably I can illustrate this on the board. In the construction as shown in *A*, or as shown in Fig. 2.

If the *B* construction issued, the leakage would be constant, although it would be only of a very small amount, "less than one milliampere when new, and contrary to expectations, continued service does not increase it."

However, if there should be a gradual deterioration of these pellets, or if the leakage remains the same as when new and you had a great many of these arresters, for example, several hundred connected on a distribution circuit, there will be a condition of a multitude of slight grounds on this circuit. This would cause considerable annoyance when every effort is being made to keep the system clear of grounds and confusion would result if each circuit had a ground detector connected to it.

As I understand it, this is primarily a distribution type of arrester used for the protection of transformers at industrial plants and at small pumping plants, for example, such as exist in considerable numbers in Southern California.

The statement is made that the tests of the pellet arresters show practically the same characteristic as that of the oxide film cells with which we are so familiar. However, is there any quantitative value which could be placed upon them? The standard oxide film cell has a certain discharge capacity. A column of pellets has another certain discharge capacity, depending upon the diameter and the length. What is the relation between the two? In other words, would it be possible to substitute a number of those pellet type arresters for one of the oxide film arresters to protect the transformers at a large capacity substation? At some of our smaller substations it is found more convenient to mount an arrester on a pole. If several of these distribution type pellet arresters could be mounted in parallel in place of the ordinary oxide film arrester, probably the construction would be much cheaper.

I would appreciate very much hearing from Mr. Lougee, or any of the discussers as to whether they have any information on this point.

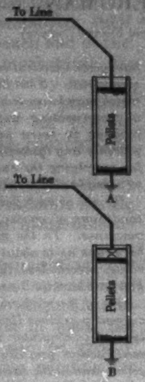

FIG. 2

N. A. Lougee: Mr. Atherton has asked several questions relative to the protective value and life of the pellet arrester. While it may be of interest to know that in spite of so many miniature cells in series, the 60-cycle or low-frequency voltage breakdown is very sensitive, it is the impulse breakdown which we are interested in with lightning arresters. The curve shown by Mr. McEachron gives the maximum voltage drop across the arrester during high-voltage impulse discharges with large values of current. This voltage, measured by a sphere gap, records the maximum voltage appearing across the arrester, and which may be either the initial breakdown voltage or the voltage drop across the arrester during discharge. The results which have been obtained as illustrated by this curve show unusually low voltages, insuring a high degree of protection. As stated in the paper the sensitivity of the arrester, although composed of many films in series, may be due to the fact that the litharge film acts principally as a spacer and not as a solid dielectric.

Mr. Atherton's reference to a former paper may be somewhat misleading. Some of the tests referred to in this paper were surges of arcing ground characteristics and were used to help determine the life of the regular *OF* arrester. The lightning generator has since been developed and is the criterion, of course, in all types of tests. The pellet type has successfully withstood thousands of discharges on this circuit. This older test referred to, the arcing ground, is still of value, as there are very few types of lightning arresters which can successfully withstand this sort of test for any length of time.

Extensive laboratory tests show that testing will not be required. This is apparently due to the number of pellet cells in parallel and because they are affected even less by discharges than the regular *OF* cells.

Mr. Stauffacher asked if a series gap is used. It is used, even though the leakage current is extremely minute, and it is used principally to prevent grounding the circuits and to eliminate the small ground currents which would occur. Although the pellet type was developed primarily for low voltage distribution circuits, extensive tests prove that it is the equal of the *OF* type, and we see no reason why it can not be used to protect any type of apparatus and on any type of circuit.

A Continuous-Current Generator for High Voltage

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***Review of the Subject.**—Of all electro-dynamic machines, the direct-current machine probably is the best understood. It was the first machine to be commercially utilized and it is an interesting matter from an engineering standpoint to follow the evolution which this type of machine has passed through. The first great improvement to be incorporated in this type of machine was the application of commutating poles. As is the case in any radical development, engineering opinion was much divided as to the usefulness of such a construction. At first it was held that commutating poles were only useful where the service was severe, as, for example, in adjustable speed motors covering a large speed range. Today, however, it is well realized that commutating poles are advantageous in all kinds of direct-current machines, except in very small machines of fractional horse power output.*

It is well understood in the art that commutating poles alone do not offer a perfect solution of the direct-current problem, since there exists in such machines troublesome flux distortions. Many attempts have been made to overcome this by aid of distributed windings, and in this paper is described a form of such windings, which have given excellent results both from an operating and an economical standpoint. This distributed winding consists of two parts, one of which may be considered a counterpart of the armature, and is called a compensating winding. This winding opposes the armature reaction in every point around the circumference and effectively prevents any distortion of the flux. There is also a second winding for supplying the excitation and as is described in the paper, this winding is also distributed.

By aid of such field windings, it is possible to meet very extreme conditions of service, for example, as is met with in high-voltage machines. In the machine that is described, the voltage between bars runs up to about 100 volts, or nearly four times the conventional value; nevertheless, the commutation is perfect. Machines rated 15 kw. have been built successfully up to 15,000 volts.

One of the mechanical difficulties in building high-voltage, direct-current machines lies in the building of the commutator. A novel type of commutator is described in the paper. This commutator construction provides simple means for holding the segments together, and at the same time secures high insulation qualities.

A PROBLEM of long standing in electrical engineering, which offers many difficulties both from an electrical and from a mechanical standpoint, is that of producing continuous-current machines for very high voltages. The application of high-voltage continuous current to power transmission has been seriously considered, and to a small extent carried out abroad by Thury. The Thury system consists of a number of series-excited generators coupled in series, each generator being designed for a moderate voltage of a few thousand volts. While this system has been actually carried out in a few instances abroad, it has not had any application in this country, and since the generators are only of moderate voltage, their structural features, viewed from the standpoint of high voltage, do not appear to be of any great interest since the voltage that we are considering ranges as high as from 12,000 to 20,000 volts per generator.

In the early development of arc lighting, machines for high voltage were produced to give unidirectional, but pulsating current. The voltage ran as high as 10,000 volts per circuit. Examples of this type are the well-known Thomson-Houston arc generator and the Brush arc machine. These machines cannot, however, be considered continuous-current machines since they produce a pulsating current. They are of the open coil type and come nearer to that type of apparatus known as rectifiers. A very marked arcing occurs on the rectifying segments, and since the current is pulsating, this type of machine cannot be used for any purpose requiring a non-pulsating voltage.

Lately, a demand has been created for continuous current machines of high and steady voltage for wireless work. By means of vacuum tubes, the direct current is converted into high-frequency alternating current. This system has proved to be successful and possesses a great many advantages. The purpose of this article is not to discuss the system as a whole, but to describe the design and construction of continuous-current generators built for wireless applications and ranging in voltage from 12,000 to 20,000 volts. It is believed that this type of machine possesses a number of novel features of general interest.

Early experiments with high-voltage, continuous-current machines for wireless work have shown that under certain conditions, difficulties arose due to pulsations in the current wave. At first, it was thought that this was due to the fluctuations inherent in any commutator machine, since the resultant voltage in such machines is, in reality, a rectification of a large number of coils. However, these fluctuations often became much larger than the theoretical value, and an investigation disclosed that these fluctuations were mainly due to faulty commutation. If the current is not properly commutated, it is a well-known fact that the extra currents react magnetically upon both the exciting and commutating circuits, resulting in changes in the induced voltage. It is, therefore, essential that the commutation should be absolutely perfect.

Another difficulty that is met with due to the characteristics of the tubes, is the possibility of dead short circuits. The machine must, therefore, be built to withstand short circuits without flashing, since flashing at this high voltage may seriously damage the commutator and brush rigging.

Another difficulty in designs of this nature lies in the mechanical limitation of the number of segments in the commutator. It is generally considered that in con-

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tinuous-current machines, the average voltage between bars should not exceed 25 volts. In order to limit this voltage, two armature windings are employed, each winding connected to a commutator at each end of the armature. An investigation of the mechanical dimensions of a 15-kw. generator showed that by employing the largest number of segments that were practical per commutator, the average volts per bar become about 90 volts, or nearly four times the conventional value.

One of the serious questions is, therefore, whether it is possible to build a continuous-current machine having 90 volts per bar which will withstand instantaneous short circuit without flashing.

It is well understood that where commutation conditions are severe, as in motors which must operate over a wide range of speed, or in machines which have to endure heavy and sudden fluctuations in load, the operation of direct-current machines can be greatly improved by employing a compensating winding for neutralizing the armature reaction. A well-known form of a compensated machine consists of a concentrated form of exciting winding with definite poles and a compensating winding laid into the pole faces. Experiments have shown that a still better operation can be obtained if both the exciting and the compensating windings are distributed. Windings of this type were, I believe, first proposed by Deri, but, as far as I am aware, this type of winding has not been employed to any large extent in the industry due to the additional expense of such windings, both due to material and labor. The writer has, however, found that by distributing the compensating and the exciting windings in a certain way, it is possible to obtain results which are electrically perfect, mechanically simple and economical with respect to material.

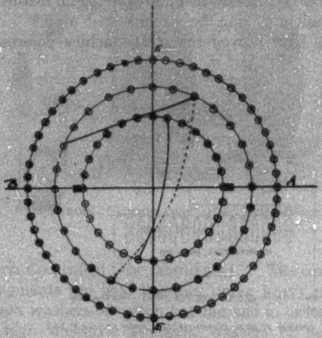


Fig. 1

In order to explain the arrangements proposed by the writer, reference is made to Fig. 1, representing a two-pole machine. The inner circle represents the conductors of the armature winding, the middle circle represents the conductors of the compensating winding, and the outer circle represents the conductors of the exciting winding. The current distribution in the conductors is shown by means of the usual convention in which crosses and dots applied to the conductors indicate respectively that the current is flowing away from or toward the reader. The direction of the armature reaction is obviously indicated by the line *A B*. The compensating winding is so arranged that the current in each conductor is flowing in the opposite direction to the current in the opposite conductor in the armature. The compensating winding accordingly produces a resultant magnetomotive force in the direction *B A*, opposite to the armature reaction. The exciting winding should be so arranged that its magnetic polarization is along the line *E E*, *i. e.*, displaced 90 electrical degrees in space from the line *A B*.

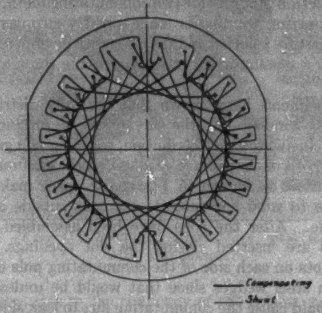


Fig. 2—Field Winding for 12,000-Volt, d-c. Generator

Referring again to Fig. 1, it is indicated that the armature winding is a full-pitch winding. The compensating and the exciting windings would, most naturally, also be laid out as full-pitch windings in which case the end connections would be represented by the dotted line in Fig. 1, in such case the compensating winding would be a counterpart of the armature winding mechanically as well as electrically. The writer, has, however, found that exactly the same distribution of the field winding can be obtained by the use of a 50 per cent pitch winding, one single coil being indicated by the full drawn line in Fig. 1. In Fig. 2 all of the coils are shown, and it will be observed that in each slot there is a compensating as well as an exciting coil, and that all of the coils together form a mechanically single winding similar to lap windings as they are often used in the field of induction motors. It is interesting to note that the armature reaction of a full-pitch armature winding is neutralized by a completely distributed winding of a half pitch. It is obvious that by the use of a half pitch winding in the field, a large economy is obtained as compared with a full-pitch winding, because both windings exert exactly the same amount of magnetomotive force by the use of the same number of turns, but the length of the turns of the half pitch windings is far shorter than in the full pitch winding. From Fig. 2 may be seen that the teeth of the field are evenly distributed except that one tooth on each side of the commutating tooth is left out, thereby creating a wide neutral zone.

An examination of Fig. 2 reveals the fact that the construction provides for two commutating teeth. The compensating winding is made slightly stronger than the armature reaction and the difference in ampere turns sets up a commutating flux through the commutating teeth.

Experiments have shown that the fully distributed exciting winding has the advantage of producing a nearly sine wave flux distribution, resulting in a gradual building up of the voltage gradient on the commutator.

The slots as shown in Fig. 2 are open, making it possible to wind and insulate the coils outside of this machine. After the coils have been assembled, steel wedges are inserted closing the slot openings. The wide slots on each side of the commutating pole do not contain steel wedges since that would be undesirable as it would cause the commutating flux to leak sideways instead of penetrating the armature.

The most important feature of this new machine is illustrated in Fig. 2 and the advantage of this construction may be summarized as follows:

First: A mechanically simple single-coil winding is used, which winding is economical with respect to material, due to the use of a 50 per cent pitch.

Second: A single compensating winding is used both for the purpose of neutralizing the armature reaction and producing a commutating field.

Third: The exciting winding is fully distributed giving the advantage of being placed close to the armature, thus securing a minimum of leakage, and also giving a nearly sine wave flux distribution.

Fourth: Wide neutrals are provided so as to remove the exciting flux from the coils undergoing commutation.

The armature, as has already been stated, carries two independent windings, each being connected to a commutator. As the bottom winding naturally has more self induction than the top winding, it was found that if the compensating winding was adjusted to give black commutation for the top winding, the brushes on the commutator belonging to the bottom winding would spark. Vice versa, if the compensating winding were adjusted properly for the bottom winding, the top winding would spark. It is obvious that the bottom winding, which possesses the larger self induction, reverses more slowly than the top winding and it was found that this difficulty could be overcome by the use of thicker brushes for the bottom winding than for the top winding. The brushes on the commutator connected to the bottom winding were about one-third thicker than the brushes belonging to the top winding.

In order to further illustrate the field winding, in Fig. 3 are shown the field coils developed in a plane. In Fig. 4 is shown the connection diagram of the machine, and as may be seen, a separate exciter is used in preference to self excitation. The reason for using a separate exciter is two-fold:

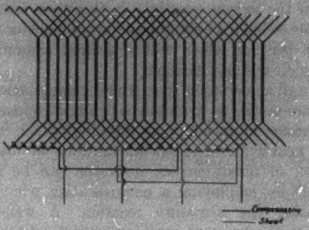
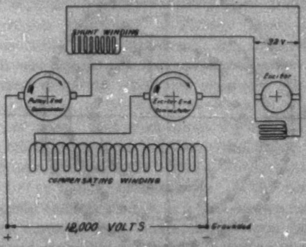


Fig. 3—Internal Field Connections for 12,000-Volt, d-c. Generator

First: It would be difficult to wind and insulate a shunt field for 6000 volts and above, which is the voltage on one commutator.

Second: At a sudden variation of the load, the tremendous induction of the shunt field would cause a discharge through the armature setting up disturbances and possibly making the machine flash over.

Tests have proved that this machine possesses excellent operating qualities. The commutation is perfect up to five times load, which load can be thrown on and off without causing any sparking or the slightest amount of flashing. The excitation can be thrown on and off without any resistance being inserted in the exciting winding. Instantaneous short circuits through the tubes, which have repeatedly occurred, do not disturb the machine in the least.



LOOKING AT BOTH COMMUTATORS FROM EXCITER END

Fig. 4—Connection Diagram for 12,000-Volt. d-c. Generator

During the construction of the first sample machine, serious difficulties arose in the building of the commutators. The first commutators were constructed in accordance with standard practise, the commutator bars being clamped in place by mica cones, the cones being moulded from what is known as pasted mica. It was found that these mica cones would not withstand the required testing voltage, which was set as high as 40,000 volts.

This difficulty was finally overcome by a new method of building commutators, the construction of which is shown in Figs. 5 and 6. The idea originated from the construction of a special gear made by the General Electric Co., called "the fabroil gear." The commutator segments are held together under heavy pressure between washers of cotton batting and in order to obtain high dielectric strength, washers of mica are interspersed between these cotton washers. The commutator is put up under a very heavy pressure, about 5 tons per square inch, amounting to a total pressure on the particular commutator under consideration of 120 tons. As cotton possesses the characteristic of resiliency under heavy pressure, the pressure is maintained at all times and the commutator, therefore, is tight under all conditions. Therefore, there need be no provisions made for tightening the commutator after it is once completed. This construction gives a three-fold advantage:

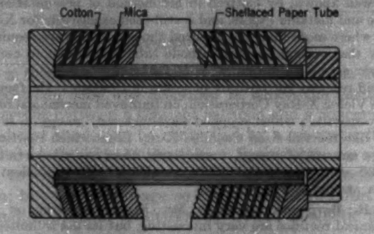


Fig. 5—Fabroil Commutator for 12,000 Volts

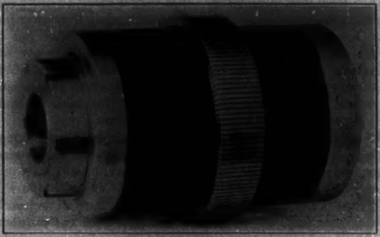


Fig. 6—Fabroil Commutator for 10,000-Volt, d-c. Generator

First: We obtain a large and clean creeping distance to ground.

Second: The multiplicity of mica washers provides an excellent insulation since these washers are placed in series with the voltage, a factor which is not obtainable by constructions heretofore employed in commutators.

Third: The commutator, being under a continuous pressure, due to the resiliency of the cotton, needs no provisions for tightening after completion.

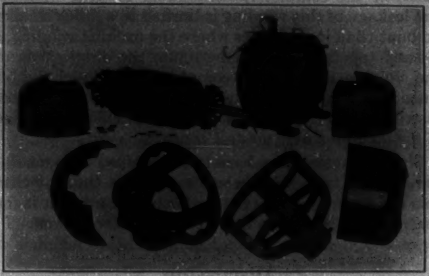


Fig. 7—Type C Y-70-15 kw., 12,000-Volt, 1800 Rev. Per Min. Generator

In order to give an understanding of the appearance of a finished machine, photographs of the disassembled parts, as well as the assembled machine, are shown in Figs. 7 and 8. This machine is rated 15 kw., 12,000 volts, 1750 rev. per min.

From the results obtained from tests, some interesting observations may be made. The very fact that the volts between bars run as high as 90 volts, shows that the individual armature coils contain an unusually large number of turns, which means that the reactance voltage is considerable. Nevertheless, the commutation is perfect. As has already been stated, when conditions of operation are serious, a distributed compensating winding should be used, which statement is

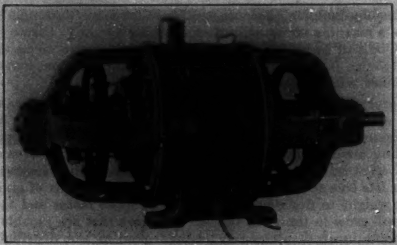


Fig. 8—Type C Y-70-15 kw., 12,000-Volt, 1800 Rev. Per Min. Generator

fully vindicated by the results obtained in this machine. In machines with commutating poles in which no compensating winding is employed, the stability is imperfect due to the fact that the armature reaction is only compensated for along one single line. In machines with fully distributed compensating windings, the armature reaction is neutralized in every point and no disturbances occur due to shifting of the flux. Because of the completely distributed exciting winding, the leakage of this winding is limited to a much smaller amount than in machines where the exciting winding is massed together, and furthermore, the distribution of the voltage around the commutator is much better since this distribution of the winding gives a favorable distribution of the flux securing the particular advantage of a wide and stable neutral zone.

Since great successes have been obtained with generators of this type for wireless work, it is thought that this will open up new avenues for progress and it is for that purpose that this description has been given.

Discussion

P. L. Alger: The chief interest of Mr. Bergman's paper is in the commutation of the machine, that is, the chief theoretical interest. Mr. Bergman is building a machine for proper commutation. To get the commutation we must have low reactance and we must have a very definite commutating flux to make the voltage induced by the flux exactly overcome the voltage of the changing current. Unless a machine is very perfectly compensated, you cannot hold that flux just right because when the load changes, the armature reaction makes the neutral point shift; so the first thing is to obtain very perfect compensation. Mr. Bergman has done this. Another thing that helps commutation is his distributed field winding, as by this means he has a relatively gradual rise of flux on leaving the neutral, as compared with the salient pole winding.

Thus, by building a machine with perfect commutation he has made a machine which will stand anything almost in the way of speed or load variation without trouble and it appears to me that this scheme is also applicable to low-voltage continuous-current machines with similar advantage.

It seems to me the philosophy of the matter is that the machine with salient poles is one which has maximum space factor and maximum flux per pole, whereas Mr. Bergman's distributed winding machine is one which has a maximum fixity of wave form; that is, the flux is exactly placed by the winding instead of being left free to shift to and fro. I believe this idea of giving more weight to flux distribution than to space factor and total flux will grow in time and that all machines that are capable of compensation will ultimately change over to the distributed winding instead of the salient pole type.

H. Lemp: Mr. Bergman mentioned another form of producing unidirectional currents for high voltages, referring to kenotron or vacuum tubes, but he omitted to make any reference whatever to the mechanical synchronous rectifier. While I do not wish to put in a plea for myself, I would like to plead for a child of my brain which has survived in the form of a mechanical rectifier for producing high-voltage unidirectional currents, and is particularly useful for X-ray work.

With the advent of the X-rays, there arose a demand for a continuous source of excitation of high voltage susceptible of furnishing large quantities of unidirectional currents; various forms were tried, and the Holtz influence machine which was at that time considered the most desirable source of excitation, was found ineffective on account of lack of sufficient energy behind it, particularly in wet weather, and it was then that the mechanical rectifier was developed by me in order to produce large quantities of these unidirectional currents unaffected by weather conditions and of sufficient high frequency to give a continuous non-flickering illumination, so necessary for fluoroscopic examinations.

This type of mechanical synchronous rectifier is now used, as is well known, by Cottrell in his method of precipitation of smoke and metallic vapors, and is still used exclusively for X-ray work where more than one hundred thousand volts are required. Below that voltage the hot cathode tube is sufficient to rectify its own supply.

I had recently the pleasure of seeing in Chicago at the factory of the Victor X-Ray Corporation, an improved mechanical rectifier in operation which supplied simultaneously energy to something like ten full-sized Coolidge X-ray tubes which represents a large amount of energy, particularly when one considers that 200,000 volts are used, which is substantially equivalent to sparks of from 12 in. to 14 in.

S. R. Bergman: I think Mr. Lemp's remarks on the mechanical rectifier are very interesting, but its use is limited to certain applications; for example, for X-ray work and for precipitation of smoke and metallic vapors.

The mechanical rectifier, however, for producing actual power does not seem very promising. It is very difficult to drive such a rectifier in exact synchronism, particularly when the power factor of the load changes. Since the mechanical rectifier not only changes the direction of the current, but also the wave form, particularly if we wish to derive a steady current, it is obvious that there exists a large differential current which causes sparking, which, particularly on short circuit, becomes so violent that it generally puts the rectifier out of business.

The mechanical rectifier generally gives rise to pulsations, in that respect, showing a similarity to a direct-current generator having two segments per pair of poles corresponding to a pulsation of 100 per cent. One of the objects of the machine which I described in my paper is to give as steady a current as possible. In a d-c. machine, the pulsations decrease with the number of segments per pole; for example, with 90 segments per pair of poles, the pulsations amount to 0.03 per cent. As a matter of fact, the pulsations due to the segmental arrangement of the commutator in a d-c. machine are generally smaller than the variation caused by the pulsation of the flux due to the variation of the reluctance of the magnetic circuit. It is a well-known fact that due to the presence of teeth in the armature, a certain amount of pulsation of reluctance takes place causing variation of the flux. Although, by choosing the number of teeth in the armature properly as compared with the number of teeth in the field, it is possible to limit this variation so as to make it unobjectionable. In a machine of the character which I describe, we manage to keep the pulsations down to 0.1 per cent. I believe it would be extremely difficult to make a mechanical rectifier having so small a variation.

The Axially Controlled Magnetron

A New Type of Magnetron, Controlled by Current Through the Filament

BY ALBERT W. HULL

General Electric Company

Review of the Subject.*—In vacuum tubes with straight filaments of large diameter, it is found that the magnetic field of the heating current exercises a restraining effect on the escape of electrons, equivalent to the action of the grid in the pliotron, or the impressed magnetic field in the magnetron. This new valve principle may be utilized to control the output of the tube for practical purposes, such as changing high-voltage direct current into alternating current; and leads to a very simple and efficient type of tube.*

CONTENTS.

Review of the Subject. (85 w.)
Introduction. (120 w.)
Theory. (450 w.)
Table I.—Critical Voltages for Tungsten Filaments at 2500° K. (225 w.)
Experimental Tests. (1650 w.)
Applications. (750 w.)

INTRODUCTION

In a former number of this journal[1] a simple two-electrode tube was described in which the electron current between a straight cathode and a concentric cylindrical anode can be controlled by a magnetic field parallel to the axis of the tube. Such tubes are very easy to construct in comparison with three-electrode tubes; but the advantage so gained is partly offset by the requirement of an externally wound magnetic field coil.

The purpose of this paper is to show that in the case of large magnetrons such as are of interest for engineering applications, the magnetic field of the heating current which flows through the filament is sufficient for controlling the electron flow, so that the external magnetic field coil is unnecessary.

THEORY

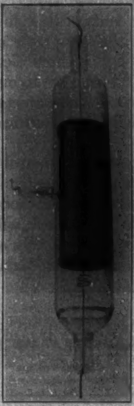
Fig. 1 show a typical magnetron, consisting of a straight filament in the axis of a concentric cylindrical anode. The lines of magnetic force produced by the heating current are circles about the filament. Electrons passing from cathode to anode across these lines of force are deflected at right angles to the lines, as illustrated in Fig. 1B. The deflection is greater the larger the filament current, and with a sufficiently large filament current the electrons miss the anode entirely and return to the cathode (Fig. 1C).

Calculation shows that, with a given potential difference, E, between cathode and anode, the electrons should all reach the anode or fail to reach it, according as the current I through the cathode is greater or less than a critical value, given by the equation

$$E = 0.01882\, I^2 \left(\log_{10} \frac{D}{d}\right)^2 \qquad (1)$$

D, d = diameters of anode and cathode respectively.
I = current through cathode in amperes.
E = critical potential difference between anode and cathode, in volts.

The current I in Eq. 2 may be expressed in terms of the diameter of the cathode, for any given temperature.[2]



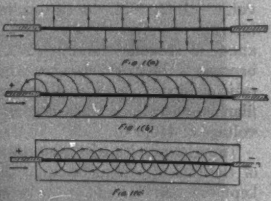


Fig. 1—Small Magnetron in Glass

(a) Longtudinal cross-section through the filament showing paths of electrons with no filament current.

(b) The same for weak filament current.

(c) The same for strong filament current

Thus for tungsten cathodes at 2500 deg. K. Eq. (2) becomes:

$$E = 0.000716\, d^3_{mils} \left(\log_{10} \frac{D}{d}\right)^2$$

or

$$E = 44100\, d^3_{cm} \left(\log_{10} \frac{D}{d}\right)^2 \qquad (2)$$

(tungsten cathode of dia. d at 2500 deg. K. in axis of cyl. of dia. D)

For the general case of a tungsten cathode at any temperature near 2500 degrees absolute, the critical voltage may be expressed with sufficient approximation by the eq.:

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[1] JOURNAL of the A. I. E. E., Sept. 1921, pp. 715-23; also *Phys. Rev.*, 18, 31-57, 1921.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

[2] Characteristics of tungsten filaments, Langmuir, *Phys. Rev.*, 7, 302-30, 1916; *G. E. Rev.*, Mar., 1916.

$$E_{volts} = 44100\, d^3_{cm} \left[\log_{10} \frac{D}{d}\right]^2 \left[1 + 0.00143\,(T-2500)\right] \tag{3}$$

Eq. 3 shows that the critical voltage, that is, the voltage below which the magnetic field of the heating current is able to hold back the electrons, increases as the cube of the filament diameter, for the same temperature and relative size of electrodes. For example, in the "peanut" pliotron, *UV*-199, which has a filament of 0.0006 inches diameter, surrounded by a grid of approximately 0.06 in. diameter, operating at 1900 deg. K., the critical voltage is 0.0003 volts. For a 0.0085 in. filament, such as is used in the 50 watt pliotron *UV*-203A, the critical voltage (assuming the same ratio of anode to cathode diameter) is $\left(\frac{85}{6}\right)^3$ or 2800 times as great, viz. 0.84 volts[3]. Increasing the filament diameter to 0.085 inches would again increase the critical voltage 1000 fold, to 840 volts. Another ten fold increase of cathode diameter, with the same ratio of anode to cathode diameter (100 to 1), is impractical because of the size of the anode; but taking a ten to one ratio of anode to cathode diameter, a 0.85 inch diameter filament operating at 2500 deg. K. would have a critical voltage of 440,000.

The critical voltage increases less rapidly than the cube of the cathode diameter when the anode diameter is kept constant, instead of the ratio of anode to cathode diameter. A few examples, tabulated in Table I, will give an idea of the cut-off voltages for different sizes of filament heated by direct current, in a two-inch diameter anode.

TABLE I
CRITICAL VOLTAGES FOR TUNGSTEN FILAMENTS AT 2500° K

Diameter of Anode cm.	Diameter of Filament cm.	Critical Voltage volts
5.00	0.0025	0.0075
5.00	0.025	3.6
5.00	0.100	127.
5.00	0.250	1140.
5.00	1.00	21,600.
5.00	2.50	62,300.

It is seen that for all cathodes used in standard tubes at the present time, the effect of the magnetic field of the heating current is entirely negligible. For large cathodes, on the other hand, such as are needed for power tubes, the magnetic field of the heating current is able, even at the highest voltages used in engineering, to inhibit entirely the electron current between cathode and anode; *i. e.*, to give the tube infinite resistance. It is thus a factor that must be considered in the construction and operation of large tubes, both as a possible limitation and as a means of useful control, performing the function of the grid of the pliotron. Experimental tests of this method of controlling electron current, and possible applications of it, are described in the following pages.

EXPERIMENTAL TESTS

According to the theory outlined above, the electron current from a straight filament to a concentric cylindrical anode should be zero for all voltages below the critical value, given by Eq. 1; while for all voltages above this critical value the magnetic field should have no effect, *i. e.*, the electron current should be limited only by the temperature of the filament or the mutual repulsion of the electrons in the space (space charge limitation.)

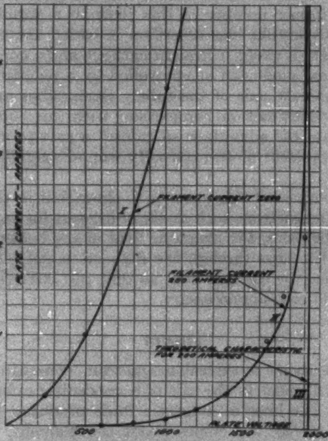

FIG. 2—VOLT-AMPERE CHARACTERISTIC OF TUBE SHOWN IN FIG. 1

Filament diameter 100 mils, anode diameter 4 in.

Fig. 2 shows the currents observed, as a function of voltage, in a tube containing a 100 mil (0.25 cm.) diam. filament in the axis of a cylindrical anode 4 inches (10 cm.) in diam. by 12 inches long. Curve 1 shows the ordinary space-charge characteristic observed in the absence of magnetostriction. These values were obtained by opening the filament switch, and observing the plate current before the filament had time to cool. Immediately upon opening the filament switch the plate current rises to the value set by space-charge, remains constant for an instant, then falls rapidly as the filament cools to the point of temperature limitation. Curve 2 shows the currents observed when the heating current of 200 amperes was flowing through the filament. The characteristic predicted by theory is represented by the vertical straight line at 1930 volts (Curve 3).

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[3] Both of these values of critical voltage are meaningless, as they are scarcely larger than the average initial velocity of the electrons, and much smaller than the voltage drop in the filament.

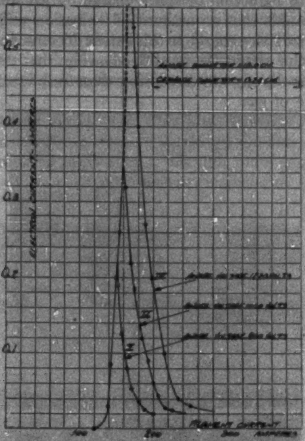

FIG. 3—VARIATION OF PLATE CURRENT WITH FILAMENT CURRENT AT CONSTANT ANODE VOLTAGE

It is seen that the magnetic field of the filament current has a profound effect on the characteristic, reducing the plate current to essentially zero at all plate voltages

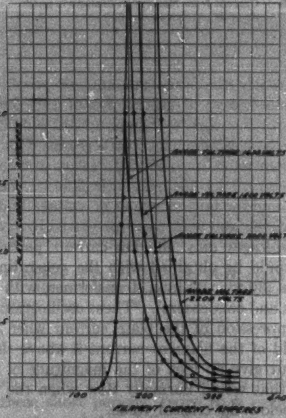

FIG. 4—VARIATION OF PLATE CURRENT WITH FILAMENT CURRENT AT CONSTANT ANODE VOLTAGE
Higher Voltage than in Fig 3.

below about 1400 volts. The fact that the transition from zero current to full space charge current is not abrupt, may be partly accounted for by the crookedness of the filament, which was bowed about an inch out of center at its middle point; and partly by deflection of a small fraction of the electrons by collision with gas molecules. All the characteristics obtained show slight transition currents of this kind, but their magnitude is always small compared with the current under control.

Fig. 3 shows the variation of plate current with filament current at constant anode voltage for the same tube (filament 100 mils diameter, anode 4 inches diameter by twelve inches long. Curve 1 was taken with 800 volts on the anode, curve 2 with 1000 volts, and curve 3 with 1220 volts. The left hand, ascending branch is the same for all voltages, and depends only on the temperature. It represents the temperature-limited saturation emission from the filament, and is given accurately by the equation.[4]

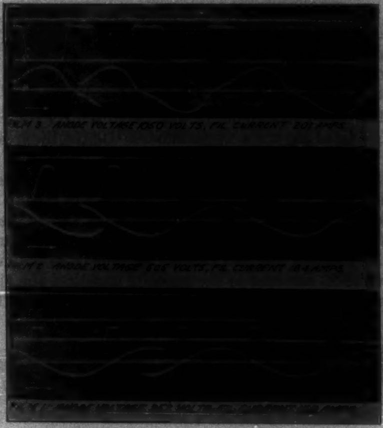

FIG. 5—THREE FILMS SHOWING THE CONTROL OF PLATE CURRENT BY 60-CYCLE FILAMENT CURRENT IN A SMALL MAGNETRON, WITH CONSTANT ANODE VOLTAGES OF 250, 505, AND 1050 VOLTS DIRECT CURRENT

Record A — Plate current of rectangular form with the zero value at A.

Record A to 1 — Deflection, for calibration, given by a plate current of 1.4 amp. in film 1; 4.0 amp. film 2; and 6 amp. film 3.

Record B — 60-cycle filament current.

Note that the plate current flows only when the filament current is below a definite value.

$$i = 60.2\,T^{2} e^{-\frac{52600}{T}} \qquad \frac{\text{amperes}}{\text{cm}^{2}}$$

The right hand descending branches are due to magnetostriction. It is seen that for each constant anode voltage the electron current begins at a definite heating current (minimum temperature for emission) and increases rapidly with heating current up to a critical point. This critical point is the "critical filament current" given by Eq. 1. When the heating current is raised above this critical value, the electron emission falls rapidly to essentially zero because of the magnetic effect of the heating current.

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[4] Dushman, *G. E. Review*, 16, 153-60, Mar. 1923 and *Phys. Rev.*, Apr. 1923.

Fig. 4 shows a similar series of characteristics with higher plate voltages and larger currents. At the highest filament current, 325 amperes, the filament is at 3300 deg. K., and is capable of emitting approximately 2000 amperes of electrons. The actual emission at 1600 volts is only 20 milliamperes.

Fig. 5 shows an oscillographic record of the plate current in the same tube when 60-cycle alternating current is used to heat the filament. In this case the filament temperature remains constant while the instantaneous value of the heating current changes, so that the full space-charge current flows during that part of each cycle in which the heating current is below the critical value, and no current during the part when the field is above the critical value.

The lower film (film 1) was taken with a constant potential of 250 volts on the anode. During the greater part of each cycle the filament current is above the critical value for 250 volts, and the plate current is zero, as it should be. When the filament current falls to the value 73 amperes, plate current begins and increases rapidly to the maximum value allowed by space charge. As soon as the filament current again reaches the same value of 73 amperes in the opposite direction, the plate current begins to fall, and drops rapidly to zero. The plate current is thus interrupted twice each cycle, since both directions of filament current are alike in their magnetostrictive effect.

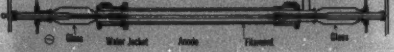


Fig. 6—Construction of High-Power Magnetron

The second film was taken with 505 volts on the plate. The space current is larger and flows for a longer part of each cycle, beginning and ending when the filament current reaches the value 101 amperes. The critical value given by Eq. 2 is 102 amperes. The filament current in this test (184 amperes) was larger than that used for the first film, and the curve shows a slight distortion due to saturation in the filament transformer.

For the upper film (film III.) a plate voltage of 1050 was used, and a larger filament current (201 amperes). The filament current curve shows bad saturation distortion, so that the distance from the point where it crosses the axis to the point where it reaches its critical value is very different above and below the axis. In spite of this, the plate current begins and ends accurately at the critical filament current of 157 amperes, which agrees with the theoretical value of 149 amperes within the error of measurement.

The observed plate currents in each of these oscillographic tests are about 3 times as large as the value to be expected from the space charge law. The increase is due to reduction of space charge by gas ionization. It is notable that an amount of ionization sufficient to neutralize space charge to this extent does not noticeably affect the magnetic cut-off, showing that the magnetron is slightly less susceptible than the pliotron to the presence of gas.

3. A series of tests with much larger filaments has been carried out by Mr. J. H. Payne. Exact measurements are more difficult in this case, because of the inability of ordinary generating systems to deliver intermittently the large amounts of power required, and the difficulty of getting rid of this power when delivered. For low voltages, up to 2000 volts, the tests are very satisfactory (oscillograms, Figs. 7 and 8). Electron currents up to 50 amperes are seen to be under perfect control by the magnetic field of the filament current; and the critical values of voltage and filament current agree well with those predicted by theory. At higher voltages only the low-current portion of the curves could be obtained with the generating system available (30 kv., 20 kv-a. transformer, with kenotron rectifiers and condensers). These oscillograms (Fig. 9) show suppression of the plate current when the magnetic field of the filament current is above the critical value, but give no idea of the magnitude of the plate current that would normally flow when the magnetic field is below the critical value.

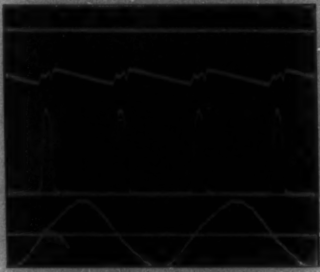


Fig. 7—Control of Plate Current by 60-cycle Filament Current in a High-power Magnetron. Filament dia. = 1 cm. Anode Dia. = 4.6 cm.

Record A — Voltage between filament and anode (deflection downward)
Record B — Current from filament to concentric anode.
Record C — 60-cycle current in the filament.
Note that the current between filament and anode is zero except when the filament current is below a critical value.

The tube used for these tests is shown in Fig. 6. The anode is a copper tube 1 13/16 inches (4.60 cm.) in diameter by 30 in. (76 cm.) long, surrounded by a water jacket. The cathode is a tungsten rod 0.400 inches (1 cm.) in diameter, 22 inches (56 cm.) long. Upon each end of the cathode is tightly swaged a short bar of molybdenum, which is threaded so that it can be screwed into the copper lead. A short section of glass at each end insulates the filament from the anode.

The results of these tests are shown in Figs. 7, 8 and 9. Fig. 7 shows a test at 1700 volts anode potential. This potential was furnished by a 20-kw., 3000-volt d-c. generator, with 50 μf. across its terminals to stabilize the voltage during the sudden starting and stopping of the current. In spite of this, the voltage dropped 25 per cent during each current pulse, and then rose gradually during the remainder of the cycle, as shown in curve *A*. Curve *B* shows the electron current that flowed from cathode to anode and thence in the plate circuit. It is seen that this current is completely suppressed except during the short fraction of each cycle when the filament current is below a critical value. During these short periods, twice each cycle, it rises to 36 amperes, and then falls abruptly to zero as the filament current again reaches the critical value. The filament voltage, which is nearly in phase with the filament current, is shown in curve *C*.

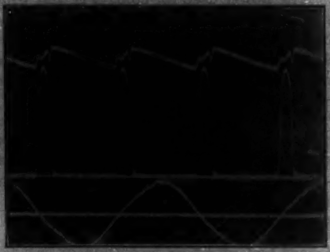


Fig. 8—Same Condition as Fig. 7 Except the Voltage and Current Between Filament and Anode are Larger

Fig. 9—Control of Plate Current by 60-cycle Filament Current in High-Power Magnetron. Filament dia. = 0.87 cm., Anode Dia. = 7.5 cm.

Record A — Plate voltage (deflection downward).
Record B — Plate current (deflection upward).
Record C — 60-cycle filament current.

Fig. 8 shows a similar test at 2100 volts. The anode current in this case rises to 45 amperes.

Fig. 9 shows a test at 8000 volts. The electron current (curve B) is limited by filament temperature to 1.7 amperes, and even with this small current the anode voltage (curve A) falls from 8000 to 3000 volts during the short fraction of each cycle when power is being drawn. The small residual current of ¼ ampere, due to gas and end leakage, shows prominently on the large scale of the oscillogram. It is entirely negligible however, compared with the normal useful current of 50 amperes which the tube can control. In this test the cathode was smaller, viz., 0.350 inches diameter, and the anode 3 inches diameter.

Preparations are being made for tests at higher voltage with larger amounts of power. A few tests already made at 20,000 and 30,000 volts, show that the magnetic control is effective at these voltages, but the currents available were too small for satisfactory oscillograms.

Applications

The magnetron, like all high vacuum thermionic tubes, is a high resistance device, so that its use is limited to either small power or high voltage. The type of magnetron here described suffers the further restriction that it is operable only with large diameter filaments, and hence is adapted only to high power applications. For these applications, however, it appears to be the simplest and most efficient tube that has yet been studied.

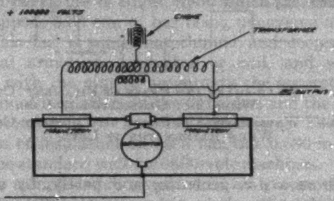
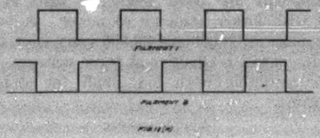


Fig. 10—Circuit for Conversion of Direct-current to Single-phase Alternating Current by Means of Magnetrons

A single example will serve to illustrate the method of operation and the efficiency to be expected. The problem chosen is the conversion of high-voltage direct current into alternating current. Let the direct-current voltage be 100,000 and the alternating-current voltage anything that is desired. A tube is chosen similar to that shown in Fig. 5, having a cathode of 2.0 cm. (0.79 in.) in diameter by 1 meter long, and an anode 8.0 cm. (3.1 in.) in diameter. A current of 4400 amperes heats the filament to 2500 deg. K., at which temperature its electron emission is 170 amperes; and the magnetic field of this heating current is capable of making the magnetron a non-conductor at 200,000 volts.

For conversion to single-phase alternating current two of these tubes may be connected as shown in Fig. 10. Current passes from the positive d-c. terminals, through a choke coil, to the middle point of the high-voltage winding of a transformer, whose terminals are connected to the anodes of the two magnetrons. The cathodes are connected to the negative d-c. terminal, and are heated by square-wave current of the frequency of the desired alternating current. This square-wave current may be obtained in a variety of ways. For example, it may be derived from a d-c. generator and commutator, giving the wave form shown in Fig. 11A; or from a quarter-phase alternator of proper design, giving the wave form shown in Fig. 11B; or it may be derived partly from a d-c. generator and partly, by suitable coupling, from the alternating-current output, provided the output circuit contains a frequency-determining element, or is tied to a system of fixed frequency.



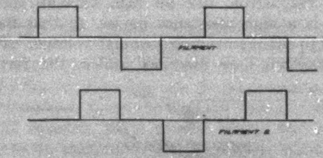


FIG. 11A— WAVE-SHAPE AND PHASE OF FILAMENT CURRENTS WHEN D-C. GENERATOR IS USED

FIG. 11B— WAVE-SHAPE AND PHASE OF FILAMENT CURRENTS WHEN ALTERNATOR IS USED

The operation is as follows: Current from the d-c. line flows through the choke and the transformer winding to the magnetron anodes, and then through one of the magnetrons—the one whose filament current is zero—to the negative d-c. terminal. If the load is adjusted for 100 amperes in the d-c. line, a voltage of 3000 volts will be required to carry this current through the tube, the remaining 97,000 volts being supported by the reactive drop in the transformer.

Just before the magnetron that has been carrying current becomes a non-conductor by the establishment of filament current through it, the filament current in the other tube becomes zero and the load is transferred to it. The slight disturbance incident to this transfer is absorbed by the choke coil.

The total power converted by this pair of tubes is 10,000 kw. (100 amperes × 100,000 volts); and the efficiency, exclusive of transformer losses, is 96 per cent (150 kw. per tube dissipated at anode, 50 kw. in filament). The power used for control is that which heats the filament, and the total amount of power needed for this purpose, in case it is furnished by a separate generator, is 100 kw., or 1 per cent of the output. The calculated life of the filament, which is taken arbitrarily as the time necessary to reduce its diam. by 5 per cent by evaporation, is 60,000 hours, or approximately 7 years of continuous operation. The life will be shorter than this if there is appreciable disintegration of the filament due to bombardment with gas ions. The amount of this disintegration cannot be predicted from data at present available.

The simple single-phase conversion system just described gives square alternating-current waves. This is an advantage, if the alternating current, after being stepped down to the desired voltage, is to be again rectified and used as direct current. Sine waves of any required degree of purity may be obtained by the use of an appropriate number of these single-phase units, connected according to well known multiphase circuits. A system containing 6 single-phase units (12 tubes) would give an output of 60,000 kw. of fairly good wave-form, with an efficiency of 96 per cent.

It is seen that the power capacity of these tubes is ample for conversion purposes, and their efficiency, so far as can be predicted at present, compares favorably with that of other types of conversion apparatus. Unquestionably many problems remain yet to be solved before high-voltage operation of these tubes is practical; but there is no known obstacle or limitation in sight. It is hoped that actual conversion tests, as well as static characteristics, at high voltages, will soon be available.

I wish to acknowledge the very able assistance of Mr. F. R. Elder and Mr. E. F. Hennelly in carrying out these tests.

Discussion

For discussion of this paper see page 953.

Gaseous Ionization in Built up Insulation

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Review of the Subject.—*A study of a series of 6600-volt armature bars with different types of insulation, with special reference to the influence of entrapped air spaces on the dielectric losses, power factor, and life of the insulation. The variation of the losses between the green and dried states was studied.*

Gaseous ionization losses begin at about 4000-volts, above which the losses increase more rapidly than as the square of the voltage. In well wrapped mica folium insulation from 10 per cent to 40 per cent of the total dielectric losses are due to internal ionization. Ionization losses, while not being of sufficient amount to materially increase the temperature of the coil, do, however, cause progressive deterioration of the insulation. The total dielectric losses in mica folium and paper insulation increases with decreasing mica content. Reducing the mica folium by 75 per cent and substituting shellaced paper approximately doubles the loss. Omission of the final 25 per cent of mica increases the original loss ten times. The loss in an all paper bar is probably almost entirely due to conductivity. The principal function of mica in this type of insulation is the reduction of the conductivity, and withstanding the action of internal ionization.

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THE presence of entrapped air or other gases in laminated insulation of all kinds, and their breakdown under high-voltage gradient have been suspected for some time. Simple experiments[1] will show visually the sudden breakdown of air in very thin layers. Loss and power factor curves on cable insulation[2], show more or less sharp rises or breaks with increasing voltage, and these have been attributed to the setting in of ionization in entrapped gases. Sheet insulation[3] separated by air layers shows these sharp breaks with great clearness, and in this case there is no doubt as to the cause. The insulation of the armature bars of high-voltage alternating-current generators often shows pitting and other evidence of internal brush discharge, and it is well known that such machines often develop invisible spark discharge in the short air spaces between armature insulation and core.

The high-voltage armature bar presents perhaps the most important of all problems of insulation. It is the weakest point of the machine, and the consequences of its failure are probably more costly or diastrous than in any other case. Owing to the exceptionally high values of voltage gradient which occur in this type of insulation, to the laminated form in which it is applied, and to the shape of the cross section of the usual armature coil, the conditions are highly favorable for the presence of internal ionization. It is important therefore that we know the values of voltage at which the ionization begins, its amount as affecting the dielectric losses, especially its influence on the life of the insulation, and other kindred matters. The following experiments and tests were undertaken in the effort to answer some of these questions.

Types of Insulation Tested

The test samples consisted of wrought iron bars 96.5 cm. long, and cross section 5.08 cm. by 1.27 cm., with standard curvature of edges, upon which the insulation was wrapped, 5.08 cm. of bar being uncovered at each end. This bar is somewhat thinner and deeper than the usual armature conductor or coil, and was so selected with the aim to accentuate the separation of the layers of insulation on the flat sides, and the consequent occurrence of internal ionization. The total thickness of insulation on each bar was 0.090 in. (2.28 mm.), the standard wall for 6600-volt machines.

Thirty samples were prepared in ten groups of three bars each. The three bars in each group were identical as to makeup and assembly. The ten groups were divided into two sets, one having shellaced paper and the other having treated cloth as the principal supporting binder. The five groups in each set contained progressively increasing amounts of mica, the paper and cloth fillers being varied in amount, so as to preserve the same uniform thickness. Owing to the facility with which shellaced paper can be handled in rolls and under tension it was possible to assemble the paper set under high temperature and to apply pressure by ironing, to the successive layers. This resulted in an extremely compact cell of insulation with no apparent lifting of the flat sides. Similar methods could not be followed in assembling the treated cloth set of bars, and although they were wrapped as carefully as possible under tension, on completion a few of them showed a slight yielding to pressure on their flat sides. It is obvious that these two methods of assembly promise thicker layers of entrapped air for the treated cloth set. Important corresponding differences in the two groups are shown by the tests. The makeup of the respective groups of bars is shown in the following Table I.

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1. A list of all references will be found at the end of the paper.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 25-29, 1923.

In addition to the wrapped samples described above, a sample of the insulation of each set of three bars was made up in the form of a flat sheet 44.72 cm. square, effort being made by pressure and temperature to produce a uniform, compact, composite, sheet free from entrapped air. The purpose here was to make a comparative study of the wrapped and the flat insulation, having in mind the greater probability of air layers in the former.

For the electrical tests described below a tin foil electrode 22.32 cm. long, was applied to the central section of each bar. Each end of this electrode was flanked by a guard electrode 5.4 cm. wide, and separated from the central electrode by 0.16 cm. Each of the flat samples was provided with a central circular tinfoil electrode 15.24 cm. diameter, surrounded by a guard ring 2.54 cm. radial width; and on the opposite side an electrode of tinfoil of diameter equal that of the guard rings. The tinfoil was first coated on one side with shellac, dried, and then applied to the insulation with a hot iron. This resulted in a very intimate contact free from blisters or other evidence of entrapped air.

METHOD OF TEST

The purposes of the investigation were to determine, if possible, the values of voltage, or potential gradient, at which ionization begins, the increase in dielectric losses due to ionization, and their influence on the structure and the life of the insulation. For detecting the beginning of ionization either direct visual observation, or some conductivity method as used in the corona voltmeter, is the most sensitive; but obviously neither can be used in the present case. Moreover owing to the wide variation in area and thickness of the entrapped air spaces it is probable that ionization begins at different values of voltage in different parts of the insulation, and so would show a gradual rather than a steep curve of rise by any method. Since the problem requires the measurement of the dielectric losses, it was decided to rely also on the loss measurements for indication of the beginnings of ionization. Ionization should cause more or less sharp breaks in both loss and power factor curves. There is required, therefore, a method of measuring relatively small amounts of power, say of the order of tenths of a watt, as found necessary in some cases, at low power factor, and at voltages in the range 1000 to 20,000 volts. Of the several available methods that have been used for this type of measurement, it was decided to use the quadrant electrometer in wattmeter connection, that is with voltage on the needle, and the quadrant connections taken from the terminals of a relatively low resistance in series with the dielectric to be measured. In order to simplify the problem of the insulation of the needle, it was determined to use the method of applying to the needle known fractional values of the total voltage, by means of taps taken from the high-tension winding of the transformer. This method has been described and used by Miles Walker[4], C. E. Skinner[5], Rayner,[6] Russell[7], Orlich[8] and others. The principal connections are shown in Fig. 1.

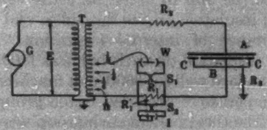

FIG. 1—CONNECTIONS OF ELECTROSTATIC WATTMETER

W is the quadrant electrometer with its needle connected, as desired, to several different points on the high-tension winding of the transformer. The quadrant terminals are connected through a reversing switch S_1 to the terminals of the non-inductive resistance R_1. One terminal of the transformer winding is connected to ground and the other through the non-inductive resistance R_2 to the central bar of the specimen under test, *A*. The central electrode *B* of the specimen is connected to ground through the resistance R_1. The guard electrodes *C C* are connected together and to ground through the resistance R_3. *I* is a Dolacelek quadrant electrometer which is connected through the reversing switch S_2 to the terminals of the non-inductive resistance R_1^1. This instrument serves to read the current to ground from the central electrode *B*. The current and watt readings are taken independently permitting the use of the same resistance for the different values R_1 and R_1^1. The resistance R_2 may be used for correcting the wattmeter reading for the error caused by the resistance R_1, or for the purposes of calibration, as described below. The resistance R_3 may be used for adjusting the difference of potential between the electrodes *B* and *C* to zero. However, in most of the observations, owing to the low values of potential drop over R_1, R_3 could be varied from zero over a wide range without influence on the wattmeter readings, and the electrodes *C* were usually connected directly to ground.

The wattmeter electrometer, especially constructed for this work, had cast brass quadrants of the usual cylindrical box type with an inside diameter of 6 in. (15.24 cm.) and a height of 1½ in. (3.81 cm.). They were mounted on hard rubber chairs with a clearance of 0.381 cm. to the base of the instrument. The needle was of 0.158 mm. sheet aluminum. The suspension was of phosphor bronze strip 20 cm. long. The needle system was insulated by a corrugated hard rubber bushing 5 cm. long and was tested to 2000 volts. During most of the experiments the maximum voltage on the needle did not exceed 1500 volts. Below the quadrants the needle carried mica damping vanes immersed in light oil. The whole instrument was enclosed in a metal case connected to ground and provided with a glass covered opening for taking readings, by means of a beam of light and ground glass scale at about one meter's distance. The resistances R_1, R_2 and R_3 were all placed in metal boxes connected

TABLE I

Bar Nos.	Shellaced Paper		Treated Cloth		Mica Folium		Shellaced Paper		Total Inches
	Layers	Inches	Layers	Inches	Layers	Inches	Layers	Inches	
B-1 (a) 31-32-33					*7 + 4	0.083	2	0.007	0.090
(b) 34-35-36	6	0.020			10	0.063	2	0.007	0.090
(c) 37-38-39	12	0.039			8	0.044	2	0.007	0.090
(d) 40-41-42	18	0.058			4	0.025	2	0.007	0.090
(e) 43-44-45	25	0.090							0.090
B-3 (a) 46-47-48					*7 + 4	0.083	2	0.007	0.090
(b) 49-50-51			2	0.018	10	0.065	2	0.007	0.090
(c) 52-53-54			5	0.045	6	0.038	2	0.007	0.090
(d) 55-56-57			7	0.063	3	0.020	2	0.007	0.090
(e) 58-59-60			10	0.090			2	0.007	0.098

*7 sheets of mica folium and 4 sheets of 0.010 mica plate.

TABLE II
CALIBRATION OF WATTMETER
NON-INDUCTIVE LOAD

E	$R_1{}^1$	Zero	Reading	Defln.	R_1	Zero	Reading	Defln.	$\frac{N\phi}{R_1} \times 10^3$	K
40	2000	−1.4	+7	8.4	300	0	+ 4.0 − 4.35	4.175	1.391	166
	$I = 3 \times 10^{-3}$ amperes $R_2 = 256{,}986$ ohms $I^2 R_2{}^2 = 2.31$ watts $N = 1$				600		+ 8.25 − 8.5	8.375	1.395	
					900		+12.4 −12.7	12.55	1.394	
60	1000	−1.35	+3.6	4.95	200	+.05	+ 6.3 − 6.4	6.35	3.17	166
	$I = 4.55 \times 10^{-3}$ amperes $R_2 = 256{,}986$ ohms $I^2 R_2 = 5.27$ watts $N = 1$				400		+12.75 −12.75	12.75	3.188	
					600		19.2 −18.9	19.05	3.17	
30	600	−1.25	+2.95	4.2	100	+.2	+ 2.8 − 2.25	2.505	30.06	168
	$I = h\,6.98 \times 10^{-3}$ $R_2 = 1.039 \times 10^6$ ohms $I_2 R^2 = 50.81$ $N = 12$				200		+ 5.3 − 4.8	5.05	30.24	
					300		+ 7.82 − 7.4	7.56	30.24	
					400		+10.3 − 9.95	10.125	30.375	
50	400	−1.2	+3.92	5.12	50	0	+ 3.6 − 3.5	3.55	85.20	166
	$I = 11.58 \times 10^{-3}$ $R_2 = 1.039 \times 10^6$ $I^2 R_2 = 140$ watts $N = 12$				100		+ 7.12 − 7.	7.06	84.72	
					150		+10.75 −10.5	10.62	84.96	

to ground. Transformer and test specimen were placed close together and the leads to the instruments were as short and as widely separated as possible, having in mind the importance of removing the needle terminal of the instrument from the electrostatic influence of the high-voltage connection. The transformer had a capacity of 3 kv-a. and a ratio of 100 to 25,000 volts. The high-tension winding was in twelve sections and points were brought out for the intervals 1/12, 1/6, ¼ and ½ of the total winding. One end of the primary and one end of the secondary were connected together and to ground. The 5 kv-a., 110-volt, 60-cycle generator G has a surface wound armature and gives a smooth wave. It is driven by direct-connected 7½ h. p., d-c. motor, which throughout the experiments was supplied from a large storage battery, insuring closely constant frequency at 60 cycles. The frequency could be read at all times by a direct-reading frequency meter showing differences of less than ¼ of a cycle. The test specimen A was placed on insulators in a deep metal trough connected to ground and careful tests made to insure the absence of electro-static influence of the high-tension winding on the electrode B and its connection to ground.

CALIBRATION OF INSTRUMENTS

The usual formula for the loss W as determined by the quadrant electrometer wattmeter is

$$W = N\,K\,\theta/R_1 + \frac{N-2}{2}\,R_1\,I^2 \qquad (1)$$

in which N is the number of times the needle voltage is contained in the total voltage, θ the deflection of the instrument in centimeters of scale, R_1 the resistance in ohms between quadrants, I the current in R_1, and K the constant of the instrument.

The constant K may be determined by measuring the power in a non-inductive circuit. The formula as given may be used with safety for all values of loss when the power factor is relatively high. The second right-hand term disappears for $N = 2$ and its computation for any value of N may be avoided[9] by proper adjustment of the value of the resistance R_2. A more convenient method, however, is to take two readings with different values of R_1 and to extrapolate for the value of $\frac{N\,K\,\theta}{R_1}$, corresponding to $R_1 = 0$.

For low values of power and of power factor, such as usually occur in dielectrics of good insulating power, and such as are to be expected in the present work, the large value of R_1 necessary for a satisfactory reading, may introduce errors due to the passage of the quadrant charging current over R_1, and a change in phase of the current in R_1. The precise correction for this type of error involves a number of constants both of the instrument and of the sample under test, and does not appear to have been worked out in a readily available form. However, Orlich[9] concludes that for ordinary frequencies this error is proportional to R_1 over a considerable range. Thus correction for the loss in R_1, and for the quadrant charging current in R_1, are both proportional to R_1, and so may be corrected for by readings for two values of R_1 and extrapolation to the value for $R_1 = 0$. The test of the validity of this method is found in taking readings for three or more values of R_1; if values of $\frac{\theta\,N\,K}{R_1}$ fall in a straight line the method is reliable. This test and method have been followed in the present work. In calibrating and in all the loss measurements observations have been taken for two or more values of R_1. For each value right and left-hand readings of the electrometer were also taken by reversal of the switch S_1.

The wattmeter was first calibrated for unity power factor by comparison with the watt input to a non-inductive load, current being measured by the Dolacelek quadrant electrometer, (calibrated for volts, in terms of laboratory standards), and voltage by means of standard voltmeters on the low-tension side of the step-up transformers, whose ratios were checked with potential transformers. The load consisted of 1.5 megohms of fine manganin ribbon wound on mica cards, 6000 ohms each, recommended by the Bureau of Standards for pure reactance-free characteristics. This resistance was inserted at R_2 in Fig. 1, the insulated armature bar being omitted. Current and voltage

TABLE III
SUMMARY OF CALIBRATION OF WATTMETER

N	E	V	V_n	Watts	Power Factor	K
6	30	7500	1250	0.43	0.011	170
6	30	7500	1250	0.6	0.015	170
12	30	7500	625	0.41	0.011	166
12	30	7500	625	0.58	0.015	167
12	50	12500	1040	2.85	0.027	168
12	30	7500	625	2.22	0.11	167
12	50	12500	1040	7.67	0.13	166
1	40	800	800	2.31	1.00	166
1	60	1200	1200	5.27	1.00	166
2	40	800	400	2.32	1.00	167
2	60	1200	600	5.27	1.00	167
4	20	5000	1250	44.6	1.00	164
4	25	6250	1560	70.8	1.00	166
6	30	7500	1250	50.3	1.00	163
12	30	7500	625	50.8	1.00	168
12	50	12500	1040	140.0	1.00	166

E = Primary Volts
V = Volts on Load
V_n = Volts on Needle
K = Constant of Instrument

TABLE IV
CALIBRATION OF WATTMETER FOR LOW POWER FACTOR

R_1^1	Zero	Reading	Deflection	R_1	Zero	Reading	Deflection	$\frac{\theta N}{R_2} \times 10^3$
700	−1.95	+1.05	3.00	3,000	−1.3	−1.45 −1.1	−0.175	−0.7
	$R_2 = 0$ Watts = 0.25 Power factor = 0.007 $E = 30, N = 12$			1,500		−1.22 −1.32	+0.05	+0.4
				6,000		−2.65 +0.15	−1.4	−2.8
				9,000		−4.9 +2.6	−3.75	−5.0
				12,000		−8.25 +6.1	−7.175	−7.175
				0				+1.5
700	−1.90	+1.10	3.00	1,500	−1.2	−0.92 −1.28	+0.18	+1.44
	$R_2 = 6{,}287$ ohms $I_2\,R_2 = 0.162$ watts Total watts = 0.41 Power factor = 0.011 $K = 166$ $E = 30, N = 12$			6,000		−2.0 −0.15	−0.925	−1.85
				9,000		−4.05 +1.95	−3.00	−4.00
				12,000		−7.15 +4.95	−6.05	−6.05
				0				+2.48
700	−1.7	+1.3	3.0	1,500	−1.1	−0.80 −1.40	+0.30	+2.5
	$R_2 = 12{,}745$ ohms $I_2\,R_2 = 0.167$ watts Total watts = 0.582 Power factor = 0.015 $K = 167$ $E = 30, N = 12$			6,000		−1.50 −0.65	−0.425	−0.85
				9,000		−3.3 +1.2	−2.25	−3.0
				12,000		−6.2 +4.2	−5.2	−5.2
				0				+3.48

were read at I and E, and right and left deflections of W taken for each of two or more values of R_1. This was done for values of N, 1, 2, 4, 6, and 12. The observations for $N = 1$ and $N = 12$ are given in Table II, and the summary of all the readings in the last division of Table III. The $I^2 R$ loss is computed and equated to $\frac{\theta N K}{R_1}$ giving the value of K the constant of the instrument. The frequency throughout was 60 cycles.

For calibration of the wattmeter for low values of loss at low power factor a 0.002 microfarad Moscicki condenser unit was used. These condensers are of Leyden jar type filled with, and immersed in oil, and with other features of design for preventing brush discharge from the edges of the inner and outer electrodes. The voltage rating was 15,000 volts. The condenser, in series with the variable resistance R_2, was substituted for the sample bar, with the connections of Fig. 1. For each of three values of R_2 a series of observations was taken with different values of R_1. Correcting each series by extrapolation to $R_1 = 0$, the successive increments of $\frac{\theta N K}{R_1}$, together with the corresponding losses in R_2, as determined by the current measurement, will give the desired values of K. Table IV shows one set of observations with the resulting values of K. In Fig. 2, $\frac{\theta N}{R_1}$ is plotted as a function of R_1, showing the linear relation necessary for correcting the errors due to the resistance R_1 and to the capacities between the several elements of the instrument. These data also permit computations of the constants of the Moscicki unit. The approximate values are: capacity 0.002 microfarad, reactance at 60 cycles 1.33×10^6 ohms, effective resistance 1.04×10^4 ohms, loss at 7.5 kv., 0.25 watts, power factor 0.007.

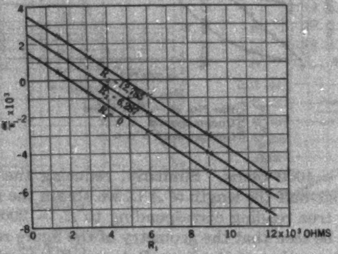

FIG. 2—CALIBRATION OF WATTMETER FOR LOW VALUES OF POWER FACTOR, $E = 30$, $N = 12$

No values of power factor as low as 0.007 were encountered in the tests, although values of loss lower than 0.25 watts were often met. A further check of the constant K was obtained by substituting one of the test bars for the Moscicki condenser in the foregoing method. The results were in all respects similar in character to those described and gave values of K shown in the second division of Table III. Throughout the tests the value for K has been taken as 166.

It appeared desirable to check the calibration of the wattmeter from time to time. For this purpose, in order to avoid the setting up anew each time, of the condenser and its auxiliaries, with the precautions for screening, one of the test bars was used as a secondary standard. This bar was immersed in compound, and after comparison with the original calibration, the reading of its loss and charging current at a definite value of voltage, at regular intervals, was used as a check of the constancy of the wattmeter. No variations were found during the course of the work.

CORRECTION FOR TEMPERATURE

The dielectric losses in insulation vary with the temperature. The law of this variation for one or two of the test bars was investigated and is discussed hereafter. However, since it is a difficult, if not impossible, matter to determine the temperature of the insulation, it was necessary to make the measurements at as nearly as possible constant temperature. In the experiments relating to the variations of dielectric loss among the various samples, this was accomplished by working at atmospheric temperature in a room comparatively free from temperature variation. Most of the observations were made at temperatures between 21 deg. cent. and 25 deg. cent. and in nearly every case the temperature variation for the observations on a single specimen was less than 1 deg. In this method therefore a small error is introduced in comparing actual values of losses of two specimens. The observations on individual specimens are free from appreciable temperature error, except perhaps at the very highest values of voltage. At high values of voltage in the case of nearly every specimen the loss is sufficiently high to cause an elevation of temperature during the time interval required for the taking of readings. The wattmeter was heavily damped and required two minutes to come to rest for any one reading. Taking right and left deflections for each of two or more values of the resistance R_1 therefore, requires a minimum of eight minutes for the making of one loss measurement. An additional two minutes is required for making the measurement of charging current, giving therefore a total of ten minutes for each reading, during which time voltage is applied to the specimen. In the upper range of voltage the loss during this interval is sufficient to increase the temperature and so make the loss readings higher than that pertaining to the initial condition of the specimen. In order to meet the error thus caused the following method was devised:

Beginning at atmospheric temperature and taking right and left deflections of the wattmeter at two-minute intervals, it was found that for the higher voltages the wattmeter readings increased linearly with the time over a considerable interval, 30 minutes in some cases, and less than this only in the cases of specimens having abnormally high loss and in the neighborhood of breakdown. This linear relation is doubtless due to the facts that the increase in loss is proportional to the temperature, and that the temperature rise is uniform with the time for a slow total rise, radiation being practically negligible. This linear relation of loss and time makes it a simple matter to correct for the elevation of temperature during observations by means of the straight line diagram used for extrapolating the value of $\dfrac{\theta N K}{R_1}$ to its value for $R_1 = 0$. By taking the observation for the second value of R_1 immediately after the observation for the first value of R_1, and by having these two values in the ratio 2 to 1, the temperature correction is automatically applied by the straight line passing through the two observations, making its intersection with the vertical axis $R_1 = 0$. The method is illustrated by the readings of Table V and the corresponding diagram of Fig. 3. A is the loss reading corresponding to $R_1 = 2000$ ohms, based on right and left deflections requiring a total of four minutes from the application of voltage. B is the loss reading corresponding to $R_1 = 4000$ ohms based on right and left deflections and read eight minutes after the initial application of voltage. C is the loss reading corresponding to $R_1 = 2000$ ohms and taken four minutes after the reading B, or twelve minutes after the initial application of voltage. The length of the line $A\,C$ therefore is the increase in loss due to the heating of the bar in an interval of eight minutes. Taking this value from that corresponding to the point B, we obtain the point E, which represents the true reading corresponding to $R_1 = 4000$ ohms. Similarly, reducing the reading A by one-half the length of $A\,C$, we obtain the point D, or the true reading corresponding to $R_1 = 2000$ ohms. It will be seen that the geometry of the figure results in the intersection of the lines $B\,A$ and $E\,D$ in the same point on the vertical axis and this point gives the true value of $\dfrac{\theta N K}{R_1}$ corrected for temperature resistance loss in R_1 and also for quadrant charging current. The method obviously does not correct for absolute temperature differences, but only reduces the observations to that temperature obtaining when voltage is first applied. In those cases in which the loss was sufficiently high to cause an appreciable change in temperature during one observation it was not possible to take an observation at the next higher voltage until after the bar had cooled to the initial temperature.

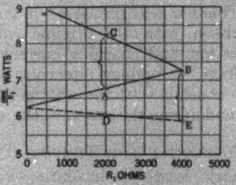

Fig. 3—Correction for Temperature Rise

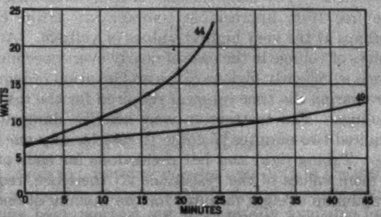

Fig. 4—Increase of Loss Due to Temperature Rise No. 44-75 kv., No. 49-12.5 kv.

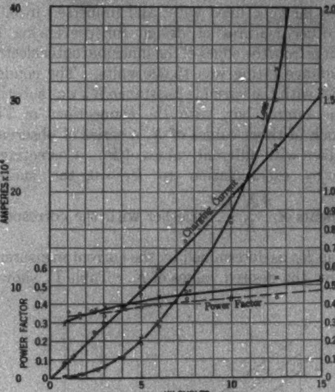

Fig. 5

Bar No. 48 — Not Dried
Uncompressed x x x
Compressed o o o

In these cases twenty-four hours were allowed to elapse between successive observations on the same bar. Generally this temperature change due to dielectric loss was limited to the higher values of voltage, except in the cases of those bars containing little or no mica. Fig. 4 shows the increase in loss, at constant applied voltage, with time, in the case of bar No. 49 having large mica content, and bar No. 44 consisting of shellaced paper with no mica. It will be seen that at 7.5 kv. No. 44 shows a rapidly increasing loss which if continued would soon result in breakdown, while No. 49 at 12.5 kv. shows a practically linear increase of loss at a much lower rate and over a much longer time.

OBSERVATIONS

Figs. 5 and 6, plotted from the readings of Table VI show the dielectric loss, charging current, and power factor as functions of the voltage applied to the insulation of bar No. 48. This insulation consisted entirely of mica folium except for two layers of shellaced paper on the outside. The insulation was applied to the bar by the rolling-ironing process under temperature and pressure, resulting in a structure as compact and as free from entrapped air as possible. It was expected, in this method of investigation, that the beginning of internal ionization, or corona, would be indicated by a break in either the power factor or loss curves. It will be noted from Fig. 5 that the power factor curves show no such break. The charging current, however, shows a slight departure from the straight line relation, but not in sufficient amount to use it as an indication of the point at which corona begins. In Fig. 6 the voltage loss curves are plotted on logarithmic coordinates. The relation, when so plotted, between voltage and loss is obviously linear, indicating that the loss increases as a simple power of the voltage. However, the slope of the linear relation changes in the neighborhood of 3 kv. Later on it will be seen that this break is much more pronounced in other samples.

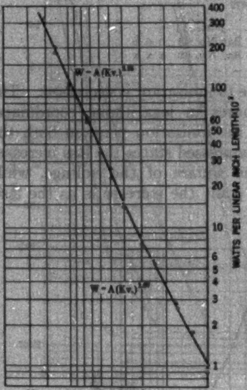

Fig. 6
Bar No. 48
Uncompressed x x x
Compressed o o o

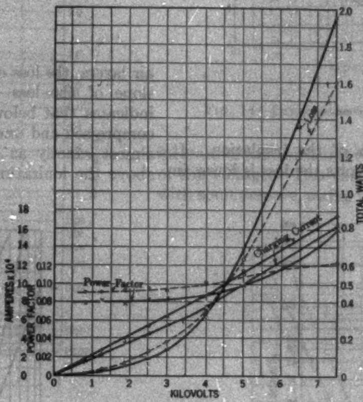

Fig. 7
Bar No. 50
High corona loss
Uncompressed x x x
Compressed o o o

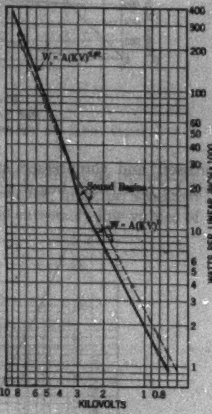

Fig. 8
Bar No. 50
Compressed o o o
Uncompressed x x x

Early in the tests it was noticed that some of the bars developed the sound of internal sparking, or ionization, as the voltage was raised, thus giving direct evidence of the presence of internal ionization. The effect of compressing these bars was studied and it was found that very substantial reductions in the losses resulted. This method was used for a study of the amounts of internal ionization in the various bars and the results of this study are given in a later paragraph. In Figs. 5 and 6 it will be noticed that there are two sets of points corresponding to the uncompressed and compressed states of the insulation. The reduction in loss, as a result of compression, is scarcely noticeable in the case of bar 48, showing that the corona loss in this bar is very small. This is also indicated by the slight change in the slope of the logarithmic curve in passing from the voltage range in which corona loss is practically absent, into the upper range where it is present.

Figs. 7, 8, 9 and 10 give similar curves for bars No. 50 and No. 34. Each of them has approximately 75 per cent of micarta folium the additional 25 per cent being treated cloth in the case of No. 50, and shellaced paper in the case of No. 34. Figs. 7 and 8 for No. 50 give two sets of curves corresponding to the uncompressed and compressed states of the insulation. It will be noted that below 4 kilovolts the loss is lower in the uncompressed than in the compressed state, but higher for the upper ranges of voltage. This is obviously due to the fact that when uncompressed the air layers have the effect of lowering the potential gradient in the insulation and thus lowering its loss. Above the ionization voltage, however, the loss is high, due to the ionization. On compression the potential gradient in the insulation is increased, resulting in an increase of dielectric loss below 4 kilovolts, while above that value, owing to the decrease in thickness of the air layers, the loss due to ionization is decreased. The slope of the loss curves in logarithmic coordinates indicates that below the ionization point in both the compressed and uncompressed states the loss increases almost exactly as the square of the voltage, while above the ionization point the increase is as the 2.67 power of the voltage. On compression bar No. 34 shows practically no change of dielectric properties. The logarithmic curve, however, shows a slight change

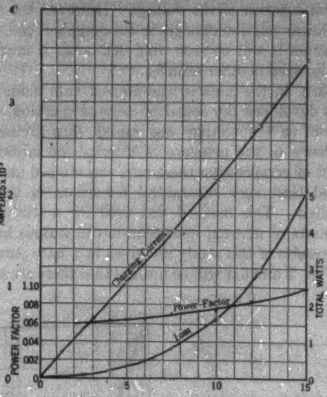

Fig. 9
Bar No. 34
Low corona loss

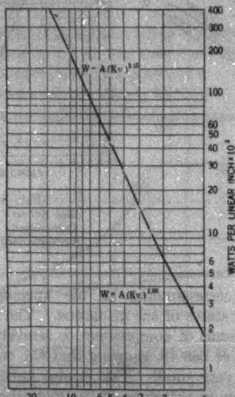

Fig. 10
Bar No. 34
Low corona loss

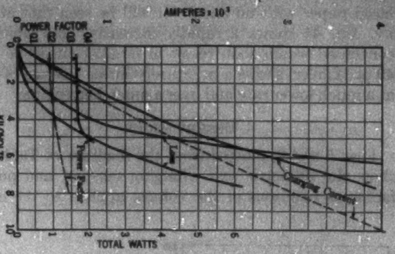

Fig. 11
Bar No. 43 o o o
Bar No. 59 x x x

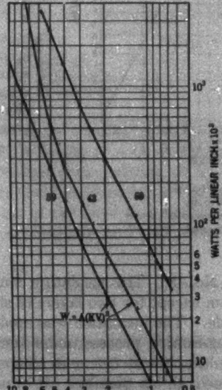

Fig. 12
Loss, Bar No. 43 x x x
Loss, Bar No. 59 o o o

in slope in the neighborhood of 3 kilovolts, indicating the presence of a slight ionization loss above that value.

Figs. 11 and 12 show the results on bars No. 43 and No. 59. The former is made up entirely of shellaced paper and the latter entirely of treated cloth, there being no mica in either. These bars are conspicuous for the very high values of their dielectric losses. However, they apparently have relatively very low internal ionization, as indicated by the slight change of slope in the logarithmic curves. It will be noticed that in the case of No. 43 the slope of the logarithmic curve above 4 kilovolts is changing rapidly. In this region the internal loss is so high that the temperature of the insulation, and therefore the loss, are increasing rapidly; in fact this bar broke down at 7.5 kilovolts at the end of six minutes. The slope of the logarithmic curves below 4 kilovolts indicates that the loss increases almost exactly as the square of the voltage.

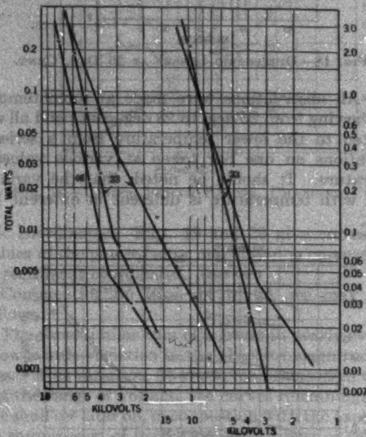

FIG. 13—LOSSES BARS NO. 33, 46.
Before compression x x x
After compression o o o

The break in the logarithmic loss curves in the neighborhood of 3 or 4 kilovolts and the effect of compression on the internal losses are also well shown in Fig. 13, giving observations on bar No. 33, which has full mica folium content. The observations of Fig. 13 were made after the bar had been subjected continuously for seven months to a temperature of 125 deg. cent. and also to 10,000 volts for 16 out of every 24 hours. The result of this treatment was that the bar was thoroughly dried out, its losses reduced to very low values, but that the insulation had expanded so that the thickness was considerably increased and the air layers obviously also increased in thickness. The logarithmic curves show again the change in the gradient in the neighborhood of 3 or 4 kilovolts, the very steep slope of the curve before compression, the increase of loss occasioned by compression for low voltages, and its decrease for high voltages. The slope of the curves below the ionization point indicates an increase of loss with the voltage gradient as the square of the voltage. This case, however, must be accepted with some reserve, owing to the extremely low values of loss and the probable error in the wattmeter reading. The total loss being measured here is of the order 0.003 watts at 0.03 power factor, requiring abnormally large values of the resistance R_1. The uncertainties thus introduced are indicated by the irregular location of the plotted points in Fig. 13.

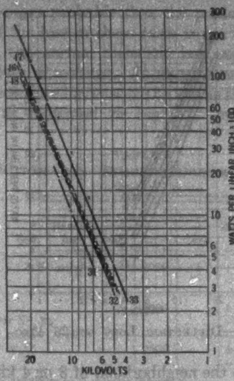

FIG. 14—DIELECTRIC LOSSES AT 25 DEG. CENT.

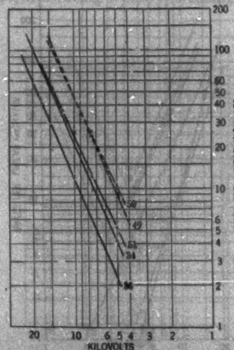

FIG. 15—DIELECTRIC LOSSES AT 25 DEG. CENT.

LOSS DUE TO INTERNAL IONIZATION

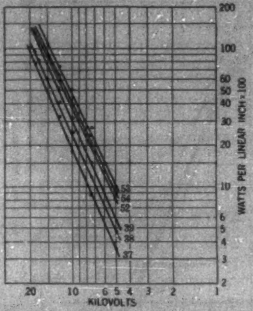

FIG. 16—DIELECTRIC LOSS AT 25 DEG. CENT.

It is suggested from the foregoing that the dielectric losses in the various types of insulation studied, all of them having the thickness of standard 6600-volt insulation, vary approximately as the square of the voltage up to the neighborhood of 3 or 4 kilovolts and that beyond this value the losses vary as some higher power of the voltage.

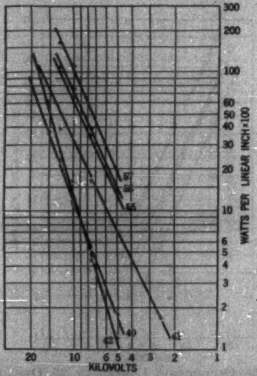

FIG. 17—DIELECTRIC LOSSES AT 25 DEG. CENT.

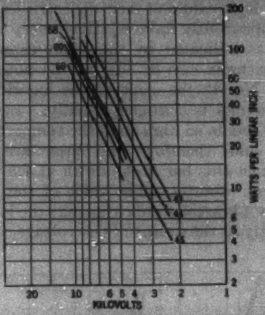

FIG. 18—DIELECTRIC LOSSES AT 25 DEG. CENT.

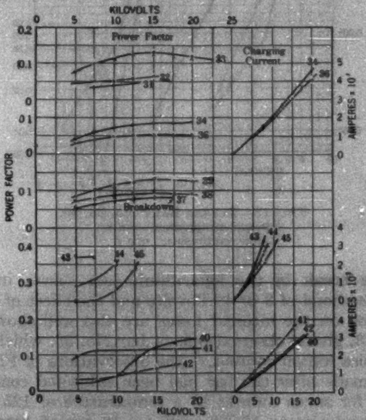

FIG. 19—LOSSES, POWER FACTOR, CHARGING CURVE

The presence of internal sound at higher values of voltage, and the effect of compression of the sample in reducing the losses, and also the sound, are strong indications that this increase in loss at higher voltage is due to internal ionization. In further study therefore of this question, loss readings were taken on the whole series of bars at several different values of voltage from 5 kilovolts upwards to values limited either by sparkover between the central conductor and the outside electrodes, or by the temperature or sharp increase of the loss curve, indicating that the sample was approaching breakdown. No two observations on the same bar were made in one day, this being done to insure a minimum of error due to difference in temperature. All observations were made at room temperatures varying from 25 deg. to 28 deg. cent., and all were corrected to the lower temperature from a series of observations on one bar taken at various values of temperature. It should be noted, that the variation of loss with temperature is different in different bars and that the whole question of temperature variation is apparently dependent upon the condition and life of the insulation. However, in the condition in which the present measurements were made, losses increased uniformly with the temperature, and as the range of variation was small, it is certain that no serious error is introduced here on this account.

The results of the measurements are given in Figs. 14, 15, 16, 17 and 18, in which watts per inch of length of bar are plotted as functions of the voltage in logarithmic coordinates. Figs. 19 and 20 give corresponding values of the power factor and charging current plotted in rectilinear coordinates.

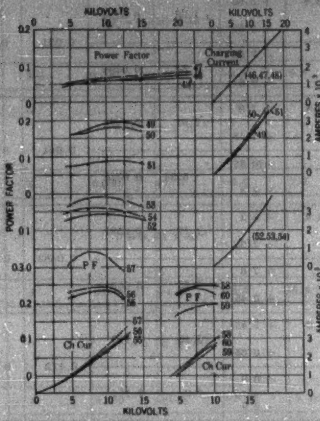

Fig. 20—Losses, Power Factor, Charging Curve

Commenting on Figs. 14 to 20, we may note as follows:

The three similar bars of a group in nearly every case show a wide variation in the absolute magnitude of the loss. This variation, however, maintains itself throughout the range of voltage, the curves remaining parallel. We shall see later on, in connection with the experiments on compression of insulation, that a part of this variation, at least, is due to a corresponding difference in the magnitude of the loss due to internal ionization.

Taking the average values in the various sets, the losses are in general higher the less the mica content of the insulation. The relation, however, seems to follow no regular law. The curves for these average values for the set *B*-3 are given in Fig. 21. The extreme difference in loss is between group 46-47-48 and group 43-44-45, the ratio of the losses being 1 : 10.

With the exception of one group, the losses are higher for the bars containing treated cloth than for those containing shellaced paper. The exception is the group in which there is no mica, and in this case the shellaced paper shows a considerably higher loss (bars 43-44-45). The difference in the methods of assembling the two types of insulation, and subsequent experiments on the influence of compression, seem to indicate that the higher losses in the bars containing treated cloth are due to less compact structure and therefore higher ionization losses. This does not explain the higher losses of the samples made up entirely of shellaced paper. However, neither of these groups contains any mica, the absolute values of the losses are much higher, and it appears probable that the predominating component of the loss in these cases is not internal ionization, but the normal dielectric loss pertaining to the lower values of voltage. It may be noted in passing that the accurate increase of the loss as the square of the voltage, as indicated in Fig. 12, suggests that the dielectric losses in the absence of mica are due entirely to the conductivity of insulation, that is, the losses are of the I^2R character.

TABLE V

CORRECTION FOR TEMPERATURE RISE DUE TO INTERNAL LOSS

E	$R_1{}^1$	Zero	Reading	Defln.	R_1	Zero	Reading	Defln.	$\frac{\theta N K}{R_1}$
70	1000	−3.4	+0.4	3.8	2000	−2.7	+ 3.7 − 9.9	6.8	6.77
	Bar No. 34 $I = 4.00 \times 10^{-3}$ $N = 12$				4000		+11.3 −17.9	14.6	7.27
					2000		+ 5.1 −11.4	8.25	8.20

All readings at 2 minute intervals.
Voltage on sample continuous.

The logarithmic curves are apparently all straight lines and moreover these lines all have very nearly the same slope, indicating a loss increasing approximately as about the 2.3 power of the voltage. Exceptions to these statements are found in the cases of bars 40 and 42. These bars, however, are believed to have been defective, as they heated rapidly at higher voltages, and, in fact, punctured at 15 and 17.5 kilovolts respectively. Also, by noting the points of the lines pertaining to

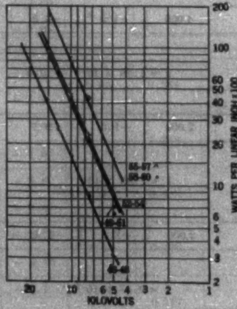

Fig. 21—Average Losses by Groups

TABLE VI
LOSS MEASUREMENTS ON BAR NO. 48, AT 25.6 DEG. CENT
INSULATION NOT COMPRESSED

										ONK		
E	KV	$R_1{}^1$	Zero	Rdg.	Defln.	R_1	Zero	Rdg.	Defln.	R_1	Watts	Power factor
15	1.67	10,000	+1.8	−1.8	3.2	20,000	+ 3.8	+ 5.62	1.58	0.0254	0.021	0.034
	$I = 0.366 \times 10^{-3}$							+ 2.55				
						40,000		+ 7.68	3.54	0.0294		
								+ 0.6				
20	2.5	10,000	+1.4	−4.2	5.6	20,000	+ 3.8	+ 7.0	2.91	0.0483	0.044	0.036
	$I = 0.46 \times 10^{-3}$ $.48 \times 10^{-3}$							+ 1.18				
						40,000		+10.38	6.34	0.0526		
								− 2.3				
12	3	8,000	+1.35	−3.8	5.15	20,000	+ 3.9	+ 5.8	1.77	0.0587	0.0583	0.033
	$I = 0.58 \times 10^{-3}$							+ 2.25				
						40,000		+ 7.6	3.57	0.0591		
								+ 0.45				
16	4	5,000	+1.35	−2.15	3.4	20,000	+ 3.9	+ 7.45	3.40	0.113	0.112	0.037
	$I = 0.75 \times 10^{-3}$							+ 0.65				
						40,000		+10.9	6.9	0.114		
								− 2.9				
20	5	5,000	+1.35	−4.2	5.55	20,000		+ 9.6	5.5	0.182	0.182	0.038
	$I = 0.96 \times 10^{-3}$							− 1.4				
						40,000		+15.4	11.0	0.182		
								− 6.6				
AT 26 DEG. CENT.												
30	7.5	4,000	+1.5	−6.3	7.8	10,000	+ 3.82	+ 8.82	5.1	0.508	0.513	0.047
	$I = 1.44 \times 10^{-3}$							− 1.38				
						20,000		+13.74	10.11	0.503		
								− 6.48				
40	10	2,000	+1.5	−2.1	3.6	5,000	+ 3.82	+ 8.9	4.775	0.952	0.859	0.044
	$I = 1.95 \times 10^{-3}$							− 0.65				
						10,000		+14.6	10.5	1.045		
								− 6.4				
50	12.5	2,000	+1.5	−4.4	5.9	5,000	+ 3.8	+ 7.7	4.05	1.595	1.67	0.054
	$I = 2.47 \times 10^{-3}$							− 0.4				
						10,000		+11.22	7.61	1.515		
								− 4.0				
60	.15	2,000	+1.5	−7.2	8.7	5,000	+ 3.8	+10.05	6.00	2.39	2.44	0.053
	$I = 3.65 \times 10^{-3}$							− 1.95				
						10,000		+15.5	11.75	2.34		
								− 8.0				

TABLE VI (*Continued*)
LOSS MEASUREMENTS ON BAR NO. 48, AT 25 DEG. CENT.
INSULATION COMPRESSED

E	K V	R_1^1	Zero	Rdg.	Defln.	R_1	Zero	Rdg.	Defln.	$\frac{O\,N\,K}{R_1}$	Watts	Power factor
15	1.67	10,000	+1.5	−1.5	3.0	20,000	+ 3.7	+ 5.35 + 2.02	1.68	0.0279	0.0224	0.033
	$I = 0.356 \times 10^{-3}$					40,000		+ 7.28 + 0.2	3.54	0.0294		
20	2.5	10,000	+1.5	−3.8	5.3	20,000	+ 3.7	+ 6.76 + 0.88	2.94	0.0488	0.0448	0.037
	$I = 0.47 \times 10^{-3}$					40,000		+10.32 − 2.54	6.43	0.0533		
12	3.	10,000	+1.5	−6.0	7.5	20,000	+ 3.7	+ 5.74 + 1.92	1.191	0.0634	0.064	0.038
	$I = 0.56 \times 10^{-3}$					40,000		+ 7.84 + 0.38	3.73	0.0628		
16	4	5,000	+1.5	−1.9	3.4	20,000	+ 3.7	+ 7.06 − 0.08	3.57	0.118	0.118	0.038
	$I = 0.76 \times 10^{-3}$					40,000		+10.66 − 3.6	7.13	0.118		
20	5	5,000	+1.5	−3.85	5.35	20,000	+ 3.7	+ 9.6 − 2.68	6.14	0.204	0.214	0.045
	$I = 0.944 \times 10^{-3}$					40,000		+15.42 − 7.96	11.60	0.194		
24	6	5,000	+1.5	−6.2	7.7	10,000	+ 3.8	+ 6.65 + 0.74	2.94	0.293	0.301	0.044
	$I = 1.14 \times 10^{-3}$					20,000		+ 9.36 − 2.08	5.72			
AT 25.6 DEG. CENT.												
24	6	5,000	+1.3	−6.7	8.0	10,000	+ 4.0	+ 6.6 + 1.3	2.65	0.264	0.279	0.039
	$I = 1.17 \times 10^{-3}$					20,000		+ 8.98 − 1.0	5.0	0.249		
30	7.5	4,000	+1.3	−6.8	8.1	10,000	+ 3.9	+ 8.6 − 0.4	4.5	0.448	0.468	0.042
	$I = 1.475 \times 10^{-3}$					20,000		+12.75 − 4.45	8.6	0.428		
40	10	2,000	+1.2	−2.5	3.7	5,000	+ 4.	+ 6.1 + 2.1	2.0	0.707	0.833	0.0425
	$I = 1.95 \times 10^{-3}$					10,000		+ 7.9 + 0.25	3.82	0.761		
50	12.5	2,000	+1.25	−4.7	5.95	5,000	+ 3.9	+ 7.54 + 0.5	3.52	1.402	1.5	0.048
	$I = 2.5 \times 10^{-3}$					10,000		+10.6 − 2.5	6.55	1.305		

the 46-60, or treated cloth group, there is, in several cases, a suggestion that the rate of increase of loss falls off slightly at the higher values of voltage. This tendency, if a true effect, is apparently absent in the other group. It appears probable from these experiments therefore that the losses due to internal ionization increase as some power of the voltage higher than the second, and that this loss, combining with a normal dielectric loss curve varying as the square of the voltage, yields the steeper loss curve of the higher ranges. It is probable also that the absolute values of the losses due to ionization depend upon the thickness and volume of the internal air spaces. This factor probably contributes more than any other to the wide separation of the loss lines of the several similar samples of the same group.

The drooping power-factor curves pertaining to the treated cloth bars shown in Fig. 20, are due to this slight falling off of the values of loss at the higher voltages in conjunction with a continually increasing charging current.

In considering all of the foregoing, it should be remembered that the various samples were all in a green state; that is, they had not been subjected to any drying process other than that of standing perhaps six or eight weeks after the original rolling and pressing. However, the absolute values of the initial losses, as determined for measurements on three bars, did not differ greatly from those obtaining after a long heat run. During the process of heating, the losses increased to high values and then decreased again, as will be shown in a later section of this paper.

THE INFLUENCE OF COMPRESSION ON THE DIELECTRIC LOSS OF ARMATURE INSULATION

In a number of the bars in which the losses increased markedly above 4 kilovolts, it was noticed that at about this value of voltage there also began a sound indicating internal discharge. This sound increased with the voltage and a rough compression of the bar resulted immediately in a lowering of the volume of this sound. A complete series of tests was therefore made on the whole series of bars. All of these tests were made at 7.5 kilovolts, 60 cycles. Observations were made first with the bar under no compression, then with the bar tightly compressed by means of two 1/32 in. steel plates 8 in. long and of the exact width of the bar, placed on opposite sides of the sample, and clamped tight by means of 15 set screws uniformly placed in a specially constructed wrought iron clamp. Measurements were made in each case immediately before and after compressing and in a number of instances after standing 24, 48 and 72 hours respectively. The charging current was measured in each case. No measurement of the degree of compression was attempted, but the amount was approximately the same in each case. A few tests were also made at extreme values of compression and in several cases after removal of the compression to note the recovery. All of the observations were corrected for temperature changes. The results of the tests are given in the following Tables VII and VIII, in which the values of loss are given in total watts for 8 in. length of bar.

TABLE VII
INFLUENCE OF PRESSURE ON LOSS DUE TO INTERNAL CORONA
Treated Cloth and Mica Group

Bar No.	Loss Watts				Total decrease per cent	Charging currents $\times 10^3$	
	No compression	Compressed				Before Compression	After Compression
		Start	24 hrs.	48 hrs.			
46	0.549	0.544	0.533		2.95	1.4	1.41
47	0.586	0.522		0.517	11.8	1.43	1.47
48	0.494	0.479	0.462		6.5	1.41	1.44
49	2.00	1.35	1.25		37.5	1.40	1.58
50	1.95	1.30	1.30		33.3	1.57	1.77
51	0.92	0.88	0.88		5.4	1.58	1.62
52	* .687	0.502	0.502		27.0	* .98	1.05
53	2.14	1.6	1.53		28.5	1.5	1.66
54	1.9	1.4		1.37	27.9	1.55	1.65
55	3.13	2.31		2.11	32	1.66	1.9
56	3.32	2.43		(2.26	32	1.71	1.93
57	4.31	3.29	3.03	8 days)	29.7	1.73	2.1
58	4.03	4.05		3.88	3.75	2.24	2.52
59	2.8	2.85	2.9	(72 hrs.)	3.5 incr.	2.00	2.16
60	3.96	3.82	3.4		14.2	2.11	2.36

(24 and 48 hrs. readings corrected for temperature differences.)
*5000 volts.

In considering the question of internal air spaces it was thought that the bending of mica around the corners of the armature bar would to some extent have the effect of causing the flat sides to spring outward, causing further increase of the thickness of entrapped

TABLE VIII
INFLUENCE OF PRESSURE ON LOSS DUE TO INTERNAL CORONA
Shellaced Paper and Mica Group

Bar No.	Loss Watts				Total decrease per cent	Charging currents $\times 10^3$	
	No compression	Compressed				Before Compression	After Compression
		Start	24 hrs. corr.	48 hrs.			
31	(not tested here)						
32	(secondary standard; in compound: not tested)						
33	0.97	0.83		0.83	14	1.44	1.53
34	0.676	0.672	0.658		4.1	1.53	1.54
35	(punctured)						
36	0.42	0.41	0.41		2.4	1.47	1.47
37	(punctured)						
38	0.816	0.75	0.75		8.1	1.53	1.54
39	0.937	0.855	0.81		13.6	1.34	1.40
40	(punctured)						
41	0.97	1.00		1.00	+2.	1.48	1.63
42	(punctured)						
43	10.2		9.8		3.9	3.23	3.3
44	8.9		7.44		16.4	2.75	2.69
45	5.82		4.68		19.6	2.3	2.1

air layers. Provision was therefore made for testing the various types of insulation when assembled in flat sheets rather than when wrapped on armature bars. One sheet of this character, corresponding to each group of three armature bars, was prepared. The same materials were included in each case and the assembled sheet was compressed to the same thickness as that of the bar insulation, and allowed to dry.

These sheets were provided with electrodes of tinfoil, coated with shellac, dried and melted into the permanent contact with the sample by ironing. The upper electrode was of circular form, 6 in. in diameter, surrounded by a guard ring of 1 in. radial width and a radial separation from the inner electrode of 1/16 in. The lower electrode was 8½ in. in diameter. Provision was made for compressing the samples. Measurements were made before and after compression, as in the case of the armature bars. Table IX gives the result of tests carried out in all respects in exactly the same way as those pertaining to the bars.

TABLE IX
INFLUENCE OF COMPRESSION ON LOSS DUE TO INTERNAL CORONA IN FLAT SHEETS

Plate No.	Watts per Sq. Inch.							Bars with same insulation
	No. comp.	Compressed	Per cent decr. on compr.	After 24 hrs. compr.	Per cent decr. 24 hrs. compr.	Compr. relieved	Per cent incr. on release	
11	0.0134	0.0132	1.5	0.0114	13.	0.0128	12.	31-33
12	0.0257	0.0251	2.2	0.0248	1.2	0.0250	0.8	34-36
13	0.0527	0.0537	1.9 incr.					37-39
14	0.1110	0.1092	1.6	0.1073	1.7	0.1131	3.4	40-42
15	0.0924	0.0906	0.95	0.0775	14.5	0.0806	4.0	43-45
16	0.0130	0.0125	3.8	0.0095	24.0	0.0096	1.1	46-48
17	0.0276	0.0277	12 days 0.36 incr.	0.0204	25.5	0.0230	10.3	49-51
18	0.0385	0.0377	48 hrs. 2.1	0.0378	48 hrs. 0.3 incr.	0.0408	7.9	52-54
19	0.0487	0.0476	42 hrs. 2.3	0.0430	42 hrs. 9.7	0.0443	3.0	55-57
20	0.0460	0.0447	3.5	0.0445	0.5	0.0451	1.3	58-60

The following conclusions seem to be warranted from the above results:

All bars show a diminution of loss with increasing mechanical compression except bar 41, which shows a small increase.

The amount of decrease for the bars made up of shellaced paper and mica folium is in all cases less than in those made up of varnished cloth and mica.

For the paper bars, all except 43, 44 and 45 have a relatively small initial loss and the per cent amount of decrease is about that of the standard bars 31-33 and 46-48. Bars 43-45, no mica and all paper, have extremely high initial losses, at once indicating the importance of mica; for example, there is a tenfold increase in loss between No. 41 and No. 43. As suggested elsewhere, the law of the variation of loss with the voltage in 43-45 seems to be purely that due to conductivity. In this group the reduction of loss by compression is somewhat higher than in others of the paper group.

In the treated cloth group the maximum reduction is 37.5 per cent and all treated cloth and mica bars, with one exception, show a reduction greater than 27 per cent, with a somewhat general indication of less reduction in loss for less mica content. Bars 58-60, all treated cloth, no mica, show less losses than the all-paper bars and the relatively low reduction on compression of from 3.5 per cent to 14 per cent. Here again it is suggested that the principal loss in these bars is due to conductivity and that the loss due to internal ionization is relatively unimportant. This is probably due to the absence of mica and the close adhesion between layers of treated cloth.

Considered in groups, the least reduction in loss occurs in bars 31-33 and 46-48, both having standard mica folium insulation, wrapped and compressed under temperature. No. 33 stands out with a considerable reduction in loss under compression. A further study of this bar is described hereafter.

In each group the greatest percentage reduction of loss is shown by the bar having the highest initial value of loss.

Tests at extreme values of compression show only a very slight further increase of loss over those given in the tables. Tests on relieving the compression show an immediate recovery of about one-half the total reduction of loss. No tests of further recovery with lapse of time were made. The further reduction of loss under compression after standing 24 hours or longer is in general relatively small.

The charging current is invariably increased by compression, in spite of the decrease in loss. This is obviously explained by an increase of capacity due to the elimination or lessening of the thickness of the air layers. Due to the same cause, the loss is decreased, in spite of the increased voltage gradient in the body of the insulation.

The experiments on the sheet form of the insulation all show the presence of entrapped air, causing increase in dielectric loss. The losses per square inch are lower than those of the corresponding armature insulation but of the same order of magnitude, (*e. g.* sheet No. 11, 0.0114, bar No. 33, 0.018). The decrease of loss under mechanical compression is generally less than in the armature bars, indicating some influence of the bending of mica in increasing internal ionization loss. The influence of prolonging compression is to still further reduce the loss, and of the removal of pressure a recovery of the loss to approximately the original value.

In view of the foregoing, it appears certain that all of the types of armature insulation measured have in them a greater or less proportion of loss due to internal electric discharge, or ionization based on the following facts:

(a) Bars having the highest losses always give a sound of internal discharge which is more or less reduced by compression.

(b) The reduction of loss on compression is greatest for the highest losses, indicating also that variations in the values of loss in a single group are to some extent due to the ionization losses.

(c) Bars with no mica show the least reduction in losses.

(d) The charging current is always increased by compression, in spite of the corresponding decrease in loss.

Mica therefore has the effect of reducing the losses in paper and treated cloth insulation by as much as the ratio of 10:1. It also tends to increase the losses due to internal ionization. However, this increase of ionization losses is reduced to a small, if not negligible, amount by the method of wrapping and pressing under temperature, used in bars 31-33 and 46-48.

INFLUENCE OF SUSTAINED HIGH TEMPERATURE AND VOLTAGE ON THE INTERNAL LOSSES

As has already been pointed out, none of the bars under observation has been subjected to any special drying out process. The variations in loss, with comparatively small differences in temperature, are attributable to this condition. It was realized that the sustained voltage and high temperature obtaining in operating machinery might well introduce modifications of the values and conclusions described above. Provision was therefore made for a sustained run at high voltage and high temperature on three typical bars. For this purpose a special Fries constant temperature oven was obtained and equipped with insulating bushings, for 20,000 volts, and also for the necessary connections to the central electrodes and guard ring electrodes of three armature bars, as already described. These were assembled on insulating stands and in a grounded metal screen inside of the oven. Some trouble was encountered at the higher voltages at various stages of the work at atmospheric temperature, with brush discharges over the surface of the insulation between the guard ring electrodes and the exposed ends of the armature bar. Realizing that at higher temperatures this trouble would be greatly increased, the bars, before placing in the oven, were dipped in a special compound obtained through the courtesy of the Standard Underground Cable Co., this compound being still stiff at a temperature of 150 deg. cent.

After assembly as above, measurements were taken of dielectric loss, charging current, and insulation resistance, every one or two days, over a period of six and a half months.

The bars selected for this purpose were two of the standard mica folium bars, namely No. 33, for high internal loss, and No. 46, for low internal loss, and bar No. 49. At first the third bar was No. 58, containing no mica and having high internal loss. Within a few days, however, this bar broke down under only a moderate elevation in temperature, doubtless due to the cumulative influence of its internal losses in still further increasing its temperature. There was therefore substituted, as a third bar, bar No. 49, having less mica than the others and also having relatively an extremely high internal loss.

The results of this series of tests can be best seen by the curves in Fig. 22. Starting December 5th, the losses were measured at atmospheric temperature and the oven was then set for 80 deg. cent. It was operated at this temperature until early in January, when it was necessary to repair a contact point of the electric temperature control, which necessitated opening the oven for a brief period, during which the temperature dropped to 25 deg. cent. It was then set for 111 deg. cent., at which value it operated until January 24th. For the remaining 4½ months the temperature was maintained at 127 deg. cent. During the entire run 10,000 volts was applied to all three of the central conductors, with grounded outer electrodes, for 16 out of every 24 hours. The temperature, however, was never interrupted. The loss measurements were made as described above, and the power factor computed from the values of loss and charging current. The insulation resistance was measured, using a primary battery of 215 volts, a Weston galvanometer having a sensitivity of 10^{-9} amperes per division, checking calibration at every series of readings. The insulation resistance, however, particularly in the later stages, was so high that the galvanometer deflections were of the order of only 2 or 3 millimeters. Consequently the values of insulation resistance vary considerably. The general trend of the curves, however, is probably reliable.

The tremendous increase in internal losses on the first application of temperature should be noted. They reached 10 or 12 times their initial values. Within two or three days, however, they began to decrease sharply in the process of drying out and the decrease continues steadily with continued application of temperature. The brief reduction to 25 deg. cent. showed an immediate influence of reduction of loss and each succeeding increase in temperature was accompanied by a temporary increase of loss, followed by a further decrease as the process of drying continued.

It is especially interesting to consider the influence of the internal losses on the position of the complete loss curves. In the beginning No. 33 of standard mica folium insulation showed, by the compression tests, an internal loss relatively high in its own group. Throughout the tests this bar maintains its position of having the highest loss.

Bar No. 46 showed the smallest reduction of loss under compression, indicating a minimum of ionization loss. No. 49, on the other hand, showed a high internal ionization loss. Notwithstanding the fact that the initial loss of No. 46 was considerably higher than that of No. 49, about one-quarter of the way through the run No. 46 falls below No. 49, which latter bar shows a slowly increasing loss approximating to that of No. 33 at the end of the run. Thus No. 33 and No. 49, both having relatively high internal ionization loss, show a slow but steady increase in loss under sustained temperature and voltage. No. 46, on the other hand, having very low internal ionization loss, reached a minimum value at 127 deg. cent., and held it throughout the run. These properties of the loss curves are followed in general by the curves for power factor, that is, the power factors of Nos. 33 and 49 are gradually increasing, while that of No. 46 holds to a constant value. The curves for insulation resistance are not so definite, for while No. 49 seems to hold at a low value, Nos. 33 and 46 appear to be increasing, the former at a higher rate. A possible explanation of this fact is the continued expansion of the sides of No. 33, giving higher losses, and at the same time increasing insulation resistance.

The run was terminated by a failure of one of the control circuits, but the curves show quite clearly that the influence of sustained temperature and voltage on the insulation of armature bars having internal ionization losses is to still further increase those losses, with the suggestion therefore of a tendency toward the shortening of their life. On the other hand, in a tightly wrapped bar having low internal loss, there is no indication during this 6½ months run at 125 deg. and at about 50 per cent excess voltage, of any increase of loss or tendency to a shortening of life.

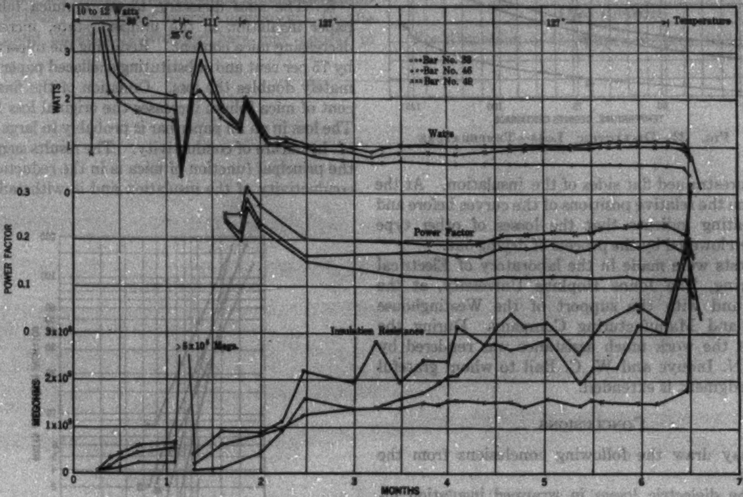

FIG. 22—WATTS LOSS AT SUSTAINED HIGH TEMPERATURE AND HIGH VOLTAGE

On opening the oven it was found that the insulation on the bars had expanded considerably on the sides. It is noteworthy, however, that this expansion was limited to the region covered by the outer electrodes. Since this is the only region where the full voltage gradient occurs in the insulation, it appears that the internal ionization is the most important factor in the expansion or spreading of the insulation. The losses in these bars were not especially high, and there is nothing to suggest that the temperature under the electrodes was any higher than that in the remainder of the insulation; the temperature of the oven was uniformly maintained within 1 or 2 deg. In practise this expansion of the sides of the insulation would be limited to small amounts, by the sides of the slot.

During the process of cooling the oven after interruption and by a few measurements taken subsequently, the loss temperature curves on these bars, shown in Fig. 23, were taken. They are of interest as showing that in the neighborhood of 127 deg. the losses for all three of these bars tend to increase rapidly.

The variation of the loss with voltage on these three bars, after removal from the oven and at atmospheric temperature, was also studied and the results for bar 33 are given in Fig. 13. Another curve in Fig. 13 gives the result of compressing this bar. It is seen again that the effect of compression is to increase the losses at the lower values of voltage, decreasing them in the upper range. The influence of the expansion under temperature and voltage is also indicated in the curves of Fig. 24, which show the great increase in the slope of logarithmic loss-voltage lines after the heat run. Before drying out, the exponents of the voltage represented by these lines were 2.43, 2.16 and 2.3 for the three bars 33, 46 and 49, and after the six months' run these exponents were 3.95, 4.38 and 3.95 respectively. Thus the internal ionization loss is greatly increased after the long heat run, due to the expansion of the unrestrained flat sides of the insulation. At the same time the relative positions of the curves before and after heating indicate that the losses of other type have been lowered by the process of drying out.

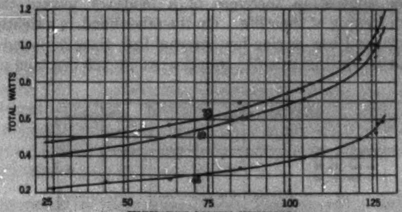

Fig. 23—Dielectric Loss—Temperature

The tests were made in the laboratory of Electrical Engineering, The Johns Hopkins University, at the request and with the support of the Westinghouse Electric and Manufacturing Company. During the course of the work much assistance was rendered by Messrs. N. Inouye and W. C. Ball to whom grateful acknowledgment is extended.

Conclusions

We may draw the following conclusions from the foregoing:

1. The dielectric losses in wrapped insulation increase as the square of the voltage in the lower range. In the 6600-volt insulation, on which the tests were made, this simple relation extends to 3 or 4 kilovolts, corresponding to an average potential gradient of about 16,000 volts per cm. In cases of bars having a large proportion of mica, particularly if dried out, and expanded by temperature, there is some evidence that the exponent of the voltage may fall below 2.

2. Above 4 kv. the loss also increases directly a as power of the voltage, higher, however, than 2. For the most tightly wrapped mica folium bars the exponent may be as low as 2.15. For bars high in mica, dried and expanded under temperature it reaches 4.4. The values for the whole series of bars in the undried state lie between 2.15 and 2.6 with an average value about 2.3.

3. The increase in loss above 4 kv. is due to gaseous ionization in entrapped air spaces. This is shown by the simultaneous beginnings of internal sound and loss increase, and the reduction of each by compression of the insulation. Further, complete voltage—loss curves for the uncompressed and compressed states show an increase of loss below 4 kv. and a decrease at higher voltages following compression. Notwithstanding the decrease of loss, in the higher range the charging current is at the same time increased.

4. The internal air spaces are increased to some extent by the bowing of the mica on bending, but more particularly by loose wrapping and expansion of the insulation under temperature. Bars wrapped under temperature and pressure show the lowest ionization loss but the run of the bars in each group of three is not uniform, occasional samples showing high ionization loss.

5. The *total* dielectric losses, in mica folium and paper insulation in the undried state, increase with decreasing mica content. Reducing the micarta folium by 75 per cent and substituting shellaced paper approximately doubles the loss. Omission of the final 25 per cent of mica folium increases the original loss 10 times. The loss in an all paper bar is probably in large measure of the nature of conductivity. The results suggest that the principal function of mica is in the reduction of the conductivity of the insulation and in withstanding the action of internal ionization. Bars in which treated cloth is substituted for mica show great increase in loss, resulting in internal temperature rise and ultimate breakdown. The breakdown of all types of insulation investigated, including those containing large proportions of mica, does not occur suddenly, but is the result of large increase of loss, and consequent temperature elevation.

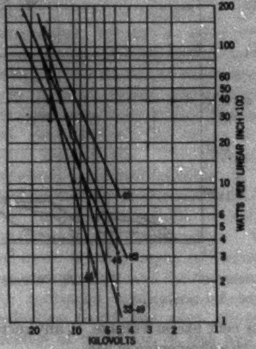

Fig. 24—Dielectric Losses
Before drying o o o o
After drying x x x x

6. The loss in well wrapped mica folium insulation of 6600-volt rating, is, at 6600 volts conductor to

ground, about 0.0017 watts per sq. cm. (0.02 watts per cu. cm.). Of this probably not more than 10 per cent is due to internal ionization. On the other hand bars of the same type and prepared by the same process may show losses 10 per cent to 40 per cent higher than the above figures due to internal air spaces and ionization.

7. Wrapped insulation prepared in factory processes, shows at first rapid increase of loss under increase of temperature; this is followed, under sustained temperature, by a slow decrease to values considerably below those at start. The initial increase of loss may be 10 to 12 times the starting value. The consequent internal temperature rise, if voltage is applied during this period, may lead to a further cumulative increase of loss resulting in breakdown.

8. The life of well wrapped mica folium insulation is not seriously affected by internal ionization. On the other hand imperfectly wrapped bars may appear in factory processes and in these there results a slow increase of loss and power factor. In two bars studied, under voltage and temperature continuously applied, this increase was about 10 per cent in three (3) months.

REFERENCES

1. F. Dubsky, TRANS. A. I. E. E., XXXVIII, 1919, p. 537.

2. W. S. Clark and G. B. Shanklin, TRANS. A. I. E. E., XXXVI, 1917, p. 447.

3. J. E. Shrader, JOUR. A. I. E. E., 1922.

4. Miles Walker, TRANS. A. I. E. E. XIX, 1902, p. 1035.

5. C. E. Skinner, TRANS. A. I. E. E. XIX, 1902, p. 1042.

6. E. H. Rayner, *Jour. Inst. El. Engrs.*, p. 49, 1912.

7. A. Russell, Alternating Currents, Vol. I (1914) p. 319.

8. Orlich, Zeits, Instkunde, 29, 1909, p. 33.

9. Hallo and Lande, Ressingen und Messinstrumente 1906, p. 246.

Discussion

For discussion of this paper see page 953.

The Effect of Transient Voltages on Dielectrics—III

(An Investigation with very High Lightning Voltages.)

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Review of the Subject.—*During a thunder storm lightning voltages that reach the transmission line appear across insulators, transformers and other apparatus at the extremely rapid rate of millions of volts per second. With this rapid rate of application the voltage may reach a very high value in a microsecond (millionth of a second). Hence, since there is always a delay or lag in the breakdown of insulation, quite peculiar effects result from these voltages. For instance, some remarkable phenomena that take place are: Much higher lightning voltages are usually required to jump a given distance than voltages at normal operating frequency; conductors at normal frequency voltages are often good insulators for lightning voltages; water may be punctured like oil; the wet and dry spark-over voltage of insulators are equal; the lightning discharge has a decidedly explosive effect, etc. In addition to the characteristics just mentioned, a study has also been made of the change in voltage and shape of a lightning wave as it travels over a transmission line at the velocity of light.*

In order that a laboratory study may be of a practical as well as a theoretical interest, it is necessary to be able to reproduce lightning voltages in the laboratory on a large scale and of known characteristics. This investigation was started some years ago with a 200-kv. generator. The generator has been added to from time to time until now, 2,000,000 volts are available and single lightning strokes can be obtained that increase at the rate of 50 million million volts per second. The power is of the order of millions of kilowatts. It is believed that this generator closely approximates voltage and other conditions that usually occur on transmission lines. The lightning voltages used in this investigation were far in excess of any heretofore produced in a laboratory. This impulse generator discharge must not be confused with that produced by an oscillator. The lightning generator, unlike the ocillator, discharges with a loud sharp report or crack.

The photographic study shows the lightning spark-over of insulator strings that are of such a length as rarely to spark-over in practise even in bad lightning country. The photographs of the flashes show all the characteristics of lightning such as a zig-zag path, side flashes, etc.

The study of the travel of the lightning wave on transmission lines is of interest. It indicates for instance, a certain protective effect of corona and shows that under certain conditions inductance coils may increase the lightning voltage four fold. Certain phases of the ground wire have also been studied.

FORMER INVESTIGATION—THE LIGHTNING GENERATOR

THE first paper of this series, published in 1915, described a generator for producing lightning impulses of a predetermined shape and voltage.[1] The maximum voltage of this generator was about 200 kv. Quite complete data were given at that time on the time lag and lightning spark-over voltages of various gaps, line insulators, etc. Data were also given on the lightning breakdown voltage of oil, air and solid insulation, corona produced by lightning, etc. It was found convenient to term the ratio of the lightning spark-over voltage to the continuously applied or 60-cycle voltage the "impulse ratio." This term has since come into general use.

The second paper dealt more particularly with lightning arrester gaps.[2]

The voltage and capacity of the impulse generator has been increased from time to time as higher exciting voltages have become available. During the past year the voltage has been increased to about two million maximum, while the power may be millions of kilowatts. Two million volts is higher than most lightning voltages that are induced on transmission lines. Tests can now be made in the laboratory at voltages and energy approximating operating conditions. It seems desirable, therefore, to report progress at this time.

The methods of making the tests and predetermining the characteristics of the impulse have already been adequately described in the first paper of this series. For convenience, a very brief description will be given here. The circuit is as shown in Fig. 1A. The gap *G* is set at some desired voltage. The transformer voltage is increased until discharge occurs. At that instant the condenser *C* is charged up to a voltage corresponding to the gap setting. A dynamic arc forms at *G* and holds. This acts as a switch and the condenser discharges through the known inductance *L*

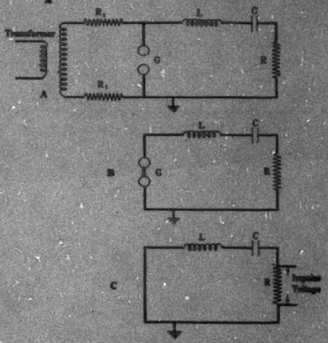

FIG. 1—IMPULSE GENERATOR CIRCUIT

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1. Peek, Jr., F. W.—"The Effect of Transient Voltages on Dielectrics,—I." TRANSACTIONS, A. I. E. E. 1915.

2. Peek, Jr., F. W.—"The Effect of Transient Voltages on Dielectrics,—II." TRANSACTIONS, A. I. E. E. 1919.

Presented at the Annual Convention of the A. I. E. E., Swampscott, Mass., June 26-29, 1923.

and resistance *R*. The circuit that produces the impulse is shown in Fig. 1B or is, in effect, that shown in Fig. 1C. The condenser discharging through the known resistance and inductance causes a transient current that can be readily calculated. This current produces a transient voltage drop across *R*. This is the impulse voltage used in the test.

FIG. 2—2,000,000 VOLT IMPULSE OR LIGHTNING GENERATOR

The lightning generator in its present form in the "High-Voltage Engineering Laboratory" at Pittsfield is shown in Fig. 2. The condenser *C* of Fig. 1 is shown supported on post insulators. It will be noted that *C* is made up of a number of cells or frames with ten plates in series in each frame. The plates are made of glass coated on both sides with tin foil. The insulated stands as shown will hold fortyeight frames or cells or four hundred and eighty plates. More cells can be added at will. In fact, it is planned to increase the number of plates to one thousand.

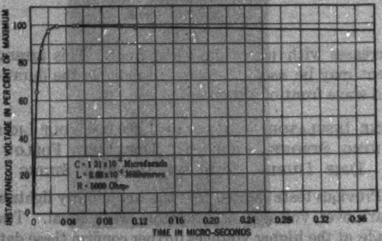

FIG. 3—WAVE SHAPE OF LIGHTNING USED IN THE TESTS

Wave shape No. 1. Two million-volt impulse generator.

The capacity per plate is 0.0112 microfarads; per cell, 0.00112 microfarads. The cells are readily arranged in multiple and series combinations as required. Three cells in series on each side operate satisfactorily at a million and a half volts maximum to ground. In the "set-up" shown, a series of tests were made up to 1500 kv. max. The energy available for the discharge is proportional to $\frac{E^2 C}{2}$. The inductance *L* is that formed by the rectangular circuit. The resistance *R* is a water tube of 5000 ohms. This particular arrangement gives the wave shown in Fig. 3.

FIG. 4—1,500,000-VOLT LIGHTNING STROKE BETWEEN POINTS

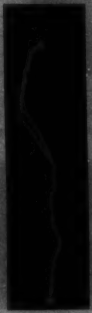

FIG. 5—1,500,000-VOLT LIGHTNING STROKE BETWEEN POINTS

Note zig-zag path and characteristic side flashes of lightning.

FIG. 6—THREE 1,500,000-VOLT LIGHTNING STROKES BETWEEN POINTS

Note that successive strokes do not follow the same path.

NEEDLE GAPS—DISCHARGE HAS ZIG-ZAG PATH AND CHARACTERISTIC SIDE FLASHES OF LIGHTNING—SAME LAWS FOLLOWED AS AT LOWER VOLTAGES

Fig. 4 shows a discharge of 1500 kv. max. between points. This type of discharge must not be confused with the high-frequency discharge produced by an oscillator. This discharge takes place with a loud explosive report. It will be noted that the general appearance of the spark resembles a lightning flash. In addition to the "zig-zag" path a close examination of Fig. 5 will show characteristic side flashes. In Fig. 6 three discharges are produced on the same plate. It will be noted that no two occur in the same place. The lightning spark-over curve for needles is given in

Fig. 7. Confirming data given in the previously mentioned papers the impulse ratio is high for points.

SPHERES—SAME LAWS FOLLOWED AS AT LOWER VOLTAGES. SERIES RESISTANCE INTRODUCES TIME LAG

Figs. 8 and 9 show the discharge between spheres. The great thickness of the spark (over 3 in. in diameter) shows the large current (approximately 10,000 amperes) in this discharge. In measuring the maximum lightning voltage with sphere gaps it is important to have no appreciable series resistance. Series resistance will give the sphere the characteristic of the needle gap. It takes time to charge the capacity of the sphere through the high resistance.

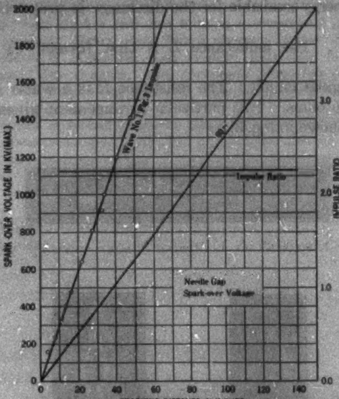

FIG. 7—LIGHTNING SPARK-OVER CURVE BETWEEN POINTS

FIG. 8—LIGHTNING DISCHARGE BETWEEN SPHERES
Note thickness of the discharge.

FIG. 9—LIGHTNING DISCHARGE BETWEEN SPHERES
Note thickness of discharge.

This is illustrated by the following data:

Applied lightning voltage kv. (max.)	Series resistance ohms	12.5 cm. spheres gap cm. setting	60-cycle spark-over kv. (max.)
106	0	3.95	106
106	2,500	3.60	98.5
106	5,000	3.30	92.0
106	10,000	2.80	80.0
106	20,000	2.30	66.0
106	30,000	2.05	59.1

Constant lightning voltages were applied to the sphere and series resistance. As the series resistance was increased it was necessary to reduce the gap as indicated in order to obtain spark-over with the same applied impulse. The effect of the series resistance increases with increasing wave front. This arrangement may be used, therefore, to indicate the duration and wave front of a transient.

FIG. 10—DRY 60-CYCLE SPARK-OVER OF A SHIELDED STRING OF INSULATORS
Arc clears the string

LINE INSULATORS—EFFECT OF 1,200,000 VOLT LIGHTNING STROKE—LIGHTNING DOES NOT FOLLOW DRIP DURING RAIN ON SHIELDED STRING

Previous tests showed that the wet and dry lightning spark-over voltages of insulators were equal. Tests made at the higher voltages further confirm these data. These tests are, however, particularly important from the practical standpoint since they give the lightning spark-over characteristics of very long strings of insulators. Laboratory tests have been made at higher lightning voltages than usually occur on operating lines. Fig. 10 shows the characteristic dry 60-cycle arc-over on a shielded string; the corresponding lightning spark-over is shown in Fig. 11. Fig. 12 is of great practical importance. It shows that the lightning spark-over clears the string during a very heavy storm. The great display is due to the illumination of the drops. The drops appear stationary in space because the photograph was taken by the light of the spark lasting less than a millionth of a second. It was found that the wet and dry lightning spark-over voltages were not reduced by the shield. Fig. 13 shows a non-shielded string struck by lightning.

FIG. 11—1,200,000-VOLT LIGHTNING SPARK-OVER OF A SHIELDED STRING OF INSULATORS—DRY

Spark clears the string

A few years ago the author's attention was called to some suspension insulator units of the cemented type that had been punctured through the head, apparently the result of a direct stroke. In these insulators, not only was the porcelain in the head shattered, but the metal cap had the appearance of having been punctured by a bullet shot through from the inside. Recently, in puncturing a glass plate in the laboratory a heavy foil coating was punctured in the same manner. This seems to be due to an "explosion" between the insulation and the foil. The foil is shown in Fig. 14.

FIG. 12—1,200,000-VOLT LIGHTNING SPARK-OVER OF A SHIELDED STRING OF INSULATORS

During a heavy rain. Note rain drops illuminated by the flash. The illumination differs from that in Fig. 11. The spark clears the string.

As in the case of spheres, if a resistance is placed in series with an insulator the lightning voltage necessary to cause spark-over is greatly increased. The following insulator spark-over data are given as an example.

FIG. 13—1,200,000-VOLT LIGHTNING SPARK-OVER OF A NON-SHIELDED STRING OF INSULATORS—DRY

FIG. 14—METAL "PUNCTURED" BY LIGHTNING

LIGHTNING SPARK-OVER VOLTAGE—INSULATOR IN SERIES WITH RESISTANCE

Series resistance ohms	60-cycle spark-over voltage kv. (Max.)	Lightning spark-over voltage kv. (Max.)
0	119.0	127.0
5,000	119.0	190.0
10,000	119.0	235.0
20,000	119.0	320.0
30,000	119.0	420.0
50,000	119.0	600.0

The resistance has the effect of increasing the time lag. In these tests the gap (insulator) was kept constant and the impulse spark-over voltage measured with increasing resistance. In the sphere gap tests above, the impulse was kept constant and the gap varied.

Incidentally, these tests show how useless a high-resistance arrester is.

PROPAGATION OF LIGHTNING ON TRANSMISSION LINES

DECREASE IN VOLTAGE DUE TO CORONA AND OTHER LOSSES. CHOKE COILS CAUSE DANGEROUS VOLTAGES—GROUND WIRE—REFLECTIONS

A study is being made of the propagation of lightning on transmission lines. It is possible at this time to report only the preliminary study which seems to be of considerable practical interest. In these preliminary tests an impulse with a front equivalent to that of a 5,600,000 cycle sine wave was applied to a short transmission line. The length of the front of this wave from zero to maximum was approximately 36 feet (12 meters). The maximum length of line used was made up of two parallel wires each 280 feet (85 meters) long and spaced 3 ft. (0.92 meters) apart. The diameter of the wire was 0.04 in. (0.102 cm.). The object of the small wire was to cause a high corona loss, and thus get the attenuation effects of a much longer line. Corona produced by these transients could readily be seen and the eye could differentiate between the positive and negative wire.

The general connections used in studying wave propagation are shown in Fig. 15. The data will be found in Table I. Fig. 15 shows the arrangement for test 3 of Table I. The impulse was applied to the line and the voltage measured at the start, in the center of the line, and at the end. Both needles and spheres were used to measure the voltage. The sphere measures the correct maximum voltage, while the needle always indicates a lower voltage. The steeper the wave the lower the voltage that is indicated by the needle. The ratio of the sphere and needle voltages, or the impulse ratio, is thus a measure of the steepness of the wave. The higher the ratio the greater the steepness. Referring to Fig. 15, test 3, a 133-kv. impulse was applied. The needle gap indicated 56 kv. or an impulse ratio of 2.4. At the center of the line the maximum voltage of the wave was reduced 18 per cent while the wave front was flattened out as shown by the impulse ratio of 1.7. It is interesting that the needle gap indicated a higher voltage in the center than at the start. If the voltage decreased at the above rate it would have a value of 80 kv. at the end. Actually, approximately double this value was measured; in striking the end the voltage doubled in value as would be expected. The wave front was also steepened at the open end as shown by the impulse ratio.

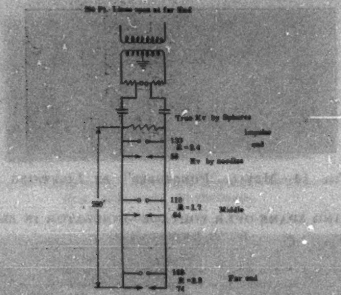

FIG. 15—GENERAL CONNECTIONS USED IN STUDYING PROPAGATION OR TRANSMISSION LINES

Figures give voltage as measured by spheres, impulse ratio and voltage as measured by needles.

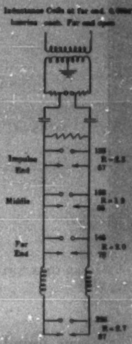

FIG. 16—LIGHTNING PROPAGATION ON TRANSMISSION LINES INDUCTANCE AT END OF LINE

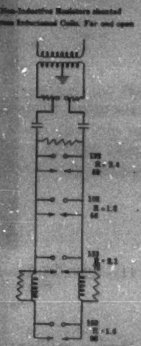
Non-Inductive Resistors shunted across Inductance Coils. Far end open
R = 2.4
R = 1.8
R = 2.1
R = 1.6

FIG. 17—LIGHTNING PROPAGATION ON TRANSMISSION LINES INDUCTANCE AT END OF LINE SHUNTED BY RESISTANCE

In test 4, the line was short-circuited at the far end, while in test 5 the far end was closed by a resistance approximately equal to the surge impedance. In both cases, the voltage and impulse ratio at the start and in the center of the line were approximately the same as for the open line.

Of great practical interest is test 7, shown diagrammatically in Fig. 16 where inductance coils were placed at the end of the line. In this case the wave almost doubled up on reaching the start of the reactor. At the far side of the reactor a very high voltage appeared. This voltage, 238 kv. was almost double the impulse applied to the line and probably three times the voltage that reached the start of the reactor. Under these conditions the reactor would be dangerous. By shunting the reactor by a resistance, test 9, Fig. 17, the voltage was greatly reduced. A greater reduction in voltage is found when the far end is shunted by a capacity.

Tests made by applying the impulse between one wire and the ground are given in Table II. A comparison of tests 12 and 13 is of interest. Test 13 is similar to test 12 except that the wave was split at the start

TABLE I
PROPAGATION OF LIGHTNING ON TRANSMISSION LINES

Test number	Arrangement Parallel wires	Start			Middle			Start of Inductances or Resistances			Far End		
		Voltage (Max.)		Impulse ratio	Voltage (Max.)		Impulse ratio	Voltage (Max.)		Impulse ratio	Voltage (Max.)		Impulse ratio
		Sphere	Needle		Sphere	Needle		Sphere	Needle		Sphere	Needle	
1	Line 6′ (1.84 meters) long—open at far end	..	..	..	..	..	..		None		191	71	2.7
2	Line 140′ (43 meters) long—open at far end	137	56	2.4	..	..	..		None		194	78	2.5
3	Line 280′ (86 meters) long—open at far end	133	56	2.4	110	64	1.7		None		168	74	2.3
4	Line 280′ (86 meters) long short circuited at far end	139	57	2.4	100	51	2.0		None		0	0 Short circuited	0
5	Line 280′ (86 meters) line short circuited at far end by resistance = approximate surge imped. = (850 ohms)	132	54	2.4	99	48	2.1	Resistance = surge impedance 85	47	1.8		End closed	
6	Line 280′ (86 meters) line open with 425 ohm resistance in series on each line	133	54	2.5	104	59	1.8	163	71	2.3	160	71	2.3
7	Line 280′ (86 meters) line open at far end with an inductance coil in series in each line	133	57	2.3	103	55	1.9	146	72	2.0	238	87	2.7
8	Line 280′ (86 meters) closed at far end with an inductance coil in series in each line	134	55	2.4	103	54	1.8	148	64 L = .47 mile henries	2.3	0	0 End closed	0
9	Line 280′ (86 meters) open at far end. Inductance shunted by resistance	132	55	2.4	102	56	1.8	153	72	2.1	150	93	1.6
10	Line 280′ (86 meters) end closed by a condenser	136	60	2.8	102	54	1.9	..	..	..	92	44	2.1
11	Single 280′ (86 meters) line ground return-end open	116	51	2.3	..	..	..	..	..	..	158	61	2.6

Size of conductor 0.04″ (0.102 cm.) Spacing 3′ (91.5 cm.). 5600 Kc. Wave.

TABLE II
PROPAGATION OF LIGHTNING ON TRANSMISSION LINES

Test number	Arrangement	Start			Far End		
		Voltage (Max.) to ground		Impulse ratio	Voltage (Max.) to ground		Impulse ratio
		Sphere	Needle		Sphere	Needle	
12	Single wire 280′ (86 meters) long-open at far end. Impulse between line and ground	133	56	2.4	151	65	2.3
13	Two wires connected at start 280′ (86 meters). Volts between wire and ground	133	56	2.4	113	62	1.7
14	Single wire 280′ (86 meters) single ground wire 3′ (.91 meters) distance grounded at far end	133	56	2.4	146	64	2.3
15	Same as (14) ground wire grounded at start and end	133	56	3.4	148	66	2.2
16	Two parallel wires each 280′ (86 meters) connected together at far end. Impulse sent out on one and back on other at total of 560′ (172 meters)	133	56	2.4	136	61	1.9

Size of conductor 0.04″ (0.102 cm.). Distance to ground 25′. 5600 Kc. Wave.

and sent over two wires. The result was a considerable reduction in voltage at the far end.

When the impulse was applied between one wire and ground it was observed that the attenuation was greater when the wire was positive than when it was negative. That is, the voltage was always higher at the end of the line when the wire was negative.

The effect of a ground wire in absorbing energy and thus reducing the lightning voltage is indicated by comparing tests 12, 14 and 15, Table II. This effect is quite small. However, this is only one phase of the ground wire question; it is in no way a measure of the value of the ground wire in reducing induced voltages or of protecting from direct strokes.

That phase of the problem is being investigated.

The Dielectric Strength of "Conductors"

Conductors at Operating Frequencies may be Insulators for Lightning. Importance of Low Resistance Arrester

When voltage is gradually applied between metallic terminals separated by an insulating material, there is no appreciable flow of conduction current. If this voltage is increased so that the stress is of the order of 30 to 200 kv. per cm. of insulation, puncture suddenly occurs at some point; the insulation breaks down and conducts through the arc. If the insulation is replaced by conducting material the current increases with increasing voltage, and the conductor either melts or boils away before a small portion of a stress of 30 kv. per cm. can be placed across it. By suddenly applying the voltage very high stress can be obtained across conductors without appreciable heating. Certain conductors can thus be subjected to the same voltage stresses as insulations. By means of the impulse generator it was found possible to puncture water.

In this test the voltage of the impulse applied to the water was increased until disruptive discharge or spark took place. The water was actually "punctured." The appearance of the spark was very much like a disruptive discharge in air or oil and was quite violent. Tests were made with both point and sphere electrodes. It was possible to cause voltages as high as 165 kv. across 1.50 cm. (0.6 in.) of water. Since the time of application was about one micro-second (10^{-6} seconds) there was no heating. Applied in the usual way over a few seconds not 1 per cent of these voltages would be possible without causing the water to boil out.

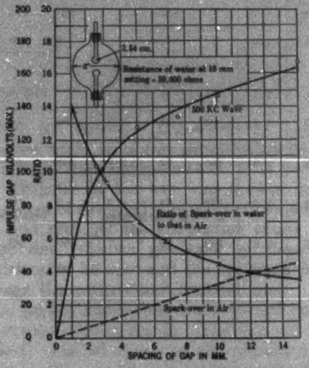

Fig. 18—Disruptive Strength of Water between 2.54 cm. Brass Spheres

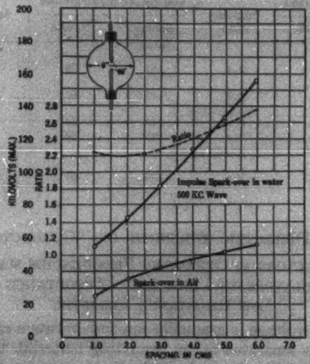

Fig. 19—Disruptive Strength of Water between 60-degree Points

The results of the tests are given in Table III and plotted in Figs. 18 and 19. The spark-over voltages for the same electrodes with the globe filled with air were also measured and are given on the curves. It will be noted that the disruptive strength of water is much greater than that of air.

TABLE III
TRANSIENT DISRUPTIVE STRENGTH OF WATER
2.54 cm. Spheres

Gap cm.	Impulse kv. (Max.) water	Impulse kv. (Max.) air	Ratio
0.1	49.5	3.5	14.1
0.2	86.0	7.0	12.3
0.3	105.5	11.0	9.6
0.5	126.0	17.5	7.2
0.7	137.0	24.0	5.7
1.0	149.0	33.0	4.5
1.3	159.0	42.0	3.8
1.5	165.0	46.0	3.6
	Resistance 20,000 ohms		
	60 deg. Points 1/8 in. Rod		
1.0	56.5	25.0	2.25
2.0	72.0	35.3	2.15
3.0	92.0	41.0	2.25
4.0	113.0	46.5	2.42
5.0	134.0	51.5	2.61
6.0	156.0	56.5	2.76
	Resistance 40,000 ohms		

Note. Tests made with single half cycle of a 500-kilocycle wave. This wave was, therefore, not as steep as waves used in some of the other tests.

The results of this test are in agreement with the tests above which showed the wet and dry spark-over voltages of insulators to be equal. The test also shows that "insulation" that is not good or is, in fact, a conductor at operating voltages may be very good insulation at lightning voltages. This again emphasizes the importance of low-resistance lightning arresters.

CONCLUSIONS

Lightning voltages are now available in the laboratory which exceed those usually induced on transmission lines. This conclusion is based on the fact that insulator strings of the length flashed over in this investigation rarely spark-over in practise due to lightning. An investigation on lines in Colorado also showed that lightning voltages in excess of 400 kv. were rarely induced on lines. This, of course, is not a direct measure of the voltage of the lightning bolt. The lightning voltages used in the investigation were far in excess of any heretofore produced in a laboratory.

The lightning spark-over of various gaps follow the same laws at these extremely high voltages as at the lower voltages.

It usually takes a higher lightning voltage to "jump" a given gap than a low-frequency voltage. When it is considered that two million volts bridge only a few feet, the voltage of the lightning bolt from cloud to cloud or cloud to ground must be exceedingly high.

A photographic study shows that the laboratory lightning has the characteristic zig-zag path and side flashes of cloud lightning. The discharge also takes place with an explosive report.

The lightning spark-over of insulators is not greatly affected by rain or weather conditions. On a shielded string of insulators the spark may be made to clear in both fair and rainy weather. The lightning spark-over voltage is not reduced by the shield.

Certain materials that are conductors of moderate resistance at normal frequency voltages may be good insulators for lightning voltages. Tests in this direction indicate at once how useless a high-resistance lightning arrester is.

In measuring lightning voltages resistance must not be used in series with the sphere gap. Resistances so placed give the sphere gap all of the time lag characteristics of the needle gap and the spark-over voltage varies with the wave front.

The investigation of the change in characteristics of lightning waves as they travel along a transmission line shows a decrease in voltage and flattening of the wave front due to corona and other losses. The waves tend to double up or increase on striking an open line or an inductance. An inductance may be a real source of danger. Under certain conditions the voltage on the *far* side of an inductance may be increased to three or four times the voltage that reaches the inductance. In general, inductance to be safe should be shunted by resistance.

With these high lightning voltages and currents it is possible to investigate the protective value of ground wires and also of lightning rods. This investigation is under way.

I acknowledge the assistance of Mr. W. L. Lloyd in conducting the laboratory work.

Discussion

For discussion of this paper see page 953.

Two Photographic Methods of Studying High-Voltage Discharges

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Review of the Subject.—*This paper presents some of the results of two methods of drawing out the alternating-current corona discharge along a time axis.*

The first method consists in photographing, with the usual camera, the discharge from a needle point revolved by the alternator which is the source of supply for the high-voltage transformer. The needle is revolved inside a porcelain tube whose outside surface is made conducting and grounded.

The second method makes use of a special camera using mirrors revolved synchronously by the alternator itself. This camera is equipped with a shutter so arranged that photographs of sparks may be taken using one sweep of one of the mirrors.

Photographs have been taken showing the discharge between needles, both with and without a solid dielectric placed midway between the points.

When drawn out along the time axis certain characteristics of corona discharges may be seen, even when the discharge is extremely weak for the exposure may be continued for any length of time with a reoccurring phenomenon.

A few photographs giving same conception of the different discharges under varying conditions are given.

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INTRODUCTION

FOR several years the Engineering Experiment Station at Purdue University[1] has been investigating the problem of the fixation of atmospheric nitrogen using various forms of discharge at high voltage.[2] During the progress of this investigation, the need for further knowledge concerning the mechanism of these discharges became increasingly apparent. It has been shown that the chemical effects produced by these discharges are erratic,[3] this being particularly so when using the corona or so-called "silent discharge." The nature of the gas and even the solid dielectric in the electric field seems to play an important part in determining the nature of the discharge. As a rule, engineers have been more interested in the prevention of corona than in its production, which involves a viewpoint somewhat different from that of the author.

Many investigators have been concerned with the discharge from the static machine or the induction coil, through gases under varying conditions.[4] Farwell[5] used a number of direct-current generators in series and

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Presented at the Annual Convention of the A. I. E. E. Swampscott, Mass., June 26-29, 1923

studied the characteristics of the corona using different diameters of wires inside concentric tubes. Much important work has been done in recent years in connection with the alternating-current corona discharge by Peek, Whitehead, Ryan, and others.

The present investigation of electric discharges in air deals with two methods which are somewhat different from any other known to the author. These two methods will be described separately.

PART I

Method Using the Revolving Electrode

APPARATUS EMPLOYED

If a needle point is revolved synchronously with the particular alternator which produces the potential for corona discharge, the successive discharges of each alternation of the potential will appear to the eye as stationary in space and discharges which are scarcely visible to the eye may be photographed if the exposure is made sufficiently long.

FIG. 1—REVOLVING ELECTRODES (SLIGHTLY WITHDRAWN) AND PORCELAIN TUBE USED IN MAKING PHOTOGRAPHS OF CORONA

a.—Wooden shaft extension of alternator
b—Chuck and discharging needle point
c—Grounded tinfoil covering of porcelain tube
d—Insulating disk to prevent arc over the end of the porcelain tube.

The apparatus for producing the corona is shown in Fig. 1. The wooden shaft shown at the left of the photograph is connected to an extension of the shaft of an 8-pole, surface wound, alternator rated at 10-kv-a., 110/1100 volts, 133 cycles. This alternator at frequencies below 60 cycles produced a voltage wave which departed considerably from the true sine wave. The unfortunate effect of this irregular form of wave was to render the photographs of discharges somewhat difficult to analyze. This machine was, however, the only one available. The alternator was connected to a 50-kv-a., air-cooled transformer capable of producing 100 kv. from line to ground or 200 kv. between lines.

The wooden shaft extension of the main shaft of the alternator carries at its outer end an adjustable chuck in which is fastened a No. 14 iron wire bent at right angles to the shaft in the form shown in Fig. 1. The end of the wire was carefully pointed and polished. When revolved, the needle point traces a circle 1.78 inches (4.52 cm.) in diameter. The plane of this revolving point was made to coincide with the plane of the end of a porcelain tube which was 3 inches (7.62 cm.) inside diameter and 4 inches (10.15 cm.) outside diameter and several feet in length. Part of the outside of this porcelain tube was covered with tinfoil which was grounded. The nearest approach of this foil to the plane of the revolving needle was one inch (2.54 cm.). Precautions were taken to limit the form of discharge to either glow or spark. To prevent a discharge from taking place directly from the needle point to the tinfoil, a pressboard collar was sealed over the edge of the tinfoil in the manner shown in the photograph (Fig. 1). Sealing wax was used to close the space between the disk and tube.

A connection was made from the high-voltage terminal of the transformer to the revolving needle by the use of a wire looped loosely around the chuck which held the needle.

When the voltage was raised sufficiently high, eight discharges on the circumference of the circle traced by the needle point could be plainly seen. These corresponded to the eight poles on the alternator, being alternately positive and negative. All of the photographs shown in the paper were taken from the rear of the tube. By shortening the porcelain tube to about 12 in. (30.5 cm.) in length, it was possible to take the pictures full size.

The photographs were all taken in a dark room, the exposure being 60 minutes. Standard plates made by the Eastman Company were used for the photographs.

The frequency was varied by changing the speed of the driving motor, and in most cases its value was determined from the alternator speed.

A preliminary test to determine the polarity of the discharge was made in a very simple manner. With full field excitation, the generator shaft was revolved slowly by hand while note was made of the position of the shaft that gave positive and negative deflection on a 750-volt permanent-magnet type of voltmeter which was connected between the needle point and the ground.

RESULTS OF TESTS

Successive photographs were taken at four frequencies: 15, 30, 60 and 120 cycles. At each frequency three successive voltages were applied: 5, 8.5 and 15 kv. However, only photographs of discharges at 60 and 120 cycles are shown here (Fig. 2).

The voltage impressed on the needle point by the transformer was determined by the readings of a tertiary coil which had been previously calibrated against a standard 25-cm. sphere gap. A calibration curve was taken for 30, 60 and 120 cycles and the results for the three were found to be identical.

The difference between the positive and negative discharge was recorded on the photographs. When the needle was positive it showed more or less brush discharge in a radial direction. When the needle point was negative the tiny glow at its point traced on the photographic plate the arc of a circle. With the aid of a stroboscope, Peek[4] has obtained photographs of the positive and negative discharges which show the general characteristic observed here, namely the positive brush discharge and the negative point discharge.

When the needle was positive a dark space between the brush and the point discharge in the radial direction on the photograph could be plainly seen in Fig. 2 at 8.5 kv., 120 cycles. Similar dark spaces were found at the lower frequencies.

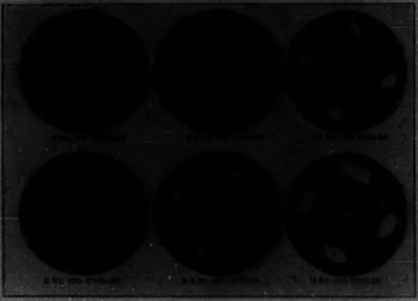

Fig. 2

At 15 kv. the discharge at 15 cycles (not reproduced here) was about as strong as that at 120 cycles, but the 30-cycle discharge showed no brush at all. At 8 kv. and 60 cycles there was no brush discharge recorded. The reason for this anomaly was not apparent.

It is interesting to note that in Fig. 2 the positive brush apparently forms in the air before the needle point itself shows any glow. This effect is due, partially at least, to the natural spreading out of the brush in a cylindrical form and also, perhaps, to the motion of the needle.

PART II

Method Using the Revolving Mirror

The preliminary study using the revolving point showed clearly some of the interesting characteristics of the corona discharge, but a method of observation in which the needle point did not move would be advantageous in some ways. As a substitute for the revolving electrode the attention naturally turned to the use of a camera employing revolving mirrors driven synchronously.

*The Camera and Revolving Mirrors.** The essential parts of the camera are shown in the diagrammatical sketch (Fig. 3).

Four "first-surface" mirrors are mounted as indicated at *M* and driven by the gear *G* mounted on the same shaft as the mirrors. Gear *G* is driven by pinion *P*, the ratio being 8 to 1. The light from the discharge to be photographed passes through the glass lens *L* and is reflected from the mirror surface *M* to the surface *F* on which a sensitized film has been placed. The surface *F* was constructed of heavy celluloid, supported on wooden side pieces shaped to such a curvature that a sharp image is obtained with any angle of the shaft carrying the four mirrors.

The camera was mounted on a stout tripod with braced legs to prevent vibration, and a considerable weight was hung underneath to increase its stability. One end of the pinion shaft was provided with a universal joint coupling, thus, not only allowing a positive drive, but also preventing the mechanical vibration of the alternator from being communicated to the camera. The camera was provided with magazines for the film, and spring clamps for holding the film taut and in position.

In order to obtain photographs of sparks where only one exposure of the film was desired, use was made of a shutter operated by a contact arrangement mounted on the mirror shaft.

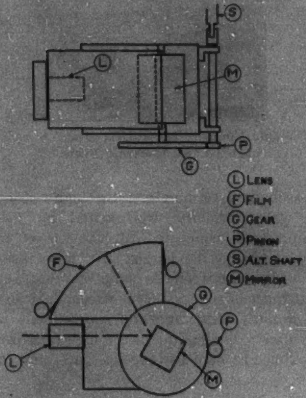

Fig. 3—Sketch of Corona Camera

On one of the curved wooden surfaces supporting the film, a pin was placed in such a manner that the position of the film could always be determined, since the pin passed through a hole previously punched through the edge of the film.

All photographs were made either at night, or in a light-tight room which was later found desirable to build around the apparatus. To prevent the possibility of fogging the film, the duplex paper provided by the film manufacturers was kept over the film. In addition to this, the camera was always covered with a black cloth when in use.

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*The success of this camera is largely due to the skillful work of Messrs. George, Pugh, and Maupin, all of Purdue University.

The water-cooled glass dielectric which was used for most of the tests is shown in Fig. 4. Two plates of double weight window glass approximately 0.118 in. (3 mm.) in thickness were sealed to wooden spacing pieces with a special insulating compound. The total thickness of the dielectric was 0.695 in. (1.765 cm.) including the space occupied by the cooling medium. Cooling water from the city supply was circulated at a rather slow rate,—a glass tube 5 ft. (1.5 m.) in length and 0.39 in. (1 cm.) in diameter being used to convey the water to the dielectric. The level of the water was kept constant by means of the overflow pipe which discharged the water in a discontinuous stream. This arrangement, in effect, grounded the water between the two glass plates through a high resistance.

The needles used were made of steel rod 0.127 in. (0.323 cm.) in diameter, and were adjustable. They were carefully pointed and polished. Considerable care was taken to get the points accurately in line and perpendicular to the dielectric surfaces.

FIG. 4—WATER-COOLED GLASS DIELECTRIC SET PERPENDICULAR TO THE ELECTRIC FIELD

The secondary terminals of the high-tension transformer whose primary was excited by the 8-pole alternator, were connected by the use of bus bars directly to the needle points. No resistance of any kind was placed between the points and the transformer. The resistance of the transformer secondary was 33,390 ohms, and the inductance by short-circuit test was 1920 henries. The middle of the high-tension winding was grounded as were also the alternator and the metal parts of the camera.

RESULTS OF TESTS

It was found by trial that a film exposure of 45 minutes was necessary to secure a satisfactory image of the corona discharge at low potentials, and this length of exposure was therefore used with all of the corona photographs regardless of the intensity of the discharge.

Data on the following points were obtained: Effect of changing the potential, frequency, spacing of electrodes, dielectric material and electrode shape.

For many of the corona photographs the voltage was raised to a point so near the sparking potential that occasionally a spark would pass. From the appearance of such sparks on the developed film, it was concluded to make some investigation of sparks. On account of the high resistance and reactance of the transformer, such tests could be easily made without the possibility of an arc following.

In Fig. 5 may be seen the discharge as it appears to the eye. This was taken with the pinion driving the mirror shaft disconnected from the alternator, the mirrors thus being stationary. The discharge seems to consist of two parts, the neck or stem and the brush which extends to the glass dielectric. This characteristic form has been noted by others, and some very good photographs were obtained by Peek,[7] whose only dielectric was air. Thus to the eye the discharge appears essentially the same whether the solid dielectric is present or not.

FIG. 5—DISCHARGE BETWEEN POINTS AS IT APPEARS TO THE EYE

Taken with the camera mirrors stationary. The potential is 113 kv. at 60 cycles. (The edges of the glass dielectric are plainly outlined by the parallel reflections appearing as bright lines.

FIG. 6—CORONA DISCHARGE AT 11 KV., 60 CYCLES, DRAWN OUT ALONG A TIME AXIS BY THE USE OF THE SYNCHRONOUSLY DRIVEN ROTATING MIRROR CAMERA. 16 CM. SPACING

When these discharges are drawn out along a time axis at right angles some additional features can be seen. The stroboscopic method, as used by Peek showed plainly the difference between the positive and the negative discharges, but did not show how they vary with time. Fig. 6 shows the discharge at 60 cycles and Fig. 7 at 120 cycles, the voltage in each being kept as high as possible without sparking.

As in the photographs showing the discharge with the revolving point, the negative appears as a line, which is the trace of a concentrated glow on the needle point, while the positive discharge is accompanied by the characteristic brush extending from the needle points to the surface of the glass dielectric (Figs. 6 and 7). At all frequencies studied, the negative discharge seems to take place throughout the entire 180 degrees. This condition was made use of in determining the location of the position of the degree scale, for it was possible, on the film, to erect a perpendicular at the end of the negative on one needle to the beginning of the negative on the other. The degree marking was drawn on a separate film, and by the use of the punch marks, which determined the location of the film on the camera, the degree marks could be located in the same position for each frequency.

The positive discharge shows a distinct dark space between the end of the neck and the brush. The shape is somewhat different at the different frequencies, the change being most marked at 120 cycles (Fig. 7) where there is evidence of a double dark space. On both films there is a point discharge from the needle point before the neck forms and the brush appears. With each frequency this point discharge continues a short time after the neck has disappeared. Oscillograms of the current flow between a wire and a concentric cylinder taken by Bennett[8] show the presence of a slight flow of current previous to the point of "copious ionization." It should also be noticed that the discharge is not symmetrical with respect to the mid-points—90 and 270 degrees.

FIG. 7—CORONA DISCHARGE AT 104 KV., 120 CYCLES. 15 CM. SPACING

The negative discharge is much the same for each frequency and shows in each case a stronger discharge during the time that the positive neck exists.

In studying these photographs, it is important to remember that there is considerable overlapping of the brush discharge since it has appreciable width.

EFFECT OF ELECTRODE SHAPE

With a brass sphere 24 mm. in diameter substituted for the lower needle, and with a spacing of 15 cm., photographs were taken showing the discharge at different frequencies. The discharge from the needle point is not appreciably changed by the substitution of the sphere for the lower point. The discharge from the sphere itself seen in the lower half of Fig. 8 is interesting. The negative is no longer a point discharge but extends out some distance. A brush appears at the beginning and at the end of each negative discharge from the sphere, the first brush with a dark space and the second without one. This effect is very plain on the film but may be lost in reproduction. The positive has no dark space and shows no point discharge. The neck forms abruptly and ceases in the same manner. Both the positive and the negative discharge are shorter in degrees than with needle points.

FIG. 8—SHOWING THE DISCHARGE BETWEEN A NEEDLE (UPPER) AND A SMALL SPHERE (LOWER ELECTRODE) AT 96 KV. AND 60 CYCLES. 15 CM. SPACING

It should be mentioned in connection with these tests with the sphere and needle that the discharge from the sphere looks very different from that of the needle, consisting of much coarser streamers and tending more toward the condition of sparks than is the case with needles.

FIG. 9—DISCHARGE WITH AIR DIELECTRIC ONLY. 43 KV., 120 CYCLES. 15 CM. SPACING

TESTS WITH AIR DIELECTRIC ONLY

With the glass dielectric removed it was only with difficulty that photographs of corona could be obtained, for the spark point is close to the corona point with this spacing. Photographs were taken at 30, 60 and 120 cycles. Fig. 9 shows the discharge at 120 cycles. The positive brush has extended clear across to the negative. The positive dark space is plainly seen in this photograph. The positive brush starts at the same time the glow begins on the positive point. The brush extending to the negative does not form until the 90 degree point has been reached.

PHOTOGRAPHS OF SPARKS

Quite a large number of photographs of sparks were taken. The characteristics seen in Fig. 10, which represents only one sweep of one mirror, are observed at all of the different frequencies studied. When using the glass or a bakelite dielectric, the sparks from the positive needle point are straight for a short distance out from the point and then become quite crooked, deviating considerably from a straight line connecting the needle points. On the other hand, the negative in many cases passes in a straight bold path to a point close to the dielectric, where it jumps to one side before reaching the dielectric.

It should be noticed that the negative spark widens out at the same average distance from the needle point that the positive begins to deviate from a straight line. This distance at 60 cycles is about 17 mm.

FIG. 10—PHOTOGRAPH OF SPARKS: STEEL POINTS, 15 CM. SPACING GLASS DIELECTRIC, SINGLE EXPOSURE, 116 KV., 60 CYCLES

Sparks from a small sphere (2.4 cm. dia.) are nearly as straight when the sphere is positive as when it is negative.

SPECIAL TESTS

Placing an air condenser in series with one of the needle points allowed a much larger discharge without the formation of sparks. The dark spaces, although present, are considerably modified in appearance.

A photograph was taken without a solid dielectric with one of the needles turned through 90 degrees about the point as a center. Sparks from the end of this needle follow the direction of its axis for a short distance.

When a grounded plate was placed in the cooling water, it was found that it was not possible to produce any appreciable corona discharge without the formation of sparks, (15 cm. spacing). This calls attention to the fact that with a dielectric between two needles, the dielectric will not be at ground potential if a discharge is taking place, but will have a potential closer to the positive than to the negative point. This effect is quite appreciable for with grounded cooling water, only 68 kv. is required to produce sparks, while 115 kv. is necessary when the ground plate is removed. This has an important bearing in several applications; for instance, in an ozonizer, the maximum discharge will be obtained when the dielectric is not in contact with either electrode.

When a bakelite dielectric was substituted for the glass, the general form of the discharge was unchanged, but differences in detail were noted particularly in connection with the shape of the dark space.

A peculiar discharge when the needle is negative is found to have occurred with many of the photographs taken with the corona camera. A luminous point formed back along the needle a distance of about 1 cm. from the point. This point discharge moves in toward the point as the cycle progresses, the total movement being three or four mm. The clearness of the image on the films shows that this movement occurs synchronously when the needle is negative. This has occurred with both needles even after having been repolished. When photographing a wire, Whitehead[9] found a spiraling effect which may be of the same nature.

SUMMARY

Two methods which show the variation of the discharge with time have been developed. In the first method, the discharging point is moved synchronously with the alternating current.

The second method, by the use of synchronously revolved mirrors, draws out the discharge along a straight time axis.

It has been found by the use of these two methods that a region exists, between the positive brush and the brilliant discharge from the end of the needle, which does not affect the photographic plate.

It has also been found that the appearance of the positive discharge is considerably modified by changes in circuit conditions.

Sparks from needle points to a solid dielectric are found to possess definite characteristics depending on whether the needle point is positive or negative.

CONCLUSION

A large number of photographs under many different conditions have been taken, using the methods described in this paper. It is hoped that these may soon be published in a Bulletin of the Engineering Experiment Station.

Much additional information should be obtained using quartz lenses and studying the discharge with different gases and at varying pressures. This work is being continued at Purdue University with the idea of learning more about the mechanism of this discharge.

The author wishes to express his appreciation to his associates in conducting this work, especially to R. H. George, Research Assistant, and E. Pugh, Instructor; both of Purdue University. To Dr. Anderegg, and to Prof. C. F. Harding under whose general direction the work was done, thanks are due.

REFERENCES

1. C. F. Harding and K. B. McEachron, TRANS. A. I. E. E., Vol. XXXIX, 1920, p. 761.
2. Bulletin No. 9, Eng. Exp. Sta., Purdue Univ., 1922.
3. *Loc. Cit.* (2) p. 55.
4. See J. J. Thomson "Conduction of Electricity through Gases."

   Also J. S. Townsend, "Electricity in Gases."
5. S. P. Farwell, TRANS. A. I. E. E., Vol. XXXIII, 1914, p. 1692.
6. F. W. Peek, Jr., TRANS. A. I. E. E., Vol. XXXI, 1912, p. 1088.
7. F. W. Peek, *Loc. Cit.*, (6).
8. Edward Bennett, TRANS. A. I. E. E., XXXII, 1913, p. 1800.
9. J. B. Whitehead, TRANS. A. I. E. E., Vol. XXXI, 1912, p. 1105.

Discussion

THE AXIALLY CONTROLLED MAGNETRON (HULL), GASEOUS IONIZATION IN BUILT-UP INSULATION (WHITEHEAD), EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS —III (PEEK), and TWO PHOTOGRAPHIC METHODS OF STUDYING HIGH-VOLTAGE DISCHARGES (McEACHRON).

SWAMPSCOTT, MASS., JUNE 29, 1923

F. B. Jewett: I wish to touch upon a point which Mr. Peek has referred to in his concluding remarks and call attention to the curious phenomena which have been observed from time to time in connection with these very high energy transients, which sometimes occur in connection with lightning discharges.

Very frequently in examining pieces of rubber covered wire, such as that used in ordinary telephone work for connecting up subscribers' instruments, pieces of wire through which heavy current discharges have passed, the most curious things have been noticed. In the first place you frequently find long lengths of this insulated wire where the wire itself has not only been completely disintegrated, but where casual observation indicates that the metal of the wire has almost completely disappeared without any visible indication of the rubber coating having been seriously ruptured. There are ruptures, of course, and that has given rise to suggestions such as Mr. Peek has just made as to what curious things might be produced by these enormous mechanical forces which must be for a very short instant of time developed in the wire itself. With the apparatus which Mr. Peek has had in hand and similar apparatus, I rather expect that in the next few years we will learn a great many things about matters which in the past have been more or less the subject of speculation.

For instance, if the metal of the wires were not copper—were not a pure metal, but an alloy—would these enormous forces applied at a time when the metal was in a molten state, or near a molten state, produce extreme examples of those processes which have been developed for producing very high grade alloys from relatively low grade material?

I just mention that as a matter of curiosity because there are obviously a lot of things that we want to know about, which may or can take place under the conditions of these enormous energies dissipated in a very small space of time.

H. Goodwin: There has long been a feeling among practical men that wood arms have some value in connection with the insulation of high-voltage lines, that is, from 11,000 to 66,000 volts. In general, technical men have scoffed at the idea of the wood having any real value as insulation for these voltages. Further, it has been considered that when these arms are wet their resistance is very much reduced, and any insulating value they might have when dry is entirely eliminated. Have your tests gone far enough to show that there may be some insulating value in the wet wood cross-arm in series with the insulators when subjected to a high-voltage steep front wave? Some of your data would seem to indicate that the wood arm may have considerable value, but we appreciate how very unsafe it is to draw sweeping deductions from special tests.

What you have developed on the increase of voltage due to the use of choke coils is of very great interest and confirms claims made by others.

C. J. Fechheimer: I think perhaps that the causes of insulation failure in large high-voltage generators may be attributed to three sources, the first being too high temperature, which may cause the insulation to char and to chafe. This is well known. We have had a great deal of discussion in the Institute on that subject.

The second is mechanical failure which may arise from various causes; with long core machines there is possibility of electrical failure from alternate expansion and contraction, due to changes in temperature. That has not been discussed, so far as I know, at any of our meetings.

The third is the possibility of failure from corona, either external to the insulation or internal.

With regard to the second source, I may say that at the Westinghouse Company we are conducting a large number of tests to determine how serious may be the danger of failure from alternate expansion and contraction and we hope that within the course of a year or less we shall be able to present before this Society the results of our investigation. At present, I would rather not speak of that as it does not pertain to the subject.

On the subject of the possibilities of failure from corona that Dr. Whitehead's covers, there are two or three points to which I want to call attention.

I think the principal conclusion which one may reach is that the danger from failure in well insulated coils having a large percentage of mica is extremely small. Those who have been skeptical in this regard may now feel quite safe that their machines will not fail. I know that some years ago when mica was not used to the extent that it is today, treated cloth or paper were the chief constituents of the insulation; the insulation failed undoubtedly from internal corona or possibly external corona.

Even when coils are insulated with mica we have found that after they were in service for a number of years, the insulation on the outside—which is usually paper for mechanical protection—was pitted. But immediately after removing the external layer of paper, where the mica appears it was found there was no pitting whatsoever; the mica was in as good condition as when it was placed upon the coil. In other words, for ordinary service up to, say, around 15,000 volts, there is practically no danger from breakdown due to corona.

That is one conclusion which one might infer from the paper, but the other conclusion is that the corona loss with mica insulation is very much less than with fibrous material. In other words, we have eliminated danger from breakdown when mica insulation is employed, by two means; we have first reduced the internal corona loss, and then we have, since we use mica, practically eliminated any further danger of breakdown because mica will successfully withstand corona discharge for indefinite periods.

J. F. Peters: Mr. Peek's paper indeed is very interesting in showing us the behavior of very high-voltage circuits and the apparatus described in the paper undoubtedly will be of very great value in obtaining data for high-voltage engineering. I feel, however, that some of the conclusions drawn from the tests have been made rather loosely.

For instance, in the second column of the third page, where the author is discussing the impulse ratio of sphere gaps with various series resistances he states, "The effect of the series resistance increases with increasing wave front. This arrangement may be used, therefore, to indicate the duration and wave front of transients."

It has been very definitely proven that the delay of a sphere gap with series resistance is due to the time required to charge up the spheres as a condenser through the series resistance. That being the case, the delay is fixed almost entirely by the duration of the surge and is practically independent of the wave front. The wave front and duration to a very limited extent are proportional, but the duration can be varied over a very wide range without materially affecting the wave front.

For instance, taking the curve shown in Fig. 3 and the constants there given, the equation of that curve is

$$E_0 \; (\epsilon^{-0.152\,t} - \epsilon^{-0.173\,t})$$

where ϵ is the base of naperian log and t is time in microseconds. ϵ to the minus 173, t fixes the wave front almost entirely while the other factor ϵ to the minus 0.152 t—fixes the time or tail of the wave.

If the capacitance there used were double and R and L remained the same, then the exponent fixing the wave front would be 173.1 as compared with 173, whereas the exponent fixing the length or time of the surge would be 0.075 instead of 0.152. In other words, the duration is practically doubled, where the wave front is not modified or is modified very slightly.

That being the case, it is obvious that the impulse ratio would be different for the conditions with a larger condenser than it is for the small condenser. So that the spark gap for the given resistance really gives wave front only for some particular quantity of charge.

In connection with the author's investigation of strings of insulators in series with resistances shown on the fourth page, which have characteristics similar to the needle gap, the author states at the top of the first column on the fifth page: "Incidentally these tests show how useless a high-resistance arrester is."

Although I entirely agree with the author on the ineffectiveness of a high-resistance lightning arrester—it is not clear how these tests prove the point. In the first place, a resistance type arrester as usually constructed consists of combinations of resistance and spark gaps in series and has characteristics similar to those shown on the third page, and for moderate resistances the impulse ratio would not be serious. My belief in the ineffectiveness of the high-resistance type arrester is not due to its large impulse ratio but is due to its limitation of current that it will drain from the system when it does function.

Referring to Mr. Peek's conclusion, his last conclusion in the first column of the eighth page is: "In measuring lightning voltages resistance must not be used in series with sphere gap. Resistances so placed give the sphere gap all of the time lag characteristics of the needle gap and the spark-over voltage varies with the wave front."

Whether a resistance in series with a spark gap is at all objectionable or not depends on the relative value of that resistance as compared to the capacitance of the spheres. A certain amount of resistance may be desirable. We all know that if a voltage is abruptly applied to an inductance and capacitance in series where the circuit is not critically damped, the voltage on that condenser will overshoot. A sphere gap with no series resistance is just such a circuit.

The author shows by some of his tests that an inductance located at the open end of a line gives a large increase in voltage due to reflections, and in connection with those tests he finds that by shunting these inductances by resistances, the reflections are very much reduced, and he concludes that "In general, inductances to be safe should be shunted by resistances."

The usual installation of inductance (reactances) is not at the open end of lines. The only inductances that have a legitimate right to be located at the open end of a line are those used in connection with lightning arresters, and when used in connection with lightning arresters, the better reflector they are the more effective will be the lightning arrester.

R. H. Marvin: I was particularly struck by Mr. Peek's experiments with regard to resistance in series with the sphere gap.

It is, I think, generally recognized that wooden construction is of great value in reducing lightning disturbances. I think the general feeling has been—I know my own has been—that its value in this case was not so much in preventing flashover, as in preventing a dangerous arc from following, or hastening the extinguishing of the arc after its occurrence. It would appear from these experiments that we have a still further action in actually preventing the flashover of the insulator and so reducing the number of disturbances. Of course, it is true also that we may have this action in limiting the current after the arc has started.

It is also interesting in this connection to consider the effect of ground resistance. We all know that the resistance of the earth varies widely with the amount of moisture in the ground, and it seems entirely possible that even with metal construction the resistance of the earth may have some effect in determining flashovers from lightning voltages.

J. Slepian: The phenomenon of time-lag in the electrical breakdown of dielectrics has been rightly attributed to the necessity of a certain energy input for converting the dielectric from the insulating to the conducting state. Since electrical energy input is determined by the product of potential, current and time, we see that in all cases time is necessary for breakdown.

The sphere gap in air is known to be extremely fast in its breakdown, and for most purposes may be considered instantaneous. However, we must not forget that actually a finite time is required, and when, as Mr. Peek does in his paper, we are considering things happening in 10^{-8} seconds, we must inquire whether the sphere gap is sufficiently "instantaneous" for its time of breakdown to be negligible I shall show that to bring a sphere-gap into a highly conducting state something of the order of a micro-second is necessary. This being the case, the calculated steep front of the wave shape shown in Fig. 3 of the paper has no actual significance. The gap G of Fig. 1 does not break down fast enough for the voltage on the resistor R to build up at the rate indicated in Fig. 3.

The theory of conduction in gases is now in a highly developed state, and we may make calculations in this field with great confidence in the order of magnitude. I shall now calculate the time of breakdown of the gap G.

To begin, what energy is necessary to make the gap conducting? Before this can be answered we must state more precisely the degree of conductivity the gap is to attain. I assume that in Fig. 3 a four cm. gap was used which breaks down at about 100,000 volts. With a resistance of 5000 ohms, this will give 20 amperes after complete breakdown. Since in calculating the curve of Fig. 3 the volts taken by the gap are assumed negligible against the 100,000 volts, I shall assume that 1000 volts are taken by the gap when carrying the 20 amperes. This defines the degree of conductivity attained by the gap.

The current in the gap is carried by ions, that is, positively and negatively charged particles into which formerly neutral molecules of the air have been broken. The negative ions will be electrons which have a very much greater velocity than the positive ions. Hence, practically all the current will be carried by the negative ions.

Since the current will be given by the product of the charge per cm. of gap carried by the electrons and the velocity with which the electrons move, if we know the velocity, we can determine the charge. Referring to Townsend, "Electricity in Gases" p. 180, we find that the velocity for a gradient of 250 volts per cm. is about 700,000 cm. per sec. This gives

$\frac{20}{700,000} = 2.9 \times 10^{-5}$ coulombs per cm. or for the whole 4 cm. of gap 11.1×10^{-5} coulombs as the charge carried by the electrons.

Now, to separate an electron from a neutral molecule requires a certain amount of energy. This energy has been fairly accurately determined, and is commonly expressed as the voltage, called the ionizing potential, through which an electron must fall to attain this energy. The ionizing potential for air is about 15 volts. Hence the energy necessary to produce the 11.1×10^{-5} coulombs of electrons in the gap is $15 \times 11.1 \times 10^{-5} = 1.67 \times 10^{-3}$ joules. This is about 1/12 of the energy which leaves the condenser in the first two hundredths of a micro-second, as shown in Fig. 3.

Now although 1.67×10^{-3} joules is the energy necessary to produce the assumed degree of conductivity of the gap, this is not a measure of the total energy which must be put into the gap, for the larger part of this energy goes into produicng heat, and only a small part into producing ionization. To get an idea of the relative proportion, we must consider the mechanism of ionization in more detail. An electron will ionize a molecule if it collides with it after having fallen freely through fifteen volts of potential. Now with the largest mean gradient in the gap, namely, with the initial 100,000 volts acting on 4 cm., or a mean gradient of 25,000 volts per cm., this means that the electron will have to fall freely through a distance of $\frac{15}{25,000}$ or 60×10^{-5} cm. But the mean free path of an electron in air is only about 5.7×10^{-6} cm. Hence very, very few electrons will fall freely through the necessary 60×10^{-5} cm. Practically all of them will collide with molecules before they have fallen through this distance and give up there energy as heat instead of as ionization. The actual proportion which will produce ionization may be determined by the formula given in Townsend, "Electricity in Gases," pp. 292 - 293. This gives, on substituting,

$$e \quad \frac{60 \times 10^{-5}}{5.7 \times 10^{-6}} = 10^{-4.6}$$

Thus an electron will collide with molecules $10^{4.6}$ times with an energy corresponding to a fall through the mean free path, for each time it collides with a molecule with energy sufficient to ionize, corresponding to a fall through the distance 60×10^{-5} cm. We conclude then that

$5.7 \; 60 \times 10^{4.6} = 4 \times 10^{3}$ times as much energy will go into heat, as goes into ionization. If this proportion were kept up, $4 \times 10^{3} \times 1.67 \times 10^{-3} = 6.68$ joules would have to be put into the gap to produce the desired ionization, and at 10 amperes, 100,000 volts, this would require, 6.68×10^{-6} seconds.

However, after some ionization is produced conditions become more favorable for further ionization, since portions of the gap become charged and cause the electrostatic field in the gap to be distorted, the gradient being increased near the electrodes, and reduced at the center of the gap. In the stronger fields a larger part of the energy spent will go into ionization, so that the total energy which must be put into the gap will be reduced. This does not help out very much on the time of breakdown, however, as the ions which are produced in the stronger portions of the field must move into the weaker portions before full conductivity can be obtained. Now, for a gradient as high as 25,000 volts per cm. the velocity of the electron will be nearly 10^{8} cm. per sec., but the velocity of the positive ion is very much less, so that time of the order of micro seconds must elapse before the ions move from the strong field, where they are generated rapidly, to the weak field.

More convincing, perhaps, than the above calculations, are actual experiments. A. Leontiewa, in *Physikalische Zeitschrift*, Jan. 15, 1922, p. 33, describes experiments which show a lag in the breakdown of sphere gaps distinctly greater than 10^{-8} seconds. In these experiments however, breakdown meant merely a luminous discharge, and a discharge of only a microampere may be luminous. To attain conductivity sufficient to discharge amperes, as in commercial lightning arresters, or as in the gap *G* of Mr. Peek's tests will require considerably longer times.

This brings us now to the matter of spark gaps for measuring transient voltages. Since the current necessary to produce a luminous discharge is so small, a measuring spark gap does not need to develop the high degree of conductivity assumed above, and its time lag is much shorter, of the order of 10^{-8} seconds. This may be considered as instantaneous when the gap is used for measuring surges initiated by discharge gaps in the laboratory, or lightning strokes or insulator flashovers in the field, which require 10^{-6} sec. or more to develop their full conductivity.

Mr. Peek states that for most accurate work, no resistance should be used in series with measuring spark gap. I do not agree with him there in general as the leads to the spark gap necessarily have inductance, and as is well known, if a voltage is suddenly applied and maintained on an inductance in series with a condenser, the voltage on the condenser will rise to nearly twice the applied voltage. A somewhat similar question would be as to what is the best value of damping to put in the movement of an ordinary voltmeter to read accurately voltages which are suddenly applied and held on for only a brief time. If there is no damping at all, the movement will come most rapidly to the proper position, but it will not stop there. It will overshoot, and momentarily indicate too high values. The amount by which it overshoots will depend on the rate at which the applied voltage builds up. If the applied voltage builds up in a time small compared to the natural period of the movement, it will overshoot to twice the correct displacement. If the applied voltage builds up in a time large compared to the natural period of the movement, the overshooting will be negligible.

If too much damping is used, the movement will be sluggish, and will not reach the proper deflection during the time of application of the voltage.

To reach the proper deflection most rapidly and without overshooting, the movement should be critically damped.

Similarly, for a measuring spark gap it is best to use that value of series resistance which will critically damp the oscillations attendant upon charging the capacitance of the sphere gap through the inductance of the leads. When, as was most likely in Mr. Peek's circuits, the time of building up of the voltage is long compared to the natural period of the measuring sphere-gap oscillations, not much error will result in omitting this series resistance, but in other cases the error may be considerable. It is not unusual in the laboratory to find an undamped measuring gap indicating higher voltages than the gap initiating the impulse.

H. J. Ryan: In the high-voltage field, generally speaking, breakdowns of solid, liquid, or gaseous dielectrics are due to many factors, and often in very complicated relations. Exact knowledge of all circumstances is, therefore, of the highest importance. Systematic work under definite conditions is rapidly erecting a sufficiently complete understanding of high-voltage phenomena to assist greatly in placing causes of accidents of a high-voltage character in practise.

There is a duty imposed upon those who undertake to profit by these papers. They should remember in connection with Mr. Peek's paper what has been said therein regarding the conditions involving the time element. Where the time in action is so short, as was always the case in those studies, the results show that the discharges through the air require extraordinary voltages, having impulse ratios of about two, and values of 20 kilovolts, effective, per inch in air at common densities. On the other hand, the oscillator that delivers 120, 60,000-cycle wave trains per second produces discharges through air at ordinary densities with impulse ratios that begin at 1.5 for short distances, say *one foot*, and diminish with ever increasing rapidity to 0.67 at *eight feet*. In these circumstances the logarithm of the discharge distance is found to be proportional to the discharge voltage. The logarithms of numbers grow very slowly. The discharge lengths grow very rapidly, therefore, with voltages.

Finally, when we came to that type of discharge that is established through air with high voltages at continuous high frequencies, wherein some of the ionization that is established holds over from voltage crest to crest, the nature of this action is further developed and amplified. In comparatively short discharges the voltage per inch required to break down an air column may fall to the low value of 1.5 or 1 kilovolt. In these circumstances I am convinced that the value of voltage in relation to increase of discharge distances will continue to fall. If one keeps up the action, increases the distance, follows it up with the requisite amount of voltage to accomplish breakdown, I am convinced that the value of voltage per inch of gap will fall below one kilovolt—a ratio of voltages from one extreme of circumstances to the other of 20 to 1, or more.

It is important, therefore, for those who study these papers and their discussions, to remember the conditions their authors carefully specified, under which they obtained their results.

D. D. Clarke: My own great interest in Mr. Peek's paper lies, perhaps, in the new tool that his work has given us in studying what happens on a transmission line and on distribution lines during lightning storms. Those of us who have been more or less responsible at times for keeping lines going, and the people of widely distributed towns in service during lightning weather, very greatly appreciate any move toward solving the peculiar problem of stopping the trouble.

There are several things in a lightning stroke as indicated on the line which are not entirely new to the operating man, but which might be divided into a few classes of phenomena. There is (1) the spit-over, which is not due to a direct stroke, but over which the power current may flow with destructive results.

There is (2), the direct stroke which may melt conductors and drop the lines.

And there is (3), the burning of poles by the power current which may drop the line.

Lines are classified in general as wood pole lines and metal supported lines. Outdoor substations are of course metal supported. The great interest in this paper, to me, comes particularly in the wood pole line construction. Occasionally on such work we find that the poles are split. In the three weeks preceding my attendance at this meeting, we had three distinct cases of such split poles. These are always accompanied by more or less splintering of the arm from the pin of one or more insulators, to the brace, showing that a very steep wave front has existed. Again, the poles in these three cases were distinctly split, but in no case dropping the line. In one of these cases not only one pole was split but four successive poles showed splinters.

The splitting of poles is dangerous to the operation of the line because it is liable to drop the line. It seems very probable that continued investigation with this new tool—the impulse generator—will lead to a solution of this problem and at the same time allow us to retain the very important added flashover value of the line as a whole due to the wood construction.

On a low-frequency oscillator we have demonstrated repeatedly that the wood, whether wet or dry, has a very distinct advantage in increasing flashover voltages of pole-line construction.

The flashover value of three or four feet of wood is so high as to make comparatively negligible the flashover value of the insulator itself. After the insulator flashover is over, the wood has sufficient resistance ordinarily to put out any ensuing arc.

Again, this paper strikes home to the operating man in protecting his substations. The substation is protected ordinarily by a lightning arrester from destructive insulation failure either due to flashover or puncture of transformer coils. The great problem with the operating engineer is to apply the lightning arrester so as to give it a chance to perform its function at all times.

The performance of lightning to date has been a very mysterious quantity. The man has quite a problem on his hands whose duty it is to investigate failure during a lightning storm in a substation which has been equipped with the most complete protective apparatus that modern science has put on the commercial market. He finds that the lightning arrester is set at a reasonable value, that the ground resistances are as low as they can be made on isolated construction—and yet it fails.

I feel that the experiments as indicated in Mr. Peek's paper have lent a way to solving to best advantage that problem.

D. M. Simons: I have been particularly interested in Dr. Whitehead's method of measurement, apart from the results which he has obtained. I do not think that the quadrant electrometer or electrostatic wattmeter has had sufficient recognition, and I would like to say a few words about this method of measuring dielectric losses.

A great many people seemed to have obtained an impression that the electrometer is a very fragile and delicate instrument. We have had one in service for 10 years without an accident and without even changing its suspension, and have found it a very valuable and rugged instrument.

Dr. Whitehead says that some of the errors in the electrometer such as that due to the charging current between the needle and the quadrant and others have not yet been worked out. We have done a lot of work on this subject and I hope that we can soon present some of this matter in definite equation form, which may help to add to the utility of the instrument.

We have been able to measure dielectric losses of an extremely low value, down to milliwatts, at high voltages and at very low power factors and to check our results very thoroughly. I merely want to add these remarks to emphasize what I believe is the very great importance and value of this method of measuring extremely low dielectric losses.

C. F. Harding: Mr. McEachron's paper will be recognized, I think, as confirmatory evidence of the nature of the corona discharge, particularly at the positive and negative electrodes respectively, which was presented some years ago by Mr. Peek and others. It might be well to emphasize the fact, however, that the two methods outlined in this paper provide opportunity for additional study of the variation of such corona throughout the cycle, with respect to time.

Also, it might be well to point out that in both of these cases the corona under investigation is simply that portion which affects the eye or the photographic film. Some recent investigations seem to indicate that corona formation may seriously affect insulation and produce the higher oxides of nitrogen, which in connection with moisture cause insulation to deteriorate, even though that corona does not affect the photographic plate or is not visual. That is particularly true in other portions of the spectrum than the ultra-violet, since the latter may be studied by means of quartz lenses.

Referring briefly to the next to the last paragraph on the second page of his paper, Mr. McEachron states that "At 15 kv. the discharge at 15 cycles (not reproduced here) was about as strong as that at 120 cycles, but the 30-cycle discharge showed no brush at all. At 8 kv. and 60 cycles there was no brush discharge recorded."

It may be that at certain voltages we have critical frequencies at which this corona effect takes place which may, at those frequencies, cause serious deterioration of insulation.

I wish to emphasize also the fact that there were a number of rather important developments brought out by Mr. McEachron in this work that will be published in a bulletin of the Engineering

Experiment Station of Purdue University in the near future, which were not included in this paper.

R. B. Williamson: The factors influencing internal ionization, or internal corona, in built-up insulation as used on the coils of high-voltage generators are of great importance both to the designing engineer and also to the companies operating such generators. There has been a great deal of investigation and discussion during the past few years relating to the effect of temperature on various types of insulation and also the dielectric strength of insulations, but the questions of both internal and external corona have hardly received the attention that their importance warrants. A contribution such as the present one is therefore very timely and the results shown are of great interest and value.

For high-voltage machines, say 6600 volts and over, the thickness and composition of the insulating wall has as a rule been determined for the most part by considerations of dielectric strength, and operating temperature. However, this question of corona in such high-voltage machines is in many instances really the most important determining factor in fixing both the thickness of the insulating wall and the character of the insulation. The tests described in this paper show the superiority of mica insulation, and experience extending over many years has also shown that mica resists the attacks of corona very much better than fibrous insulation. There are many cases, as in high-voltage moderate speed, waterwheel generators, where it is not necessary to use mica insulation so far as heat resisting qualities are concerned, because such machines, if provided with effective ventilation, will operate at temperatures well within the limits allowable for Class A (fabric) insulation. At the same time I believe that such high-voltage machines should be provided with mica insulation to take care of the corona action even though mica is not needed so far as thermal considerations are concerned.

It is interesting to note in these tests that evidences of corona appeared at an average potential gradient of about 16,000 volts per cm. or roughly 40 volts per mil. In an article published about ten years ago in England, by Fleming and Johnson on the chemical action of corona on coil insulation, numerous cases were cited to show that there was danger of such action in ordinary insulation if the volts per mil exceeded 35, and the authors recommended that the thickness of wall be made such that the potential gradient between conductor and ground be not over 35 volts per mil. It was recognized that dense hard mica insulation could be worked higher than this but the figure they arrived at checks up quite well with the potential gradient at which corona became evident in the present tests.

At the bottom of the seventeenth page, it is noted in the baking tests, that all the samples showed a swelling at the points where the electrodes were applied, and the internal ionization apparently was the most important factor in causing the insulation to bulge out. When the coil is placed in the slot this action is limited by the sides of the slot but in order to get the coils in place there must be some clearance and a slight spreading in service is almost certain. Besides, at the ventilating ducts the insulation is free to expand and it is at this point, unfortunately, that the effects of corona are frequently most in evidence because of the sharp corners at the edge of the ducts. While, therefore, it is possible to make coils having a dense hard mica insulation, there is always the probability of some bulging under long continued operation. Moreover, longitudinal expansion and contraction of the copper will loosen the insulation and a certain amount of flexibility is highly desirable to allow the coil to adjust itself to varying temperature conditions and incidentally to facilitate winding.

Since, therefore, with insulation built up as on stator coils, it is more than likely that some voids will be formed under long continued operation even though these be small, we believe that such coils should be insulated with as high a percentage of mica as practicable even though the operating temperatures may not demand it, because mica is the best material we have available to withstand the corona. In addition, while a certain thickness of insulating wall may be sufficient, so far as dielectric strength is concerned, it may frequently be advisable to make it thicker than this, in order to reduce the potential gradient to a point where internal ionization will not be present to any injurious extent. For low-voltage machines the thickness of insulation necessary for mechanical reasons is usually such that the gradient as a matter of course, is so low that internal corona is not the determining factor, and for windings of this kind, a fabric insulation is satisfactory provided the machine is worked within the temperature limit allowable for Class A insulation.

D. E. Howes: Dr. Whitehead's results and conclusions should serve to give users of high-voltage generators confidence in the present structure of windings, and to point out to the manufacturer the importance of careful inspection and construction.

I would like simply to give the results of an investigation to show the potential gradient necessary to produce corona in air spaces of different thicknesses. That corona exists in small air spaces has been most conclusively shown by Dr. J. E. Shrader in a paper on "Corona in Air Spaces." If two dielectrics, 1 and 2, are in series with faces parallel to the electrodes, the gradient in one in terms of the total voltage applied is,

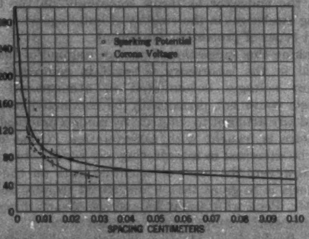


Fig. 1—Sparking Potential and Corona Voltage

$$g_1 = \frac{E}{d_1\left(1 + \frac{d_2\,K_1}{d_1\,K_2}\right)}$$

where the d's and K's are respectively the thickness and specific inductive capacities of the two dielectrics and E the total voltage. In this manner the gradients necessary to produce corona were calculated from data obtained by the method described by Dr. Shrader. The results are shown in the accompanying curve, Fig. 1 where the gradient is plotted against thickness of air space for values of the latter from 0.004 cm. to 0.026 cm. For reference, the sparking potential in air between 1 cm. radius spheres is also shown. This indicates that, as would be expected, the potential gradient necessary to produce corona in small air gaps follows the same law as sparking potential, and does not remain at the conventional value 30 kv./cm. The values fall slightly below the reference curve but this is to be expected when we consider the probable smoothness of the surface of separation.

We see that for very small separations the gradient required may be very high and in all probability the dielectric would rupture first. The curve emphasizes the conclusions of Dr. Whitehead's paper, that built up insulation must be well pressed together, and this is especially true where mica with a high specific inductive capacity is used. It would be desirable if a flexible impregnating compound could be used, but this probably is not feasible at the temperatures and conditions prevailing in an armature winding.

H. S. Warren: These very interesting tests which Mr. Peek has described have suggested to me the possibility of employing his equipment in a field somewhat outside its intended range. I refer to the efficacy of lightning rods for the protection of buildings against lightning.

The subject of protecting buildings—particularly farm property and buildings containing oils and explosives—has received considerable attention but has not been given the intensive scientific study which it deserves. In consequence, while the lightning rod is fairly well established in the opinion of those who actually provide such installations, there is room for legitimate doubt as to the necessity and sufficiency of the methods now commonly employed. This doubt is due to a realization that the whole subject of protecting buildings against lightning rests on an inadequate foundation because of the lack of exact knowledge of the physical principles underlying lightning phenomena.

That this should be so is not surprising in view of the difficulties presented by a scientific investigation of the subject. To conduct in the field such an investigation, comprehensive enough to lead to conclusive results, is a most formidable undertaking.

I wish, therefore, to suggest for Mr. Peek's consideration the possibility of utilizing his apparatus to make laboratory tests in the endeavor to throw some light on the behavior of lightning discharges in the vicinity of buildings with and without different types of protective equipment.

In making this suggestion, I am not unmindful that lightning produced in the laboratory is small in comparison with the maximum voltages encountered in atmospheric electricity. Such tests could not presumably be conducted on a full-scale basis and any attempt to simulate by models practical conditions of exposure is fraught with uncertainties. Great care would have to be exercised in interpreting any results obtained in that way. However, the need of scientific data on the protection of buildings is so great that I am moved to raise the question. I hope Mr. Peek will take it up and see if something cannot be done which will be useful in that application.

F. W. Peek, Jr.: Messrs. Goodwin and Clarke are quite right in that wooden poles even when wet add materially to the line insulation for lightning voltages.

Referring to Mr. Peters' discussion, the table on page 625 gives the effect of various values of resistance in series with the sphere for a given wave. Obviously the effect will vary with the capacity of the sphere. The "instantaneous" voltage can only be measured when there is no series resistance or when it has a very moderate value. I have sometimes used a moderate value of resistance to suppress local oscillations. Generally this is not necessary. The duration of the transient is readily determined by measuring both the "instantaneous" voltage and the voltage with a high known resistance in series with the sphere. A small known inductance in series with the circuit shunted with a sphere gap offers a means of determining the wave front.

A lightning arrester with high impulse ratio or high series resistance in undesirable. The high series resistance has the effect of reducing the discharge capacity and also of increasing the impulse ratio.

The tests with the inductance coils show, I believe, that an inductance without shunt resistance may be dangerous.

Mr. Slepian's calculations are very interesting. I have made similar calculations. The difficulty in this, before the calculations are started certain assumptions must be made. While the calculations may be correct, there may be some doubt as to the assumptions upon which they are based. However, such work is often of great help in the final solution of a problem.

I naturally do not claim the accuracy for these measurements as that which could be obtained by leisurely reading the voltage with a meter in a d-c. circuit. Nevertheless I believe the accuracy is fair considering the difficulties.

Professor Ryan's word of caution is timely.

Dr. Jewett's observations add to the many peculiar phenomena caused by lightning voltages. I have observed that when voltages from the lightning generator are applied to a wooden post, it is blown apart violently. Examination of the two parts show a tiny hole lengthwise through the post where the wood has entirely disappeared. The wood has apparently been turned into a gas and tremendous pressures are thus produced. This effect is readily obtained on kiln dried wood.

Answering Mr. Warren's question, I am engaged at present in an investigation to try and determine the value of the lightning rod.

K. B. McEachron: There has been considerable comment among those who discussed Mr. Peek's paper concerning the explosive effect which appeared in the pieces of wood, poles and so forth. I don't wish to attempt any explanation, but I do want to call your attention to something which may help to explain it.

In J. J. Thompson's book on the "Conduction of Electricity Through Gases," which was published quite a good many years ago, there will be found a description of some experiments on the pressure developed in the electric spark. I remember being much surprised when I found that pressures as high as 600 atmospheres could be expected from the electric spark.

When it is remembered that in those experiments the spark was very small compared to the sparks with which Mr. Peek is working, or the sparks which we get in lightning discharge, it is clear that mechanical pressures may be produced which are sufficient to account, at least in part, for the stresses set up when the wood or other material is torn apart.

The reason why sparks produce oxides of nitro much better than the more gentle corona discharge, may be due to the explosive effect, combined with the difficulty of breaking up the nitrogen molecule.

Mechanical Electrical Construction of Modern Power Transmission Lines.—Insulators for High-Voltage Lines

BY C. B. CARLSON and W. R. BATTEY
Associate, A. I. E. E.
Both of the Southern California Edison Co.

***Review of the Subject.**—The paper deals with the investigations for economics followed by the design assumptions based on the economical findings; the limitations required by electrical clearances; the testing of the structures; the design assumptions of the cable attachments; the unusual requirements of loading for attaching cable to structures; the extensions required by nature of country traversed; and the erection problems.*

* * * * *

In handling the subject assigned, a complete description of the problems encountered in the design of the 220-kv. transmission line from the Laguna Bell Substation to Eagle Rock Substation of the Southern California Edison Co. will form the subject matter for this discussion.

The design was started following a request from a committee assigned to investigate the electrical features of a line of this voltage. The questionnaire outlined investigation on the use of the following five sizes of conductors:

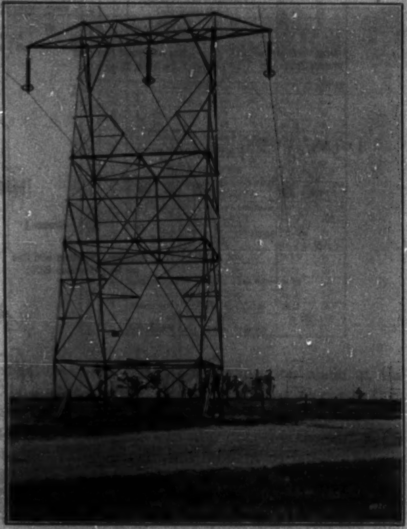
605,000 circular mill steel reinforced aluminum cable
1,000,000 " " " " "
1,500,000 " " " " "

Copper cables of equal conductivity to the last two sizes.

There were several clearances to ground assigned for investigation together with a strict adherence to the recently adopted order of the Railroad Commission regulating high-voltage transmission lines.

Full Size Tower Test

The first preliminary study eliminated from consideration the majority of the materials and limitations and further study was requested on the use of the following:



Cable—666,600-circular mill steel-reinforced aluminum, made up of 7 strands of extra high strength steel and 54 strands of aluminum having the following characteristics:

Diameter—1 inch
Ultimate tension—23,430 lb.
Elastic limit—16,180 lb.
Area—0.59 Square inches
Weight—0.86 lb. per lineal foot

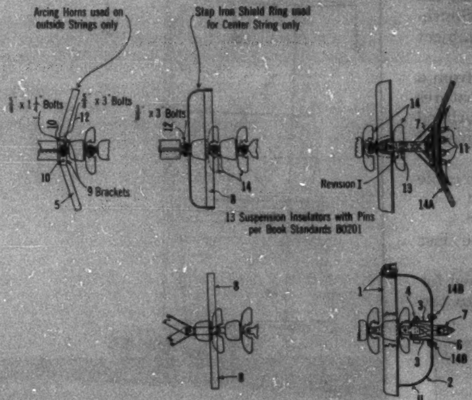
Further, a single ground wire running over the top of the towers, was desired to provide the grounding for the line.

Having now fixed on the cable, a complete set of assumptions was made on which to base an economical study of height of tower and span.

The calculations showed that with present-day costs and the assumed tensions that a span between 1150 and 1300 ft. would give the most economical line.

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

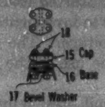
Shortly before the need of ordering steel for the supporting structures, the Southern California Edison Co. needed a large quantity of steel for extending the heigth of the towers of their present 150,000-volt transmission line to meet the conditions of the new legislation regarding ground clearances. It was found that by considering the higher stresses permissible by the use of high-elastic-limit steel as manufactured by a Pacific Coast manufacturer that a saving could be made, so the towers for the 220-kv. lines were designed on the basis of this steel and other conditions of design which were as follows:


Arcing Horns used on outside Strings only
Stap Iron Shield Ring used for Center String only
⅝" x 1¼" Bolts
⅝" x 3" Bolts
⅝" x 3" Bolts
10
12
12
10
9 Brackets
5
14
8
14
7
11
Revision I
13
14A
13 Suspension Insulators with Pins per Book Standards B0201
8
8
1
4
3
14B
7
6
14B
3
2
11

SUSPENSION INSULATOR STRING FOR STANDARD TOWER

Note:- Omit Item No. 8, Strap Insulator Shield Ring, when Arcing Horn Items No. 5-9-10 are to be used.
For each Standard Tower use Item No. 8, Strap Shield Ring for Center Insulator String and Arcing Horns Items No. 5-9-10 for Outside Insulator Strings

LIST OF MATERIAL FOR ONE COMPLETELY ASSEMBLED GROUND WIRE CLAMP FOR STANDARD TOWER			
Item No.	No. Pcs. Req'd	Description	Detail Drwg. No
15	1	Ground Wire Clamp Cap	29-Dr. 30074-110
16	1	Do Do Base	30 Do
17	4	Do Do Bevel Washer	32 Do
18	4	½" Sq. Hd. Bolts-2 hex. nuts & Lock Washer	17-Dr. 30074-132



GROUND WIRE CLAMP FOR STANDARD TOWER

	LIST OF MATERIAL FOR ONE COMPLETELY ASSEMBLED SUSPENSION INSULATOR STRING FOR STANDARD TOWER			
	Item No.	No. Pcs. Req'd.	Description	Detail Drwg. No
II	1	1	Suspension Shield Ring with Insert Bolts	*1 Dr. 30114 109-111
II	2	1	Supporting Arm for Shield Ring	* 2-Dr. 40001-347
II	3	2	Suspension Lugs	* 1-Dr. 40001-347
II	4	1	⅝" x 4½" Bolt Hex. Nut & Washer	* 4-Dr. 40001-347
	5	1	Arcing Horn	* 6-Dr. 13015-216
	6	1	Suspension Clamp Clevis	* 38-Dr. 30074-132
	7	2	Suspension Clamp-Half	* 39-Dr. 30074-130
	8	1	Strap Insulator Shield Ring	* 15-Dr. 24133-428
	9	2	Arcing Horn Brackets	* 2-Dr. 13015-215
	10	2	⅝" x 1¼" Bolts- with Hex. Nuts & Lock Washer	
	11	8	⅝" x 3½" Car. Bolts- 1 hex. nut & Lock Washer	* 20-Dr. 30074-132
	12	1	⅝" x 0 3" Bolt Sq. Hd. Hex. nut	
II	13	1.	Insulator Pin	El. Stds. B0201
	14	13	Suspension Insulators with Pins	El. Stds. B0201
	14A	1	14 Ft. length of .05 x .3 Alum. Armor Wire	
II	14B	2	⅝" x 1½" Bolts Hex. nuts	* 3-Dr. 40001-347

Span 1250 ft.

Tension in cable—12,000 lb. under maximum conditions

Electrical clearance to tower members—6 ft. under wind conditions

Insulators

Suspension—13—10-inch disks.

Dead-end double-string with 15 insulators in each

Shields—Aluminum castings supported on end of insulator strings and surrounding the bottom insulator.

Temperature range—10 deg. fahr.

Clearance to ground at minimum point—30 ft. at 120 deg. fahr.

Wind on Tower—10 lb. per square foot on 1½ area of one face

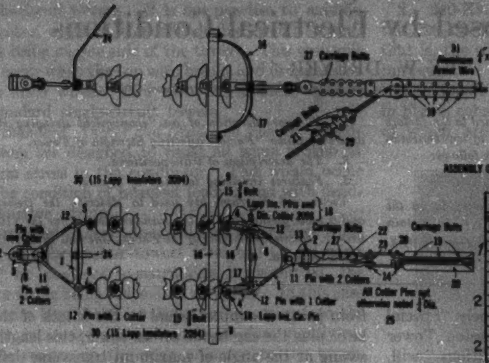
Wind on Line—8 lb. per square foot on projected area of wire

Footings—Earth with frustum of cone at 30 deg. for uplift resistance. Bearing according to soil.

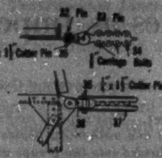
Full-sized tests were made at the steel company's testing frames as shown on accompanying photograph marked "Full Size Tower Test." The results of these tests, with the exception of a few minor details of connection, justified the design and were considered representative of the conditions of actual loading in the field under broken wire and maximum tension conditions with the required factor of safety. An illustration of the standard tower is included herewith showing the framing systems.

Extensions were also then designed to meet conditions of country through which the line had to pass These extensions were made vertical from the same footing spreads as the tower itself, thereby making the setting of stubs uniform throughout. Sidehill extensions both parallel and normal to the line were made, as considerable of the line traverses a mountainous country. Special footing extensions were made to take care of side slopes where sufficient resistance-to-uplift was not obtainable.

The committee having the features of the design of this line in hand required a factor of safety of 3 in the strength of the attachments of the cable to the towers. Due to limitations of clearances this required the hardware to be made of cast steel. The entire


24
14
17
27 Carriage Bolts
31 Aluminum Armor Wire
Carriage Bolts
21
20
19
30 (15 Lapp Insulators 2204)
9
15 Bolt
Lapp Ins. Pins and Dia. Cotter 2056
18
7 Pin with one Cotter
12
5
2
26
1
8
10
11 Pin with 2 Cotters
16
4
13
Carriage Bolts
22
23
28
19
11 Pin with 2 Cotters
14
20
All Cotter Pins not otherwise noted Dia.
25
12 Pin with 1 Cotter
18 Lapp Ins. Co. Pin
12 Pin with 1 Cotter
15 Bolt
30 (15 Lapp Insulators 2204)
9

ASSEMBLY OF DEAD END INSULATOR STRING FOR ANGLE & ANCHOR TOWER


32 Pin
33 Pin
x 1 Cotter Pin 35
34
Carriage Bolts
35
x 1 Cotter Pins
36
37

ASSEMBLY OF GROUND WIRE CLAMP FOR ANGLE & ANCHOR TOWERS

LIST OF MATERIAL
FOR ONE COMPLETELY ASSEMBLED GROUND WIRE CLAMP

Item No.	No. Pcs Req'd.	Description	Detail Drwg. No.
32	1	[illegible] Dia. Pin - Mach. Steel	No. 15 Dr. 30074-132
33	1	Do Do	No. 16 Do
34	8	[illegible] Carriage Bolts - 1 Hex. Nut - 1 Lock Washer	No. 23 Do
35	3	[illegible] x 1 [illegible] Brass Cotter Pins	No. 25 Do
36	1	Ground Wire Half Clamp R. H.	No. 27 Dr. 30074-110
37	1	Do Do Do L. H.	No. 28 Do
38	1	Do Do Clamp Clevis	No. 31 Do

LIST OF MATERIAL FOR ONE COMPLETELY ASSEMBLED DEAD END STRING FOR ANCHOR & ANGLE TOWER

Item No.	No. Pcs Req'd.	Description	Detail Drwg. No.
1	2	Dead End Yokes S. C.	No. 1 Dr. 30074-109
2	2	Yoke Attachment Clevis S. C.	No. 2 Do
3	1	Adapting Clevis for Vertical Pins S. C.	No. 3 Do
4	2	Lower Insulator Clevis S. C.	No. 4 Do
5	2	Upper Do Do Do	No. 5 Do
6	2	[illegible] x 4 Mach. Bolts - Hex. Nut & Cut Washer	No. 6 Dr. 30114-110
7	1	1" Dia. Pin drilled for 1 Cotter Mach. Steel	No. 7 Dr. 30074-132
8	1	Do Do Do 2 Do Do	No. 8 Do
9	2	Aluminum D. E. Shield Ring Part	No. 1 Dr. 30114-110
10	2	Do Do Do	No. 2 Do
11	2	1" Dia. Pin drilled for 2 Cotters Mach. Steel	No. 11 Dr. 30074-132
12	4	[illegible]" Do Do 1 Do Do	No. 12 Do
13	1	1" Do Do 2 Do Do	No. 13 Do
14	4	[illegible]" Do Do 2 Do Do	No. 14 Do
15	4	[illegible] x 3[illegible] Carriage Bolts - Hex. Nut & Cut Washer	No. 5 Dr. 30114-110
16	2	Supporting Arm for Double String Shield Ring	2R Dr. 30074-157
17	2	Do Do Do	2L Do
18	2	Lapp Insulator Pins #2056	Dr. 40001-L 187
19	10	[illegible] x 2[illegible] Carriage Bolts - 1 Hex. Nut & Lock Washer	No. 19 Dr. 30074-132
20	2	Half Aluminum Cable Clamp S. C.	No. 33 Dr. 30074-131
21	8	[illegible] x 3[illegible] Carriage Bolts - 1 Hex. Nut & Lock Washer	No. 21 Dr. 30074-132
22	1	Core Clamp-Right hand half C. S.	No. 34 Dr. 30074-131
23	1	Do Left Do Do	No. 35 Do
24	1	Arcing Horn Steel	No. 24 Dr. 30074-132
25	22	[illegible] x 1[illegible] Cotter Pins - Brass	No. 25 Do
26	2	[illegible] x 1[illegible] Do Do	No. 26 Do
27	10	[illegible] x 4[illegible] Carriage Bolts 1 Hex. Nut & Lock Washer	No. 23 Do
28	2	Splice Plates for D. E. Clamp	No. 36 Do
29	2	(Nelson) Jumper Loop Splice Alum. Casting	No. 37 Do
30	30	Lapp Insulators 2204	Dr. 40001-L 187
31	1	20 Ft. length of 0.05 x 0.3 Alum. Armor Wire	

requirements for hardware were designed and tested for loads three times the maximum tension.

A complete assembly of both the suspension and dead end strings is shown on plates marked "Assembly of Insulator String for Anchor Tower" and "Assembly of Insulators for Standard Tower." It is to be noted that to develop, on a factor of 3, the tension in the insulators that a special type of high-strength disks and pin connections was necessary.

The towers, except those in rather inaccessible places, which were assembled piece by piece, were erected by assembling the tower on the ground in two parts dividing the face normal to the line and bolting with loose bolts each leg to the footing studs. Then by means of two gin poles set aside the footings and a winch set on a truck and operated by its motor, the two halves were raised and connection made at the center gusset plates. The bridge of the tower was then hoisted in place.

The line stringing was also done by the use of the motor-operated winch and brought to tension by use of the sag method.

The unusual features in transmission line design such as the stringing tension, the spans, and the high voltages, made many difficult problems, but as the line is just now in process of erection, the complete justification for the assumptions cannot be completely told.

Discussion

For discussion of this paper see page 966.

Special Features in the Design of Transmission Tower Lines as Imposed by Electrical Conditions

BY W. DREYER

Asst. Engineer, Division of Civil Engineering, Pacific Gas & Electric Company, San Francisco, California

***Review of the Subject.**—This paper is confined to the factors which affect the structural and mechanical phases of transmission line design. These are divided into two groups as follows:*

1. Factors affecting design of tower.

2. Features involving location of towers.

The writer has in general refrained from commenting on the various items listed. Exceptions are made, however, in certain features which in his opinion deserve particular emphasis. Briefly, these are:

1. The consideration of an "economical stringing stress" as described under "Factors affecting Strength of Tower."

2. The description of transpositions.

3. Features involving location of towers.

Views and diagrams are used to describe the transpositions and a table giving data on the standard towers for the 110-kv. and 220-kv. transmission lines of the Pacific Gas and Electric Company is also included.

THE title assigned to this paper, namely "Special Features in the Design of Transmission Tower Lines as Imposed by Electrical Conditions," will, if strictly interpreted, cover a field somewhat broader than the scope of the writer's knowledge, which is confined strictly to the structural and mechanical side of the transmission problem. The paper will, therefore, be limited to those features which affect either the design of the supporting structures, or the location of these structures in the field.

The writer will attempt to enumerate all of the factors, giving a brief description of the more important points and describing how the problems are handled in the transmission lines of the Pacific Gas and Electric Company. The features of greatest interest,—transpositions,—are described by illustration. A table is also included which gives data on the standard towers for the 110-kv. and 220-kv. transmission lines of the company.

FACTORS AFFECTING DESIGN OF TOWER

The factors which affect the design of the standard line towers may be divided into two groups, one affecting the shape and the other the strength of the supporting structure. These factors are briefly enumerated below:

(a) *Factors Affecting Shape of Tower:*

1. Type of tower—single vs. double circuit.
2. Permissible distance between phases.
3. Permissible distance between circuits.
4. Length and allowable transverse deflection of insulator string.
5. Sag of wire inside tower.

An allowance for sag inside of the tower based on a 20-deg. angle of pull will usually be sufficient for obtaining proper clearance between the wire and the members of the tower.

6. Minimum clearance between conductor and members of tower.

This clearance should be maintained, using a length from point of support equal to the length of the insulator plus the sag inside of the tower, this length being swung at the angle of maximum transverse deflection. This investigation will, particularly for single-circuit towers of the flat top type practically establish the shape of the transverse framing of the structure.

(b) *Factors Affecting Strength of Tower:*

1. Size and maximum stress of conductor.

The size of the wire on trunk transmission lines is usually selected to best suit electrical conditions. The maximum stress in the wire can, however, be controlled by the tension at which the wire is strung.

The usual practise in tower line construction is to string the conductor so that it will be stressed nearly to its elastic limit when under maximum loading. The writer believes that following this practise blindly will give neither the most economical nor the most satisfactory transmission line. Investigations have been made by our company to determine the conductor stress which would result in the lowest cost for supporting structures, consideration being given to both the towers and their foundations. This investigation showed that a maximum tension of 17,000 lb. per sq. inch for copper 250,000 cir. mil to 500,000 cir. mil in size resulted in the lowest cost of completed line. The use of this tension instead of 27,000 lb. per sq. in., resulted in a saving of about 20 per cent in the cost of towers and foundations. For the Pit River 220-kv. line this represents a saving of nearly $1200 per mile. In addition, this tension, which is only about 60 per cent of the elastic limit of medium drawn copper, greatly reduces the load on dead-end insulator strings, consequently reduces the number of dead-end insulator failures, and therefore gives a more satisfactory line.

2. Standard Span.

The standard span is determined by economic considerations and for steel towers on concrete foundations, greatest economy will result with spans 800 to 1200 ft. in length.

3. Type of Tower.

(a) *Double-Circuit vs. Single-Circuit.* It is customary to consider more broken wires for double-circuit than

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

for single-circuit towers. It is our practise to assume three broken wires on standard double circuit towers, all wires being on one side of the tower. This case will usually occur when the top wire breaks and drops across the two lower wires. Two wires are assumed broken with standard single-circuit towers.

(b) *Standard, Anchor and Dead-End Towers.* Dead-end towers are designed to resist all wires broken at maximum stress in the conductor. This would occur under the condition of heaviest loading and with dead-end insulator strings.

For standard towers (designed for two or three wires broken) and for anchor towers (designed for all wires broken) no dead-end construction is permitted, the insulator string being suspended vertically. In this case when a wire breaks the insulator strings swing into the catenary, increase its length and therefore greatly reduce the tension in the span adjacent to the break. For our standard 110-kv. double-circuit lines the resultant stress is 78 per cent and for the 220-kv. double-circuit line, it is 66 per cent of the maximum conductor stress.

All towers of the Pacific Gas and Electric Company are tested to 50 per cent overload.

(c) *Transposition Structures.*

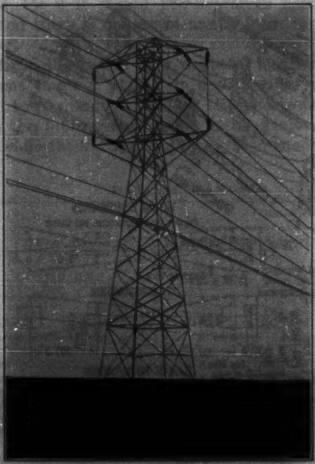


Fig. 1—110,000-Volt Transposition Tower

1. 110-*Kv. Transmission Lines.* Transpositions on the 110-kv. lines are of the conventional "Dead-end and jumper" type, where the transposing is done on a tower. Our standard 110-kv., double-circuit transposition tower is shown as Fig. 1. The only additions to the standard tower are extensions to the upper and lower arms.

2. 220-*Kv. Transmission Lines.* Transpositions on the 220-kv. Pit River line are of the "rolling type," that is, the wires are transposed without dead-ends or jumpers.

The double-circuit transposition structure, as shown in Fig. 2, consists of two standard line towers the cages of which are connected by two latticed bridges. A diagrammatic plan showing the transposition scheme is shown on Fig. 3. It will be noted that the crossing wire of each circuit is pulled out of the vertical plane to an insulator suspended from the bridge, and that the transposition is completed in two spans.

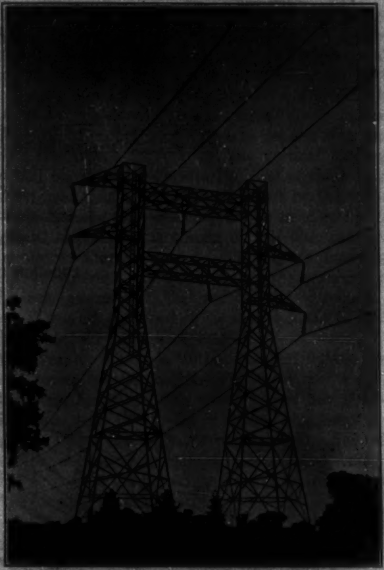
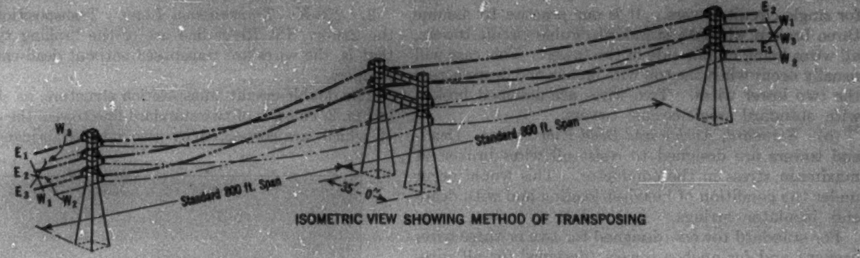
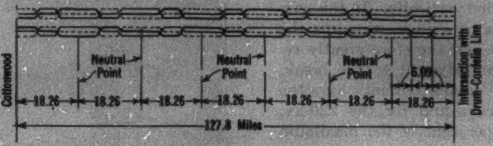


Fig. 2—220,000-Volt Double-Circuit Transposition Tower

The single-circuit transposition, shown diagrammatically on Fig. 4, requires three spans for completion. This was done to bring the crossing points of the wires on the towers rather than at the center of the span. The intermediate towers are standard line towers with an extension above them, and with a portion of the cross-arm removed. In order to reduce the side deflection of the insulators to a minimum, the intermediate towers are offset as shown on the plan. This results in the wires crossing in straight lines, with the insulators on the intermediate towers hanging vertically. Fig. 5 shows one of the completed transpositions; it will be noted that there is no deflection of the insulators except on the end towers, which is unavoidable.


E2
W1
W3
E3
W2
W3
E1
E2
E3
W1
W2
Standard 800 ft. Span
35'-0"
Standard 800 ft. Span

ISOMETRIC VIEW SHOWING METHOD OF TRANSPOSING


Cottonwood
Neutral Point
Neutral Point
Neutral Point
6.09
Intersection with Drum-Cordelia Line
18.26
18.26
18.26
18.26
18.26
18.26
18.26
127.8 Miles

LOCATION OF TRANSPOSITIONS

Fig. 3

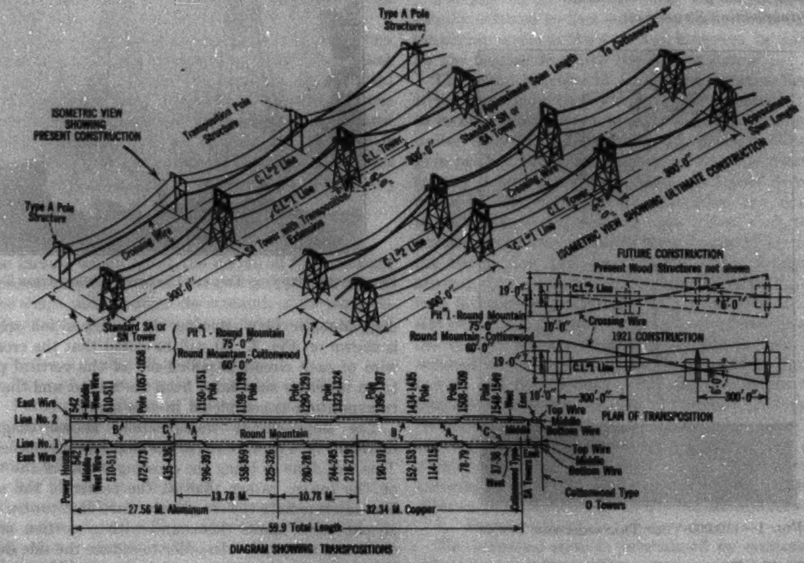

Type A Pole Structure
To Cottonwood
ISOMETRIC VIEW SHOWING PRESENT CONSTRUCTION
Transposition Pole Structure
Approximate Span Length
Standard SN or SA Tower
C.L. Tower
C.L.#2 Line
C.L.#1 Line
300'-0"
SA Tower with Transposition Extension
Type A Pole Structure
Crossing Wire
Approximate Span Length
300'-0"
Crossing Wire
C.L. Tower
6'-0"
ISOMETRIC VIEW SHOWING ULTIMATE CONSTRUCTION
C.L.#2 Line
C.L.#1 Line
300'-0"
Standard SA or SN Tower
Pit #1 - Round Mountain 75'-0"
Round Mountain - Cottonwood 60'-0"
FUTURE CONSTRUCTION
Present Wood Structures not shown
19'-0"
C.L.#2 Line
6'-0"
10'-0"
Crossing Wire
1921 CONSTRUCTION
19'-0"
C.L.#1 Line
10'-0"
300'-0"
6'-0"
300'-0"
PLAN OF TRANSPOSITION
East Wire
Line No. 2
Line No. 1
East Wire
Power House
542
Middle
West Wire
510-511
Pole 1057-1058
1150-1151 Pole
1198-1199 Pole
Pole 1290-1291
Pole 1323-1324
Pole 1396-1397
1434-1435 Pole
Pole 1508-1509
Pole 1548-1549
West
East
B
C
A
Round Mountain
B
A
C
Middle
Top Wire
Middle
Bottom Wire
Top Wire
Middle
Bottom Wire
542
Middle
West Wire
510-511
472-473
435-436
396-397
358-359
325-326
280-281
244-245
218-219
190-191
152-153
114-115
78-79
37-38
West
Cottonwood Type
SA Towers
East
Cottonwood Type O Towers
13.78 M.
10.78 M.
27.56 M. Aluminum
32.34 M. Copper
59.9 Total Length
DIAGRAM SHOWING TRANSPOSITIONS

Fig. 4

Features Involving Location of Towers

(a) *Stagger or Offset of Towers on Twin Lines.*

Where twin tower lines traverse country that is not flat, it is impossible to set the structures directly opposite each other. The amount of offset that will be permitted should be based on maintaining a minimum clearance between adjacent wires equal to the distance between phases on the flat-top, single-circuit towers. The horizontal projection of the catenary as deflected by wind, furnishes the basis for the determination of allowable stagger.

(b) *Ratio of Horizontal and Vertical Loads to Limit Deflection of Insulator.*

In designing the towers, clearances are investigated under the assumption that the insulators are permitted to swing transversely to the line at some angle (usually between 30 deg. and 45 deg.) from the vertical. In locating the towers in rough country it is necessary to see that this maximum angle of deflection is not exceeded, for if it should be, clearances become impaired and there is danger of arcing from the line to the tower.

Fig. 5—220,000-Volt Single-Circuit Transposition Tower

A typical illustration of a doubtful location is one where one tower is placed in a depression with the adjacent towers on high ground. The horizontal wind load on the intermediate tower is unaffected by the profile, while the vertical load may be reduced almost to nothing. In this case the insulator would probably deflect toward the tower beyond the allowable angle, and would create a situation which would ultimately cause an interruption to service. Where this situation exists, there are three remedies available, one to extend the tower in order to increase the amount of vertical loading, and if this cannot be economically done, to resort to either a tie-down or a dead-end.

On the 220-kv. Pit River tower line it was considered highly desirable to reduce the number of dead-end strings, it being felt that these points offered the greatest opportunity for trouble, which would mean cutting out of service a line carrying in excess of 200,000 kilowatts. As tie-down strings were also considered undesirable the only alternative to dead-ending was to extend the tower to increase the vertical loading at doubtful points. The additional cost of the extension to the tower was always largely offset by the value of the insulators saved (about $110 per dead-end string). With the single-circuit towers of this line, the saving in insulators alone would provide over two tons of steel in the extension. At other points the elimination of dead-ends permitted the use of the standard line tower (Type SA) instead of the anchor tower (Type SC) with an actual saving of money.

As a result of this investigation, and by the liberal use of tower extensions, the Pit River line, some 200 miles long, contains one tower which has dead-end insulator strings to take care of the irregularity of the profile. There are only 21 points on the line where dead-end strings are required, 2 of these being the terminal points, 18 at horizontal angles and one because of the profile of the ground.

DATA ON STANDARD TOWERS OF PACIFIC GAS AND ELECTRIC COMPANY

Item	220-Kv. Tower Lines				110-Kv. Tower Lines			
	Single-Circuit Towers		Double-Circuit Towers		Single-Circuit Towers	Double-Circuit Towers		
	Type SA	Type SC	Type M	Type 0-98	Type S	Type AH	Type BH	Type CH
Height								
Overall	55½′	55½′	97′	98′	49½′	83′	83′	83′
To Lower Cross-Arm	51½′	51½′	62′	63′	48′	58′	58′	58′
Width								
At base	20′ x 20′	20′ x 20′	20′	20′	14′ x 14′	16′	16′	20′
At cross arm	4′ x 20′	4′ x 20′	6′	6′	2′ x 14′	4½′	4½′	4½′
Conductor Separation								
Between phases	19′	19′	15′	15′	13½′	10′	10′	10′
Between circuits	..	..	24′	24′	..	17½′	17½′	17½′
Conductor								
Max. size (copper)	500,000 cm.	500,000 cm.	500,000 cm.	500,000 cm.	3-0	250,000 cm.	250,000 cm.	250,000 cm.
Loading	8 lb. wind ½″ ice	8 lb. wind ½″ ice	8 lb. wind no ice	8 lb. wind no ice	6 lb. wind 1½″ snow	8 lb. wind no ice	8 lb. wind no ice	8 lb. wind no ice
No. broken	3	3	3	6	2	3	3	6
Span								
Normal	500	500	800	800	400	800	800	800
With angles	...	1000	...	800	...	...	800	800
Max. on tangents	750	1500	1000	1800	400	1000	2000	2500
Max. contributing weight	1500	2100	1050	1800	400	2000	2500	5600
Max Angle	None	22½°	None	22½°	None	None	15°	45°
Type of Conductor Support	Suspension	D. E. or Susp.	Susp.	D. E. or Susp.	D. E. or Susp.	Susp.	D. E. or Susp.	D. E. or Susp.
Weight of Tower	5100 lb.	7270 lb.	8440 lb.	12,950 lb.	4250 lb.	5080 lb.	6200 lb.	7500 lb.
Foundations								
Concrete (cu. yds)	6.28	11.8	7.8	15.2	11.2	6.72	8.72	12.9
Excavation (cu. yds)	18.1	60.4	25.5	84.5	32.5	22.4	31.2	44.4

Discussion

MECHANICAL ELECTRICAL CONSTRUCTION OF MODERN POWER TRANSMISSION LINES—INSULATORS FOR HIGH-VOLTAGE LINES[1]

(Carlson and Battey),

SPECIAL FEATURES IN THE DESIGN OF TRANSMISSION TOWER LINES AS IMPOSED BY ELECTRICAL CONDITIONS[2] (Dreyer),

Del Monte, Cal., October 3, 1923.

E. V. Pannell: Mr. Dreyer, on the fourth page of his paper, makes particular reference to the matter of excessive swinging of insulator strings where the vertical loading disappears. On the subject of dangerous swing of insulators where the horizontal load is unbalanced, the facts seem to show that the tie-down string is somewhat unsatisfactory, but Mr. Dreyer makes no mention of the possibility of using weights and I would like to know whether he has made any experiments in this respect. The use of a weight in place of a tie-down string has the advantages that a vertical component is provided at the same time the flexible characteristics of the line are not impaired and the arrangement would seem to be inherently superior to imposing dead-end stresses on a point which is not designed as a dead-end. In several installations in this country weights are being used instead of tie-downs and I also know of work in Japan where weights have superseded tie-downs with greater satisfaction. It is necessary, of course, that clearances shall not be impaired but if this condition is satisfied I believe weights will be found the most satisfactory device.

H. Michener: On the Big Creek lines, consideration was given to the use of weights instead of tie-downs; it was finally decided that it took too many tons of cast iron. The tie-down strings on the Big Creek lines have given no more trouble than any other strings on the line, so we do not feel that they are a great hazard.

R. J. C. Wood: It seems to me that we may go too far in trying to eliminate the dead-end strings. We would like to get away without any of them largely on account of the cost, but it is a fact that on the Big Creek lines we have had less trouble on the dead-end strings than on any of the others. I think the exact record is two cases of trouble on dead-end strings in the whole operation of the line.

M. Hillebrand: I would like to ask Mr. Wood to amplify that last statement.

The reason for the elimination of dead-end strings, to my mind, holds as much today, possibly not just as much, but the fundamental reasons are the same as they always have been. That is, that the mechanical load on the dead-end string is higher than elsewhere. This tends to cracking and exists to a greater extent in the dead-end insulator than on the suspension strings. It is in order to reduce the cracking of the insulators that it has been so often recommended to eliminate dead-end strings wherever possible.

Now, the troubles that Mr. Wood refers to, unless I am mistaken, are in the nature of flashovers through the air and around the insulator. On the other hand, I believe it is true that their records show that their insulator failures on dead-ends have been at much higher rates than those on the suspension string. Is that true, Mr. Wood?

R. J. C. Wood: Yes, it is higher.

Mr. Hillebrand: I would like to ask Mr. Dreyer just what is derived from the use of a lower string tension. That is, was it due to a lighter tower, resulting from a lower design in order to maintain a minimum clearance?

C. B. Carlson: On the point of tension, it might be added that the span, as we have made it in our line, is considerably larger than those heretofore used. This might bring that point back again to a higher tension rather than lower tension. That is, with a greatly increased span maintaining all the time a factor of thirty feet from the ground as a constant, the calculations might revert back to an advantage towards a little higher tension insofar as that is concerned.

W. Dreyer: Answering Mr. Hillebrand's question as to how the lower tension affects the tower design, the towers become lighter and the footings become lighter, almost in direct relation to the movement of the loading. The product of the loads on the tower times their distance from the ground, approximately gives the law of variation in the weight of the towers. Thirty feet of the height of the towers is put in to take care of minimum ground clearance. Seven feet more is necessary to take care of the length of suspension insulators on 220-kv. lines, giving a total height of thirty-seven feet which is necessary without any consideration of conductor stress.

In order to show how the problem would work out on a line of 220-kv. potential using 500,000 cm. copper and insulators seven-ft. long, the following table comparing maximum stresses of 27,000 and 17,000 lbs. per sq. inch respectively.

Stress—Lbs. per Sq. Inch			Sag. at Min. Stress	
Max.	Min.	Broken Wire (Allowing for insulator)	Span = 800 ft.	Height to Low Wire
27,000	19,700	15,000	16 ft.	53 ft.
17,000	13,000	11,200	24 "	61 "

Using the above table as a basis, the following table shows the relation between heights to center of application of leads and overturning moments for both single and double-circuit towers.

	Single-Circuit Towers				Double-Circuit Towers			
Max. Stress	Height to Center of Load	Ratio of Height	Ratio of Load	Ratio of Moment	Height to Center of Load	Ratio of Height	Ratio of Load	Ratio of Moment
27,000 80'	58	1.00	1.00	1.00	68	1.00	1.00	1.00
17,000 80'	61	1.15	0.75	0.86	76	1.12	0.75	0.84

The ratio of moment, which is nearly in proportion to the ratio of weight of towers and size of footings, shows a saving of 14 per cent for the single-circuit and 16 per cent for the double-circuit towers. This analysis is only approximate, but it indicates the existence of a point of economy for the stress in the conductor.

Regarding the affect of torsional stresses the square tower is not materially affected in weight, the torsional stresses being taken care of almost entirely by the web members.

Regarding the use of weights to prevent excessive swinging of insulator, as suggested by Mr. Pannell, we have given consideration to such an expedient. The objection was the additional electrical clearances required, which would either make it necessary to use special towers with greater clearances for the particular cases involved, or else design all towers with excessive clearance, which would be uneconomical.

110-Kv. Transmission Line for Oak Grove Development of Portland Railway, Light and Power Company

BY H. R. WAKEMAN and W. H. LINES

Member, A. I. E. E.

Both of the Portland Railway & Light Co.

Review of the Subject.—*This paper covers the design of the 110-kv. transmission line to transmit energy from the new Oak Grove development of the Portland Railway, Light and Power Company to the city of Portland, Oregon, a distance of 54 miles.*

The company now has three hydroelectric developments aggregating 52,000 kw. supplied from the same general watershed from which energy is transmitted to Portland over five 60-kv. circuits, only one of which is on steel towers. The problem was to design a transmission line to most economically handle the three 30,000-kv-a. units as installed at the new development and to tie in with the existing transmission system. The following factors were considered in determining the design:

1. *Transmission voltage*
2. *Conductor capacity*
3. *Maximum conductor tension*
4. *Assumed maximum mechanical loading conditions*
5. *Type of supporting structures*

Consideration was also given to the physical characteristics of the country through which the line must be built. For 14.8 miles the line will be located in the United States Government Forest Reserve, which is very heavily timbered, and accordingly cost of clearing will be very high, averaging $15,000 a mile, even after credit for merchantable timber is allowed. This fact, outside of other considerations, required a minimum number of circuits. To reduce clearing costs as much as possible, the line was located along the river gorge necessitating 55 angles, a few as great as 60 deg., and 28 river crossing spans, varying from 400 to 1800 feet.

After considering various types of supporting structures, three types of double circuit, heavy rigid steel towers were decided upon as follows:

Type A towers, to be used on tangents up to 800-foot spans, also on curves up to 10 deg., the spans not exceeding 500 feet; suspension insulators to be used.

Type B towers, to be used on tangents up to 1700-foot spans, also on curves from 8 to 30 deg., the spans not exceeding 800 feet for the larger curves; suspension insulators to be used.

Type C towers, to be used at dead-ends and curves over 30 deg.

Towers are designed for 250,000 cir. mil. copper conductors, and maximum loading conditions of one-half inch of ice and eight pounds per square foot wind pressure at 0 deg. fahr. were assumed. Each type of tower was designed to withstand the maximum loading conditions plus 50 per cent without exceeding the elastic limit of the steel, and the towers were subjected to an actual field test. Special steel having an elastic limit of 45,000 pounds per square inch and a minimum elongation of 22 per cent was used in the towers.

In order to reduce the number of insulators and deadened positions, use was made of brackets pivoted to the cross arms so that they would be free to swing in the line but rigid across the line.

12-inch brackets are provided for the Type A towers on 8 deg. curves and 24-inch brackets on Type B towers with 30 deg. curves so as to maintain a three-foot clearance between the conductor and steel on that side of the tower where the conductors pull the insulator strings up and toward the cage. The use of these brackets reduced the total cost of the insulators and hardware fully 25 per cent.

Tables are included giving resultant simultaneous loading and weights and dimensions for each type of tower.

THIS paper is one of several dealing with the review of the problems encountered in the design and construction of the high-tension tower lines completed during the past two years. The Portland Railway, Light and Power Company has not constructed any lines of importance in this class during the period in question, but has recently completed the design and started field work on a transmission circuit of considerable capacity to serve its new Oak Grove development. This project is situated on the Clackamas River, a tributary of the Willamette, about 54 miles from the city of Portland. The ultimate capacity of 90,000 kv-a. will be provided by three units of 30,000 kv-a. each, the first of which is scheduled for completion in September, 1924, the others to follow as the system load requires.

At the present time the company has three hydroelectric developments, aggregating 52,000 kw. in capacity, fed from the same general watershed, two of which are located on the Clackamas River, 19 and 22 miles respectively below the Oak Grove development, and one on the Bull Run River, a tributary of the Columbia. Transmission from these plants is supplied by five circuits at 60 kv. grounded neutral, only one of which is on steel towers. Four of these circuits converge at Lents Junction on the outskirts of the city proper.

Briefly the problem was to design the most economical transmission system to handle the successive units of the Oak Grove development and to tie in with the physical and electrical characteristics of the present lines. This involved the consideration and proper economic selection of the following major variables:

1. Transmission voltage.
2. Conductor capacity.
3. Maximum conductor tension.
4. Assumed maximum mechanical loading conditions.
5. Type of supporting structures.

Naturally all of these variables are intimately related and in addition are effected by the physical conditions obtaining along the line. To arrive at the proper economic solution due consideration was given all the physical conditions and all possible combinations of the five major variables mentioned above.

The first step was a reconnaissance and a preliminary location of the line in the field. Then a party was organized and a sufficient topography taken to enable

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

the selection of the best profile. The data thus obtained brought out the following physical facts affecting the design:

The total length from generating station to Lents Junction, where it is proposed to tie with existing 60-kv. circuits, is 45 miles. Of this distance 14.8 miles is in U. S. Government forest reserve and very heavily timbered. In order to secure a right-of-way from the Forest Service, the company was forced to agree to several somewhat stringent regulations as to the methods to be used in felling timber and clearing up, and these requirements, plus the price paid for stumpage, will result in an average cost of about $15,000 per mile to clear and clean up a 400-foot strip through the reserve, even after proper credits are allowed for merchantable timber, which it is proposed to market. In order to avoid as much as possible all this excessive cost, the line was located along the river gorge, which, in turn, required many horizontal angles and river crossings. In addition, it became apparent that a minimum number of circuits should be adopted in order to avoid excessive clearing costs. In the upper 19 miles of the line, namely, from the generating station to Faraday, at which point existing transmission circuits start, there are 55 angles, a few of which are as great as 60 deg., 28 river crossings with spans varying from 400 to 1800 feet, and numerous grades, some over 75 per cent.

TRANSMISSION VOLTAGE AND CAPACITY

It was determined, after proper consideration of the above physical facts, that for the ultimate development of 90,000 kv-a. two circuits each of 250,000 circular mil hard-drawn, stranded copper, at 110 kv. grounded neutral, were respectively the economic number, size and voltage to employ.

MAXIMUM CONDUCTOR TENSION

This was a problem to which considerable study was given. The limitations imposed by the mechanical strength of available insulators and hardware were a considerable factor in the decision to use 4000 pounds maximum conductor tension, equivalent to a stress of 20,000 per square inch or one-third of the ultimate strength of the conductors in this line.

ASSUMED MAXIMUM LOADING CONDITIONS

After an investigation of the possible sleet, snow and wind conditions in the country traversed by this line, it was determined to use class B loading, namely, a maximum conductor tension equivalent to the load of the conductors plus one-half inch of ice and 8 pounds per square foot wind pressure on the ice-coated conductors at temperature of 0 deg. fahr.

TYPE OF SUPPORTING STRUCTURES

Before a decision was reached on this point, there were placed on the profile, to scale and by means of properly constructed sag templates, four types of supporting structures designed for the assumed loading conditions and appropriate for the physical topography of the country. Thomas sag curves were used in computing the relations between stress and sag under various loading conditions. It was found that the maximum sag occurred on the line under summer loading conditions, at a temperature of 120 deg. fahr. The following are the four types of supporting structures which were considered:

1. Single wooden poles, average spans 300 feet, single-circuit capacity.
2. Double wooden poles (H frames), average spans 450 feet, single-circuit capacity.
3. Light steel towers (combination of flexible frames and anchor towers), average spans 500 feet, double-circuit capacity.
4. Heavy rigid towers, average spans 700 feet, double-circuit capacity.

Detailed estimates of material and labor were then prepared for each of the four types and these resulted in favor of the fourth, namely, heavy rigid towers, average spans 700 feet.

A brief consideration of the following points will perhaps explain this result:

The cost, coupled with the possibility of failure of modern high-voltage insulators and hardware, tended to force the adoption of longer spans and thus fewer insulators. In the case of wooden poles, the item of hole digging for poles and guy stubes, mostly in rock, was considerable. Guying costs to reinforce pole strength at the many horizontal and vertical angles in the line were excessive. River crossing and other spans of greater than the average length required the adoption of special and expensive structures to provide the required strength. The difficulty of placing material along the line was also a considerable factor in making the estimates high, particularly with the wooden pole structures with their attendant extra weight per mile of line.

From a study of the loading conditions imposed on the towers by the varying span lengths and the horizontal and vertical curves, it was decided to provide three types of towers to meet all these loading conditions, as follows:

Type A Towers. To be used on tangents up to 800-ft. spans, also on curves up to 10 deg. with spans not exceeding 500 feet; suspension insulators to be used.

Type B Towers. To be used on tangents up to 1700-foot spans, also on curves from 8 deg. to 30 deg. with spans not exceeding 800 feet for the larger curves; suspension insulators to be used.

Type C Towers. To be used at dead-ends and curves over 30 deg.

The towers were designed to withstand maximum loading conditions embraced in each of the above classes, plus 50 per cent as an assumed margin of safety, without exceeding the elastic limit of the steel. In Types A and B, where suspension insulators are used, consideration was given the fact that with a broken conductor on one side of the tower the insulator string will swing into the line, increasing the sag, and thus reducing the conductor tension and tower loading. At all horizontal angles the towers will be set to bisect the line angle.

Following is a summary of the resultant simultaneous loading assumed for each type of structure and applied at points of insulator support:

Loading Conditions	Type A	Type B	Type C
Number of conductors assumed broken	3	6	6
Maximum horizontal curve	10 deg.	30 deg.	60 deg.
Maximum span length when combined with curve	500	800	800
Vertical load—dead weights of conductor	6 at 1,800#	6 at 1,200#	6 at 1,200#
Horizontal load in line component of conductor tension	3 at 4,000	6 at 3,500	6 at 5,200
Horizontal load across line component of conductor tension + wind on cond.	6 at 1,400	6 at 2,200	6 at 3,700
Horizontal load across line wind on tower	1 at 1,600	1 at 1,600	1 at 1,600
	32,800#	43,000#	62,000#

The type A tower on tangents will accommodate spans to 800 feet.
The type B tower on tangents will accommodate spans to 2400 feet.

The design of the towers was also checked for maximum possible torsional load, namely, with three conductors broken on one side of the cage.

Special steel having a guaranteed elastic limit of 45,000 pounds per square inch and a minimum elongation of 22 per cent was used in the towers. Actual tests of samples taken from the tower stock showed yield points from 48,000 to 53,000 pounds per square inch.

As a positive check on the design, each type of tower was subjected to an actual field test, and all of them withstood the prescribed loads without permanent deflection of any member. We are reliably informed that these towers are the lightest ever built for equivalent loading conditions, which fact can be explained in part by accurate design and in part by the use of high elastic limit steel. The following tabulation of comparative dimensions and weights may be of interest:

	Type A	Type B	Type C
Cage	4 ft.-6 in.	4 ft.-6 in.	4 ft.-6 in.
Length of crossarm, cage to insulator support	7 ft.-0 in. & 9 ft.-0 in.	7 ft.-3 in. & 9 ft.-3 in.	7 ft.-3 in. & 9 ft.-3 in.
Horizontal separation of conductor at supports	18 ft.-6 in. & 22 ft.-6 in.	18 ft.-9 in. & 22 ft.-9 in.	18 ft.-9 in. & 22 ft.-9 in.
Vertical separation of conductor at supports	10 ft.	10 ft.	10 ft.
Height of lowest arm above ground	48 ft.	48 ft.	45 ft.
Overall height above ground	72 ft.	72 ft.	69 ft.
Weight of tower exclusive of extension and footings	4325 #	5355 #	5635 #
Weight of footings (all steel)	550 #	1320 #	2780 #
Depth of footings in ground	7 ft.-6 in.	6 ft.-6 in.	7 ft.-6 in.
Area of footing at bottom	3 ft. x 3 ft.	4 ft.-8 in. x 4 ft.-8 in.	5 ft.-6 in. x 5 ft.-6 in.

In order to eliminate any possibilities of the conductors at different elevations swinging together when a lower conductor drops a heavy ice load before the one above, the middle conductors were offset two feet beyond the top and bottom ones.

Steel footings were decided upon on account of the difficulty and expense of handling and mixing concrete along the line. The footings are designed to resist maximum uplift 10 per cent greater than that occasioned by the assumed test loading conditions.

The towers are designed to permit hillside and square extensions at 5, 10, 15 and 20-foot intervals and any combinations of these. Some idea of the nature of the country can be had when notice is taken of the fact that approximately 70 per cent of all the towers in this line are provided with an extension of some sort.

A maximum clearance, conductor to steel, of 3 ft. 0 in. was specified for all possible positions of the conductor.

An interesting detail in connection with this line was the decision to employ a string of insulators in suspension position on horizontal curves up to 30 deg. In order to maintain the specified clearance of 3 ft. conductor to crossarm, it became necessary to drop the point of insulator support 12 in. on Type A towers (8 deg. curve) and 24 in. on Type B towers (30 deg. curve) below the crossarm on that side of the cage where the conductor pulls the insulator string up and toward the cage. This was accomplished by the use of brackets attached and pivoted to the crossarms so that they would be free to swing in the line but rigid across the line. The use of this device saved fully 25 per cent of the total cost of insulators and hardware and further reduced the number of actual dead-end positions, always a possible source of insulator failure, to a minimum.

Insulators are attached to crossarms by means of forged steel hooks with an elastic limit of 6000 pounds. It was thought best to limit the strength of this connecting member so that if any load greater than the assumed came on the conductors, the connection would fail rather than the tower.

The two transmission circuits above described were designed to transmit the ultimate development of Oak Grove, 90,000 kv-a., at 110 kv., a distance of 45 miles to Lents Junction where it is planned to tie them through auto-transformers on to a 60-kv. switching bus. The existing 60-kv. circuits from other hydroelectric plants will tie on to the same bus. Two synchronous condensers of 15,000 kv-a., each supplied by tertiary windings from the auto-transformers, are planned at this point for power factor correction and voltage regulation of all incoming lines.

Discussion

For discussion of this paper see page 974.

Transmission Line Construction in Crossing Mountain Ranges

BY M. T. CRAWFORD

Fellow, A. I. E. E.
Superintendent Distribution, Puget Sound Power and Light Company

Review of the Subject.*—In crossing mountain ranges with a recent transmission line in the Pacific Northwest severe climatic conditions were encountered. Snow lies from ten to thirty feet deep, wind attains high velocities and the temperature ranges between wide limits. A strongly built steel tower line successfully met these conditions. At one point however, a somewhat unusual formation of frozen fog was found to successively build up and drop off of wires and towers producing stresses greater than it would have been reasonable to design a line to withstand, and as a result failures occurred.*

Conditions in the extreme loading area are outlined and the assumption made that the most severe stresses were largely of an intermittent nature. The subsequent modifications in the construction are discussed, which were designed to more efficiently stress the materials under intermittent strains, providing an increased flexibility by the substitution of suspension for strain forms of wire support.

Where line construction fails from excessive dead loading a stronger design throughout is no doubt the proper remedy. If however this loading is only excessive in one span at a time it may be relieved by longitudinal flexibility. While not submitting the principle of the elimination of strain forms as universally applicable in transmission line construction, it is hoped that this paper will elicit a discussion from authorities on the subject that will define its range of application.

* * * * *

AMONG recent extensions of the Puget Sound Power and Light Company is a 120-mile 110-kv. transmission line from the White River generating station to Wenatchee. This line was built up the Green River valley, over the Cascade Range near Stampede Pass at about 4000 feet elevation, down through the Yakima River valley and over the Wenatchee Range at an elevation of nearly 6000 feet, and down into the Columbia River valley to the City of Wenatchee. The engineering features of this line were worked out with admirable care and skill, and the construction embodies exceptional mechanical sturdiness throughout.

Fig. 1—Light Frost Formation Early in the Winter on Tower No. 825

On account of the heavy snows and inaccessibility of sections of the line over the two mountain ranges, additional mechanical strength was provided where the elevation exceeded 3000 feet. Type S. A. 55-foot steel towers were used for line supports, 4/0 B. & S. hard-drawn seven-strand copper cables for conductors, with O. B. No. 25620 cap and pin-type insulators, six-unit strings in suspension and seven-unit strings in strain. Spans were normally from 400 to 700 feet, with conductors pulled to a tension which would stress the wire to its elastic limit of 4250 lb. at 0 deg. fahr., 8 lb. wind pressure and one-half inch of ice coating.

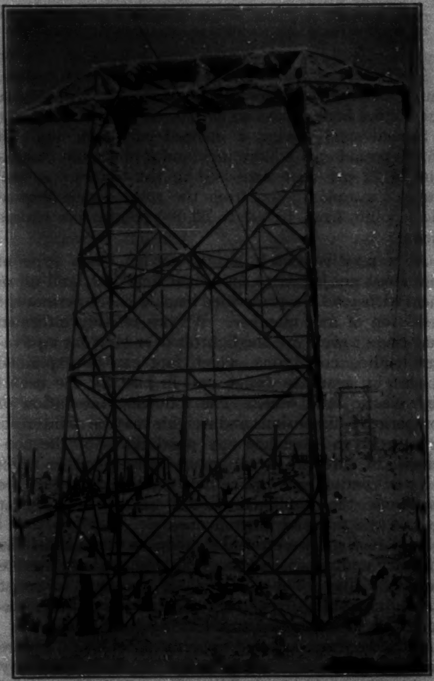

Fig. 2—Heavy Frozen Fog Formation Later in Winter, Tower No. 823

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

Although the Wenatchee Range crossing was at a higher elevation, the climatic and topographic conditions over the Cascade Range were much more severe from the standpoint of line operation. The summit of this range marks a radical change in climatic conditions, from the moderate temperatures and damp winters of the Puget Sound area to the wider range in temperature and dry climate of Eastern Washington. In crossing the Cascade Range at Stampede Pass, it was necessary to locate the line at a bare exposed point, where the summit was clearly defined. It was found that at times during the fall and winter months a heavy frost formation builds up on exposed surfaces near this summit, due to the wind carrying moisture-laden clouds of fog up from lower elevations, which freezes as it strikes the cold surfaces, building out rapidly toward the wind. Formations up to a maximum of about two feet in thickness in one direction were observed. As soon as the weight of the frosty mass becomes great enough it falls off and the building-up process starts again. The weight was estimated at from 5 to 10 lb. per cubic foot. Observations were taken from time to time during the first winter and advice sought from officials of the Weather Bureau. The indications were that the conditions bringing about this formation are apt to recur a number of times per year, but usually within a limited area, as the contact with cold air soon turns the fog clouds into snow. This formation has been found to occur along only about one mile of line near the summit, from observations made so far. The photographs were obtained after the atmosphere cleared and the formation had dropped off of the wires.

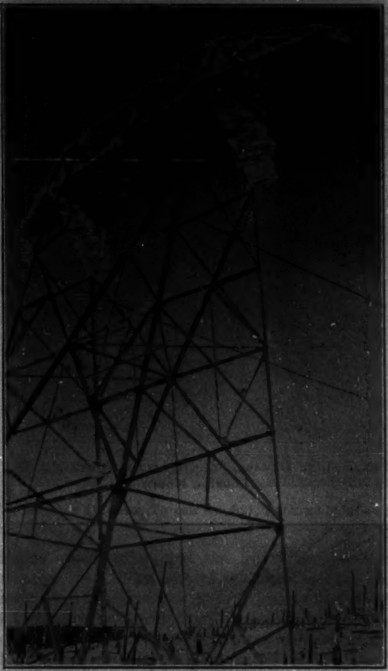

Fig. 3—Heavy Frozen Fog Formation Later in Winter, Tower No. 823

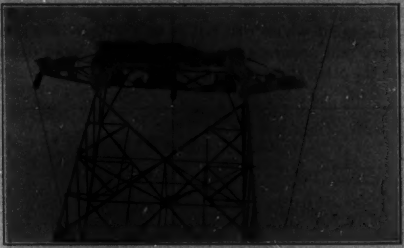

Fig. 4—Heavy Frozen Fog Formation Later in Winter, Tower No. 826

One break occured in the line during the first winter, which is considered too many for trunk line operation by an organization that has high standards of service. A careful study was therefore made of all the factors involved in order to plan reinforcement work. As shown in the photographs and profile, the south wire was pulled in two in the second span east of the summit, but no insulator strings failed on this wire, although the wire slipped through the clamp at the next dead end east of the break. The other two wires were not broken, but three insulator strings pulled in two and one clamp slipped at the dead end points adjoining. It should be especially noted that there were several suspension towers in between these failures, but no

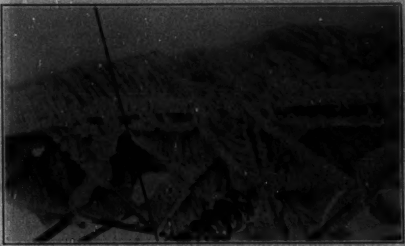

Fig. 5—Close-up of Formation on Tower No. 826

evidences of insulator damage or wire slipping were found at suspension points. Two of the insulator failures were of a character typical of the failure of cap and pin-type insulators under extreme tension, the porcelain parting well up under the cap with a fracture at right angles to the tensile stress. The third failure was in the lip of the cap casting. The wire failure was typical of a tensile break, six of the strands being drawn out to a reduced diameter before parting. The seventh strand had a brazed lap joint at this point which was put in at the time of manufacture and apparently parted somewhat more easily than the other strands. The effect of this was probably to reduce the strength of the cable at this point a few per cent, which accounts for the wire parting and not the insulators on this conductor.

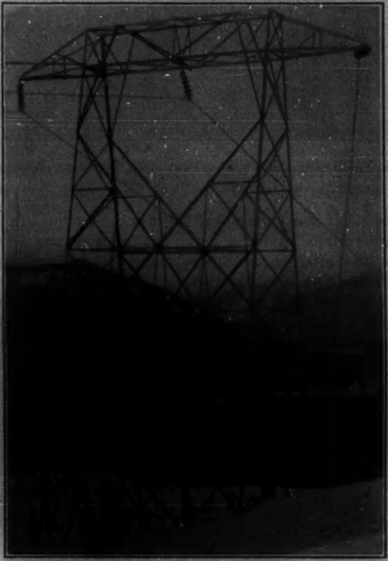

Fig. 6—Line Damage, Tower No. 826, Looking Northeast Showing Broken Wire

The towers were not damaged by the frost formation. Later in the winter heavy snow falls packed down to a depth of eight to ten feet, very hard on top. The spring melting underneath and the settling of this packed crust damaged the tower members badly in places near the ground and sheared off bolts.

The ultimate strength of this conductor is about 8500 lb. and of some of the insulator disks probably slightly less. Approximate computations indicate that a quiescent dead load on the wires in these spans would not have equalled this ultimate strength with the largest formation that evidence could be found of adhering to the wires. This formation builds out almost horizontally toward the wind from the conductor, forming in a narrow elliptical cross-section up to about 3 by 8 inches before dropping off, weighing possibly one and one-half to two pounds per lineal foot. It is believed that the building up and falling

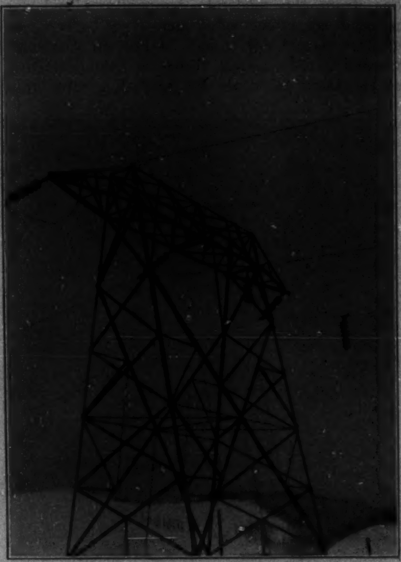

Fig. 7—Line Damage, Summit Angle, Tower No. 825, Looking North

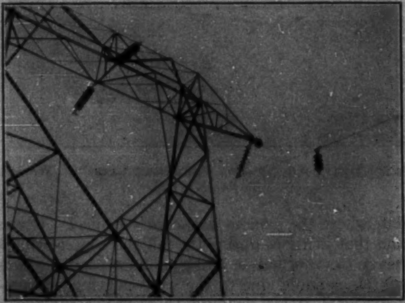

Fig. 8—Line Damage, Tower No. 822, Looking Northwest

off of the heavy frost formation on the conductors resulted in intermittent strains and jerks of a very severe character near dead end points which exceeded the strength of the wire, and that where the insulator strings failed or clamps slipped at dead end points, this tension was relieved thereby, but where the insulators held rigidly the conductor was broken. It should be noted that the damage along the two wires that stayed up through the winter was such as to virtually convert the construction to a suspension form through the loaded area.

In the heavily timbered Puget Sound region it has been found expedient to build some less important lines with only a narrow swath cut through the timber, the tall firs towering above the wires on each side. Branches and sometimes entire trees will often be blown across the line, producing a suddenly applied extreme loading. Many years of experience with this sort of trouble has shown that the most flexible types of line construction are the least damaged. For example, one 110-kv. line of this class is built on a single wood pole line with light crossarms and suspension construction with few dead ends. Trees and branches will often fall on the wires without breaking them, although they may be carried to the ground. The tension is relieved by pulling slack out of spans some distance on each side, with possibly a few broken crossarms. On lines where wires are rigidly held the damage is much more severe.

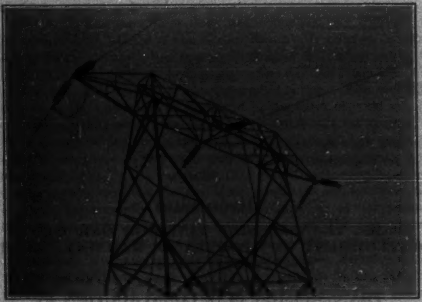

Fig. 9—Line Damage, Tower No. 821, Looking Northwest

Conclusions reached involved a redesign of certain features of the construction through the exceptional loading area, to provide:

1. A reduction in working tension on insulator strings which will safeguard against failure, as well as improve the electrical reliability.

2. A flexibility in wire supporting mechanism to allow some longitudinal movement of wires at all supports, which will permit the easing off of temporary over stresses from one span into adjoining spans.

3. Additional strength in tower members near the base.

It is believed that the substitution of the suspension form of support for all dead ends has taken care of the first two requirements. The insulator string under suspension conditions carries only the resultant of the respective wire stresses in each direction, and the dead end loading is largely shifted from porcelain to copper, reducing insulator tensions without increasing the number of insulators on the line. The suspension unit installed in places of former strain construction was made up of two strings of insulators in parallel with strain yokes at each end, which reduced the tension duty in the porcelain by half. This yoked double string unit was installed to hang approximately in vertical suspension, and the wire so pulled as to

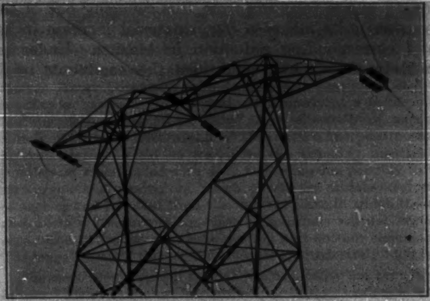

Fig. 10

Angle strain tower No. 825 at Summit, with one wire changed to suspension form of support and two yet to be changed. Wire was hung in free running sheaves for over a mile before placing in clamps to insure balanced stresses at supports.

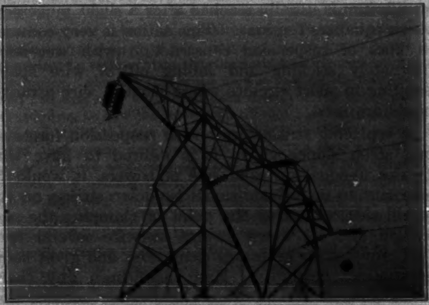

Fig. 11

Straight Strain Tower No. 822, with one wire changed to suspension form and two yet to be changed.

give normally equal tensions on each side. At such times as abnormal loads come on one side these suspension units will swing out into the position of strain units. The necessary spacing out was provided at tower fastening and wire clamp to allow this with proper clearances.

Computations indicate that a slight increase in the length of arc resulting from this swinging out of suspension units makes a very material increase in sag and corresponding reduction in tension. Under extremely unbalanced conditions the conductor might even sag down until it touched the snow in the center of the span. This movement is restricted, however, by the amount of sag that can be pulled out of the several adjoining spans in each direction, and an equilibrium is usually reached after a slight movement which equalizes tensions. This action is very common on lines of suspension construction with wet snow alternately sticking and falling off of wire spans, resulting in short circuits where vertical wire arrangements are used.

In replacing strain forms with suspension forms on long steep slopes where it is desired to have wire stresses unequal on each side of towers, it would be necessary to install additional tie down strings on the up-hill side, offsetting the point of clamping the suspension and tie down strings to wire several feet. This provides for necessary clearances, and gives some longitudinal flexibility. Had there been any sharp line angles in this section, the flexible construction could have been provided by installing additional towers so as to make a round turn with short spans, the suspension strings swinging out nearly horizontal. This form of corner construction has been successfully used on suspension insulator lines along county roads and railroads where the right of way turns on a wide radius. It involves the use of bracket supports for inside insulator strings to obtain clearances.

The snow damage to tower leg members will be prevented by the installation of additional pieces of heavier angle stock alongside existing members, with the corner of the angle facing upwards to form a roof effect and provide a cutting edge to break the snow.

If still more severe winters are encountered in the future which this construction will not withstand, a stronger conductor with more insulator strings yoked in parallel and possibly additional towers may be needed, but the principles of mechanical design are submitted as obtaining the best balance in the stressing of materials under such extreme loading conditions.

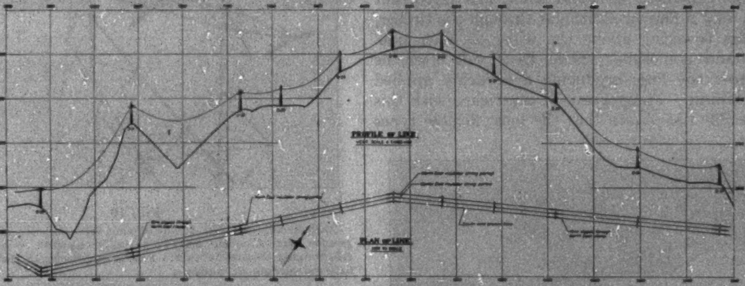

FIG. 12—TRANSMISSION LINE CROSSING CASCADE RANGE, NEAR STAMPEDE PASS
Showing details of damage by heavy frost formation.

Discussion

110-KV. TRANSMISSION LINE FOR OAK GROVE DEVELOPMENT OF PORTLAND RAILWAY, LIGHT AND POWER COMPANY[1]

(WAKEMAN AND LINES),

TRANSMISSION LINE CONSTRUCTION IN CROSSING MOUNTAIN RANGES[2] (CRAWFORD),

DEL MONTE, CAL., OCTOBER 3, 1923.

R. J. C. Wood: We have encountered some little trouble in crossing mountain ranges in snow countries, supposedly due to the loss of the snow or sleet load from a span and the conductor coming in contact with the one above it. On the double-circuit steel tower line, in question, the conductors are offset a foot and a half from the vertical plane of the one above it. We have had considerable discussion as to just what happens to cause these conductors to come together. One theory being that the sleet falls off the one span and still remaining on the adjacent span, the lighter span is pulled up into the one above it.

Another theory is that the sleet falls off a portion only of a long span and that gives it a deformed shape sufficient to permit the two conductors to come together.

The third theory is that when the sleet falls off it sets up waves of sufficient amplitude to produce contact.

It will be interesting to know if anybody else has made observations on this subject and can give us information on just what happens when the sleet falls off.

J. P. Jollyman: We do not cross any high ranges at high altitudes or any locations subject to extremely severe climatic conditions.

We have had some sleet trouble. It has been generally thought, though not actually observed, that the sag in the conductors is greatly interfered with by the inequality in the loading of sleet.

It is now believed that the conductors load up uniformly and when the sleet starts to melt the trouble begins.

We had one interesting report where sleet accumulation on spans was found to slide to the lower part of the span and pile up in very great weight at the bottom of the span. We had one or two cases of trouble which indicated that such an event had happened. The only precaution that has been taken in country known to be subject to sleet is to employ the horizontal configuration, or to employ the displaced conductor arrangements of the vertical circuit. It has been our experience thus far in points not subject to very heavy loading, that a 20-foot vertical displacement seems to be free from trouble, whereas, a 10-foot is not. In a configuration using three conductors normally spaced ten feet apart vertically, the center conductor is displaced either outward or inward two or three feet and thus far it has been satisfactory.

R. J. C. Wood: Referring to the 20-foot vertical spacing, what length of span is that?

J. P. Jollyman: We would not ordinarily regard the horizontal displacement as effective in spans of, over 500 or 600 feet. Where we have longer spans, and where we have any idea that sleet conditions will occur, we would feel it necessary to change from a vertical to a horizontal configuration, to be entirely safe from the difficulties attending on the unloading of sleet.

W. S. Jennens: Sleet does not occur on our lines on the plateau or high points in the Rocky Mountain district, in which our system is located, but mostly in the low hills adjacent to the Bear River and at those points where the cold air from the other side of the hills strikes the moisture-laden air. The condition known as sleet by most of us, is not in the form of solid ice, but chiefly in a compact form of frost. Sometimes, of course, we have damp or extremely wet snow which, in falling from the conductors, gives us the same condition as you gentlemen know as "sleet-jump." Fortunately the condition prevails mostly in short sections of the line and if the condition can be discovered in time, lines mostly concerned are taken out of service long enough to permit the maintenance crews to dislodge the wet snow by jarring the conductors. From the observation of these crews, we have concluded that there is no definite way in which the conductors act when the sleet jumps before the snow actually falls. We will find adjacent spans having entirely different appearances, one being loaded with snow, the conductors will sag down and the other span without snow, the conductors will be stretched tight and consequently rise. If, therefore, no equal loading of the conductors in the same span prevails, it will readily be seen that the conductors will come in contact with one another without the sleet having jumped. On the other hand, when the sleet does not let go through any unbalance of adjacent spans, conductors may rise in accordance with their previous loaded condition, and come in contact with each other in the third or even in the fourth span, particularly if the latter be below on a hillside where the tendancy of the conductor is to run down towards the bottom of the slope. In some cases there will be a wave motion of the conductors, as pointed out by Mr. Wood, due to snow falling in one span and its jarring effect communicated to the conductors of other spans. The result of change of tension in the various contiguous spans causes the wave of the conductor to run along the line and if two or more conductors do not act similarly, they will come in contact with each other. Where they are out of step, so far as the wave motion is concerned, it is evident that on 130-kv. line that contact made thus will quickly burn the conductor through. We find that the conductors burn chiefly in the center of the span.

Each year's condition has become worse and worse, and during the past summer we have off-set the center conductor of each circuit arranged in a vertical configuration to a distance of approximately three feet, only however in the locations where previous observations have shown the sleet to have formed to any considerable degree. We expect in the future to complete this off-setting in order to eliminate the chances of failure in any locality throughout the length of the line. On account of the design of our steel tower being such that it greatly weakens the same if we lengthen the middle crossarm or reduces the clearance to ground at the tower should we shorten the arm, we have followed the scheme in use on the Pacific Gas & Electric Company's lines, using an additional string of insulators on the middle arm to pull the conductor towards the tower.

We are not certain whether our three-foot off-set is sufficient and would like to ask Mr. Jollyman, if, in his opinion, it is enough for operation at 130-kv.?

R. H. Halpenny: On one tower line of The Southern Sierras Power Company, some trouble resulting from sleet on conductors has been experienced, particularly at a point where the line crosses a mountain range at an elevation of approximately 4000 feet. No trouble of this nature has been noticed, however, which could be traced to the swinging together of conductors when loaded with sleet, this being due possibly to the arrangement of conductors, which offsets the middle conductor of each circuit five feet from the vertical plane in which the top and bottom conductors lie, with a resultant normal vertical separation of twenty feet between top and bottom conductors. The longest span in this section of line is approximately 1400 feet.

The most severe kind of trouble in this locality has been tower failure. Line conductors have failed due to sleet loading in a few instances but an excessive dead load of this nature has ordinarily resulted in the deformation of the cross arms or some other tower member.

This trouble has been overcome to a great extent by additional bracing of tower members showing a tendency to fail under such loading, also in some cases by the addition of towers at points liable to be most heavily loaded.

John B. Fisken: This discussion takes me back to the Vancouver Convention in 1913, at which convention the same subject was discussed. The question has been asked, and the desire has been expressed for actual testimony, as to what occurs. I can't of my own knowledge give any such testimony, and, as a general rule, I don't always believe all that the average farmer says, but we have one farmer in our region who I believe told us the truth. He gave us, some years ago, a very full description of what he actually saw. The conditions were vertical arrangement of conductors on a two-circuit tower line, the wires in vertical planes having horizontal separations of about six inches. He said that while he was passing the line one day, and this was after we had all been driven nearly crazy with the shutdowns, he saw the frost drop off one wire and that wire fly up and strike the one above it. This is not sleet loading but frost that forms and drops off when the day gets warmer. After that we separated the vertical planes of the wires horizontally by moving the top conductor in towards the tower, and lengthening the middle crossarm and moving the conductor from the tower. This gave horizontal separations between vertical planes through the conductors of top to center, 5 feet, 10 inches, center to bottom 3 feet, 5 inches and top to bottom, 2 feet, 5 inches. Since then we have had no trouble at all and I am satisfied that all of our troubles were due to the same thing, the unloading of the conductor in one span and the excessive weight of the adjoining spans causing it to hit the wires above it.

Harold Michener: Referring again to the line of which Mr. Wood spoke—I was over it on Saturday and Sunday, last, trying to determine what we could do to it. We selected a span about 890-ft. long that was held by suspension insulators at each end, calculated the load for the span with half an inch of ice, and concentrated one-fourth this load at the one-eighth and one-fourth at the three-eights points of the span. This pulled the center wire down about one foot below the bottom wire and showed that if the bottom wire should be unloaded and the center wire loaded, the center wire would go down into the bottom wire for more than three-eights of the length of the span. That was only an indication of what might happen in a territory where we had not assumed any sleet load in designing the line, and we are of the opinion that in the territory where the design assumptions include a sleet load, and, for that reason, a lower stringing tension, and where the spans are longer, the top wire would come down to the bottom wire if the top wire were loaded in one-half the span and the bottom wire unloaded. The vertical distance between top and center and bottom wires at the towers, is six feet, three inches.

H. H. Schoolfield: We have observed a very peculiar phenomena on one of our transmission lines which might be of interest to the engineers present. I have never yet been able to find a satisfactory explanation for it. Perhaps some of you gentlemen can enlighten me.

We have a 66,000-volt line, with suspension insulators, and conductors of three-strand copper clad steel. In connection with this we have a seven-strand steel ground wire. There are several places in the line where we get frost conditions similar to those described by Mr. Fisken. I call it frozen fog. It builds up on the copper clad conductors to a considerable extent and we have experienced the same trouble of wires swinging together; the lower wire flying up and hitting the upper wire. The peculiar fact is that the frost does not accumulate on the steel ground wire. I don't know why it is and I can't find a satisfactory explanation for it, but it is a fact that the frost will build up to considerable extent on the copper clad conductors, but does not build up at all on the steel ground wire.

L. J. Pospisil: In addition to what Mr. Fisken has said in regard to his experience, I want to say that in the year 1910, I believe it was, there was an effort made to reproduce the conditions, of what occurred from actual experience, by constructing two spans of a sample line and reproducing the loadings along the conductors by means of weights, and then releasing the weights and observing what took place. At that time Mr. Geiser wrote a paper on those experiments, which I think was given at the Vancouver Convention in 1913.

H. T. Plumb: The Utah and Idaho power companies have had some trouble with frost conditions; but along most of the lines it has caused no trouble. The frost referred to is a wonderful formation; it happens only in a very still valley where no wind is blowing; it builds up in the night, and disappears as soon as the sun begins to shine when it drops off. It adds considerable weight to the conductors, and their diameter may become four or even six inches. An explanation of why it might form on copper conductors more than on iron conductors is that the iron is probably smaller in diameter and the amount of growth seems to be dependent on the diameter of the thing to which the frost fastens itself.

Not long ago one of the smaller power companies in Utah had difficulty with a very wet sleet bridging over insulators and causing them to ground, or flash across. It was a sleet so wet that it could hardly be called ice and yet it would stick and build up on the lee side of an insulator.

M. T. Crawford: Experience with wires arranged in a vertical plane has been universally unsatisfactory in our territory through snow country and it should be noted that, in the line described in this paper, a horizontal arrangement was used. Longitudinal movements of the wire accompanying the vertical movements sufficient to cause short circuit with a ten-foot vertical displacement would necessarily produce severe stresses at dead end points, and it was to avoid damage from these stresses that the special form of suspension construction was employed. Since writing this paper the line has gone practically through another winter without a repetition of the failure described although ice and frost formations have been equally severe.

Insulation Design of Anchors and Tower Supports for 110,000-Volt 4427-Foot Span Over Carquinez Straits

BY L. J. CORBETT

Member, A. I. E. E.

Pacific Gas & Electric Company, San Francisco, Cal.

Review of the Subject.*—The first crossing of the Carquinez Straits by a high-tension transmission line was one of the stepping stones in the progress of the electrical industry. The pioneers built well, and the march of events, with its demands for greater and greater blocks of power, found the crossing structure adequate for the new requirements.*

Constructed in 1901 for one 60,000-volt circuit with a spare cable, two additional cables were added in 1914, making two 60,000-volt circuits. In 1922, coincident with the replacement of 60,000-volt pole lines by a 110,000-volt tower line, the crossing was modified to allow 110,000 volts on the two circuits, thus again practically doubling the transmission capacity.

The original design made use of insulating materials in compression. A clear record for over 20 years led the company to adhere to the compression type in the reinsulation for the higher voltage, thus making the crossing unique among long high-voltage spans. The methods by which some of the problems were solved, the hinged anchor structure, the sturdy insulated support for the towers, the movable top for the middle tower support to allow for relative motion and the supplementary cable system to obviate crystallization at the supports, are described in the paper.

WHEN the Bay Counties Power Company installed the 4427-foot span across the Carquinez Strait on San Francisco Bay to operate at 60,000 volts it was considered a noteworthy achievement. So carefully and so thoroughly had all the features been investigated that it established and maintained a record for many years under operation by that company and its successor, the Pacific Gas and Electric Company.

As at first constructed in 1901 there were four cables—one for each phase of the three-phase circuit, and a spare cable. The steel towers were fitted with wooden crossarms supporting saddles of wood and iron upon six pin-type insulators. The anchor insulators, two in series, each consisted of a steel link against micanite in compression in a container filled with oil. These insulators have given complete satisfaction at 60,000 volts for the past 22 years.[1]

In 1914 two additional cables were added, making two circuits at approximately 20-feet separation with cables at 10-feet spacing in each circuit. The crossarms were changed from wood to steel and wood was also eliminated from the saddles. The old anchor insulators were left in but on the two cables added, a steel structure pulling against six pin-type, 60-kv. insulators in compression was installed at each cable end. These have given no trouble whatever since their installation in 1914.

To care for the growth of power load in the San Francisco Bay district and to allow entry for the power from the company's developments on the Pit River, a substation was built about 50 miles northeast of San Francisco where the power is stepped down from 220,000 to 110,000 volts and brought down to the Bay district over new 110,000-volt tower lines replacing the old 60,000-volt pole lines which were no longer adequate. The crossing, mechanically suitable, had to be reinsulated for the new voltage.

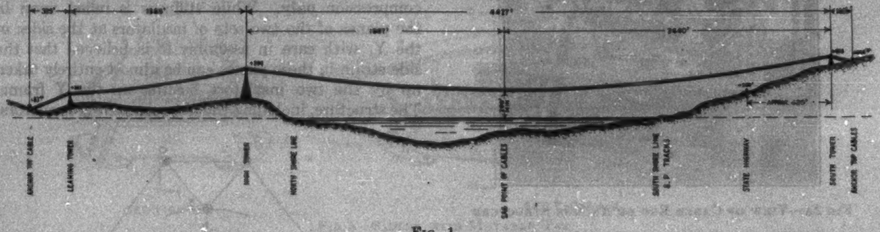

Fig. 1

So successful has the compression type of insulation proved on this crossing that it has been adhered to in the work carried out during 1922, notwithstanding the fact that the tension type of insulation has been developed to a fair degree of reliability and has been adopted on a number of important spans.

Fig. 1 is a profile of the complete cable system. The cables merely rest upon the supports at the three towers, the direct tension being held by concrete anchor blocks at the ends. The South Tower, 140 feet from the extreme anchor block, is 64 feet high to the top crossarm. The Main Tower on the north

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[1] A description of the original installation will be found in the *Engineering News* for October 3, 1901.

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

side of the strait is 4427 feet from South Tower and is 225 feet high. The North or Leaning Tower is 1385 feet farther on and is 84 feet high. The north anchors extend 325 feet beyond this tower. Due to the differences in elevation of the bases the elevations of the supports of the top cable are 478, 394 and 201 feet on the respective towers. The leaning tower is so called because its upper portion is inclined at an angle of 13 degrees from the vertical in order to resist to best advantage the resultant stress at its location part way down the catenary.

Analysis of the crossing showed that the unequal expansion of the 4427-foot span and the 1385-foot span would cause unbalanced horizontal stresses in the direction of the cable unless provision was made to allow the small amount of relative motion necessary to keep the horizontal stresses on both sides of the tower equal.

FIG 2A—VIEW OF CABLE END OF ANCHOR STRUCTURE

ANCHORS

For the anchors was adopted in part the method used on the cables which were added in 1914. The end of the cable was passed around a 24-inch sheave and securely clamped. From the sheave axle the pull is transmitted through a system of springs to a round bar which forms an axle at the end of the stem of a deep Y-shaped steel frame. The arms of the Y are braced across, and at the extremity of each arm is a round bar which acts as an axle for one end of a set of three pillar insulators on each side. The insulators are inclined outward from the arms of the Y and at their extremities additional axles transmit the strain to two columns which form part of a steel cage which surrounds the insulator system and converges to a point at which it is attached by a hinge to the concrete anchor block. The weight of the Y-shaped frame is supported by two additional pillar insulators with universal joints at top and bottom. Each pillar insulator has a simplified jack at its base by which the strain can be approximately equalized in each set. These features are shown in Figs. 2A and 2B. The anchors are housed in steel and corrugated iron structures affording ample clearances.

FIG 2B—VIEW OF GROUND END OF ANCHOR STRUCTURE

The aim of the design is to relieve the insulators entirely of bending moment, leaving them to resist compression only. While stiffness is relied upon in the planes of the two sets of insulators at the sides of the Y, with care in assembly it is believed that the side strain in these planes can be almost entirely taken up by the two insulators holding up the Y frame. The structure, including the Y frame and six insulators, rocks into equilibrium on the five hinge bars so that the load is firmly placed on the two sets of insulators.

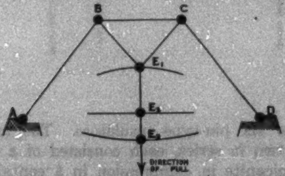

FIG 3—ANALYSIS OF FIVE-HINGED ANCHOR STRUCTURE

This five-hinged system was decided upon after analysis and trial of a number of possible arrangements. A rigid rectangular structure, while satisfactory for 60,000-volt insulation with only one insulator in depth, would not serve for 110,000-volt insulation with pillar insulators three times as long, as the side strains would introduce bending moments which it was thought best to avoid, especially since with such a structure, any yielding to the strain would increase the moment and probably result in complete collapse.

An analysis of the hinged system is shown in Fig. 3. Hinges *A* and *D* are fixed in position, while hinges *B*, *C* and E_3 are free to shift. The cable is attached at E_3.

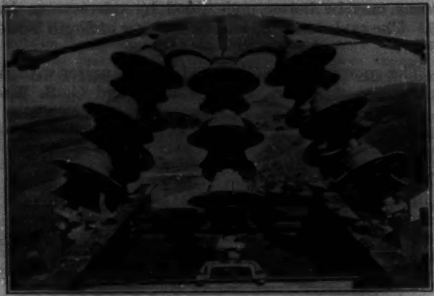

Fig 4—View of Tower Support on Tower Adjacent to Anchor

The saddle is electrically connected to the cable to obviate possible burning at the support.

The location of *E* along the stem of the *Y* is extremely important. If the center hinge were set at some point high up on the stem such as E_1 where the axis would trace a curve convex upward, any displacement would cause *E* to run down the curve and the structure would "jacknife" or turn inside out with great rapidity. This was very neatly illustrated by means of a model of a single element made of cardboard in which the pull of the cable was simulated by an elastic string.

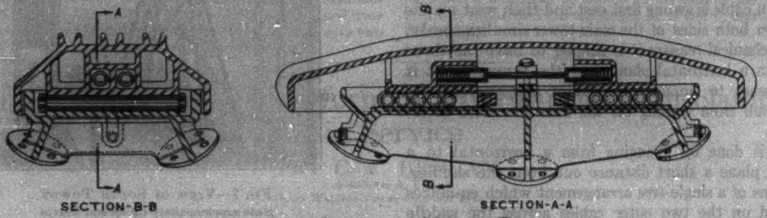

Fig 5—Saddle with Movable Top

Used on main tower between 4427-ft. and 1385-ft. spans.

Part way down the stem was a point E_2 which traced a straight line. Attachment to this point would render the cable indifferent as to its position, judging by the model, and such a structure would of course be unsafe to use where clearances must be maintained on both sides of the cable.

When attached at the position E_3 well down the stem of the *Y* the curve traced is concave upward, therefore the pull of the cable tends to keep it in the center, the low point of the curve. It will be a stable structure even though containing five hinges, and the load will be divided equally between the insulator systems *B A* and *C D*.

TOWER SUPPORTS

At each support on the towers a cast steel saddle is mounted on six pillar insulators consisting of three units each, with a jack at the bottom of each pillar to permit equalization of load.

In order to resist side strains caused by wind on the cable and unequal strains in the direction of the line without introducing objectionable bending moments in the insulators, they are mounted in two inclined planes at 22½ deg. from the vertical, and the two outer insulators on each side are inclined at the same angle relative to the center insulator in that plane. (See Fig. 4). The insulators are mounted on cast steel brackets on the 15-inch I beams which form the crossarms of the towers and on two H beams which span between them.

At South Tower and the Leaning Tower, both adjacent to the anchorages, the saddles are stationary and any unequal stresses in the direction of the line will be transmitted to the towers by the rather sturdy insulator structure. These towers are so close to the anchorages, it is believed that such stresses will be very small.

At the high Main Tower, however, with spans of 4427 feet on one side and 1385 feet on the other, unbalanced stresses do result with changes in temperature. For this tower, the saddle is made with a movable top which glides back and forth on rollers, (See Fig. 5). A part of this motion is free, but the last inch in each direction is against buffer springs to transmit any such unbalanced strain to the tower gradually. The rollers are immersed in a thin grease to prevent rust as well as for lubrication. It is found that this motion is freely exercised by the saddle top with daily temperature changes. The limit was approached during construction when a man was sent out in a suspended car to paint the cable, as is the company's practise to do at intervals.

SUPPLEMENTARY CABLES OVER TOWER SUPPORTS

The cables on the long spans are subject to serious vibration which in time would cause crystallization of the steel at the tower supports unless precautionary measures are taken. This vibration changes with the velocity of the wind and is of two types,—one is evidenced by a continuous hum, the other is in the nature of an impulse which travels back and forth across the span at intervals of a few seconds. To afford cushioning for these vibrations at the tower supports and to add to the strength at these important locations, supplementary cables are used over the saddles. By thus adding to the weight of the main cable before it reaches a support, the vibration is absorbed gradually and its concentration at one point is avoided. Two supplementary cables of the same size and quality as the main cable are used at each tower support and arranged with turnbuckles so that all or part of the tension can be taken off the main cable.

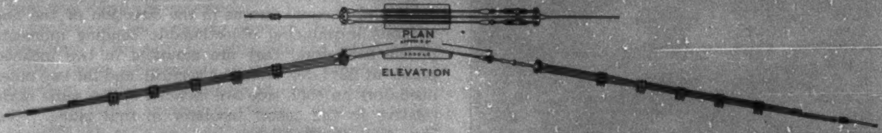

FIG 6—SUPPLEMENTARY CABLES OVER TOWER SUPPORTS

As the wind alternates from sea breeze to land breeze, the main cable is swung first east and then west on the spans on both sides of the main tower simultaneously. For mechanical reasons it is simpler to have the three cables in a horizontal plane over the support, but it is necessary to equalize the strains and prevent the entire pull from coming on the outside cables alternately.

This is done by changing from a horizontal to a vertical plane a short distance out from the saddles, by means of a single-tree arrangement which equalizes the load on the two outer cables across the saddle and permits the main cable in the center to go free.

The supplementary cables across the saddles are short sections with sockets on both ends. The turnbuckles are joined to these portions for tightening as much as desired to equalize the load and relieve the main cable of a part or all of the strain. These short sections and turnbuckles go between the equalizing plates at their outer holes at each end, the main cable passing through between the plates. The center holes of the plates engage pins in open sockets on the ends of cables which parallel the main cable in a *vertical* plane. These cables are also of the same character as the main cable and are attached to it and to each other by five clamps which hold all three cables at suitable spacing. The ends of the supplementary cables beyond the clamps are attached to the main cable by serving or Crosby clips, the lengths being varied on the different spans to provide suitable cushioning for the vibration. The distances to which the cable ends extend vary from 21 to 55 feet from the centers of the towers, one cable however being terminated at a lesser distance in each case. The arrangement may be better understood by reference to Fig. 6. The cables may be seen in the view of South Tower, Fig. 7.

The equipment was designed by the writer with the assistance of Messrs. A. T. Church and G. S. Tune of the drafting room staff and under the supervision of Mr. J. P. Jollyman, chief of the division of hydroelectric and transmission engineering for the company.

FIG 7—VIEW OF SOUTH TOWER
Note supplementary cables and clamps

Discussion

L. M. Klauber: I wish to point out one fact, namely, that a line of such importance, crossing a water way open to traffic where great clearances are necessary, would naturally involve special structures and relatively high costs. One not familiar with long spans might deem it essential that whenever such spans are constructed, regardless of the character of the line, equally expensive structures are essential. I would like to point out that for spans of this magnitude in less important lines, a much less expensive construction may be used with entirely satisfactory results.

For instance, it was recently necessary for our company to construct a line serving a load of only 50 kw. We have in that line one span which in length approaches this one within one hundred feet; it is approximately 4300 feet long and yet the construction is inexpensive, utilizing wood poles for towers. Of course, it is obvious that this line does not cross a waterway through which ship traffic must be accommodated and thus great clearances are unnecessary. On the contrary it crosses a rather wide canyon and had it not been for a spur of a hill that came up in the center, the span would have been 7600 feet instead of 4300 feet long. As it was we had to locate a tower in the middle on the spur between the 4300 foot span and a 3300 foot span. The line including these spans is no more expensive than an ordinary pole line following the road would have been.

H. H. Schoolfield: We have some long spans but nothing like Mr. Klauber mentioned. We have one about 2800 feet long on wood pole construction and it is no more expensive than following down the canyon.

J. P. Jollyman: The detail of the design of the re-insulation of the Carquinez crossing is not offered as a sample of how long span construction should ordinarily be made. As has been pointed out, this crossing is one over a navigable stream. The crossing was designed over twenty years ago and represents a very remarkable achievement for that period.

It is rather seldom that we find spans of that length with conductors of that character supported on rigid insulators; most of our new work is with suspension or strain insulators where the rigid points of support are at the tower and not at the conductor. In the Carquinez span, the long time success of the compression theory of insulation led us to conclude it would be a wise thing to continue and for that reason the insulators for the higher voltage were determined to be of the rigid type. A rigid form of support introduces the very pressing problem arising from the vibration of the conductor and our endeavor to terminate that vibration at a fixed point. Experience has shown the necessity of absorbing the vibration before it strikes the rigidly supported point, or in a short time, crystallization of the conductor will occur. A rather striking experience was had in this crossing. A few years ago the crossing was worked over and was changed from four wires to six wires in order to give two complete circuits. At that time it was known that some reinforcements of the original cables over the rigid supports was necessary and an attachment was put on designed to take most of the strain off the main cable. Within a few weeks crystallization occurred and a strand broke at the attachment. The attachment was several feet out from the original point of the support, but it was a little too heavy; so in redesigning the work we have taken particular precautions to add weight gradually as the point of rigid support is approached.

First we add one cable for reinforcement and a few feet further on we add a second. These cables are then attached to the main cable by means of relatively light clamps sufficient in number to take the entire strain and amply spaced, and finally the main cable with its reinforcing cables changed from the vertical to the horizontal plane, is brought to the point of rigid support. The construction has been up over a year. We have had no mechanical trouble with it whatever, or any electrical trouble except with some windows at the entrance to the anchor houses. We installed those windows thinking that they might be of some use, but they proved to be the weak point in the construction and have been removed. It is not our intent to replace the windows. We have had no mechanical trouble and I believe our experience has been sufficient to justify the assumption that the mechanical design is going to function correctly.

L. J. Corbett: There is little to add as no questions have been raised. The important link this particular crossing forms in the transmission system: the fact that it crosses a navigable waterway, a main line railway and other power and communication circuits, and the adherence to the compression type of insulation have resulted in the rather elaborate construction described. Our company has other long spans on lines in mountain districts of the type described by Mr. Klauber and Mr. Schoolfield.

I wish to emphasize the importance of the proper design of the five hinged anchor structure of Fig. 3. E_3 is the only point at which a hinge can be located on the stem of the Y. The entire Y frame between the hinges E_1, B and C, must be absolutely rigid.

Test Results on the Performance of Suspension Insulators in Service

BY C. F. BENHAM
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Review of the Subject.*—Megger test results on suspension insulators on the lines of the Great Western Power Company covering records since 1908 giving the percentage of depreciation by districts, viz., mountain, valley and coast conditions; also results covering the different types.*

* * * * *

THE suspension insulator has been the subject of much discussion, particularly during the last decade, when the faults of most of the products of earlier manufacture were brought to light and the great increase in their use, due to the general tendency toward higher voltages, considerably extended the interest in their behavior.

Extensive investigation has been carried on as to the cause of their deterioration, which was so pronounced in the older units and from which the later output is not entirely free. Many valuable data have been collected, from observation of insulators in actual service,—and experimentally as well,—and many conclusions have been drawn. At the present time the most generally accepted theories attribute failure to improperly fired porcelain,—either porous from underfiring or brittle from overfiring,—and even more to the mechanical stresses set up by the unequal expansion of the various elements of the usual cap and pin type of unit.

Most of the manufacturers have profited from the light which has been thrown on the subject; by a

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

combination of better porcelain through more scientific mixing and heat control of the firing and the elimination of a large part of the temperature stresses through better design and methods of assembly, they have produced an insulator which is fairly long lived, as compared to the article supplied fifteen years ago.

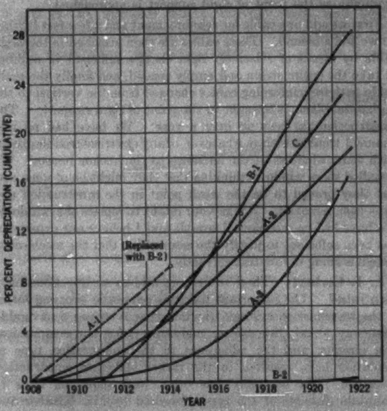


Fig. 1—Deterioration of Suspension Insulators under Different Climatic Conditions

Records are available covering the performance of a fairly large number of suspension units in use in the high-tension lines of one company,—a majority of which have been in service for about fifteen years,—and most of the balance from eight to twelve years.

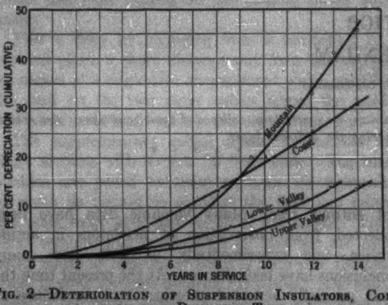


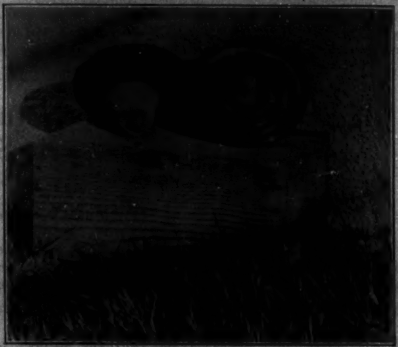
Fig. 2—Deterioration of Suspension Insulators, Comparison of Different Types

Curves designated by same letter are products of same manufacturer.

After about six years of operation, the frequency of insulator failures was increasing at an alarming rate, and interruptions to service became so numerous that it was necessary to adopt a system of field testing to eliminate the faulty units.

From the indications of the first testing it was deemed advisable to replace one type of insulator completely—as well as the defective units of all types—using a new insulator of improved design and material. Subsequently other types were replaced as a whole and all insulators in certain sections where the rapid depreciation coupled with the inaccessibility made testing very costly. As a result failures have been reduced to only one or two sporadic cases in a year, at the most, with one period of twenty months in which none occurred at all. Testing intervals have also been increased to a fairly reasonable period, varying from one year in a few sections to from two to three years for the majority. This is governed partly by the type of insulator preponderating, but more especially by the climatic conditions prevailing, which vary considerably over the entire system.

Insulator Shown in Curve *B*-1



Insulator Represented by Curve *B*-2

Periodic testing has been carried on for the past nine years, and an analysis of the test records, with reference to the effect of different climatic conditions and also comparing the behavior of various types, is presented graphically in Figs. 1 and 2.

Fig. 1 brings out the decided effect of temperature changes on the life of an insulator, and is rather conclusive proof that expansion stresses are a more serious problem than porosity. The four geographic divisions selected are quite sharply defined with distinct characteristics as to climate; while they vary greatly in extent, each is large enough to include a representative number of each of the various insulators and hence affords a fair comparison. The mountain and coast hill sections are subject to almost daily ranges of temperature of considerable magnitude, while in the upper and lower valley sections the only variations of any consequence are seasonal, though the daily range in the lower valley is perhaps a trifle greater on account of the winds. Again, the mountain and upper valley districts are comparatively dry, while the lower valley and coast sections are quite humid, the one being in a river delta and the other subject to fogs. Thus we have four distinct combinations of temperature cycles and humidity.

INSULATOR SHOWN IN CURVE *A*-3

Fig. 2 covers the general performance of several different types of insulator, and is particularly interesting in the manner in which it brings out a comparison between different designs and stages of development of the same manufacture. The slight tendency of Curve *B*-1 to show a decrease in the deterioration can be accounted for in the fact that it includes insulators of two different ages. It has not been possible to segregate them generally in the test records, as they are alike in appearance, but a few specific cases indicate that the newer insulator has a much greater life due to improvements in porcelain and details of assembly. The proportion of older insulators is gradually being reduced through replacement, with a resulting decrease in the rate of depreciation of the combined group.

The most striking feature is the very slight deterioration of the insulator shown in Curve *B*-2, which represents a fairly recent output of a design which is standard at the present time. Actual figures are perhaps more conclusive. Of some 10,000 units subjected to test, no failures developed in six years, and only nine in the seventh year and 10 in the eighth year or a total of only 19 units out of 10,000 after eight years of service. This would rather indicate that the insulator question is no longer a very serious problem, at least under the conditions which prevail among those in the foregoing study.

Discussion

J. A. Koontz: I think there is one point that Mr. Benham has mentioned which should be emphasized, *viz.*, that if the loading on the dead end insulator is kept within reason, there is no greater depreciation on the dead end insulator than on insulators in suspension. A great many engineers feel that insulators at the dead end points are quite a hazard. In fact on the lines in question all the early type insulators used at the dead end positions had to be removed, but when they were replaced by later designed units, the trouble ceased, as shown by the curves, and from tests, we find no more failure in the dead end position than when the insulators are used in suspension. With this in view, we feel that from a depreciation standpoint, we do not have to worrry about the dead end insulators, so long as the mechanical loads are not excessive.

J. E. Woodbridge: I might speak of the progress of the insulation art. So far as the suspension insulator is concerned, it has been practically developed within the last ten years. I say ten years because the difficulties with suspension insulators were first noted about ten years ago. They had then been used about five years, but nobody dreamed that they had qualities differing from those of the pin type. About the year 1913 suspension insulators began to deteriorate very rapidly; there were burnouts, punctures and flashovers, and we went out on the lines with meggers and discovered that the insulators were practically all to the bad, that they were depreciating at a high rate and that total replacement would be required within a few years. Of course, that developed a new problem and we were up in the air for some time. Some blamed the porcelain, some the metal parts, and some the cement—everybody had a different idea and each engineer proposed a different cure for the other man's diagnosis, but the whole problem is now crystallizing, as shown in this paper, to a definite knowledge of what the troubles are.

Telephone Transmission Over Long Distances

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***Review of the Subject.**—In this paper are pointed out some of the similarities and contrasts of power transmission and telephone transmission over long distances. The problems of telephone transmission on open-wire lines are illustrated by a discussion of the methods by which the over-all efficiency of the transcontinental telephone circuit has been greatly improved. A brief discussion is given of recent important developments in telephone transmission through cables over long distances. An outline is given of the results obtained in the commercial application of carrier telephone and telegraph systems. Finally, a demonstration talk between Havana, Cuba, and Avalon on Catalina Island off the Pacific coast, is described as an illustration of what can be done with the commercial telephone system in its present stage of development.*

* * * * *

At a convention where the main topic of discussion is the transmission of power over long distances, it would appear interesting to review some of the problems involved in the transmission of telephone currents over long distances. This review does not contribute very much which could be used directly in solving the power problems but serves to point out some interesting similarities and some very important differences between these two branches of electrical art.

In essence, both consist in the transmission of alternating currents over very long electrical circuits and in both, therefore, the problem of reducing the losses of electric power in transmission is very important. In the case of power transmission, however, the commodity delivered is the power itself and, therefore, for commercial success a large percentage of the power transmitted into the line must be delivered at the output. In the telephone problem, on the other hand, the commodity delivered is communication and the delivery of electrical power is only a means to this end. The efficiencies of the telephone transmitter and of the telephone receiver are such that under many conditions satisfactory communication can be given when only a small fraction of 1 per cent of the transmitter output is delivered to the receiver at the distant end of the line, the rest being absorbed largely in line losses. Furthermore, in many cases the power delivered does not come directly from the transmitting end but comes only from the nearest repeater station on the line.

The power engineers are free to choose, with a view to transmission efficiency and other features of economy, the frequency of alternating current to be used. As a result, a relatively low frequency is always chosen. In telephone transmission, on the other hand, the frequency of transmission is necessarily high, for it cannot be lower than the important harmonic components of the complex waves which constitute speech. How complicated these waves are is illustrated by Figs. 1 and 2. Fig. 1 shows oscillograms of telephone currents corresponding to the vowels "o" and "e". The most prominent oscillation in the vowel "o" is, roughly, 800 cycles a second and the most prominent oscillation in the vowel "e" is about 1900 cycles. Fig. 2 is an oscillogram of the word "Pacific." The pronunciation of this word occupies less than a second, but the oscillations are so complicated that it has been necessary to crowd them together very closely in order to get the whole word in one figure. These two figures are illustrations of the fact that the important harmonic components of telephone currents cover a frequency range from 200 cycles to well over 2000 cycles and that components as high as 3000 or 4000 cycles in frequency contribute somewhat to the intelligibility of speech. By using the speech currents to modulate a carrier current the frequencies transmitted over the line can be raised as may be desired, but no feasible method has been suggested for lowering them materially.

Fig. 1—Oscillograph Records Showing the Fluctuations in the Electrical Current when the Vowels "O" and "E" are Spoken into the Telephone Transmitter, the Speed of the Film was about 10 Feet per Second.

This difference in frequency between power and telephone currents is an important difference from the standpoint of transmission, because the losses per unit length go up rapidly with the frequency. Furthermore, the wave lengths are shorter for higher frequencies, so that long telephone lines may be many wave lengths in length.

Another fundamental difference between the power and telephone transmission systems arises from the difference in type of service which they perform. In

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

the case of the power system all customers can be served from the same circuit and the tendency is, therefore, towards interconnection and toward the development of large systems consisting of a relatively small number of very large units. In the telephone system, on the other hand, an independent channel of communication must be given to each pair of talkers. This necessity has led to great efforts to find ways to make a moderate amount of copper provide a large number of circuits either by the use of small conductors or by the superposition of a number of independent channels of communication on one pair of wires.

In the telephone system also the amount of power is necessarily small. When talking in an ordinary tone of voice the power delivered to the telephone transmitter in the form of acoustic waves by the talker is in the order of millionths of a watt. The telephone transmitter amplifies this power by a large ratio, so that the power delivered to the telephone lines has peak values of the order of 0.001 to 0.01watt.

Fig. 2

Both power and telephone transmission systems have important insulation difficulties. In the telephone system, however, the difficulty is of course, not in the strength of the insulation to withstand voltage stresses but is to prevent, as far as practicable, the absorption of power in or over the surface of the insulation at the relatively high frequencies of the alternating currents transmitted.

Transcontinental Telephony

As a result of the different fundamental requirements and different technical conditions, the development of the two industries has led to the transmission of telephone currents over considerably greater distances than power currents. A conspicuous example of long distance telephone transmission is the much used service between the Atlantic and Pacific Coasts. The circuit between New York and San Francisco is about 3400 miles (5500 kilometers) long, largely in open-wire construction of copper weighing 435 pounds to the wire mile. (600 kg. per km.). Recent improvements in this circuit have very greatly increased its over-all transmission efficiency, and it is believed that a brief discussion of these improvements will be of interest as illustrating the technical problems involved in telephone transmission over very long open-wire lines.

As originally constructed, this line was loaded throughout and was provided with six repeater points between New York and San Francisco.

Loading[1] is the means by which in telephone practise the electrical efficiency of the line is increased by using lower currents and higher voltages to transmit a given amount of power. In a power circuit the voltage can be raised by simply changing the ratio of terminal transformers. In a long telephone circuit the voltage cannot be raised in that way. The ratio of voltage to current at the transmitting station depends not on the impedance of the receiving station, but as the telephone line is electrically long, it depends upon the characteristics of the line itself. Therefore, to raise the impedance, the characteristics of the line itself must be changed.

There are several ways in which this can be done. For the transmission of a single frequency, it can be done very effectively by connecting inductive loads across the circuit at regular intervals. For telephone transmission, however, where a uniform efficiency of transmission over a wide range of frequencies is necessary, the result is accomplished by the use of series inductance loads, designed for very low energy losses and located regularly at eight-mile intervals throughout the line.

The extent to which the efficiency can be improved is rather narrowly limited in open wire by the insulation losses in the line which are, of course, increased as the voltage is raised and in part offset the decrease in series resistance losses due to decreased current. By means of loading, however, it is possible on a circuit such as the transcontinental telephone line to raise the voltage by about 80 per cent. and reduce the losses per unit length by a factor of about 2.2 in dry weather.

The benefits from the use of loading are insufficient to make transcontinental telephony commercially practicable without the use also of telephone repeaters[2] which receive the attenuated telephone currents after transmission over a few hundred miles of line and deliver to the adjacent section of line greatly amplified currents of the same wave shape. The results obtained on the transcontinental circuit by the combination of loading and repeaters are shown in the upper part of Fig. 3 which is drawn to represent the amount of energy at different points of the circuit when 1000 microwatts are delivered to the circuit at the San Francisco end. The power rapidly decreases along the circuit, due to line loss, at Winnemucca is amplified to 700 microwatts, falls off again rapidly, and so by successive stages of attenuation and amplification 10 microwatts are at last delivered to the local telephone circuits at New York.

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[1] For detailed information regarding the loading of telephone circuits, see paper entitled: "Commercial Loading of Telephone Circuits in the Bell System," by B. Gherardi, Transactions, American Institute of Electrical Engineers, 1911.

[2] For discussion of the design and action of telephone repeaters see paper entitled: "Telephone Repeaters," by B. Gherardi and F. B. Jewett, Transactions, 1919, Vol. XXXVIII.

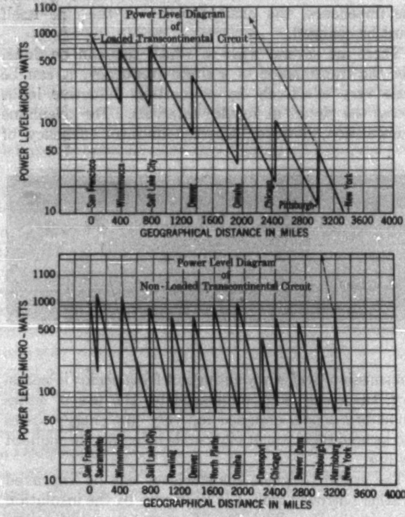

Fig. 3

The improvements in the transcontinental line mentioned above have been brought about by removing the loading from the circuit and modifying the characteristics of the repeaters. Also, in view of the much larger line losses to be made up by the repeaters and in order to stay within economical upper and lower limits of power output and input of the repeaters, the number of repeaters was doubled. This change, however, was not essential and would not have brought about the improvements in transmission without the other changes. The energy level at different points in the circuit under the improved conditions is indicated in the lower part of Fig. 3, and it will be seen that now instead of 10 microwatts, 70 microwatts are delivered at New York when 1000 are transmitted into the San Francisco end of the line.

In view of the benefits which can be obtained by loading, it may seem surprising that improvements in the efficiency of this circuit were obtained by removing the loading. The explanation of this result requires the discussion of some very interesting transient phenomena which are important in very long distance transmission.

When a train of oscillations is launched upon a long electrical circuit, it travels along the circuit attenuating in magnitude but without reflection so long as the impedance characteristics of the circuit are uniform. When a non-uniformity in line impedance is reached a part of the wave is reflected and the reflected wave travels back towards the transmitting end, being of course attenuated in the process. By this process of successive reflection the steady state of transmission is produced, although in the process there are of course other transient components of the currents which do not concern us here. In open-wire telephone circuits a few hundred miles in length, or less, the steady state is established very rapidly and the reflected currents are not noticeable. In very long circuits, however, which are made of high over-all efficiency by the use of repeaters or otherwise, the reflected current may have sufficient volume and time lag to be noticeable and even to be heard as distinct echoes. Hence they have been named "Echo Currents."

Echo currents, of course, may be heard by the listener as well as by the talker, for the reflected current, striking a second irregularity, is reflected again towards the listener.

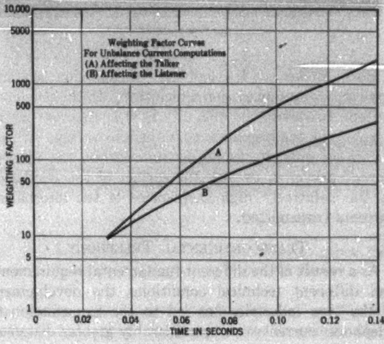

Fig. 4

In order that the transmission may be satisfactory the conditions of the circuits should be such that this echo current is not noticeable, having either a very short time interval or a small magnitude. It has been found that there is a definite relation between the time lag of an echo and the maximum amount of echo current which can be permitted without appreciable effect on the clearness of speech. This is indicated in Fig. 4 in the form of relative weighting factors for echo currents of different time lags, showing for both talker and listener and for different time lags the reciprocal of the relative maximum amount of power in the echo for no material interference with conversations.[3]

In the transcontinental line, which is chosen as our example, the most important echoes heard by the talker come from the irregularities at the distant end of the line where there is a marked change in the characteristics of the line due to the change from toll line to local construction. With the loaded circuit the velocity of propagation is about 55,000 miles (88,000 km.) per second, and the time lag of this echo is about 0.11 second. On the non-loaded circuit, however, the velocity of propagation is much faster, being about 180,000 miles (290,000 km.) per second, and the time lag of the echo from the distant end is only about 0.04 second. The corresponding weighting factors are 670 and 15, which means that on the non-loaded line this echo could be about 44 times as loud as on the loaded circuit for the same effect of the terminal irregularities on transmission.

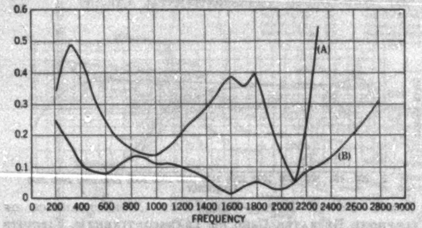

FIG. 5—DIFFERENCE BETWEEN LINE AND REPEATER IMPEDANCE EXPRESSED AS A FRACTION OF THE SUM OF THESE IMPEDANCES.
(A) THE LOADED TRANSCONTINENTAL CIRCUIT
(B) THE NON-LOADED TRANSCONTINENTAL CIRCUIT

The magnitude of the echo for a given irregularity depends upon the over-all transmission efficiency of the circuit, and the increase in permissible volume of echo current, therefore, means that the over-all efficiency of the circuit may be increased without interference from the echo currents.

So far we have been discussing the effect of reflected currents due to terminal irregularities. This, however, is not the whole story. On a line provided with repeaters there are irregularities not only at the terminals but also at the repeater points, due to the impracticability of making the impedance of the repeaters identical with that of the lines at all frequencies. In making the line non-loaded, it was found possible by modifying the telephone repeaters to improve the similarity of impedance of the repeaters and the line, thus reducing the amount of reflected current at these points. The extent to which this was practicable is indicated in Fig. 5 which shows the ratio of difference to sum of line and repeater impedance over a range of frequencies for the loaded circuit condition and for the non-loaded circuit condition.

The result of these two improvements, namely, increasing the velocity of transmission over the line and decreasing the amount of the irregularities is indicated in Tables I and II. Table I summarizes the effect of echo currents on the loaded circuit. This table shows for the worst current path the percentage of transmitted and received energy appearing as echo current, the time lag in seconds, the corresponding weighting from Fig. 4 and the product of energy and weighting.

TABLE I
LOADED TRANSCONTINENTAL CIRCUIT OVER-ALL TRANSMISSION EFFICIENCY 1 PER CENT

Unbalances Affecting Talker

	No. of paths	Length in miles	Time in seconds	Weighting factor (*a*)	Energy ratio (*b*)	Weighted index (*a* × *b*)
Worst Path........	1	6000	0.107	670	0.0001	0.067
Total-All Paths.....	6	Varying for Different Paths				0.248

Worst Path includes all repeaters, *i. e.* is the over-all path.

Unbalances Affecting Listener

	No. of paths	Length in miles	Time in seconds	Weighting factor (*a*)	Energy ratio (*b*)	Weighted index (*a* × *b*)
Worst Path........	1	1040	0.0187	4.35	0.0035	0.015
Total-All Paths.....	21	Varying for Different Paths				0.160

Worst Path includes end repeater and next adjacent repeater.

TABLE II
NON-LOADED TRANSCONTINENTAL CIRCUIT OVER-ALL TRANSMISSION EFFICIENCY 7 PER CENT

Unbalances Affecting Talker

	No. of paths	Length in miles	Time in seconds	Weighting factor (*a*)	Energy ratio (*b*)	Weighted index (*a* × *b*)
Worst Path........	1	6500	0.037	15.3	0.0019	0.029
Total-All Paths.....	12	Varying for Different Paths				0.191

Worst Path includes all repeaters, *i. e.* is the over-all path.

Unbalances Affecting Listener

	No. of paths	Length in miles	Time in seconds	Weighting factor (*a*)	Energy ratio (*b*)	Weighted index (*a* × *b*)
Worst Path........	1	6200	0.035	11.8	0.00072	0.0085
Total-All Paths.....	78	Varying for Different Paths				0.199

Worst Path includes all repeaters, *i. e.* is the over-all path.

This detail is shown only for the worst echo current but has been computed for all the echo current paths, and the sum of the products of energy ratios and weighting factors is given in the table. This total gives a good means for comparing the echo-current effect in different circuits.

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[3] For further discussion of echo currents, see Mr. A. B. Clark's paper on "Telephone Transmission on Long Cable Circuits," JOURNAL A. I. E. E., January, 1923.

Table II shows the corresponding figures for the non-loaded transcontinental circuit after improvement of the repeaters and establishment of the higher over-all efficiency. It will be noted that in spite of the facts that the amount of energy represented by the worst echo current paths is greatly increased and that the number of echo paths is also increased, the much shorter time lag due to the higher velocity of propagation results in a total weighted echo current of about the same magnitude as the total for the loaded circuit at a very much lower efficiency.

TABLE III

LOADED TRANSCONTINENTAL CIRCUIT OVER-ALL TRANSMISSION RFFICIENCY 7 PER CENT

Unbalances Affecting Talker

	No. of paths	Length in miles	Time in seconds	Weighting factor (*a*)	Energy ratio (*b*)	Weighted index ($a \times b$)
Worst Path........	1	6000	0.107	670	0.0055	3.68
Total-All Paths.....	6	Varying for Different Paths				5.24

Worst Path includes all repeaters, *i. e.* is the over-all path.

Unbalances Affecting Listener

	No. of paths	Length in miles	Time in seconds	Weighting factor (*a*)	Energy ratio (*b*)	Weighted index ($a \times b$)
Worst Path........	1	5200	0.093	97	0.0054	0.52
Total-All Paths.....	21	Varying for Different Paths				1.58

Worst Path includes all repeaters, *i. e.* is the over-all path.

For comparison, Table III is made up for the loaded circuit, assuming it to be operated at the same over-all efficiency as the non-loaded circuit, *i.e.*, 7 per cent. This table shows how much greater the effect of the echo currents would be at that equivalent.

In addition to the improvements discussed above, the changes in the transcontinental line made it practicable to improve the quality of telephone transmission by increasing the degree of uniformity of transmission of the different frequencies within the important telephone range. For long stretches of line the non-loaded open-wire transmits current with much more uniform efficiency than the loaded open wire. This is illustrated in Fig. 6 which shows the percentage of output to input power at different frequencies in the telephone range for a 281 mile (452 km.) section of non-loaded open wire 0.165 inches in diameter (435 pounds per wire mile) and a 563 mile (907 km.) section of loaded open wire of the same size. The distances chosen are in each case an average repeater section.

Furthermore, in modifying the telephone repeaters, changes were made to improve the uniformity of amplification given by the repeaters over a wide range of frequencies. Fig. 7 shows the amplification frequency characteristic of these repeaters before and after their modification.

These improvements in the transmission characteristics of the line and repeaters made it possible to get a very good over-all transmission frequency characteristic in spite of the very great length of this line.

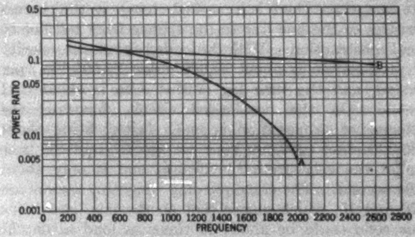

FIG. 6—POWER RATIO—FREQUENCY CHARACTERISTICS OF AVERAGE REPEATER SECTIONS ON TRANSCONTINENTAL CIRCUIT
(A) LOADED TRANSCONTINENTAL CIRCUIT
(B) NON-LOADED TRANSCONTINENTAL CIRCUIT

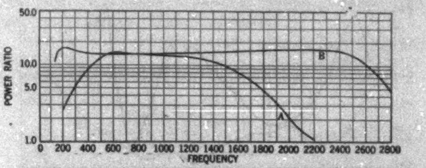

FIG. 7—POWER RATIO—FREQUENCY CHARACTERISTICS OF TELEPHONE REPEATER USED ON TRANSCONTINENTAL CIRCUITS
(A) LOADED TRANSCONTINENTAL CIRCUIT
(B) NON-LOADED TRANSCONTINENTAL CIRCUIT

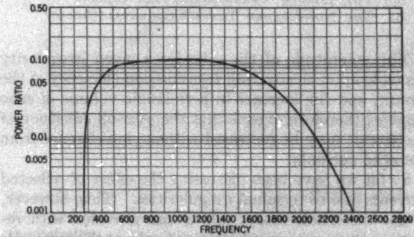

FIG. 8—POWER RATIO—FREQUENCY CHARACTERISTICS OF COMPLETE NON-LOADED TRANSCONTINENTAL CIRCUIT

The result expressed in terms of net over-all efficiency is indicated in Fig. 8.

An incidental advantage of the change from loaded to non-loaded construction is greater independence of weather conditions. The loaded circuit being a higher voltage circuit, varies much more in efficiency between wet and dry weather conditions than the non-loaded circuit. On the other hand, the non-loaded circuit involves twice as many telephone repeaters and the over-all efficiency of the circuit depends on a very exact maintenance of the amplification of each of these repeaters. This requires very carefully planned and faithfully executed maintenance routines. It should be said that the results which have been obtained over this circuit have been exceedingly satisfactory.

TOLL CABLES[4]

The transmission of telephone currents through cables has always been difficult in comparison with the transmission over open-wire lines. One factor of difficulty is the much greater loss in the cable circuits per unit of length. This is due in part to the close proximity of the two sides of the circuit and in part to the fact that economical construction in cable requires relatively small conductors. The telephone repeater has been developed to a point where a practically unlimited number of them can be used in tandem in a circuit without distortion of the telephone currents. This has, therefore, removed the limitations which were set by high attenuation losses in the cable conductors. Although there are other important limitations and there have been large difficulties to overcome, cable transmission has been made practicable up to distances of at least 1000 miles (1600 km.).

As a result, the development of toll cables has become an exceedingly important phase of long distance telephone development and during the next few years it is expected that toll cables will be built in the Bell System at the rate of more than 500 miles (800 km.) a year. The extent of present and prospective use of this type of construction is illustrated by Fig. 9 which indicates important toll cable routes in the Northeastern part of the country. In this section, because of the congestion of population and business, the toll cable development has been most pronounced, and as you will note, the cable which was completed some years ago up and down the Atlantic Coast, between Boston and Washington, has been supplemented by cable stretching westward now as far as Cleveland. This cable will be extended as far as Chicago as rapidly as the work can be carried out and will provide high grade telephone circuits entirely in cable between Chicago and the Atlantic seaboard cities.

An outstanding advantage of the cable type of construction is the ability thereby to concentrate very large numbers of circuits along a single route. Typical aerial toll cable construction is shown in Fig. 10 which shows a section of the cable between Pittsburgh and Cleveland, carrying about 260 circuits. In open-wire construction 6 or 7 very heavy pole lines would be required to provide this number of circuits. In many places where cable construction is now being used the available highway routes for pole lines are largely occupied and purchase of numbers of rights-of-way would be exceedingly expensive, so that in many cases the toll cable development furnishes almost the only practicable method of providing for the large numbers of circuits which the great development of toll business is requiring.

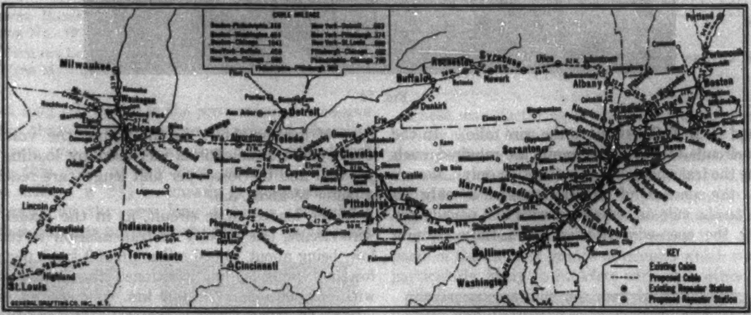
CABLE MILEAGE
Milwaukee
Waukegan
Chicago
Rockford
Joliet
Bloomington
Lincoln
Springfield
Vandalia
Highland
St. Louis
Indianapolis
Terre Haute
Ft. Wayne
Logansport
Detroit
Ann Arbor
Toledo
Findlay
Lima
Dayton
Cincinnati
Cleveland
Cuyahoga Falls
Canton
Cambridge
Wheeling
Zanesville
Pittsburgh
New Castle
Fairmont
Erie
Dunkirk
Buffalo
Batavia
Rochester
Syracuse
Newark
Utica
Johnstown
Schenectady
Albany
Elmira
Binghamton
Kane
Du Bois
Williamsport
Scranton
Hazleton
Harrisburg
Reading
Altoona
Johnstown
Bedford
Shippensburg
Cumberland
Baltimore
Washington
Philadelphia
New York
Atlantic City
Kingston
Springfield
Hartford
New Haven
Providence
Boston
Fall River
Portsmouth
Concord
Portland
KEY
Existing Cable
Proposed Cable
Existing Repeater Station
Proposed Repeater Station
GENERAL DRAFTING CO. INC., N.Y.

FIG. 9—LONG TOLL CABLES, EXISTING, PROPOSED AND IMPORTANT BRANCHES

Another advantage of cable construction is the fact that where underground sections are necessary in passing through cities no irregularity is caused in the constants of the circuit as is the case with open-wire construction. With the general use of repeaters in connection with long toll circuits these irregularities in type of construction are important factors in limiting the efficiency.

Another important advantage of cable construction is its relative immunity from the effect of weather and particularly from the effect of sleet. The damage which can be done by sleet storms is illustrated in Figs. 11 and 12. The cable construction can be made very substantial and capable of withstanding severe conditions of sleet.

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[4] For a more detailed discussion of modern toll cable developments, see paper entitled: "Philadelphia-Pittsburgh Section of the New York-Chicago Cable" by J. J. Pilliod, JOURNAL A. I. E. E., August, 1922, and "Telephone Transmission on Long Cable Circuits" by A. B. Clark, JOURNAL A. I. E. E., January, 1923.

FIG. 10

The large number of circuits provided by one cable are obtained by the use of very small conductors, the gages in common use being 19 B. & S. (20 lb. to the wire mile) and 16 B. & S. (40 lb. to the wire mile). In the longest circuits two 19-gage circuits are required, each carrying the transmission in one direction only. Nevertheless, the amount of copper required is only 80 pounds to the mile as contrasted with 870 pounds per mile for the open-wire circuits which these circuits replace.

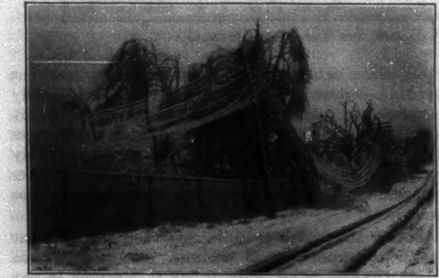

FIG. 11

In contrast to the recent developments on open-wire lines discussed above, toll cable circuits are always loaded. A non-loaded cable circuit consists largely of resistance and capacity, and as a result there are very wide differences in efficiencies of transmission at different frequencies. By loading, this variation is reduced with a corresponding improvement of the quality of transmitted speech and at the same time the efficiency of transmission is raised. This is illustrated in Fig. 13 which shows the transmission efficiency at different frequencies of 1 mile (1.6 km.) of 19-gage toll cable circuit non-loaded and when provided with the type of loading most used for toll cables. With the type of loading shown in the figure, the voltage for 1000-cycle transmission is increased by loading by 70 per cent and the losses per unit length are reduced by a factor of about 3.6.[5]

In the loaded cable circuit, as in the loaded open-wire circuit, the velocity of transmission is relatively low, being about 10,000 miles (16,000 km.) per second for the type of loading mentioned above, as compared with 180,000 miles (290,000 km.) per second on non-loaded open-wire lines. Therefore, for circuits more than a few hundred miles long, care must be taken to avoid excessive echo currents. On the very long toll cable circuits, say over 500 miles (800 km.), a lighter weight of loading is used, whereby, because of the

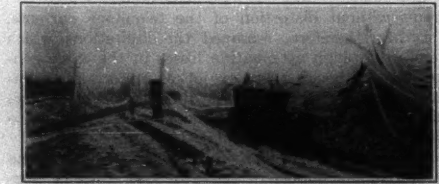

FIG. 12

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[5] In part, this improvement is due to improvement in power factor, in addition to the improvement caused by higher impedance.

lesser inductance per mile inserted in the cable, the velocity of propagation is as high as 20,000 miles (32,000 km.) per second. In addition to the echo currents, care must be taken in the design of the cables to avoid interference with speech by the transients produced by the periodic structure of the loaded cable circuit.

One of the interesting problems involved in the design of long toll cables is the prevention of excessive crosstalk, that is, of excessive transfer of electrical energy from one circuit to another. The difficulty of avoiding this with a very large number of circuits crowded within a 2⅝-in. sheath, is increased in the long circuits by the very long distances throughout which these circuits parallel each other and the frequent large amplifications of both transmitted current and crosstalk at repeater stations along the line. An adequate discussion of this problem would require a paper in itself. In this paper we must be content to note merely a few of the more important methods which have been developed.

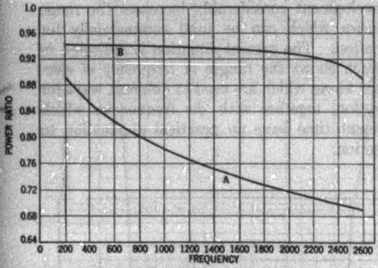

FIG. 13—POWER RATIO—FREQUENCY CHARACTERISTICS FOR NO. 19 GAGE SIDE CIRCUITS

(A) WITHOUT LOADING
(B) WITH MEDIUM HEAVY HIGH CUT-OFF LOADING

FIG. 14

The problem starts with the construction of the cable, and manufacturing methods have been carefully worked out to give the greatest possible degree of symmetry in the construction of the two insulated conductors which are twisted together to form a pair and of the two pairs which are twisted together to form a quad. The construction of the various pairs and quads, also, is such as to properly coordinate all of the circuits which will be near each other in the cable.

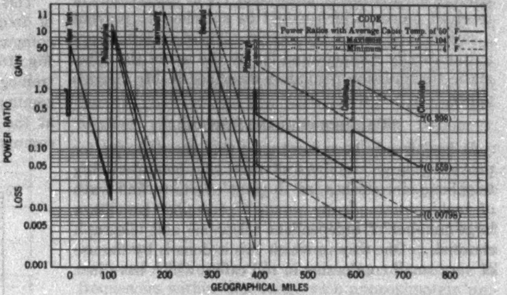

FIG. 15—TRANSMISSION ON LEVEL DIAGRAM OF NEW YORK—CINCINNATI, NO. 1

These manufacturing precautions are supplemented by very careful tests made when the cables are installed, and by splicing procedure by means of which, based on the results of the tests, the induction between circuits is still further greatly reduced. As a precaution against slight series unbalances, all joints are soldered. Further precautions include the segregation into different parts of the cable of groups of circuits which would be particularly likely to interfere with each other.

Similar precautions are used in the design, manufacture and installation of loading coils and other apparatus used with the circuits.

The toll cables require telephone repeaters at intervals of 50 or 100 miles (80 or 160 km.), and this has led to much work in the development of economical repeaters and auxiliary equipment for use with cable circuits.[6] Fig. 14 shows a considerable group of telephone repeaters in the repeater station at Bedford, Pa., and illustrates the degree of condensation which has been worked out in the present types of apparatus developed for this service.

The telephone cable benefits not only traffic between points along its route, but at important points connects to open-wire lines along other routes. This use of the toll cable is well illustrated by Fig. 15, which shows the relative power levels at different points of a typical circuit now in use in the New York-Chicago cable route. This circuit is in cable between New York and Pittsburgh and takes open-wire between Pittsburgh and Cincinnati, forming a New York-Cincinnati circuit. This figure has been drawn to represent the ratio of power at any point in the circuit to power transmitted to the circuit at New York and shows the variations in the amount of power transmitted due to variations in the resistance of the cable with temperature. It is to be noted that these resistance variations would introduce a variation of more than 7:1 in the amount of received power, which would be sufficient to prevent a satisfactory use of the circuit. These variations are automatically compensated for by the use of the automatic transmission regulators described in Mr. Clark's paper.

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[6] Refer to paper by Mr. Pilliod noted above and to paper entitled: "Telephone Equipment for Long Cable Circuits," by C. S. Demarest, A. I. E. E. Convention, Swampscott, June, 1923.

CARRIER-CURRENT SYSTEMS

It has already been mentioned that the necessity for providing large numbers of mutually non-interfering telephone circuits has led to great efforts to find ways of making a moderate amount of copper provide a large number of independent telephone and telegraph channels. One way in which this is done is by the use of very small conductors in toll cables which have just been discussed. This is in general economical only where the traffic is heavy.

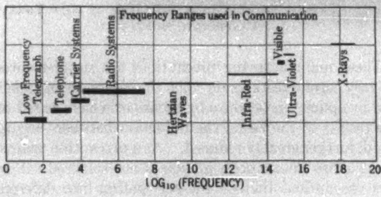

Fig. 16

Another way of obtaining the same result is by requiring each pair of conductors to transmit a wider range of frequencies and using different portions of this range for independent channels of communication. A telephone conversation occupies the range between 300 cycles and something over 2000 cycles. It has long been the custom to use the range below 300 for telegraph and signaling purposes. The range above two or three thousand, however, was not commercially useful until the development of carrier-current systems.[7]

In the carrier-current telephone system the voice frequency telephone currents are made to modulate a higher frequency current. The frequencies of the modulated currents used in the carrier system represent the same width of band arithmetically as the original telephone frequencies, but all are shifted in magnitude by the frequency of the carrier current. That is, a telephone band of 300 to 2000 cycles, when used to modulate a 15,000-cycle carrier current, produces a band of frequencies ranging between 15,300 and 17,000 cycles in addition to other bands which are not used in existing systems and which need not be considered in this discussion. The principle of modulation is the same as is employed in radio telephony, but in the carrier system the modulated waves are carried along wire circuits, rather than radiated into space.

This development considerably increases the range of frequencies which can be used commercially for communication. The present situation is indicated in Fig. 16 which shows the ranges of frequency of electromagnetic waves which are used in different ways for communication, together with the frequencies which at the present time have no practical application to communication.

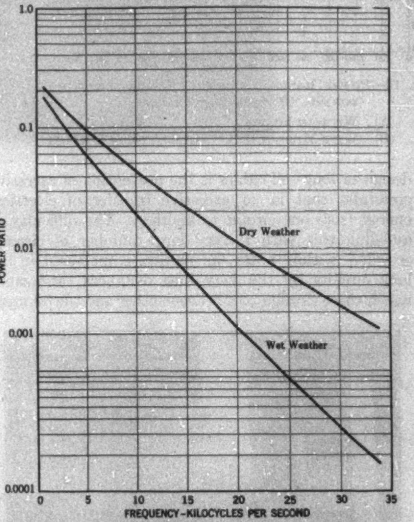

Fig. 17—Typical Power Ratio-Frequency Characteristics of 165-In. Diameter Copper Open Wire, 200 Miles Long

The question may naturally arise why the lower frequencies have been chosen for the wire carrier systems developed for commercial use. A very important argument for using the lower frequencies in commercial telephone practise is that a number of carrier systems covering the same frequency ranges are used on different pairs carried on the same pole line. The crosstalk between the pairs is prevented by specially designed systems of transpositions in the circuits. The difficulty of preventing interference between the two circuits, however, goes up more rapidly than the frequency of the currents, so that at frequencies very much higher than those now used it would probably be impracticable with present forms of construction to avoid excessive interaction between the circuits.

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[7] Refer to paper entitled: "Carrier-Current Telephony and Telegraphy," by E. H. Colpitts and O. B. Blackwell, A. I. E. E. Transactions, Vol. XL, 1921.

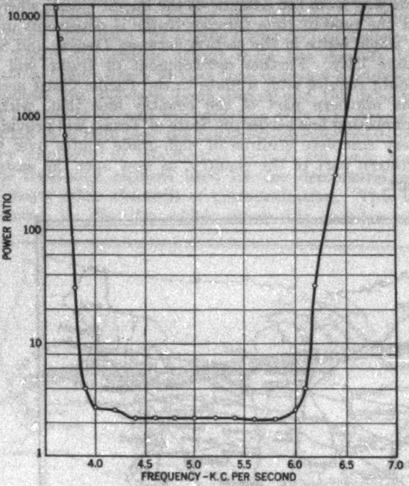

Fig. 18—Attenuation of the 4-6 K. C. Band Filter

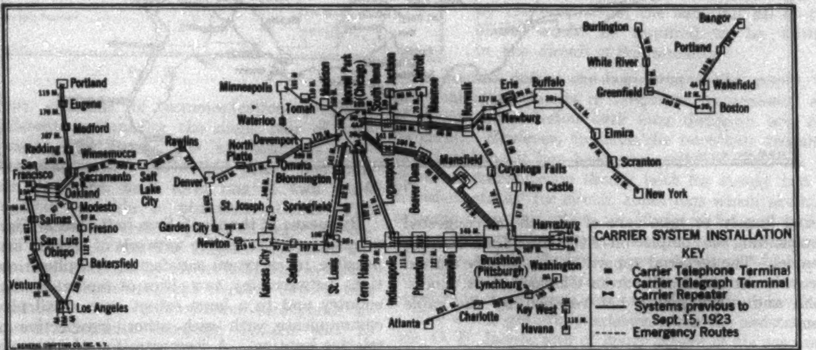
Portland
Eugene
Medford
Redding
Winnemucca
San Francisco
Sacramento
Oakland
Modesto
Salinas
Fresno
San Luis Obispo
Bakersfield
Ventura
Los Angeles
Salt Lake City
Rawlins
Denver
North Platte
Garden City
Newton
Kansas City
St. Joseph
Sedalia
Minneapolis
Waterloo
Tomah
Davenport
Omaha
Bloomington
Springfield
St. Louis
Madison
Morrell Park (Chicago)
South Bend
Jackson
Detroit
Terre Haute
Indianapolis
Logansport
Beaver Dam
Phoneton
Zanesville
Monroe
Norwalk
Mansfield
Erie
Buffalo
Newburg
Cuyahoga Falls
New Castle
Brushton (Pittsburgh)
Harrisburg
Washington
Lynchburg
Charlotte
Atlanta
Key West
Havana
Elmira
Scranton
New York
Burlington
White River
Greenfield
Bangor
Portland
Wakefield
Boston
CARRIER SYSTEM INSTALLATION
KEY
Carrier Telephone Terminal
Carrier Telegraph Terminal
Carrier Repeater
Systems previous to Sept. 15, 1923
Emergency Routes
GENERAL DRAFTING CO. INC. N.Y.

Fig. 19

Another important reason for using the lower carrier frequencies is the relatively great effect on high-frequency currents of short sections of cable inserted in the open-wire line. These effects can be reduced to a considerable extent by loading but the difficulties of loading efficiently increase rapidly with the frequency. The types of loading which were designed for voice frequency currents do not transmit currents of more than 3000 cycles frequency and for the carrier frequencies used in commercial telephony special loading with a very close spacing of loading coils has been developed.

Another way in which the lower frequency is advantageous is indicated in Fig. 17 which shows the ratio of output to input power at different frequencies under typical wet and dry weather conditions, for 200 miles (320 km.) of metallic circuit composed of copper wire 0.165 inches in diameter (435 pounds per wire mile). It will be noted that this loss increases rapidly with increasing frequency and furthermore that the variation of the loss between the wet and dry weather conditions also increases with frequency.

One of the problems involved in using low-frequency carrier currents for telephony arises from the fact that the width of the band of frequencies is appreciable compared with the frequency of the carrier current. For example, the carrier band of lowest frequency used in present Bell System practise is the band between 4000 and 6000 cycles. It is necessary to transmit all frequencies within that range with approximately uniform efficiency and to sharply cut off frequencies outside the range. A sharply tuned circuit would obviously be of no use, as it would very greatly distort the speech. This problem is beautifully solved by the invention of the electrical filter which provides a path for transmitting with almost uniform efficiency any selected band of frequencies and excluding all others. The action of an electrical filter is illustrated in Fig. 18 which shows the measured transmission characteristics of one of the filters in use in a carrier-current system.

The use of carrier systems saves the installation of additional copper conductors, but to offset this requires the use of relatively expensive terminal apparatus and of repeaters at frequent intervals. The expense of terminal apparatus is such that carrier telephone systems are economical under present conditions only on the longer circuits, or in cases where their installation makes possible the deferment of a large expense such, for example, as a new toll cable. The extent of use of carrier systems in the Bell System is indicated in Fig. 19. Each line in this figure represents a system, that is, in the case of carrier telephone, three or four telephone circuits, and in the case of carrier telegraph, usually ten telegraph circuits. The total carrier circuit mileage now in use is about 20,000 miles (32,000 km.) of carrier telephone and 88,000 miles (140,000 km.) of carrier telegraph. The considerable number of carrier systems between Chicago and eastern points are required largely to take care of growth on congested routes pending the completion of the New York-Chicago toll cable. The terminal apparatus can then be shifted to provide carrier circuits from Chicago to St. Louis, Omaha and other points not reached by the cable system.

CONCLUSION

The developments which are briefly discussed above have made it possible to realize in a large measure the goal of a universal telephone service throughout the country, making use of a network of very long telephone circuits covering the country. A number of the more important routes are indicated in Fig. 20. It is of interest to note that the growth in traffic has so loaded up the central transcontinental route as to make desirable the establishment of a southern route from Los Angeles across Arizona and New Mexico. This route is now under construction and will be in service by the end of the year. With the completion of this line there will be at least two independent routes all the way from the Atlantic to the Pacific Coast, whereas at the present time through traffic is dependent upon a single route in the section between Denver and Salt Lake City. Further development of traffic will no doubt require later a route to the Pacific Coast across the northern part of the country and that, with a connecting line across the State of Texas, will provide a very complete gridiron of high-grade routes over the western part of the country, as now over the eastern part.

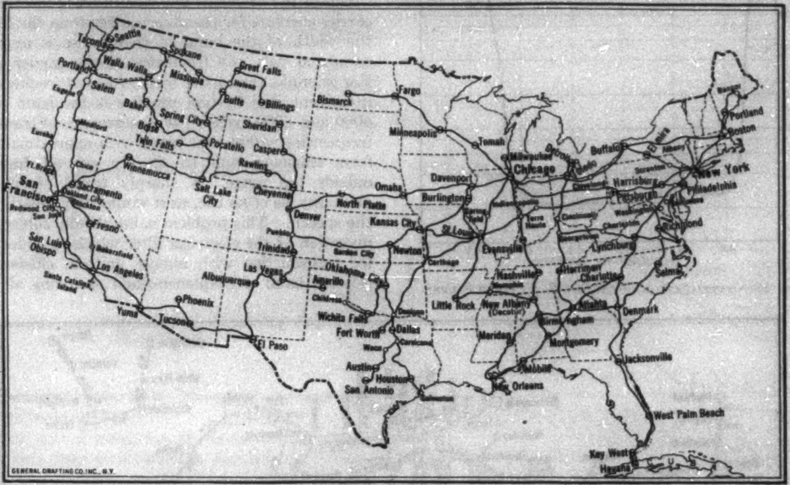
Seattle
Tacoma
Spokane
Portland
Walla Walla
Missoula
Great Falls
Helena
Salem
Eugene
Baker
Butte
Billings
Bismarck
Fargo
Spring City
Boise
Sheridan
Medford
Twin Falls
Pocatello
Casper
Eureka
Rawlins
Ft. Bragg
Chico
Winnemucca
San Francisco
Sacramento
Oakland City
Stockton
Redwood City
San Jose
Salt Lake City
Cheyenne
North Platte
Denver
Fresno
San Luis Obispo
Bakersfield
Pueblo
Trinidad
Garden City
Los Angeles
Santa Catalina Island
Las Vegas
Albuquerque
Amarillo
Oklahoma City
Childress
Phoenix
Yuma
Tucson
El Paso
Wichita Falls
Fort Worth
Dallas
Waco
Austin
Houston
San Antonio
Minneapolis
Tomah
Milwaukee
Chicago
Detroit
Toledo
Buffalo
Elmira
Albany
Bangor
Portland
Boston
Scranton
New York
Davenport
Omaha
Burlington
Cleveland
Harrisburg
Philadelphia
Pittsburgh
Indianapolis
Springfield
Terre Haute
Cincinnati
Washington
Kansas City
St. Louis
Charleston
Richmond
Newton
Evansville
Lynchburg
Carthage
Nashville
Harriman
Charlotte
Salma
Memphis
Little Rock
New Albany (Decatur)
Atlanta
Denmark
Denison
Birmingham
Meridan
Montgomery
Corsicana
Jacksonville
Mobile
New Orleans
Galveston
West Palm Beach
Key West
Havana
GENERAL DRAFTING CO. INC., N.Y.

FIG. 20

By means of the trunk lines indicated in Fig. 20 and a much more extensive network of shorter lines, it is possible to carry on satisfactory telephone conversations between any two cities of moderate size in the country and to a large extent even small places can communicate with each other, irrespective of their relative location. A demonstration which was given in connection with the formal inauguration of service between the United States and Cuba in April, 1921, is a striking illustration of what can done.

The service to Cuba was established by means of the extension of the wire lines to Key West along the viaduct of the Florida East Coast Railroad and the installation of long submarine telephone cables between Key West and Havana. The construction of the cables involved a considerable number of very interesting problems.[8] The demonstration referred to was a conversation between Havana and Catalina Island. This circuit was about 5500 miles (8800 km.) long, included the submarine cables lying at the bottom of the Florida Straits, open-wire lines extending up the Atlantic Coast to New York, across the continent to San Francisco and down the Pacific Coast to Los Angeles, and finally the unique circuit from Los Angeles to Catalina Island which at that time included a wireless telephone link between Long Beach and the Island.

A feature of interest in connection with this demonstration is to note that the telephone circuits involved carry regularly in commercial service not only the voice frequency channel used for the demonstration but many other channels of communication. This is indicated in Fig. 21, which shows the number of independent channels of communication provided in the different sections by the single pair of conductors used. The heavy line represents a telephone conversation and the dotted heavy line indicates that the wires used form one of two pairs which together provide a phantom circuit. The heavy lines between Harrisburg and Chicago and between San Francisco and Los Angeles represent additional channels obtained by carrier telephone systems. The light lines indicate telegraph circuits, two circuits being obtained over the pair of wires throughout by direct-current composite telegraph system and 10 additional telegraph circuits being obtained between San Francisco and Chicago by carrier telegraph systems.

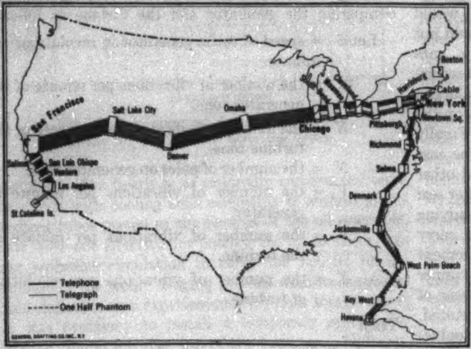

Fig. 21

It is interesting to note that the sound is transmitted over this 5500-mile circuit in less than one tenth of a second. This emphasizes the necessity for using electrical means for the transmission of speech over great distances. If the means were acoustic and transmission was through the air, it would take seven hours for the sound to be transmitted from one end of the circuit to the other.

The essential part which amplifiers at intermediate points play in giving service over these very long circuits is evident. The Havana-Catalina circuit passed through 25 amplifying stations. The impossibility of getting equivalent results by amplification of the terminals is perhaps best illustrated by an example. In talking from San Francisco to Havana, for example, with the transmitter delivering 1000 microwatts at San Francisco, the power delivered at Havana is about 25 microwatts. If there were no intermediate amplifiers, and assuming for the moment that the circuit could carry unlimited power without burning up, it would be necessary in order to deliver 25 microwatts at Havana that power sufficient to light an incandescent lamp would be flowing in the circuit at some point in North Carolina, and at Philadelphia the power would amount to five kilowatts. It is estimated that the total mechanical and electrical power generated in the world is equivalent to that required for about 20 billion electric lamps, and this power would have to be flowing in the circuit a little east of Denver. This power is, however, only about 1/200,000 of the power which is received by the earth from the sun, but all of this power would be required to be flowing in the circuit at Sacramento.

As pointed out at the beginning of this paper, the practical requirements of telephone transmission and of power transmission over long distances are very different. However, to solve the telephone transmission problems it has been necessary to work out the electrical transmission theory both for steady and for transient states[9] for circuit conditions which are more extreme than any which are likely to be met with in power transmission, and it may be that the solution of these purely electrical parts of the problem will contribute to some extent to the power transmission problem in cases where very long distances are involved.

The writer gratefully acknowledges the assistance of Mr. O. B. Jacobs and of many others of his associates in the Departments of Development and Research and of Operation and Engineering in the American Telephone and Telegraph Company in the collection of data used in this paper.

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[8] Refer to paper entitled: "Key West-Havana Submarine Telephone Cable System," by Messrs. Martin, Anderegg and Kendall, A. I. E. E. Transactions, Vol. XLI, 1922.

[9] Refer to paper entitled: "Theory of Transient Solutions of Electrical Networks and Transmission Systems," by John R. Carson, A. I. E. E. Transactions, Vol. XXXVIII, 1919.

Experience with Bearings and Vibration Conditions of Large Hydroelectric Units

BY JOHN HARISBERGER
Member, A. I. E. E.
Puget Sound Power & Light Co.

IN 1905 there was installed at Snoqualmie Falls, a 9000-h. p. single-runner, single-discharge Francis turbine. The end thrust of this runner gave considerable trouble, since the balancing of the run in a turbine of this type had not been perfected at that time. This experience determined that any future horizontal hydraulic turbine should be of the single-runner, double-discharge type unless a satisfactory solution was found for balancing the runner in a single-discharge turbine.

In 1910 a 12,000-h. p. single-runner, double-discharge turbine was installed under a 280-ft. head. When this turbine was put in operation it was found that it would begin to vibrate badly at about ¾ load and would not stop vibrating until the load was reduced to ¼ gate opening. This vibration was similar to that caused by running a high-speed turbine with a badly unbalanced runner. Numerous experiments were tried to stop the vibration, but without satisfactory results. Finally water was piped from the penstock to the turbine case to be used in the pocket between the runner and other parts, such as the side covers and inner and outer seal rings, resulting in an increased pressure and a steadying effect on the runner. It was then possible to carry full load without vibration. With this arrangement we were able to adjust the clearance between runner and seal rings very closely, so that very little wear of the seal rings and periphery of the runner resulted.

In 1911 two 20,000-h. p., single-runner, double-discharge turbines were installed in our White River station, under a head of 440 ft. The experience with these units was similar to that at Snoqualmie Falls, and the same remedy proved effective. A third unit was installed in this station in 1917, and an attempt was made in its design to overcome the vibration, such as occurred in the previous units, without the introduction of pressure water to the side covers. Since we were satisfied that the main cause of vibration was the long distance between bearings made necessary by the two draft tubes, (this being over 20 ft.), the change in design consisted principally of a larger and stiffer shaft. Nevertheless it was found necessary to apply water pressure between seal rings and side covers and runner as on the previous units.

No. 3 turbine did not come up to the capacity expected, so another runner of somewhat different design was installed, which exceeded our expectations as to capacity. With this new runner, we found that a decided hum, or vibration, developed, beginning at about $^3/_{10}$ gate opening and continuing to a little beyond $^5/_{10}$ gate opening with a maximum intensity at about $^4/_{10}$ gate opening. At its maximum, this vibration in the runner is quite annoying and is sufficient to start vibrations in any loose metal parts, such as valve wheels, covers, or balancing rings, so care must be taken to fasten these parts to keep them from wearing rapidly. We have not obtained any expression from the manufacturer of the turbine as to the cause of this hum, but we came to the following conclusion. The tone of this hum is somewhat above the 60-cycle hum. A study of the frequency of the vibration was made by comparing the generator and the turbine as follows:

Let S = speed of turbo generator in revolutions per minute.
C = the number of vibrations per minute of the generator tone.
E = the number of vibrations per minute of the turbine tone.
N = the number of poles on generator field.
V_1 = the number of vibrations per minute of generator.
V_2 = the number of vibrations per minute of the turbine.
X = the number of vibrations per revolution of turbine.

Then

$$V_1 = N S$$

$$V_2 = \frac{V_1 E}{C} = \frac{N S E}{C}$$

$$X = \frac{N E}{C} = 25$$

In this particular case

$$N = 20$$
$$S = 360$$
$$C = 60 \times 256$$
$$E = 60 \times 320$$

As there are 25 buckets on the turbine runner and a vibration occurs only over a limited range of gate opening, vibrations have been considered as due to the relative entrance angle of the runner-blades to the flow angle of the entering water. This flow-angle is varied at different gate openings by the angle of guide vanes and the velocity of the water as it enters the runner, the speed of the runner remaining constant.

Fig. 1, the efficiency curve of this runner, shows a

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

decided sag in the curve between points where this vibration takes place, owing no doubt, to the deflection of the entering water caused by the blades.

The original bearings furnished with two of our large generators were babbitted with a lead base babbitt which was unsuitable for heavy service. Before satisfactory operation could be obtained these bearings had to be rebabbitted with babbitt suitable for heavy pressure. These bearings were 15 by 45 in. with five brass oil rings, and carried a rotor complete, with shaft, weighing 120,300 lb., and operating at 360 rev. per min. The bearings as originally constructed were equipped with cooling coils consisting of several longitudinal tubes placed at the bottom of the oil reservoir, and were not effective for cooling the oil as there was not sufficient circulation to carry the hot oil to the bottom of the well. During the hot weather considerable difficulty was experienced with high temperature. It was necessary to install a temporary system for feeding the oil into the top of the bearings. Different kinds of oils were tried out with the hope of reducing the bearing temperature, and also copper cooling coils were installed in oil wells immediately below the bearings. The cooling coils so reduced the oil temperature that it was not necessary to continue feeding oil into the top of the bearings. The third unit installed in this station, was somewhat larger than the previous two, having bearings 17 by 34 in., with 4 oil rings, and carrying a rotor weight of 136,000 lb., and operating at 360 rev. per min. The cooling coils of these bearings are imbedded in the babbitt. These bearings were so successful that bearings of this type have replaced the original ones in turbines No. 1 and 2. The only difficulty experienced since the changes were made in the bearings, as mentioned above, was that the shaft, on being started after having been shut down for a number of hours, cohered to the babbitt and wiped a spot in it on the lower half of the bearing. In some instances this effect was indicated by smoke rising from the bearings just as the shaft started to turn over but subsequently the bearings continued to operate without further trouble. When after trouble of this kind the bearing was dismantled, a small spot was found wiped near the bottom of the bearing, but the cohesion was not sufficient to pull the babbitt all the way around. On other occasions wiping was more serious and as a result it was necessary to shut down the turbine and scrape the bearing. As a precautionary measure in starting the turbine, a small amount of clean oil is poured on the bearing to supply lubrication during the first revolution. To overcome completely this tendency to cohere, we are installing a high-pressure oiling system to force oil under the shaft before starting up.

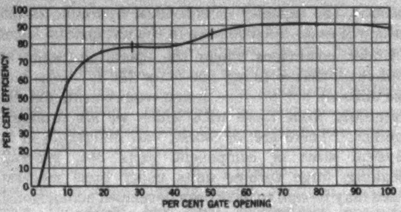

Fig. 1—Gate Opening Efficiency Curve
Turbine No. 3, White River Generating Station

Discussion

For discussion of this paper see page 1004.

Waterwheel Construction and Governing

BY E. M. BREED

Pelton Water Wheel Co., San Francisco, Cal.

THE design and manufacture of hydraulic turbine equipment has been very considerably accelerated within the past few years; more particularly since the World War, as the lessons it taught us in regard to diminishing fuel sources and the interruption to fuel supply through labor disturbances, both directly in fuel production and indirectly in its transportation, have stimulated in a very decided manner the investigation and development of the water powers of the nation. The remarkable strides made in the long-distance, high-tension transmission of power have rendered economically available water power sources formerly considered impossible and, particularly, in the West where long distance power transmission and super power zones have now become realities.

This very considerable increase in the actual and prospective use of hydraulic prime movers has resulted in the past few years in pronounced activity in study and experimentation, to improve the design features of existing types of hydraulic turbines and in the development of new types. This progress is evidenced, not only in the turbines themselves but in their auxiliary equipment, as well as in the water conduits above and below the turbine proper. The rather gratifying results of this investigation and study have introduced a new period in turbine design, characterized by a desire to design the hydraulic equipment to fit the physical conditions and load requirements existing, rather than an attempt to modify the physical conditions to suit certain recognized types of turbine design, based on accepted "type characteristics."

This present healthy condition in the field of turbine design, gives the hydraulic engineer a very considerable latitude in the choice of type and speed of turbine, and in the general arrangement of the hydraulic equipment. As a result, the collaboration of the turbine manufacturer and the power company engineers in the preliminary study of any certain development becomes more desirable than ever, thus emphasizing the community of interests between the manufacturer and the power company. This broadening of the field of possible types and designs of turbines, becomes apparent in three principal directions; a very considerable increase in speeds available for low and medium head turbines; a considerable extension of the maximum heads for which turbines can be designed, and an increase in the capacity of unit considered economically feasible.

The record, as far as size of unit is concerned, is set by the Niagara Falls Power Company which placed contracts last year for three single-runner vertical turbines each of 70,000 h. p. capacity. The engineering fraternity is inclined to expect such records to be set, from time to time, at Niagara Falls as conditions are such there as to warrant the installation of large units, but there have been a number of other installations throughout the country of large size turbine units, and the time seems to have passed when turbine units of less than 40,000 or 50,000 h. p. occasion any comment. It is not too much to say that should the economics warrant, the capacity of individual hydraulic turbine units will reach 100,000 h. p. in the not distant future.

Under conditions where the operating heads are moderate, that is to say 300 ft. or 350 ft., these large size units do not produce design requirements differing materially from those with which we are ordinarily familiar. There are, of course, vast differences in the physical sizes of component parts but, generally speaking, large turbine units under these moderate heads may be said to have been developed homologously from designs of lesser capacity. The 70,000-h. p. units referred to, will be of the same general design as the 55,000-h. p. units manufactured by the same company for the Queenston Plant of the Hydraulic Power Commission of Ontario, and as the 37,500-h. p. units manufactured for Station No. 3 of the Niagara Falls Power Co. They will operate at 107 rev. per min. under 213.5 ft. head. Some idea of the physical dimensions may be gained from the fact that the casing inlets will be 14 ft. in diameter; the power water being controlled by means of 21 ft. by 14 ft. Johnson needle-type valves. The over-all diameter of the cast steel spiral casing will be approximately 42 ft. and the outside diameter of the cast steel runner will be approximately 15 ft. 6 in.

In general, however, it may be said that size in a unit is not an advantage in itself, as seems to be sometimes considered. The increase in efficiency, if any, resulting from an increase in capacity beyond say, 30,000 h. p. to 35,000 h. p., is probably very small, and the decrease in cost per h. p. in most installations of extremely large size units is problematical when the increase cost of foundations, super-structure and auxiliary hydraulic equipment is considered. On the other hand, there are probably only a few power systems in the country where units of 50,000 h. p. or larger, can be used to advantage and a very careful study should certainly be made of the present and expected load requirements, stream flow conditions and expected shape of the efficiency curve below half load before large capacity units are decided upon. In many cases it will probably be found that large capacity units would be operating a considerable percentage of the time on a low part of the efficiency curve. This not

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

only means inefficient use of the water resources, but also rapid deterioration and, consequently, high up-keep charges on the runner, wicket gates and adjacent parts, with frequent removal of the unit from service.

The next most important forward step in turbine construction as marking the progress of the art, is found in the development on the Pacific Coast of what may be termed high-head turbine design. Indeed, from the viewpoint of engineering significance, the high-head turbine should probably be named first as the large number of high-head power sites available in the Rocky Mountains and West, where some seventy per cent of the total undeveloped power of the nation exists, gives rise to the expectation that a considerable proportion of turbine development in the future will be of the high-head type. The development, at their Kern River No. 3 Plant, by the Southern California Edison Company pioneered the field of high-head turbine construction with an installation consisting of two, 25,000 h. p. vertical turbines operating under an effective head of 810 ft. Many problems not ordinarily met in low-head turbine design required solution. Since the plant has been successfully operating for some three years, it may be definitely said that the solution of the high-head turbine has been proved.

The Portland Railway Light & Power Company has at the present time under construction for its Oak Grove Plant a high-head reaction turbine of 35,000 h. p. capacity to operate at 514 rev. per min. under an effective head of 850 ft. This plant when put into operation will be the highest head turbine installation known, this distinction at the present time being held by the two turbines of the Southern California Edison Co., previously described.

While in the larger low-head installations lignum-vitae bearings, water-lubricated, are still extensively used, the trend is nevertheless toward babbitt-lined bearing, oil-lubricated, and in the high-head turbines these have been used exclusively. It will be obvious that in the high-head installations the clearance space between the runner and the stationary parts of the turbine casing must be kept at a minimum if the leakage and consequent errosion of the high-velocity water is to be properly controlled. It is customary, therefore, to keep the high-head turbine runners rigidly in their position of alinement and babbitt-lined bearings, in which the space allowed for the maintenance of the lubricating oil film is kept as small as possible, are essential. Coincident with the necessity for keeping the runner clearances at a minimum, is the requirement for some kind of runner seal which with such close clearances will not leak, seize, nor wear. These needs have been met by the company, with which the writer is connected, through the development of rubber seal rings and several units have been in successful operation for a couple of years under very difficult silt and sand conditions, using this type of seal ring.

A great deal of attention and study has been given to the problems arising in high head turbine design, due to the increased velocities in the water resulting from operation under these higher heads. The utmost care must be taken in the design of turbine casing, speed ring, guide vanes, etc., to prevent as far as possible, formation of eddies and disturbance of the stream line flow. The volute casing is so designed that the cross-sectional area of the volute passage will decrease when passing around the circumference of the turbine at a rate sufficient to provide a continually increasing velocity in the water, inversely proportional to the radial distance from the axis of rotation of the runner. This will deliver the power water to the guide vanes under the same velocity and pressure head at all points of the guide vane circle, and it is interesting to note that the use of the Overn disk-type guide vanes, as developed by one of the Eastern hydraulic manufacturers, tends to reduce hydraulic losses at this point to a negligible quantity through the thin stream line shape permitted by this design of vane.

With the increase in the size of turbine units, the problem of governing has undergone very considerable change. The greater capacities of single-turbine units have come about to a considerable extent, from the fact that they are being installed by companies which are operating large systems and which can, therefore, effectively absorb the output of these single high-powered units en bloc, with the normal load fluctuations of their systems being easily taken care of by other units or plants of smaller capacity tied into the same transmission net work. Therefore, from the viewpoint of regulation, the governing problem on the larger units is not severe. On the other hand, the importance of perfection in the governing mechanism in these larger units can very properly be considered by realizing the responsibility placed upon it and its power to do extreme damage if it fails to function properly. For instance, the length of governor stroke on a large turbine operating under moderate head may not be in excess of 9 in. to turn on the load from zero to maximum capacity. Obviously, the rate with which this vast amount of energy may be turned on or shut off must be absolutely under control. These considerations alone have led to studies in governor design that, without question, are producing a much higher type of construction than in the past. Facilities have been provided for changing from governor control of the turbine unit to hand control by the movement of a single lever. Governors are also usually equipped with devices for automatic emergency operation to close down the plant should almost any unusual condition develop, either electrically or mechanically. Complete switchboard or remote control in the matter of synchronizing, limiting or adjusting of load and emergency shut-down are now included in the modern governor.

It has become quite general practise to install individual oil-pressure sets for each governor and turbine unit, as this gives an independence in operating control and also largely helps to keep the physical size of the pressure-oil units within such limits as will make them fit harmoniously into the power house plan. Advocates of the central oil-pressure systems usually cross-connect the individual oil pressure sets by means of suitable piping so that any one oil-pressure set may be used for any turbine. Where conditions permit, individual oil-pressure sets are made large enough not only to provide pressure oil for their own turbines but also to operate an adjacent turbine under emergency conditions.

Pressure regulators, or relief valves, have undergone considerable change of design by reason of the large water quantities that now must be handled and on occasion also because of the high heads under which they must discharge. Mechanically-operated relief valves which take their motion directly from the gate-shifting mechanism on the turbine are essential and the entire pressure regulator structure should be so designed as to safely withstand the full effort of the governor so that should an obstruction block the opening of the relief valve, the turbine guide vanes will be effectively prevented from any further closure, as otherwise a serious ram or rise of pressure in the pipe line would result. It is also essential that the pressure regulator shall have a discharge characteristic which will approach as closely as possible the discharge characteristic of the turbine guide vanes, as otherwise the inverse movement of the guide vanes and relief valve will cause variations of the velocity of flow in the pipe line which may result seriously.

Another development of importance in the turbine field is the rapidly increasing use of the so-called high-speed or propeller type runner. The Francis type runner, which has been the standard for many years, has been developed and perfected so that for low head work, specific speeds as high as 80 or 85, in the English system, can be gotten with fairly high efficiencies, but as these appear to be about the limit to be expected, a great deal of experimentation and study has been given to the development of a new high-speed type of runner. Some of the advantages gained by the use of the propeller type or diagonal runner are:

First. A reduction in the first cost of the hydroelectric unit, due to the smaller dimensions resulting from the higher speed, and a reduction in the cost of the super-structure of the power house, cranes, etc. owing to the lighter weight of the rotating element and other parts to be handled.

Second. The mechanical strength of the diagonal runner is improved over that of the Francis type as the overhang of the vanes is less and the connection between runner hub and vane considerably longer, resulting in smaller bending stresses in the vanes. As the vane is unshrouded the construction permits of the use of separate vanes which can be bolted to a central hub, allowing the replacing of a single vane, if accidentally damaged.

Third. The design of the diagonal runner is much simpler than that of an equivalent Francis runner, thus eliminating to a very considerable extent the possibility of erosion, cavitation, etc., and providing a more direct passage for the water. The runner usually consists of from four to eight vanes, in comparison with the Francis type, having from fourteen to eighteen vanes, so that the clear openings through the vanes are considerably greater, preventing to a very considerable extent, the possibility of clogging with foreign material.

A considerable number of these high-speed runner units have been installed in the past two or three years and very satisfactory results obtained. Numerous tests have been made both of the model runners and of complete units installed, and efficiencies of 88 per cent to 90 per cent can reasonably be expected from this type of runner when designed for specific speeds ranging from 110 to 150, English system; the efficiency dropping off somewhat as the upper range of specific speed is reached. Possibly the most notable installation thus far made with this type of runner, is that of the Manitoba Power Company, installed last year consisting of three 28,000 h. p. vertical hydroelectric units with propeller type turbines, designed to operate at 138.5 rev. per min. under a net effective head of 56 ft. This corresponds to a specific speed of 153, English system. The runners of these turbines contain six blades and have an over-all diameter of approximately 16 ft. These units have indicated very clearly the remarkable characteristics of this type of runner, in particular, its ability to operate at normal speed under greatly reduced heads, with a considerable power output, the range of effective head being from 28 ft. to 56 ft. It is interesting to note, in this connection, that the runner of the Manitoba Power Company turbines is within a few inches of the same diameter as the Francis type runner of the turbines installed at the Cedars Rapids Plant on the St. Lawrence River, which were manufactured by the same company eight years earlier, but that the specific speed of the Manitoba turbines is practically twice that of the Cedars Rapids units, notwithstanding the Cedars units represented the highest specific speed obtainable at that time.

The subject of draft tube design is one which has caused a great deal of discussion in recent years and on which there is a considerable diversity of opinion. Numerous tests have been conducted for the purpose of determining the relative values of the different types, but as most of these tests have been made with model runners and tubes where slight variations in construction and setting of the tubes produced widely differing results, there still exists a great difference of opinion as to the most efficient form.

Discussion

For discussion of this paper see page 1004.

A Study of Irregularity of Reaction in Francis Turbines

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***Review of the Subject.**—Irregularity of reaction in hydraulic machinery causing vibration has up to the last few years been of relatively little importance and treated as a more or less necessary evil.*

With the advent of larger units, particularly for high heads, it became of prime importance that there be little or no vibration. In cases where vibration occurred, a cut and try system of remedying it was usually resorted to and as little publicity as possible given the procedure.

For the study of such phenomena as vibration caused by irregularity of reaction manifesting itself in several impulses per second the electrical methods long since developed are admirably adapted and this paper gives a brief description of a successful method of study.

DURING the last few months there has arisen an occasion to study the hydraulic conditions causing vibration in some of the Francis type turbines on the system of the company by which the writer is employed.

Preliminary reconnaissance showed that these vibrations had a period in the order of the electrical frequencies commonly used and that for their study special equipment must be used which, so far as is known, had not been developed.

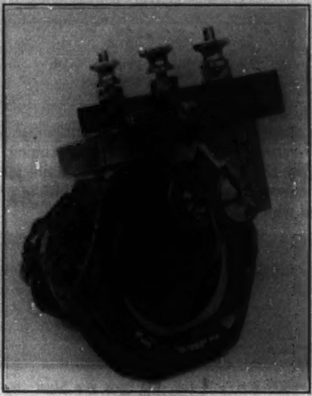

FIG. 1

There was accordingly developed somewhat hurriedly from parts available a device for converting pressure variations to electrical impulses and then viewing or recording these by means of an oscillograph.

As shown in Figure 1 a 1½ in. pipe union was used to hold a diaphragm of spring steel of a thickness such that at the maximum pressure used its deflection is still approximately proportional to pressure change. (Several well known gas engine indicators operate on this principle and it is known to be trustworthy.) Rigidly attached to its center is an arm bearing on its outer end the center elements of two microphones of a variety used commonly in amateur wireless work.

The outer elements or shells of these microphones are rigidly attached by an insulating member to the union in such a manner that the supports are used as current-carrying terminals.

A third terminal grounded to the union makes electrical connection through the arm to the center elements of the microphones.

With the connections as shown the device forms an elementary bridge with microphones and batteries as members and an oscillograph element as a detector. The 2-volt batteries forming two arms of the bridge are so connected that they tend to cause current to flow through the oscillograph element in opposite directions so that as the diaphragm moves due to pressure changes we have a curve resembling an alternating-current curve traced on the oscillograph film proportional to and in correct sequence with the pressure changes back of the diaphragm. The device can be attached to any point that an ordinary pressure gage can and a ½ in. connection if comparatively short is more than ample to actuate it.

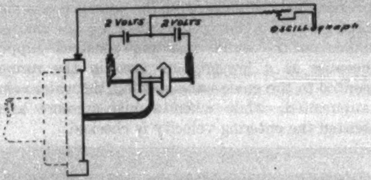

FIG. 2

Using one of the portable oscillographs now available studies have been made on several turbines under varying conditions of load, head, gate opening, etc., with a view of determining the fundamental reason for vibration. It is known that the pressures across the face of a turbine runner opening vary both from top to bottom and from trailing to leading sides and

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

are dependent upon design characteristics as applied to running conditions.

As a check on probable values, tests were made using a Pitot tube traveling with the runner and passing to the center of the draft tube and out through a packing gland either to a pressure gage or the pressure indicator described above.

By filling this rotating tube with air under pressure and then closing, the supply air is trapped under the pressure of the water at the end of the tube and centrifugal action and leakage are minimized. In this way the whole face of the runner entrance may be explored under actual running conditions.

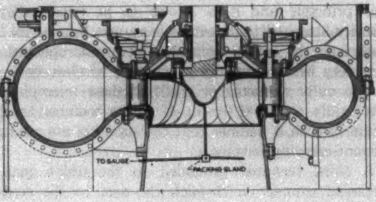

Fig. 3

Having established the fact that there is a difference in pressure on the water over the entrance of a runner bucket gives us a clue to the reason for the changes in pressure on the scroll case and penstock supplying it because as a low-pressure area in the runner is presented to the guide vane opening, the water velocity is augmented, while when a high-pressure area is presented the entering velocity is checked.

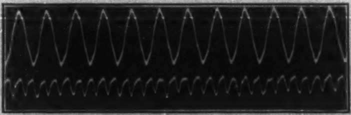

Fig. 4—5000 Kw., 220 Lb., 450 Rev. per Min., 19 Runner Vanes, 20 Guide Vanes. Hum Apparent in both Power House and Pen Stock. 15 Ohms in Vibrator

This difference in pressure has been measured where the difference between leading and trailing sides of the runner entrance pressures was approximately 10 per cent of the total head on the turbine.

If we have a turbine in which the number of runner vane openings $(n-1)$ are one less than the number of guide vane openings (n) the openings will register successively around the scroll case each $1/(n-1)$ revolution as a vernier backward from the direction of rotation.

This gives $(n-1)$ impulses per revolution or the revolutions per second times the number of runner vanes in impulses per second.

A difference of two in the number of guide and runner vanes gives two impulses traveling at half speed around the scroll case while combinations such as 20 and 17 gives irregular impulses. These combinations are best studied on small drawings to scale on cardboard, the runner portion of which may be rotated by hand.

Fig. 4 shows a pressure curve of a turbine having 20 guide vanes and 19 runner vanes with a timing wave of 60 cycles. This gives the Vernier action described above and the machine has a decided hum and a vibration which persists with gradually lessening intensity up the penstock 3600 feet to the forebay where it is scarcely perceptible.

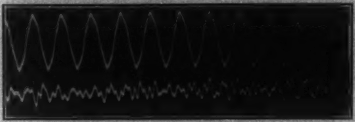

Fig. 5—5000 Kw., 220 Lb., 450 Rev. per Min., 17 Runner Vanes, 20 Guide Vanes. No Apparent Noise or Vibration 15 Ohms in Vibrator

Fig. 5 shows the curve from an identical scroll case with 17 runner vanes and a 60-cycle timing wave. In this case there is apparent neither hum nor vibration yet the curve shows just as high a hydraulic pressure variation which is, however, not regular.

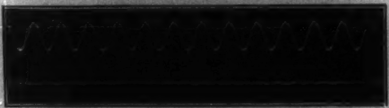

Fig. 6—5400 Kw., 238 Lb., 400 Rev. per Min. No Noise or Vibration. 15 Ohms in Vibrator. 20 Runner Vanes, 24 Guide Vanes

Fig. 6 was taken on a turbine of somewhat older design and different specific speed and is almost ideal as regards hydraulic vibration. Just a slight wave may be traced which corresponds to the frequency we would expect.

Fig. 7 is an example of a combination of 18/16 and in operation this machine has a perceptible hum and vibration which persists up to the forebay 1600 feet from the turbine.

There are some conditions under which this is more pronounced than others due to the manner in which the water enters the runner.

Fig. 8 is a high-speed film on large ordinates of a record of a turbine with a 20/19 combination of guide and runner vanes taken to study the effect of the guide vanes; a ripple corresponding to each guide vane appearing in the pressure wave.

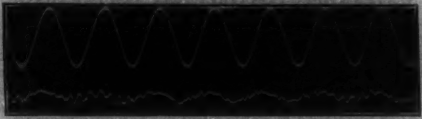

Fig. 7—4000 Kw., 3.34 Gate, 83 Lb. Pressure, 3.5-In. Vacuum, 15 Ohms in Vibrator, 18 Guide, 16 Runner Vanes

Fig. 8—Film 8-A Vib. No. 2-3w., No. 1 Machine Full Load. 60 Cycles Timing Wave

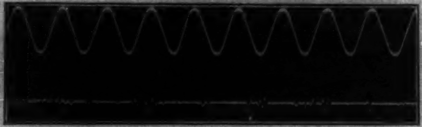

Fig. 9—Film 12-c. Vib. No. 2-15w. Velocity Pressure at Inlet Side of Runner on Axis of Guide Vanes. No. 2 Machine 22,500 Kw. 40 Lb. Back Pressure on Pipe. Timing Wave 60 Cycles

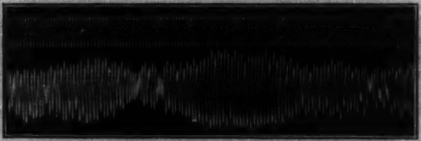

Fig. 10—Film 5-c. Vib. No. 2-15w. No. 1 Pipe Line just Above Pier at End of Tunnel. No. 1 Turbine, Full Load No. 2 Off. Timing Wave 60 Cycles

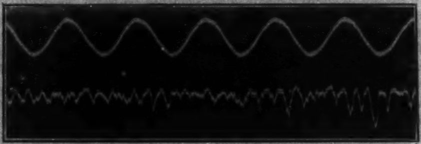

Fig. 11—5000 Kw., Penstock Pressure, 218 Lb., 360 Rev. per Min., 15 Ohms in Vibrator

Fig. 9 is a record of a traverse around the scroll case at the center of the runner opening taken with the rotating pipe referred to above to see if pressure conditions were uniform around the entire circumference.

Little ripples caused probably by the scroll case guides, that is, the fins back of the guide vanes at points around the scroll case to direct the water into the guide vanes are evident though of no particular importance.

Fig. 10 shows the condition in a large somewhat elastic pipe line connected to a turbine having a uniform vibration. Here the pressure waves are influenced by reflection at the natural period of the penstock, by draft tube variations and by irregularity of flow in the penstock itself.

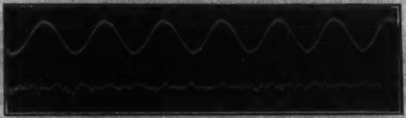

Fig. 12—12,600 Kw., 202 Lb. Penstock Pressure, 360 Rev. per Min., 15 Ohms in Vibrator

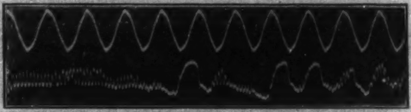

Fig. 13—3240 Kw., 60 Cycles, Pressure Scroll Case 59 Lb., Penstock 72.8 Lb. Draft Tube 4½, Gate 6/8. Relief Valve Closed. 20 Guide 16 Runner Vanes

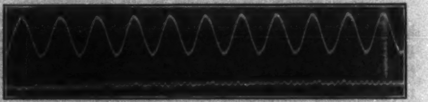

Fig. 14—4470 Kw., 60 Cycles. Pressure Scroll Case 65 Lb., Pressure Penstock 65 Lb., Draft Tube 3½, Gate 7/8, 20 Guide, 16 Runner Vanes

Fig. 11 shows a turbine in which the flow conditions are critical at a certain gate opening, that is, the pressure difference across the runner inlet face is greater at this particular opening than at others, though not serious on any.

Fig. 12 shows the same machine as Fig. 11 with a greater gate opening.

Fig. 13 shows a curve for a turbine on which abnormal conditions of head and load were imposed. The flow relations were unstable and were manifest in irregular vibrations or "shivers" of the unit.

Fig. 14 shows the same turbine under more nearly normal conditions.

The mechanical vibration was about in the proportion of the curves and there was very little hum on account of the irregularity of pressure variations and the lack of elasticity in the equipment.

All of the curves shown have a 60-cycle timing wave for comparison.

OPERATION OF APPARATUS

In taking such records the so-called auxiliary data are of as much importance as the record itself and all conditions should be faithfully recorded in order to study and fully appreciate all of the changes made evident by a film of pressure variations.

Only ordinary precautions are necessary in using the apparatus such for instance as making sure that there is no trapped air behind the diaphragm, that all electrical connections are good and that the oscillograph is properly adjusted and handled.

The microphones are adjusted originally by disconnecting one microphone lead and observing the shift in the zero line caused by current through the one connected and then repeated with the other, adjusting until the shift is the same for each. The adjustment in the present device consists in moving the microphone shell in or out a small distance with respect to the *central* part fast to the diaphragm.

It is but a few minutes work to set up and observe the waves and changes in pressure waves may be watched while operating conditions are changed. Once adjusted, different points may be explored wherever a half-inch pipe connection can be made.

Nearly all of the cases studied have to do with the manner in which the water enters the runner and are fundamental in turbine design.

It is thought that the shock due to sudden changes in velocity of moving water has not been fully appreciated in many cases.

Using the ordinary formulas for surges and water hammer as given in Prof. Durand's Hydraulics of Pipe Lines it is seen that with 80 cycles per second, that is, 160 changes, there is required somewhat less than four one hundredths of a foot change in velocity per second to cause 100 lb. pressure change, provided all of this change were effective.

Elasticity in the scroll case and penstock and large clearances between guide and runner vanes together with the combinations of guide and runner vanes which set up conflicting pressures all tend to prevent the difference in pressure across a runner opening from causing high-pressure variations in the scroll case and penstock.

Certain design characteristics cause most of the energy in the water entering the runner to be in the form of velocity and therefore incapable of causing any of the phenomena observed. In other designs more reaction and therefore more pressure is necessary and greater care must be used in entrance angles and clearances.

The subject is as yet very new and very little concerning it is known. It is only hoped that such work as has been done may be of assistance in further study.

Discussion

EXPERIENCE WITH BEARINGS AND VIBRATION CONDITIONS OF LARGE HYDROELECTRIC UNITS[1]

(HARISBERGER),

WATERWHEEL CONSTRUCTION AND GOVERNING[2]

(BREED),

A STUDY OF IRREGULARITY OF REACTION IN FRANCIS TURBINES[3]

(WILKINS) DEL MONTE, CAL., OCTOBER 3, 1923

R. Treat: The device which was developed by Mr. Wilkins, appeals to me as being applicable to investigations of other similar problems. It appears to be a very simple, and it is hoped, reliable device which may be used for determining among other things, the pressures in the interior of oil circuit breakers at the time of opening the circuit and the stresses which are brought to bear upon bus insulators at times of short circuit.

R. L. Hearn: As one of the staff to install and put in operation the first two 55,000 h. p. units in the Queenston-Chippewa station, I had an opportunity to closely watch the operation of these units for some time after they were put in service. While we did not find the vibration in these units to be of such magnitude as to be dangerous, yet it was there to such a degree that it has made us give this problem consideration. We found the vibration to be worse at part gate and were inclined to consider the draft tube to be the prime cause due to the unstable condition of the water column in the draft tube under part load. Mr. Wilkins has shown in his paper that vibration can come from other sources.

I would like to ask Mr. Harisberger whether he has considered the affect of the draft tube on the vibration found in the units which he discusses in his paper. I have in mind the condition we found with two different types of draft tubes which were installed on the Queenston plant on two wheels of identical design. One draft tube was of the bent tube type, while the other was a Moody type draft tube. In comparing the vibration in these two units we found very little difference at full gate opening, but at part gate there was considerably more vibration with the bent tube than with the Moody tube. This difference we contributed to the design of the draft tube.

In many of the recent large installations of vertical type Francis turbines, the upper portion of the draft tube immediately adjacent to the runner has been constructed of cast iron and has been so designed that it can be removed, thus allowing the dismantling of the runner from the lower side of the machine.

In many cases vibration has caused the cast iron section of the draft tube to loosen from the turbine casing and I believe it is the consensus of opinion, that it is advisable to concrete in this section of the tube in order to absorb the vibration and thus protect the draft tube from being shaken loose.

In order that the work of investigating problems, such as we have just been discussing, might be more effectively carried on, we should develop a spirit of cooperation between the operating companies and the manufacturers. So often in the past, it has been the case that the manufacturer delivers his equipment and after acceptance by the purchaser obtains very little knowledge as to the performance of this equipment during its operating life.

E. R. Stauffacher: I would like to ask a question in regard to the interpretation of a curve, Fig. 4 of Mr. Wilkins' paper. In the later part of the notation underneath the figure the statement is made: "Hum apparent in both Power House and Pen Stock." By reference to the curve it can be seen that it is quite regular.

In Fig. 5, you will notice: "No apparent noise or vibration." By referring to the curve you can see that it is quite irregular.

Am I right in interpreting these curves to mean that if the curve is regular, you have a source of danger, whereas, if it is irregular there is no danger?

Also, in reference to Mr. Harisberger's paper, as to the technique of determining the tone of the generator and the tone of the turbine, what method to determine pitch was used? Was it by means of a tuning fork, or just by the ear of this musical operator?

R. J. C. Wood: I would like to ask Mr. Wilkins whether any experiment was made by varying the speed of the machines slightly above and below the point at which he obtained these diagrams that he gives us, to see whether there was any resonance that might be in some portions of the casing or a water column of a length that would respond to certain frequencies, just as in some automobiles there is a certain speed at which the machine tries to vibrate itself to pieces, and below that speed you are comfortable, and also above it?

John Harisberger: In reply to the question as to how the tone was arrived at, this was done by the chief operator of this particular station by tuning in on a keyboard of a piano in the club house, which is near the power house, so the hum can be distinctly heard.

As to Mr. Hearn's question—Was anything done, or any attempt made to correct the difficulty by changing the draft tube?—No, this particular unit has a single-runner double discharge turbine with boiler riveted steel draft tubes similar to that of the two other units which have given no trouble. As the vibration occurs only at partial gate, in the neighborhood of 3/10 to 5/10 opening, and as it is rarely that the unit is operated at this gate opening, we did not think it necessary to go to much trouble or expense to correct the difficulty. In fact, we have not had any very definite information to work from until Mr. Wilkins' paper was presented, which seems to me gives us something well worth considering.

I would like to ask Mr. Wilkins if all the runners considered in his paper were Francis runners?

John Sturgess: At one period in our forward progress some particular problem seems to be right in the lime light and demands first attention, and during that period some of the other problems are side-tracked.

Now, I feel that is particularly true in the case of vibration, because many years contact with hydraulic turbines leaves the impression on me that this problem was relatively more fully solved several years ago than it is today. It is almost an example of partial retrogression.

In 1898 I spent three weeks on top of a hydraulic turbine at Niagara Falls, where the vibration was quite severe. At that date, 1898, hydraulic calculations and engineering were very inadequate. The wheel was designed for 2500 h. p., but it accidently developed 5000 h. p. The hand wheel for opening the gate was 18 in. diameter but before the gate could be opened it was necessary to strap to the hand wheel a 12-ft. beam with block and tackle at the outer end. While there was severe vibration in this unit, it was no worse, if as bad, as on some recent units, and I do not feel that this apparent retrogression is to be entirely accounted for by size of unit, or the head, for some of the new units were not materially different from the 1898 unit in respect to size or head. There has been recent discussion regarding synchrony between vanes and gates, but this was the subject-matter of discussion from ten to twenty years ago.

I would like to express a thought in regard to that part of Mr. Breed's paper illustrating the propeller type turbine, which, by the way I believe, if proper credit is to be given the original proponent, should be called the Kaplan turbine. It is obvious that the propeller type is nothing but the continuation to its logical conclusion of a process we have all gone through in developing high specific speed runners,—diminishing the inflow element and amplifying the axiflow element. It is easy to see that the complete elimination of the inflow element was the logical conclusion of this process. Yet it is called a "new" turbine.

Looking ahead, I expect to see for this type, before it is finally established as a standard, an entirely new type of gate developed. The swivel gate, while obviously correct in connection with the inflow turbine, is illogical when applied to the axiflow type.

L. A. Barnes: In regard to the 2.5 in. which Mr. Wilkins took off the runner. I assume he took that off in small amounts at a time. If he did not I would like to ask how he determined that amount? And, if he did take that off a little at a time I wonder if he could tell the success that he had in the successive cuts, that is, whether it was progressive or whether he arrived at it all at once.

R. Wilkins: In answer to Mr. Stauffacher, Curves No. 5 and No. 6 were taken on identical scroll cases with different runners and under the same head on the same pipeline. On No. 4 there was sufficient noise to make talking difficult, that is, the hum was so loud that you had to raise your voice to be understood in the power house; this gradually decreased up the pipeline yet you could hear the hum at the penstock. With the seventeen vane runner in an identical scroll case just beside it on the same pipeline, there was no noise that you could hear at all There was no vibration that you could feel with your hand, yet the hydraulic pressures inside the pipeline were just as violent as those in the other unit. Both these, however, are considerably smaller than some of the other curves. Fig. 13, for instance, gives no hum; it is not regular enough to give a hum and it didn't give an actual noise, yet that unit vibrated so that we were afraid to run it for fear it would shake out the bearings; it was put under different conditions and it toned down as is shown in one of the other curves. I would not say that it entirely stopped but the pulsations were not audible or perceptible, that is, it wasn't dangerous. These pressure pulsations in pounds per square inch are quite high and we don't know just what they do; which is the reason for the investigation, because of the things that we were afraid they might do.

In answer to Mr. Wood's question: As far as the frequency of the system would permit, curves were taken from as low to as high as we could get with the load. Without load these pressure variations are light because there is little water entering the runner and as you decrease the speed you change the conditions under which that turbine operates at a different head and it depends on head and load conditions where it is the worst. If it vibrates at all it vibrates over all speeds, but there is a place where conditions are the worst and this depends on the design, principally, of the runner.

In answer to Mr. Harrisberger: All runners, as stated in the title of the paper, were Francis runners. In fact, all the runners we have on the system are Francis runners and the pressure that is built up in them depends on their design and the amount of clearance between them and the guide vanes. It is the way the water enters the runner that causes the variation in pressure. If there can be obtained between the runners, even on runners not correctly designed, sufficient clearance to equalize part of the pressure, vibration will be cut down. On the runners in which we tried to correct the vibration we cut back as far as we dared. On a runner 105 inches in diameter, we cut back about five inches, $2\frac{1}{2}$ inches on each side.

R. J. C. Wood: I would like to know whether Mr. Wilkins noticed any relation between the shape of the guide vanes in these different units and the degree of vibration hum, or the oscillations?

R. Wilkins: The guide vanes, so far as we could determine, had very little effect on the water as it entered the runner. We tested practically all of the types of guide vanes available, and found no difference.

In answer to Mr. Barnes: The cuts taken off were as much as we possibly dared take off, but I don't think that we have enough off yet. There were no preliminary cuts and there have been none since. It is quite a job to disassemble a unit of that size and to make the cut at the same time; it is quite a loss to a power company to have to put out of service a unit of 35,000-kw. capacity on which such work is being done. We did the best we could at one guess.

Recent Hydroelectric Developments of Southern California Edison Company

BY H. L. DOOLITTLE
Southern California Edison Co., Los Angeles, Calif.

THE important water power plants constructed by the Southern California Edison Company in the last two years are Kern River No. 3, Big Creek No. 8, additional high-head impulse units in Big Creek plants Nos. 1 and 2, and the new Big Creek No. 3 plant which will be placed in operation the latter part of this summer. This paper will review some of the outstanding features of interest that have been incorporated in this work.

THE KERN RIVER NO. 3 PLANT

In this plant are installed two 22,500-horse power vertical turbines operating under head of 800 feet. To date these are the highest head reaction turbines that have been constructed, although a contract is now under way for another company for similar units to operate under total head of 850 feet. A recent inspection of the Kern River No. 3 runners showed them to be in good condition after nearly two years of operation.

One novel feature of these units is that they were designed to operate at a frequency of either 50 or 60 cycles, this being due to the fact that the load to be supplied by this plant is in 60-cycle territory part of the year and in 50-cycle territory for the remainder. Every precaution was taken to make the changing of runners as easy as possible and a design was finally adopted which accomplished this result very satisfactorily. Acceptance tests of the equipment show that an efficiency of 90 per cent was obtained and that the 50-cycle runner gave practically the same efficiency when operating at either normal speed of 500 rev. per min. or the 60-cycle speed of 600 rev. per min. There was, however, a somewhat lower maximum output when operating at the higher speed. The 60-cycle runner showed the same efficiency at normal speed but was considerably less efficient at the 50-cycle speed.

While the good condition of the runners after this long period of operation is mainly due to proper design it is also partly due to the absence of sand and silt in the water. Realizing that sediment of any kind would be very undesirable with these high head turbines an elaborate sand box was constructed for precipitating all foreign matter from the water. This sand box is 80 feet wide and about 500 feet long, reducing the velocity to about one-half foot per second. After several months of operation the sand box was inspected and it was found that about six inches of fine silt had been precipitated in the lower half of the box. This silt was so fine that forty-five per cent of it would pass a 150-mesh screen.

BIG CREEK NO. 3 POWER PLANT

The Big Creek No. 3 power plant which is now under construction will consist at present of three 35,000-horse power turbines operating under a head of 740 feet. There is nothing unusual about the design of the main units, although several special features have been worked into other parts of the plant.

At the intake to the tunnel for this plant a cylindrical gate 22 feet in diameter and 90 feet high has been installed for shutting off the water to the tunnel line. This gate is motor-operated and is installed in a concrete tower located two or three hundred feet up stream from the dam.

A surge chamber is being constructed at the lower end of the tunnel. This surge chamber is excavated from solid rock and has an hour glass section, the bottom diameter being 60 feet, the central shaft 25 feet and the top diameter 75 feet. This hour glass section was adopted on account of the wide variation in water level at the intake under normal operating conditions. The large section at the bottom provides for a load suddenly applied and the large section at the top will take care of any surge created by loads rejected.

Connecting to the outlet of the tunnel is an 18-foot steel pipe which terminates in a manifold providing connections for six 7-foot, 6-inch diameter penstocks. The design of this manifold, which will operate under a total head of approximately two hundred feet, offered many difficulties. After much study a design was finally developed which consists of two plate steel spheres joined by a short section of pipe. To one of these spheres the 18-foot pipe and two of the penstocks connect, the other sphere providing connections for four penstocks. The adoption of this type of manifold permits taking care of the excessive stresses in a very simple manner and has successfully solved a rather difficult problem.

Several novel features have been incorporated in the penstocks for this power plant. The entire pipe lines will be of forge welded steel. As the pipes are to be installed above ground, expansion joints are used between anchors. These expansion joints are made so that the portion of the pipe on which the packing bears is copper-plated to a thickness of about 1/16 of an inch. This will provide a perfectly smooth and non-rusting surface for the packing and it is expected that all future trouble due to leakage will be eliminated.

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

In a pipe line of this sort the friction on the supporting piers make necessary extremely heavy anchors at the bends. In order to reduce the size of the anchors to relieve the pipe of excessive longitudinal stress and also to facilitate erection, these pipes will be installed on cast iron rockers. Tests have shown that the friction will be reduced so greatly that a very large saving can be made in the cost of anchors and supporting piers. These rockers will be installed every 40 feet instead of every 20 feet as has been the custom heretofore with ordinary concrete piers.

In the power plant one of the important improvements that has been adopted is the elimination of an exterior lubricating oil system consisting of filters, supply tanks, pumps, piping, etc. It is felt that much more reliable operation will be obtained by simplifying the lubricating oil system to the greatest extent. The bearing on the water wheel and the generator bearings will be provided with gear-driven oil pumps which will pump the lubricating oil from a small sump below the bearings to the top of the bearings and thus keep it circulating continuously. Motor-driven pumps will also be installed, arranged to be started automatically in case the gear-driven pumps should cease to operate.

In the general arrangement of the station it is felt that a great improvement has been made, from an operating standpoint, by the elimination of the basement. The main floor of the generator room will be on two levels; about half of the generator room will have a floor on a level with the generator base and the other half will be on a level with the turbine. This will permit the maximum amount of equipment to be in view of the operators at all times.

VENTURI METERS

It is now this company's practise to install Venturi meters in all new hydraulic plants. The Kern River No. 3 plant was equipped with Venturi meters at the time the plant was constructed. Venturi meters will be installed on the three units for the Big Creek No. 3 plant and similar meters have already been purchased for the six units in the Big Creek No. 1 and Big Creek No. 2 power plants. The importance and value of a reliable meter for measuring the water supplied to a water wheel has been proved in many ways. At Kern River No. 3 the Venturi meters are constantly used by the operators in their effort to obtain the maximum kilowatt hours output from the water available. Some recent tests on one of the Big Creek impulse units showed that the difference in output between a new and a somewhat worn nozzle was in the neighborhood of $1,000.00 per month. As this amount is several times the cost of installing new nozzles it emphasizes the value of having a continuous check on the efficiency of the wheels.

DRAFT TUBES

The Kern River No. 3 turbines are equipped with the regular quarter bend draft tube. The turbine in the Big Creek No. 8 plant has a spreading type of draft tube while the new turbines for the Big Creek No. 3 plant will have a modified spreading type of tube which is supposed to embody the advantages of the regular spreading type of tube but is somewhat simpler to construct. As far as our experience goes we have been able to notice very little difference in the operation between any of the types of draft tubes that we have. This is undoubtedly due to the fact that all our turbines operate at very high heads so that any increase in efficiency due to the more modern tube is difficult to detect.

IMPULSE WHEELS

The impulse wheels installed by the company in the last two years do not embody any novel features, being duplicates of others that have been in operation for many years. We do have under way, however, a preliminary study of a new plant that will operate under a total head of 2400 feet. At the present time preliminary plans are being worked out for double over-hung impulse units for this plant which will have a capacity of 75,000 horse power per unit. These wheels will undoubtedly be the largest impulse units ever constructed from the standpoint of total output but on account of the high head will probably not be any larger in physical dimensions than the wheels in the Caribou plant of the Great Western Power Company. The chief difficulties to be overcome in the design of a unit of this size are the penstock and the water wheel and generator bearings.

RUBBER SEAL RINGS

For several months we have had in operation a small hydraulic turbine equipped with rubber seal rings. These rings have not been in operation long enough to definitely determine their practicability but it appears that when this type of ring has been fully developed it will have great possibilities. Arrangements are being made to try out these rubber rings on one of the turbines in the Kern River No. 3 plant. This trial will give some valuable information as it will be the first time this type of ring has been installed on a unit of this size.

Upper Falls Development of The Washington Water Power Company in Spokane, Wash.

BY L. J. POSPISIL
Associate, A. I. E. E.
Washington Water Power Co., Spokane, Wash.

Review of the Subject.—*This describes a late hydroelectric development in the center of a city having a single vertical shaft generator and delivering its output to the busses of an existing distribution substation 350 ft. distant, the excitation and load control of the new generator being from the substation. The principal features of the development are the auxiliary devices to assure reliable operation of a remotely controlled unit and provision for an automatic shutdown of the unit in case of major trouble; also the automatic by-passing of a large portion of the water used by the wheel if the load is greatly reduced or dumped. Other features are a 20-ft. diameter reinforced concrete penstock, a Moody spreading draft tube and a large concrete volute casing, reinforced against 110-ft. head; also 60 ft. by 15 ft. gates remotely operated, the operator having before him devices indicating gate travel and elevation of water level being controlled.*

* * * * *

BEFORE the main features of the development that forms the subject of this paper are described, some mention should first be made of the prior developments to which the Upper Falls is an important addition.

The four earlier developments of The Washington Water Power Company totaling 125,000 h. p. as well as the one described, are all located on the Spokane River and utilize succesively the same water with only a small additional inflow between plants.

Spokane River has its source in Lake Coeur d'Alene, a body of water some 42 square miles in area, where a storage and draw down of 5½ feet is possible. A few miles below the outlet of this lake is the Post Falls station which utilizes a head of 50 to 55 feet. At this station, the second one constructed by the company, are located the flood gates by which the lake level, except at times of flood, is controlled.

At Spokane, the next station downstream from Post Falls, there is a rare situation where a city of considerable size has been built up around a series of falls. Here the initial hydroelectric power developments, utilizing about 53 per cent of the total available head within the city, were begun in the very early days of the electrical industry and completed in 1904. This development, known as Monroe Street station makes use of the lower 73 ft. of the available head. The development of the upper head of 64 feet is the subject of this article.

The other two power plants of the company are at Long Lake, utilizing a head of 172 feet, and at Little Falls, which is immediately below Long Lake, where the head is 73 feet.

In addition, Spokane River water operates a part of the pumping equipment of the city of Spokane and the hydroelectric station of the Spokane & Eastern Ry. & Power Company located between Spokane and Long Lake. Unregulated flow of the Spokane River varies from a minimum of about 1200 cu. ft. per second occurring usually in October to a maximum so far recorded of approximately 50,000 cu. ft. per sec. The mean low-water flow is about 1400 cu. ft. per sec.

Lake Coeur d'Alene in turn is supplied from mountain streams having their sources up to 6000 ft. in elevation. Precipitation is mostly in the form of snow and the annual flood, which reaches its peak between May 10th and June 5th is the result of melting snow and is usually over by the 15th of July. There is usually very little precipitation in the drainage basin between July 10 and September 15th and the inflow into Lake Coeur d'Alene from July 15th until the fall rains arrive is the yield from ground storage.

The Spokane plants have practically no storage in their forebays but there is most excellent storage in the forebay of the Long Lake station. Long Lake serves as storage also for Little Falls, which is hydraulically in series, and has power units of the same water capacity as the turbines at Long Lake.

After the passing of the summer flood the gates at Post Falls, which control the level of Lake Coeur d'Alene, are closed and a practically uniform load is carried by the units in this station. Because of lack of storage in Spokane the uniform flow released from the lake passes through unchanged to the forebay of the Long Lake station. This station and the one immediately below it take care of the daily variation in the load carried by the system.

Among the early installations at the Monroe Street station for lighting and power was the so called Edison 3-wire d-c. system to supply the principal business district, and 600-volt., d-c. current for street railway service, the generators being driven by direct-connected water wheels. These have been added to as the demand grew and since 1904 about 4300 kw. of direct-current energy has been produced at this station by water-driven machinery.

A dependable supply of direct-current energy for the Edison system and street railways having thus been in part assured by its production near the center of load, there still remained the problem of a dependable source of a-c. energy for power outside the economic limits of the Edison system and for residential lighting.

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

Up to the time of the complete Upper Falls development this a-c. energy was mainly supplied from outside plants by transmission lines which for the most part center in Post Street substation where there are also located the step-down transformers, additional motor-generators, lighting and power feeders, and feeders of the series street lights. To assure for these classes of services a dependable supply of a-c. energy its generation close to the point of feed was needed.

When in 1920 it became apparent that an additional supply of power would be required in the near future it was agreed at once that the next development should be made in Spokane where all necessary water rights had been acquired many years previously and where is also located the principal loads on the company's system. Some physical features of the situation may be gathered from the map of Fig. 1, on which the location of Monroe Street station, Post Street substation and the company's office building are also shown. A profile, taken along the main and middle channel of the river, giving some idea of the elevations that exist here, is shown in Fig. 2.

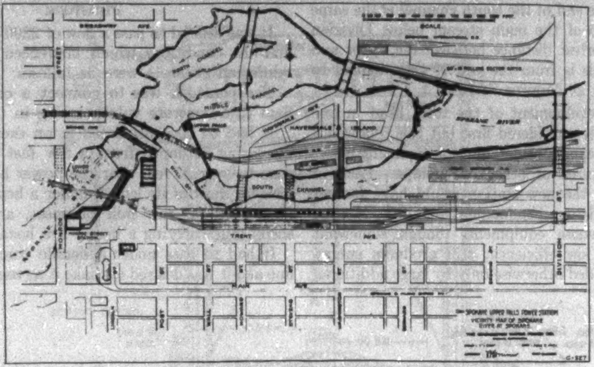
BROADWAY AVE
SCALE
STREET
NORTH CHANNEL
MIDDLE CHANNEL
UPPER FALLS STATION
HAVERMALE AVE
HAVERMALE ISLAND
SPOKANE RIVER
LOWER FALLS
SOUTH CHANNEL
TRENT AVE
MAIN AVE
MONROE
SPOKANE
POST
MILL
HOWARD
STEVENS
WASHINGTON
DIVISION ST.
SPOKANE UPPER FALLS POWER STATION
VICINITY MAP OF SPOKANE RIVER AT SPOKANE
G-327

FIG. 1

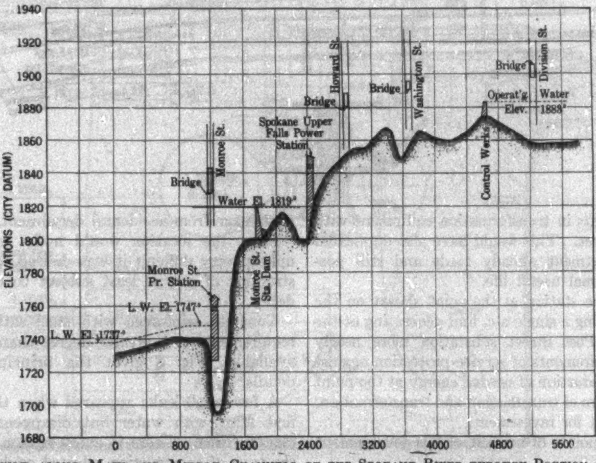

FIG. 2—PROFILE ALONG MAIN AND MIDDLE CHANNELS OF THE SPOKANE RIVER THROUGH PORTION OF SPOKANE.
SCALE: HORIZONTAL, 1 IN. = 400 FT. VERTICAL, 1 IN. = 20 FT.

Originally the south channel rejoined the main stream at the junction of the middle and north channels but this was dammed in the early history of the city and the water utilized to furnish the drive for a sawmill. This water-driven equipment remained in operation up to the time of the Upper Falls development. The water level in the end of the south channel is the same as at the branch of the main stream below Division Street. The shortest distance between the upper and lower water levels is from the end of this channel to the main stream at Monroe Street.

Making the development of the total head in one step was naturally considered first but this required a rather large tunnel of considerable length passing under important structures of other ownerships. Lack of room needed for the new station would have further required immediate abandonment of the Monroe Street station whose equipment, though somewhat obsolete and of low efficiency, still produces energy that can be provided otherwise only by large additional capital investments in transformation equipment with its attendant losses. This would have also duplicated in part an investment already made and still possessed of additional useful life.

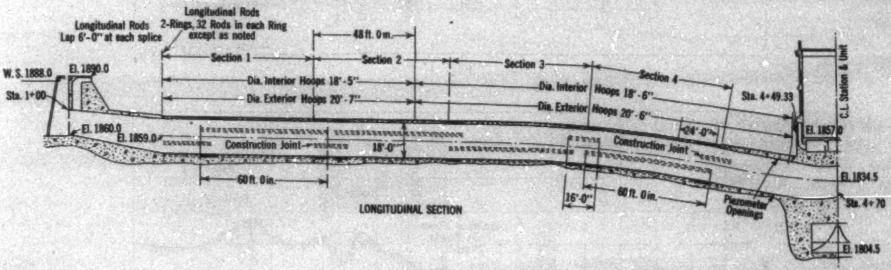
Longitudinal Rods
Lap 6'-0" at each splice
Longitudinal Rods
2-Rings, 32 Rods in each Ring
except as noted
48 ft. 0 in.
Section 1
Section 2
Section 3
Section 4
W. S. 1888.0
El. 1890.0
Sta. 1+00
Dia. Interior Hoops 18'-5"
Dia. Exterior Hoops 20'-7"
Dia. Interior Hoops 18'-6"
Dia. Exterior Hoops 20'-6"
Sta. 4+49.33
C.L. Station & Unit
El. 1860.0
El. 1859.0
Construction Joint
18'-0"
24'-0"
El. 1857.0
Construction Joint
60 ft. 0 in.
El. 1834.5
16'-0"
60 ft. 0 in.
Piezometer Openings
Sta. 4+70
LONGITUDINAL SECTION
El. 1804.5

Fig. 3

Location of the station at the point shown on the map and containing a single a-c. unit generating at the bus voltage of Post Street substation more nearly satisfied the requirements of service protection against interruptions, production of needed energy at the point of feed, a minimum of transmission and transformation, at the lowest cost for investment.

The principal points of interest of this development are as follows:

1. A reinforced concrete penstock 18 ft. in diameter and 370 ft. long.

2. A motor-controlled Johnson valve electrically interconnected with water wheel to open automatically and by-pass a large part of the river flow should the wheel load fall below a predetermined amount.

3. A vertical type water wheel having a concrete volute casing and Moody spreading draft tube.

4. Supply of excitation current and complete control of unit from a distribution station located 350 feet away.

5. Control works designed to pass a flood of 60,000 cu. ft. per sec. and partly under control of the system operator.

PENSTOCK

It was desired to pass a flow of 2200 cu. ft. per second and the conditions which influenced the selection of reinforced concrete were as follows:

The penstock was to connect a concrete headgate dam to a concrete volute casing in the power house.

It had to be constructed in an excavated rock cut, increasing in depth from a few feet at the headgate dam to over 20 feet near the power house. It had to pass for most of its length under a heavy wooden platform used for saw mill purposes, and under lumber sheds thus carrying a heavy fire hazard.

It had to pass under a double track steam railway line and it was desired to make it as resistant as possible to damage from accidental derailment.

Since the location would make maintenance and upkeep very difficult it was desired to make the construction of a type least subject to depreciation and decay.

Concrete reinforced with steel satisfied the above requirements to a greater degree than any other type available. Fig. 3 gives the principal construction details.

A few small leaks appeared when the penstock was first filled with water but disappeared after a few months time. There is every reason to believe that the type of construction selected could not have been better.

JOHNSON VALVE

It has been mentioned above that for a large portion of each year this station would utilize the full regulated flow of the stream. The tailrace of this station is the forebay of the older Monroe Street station, with pondage that would be exhausted in a few minutes time.

In case of a sudden shut down of the Upper Falls single-unit Monroe Street station would cease operating for want of water before the gates at the control works could be opened and the released water reach the forebay through the main river channel.

To prevent this contingency a 9-ft. diameter Y branch was led off from the main penstock within the power house foundations to a 62-in. Johnson discharge regulator located so as to discharge into the tailrace of the station. Under the existing head it is rated to pass 1000 cu. ft. per second and this will keep Monroe St. station operating until additional water released from the control works arrives in sufficient volume.

Location of the valve, its shape and the supply line branching from the main penstock may be seen in the cross-section of Fig. 4, where this valve is shown open.

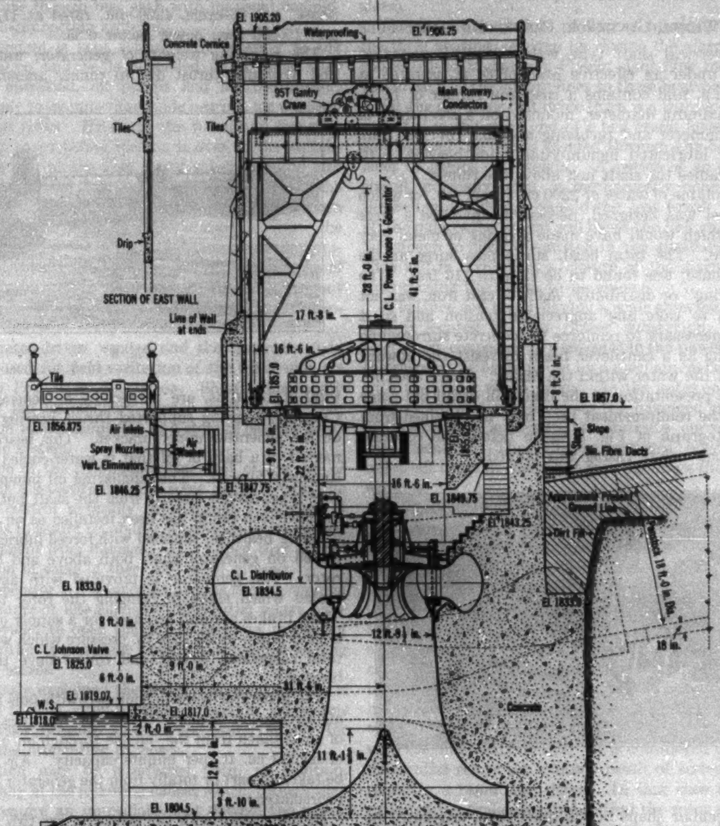
El. 1905.20
Waterproofing
El. 1906.25
Concrete Cornice
95T Gantry Crane
Main Runway Conductors
Tiles
Tiles
Drip
SECTION OF EAST WALL
C.L. Power House & Generator
28 ft.-0 in.
41 ft.-6 in.
Line of Wall at ends
17 ft.-8 in.
16 ft.-6 in.
El. 1857.0
8 ft.-0 in.
El. 1857.0
Tile
El. 1856.875
Air Inlets
Air Washer
Spray Nozzles
Vert. Eliminators
9 ft.-3 in.
22 ft.-6 in.
El. 1865.62
Slope
Steps
3in. Fibre Ducts
El. 1846.25
El. 1847.75
16 ft.-6 in.
El. 1849.75
Approximate Present Ground Line
El. 1843.25
Dirt Fill
Penstock 18 ft.-0 in. Dia.
C.L. Distributor
El. 1834.5
El. 1833.0
El. 1833.0
8 ft.-0 in.
12 ft.-9½ in.
18 in.
C.L. Johnson Valve
El. 1825.0
9 ft.-0 in.
6 ft.-0 in.
31 ft.-6 in.
El. 1819.07
W.S.
El. 1818.0
El. 1817.0
2 ft.-0 in.
Concrete
11 ft.-1½ in.
12 ft.-6 in.
3 ft.-10 in.
El. 1804.5
El. 1802.0

Fig. 4

Operation of the Johnson valve is from a motor-driven Dean Control unit, the valve starting to open when the servo-motor piston has closed to within 0.25 of its travel and the valve will start to close when this piston has reached 0.4 of its travel in opening. Closed or open positions of the valve are indicated by green and red lights respectively in the generator room of the power station and also in the System Operator's room in the company's office building. If it is desired to shut down the wheel without opening the valve this is accomplished by simply opening a switch in the motor circuit of the Dean Control unit. This arrangement functions splendidly and has been tried out a number of times within the past 9 months.

WATER WHEEL, GOVERNOR, CASING AND DRAFT TUBE

The vertical shaft type water wheel, designed to operate under an effective head of 64 ft. is rated at 14,250 h.p. and contains a cast iron runner of 11 ft. 7 in. maximum diameter, mounted on a steel shaft of 21 in. diameter and revolving at 105.8 rev. per min. A water lubricated lignum-vitae guide bearing, 6 ft. long, steadies the shaft just above the runner.

The volume of water of 2200 cu. ft. per sec. for which the wheel was designed, necessarily involved a large casing which would have made a casing of metal very expensive. The total head, static plus surge at the power house, was found to be close to 110 feet. The "speed ring" or distributor, made of cast iron, was not designed to take any appreciable tension and so it became necessary to reinforce the concrete surrounding the casing as a cantilever beam to resist the upward thrust of the water within the casing. Space does not permit a presentation of the design in detail but some idea of the reinforcement used may be obtained from the photograph of Fig. 5 taken before the concrete around the volute casing was poured.

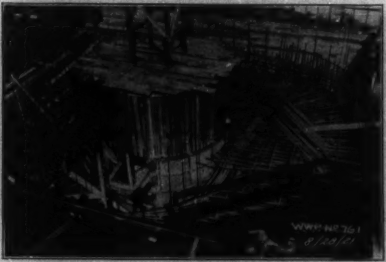

FIG. 5

The peculiar shape of the draft tube, originated by Lewis F. Moody, Consulting Engineer for The Wm. Cramp & Sons Ship & Engine Building Company, builders of the water wheel, may be gathered from the cross-section of Fig. 4. The outlet of the draft tube is through two rectangular openings 19 ft. wide and 12½ ft. high separated by a pier 6 ft. thick.

The governor is the latest design double-floating, lever-type, belt-driven. It is provided with the Taylor Control System which permits of the change from governor control to hand control, and the reverse, to be done by the oscillation through a small angle of a single lever which can be done in the fraction of a second.

GENERATOR, EXCITATION AND CONTROL

The generator, built by the General Electric Company is of the vertical shaft revolving field type, 3-phase, a-c., 60-cycle, 4200-volt, rated at 11,750 kv-a. and 10,000 kw., power factor 0.85.

The revolving weights of generator, water wheel, and hydraulic thrust due to runner amounting to a total of 188 tons, are carried by a General Electric Company's spring supported thrust bearing mounted on the generator upper bracket. This bearing is immersed in a bath of oil that is water-cooled and continuously filtered. The circulating oil pump is of the gear type, driven directly from the shaft but a similar motor-driven pump has been installed as an auxiliary.

FIG. 6

Guide bearings, supplied with forced lubrication, are placed on generator shaft both above and below the rotor. The generator is provided with air-operated brakes which assist in bringing the rotor to a stop. These brakes are also arranged for a supply of oil from a hand pump up to 1500 lb. pressure which will lift the total suspended weight and thus relieve the thrust bearing.

The generator rotor is equipped with fans for forced air circulation through the stator windings. Supply of air is taken from the outside through an air washer of 50,000 cu. ft. per minute capacity. Air may also be taken in part or totally from the generator room and recirculated.

In the station with the generator are two gear-type oil pumps one of which is driven by an a-c. motor and the other by a d-c. motor, supplied from the Edison System, thus assuring reliability of fluid pressure supply to the governors; also oil pressure and sump tanks for the servo-motors, and a motor-driven, two-stage air compressor connected to a receiver. Air is used in the oil accumulator tank and for the air brakes under the generator rotor.

Here are also located an indicating thermometer showing the temperature of the thrust bearing and a flow meter showing the quantity of water supplied to the lignum-vitae guide bearing.

The generator mains consist of three, 1,250,000 cir. mil. lead-covered, paper-insulated cables per phase laid in fiber duct embedded in concrete. As there are duplicate feeder busses in Post Street substation to which this unit is connected there are also two circuit breakers, one to each bus, each with its usual complement of disconnecting switches.

An attendant is kept continuously on duty at the power station and he is depended on to look after governor operation, oil pumps and air compressor, water supply to lignum-vitae guide bearing, air washers, and similar tasks. He starts the wheel and brings it up to part speed before turning it over to the switchboard operator at the Post Street substation and also has control of the Johnson valve separate from the interlock with water wheel gates. A Klaxon horn actuated from a pressure gage is used to warn the attendant should the supply of water to the lignum-vitae guide bearing fail for any reason. By means of a controller located in the power station the attendant can also close the head gates of the penstock that supplies this unit.

The motor-driven exciter and the generator field rheostat used for field excitation of the main unit are located at Post Street substation. Should this supply of energy fail an auxiliary supply may be had from the Edison mains connected to a storage battery floating on the system.

Location of the main generating unit of the size of this one in a separate building some distance from the point of control naturally necessitated a number of safeguards to assure reliability of service and to protect the equipment from damage. It is hoped that a brief description of some of them will be of interest.

The generator is Y-connected and, as is now common on generators of like capacity, differential relays have been provided to open the circuit breakers in case of trouble within the machine itself or in the cables leading to feeder busses.

As already explained the power station attendant is relied on to look after governor operation, oil pumps for governor pressure system, and water supply for cooling and lubricating purposes.

Operation may be interferred with, in addition, through any of the following causes which, though they do not require an immediate shut downof the wheel, need to be brought at once to the attention of the switchboard operator at Post Street substation.

Heating of the thrust bearing or guide bearings; failure of supply of lubricating oil to generator guide or thrust bearings; the breaking or slipping off from pulleys of the governor belt.

Warning is given through the ringing of a bell and the lighting of a signal lamp at the Post Street substation switchboard.

Thermal relays, normally closed, are inserted in each of the three generator bearings. A small current is kept circulating through this relay circuit and keeps open, by means of a solenoid switch, an auxiliary circuit containing the bell and light. Should any of the bearings become over-heated this relay will open the circuit. This will de-energize the coil holding open the switch in the auxiliary circuit and thus give the alarm.

Warning of the failure of flow of lubricating oil to the three bearings is given by closing the alarm circuit connected to Richardson-Phenix sight flow indicators, one in each of the supply lines to these bearings.

On the governor belt drive there is a tightener pulley mounted in the small end of a bell crank lever. Pressure against the belt is exerted through the bell crank lever by means of a weight. A switch of an auxiliary circuit, normally open, is mechanically connected to the bell crank lever. Should the belt break or climb off the pulleys the weight will move the bell crank lever far enough to close the switch in the auxiliary circuit. This will in turn effect a closure of the alarm circuit, thus causing the alarm to be given.

A balance coil meter having a scale showing percentages is located in Post Street substation. This is electrically connected to a rheostat in the power station, the rheostat being actuated by a mechanical connection with the water wheel gates. As the gates open resistance is cut out which causes the balanced coil meter at Post Street substation to indicate corresponding gate openings. Thus the switchboard operator can tell at all times at just what gate openings the wheel is working.

It has been mentioned that air-actuated brakes are provided to assist in bringing the rotor to a stop. A rapid or slow application of the brakes may be made. There are two parallel air supply lines, each one controlled by a solenoid-operated valve, leading to these brakes. One of these lines contains a small orifice which has a throttling effect on the air supply. This will cause a slow application of the brakes and a relatively slow shut down which does not cause rapid wear of the brake shoes. This is normally used. The other supply line contains no restriction and its solenoid-operated valve is opened by a solenoid in a circuit interlocked with the differential relays. It is considered that if the differential relays act it is the result of serious trouble requiring a rapid shut down. In that case the application of the brakes is rapid so that the rotor is brought to rest very quickly. Provision is also made for applying the air to the brakes by means of a hand-controlled three-way valve.

CONTROL WORKS

In order to obtain the maximum possible head throughout the entire year it was necessary to install gates to maintain a practically constant forebay level and pass a flow varying from nothing to an amount in excess of the maximum measured since records have been kept. Since the forebay extends through a portion of the city it was desired to have means showing the water level at all times and for quickly passing a large volume of flow should the water wheel suddenly lose its load for any reason.

The arrangement adopted was the installation of two motor operated "Rolling Sector" type gates each 60 ft. wide and 16 ft. high, operated either from a control house located on one of the abutments or by the system operator in the office building. Also the provision of four 40-ft. by 11-ft. openings, each closed by five vertical lift gates supported by adjacent piers and four top-hinged girders. One of the rolling sector gates is normally held in reserve while control for a flow up to about 9000 second feet is exercised by the second rolling sector gate. Flows in excess of this are normally passed by clearing the 40-foot openings as needed. A plan of the Control Works is given on the map of Fig. 1 and in Fig. 6, taken when a moderate flood was passing, shows the works with one sector gate open and the other closed.

Each sector gate is operated by a 15-h.p. series d-c. motor mounted in the middle of a bridge spanning between gate piers and controlled either from a drum controller or from a contactor panel, both being contained within a house on one of the gate abutments. Connecting one gate motor through selector switches to drum controller connects at the same time the other gate motor to control by contactor panel. Both systems of control give dynamic braking in lowering and in the off position.

Motor rheostats have been adjusted so that the current on the first point of the rheostat is about three times the value of normal load current. This assures operation even if the contactors should fail to short out the succeeding resistance steps. In lowering there is only one resistance adjusted to suit over-haul of gate.

The contactor panel may be operated from either of two parallel double-throw knife switches one of which is mounted on the contactor panel itself and the other one is in the system operator's room in the office building located some 3000 feet from the gates.

The system operator has mounted above the remote control switch a gate position indicator which shows at once any travel of the gate. Should the indicator fail to show a change in gate position a few seconds after the control switch is closed it is evidence that motor has failed to start and the switch can be opened to prevent burning out of motor and shortly another trial made. Should the supply of power fail for any reason a double acting breaker opens the control circuit and closes the motor circuit for dynamic breaking. This breaker may be reset by a switch in the system operator's room as soon as power is restored and the gate again operated in the direction desired.

There are two power supply circuits for the motors partly under ground and partly overhead, one directly from Post Street substation and one from the Edison mains serving the district adjacent to the Control Works. This arrangement has been found satisfactory in practise and has made it possible to dispense with continuous attendance at the Control Works.

The Upper Falls development was designed and constructed under the general direction of V. H. Greisser, Chief Engineer, assisted by H. L. Melvin, in charge of electrical features and the writer in charge of mechanical and structural work.

High-Voltage Switches, Bushings, Lightning Arresters

Experiences of the Southern California Edison Company on its 60,000, 150,000 and 220,000-Volt System

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Review of the Subject.—*This paper describes the experiences of the Southern California Edison Company with oil switches on its 60-kv., 150-kv. and 220-kv. systems, with air break switches on its 220-kv. system, with bushings on its 60-kv., 150-kv. and 220-kv. systems, and with lightning arresters on all voltages from 11 kv. up.*

In some instances the experiences do not lead to unquestionable conclusions. This is particularly true in the case of lightning arresters. More nearly complete information in regard to actual occurrences will aid in drawing definite conclusions.

* * * * *

SWITCHES

For many years the Southern California Edison Co. has been equipping its 60-kv. system very largely with western made oil switches. As the system has grown it has been necessary to increase the capacity of these switches. The first switches were made with three rectangular tanks, each tank having two horizontal breaks in series. The next step consisted of increasing the size of each rectangular tank and putting four breaks in series. And now we are installing switches, each consisting of three cylindrical tanks with round top and bottom and each tank having six or ten horizontal breaks in series. The rupturing capacity of these switches has never been known, but they have handled the job with a fair degree of satisfaction. Of course, the changes in design have been forced by increases in system capacity, which have proved the type in use to be inadequate. And it is not known how long the present type will be adequate.

In addition to the switches of local manufacture there are a considerable number of 60-kv. switches on the system which were furnished by eastern manufacturers. These switches, in general, have given good results, but it seems impossible to draw any reasonable conclusions in regard to whether the performance of the locally made switch is better or worse than the eastern made switches.

It would be very desirable to make comparative tests of oil switches under conditions similar to those under which the switches operate, but these tests would be very difficult to arrange.

It seems quite certain, however, that the capacity of the 60-kv. system can not be indefinitely increased. Although it may be physically possible to make oil switches which would not blow up when opening automatically under severe short circuits, their cost would be greater than the earnings would justify. This situation will probably be avoided by dividing the system into two or more partial systems and by the judicious use of the current-limiting reactors.

The 150-kv. oil switches on the Big Creek system have given good satisfaction as non-automatic switches. The method of operation has been such that automatic opening has not been required. The greater number of these switches have been in operation since 1913. They were furnished by two large eastern manufacturers.

The 220-kv. oil switches which were put into operation at 220-kv. on May 6th, 1923, have not been allowed to operate automatically up to the time of writing (June 20th, 1923). However, it is intended that they shall operate automatically to drop out a section of line which may be in trouble.

FIG. 1

These switches were furnished by two large manufacturers. Aside from minor defects due to careless workmanship, or careless details of design, these switches have given satisfaction during the brief period they have been in operation.

There are three types of air brake switches in use on the 220-kv. system, namely, disconnecting switches for the oil switches; line-sectionalizing switches, and line-paralleling switches, the two latter types being used at line crossover stations other than those equipped with oil switches.

The disconnecting switches on either side of the 220-kv. oil switches are mounted on the tops of insulator tripods which in turn are carried on the top of steel frame-work. (See Fig. 1). Each leg of the

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

tripod consists of fourteen insulator units and has a total length of seven feet. The insulator unit is made up by cementing pillar type hardware to the porcelain of a standard 10-in. suspension insulator and the units are bolted rigidly together.

The switch blade is 10 ft. 6 in. long from the hinge to the jaw end and is composed of two pieces of 1¼-in. copper pipe joining at the jaw end and spread apart at the hinge end. The blade also extends back of the hinge to form a lever to which the operating rod is attached. The jaws are solid without any spring. The contact part of the blade is formed of two paralled copper straps which have considerable spring in them. These copper straps are drawn together or forced apart by a cam mechanism which in turn is operated by a rod which extends above the blade of the switch, past the hinge end to the operating rod. The operating rod consists of a fourteen-unit pillar type insulator and a rod extending to the ground, where a bell crank and operating handle complete the mechanism.

In opening the switch the first few inches of movement of the operating handle pulls the blade straps together enough to free the blade in the jaws. Further movement of the operating handle swings the blade upward. In closing, the blade reaches its closed position and then the last few inches of travel of the operating handle forces the blade straps out against the jaws. The blade is counterweighted to make the operation easy. The clear opening of the switch is approximately 8 ft. 1½ in.

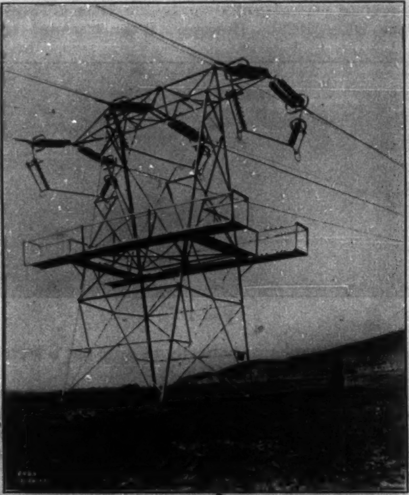

Fig. 2

The line-sectionalizing switches take the place of the jumper loops in a dead-end tower and are in reality two switches in series, each switch blade when closed short-circuiting a string of eight pillar type insulators, the string length being about 4-ft. (See Fig. 2). This gives two 4-ft. openings between a live section and a dead section of line.

These switches are not to be operated except when the line is dead. The operating is done by means of a switch pole from a platform built into the tower.

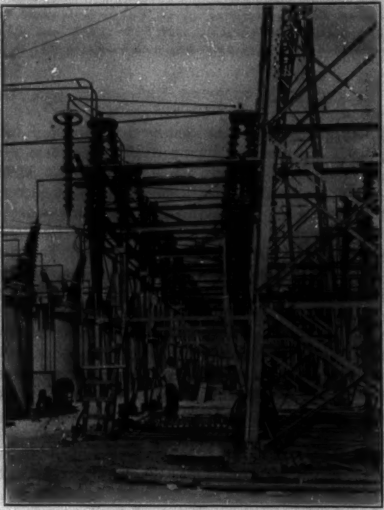

Fig. 3

The paralleling switch, Fig. 3, is placed between the lines with a set of sectionalizing switches in each line on each side of it, thus making four sets of sectionalizing switches and one set of paralleling switches for each crossover station.

Each single-pole paralleling switch consists of a pipe switch blade which bridges across a horizontal diamond of four strings of strain insulators (standard suspension insulators). The grounded corners of this diamond are supported on steel poles. The other corners are carried by the conductor of the crossover connection. The switch is operated through a 6-ft. column of twelve pillar-type insulators and a rod reaching to a bell crank and operating handle located on the ground. The switch blade opens upward and will break the current passing across it when the lines are still in parallel at another point, provided that the wind is not blowing too hard. In one instance when the wind was blowing hard the arc went to ground on the supporting structure, doing no more physical damage than the breaking of the porcelain on one insulator unit and the burning of one shield ring.

BUSHINGS

The question of bushings for the 60-kv. oil switches of local manufacture is rather simple, for the reason that the bushing does not support any part of the switch mechanism. Two of the insulator manufacturers make bushings which will fit these switches and they are giving very good results. The other 60-kv. oil switches as well as the 150-kv. and 220-kv. oil switches must have bushings made particularly for those switches, since the stationary contacts are carried by the bushings.

On the 150-kv. and 220-kv. apparatus there are the oil-filled bushings, or compound-filled on the older equipment, and the condenser type bushings. There have been a few cases of breakdown on both types. The compound-filled bushing is in general disrepute and the transformers which originally had compound-filled bushings are now equipped with oil-filled bushings. The 150-kv. oil switches, however, still have their original compound-filled bushings.

There continues to be trouble in preventing the oil from leaking out of the oil-filled bushings in spite of assurances from the factory that the problem has been satisfactorily solved. This is not serious but somewhat annoying. The same trouble is encountered in keeping all joints tight in the conservator type transformers.

There seems to have been some real progress made in the method of packing large bushings for shipment. Two or three years ago it seemed almost impossible to get a shipment of 150-kv. or 220-kv. bushings from the eastern factories without a very high percentage of breakage. For the 220-kv. equipment delivered this year, approximately 260 bushings for 220-kv. and 150-kv. were received, all in perfect condition. It is to be hoped that this is really an advance in the art of packing and not merely a piece of good luck. One manufacturer crated these bushings in a vertical position with four or six in a crate. The bushings were well braced and completely housed. The other manufacturer crated each bushing separately with the bushing riding in a horizontal position, but it was rigidly supported at five points along its length.

LIGHTNING ARRESTERS

It has been the general policy of this company to install lightning arresters at all its stations on all voltages from 11-kv. to 150-kv., inclusive. Electrolytic arresters have been used almost exclusively until the last year or two. These arresters have been installed in various ways, sometimes on each line connecting to the station and sometimes on the station busses only; sometimes with reasonably straight connections and sometimes with very tortuous connections between the lines and the arresters, sometimes inside, sometimes on the roof, and sometimes on the ground outside of the building. The experience with all this variety of installations has shown that certain things should not be done.

1. Electrolytic arresters should not be installed inside of a building because of the fire hazard. Several bad fires have been caused by burning oil being thrown from arrester tanks at times of heavy discharges.

2. That electrolytic arresters should not be installed on a roof of a building because of the fire hazard and also because of the deteriorating effect of oil on a composition roof.

It is generally believed that the connections from the line to the lightning arresters should be as straight as possible. However, there are a very few cases reported in which the lightning did not follow the conductors provided for it, but instead, jumped to ground from the end of the bus, or from some sharp angle in the conductors.

As to the actual value of lightning arresters for the protection of station equipment from excess voltage due to lightning or to surges in the lines, it is hard to judge for two principal reasons; first, there is no adequate device available to record the discharges which pass through an arrester and, second, there are so few cases of lightning.

From the observations of station operators it is known that discharges occur when there is no lightning storm; but it is not known how many such discharges occur, nor whether any damage would be done if the arresters were not there to let the discharge pass to ground. Nor in the case of a lightning storm is it possible for an operator to observe and record more than a small per cent of the discharges that probably occur. There is a great need for a discharge recorder, which will give a reliable record of the time, and if possible the magnitude, of each discharge through an arrester and which will be sufficiently simple and inexpensive so that it can be installed on a large number of arresters. Such records, taken over a few years, would go a long way toward settling the controversy as to whether lightning arresters justify their expense.

It is quite generally conceded throughout the company that lightning arresters are a really worth while protection on circuits of 15-kv. and lower, particularly in cases of higher voltage lines coming in contact with lower voltage lines. There have been several such instances in which the arresters performed their functions properly and were not injured in doing so. The arresters are sometimes damaged by the higher voltage power currents as they are occasionally by the power current which follows a discharge due to lightning.

On the 60-kv. system the case in favor of lightning arresters is not quite so clear as it is on the lower voltages. The stations having transformers are almost universally equipped with them. Here again the inadequacy of records makes it almost impossible to say whether arresters are or are not worth the money invested in them. The fact that they are installed shows where the predominant opinion lies.

The 150-kv. system was completely equipped with electrolytic arresters until about a year ago, with the exception of a considerable period some years ago when the arresters at Big Creek No. 2 were out of commission. Since about a year ago the arresters have been taken off the lines as the 220-kv. construction work made that necessary, and the fire at Eagle Rock eliminated those at that station. The fire brought forth the decision to permanently abandon the use of lightning arresters on the 150-kv. stations at Big Creek No. 1, No. 2 and at Eagle Rock, where the arrester tanks were inside the buildings. It had previously been the intention to leave these arresters in service, although the lines had been changed to 220-kv. It has not definitely been decided whether or not the outdoor arresters at Vestal will be re-installed.

On the 150-kv. lines there have been only two or three cases of trouble due to lightning, one of these was at Eagle Rock after the 150-kv. arresters had been removed. A 150-kv. insulator inside the building flashed over during a storm which was causing trouble on the 60-kv. and 15-kv. lines out of the same station. Another case of trouble attributed to lightning occurred on the 150-kv. bus connection inside the building at Big Creek No. 1, when the arresters were connected to the lines and apparently in perfect working conditions. At Big Creek No. 2, it was reported that the arresters discharged heavily at a time when lightning struck so near that the operators thought the building had been struck.

The arresters on the 150-kv. lines always discharged when 150-kv. line switching was being done. The statement is often made that arresters are not needed on 150-kv. and 220-kv. lines as a protection against lightning, but what will these voltage surges, which cause the lightning arresters to discharges during switching, do if the arresters are not there? The answer is, even though these voltages are high enough to jump the arrester gaps, they are not high enough to damage the station equipment. The fact that Big Creek No. 2 operated for many months without arresters indicates this. And some recent studies made with a surge recorder on the 150-kv. lines indicate that switching of a 100-mile section of line does not produce surges of more than 200 per cent normal voltage. These studies were made at Eagle Rock while the 150-kv. arresters were in service, however.

At Vestal, near the middle of the 150-kv. line, there were two cases of 150-kv. transformer insulation breakdown. The transformers were old and the insulation was admittedly of a quality inferior to that used in modern transformers. Yet the extreme thickness of insulation punctured made it difficult to believe that inferior insulation could be the cause of the failure. Perhaps high voltage did it, but the 150-kv. lightning arresters were in service, having very advantageous connections to the 150-kv. lines.

On the 220-kv. lines there are no lightning arresters and there probably never will be.

In regard to the types of arresters used by this company, the electrolytic arresters, which, until the last two or three years, have been considered the only arresters at all adequate for station service, have several faults, one of which is that they require expert maintenance, which often they do not get. Another fault is the fire hazard.

The oxide film arrester has become popular and this company is installing many of them on voltages of 60-kv. and lower. Thus far they have given no trouble and it is assumed that they will give at least as good protection as the electrolytic arresters. There is apparently no fire danger from them and this is greatly to their advantage. The maintenance promises to be very low. There has been one case of a discharge throwing some of the active material out of a stack of the disks. Examination showed that there had been many discharges through this stack. This discharge occurred at the time of a heavy short circuit on the system and the arrester was located in the heart of the system.

There is only one installation of the autovalve arresters of the distribution type on the system. This is on a 15-kv. line on Mt. Wilson at an elevation of 5000-ft., and though it is not definitely known to have functioned, at least one lightning storm has occurred on that part of the system since its installation.

The graded resistance type of lightning arresters have been installed on 11-kv. lines at several stations during the last two years. These have given no trouble to date and have functioned satisfactorily in several instances.

There is one installation of the Water Column Surge Arrester on the system. It functions properly when an arc is formed intentionally across the horn gap, but that is all that is known of it as yet.

Discussion

For discussion of this paper see page 1025.

High-Voltage Circuit Breakers

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***Review of the Subject.**—The 150-kv. system operated for some years by the Southern California Edison Company has not made use of automatic operation of circuit breakers. The multiplication of generating stations and connections which accompanied the change to 220-kv. makes desirable the automatic sectionalizing of the line in cases of trouble and, therefore, requires circuit breakers for automatic operation similar to that obtained on more moderate voltage systems. Circuit breakers have been manufactured for this service with high rupturing capacity and the designs have followed closely the standard designs for more moderate voltages.*

IT has been standard practise for some time to take care of fault conditions on moderately high-voltage lines of about 110,000 volts by the automatic operation of the high-voltage circuit breakers.

On the 150,000-volt circuits which have been operated for several years by the Southern California Edison Company high-voltage circuit breakers have been in service but have not been tripped automatically on the occurrence of trouble but, instead, there has been utilized a scheme of clearing high-voltage trouble by means of lowering the line potential.

In the case of flashovers this scheme has been efficacious. The operation has been entirely manual, the power house operator cutting in field rheostat by hand upon obtaining indication of ground current as shown by an ammeter connected in the neutral of the step-up transformers at the power house. With two power houses in service it has been necessary for the operators at the two plants to cut in field rheostat at the same time and approximately at the same rate. This has been found to be quite feasible.

With the growth of the system, involving increases in the number of power plants and generating units, and with the growing importance of absolute continuity of service, it has been considered desirable to carry out a scheme of breaker operation similar to that used on the moderate voltage systems.

The 220,000-volt double-circuit line, which has this year been put in service by the Southern California Edison Company, has, therefore, been provided with automatic high-voltage circuit breakers and a relay system to give proper sequence of breaker operation on the occurrence of trouble. The lines from the Big Creek Power Houses to Los Angeles are sectionalized at two intermediate points by means of 220,000-volt circuit breakers and upon the occurrence of a flashover or other trouble on any section the particular section in trouble is separated from the rest of the system by the operation of the sectionalizing circuit breakers.

The Pacific Gas & Electric Company, in laying plans for their 220,000-volt lines, running from Pit River to the San Francisco district, have likewise provided high-voltage circuit breakers and relays so as to give automatic operation in case of line trouble.

The desirability of this automatic breaker operation is self-evident, as it insures a maximum speed in the clearing of troubles and a minimum loss of service. With two parallel circuits there should be no interruption of power at the receiving end, under any circumstance, unless both lines are simultaneously in trouble. Operation of the more moderate voltage lines, in various parts of the country, has demonstrated that dependable relay opeartion for the automatic opening of circuit breakers is obtainable.

The problem confronting the engineers, in carrying out this method of operation, is therefore largely the design and manufacture of circuit breakers which would successfully handle the rupturing capacities required on these high-voltage lines. Such design and manufacture has been carried out and the circuit breakers for this service are now installed. In carrying out the design, it is noteworthy that no radical departure was made from the construction used in breakers for more moderate voltage service.

Circuit breaker tanks have been made larger and heavier in order to give the greater clearances required by the high voltages and to care for the heavy rupturing capacities called for. The actual contact parts for the 220,000-volt circuit breakers are practically identical with those of the larger moderately high voltage breakers. Solenoid operating mechanisms energized by direct current are employed in both types of breakers.

Bushings, for carrying the high voltage into the breaker tanks, are a continuation of the standard lines of design, both condenser type and the oil-filled type of bushing now being in service at 220,000 volts. All of the bushings have been made of the outdoor type with porcelain shields provided to protect against the weather. Bushing-type current transformers provide means for current measurement and relay protection. The bushings are made interchangeable with transformer bushings for the same voltage service and the insulation tests on breakers have been brought into line with the reduced tests on transformers, arrived at by consideration of the effect of the solidly and permanently grounded neutral of the system.

It has been necessary to determine the rupturing capacities of these high-voltage circuit breakers by calculation from certain determining data such as the strength of tanks, the amount of oil, the depth of the

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

contacts under the oil and the speed of moving parts or arc extinction and up to the present it has not been possible to make actual tests of these rupturing capacities.

Such tests, however, have been made on the similarly designed breakers used on 110,000-volt circuits and these tests have contributed empirical data which gave assurance that the 200,000-volt circuit breakers will take care of the rupturing capacities specified. These rupturing capacities are considerably in excess of any current which they may be expected to be called upon to open until the systems on which they have been applied have grown still larger and more generating capacity has been installed.

Discussion

For discussion of this paper see page 1025.

Magneto-Mechanical Loads on Bus Supports

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Review of the Subject.—*Current progress in electrical development and the increasing capacity of power systems has brought into prominence many factors of mechanical strength that were previously negligible in the design of electric stations and circuits. Heavy currents, that may develop during short circuits in stations that are connected to large generating systems, make it essential that bus and cable supports be strong mechanically as well as adequately insulated. Formulas are given for estimating the forces that may be developed under various conditions with different arrangements of conductors and supports.*

The mechanical strength of bus and cable supports should be guaranteed by manufacturers and tested as well as the insulating qualities. In order to facilitate the selection of bus and cable supports, it is desirable that uniform terms for expressing the various kinds of loads, standard methods of conducting tests and a minimum factor of safety for mechanical strength be adopted.

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THE successive improvements in the insulation and control of high-voltage circuits are the means of ever increasing the scope and capacity of electric power systems. This progress demands the solution of innumerable problems and naturally most of the factors entering these problems are of an electrical nature. However, one outstanding feature of the electrical development of the past decade or so, has been the increasing prominence of mechanical factors that were previously negligible in electrical station and circuit design.

Special attention is now given to the matter of anchoring and bracing generator and transformer windings. Safety catches are now a necessity on most disconnecting switches. And the oil circuit breaker is finally rated on the basis of its capacity to perform its originally intended function, namely to interrupt current. Numerous other cases might be cited but the above indicate the growing importance of mechanical features in the design of electrical apparatus and circuits.

In small stations, and especially in those that are blessed with double busses, the failure of a bus support may not involve a serious interruption of service nor a considerable expense for repairs. But in large stations, where every interruption of service is serious and the breakdown of a bus is practically certain to entail costly repairs, it is doubly essential that busses be strong mechanically as well as adequately insulated.

As a rule bus supports must be spaced at such frequent intervals in order to prevent sagging of the conductors and the usual types appear so rugged compared to the weight of the conductor supported that, to a casual observer, there seems little reason to question their adequacy as mechanical structures. However, a brief computation of the magneto-mechanical forces, that may develop between parallel conductors in medium sized and large stations especially during short circuits, shows that these forces are far from negligible.

This has been recognized for some time and formulas and curves for estimating the forces involved have been published[1] so it does not seem necessary to present a complete discussion of the fundamental principles at this time, but merely to give some convenient formulas for the more common cases. The present purpose is rather to emphasize the fact that it is becoming more and more important to ascertain the mechanical strength of bus and cable supports for inside station wiring and to point out that there is room for standardization in this respect.

The magneto-mechanical phenomenon produced by

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[1] C. P. Steinmetz, TRANS. A. I. E. E., 1911, p. 367.
H. B. Dwight, *Electrical World*, Sept. 1917, p. 522.
S. G. Leonard and and C. R. Riker, *Electric Journal*, Dec. 1917, p. 491.

electric currents in parallel conductors is fundamentally the same as the basic reaction in an electric motor. It can be thought of as the attraction, or repulsion as the case may be, between the current in one conductor and the magnetic field set up by the current in the other conductor or, perhaps more conveniently, as the force between the magnetic fields set up by the two currents.

In the design of motors and similar machines involving alternating currents the chief interest lies in the average force developed. But, in dealing with the mechanical requirements of bus supports, we are interested primarily in the maximum instantaneous force that may be developed.

The instantaneous force, corresponding to instantaneous currents, i_1 and i_2 in parallel conductors, is

$$S = \frac{5.40\, i_1\, i_2}{10^7 D} \text{ lb. per ft. of parallel}$$

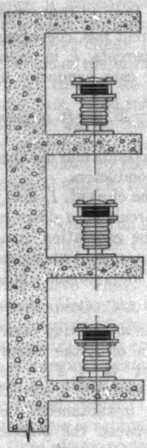

FIG. 1—TYPICAL CROSS-SECTION OF THREE-PHASE BUS

where D is the interaxial distance in inches between the conductors. This is the general formula for the determination of the magneto-mechanical force exerted upon a conductor of electric current due to the proximity of another parallel current. In parallel conductors, currents flowing in the same direction attract each other, tending to draw the conductors together, and currents flowing in opposite directions repel each other, tending to force the conductors apart.

The derivation of the formulas that apply for certain arrangements and various conditions on direct-current and alternating-current busses are given in the Appendix in sufficient detail so that formulas may be derived readily for special arrangements of conductors or groups of busses and for other than single-phase and three-phase circuits.

It is seen from the formulas in the Appendix that, for one three-phase bus having its conductors arranged in a plane, the maximum stress on a conductor will be developed when two adjacent conductors are short-circuited. Then

$$S = \frac{2.62\, I_s^2}{10^6 D} \text{ lb. per ft. of parallel (repulsion).}$$

For example, in a station where the symmetrical three-phase, short-circuit current for a short circuit on the main bus is 30,000 amperes, and the conductors are arranged in a plane 18 inches apart on centers, the force of repulsion between two adjacent conductors when they are short-circuited may become

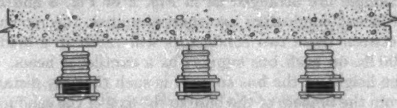

FIG. 2—TYPICAL CROSS-SECTION OF THREE-PHASE BUS

$$S = \frac{2.62\,(30{,}000)^2}{18 \times 10^6}$$

$$= \frac{2.62 \times 900}{18}$$

$$= 131 \text{ lb. per ft. of parallel.}$$

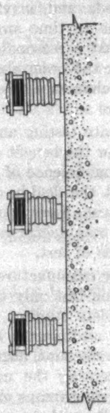

FIG. 3—TYPICAL CROSS-SECTION OF THREE-PHASE BUS

If the bus supports along each conductor are spaced 27.5 inches apart, the corresponding force on each support will be

$$F = \frac{27.5 \times 131}{12}$$

$$= 300 \text{ lb.}$$

If the conductors of the three-phase bus in the above example are arranged as in Fig. 1 and two adjacent conductors are short-circuited, the repulsion between these two conductors will result in a tensile force of 300 lb. on each bus support of the upper one, and a compressive force of 300 lb. on each bus support of the lower one; and the bus supports should have ultimate tensile and compressive strengths of not less than 1200 lb. if the factor of safety is 4.

On the other hand, if the conductors in the above example are arranged as in Fig. 2 or Fig. 3 and two adjacent conductors are short-circuited, the repulsion between these two conductors will result in a load of 300 lb. on each bus support as a cantilever beam. If the height of the bus support is such that the distance from the bottom of the base to the axis of the conductor is, say, 7.5 inches, the cantilever load will be

$$7.5 \times 300 = 22{,}500 \text{ pound-inches.}$$

And, if the factor of safety is 4, the ultimate strength of the bus support as a cantilever beam should be not less than 90,000 pound-inches.

After determining the magnitudes and character of the loads involved and the ultimate strengths required, the next step is to select bus supports of adequate mechanical strength. In this, reliance must be placed upon the experience of the manufacturers with their various types and sizes of supports. But it is clear that mechanical strength, as well as insulating qualities, should be guaranteed; and, in view of the inevitable variations from time to time among porcelains, it is only fair that at least a reasonable percentage of the supports covered by the order should be tested under specified types of mechanical loads up to the guaranteed ultimate strength.

In order to facilitate testing and to put the matter on a uniform basis for the benefit of the manufacturers as well as for the convenience of purchasers, it seems highly desirable that the field be canvassed and standards adopted because there now are considerable divergencies among manufacturers as to the meanings of terms and methods of test.

For instance, some manufacturers test the complete bus support and some test only the porcelains. It is clear that the complete support should be tested because metallic bases and clamps have been known to fail earlier than the porcelains, also because the method of clamping or cementing the metallic parts to the porcelain sometimes determines the breaking point of the porcelain especially under a cantilever load.

Tensile and compressive strength should mean strength in carrying loads applied along the line joining the centers of the base and bus clamp, and not refer to tension or compression along the axis of the bus.

At present, some manufacturers express the ultimate mechanical strength as a cantilever beam in pound-inches and others express it in pounds without giving the length of the lever arm or radius. It is clear that the former is the preferable way because the strength of the bus support should be independent of the dimensions of the bus except in so far as the latter affects the size of the clamp.

Whether the factor of safety should be 4, based on the ultimate strength of the support, is somewhat a matter of opinion and judgment. But, as in machine and structural design, it is desirable to have a standard minimum factor of safety. In view of the metallic parts used and the variations in the quality of porcelain, it does not seem wise to use a factor of safety much less than 4.

There are undoubtedly other phases of the subject that might be standardized but only a thorough canvass of the field can bring out all of them.

Appendix

D-C. CONDUCTORS

In a direct-current bus, consisting of two conductors carrying equal currents, *I*, in opposite directions, the stress exerted by each conductor upon the other is

$$S = \frac{5.40\, I^2}{10^7\, D} \text{ lb. per ft. of parallel (repulsion)}$$

where *D* is the perpendicular distance in inches between the axes of the conductors.

The value of the current, *I*, that will develop in the case of a short circuit on a direct-current bus, will be determined by the generating capacity connected to the system and the equivalent resistance of all the paths feeding into the short circuit combined.

A-C. CONDUCTORS

In the case of an alternating current, having a root-mean-square value of *I*, the maximum instantaneous value of current is $\sqrt{2}\, I$. Consequently, in a single-phase bus, carrying a normal root-mean-square current, *I*, the instantaneous magneto-mechanical stress exerted between the busses pulsates between zero and the maximum value,

$$S = \frac{5.40\,(\sqrt{2}\, I)\,(\sqrt{2}\, I)}{10^7\, D}$$

$$= \frac{1.08\, I^2}{10^6\, D} \text{ lb. per ft. of parallel (repulsion).}$$

SHORT-CIRCUIT REACTANCE

The current, that will develop at a short circuit, depends on the rated kv-a. of the synchronous equipment connected to the circuit and upon the impedance of the combined circuits feeding into it.

Obviously, it is tedious to determine the combined impedance of the numerous groups of series and multiple circuits that make up a generating and distribution system. However, it is usually true that the reactances involved are large compared to the respective resistances so that the impedance is only slightly greater than the reactance numerically. Consequently, to simplify the calculations it is usually sufficiently accurate to neglect the resistances and to use the reactances in place of the impedances.

This approximation is customary in dealing with short-circuit problems and the term "reactance" is used in this sense hereinafter. It gives values of short-circuit current slightly larger than actually obtain and consequently the results are on the safe side. Of course, in cases where the resistances are large compared to the respective reactances or economies may be effected that will warrant the extra labor of the more precise determination, the total impedances, including resistances, should be used.

A-C. SHORT-CIRCUIT CURRENT

The maximum instantaneous value of current, that will be produced when a synchronous generator is short-circuited, depends upon the instantaneous value of the voltage when the short circuit takes place.

If the short circuit occurs at the instant of maximum voltage, the short-circuit current wave will be symmetrical about the time axis. And the effective value of the short-circuit current will be that corresponding to the rated kv-a. of synchronous equipment, that is connected to the circuit, divided by the transient reactance expressed as a decimal (that is, the per cent reactance divided by 100).

But, if the short circuit occurs at an instant when the voltage is not at its maximum, the current wave will be symmetrical not about the time-axis, but about an exponentially decaying curve which finally becomes tangent to the time-axis. Under the worst condition, namely, short circuit occurring at the instant of zero voltage, the axis of symmetry of the current may be so high as to produce a maximum instantaneous value of current approximately 1.8 times that which arises in case of a symmetrical short circuit.

In the case of a three-phase synchronous generator, if a short circuit of all three phases occurs at the instant of zero voltage in phase *A*, the factor of asymmetry for phase *A* is approximately 1.8, and that for phases *B* and *C* is approximately 1.35.

Since we are most often concerned with the maximum currents that may arise during a short circuit, these factors 1.8 and 1.35 are usually included in the formulas pertaining to short circuits of alternating-current conductors.

SINGLE-PHASE CIRCUIT

The symmetrical short-circuit current of a single-phase circuit is

$$I_s = \frac{100\, I}{X}$$

where *I* is the total rated current of all the synchronous machines connected to the circuit reduced to terms of the circuit voltage; and *X* is the per cent reactance of the circuits (including equipment), that feed into the short circuit, based on the total rated generating capacity of the synchronous machines.

The maximum peak value of current, in case of a short circuit occurring at the instant of zero voltage, then is

$$1.8\sqrt{2}\, I_s.$$

And the stress on each of the two parallel conductors spaced *D* inches apart and carrying this current, is

$$S = \frac{5.40 \times 1.8\sqrt{2}\, I_s \times 1.8\sqrt{2}\, I_s \cos 180^\circ}{10^7\, D}$$

$$= \frac{-3.50\, I_s^2}{10^6\, D} \text{ lb. per ft. of parallel (repulsion).}$$

THREE-PHASE CIRCUIT

The symmetrical three-phase short-circuit current of a three-phase circuit is

$$I_3 = \frac{100\, I}{X}$$

where *I* is the total rated three-phase current of all the synchronous machines connected to the circuit reduced to terms of the circuit voltage; and *X* is the per cent reactance of the circuits (including equipment), that feed into the short circuit, based on the total rated kv-a. capacity of the synchronous machines.

(1) SHORT CIRCUIT OF TWO ADJACENT CONDUCTORS

The maximum peak value of current, in case of a short circuit occurring between two adjacent conductors of a three-phase circuit at the instant of zero voltage, is

$$1.8\sqrt{2}\left(\frac{\sqrt{3}}{2}\right) I_3.$$

And the stress on each of these two conductors (if parallel and spaced *D* inches apart) then is

$$S = \frac{5.40 \times 1.8\sqrt{2}(\sqrt{3}/2)\, I_3 \times 1.8\sqrt{2}(\sqrt{3}/2) I_3 \cos 180^\circ}{10^7\, D}$$

$$= \frac{-2.62\, I_3^2}{10^6\, D} \text{ lb. per ft. of parallel (repulsion).}$$

(2) SHORT CIRCUIT OF THREE CONDUCTORS

In calculating the currents that develop during the short circuit of all three conductors of a three-phase circuit, it is of course necessary to take into account the time-phase of the three currents. Assume the current in phase *A* is I_3, that in phase *B* is $I_3 \underline{/120^\circ}$ and that in phase *C* is $I_3 \underline{/240^\circ}$. The total magneto-mechanical force exerted on each conductor is a resultant of the forces exerted by each of the other conductors. It is necessary to solve first for the forces exerted between conductors by pairs and then obtain the resultants by combining these individual forces, taking into account the angles between their directions.

The forces depend on the distances between the conductors and consequently upon their relative positions. The two most common configurations of three-phase conductors are: (a) at the corners of an equilateral triangle, and (b) in a plane. Formulas are given below for these two common types of arrangement. Equations corresponding to other configurations can readily be derived.

(*a*) *Three Conductors in Equilateral Triangle*

$$\begin{array}{ccc} & A & \\ B & & C \end{array}$$

Short-circuit occurring at instant of zero voltage in phase *a*.
Between Conductors by Pairs:

$$S_{AB} = \frac{5.40 \times 1.8\sqrt{2}\, I_3^2 \times 1.35\sqrt{2} \cos 120^\circ}{10^7 D}$$

$$= \frac{-13.1\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Repulsion)}$$

$$S_{BC} =$$

$$\frac{5.40 \times 1.35\sqrt{2}\, I_3 \operatorname{Cos} 120^\circ \times 1.35\sqrt{2}\, I_3 \cos 240^\circ}{10^7 D}$$

$$= \frac{+4.92\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Attraction)}$$

$$S_{CA} = \frac{5.40 \times 1.35\sqrt{2}\, I_3 \cos 240^\circ \times 1.8\sqrt{2}\, I_3}{10^7 D}$$

$$= \frac{-13.1\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Repulsion)}$$

On each Conductor:

$$S_A = \frac{(13.1 + 13.1)\, I_3^2 \cos 30^\circ}{10^7 D}$$

$$= \frac{22.7\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Upward)}$$

$$S_B = S_B' + j\, S_B''$$

$$S_B' = \frac{(-13.1 \sin 30^\circ + 4.92)\, I_3^2}{10^7 D}$$

$$= \frac{-1.63\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Toward Left)}$$

$$S_B'' = \frac{-j\, 13.1\, I_3^2 \cos 30^\circ}{10^7 D}$$

$$= \frac{-j\, 11.35\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Downward)}$$

$$S_B = \frac{I_3^2}{10^7 D} \sqrt{1.63^2 + 11.35^2}$$

$$= \frac{11.47\, I_3^2}{10^7 D} \text{lb. per ft.} \quad \text{(Resultant)}$$

$$S_C = S_C' + j\, S_C''$$

$$S_C' = \frac{(13.1 \sin 30^\circ - 4.92)\, I_3^2}{10^7 D}$$

$$= \frac{1.63\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Toward Right)}$$

$$S_C'' = \frac{-j\, 13.1\, I_3^2 \cos 30^\circ}{10^7 D}$$

$$= \frac{-j\, 11.35\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Downward)}$$

$$S_C = \frac{I_3^2}{10^7 D} \times \sqrt{1.63^2 + 11.35^2}$$

$$= \frac{11.47\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Resultant)}$$

(*b*) *Three Conductors in Plane*

C B A or *A B C*

Short-circuit occurring at instant of zero voltage in phase *a*.
Between Conductors by Pairs:

$$S_{AB} = \frac{5.40 \times 1.8\sqrt{2}\, I_3 \times 1.35\sqrt{2}\, I_3 \cos 120^\circ}{10^7 D}$$

$$= \frac{-13.1\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Repulsion)}$$

$$S_{BC} =$$

$$\frac{5.40 \times 1.35\sqrt{2}\, I_3 \cos 120^\circ \times 1.35\sqrt{2}\, I_3 \cos 240^\circ}{10^7 D}$$

$$= \frac{4.92\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Attraction)}$$

$$S_{CA} = \frac{5.40 \times 1.35\sqrt{2}\, I_3 \cos 240^\circ \times 1.8\sqrt{2}\, I_3}{2 \times 10^7 D}$$

$$= \frac{-6.55\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Repulsion)}$$

On each conductor:

$$S_A = \frac{(13.1 + 6.55)\, I_3^2}{10^7 D}$$

$$= \frac{19.65\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Toward } A\text{)}$$

$$S_B = \frac{(-13.1 - 4.92)\, I_3^2}{10^7 D}$$

$$= \frac{-18.02\, I_3^2}{10^7 D} \text{ lb. per ft.} \quad \text{(Toward } C\text{)}$$

$$S_C = \frac{(-6.55 + 4.92)\, I_3^2}{10^7 D}$$

$$= \frac{-1.63\, I_3^2}{10^7 D} \text{lb. per ft.} \quad \text{(Toward } C\text{)}$$

Discussion

HIGH-VOLTAGE SWITCHES, BUSHINGS, LIGHTNING ARRESTERS—EXPERIENCES OF THE SOUTHERN CALIFORNIA EDISON COMPANY ON ITS 60,000, 150,000 AND 220,000-VOLT SYSTEM (Michener), HIGH-VOLTAGE CIRCUIT BREAKERS (Copley), MAGNETO-MECHANICAL LOADS ON BUS SUPPORTS (Robinson),

Del Monte, Cal., October 3, 1923

J. S. Thompson: I would like to have you consider what we believe is a step in the right direction in regard to oil circuit breakers. Our factory has developed an oil circuit breaker with six breaks per pole, which is not a new idea at all. We have done it in a novel way in that we have not increased the number of pieces of insulation between the conductors and the ground, but have simply introduced an auxiliary insulation between breaks. This auxiliary insulation is far from sufficient to maintain itself against the line's voltage, but it is only subjected to an impressed voltage at the instant during which the breaker is between the closed and the open position; it is of bakelite and relatively small and failure of this auxiliary insulation simply would cut out certain of the breaks in the circuit. We have such an installation on a 60,000-volt line. This breaker after breaking thirteen short circuits produced by the test of various fuses which failed to open the circuit, was then tested on direct dead short circuit. After having been tested once with no evidence of any form of distress, the engineers took the bold step of putting this breaker on the loaded system of the Pacific Gas & Electric Company, which was at that time being contributed to by Pit River, Drum Electra, Oakland and San Francisco. The breaker was subjected to short-circuit operations with this power concentrated on it (though I will have to say that the power was brought to this distributing point by a 110-kv. system and transformed to 66-kv.), so you will see that the breaker was put to a thorough test. It operated at 2000 amperes, which immediately dropped to 1350 or 1400. A Westinghouse relay closed the circuit in about five cycles and then there were consumed ten cycles for the mechanical operation of the tripping coil, and the movement of the blade out of contact, and nine cycles of diminishing amperes while the arc was being drawn—then five cycles between the rupturing of the arc and the end of the travel of the oil circuit breaker.

So it took a total of just thirty cycles for the switch to range from the impression of the short through to the actual coming to rest of the switch. That was repeated again after the circuit breaker was opened and very minor injuries were shown, simply a certain amount of pitting of the arcing tips such as might be expected. The circuit breaker is in shape to be closed in again on a similar load.

R. W. Sorensen: Until very recently oil switches have found very little place in my technical experience, but within the past few weeks a problem involving the use of oil switches made it desirable to search our technical literature for articles expounding the theory of oil circuit breakers.

I found articles discussing mechanical features, test data, and applications, but almost nothing seems to have been printed in American technical literature as to the theory of oil circuit-breaker design.

Eventually I discovered in a recent number of the "Revue Generale de L'electricite" an article on "Oil Switch Design" by Mr. P. Charpentier who signed himself Chief Designer of Service Apparatus in a factory at Jeumont, France, and a series of papers by D. R. Davies published in the London Electrician, the first of which appeared in a number published June 16th, 1922.

These papers present the subject of oil circuit breakers entirely from a theoretical point of view but the language of the authors indicates men familiar with the manufacture and use of oil circuit breakers.

Mr. Davies' paper includes this definition 'Breaking capacity may be defined as the maximum kilovolt amperes which the circuit breaker can break under prescribed conditions, at stated intervals, a specified number of times."

I have found it interesting to attempt to apply the theories advanced in these papers to some oil switches now made by one of our well known oil switch manufacturers, and these results I should like to call to your attention, not because of any conviction as to their accuracy in determining from the dimensions, and speed of operation of a switch just what rating it should have, but rather to stimulate a research program which will develop our knowledge of the theory of switching, and thereby give us a more definitely rated product.

Mr. Davies' first article which has to do largely with the carrying capacity of switch parts as determined by their thermal properties, and eddy current set up in them, needs no discussion at this time.

The facts pointed out in his second paper are the size of arc as determined by the current to be interrupted and its persistance as determined by voltage, switch speed, and stored energy in the circuit.

Also there are many other important phenomena discussed in this paper but we have time to glean only some apparent evidence as to the futility of long breaks, the uncertainty of reliance upon explosion chambers, and an important reference to Dr. Bauer's report to the Swiss Commission. From Dr. Bauer's report we have two important constants, one the energy factor of 0.07 as a maximum for the relation between kv-a. interrupted and the amount of energy which must be dissipated in the arc; the other is the figure 46.5 which represents the number of cubic centimeters of gas liberated for each kw. second of energy which must be taken up by the oil. These data were selected not because they are of necessarily greater importance than the other data given, but rather because they are likewise included in the formula by Mr. Charpentier, the application of which I wish to call your attention.

Time of break is fundamental, and according to Charpentier is equal to w/v where U is voltage per break and V is average velocity of blade in cm. per sec. This constant was used in the Charpentier formula for standard switches with two breaks in series, but if applicable directly to two breaks, should certainly also indicate much advantage for several breaks in series, even though some constant must be applied to allow for a possible uneven distribution of the voltage over the several breaks.

For example, last week I applied the Charpentier formula to several 45-kv. switches of standard make with the following results. Two break switch theoretical rating 3850 amperes. This switch is known to have interrupted successfully a number of times a short of 1,000 amperes, but finally it failed to properly care for a short of about this value. This shows a use of the formula not intended or else an omission of some factor that should be included. A four break switch of the same design shows by calculation a rupturing capacity of about 11,000 amperes instead of the 3600 which experience shows to be its probable rating. Using eight breaks the calculated rupturing capacity is 30,000 amperes.

We have however, a better check on a 30,000 ampere switch, from a known record of the operation of a 45-kv. switch with two breaks used on a 16.5-kv. circuit. The calculated breaking capacities for this switch 10,000 amperes, 27,600 amperes, and 103,000 amperes, for two, four, and eight breaks respectively. The four break switch actually did successfully rupture 20,000 amperes without damage.

Two sets of observations I have made also seem to check the value of the multi-break switch. One is the addition of more series breaks, without any other changes has transformed faulty operating switches into perfectly satisfactory ones.

The second is that when two switches of like design but with different number of breaks are in series on a short circuit, the switch with the greater number of breaks shows less carbon deposit from the oil and less burning of the switch clip, than does the one with the smaller number of breaks.

J. P. Jollyman: What kind of oil was used in the test?

J. S. Thompson: We used a mineral sea oil, that is, one that is as free from water and acid as possible. We purposely avoid attempting to furnish a high grade oil. So, we use a mineral seal oil.

H. Michener: I would like to ask what the test value of that oil was when the experiment started?

J. S. Thompson: I don't think I am able to answer that because I don't think we tested that particular supply of oil. This oil, you know, deteriorates in a breaker.

A. J. Bowie: I would like to ask if there was any explosion chambers on this 220,000-volt breaker; also how many breaks they had and what was the most severe test to which the breakers had been subjected?

R. M. Spurck: In Mr. Copley's paper reference is made to the practise of lowering the transmission voltage by field control in order to extinguish short circuits or flashovers. Such procedure, although effective, will, or has the disadvantage that it generally interrupts service and is liable to cause falling out of synchronous or induction apparatus in the area of the lowered voltage. The newer practise, namely, that of using automatic oil circuit breakers in combination with suitable relay schemes, makes it possible in net works or parallel lines, to automatically drop a defective section of the line without interrupting the service.

Oil circuit breakers are available for interrupting any voltages and currents that are commercially required. These high-voltage breakers are necessarily expensive. This expense is due, primarily, to the large clearance required for insulation. That is, a breaker designed to give the insulation for a 220,000-volt system, will in general, have an interrupting capacity comparable with normal requirements of such a system. In spite of the expense of these breakers, it is believed that their use is justified for sectionalizing purposes because the fixed charges and operating expense on them is small compared with the benefits derived from continuity of service.

One of the types of breakers in operation on the 220,000-volt system on the Pacific Coast utilizes the explosion chamber principle. The advantages of the explosion chamber principle are many and among them are the following: In the inherent speed characteristic, that is, the pressure in the chamber acting on the moving contacts as a piston accelerates the opening speed by the amount it can load. The effect of the increased speed of the opening is also obtained by the pressure inside the chamber causing a rapid movement of oil through the breaking contacts. It is possible, by changing the clearance between the opening in the explosion chamber and the moving contacts to regulate the rate of liberation of gas from the explosion chamber to the outer tank and thereby control the pressure in the outer tank. The ability to control this pressure permits the maximum interruption to be obtained from a given structure. The expulsion of gas into a body of cooled oil cools the gas, reducing its value and causing the breaking capacity to be increased.

The interrupting rating of a flow exchange type breaker has been determined by tests not only in the field, but also in the factory. The factory tests have been made under the worst conditions, namely, that of opening the circuit of a fully excited generator directly at the generator. The factory equipment consists of a 27,600-kv-a. generator complete with high-voltage transformers, reactors, oscillographs, and speed and pressure measuring devices. Such apparatus, for instance, enables us to determine the magnitude of the pressure in the tanks, or explosion chamber, the rate of pressure formation, the arc length and other factors affecting the rupturing capacity. From this definite data and conversion factors obtained therefrom, calculations can be made for other breakers.

In connection with what Mr. Sorensen said, I might add that all of the theories so far published that I have seen have constants in them that have not been adequately determined. For instance, taking a 15,000-volt breaker, opening a low current, the arc duration will be from five to six half cycles. However, if the same breaker is called upon to interrupt 35,000, 40,000 or 45,000 amperes, the arc duration drops down to one-half cycle.

A. J. Bowie: I would like to ask Mr. Michener what the short circuit would be on a 220-kv. line?

H. Michener: I don't know.

E. R. Stauffacher: I would like to emphasize something that Mr. Michener said in regard to the operation of the 220 oil circuit breaker. That is we do not know what amount of current the breaker had to interrupt. It might be stated that the current balanced relays were set to operate when an unbalance of 360 amperes occurs. The probable current interrupted was in the nature of 800 or 900 amperes.

I would like to emphasize as strongly as possible that there is a great need for a discharge recorder on lightning arresters. That is, a discharge recorder which will give a reliable record of the time, and if possible, the magnitude of each discharge, and which will be sufficiently rugged and inexpensive so that it can be installed on a large number of arresters. Such a record taken over a few years, would go a long ways toward settling the controversy as to whether lightning arresters justify their expense.

Time and again, we in the Southern California Edison Company, have wished we had some sort of a recorder available for the studying of the arresters. We have developed a discharge recorder which will give the time, but which does not give the quantity of current which was discharged. In fact, the instruments used in the past have been sometimes destroyed, at other times the chart has been destroyed.

If the manufacturers could see fit to attack this problem and develop some kind of an inexpensive recorder, I believe it would put the question of the application of lightning arresters on a much more definite and reliable basis.

O. R. Schurig: In the paper are given formulas expressing the electromagnetic force due to short-circuit currents in parallel conductors. It appears from the paper that the formulas are intended to be used for the calculation of the loads acting on the busbar supports, during short circuits, for the purpose of obtaining the strength required for the supports. In other words, formulas defining the electromagnetic force for a given value of current appear to be represented in the paper as defining, at the same time, the load acting on the support. While it may seem from mere superficial analysis that, in a busbar structure having a number of equal spans as a continuous beam, each support would be acted on by a load equal to the electromagnetic force per span,—a more thorough examination will show that the maximum load acting on the bus support will often differ materially from the maximum electromagnetic force per span. The reasons are (1) that the electromagnetic force is suddenly applied having periodic variations, and (2) that the busbar span and the supports have each their own natural frequencies and consequently respond differently to applied electromagnetic forces of different frequencies. Consequently, the formulas in the paper, give, in general, only roughly approximate values of actual loads acting on the bus supports during short circuits. Experiments, the results of which will be published later, have shown that the maximum load which the support has to withstand is in many cases, much smaller than the electromagnetic force (per span) calculated for the peak value of short-circuit current, but may rise well above the electromagnetic force so calculated when the busbar is in resonance with the electromagnetic force. Both resonant and non-resonant structures occur among the customary designs.

The difference between the electromagnetic force and the load acting on the support will be seen more clearly from the following considerations:

When an a-c. short-circuit current is suddenly caused to flow in a pair of parallel busbars, the two bars carrying equal and opposite currents, a rapidly varying pulsating force tending to push the bars apart is suddenly applied to the two conductors. The force is called the electromagnetic force. It is the driving force which tends to set the busbars into motion. The deflection of the busbar, and hence the stress set up in the busbar material as well as the load on the support, depend not only on the magnitude of the driving electromagnetic force, but also to a large extent on the ability of the busbar to respond as a vibrating body to the rapidly recurring force pulses. The stresses caused in the busbar cross-section during its vibratory motion increase with increasing deflection of the busbar span and reach a maximum at the instant of its maximum deflection. In stress determinations it is convenient to consider the equivalent uniformly distributed dead load per span of busbar, to be called briefly the "equivalent busbar load," which may be defined briefly as the uniformly distributed dead load (per span) which creates the same busbar deflection (relative to supports) as that produced by the driving electromagnetic force in the busbar span at any instant during the vibratory motion. Thus the character of the equivalent busbar load is indicated by the character of the busbar deflection.

In order to form a picture of the busbar deflections, it is necessary to consider the various components of the electro-magnetic force causing the deflections. It can easily be shown that a displaced sine-wave short-circuit current produces, at the beginning of the short circuit, an electromagnetic force of the following three components: a direct component, an alternating component at current frequency, and an alternating component at twice current frequency. It follows than that a busbar of very low frequency with respect to the current frequency, will respond mainly to the direct force component, but only very feebly to the alternating force components, and will, therefore, vibrate almost purely at its own natural frequency, approximately as if the alternating electromagnetic force components were entirely absent. Thus the maximum busbar deflection, and hence the maximum busbar load, may not occur until several (current) cycles after the peak current has occurred. Hence the maximum load is but remotely dependent on the initial peak electromagnetic force.

On the other hand, a busbar of high frequency, with respect to the current frequency, will respond to all the electromagnetic force components with practically equal fidelity, in much the same manner as a high-frequency oscillograph vibrator traces true images of composite low-frequency current waves. Now, the maximum busbar load will be practically equal to the maximum electromagnetic force per busbar span.

Should, however, the busbar be at resonance with the electromagnetic force, *i. e.*, having a natural frequency equal to the frequency of one of the alternating components of the electromagnetic force, the busbar will show a decided preference for the vibrations at the resonant frequency. Cumulative vibrations will occur, limited in magnitude only by the relatively low motional resistance of the busbar span. Thus the maximum equivalent busbar load may occur considerably later than the maximum short-circuit current, but may reach a value materially greater than the peak electromagnetic force.

The above considerations indicate that the maximum busbar load will frequently differ substantially from the peak electromagnetic force, both in regard to magnitude and time of occurrence. The maximum support load will not generally be exactly equal to the maximum busbar load, because the support also has a natural frequency of its own, largely independent of the busbar natural frequency, and will therefore respond selectively to the various frequencies of the driving-force components. Consequently, the support load will differ, in a great many cases, from the maximum electromagnetic force.

To summarize: The formulas in the paper give definite values of electromagnetic force. The maximum load acting on the supports during short circuits very often differs materially from the maximum electromagnetic force, the relationship depending on the natural frequencies of the busbar and of the supports as well as on the short-circuit current, as will be more fully brought out in a subsequent paper. Formulas for electromagnetic force, such as those presented in the paper, give as a rule only a very rough approximation to the support loads due to short-circuit currents.

E. G. Allen: Mr. Robinson calculates the force between conductors under a three-phase short-circuit which occurs at the instant of zero voltage in phase *a*. Under these conditions he finds the force at the moment the current in conductor *a* is a maximum which he calls $1.8\sqrt{2\,I_s}$, evidently allowing 10 per cent decrement in the first quarter cycle. At this instant the current in *b* and *c* would be $-0.9\sqrt{2\,I_s}$, as the current in *b* equals that in *c* and the sum of currents in *a*, b, and *c* equals zero. Under these conditions, the instantaneous force between conductors is—

$$S_{AB} = \frac{5.40 \times 1.8\sqrt{2\,I_s} \times 0.9\sqrt{2\,I_s}}{10^7\,D} = -\frac{17.5\,I_s^2}{10^7\,D}$$

.A
.B
.C

$$S_{BC} = \frac{5.40 \times .9\sqrt{2\,I_s} \times 0.9\sqrt{2\,I_s}}{10^7\,D} = +\frac{8.75\,I_s^2}{10^7\,D}$$

$$S_{CA} = \frac{5.40 \times 0.9\sqrt{2\,I_s} \times 1.8\sqrt{2\,I_s}}{2 \times 10^7\,D} = -\frac{8.75\,I_s^2}{10^7\,D}$$

$$S_A = -\frac{26.25\ I_s^2}{10^7\,D}$$

$$S_B = +\frac{26.25\ I_s^2}{10^7\,D}$$

$$S_C = 0$$

Although the above equations show the force at time of maximum current in *a*, they do not give the condition of maximum stress. Under the short circuit conditions assumed, the force in the middle conductor is a maximum 160 deg. after the moment of short circuit, when it is, without decrement $\frac{35.7\,I_s^2}{10^7\,D}$. The maximum force in the outer conductor is slightly less and occurs 164° 28′.65 after the moment of short circuit. However, analysis shows that the stress in conductor *a* is still greater when the short circuit occurs 15 deg. before the current in *a* is a maximum, and that the maximum force occurs 180 deg. after the short circuit. Under these conditions the instantaneous force on *a*, without decrement, is equal to $\frac{34.8\,I_s^2}{10^7\,D}$ and on *b*, $\frac{37.4\,I_s^2}{10^7\,D}$.

If 10 per cent current decrement is allowed, these values will be reduced to 81 per cent of their given value, but it should be borne in mind that these are instantaneous values of the force upon the conductors. The average values over the cycle are much less. The difference is a pulsating force tending to put the bus bars into vibration. The inertia of the bar is generally such that if the average force were withstood, the pulsating force would cause vibrations amounting to but a small fraction of an inch. This leads to the thought that stresses set up by short circuits might be greatly reduced if some sort of flexible support were provided which would permit the pulsating forces to be absorbed by the inertia of the bar.

The analysis leading to the above is briefly as follows:

Neglecting decrement of current, the equations of the unsymmetrical short-circuit currents are:

$i_1 = A\,[\cos(\theta + \alpha) - \cos\alpha]$

$i_2 = A\,[\cos(\theta + \alpha - 120) - \cos(\alpha - 120)]$

$i_3 = A\,[\cos(\theta + \alpha - 240) - \cos(\alpha - 240)]$

Where A = max. value of symmetrical current

= elapsed time (degrees) between moment of short circuit and moment under consideration.

= time (degrees) between moment of max. current and moment of short circuit.

$$\text{Force in middle bus} = \frac{5.40}{10^7 S}\,(i_2 i_3 - i_1 i_2)$$

$$\text{Force in outer bus} = \frac{5.40}{10^7 S}\left(i_1 i_2 + \frac{i_1 i_3}{2}\right).$$

Where S = distance between bus bars in a plane—

From the above:

$i_2 i_3 - i_1 i_2 = \sqrt{3}/2\,A^2\,[\sin(2\theta + 2\alpha - 240) - 2\sin(\theta + 2\alpha - 240) + \sin(2\alpha - 240)]$

This is a maximum or minimum with respect to α when

$\alpha = -15 - \theta/2$ or $75 - \theta/2$ or $165 - \theta/2$

It is maximum or minimum with respect to θ when

$\theta = 40 - 4\,\alpha/3$ or $160 - 4\,\alpha/3$ or $280 - 4\,\alpha/3$

For maximum with respect to both α and θ

$\theta = 180$ deg.

$\alpha = -15$ deg. or 75 deg. or 165 deg. or 255 deg.

Under any of these conditions

$$i_2 i_3 - i_1 i_2 = 2\sqrt{3}\,A^2 \text{ and } F = \frac{18.72\,A^2}{10^7 S}.$$

If I_3 = root mean square current $= A/\sqrt{2}$

$$F = \frac{37.4\,I_3^2}{10^7 S}$$

L. H. Robinson: I agree with Mr. Schurig that the method submitted in my paper, gives only approximate results and consequently I prefer to use a liberal factor of safety. If vibrations of the bus and its supports, due to currents of ordinary frequency (25, 50 and 60 cycles per second) with common sizes of bus bars, lengths of span, etc., may augment or partially neutralize the external force; formulas correlating these factors with the bus currents, and experimental confirmation of these formulas will be valuable and useful.

H. Michener: Twelve of the twenty-eight 220-kv. oil switches referred to in the paper have been operating on balanced relays since early in September. These switches have given very satisfactory performance. A ground on a section of line thus protected causes only a momentary drop in system voltage of approximately 10 per cent. After as many as five automatic operations under short circuit to ground conditions and perhaps fifty non-automatic operations for routine line switching, inspections have shown only slight burning on the contacts and some minor mechanical defects.

The longest section of line being switched out automatically up to the present time is 98 miles. About March first, a section of the line 166 miles long will be arranged for automatic isolation on balanced relays.

The actual currant values which occur at the time of a short circuit to ground are very problematic. Using the calculating table which has been constructed for the purpose of studying short-circuit currents on the system, it has been determined that with the present generator capacity the maximum current value on the 220-kv. lines is about 5000 amperes in case of a three-phase short circuit. This is the instantaneous current. In about two-tenths of a second the current is reduced to about half that value.

A. W. Copley: With reference to Mr. Bowie's question as to the construction of the 220-kv. circuit breaker which I described, I will say that this breaker is built with two breaks per pole and without explosion chambers.

A high speed of contact separation is obtained by means of arcing contacts held to the stationary contact parts by a spring and latched to the moving contact parts in such a way that the arcing contacts maintain connection until the main moving contacts are open several inches. The latch then releases allowing the arcing contacts to be quickly retrieved upward by means of a spring while the moving contacts are at the same time moving downward at high speed.

Mr. Bowie also asks concerning the tests for the determination of interrupting capacities. Tests at full rupturing capacity on the 220-kv. breakers naturally have not been made because of the inability to obtain the necessary combination of high energy and high voltage for adequately determining such interrupting ability. However, tests have been made on similarly designed breakers for somewhat lower voltages and these have demonstrated their ability to more than meet the guaranteed interrupting capacity. It is fully expected, therefore, that the rated interrupting capacity of the 220-kv. breakers will be found to be conservative when the systems to which they are connected have grown to the point where this rating is reached by the requirements of service.

Mr. Spurck has described another design of circuit breaker for 220-kv. service in which the explosion chamber is utilized. While the value of this device may be open to question in theory, the only complete answer as to the dependability of the breakers will be obtained by service records at a later date. Testing interrupting ability by means if a special generator and other testing equipment is of very great value and should be followed out further. But, it is evident that obtaining of the necessary energy and voltage to make adequate tests on a breaker with an interrupting capacity of more than 1,000,000 kv-a. at 220 kv. is hardly possible at this time.

The theories as to the mathematical determination of oil circuit breaker interrupting capacity as described by Professor Sorensen, are of considerable interest. But the best guide at present in determining capacities is obtained by actual test and operating experience with the various types of breakers. I believe that the factors entering into the determination of rupturing capacity are so numerous and complex that it will be difficult to express the result in a mathematical formula. It is of value, however, to continue to work toward this end along with tests and operating experience from which the final answer must be obtained.

It might be inferred from the discussion by Mr. Thompson that high quality of oil is not of vital necessity in high-voltage circuit breakers. While there is some evidence in favor of such a conclusion, there is also much reliable evidence to the effect that a poor quality of oil has caused many switch failures. It is therefore premature, to say the least, to ignore this factor and until much more is known about breaker operation on high voltages no oil but the best should be used.

High-Voltage Insulation

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I. General

THE most important chapters in electrical engineering are those dealing with efficiency, heating, magnetism and insulation.

In the field of insulation, our knowledge is most backward.

In regard to efficiency, with efficiencies of electrical apparatus of 90 up to over 99 per cent, no further radical progress appears feasible.

In magnetism, the losses in an alternating field have been reduced so far, that they have ceased to limit, by their heating effect, the size of apparatus, but are merely a factor in the efficiency. An increase of the saturation density would decrease the size of apparatus, but is excluded by its inherent chemical limitations.

Heating specifications are not made for other classes of apparatus, such as prime movers, etc., and if an important subject in electrical engineering, it is almost entirely because the insulation of the apparatus is destroyed by the higher temperatures. Thus the problem of the heating of electrical apparatus is essentially one aspect of the insulation problem.

In our high-voltage apparatus, cables, etc., we operate the insulation at voltage stresses which rarely exceed much the disruptive strength of air, though laboratory tests often show this insulation to have a disruptive strength of 10 to 20 times that of air.

All phenomena of nature are very complex. Therefore, in calculating a phenomenon or designing an apparatus, we must approximate by neglecting "secondary terms," and take care of these by an allowance, a margin or a factor of safety. Obviously, the more completely a phenomenon is known and understood, the closer it can be calculated, in other words, the less is the margin or safety factor required in order to allow for the unknown stresses, etc. The margin or safety factor, which experience shows as necessary, thus is an indication of the exactness of our knowledge of a phenomenon. For example when dealing with magnetic phenomena, with efficiency, with heating, etc., we have to allow a margin of a few per cent only. In testing the insulation of apparatus however, the A. I. E. E. standards specify a test voltage more than twice the delta voltage, though the normal stress is the Y voltage. That is, we require a safety factor of over 3.46, a margin above normal of over 246 per cent.

The insulation problem has become of increasing importance with the rapid advance of electrical engineering into higher voltages. Not many years ago 44 kv. was the highest transmission voltage for reliability of operation of overhead lines. Now we have reached 220 kv. Then 12 kv. was the highest satisfactory cable voltage; now we have reached 22 kv. and a few cables of 33 kv. and higher, but cables at 33 kv. are still semi-experimental. The comparison shows that the advance in pushing cable voltages up to higher values, has been slower than with overhead lines, and our knowledge of liquid and solid insulation, such as come into consideration in the cable wall and the machine insulation, is materially less advanced than that of air as dielectric. With regard to air, a good working theory has been established, by considering the dielectric strength of air as analogous to the mechanical strength of structural materials. The theory recognizes a definite dielectric strength of air, or a disruptive breakdown gradient, of 30 kv. per cm. at normal air density. Puncture occurs when this dielectric strength is exceeded, just as mechanical disruption occurs, when anywhere in a mechanical structure the stresses exceed the elastic limit of the material. This conception of a definite breakdown strength then was extended to liquid and solid dielectrics, but with these, it failed to give a satisfactory explanation of the mechanism of the breakdown, more particularly of the all-important feature of the time lag of disruption. And even with air, the theory of a constant breakdown gradient, as modified by the conception of the energy distance, is satisfactory only within a certain range.

II. Air as Dielectric

The present practically universally accepted theory of a dielectric strength of air at and near atmospheric pressure, as most completely developed by Mr. F. W. Peek, Jr., is:

Air has a definite and constant "dielectric strength," at which it ceases to be an insulator and becomes a conductor, that is, breaks down electrically.

The dielectric strength of air is proportional to the air density, and is 30. kv. per cm. at normal air density of 0 deg. cent. and 76 cm. barometer.

The dielectric breakdown (or puncture) of air does not occur as soon as the voltage gradient in the dielectric field exceeds the dielectric strength at any point, but the voltage gradient in the field must exceed the dielectric strength over a finite distance, the so-called "energy distance."

The energy distance depends on the convergency of the electric field at the place where the breakdown occurs, and is the less, the more convergent the field.

The energy distance between parallel cylinders (wires) of radius R is $0.3\sqrt{\delta R}$; between spheres of radius R it is, approximately; $0.54\sqrt{\delta R}$, where δ = air density (with normal air density as unity).

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Thus the breakdown gradient at the surface of two parallel cylinders of radius R is:

$$g_o = 30\,\delta\left(1 + \frac{0.3}{\sqrt{\delta R}}\right)$$

at the surface of spheres it is

$$g_o = 30\,\delta\left(1 + \frac{0.54}{\sqrt{\delta R}}\right)$$

For gaps of a length less than the energy distance, a higher breakdown gradient is required, the more so, the shorter the gap. The law is unknown.

The space in the dielectric field in which the dielectric strength is exceeded, becomes conducting, and, if adjacent to the terminals, becomes a part of the terminal. This changes the configuration of the dielectric field and if thereby the dielectric strength is exceeded in other parts of the dielectric field, these also break down and become conducting, until either the breakdown extends from terminal to terminal, the air gap punctures and short-circuits by a spark discharge (which tends to develop into an arc), and the current rises and the voltage drops to the short-circuit values of the supply source. Or the breakdown in the dielectric field limits itself, producing a new configuration of the dielectric field, containing conducting regions, in which the air is broken down and luminous by "corona," separated by insulating air spaces below the breakdown gradient. In this case the discharge current is limited to the small "corona current." A further increase of voltages then may extend the breakdown across the terminals and change the limited discharge into an unlimited spark discharge.

Between needle points, corona always precedes the spark discharge; between spheres at moderate distance (less than 3 to 4 times the radius of the spheres), spark discharge occurs without corona.

The disruptive discharge of single gaps in air, is instantaneous, that is, no time lag is produced in the spark gap, if no corona precedes the spark discharge, and the gap length is greater than the energy distance. When with rising voltage corona precedes the spark discharge, a time lag exists. With gaps of less than the energy distance, and multigaps, a time lag also seems to exist. The combination of resistance in series and capacity in shunt to an instantaneous spark gap gives the gap a time lag owing to the time required to charge the capacity by a current limited in value by the resistance. Without a capacity in shunt, an otherwise instantaneous sphere gap in series with a resistance lags by the time of charging the capacity of the spheres themselves, through the series resistance. For high values of resistance, this may become quite considerable, especially with large spheres.

This standard theory of dielectric strength of air at and near atmospheric pressure is fairly satisfactory for gaps ranging between the energy distance and a larger value where corona begins.

For gap lengths less than the energy distance, the standard theory does not apply.

For gaps in which corona precedes the spark discharge, the law does not seem to satisfactorily agree with experience. Between spheres, corona appears only at gap lengths materially larger than those in which it should appear according to the theory. Also the disruptive voltage at which spark discharge occurs between needle points, is materially lower than calculated. This may partly be accounted for as follows:

(a) The space filled by corona is not a perfect conductor, but has a finite resistance, which depends on the corona density, thus on the voltage, frequency, etc.

(b) It is practically an impossibility to calculate accurately the final dielectric field and its voltage gradients, as resulting from the partial short circuit by the corona, of regions of the initial dielectric field, even if the resistivity of the corona regions were neglected.

(c) The nature of the corona-filled space is that of an unstable third class conductor, which tends to produce local oscillations and other transients, and thereby rapid variations of the resultant field and premature breakdown. Incidently, when high-frequency oscillations and other low-power transients of unknown origin appear in a system, it is advisable to look for the existence of corona somewhere in the circuit, as possible cause.

The theory of the dielectric strength of air at and near atmospheric pressure, outlined above, is based on the distribution of dielectric stresses and therefore depends on the configuration of the dielectric field. As the dielectric field is essentially determined by the shape of the terminals, the latter thus is of fundamental importance. At the same time, in the "conduction of gases in a vacuum" a theory has been developed, based on convection phenomena by carriers, the ions, and in this theory the configuration of the dielectric field and the shape of the terminals is practically not considered, and the two conceptions thus apparently are out of line with each other, though the phenomena of the Geisler tube and those of air at normal pressure should differ quantitatively only, by the different value of air density δ.

Approximately, and within a limited range, the voltage e required to break down a gap of length l at air density δ, is constant as long as the product $l\,\delta$ is constant, that is, for constant quantity of air within the gap. In other words, for different air densities δ, the gap length l at constant voltage e varies inversely proportional with the air density δ. The air densities in vacuum tubes usually vary from one thousandth to one millionth of normal air density and less, and even the longest vacuum tubes usually therefore correspond at atmospheric pressure to gaps of very small length, greatly within the energy distance. In other words, the phenomena of vacuum tube conduction correspond to the phenomena occurring in very small gaps at atmospheric pressure, where the conventional theory does not apply, and the configuration of the field and the shape of the terminals become of lesser importance. Inversely, the voltage phenomena of the Geisler tube, may also appear in the dielectric strength of large gaps at atmospheric pressure, as terminal phenomena, though then secondary in magnitude to the gradient voltages of the dielectric field.

The conventional theory considers only the voltage gradient or intensity of the dielectric field as determining the dielectric strength, but not the direction, that is, assumes the same breakdown strength for either polarity of the impressed voltage. This resulted from the use of alternating voltages in determining the laws of the dielectric strength of air, due to high direct voltage of considerable power being unavailable until recent years. It is very far from correct, however, and in an unsymmetrical field, the disruptive strength with the voltage in one direction, may be more than twice that with the voltage in opposite direction. As illustration, Fig. 1 shows the striking distances between a needle and a 12.5 cm. sphere, for various gap lengths. Curve I gives the results with the needle positive, curve II with the needle negative, and curve III with alternating voltage. As seen, I and II differ greatly, II being more than twice the voltage of I. The alternating curve approximates I, as is to be expected. In the voltage range between I and II, such an unsymmetrical gap rectifies.

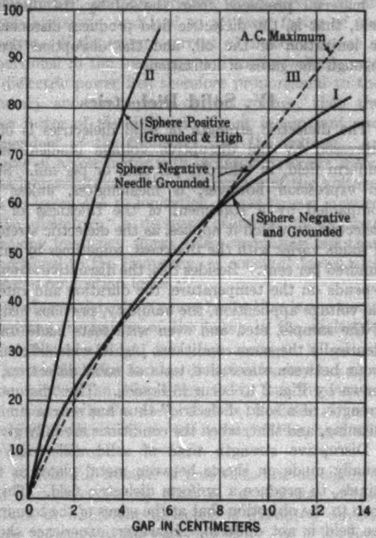

Fig. 1

Investigation shows that it is the form and intensity of the dielectric field at and near the positive terminal which has the greatest effect on the disruptive voltage of the air gap, while the field at and near the negative terminal is of secondary importance. Thus if at constant gap length the positive terminal is gradually changed in steps from a sharp point to a flat plate, while the negative terminal remains unchanged in form, the disruptive voltage changes correspondingly over a wide range. A corresponding change in the physical form of the negative terminal however, with the positive terminal remaining unchanged in form, varies the disruptive voltage relatively little.

In Fig. 1, curve I approaches the striking distance curve between needles, as modified by the different distribution of voltage gradients due to the large negative terminal, and curve II similarly approaches the striking distance curve between 12.5 cm. spheres.

This suggests the explanation that the disruptive discharge is due to carriers of current produced by the field at the positive terminals, the positive ions possibly.

Tests made at atmospheric pressure on very small gaps—down to fractions of microns—show that with decreasing gap length the voltage does not indefinitely decrease, but reaches a finite minimum value, of about 320 volts, and becomes apparently constant at this value. This would be the minimum voltage required to disrupt an air gap, no matter how small. It means that with decreasing gap length, the voltage gradient of the air film rises to very high values, and such thin air films have an extremely high dielectric strength; gradients of 6.2 million volts per cm. have been reached.

Inversely, with increasing gap length, the voltage increases with the gap length, and proportional thereto, from an initial value of about 320 volts. The increase with increasing gap length, however, is not at the rate of the dielectric strength of air, 30 kv. per cm., as might be expected, but is at approximately twice this rate; about 60 kv. per cm., up to voltages of about 4500. Then the increase of voltage with increasing gap length decreases and approaches the normal value of the breakdown strength, 30 kv. per cm.

The voltage e required to break down an air gap of length l and uniform dielectric field intensity thus would consist of

(1) A constant voltage $e_1 = 320$ volts $= 0.32$ kv.

(2) A voltage gradient g_2 of about 60 kv. per cm. over a short length $l_0 = 0.07$ cm., giving a constant voltage $e_2 = 60\, l_0$ kv.

(3) The voltage gradient of dielectric strength, $g_0 = 30$ kv. per cm., for the rest of the gap, $l - l_0$, or $g_3 = 30\,(l - l_0)$; thus giving a total voltage, approximately:

$$\begin{aligned} e &= e_1 + e_2 + e_3 \\ &= 0.32 + 30\, l_0 + 30\, l \end{aligned}$$

or, if $l < l_0$, simply

$$e = 0.32 + 60\, l$$

In a non-uniform field, the breakdown would occur from the positive terminal up to the distance, to which the breakdown gradient $g_0 = 30$ extends, and this space is filled with corona.

In Appendix I is given a suggestion of the mechanism of the dielectric breakdown of air by carriers issuing from the positive terminal.

III. Liquid Dielectrics

Oil and similar materials (as petrolatum in the high-potential cable impregnation) are the most important liquid insulating materials and are depended upon for the insulation of the highest voltage electrical apparatus, the transmission power transformer. The fairly satisfactory agreement of the behavior of air as insulating material, with the theory of a definite dielectric strength, modified by the conception of the "energy distance" led to the application of the same theory to liquid dielectrics. Here, however, the agreement with experience is not satisfactory; and tests show that oil does not have a practically available definite dielectric strength, but successive tests made with all precautions with the same oil in the same gap (using such a large quantity of oil as to exclude the effect of deterioration) differ from each other by values many times greater than the possible errors of observation. The upper curve on Fig. 2 shows the deviation from average of 100 successive tests of the disruptive voltages of an oil gap, and the lower curve shows the corresponding results with an air gap. Plotting the number of times each deviation from average occurs, in A, as function of the value of deviation, gives a probability curve.

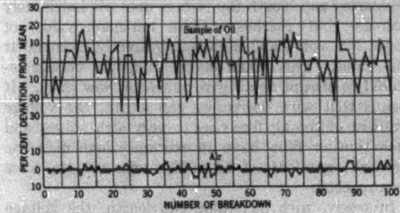

Fig. 2

The most satisfactory explanation of the mechanism of the breakdown of oil and other liquids seems to be that based on the assumption of lack of homogeneity, namely that the breakdown of an oil gap is the result of materials of different conductivity or specific capacity being sucked into the gap by the dielectric field, or being produced in the gap by it, and so causing a distortion of the field, with local high densities, at which disruption begins. This can be visually shown by dropping a minute quantity of water or other foreign material into an horizontal sphere gap in oil, at moderate voltage between the spheres, and observing the resultant motions and deformations. It is even somewhat questionable whether liquid dielectrics have a definite dielectric strength at all, or whether disruption does not always occur through gases produced from the dielectric by the electric stress. A discharge through oil seems always accompanied by the appearance of gas bubbles, and while the gas bubbles may not be the cause, but the result of the discharge, there are some differences in the shape of the striking distance curve of an oil gap from that of an air gap, which look as if the disruptive discharge in air is the result of the action of the dielectric field on the air, while the disruptive discharge in oil is the result of the action of the dielectric field on a material produced from the oil by the dielectric field, that is, the dielectric field produces dissociation or ionization of the oil, and the disruption occurs through the gases of ionization.

IV. Solid Dielectrics

The dielectric strength of solid dielectrics is often expressed by the disruptive voltage gradient in a uniform field, in kilovolts per cm., or per mil. Such an expression however, is meaningless, unless accompanied by a statement of the thickness of the dielectric, to which it applies, as the dielectric strength of solids varies with the thickness, sometimes by many hundred per cent. Besides this, the disruptive strength depends on the temperature, the duration and rate of the voltage application, the humidity, previous history of the sample, etc., and even with tests made under identically the same conditions, just as wide differences occur between successive tests of solid dielectrics, as shown by Fig. 2 to occur in liquids. The "disruptive strength of a solid dielectric" thus has only a limited meaning, and that, when the conditions are fully given.

Disruptive strength tests of solid dielectrics are usually made on sheets between metal plates as terminals, to produce a uniform dielectric field. This is open to the objection that at the edges of the terminals the field is not uniform. However, experience shows that puncture does not always occur at the edges of the terminal plates, but often inside, and by throwing out the tests in which the disruption takes place at the edge of the terminals, and averaging those in which the disruption occurs well inside of the terminals, fairly representative results are secured.

Two phenomena come into consideration in solid dielectrics: The electric conductivity and the dielectric losses in an alternating field.

The dielectric has an appreciable though usually very low conductivity. That is, under an impressed voltage, a slight current is conducted through it, causing a loss of power and thereby a heating of the dielectric. The power consumed by ohmic conduction in the dielectric usually is extremely small, at least at atmospheric temperature. The conductivity usually increases very greatly with the temperature, often approximately exponentially, (for instance, increases ten-fold for every 25 deg. temperature rise, so that at high temperature the dielectric may become a good conductor). Often, the conductivity also increases with increasing voltage. In an alternating field, the resistance loss usually is very small compared with the other dielectric losses.

Under an alternating voltage, losses occur in the dielectric, more or less of the nature of a dielectric hysteresis. While probably mainly proportional to the frequency, some of these losses may increase faster than the frequency, some at a lesser rate, giving in the latter case a dielectric loss at zero frequency.

These dielectric losses may be represented by an energy current and an effective conductivity of dielectric hysteresis, and lead to a power factor of the dielectric. If the energy current of dielectric losses is proportional to the frequency and to the voltage, and the dielectric power loss therefore proportional to the frequency and the square of the voltage, then the power factor of the dielectric circuit is constant and independent of the frequency and the voltage. An increase of power factor with increasing voltage shows losses increasing faster than the square of the voltage, such as losses due to ionization of gas spaces inclosed in the dielectric; a decrease of power factor with increasing frequency shows losses increasing less than proportional to the frequency, or constant losses such as ohmic conduction losses.

The observation of the power factor thus is one means of judging the nature of the losses in the dielectric.

A dielectric of high disruptive strength in general has low conductivity and low dielectric losses, thus a low power factor in an alternating field; but the reverse is not necessarily the case, that is, a dielectric may have low conductivity or low power factor, and still be of poor disruptive strength.

It is doubtful whether a true dielectric strength of solid insulation exists, that is, a definite voltage gradient at which the dielectric is disrupted directly by the intensity of the dielectric field; or if it exists, it is so far beyond the disruptive strength observable, as to be of no practical consideration. That is, puncture of a solid dielectric probably in practise always occurs as the result of a more or less rapid progressive deterioration, far below the true dielectric strength, and the latter may be of moment only under direct lightning condition, for instance, in the puncture of a sheet of insulation by a powerful Leyden Jar discharge (giving the characteristic appearance of an internal explosion).

The mechanism of the breakdown of a dielectric under electric stress, may probably be any of a number of possibilities, thermal, mechanical, chemical, physical, etc.

MECHANISM OF THERMAL BREAKDOWN OF THE DIELECTRIC AS THIRD CLASS CONDUCTOR

Many, if not most of the dielectrics in reality are third class conductors, that is, have in some range of their voltampere characteristic such a high negative temperature coefficient of conductivity, that in temperature equilibrium the voltage decreases with increasing current.

Suppose a uniform sheet of solid dielectric is exposed to a constant direct voltage between two conducting terminals. Due to the slight conductivity of the dielectric, a current then flows through it at a uniform, though very low current density. This consumes electric energy, converting it into heat, and gives a slight temperature rise of the dielectric. Now, no material can be absolutely uniform and homogeneous, and thus in this sheet of dielectric there will be some spots of a slightly higher conductivity. However slight the difference, at such a spot the current density must be slightly greater, thus the energy consumption and the heat production slightly greater, giving locally a slightly higher temperature rise. However little this may be, if the conductivity of the dielectric increases very rapidly with increasing temperature, it will lead to a slightly higher current density, a correspondingly higher energy consumption and heat production and thus temperature rise, and so on. Two possibilities then exist, either the heat which can be conducted away from such a "hot spot" due to its temperature rise, is more than the heat produced by the increased conductivity due to the temperature rise. Then the temperature of the hot spot finally limits itself, and stable thermal conditions pertain. Or the heat which can be conducted away from the hot spot into the dielectric, by the temperature rise, is less than the heat produced in the dielectric by the increased conductivity due to the temperature rise. Then the conditions are unstable, that is, the temperature continuous to rise and the conductivity to increase indefinitely, until the energy concentration at the hot spot becomes destructive, and the dielectric is destroyed by "puncture."

The successive steps of this phenomenon have been observed by limiting the current density by suitable terminals and can be calculated from tests made on the conductivity of the dielectric at various temperatures and voltages.

As the energy consumption by the conduction current varies with the square of the voltage, there is thus a definite voltage—for a given set of conditions—at which instability is reached and puncture results, and this voltage is the "breakdown voltage," its voltage gradient the "dielectric strength" of the material under the conditions of test. This voltage depends upon the initial temperature, and considerably decreases with increasing temperature; at higher temperature, a lesser temperature rise is necessary to reach unstable conditions, and at the higher initial temperature the phenomenon starts with a higher conductivity, that is, greater power consumption and heat production. The heat conductivity of the dielectric and of the terminals and other surrounding objects, and their ability to quickly absorbing heat, that is, their specific heat, are essential factors in determining the value of the puncture voltage. An essential factor also is the relation between temperature and conductivity of the dielectric, etc., so that the "dielectric strength" thereby determined, is very far from a constant for the material.

Suppose the "hot spot" is filamentary in shape. That is, the diameter of the spot of slightly higher conductivity (or temperature) at which the breakdown starts, is small compared with its length in the direction of the lines of force, so that heat conduction from it is essentially into the dielectric. Then a change of thickness of the dielectric will not affect the heat conduction, and in this case it follows that the disruptive voltage is proportional to the thickness of the dielectric, its dielectric strength independent of the thickness.

Suppose however the "hot spot" is plate-shaped, that is, its dimension parallel to the lines of force (from terminal to terminal) is small compared with its diameter parallel to the terminals, so that the heat conducted from it flows into the terminals. Then an increase of thickness of the dielectric gives an increase of heat to be conducted through a longer distance through the same area, thus instability is reached at a lower voltage gradient. In this case it is found that the puncture voltage is proportional to the square root of the thickness of the dielectric, and the dielectric strength inversely proportional to the square root of the thickness.

In general then, in dielectric breakdown by temperature instability, depending on the path of the heat conduction, the puncture voltage varies between the first and the 0.5th power of the thickness of the dielectric, and the dielectric strength or breakdown gradient varies between independence of the thickness, and the—0.5th power of the thickness of the dielectric.

This phenomenon of dielectric breakdown by thermal instability is the result of the increase of energy loss in the dielectric, with increasing temperature, and occurs wherever the losses greatly increase with the temperature. While its mechanism has been illustrated above on ohmic conduction in a direct-voltage field, the same phenomenon occurs in the same manner in an alternating field, and as the result of the specific losses of the nature of a dielectric hysteresis, as far as these losses increase with the temperature. The increase—and usually very rapid increase—of the power factor with the increase of temperature of most dielectrics shows that the losses in the dielectric in an alternating field, increase with the temperature, and thermal instability leading to puncture at "hot spots" thus results.

MECHANISM OF DISRUPTION DUE TO MECHANICAL INSTABILITY

Suppose a dielectric encloses a particle of a higher conductivity or higher specific capacity. An electric field then exerts a mechanical force on this particle, which tends to elongate it in the direction of the electric field, and to compress it at right angles thereto, that is, tend to form it into a filament short-circuiting the dielectric field. With liquid dielectrics, this can be observed visually by dropping a little water into a horizontal sphere gap in a transparent oil. Each droplet, as it is sucked into the field between the spheres, lengthens into a filamentary conductor which bridges between the terminals, and then is destroyed with a flash by the heat of the current conducted by it. A similar phenomenon occurs with moisture in a solid dielectric, except that the puncture usually is permanent; occluded moisture moving under electrostatic forces through the pores of the solid dielectric, may form a conducting bridge between the terminals and by the heat of the current in the moisture filament start destruction of the dielectric. Or the moisture-thread may partially bridge between terminals and locally short-circuit the field, which will cause excessive dielectric stresses in the part of the gap in series with the moisture filament. In these over strained portions of the field, destruction starts by thermal or chemical instability, etc.

In impregnated insulation, where the impregnating material is liquid or viscous, motions of the impregnating material through the impregnated material may result from the mechanical forces caused by differences in specific capacity or conductivity of the materials, leading to a redistribution of the impregnating material. Conductivity and specific capacity here act in the same direction, in general. The differences in specific capacity of different dielectrics are however relatively small; rarely more than 1 in 6 in materials which are suitable for apparatus insulation; while the same constituent materials may differ in conductivity over an enormous range. The final effect of conductivity as compared to specific capacity is likely therefore, to be much greater. On the one hand, the mechanical forces due to differences in specific capacity appear instantaneously, and thus are present also in alternating fields. On the other hand the differences due to difference in conductivity appear gradual, being accompanied by the formation of local internal electrostatic charges, and may require seconds or even minutes for their completion. These local changes would not be present or only partly present in alternating fields, and the mechanical actions of alternating fields thus differ from those of continuous fields. This can conveniently be observed visually on sphere gaps in oil.

MECHANISM OF DISRUPTION BY CHEMICAL DETERIORATION

Suppose the dielectric contains a particle of higher specific capacity or higher conductivity. Let us assume, at first, this particle to be spherical in shape, and of relatively infinite specific capacity or conductivity, that is, short-circuiting the field (for instance, an oil of 10 megohms resistivity would fulfil this condition in a dielectric of 1000 megohms resistivity). Then the density of the lines of dielectric force inside of the spherical particle is three times that in the surrounding dielectric, and—at least temporarily, until a redistribution of internal charges has occurred—the density of the lines of dielectric force and thus the voltage gradient in the dielectric outside the poles of the spherical particle would be three times normal. It would be less than three times normal, if the specific capacity or conductivity of the spherical particle differs less than given above from the surrounding dielectric. But it would be greater if the particle is not a sphere, but more irregular in shape, and would assume much higher values at the edges and at points of the particle. Thus in the dielectric adjacent to edges or points of an enclosed particle of higher specific capacity or conductivity, very high-voltage gradients occur, and may be far beyond the dielectric strength of the material, and lead to local breakdown of the material by what may be called "electrostatic cutting edges." With organic insulation, the effect usually would be carbonization by high local temperatures, a chemical change which in general increases the conductivity. The mechanical electrostatic forces brought about hereby are in the direction of the field, so that the shape of the product of chemical deterioration tends toward the form of a conducting needle, with excessive voltage gradients in front of it, gradually piercing the dielectric until final puncture occurs between the terminals. Thus in a laminated insulation consisting of very many layers, a foreign particle in one of the layers though originally forming only an insignificant part of the total thickness of the dielectric, may gradually but cumulatively, in the course of time, pierce and destroy the insulation by its electrostatic cutting edges, the average voltage gradients within the dielectric, being still very low compared with the tested "dielectric strength" of the material. The destruction will be the faster, the higher the local voltage gradient. This seems to be the main reason why such excessive margins are required in high-voltage insulation, to give reasonable length of life.

IONIZATION

The specific capacity of the common solid dielectrics is from 2 to 8 times that of air. Thus if air pockets are contained in the dielectric, the electric stress in the air is much higher than in the solid, and as the break down gradient of air is low, it breaks down with the formation of "corona," giving heat and chemical action, and such ionization due to enclosed air probably is a frequent cause of disruption. It is guarded against by excluding the air by impregnation with a material of higher dielectric strength and higher specific capacity, such as oil. It means however, that the process of impregnation to be effective must be perfect.

It is interesting to note that some of the mechanisms of breakdown, as thermal instability and mechanical forces, are reversible, that is, at the withdrawal of the voltage, the original condition may gradually return, leaving the dielectric undamaged, while the chemical deterioration by the electrostatic cutting edges or by ionization is irreversible, that is, what damage has been done is permanent and remains at the withdrawal of the voltage, and at the next application of voltage, the deterioration progresses further.

In non-homogeneous dielectrics, such as laminated insulation, due to the differences in the ratios of the respective specific capacities and the specific conductivities of the component dielectrics, gradual changes may occur in the distribution of the voltage gradients through the dielectric, with the formation of internal charges, which continue for some time, thus resulting in a corresponding change of the dielectric stresses.

V. Time Lag of Insulation

The two important properties of insulation are its dielectric strength and its time lag. The former means that there is a limit in the voltage gradient up to which an insulation can hold back the voltage, and when this voltage gradient, the "dielectric strength" or "disruptive strength" of the insulation, is reached, disruption occurs and the insulation ceases to insulate, becomes a conductor.

Experience shows the following condition: At least with very many insulations, the voltage at which disruption or breakdown occurs, depends on the time or duration of voltage application. The lower the voltage, the longer the time it has to be applied. There is a minimum voltage, which continuously applied, still just disrupts the insulation, and inversely, the higher the voltage, the shorter is the time during which it has to be applied.

This time, during which a given voltage has to be applied to cause disruption, is called the "time lag" of this voltage. The ratio of this particular voltage of brief application to the voltage which permanently applied breaks down the insulation, is called the "impulse ratio" of the time of voltage application.

Time lag bears to dielectric strength, in electrical engineering, a relation similar to the one between elasticity and mechanical strength in mechanical engineering; if it were not for elasticity, there would be no mechanical engineering. A pebble dropped on an armor plate would shatter it,—since theoretically, at the point of impact, without elasticity infinite mechanical forces would be produced.

So without the phenomenon of time lag, there would be no electrical engineering, as there would be no possibility of insulation, since every insulation, even a low-voltage lighting circuit, is theoretically constantly exposed to transients of infinite voltage (though negligible energy), that is, far beyond its possible dielectric strength, by the inductive and capacity effects of any change of circuit conditions, and thus saved only by the time lag of its insulation.

Just as the relations between mechanical strength and elasticity give the wide variety of structural materials, on which the mechanical engineer depends, and which we denote by brittle, tough, ductile, elastic, rigid, flexible, yielding, etc., etc., so the relation of time lag to dielectric strength gives us insulating materials of widely different properties and correspondingly widely different uses—but our knowledge in this field is unfortunately still very limited.

To illustrate the importance of the time lag: Experience as well as calculation shows that in 2300-volt primary distribution circuits during thunder storms, potentials of short duration of the magnitude of hundred thousand volts are not infrequently produced by the setting free, by the lightning flash, of the bound charge of the atmospheric electrostatic field. The lighting transformers distributed over these circuits are not, and economically cannot be insulated to stand this voltage continuously. Hence, they must depend on the time lag of their oil insulation to stand the lightning voltage until it is dissipated or discharged by the lightning arrester. Inversely, the time lag of the lightning arrester must be so short, and its discharge rate so high, as to discharge the lightning voltage in a time less than the time lag of the transformer, bringing the voltage down to values safe for the transformer.

From the pnenomenon of time lag it results that the rate of voltage application has a discriminating effect. Suppose two insulations are used in parallel, the one of lower dielectric strength but higher time lag than the other. (As for instance, the oil insulation of a transformer and the surface air insulation of its entrance bushings). At very rapid voltage application, the voltage may rise beyond the dielectric strength of the stronger insulation of shorter time lag, in less than the time lag of the weaker insulation, and the former thus punctures, while with a slower voltage application the weaker insulation of greater time lag would puncture. Thus under lightning conditions, the transformer bushings may flash over, short-circuit and blow the fuses without any damage to the transformer, while under high potential test the oil insulation may puncture far below the voltage at which the bushings flash over.

To illustrate the importance of this discriminating effect of rate of voltage rise: The insulation of high-voltage transmission lines depends on sectional or string insulators. With these, it is very desirable that in case of a failure the insulator disks should flash over rather than puncture. The transmission insulators thus are designed for a puncture voltage much higher than the flashover voltage, and 60 cycles tests as well as high-frequency tests show such insulator strings to flash over and not to puncture. Nevertheless lightning punctures them not infrequently. The time lag of flashover is greater than that of puncture, due to the relatively high capacity of the insulator string. Therefore, under lightning conditions, that is, very rapid application of voltage of considerable energy, the voltage reaches puncture values in less than the time lag of flash over, while in low-frequency tests flashover limits the voltage, and in high-frequency tests the rate of voltage application is reduced, that is, the wave front flattened by the capacity of the insulator string, unless there is very great power back of the voltage, so as to maintain it against the short-circuiting effect of the capacity of the testing appliance (so-called "lightning generator").

TIME LAG OF SOLID AND LIQUID INSULATION

When the mechanism of the dielectric breakdown of the insulation consists of a cumulative thermal, mechanical or chemical effect, as discussed above, it inevitably involves a time lag, and usually a considerable one.

This time lag may be as short as a fraction of a second, such as occurs in the electrostatic field of an oil gap sucking in a moisture droplet, stretching it into a filamentary conductor, bridging between the electrodes and flashing over. In the formation of hot spots in solid insulation, it may take minutes and hours, until the process leads to the final disruption, and may extend to days and years in the chemical action of ionization, etc., so that here the time lag of the insulation breakdown gradually merges into the aging or deterioration of the insulation.

For instance, under overload a cable may get overheated and some hot spots form in the insulation and finally, after some hours, lead to a puncture. If however the load is taken off before the final breakdown, the cable cools and the hot spots disappear and leave the insulation undamaged, and we say that the cable has been saved by the time lag of insulation and no harm done by the temporary overload, because in this case of approaching thermal breakdown the process is reversible.

If however, under high-voltage test, ionization occurs in the cable and by chemical action begins to destroy the insulation, we also may say that the cable is saved by the time lag of chemical breakdown, if the overvoltage is taken off before failure has occurred. However, as this process is not reversible, some damage has been done, and at the next overvoltage further damage is done and adds itself, until final disruption occurs.

In some respect, we may thus consider the gradual and slow aging and deterioration of the insulation during the years of use, which limits its final life, as a progressive breakdown, and the entire life of the insulation as the time lag of breakdown; however, this rather extends the meaning of the term.

Electrical Time Lag

Air apparently has no time lag, at least, no appreciable time lag, and the dielectric breakdown of the air gap between spheres at a distance greater than the energy distance and less than the corona distance, with a negligible impedance between spheres and the source of voltage supply, is as nearly instantaneous as can be measured. Time lags (or impulse ratios) with air gaps therefore are due either to the configuration of the gap, or due to the conditions of the supply circuit.

A sphere gap with a considerable resistance in series has an appreciable time lag, the greater, the higher the resistance, due to the time required to charge the capacity of the spheres over the resistance.

By shunting an instantaneous sphere gap by a small capacity C, with a non-inductive resistance r in series, any desired time lag can be produced by the proper choice of C and r, and such a combination thus forms an adjustable time gap, convenient for the testing of time lags.

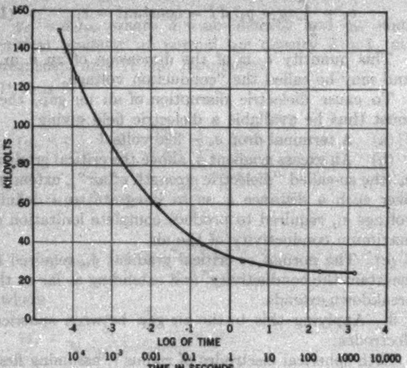

Fig. 3

This raises the question of measuring the time lag. The usual way of expressing a time lag is by the impulse ratio. This however is not a constant, but a function of the duration of the voltage application, as shown by the curve Fig. 3, which gives the impulse ratio of a cable, for duration of the voltage application from continuous down to a microsecond.

An approximate expression of the electrical time lag by a constant may sometimes be given as follows: Assuming that proportionality exists,—a usual assumption with such phenomena—that is, that the change of the condition in the dielectric is proportional to the difference between the existing condition and the final condition at disruption. Then the phenomenon would be exponential, that is, start at maximum rate and gradually decrease, thus theoretically take infinite time. It is customary to consider as the duration of such an exponential phenomenon (for instance the duration of an exponential transient) the time which would be required if the phenomenon continued to its end at constant initial intensity. Assuming this for the time lag, then the duration T would be the time lag at impulse ratio $U = \epsilon$.

In the adjustable time gap consisting of an instantaneous sphere gap in air, shunted by a capacity C and in series with a resistance r, the duration T, as defined above, then is:

$$T = r\,C,$$

thus has a physical meaning as the rate of condenser charge.

The capacity of two spheres of ratio R, at distance l from each other, is, if:

$$b = \frac{l}{R}:$$

$$C = \frac{r}{8\,S^2}\,10^9 \left\{ 3 - \frac{l}{R} + \sqrt{9 + 2\frac{l}{R} + \frac{l^2}{R^2}} \right\}$$

$$= 0.139\,R\,10^{-12} \left\{ 3 - b + \sqrt{9 + 2\,b + b^2} \right\}$$

and a sphere gap of 12.5 cm. spheres, at 5 cm. distance, in series with 1000 ohms resistance, would have a time lag of 0.005 microseconds.

Two 25-cm. spheres in air, at 0.7 cm. distance, have a time lag of about 0.5 microsecond. (Within the energy distance).

Two 1-cm. spheres in air at 3 cm. distance, have a time lag of about 1 microsecond. (Corona)

A needle gap of 5 cm. in air has a time lag of the magnitude of 1 to 2 microseconds.

A sphere gap of 2 mm. in oil, between 2.5 cm. spheres, has a time lag of about 20 microseconds.

Whether, and how far and in what manner the time lags depend on configuration, size, voltage, etc. is still largely unknown, and the entire field, though of high importance, is almost entirely unexplored.

Appendix

The Dielectric Breakdown of Air as a Conductivity Phenomenon

1. Assume that the dielectric breakdown of air, and the conductivity produced by it, are due to conducting particles or carriers, that is, particles which carry electric charges and thereby conduct the current.

We may assume that in free air, there are always some such carriers present. In an electrostatic field, they are set in motion with an acceleration a proportional to the dielectric field intensity g, and therefore in the time t acquire the velocity v, in a uniform field:

$$v = g\,t \tag{1}$$

In a non-uniform field, the velocity would be given by:

$$v = \int g\,d\,t \tag{2}$$

Let us assume that there is a critical velocity v_0, at which these carriers produce additional carriers by collision with the atoms or molecules of the air or gas.

The time t_1 required to reach collision velocity v_0 then is,

$$t_1 = \frac{v_0}{g} \tag{3}$$

and the distance, within which this is reached:

$$l_1 = \frac{t_1\,v_0}{2} = \frac{v_0^2}{2\,g} \tag{4}$$

thus:

$$l_1\,g = \frac{v_0^2}{2} = \text{const.} \tag{5}$$

or, in a non-uniform field:

$$\int g\,d\,l = \frac{v^2}{2} = \text{const.} \tag{6}$$

This is of the dimension *voltage*. That is, a definite voltage,

$$e_0 = \frac{v_0^2}{2} \tag{7}$$

is required to raise the free conducting carriers of the air to the collision velocity v_0, at which they produce additional carriers from the atoms or molecules.

The voltage e_0 may be considered as of the nature of a terminal drop, which must be present before the formation of conducting carriers can take place.

It then would be the minimum voltage, approached for minute air gaps:

$$e_0 = 320 \text{ volts.}$$

2. As the conductivity of the air rapidly disappears with the withdrawal of the dielectric field, these conducting carriers must be very short lived.

Let then g_0 be the voltage gradient, at which the rate of production of conducting particles equals their rate of decay, and the number of conducting particles, that is, the conductivity of the air, remains constant (whatever may be its value, whether high or low).

$$G = g - g_0 \tag{8}$$

then is the rate of increase of conducting particles, or the "ionizing force," and

$$\begin{aligned} c &= G\,l \\ &= (g - g_0)\,l \end{aligned} \tag{9}$$

or in general, in a non-uniform field:

$$c = \int_0^l (g - g_0)\,d\,l \tag{10}$$

is the total production of conducting particles in the distance l, or the *ionic density* or *conductivity* produced by the gradient g in the distance l.

If we assume that the conducting particles or carriers are produced from the gas molecules by collision, the ionic density or conductivity c must reach a finite maximum value, at which "complete ionization" of the gas has occurred, and the conductivity thus reached its maximum or "short circuit" value c_0.

Dielectric breakdown thus would be characterized by

$$c_0 = (g - g_0)\,l_0 = \text{constant} = e_1 \tag{11}$$

or, in a non-uniform field

$$c_0 = \int_0^l (g - g_0)\,d\,l = \text{constant} = e_1 \tag{12}$$

This quantity e_1 is of the dimension of an e. m. f. and may be called the "conduction voltage."

To cause dielectric disruption of an air gap, there must thus be available a dielectric field giving

(a) A terminal drop $e_0 = 320$ volts.

(b) An excess gradient g, above the critical gradient g_0 (the so-called "dielectric strength of air"), extending over such a distance l_0 as to integrate to a definite voltage e_1, required to produce complete ionization or maximum conductivity of the air.

(c) The normal or critical gradient g_0, required to maintain the conductivity, and extending as far as the breakdown extends.

3. Applying this to the air gap between spherical electrodes:

With spherical electrodes of radius r, assuming first, that the other electrode is of such shape or so far away as not to distort the field, if we denote

g_r the gradient at the surface of the sphere of radius r.
r_0 the radius, at which the gradient has decreased to g_0

we have, at radius x,

$$g_x = \left(\frac{r_0}{x}\right)^2 g_0 \tag{13}$$

and

$$\begin{aligned} c_0 &= \int_r^{r_0} (g - g_0)\,d\,x = g_0 \int_r^{r_0} \left\{ \left(\frac{r_0}{x}\right)^2 - 1 \right\} d\,x \\ &= g_0 \left(\frac{r_0^2}{r} + r - 2\,r_0 \right) = \frac{g_0}{r}\,(r_0 - r)^2 \end{aligned}$$

it follows, solving for r_0

$$r_0 = r \left\{ 1 + \sqrt{\frac{c_0}{g_0\,r}} \right\} \tag{14}$$

$$= r \left\{ 1 + \frac{m}{\sqrt{r}} \right\}$$

and the voltage gradient at the surface of the electrode sphere is

$$g_r = g_0 \left(\frac{r_0}{r} \right)^2$$

$$= g_0 \left\{ 1 + \frac{m}{\sqrt{r}} \right\}^2 \qquad \textbf{(15)}$$

or approximately

$$g_r = g_0 \left\{ 1 + \frac{2\,m}{\sqrt{r}} \right\} \qquad \textbf{(16)}$$

where

$$m = \sqrt{\frac{c_0}{g_0}} \qquad \textbf{(17)}$$

$g_0 = \delta\, g_{00}$, where δ = air density and g_{00} equals critical gradient at normal air density $\delta = 1$, at 0 deg. cent. and 76 cm. barometer.

It thus is

$$r_0 = r \left\{ 1 + \frac{m_0}{\sqrt{\delta\, r}} \right\} \qquad \textbf{(18)}$$

$$g = g_{00}\, \delta \left\{ 1 + \frac{m_0}{\sqrt{\delta\, r}} \right\}^2 \qquad \textbf{(19)}$$

$$= g_{00}\, \delta \left\{ 1 + \frac{2\,m_0}{\sqrt{\delta\, r}} \right\} \qquad \textbf{(20)}$$

where

$$m_0 = \sqrt{\frac{c_0}{g_{00}}} \qquad \textbf{(21)}$$

and c_0 is the (total) conduction or ionizing voltage, g_{00} the critical gradient at unit air density, and δ the air density. Equations (**14**), (**16**), (**18**) and (**20**) are the equations of energy distance and of surface gradient, given by Peek, and are here derived from the conception of the dielectric breakdown as a conductivity phenomenon.

4. With small gaps between plates or spheres of a diameter large compared with the gap length, the dielectric field and the gradient are uniform throughout the gap, and the maximum gradient, or gradient at the electrode surface, equal to the mean gradient throughout the gap.

$$g = e/l$$

and the conduction voltage required to produce maximum conductivity is by (**11**) given as

$$e_1 = (g - g_0)\, l \qquad \textbf{(11)}$$

(a) Assume the gap length is so small, that the ionic density increases throughout the entire gap, that is, the gap length l is less than the length l_0, within which the density of conducting particles increases from the low density existing in the free air, to the maximum density, where maximum conduction is produced, and the dielectric gradient thereby has dropped to g_0, so that no further increase of ionization occurs.

The conductivity throughout the gap then varies from the low initial value to the maximum value of gradient g_0, and its average is half the maximum value, and the average gradient throughout the gap thus is twice the minimum value g_0.

That is, in a short gap, the average gradient must be $2\,g_0$, and the voltage consumed by the gradient (which might be called "stream voltage," in distinction from the voltage e_0 of (**7**) or the "terminal drop") thus is:

$$e_1 = 2\,g_0\,l \qquad \textbf{(22)}$$

this gives as the total voltage consumed by the dielectric breakdown of a very small gap of length l,

$$\begin{aligned} e &= e_0 + e_1 \\ &= e_0 + 2\,g_0\,l \end{aligned} \qquad \textbf{(23)}$$

b. Assume the gap length l to be greater than the ionizing distance l_0, that is, the distance in which equation (**1**) produces maximum conductivity. Dielectric breakdown of the gap then requires the voltage

$$e_0 = (g - g_0)\, l_0$$

as the voltage producing the conducting stream, or the "terminal drop" (though not limited to the terminal alone).

$$e_1 = 2\,g_0\,l_0$$

as the voltage consumed by the average gradient $2\,g_0$ in the distance l_0, in which the conductivity is produced by (**1**)

$$e_2 = g_0\,(l - l_0)$$

as the voltage consumed in the rest of the gap $(l - l_0)$ by the gradient g_0 required to maintain conductivity.

The total voltage thus is,

$$\begin{aligned} e &= e_0 + e_1 + e_2 \\ &= e_0 + 2\,g_0\,l_0 + g_0\,(l - l_0) \\ &= e_0 + g_0\,l_0 + g_0\,l \\ &= E_0 + g_0\,l \end{aligned} \qquad \textbf{(24)}$$

where

$$E_0 = e_0 + g_0\,l_0 \qquad \textbf{(25)}$$

is the total excess voltage over that consumed by the critical gradient g_0 in the gap l.

c. Assume now that the gap is larger and the field is not uniform.

As g_0 is the minimum gradient of ionic conductivity, that is, the gradient required to maintain constant conductivity, the conductivity of the gap space can extend only up to the distance from the electrodes, within which the gradient g is greater than g_0.

The dielectric breakdown thus extends up to the distance of the gradient g_0, and within this distance l', the voltage consumed must exceed the value of breakdown gradient g_0 *times* distance l', by the total ionizing voltage E_0 of (**25**), that is, must be

$$e' + E_0 + g_0\, l'$$

or, since

$$e' = \int_0^{l'} g\, d\, l$$

it is

$$E_0 = \int_0^{l'} (g - g_0)\, d\, l = e_0 + g_0\, l_0 \qquad (\mathbf{26})$$

that is:

The voltage c in paragraph 3, which gives the constant m in Peek's equation (**14**),etc., is not merely the excess voltage e_0 required to produce the conducting stream, but is the sum of this voltage plus the excess voltage $g_0\, l_0$ required by the excess gradient in that part of the field, in which the conductivity has not yet reached full value.

$$E_0 = c \qquad (\mathbf{27})$$

5. Consider now numerical values.

From test values of small gaps between large spheres and between a large sphere and a plate, we find,

$$e = 320 + 59\, l$$
$$= e_0 + 2\, g_0\, l$$

This gives

$$e_0 = 320 \text{ volts}$$

$2\, g_0 = 59$ is in good agreement with $g_0 = 30$, as the temperature correction has not been applied.

For medium gaps, (case 4b,) the tests give,

$$e = 2370 + 30\, l$$

Thus

$$E_0 = 2370 = c_0$$

This gives by (**17**)

$$m = \sqrt{\frac{2.370}{30}} = \sqrt{0.079} = 0.28$$

in good agreement with Peek's value: $m = 0.27$ or

$$g = g_0 \left\{ 1 + \frac{0.56}{\sqrt{r}} \right\}$$

while Peek gives

$$g = g_0 \left\{ 1 + \frac{0.54}{\sqrt{r}} \right\}$$

The limit l_0 between equation (**23**) and (**24**) is

$$e = e_0 + 2\, g_0\, l = 0.320 + 60\, l_0$$
$$e = E_0 + g_0\, l = 2.370 + 30\, l_0$$
$$l_0 = 68 \times 10^{-3} \text{ cm.}$$

which is in close agreement with the experimental data given in the tables and figures, of $l_0 = 69$.

In Figs. 4, 5 and 6 are given numerous test values taken under various conditions, as indicated, with the theoretical curves shown by the drawn lines.

6. The polarity of the charge of the conducting particles or carriers, which give the dielectric breakdown of air gaps at atmospheric pressure, can be determined by the action of unsymmetrical gaps at direct voltage. Consider an unsymmetrical air gap such as that between a needle point and a plate, or between a small and a large sphere. In such a gap, with increasing applied voltage, the critical gradient g_0 is first reached and thereby the production of conducting particles started at that terminal, which has the greater curvature and thus the higher gradients, the point or small sphere. If then the polarity of this terminal is that of the conducting carriers, these move outwards away from the terminal, towards the opposite terminal, and thereby short circuit the gap and cause disruption as soon as the average gradient is sufficiently high to maintain the conducting stream across the gap. If however, the larger terminal—plate or large sphere—is of the polarity of the carriers, a higher voltage across the gap is required before the critical gradient is exceeded and the swarm of conducting carriers started at this larger terminal and disruptive discharge occurs. At the smaller terminal, in this case, the formation of conducting carriers has started already at lower voltage, but these carriers do not move away from the terminal and across the gap, and thus do not cause

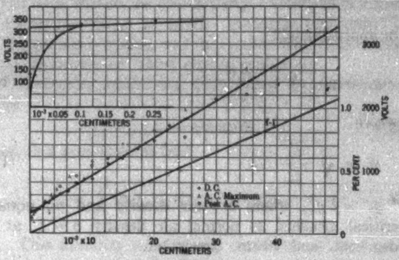

FIG. 4

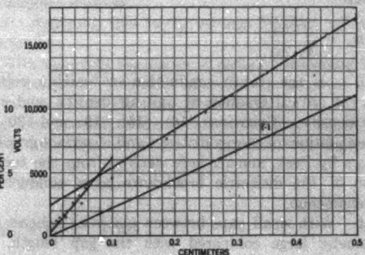

FIG. 5

break down, but move towards the terminal near which they started, and form a local corona in the space at and near this terminal.

Thus with unsymmetrical gaps, the direction of the polarity of the applied voltage should make a difference in the disruptive voltage, and this voltage be higher with the voltage in one direction, than with the voltage in the opposite direction. As the average gradient in the space in which the conducting particles are formed, was shown to be about twice the breakdown gradient of air, the disruptive voltage of such an unsymmetrical gap should be about twice as high with the voltage of the carriers on the large electrode, than with the voltage of the carriers in the opposite direction. Fig. 1 shows this, and also shows that the disruptive voltage is lower with the small terminal (the needle point in this case) positive.

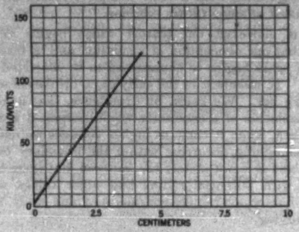

Fig. 6

It follows from this, that the conducting particles or carriers, which carry the disruptive discharge in air at atmospheric pressure, are *positively* charged, that is, they may be the *positive* ions, but cannot be the negative *electrons.*

7. Dielectric conduction thus may be either ionic, or electronic. The relation possibly is the following:

At atmospheric pressure the voltage required for ionic conduction, that is, disruptive discharge, is much lower than the voltage required for electronic conduction, and the conduction, that is, the disruptive discharge through air, is ionic, that is, by positive carriers. With decreasing air pressure, the voltage required for ionic conduction decreases, approximately proportional to the air pressure, and the conduction thus remains ionic. At some low pressures a minimum value of voltage (or rather voltage gradient) is reached. Then, with further decrease of air pressure, the voltage of ionic conduction again increases, due to the decrease of carriers, and finally passes beyond the voltage of electronic conduction. The phenomenon then changes its character, ionic conduction ceases and electronic conduction begins. At high vacuua, the conduction thus is electronic.

There is a difference between ionic and electronic conduction: Ionic conduction permits practically unlimited current, and when it begins, thus short-circuits the gap and drops the voltage. Electronic conduction however is limited in current, and when it occurs, the voltages can be maintained and still further raised. At such high vacuua, where the voltage of ionic conduction has risen beyond that of electronic conduction, and the conduction become electronic, the voltage can be raised beyond the value of ionic conduction through the traces of residual gases, and then ionic conduction again begins and "short-circuits" the electron tube, thus limiting electronic conduction. Therefore a practically perfect vacuum is needed for pure electronic conduction in the modern high-power electron tube.

On the other side, with pressures higher than atmospheric, the disruptive voltage increases with increasing pressure, and approximately proportional thereto, as shown by experiment. It may be expected then, that at some high pressures the voltage of ionic conduction increases beyond that of electronic conduction, and the latter limits the increase of disruptive strength of gases at high pressures. While this field has been little investigated, experiments indicate that the dielectric gradients of air and gases with increasing pressures reach a limiting value somewhere at 1000 kv. per cm.

Discussion

R. W. Sorensen: My first point is in connection with the statement that we use our insulations under stresses which rarely exceed the breakdown voltage of air, though tests show a strength of 10 to 20 times that of air for many of the insulations used. This plea for a more strenuous use of insulating materials is interesting in the light of a request in another paper given this morning, in which the author recommends a decrease in the test voltage applied to transformers with one terminal grounded.

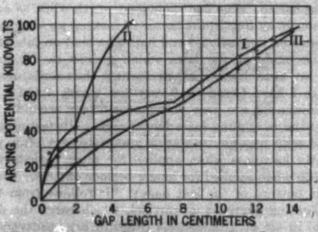

Fig. 1—Arcing Potential vs. Gap Length

Between Needle and Sphere, 12.5 cm. dia.
Curve I—Direct Current, Sphere Negative
" II— " " Positive
" III—Alternating Current, Maximum Values.

In Fig. 1 of the paper the curves show the interesting fact that a given potential will cause spark over between a point and a sphere for much greater spacing when the sphere is negative, than will be the case with the needle points negative and the sphere positive. The results shown in Fig. 3 have been duplicated by Messrs. Otis and Mendenhall, two students at California Institute of Technology as shown by the curves in Fig. 1 accompanying this discussion. In making these tests the high-voltage alternating current was rectified by means of a two-segment commutator driven by a two-pole synchronous motor.

The a-c. curve is very much like that obtained by Hayden and Steinmetz, but the d-c. curves differ in shape because of the pulsating current obtained with the commutator, whereas Hayden and Steinmetz had very steady current delivered by a four kenetron rectifier. An explanation of the dip in these curves would be very interesting.

In John S. Townsend's "Ionization of Gases by Collision" is found this statement "When the point is negative, the strong field is near the negative electrode, so that the potential required to produce a discharge is less than when the point is positive." Also in "Conduction of Electricity through Gases," 1906 edition, by J. J. Thompson on page 498 we find this statement "............this minimum potential depends upon the sharpness of the point, the pressure and nature of the gas, and the sign of the electrification of the point, being less if the point is negatively than if it is positively electrified." Hence we have from these authorities statements which, at first reading, appear quite contrary to those given in this paper.

There is, however, some confusion as to the definition of the term "spark discharge," in the texts referred to the term does not seem to apply to an arc current and the voltage required to cause it, but means the point at which a leak discharge only and not a complete arc over occurs between the electrodes. In fact in one treatise on the subject of ionization this definition appears: "Sparking potential may be defined as the potential which is required to maintain a very small current in the gas."

Dr. Millikan has explained the Hayden-Steinmetz results on the basis of the increased difficulty experienced in extending the ionization envelope with negative points, as compared to that phenomena for positive points, hence the required higher potentials for a breakdown over a given distance.

In discussing the mechanism of "thermal breakdown of dielectrics" the paper follows the work of Mr. Wagner as published in the JOURNAL for December 1922, but Wagner does not deduce the same law. Also some tests we have made in our laboratory do not conform the deduction that puncture voltage is proportional to the square root of the thickness of dielectric.

To emphasize "time lag" is indeed worth while as our experience shows it an important factor in making an analysis of voltage stresses on insulations, and indicates that an intensive study should be made of the laws which govern it. It may be of interest to note here that in testing thousands of porcelain insulators I have found that very few which stand a potential test at minimum arc over voltage for fifteen seconds, fail when the potential is applied for a longer time. In testing apparatus insulated with organic insulating materials very often potentials apparently harmless when applied for short periods will cause breakdown when applied for longer periods of time.

At the bottom of the last page: "It follows from this, that the conducting particles or carriers, which carry the disruptive discharge in air at atmospheric pressure, are positively charged, that is, they may be the positive ions, but cannot be the negative electrons."

If an atom is ionized and you have your positive ions turned loose what becomes of the negative ions, where do they go?

Then follows: "At atmospheric pressure the voltage required for ionic conduction, that is, disruptive discharge, is much lower than the voltage required for electronic conduction." This is explained further on in the paper. If you have a perfect vacuum, as stated, how would you have the pure electronic conduction? I presume tubes in which there are heated filaments have been used.

C. P. Steinmetz: The paper deals with a subject which has assumed, in the last year, a still greater importance than it had before, that is, the problem of high-voltage insulation and mechanical breakdowns.

This is such a vast field and so much work has and is being done, and can be done, that the paper must necessarily be only a general part of the preliminary announcement of the results and investigations, with a more complete record of results to be announced at some future occasion.

There is, however, one feature which begins to get clear, namely, that our conception of insulation and of breakdown of insulation again begins to change and to be subjected to multiplication. To members in the early days, insulation was merely a boundary bar. We knew, by experience, that a tenth of an inch of insulation of a conductor would protect it against 2300 volts. Then, when it came to higher voltages we realized that there is something occurring within the boundary bar, of importance, and it is not merely the material, but there is a dielectric field with potential radius and other things within the boundary bars which require consideration and study, and which we are studying.

Now, it seems that our views are just beginning again to get a multiplication with respect to at least the failure of insulation and the mechanism of breakdowns. It seems to be clear that the mechanism of breakdowns, under the failure of insulation on high voltage is a phenomena of instability. In other words, it is not that insulation fails, that dielectric breaks down, when electric stresses are beyond limits and value, but it is said that under conditions very much lower than those gradients in lightning conditions of instability occur which gradually brings about the multiplication and changes leading ultimately to a breakdown of insulation. It is, therefore, a condition of instability of constants of material which instability brings about, largely upon lesser changes, which leads either to destruction or breakdown. Therefore, the mere reduction or stress on the insulation is not a factor which saves breakdowns, but that the new problem of insulation seems to assume the shape of arranging of designs in the dielectric field of insulation, so as to get the condition of stability and not instability. That is the future which seems to impress itself upon us. The more we study the problems of insulation we find that it is not existing stresses that cause this, but largely it is the result of stability, or instability.

Now, that is not only true in solids, but probably in the air. In this respect we could point out the same idea that the discharge makes its own gradient. Now, you have the same conception there, that it is the discharge which is taking place in the dielectric field about conditions which are unstable. We all know that if we had 2,000,000 volts spread over, I don't know how many thousand inches, there could be a gradient that would be so low there would be no puncture and for that discharge under such conditions. By the discharge making its own gradient there would be produced a dielectric condition of instability which would finally lead to self-destruction.

Waterwheel Generators and Synchronous Condensers for Long Transmission Lines

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Review of the Subject.—*It is the intention of this paper to bring out some of the special conditions which affect the operation and design of machinery for long transmission lines and the discussion is confined largely to waterwheel generators and synchronous condensers.*

Under the subject of generators, the following points are discussed: Leading current drawn by transmission lines and its effect upon the stability and mechanical design of generators; characteristic curves of generators designed for operation at leading power factor; special winding connections which increase the capacity of generators for leading power factor operation for temporary periods without increasing their weight and cost; description of general construction and ventilation of a 28,000-kv-a. vertical waterwheel generator.

The following points are discussion with reference to synchronous condensers: Operation at leading and lagging power factor and how it affects the stability cost and weight of the machines; special winding connections and their effect on stability; general mechanical construction with particular reference to damper winding design and the use of sheet steel end bells; the importance of losses and a curve showing values it is possible to obtain on machines with the latest improvements in design; starting kv-a. and how it may be minimized by the use of oil pressure in the bearings.

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Synchronous Condensers. (40 w.)
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Losses. (450 w.)
Starting Kv-a. (225 w.)

WHILE the design of waterwheel-driven generators and synchronous condensers for long transmission lines is, in general, the same as for other applications, there are often special conditions imposed on machinery for long transmission lines which make the design special and of greater cost and, in the case of large high speed units, often difficult. A better knowledge of these operating and design difficulties by both operating and designing engineers might enable them to modify the conditions in such a manner as to obtain a lower cost and better operating installation as a whole. Some of the special conditions applying to the design of machinery for long transmission systems are discussed in this paper with this idea in view.

LEADING CURRENT AND STABILITY OF GENERATORS

The effect of the leading current taken by a long transmission line upon a generator is shown by curves in Fig. 1. Curves 1, 2 and 3 represent the saturation curves for no-load, full-load, 0 per cent power factor lag and full-load 0 per cent power factor lead, respectively. The shape of curve 3 below the point *c* is of little practical value and is out of the range of practical operation. Assuming that an external power factor of 0 per cent lead could be maintained and no losses in the circuit the curve 3 would extend down to the point *e* and have a shape similar to curve 2. This assumption, of course, never exists. It would mean condenser capacity approaching infinity and no resistance. In actual practise the resistance and losses in the circuit would tend to give curve 3 a shape indicated by *a d f*. However, this means that the machine must produce torque and that its power factor is not 0. With such low excitation as indicated by the point *d* even a salient pole generator cannot supply the required torque for the losses of the machine and line and will "slip a pole" or "pull out". Therefore, in practical operation curve 3 cannot follow either the shape *a d e* or *a d f* below some point such as *c*. If in practise

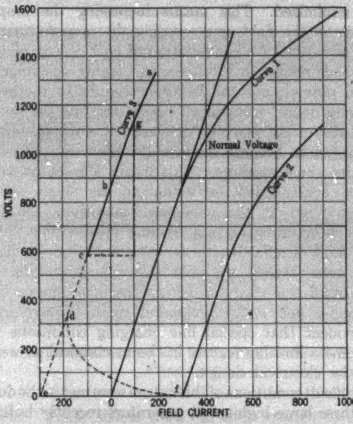


FIG. 1—REPRESENTATIVE SATURATION CURVES

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

it is attempted to operate the machine on curve 3 below point *c*, the generator will slip a pole as stated above and the excitation which was negative will become positive with respect to the voltage on transmission line (since it is reversed) and the voltage will rise from the value *c* to the value *g*. This, of course, is a dangerous condition and care must be taken in the design of generators which must supply high values of leading current to prevent the necessity of the field excitation being reduced to such low values that the generators will not have sufficient synchronizing power or stability. It is not considered good practise to go below zero excitation even on salient pole machines where the irregularity of the magnetic circuit permits the development of torque even at reversed excitation. On machines with uniform reluctance of the magnetic circuit in the rotor, such as steam turbine generators, stable operation cannot be had at any reversed field excitation. The term "stability" is rather general, but it is used here to refer to the ability of synchronous apparatus to remain in synchronism without hunting or pulling out of step.

EFFECT ON MECHANICAL DESIGN

To increase the stability of generators, supplying leading current the field, strength must be increased with respect to that of the armature. In other words, the design of the machine must be such that the effect of the leading current in the armature on the field will be minimized. This means increasing the ampere-turns on the field or decreasing the ampere-turns on the armature or a combination of both. Any of these methods results in a heavier and higher cost machine than normal, but the latter procedure or method is usually more economical. Not only is the weight and size of the machine increased as a whole, but the rotor parts become abnormally large in proportion to the whole machine. This means a more expensive rotor construction to withstand the high stresses encountered at the overspeed condition. On large high speed units this may become quite serious. After the limit of axial length as determined by ventilation has been reached, the only other alternative to meet abnormal line charging conditions is to increase the diameter and hence peripheral speed. Therefore, it is evident that special line charging conditions may become a limiting feature in the mechanical as well as in the electrical design.

Difficult conditions of this nature were met in the design of three large high-speed generators recently built for the Southern California Edison Company. So far as the writer knows, these generators are the largest waterwheel machines ever built to run at so high a speed. They have 50-cycle ratings of 28,000 kv-a. and 33,500 kv-a. at guaranteed temperature rises (by detector) of 60 deg. cent. and 90 deg. cent., respectively, at 428 rev. per min., and they have 60-cycle ratings of 31,300 kv-a. and 37,500 kv-a. at guaranteed temperature rises of 60 deg. cent. and 90 deg. cent. respectively at 514 rev. per min. They are required to meet an overspeed of 85 per cent at either frequency. At the 60-cycle overspeed (951 rev. per min.) the peripheral speed of the rotor is nearly 28,000 feet, or over 5 miles per minute. This is unusually high for a salient pole machine (although quite common in cylindrical rotors of steam turbine driven generators) and a very careful design of the pole and spider proportions was necessary in order to equalize the rotor stresses and work the material in the most effective manner. The rotor spider is built of rolled steel plates, approximately 2½ inches thick carried on a through shaft. A photograph of the spider is shown in Fig. 2. The poles are built of 1/16-inch rolled steel punchings which are riveted and bolted together between cast steel end plates. This built-up construction of the rotor spider insures material with uniformity and reliability which is unquestioned. The construction

Fig. 2—Plate Spider for 28,000 Kv-a. Vertical Waterwheel Generator

Fig. 3—Rotor of 8125 Kv-a. 6600-Volt, 50-Cycle, 600-rev. per Min. Waterwheel Generator

of the rotor is similar to that shown in Fig. 3, except that two coil braces were necessary between poles on account of the high speed and great length of the field poles and coils.

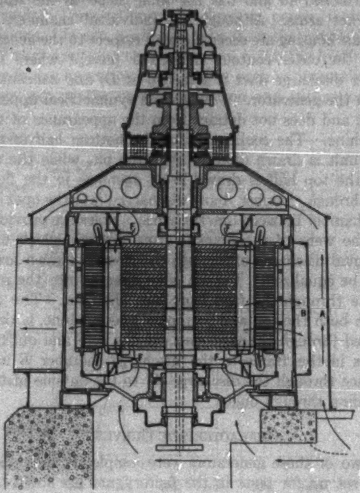


Fig. 4

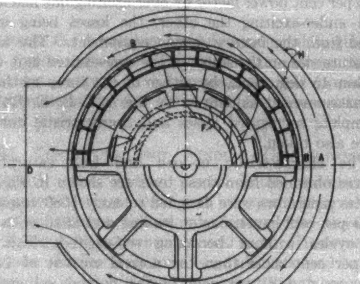


Fig. 4a

The field coils are made of copper strap formed on edge. The bulge or swell at the corners caused by the bending process was pressed out by means of a hydraulic press. In this operation steel plates were placed between turns as shown in Fig. 5 to prevent the skewing of the coils. After being smoothed up by filing where necessary the individual turns were thoroughly coated with an air drying shellac and permitted to dry. Two insulating strips of asbestos each coated on one side with shellac were then placed between turns and the coil heated to near a red heat (by circulating current) in order to drive out all solvent and volatile matter. During this process the coil was held in shape by heavy metal sizing plates, pressed to size and allowed to cool. Particular attention was paid to the cell insulation between the coil and pole. It is made of several thicknesses of 10-mil flexible and hand built mica sheets. These sheets were cut wide enough to cover the inside of the coil and to be turned down over the top and bottom edges as shown in Fig. 5a. The mica is the darker material extending above the coil and is reinforced all around by one thickness of heavy asbestos cloth which is indicated by the light portion of the material extending above the coil. Both the asbestos cloth and mica are turned down over the top and bottom edges of the coil, but the

Fig. 5—Pressing Out the Swell or Bulge at the Corners of a Strap Field Coil

Fig. 5a—Assembling of Coil Insulation on Strap Wound Field Coil

asbestos cloth was made sufficiently long to be turned back under and between the mica and the top and bottom of the coil. The cell was then given a heavy coat of bakelite and moulded to the coil while heated and held in place under pressure with metal plates. In this process the bakelite becomes hard and forms a solid moulded insulation around the coil. Furthermore, with the asbestos cloth turned back under the ends of the mica cell no raw edges are exposed. When the coils were assembled on the poles they were further protected by micarta washers at the top and bottom.

The drawings in Figs. 4 and 4A show a cross-section and plan view of the machine with exciter and Kingsbury thrust bearing, and a photograph of one unit without the air housing is shown in Fig. 6. The general construction of the machine is of conventional design. The lower guide-bearing bracket is insulated from the generator frame to prevent the circulation of bearing currents through the guide and thrust bearings. The lower end plate, supporting the lower finger and core punchings, is cast integral with the frame. This construction facilitates building and pressing the core.

FIG. 6—28,000 KV-A., 11,000-VOLT, 3-PHASE, 50-60 CYCLE, 428-514 REV. PER MIN. VERTICAL WATERWHEEL GENERATOR

It was necessary that all the cooling air (for both top and bottom of the machine) be brought in at the base of the generator. On low-speed machines which permit the use of open-type spiders through which the air for the upper part of the generators may be drawn,—this condition causes no difficulty. However, on machines such as these where the size of the rotor and the stresses make it advisable to use a solid plate construction, the cooling air obviously cannot be brought up through the rotating part. A special air housing as indicated in Figs. 4 and 4A has been used in this case for conveying the cooling air to the top of the machine and for collecting and discharging the hot air from the frame. The upper part of the housing is attached to and has the same slope as the upper bracket arms. Although the individual chambers of the air housing are eccentric with respect to the generator, the outer contour is circular (except where the outer discharge duct is attached at *D*) and concentric with the generator. This gives a symmetrical appearance and does not detract from the appearance of the machine. The cooling air for the bottom half of the the unit is drawn directly from the pit, while the air for the top half is drawn in at openings in the floor up through chamber *A* and discharged into the opening or chamber between bracket arms. Holes are provided in the webs of the bracket arms through which any unequal discharge of air into this space between arms can be equalized. The air goes directly into the machine through the fans from this chamber. The air from both the top and bottom of the machine, is then forced through the vent ducts in the core and out the holes in the frame to the discharge chamber *B* and thence through the discharge *D* to the outside of the building through ducts.

CHARACTERISTIC CURVES

Two of these generators were completely assembled and set up for tests in the factory, and are about the largest units ever to receive a complete factory test. With two units assembled, it was possible to obtain temperature tests at normal voltage, current and frequency at 0 per cent power factor by over-exciting one machine and under-exciting the other, the losses being supplied from the factory testing equipment. The temperature rise on the stator of the over-excited unit was within 45 deg. cent., by either of the three methods of measurement as defined by the A. I. E. E. Rules. Complete segregated losses and characteristic curves were also taken.

The characteristic no-load and load saturation curves obtained from these tests are shown in Fig. 7. These generators were designed to carry 1680 amperes at 0 per cent power factor leading and 8910 volts at 50 cycles without becoming self-excited. This is 14 per cent more than the normal current of 1470 amperes and 81 per cent of the normal voltage of 11,000. They were also designed so that with a constant separate d-c. excitation corresponding to 2000 volts no-load, they would not be self-exciting above 8910 volts when delivering 1400 amperes 0 per cent power factor leading. Both of these conditions are unusually severe. Curve 3 shows the former condition and curve 5 the latter, and it may be seen from these curves that both conditions were met with a reasonable margin. The condition shown by curve 5 is the more severe and is, of course, the determining one.

If the generator had to operate at leading power factor only, it would be quite easy to make the machine stable without exceeding normal costs and weights. However, it is the lagging power factor operation that imposes the severe condition when the machine is especially proportioned for leading power factor conditions. It can readily be seen from the curves in Fig. 7 that anything that is done to make the leading power factor operation more stable, such as increasing the air gap or the normal induction and saturation, will increase the demand on the field winding at lagging power factor operation. This means an increase in the field temperature, or an increase in the field copper, or an increase in the size of the machine, or a combination of these.

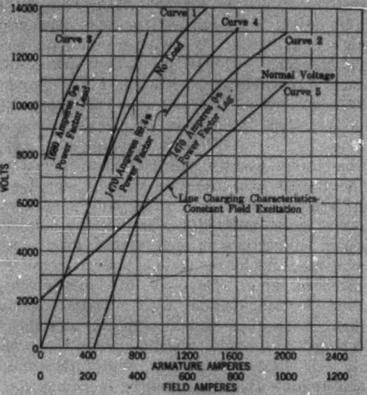
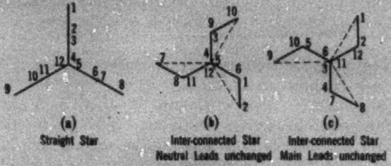


FIG. 7—TEST CURVES ON 28,000-KV-A., 11,000-VOLT, 3-PHASE 50-CYCLE, VERTICAL WATERWHEEL GENERATOR

SPECIAL CONDITIONS AND WINDING CONNECTIONS

Ordinarily, the leading power factor operation is encountered only at short and infrequent intervals when a transmission line is unloaded and the power factor becomes lagging when the load is on. However, in the case of a line built for a large ultimate load but that has a small initial load and generating capacity the power factor even at maximum kw. load of the generator may be leading. This means that the generator must be designed for a very stable operation at leading power factor and must also be able to take care of the lagging power factor operation in the future when the ultimate load builds up on the line. For the machine to deliver power under leading power operation requires that it be designed with more stability than when merely charging the line to insure that it will not pull out of step on account of the torque it must develop. Therefore, a unit for this class of service may become unduly large and expensive. In cases like this the use of a machine of normal design with some special connection of the armature winding, such as an inter-connected star, during the period of light load on the line is suggested. In this scheme, each leg of the ordinary star winding is opened at the mid-point and each half of each leg is connected in series with half of another leg which has a phase relation of 120 degrees with respect to it. Diagrams of this connection are shown in Fig. 8. Diagram (*a*) represents the standard star connection, diagram (*b*) represents the interconnected star with the neutral leads unchanged, and diagram (*c*) represents the interconnected star with the main leads unchanged. This is equivalent to reducing the armature conductors so that the voltage generated for a given flux will be 86.7 per cent of that for the straight star connection. Therefore, to get the same voltage, the field strength must be increased while the demagnetizing effect of the armature is decreased. The reactance is practically unchanged. So long as the power factor remains leading this expedient increases the stability of the machine without any extra duty on the field because of the inherently small field excitation required under such conditions. However, when the load on line increases to the extent that the power-factor becomes sufficiently low and lagging to overload the field with the machine operating at the high induction the armature must be reconnected for straight star and the machine operated under the normal lagging condition for which it was designed. With the lagging power factor there is, of course, no occasion or necessity for increased stability.

Such an expedient as the above scheme might, in some cases, be used on generators when charging the line on starting up the system, even though the normal operation be at lagging power factor. For instance, assume there are several generators on a system and it is necessary that each machine be able to charge the line. Instead of increasing the size of the machines to make them inherently of such proportions as to be able to charge the line it might be possible to use machines of normal or of less abnormal design by bringing out the leads on the machines to a panel in such a way as to permit the connection shown in diagram (*c*) Fig. 8 during the period prior to the building



(a) Straight Star

(b) Inter-connected Star Neutral Leads unchanged

(c) Inter-connected Star Main Leads unchanged

FIG. 8—INTERCONNECTED STAR

up of the load. After the load built up to the limit of the field capacity of this one machine, one or more of the other machines could be put on the line to carry the load while the first machine was disconnected from the line and the armature connection changed to the normal star connection. This machine could then be put back on the line, if desired. The feasibility of this method of operation will depend on local conditions. If the occasions for charging the line were very seldom and if the leads were brought out of the machine to a panel with a switching arrangement that would permit a quick change in connections, it is possible that it would be more economical and satisfactory in some cases to use this or some other special connection than to increase the size and cost of the generator to meet the special and infrequent line charging condition. Connecting the winding in interconnected star increases the line charging capacity of a normal machine about 30 per cent.

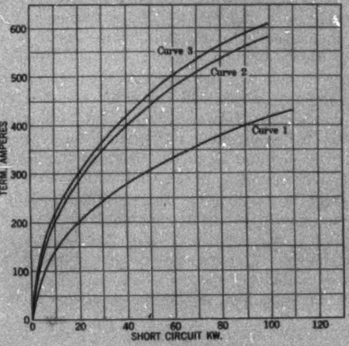


Fig. 9—Short-Circuit Loss Curves on 1080-Kv-a., 2400-Volt, 3-Phase, 60-Cycle, a-c. Motor

Care must be taken in the use of special connections of the winding such as interconnected star to insure the proper distribution and relation of the armature reaction set-up by the current in the armature winding. With the ordinary connection shown in diagram (*b*) or (*c*), Fig. 8, the armature reaction is not evenly distributed around the machine. This is equivalent to a resultant single-phase action and gives a negative sequence component to the rotation of the armature m. m. f. This results in excessive losses in the pole faces and damper winding, if any is used. However, by means of a special connection for interconnected star the desired result can be obtained with very little increase in losses. In this special connection the phase groups are split and the interconnection between phases made every pole. It results in a much more even and uniform distribution of the armature m. m. f. than does the usual interconnected star and hence does not produce the bad effects found in the latter connection. This difference in the distribution of the armature m. m. f. for the straight star, the usual interconnected star, and the special interconnected star can be readily seen by plotting the armature reaction or m. m. f. for the several conditions. The m. m. f. for the usual interconnected star is irregular in shape for a given pole and varies in shape and magnitude from pole to pole, while the shape of the m. m. f. for the special interconnected star is very similar to that of the straight star. This analysis is also borne out by actual test results of short-circuit loss on a 1080-kv-a., 900-rev. per min., 2400-volt, 60-cycle machine with the usual and special interconnected star windings as shown in Fig. 9 by curves 1 and 2, respectively. Curve 3 shows the short-circuit loss for a straight star connection. It is readily seen from these curves that there is quite an extra loss for the usual interconnected star and that there is very little additional loss over the straight star for the special interconnected star. The only objection to the special interconnected star is that the number of changes in connections is greater than for the usual interconnected star by the ratio of the number of poles in a given machine. Therefore, machines which are to operate for long periods with interconnected star and then changed permanently to the straight star should have the special connection, while those which operate only for short periods with interconnected star and require that the changes in connection be made quickly, and often, should have the simpler connection. This extra loss for short periods would not be serious and the standard connection would permit the 9 leads to be brought out of the machine to a panel for rapid connection as referred to above.

Overspeed and Flywheel Effect

A reasonable overspeed is, of course, necessary for any waterwheel generator. However, it is often possible to influence it considerably by the design of the waterwheel and it should be kept as low as the economical design of the waterwheel will permit. Unusually, high overspeed, especially in the case of large high speed machines, often necessitates difficult and expensive mechanical construction and may even require a reduction in the normal running speed.

There seems to be a tendency to base the hydraulic design on a standard speed regulation and corresponding flywheel effect, regardless of the magnitude of the probable load changes. In the large units and stations usually involved in long transmission systems sudden load changes seldom amount to any considerable percentage of the connected generator rating. Under such conditions a larger percentage regulation (with 100 per cent load change) can be permitted than in the case of smaller systems in which sudden load changes may be a large percentage of the connected generator rating. It appears to the writer that sufficient consideration is not given to actual operating conditions and that over conservative values of flywheel effect are sometimes specified on units which actually operate with very small changes in load. Flywheel effect, of course, does not become a factor in the generator design unless it exceeds the value determined by normal design. Abnormal flywheel effect always means increased cost. Either a separate flywheel must be furnished, the weight of the spider increased or the diameter of the machine increased. Even if the increased flywheel effect can be obtained at an increased diameter without increased weight, the larger diameter machine is generally more expensive. Also if the rotor weight is increased on a given diameter the mechanical parts, especially in the case of a vertical unit, must be made heavier and more expensive. In the case of large high speed units and where it is not possible to get the required flywheel effect by increasing the spider weight of a normal machine nor permissible to use a separate flywheel and it becomes necessary to increase the diameter, the stresses may increase to the extent that special and expensive construction or material may become necessary.

SYNCHRONOUS CONDENSERS

The great increase in the size and length of transmission systems in recent years has greatly increased the demand for, and importance of, synchronous condensers for correcting power factor and regulating and stabilizing the voltage of the lines.

LEADING AND LAGGING OPERATION

The majority of condensers are designed primarily for leading power factor or over-excited operation. However, on long lines where the leading current drawn by the line may be sufficient to increase the receiver voltage at light loads the condenser may be called upon to operate at a lagging power factor or under-excited, in order to maintain the normal receiver voltage. The effect of lagging current on a condenser normally designed for leading power factor operation is similar to that of leading current on a generator normally designed for lagging power factor operation and previously referred to. Anything that is done to a normal machine without increasing its weight or cost, to improve its stability under lagging power factor operation increases the duty on the field at the leading power factor condition. A condenser may be made with normal proportions and at normal costs to operate at full kv-a. leading or at full kv-a. lagging, but not at both. A machine designed to operate at both leading and lagging power factor (above a normal amount) must have increased proportions and cost. A normal condenser designed for leading power factor operation can carry about 50 per cent, of its rated capacity lagging. Although it is possible to operate condensers of salient pole construction at considerable reversed excitation (about 50 per cent of no-load excitation) and get still more lagging capacity it is not considered good practise to operate below zero excitation on account of possible instability. With excitations below zero the machine is likely to become unstable and drop out of step during minor line disturbances.

SPECIAL CONNECTIONS AND STABILITY

The special armature connections referred to in the discussion of generators might also be used to increase the stability of condensers at lagging power factor operation in the case of a line operating at a small initial load requiring lagging condenser capacity and that will have a large ultimate load requiring leading condenser capacity. This would require that the special connection be used initially and that the machine be reconnected, straight star, for the ultimate and normal condition. Although lagging capacity increases the cost of a condenser which normally operates at leading power factor it increases the stability of the machine and is quite desirable at extreme overloads on the system when the regulation limit or pull-out point of the line is approached.

FIG. 10—14,285 KV-A., 6600-VOLT, 3-PHASE, 60-CYCLE, 450-REV. PER MIN. WATERWHEEL GENERATOR

CONSTRUCTION

Synchronous condensers are usually built at the highest economical speed and resemble horizontal water wheel generators in general appearances and construction. They are usually designed for very little (about 25 per cent) overspeed as under usual conditions no overspeed need be anticipated. However, occasionally manufacturers are called upon to proportion the machines to meet the overspeed of the generators to which the condensers will be connected. In such cases the condensers are usually on isolated lines or in the power station with the generator. A 15,000-kv-a., 6600-volt, 3-phase, 450-rev. per min. synchronous condenser being built for the City of Seattle is required to meet such a condition. The condenser will be the same as the generator shown in Fig. 10 except for the frame and bearings.

A new type of damper winding is employed on account of the unusual overspeed of this condenser which is described later.

DAMPER WINDINGS

In large condensers, the damper winding constitutes one of the most difficult design problems and has given considerable trouble in condensers in operation. The early design employed a complete ring without joints (on account of stress conditions) with bars bolted to the rings. This design is satisfactory in smaller units and when the current density at the joint between bar and ring can be moderate. In larger units some trouble has been experienced due to burning between the bars and end rings and this has led to the development of design with welded joints. Welding the bars to a continuous end ring is not a wholly satisfactory solution on account of the impossibility of removing a pole or field coil without destroying the damper winding. Bars cannot be welded to a sectional end ring (except for moderate stress conditions) on account of the effect of the welding heat on the strength of the end ring segments. A construction has been worked out that has the advantage of the welded joint and the strength of the continuous end ring without the disadvantages mentioned above. The construction is shown in Figs. 11, 12 and 12A. The bars are welded to a copper segment of angle section and these segments are bolted to the continuous copper ring. This ring is rolled copper strap with a brazed scarf joint located opposite one of the poles. The ring is on the edge and hence has maximum mechanical strength as well as ample contact surface at the various sections to which it is bolted.

FIG. 11—PARTIALLY ASSEMBLED, 4-POLE ROTOR, SHOWING WELDED DAMPER CONSTRUCTION

FIG. 12—ROTOR OF 5000 KV-A., 11,000-VOLT, 60-CYCLE, 720-REV. PER MIN. SYNCHRONOUS CONDENSER WITH WELDED DAMPER WINDING

This construction has proved quite satisfactory and is generally applicable to high-speed condensers. Machines with this damper construction have been run at very considerable overspeed in factory tests. It is believed that this design constitutes a distinct advance in damper winding construction and removes one of the limits that previously existed in the design of large high-speed condensers.

FIG. 12A—ANOTHER VIEW OF ROTOR IN FIG. 12

END BELLS

As the necessity for locating condenser substations in residential and business districts increases, the question of noise becomes of increasing seriousness. With the high speed to which it is necessary to go to obtain the most economical design, the moderate and large condensers are inherently noisy even though the fans and other projecting parts may be specially proportioned to minimize the noise. Therefore, where noise is objectionable, it is advisable to totally enclose the condenser so as to contain and suppress the noise within the machine. To further suppress or muffle the noise, the enclosing parts, such as the frame and end bells, should be designed so that ducts leading to the outside of the building may be attached for bringing in the cooling air and discharging the hot air. Even in uninhabited or mill districts where noise will not be objected to by the surrounding community it is often advisable and profitable to enclose the condensers to provide additional comfort for the station operators and attendants. Enclosing has sometimes been objected to in the past on account of additional weight and difficulty of handling the heavy cast iron end bells. However, this objection is largely eliminated by the use of sheet steel end bells. With the improvements in their design in recent years sheet steel end bells do not detract from the appearance of machines on which they are used and on account of their decreased cost and weight and the ease of handling their use is strongly recommended. Fig. 13 shows a totally enclosed 5000-kv-a., 11,000-volt, 720-rev. per min. condenser with totally enclosed frame and double sheet steel end bells. The frame has a rectangular opening at both top and bottom and the depth of the frame is such that either opening may be closed and all the hot air carried around the frame and discharged at the other opening through ducts, if any are provided.

Fig. 13—5000-Kv-a., 11,000-Volt, 3-Phase, 60-Cycle, 720-Rev. per Min., Synchronous Condenser with Sheet Steel End Bells

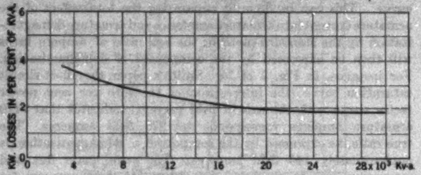


Fig. 14—Synchronous Condenser Losses

The outer sheet steel end bells are arranged so that ducts may be readily attached. Even on machines which are semi-enclosed and do not have the outer end bells, the use of sheet steel for the inner bells is recommended for reasons given above.

Losses

In the case of generators there is usually very little doubt as to their necessity. If a certain load is to be furnished the required generator capacity is rather definite. However, in the case of condensers the required capacity is not so definite and their economical necessity is often debatable. Condensers have, therefore, had to justify their use by results and in the process great importance has been placed on low losses. It has, therefore, been necessary to reduce them to a minimum. This has been made possible by the use of improved core plate material, improved fan design and refinements in general design. The curve in Fig. 14 shows the low losses it is possible to obtain on synchronous condensers designed for 11,000 volts or less and with all modern improvements and refinements in design. Guarantees will usually be about 10 or 15 per cent above this curve. As an example, the segregated full-load losses for the two 20,000-kv-a. 11,000-volt, 3-phase, 60-cycle, 600-rev. per min. condensers built for the Pacific Gas and Electric Company are given below.

Fig. 15—Two 20,000 Kv-a., 11,000-Volt, 3-Phase, 60-Cycle, 600-Rev. per Min. Synchronous Condensers

SEGREGATED FULL-LOAD LOSSES ON TWO 20,000 KV-A. 11,000-VOLT, 60-CYCLE, 3-PHASE, 600-REV. PER MIN. SYNCHRONOUS CONDENSER

	Kw. Losses	
Segregation	No. 1	No. 2
Armature I^2R Loss at 75 deg. cent.	57.3	58
Load or Stray Power Loss (100 per cent of tested sh. cir. loss)	46.2	45
Field I^2R Loss at 75 deg. cent.	63.3	60
Core Loss	118.0	113
Friction and Windage Loss	115.0	115
Total Losses	399.8	391

A photograph of the two machines is shown in Fig. 15. They are installed on the Mount Shasta system and are designed for a lagging capacity of 12,000 kv-a. Fig. 16 is a photograph of the stator of one of the units showing the armature winding and coil bracing. The connections between armature coils and groups were transposed in such a manner as to reduce the eddy current loss to a low value. This is partially responsible for the fact that the kw. losses on this unit were reduced to 2 per cent of its kv-a. rating. A photograph of the rotor of one of the units showing the coil bracing, fans, etc. is shown in Fig. 17. The shaft has a forged half coupling to permit its possible use as part of a frequency changer set in the future.

STARTING KV-A.

By the use of oil pressure in the bearings during starting and with improvements in damper design it is possible to start synchronous condensers of 5000 kv-a. capacity and over on 15 per cent of normal kv-a. To obtain such low starting kv-a., however, requires a very low starting voltage, and unless low starting kv-a. is extremely important, it is advisable to increase the starting kv-a. to 25 or 30 per cent and obtain more margin in starting voltage, reduce the difference in the starting and running voltage, and reduce the time required to reach synchronism.

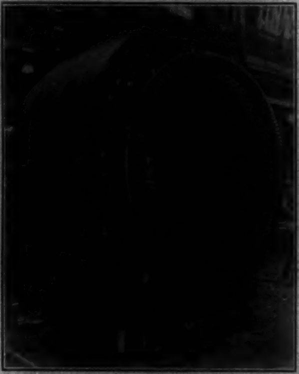


FIG. 16—STATOR OF 20,000 KV-A. SYNCHRONOUS CONDENSER

FIG. 17—ROTOR OF 20,000-KV-A. SYNCHRONOUS CONDENSER

FIG. 18—TWO-CYLINDER OIL PRESSURE PUMP AND MOTOR

Tests on one of the 20,000-kv-a. condensers referred to above showed that it would start on approximately 2800 kv-a., at 19 per cent voltage and come up to speed in three minutes. However, at the minimum auto-transformer starting voltage tap, of approximately 27 per cent voltage, the machine takes approximately 6000 kv-a. and comes up to speed in a little over one minute.

The latest equipment for furnishing the oil pressure during starting consists of a small two cylinder pump driven by a one horse power motor. The pressure usually required for starting varies from 400 to 800 lb. per sq. in. However, the equipment is capable of furnishing a sufficient quantity of oil at 1000 to 1200 lb. per sq. in. when necessary. A photograph of the pump is shown in Fig. 18.

Discussion

E. B. Shand: It is desired to draw attention to the conditions of line charging by an a-c. generator. One method which has been frequently used to determine this effect consists in taking the voltage characteristics of the generator when excited by the armature current directly from the no-load saturation and the short-circuit saturation curves. By plotting the volt-ampere characteristics for the capacity effect of the external circuit, the intersection of the two curves will give the value to which the terminal voltage will rise, with the field un-excited. This method involves quite an appreciable error, due to the fact that when the generator is excited from the field, the voltage is generated only by the flux in the magnetic circuit of the armature, but when excited from the armature an additional voltage is generated due to the leakage fluxes which are not present in the other case. It is evident, therefore, that the relation between voltage and ampere turns will differ under the two conditions of excitation.

To evaluate the effect of capacity loads in a more accurate manner, the actual effect of ampere turn relation between the armature and the field should be used in place of the modified relation as obtained directly from the short-circuit saturation curve. This can be done by separating the total effect of the armature current into armature reactance and armature demagnetization by the standard method; the real ratio of ampere turns will then be expressed by the component of demagnetization. For instance, in Fig. 7 of Mr. Smith's paper the short-circuit saturation curve (the intersection of the zero per cent power factor curve with the axis of field current) indicates that 1470 armature amperes corresponds to 220 field amperes. The true ratio, as expressed by the component o demagnetization, is approximately 145 field amperes. The same main flux and saturation will be produced by either 145 field amperes for 1470 armature amperes. The additional component of voltage for the same latter current, and due to the armature leakage fluxes will be approximately 2200 volts. These fluxes are largely in air paths and therefore, are practically independent of saturation. The terminal voltage of the generator under the conditions assumed, may be expressed as the voltage taken from the no-load saturation curve corresponding to $(I_f K + I_a)$ field amperes, plus $I_a \times x_a$,

where I_f = actual field current.
I_a = armature current.
x_a = armature leakage reactance.
K = relation between armature current and field current as expressed by the component of demagnetization.[1]

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[1] This constant K may be calculated directly from the design constants of a machine when these are available.

Fig. 1 gives the general construction for determining the voltage when a generator is connected to an open-circuiting transmission line or similar circuit with a high capacity effect. The line characteristic curve is merely the relation between the terminal voltage and the leading reactive current in the external circuit. Curve (2) is curve (1) less the armature reactance drop. The intersection of (2) with no load saturation curve furnishes the point of operation required. The actual terminal voltage will be the point (*P*).

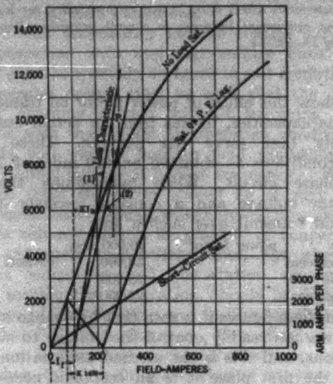

FIG. 1

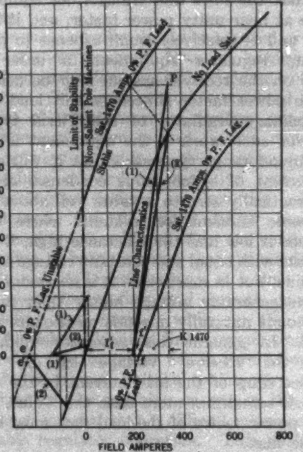

FIG. 2

Fig. 1 of Mr. Smith's paper gives two alternatives for the load saturation for zero per cent leading power factor at low voltages. Although of little practical importance, it may be of slight interest to investigate this point by means of the type of diagram described above. This diagram will be modified slightly to represent a condition of adjusting the d-c. excitation so that on varying the capacity of the external circuit, the armature current will be maintained constant. In Fig. 2, therefore, $K\ I_a$ will be constant while I_f will be assumed a variable, as well as the slope of the line characteristic curve. As the external capacity increases as indicated by the decrease slope of the line characteristic, the d-c. excitation must decrease and eventually reverse. When curve (2) coincides with the saturation curve, stability will cease so that with reversed excitation, the whole zero per cent power factor curve must be considered hypothetical, this, of couse, neglecting the effect of salient pole construction. Under this condition of instability, the voltage will naturally drop to zero and build up in the opposite direction which is indicated by the relative positions of the two curves at the intersection. Assuming, however, that the voltage could be stabilized by some means, it will be seen that the load saturation curve must approach the point *e*, which represents the condition of an infinite capacity of the external circuit.

By rotating this diagram through 180 deg. it may be seen that the point *e'* may be considered as the equivalent of *f'* under the condition that the reverse excitation had been employed in taking the load saturation curve at zero per cent lagging power factor. It can, therefore, be considered that the points corresponding to short circuit and to an infinite capacity may approach either of the points *e* or *f*, depending upon the direction of excitation. In the case of the circuit of high capacity, however, the operating condition is absolutely unstable and not possible. This seems to represent the relations involved rather than the curve *d f* of Mr. Smith's figure.

It might be suggested that there is probably only one method to actually obtain from test the values for the part of the curve which is unstable. By coupling two generators together so that their voltages are exactly in phase and consequently, that they can exchange no power by driving the machines from a separate source and properly adjusting their excitations the complete curve can be obtained for either condition of reactance or capacity, and moreover, the two curves will be continuous at the points *e* and *f*. The necessary stability is, of course, acquired through the mechanical interconnection of the two machines.

M. W. Smith: Although the criticism in the first paragraph of Mr. Shand's discussion may apply to the usual method, it is not applicable in this particular case. In determining the voltage obtained at the terminals of the generator when delivering 0 per cent, leading power factor current, the effect of both the armature re-action and armature reactance were taken into account. Furthermore, in Fig. 7, the two abscissas coordinates for "Armature Amperes" and "Field Amperes" are not interchangeable as apparently assumed by Mr. Shand, *i. e.*, they do not apply to the same curves. The abscissa for "Armature Amperes" applies only to curve five. Curve five, shows the values of terminal voltage, that will be obtained with various values of 0 per cent leading power factor current in the armature, with a constant d-c. excitation, and the effect of the armature reactance, as well as, the magnetizing effect of the armature, was included. The abscissa "Field Amperes" applies only to curves 1, 2, 3, and 4.

I wish to correct Mr. Shand's impression, that Fig. 1, gives two alternatives for the 0 per cent leading power factor curve at low voltage. As stated in the paper, if 0 per cent leading power factor could be maintained, the curve would extend to "*e*" at 0 voltage. It is further stated that it is not practicable to maintain 0 per cent power at low voltages on account of the increasing effect of the resistance in the circuit as the voltage is decreased and that in actual operation the machine would tend to follow the portion of the curve shown by *d. f.* The portion *d. f.* is, therefore, not a 0 per cent power factor curve, but represents the tendency of the machine to change over to points on higher leading and finally lagging power factor curves.

Both Mr. Shand and I agree that it is not practicable to operate generators at such low voltages and further discussion on this academic point is hardly justified.

Methods of Voltage Control of Long High-Voltage Lines by the Use of Synchronous Condenser

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Review of the Subject.*—The following points are covered in this paper:*

Inherent characteristics of high-voltage lines and high-voltage transformers.

Effect of low power factor load on high-voltage lines as to limiting capacity and uses of condenser for increasing line output and obtaining good voltage control.

Characteristics which must be met by generators, condensers and regulating equipment in order to control properly voltage over all operating conditions.

THE long high-voltage transmission line is inherently very poor as regards voltage regulation. The inductive reactance is high, due to the wide spacing of wires required for the high voltage, or to the long spans that are necessary to reduce the cost of structures and insulation. Transformers must always be considered with the line when voltage regulation is concerned, and they must be designed with high reactance for the large power systems so that they can withstand the short-circuit stresses without mechanical damage, and reduce the short-circuit current to a value to give them sufficient heat storage that they will not be destroyed during system trouble. The wire must have a large diameter to avoid corona loss, but in general, this conducting material of the large cables is utilized to advantage in keeping the energy loss at a low figure. These cables, however, subjected to a high voltage, cause heavy charging currents to flow.

With these points in mind, namely, a system of high reactance, which at no-load carries a heavy charging or leading current, it is readily seen that the voltage generated by the flow of this current through the inductive reactance is added to the impressed voltage and thus the receiver voltage on the long transmission line is higher at no-load than the generator voltage.

The amount of this rise in receiver voltage at no-load will naturally depend on the design of the line and connected transformers, but with present 100-kv. systems this value may be as much as 8 per cent above the generator voltage, and the highest voltage lines met with may have an increase in voltage of almost double this amount under no-load conditions.

The load met with in practise is almost universally of a lagging nature. When this load with its lagging current is carried by the transmission line, we meet with the reverse conditions to that of an empty line with charging current, as the e. m. f. generated in this high reactance system when carrying lagging current, opposes the generator voltage, and the drop between generator and receiver ends of the line becomes quite excessive at low power factor and heavy load.

To show this effect of power factor on transformers and line capacities, figures from an actual curve will probably best serve the purpose. These results cover conditions as obtained on 190 miles of 160-kv., three-phase circuit. When maintaining the same generator voltage and a constant receiver voltage, we could transmit over such a line:

At unity	power factor	60,000	kw.	
At 90 per cent	" "	35,000	"	(current lagging)
At 80 per cent	" "	28,000	"	" "
At 70 per cent	" "	21,000	"	" "

From these figures it is seen that by change of power factor we can readily control the voltage to suit load conditions, and this is the method used on long lines, for in the first place it is an economic necessity to provide ample synchronous condenser equipment in order that the lines and transformer equipment may operate at near unity power factor, and it is a simple matter and of relatively small expense to provide the necessary regulator equipment in order that the voltage on the receiver system may be held constant and the power factor shifted to suit load conditions. The ordinary regulator of the Tirrill type is well suited for such work if the equipment is of the so-called broad range type such that it will control the field current of synchronous condenser or generator from practically zero to a value well above normal.

In some transmission systems where the ohmic resistance is low and the reactance high, it is possible to operate with equal and constant voltage at both ends of the line, but in most cases it is necessary to operate with the generator voltage higher than the receiver. The amount naturally depends on the size and length of line. In general this generator voltage should exceed the receiver voltage by at least the resistance drop of the load current in the connecting lines, and sometimes more than this value, depending on the power factor of the load that must be handled over the line.

In order to reduce the line losses, particularly at light load, line drop compensators can be used on the generator voltage regulators to advantage, as by proper setting of these power house regulators, they will increase the generator voltage as the load increases to take care of the line drop, and at light-load or

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no-load the voltage will be automatically lowered so that the receiver condensers will not be called upon to supply such large amounts of lagging line current at light loads. These line drop compensators will not only increase light-load losses but will also permit of use, in general, of condensers which are of slightly less cost, as a condenser will probably only have to supply one half or less of its rating in lagging line current at no-load conditions.

In order that satisfactory voltage control may be had in connection with any high-voltage lines, the apparatus must be specially designed to suit conditions.

The generator design must be such that the units will carry charging current without becoming self-exciting, or such that under all operating conditions, these units will require positive field excitation to give their normal voltage at normal frequency. When practicable, these generators should be of such size and so designed that one unit will charge a transmission line. This will not always be possible in connection with the extremely high-voltage lines, but where a single unit cannot be used, the station, or certain groups of machines in the station, should be designed to take care of line charging and testing, so that this may be handled with dispatch. One disadvantage in using more than a single machine for testing purposes is that when these units are operating at very low values of field current they have low synchronizing torque, and it is sometimes difficult to keep a number of units in step.

The synchronous condenser should be selected so that they will furnish their maximum kv-a. of lagging line current with field excitation above the residual voltage of their exciters. Condensers that are operated from the secondaries of transformers or tertiary windings of auto-transformers where power is not supplied from the same winding or from the condenser bus, should be carefully selected for operation with their respective transformer designs, so that the voltage variation obtained, due to power factor changes through the transformers, will not effect the condenser output, as it must be borne in mind that the condenser will have to furnish its highest voltage at a time when it is operating with its heaviest field current, or when furnishing leading line current, while on the other hand, when furnishing lagging line current and extremely low fields, the condenser will in turn be operating at well below normal voltage, which may effect the kv-a. output of the machine or reduce the field excitation to such value that it is difficult or impossible to obtain stable regulator operation. This variation in voltage in modern transformer design, may easily amount to 15 per cent.

Automatic voltage regulators are difficult to adjust to control the voltage of long lines under no-load conditions and obtain stable operation. This is due to the extremely low values of exciter voltage required on the generator and condenser fields and to the fact that quite a portion of the excitation is supplied by the leading current of the line, and the regulator is hence only controlling a portion of the total excitation.

This condition can be greatly improved by installing proper relays so that at suitable exciter voltage the condenser field rheostats and generator field rheostats are gradually cut into the circuit before this low-exciter voltage is reached. This resistance acts as a governor, tends to reduce surging and permits the exciter to operate at a higher voltage.

To obtain the low-exciter voltages necessary it requires very high resistance in the exciter fields over which the regulator contacts must operate if speed is obtained under no-load or line charging conditions. When using this large amount of resistance, very sluggish and poor regulator operation is obtained under heavy load conditions, so at load conditions it is necessary to shift the amount of resistance in the exciter field and make sure that all resistance is cut out of the generator and condenser fields. This can be taken care of by utilizing a proper type relay such that it will not only cut in the resistance on light load conditions, but will cut out this resistance in generator and condenser fields when the load increases to the proper value, an additional relay should be used connecting a resistance in shunt with the exciter field rheostat under heavy load conditions. Thus, by suitable adjustment of this relay, it can be made to cut in or out of the exciter field at suitable values in order to obtain the most stable operating conditions.

Voltage regulation of long lines is thus taken care of mainly by power factor control. The synchronous condenser not only serves to correct the power factor and in many cases reduce line losses, but it also gives a steadying effect on the system, which is difficult or impossible to obtain by any other method of voltage control. Short lines can be operated without condenser equipment, but it is difficult to obtain good voltage regulation by generator regulators alone when the load power factor is poor. For satisfactory operation of long lines it is essential that automatic voltage regulators be used at both generating stations and on the condensers at the receiving stations, and in general line drop compensators can be used to advantage in connection with the generator regulators.

Discussion

A. W. Copley: In amplification of Mr. Koontz's paper and carrying out the thought a little farther, it may be pointed out that there is a very definite power limit on a transmission line on which the values of generator and receiver voltage are fixed. The addition of synchronous condenser capacity beyond a certain limit does no good as far as increasing the capacity of the line is concerned. For instance, on a 200-mile line with 200 kv. maintained at both ends, the addition of synchronous condenser capacity beyond the point which allows a load of about 220,000 kw. at the receiver end, does not increase the capacity for transmitting power. The power limit of such a line being in the neighborhood of 220,000 kw., it can be seen that, at least on projected lines, the limit is being approached. It is, however, possible to increase the power limit of the line by making, in effect, two lines in series. For instance, on a 400-mile line, with synchronous condenser capacity at the receiver end, the limit of capacity is around 120,000 or 125,000 kw. By breaking the line in two parts of 200 miles each, the power limit of the upper end of the line is about 220,000 kw. and with that power delivered to the middle of the line, the balance of it again has the capacity of 220,000 kw. minus the losses, which bring the delivered power down to about 200,000 kw. That is, by the introduction of condenser capacity in the middle as well as at the receiver end, the line capacity is increased from 125,000 to 200,000 kw.

One other point mentioned by Mr. Koontz which is of much importance, is the method used for the control of the exciter voltage at times when lagging current is being taken by the generator or condenser. When the line is being charged and no load being carried, the condenser must be operated with a very light field. The same holds true of the generator field under the same conditions. With the ordinary generator voltage regulator, the voltage across the generator or condenser field can be controlled only down to about the residual voltage of the exciter. By cutting resistance directly in the field of the condenser or generator the voltage across the field can be reduced still further and it is now possible to do this automatically by means of a face plate regulator which is operated as an auxiliary to the usual generator voltage regulator. Such a regulator has been applied on the generators of the Big Creek No. 3 Plant of the Southern California Edison Company. It allows the lowering of the voltage across the generator field to something like ten per cent of the residual voltage of the exciter and thus there is obtained a considerable increase in the lagging current capacity of the machines while under the control of the regulator.

R. J. C. Wood: I didn't quite understand what Mr. Koontz said about the generator voltage being low under certain conditions of operation, when the load was low. We have made calculations on the Big Creek System 250 miles long—that line, with 100,000 kw. per circuit, transmitted from Big Creek No. 3 plant, including the effects of the transformers on the line, which take certain magnetizing currents. It developes that the variation of the voltage on the generator will be very slight between zero load and full load. As I remember the figures the 11,000-volt machine requires slightly under 11,000 volts at zero load and in the neighborhood of 11,500 or 11,600 at full load. So, the variation at the generator is slight and the demands on the regulator will not be heavy.

About what Mr. Copley said relative to putting synchronous condensers in the middle of the line, that, of course, gives you a new lease of life. You get power at the end of the first section of the line at a certain voltage and at power factor desired and start all over again in the next section of the line. The method has its limitations and would not be used on a comparatively short line.

The calculations for the Big Creek 250-mile line indicated that there was no advantage in putting condensers in the middle of that line. The installations of a synchronous condenser at the middle point causes a loss of, roughly speaking, one-half of its capacity. A great many of these calculations are often made for the line only and the effects of transformers are not taken into consideration. This should be done, as illustrated by a test we made when we were first getting ready to energize the line at 220 kv. One of the preliminary steps was to test the transformers and equipment of the Big Creek No. 8 Plant. We had generators and transformers at the Big Creek No. 8 Plant, a 220-kv. line about 105 miles in length, from Big. Creek No. 8 to Vestal, and in addition auto transformers at Big Creek No. 1, No. 2 and also at Vestal, if I remember rightly. The result was that we got about 600 amperes on the generator at considerably less than full voltage and when we were up to full voltage on the line only 300 amperes, on account of the large amount of magnetizing current taken by the transformers on the line, so that you have a very good regulator in the magnetizing current of transformers; if the voltage tends to increase greatly on account of loss of load, the magnetizing current will increase rapidly and neutralize the charging current on the line and prevent excessive voltages. Magnetizing current may be taken at about 4 per cent of full-load current at normal voltage; at 110 per cent voltage it will be about 12 per cent, and at higher voltage—the manufacturers conceal what it is going to be. They need not conceal it; for our particular purpose it is a good thing to have.

J. A. Koontz: The statement, that there is a wide variation in voltage at the generating station, should refer to the exciter voltage. The generating voltage usually only varies some 8 or 10 per cent depending on how the lines are operated. On the lines of the Great Western Power system, this variation amounts to 10 per cent. The impression I intended to convey was that synchronous condensers have to operate over a rather wide range of terminal voltage, particularly when they are operated from high-reactance transformer windings which supply no other load. The condensers require minimum field excitation when the machine is supplying the minimum terminal voltage, and maximum field excitation at maximum terminal voltage, hence extreme exciter voltage variation. I have a case in mind where the condensers deliver only 9800 volts at minimum field and 12,200 volts under maximum field conditions. It is this change in machine terminal voltage, together with the magnetizing or de-magnetizing effect of the armature, which necessitates extreme field ranges and makes difficult regulator operation.

Economic Considerations of the Power Factor Control of Long High-Voltage Lines

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***Review of the Subject:**—In the early days of the electric industry when transmission lines were short, power factor control was not considered necessary. At the present time, with transmission lines of 200 miles and more, and with loads whose power factor is very poor owing to the universal use of the induction motor, power factor control is being forced upon the transmission engineer as a necessity if power is to be transmitted economically.*

By taking a few typical examples of transmission lines, the effect of power factor upon efficiency of transmission is shown very clearly.

The most economical line power factor for any ordinary commercial transmission line is shown to vary by a comparatively small per cent and to be slightly less than unity. At the same time there appears a very wide range of load delivered over a line without an undue rise in cost of transmission.

It is quite generally accepted that some degree of power factor correction is advisable over long high-voltage lines but the amount of this correction has been considered a special problem to be worked out for each particular line.

By constructing curves of kilowatt capacity and capital cost at various power factors for several typical lines, the writer has endeavored to show that the power factor for lowest transmission cost varies through a narrow range.

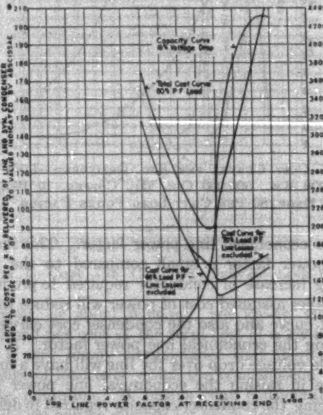

FIG. 1—COST AND CAPACITY CURVES—LINE A 200 MILES, 200 KW. 500,000 CIR. MILS COPPER, 10 PER CENT VOLTAGE DROP

The lines taken as typical are the following:

a. 200 miles, 2 circuit tower line, 500,000-cir. mil copper conductor, 200-kv. at receiving end, 10 per cent drop in voltage, 5 per cent transformer reactance at receiver end. Cost per mile, $35,000.

b. 100 miles, 2 circuit tower line, 250,000-cir. mil copper conductor, 100-kv. at receiving end, 10 per cent drop in voltage, 5 per cent transformer reactance at receiver end. Cost per mile, $15,000.

c. 50 miles, 2 circuit tower line, 250,000-cir. mil. copper conductor, 100-kv. at receiving end, 5 per cent drop in voltage, 5 per cent transformer reactance at receiver end. Cost per mile, $15,000.

Figs. 1, 2 and 3 each has a curve showing the kw. capacity of the different lines as the power factor at the receiving end varies. It will be noticed that the kw. capacity rises very rapidly around unity power factor.

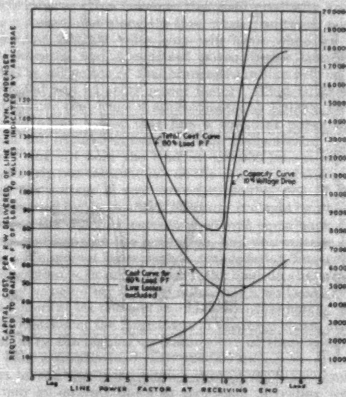

FIG. 2
COST AND CAPACITY CURVES - LINE "b"
100 MILES 100 K W 250000 C.M. COPPER 10% VOLTAGE DROP

FIG. 2—COST AND CAPACITY CURVES—LINE B 100 MILES, 100 KW., 250,000 CIR. MIL COPPER, 10 PER CENT VOLTAGE DROP

Fig. 4 shows curves of capital invested per kw. delivered over line *a*. Curve marked "Line Cost" shows capital investment in line only. Curve marked "Condenser Cost" shows capital investment in synchronous condensers required to bring the load power factor, assumed to be 80 per cent, to the line power factor which

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is required for transmitting the load at the designated voltage drop.

"Condenser Cost," covers condensers installed on transformers necessarily in use for other purposes, and includes condenser losses at 3 per cent and at 4 mils per kw-hr. and depreciation at 3 per cent all capitalized at 6 per cent per annum. Itemized as follows:

Synchronous condenser installed per kv-a. =	$9.00
Power required to operate at 3 per cent loss for one year = 0.03 × 8760 = 262.8 kw-hr. 262.8 kw-hr. at $0.004 =	$1.05
Depreciation at 3 per cent on $7.00 =	.21
Cost of operating power and depreciation	$1.26
Capitalized at 6 per cent per annum =	21.00
Total condenser cost per kw. delivered =	$30.00

Line and condenser cost curve is the sum of these two curves.

Line loss cost curve shows the $C^2 R$ line losses per kw-hr. capitalized at 6 per cent per annum.

Total cost curve is the sum of all the above costs.

Two of these curves, total cost and line and condenser costs are shown in Fig. 1 plotted to line power factor as abscissas in place of kw. delivered.

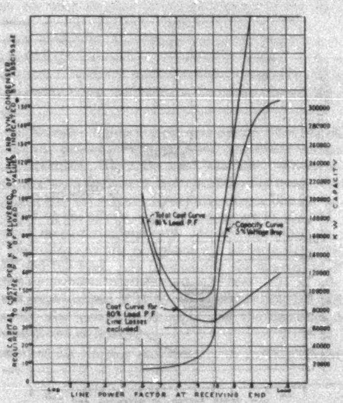

FIG. 3—COST AND CAPACITY CURVES—LINE C 50 MILES, 100 KW., 250,000 CIR. MIL COPPER, 5 PER CENT VOLTAGE DROP

Total cost curve shows that, for a minimum transmission cost on this line, the line power factor should be about 97 per cent lagging.

Similar curves are shown in Figs. 2 and 3 for lines *b* and *c*. They indicate that for minimum transmission costs the line power factors should be about 95 per cent and 91 per cent lagging respectively.

Fig. 5 shows curves of line *b* at two different line drops; 0 per cent, and 15 per cent, which indicate the marked effect these differences have on the most economical line power factor.

Fig. 1 has a second cost curve with line losses excluded for a load power factor of 70 per cent. Comparing it with the 80 per cent load power factor show that the load power factor has a very slight effect on the most economical line power factor.

An increase in the size of conductor chosen will decrease the line loss cost curve and increase the line cost curve to a less degree; the net result being to raise the economical line power factor. This increase of the power factor is small compared to the corresponding increase of conductor required to produce it.

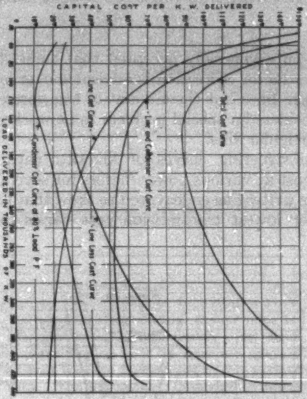

FIG. 4—COST CURVES PER KW. DELIVERED—LINE A 200 MILES, 200 KW., 500,000 CIR. MIL COPPER, 10 PER CENT VOLTAGE DROP, 80 PER CENT LOAD POWER FACTOR.

A smaller assumed cost of power will reduce the line loss cost curve and also the condenser cost curve, thus increasing the economical line power factor. A larger allowable voltage drop over the line will have a marked influence in raising the economical line power factor by reducing the condenser capacity required to deliver a stated amount of power.

Some of the smaller factors such as attendance for operating condensers have been neglected as well as the advantage gained by voltage regulation obtained by use of the condensers, which displaces regulation by induction regulators or other regulators with their attendant losses. Also the saving in generator size and losses due to the use of condensers has been neglected.

The foregoing considerations have all been based on a 100 per cent load factor which unfortunately is not often met with. However, by taking a load which will give the average losses of the real load the above considerations will apply very closely The condenser capacity required would be enough greater to maintain the proper voltage at the peak load. The condenser losses would be only slightly greater. The condenser cost would be materially increased but this is roughly only about one third of the "condenser cost."

While the economy of transmission changes very rapidly with a small change in line power factor it changes very slowly over a wide range of kw. delivered, on both sides of the minimum cost point. This can be clearly seen in the "total cost curve," Fig. 4.

A change of load delivered from 125,000 kw. to 225,000, *i. e.* from 91 per cent power factor to unity means only a change in capital cost per kw. delivered of $4.50. Assuming an interest depreciation and operation charge of 12 per cent this means 54 cents per 8760 kw-hr. or about 0.06 mil. per kw-hr. which is a very small per cent of the total cost of a kw-hr.

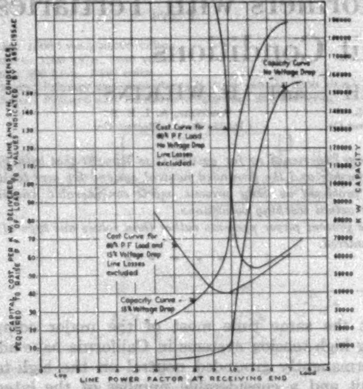

Fig. 5—Cost and Capacity Curves—Line B

It is a very happy circumstance that there is this wide range of load over which the transmission cost is near the minimum. It means that considerable increase of load can be allowed for when designing a new line without sacrificing present economy. Likewise the load on an existing line can be taken care of economically up to the point where it will justify an additional line.

CONCLUSION

On long high-voltage transmission lines it is economy to install synchronous condensers to correct the line power factor very close to unity and if it is necessary to correct slightly more than this on account of voltage regulation the economy of the transmission line will not be greatly effected.

Discussion

F. G. Baum: Our vision of the future transmission system of the United States is a 220,000 volt system, at least for most of the country, and for most of the time that we will be here.

In the states east of the Mississippi River, the average transmission line distance will be around 250 or 300 miles. That is not a long transmission. In our Western country, however, we are already up to that distance, both in the north end and the south end of the state. In not very many years the southern part of the state will probably have to go over to the Colorado for its power, which means a 500-mile transmission, and the northern part of the state will have to go up to the Columbia Basin, which means again approximately a 500-mile transmission.

If we consider a circuit of 500 miles and apply voltage at one end, as shown in the paper by Mr. Peek a year ago, approximately 110-kv. at one end of the line will give you approximately 220-kv. at the other. Of course, you cannot have such a line and have it safe. So, we must have some way of holding the voltage down or stabilizing the line.

If we have the charging current at one end of the line and the capacity current is traveling 500 miles, then in that capacity current travel of 500 miles it passes over 500 miles of reactance and that is what raises the voltage of the lines.

Let us look at what are the natural characteristics of a 220,000-volt line. Let us say we have a 600,000 circular mil. copper line of any length and let us apply a load on one end of the line. I don't care whether the line is 500 miles long or 1000 miles long. Let us take the load at unity power factor and see what the characteristics are. Now increase that load gradually and take a voltage measurement along the line, when you reach a load of 120,000-kw. at 220,000 volts, you will have what we call a constant potential and a constant power factor transmission line. That is, all points of that transmission line will be at the same potential and at unity power factor. That is, there is a load for that line that when it approximates at 120,000 kw. for a 220-kv. line which makes that line a constant potential line; that I call the natural capacity of that transmission line. Of course, we can not hold the load at 120,000 kw. at all times. Suppose we trip out the switches at the end of that transmission line, then the voltage is approximately double due to the rise in voltage caused by the charging current; it would be worse than that because the charging current itself will come back to your generators and increase the excitation, and the voltage limit is unstable. We cannot have such a transmission line for satisfactory operation. In order to vary the load on that transmission line (let us again say it is 500 miles long), we locate synchronous condensers at the end and at one or two other points, depending on the location of the large loads in the large cities, etc. We may then vary the lagging or leading kv. of those condensers. Say we want a smaller load than 120,000 kv-a.; we change the condition by drawing some lagging current on those condensers and we may go entirely down to zero load, providing we make the charge kv-a. of a given section of the line come from the condensers connected to that section. That is, the charging kv-a. of the line is circulated merely through the condenser and does not travel through the entire transmission line. (I believe Prof. Ryan also showed that in a new way at that special meeting last year). When you want a higher load than 120,000 kv-a. over that line you raise the supply leading kv-a. or take it from the line wherever, or whichever way you want, and you get an economical capacity around 120,000 to 160,000 kw. and an overloading capacity of around 200,000 kv-a.

What would be the advantage of such a system? The generators would all be standard and would be uniform. That is, the excitation, in place of varying from, say, 100 amperes up to 300 amperes from no load to full load for 80% power factor, would vary from 100 to 200 amperes for unity power factor.

The generator designers would have an easier job and some of the men I have talked to hope to see that day in order to relieve them of the trouble of designing from 80 to 100% power factor. Also, the transformers along such a line would be all of one voltage and standard and there would be no necessity for voltage taps. In operating such a system with those condensers tied to it, there would be a stabilizing effect produced by the condensers. The transformers would help to stabilize the line. But, a rising voltage without the condensers on the transmission line, tends to produce a further rising voltage and, therefore, produce instability; whereas with the condensers, the condensers react, and have a tendency to lower the voltage and to cause the line to operate just in the opposite way, giving you a stabilized transmission system. And, it is such a transmission system as I have in mind to cover the country.

Mr. Joslin has shown in his paper the economic value of the condensers for 220 kw. transmission.

This is also shown on pages 15 and 18 of the Atlas of U. S. A. Electric Power Industry. The addition of the condensers does not materially increase the cost of the transmission, but the addition gives a safe line, and a line that may be extended as far as economy dictates. With the condensers we also obtain increased capacity so that the cost per kw. for transmission is reduced.

I especially urge electrical engineers to study the characteristics of the constant potential 220 kv. transmission with condensers for voltage control. I think they will find that this gives a system of transmission commensurate with the needs of the industry at this time.

J. P. Jollyman: It is fortunate that the engineer, taking advantage of the information that Mr. Joslin has presented, can say in some cases, "Spending a half-million dollars for synchronous condensers will do more good than spending it for additional transmission circuits."

I think the economic phase of the problem has not been emphasized enough in the past.

Performance of Auto Transformers with Tertiaries Under Short-Circuit Conditions

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Review of the Subject.—*The problem of operating many of the present day transformers designed and built to function in sizes and under conditions unheard of only a few years ago, is now of prime importance to many of the larger power companies.*

A statement of one type of the operating troubles encountered in the larger sizes is discussed in this paper and data taken as far as practicable under operating conditions is given.

From experience so far gained it is thought that both the autotransformer and the grounded neutral system are here to stay and such problems as they present merit considerable investigation under actual working conditions.

The present paper presents rather than solves one type of trouble encountered.

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In July, 1914, the California Railroad Commission published the first report of the Joint Committee on Inductive Interference.

In 1918 General Order No. 52 superseded General Order No. 39, following the final report of the same committee. In this order under which California public utilities now operate is the following clause, "no star-connected transformers or auto transformers shall be employed with a grounded neutral on the side connected to a three-phase power circuit involved in a normal parallel, unless low-impedance, delta-connected secondary or tertiary windings or equivalent means are used for suppressing the triple harmonic components of the residual voltages and currents introduced by the transformers".

In complying with this order, and to reduce the cost on a winding used solely to suppress the effects of causes which could not be removed, most companies have used tertiary windings of capacity less than the rating of the transformers, that is, the current-carrying capacity of the tertiary winding has been reduced by reducing its copper cross-section.

At the same time most of the major companies using high-tension systems in California have adopted a permanently grounded neutral on the high-tension circuit and a considerable number of the later-built transformers have been auto transformers, designed specifically to operate only with one terminal of the high-voltage winding grounded.

In present designs some of the tertiaries have a current density 20 per cent higher than the primary of the same transformer for the nameplate rating. This relatively high resistance further increases the tertiary temperatures under short-circuit conditions. Having this condition in mind it becomes an operating problem of when and how to protect the tertiary winding against destructive overheating during line short circuits.

Tertiary windings are used, in addition to the prevention of inductive interference, for:

1. Providing a path for the flow of third harmonic current, necessary for the proper magnetization of the core for a sine wave e. m. f. across the main windings.

2. Stabilizing the neutral and providing a sufficient flow of current in the line-or windings at times of

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single-phase short-circuit conditions to give proper operation of relays and circuit-breakers.

3. Providing a winding for load connections or a synchronous condenser.

In practise the size of the tertiary is fixed by the short-circuit kv-a. rather than the third harmonic component of magnetizing current, *i. e.*, this winding must carry the short-circuit kv-a. long enough to allow circuit opening devices to operate. The time required to open the circuit is usually short, of the order of 5 seconds or less, and therefore very little radiation of heat takes place, consequently the copper must store its $I^2 R$ losses by thermal capacity during this period.

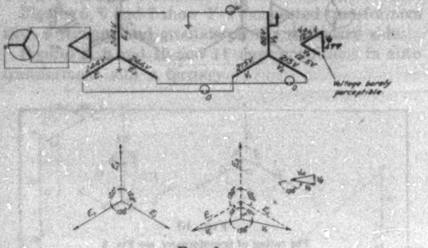

FIG. 1

Rating
Primary: 1500 kv-a., 40,000 volts
Secondary: 1500 kv-a., 6680 volts
Tertiary: 300 kv-a., 2340 volts
Impedance: Primary and Secondary: 5.7 per cent at 1500 kv-a.
Impedance: Primary and Tertiary: 20 per cent at 1500 kv-a.

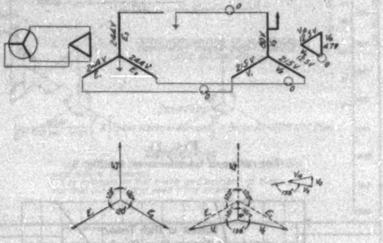

FIG. 2
For rating of transformer, see Fig. 1

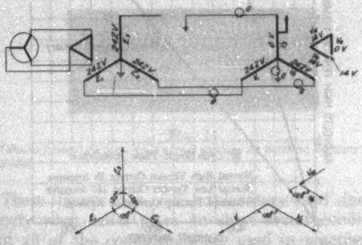

FIG. 3
For rating of transformer, see Fig. 1

There is given later a curve for determining approximately the relations of size of copper, current, temperature, and time for this condition.

For short-circuit conditions, the size of the tertiary is determined by the value of short-circuit current that it is desirable for protective reasons and the duration of the heavy current.

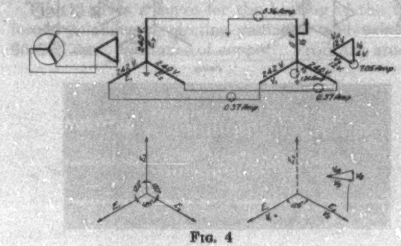

FIG. 4
For rating of transformer, see Fig. 1

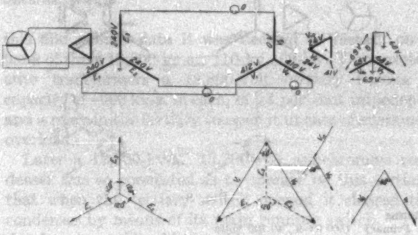

FIG. 5
For rating of transformer, see Fig. 1

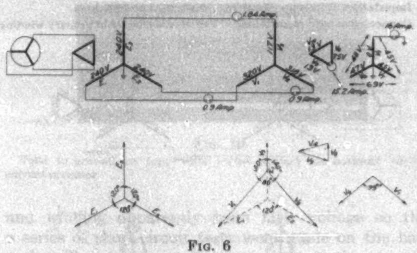

FIG. 6
For rating of transformer, see Fig. 1

From 300 per cent to 400 per cent of normal current is usually desired for operation of protective relays and this abnormal current requires a tertiary winding constructed to have from 25 to 30 per cent of the capacity of the main transformer windings.

The reactance in the tertiary circuit is then made such as to limit the current to the above values for the voltage applied to a short-circuited transformer.

It is unsafe to use less reactance on account of danger from burnout and not wise to use more because the amount of the short-circuit current desired would not be obtained.

The kw. capacity and reactance of the tertiary are interdependent and with one value given the other is fixed.

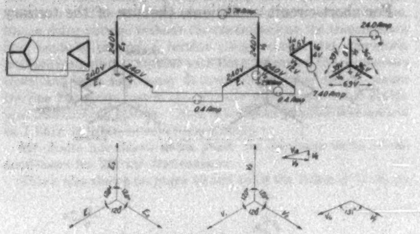

FIG. 7
For rating of transformers, see Fig. 1

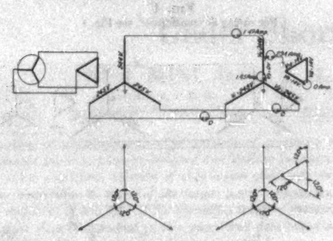

FIG. 8

Rating
Primary: 1500 kv-a., 40,100 volts
Secondary: 1500 kv-a., 6680 volts
Tertiary: 300 kv-a., 2340 volts
Impedance: Primary ϵ, Secondary: 5.7 per cent at 1500 kv-a.
Impedance: Primary ϵ, Tertiary: 20 per cent at 1500 kv-a.
Connected as auto transformer. Tap in exact center of primary winding.

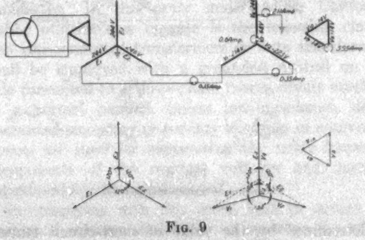

FIG. 9
For rating of transformers, see Fig. 8

In operation a few specific schemes only are of interest because, for operating reasons, only certain connections are used; and odd or infrequent connections if determining the design characteristics, should be treated as special cases, as should all cases where a synchronous condenser operates from a tertiary.

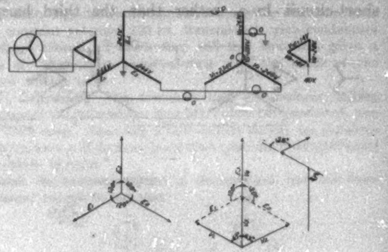

FIG. 10
For rating of transformers, see Fig. 8

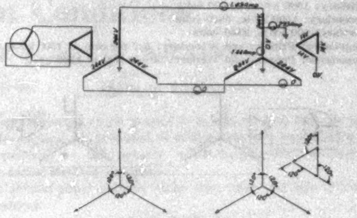

FIG. 11
For rating of transformers, see Fig. 8

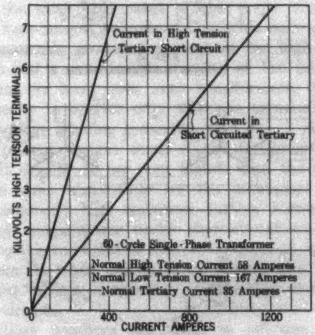

FIG. 12

To show approximately what takes place for different transformer connections a bank of transformers having a tertiary was tested at less than normal voltage and the several voltages and currents read as shown in the following figures.

Fig. 1 shows a ground on the high-tension with the neutral ungrounded tertiary closed.

Fig. 2 shows the tertiary open.

Fig. 3 shows the neutral grounded tertiary open.

Fig. 4 tertiary closed.

Figures 5, 6 and 7 show *Y*-*Y*-connected transformers with a tertiary and grounds on the secondary side.

Figures 8, 9 and 10 and 11 show conditions in auto transformers with a tertiary.

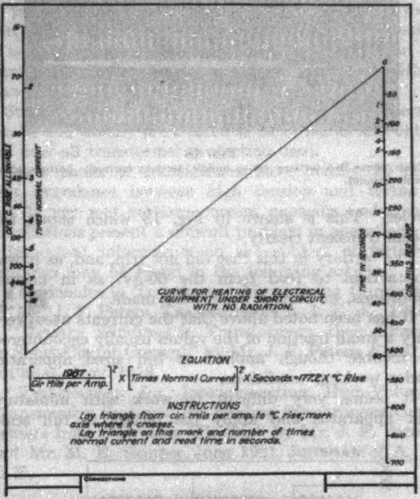

Fig. 13

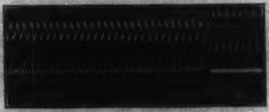

Fig. 14

Volts to ground on top; volts across tertiary on bottom; tertiary current in center

These figures are self explanatory and indicate clearly what takes place on short-circuit conditions. Not all of the connections are used in practise, but from a study of the diagrams given in the several figures an idea can be gained as to what happens in practise.

Fig. 12 gives a curve from a bank of transformers in which the tertiary has a rating of 20 per cent of the transformer rating, and an impedance such that, with the tertiary short-circuited, from 15 to 20 per cent of full voltage is required to circulate full current through the high-voltage winding. For this test one high-voltage winding was short-circuited and gradually increasing 3-phase voltage applied, measuring the current in the tertiary and short-circuited high-voltage winding. For 300 per cent high-voltage current there will flow approximately 14½ times normal current in the tertiary.

Fig. 13 gives a curve for the heating of the transformer windings neglecting radiation and using the 60 deg. cent. resistance of copper. From the study of this and similar data it was decided to install, on a bank of three 12,000-kv-a., 110-kv., 60-kv., *Y*-connected auto transformers a 13,800-volt tertiary having a capacity of 4000 kv-a. in each, of 13 per cent impedence and a relay in the tertiary to open it in case of sustained overload.

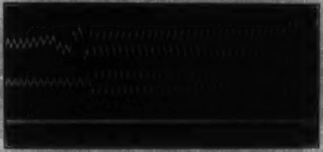

Fig. 15

Volts to ground on top; volts across tertiary on bottom; tertiary current in center

Later a 12,500-kv-a., 13,200-volt synchronous condenser was so connected as to operate on this tertiary that when the tertiary switch opened it cleared the condenser by means of its main running switch. This developed considerable trouble in the tertiary switch and winding apparently from high voltage so that a series of short-circuit tests were made on the bank and oscillographic records taken of the tertiary current and voltage.

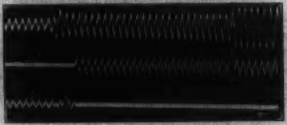

Fig. 16

Volts to ground on top; volts across tertiary on bottom; tertiary current in center

By using a comparatively long 100-kv. line with a limited amount of available power it was possible to short-circuit the 60-kv. side of one auto transformer to ground.

Fig. 14 gives a curve of the voltage from one condenser lead *i.e.* one corner of the delta to ground together with the tertiary current when one phase of the 60-kv. was grounded.

There is shown a somewhat violent change in tertiary current for the first 1½ cycles after the short circuit occurred from the usual triple harmonic magnetizing current to a badly peaked 60-cycle wave which then continues as long as permanent short-circuit conditions remain.

Fig. 15 shows the 60-kv. ground thrown on with the condenser drawing approximately 30 per cent normal armature current leading. This shows the same violent current fluctuation for approximately the same time and thereafter a more gradual change until the condenser cleared by overload in about ½ second.

This tertiary current wave form during short circuit shows the effect of the condenser as compared with the normal running conditions at the start and the short-circuit conditions without the condenser at the latter end of the film.

The voltage from one corner of the delta to ground changes as the short circuit was applied from an approximate sine wave to a wave showing the effects of the disturbed magnetic conditions and finally as the condenser clears to the static voltage of the winding to ground.

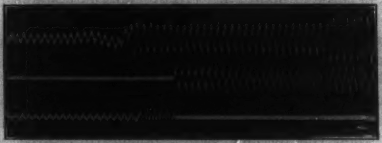

Fig. 17

Volts to ground on top; volts across tertiary on bottom; tertiary on bottom; tertiary current in center

In Fig. 16 the condenser boost was raised until it was drawing somewhat less than 25 per cent normal current (532 amperes) with the 60 kv. grounded as before. The transient changes for the first few cycles are the same, but this test was so arranged that the tertiary relay opened the tertiary switch which in turn tripped the main condenser switch as in normal operation.

For about 4 cycles between the time of opening of the tertiary switch and the clearing of the condenser there appears a badly distorted voltage of some 4500 volts across the tertiary switch.

Fig. 17 is the same with an added load on the condenser and shows a little more clearly the action between the time of opening the tertiary switch and clearing the condenser.

The values shown are not alarming except as they show abnormal characteristic tendencies. The short-circuit current was limited by limiting the supply to a very small fraction of what would normally be available and it is very difficult if at all possible to predict what would happen with current values 10 to 15 times those used which are known to exist in times of trouble.

There would also probably be a more violent disturbance for a ground on the 100-kv. fed from a 60-kv. source but the system connections were such that the supply could not be limited and it was not attempted. A ground was however thrown on the 100-kv. line without any other source than the 100-kv. itself and shows the same characteristics as regards sudden magnetic and current changes in the first two cycles. This is shown in Fig. 18 which shows the initial transient clearly.

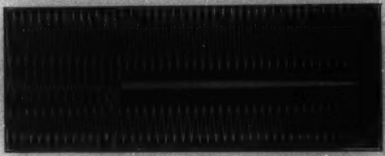

Fig. 18

Top, phase B-A tertiary volts; middle, tertiary current; bottom, phase A volts ground.

The tertiary in this case did not trip, and, as it was impractical to feed from the 60-kv. as in normal conditions, no further tests were made.

It has been noted above that the currents used were only a small fraction of the values usually encountered in trouble though applied to full sized apparatus under working conditions.

It seems very difficult to work with miniature test apparatus and apply the results to full scale working conditions because of the difficulty of securing the same characteristics in each case.

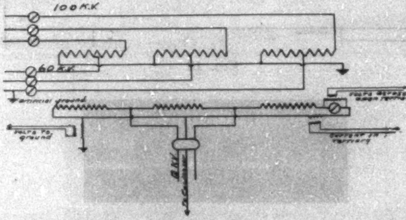

Fig. 19—Diagram of Connections

From a study of the conditions encountered it was thought that it would be advisable:

1. To permanently close the tertiary, leaving its current transformer in service.

2. Connect the tertiary relay so that it clears the 100-kv. and 60-kv. transformer bank switches at about 300 per cent normal current in the tertiary in approximately 10 seconds.

3. This presupposes that the line protective switches

are to operate as they have in the past to clear trouble selectively in about two seconds.

In the original scheme where the tertiary opened and tripped the synchronous condenser all of the switches were opened during the first ½ second after a short circuit occurred, that is on the definite time portion of the induction type relay curve.

It seems advisable to delay the action until the high-current values had subsided to more nearly a steady state and then clear the bank of transformers completely as a last resort. On banks of transformers in which the tertiary impedance is in the order of 20 per cent and the current-carrying capacity 25 per cent or above, with a primary impedance of 5 per cent it is thought that the tertiary will withstand for a short time any overload that the primary can withstand.

Such combinations, unless supplying condensers, can be and usually are treated as delta banks and only the over-all transformer protection used.

Some banks of transformers have neither 20 per cent impedance between high tension and tertiary nor 25 per cent tertiary current capacity and these installations present a difficult problem in operation.

Important information on the subject of tertiary windings may be found in the following articles:

"Harmonics in Transformer Magnetizing Currents" by Mr. J. F. Peters, TRANSACTIONS A. I. E. E. 1915.

"Tertiary Windings in Transformers, their Effects on Short-Circuit Current" by Mr. J. F. Peters, Electric JOURNAL, Nov. 1919.

"Transformers for Interconnecting High-Voltage Transmission Systems; for Feeding Synchronous Condensers from a Tertiary Winding" by Mr. J. F. Peters and Mr. M. E. Skinner, June 1921, JOURNAL of A. I. E. E.

Discussion

L. F. Blume: The subject of the paper by Messrs. Mini, Moore and Wilkins involves many interesting problems because the transformer connection to which the authors mostly refer is a mongrel which has inherited its pecularities from three distinct sources. For, in this one transformer connection are combined the various aspects of (a) an auto transformer, (b), the *Y* connection, and (c) the 3-winding transformer. I will point out briefly for purposes of emphasis, a few of the salient characteristics of only one of them, namely, the 3-winding transformer.

Wherever transformation of power is desired between three circuits, the practise of using three windings of different voltages is growing on account of the resulting economy, not only in first cost, but also in operation. By using one transformer bank rather than two, double transformation of power is avoided and the total amount of transformer kv-a. for a given service is correspondingly reduced. A particularly interesting and important illustration is in high-voltage transmission where synchronous condensers are used for power factor control, in which case the synchronous condenser sometimes is neutralizing the lagging reactive current of the low-voltage system and at other times it is neutralizing the leading charging current of the high-voltage line. The joining together in one bank, of the transmission line, the low-voltage circuit and synchronous condenser for the purpose of transforming directly from any one circuit to another by one transformation is the unique function of the three-winding transformer.

In addition to this, however, it is also being frequently used where the *Y-Y* connection is primarily desired, the third winding being added merely to correct the inherent weakness of the *Y-Y* connection. These inherent weaknesses resulting sometimes in abnormal voltages either in the connection or in the line, or sometimes in interference with parallel telephone lines and also in various operating difficulties, may be grouped under the rather broad generalization of neutral instability. This use of the tertiary delta in itself, is good evidence of the superiority of the *Y*-delta connection, for by the addition of the delta the stable characteristics of the *Y*-delta connection is secured. It is desirable to point out in this connection that if the tertiary winding is not needed for loading purposes the *Y*-delta connection is simpler and cheaper, and therefore, the only legitimate reason for adding the third winding is found in those cases where the system conditions are such that a *Y* connection for both the primary and secondary windings is essential.

Thus it is seen that the 3-winding transformer is being employed for two distinct reasons, the one being based on economy both in first cost and operation; and the other being for the purpose of securing specific improvements in operating characteristics. The one purpose may be called the economical use, and the other, the non-economical use of the 3-winding transformer.

However, whatever may be the reason for the use of the three windings, the majority of them are connected in *Y-Y* delta, with the delta tertiary having a capacity considerably less than the rating of the main windings. In these cases the size of the tertiary winding may not be determined by the load which it is to deliver, but rather by other characteristics such as the short-circuit current. For example, a short circuit from line to neutral on either of the high or low-voltage circuits results in a short-circuit current circulating in the delta limited only by the internal reactance of the tertiary winding. In this case the controlling factors in design as pointed out by the authors, are the amount and duration of the short-circuit current.

It may be well to add that when a tertiary winding is to be designed for a given short circuit and for a given length of time, these other terms, kv-a. rating and reactance become superfluous and are arbitrary.

Where the tertiary windings constitute an important protective link in a system, it is highly desirable that they be sufficiently liberally designed so as to possess an ample margin of safety from the point of view of the mechanical forces and the thermal effects resulting from short-circuit currents. The certainty that the over loading of tertiary windings will be much more frequent by short circuits and also of longer duration than in ordinary transformer windings is an additional reason for a conservative policy. Moreover in most instances a larger tertiary costs comparatively little more than a small tertiary. For example, comparing a 50 per cent with a 25 per cent capacity tertiary winding, the former involves only 5 or 6 per cent increase in transformer cost.

To determine the short-circuit and regulation performance of a three-winding transformer it is essential to have a clear conception of the manner in which the reactance between the windings affects the current division, regulation and short-circuit currents. The determination of these characteristics may prove to be very perplexing and unnecessarily complicated unless the fundamental relations are clearly understood. These relations, however, are fortunately quite simple.

In a two-winding transformer the internal impedance is usually expressed in terms of one value. The leakage reactance between primary and secondary, and the resistances of the two windings are lumped into one and thus one value of impedance is sufficient to express the voltage consumed internally by the transformer. From time to time academic discussions arise concerning how much of the reactance belongs to the primary and how much to the secondary but no practical use is served in attempting to make this distinction and no matter how the reactance is assumed to be divided between high-voltage and low-voltage windings, the performance of the transformer remains unchanged. Therefore, it is common practise to use only one value to express the impedance of a two-winding transformer. The reason that this may be done is because the high and low-voltage currents are equivalent and in phase.

On the other hand in 3-winding transformers the current flowing in one winding is not necessarily equivalent to, nor in phase with, a current in one of the other windings, and for that reason resistances and reactances cannot be so simply expressed. It is convenient, and in fact necessary, to consider that every winding possesses a reactance belonging to itself just as distinctly as its resistance, and that they are connected together in the equivalent circuit in *Y*. Having segregated in this manner the impedances into a primary, a secondary and a tertiary value, regulation drop can be calculated for each one separately by using the ordinary formula for regulation, and also by means of the simplest algebra, current division for any condition of loading or of short-circuit condition can be readily determined.

Thus there are two new problems introduced by the 3-winding transformer. First, the determination of the size of the tertiary winding may not be obtained in the ordinary manner but by one or more of the unique functions which it is called upon to perform. Second, the three windings are so related to each other that impedance and regulation cannot be so simply expressed as in ordinary transformers, although by means of a simple equivalent circuit, equivalent impedance values can be determined for each winding and these values used in the ordinary formulas to obtain regulation or current division for any given condition.

A. Boyajian: There is an implied recommendation in the paper by Messrs. Mini, Moore and Wilkins that tertiary windings should have larger capacity or larger margin of safety; a suggestion which deserves serious consideration, especially in view of the relatively small cost that it involves.

Relative Cost of Tertiary Windings: In three-winding transformers a larger tertiary costs only a trifle more than a small tertiary. For instance, a transformer with a 50 per cent tertiary winding has a total kv. rating which is only about ten per cent larger and a cost which is only about five or six per cent greater than a similar transformer which has only a 25 per cent capacity tertiary. That is, the capacity of the tertiary winding can be doubled with only a five or six per cent increase in the cost of the transformer. Although it seems to be common practise to rate tertiary windings in the neighborhood of 25 per cent (excepting those which have to carry large loads normally), I happen to know that in a number of cases a manufacturer furnished tertiary windings with capacities much in excess of the capacity specified in the contract, simply because the additional expense involved was very small while the greater security to the transformer resulting therefrom was very large.

Tertiary windings cut a somewhat larger figure in the case of auto transformers than in transformers on account of the fact that the economy in the primary and secondary circuits accomplished by their auto-transformer connection does not apply to the independent tertiary winding. For instance, in a two-to-one auto-transformer, the cost of fifty per cent capacity tertiary may be about 10 per cent more than that of a 25 per cent capacity tertiary. But even in this case the extra cost is small, the capacity of the tertiary being doubled with only about 10 per cent increase in cost.

In as much as a tertiary winding is an important protective link in a system, its capacity should be specified liberally, though not wastefully, and this naturally raises the question as to what is the proper capacity for tertiary windings.

Factors Controlling Tertiary Capacity: The authors have properly recognized that the tertiary capacity and the tertiary reactance are interdependent and that when one is given the other is fixed thereby. The reactance in this case can of course only mean the net equivalent reactance as effective in the limitation of the short-circuit current. In other words, the capacity of the tertiary winding is determined by its short-circuit load. I wish to point out, however, that the *duration* of the short-circuit load is just as important a factor as the *magnitude* of the short-circuit load in the determination of the necessary tertiary capacity. This point probably can be best brought out by a comparison as follows:

Transformers are as a rule guaranteed for a two-second duration of short circuit. If we arbitrarily assume 5 per cent reactance as an average conservative lower limit of reactance for convenience in this illustration, it means that the conductor can carry twenty times its rated current for two seconds without exceeding a certain permissible temperature rise. From the standpoint of temperature rise based on heat storage, we have the following approximate equivalents:

20	times normal current for	2 seconds.
= 14	" " " "	4 "
= 11.5	" " " "	6 "
= 10	" " " "	8 "
= 7.8	" " " "	10 "

That is, a normal transformer winding will carry 7.8 times (call it 8 times) normal rated current for 10 seconds, on a conservative basis.

If the duration is assumed fixed, the capacity of the tertiary varies directly as the current. Based on the 10 seconds time for tertiary relay operation assumed by the authors we have the following table of values:

Duration of short-circuit assumed 10 seconds.

Short-Circuit Current	Tertiary Capacity in Per cent of Primary
8 times normal primary	100
6 "	75%
5 "	62%
4 "	50%
3 "	37%

This table shows that if a tertiary winding which is to carry four times rated primary current for 10 seconds is to have the same margin of safety in temperature rise as a 5 per cent reactance transformer designed to operate at normal conservative densities, it must have about 50 per cent of the capacity of the primary. Otherwise, it will have a smaller margin of safety.

It may also be interesting to note a table of tertiary capacities for various currents for a shorter duration, say 6 seconds.

Duration of short circuit 6 seconds.

Tertiary Short-Circuit Load	Tertiary Capacity in Per cent of Primary
11.5 × normal primary	100%
10 "	87%
8 "	70%
6 "	52%
4 "	35%
3 "	26%

It is not the object of this discussion to recommend any particular tertiary capacity nor the magnitude or duration of the short-circuit currents, but to indicate how the relative merit of tertiary capacity can be estimated in terms of customary values applying to the major windings and to emphasize the wisdom of specifying tertiary capacities ungrudgingly in view of the very small additional cost involved.

A further circumstance that makes a conservative policy towards tertiary windings desirable is the certainty that short-circuits which will load-up tertiary windings to their full capacity will be much more frequent than short-circuits which will load up the major windings to their full capacity. In other words, line-to-ground short-circuits are bound to be much more frequent than line-to-line short-circuits, and, hence, tertiary windings will have their ordeal oftener than the major windings.

Relation of Reactance to Short-Circuit Currents and Tertiary Capacity. Reactance in connection with tertiary windings usually means the reactance between the tertiary and the primary circuits. This reactance, however, does not directly and completely determine the short-circuit currents of the tertiary winding for all the various possible system connections and location of short-circuit. These are so varied that time would not permit their discussion now, but in any practical case, it is necessary to consider the various possible conditions, especially two of them, *viz.*, (a), the condition for the sake of which the tertiary winding is to be provided, together with, (b), the condition which imposes the severest duty on the tertiary winding. For instance, in a system grounded at a number of points for purposes of selective relay operation, a short-circuit across the terminals of the tertiary winding is likely to be a much more severe load to it than a line to ground short-circuit on the primary or secondary lines. In such an instance, if it is not absolutely necessary that the tertiary should furnish a load normally, it would be better policy not to bring out tertiary leads and thus avoid the extra risk that would otherwise be incurred.

Test Data: The authors present a number of interesting test data in curves and vector diagrams. All of these could have been predicted accurately by calculation, and the following comments may be interesting and helpful in their interpretation.

1. Figs. 1 and 2 of the authors are the experimental verification of two theorems or principles, *viz.*:

(a) Y-delta (or delta-Y) connected transformers operate equally well on unsymmetrical as well as symmetrical line voltages, that is, lack of balance or symmetry in the line voltages does not produce any circulating current in the delta.

(b) The leg voltages of a Y-connected bank of three similar transformers are equal to two-thirds of the corresponding median of the triangle of line voltages. On the side of the vertical leg the median of the line voltages is 244/2, that is, 122 volts, and $^2/_3$ of this, that is, 81 volts, should appear across that leg. The observed voltage is 80 volts, which may be considered as a complete commercial check. The fact that the observed voltage on the delta side on the corresponding leg is 4.7 volts proves by ratio that the observed voltage on the Y side must have been nearer 81 volts than 80.

2. In Fig. 3 excitation is applied only to the lower two legs (V_1, V_2). The vertical leg is short-circuited, but this short circuit does not affect the circuit in any way because the delta is open. The phenomena are capable of prediction by very elementary considerations, but presumably the authors included this in their tests for comparison with the case of Fig. 4.

3. Fig. 4 is the same as Fig. 3 except that the delta is closed. The vertical leg is disconnected from the generating system (note the break in the upper transmission line) and is short-circuited. The object of this line-break was no doubt to prevent a direct dead short circuit on the left hand bank of transformers at the generating end, because the break does not influence the phenomena of the righthand bank (the bank under test) in any way. The formula for the short-circuit current in this case is:

$$I_s = \frac{\% E}{3 \times \% I Z} \times \text{rated current}$$

An interesting fact about the short-circuit current of this case is that if the step-up transformers were not grounded, only the step-down bank being grounded, (in which case the upper line break would be dispensed with) the coil currents in the short-circuited step-down bank would be three times as large as the figures given above. This is a good illustration of the influence of system connection and number of neutral grounds on the short-circuit currents.

The voltages on the delta side by ratio from the primary Y side would be 14 volts. The short circuit current being only one third of that corresponding to a short-circuit on a unit as a single-phase unit, the voltage on the delta side of the short-circuited leg will be one third of the excitation, namely, one third of 14 volts, or, 4.7 volts. The measured value is 4 volts, and the small discrepancy may probably be accounted for by reading the voltmeter very low in the scale, as would be the case if the scale was of the order fo 50 volts or more. The voltages on the other legs would be 12.3 volts, the measured value being 12 volts which may be considered a sufficiently close check.

4. Fig. 5 is the verification of the simple statement that if one leg of a Y-Y bank is short-circuited, the other two legs take up the full line voltage in open-delta. The corner of the tertiary delta winding is open, at which point 41 volts, which is 1.73 times the voltage of the excited legs, is observed, being the vector sum of the two excited legs with a 60 deg. angle, as the authors have indicated in the vector diagram. When the open corner of the tertiary delta is closed, this voltage is short-circuited and we have the case shown in Fig. 6.

5. The calculation of the currents in Fig. 6 would require a knowledge of the impedance between the tertiary and the secondary. This is not given, but in as much as the currents are given, it is possible to reason back from them to the value of this reactance which figures out to be about 13.5 per cent and which we may apply to the solution of other cases, as for instance the case of Fig. 7.

6. In Fig. 7 short-circuit current could be furnished with the upper transmission line on open circuit, that is, the vertical leg not taking any part whatever. Short-circuit current could also be furnished with the lower two transmission lines on open circuit, that is, the lower two legs not taking any part whatever. This means that we have in effect two sources in parallel furnishing current to the short circuit. What the actual short-circuit current will be and how it will distribute itself in the network may be calculated as follows:

Let us call the upper vertical leg, on primary side, circuit A; the lower two legs together, B; and the line-to-neutral circuit on the secondary side, C.

When the upper leg circuit (A) alone is excited, lower legs (circuit B) being disconnected, the impedance effective from A to C is

$$\% I X_{ac} = \% I X_{ps} = 5.7 \text{ per cent}$$

When the upper leg (circuit A) is disconnected, and the lower legs (circuit B) alone furnishes the load in C, the effective impedance is

$$\% I X_{bc} = 2 \times \% I X_{pt} + \% I X_{st} = 53.5 \text{ per cent}$$

Evidently, the impedance from A to C being about one-tenth of that from B to C, we can say as a first approximation that the former will furnish 10 times as much kv-a. into the short circuit as the latter. Since the voltage of the former is twice that of the latter, its current will be five times as much to make its kv-a. ten times as much. Considering the test data we find that the current in circuit A (vertical leg) is 3.78 amperes, and that in circuit B (lower two legs) is 8 amperes total, the ratio being about five to one. The effective impedance of the two circuits A and B in parallel for the circuit C will be approximately

$$\% I Z_e = \frac{1}{1/5.7 + 1/53.5} = 5.2 \text{ per cent}$$

$$I_e = \frac{\% \text{ Excitation}}{\% I Z_e} \times \text{Rated current}$$

$$= 0.6/5.2 \times 225 = 26 \text{ amperes.}$$

The observed current is 24 amperes, about 8 per cent lower, which may perhaps be accounted for partly by the impedances introduced into the circuit by the meters, leads and grounds, and partly by the approximateness of the data on which the calculation is based.

The exact calculation of problems like this involves some methods into a discussion of which we cannot enter now but which will be found described in a paper entitled "Theory of Three-circuit Transformers" to be published in a forthcoming issue of the JOURNAL of the Institute.

8. Of the tests with auto-transformers, Fig. 10 is of particular interest illustrating the reason for and widsom of the two simple general maxims for the operation of auto transformers, *viz.*: "If the system is grounded, ground the neutral of the auto-transformer; if the system is isolated, isolate the neutral of the auto-transformer." In Fig. 10, the delta is open, so that the unit is acting as a straight Y auto-transformer, and it is found that if the system is grounded and the auto-transformer is isolated, a line ground on the secondary over-excites that leg in the ratio $E_1/(E_1 - E_2)$ and shifts the neutral. In this case, the low voltage being 50 per cent of the high voltage, this ratio was 2. If the low voltage had been 90 per cent of the high voltage, this ratio would be 10, that is, the leg whose line is grounded would be overexcited 10 times. This would of course overexcite the other legs also. If the primary is the low-voltage side, a high-voltage line ground not only overexcites but also reverses the voltage of the corresponding leg.

Transformers for High-Voltage Systems

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Review of the Subject.*—The designs of transformers for 220-kv. systems do not represent radical departures from designs which have become standard for lower voltages. The solidly grounded neutral system is used and advantage is taken of this to use graded insulation and reduced voltage test. Auto-transformers are used to connect between the 220-kv. system and existing high voltages and are connected star-star with delta tertiary. The tertiary may or may not be used to supply synchronous condensers but it is not used for the supply of power load.*

* * * * *

WHEN considering the use of extra high voltages for the transmission of electrical energy, the apparatus that first claimed the attention of the engineers consisted of transformers, switches and line insulators. Other apparatus, such as generators and synchronous condensers, have generally been considered as being unaffected or only slightly affected in their design because of their application in connection with high-voltage transmission. The problem of transformer design for 220,000-volt operation has been met and solved, however, without the encountering of any unforeseen difficulties or complications.

The application of transformers to 220,000-volt service has brought about a change in the generally prevailing ideas in regard to transformer construction in that the considerable savings obtained by the use of graded insulation on the 220,000-volt transformers has centered attention on similar savings which could be accomplished on transformers for the more moderate voltages. Except for this point, design problems on the 220,000-volt transformers have not been appreciably different from those which had been previously met in standard high-voltage transformers and design. The same types of design have been used, transformers of both the shell and core forms having been built and are now in operation on 220,000-volt circuits. The quality of insulating materials and the insulating oil have not been affected by the advent of the high-voltage. The same types of terminal bushings and the same principles of design have been followed.

It was universally recognized among engineers that a solidly grounded neutral should be used on the 220,000-volt system. This point being established, it became desirable to take advantage of the savings that could be effected in transformer manufacture by using only one high-voltage terminal and by grading the insulation from the maximum at the line end to a minimum at the neutral end of the high-voltage winding. Such grading of the insulation does not, however, affect the providing of extra insulation between turns or the turns immediately adjacent to the ends of the winding, and such extra insulation has been provided, both at the line end and at the ground end. This practise has been followed in recognition of the fact that surges or steep wave front potentials might be experienced at either end of the winding even when one end is connected solidly to ground.

In carrying out the design of the transformer for use on a system on which the neutral is solidly and permanently grounded, it has seemed to be possible and desirable to recognize the lower electrical stresses to which a transformer so applied would be subjected when compared with a transformer applied on a system in which the neutral is not necessarily permanently grounded. It should, therefore, be possible to reduce the insulation test to which the transformer is subjected.

It is evident that it would be impracticable to apply the voltage test specified in the Institute Rules between the whole winding and ground because of the grading of the insulation. The logical method of applying the insulation test is to ground the neutral end of the winding on the transformer core and to

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

induce the test voltage in the winding. This gives the maximum voltage test at the line end and graduates it to zero at the neutral end. The dielectric test for a transformer as specified by the Institute Rules is two times the line-to-line voltage plus 1000 volts. The maximum operating voltage which might obtain on a transformer used on a line without solid grounding of the neutral would occur when one line became grounded and would, therefore, amount to the line-to-line potential.

The voltage test specified by the Institute gives a factor of safety of two on this voltage or, in other words, gives a margin equal to line-to-line voltage above the maximum normal obtainable voltage. When a transformer is used on a solidly grounded neutral system and is provided with graded insulation, the practise has been followed of testing with this same margin but as the maximum normally obtainable voltage in such a case is only the line-to-neutral value, the test given is the line-to-neutral value plus the line-to-line value or 2.73 times the line-to-neutral voltage.

As has been suggested, the attention brought to the graded-insulation, single-terminal transformer, by its use on 220,000-volt circuits, has brought about similar arrangements for more moderate voltages and recently transformers for 165,000-volt systems and also for lower voltages have been constructed on this principle. The savings accomplished are larger and more important on the very high-voltage transformers, but there is also a very real saving on transformers built for lower voltage operation.

On the transmission systems now arranged for 220,000-volt operation, special conditions have called for the use of auto-transformers connected to the high-voltage line. The Southern California Edison Company had been operating their Big Creek lines at 150,000 volts and the two original Big Creek plants and the Eagle Rock step-down substation were provided with transformers and switching equipment to give or to utilize this potential. It was considered economy, therefore, to allow this 150,000-volt apparatus to stand and to obtain the 220,000-volt potential by auto-transformation. This necessitated star-star connection between the two voltages and the use of tertiary windings to supply the third harmonic component of magnetizing current and to provide a comparatively low impedance path for the flow of the unbalanced current demanded in the case of a ground on the star-connected system. The importance of maintaining this tertiary winding in service under all exigencies was considered by the engineers of the Southern California Edison Company to be enough to warrant the exclusion from this winding of any other load such as low-voltage local distribution or synchronous condenser current.

The newer Big Creek power plants and the new Laguna Bell substation, near Los Angeles, are equipped with two-coil transformers connected in star on the high-voltage, and delta on the low-voltage side as no intermediate 150,000-volt potential is required.

The Pacific Gas & Electric Company have had a 100,000-volt system in use in the San Francisco district for many years and it was planned to bring the power from Pit River into the district and to transmit it to the 100,000-volt system already in existence. Thus transformation from 220,000 volts to 100,000 volts became necessary and auto-transformers provide the most economical means of making such transformation. These auto-transformers are located at the Vaca substation. They are connected star-star and provided with a delta tertiary winding which cares for the third harmonic component of magnetizing current, unbalanced ground current in case of line faults, and at the same time supplies the synchronous condensers used for line regulation.

It is not practicable to use the tertiary winding for supplying local distribution and at the same time synchronous condenser current if the field of the synchronous machine is varied in such a manner as to give a constant potential on the high-voltage side of the transformer. This prohibition is on account of the voltage variation which must obtain on the tertiary winding under this condition.

Discussion

E. A. Smith: It would be quite interesting if Mr. Copley would give some more information on these 220,000-volt circuit breakers, such as the necessary data on the rupturing capacity, size of tanks, amount of oil, the grade and the insulating factors, what sort of contacts used and the depth of contacts under the oil.

Also the general tests and curves covering all the characteristics of flashover and safe operating limits.

R. W. Sorensen: Mr. Copley's paper sums up nicely the present-day accomplishments toward the consummation of the 220,000-volt, 1100-mile California bus-bar system as proposed at the 1919 Pacific Coast Convention held in Los Angeles. At that time I hardly thought the plan presented would in any way be realized in less than a decade.

In paragraph two, Mr. Copley speaks of graded insulation being used. Descriptive articles I have read and a rather casual inspection of some of these transformers I have seen out of the tank did not indicate to me a truly graded insulation. To my way of thinking, the term "graded insulation" means a tapering insulation. I am under the impression that, with the exception of the end turn insulation, the insulation between turns is uniform throughout the winding. Also I did not find grading between high-tension windings and core legs. What I have understood is that the large insulation barriers used in the more familiar types of construction, between coils and yokes have been left out, a condition made possible by a plan of construction which permits a single high-voltage lead to be brought out from the center of the coil stacks. This form of construction is not original with the 220,000-volt transformers, as it was first made known by the W. J. Wooldridge patents, so old they have expired, and afterwards used in the Fortescue plans for stress distribution, published about ten years ago. In practise, it has been tried out on many voltages for several years. It is not a scheme necessary for engineering reasons, but is largely one of economy, determined not only by the voltage, but also the rating of the apparatus. I think it would be a mistake to create the impression that transformers for this voltage can be built only with one insulated lead and a solidly grounded winding.

This brings us to a point in paragraph three. I rather doubt the advisability of saying engineers are universally of the opinion that 220,000-volt systems should be provided with solidly grounded neutrals only. As such a system grows it may be necessary to use Peterson coils, or a resistance of some type in the neutral of 220,000-volt systems and also for systems for other voltages, even though we have by long practise become accustomed to the use of solidly grounded neutrals on the Pacific Coast.

It seems to me ample provision for giving single terminal transformers a voltage test made in Rule 6362 of the A. I. E. E. Standards which reads, "Under certain conditions it is permissible to test transformers by inducing the required voltage in their windings in place of suing a separate transformer. By 'required voltage' is meant a voltage such that the line end of the winding shall receive a test to ground equal to that required by the general rules."

Further, the Standardization Rules call for a specific test for three-phase work of two times line voltage, because many years of practise show this to be desirable. This of course gives a ratio of transformer test voltage to transformer working voltage of 3.46 in a Y connection and of 2.00 for a delta connection. Also, Rule 6321 (*e*) specifically states that single-phase systems permanently grounded used for more than 300 volts shall be tested at 2.73 times circuit voltage plus 100 volts. If this difference is unwarranted, the matter should be corrected by a change in the Standardization Rules and not by an avoidance thereof.

The economy and also the difficulties encountered in the use of auto-transformers are well known. Therefore, to avoid most of these difficulties auto-transformers, when Y-Y as they usually are, are provided with tertiary windings. The capacity of the tertiary winding is determined by its ability to take care of short circuits. The tertiary winding by providing a path for the third harmonic component of the exciting current, stabilizes the neutral, and should a ground occur on the system it provides a path through which current can flow until a balance is obtained, but if there were no tertiary winding there would be no current flow, as indicated in the seventh line from the bottom of column one of the 2nd page of Mr. Copley's paper. For my part I can see no particular reason to assume the tertiary more immune from trouble when left idle than when used to supply power, as power load, or to a synchronous condenser, providing proper precautions are taken against faulty connections and overload. Of course, if there is a large leakage reactance factor between tertiary and secondary winding of the transformer, the secondary voltage could not be expected to stay constant if the regulator were set to regulate on the teritary or the primary, but I see no reason why a condenser can not be supplied with power by the tertiary and yet be made to regulate either the secondary or primary voltage as desired.

A. W. Copley: Professor Sorensen is correct in his statement that the insulation between turns in a transformer with "graded insulation" is uniform throughout the body of the winding except for the end turn padding. The grading is, however, accomplished in the insulation from coils to ground as it is the potential from coils to iron that varies from zero at the ground end to a maximum at the line end of the winding. In shell type construction the grading is done in the group insulation.

The fundamental reason for grading is economic and its justification is the fact that with one end of the winding solidly grounded there is no necessity for taking up space by also insulating this end for full voltage. For the same reason there is no necessity for a high-voltage bushing on the ground end of the winding. The only systems now operating at 220 kv. use the solidly grounded neutral and I have yet to hear engineers seriously propose to operate at such voltages with neutrals either free or grounded through resistance or reactance.

As to the test voltage value for transformers having graded insulation, I explained in my paper the reason for the acceptance of a value of 2.73 times the voltage from line to neutral. Although this is a lower value than is called for by a strict interpretation of the Institute Standards, I believe it is justified and consistent with the values used on transformers with straight insulation which might be used on free neutral systems. The present standards take no cognizance of the graded insulation transformer in the test values given.

The desirability of taking load from transformer tertiaries in addition to the condenser current is largely a question of voltage regulation. The condenser may be used to regulate the voltage on any one winding but not on two simultaneously. In most instances it is desired to regulate the secondary voltage and in these cases the tertiary voltage will generally vary over so wide a range as to make it unsatisfactory for supplying load.

Applications of Long Distance Telephony on the Pacific Coast

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***Review of the Subject.**—This paper deals with the general problem of providing long distance telephone service on the Pacific Coast. A description of the present toll plant is given and the applications which have been made of recent developments in telephone practise are illustrated. Reference is made to the extensive use which has been made of carrier telephone and telegraph systems, and the many special problems, such as the loading of long toll entrance cables, which these systems introduce, are pointed out. The service to Catalina Island is described to illustrate the changes which are taking place in the communication art. This service was first provided by means of a radio link to which a privacy system was later added. This radio system was later replaced by two submarine cables between the island and the mainland. The mechanical and electrical characteristics of these cables are given, together with a description of the work of laying, which was done principally by the government cable ship, "Dellwood."*

WITHIN the last few years the extensive and systematic research which has been carried on by the Bell Telephone System has resulted in developments in the art of long-distance telephony which are indeed remarkable and have placed at the disposal of the Bell companies an ever increasing variety of facilities whereby the telephone service rendered the public has been greatly extended and improved. Many of these new developments have been ably described in Mr. Osborne's paper, and in others recently presented before the Institute. It is the purpose of this paper to point out the extensive use which has been made of these newest facilities on the Pacific Coast and how the various elements have been combined under the par-

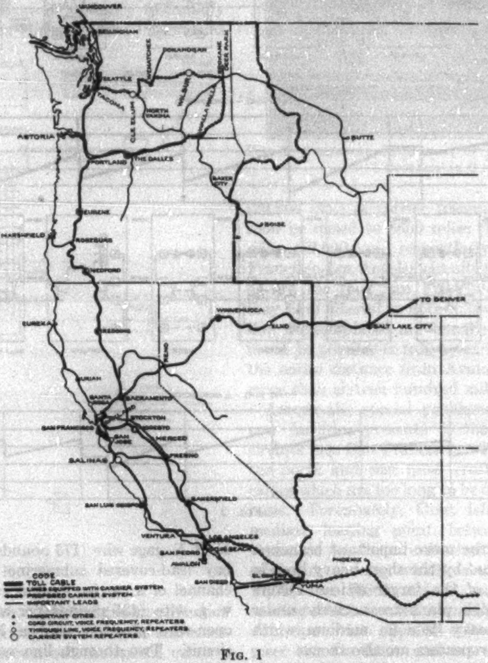
VANCOUVER
BELLINGHAM
WENATCHEE
OKANOGAN
SPOKANE
DEER PARK
SEATTLE
TACOMA
CLE ELUM
NORTH YAKIMA
WILBUR
WALLA WALLA
ASTORIA
PORTLAND
THE DALLES
BAKER CITY
BUTTE
BOISE
EUGENE
MARSHFIELD
ROSEBURG
MEDFORD
EUREKA
REDDING
WINNEMUCCA
ELKO
SALT LAKE CITY
TO DENVER
RENO
UKIAH
SANTA ROSA
SACRAMENTO
STOCKTON
SAN FRANCISCO
SAN JOSE
MODESTO
MERCED
FRESNO
SALINAS
SAN LUIS OBISPO
BAKERSFIELD
VENTURA
LOS ANGELES
SAN PEDRO
LONG BEACH
AVALON
SAN DIEGO
EL CENTRO
YUMA
PHOENIX
CODE
TOLL CABLE
LINES EQUIPPED WITH CARRIER SYSTEM
PROPOSED CARRIER SYSTEM
IMPORTANT LEADS
IMPORTANT CITIES
CORD CIRCUIT, VOICE FREQUENCY, REPEATERS
THROUGH LINE, VOICE FREQUENCY, REPEATERS
CARRIER SYSTEM REPEATERS

FIG. 1

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

ticular conditions encountered here to form a coherent toll plant.

To illustrate this, the accompanying map has been prepared. Backbone routes extend north from San Diego through Los Angeles, San Francisco, Portland and Seattle, to the international boundary where connection is made with the British Columbia Telephone Company's lines to Vancouver. Contact with the territory to the east is effected by leads to Phoenix, Salt Lake, Boise and Butte. The heavy lines on the map represent leads on which carrier systems are in operation. The medium weight lines represent other main leads not so equipped at present, while the light lines represent some of the more important branches. Toll cables are represented by the short heavy lines in the immediate vicinity of the larger cities. Future carrier facilities for which plans are actively under way are indicated by heavy dots on medium width lines. Various types of repeaters are also shown.

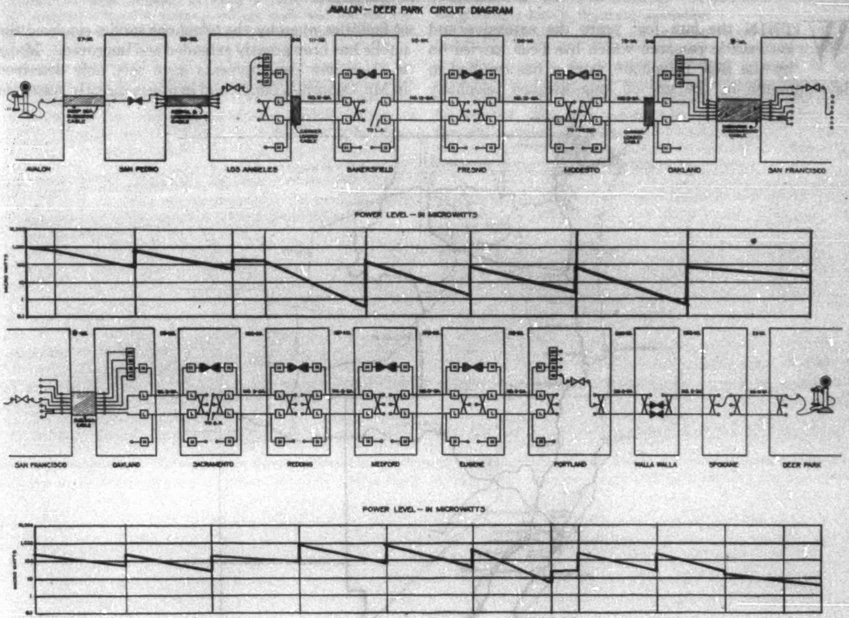

Fig. 2

In any telephone connection regardless of its length the aim is to maintain a satisfactory equivalent or standard of efficiency. All direct circuits between the large cities and towns are designed to be well within this equivalent. Where no direct connection exists, two or more circuits joining intermediate points are connected together. When the equivalent of such a built up circuit exceeds the limit which has been set, repeaters are inserted at the intermediate points to preserve the necessary over-all efficiency.

To illustrate the manner in which the several types of circuits may be combined, let us assume a connection established between Avalon on Catalina Island and Deer Park, Washington, a tributary of Spokane. The circuit arrangement together with the comparative power level at all points when the Avalon party is speaking is shown in Fig. 2. Starting from the Avalon end we encounter in succession, a deep sea submarine cable, a long loaded toll cable, the 25-kilocycle channel of a carrier telephone system operating over No. 12 N. B. S. gage wire (173 pounds per wire mile), a trans-bay lead-covered submarine cable, the 10-kilocycle channel of a carrier system operating over No. 8 B. w. g. wire (435 pounds per wire mile), a No. 8 gage open-wire phantom and a No. 12 gage open-wire circuit. Two through line voice frequency repeaters and seven carrier-frequency repeaters are employed. Voice-frequency cord circuit repeaters are inserted at Los Angeles to connect a toll cable with a carrier system, at San Francisco to join two carrier channels, and at Portland to connect a carrier channel with an open-wire phantom circuit.

In addition to the various types of circuits and amplifiers involved in this connection the power relations are also of interest. Consider for example the conditions at a time when the power delivered to the line in question by the Avalon telephone is 1000 microwatts. The over-all circuit efficiency is about 0.4 per cent (25 mile equivalent) so that four microwatts are delivered to the Deer Park instrument. Were the cord circuit repeaters omitted the power transmitted would be reduced to 0.02 microwatts. Should all the repeaters including those in the carrier systems be omitted, the received power would be 1.3×10^{-16} microwatts.

Not only is it necessary to maintain a certain over-all efficiency, but also is it required that the power be kept within certain upper and lower limits at all points along the way. Should the voice currents become too great, overloading of the repeater tubes accompanied by distortion of the speech waves will occur. If the current becomes too weak, the circuit will be noisy or experience excessive crosstalk from other lines. These upper and lower limits make it necessary to amplify the current at regular and fairly frequent intervals as is done in the circuit shown. Were all the amplification to be effected at the receiving end, the noise and crosstalk would completely drown out the speech current. In case the amplification were all applied at the sending end the speech input power would have to be about thirty billion kilowatts. Such a mode of operation is of course impossible.

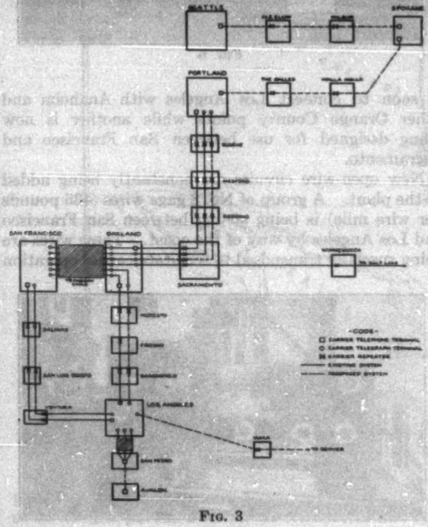

FIG. 3

One of the most noteworthy things about the Pacific Company's toll plant is the extensive use which has already been made of carrier systems. This is due largely to the fact that the great distances which separate the main centers of population are sufficient to justify the large cost of carrier terminal and intermediate repeater equipment, because of the resulting saving in line wires. Fig. 3 shows the systems now in use and those upon which work is progressing. At the present time, there are in operation 6400 channel miles (10,300 km.) of carrier telephone and 18,500 miles (29,800 km.) of carrier telegraph. These figures will soon be raised to 8600 miles (13,800 km.) and 32,300 miles (52,000 km.) respectively. All of the eight San Francisco-Los Angeles circuits now are carrier channels as are the four San Francisco-Portland circuits. It may be of interest to note that when the carrier systems now contemplated are completed it is expected that it will be possible to talk by carrier alone for practically the entire distance from Avalon to Spokane, which is more than sixteen hundred miles (2600 km.).

FIG. 4—TERMINAL EQUIPMENT OF TWO CARRIER TELEPHONE SYSTEMS AT OAKLAND

Among the special problems which the use of the new facilities presents is that of bringing the toll circuits into San Francisco. Those which enter from the north and east must cross the bay in submarine cables which are too long to be operated on a non-loaded basis. Fortunately, Goat Island provides an intermediate loading point between Oakland and San Francisco for the circuits from the east, although the minimum spacing which is obtained in this way is 12,000 feet (3660 meters), or more than twice the spacing which is used normally with cables associated with open wire. Special coils have been developed to meet this condition and are used on all the existing submarine toll entrance cables, since the provision of submarine loading coils would not have been as economical.

Although lines loaded in this special manner are satisfactory for voice frequency circuits, they cannot be used for carrier systems. It has therefore been necessary to make Oakland the carrier terminal point for all such systems operating over wires which approach the bay from the east, although the individual channels, both telephone and telegraph, are extended across the bay to San Francisco and terminated there.

Another special problem incident to the use of carrier systems is the loading of long toll entrance cables. Two of these, each about nine miles (14.5 km.) in length, occur in this territory. One extends from the Los Angeles office north to Cahuenga Pass and provides entrance facilities for both the coast and valley leads. This cable will soon be extended to Simi junction, an increase of three miles (5 km.). The second cable extends from Oakland to San Leandro and serves the valley circuits to Los Angeles. Both of these cables have been loaded at frequent intervals with special coils so as to render them suitable for use with carrier systems. A third cable 12.5 miles (20 km.) in length extending south from Seattle will also be loaded for carrier operation in the near future. Such loading is quite expensive and has an important bearing on the question of carrier circuit economy.

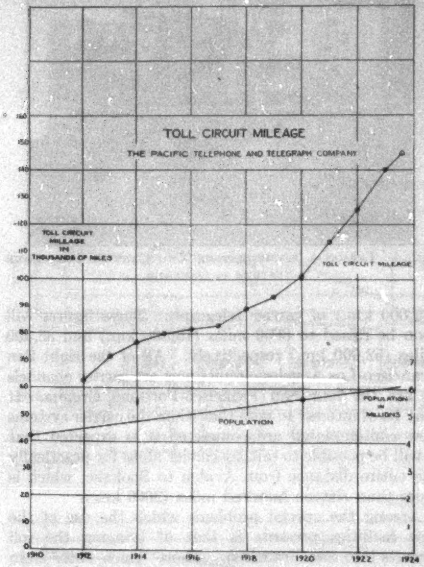

Fig. 5

One of the most difficult problems encountered in this territory is to anticipate and provide for the rapid increase in the long distance or toll business. The extent of this growth is shown on Fig. 5, which gives the total toll circuit mileage for the last ten years. For comparison the increase in population for this period is also shown. In order to care for this increase in business, extensions to the plant are constantly being made. Mention has already been made of the several carrier projects now under way. Toll cables are to be placed between Seattle and Tacoma, between San Francisco and San Jose and between Los Angeles and Long Beach to supplement those now in use. A cable is soon to connect Los Angeles with Anaheim and other Orange County points while another is now being designed for use between San Francisco and Sacramento.

Fig. 6

New open-wire circuits are constantly being added to the plant. A group of No. 8 gage wires (435 pounds per wire mile) is being strung between San Francisco and Los Angeles by way of the coast. These wires are being specially transposed to facilitate carrier operation and together with those already in place will provide facilities for several new carrier systems over this route. The new No. 8 gage circuits from Los Angeles to Denver will furnish a new and much needed outlet for eastern business and will also materially reduce the possibility of total interruption to service between the coast and eastern points.

Fig. 7

No better illustration of the rapid advance in the art of communication or the application of new methods to the telephone toll plant can be found than that provided in our territory by the service to Catalina Island.

In 1920 commercial telephone communication was established between Pebbly Beach on Catalina Island and Long Beach by means of a two-way radio system. At the two ends the connection was extended to Avalon and Los Angeles respectively by means of wire circuits. Soon after this a telegraph channel for handling commercial messages was added. This is the first and perhaps the only instance in which radio has been used as an integral part of a telephone toll system and operated under the same severe requirements as are imposed on the wire circuits.

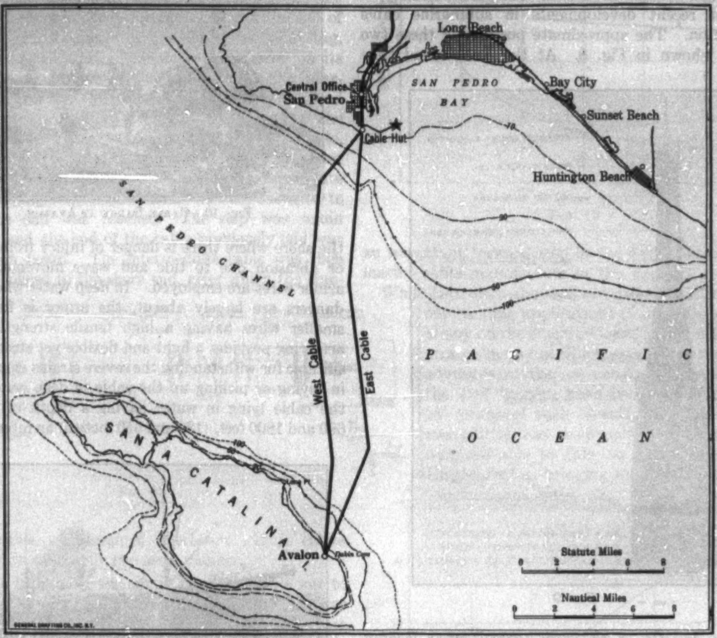

FIG. 8—LOCATION OF SOUTH CATALINA ISLAND CABLE

From the time of its installation, this circuit handled a tremendous volume of business. That it was quite efficient is proved by the fact that it was used in frequent demonstrations such, for example, as the one in 1921 when a conversation was held by way of the transcontinental telephone line between Catalina and the S. S. Gloucester one hundred miles (160 km.) off the Atlantic coast; and again in the same year when Catalina talked to Havana by way of the radio and the newly completed Key West-Havana submarine telephone cable,—a total distance of about 5500 miles (8800 km.).

One objection to the radio circuit was the lack of secrecy, and the recent increase in the use of radio sets for the reception of broadcasting rendered this increasingly troublesome. In order to obviate this difficulty a privacy device which was recently developed by the Bell System was installed on this circuit. This privacy system although it in no way impaired the quality or volume of the speech over the radio circuit, rendered it unintelligible to anyone listening with a radio receiving set of the usual type.

The radio system recently was replaced by two deep sea cables between Avalon and the mainland. The license of the radio "talk bridge" expired on August 1st and the Department of Commerce requested that its operation be discontinued, making its wave lengths available for broadcasting.

The capacity of the ether for radio messages is distinctly limited and when such services as broadcasting, ship-to-shore telegraphy and telephony, radio compasses, etc., which can be handled only by radio, have been accommodated, the available wave lengths of the ether are about exhausted. Consequently, the change from radio to cable in a case like Catalina Island, where the cable is not only feasible but more economical than radio, was for the good of the service, especially as the large volume of business which developed made it necessary to provide additional facilities.

The submarine cables were manufactured by the Western Electric Company at Hawthorne, Illinois, and in their design have been incorporated many of the most recent developments in submarine cable construction. The approximate position of these two cables is shown in Fig. 8. At the Avalon end they terminate directly in the new central office located about 800 feet (240 meters) from the shore. At the San Pedro end a small concrete hut was erected at the base of the breakwater in which to terminate the cable and house the special terminal equipment. This was selected as a landing point as the water in the cove formed by the shore and the breakwater is comparatively quiet and at this point a small sandy beach has been formed. At all other points in the neighborhood of Point Fermin the shore is very rocky and steep and the water quite rough. The distance between Point Fermin and Avalon is about twenty-three nautical miles (6087 feet or 1856 meters per nautical mile). Near the center of the channel the water reaches a depth of slightly over 3000 feet (900 meters). For the greater part of the distance the cables were laid about two miles (3.2 km.) apart. This separation was maintained so as to give greater insurance against simultaneous interruption and to render it easier to pick up one cable without disturbing the other.

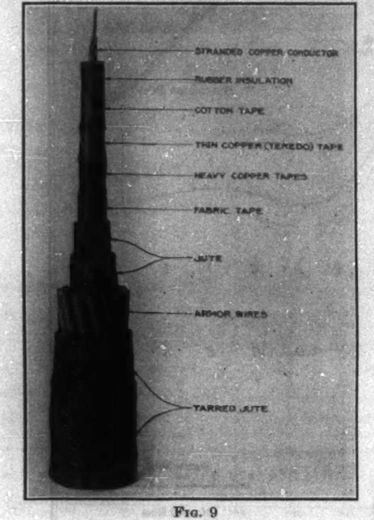

FIG. 9

The cables themselves are of the deep sea type. Such a cable provides but a single physical circuit formed by the central insulated conductor as one side and the sea water, armor wires, and copper tapes, all in parallel, as the other side. The details of construction are shown in Fig. 9. In shallow water near the shore where there is danger of injury from anchors or abrasion due to tide and wave movement, large armor wires are employed. In deep water where these dangers are largely absent, the armor is formed of smaller wires having a high tensile strength. Such armoring provides a light and flexible yet strong cable, suitable for withstanding the severe strains encountered in laying or picking up the cable in deep water. For the cable lying in water having a depth of between 600 and 1800 feet, (180 and 550 meters) an intermediate type of armoring employing medium size wires was used.

FIG. 10—CABLE BARGE AT AVALON

FIG. 11—U. S. CABLE SHIP "DELLWOOD"

The first part of each cable to be laid was the 2.5 mile (4 km.) shore end section on the Avalon side. This was done from a barge equipped with a large power-driven reel upon which the cable was first wound. At the Avalon end the barge was anchored near shore and the end of the cable pulled up through the underground ducts and into the office by means of a tractor. Following this the barge was towed out to sea, the cable being unwound from the reel. Prior to the laying, marker buoys had been placed to indicate approximately the location of the ends of these shore end sections. To the end of the cable a section of chain and a long rope was attached, the other end of which was made fast to a buoy.

The main portion of the cable was laid by the government cable ship "Dellwood." This ship, which has a length of 330 feet, (100 meters) is well provided with the usual cable handling equipment.

In laying the main portion of the cable the ship was first anchored about half a mile (about 1 km.) off the San Pedro shore opposite the cable hut and the end of the cable pulled ashore with the aid of the barge, tug, and other small craft. The ship then started on its course paying out first the shore end, then the intermediate and finally the deep sea cable, all of which had previously been spliced together. On reaching the 1800-foot (550 meter) depth line on the Avalon side, the ship was stopped, the remainder of the deep-sea cable cut off and the end spliced to the intermediate type. Laying was resumed until the buoy fastened to the shore end was reached. The ship was again stopped, and the end of the cable previously laid was hoisted on board. The intermediate cable was then cut and spliced to the shore end, after which the completed cable was dropped overboard. Both cables were laid in essentially the same manner.

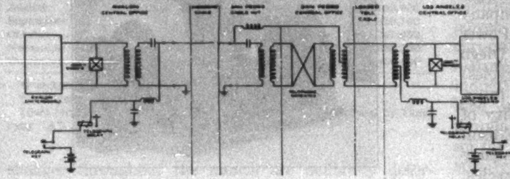

Fig. 12

As the balance of the circuit from the cable hut to Los Angeles had been provided in advance, it was possible to communicate between the ship and shore points as soon as the end of the cable was brought into the hut and the connection established there.

While the cable was being laid, a number of conversations were held between company officials on board the ship and persons in and around Los Angeles and San Francisco. As soon as the final core splice was made communication was established between Avalon and Los Angeles and two hours later a conversation was held with American Telephone and Telegraph officials in New York.

At present each cable provides one d-c. telegraph and one voice-frequency telephone circuit, the circuit arrangement being shown in Fig. 12. Figs. 13 and 14 show the impedance and attenuation respectively of the East Cable.

Although not imposing as regards total length or depth of water encountered, the performance of this cable will be watched with great interest as it marks an important forward step in the development of submarine cable manufacture in this country.

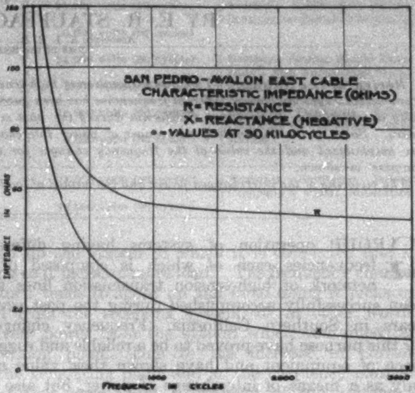

Fig. 13

While this discussion has covered only a few of the outstanding applications of long distance telephony on the Pacific Coast, it will serve to indicate the way in which some of the most recent advances in the art constantly developed by the Bell System have been employed. It is by means of such recently developed instrumentalities that the people of the Pacific Coast now are able to talk to all sections of the United States, bringing the inhabitants of this broad country into an intimate contact that cannot fail to be of great commercial and social benefit.

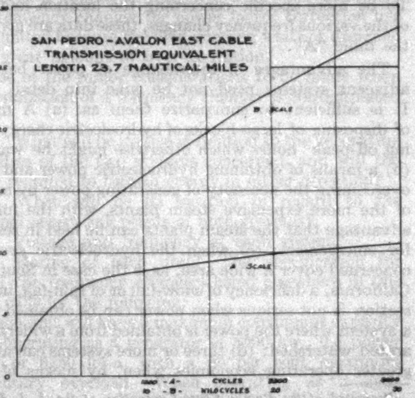

Fig. 14

Group Operation of Systems Having Different Frequencies

BY E. R. STAUFFACHER and H. J. BRIGGS
Associate A. I. E. E. Associate A. I. E. E.
Both of the Southern California Edison Co.

Review of the Subject.—*The group operation of high-tension transmission systems having different frequencies has been successfully accomplished in Southern California during the past seven years. Some of the outstanding advantages of power interchange are emphasized and the value of the frequency changer for this purpose is shown.*

The metering of the interchanged power and the method of loading a frequency-changer offer somewhat unusual problems. With a liberal field design a synchronous frequency changer is of value as a means of power factor correction.

Certain troubles with the starting winding and the bearings of some of the machines have developed as a result of severe operating conditions but the frequency-changer has proved to be a reliable and satisfactory means of interchange of power between large systems.

GROUP operation of systems having different frequencies each of which is composed of a network of high-tension transmission lines has been successfully accomplished during the past seven years in Southern California. Frequency changers for this purpose have proved to be a reliable and rugged piece of equipment and have shown their value not only as a means of interchange of power, but also as a means of power factor correction when necessary.

The position of the Southern California Edison Company is unique in that it is, with one exception, entirely surrounded with systems operating at a different frequency. The above company interchanges power with the San Joaquin Light and Power Company (60 cycles) on the north; the Southern Sierras Power Company (60 cycles) on the east; and the San Diego Consolidated Gas and Electric Company (60 cycles) on the south. In addition to this there is an interchange of power between the systems of the City of Los Angeles and the Southern California Edison Company, but without the necessity of frequency changers as both systems operate at fifty cycles. To be more specific concerning the location and size of the various frequency changes, these data are given in the table "A".

The advantages of interchange of power between adjacent systems need not be gone into detail here. It is sufficient to summarize them as: (a) A means of disposing of large blocks of hydroelectric energy during off-peak hours which otherwise might be wasted; (b) a means of obtaining hydroelectric power and thus overcoming the necessity of generating power by means of the more expensive steam plants, with the further advantage that the steam plants can be held in reserve for emergencies; (c) when the hydroelectric systems concerned cover a wide area, as is the case in Southern California, a deficiency of snow-fall or of rain-fall, in one section is not serious when power can be obtained from a system where the power is obtained from a widely separated watershed; (d) three or more systems can attain greater operating economies when, by means of frequency changers and intermediate transmission networks, one system not directly connected with another can obtain power through the lines of the intermediate system; (e) a ready means of exchanging power in case of a breakdown of a major generating plant of any of the systems so inter-connected.

To give an idea of the value of group operation of a number of systems through frequency changers, table "B" has been prepared showing the amount of power interchanged during the past year.

Frequency Changer at Capistrano Substation
5,000 kv-a., 50-60 cycles.

From a study of this table the flexibility of frequency changers as a means of interchanging power is well shown. In addition to delivering power from one company to another, the equipment can obviously be utilized as a means of delivering power to a third company.

It may be of interest to outline the method of metering when power is to be transmitted in either direction through a frequency changer. The scheme generally used is to have both sides of the machine equipped with a polyphase curve drawing wattmeter and two polyphase integrating watthour meters. Each of the integrating instruments is equipped with a ratchet mechanism so arranged that one instrument will meter energy only when power is flowing towards the frequency changer while the other will register only when power is flowing from the machine. By this combination the power delivered to either company is accurately recorded and the losses in the frequency

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Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

TABLE A

Connection Between	Size of Unit	Voltage	Location of Unit
Southern California Edison Company (50 cycles) and Southern Sierras Power Company (60 cycles)	5,000 kw.—0.8 P. F.	11,000 volts (50 cycles) 6,600 volts (60 cycles)	Southern California's Edison Company's Colton Substation
Southern California Edison Company (50 cycles) and Southern Sierras Power Company (60 cycles)	5,000 kw.—0.8 P. F.	11,000 volts (50 cycles) 6,600 volts (60 cycles)	Southern Sierras Power Company's San Bernardino Steam Plant
Southern California Edison Company (50 cycles) San Diego Consolidated Gas & Electric Company (60 cycles)	5,000 kw.—0.8 P. F.	11,000 volts (50 cycles) 6,600 volts (60 cycles)	Southern California Edison Company's Capistrano Substation
Southern California Edison Company (50 cycles) and San Joaquin Light and Power Company (60 cycles)	15,000 kw.—0.8 P. F.	15,000 volts (50 cycles) 18,000 volts (60 cycles)	Southern California Edison Company's Vestal Substation

TABLE B

POWER INTERCHANGE (KW-HR.) THROUGH FREQUENCY CHANGERS

Month	Purchased and Delivered by Company "B' to Company "A"		Delivered by Company "C" to Company "A"	Purchased by Company "D" from Company "A"
	Substation No. 1	Substation No. 2		
January	100	1,850	0*	571,300
February	0	0	4,492,800	1,070,700
March	0	127,150†	604,800	1,799,850
April	0	0	99,000	3,184,300
May	8,150	1,350*	0	2,798,200
June	1,750*	0	41,400	2,767,200
July	342,500‡	1,462,850	19,800	1,957,500
August	24,700‡	211,050‡	5,079,600	2,192,900
September		19,750	7,619,400	1,910,700
October		450	8,499,100	1,749,300
November		300	6,080,400	1,558,700
December	120,050	137,550	5,337,000	1,400,800

*Purchased by Company "A'.

†Received for transfer to Company "D".

‡Part received for transfer to Company "D" and part utilized by Company "A".

changer throughout the month are definitely determined. The losses in the machine can, of course, be borne by either of the parties according to the terms of the contract, but the method usually followed is to bill the receiving company for one-half of the losses. Since the installation of the original frequency changer set in Southern California this method of metering has been used with satisfaction to all parties concerned.

The method of loading up a frequency changer has apparently not always been thoroughly understood. There has been some question as to whether the operator at the substation where the machine is located had control over loading, whether either of the systems concerned had to lower or raise their system voltage or if any adjustment in the frequency was necessary. After a frequency changer is in operation and synchronized with both systems and it is desired to load up the machine the system which is to supply the energy calls on its governing plant to pick up load. This plant opens the nozzles or wicket gates of the generator which is governing or "taking the swings" as it is termed, and in doing so the generator attempts to speed up the system to which it is connected and in doing so it picks up more load. The frequency changer in turn tends to speed up and in so doing it picks up load from the receiving system, or in other words, delivers power from the system which contained the governing plant. Due to the increase in current at the generating plant and at the frequency changer substation some slight adjustment of the voltage is necessary either by manual or by automatic means but this is a comparatively minor operation in picking up load.

The motor end of a frequency changer can, in addition to driving the generator end, be utilized for power factor correction. The adaptability of a synchronous motor for this purpose is well known, which feature is quite valuable on systems having a larger induction motor load and a network of comparatively long high-tension transmission lines. If it is contemplated using either end of a frequency changer set for power factor correction, care should be taken to specify a liberal design of the field winding. There have been a number of instances where the usefulness of a frequency changer for power factor correction has been limited because the field winding was not designed for a great amount of over-excitation.

Some of the major considerations to be made when the installation of a frequency changer is contemplated are:

(a) What will be its size in relation to the generating capacities of the two systems concerned, and will the set be required to furnish energy in either direction?

(b) What will be its location in regard to the principal generating plants of both systems?

(c) Voltage and frequency of each system?

(d) Rating of each unit? The specification for one of the frequency changers listed elsewhere in this paper calls for the following:

"Each unit will have a maximum continuous capacity when operating as a generator of 5000 kw., 0.8 power factor or 6250 kv-a. with a 50 per cent maximum momentary overload. When operating as a synchronous motor, each unit will be required to operate at 0.8 power factor leading, corresponding output of the set being 5000 kw., 0.8 power factor. The temperature rise corresponding to the above conditions shall not exceed 50 degrees centigrade on either unit."

(e) Method of excitation, whether provided with individual exciters direct-connected, or if excitation will be provided from a motor-generator set or both, and whether an automatic voltage regulator such as a Tirrill is to be provided.

(f) *Efficiency.* If the set is to be operated continuously, the losses should be capitalized. For this reason the efficiency of the set for ¼, ½, ¾ and full loads should be specified, both for unity and some certain power factor leading for the motor and for unity and some lagging for the generator.

FREQUENCY CHANGER AT VESTAL SUBSTATION
15,000 kv-a., 15,000 volts, 50-60 cycles.

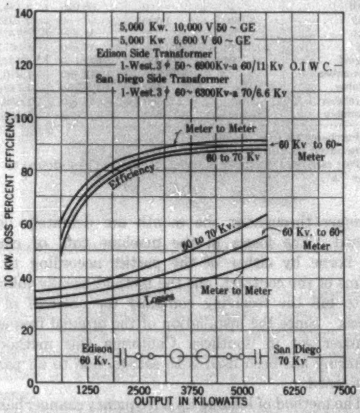
(g) *Starting Device.* Shall the set be capable of starting from either end and how will the starting be accomplished?

The experience of the Southern California Edison Company with frequency changers has been quite satisfactory during its seven years experience with such apparatus. There have been minor troubles, however, the chief of which have centered around the starting bars on the squirrel-cage winding and with the bearings.

After the first 5000-kv-a. machine had been in operation for about four years under conditions of rather severe service, it was necessary to take the machine out of service for approximately one month for the purpose of straightening the shaft. There has also been trouble with the motor bars coming loose. The original bars were replaced with bars of Monel metal. This change resulted in a different performance of the frequency changer on starting, in that the starting torque was much greater and the starting current was less, but there was more difficulty in falling into step than had been the case with the original bars.

It has also been an open question whether the trouble with the starting bars and the end rings of the frequency changer was due to the fault of the manufacturer or the result of the treatment of the machine. It must be realized that the excessive number of times of starting and stopping such a set as this will entail heavy duty on the squirrel-cage winding with the constant result of heating and expansion which will cause trouble later.

The largest machine on the system of the Southern California Edison Company (15,000 kv-a.) is a four-bearing machine and is equipped to use oil pressure in the bearings at the time of starting. Originally this machine was lined up so nicely that after the oil pressure had been applied it was possible for one man to move the rotor by applying his weight. There has been some trouble with the bearings, however, due to the fact that the conduits used for the bearing thermostat wiring accidentally short-circuited the insulating pad in the end bearings. After the bearing trouble occurred the conduit was removed and examined and evidence was found of considerable current flowing. This current flow was the result of accidentally short-circuiting the insulating pad which completed the circuit for the flow of circulating currents in the frame and shaft of the machine. The conduit was pitted and practically welded to the frame and base of the machine. There was evidence of pitting in the bearing and without doubt the circulating current was the cause of the bearing failure. Needless to say precaution has been used to eliminate the recurrence of such trouble by the same cause.



EFFICIENCY CURVES OF FREQUENCY CHANGER SET AT CAPISTRANO

In regard to the protection of the frequency changer, time limit overload relays have been used on both ends so that in case of trouble on either system the frequency changer is disconnected on the side of the system in trouble. Power directional relays can be used to good advantage for this purpose. In addition to this it is necessary to provide some means of protection against internal trouble of the frequency changer and this has been provided for at Vestal by means of two sets of power directional relays so wired that their contacts are closed when power is going towards the frequency changer. In addition to this the contacts on each set of relays are wired in series, so arranged that the tripping circuit will kick out the switches on both sides of the frequency changer. Under these conditions it can be seen that power will flow towards the frequency changer from both sides in case of a severe ground developing in the machine, and it is hoped that by this means to keep the damage to the machine windings and laminations down to a minimum.

In conclusion it may be stated that the frequency changer has proved to be a very satisfactory means of interchange of power, and that it should cause no trouble provided it is of the proper size in relation to the two systems which it connects.

Discussion

L. M. Klauber: There are two points upon which I should like to comment from the standpoint of a smaller company engaged in interconnection with a larger.

Mr. Stauffacher has told you of the smoothness of operation of frequency changes used in interconnection. On the contrary operation is not always so simple for the smaller company.

One point I might mention is that of governing. Steam turbines appear to be equipped with governors which are inherently more sensitive than those on the hydraulic machines. Therefore, a smaller system with steam turbines tied to a large system, principally hydro, through frequency changers tends to either feed into the large system or take from it a violently fluctuating block of power. For this reason it is necessary when our system is tied to that of the Southern California Edison Co., to change the degree of sensitiveness of the governors on our steam turbines. This has proved quite a problem. A method was finally devised whereby this might be done with entire satisfaction in a practical way, but unfortunately it is a method that is frowned upon by the manufacturers of the turbines. Yet, so far they have been unable to offer any other practicable solution. Our method is to place on one of the operating rods of the governor a device which we call a "load steadier" which works by friction on the governor shaft. It works, as I say, with entire satisfaction and it so arranged that the load on the turbine may be varied from the switchboard in the usual way. It may be instantly released if desired.

Recently, we have tried a second means of governing. We fix the turbine load for a certain specific amount by partially closing the hand throttle so it cannot carry more than that amount with all the governor valves wide open. The result is a decreased efficiency, approximately 2 per cent below the other method of governing, but it is thought by some to be safer. We operated for approximately a month, alternating between the two methods in order to determine the relative efficiency of each. This matter of control of governor sensitiveness is a point to which the manufacturers should give some attention.

Another matter is that of load control, which, again is somewhat difficult. Without frequent telephone communication it is difficult to carry the load at a maximum and yet within the capacity of the frequency changer. It, therefore, becomes necessary for the operators to do considerable guessing, in which they ultimately become quite expert, for if load is being taken from the line between the generating station and the frequency changing station, it is obvious they must make calculations of the load which is being taken off along the way, so that the ultimate load which reaches the frequency changer is not beyond the capacity of the machine.

P. M. Downing: My experience has been that where you attempt to operate turbines in parallel with a large hydroelectric system you must of necessity make your governors more sluggish; otherwise, your steam turbine will one moment have no load and the next moment it will be overloaded. That is not due entirely to changes of frequency on the larger system, but due to the inherent differences in governing between a steam turbine and a water power governor.

O. B. Coldwell: Our company has two frequencies, 33 ⅓ and 60 cycles. There are frequency changers in operation tying the two systems together. We have been for several years past doing away with the 33-cycle apparatus. To a considerable extent this has been accomplished by remodeling 33-cycle machinery for 60-cycle operation.

Recent Developments in Carrier-Current Communication

BY LEONARD F. FULLER

Member, A. I. E. E.

Radio Engineering Department, General Electric Co., Schenectady, N. Y.

Review of the Subject.—*Carrier-current communication over transmission lines has been developed to a point where its practicability and value are thoroughly established.*

This paper discusses several recent developments in apparatus and methods of operation.

Carrier-frequency energy flow along the line is discussed briefly to give a mental picture of the electromagnetic and electrostatic conditions existing.

Standard 50 and 250-watt apparatus are shown in the illustrations and described briefly.

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Review of the Subject. (70 w.)
Introduction. (90 w.)
Recent Developments. (1070 w.)
Energy Transfer and Flow. (480 w.)
Station Apparatus. (390 w.)
Summary. (80 w.)

CARRIER-current communication over transmission lines is being adopted rapidly not only as an auxiliary to existing wire telephones, but as the only means of direct communication for some recently constructed lines.

As our accumulated experience has increased the desirable frequencies and the range of sets of various powers over lines and networks of different types have become fairly well established.

Field experience to date has shown the practicability of transmitting carrier current over power systems with reliability and certainity. The method has been proved basically sound.

RECENT DEVELOPMENTS

The next steps in development are improvements in the apparatus, both mechanical and electrical, including those features which give greater operating convenience and flexibility.

Remote control of the apparatus is an example. Early sets were designed for control distances up to 250 ft., which is sufficient for the usual conditions encountered when the operator and apparatus are in the same building, but remote control attachments now available increase this distance to several miles. Thus a dispatcher in a city office building can control the carrier-current set miles away in an outlying suburban substation.

A modification of the foregoing is to retain the control of the carrier set at the station at which it is installed, but to have it arranged so the operator at the station can plug in the voice control on any one of a number of the company's wire phone lines. Such an arrangement allows distant stations to communicate directly with anyone in the main office.

Calling is accomplished on existing carrier-current channels either by one or more gong bells operated by a local relay, or by means of a loud speaker which permits direct voice or howler calling. This method is employed in the Great Western Power Company and Pacific Gas and Electric Company equipments, while the relay and call bell method has been generally employed in eastern installations.

Each method has its inherent advantages and disadvantages. The loud speaker cuts the time for short dispatching orders almost in half when operators are constantly within hearing distance. In such cases calling and answering calls are dispensed with. The dispatcher gives an order and the station to which it was directed repeats it. The Great Western Power Company has used this method for over a year and the Pacific Gas and Electric Company has also found it satisfactory.

If there are several carrier sets on a system, which intercommunicate frequently, it is often annoying and generally undesirable to have a loud speaker talking when its station is not concerned with the conversation. In one installation there was an objection to the loud speaker, not because of undesired conversations which did not exist in this case, but because of the line and electric storm noises which were reproduced. Relay and gong bell ringing is necessary in such cases.

Selective ringing is a further refinement which may be employed when code ringing of the gong bell would cause annoying and unnecessary disturbance at several stations of which only one was desired.

Time limit attachments are often furnished with the calling relay system to prevent electric storms and other high-frequency disturbances from ringing the gong bell. Code ringing is possible with such attachments, since the time limit feature is automatically inoperative after the first signal which must be a long dash. It may be followed by any desired dot and dash combination.

An excellent calling system is a relay with time limit attachments arranged to connect a loud speaker in circuit at the end of the long dash above mentioned. A gong bell may be added if desired. In this way the loud speaker does not reproduce undesired noises, but comes into service automatically for any speech or code calling.

A selector system may be added to such a combination. In this case the loud speaker is connected in circuit only when its station is called, but after being connected, the message may follow immediately if desired, with the resultant saving in time.

Presented at the Pacific Coast Convention of the A. I. E. E., Del Monte, Cal., October 2-5, 1923.

We are thus able to utilize the desirable features of the loud speaker system and remove those considered detrimental.

The calling relays are quite reliable when equipped with the time limit feature. They require very little energy and will operate with less than is required for satisfactory speech. Thus we can ring over any line over which we can talk.

All our installations are simplex. That is, the operator throws a tumbler switch or operates a foot pedal to connect the apparatus for talking or listening. This method has met with success wherever installed. It rarely takes an operator more than the first day to become used to the scheme of things.

The writer once saw a laborer working about a station called to the carrier phone set to receive a personal message from a man he had known in the army who happened to be visiting the station at the other end of the line. After operating the talk-listen button for him for the first few sentences no more assistance was required and the conversation proceeded vigorously.

This was a case where men who had not talked with each other for two years and who had never seen a carrier phone set before, found simplex operation so easy that they carried on an animated conversation over a 200-mile line for some time.

Duplex operation, except over short distances with sets of very low power, complicates the apparatus and decreases its reliability out of all proportion to the slight advantage gained. The users of this equipment are mainly station operators who soon grow accustomed to the man at the other end of the line and the semi-routine character of a large portion of the traffic handled.

Telegraphy is a valuable adjunct which may be added to carrier-current sets. It may be by either interrupted or continuous waves. The latter are preferable in emergency work because they will permit telegraphic communication under extremely severe line conditions.

The writer has received continuous wave telegraph signals at Pit River from the Vaca-Dixon substation 202 miles distant with all 6 line conductors grounded at several points although telephony and interrupted wave telegraphy were altogether out of the question, not a sound being heard in the phones.

For communicating over very noisy lines and during bad electrical storms interrupted waves may be used, but continuous wave heterodyne reception is probably preferable.

In starting telegraphy on an existing carrier-current telephone channel it is preferable to begin with a list of code signals for the common orders and use these frequently while the operating personnel are learning the telegraph code.

A speed of only 5 words per minute is of great value in emergencies when all other forms of communication may have failed. Most men can learn to send and receive at this speed without great difficulty. The higher speeds used in commercial telegraphy are learned easily and well only by rather young men.

ENERGY TRANSFER AND FLOW

Irrespective of the use to which the carrier-current energy is put at the receiving station, the method of transferring it to and from the line is the same.

At the transmitting station a carrier-frequency current is fed into the line. This results in a potential difference either between all line conductors and earth or between conductors. To date commercial installations have utilized the former method.

The electrostatic field due to this potential difference expands or travels down the line toward the receiving station at a velocity depending upon line constants. Usually this is not far below that of light in free space.

If the carrier frequency is 30,000 cycles per second, a half cycle will last 1/60,000 second and the static field of one half-cycle will have extended for about 3 miles along the line before the reversal of polarity due to a succeeding half-cycle commences.

If our eyes could see electrostatic fields and we stood at any point along the line, while these things were occurring, we would see nearly three miles of field slide by at about the speed of light, with the line positive with respect to earth, followed by an equal length in which the polarities were reversed.

If the line is short enough to allow the static field to travel to the receiving station while it is still being formed at the transmitter, the load drawn by the receiver will effect the power output of the transmitter. That is, the input impedance of the line will be varied by the receiver or load impedance.

This phenomenon, which is always observed at power frequencies on lines of present day length, is rarely observed at carrier frequencies. The lines are obviously too long to permit it.

Thus the energy delivered to the line at the sending station becomes wholly detached from the transmitter and is either partly absorbed by the receiver or frittered away entirely in line losses.

The most common method of charging the line at the transmitter is to erect a "coupling wire" in the vicinity of the line conductors. When this is raised to a given carrier-frequency voltage with respect to earth the conductors assume a potential corresponding to their relative positions in the static field.

Various types of coupling condensers have also been employed with mica, porcelain, and other dielectrics. Such condensers are connected between the carrier-current transmitter and the line, the current passing through them charging the line with respect to earth.

At the receiving station various arrangements capable of extracting energy from the electrostatic field may be used. Coupling wires, large wire loops, coupling condensers and combinations of them may be employed. At the present time the coupling wire method is preferable for both transmitting and receiving, due not only to its reliability, simplicity and ease of inspection, but also to its low first cost.

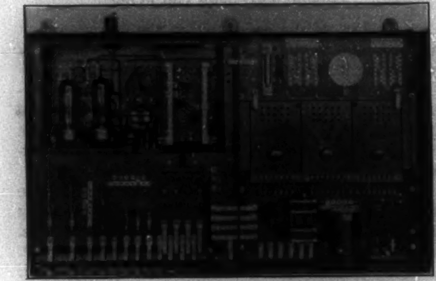

Fig. 1—Transmitter-Receiver for 50-Watt Carrier-Current Telephone
The calling relays are mounted on the section at the right.

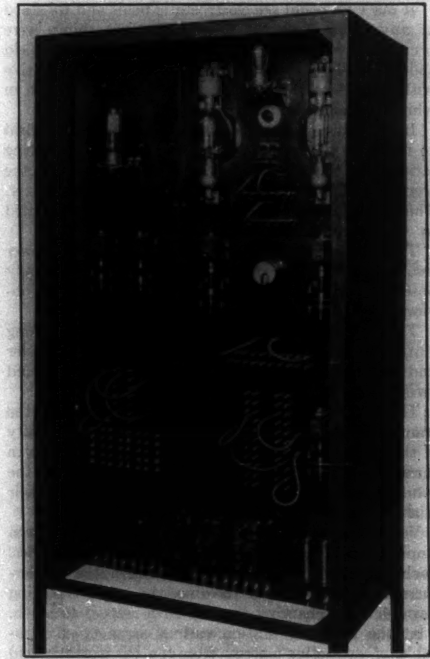

Fig. 2—250-Watt Carrier-Current Transmitter
Having master oscillator, power amplifier, voice amplifier and modulator tubes.

STATION APPARATUS

The sending and receiving station apparatus is much like that used in commercial radio communication with such modifications as are desirable for power system use.

Fig. 1 shows a typical carrier-current equipment having a power output of 50 watts. The transmitter is mounted at the left, the receiver at the right.

Power is supplied to the transmitting vacuum tubes at 1000 volts d-c. by a small motor generator driven by the station a-c. or d-c. supply as determined by local conditions. The receiving tubes are operated from storage batteries in the customary way.

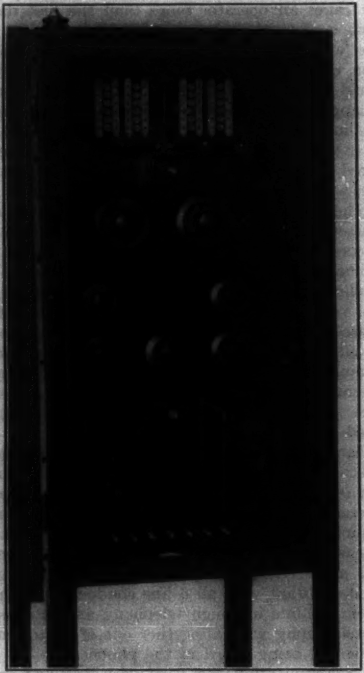

Fig. 3—250-Watt Carrier-Current Receiver
Showing tuning adjustments and calling relays.

This equipment is used for telephony over the shorter lines. Its actual range depends upon the arrangement of the system. Transformers, taps, cables and associated low-voltage lines all affect the situation. Field experience has accumulated to such an extent however, that the analysis of such networks is now readily accomplished.

More powerful sets of 250 and 500 watts carrier frequency power output are built for the longer and more difficult lines. Communication by the 500-watt

Pit River set of the Pacific Gas and Electric Company has been received over 410 miles of line.

These equipments are built in several units.

The 250-watt set comprises:

(1) A power panel on which the motor generator starter, switches, fuses and instruments are mounted.

(2) The transmitting apparatus, cabinet housing all necessary vacuum tubes, instruments, coils, relays etc.

(3) The receiver cabinet.

(4) The motor generator.

(5) The operator's desk phone set and a loud speaker when used.

Figs. 2 and 3 show portions ot this equipment.

Both the 250 and 500-watt sets are of the master oscillator type, that is, the carrier-frequency currents are generated by a small tube used to excite a larger one. The output of this power amplifier is modulated in accordance with the voice by another or modulating tube and then delivered to the coupling wire.

This method gives a substantially constant carrier frequency irrespective of line conditions and maintains a better voice quality than is possible with self-exciting tubes when forced to operate under adverse conditions.

The sets of the Great Western Power Company and Pacific Gas and Electric Company are not of the mechanical form shown in Figs. 2 and 3 having been developed in cooperation with the power companies prior to the design of the present standard equipments.

SUMMARY

In this paper I have attempted to outline briefly and without irrelevent technical or descriptive detail, the outstanding features of carrier-current work on transmission systems as we know it today.

Our present technique has the background of the older radio communication. Active commercial experience on high-voltage lines has been of shorter duration, but amply sufficient to prove the method technically and economically sound, and to demonstrate a few of the many uses the future will find for it.

Some Experiences with a 202-Mile Carrier-Current Telephone

BY E. A. CRELLIN

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ON April 11th, 1923, the Pacific Gas and Electric Company placed in operation a carrier-current telephone system between Pit River Power House No. 1 and Vaca-Dixon substation. This system utilizes the twin-circuit 220,000-volt transmission lines between the two points for a conducting medium and is for use primarily by the operating department in directing the operation of the power houses in the Pit River development together with the transmission lines which carry the energy southward to the distribution center at Vaca-Dixon substation. With the commencement of work on Pit River Power House No. 3 an increasing volume of messages is being received from the construction and auditing departments for transmission to the general offices of the company, and it is already apparent that additional carrier-current telephone sets will be required, one at the base of construction operations for Power House No. 3 and one at Claremont substation in Oakland to relieve the congestion on the wire line between Vaca-Dixon substation and the load dispatcher's office.

During the six months from October 1922 to the inauguration of the carrier-current telephone, the operation of the Pit-Vaca transmission system was directed by wire lines. In the absence of direct wire communication between the load dispatcher's office at Oakland and the power houses in the Pit River system, it was necessary to relay all messages one or more times which introduced a considerable delay and was subject to error, especially in view of the inductive interference usually encountered on telephone lines paralleling transmission lines. Since the carrier current telephone was placed in operation in April it has answered all dispatching requirements, and as the operators become more and more familiar with the peculiarities inherent in such a system its many advantages over the wire lines become more and more apparent.

The system is coupled to the transmission line through a single-wire antenna about 1800 feet long. This wire is attached to the twin vertical circuit transmission towers at a point on the center line of the towers and at the elevation of the middle arm. The main ground system for the station apparatus is also used for the carrier-current telephone ground and no counterpoise is used. The frequency adopted, after a series of tests ranging between 12,500 cycles and 85,000 cycles was 50,000 cycles. This is free from all outside interference and gives the best combination of signal strength and voice modulation.

The transmission equipment is similar to that installed in several of the high-powered broadcasting stations in some respects, and employs two 250-watt oscillator tubes, two 250-watt modulator tubes and a 50-watt speech amplifier tube. It is appreciated that this is a relatively high-powered transmitter and for all normal conditions of operation such as an amount of power is not necessary. However, communication is more urgently needed during periods of abnormal condition than at any other time and it is then that this excess power is called upon to get the message through.

Calling is accomplished by mounting a standard telephone microphone in the horn of the loud speaker which, when the calling circuit is completed, will oscillate and howl in much the same manner that the ordinary telephone will howl when the receiver is placed against the transmitter. This gives a very loud note whose pitch will depend upon the natural period of oscillation of the diaphragms and which is clearly audible in all parts of the station. Ordinarily, it is not necessary to use the calling system, as the receivers are always in service and the operator near the set so that the loud speaker simply talks at him and he starts up his set and talks back.

The system is arranged for simplex operation and all that is necessary is to operate a small telephone switch which energizes a contactor to connect either the transmitting or receiving set to the antenna, thus permitting talking or listening. Some tests made with a view to establishing the possibility of operating as a duplex system are described further on in the paper.

Normal rating of the 250 watt tubes used is based upon a filament lighting supply of 11 volts and a plate pressure of 2000 volts. With the transmission line in operation and all conditions normal it is possible to obtain an antenna radiation of 6.2 amperes with the tubes operated at rated voltage. This gives a received signal at the other end of the 202 mile transmission line considerably louder than necessary. The receiver is a standard regenerative set using two stages of audio frequency amplification to operate a loud speaker, and speech can be heard all over the power house when maximum amplification is employed. For regular service, therefore, the filament lighting supply on the transmitter is reduced to about 10.4 volts and the plate pressure to 1600 volts, which gives an antenna radiation of approximately 5.0 amperes.

The tubes are extremely sensitive to variation in

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filament voltage and it has been found by experience that 10.4 volts is about as low as it is desirable to operate. Below that point it is sometimes difficult to start oscillation and sluggish operation means delay in acknowledging receipt of a message. Communication has been carried on with a radiation as low as 2.5 amperes, but with poorer modulation. With a 5-ampere radiation at the sending station the receivers are set for minimum amplification and no difficulty is experienced in handling messages.

To one unaccustomed to listening to a loud speaker, the messages are often very hard to understand and some difficulty was at first experienced by the operators who had not had previous experience with loud speakers. However, this was easily overcome after a week or so of operation of the phones and it is now rarely necessary to repeat a message. Routine dispatching is carried on more easily with the carrier current telephone than with the wire phone due to absence of inductive interference. A considerable volume of accounting and construction department messages is handled over the carrier current telephone and this gave trouble in some instances when it was necessary to transmit unfamiliar proper names. It usually resulted in spelling and due to the difficulty in distinguishing between certain of the consonants which are similar in sound, this did not solve the problem. The method now generally used is to try the voice once and if any difficulty is experienced the name in question is transmitted by telegraph code. The operators have been supplied with a chart on which the Continental code is printed in large clear letters and those who have not memorized the code simply pick out a letter at a time and transmit it by means of the calling key. This may sound slow but in reality it is quicker and more positive than several voice repetitions of the letters composing the name in question. Naturally more speed is attained by the operators familiar with the code, and without exception all of the operators are taking a very great interest in the carrier current telephone and are practising with the code during the shift between midnight and five o'clock in the morning when other duties are at a minimum. It is desirable that the operators become used to the telegraph code and able to transmit five or ten words a minute because certain forms of line disturbance make voice transmission difficult and still permit of distinguishing the letters of the telegraph code which comes through as a high-pitched note of constant tone, readily distinguished from other noises. It is also possible to transmit the code when abnormal conditions make it difficult to secure sufficient radiation to give good voice modulation.

All operation of the carrier-current telephone to date has been carried on with one of the 220-kv. transmission lines still incomplete. The first work was done with two transmission lines complete from Vaca to Williams, a point about 50 miles north, and a single line from Williams to Pit, about 150 miles. At Williams the two lines were tied solidly together. The copper for the second line was strung in place between Williams and Cottonwood, about 90 miles, but men were constantly at work upon the line and consequently it was solidly grounded at each tower on which a man was at work. Grounding this parallel line did not affect communication in any manner on the other line.

At present two three-phase circuits are in operation from Vaca to Cottonwood approximately 140 miles with a single circuit from Cottonwood to Pit River approximately 60 miles. The circuits are tied together solidly at Cottonwood and men are now at work on the second circuit between Cottonwood and Pit. A special test was recently made with one three-phase circuit solidly grounded at Vaca and the other circuit clear, both being tied together at Cottonwood, power, of course, being off the lines. No appreciable difference from usual operating conditions was noted in ability to communicate between Pit and Vaca.

Recently one wire on the completed three-phase circuit between Cottonwood and Pit River became grounded due to failure of a piece of insulator-hardware and resultant dropping of the wire on to the tower. This left only two wires completely insulated between Pit and Vaca with a ground on the third, but no difficulty was experienced in carrying on communication.

Communication cannot be carried on with three-phase grounds at either or both ends of the transmission line. Also, all three wires were recently grounded at Hat Creek, about three miles distant from Pit toward Vaca and communication could not be carried on. At least one wire, insulated from ground, between the two points is necessary for successful communication by carrier-current telephone.

When the transmission line is dead and isolated by opening the disconnecting switches at each end of the line considerably more power output from the transmitter is required than when the line is connected to the transformers and energized at 60 cycles. This difference is probably due to the change in transmission line constants by the removal of the reactance of the transformers which in turn cuts down on the current radiated from the antenna. When the transmission line is thus isolated the station operators immediately increase the power output of the carrier-current telephone transmitter to its maximum by raising plate and filament voltages to 2000 and 11 volts respectively. Communication is then carried on as usual and no difficulty is experienced in directing operations.

In order to carry on duplex communication, that is, to be able to send and receive at the same time, it is necessary to employ two antennas, one for the transmitter and one for the receiver. Some tests were recently made to establish the effectiveness of various types of receiving antennas. At Vaca it was possible to receive messages from Pit River when using a bushing type current transformer on one of the 220-kv. line oil switches as an antenna. The leads from this current transformer are carried in lead covered cable about 300 feet to the substation wall, thence in rubber-covered wire in iron conduit to the switchboard, an additional 40 feet. The receiver was hooked on to one terminal of the line ammeter on the switchboard. Only the reactance of the ammeter coils was between the antenna and ground. The signals received were clear, but weak. It was necessary to use the head phones with all the amplification at hand to bring it in strong enough to understand. Stronger signals were received when using the secondaries of one of the 110-kv. potential transformers on the 110-kv. bus but they were not considered strong enough for use by the operators. As a matter of interest, one test was made using one side of the lighting circuit for an antenna. Very weak signals were heard which might have been brought in strong enough to be understood had a couple of stages of radio frequency amplification been available in addition to the two stages of audio frequency amplification which were employed. This reception was through three power transformers, *i. e.*, the main bank which steps down to 11,000 volts for the synchronous condensers, the station bank stepping from 11,000 to 440 volts and the lighting bank stepping from 440 to 110-220 with the mid point on the secondary grounded.

Finally a separate receiving antenna was strung at Vaca and also at Pit. These antennas were each about 250 feet long and placed in the most convenient temporary location. Excellent reception was obtained and the loud speaker could be operated at full volume from the short auxiliary antenna. The transmitter at Vaca was then lowered in frequency to about 30,000 cycles, with the Pit transmitter remaining at 50,000 cycles. With the Pit receiver tuned to 30,000 cycles it was not possible to transmit at 50,000 cycles while receiving because the transference of power between the sending and receiving antennas was so great as to blanket out the received signals. When duplex operation was tried at Vaca, the transference of power was sufficient to arc across the grid condenser which is a 23-plate variable condenser in the receiver used. It would seem that duplex transmission is not practical with two antennas when using transmitting sets of the power output necessary to successfully talk over a 202-mile transmission line, and accordingly no further tests were undertaken.

During the initial installation of the Pit-Vaca carrier-current telephone several tests were made between the Vaca-Dixon substation and the Fourth Avenue substation of the Great Western Power Company in Oakland, where a similar carrier-current telephone set is installed. There is no direct transmission line between the two substations. The Great Western feeds the Pacific Gas and Electric Company through a 60-kv. transmission line which is carried for about three miles on the same steel tower line as the 110-kv. transmission line and on the same right of way as the 110-kv. twin circuit Oakland-Big Bend transmission line to which the carrier-current telephone is coupled. The 60-kv. feeder goes into the Ridge substation of the Pacific Gas and Electric Company, and thence on to the Claremont substation. There is a parallel of some three miles or so between the 60-kv. Ridge-Claremont line and the 110-kv. Claremont-Vaca line where the two circuits are on the same right of way. At Claremont the 60-kv. and 110-kv. are tied together through auto-transformers.

Conversation is readily carried on between Fourth Avenue and Vaca, a distance of about 60 miles, and during the initial tests between Pit and Vaca, the Great Western operators were able to listen in and understand what was being said at Pit. It is believed that this is a record for distance for carrier-current telephone transmission, somewhat over 260 miles with no direct transmission lines connecting the last 60 miles to the transmitter.

The Pacific Gas and Electric Company is entirely satisfied with the results of the carrier-current telephone secured to date. Many conditions of operation have been met and the carrier-current telephone has at all times been ready to perform the duties required of it. The principal difficulty to be overcome is that of understanding the loud speaker which is simply a matter of becoming accustomed to the voice as reproduced in a loud speaker horn rather than in a telephone receiver. The operators in general seem to prefer the loud speaker to the head phones, and rely upon it at all times to receive their messages.

As stated before, the volume of traffic over the carrier-current telephone is steadily increasing and plans are already being laid for the installation of additional sets at Pit River Power House No. 3 and Claremont substation. The last named set will be remote-controlled from the dispatcher's office in Oakland and a relatively small investment in apparatus will obviate the necessity of a 260-mile metallic telephone line to handle the operation of plants in the Pit River development, and results achieved to date indicate that it will be equally reliable and a much more quiet phone to talk over.

Carrier-Current Telephony on the High-Voltage Transmission Lines of the Great Western Power Company

BY JOHN A. KOONTZ, Jr.
Great Western Power Co., San Francisco, Calif.

***Review of the Subject.**—This paper includes a brief description of the power circuits and radio equipment used in the carrier-current system; the method of calling employed, and use of this system as a trouble detector and recorder of switch operation.*

THE reports of successful tests with high-frequency telephony early interested the Great Western Power Company, and experiments were undertaken to determine whether suitable equipment could be obtained for installing carrier-current communication on our 100-kv. circuits. This equipment was later installed at the Dispatcher's office in Oakland, at the Big Bend Power Station and the Caribou Power Station.

The distance from Oakland to Big Bend via the tower line is 153 miles and from Oakland to Caribou 193 miles. Three circuits run from Oakland to Big Bend, twin circuits being carried on one tower line and a third circuit designed for 165 kv., carried on a second tower line. These two tower lines with their three circuits are located on the same 100-foot right of way for 85 miles. The twin circuits stop at the Big Bend Power House and only the 165-kv. circuit extends for the remaining 40 miles to the Caribou Station.

Under present conditions these lines are operated in parallel at the same voltage, and are in general paralleled at Big Bend, Brighton and Oakland. There are a total of nine substations tapping these lines, one of these substations being located on a short branch line.

From this brief description I think it will be readily apparent to one familiar with power carrier-current work that this is an extremely difficult system on which to place carrier-current communication. Radio engineers who are familiar with this line of work estimate that each substation is equivalent to adding at least 10 miles to the length of line and a tap line may add even more. With this in mind it is easy to see that we have an equivalent transmission distance of about 300 miles. These radio engineers acknowledge this to be the most difficult system on which they have as yet worked.

Early experiments readily demonstrated the fact that 50-watt sets were far too small for this network of circuits. 250-watt sets were then installed and it was found that at certain times this was not sufficient power to give proper communication, so these sets were modified at Caribou and the Dispatcher's office, using two 250-watt tubes in the oscillating circuit with one 250-watt modulating tube. This equipment has been in operation for about one year and has given good service. There have been a few cases of trouble but these have been few and not serious.

This system of communication apparently has many good points in connection with obtaining communication with water power plants which are situated in regions remote from the regular telephone communication. Any single telephone circuit as constructed either by the power or communication companies would be much more susceptible to mountain storms than the rugged tower circuits, and it is practically always the communication circuits that first show distress during severe storms. We were pleased to go through this last winter without a single total interruption in communication between the Dispatcher's Office and our water power stations.

At first an elaborate call bell equipment was planned and a relay was actually developed which worked very satisfactorily, but it was later found that by picking up the transmitter and placing same in the loud speakers, this particular combination would resonate to produce a howl in the loud speaker at the distant end, the frequency of which was much higher pitched than any noises around the power station and was easily detected above the power house noise.

This gave a mode of signaling without adding any additional equipment, and by simply operating the hook switch on the transmitter, code signals could be readily sent. This method of calling has proved quite satisfactory.

Because of our complex circuit arrangement we obtain at times an echoing effect in our receiving. This is possible due to the carrier current flowing along lines that have different characteristics, producing different attenuations, or due to reflection. There is, however, one very peculiar thing about this effect, and that is, often by shifting a substation from one of the twin circuits to the other the effect will be eliminated, and to date we seem to have no reasonable explanation of why this change should eliminate the trouble.

The general equipment consists of a 2000-volt, 3-wire, d-c. generator for producing suitable plate voltage for the tubes, 1000 volts being placed across the 50-watt amplifier tube and 2000 volts across the 250-watt tubes. Filaments are lighted from alternating current with rotary converters provided for supplying this current from the station storage battery

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should there be a failure in the a-c. supply at the Dispatcher's office.

The Simplex System of communication is used, it being necessary to operate a double-pole, double-throw switch to change from talking to receiving. The antenna are strung upon the towers at two of the stations, while the Caribou antenna is stretched across the canyon about forty feet from the high-voltage wires. Here, a single ⅞-in. steel core aluminum cable 930 feet in length is used, tapped at the center, giving a "T" type aerial. The wire used is the same as used on the tower circuits.

The carrier-current system has several features which undoubtedly will be of interest to the operating power men. First, it is a good detector of poor switch contacts, as the minute a switch is closed a click can be heard on the loud speakers indicating that the switch is closed, and should the contacts continue to arc, a hissing or howling noise will immediately be set up in the loud speakers. In this way, faulty switch contacts have been located and taken out of service before any trouble occurred. In fact, the equipment is so sensitive that at the Dispatcher's office slight inconvenience was experienced for several days from a hissing sound in the loud speaker. This sound was not present at the other stations and it was finally located. A potential fuse on a 60-kv. potential transformer in the station adjoining the Dispatcher's office was partially blown and the current actually jumping a small gap, this arcing causing the disturbance on the receiving equipment.

Arcing grounds on the secondary systems can be instantly detected though they are often hard to locate. Grounds on the 4-kv. system fed through two banks of transformers and the bank of transformers connecting to the 100-kv. system located 80 miles from one of our receiving sets, have been noticed and finally located. The high-frequency notes are readily detected, yet no normal frequency hum is noticeable in the receiving sets.

From present experience it would seem that this form of high-frequency communication as applied to the power line circuits, is a valuable addition to the communication system. I do not believe that it will in general replace wire lines on long high-voltage systems that necessarily cover considerable territory where public telephone communication would not be accessible for patrolmen or repair crews on maintenance work, but it is of great value in giving a reliable communication channel to the outlying stations for dispatching work, and in most cases can be utilized without very large expenditures for installation or maintenance.

This system is operating on a wave length of 5500 meters. This particular wave length was selected after careful test, as giving about the best transmission and was of suitable length so as to avoid most outside telegraph interference, this placing the station just between two of the high-powered government telegraph stations. The transmission lines make a good antenna for the longer wave lengths—3000 to 20,000 meters, but a very poor one for wave lengths as covered by the amateur or broad casting range.

The amateur operators seem to experience very little interference from our system as now operated. Only when they are situated close to the transmission lines have they reported receiving our signals, and then only at times when other radio stations were not operating. This interference has been greatly reduced by the elimination of accidental harmonics which were present during the experimental stage.

All experimental work and arrangement of equipment in connection with this system was designed and the installation directed by Dr. Leonard F. Fuller.

Mechanical Forces on Busbars
Under Short-Circuit Conditions*

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WITH the advent of the superpower station, with its concentration of enormous power and possibilities of very large short-circuit currents, the mechanical stresses to which different apparatus, including busbars may be subjected, demand serious consideration.

In the following pages the forces acting on busbars will be considered and an attempt has been made to furnish the designing engineer with a set of simple formulas by means of which he will be able to approximate with sufficient accuracy the magnitude of these forces and their distribution along the busbars. The basis of the permissible approximations has been determined by actual measurements. The combination of these forces when acting in a three-phase system will be analyzed and the characteristic action of the forces at the moment of short-circuit pointed out.

The Problem

In the problem under consideration the electric circuits are supposed to have no iron in the interlinked magnetic field. The busbar system in large stations will in general contain many different configurations and the general calculation will therefore consist of a large number of calculations for the individual configurations. Evidently then the formulas have to be of great simplicity if the labor involved is not to become prohibitive. Also mathematical precision is not required as a number of inaccurately known factors such as strength of materials, temperature, etc. entering into the construction, require the application of more or less arbitrary safety factors. As the mechanical forces acting on bus-bars may have a lever-action on their supports, it will be necessary to know their moments about any given point, requiring the knowledge of the distribution of the forces along the bars.

AVAILABLE METHODS OF CALCULATING MECHANICAL FORCES DUE TO ELECTRIC CURRENTS

Three methods for calculating these forces are known and have been referred to as:

1. The Action at a Distance Method.
2. The Energy Method.
3. A method using the magnetic field intensity at the conductor and the current strength in the conductor. The writer proposes for this the name *Current Field Method.*

The Action at a Distance Method is based upon the assumption that two elements of current can act on each other at a distance without an intervening medium[1] and thus significantly neglects the finite velocity of the electromagnetic field. It was used by Ampere in his famous researches on the action of an electric current on another electric current shortly after Oersted's discovery of the action of an electric current on a magnet, a hundred years ago.

The Energy Method, following Maxwell's theory of electromagnetism and taking account of the finite velocity of the electromagnetic field, applies the law of the conservation of energy from which the following general formula can be evolved:

$$\text{Force} = i_1\, i_2 \cdot \frac{d\,L}{d\,l} \cdot C$$

Where i_1 and i_2 are the currents in one or two electric circuits, L the total inductance of the system and $d\,l$ a virtual displacement of the part on which the force is to be calculated. C is a constant depending on the units employed.

The Current-Field method is based on a definition of the Unit of Current in the c. g. s. system, *viz.*

The unit current is that current of which an element of unit length (1 cm.) in the direction of the current, when placed in a magnetic field of unit intensity (1 maxwell = 1 line per sq. cm.) is acted upon by a mechanical force of 1 dyne at right angles to the direction of the current.

The external magnetic field due to a current in a cylindrical conductor is the same as if this current were concentrated at the axis of the cylinder or a so-called line current.

The formulas for the inductance used in the energy method and those of the magnetic force or field intensity used in the Current-Field Method are derived from the same fundamental formula, which is as follows:

The magnetic force or field strength at a point P, outside a line current i due to an element length of current $d\,y$ and distant a length c from $d\,y$ is equal to

$$i \cdot \frac{d\,y}{C^2} \sin\theta$$

Where θ is the angle between the direction of c and the line current. The magnetic force or the direction of the field is at right angles to the plane containing the line current and the point P.

The difference between the energy method and the current-field method lies in the fact that the former

*Awarded the "First Paper Prize" for the year 1922, by the A. I. E. E.

Presented at the Meeting of Toronto Section, A. I. E. E., November 8, 1922.

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[1] It is interesting to observe that the newest theory, Einstein's Theory of Relativity, again seems to dispense with the necessity of an intervening ether of space.

uses the total magnetic field interlinked with the electric circuit or system of circuits, and only indirectly, through the geometrical relation of $d\,l$ to the configuration of the system, takes in consideration the unequal distribution of the magnetic field intensity; while the latter method does so directly by using the algebraic sum of the field intensities due to the different component parts of the system.

As in general these parts do not contribute in equal amounts, the Current-Field method offers the greater facilities for approximation.

THE LITERATURE ON THE SUBJECT

The writer to whom the task was allotted to supply a set of simple formulas for the calculation of the mechanical forces on busbars, specially those acting on taps at right angles to them, found the existing literature on this subject very meagre from his point of view. Such as was available to him treated special cases while the generalizations in the standard text books seemed to be more of an academic than practical use. He therefore endeavored to develop such simple formulas by one of the three methods mentioned above, but did not succeed as he had nothing to go by to determine the amount of approximations which would be warranted, or assumptions that could be made in view of the desired result.

Moreover one of the writers[2] on the subject who used the energy method declared all other methods as fundamentally wrong and leading to erroneous results. At the time his arguments seemed to carry much weight as the writer had not then realized the common base of the energy and Field-Current methods as explained before. As for the Action at a Distance Method, Mr. H. B. Dwight has since shown this also to be mathematically correct.[3]

MEASUREMENTS OF MECHANICAL FORCES

It was therefore decided to make some actual measurements and through them either to determine the allowable approximations or assumptions or to evolve empirical formulas or curves which might facilitate the determination of these forces. These measurements were made at the Laboratories of the Hydro-Electric Power Commission of Ontario.

As the available current strength was limited, the forces to be measured would be very small. This introduced great difficulties as we were not equipped with the apparatus required for measurements which more properly belonged to the domain of a Physics laboratory. The equipment had therefore to be especially designed and constructed. After a large amount of preliminary work and trials, the following apparatus was set up.

A short piece of flat copper bar 1 inch by 3/16 inch and exactly 10 centimeters long was joined with insulating materials to a rigid brass rod provided with a steel cross-piece about 7½ centimeters long with knife edges at the ends. The latter were supported in agate cups, the whole thus forming a pendulum. The little bar made an angle of 60 deg. with the brass rod and was provided at its upper end with a brass cup in the form of a scoop for holding mercury. A number of movable weights at different points in this system allowed the regulation of the position of the center of gravity. Two short copper bars of the same cross-section as the swinging bar were firmly fixed on a wooden support and in line at each end with the swinging bar but separated by a space of 3 millimeters from it.

The lower fixed bar had at its upper end a scoop similar to that on the swinging bar. The scoops were filled with mercury (Fig. A). Bars and scoops were insulated with paints and varnishes except the nickel plated ends (cross-sections) of the bars; thus forcing the current through the mercury to follow practically the same path as if fixed and moving bars were one solid bar and the distortion of the magnetic field at the ends of the moving bar could be considered negligible. In front of and in a vertical plane passing through the center of the swinging bar a known weight was hung by a fine silk thread. This thread was suspended from a support attached to a 1½-inch aluminum tube fixed with an articulated joint to a heavy piece of wood permitting a movement parallel to the plane of swing of the pendulum. The 1½-inch aluminum tube was connected to a micrometer screw on a wooden support by means of a ¼-inch aluminum tube about 6 feet long. On the front of the brass rod of the pendulum and above the knife edges was fixed a plane mirror. Underneath the micrometer was fixed, on the same support, a telescope with scale. The different dimensions and distances of the set-up gave a lateral movement of the swinging bar of 0.000346 inches for one

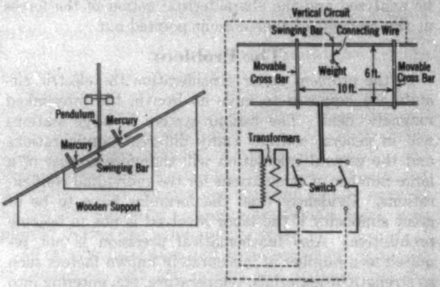

FIG. A FIG. B

MEASUREMENTS OF MECHANICAL FORCES
SKETCH OF APPARATUS AND DIAGRAMS OF CIRCUITS

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[2] See Mathematical Analysis of the Mechanical Forces on Switches and Busbars, by L. B. W. Jolley, *the Electrician*, (London), Dec. 3, 1920, p. 646, Dec. 10, 1920, p. 676.

[3] See Magnetic Force in Disconnecting Switches by H. B. Dwight, *The Electrician* (London), Sept. 2, 1921, p. 291.

scale division. After experience was gained it was perfectly possible to distinguish a movement of one-half the thickness of a division line or about 1/10 of the above amount. The swinging bar was connected horizontally to the silk thread by a fine bronze wire of negligible weight.

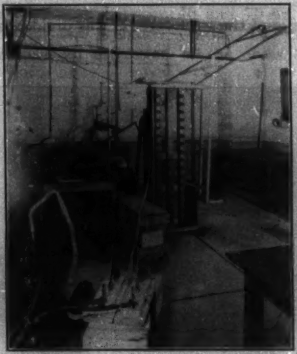


Fig. 1

By moving the support of the known weight by means of the micrometer a determinate force could be applied to the bronze wire to counteract the forces on the bar due to the current. The zero method was used throughout these experiments. Figs. 1, 2 and 3 show the described apparatus. The first measurements were made on the side of a rectangle (Fig. 1). In order to avoid as much as possible the influence of possible current distribution in the conductors it was necessary to make the dimensions of the rectangle large, resulting in very small forces to be measured. The circuit was completed as shown in Fig. B. By moving the movable cross bars in steps along the inclined bars (keeping their distance apart constant), the forces on one side of the rectangle were measured on 10-cm. sections. The current employed was 1000 amperes, 60 cycles throughout.

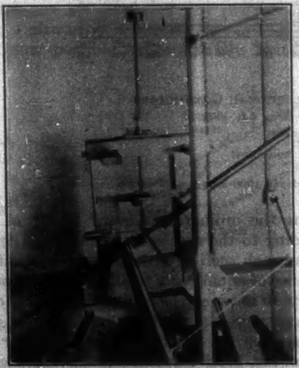


Fig. 2

Great difficulties of a mechanical and physical nature were encountered. They were mainly due to the unavoidable use of a liquid conductor (mercury) which necessitated the limiting of the possible lateral movement of the bar to a half scale division on either side of the zero line. This was done by means of adjustable stops.

The vertical circuit, also visible in Fig. 1 was used for a preliminary adjustment of the center of gravity of the pendulum. As it was not possible to adjust so accurately that a vertical force acting on the bar would have no effect in shifting the center of gravity, the final adjustment of the zero point had to be made by alining the movable and fixed bars by means of a brass gage.

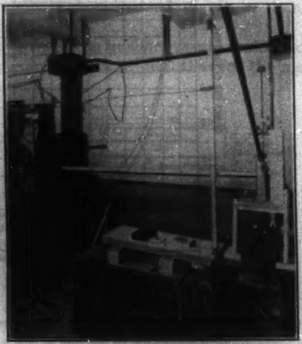


Fig. 3

The forces so measured in a preliminary series of measurements, after correction for the constants of the apparatus were plotted, and it was found that the probable curve for the forces, near the corner of the rectangle came closely to $x\,y = \text{constant}$, where x is the distance from the corner and y the force. This led to the following reasoning. The percentage of the total force near the corner, due to the current in the parallel conductor should be comparatively small, and therefore the forces due to the current at right angle, and close to the corner should give a curve closely resembling a hyperbola.

The magnetic field intensity at a point P at a distance d from a line current of length l is

$$H = \frac{i}{d} \cdot \left(\frac{a}{\sqrt{a^2 + d^2}} \pm \frac{l - a}{\sqrt{(l - a)^2 + d^2}} \right)$$

lines per square cm.

We will refer to this formula as the Fundamental Formula.[4]

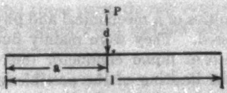
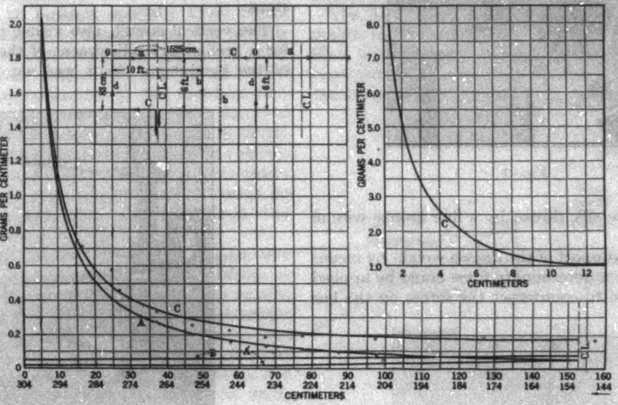


FIG. E



CURVE SHEET C

MEASUREMENTS OF MECHANICAL FORCES ON ELECTRICAL CONDUCTORS

⊙Measurements on one half of side *a* of rectangle *a*, *b*, *c*, *d* = 10 ft. × 6 ft. (Fig. 1)

†Measurements along *a* of right angle *d*, *a*

A Calculated curve of Forces on *a*, due to current in *d* (Fig. 1-2)

A' Calculated curve of Forces on *a*, due to current in *b*. (Fig. 1)

B Calculated curve of Forces on *a*, due to current in *c* (Fig. 1)

C Calculated curve of total Forces on *a* (Fig. 1). Current employed 1000 amperes 60 cycles.

If P is situated at right angles to one of the ends of l, we get:

$$H = \frac{i}{d} \cdot \frac{l}{\sqrt{l^2 + d^2}}$$

If d is very small compared with l, we have approximately $H = i/d$, or if i is constant: $H \cdot d =$ constant.

As the mechanical force with constant current is proportional to H, we have $F \cdot d =$ constant and plotted for different values of d would give a hyperbola.

From this we derived the assumption that the formulas for line-currents would express the values of the mechanical forces with sufficient approximation for our purpose subject to confirmation by actual measurement. Consequently, with the aid of the fundamental formula, curves were plotted for the mechanical forces on one side of the rectangle. These curves are given on Curve Sheet (C). After making a few additional refinements in the apparatus a new set of measurements was made.

The curve of the total force C was divided into parts by erecting the ordinates at every 10 cm. of abscissa corresponding to the successive positions of the swinging bar and each such 10 cm. divided into a sufficiently large number of equal parts to allow for the assumption that the curve between the ordinates on these divisions could be considered a straight line. By an approximate integration the point was then found for each 10 cm. where the total force on that 10 cm. could be applied to give the same total moment about a given point, or in other words, where the force per unit length would be equal to the total force on 10 cm. divided by 10.

FIG. F

The corrected values of the measured forces divided by 10 were then plotted at these points of application and are marked ⊙.

It will be seen that with the exception of points between 40 and 70 cm. the measured and calculated forces agree very well considering the difficulty of the measurements of such small forces. (From 18 to 1.60 grams.).

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[4] See Bulletin of the Bureau of Standards, Vol. 4, p. 303 and p. 309.

An investigation into the disturbing factors causing the larger deviations between 40 and 70 cm. pointed to the following two probable sources: 1st, shifting of zero point through expansion of the cross bars and bending of ¼ inch aluminum tube. 2nd, the difficulty of alining the swinging bar by means of a gage.

Fig. 4

The first cause was remedied by the insertion in the circuit of two additional mercury cups, (Fig. 4), Fig. (C). The ¼-inch aluminum tube was replaced by a fine steel piano wire and counterweight. The second one by arranging the circuit as in Fig. (D). Fig. 4 also gives a partial view of this circuit.

It was now possible by alternately switching the current through the two possible circuits to find this zero point by electrical means as the upward force on the swinging bar was the same in both cases.

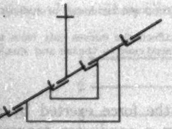


Fig. C

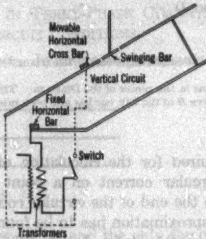


Fig. D

It will be seen that in this circuit the influence of the parallel conductor has been eliminated and that only the force due to the currents in the movable and fixed cross conductors came into play. The force due to the fixed cross conductor, which was of course, the same for each measurement, was calculated by the formula and added to the measured force and the point of application taken the same as for curve *C*. The slight error so introduced was deemed to be negligible as Curve *A* (the force due to a current at right angles) has very closely the same form as *C*. The points so obtained are marked + and it will be seen that the agreement between the calculated and measured forces is very close indeed. The additional mercury cups did not permit the measurement of points closer to the corner than on the section 20 to 30 centimeters.

The outstanding result is the close agreement of the measured and calculated forces close to the corner where unequal current distribution might be supposed to have the largest influence and especially is this true for the first point which is shown on the enlarged curve in the right hand upper corner of curve sheet *C*, which was verified by two separate series of measurements, one of 6 giving an average of 18.06 grams (uncorrected) and another of 5 giving 18.00 grams. The latter was made with a slightly different weight on the silk thread as the weight was immersed in oil to dampen the oscillations.

As regards forces on busbars we are mainly concerned with their moment and we find that for the moment of the force on a length of 4 times the width of the bar, counting from a point less than the width of the bar from the corner of the center lines, we may with sufficient approximation assume line currents. As to possible shearing forces due to large forces close to the corner, it is very probable that stresses due to uneven temperature distribution during short circuit in the corner where the bars are bolted together, will exceed in magnitude any forces due to the short-circuit currents themselves.

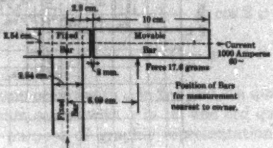
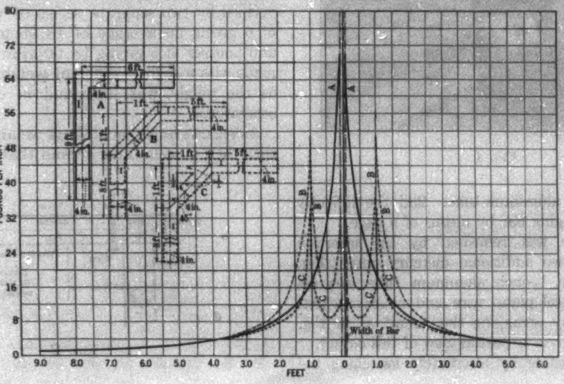


Fig. G

Calculation of Mechanical Forces

From the fundamental formula for the field strength at a point in the space surrounding an electric current I_1, the formula for the mechanical force in dynes on an element of electric current I_2 at that point is found by multiplying *H* by I_2 where I_1 and I_2 are expressed in abamperes and all dimensions are in centimeters. This force is acting at right angle to the current. With the application of proper conversion factors, this force can be expressed in any required units. By considering this force on an element of the current, the force on a determinate length of current can be found by integrating between proper limits, and again by integrating between proper limits the elementary couples, the moment or torque of the force on any determinate length of current about a given point can be found.



CURVE SHEET D

A. Bars at right angle.
B. Angle cut at 45 deg. one ft. from corner.
C. As *B* with right angle left in and current equally divided between them.

Note: The forces on the 45 deg. angle piece have been referred to the axis of the right angle bars. The triangle is supposed rigid and of sufficient strength to withstand the interaction of the currents in its sides.

Erratum: In the tracing a slight error has been made in the width of the bar in the center of the Diagram. This affects the curves only close to the bar and has no appreciable effect on the general shape of the curves. Curve *B* of the 6 ft. bar has also been mistraced close to the bar and should at that point conform with *B* on the other side.

Formulas for parallel currents, right-angle currents, currents at 135 deg. (45 deg.) and 90 deg. circular currents have been developed in this manner. These formulas were as much as possible expressed in ratios permitting the main parts to be calculated in advance and the results to be presented in curves reducing the final calculations to a few simple arithmetical operations and thus greatly reducing the necessary labor in computing these forces. These curves are given in Curve Sheets I to V. Curves of A_1 and A_2 do not lend themselves to easy interpolation and the calculation can more readily be performed by means for formula (**1c**) but they have been given because they facilitate the forming of a mental picture of the distribution of the forces along parallel conductors, and thus may be a guide to the placing of supports once the maximum forces and the required strength of the supports have been ascertained. Where a large number of supports have to be placed this will save considerable time and labor.

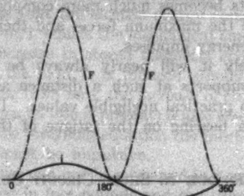
In the case where, due to the nearness of the conductor, the dimensions of the conductors and therefore the current distribution in them, would have a decided influence, approximate methods have been indicated and for the case of conductors at right angle the more rigorously correct formulas of Mr. Dwight have been given. With regard to the required accuracy and the uncertainty pertaining to the unequal current distribution in large conductors due to skin-effect and Hall-effect this unequal current distribution has not been taken into consideration.

For handy reference a few rules of approximate integration are given.

Circular Bend of 90 deg. Formulas for the forces exerted by currents in a 90 deg. circular bend and by currents tangent to the end of the bend on a point in the bend are given. These formulas can only be applied where the radius of the bend is very large compared with the width of the conductor in a radial direction.

The formulas are also given for the constants required for the calculation of the force exerted by a circular current on a point in a conductor tangent to the end of the circular conductor. In this case the approximation has to be found by replacing the circle by a 24-sided polygon. This is sufficiently accurate, giving: that twice the sine of 7.5° equals 0.26105 and twice the tan of 7.5° equals 0.26331, when taking for $l = 0.262$.

Example. As an example the distribution of the forces on a conductor of 4 inch × ¼ inch cross-section forming a right angle, with legs 6 feet and 9 feet has been calculated. Also the same forces when the right angle is cut off by an oblique conductor at 1 foot from the corner on both legs, and finally for a combination of both cases, with the assumption that the impedances of the right angle and cross piece are equal, and the currents in them in phase with each other. The results have been given in the curves on Curve Sheet (D). The current assumed is 100,000 amperes. The forces at half the width of the bar from the corner of the right angle and at the joints of the cross piece have been calculated by the approximate method by dividing the current sheets into 8 parallel and equal currents. As this force is practically independent of the length of the conductors this maximum force may be taken constant for such a conductor, it depending only on the cross-section of the conductor. Further examples of numerical calculations will be found in Appendix A.



Curve Sheet A—Single-Phase Short Circuit Force on Parallel Conductors due to Current in Return Circuit

With a direct current in the conductors the force on them will be constant. With an alternating-current measured in r. m. s. value the force will be the average or equivalent steady force but the maximum force will have twice this value if the current is sinusoidal, *i. e.,* having a peak value = r. m. s. value $\sqrt{2}$. This maximum force acts twice during one cycle.

Combination of Forces in Three-Phase Circuits under Short-Circuit Conditions

Single-Phase Short Circuit. In this case the current in the third conductor is negligible and the only forces to be considered are those of the remaining circuit and its return circuit, the two currents having a phase difference of 180 deg. This acts with double frequency. (See Curve Sheet A.)

Fig. H

Three-Phase Short Circuit. The phase angle of the short-circuit currents will remain at 120 deg. Consider three parallel conductors Nos. 1, 2 and 3, situated in the same plane, No. 2 being midway between 1 and 3 and let the force due to unit current in "1" on unit current in "2" be represented by f_{1-2} etc.

Let the phase rotation of the current be 1, 2, 3, and the impedance of the three short-circuited branches be equal, then,

If I_m is the maximum current, the currents in 1, 2, 3, at any instant are:

$$I_m \sin \varphi, \quad I_m \sin (\varphi - 120°), \quad I_m \sin (\varphi - 240°)$$

On account of symmetry we have only to consider the force exerted on "1" by currents in "2" and "3" and on "2" by currents in "1" and "3."

Then the force exerted at any one instant on conductor "1" is:

$$\begin{aligned} F_1 = f_{2-1} \{ & I_m \sin \varphi \, . \, I_m \sin (\varphi - 120°) \} \\ & + f_{3-1} \{ I_m \sin \varphi \, . \, I_m \sin (\varphi - 240°) \} \end{aligned}$$

or

$$\begin{aligned} F_1 = I_m^2 \, . \, f_{2-1} \, . \, \sin \varphi \, \{ & \sin (\varphi - 120°) \\ & + p \sin (\varphi - 240°) \} \end{aligned} \quad (16)$$

$$\text{where } p = f_{3-1}/f_{2-1} = < 1.$$

$$\text{Putting } \frac{d F_1}{d \varphi} = 0 \text{ we get}$$

$$\tan 2 \varphi = - \frac{\sqrt{3}\,(1 - p)}{1 + p} \quad (17)$$

The maximum and minimum value of F_1 may now be found by substituting the value of φ from (**17**) in (**16**). They are distinguished by the relation

$$\begin{aligned} \frac{d^2 F_1}{d \varphi^2} = I_m^2 \, . \, f_{2-1} \{ & - (1 + p) \cos 2 \varphi \\ & + \sqrt{3} \, . \, (1 - p) \sin 2 \varphi \} \end{aligned} \quad (18)$$

which indicates that

A maximum occurs with φ in 2nd and 4th quadrants
A minimum occurs with φ in 1st and 3rd quadrants

For the purpose of graphic representation formula (**16**) may be transformed to

$$F_1 = \frac{I_m^2}{4} \, . \, f_{2-1} \left\{ - (1 + p) - 2 \sqrt{1 - p + p^2} \; . \sin \left[2 \varphi - \tan^{-1} \frac{1 + p}{(1 - p) \sqrt{3}} \right] \right\} \quad (19)$$

showing that the force consists of a constant force superimposed on a sinusoidal force of double the frequency of the currents.

If the phase rotation of the currents is changed to "1, 3, 2" we find in the same way

$$F_1' = \frac{I_m^2}{4} \, . \, f_{2-1} \left\{ - (1 + p) + 2 \sqrt{1 - p + p^2} \; . \sin \left[2 \varphi + \tan^{-1} \frac{1 + p}{(1 - p) \sqrt{3}} \right] \right\} \quad (20)$$

also we get

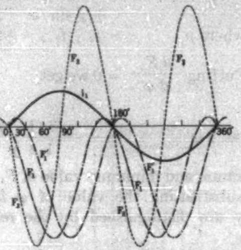
$$F_2 = \frac{I_m^2}{4} \, . \, f_{2-1} \{ \pm 2 \sqrt{3} \, . \, \sin 2 \varphi \} \quad (21)$$

Because for the middle conductor $p = - 1$.

The variations of these forces are graphically represented for $p = 1/2$ and the duration of one cycle on Curve Sheet (B).

Phase Differences of the Forces along Parallel Conductors

From formula (**19**) and (**20**) it will be seen that the phase difference between the force and the current for the outside conductors of the three-phase depends upon the ratio p of the two forces. The curves for A_1 on Curve Sheet I show that for parallel conductors at different distances from a third conductor this ratio changes from the center towards the ends of the conductors. The forces acting on such conductors at different points will therefore not reach their maximums and minimums at the same instant.



Curve Sheet B

Three-Phase short circuit. Forces on 3 parallel conductors in same plane. No. 2 midway between 1 and 2.

F : phase rotation 1, 2, 3
F': " " 1, 3, 2
p assumed = 1/2

Combination of Forces

In combining several forces acting in different directions it will thus be necessary to take into account the above phase differences. It follows that the resultant forces will change in direction as well as magnitude during a cycle, while the maximum will not occur at the same time on different points along the conductors. Therefore the moment of the Resultant Force will change in magnitude and direction during the cycle.

Vibration in Busbars

Under the influence of these double-frequency forces the conductors will have a tendency to vibrate. That this vibration may reach considerable magnitude was shown by an experiment made at these laboratories. Two busbars 3/8-inch by 1/4-inch were placed side by side with 1 inch spacers at the supports and firmly clamped. The bars formed a loop. A current of 300 amperes, 25 cycles was sent through the loop and the amplitude of the vibrations measured.

By changing the distance between supports a series of amplitude was measured and are shown in the curve on Curve Sheet E.

It will be seen that at the resonance spacing of 26 inches the amplitude experienced a very sudden increase being 70 times the deflection at 18-inch spacing.

The striking analogy between mechanical and electrical vibration can be seen by a comparison of this curve with the well known "tuning" curves of wave meters. In both cases the ordinates are functions of the square of the currents and the abscissa functions of frequency. As the external resistance to vibration of the bar is but the air resistance and therefore low, the mechanical tuning will be sharp.

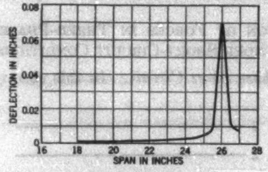
For such cases where vibration can not practically be avoided the calculation of the stresses in the busbars and supports becomes much more complicated as we have to use the maximum forces and their change in time, *i. e.*, energy impulses.

Fortunately it will nearly always be possible to space the supports at such a distance as to reduce vibration to practical negligible values. It may however have a bearing on the fatigue of the materials used.

Impulses due to Short-Circuit Currents

In our measurements a decided "Kick" of the movable bar was observed every time at the moment of closing the circuit. This could only be ascribed to the small impedance of our circuit allowing the current to suddenly build up to its maximum value, *i. e.*, the current had a steep wave front. As however, this repeated closing of the switch might have occurred at any time during a cycle it is not quite clear why the bar was invariably kicked out far beyond its steady position due to the average current.

It is well known that if a weight is suddenly applied to a bar which is prevented from bending the compression of the bar and therefore the stresses in the bar can be expressed by



Curve Sheet E

Vibration of Busbar on 25-cycle symmetrical current 300 amperes. Busbars 3/8 in. by 1/4 in. side by side. One-in. spacers at points of support.

$$y = e\,(1 + \sqrt{1 + 2\,h/e})$$

where y = maximum compression produced
h = height from which weight drops or the distance through which the force of gravitation acts on the weight before striking the bar.
e = Compression due to the static weight

If $h = 0$, then $y = 2\,e$.

From which it follows that an instantaneously applied force without kinetic energy will cause a stress double the stress produced by the same steady force.

It is difficult to conceive of a magnetic field or an electric current to have mass, although the latter is now generally regarded as an actual flow of electrons which each have an exceedingly small mass due to their electric charge. We are used to talking about electric impulses but it is difficult to conceive the mechanism which transforms them into mechanical impulses. That they must do so follows however from the law of the conservation of energy.

We know that the electric energy consumed by the induced e. m. f. equals the mechanical energy produced plus the increase in stored magnetic energy.

If then the e. m. f. is induced wholly impulsively, *i. e.*, starting from zero, the increase of stored magnetic energy (when no iron is present) is also wholly impulsive and from the foregoing equation the mechanical energy produced must be impulsive.

An impulse can not be measured in pounds but must be expressed in foot-pounds and the time element has to be brought in. The magnitude of the stresses produced in the bars will thus depend upon the steepness of the wave front of the short-circuit current.

From the above formula for the stresses due to impulses it will be seen that they may reach very high values depending upon the value h/e.

With $h = 4\,e$, the stresses will be 4 times the stresses due to the static force with $h = 24\,e$, 8 times as much, etc. Unfortunately our measuring apparatus could not be used to measure these stresses and until more is known of the real constitution of electric currents and magnetic fields we have to assume no mass for them and to accept for these stresses a value twice the stress created by the force due to the maximum value of the short-circuit current. This seems safe enough as in large stations enough impedance is put into the circuit to reduce considerably the steepness of the short current wave front.

CONCLUSIONS

To sum up the results we may say that these measurements have proved that for practical engineering purposes we may assume line currents in the busbars even very close to the corners of right angle taps, a fact which the writer thinks could not be previously assumed, and that consequently we were able to derive some very simple formulas and methods for the calculation of the mechanical forces on busbars, which formulas may also be of use in checking the mechanical strength and construction of electrical apparatus and specially of the leading-in conductors.

In addition to a bibliography in Appendix A will be found an example of calculations and in Appendix B are given the mathematical derivations of the formulas.

The writer wishes to acknowledge his indebtedness to the engineers of the laboratories of the Hydro-Electric Power Commission, who one and all were always ready with valuable advice and suggestions for overcoming the considerable difficulties encountered in these measurements; to Mr. R. Gemmell who built and erected the apparatus and to Mr. A. E. Frampton who assisted in taking the measurements.

Appendix A

PARALLEL CONDUCTORS

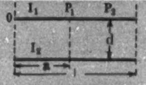


FIG. I

FOR POINTS BETWEEN PERPENDICULAR PLANES AT THE ENDS OF CONDUCTOR l

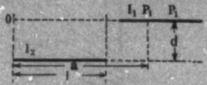


FIG. II

FOR POINTS OUTSIDE PERPENDICULAR PLANES AT THE ENDS OF CONDUCTOR l

I_1, I_2 currents in amperes

$$a/l = K \qquad d/l = n$$

Force at a point P.

$$F_P = \frac{I_1 I_2}{d}\left[\frac{K_1}{\sqrt{K^2+n^2}} + \frac{1-K}{\sqrt{(1-K)^2+n^2}}\right]$$

$$\left\{\begin{array}{r} 1.02 \times 10^{-5} \text{ grams per cm.} \\ \text{d in cm.} \\ 2.248 \times 10^{-8} \text{ lbs. per inch} \\ d \text{ in inches} \end{array}\right\} \qquad (1)$$

For case of Fig. 1

$$F_P = \frac{I_1 I_2}{d\,(\text{cm./inches})} \times A_1$$

$$\times \left\{\begin{array}{l} 1.02 \times 10^{-5} \text{ grams per cm.} \\ 2.248 \times 10^{-8} \text{ lbs. per inch} \end{array}\right\} \qquad (1a)$$

For case of Fig. 2

$$F_P = \frac{I_1 I_2}{d\,(\text{cm./inches})} \times A_2$$

$$\times \left\{\begin{array}{l} 1.02 \times 10^{-5} \text{ grams per cm.} \\ 2.248 \times 10^{-8} \text{ lbs. per inch} \end{array}\right\} \qquad (1b)$$

The values of A_1 and A_2 may be scaled from the curves of Curve Sheets I and II.

If the currents are in the same direction F_P is positive and is an attraction, if in opposite direction F_P is negative and is a repulsion.

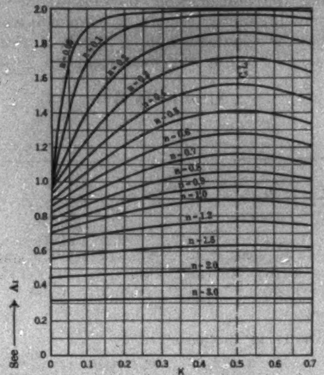
The force is at right-angle to the current at the point and in the plane containing l and P.

The force on a section $P_1 - P_2$ is:

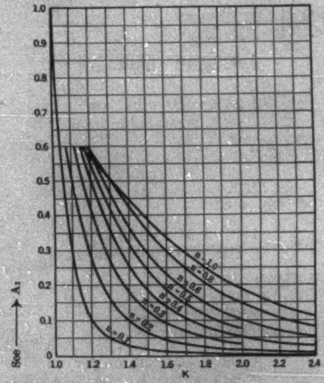
$$F_{P_1-P_2} = \frac{I_1 I_2}{n}\left[\sqrt{K^2+n^2} - \sqrt{(1-K)^2+n^2}\,\right]_{k_1}^{k_2} =$$

$$\left\{\begin{array}{l} 1.02 \times 10^{-5} \text{ grams} \\ 2.248 \times 10^{-8} \text{ lbs.} \end{array}\right\} \qquad (2)$$

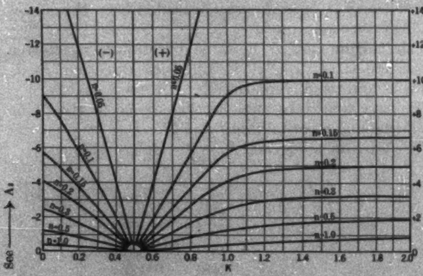
$$F_{p_1-p_2} = I_1 I_2 \times \Big[A_3 \Big]_{h_1}^{h_2} \left\{ \begin{array}{l} 1.02 \times 10^{-5} \text{ grams} \\ 2.248 \times 10^{-8} \text{ lbs.} \end{array} \right\} \quad (2a)$$



Curve Sheet I—Values of A of Formula (1.a)



Curve Sheet II—Values of A_2 of Formula (1.b)



Curve Sheet III—Values of A_3 of Formula (2A)

The values of A_3 may be scaled from the curves of Curve Sheet III.

The moment of the force $F_{p_1-p_2}$ about the point O is given by

$$M_{0/p_1-p_2} = I_1 I_2 \times \frac{l}{2} \Big[K/n \, . \sqrt{K^2 + n^2} - \frac{1+K}{n} \, . \sqrt{(1-K)^2 + n^2} - 2.303\, n \, \{ \log_{10} (K + \sqrt{K^2 + n^2}) + \log_{10} ((1-K) + \sqrt{(1-K)^2 + n^2}) \} \Big]_{h_1}^{h_2}$$

$$\left\{ \begin{array}{c} 1.02 \times 10^{-5} \text{ grams-cm.} \\ l \text{ in cm.} \\ 2.248 \times 10^{-8} \text{ inch-pounds} \\ l \text{ in inches} \end{array} \right\} \quad (3)$$

The moment of $F_{p_1-p_2}$ about any point of the conductor or its extension may also be found by dividing the proper curve (I or II) (or interpolated curve) in such number of parts that each part can be treated as a straight line. The moments of the forces of each part of the conductor can then be found by the method of approximate integration.

Note. Formula (1) can be written

$$F_p = \frac{I_1 I_2}{d} \left[\frac{1}{\sqrt{1 + (n/K)^2}} \pm \frac{1}{\sqrt{1 + \left(\frac{n}{K-1} \right)^2}} \right]$$

$$\left\{ \begin{array}{c} 1.02 \times 10^{-5} \text{ grams per cm.} \\ d \text{ in cm.} \\ 2.248 \times 10^{-8} \text{ lbs. per inch} \\ d \text{ in inches} \end{array} \right\} \quad (1c)$$

The value of each term under the brackets may then be found from curve B_1 substituting n/K and

$\frac{n}{K-1}$ for n. $\pm = K \lessgtr 1$.

If two conductors of rectangular cross-section ($h \times w$) and of length l are spaced very close together such as the bars of a built-up busbar, the mechanical force acting between them, may be computed by dividing each conductor into an equal number of strips of equal width, for instance, a_1, b_1, c_1 and a_2, b_2, c_2. If n is the number of strips the current in each strip is, if I_1 and I_2 are the currents in the bars I_1/n and I_2/n respectively, the total force or its distribution can be calculated by taking the algebraic sum of the forces acting between the different strips of one bar and the strips of the other bar with the aid of formulas (1) to

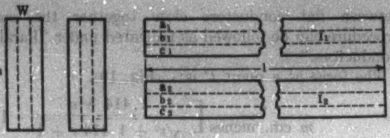


Fig. J

(3), replacing $I_1 I_2$ by $\frac{I_1 I_2}{n^2}$ and taking for d the distance of the centre lines of the strips under consideration.

Conductors at Right-Angles

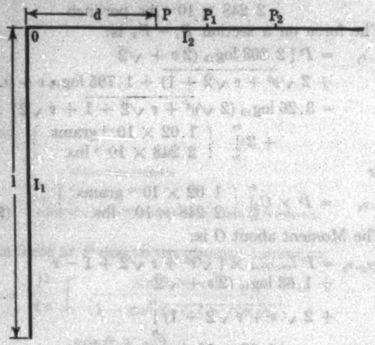


Fig. K

I_1, I_2 currents in amperes.

$d/l = n$

The Force at a point P is:

$$F_P = \frac{I_1 I_2}{d}\left[\frac{1}{\sqrt{n^2+1}}\right] \times \left\{\begin{matrix} 1.02 \times 10^{-5} \text{ grams per cm. } (d \text{ in cm.}) \\ 2.248 \times 10^{-8} \text{ lbs. per inch } (d \text{ in inches}) \end{matrix}\right\} \tag{4}$$

or

$$F_P = \frac{I_1 I_2}{d \text{ cm./inches}} \times B_1 \times \left\{\begin{matrix} 1.02 \times 10^{-5} \text{ grams. per cm.} \\ 2.248 \times 10^{-8} \text{ lbs. per inch} \end{matrix}\right\} \tag{4a}$$

The force on a section $P_1 - P_2$ is:

$$F_{P_1-P_2} = I_1 I_2 \left[\log_{10}\left(\sqrt{n^2+1} - 1\right) - \log_{10} n\right]_{n_1}^{n_2} \times \left\{\begin{matrix} 2.35 \times 10^{-5} \text{ grams} \\ 5.177 \times 10^{-8} \text{ lbs.} \end{matrix}\right\} \tag{5}$$

or

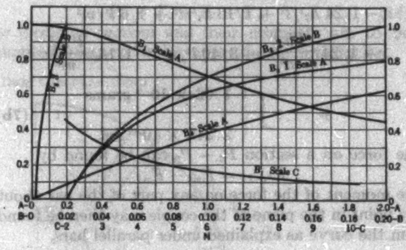
$$F_{P_1-P_2} = I_1 \times I_2 \times B_2\Big]_{n_1}^{n_2} \times \left\{\begin{matrix} 2.35 \times 10^{-5} \text{ grams} \\ \\ 5.177 \times 10^{-8} \text{ lbs.} \end{matrix}\right\} \tag{5a}$$

The moment of F_{P1-P2} about point O is:

$$M_{0/P_1-P_2} = I_1 \times I_2 \times l \left[\log_{10}\left(n + \sqrt{n^2+1}\right)\right]_{n_1}^{n_2} \times \left\{\begin{matrix} 2.35 \times 10^{-5} \text{ grams-cm. } (l \text{ in cm.}) \\ 5.177 \times 10^{-8} \text{ foot-pounds } (l \text{ in feet}) \end{matrix}\right\} \tag{6}$$

or

$$M_{0/P_1-P_2} = I_1 \times I_2 \times l_{cm./feet} \times B_3\Big]_{n_1}^{n_2} \times \left\{\begin{matrix} 2.35 \times 10^{-5} \text{ gram-cm.} \\ 5.177 \times 10^{-8} \text{ foot-pounds} \end{matrix}\right\} \tag{6a}$$



Curve Sheet IV

Values of B_1 formula (4.a)
B_2 " (5.a)
B_3 " (6.a)

Note: The values of B_2 represent the logarithm of a quotient and the curve has been drawn in 3 parts having respectively the characteristic $\overrightarrow{1}$, $\overrightarrow{2}$ and $\overrightarrow{3}$. The ordinates of the curves represent the mantissas.

The values of B_1, B_2 and B_3 may be scaled from the curves on curve-sheet IV.

If it is necessary, for greater accuracy, to take in account the current distribution in flat bars, the computation can be performed in a similar way as outlined for parallel bars by dividing the bars into strips. Or they may be calculated by formula (7a) or (7b) developed by Mr. H. B. Dwight.

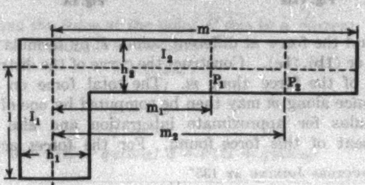


Fig. 5

Note. In these formulas the assumption has been made that the conductor l is round and the current I_1 thus may be considered concentrated at the centre of the bar.

The force on m is:

when $m < l$

$$F_m = I_1 \times I_2 \Big[2.303 \log_{10} \left\{ \frac{2\,m}{h_1} \right\} - 1/4\, m^2/l^2 + 1/16\, h_1^2/l^2 + 3/32\, m^4/l^4 - 1/16\, \frac{m^2\, h_2^2}{l^4} + \ldots \Big] \left\{ \begin{matrix} 1.02 \times 10^{-5} \text{ grams} \\ 2.248 \times 10^{-8} \text{ lbs.} \end{matrix} \right\} \qquad \textbf{(7a)}$$

when $m > l$

$$F_m = I_1 \times I_2 \Big[2.303 \log_{10} \left\{ \frac{4\,l}{h_1} \right\} - l/m - 1/24\, h_2^2/l^2 + 1/16\, h_1^2/l^2 + 1/6\, l^3/m^3 + 1/24\, \frac{l\, h_2^2}{m^3} - 3/40\, l^5/m^5 - 1/16\, \frac{l^3\, h_2^2}{m^5} + \ldots \Big] \left\{ \begin{matrix} 1.02 \times 10^{-5} \text{ grams} \\ 2.248 \times 10^{-8} \text{ lbs.} \end{matrix} \right\} \qquad \textbf{(7b)}$$

The force on a section $P_1 - P_2$ can be found by

$$F_{P_1-P_2} = F_{m2} - F_{m1}$$

The moment of the force on any part of the bar about any point in the plane of the couple may then be found from the curve as explained under parallel bars.

CONDUCTORS IN GENERAL DIRECTIONS

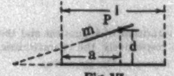


Fig. VI

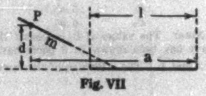


Fig. VII

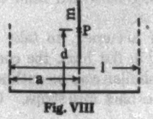


Fig. VIII

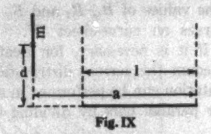


Fig. IX

Find the force at different points P by formula **(1)**, **(1a)** or **(1b)**, **(1c)**. Construct the curve of the distribution of the force along m. The total force on any distance along m may then be computed by one of the formulas for approximate integration and also the moment of this force found. For the forces acting between flat conductors close together the same procedure can be followed as indicated under "Parallel Conductors."

CONDUCTORS JOINING AT 135°

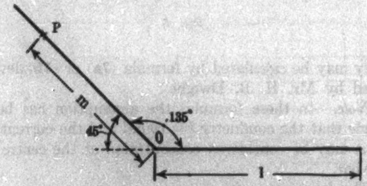


Fig. 10

I = current in amperes

$v = m/l$

The force at a point P is: (Fig. 10).

$$F_P = \frac{I^2}{m \text{ cm./inches}} \left[\frac{1.414 + v}{\sqrt{v^2 + 1.414\, v + 1}} - 1 \right] \left\{ \begin{matrix} 1.02 \times 10^{-5} \text{ grams per cm.} \\ 2.248 \times 10^{-8} \text{ lbs. per inch} \end{matrix} \right\} \qquad \textbf{(8)}$$

or

$$F_P = \frac{I^2}{m \text{ cm./inches}} \times C_1 \left\{ \begin{matrix} 1.02 \times 10^{-5} \text{ grams per cm.} \\ 2.248 \times 10^{-8} \text{ lbs. per inch} \end{matrix} \right\} \qquad \textbf{(8a)}$$

The force on a section $P_1 - P_2$ is:

$$F_{P_1-P_2} = I^2 \Big[2.303 \log_{10} \left(2\,v + \sqrt{2} + 2\sqrt{v^2 + v\sqrt{2} + 1}\right) + 1.795 \log_{10} v + 0.98 - 3.26 \log_{10} \left(2\sqrt{v^2 + v\sqrt{2} + 1} + v\sqrt{2} + 2\right) \Big]_{v_1}^{v_2} \left\{ \begin{matrix} 1.02 \times 10^{-5} \text{ grams} \\ 2.248 \times 10^{-8} \text{ lbs.} \end{matrix} \right\} \qquad \textbf{(9)}$$

or

$$F_{P_1-P_2} = I^2 \times C_2 \Big]_{v_1}^{v_2} \left\{ \begin{matrix} 1.02 \times 10^{-5} \text{ grams} \\ 2.248 \times 10^{-8} \text{ lbs.} \end{matrix} \right\} \qquad \textbf{(9a)}$$

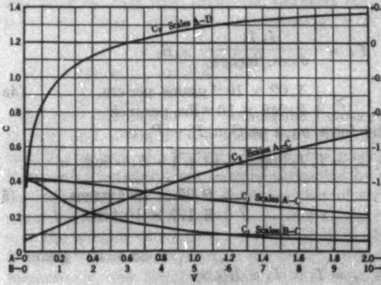
The Moment about O is:

$$M_{0/P_1-P_2} = I^2\, l_{cm./feet} \times \Big[\sqrt{v^2 + v\sqrt{2} + 1} - v + 1.63 \log_{10} \left(2\,v + \sqrt{2} + 2\sqrt{v^2 + v\sqrt{2} + 1}\right) \Big]_{v_1}^{v_2} \times \left\{ \begin{matrix} 1.02 \times 10^{-5} \text{ gram-centimeter} \\ 2.248 \times 10^{-8} \text{ foot-pounds} \end{matrix} \right\} \qquad \textbf{(10)}$$

or

$$M_{0/P_1-P_2} = I^2 \times l_{cm./feet} \times C_3 \Big]_{v_1}^{v_2} \left\{ \begin{matrix} 1.02 \times 10^{-5} \text{ gram-centimeter} \\ 2.248 \times 10^{-8} \text{ foot-pounds} \end{matrix} \right\} \qquad \textbf{(10a)}$$

The values of C_1, C_2, C_3 may be scaled from curves on Curve-Sheet No. V.



CURVE SHEET V

Values of C_1 formula (8.a)

C_2 " (9.a)

C_3 " (10.a)

90 DEG. CIRCULAR BEND

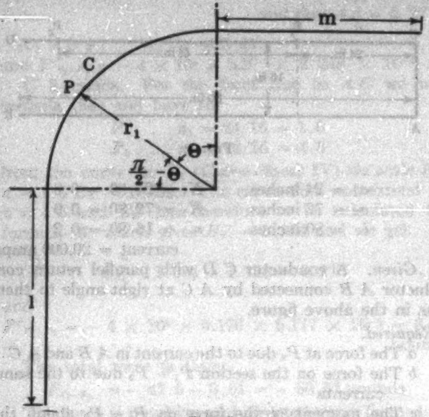


FIG. L

I = current in amperes
$l/r = p$
$m/r = q$
r supposed large compared with width of conductor

The force at P due to the circular current is:

$$F_c = \frac{I^2}{4r}\Big[\frac{\sin\theta\times\sqrt{2}}{(1-\cos\theta)\sqrt{1-\cos\theta}} + \frac{\cos\theta\times\sqrt{2}}{(1-\sin\theta)\sqrt{1-\sin\theta}} - 2.303\log_{10}\tan.\theta/4 + 2.303\log_{10}\left(\frac{\cos\theta+\sqrt{2}\times\sqrt{1-\sin\theta}}{1-\sin\theta}\right)\Big] \times \left\{\begin{array}{c}1.02\times10^{-5}\text{ grams per cm.}\\ r\text{ in cm.}\\ 2.248\times10^{-8}\text{ lbs. per inch}\\ r\text{ in inches}\end{array}\right\} \quad (11)$$

This force acts in the direction of the radius at P and away from the centre of the circle.

The force due to currents in l and m are respectively when $\frac{l}{r} = p$, $\frac{m}{r} = q$

$$F_{l\,cm./inches} = \frac{I^2}{r(1-\sin\theta)}\times\Big[\frac{p+\cos\theta}{\sqrt{(1-\sin\theta)^2+(p+\cos\theta)^2}} - \frac{\cos\theta}{\sqrt{(1-\sin\theta)^2+\cos^2\theta}}\Big]\left\{\begin{array}{c}1.02\times10^{-5}\text{ grams per cm.}\\ 2.248\times10^{-8}\text{ lbs. per inch}\end{array}\right\} \quad (12)$$

$$F_{m\,cm./inches} = \frac{I^2}{r(1-\cos\theta)}\times\Big[\frac{q+\sin\theta}{\sqrt{(1-\cos\theta)^2+(q+\sin\theta)^2}} - \frac{\sin\theta}{\sqrt{(1-\cos\theta)^2+\sin^2\theta}}\Big] \times\left\{\begin{array}{c}1.02\times10^{-5}\text{ grams per cm.}\\ 2.248\times10^{-8}\text{ lbs. per inch}\end{array}\right\} \quad (13)$$

The total force at P is:

$$F_T = F_c + F_l + F_m \quad (14)$$

The forces on sections and their moments are to be computed from curves by approximate integration.

90 DEG. CIRCULAR BEND

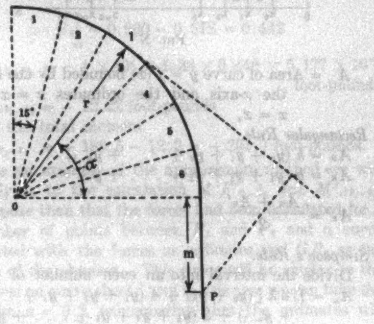


FIG. M

$l = 0.262r$
$m/r = q$
$a/l = K$
$d/l = n$

To find the force at the point P due to a current in the 90° circular part, we have

$$l = 0.262\,r$$

when

$$k = a/l$$
$$n = d/l$$
$$q = m/r$$
$$a = r\,(E + q\cos\alpha)\quad d = r\,(H + q\sin\alpha)$$

α	$\sin\alpha$	$\cos\alpha$	E	H
7.5°	0.1305	0.9914	0.262	0.009
22.5°	0.3827	0.9239	0.513	0.076
37.5°	0.6088	0.7934	0.731	0.207
52.5°	0.7934	0.6088	0.924	0.391
67.5°	0.9239	0.3827	1.055	0.617
82.5°	0.9914	0.1305	1.122	0.860

The force at P due to any of the six parts of the current can be calculated by formula (1c) and the total force will be

$$F_T = \sum_{1}^{6} F \quad (15)$$

The force on a section $P_1 - P_2$ can be found through approximate integration. For forces close to the junction of curve and straight line it will be necessary to divide the current sheet into a number of parallel currents as described before.

Note. In case the radius of the circular conductor is not large compared with the width of the conductor it will be necessary to consider the current sheet as composed of a number of parallel conductors and to replace each of these circular currents as a polygon and so compute the total forces at each point.

APPROXIMATE INTEGRATION

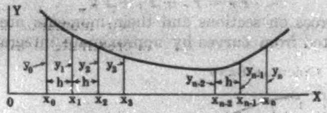


FIG. N

A = Area of curve $y = f(x)$ bounded by the curve, the x-axis and the ordinates $x = x_0$ and $x = x_n$

Rectangular Rule.

$$A_R = h\,(y_0 + y_1 + y_2 + \ldots + y_{n-1})$$
$$A_R' = h\,(y_1 + y_2 + y_3 + \ldots + y_n)$$
$$A_R'' = \frac{A_R + A_R'}{2}$$

Simpson's Rule.

Divide the interval into an even number of parts.

$$A_S = 1/3\,h\,[\,(y_0 + y_n) + 4\,(y_1 + y_3 + y_5 + \ldots + y_{n-1}) + 2\,(y_2 + y_4 + y_6 + \ldots + y_{n-2})\,]$$

Note. The approximation is closer the larger the number of intervals h.

A_R'' may be used for very close approximations

A_s requires less intervals for the same approximation

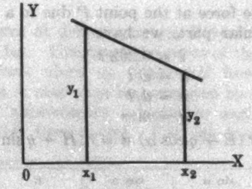


FIG. O

Straight Line.

A = the area of the line $y = f(x)$ bounded by the line, the x-axis and the ordinates $x = x_1$ and $x = x_2$

$$A = 1/2\,(x_2 - x_1)\,(y_1 + y_2)$$

If A represents a force then the moment of that force about O is given by:

$$M = 1/6\,\{\,(y_1 + 2\,y_2)\,x_2^2 - (y_2 + 2\,y_1)\,x_1^2 + (y_1 - y_2)\,x_1\,x_2\}$$

Example

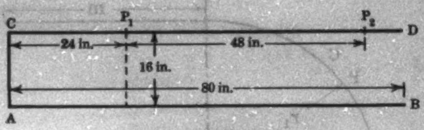


FIG. P

a_1 = 24 inches $K_1 = 24/80 = 0.3$
a_2 = 72 inches $K_2 = 72/80 = 0.9$
l = 80 inches $n = 16/80 = 0.2$
current = 20,000 amps.

Given. A conductor $C\,D$ with parallel return conductor $A\,B$ connected by $A\,C$ at right-angle to them as in the above figure.

Required.

a The force at P_1 due to the current in $A\,B$ and $A\,C$

b The force on the section $P_1 - P_2$ due to the same currents

c The moment of the force on $P_1 - P_2$ about the point C.

a. For the force due to $A\,B$ we apply formula (**1a**). The value of A_1 scaled from the curve $n = 0.2$ (Curve Sheet I) for $K = 0.3$ is 1.79. Remembering that the currents in $A\,B$ and $C\,D$ are in opposite directions we have:

$$F_{P_1}' = -\frac{4 \times 10^8}{16} \times 1.79 \times 2.248 \times 10^{-8} \text{ lbs. per inch}$$

$$F_{P_1}' = -1.005 \text{ lbs. per inch}$$

If n had such a value that interpolation would be difficult formula (**1c**) can be used and we have:

$$n/K = 0.2/0.3 = 0.667 = n'$$

$$\frac{n}{K-1} = -0.2/0.7 = -0.286 = n''$$

(The sign is immaterial as the formula contains n''^2) From Curve B_1 (curve sheet IV) we scale $B_1' = 0.832$ and $B_1'' = 0.960$ as $K < 1$ the + sign has to be used in formula (**1c**) and the term under the brackets is thus $0.832 + 0.960 = 1.792$ the same value as has been scaled before for A_1. The force due to $A\,C$ is found by formula (**4a**) where $n = 24/16 = 1.5$. From curve B_1 (Curve Sheet IV) we scale $B_1 = 0.555$.

Here the currents have different directions with regard to their junction point C and thus opposite signs and we have $F_{P_1}'' = -\dfrac{4 \times 10^{+8}}{16} \times 0.555 \times 2.248 \times 10^{-8}$ lbs. per inch or $F_{P_1}'' = -0.312$ lbs. per inch and the total force $F_{P_1} = -1.005 - 0.312 = -1.317$ lbs. per inch.

The negative sign indicates a repulsive force.

b For the force due to $A\,B$ we use formula (**2a**) and from the curve $n = 0.2$ for A_2 (Curve Sheet III) we scale for:

$$K_1 = 0.3 \;\ldots\; A_3' = -1.80$$

$$\text{and thus } A_3\Big]_{k_1}^{k_2} = 5.27$$

$$K_2 = 0.9 \;\ldots\; A_3'' = +3.47$$

and $F'_{P_1-P_2} = -4 \times 10^8 \times 5.27 \times 2.248 \times 10^{-8} = -47.5$ pounds. For the force due to $A\,C$ we use formula (**5a**) and have for

$$P_1 \ldots n_1 = 24/16 = 1.5$$
$$P_2 \ldots n_2 = 72/16 = 4.5$$

from the curve for B_2 (Curve Sheet IV) we scale for $n = 1.5$ $B_2' = 1.728$. The curve does not extend to $n = 4.5$ and B_2'' has therefore to be calculated by formula (**5**) which gives $B_2'' = 1.904$ and we get

$$B_2\Big]_{n_1}^{n_2} = 1.904 - 1.728 = 0.176$$

and

$$F''_{P_1-P_2} = -4 \times 10^8 \times 0.176 \times 5.177 \times 10^{-8} = 3.01 \text{ pounds}$$

The total force is thus:

$$F_{P_1-P_2} = -47.5 - 3.01 = -50.51 \text{ pounds}$$

This force is repulsive and directed away from $A\,B$.

If in calculating $F'_{P_1-P_2}$, interpolation should prove difficult, its value can be found by formula (**2**). This formula can be written thus:

$$F_{P_1-P_2} = I_1 I_2 \left[\sqrt{1 + (K/n)^2} - \sqrt{1 + \left\{\frac{1-K}{n}\right\}^2} \right]_{k_1}^{k_2}$$

or

$$F_{P_1-P_2} = I_1 I_2 \left[\sqrt{1 + n_1^2} - \sqrt{1 + n_2^2} \right]_{k_1}^{k_2}$$

and the values under the roots found by taking the reciprocals of the values given by curve B_1 (curve sheet IV).

c. The Moment of the force is found by formula (**3**) for the force due to $A\,B$ and we have

$K_1/n = 1.5$	$K_2/n = 4.5$
$K_1/n \sqrt{K_1^2 + n^2} = 0.54$	$K_2/n \sqrt{K_2^2 + n^2} = 4.15$
$\frac{1+K_1}{n} \sqrt{(1-K_1)^2 + n^2} = 4.74$	$\frac{1+K_2}{n} \sqrt{(1-K_2)^2 + n^2} = 2.12$
$\log_{10}(K_1 + \sqrt{K_1^2 + n^2}) = \bar{1}.81994$	$\log_{10}(K_2 + \sqrt{K_2^2 + n^2}) = 0.26055$
$\log_{10}\{(1-K_1) + \sqrt{(1-K_1)^2 + n^2}\} = 0.15473$	$\log_{10}\{(1-K_2) + \sqrt{(1-K_2)^2 + n^2} = \bar{1}.51001$
Sum of $\log_e = \bar{1}.97467 = -0.0253$	Sum of $\log_e = \bar{1}.77056 = -0.2294$
$2.303 \times 0.2 \times -0.0253 = -0.0117$	$2.303 \times 0.2 \times -0.2294 = -0.1058$
The term between brackets	The term between brackets
$[\,]^{k_1} = 0.54 - 4.74 + 0.01 = -4.19$	$[\,]^{k_2} = 4.15 - 2.12 + 0.11 = +2.14$

and we have

$$M'_{c/P_1-P_2} = -4 \times 10^8 \times 80/2\,[2.14 + 4.19] \times 2.248 \times 10^{-8} \text{ inch-pounds}$$

$$M'_{c/P_1-P_2} = -2275 \text{ inch-pounds} = -189.5 \text{ foot-pounds.}$$

The negative sign indicates a couple around C and turning away from $A\,B$. For the moment of $F''_{P_1-P_2}$ we apply formula (**6a**) and we have

$$l = 16/12 = 1.33 \text{ feet} \qquad n_1 = 24/16 = 1.5$$
$$n_2 = 72/16 = 4.5$$

from the curve for B_3 (Curve Sheet IV) we scale for $n = 1.5$.

$$B_3' = 0.518$$

The curve not extending beyond $n = 2$, we calculate B_3'' for $n = 4.5$ by formula (**6**) and find

$$B_3'' = 0.960 \text{ and thus}$$

$$B_3\Big]_{n_1}^{n_2} = 0.960 - 0.518 = 0.442$$

$$M''_{c/P_1-P_2} = -4 \times 10^8 \times 1.33 \times 0.442 \times 5.177 \times 10^{-8} \text{ foot-pounds}$$

$$M''_{c/P_1-P_2} = -12.22 \text{ foot-pounds}$$

and the total moment

$$M_{c/P_1-P_2} = -189.5 - 12.2 = -201.7 \text{ foot-pounds.}$$

As an example of the approximate method we will apply it to the calculation of $F'_{P_1-P_2}$ and M'_{c/P_1-P_2}. Suppose then that the forces had been calculated for a number of points between P_1 and P_2 and a curve plotted with the forces as ordinates and $C\,P_2$ as the axis of the abscissa. This curve will be similar to the curves on curve sheet I and in this case we can take the curve $n = 0.2$ remembering that the ordinates will finally have to be multiplied by $\frac{-4 \times 10^8}{16} \times 2.248 \times 10^{-8}$ to represent the forces in lbs. per inch. We divide the curve between the abscissa $x = 24$ in. and $x = 72$ in. into an even number of parts (in this case we will take 12) through ordinates spaced 4 in. apart or in terms of K at 0.3, 0.35, 0.4 . . . 0.85, 0.9. As the curve is symmetric on both sides of the ordinate $x = (K = 0.5)$ the value of the ordinate at $K = 0.7$, 0.75 etc., can be found at $K = 0.3$, 0.25 etc.

To find the force $F'_{P_1-P_2}$ we apply Simpson's Rule where $h = 4$ in. and we have the following values for the ordinates as scaled from curve $n = 0.2$ (curve sheet I).

$$\begin{array}{llllll}
y_0 & = 1.79 & y_1 & = 1.82 & y_2 & = 1.84 \\
y_{12} & = 1.43 & y_3 & = 1.85 & y_4 & = 1.86 \\
 & \overline{} & y_5 & = 1.85 & y_6 & = 1.84 \\
\text{Sum} & = 3.22 & y_7 & = 1.82 & y_8 & = 1.79 \\
 & & y_9 & = 1.74 & y_{10} & = 1.68 \\
 & & y_{11} & = 1.57 & & \overline{} \\
 & & & \overline{} & \text{Sum} & = 9.01 \\
 & & \text{Sum} & = 10.65 & &
\end{array}$$

and

$$F'_{P_1-P_2} = -\frac{4 \times 10^8}{16} \times 2.248 \times 10^{-8} \times 1/3$$

$$\times 4\,[3.22 + 4 \times 10.65 + 2 \times 9.01]$$

$F'_{P_1-P_2} = 47.8$ pounds
$F''_{P_1-P_2}$ as calculated before $= 3.01$ pounds
thus
$F_{P_1-P_2} = 50.81$ pounds.

To find the moment of this force we may consider the parts of the curve between ordinates as straight lines and find the moments of the forces between successive ordinates by the formula given for Appr. Intg. Finally we will have to multiply with the same factor as before in order to give the scaled values of the ordinates their proper value as forces.

The total moment is equal to the sum of the partial moments.

X_1	X_2	M
24	28	1,120
28	32	1,322
32	36	1,458
36	40	1,706
40	44	1,858
44	48	2,021
48	52	2,180
52	56	2,278
56	60	2,468
60	64	2,530
64	68	2,579
68	72	2,485
	Sum =	24,035

$$M_{c/P_1-P_2} = -\frac{4 \times 10^8}{16} \times 2.248 \times 10^{-8} \times 1/6$$

$$\times 24035 = --\ 2250 \text{ inch pounds}$$
$$= -\ 187.5 \text{ foot-pounds.}$$

$M_{c\,P_1-P_2} = -\ 187.5 - 12.2 = -\ 199.7$ foot-pounds.

Summary

F_{P_1} $= -$ 1.317 pounds per inch by formula (**1a**), (**4a**)
$F_{P_1-P_2}$ $= -$ 50.51 pound by formula (**2a**), (**5a**), (**5**)
$F_{P_1-P_2}$ $= -$ 50.81 pounds by formula (**5a**), (**5**), Approx. Met.
$M_{c/P_1-P_2} = -$ 201.7 foot-pounds by formula (**3**), (**6a**), (**6**)
$M_{c/P_1-P_2} = -$ 199.7 foot-pounds by formula (**6a**), (**6**), Approx. Met.

The distance from C of the point of the section $P_1 - P_2$ where the moments of the forces, on both sides of it to P_1 and P_2 respectively, are equal and where the total force can replace the distributed force is $M/F = 201.7/50.51 = 3.99$ feet $= 48$ inches.

Appendix B

DERIVATION OF FORMULAS

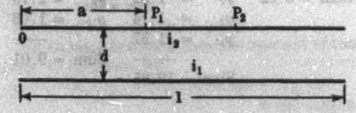


FIG. Q

From the fundamental formula for the field intensity we have for the field intensity at P_1 due to i_1

$$H_{P_1} = \frac{i_1}{d}\left(\frac{a}{\sqrt{a^2+d^2}} + \frac{l-a}{\sqrt{(l-a)^2+d^2}}\right) \text{ maxwells}$$

and for the force at P_1

$$F_{P_1} = H_{P_1} \cdot i_2 = \frac{i_1\, i_2}{d}\left(\frac{a}{\sqrt{a^2+d^2}}\right.$$
$$\left. + \frac{l-a}{\sqrt{(l-a)^2+d^2}}\right) \text{ dynes}$$

Putting $a/l = K$ and $d/l = n$, we can write at once

$$F_P = \frac{i_1\, i_2}{d}\left[\frac{K}{\sqrt{K^2+n^2}} + \frac{1-K}{\sqrt{(1-K)^2+n^2}}\right]$$

dynes per cm. (1)

$$F_P = \frac{i_1\, i_2}{d}\left(\frac{a}{\sqrt{a^2+d^2}} - \frac{a-l}{\sqrt{d^2+(a-l)^2}}\right) \text{ dynes}$$

or

$$F_P = \frac{i_1\, i_2}{d}\left[\frac{K}{\sqrt{K^2+n^2}} - \frac{K-1}{\sqrt{(K-1)^2+n^2}}\right]$$

dynes per cm.

which is the same as formula (**1**). (See Fig. II App. A).

To find the force on a part of current i_2 from P_1 to P_2 consider a small extension of a, $d\,a$ and integrate over the distance $P_1 - P_2$ or between limits a_1, a_2 we have

$$F_{P_1-P_2} = \frac{i_1\, i_2}{d}\left[\int \frac{a\,d\,a}{\sqrt{a^2+d^2}}\right.$$
$$\left. + \int \frac{(l-a)\,d\,a}{\sqrt{d^2+(l-a)^2}}\right]_{a_1}^{a_2}$$

$$\int \frac{a\,d\,a}{\sqrt{a^2+d^2}} = \sqrt{a^2+d^2} = l\sqrt{K^2+n^2}$$

$$\int \frac{(l-a)\,d\,a}{\sqrt{d^2+(l-a)^2}} = -\sqrt{d^2+(l-a)^2}$$
$$= -\,l\sqrt{n^2+(1-K)^2}$$

from which if $a_1/l = K_1$, $a_2/l = K_2$.

$$F_{P_1-P_2} = \frac{i_1\, i_2}{n}\left[\sqrt{K^2+n^2} - \sqrt{(1-K)^2+n^2}\right]_{k_1}^{k_2}$$

dynes (**2**)

The moment of $F_{P_1-P_2}$ about O is then

$$M_{0/P_1-P_2} = \frac{i_1\, i_2}{d}\left[\int \frac{a^2\,d\,a}{\sqrt{a^2+d^2}}\right.$$
$$\left. + \int \frac{a\,(l-a)\,d\,a}{\sqrt{d^2+(l-a)^2}}\right]_{a_1}^{a_2}$$

$$\frac{1}{d}\int\frac{a^2\,d\,a}{\sqrt{a^2+d^2}}=\frac{1}{d}\cdot\Big[\frac{a}{2}\cdot\sqrt{a^2+d^2}-\frac{d^2}{2}\log_e\Big(a+\sqrt{a^2+d^2}\Big)\Big]$$

$$=\Big(\frac{l}{2}\Big)\cdot\Big[\frac{K}{n}\cdot\sqrt{K^2+n^2}-n\log_e(K+\sqrt{K^2+n^2})-n\log_e l\Big]$$

$$\frac{1}{d}\cdot\int\frac{a\,(l-a)\,d\,a}{\sqrt{d^2+(l-a)^2}}=\frac{1}{d}\cdot\Big[\frac{l-a}{2}\sqrt{d^2+(l-a)^2}-\frac{d^2}{2}\log_e\Big\{\Big(l-a\Big)+\sqrt{d^2+(l-a)^2}\Big\}\Big]$$

and

$$M_{0/P_1-P_2}=i_1\,i_2\,.\,\frac{l}{2}\cdot\Big[\frac{K}{n}\sqrt{K^2+n^2}-\frac{1+K}{n}\sqrt{(1-K)^2+n^2}-2.303\,\{\log_{10}(K+\sqrt{K^2+n^2})+\log_{10}(\,(1-K)+\sqrt{(1-K)^2+n^2})\}\Big]_{k_1}^{k_2}$$

dynes cm. (3)

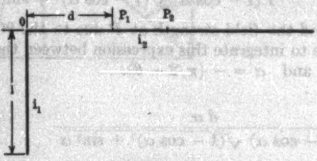


Fig. R

Here $a = 0$ and from the fundamental formula we get

$$F_{P_1}=\frac{i_1\,i_2}{d}\cdot\frac{l}{\sqrt{d^2+l^2}}$$

or if $d/l = n$.

$$F_{P_1}=\frac{i_1\,i_2}{d}\Big[\frac{1}{\sqrt{n^2+1}}\Big]\text{ dynes per cm.}\qquad(4)$$

For the force on $P_1 - P_2$ we have

$$F_{P_1-P_2}=\int_{d_1}^{d_2}\frac{i_1\,i_2}{d}\cdot\frac{l\,d\,d}{\sqrt{d^2+l^2}}$$

$$=i_1\,i_2\,.\,l\int_{d_1}^{d_2}\frac{d\,d}{d\,\sqrt{d^2+l^2}}$$

$$=i_1\,i_2\Big[\log_e\frac{\sqrt{d^2+l^2}-l}{d}\Big]_{d_1}^{d_2}$$

$$=i_1\,i_2\Big[\log_e\Big\{\frac{1}{n}\cdot\Big(\sqrt{n^2+1}-1\Big)\Big\}\Big]_{n_1}^{n_2}$$

or

$$F_{P_1-P_2}=i_1\,i_2\,.\,2.303\,[\log_{10}(\sqrt{n^2+1}-1)-\log_{10}n]_{n_1}^{n_2}\text{ dynes}\qquad(5)$$

For the moment of $F_{P_1-P_2}$ about O we have

$$M_{0/P_1-P_2}=\int_{d_1}^{d_2}i_1\,i_2\frac{l\,d\,d}{\sqrt{d^2+l^2}}$$

$$=i_1\,i_2\,[l\,.\,\log_e(d+\sqrt{d^2+l^2})\,]_{d_1}^{d_2}$$

$$=i_1\,i_2\,.\,l\,[\log_e l+\log_e(n+\sqrt{n^2+1})\,]_{n_1}^{n_2}$$

l is the unit used for measuring n, thus $\log_e l = 0$ and thus

$$M_{0/P_1-P_2}=i_1\,i_2\,.\,l\,[2.303\log_{10}(n+\sqrt{n^2+1})\,]_{n_1}^{n_2}$$

dynes cm. (6)

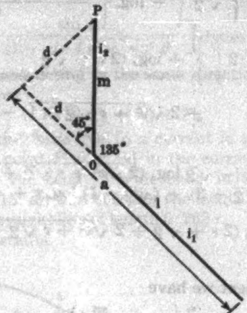


Fig. S

Here we have

$a = l + d$

$d = m/\sqrt{2}$

if $m/l = v$

then $K = \dfrac{\sqrt{2}+v}{\sqrt{2}}$

and $n = v/\sqrt{2}$

from formula (1)

$$F_P=\frac{i_1\,i_2\sqrt{2}}{m}\left\{\frac{\dfrac{\sqrt{2}+v}{\sqrt{2}}}{\sqrt{\left(\dfrac{\sqrt{2}+v}{\sqrt{2}}\right)^2+(v/\sqrt{2})^2}}+\frac{1-\dfrac{\sqrt{2}+v}{\sqrt{2}}}{\sqrt{\left(1-\dfrac{\sqrt{2}+v}{\sqrt{2}}\right)^2+(v/\sqrt{2})^2}}\right\}$$

$$=\frac{i_1\,i_2}{m}\left\{\frac{\sqrt{2}+v}{\sqrt{v^2+v\sqrt{2}+1}}-1\right\}$$

or

$$F_P=\frac{i_1\,i_2}{m}\left\{\frac{1.414+v}{\sqrt{v^2+1.414\,v+1}}-1\right\}$$

dynes per cm. (8)

For the force on a length $P_1 P_2$ we have

$$F_{P_1-P_2} = \int_{v_1}^{v_2} \frac{i_1 i_2}{l \,.\, v} \left\{ \frac{\sqrt{2} + v}{\sqrt{v^2 + v\sqrt{2} + 1}} - 1 \right\} l\, d v$$

$$= i_1 i_2 \left\{ \sqrt{2} \int_{v_1}^{v_2} \frac{d v}{v \sqrt{v^2 + v\sqrt{2} + 1}} \right.$$

$$\left. + \int_{v_1}^{v_2} \frac{d v}{\sqrt{v^2 + v\sqrt{2} + 1}} - \int_{v_1}^{v_2} \frac{d v}{v} \right\}$$

$$F_{P_1-P_2} = i_1 i_2 \left[\sqrt{2} \left\{ - \log_e \left(\frac{\sqrt{v^2 + v\sqrt{2} + 1} + 1}{v} \right.\right.\right.$$

$$\left.\left. + \sqrt{2}/2 \right) \right\} + \log_e (2 v + \sqrt{2}$$

$$\left. + 2 \sqrt{v^2 + v\sqrt{2} + 1}) - \log_e v \right]_{v_1}^{v_2}$$

or

$$F_{P_1-P_2} = i_1 i_2 [- \sqrt{2} \log_e (2 \sqrt{v^2 + v\sqrt{2} + 1}$$
$$+ v\sqrt{2} + 2) + (\sqrt{2} - 1) \log_e v + \sqrt{2} \log_e 2$$
$$+ \log_e (2 v + \sqrt{2} + 2 \sqrt{v^2 + v\sqrt{2} + 1})]_{v_1}^{v_2}$$
$$\text{dynes} \quad (9)$$

For the moment we have

$$M_{0/P_1-P_2} = i_1 i_2 \int_{v_1}^{v_2} \left(\frac{\sqrt{2} + v}{\sqrt{v^2 + v\sqrt{2} + 1}} - 1 \right) l\, d v$$

$$= i_1 i_2 \left[l \int_{v_1}^{v_2} \frac{\sqrt{2}\, d v}{\sqrt{v^2 + v\sqrt{2} + 1}} \right.$$

$$\left. + l \int_{v_1}^{v_2} \frac{v\, d v}{\sqrt{v^2 + v\sqrt{2} + 1}} - l \int_{v_1}^{v_2} d v \right]$$

$$M_{0/P_1-P_2} = i_1 i_2 \,.\, l [\sqrt{v^2 + v\sqrt{2} + 1} - v$$
$$+ \frac{\sqrt{2}}{2} \,.\, \log_e (2 v + \sqrt{2}$$
$$+ 2 \sqrt{v^2 + v\sqrt{2} + 1})]_{v_1}^{v_2} \text{ dynes cm.} \quad (10)$$

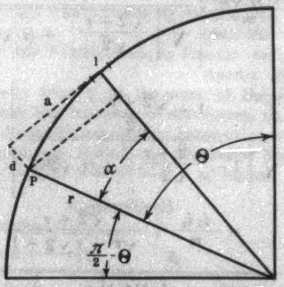


FIG. T

Consider a small l of a circular current of radius r and of unit strength. The field strength at P due to l is then

$$H = 1/d \left[\frac{a}{\sqrt{a^2 + d^2}} - \frac{a - l}{\sqrt{d^2 + (l - a)^2}} \right]$$

if l is an element of the current

$$l = r\, d\alpha$$

and

$$a = r \sin \alpha$$
$$d = r\,(1 - \cos \alpha)$$

Substituting

$$H = \frac{1}{r\,(1 - \cos \alpha)} \left\{ \frac{r \sin \alpha}{\sqrt{r^2 (1 - \cos \alpha)^2 + r^2 \sin^2 \alpha}} \right.$$
$$- \frac{r \sin \alpha}{\sqrt{r^2 (1 - \cos \alpha)^2 + (r\, d\alpha - r \sin \alpha)^2}}$$
$$\left. + \frac{r\, d\alpha}{\sqrt{r^2 (1 - \cos \alpha)^2 + (r\, d\alpha - r \sin \alpha)^2}} \right\}$$

$r\, d\alpha = 0$ compared with $r \sin \alpha$ and the first two terms under the brackets become equal thus:

$$d H = \frac{d\alpha}{r\,(1 - \cos \alpha) \sqrt{(1 - \cos \alpha)^2 + \sin^2 \alpha}}$$

To find the field strength at P due to the 90° bend we have to integrate this expression between the limits $\alpha = \theta$ and $\alpha = -(\pi/2 - \theta)$.
It is:

$$\frac{d\alpha}{r\,(1 - \cos \alpha) \sqrt{(1 - \cos \alpha)^2 + \sin^2 \alpha}}$$
$$= \frac{d\alpha}{r \sqrt{2}\,(1 - \cos \alpha) \sqrt{1 - \cos \alpha}}$$
$$= \frac{1}{r\sqrt{2}} \cdot \frac{d\alpha}{2 \sin^2 1/2\,\alpha \,.\, \sin 1/2\,\alpha .\, \sqrt{2}}$$
$$= \frac{1}{4 r} \cdot \frac{d\alpha}{\sin^3 1/2\,\alpha} = \frac{1}{2 r} \cdot \frac{d\,(1/2\,\alpha)}{\sin^3 1/2\,\alpha}$$

Thus

$$H_{P_c} = \frac{1}{2 r} \int_{\alpha = -(\pi/2 - \theta)}^{\alpha = \theta} \frac{d\,(1/2\,\alpha)}{\sin^3 1/2\,\alpha}$$

or

$$H_{P_c} = \frac{1}{4 r} \left\{ - \frac{\cot 1/2\,\alpha}{\sin 1/2\,\alpha} + \log_e (\operatorname{cosec} 1/2\,\alpha \right.$$
$$\left. - \cot 1/2\,\alpha) \right\} \Big]_{\alpha = -(\pi/2 - \theta)}^{\alpha = \theta}$$

This can be written

$$H_{P_c} = \frac{1}{4 r} \left\{ - \frac{\sin \alpha\, \sqrt{2}}{(1 - \cos \alpha) \sqrt{1 - \cos \alpha}} \right.$$
$$\left. + \log_e \left(\frac{\sqrt{2} \,.\, \sqrt{1 - \cos \alpha} - \sin \alpha}{(1 - \cos \alpha)} \right) \right\} \Big]_{\alpha = -(\pi/2 - \theta)}^{\alpha = \theta}$$

Inserting the limits and remembering that cosec 1/2 α

$-\cot 1/2\,\alpha = \tan \alpha/4$, we find for the force on a point *P* of a circular current of strength *i*, making an angle θ from one end of a 90° circular bend and due to the current in the Bend.

$$F_e = \frac{i^2}{4\,r}\left[\frac{\sin\theta\sqrt{2}}{(1-\cos\theta)\sqrt{1-\cos\theta}} + \frac{\cos\theta\sqrt{2}}{(1-\sin\theta)\sqrt{1-\sin\theta}} - 2.303\log_{10}\tan\theta/4 + 2.303\log_{10}\left\{\frac{\sqrt{2}\,.\,\sqrt{1-\sin\theta}+\cos\theta}{1-\sin\theta}\right\}\right] \text{ dynes per cm.} \quad (11)$$

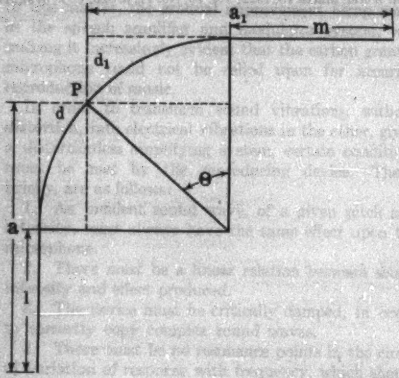


Fig. U

To find the force due to a tangent current *l* on a point *P* of the 90° circular bend.

$$F_{Pl} = \frac{i^2}{d}\cdot\left\{\frac{a}{\sqrt{a^2+d^2}} - \frac{a-l}{\sqrt{d^2+(l-a)^2}}\right\}$$

and we have

$$a = l + r\cos\theta$$
$$d = r\,(1-\sin\theta)$$

Substituting

$$F_l = \frac{i^2}{r\,(1-\sin\theta)}\left\{\frac{l+r\cos\theta}{\sqrt{r^2(1-\sin\theta)^2+(l+r\cos\theta)^2}} - \frac{r\cos\theta}{\sqrt{r^2(1-\sin\theta)^2+r^2\cos^2\theta}}\right\}$$

Writing $l/r = p$ we get

$$F_l = \frac{i^2}{r\,(1-\sin\theta)}\left[\frac{p+\cos\theta}{\sqrt{(1-\sin\theta)^2+(p+\cos\theta)^2}} - \frac{\cos\theta}{\sqrt{(1-\sin\theta)^2+\cos^2\theta}}\right] \text{ dynes per cm.} \quad (12)$$

Writing $m/r = q$, we get

$$F_m = \frac{i^2}{r\,(1-\cos\theta)}\left[\frac{q+\sin\theta}{\sqrt{(1-\cos\theta)^2+(q+\sin\theta)^2}} - \frac{\sin\theta}{\sqrt{(1-\cos\theta)^2+\sin^2\theta}}\right] \text{ dynes per cm.} \quad (13)$$

All these forces acting in the same direction, we get for the total force

$$F_P = F_e + F_l + F_m \quad (14)$$

To find the force due to a current in a 90° circular bend on a point *P* situated in the current tangent to the end of the bend: Divide bend in 6 equal parts containing 15° arc and consider the length *l* of each part as being a straight line of length 0.262 *r*. We have then for the constants

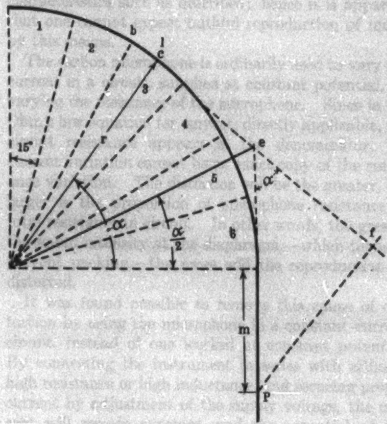


Fig. V

$a = b\,c + c\,e + e\,f$

or

$a = 0.131\,r + r\tan 1/2\,\alpha + (m + r\tan 1/2\,\alpha)\cos\alpha$

writing $m/r = q$

$a = r\,\{\tan 1/2\,\alpha\,(1+\cos\alpha) + 0.131 + q\cos\alpha\}$

$a = r\,[E + q\cos\alpha]$

$d = f\,P = (m + r\tan 1/2\,\alpha)\sin\alpha$

$\quad = r\,\{\tan 1/2\,\alpha\sin\alpha + q\sin\alpha\}$

$\quad = r\,\{\,(1-\cos\alpha) + q\sin\alpha\}$

or

$d = r\,[H + q\sin\alpha]$

The forces can now be calculated as shown and their sum will be the total force

$$F_r = \sum_1^6 F \qquad (15)$$

Bibliography

Theory and Calculation of Electric Circuits, First Ed., Chap. VI, p. 91, C. P. Steinmetz.

Electricity and Magnetism, Vol. II, Third Ed. 1892, Art. 475, page 138, Art. 502, p. 158, Art. 686, p. 318, Clark Maxwell.

Forces Acting on Disconnecting Switches, *Electrical World*, March 17, 1917, p. 504, J. Loebenstein.

The Force on Repulsion in Disconnecting Switches, *Electrical Review* (American), Oct. 20, 1917, p. 684, H. B. Dwight.

Calculation of Magnetic Force on Disconnecting Switches, presented at 36th Annual Convention A. I. E. E., June 29, 1920, H. B. Dwight.

Mathematical Analysis of the Mechanical Forces on Switches and Busbars, *The Electrician* (London), Dec. 3, 1920, p. 646, Dec. 10, 1920, p. 676, L. B. W. Jolley.

Nature and Explanation of the "Motor Effect" in the Ajax Wyatt Furnace, (Refers also to distribution of forces along conductors), *Journal* of Franklin Institute, Dec. 20, 1920, Vol. 190, No. 6, p. 817, E. F. Northrup.

The Self and Mutual Inductances of Linear Conductors, *Bulletin* of the Bureau of Standards, Vol. 4, No. 2, 1907-8, E. B. Rosa.

The Magnetic Electron, *Journal* of the Franklin Institute, Aug. 1921, Vol. 192, No. 2, p. 145, A. H. Compton.

The Magnetron, JOURNAL A. I. E. E., Sept. 1921, Vol. XL, No. 9, p. 715, A. W. Hull.

Current Distribution in Multi-Conductor Single Phase Busses, *El. World*, March 18, 1922, Vol. 79, No. 11, p. 526, C. F. Wagner.

A Diaphragmless Microphone for Radio Broadcasting

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THE broadcasting of regular programs of entertainment and instruction was begun in the Fall of 1920. It was not long before the service became quite generally used, although at first this was due to its novelty. Judged by present standards, the character of the programs rendered, and the quality of modulation attained, was susceptible to great improvement; and the development of better apparatus and methods of modulation has gone hand in hand with the broadening of the scope of entertainment offered.

At the time when the writer's attention was directed to this matter, the gradual improvement being made in the speech amplifier and modulator system, was making it increasingly evident that the carbon granule microphone could not be relied upon for accurate reproduction of music.

In order to transform sound vibrations, without distortion, into electrical vibrations in the ether, given a distortionless amplifying system, certain conditions must be met by the reproducing device. These, briefly, are as follows:

1. An incident sound wave, of a given pitch and intensity, must always have the same effect upon the microphone.

2. There must be a linear relation between sound intensity and effect produced.

3. The device must be critically damped, in order to correctly copy complex sound waves.

4. There must be no resonance points in the curve of variation of response with frequency, which should preferably be a straight line parallel to the frequency axis.

5. It is very desirable that this curve of response against frequency, be to some extent adjustable at the microphone. This is desirable in order to be able to correct for attenuation in overhead or cable lines from the signal source to the broadcasting station.

The carbon granule microphone depends for its action upon the variation of contact resistance between numbers of carbon particles. This variation is caused by movement of a diaphragm, actuated in turn by the incoming sound waves. When used for voice transmission in the usual way, an average intensity of reproduction will be maintained over indefinite periods because the speaker talks directly into the mouthpiece, causing a considerable agitation of the granules. In reproducing music comprising more than one tone, or one instrument, it is not feasible to direct the sound waves into the mouthpiece, and a much diminished sound intensity must be employed. This results in a gradual loss of sensitivity of the microphone,—the familiar "packing",—so that the first of the above conditions cannot be met. Conditions (2) and (3) are complied with, approximately, over fairly wide limits. The use of a loaded diaphragm, however, means that there will be a fundamental or natural period and its overtones, due to the instrument itself. At these frequencies the response will, by mechanical resonance, be much exaggerated. Usually this fundamental frequency is purposely placed at or near eight hundred cycles, the figure adopted as the average speech frequency. This means that below the natural frequency, the response increases with frequency, rising to a peak as resonance is reached, and falling off rapidly at higher frequencies. The timbre of the various musical instruments is due to the frequency and intensity of the various harmonics present. These occur mainly in the range above eight hundred or one thousand cycles,—the range through which the sensitivity of the carbon microphone is low and decreasing. All such microphones, to a greater or lesser extent, have undesirable characteristics such as described; hence it is apparent that one cannot expect faithful reproduction of music by this means.

The carbon microphone is ordinarily used to vary the current in a circuit, supplied at constant potential, by varying the resistance of the microphone. Since in the Ohm's law equation for current, directly applicable, the circuit resistance appears in the denominator, the current variation cannot be an exact copy of the resistance variation. The distortion will be the greater, the larger is the proportion of microphone resistance to total resistance in circuit. In other words, the greater the sound intensity at the diaphragm,—which tends to prevent packing,—the more will the reproduction be distorted.

It was found possible to remove this cause of distortion by using the microphone in a constant current circuit, instead of one worked at constant potential. By connecting the instrument in series with either a high resistance or high inductance, and securing proper current by adjustment of the supply voltage, the current will remain constant, and the potential at the microphone terminals will be directly proportional to the resistance, instead of inversely. By suitable amplification of this voltage variation, much better quality can be obtained; however, the resonance characteristics and the unsteady performance cannot be improved in this way.

Although it was found possible to correct, in some measure, the tendency to resonance, it was felt that such devices are by their very nature unsuited to the purpose in hand.

The first alternative taken up was an electrodynamic

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 14-17, 1923.

or moving coil transmitter, whose coil was connected to the center of a diaphragm clamped at its edge, which moved the coil in a strong magnetic field. If connected in a circuit of large inductance and low resistance, the movement of the diaphragm will cause alternating currents to flow in the coil circuit. These currents will be, for sound waves of constant amplitude, independent of the sound frequency. Proper corrugation of the diaphragm will annul any harmonic resonance, leaving the fundamental period alone to be corrected for. Hence the response curve will be of the same shape as an electrical resonance curve, and may be corrected in the amplifying network, by the use of suitable filters.

It was shown experimentally that such a transmitter could be produced. The compensation was satisfactory in the lower and intermediate audio ranges, but at the higher frequencies mechanical hysteretic effects in the diaphragm became prominent, and the resonance characteristics of the moving coil and connection system came into play, so that the results could not be considered satisfactory.

At about this time, advantage was taken of the condenser transmitter, developed primarily for use in measurements of sound intensity.[1] This instrument left, theoretically, little to be desired, since it had been shown[2] to be possible to construct it with critical damping and with a response characteristic practically independent of frequency. However, it developed after several months' use in broadcasting, that the problem of maintaining its insulation resistance high and constant, to avoid loss of sensitivity and introduction of ground tone, is difficult of solution. There is also a marked tendency for the diaphragm, originally tightly stretched, to sag and lower its natural frequency until it comes well within the audio range, which defeats the primary object of the construction. The high order of impedances involved, as well as the considerable amount of amplification required, make the instrument difficult to maintain in satisfactory operating condition. Even with these defects, however, the condenser transmitter represents practically the best solution of the problem of sound reproduction by use of a diaphragm. It thus became apparent that the production of a diaphragmless microphone would be a distinct step forward.

There are several ways in which sound waves may be electrically reproduced without the use of a diaphragm. In many cases, however, while fundamental frequencies and resonant harmonics are avoided, the mechanism employed is not free from inertia. This is true, for example, in the case of the inverted Goldschmidt thermophone, in which the sound waves vary the temperature and hence the resistance of a heated wire or strip. The sensitivity may be made independent of frequency, in such cases, by damping the lower ranges, but its value will then be no greater than the undamped value at the highest frequency used. The required amplification thus becomes so large that the ratio of tube noise to signal is difficult to keep small enough for quiet operation.

The variation of the impedance of an ionic stream, as the pressure is varied by the incident sound waves, offers great promise, especially if so-called cold ionization can be used. In this case there will be practically no energy loss, no heat developed, and hence no inertia at the instrument. Calculations, however, of the space current to be expected in air at atmospheric pressure, with radium or emanation as source, showed that the impedances to be expected would be of even higher order than those in the condenser transmitter circuits. Experimental tests showed the same to be true of ionization currents due to ultraviolet light.

An operative microphone was made by using the ionization from a Nernst glower. The response curve fell off very badly as the frequency was raised. It was found to be impossible to eliminate the hum caused by the commutator of the dynamo furnishing the current to heat the Nernst glower. Following this lead, it was found that the response was due in the main to change of temperature of the glower, and not to direct change of impedance due to the sound wave. This explained the mass or inertia effect found to be present.

At this time the writer's attention was brought to the relatively great change in potential across a so-called "glow-discharge," at reduced pressures, which is known to occur upon altering the length of discharge path. It was suggested that something of the same effect might be present in air at atmospheric pressure. A calculation showed that this effect would afford ample sensitivity at reasonable impedance, were it to be even one hundredth as great in open air as at the pressures employed in the published work. Tentative tests were made along this line, which seemed to show that the sensitivity in open air would not be sufficient for the purpose; also it was necessary to use a diaphragm. The writer, however, was able to show that the discharge impedance could be varied directly, without the intermediary diaphragm, by pressure variation from sound waves reaching the discharge path.

The direct-current glow discharge, at low pressure, is a fairly well known form of ionization conduction, little has been published, however, on its characteristics in open air. Since the new microphone makes use of such a discharge as its variable impedance, a brief description of the phenomena is thought to be desirable.

The application of moderately high direct potential between two electrodes separated a short distance in air, with enough series resistance to prevent formation of the usual type of heavy current arc, will cause the establishment of a peculiar low current, high voltage discharge having a characteristic glowing appearance, from which is derived the name, "glow-discharge."

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[1] E. C. Wente, *Phys. Rev.*, II, Vol. 10, p. 39, 1917.

[2] I. B. Crandall, *Phys. Rev.*, II, Vol. 11, p. 449, 1918

The order of current is from one to twenty milliamperes or more, at voltages ranging from two or three hundred to one thousand volts.

Such a discharge, when produced between electrodes of certain metals, of which copper is one of the best, is remarkably quiet and steady to the unaided ear and eye. The discharge path is similar to that produced at low pressure, except that certain portions are of much less length. There are eight portions, of which but four are of importance in the open air. Fig. 1, schematic, shows these portions, which comprise, as we go from the anode towards the cathode; the anode glow, the anode column, the Faraday dark space, and the cathode glow. Between the cathode glow and the negative electrode there is a second dark space, called the Crookes' or cathode dark space. The potential drop across this space is quite large, but its length in open air is very short.

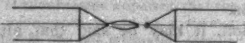

FIG. 1—ENLARGED (SCHEMATIC) APPEARANCE OF GLOW DISCHARGE

If such a discharge be connected as shown in Fig. 2, it is found that the incidence of sound waves at the gap will produce alternating potentials of equivalent frequencies across the coupling condenser *C*, whose function is to block off the direct potential drop from the grid of the amplifier tube *A*. The rectifier tube shown in the discharge circuit serves to maintain the current at a constant value as the discharge impedance changes with varying pressure. The sensitivity is surprisingly large; an amplification of ten to one will give loud signals in a head set. The discharge impedance is of the order of five hundred thousand ohms, which is low enough to cause no trouble in maintaining insulation resistance, or in adapting the device for use with standard vacuum tubes.

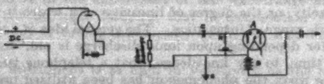

FIG. 2—DIAGRAM OF CONNECTIONS, CONSTANT CURRENT GLOW DISCHARGE, SHOWING FIRST STEP OF AMPLIFICATION

Equipment of this nature was tried out in formal broadcasting from station *KDKA*, in the spring of 1922. The discharge gap or holder used is shown diagrammatically in Fig. 3. The following defects were found to be present:

1. The curve of sensitivity as function of frequency was not flat, but fell off considerably at the higher frequencies.

2. There was considerable "ground tone,"—rumbling, hissing and popping noises not connected with the sounds being copied. These noises were not prominent enough to interfere with the modulation, but were quite disagreeable during quiet intervals. At times they were strong enough to cause overmodulation of the system.

3. As was to be expected, there was extreme sensitiveness to drafts or air currents, because no draft shields had been attached to the discharge holder. The sudden opening of a door into the studio would cause enormous variations in gap voltage. The blocking condenser *C* of Fig. 2 thus became overcharged, the potential on the first amplifier grid was made positive, and the sensitivity to sound was entirely lost until the condenser could discharge through the grid leak.

4. There was a certain amount of radio frequency regeneration.

These defects were sufficiently pronounced to make the device unsuitable for use in broadcasting. The advantages afforded, namely simplicity, durability, low impedance, and high sensitivity, in addition to elimination of the diaphragm, were so great that it was decided to see what could be done in the way of improvement. The matter of unequal response with varying frequency was taken up first, being the most serious defect. To avoid loss of sensitivity, it was desirable if possible to reinforce the weak portions of the curve, rather than to cut down the stronger portions. Unsuccessful attempts were made to do this by the use of various designs of single and double amplifying horns. It finally became necessary to make a study of the sound response of the different portions of the discharge, to find out what if any portions would give proper response. This was accomplished by the use of a perforated exploring electrode, through which the discharge was allowed to pass. As shown in Fig. 4, connection to the amplifier grid could be made from the exploring electrode and either terminal of the discharge. The most sensitive portion was found to be the positive column. Its sensitivity was so great that when using the whole gap, as in Fig. 2, but little was contributed by the other parts of the discharge. The sensitivity of the portion from the end of the positive column to the cathode terminal was found to be much less than that of the positive column, and to be independent, through wide limits, of frequency. While more amplification

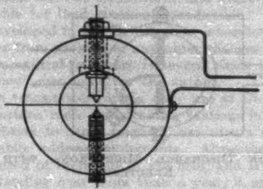

FIG. 3—FIRST GLOW DISCHARGE MICROPHONE

is required when the positive column effect is eliminated, the effect of frequency can be made negligible.

It will now be apparent that by suitably proportioning the discharge gap, and properly placing the exploring electrode, we can control, to a great extent, the shape of the frequency response curve. It should be noticed here that as the separation of the main electrodes is increased, nothing in the character of the discharge is altered, except the length of the positive column and the voltage drop across this column. The positive column gradient is roughly fifteen hundred volts per centimeter of length. If a short gap is used, with the exploring electrode close to the cathode terminal, the response will be obtained mainly from the cathode and will be practically flat. If the gap is lengthened, without altering the distance from cathode to exploring electrode, the differential action of the cathode portions and the positive column, on the voltage change at the exploring electrode, will result in partial suppression of the response on the lower audio register. An upward sloping curve will result. As the exploring electrode is moved nearer to the anode end, the positive column characteristic will become gradually more dominant, until at the limit, when the whole gap is used as pick-up, the characteristic will have a pronounced downward slope. The use of this microphone, therefore, should enable one to use a distortionless amplifier network and to control or modify the character of the modulation at will, by simple adjustments at the microphone itself.

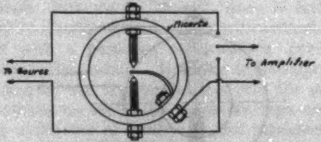

Fig. 4—Glow Discharge Microphone with Exploring Electrode

It has been found that the bad effects of drafts can be avoided without essentially altering the response to sound waves. This was accomplished by the use of properly designed draft shields placed across either end of the short tube containing the glow discharge terminals. The air current resistance of the two shields was made quite different, so that as now used, the instrument is sensitive mainly at one end only. It was found to be undesirable to completely close one end, as this resulted in diminished sensitivity as well as resonance at the natural frequency of the enclosure.

While the ratio of ground tone to signal was greatly reduced by the use of the exploring electrode described above, it was found necessary to determine the source of the noises. There resulted the development of an alloy of low heat conductivity and melting point, to be used for the electrodes. By the proper proportioning of diameter and length of electrodes, and the use of discharge currents of proper value, it proved possible to operate the device continuously over long periods of time, without objectionable ground tone. Furthermore, while the electrodes eventually fail, this occurs after about the same length of service in all cases. It is then a simple matter to design the microphone holders and units so as to be readily replaceable, and to renew them long before they become inoperative.

The development was completed by the working out of a low-current, high-voltage rectifier, with resistance-capacity filters, which permits the discharge to be struck or started by flashover, and maintains its current practically independent of discharge impedance. Units of substantially this construction have been used in the regular broadcasting programs of station *K D K A* for several months.

In conclusion, it may be said that although it is desirable and will doubtless become possible to use a microphone giving a response independent of frequency, this is not done at present. The imperfections of the existing designs of head telephones and loud speakers make it advisable to adjust the modulation, to some extent, so that the received audio signals will be more nearly faithful copies of the originals. This is readily accomplished with the new microphone, so that advantage may be taken of improvements in receiving apparatus as they appear.

Discussion

R. L. Jones: I believe we would all find ourselves in substantial agreement with Dr. Thomas in respect to the first four conditions which he sets down as those which must be met by a high quality telephonic reproducing apparatus. It is to be regretted, however, that he does not give data to show how well the glow discharge transmitter fulfills them: data as to its physical efficiency; frequency response characteristics; and its load capacity, or the degree of linearity with which it responds over a wide range of intensities. The general idea of a telephone transmitter depending upon ionic discharge is old,[1] but this is the first time, to my knowledge, that a telephone transmitter depending upon the phenomenon has been utilized practically. The world is eager for better tools of communication, and the impetus lately given by radio and public address systems stimulates interest. If the glow discharge transmitter fulfils even a part of the fundamental conditions better than other available types, it is probable, in spite of the high-voltage supply and other operating disadvantages, that it will have a certain field of use. In view of the present lack of data showing its performance under the variety of actual conditions, however, it remains for the future to reveal what place it has.

Referring to Condition 5 of the paper, the most flexible and upon the whole the most desirable system is not one where distortion in one element is compensated in another, but rather one with elements as nearly perfect in quality as possible. One of the other papers on the Convention program[2] explained the manner in which distortion in line and cable circuits can be compensated by means of network structures or attenuation equalizers associated with the lines. The possibility of designing modulators, detectors, and amplifiers having practically horizontal frequency characteristics has been demonstrated; and coming down to the terminal instruments, the trend of the development of improved transmitters and receivers is in the direction of instruments which are in themselves reproducers of high quality. In such an instrument frequency distortion is usually symptomatic of conditions which give rise to load distortion at intensities lower than it would otherwise be possible for the instrument to reproduce. This matter of load distortion is an important consideration with public address and radio systems, and one which has not received consideration generally as pointed out in the paper referred to above.

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[1] Blyth: Proceedings of the Royal Society of Edinborough, Vol. II, page 622, April 17, 1882.

[2] Use of Public Address System with Telephone Lines, by W. H. Martin and A. B. Clark.

The author of the paper has concluded that neither the condenser type nor the carbon type of telephone transmitter is suitable for such high quality pick-up work as that in connection with radio broadcasting. After relating his experience he says of the carbon transmitter, "It was felt that such devices are by their very nature unsuited to the purpose in hand." And again, after relating defects experienced with a model of the condenser type, he says, "It thus became apparent that the production of a diaphragmless microphone would be a distinct step forward." From these remarks one not intimately familiar with the communication art might fairly conclude that the type of telephone transmitter which employs a diaphragm is in a state of decline and that, with the growing requirements for high quality, the diaphragm type of instrument is moribund. In view of this it seems advisable to supplement the paper by giving some information regarding the characteristics, and the extent of use of two high quality transmitters employing diaphragms, which have been developed recently. These transmitters have had considerable use mainly in connection with high quality public address systems and with radio broadcasting work in the Bell System.

Transmitters of these two types are described in another of the Convention papers presented yesterday.[3] The condenser transmitter consists essentially of a thin steel diaphragm which is stretched and spaced at a distance of 1/1000 of an inch from a rigid plate which constitutes the second electrode. The carbon transmitter similarly employs a stretched steel diaphragm, and has a push-pull construction with two granular carbon resistance elements. It would be desirable to tune both instruments in such a way that their resonance frequencies were above the voice range. In the condenser transmitter this is actually accomplished. In the carbon transmitter, the region of resonance is designed to be in the upper part of the voice range and is so flattened out by high damping that in conjunction with the high frequency of tuning satisfactory quality of reproduction is obtained.

Neither instrument is as high in sensitivity as the ordinary telephone transmitter, and with the condenser instrument a vacuum tube amplifier is always associated as a part of the transmitting set. The quality of the latter instrument is indicated by the fact that the range of variation of its response over a region from 200 to 6000 cycles is less than three miles.[4] In the case of the carbon transmitter, the range of variation is about twelve miles. From other investigations it is known that the range of variation of ear sensitivity of normal persons frequently varies over ten miles in this frequency range, so that a reproducing system, the variations of which do not exceed this limit may be considered entirely satisfactory for high quality work.

The author mentions difficulty with maintaining insulation resistance of the condenser transmitter and a tendency for the diaphragm to sag. Due to construction of the instrument a sag even as small as one mil would cause complete short circuit and the instrument would become inoperative. In view of this fact it seems probable that the author refers to the diaphragm giving somewhat, and thereby lowering its natural period. We have had no such experience in the Bell System laboratory. Instruments have been found with their normal tuning and otherwise in good condition after approximately four years of rather hard laboratory service. In addition to a substantial number of instruments in use for various laboratory purposes, there are three of the leading radio broadcasting stations employing condenser transmitters regularly, and some of these instruments have been in service for nearly a year. While it is true that the insulation of the instrument will vary somewhat, if proper precautions are taken no more difficulty should be experienced from this source than in the case of other elements common in circuits characterized by weak currents and high amplification. It is our experience that there is less difficulty due to the insulation of the transmitter than to that of the wiring of the amplifiers used in systems such as are under discussion. Under careful handling the life of such an instrument should be as great as that of ordinary telephone apparatus. Because of the very satisfactory quality of the instrument and its great constancy, a transmitter of this type is now being favorably considered for use as a reference instrument in laboratory measurements of telephone transmission.

From the author's reference to a fundamental frequency near 800 cycles for the carbon transmitter, it is thought that he has the common form of subscribers' telephone transmitter in mind. This transmitter, which is satisfactory for commercial telephone service where speech alone is transmitted, and where hand receivers are employed, is not suitable for high quality pick-up work, where music must be transmitted, and where transmission must be suitable for loud speaker reproduction. The carbon transmitter developed in the Bell System laboratories for such purposes is free from packing or any other changes of efficiency with time. Its natural frequency is several times the "average speech frequency" mentioned by the author.

Such carbon transmitters are in daily use at about thirty of the most important radio broadcasting stations in the United States, in addition to other frequent use with a considerable number of public address equipments. Altogether there are over two hundred of them out in the field, and after practical experience extending over more than a year, it can be stated that they have made a very good record for themselves, both as to the quality of their reproduction and as to their general serviceability.

G. D. Robinson: Mr. Thomas has undoubtedly developed a valuable tool. Although his prime object has undoubtedly been the reproduction of sound frequencies lying within the ordinary range, it appears that this tool might well find use in physics or elsewhere, where the frequency is outside of the ordinary sound range. I would be obliged if Mr. Thomas would tell us anything that he knows about the entreme limiting values of frequency at which this device may be applied.

Phillips Thomas: I have to thank Mr. Jones for his very valuable contribution to the discussion. He has brought up many points, on most of which little was said in the paper.

In regard to condition 5, we are I think all in substantial agreement with Mr. Jones, that ultimately we should strive for constructive improvement all down the line, rather than correct, in the final result, inaccuracies occurring at one point by the introduction of inaccuracies at other points. It is, however, a regrettable fact that the present loud speakers, without notable exception, are far from sufficiently perfect to give satisfactory reproduction of undistorted electrical currents. The day of great novelty in radio broadcasting has passed; a large and increasing proportion of radio listeners are musically educated to a surprising degree; and these people will recognize and condemn imperfect results very quickly. The fact that a great proportion of the well-known makes of loud speakers are defective in much the same way, makes it possible to correct at the source, as stated in the paper, and give music-lovers a much nearer approach to desired results than they would receive were all the apparatus to function in a distortionless manner, with the exception of loud speakers.

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[3] Public Address Systems, by I. W. Green and J. P. Maxfield.

[4] The term "mile' as used in this discussion refers to the attenuation of one mile of standard cable at 800 cycles, corresponding to a relative power ratio of $\epsilon^{-0.218}$

I am sorry to say that we have at present but little to present in the way of quantitative data on the performance of the glow discharge transmitter as regards variation of sensitivity with frequency or with load. Such data is being gathered as rapidly as possible, and will be presented at the earliest possible moment. We do know, however, that the load characteristic is very good; also that the sensitivity to extremely weak signals is quite as good, as determined by an audibility meter, at 20 cycles as at 4000 cycles, when proper allowance is made for the characteristics of the telephone receivers used.

I am sure that Mr. Jones will agree with me that, other things being equal, the pick-up having the least actual mass of material to be moved by the sound energy will give the best results. The results secured by the use of the condenser type transmitter, for instance, are admittedly superior, when the instrument is at its best, to anything obtainable with even the most perfect double button transmitter, although this superiority may not be evident from the response curves themselves, taken one note at a time. The ability to copy involved sound signals, such as those from a large orchestra, is much greater with the condenser type than with the double button. In fact, in the absence of any better type, the condenser has been taken as the accepted reference standard. A careful comparison between a glow discharge transmitter and a standard condenser transmitter, however, is indeed a revelation. It is surprising, to say the least, how much more natural the result sounds; how every individual instrument of a large orchestra may be as readily picked out as can be done by the ear when listening at the source. We are convinced that this is due, in part at least, to the fact that the glow transmitter is, to all intents and purposes, a point receiver, without appreciable inertia, and at all times critically damped—having no natural period, high or low.

In reply to Mr. Robinson, I am obliged to repeat the statement that we have at present no quantitative data on frequency response curves from the glow discharge transmitter. We have some evidence of a qualitative nature, however, which seems to indicate a very high upper limit to the sensitivity. Although at least so far as we know extremely high frequencies are not transmitted by any existing loud speaker, we are developing an experimental device of this kind, which has a very high upper limit; and we have not been able to find, as yet, the limiting high frequency for the glow discharge microphone, with instruments of this type. As you probably know, the rattling of a bunch of keys fails entirely to be heard through the media of a transmitter and loud speaker of the usual type. Such signals come through in a satisfactory manner, when the condenser type transmitter or the glow discharge transmitter is used in connection with a loud speaker of the experimental type referred to above.

In conclusion, I would say that while there is no intention of detracting from the known good record of the better known types of transmitters, in their field, it is felt that the glow discharge transmitter is a decided step in advance. It is possible, however, that this in turn will be outdistanced by some other form of diaphragmless pick-up which will not have the disadvantage of high operating voltage with its attendant complications.

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1923

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Thirty-ninth Annual Report, for the fiscal year ending April 30, 1923. A general balance sheet showing the condition of the Institute's finances on April 30, 1923, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year; more detailed information has been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The bi-monthly meeting policy of the Board of Directors instituted in 1921 was continued through the past year with the Executive Committee functioning in the interim.

The Board held seven meetings during the year; six of these were held in New York and one at Niagara Falls, Ontario.

Information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees and the various officers, is published each month in the section of the JOURNAL devoted to "Institute Activities."

Meetings.—The policy of holding in addition to the Annual business meeting four general meetings of the Institute each year was continued. The meetings held were as follows: Annual, Pacific Coast, Midwinter and Spring Conventions.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 19, 1922. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1922 was presented. The Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1922.

In the evening the members of the Board of Directors, past-presidents of the Institute and the president-elect met at dinner and informally discussed the affairs of the Institute.

Annual Convention.—The Thirty-eighth Annual Convention was held at Niagara Falls, Ontario on June 26 to June 30, 1922. Seven technical sessions were held, including two parallel sessions and one afternoon and one evening session. A total of thirty-four papers were presented. Among a total registration of 953 were representatives from 31 states, 7 foreign countries including 4 foreign engineering societies. The annual conferences of the Sections Committee were held on the Monday preceding the official opening of the convention, thirty-nine Sections were represented. Results of the conferences were published in pamphlet form.

Pacific Coast Convention.—The Eleventh Pacific Coast Convention was held at Vancouver, B. C. on August 8th to 11th, 1922. Seven technical sessions were held at which 18 papers and 4 addresses were presented. Attendance 235.

Midwinter Convention.—The Eleventh Midwinter Convention was held in New York on February 14-17, 1923. Seven technical sessions were held and 32 technical papers presented. One evening session was devoted to a joint New York-Chicago session with direct phone connection between the two cities with loud speakers so that papers and discussions were heard in each city simultaneously. The session concluded with a lecture on European Railway practise. Thursday evening a very successful smoker was held under the auspices of the New York Section. Friday night was assigned to the Annual Dinner-Dance. Total attendance was 1200.

Spring Convention.—The second Spring Convention of the Institute was held in Pittsburgh, Pa., April 24-26, 1923. Five technical sessions were held at which nineteen papers were presented. On one afternoon and evening all those attending were the guests of the Westinghouse Electric and Manufacturing Company at an inspection trip, banquet and entertainment. Another evening was devoted to a banquet and the last day to inspection trips.

Abstracts of the reports of the chairman of many of the Institute committees and delegations are included herein under various headings.

Meetings and Papers Committee.—This Committee has solicited and reviewed the papers submitted to the Institute for presentation and publication and has arranged, in cooperation with local convention committees, the programs for the Institute conventions held during the past year.

In 1921 there was inaugurated in the JOURNAL the use of a story of the specialist's paper designed to tell briefly of the conditions of the art, the reasons for writing the paper, the new material added by experimental and analytical work, and the general conclusions reached—all in language that is found acceptable to executives and readable by the membership at large. Only a few of these stories have been written.

Progress has been slow on account of the necessity of building up a staff of engineer-editors. There are available plenty of men of academic training who can criticise the expressions used, but they are not familiar with the technical subjects. On the other hand, there are plenty of engineers who understand thoroughly the technical work, but it is difficult to find engineers in all activities represented by fifteen technical committees who are trained and willing to devote the time and energy necessary to make the specialists' articles readable to a wider range of our membership. The conditions are not hopeless. Persistence in this effort will finally result in making all the articles by specialists comprehensible to the general reader. To make improvements, a first requisite has been to induce authors to submit their papers early enough to allow time for editorial work. For the next Midwinter Convention it now seems practicable to enforce the ruling for receipt of papers three months in advance of the Convention.

One of the most important problems before the Committee is the best method of treating mathematical papers and increasing the number of readers of such papers. The Committee, recognizing the fact that only a small proportion of the Institute membership reads mathematical papers in detail, is developing a plan which it is hoped will result in emphasizing the importance of these papers by providing for their expansion into more readable form; this plan includes the publication of abstracts, including explanations that will interpret such papers for the benefit of the non-specialist, the publication of these papers in full in pamphlet form for circulation to those especially interested and the final publication in full in the TRANSACTIONS.

These mathematical papers which are often the real basis of advances in the art of electrical engineering, but which necessarily appeal to but a few specialists, will thus always be available for purposes of record and reference. By thus omitting a portion of the mathematical papers from the JOURNAL it will be possible to devote a larger proportion of space to high-class contributions upon engineering topics of more general interest.

Publication Committee.—The two main problems before the Publication Committee this year, as in recent years, were handling the material presented for publication so that the best interests of the membership and of the Institute shall be served at the minimum outlay in publishing costs, and through conferences with chairmen of other committees, Section chairmen, and thinking members widely distributed, gathering representative views and opinions of what classes of printed matter the JOURNAL should contain.

During the year various mechanical improvements have been made in the JOURNAL which have bettered the typographical makeup to an extent that has called forth favorable comment.

There is continuous pressure from various directions urging that in the monthly issues of the JOURNAL there should be a larger amount of "semi-technical" engineering material published. Various engineering groups, "theoretical" and "practical," look to the JOURNAL as an avenue of expression. Only a limited number of pages per year can be published, and the committee is using its best endeavors in cooperation with the Meetings and Papers Committee, to meet the wishes of the membership regarding the scope of the contents of the publications, keeping in mind the maintenance of the traditional high standard both in form and substance.

Sections and Branches.—The various Sections of the Institute throughout the country continued their activities during the past year upon about the same scope as heretofore as indicated by the brief reports published each month in the JOURNAL. Numerous requests were received for information relative to the formation of new Sections, indicating a strong desire on the part of the membership, so located geographically that they can not ordinarily take part in Institute meetings, to arrange for more active participation. Two new Sections were authorized and established, as follows: Southern Virginia and Springfield, Mass.

The activity among the Student Branches has been particularly marked during the past year. Many of the colleges have held a much larger number of meetings than heretofore as shown in the following table. New Branches were authorized at the University of Denver and at Northeastern University. Requests for Student enrollment were received in such volume and from such widely varying classes and types of schools that it was found desirable to revise the By-laws governing student enrollment so as to make them more definite. This was done under date of December 8, 1922.

	For Fiscal Year Ending						
	May 1 1917	May 1 1918	May 1 1919	May 1 1920	May 1 1921	May 1 1922	May 1 1923
SECTIONS							
Number of Sections	32	34	34	36	42	45	46
Number of Section meetings held	265	245	217	262	303	373	344
Total Attendance	31,299	34,614	25,837	30,741	37,823	54,378	46,672
BRANCHES							
Number of Branches	59	59	61	62	65	67	68
Number of Branch meetings held	368	268	156	360	443	439	503
Attendance	16,107	10,683	6,441	16,827	21,629	25,358	26,893

Standards Committee.—By action of the Board of Directors on June 29, 1922, the Standards Committee was reorganized, the object being to create a body which would be better constituted to handle and correlate the manifold standardizing activities of the Institute. Under the new plan of organization, the executive work of the Committee has been placed in the hands of an Executive Committee now consisting of 12 members, which meets monthly. The development of standards is carried out by working committees of the Standards Committee and by representatives of the Committee on Sectional Committees and other cooperative standardizing bodies.

The Committee has undertaken a complete revision of the A. I. E. E. standards. The scope and character of this revision is described in the April JOURNAL. It is proposed that the revised standards will be in the form of a considerable number of sections, each section containing complete standards for one type of apparatus, or for one branch of the art. Between 20 and 30 of such sections are now contemplated and the number will of course, be increased from time to time.

In connection with this revision, the Standards Committee has organized a cooperative effort with other interested organizations, with a view to arriving at a single set of standards for each type of apparatus, which set of standards can be presented to the American Engineering Standards Committee for adoption as American standards. To this end a Cooperative Committee has been formed, which now consists of one representative each of the A. I. E. E., the Electric Power Club, the N. E. L. A. and the A. M. E. S. The representatives of this Committee will present such subject matter as their societies think should properly be included in the suggested American standards, and pass judgment on the resulting compilation. The Committee already have before them a section of standards for control apparatus which has been made up in connection with the revision of the Institute standards.

In addition to the general revision, the Standards Committee has 15 working committees, who have in hand a large number of specific problems. These problems include the translation of the 1922 standards into Spanish, the development of proper standards and testing specifications for high-voltage insulators, the determination of temperature rise limits for Class "B" insulation, the preparation of standards for electric welding apparatus, metallic resistance materials and lightning arresters, the standardization of graphical symbols, and numerous other problems involving modifications in and additions to the present Standards.

About 25 members of the Standards Committee are engaged in the field of cooperative standardizing work as Institute representatives and as chairmen of Institute delegations on sectional committees and other cooperative standardizing bodies. The amount of this work in which the Institute is represented is rapidly increasing, and it is the purpose of the Standards Committee to correlate the work of the Institute's representatives, and to properly relate it to the standardizing work within the Institute itself.

American Engineering Standards Committee.—The following statistical summary of the work of the American Engineering Standards Committee, upon which the Institute is represented, is taken from the Year Book of the committee for 1922:

Member-bodies—organizations or groups of organizations whose representatives form the A. E. S. C.	23
National organizations included in the Member-bodies	33
Representatives forming the Main Committee	55
Standards approved to January 1st, 1923	29
Standards up for approval by the A. E. S. C. on January 1st, 1923	45
Projects having official status (already approved, or on which work is under way)	121
Projects for which sponsorship has been accepted	76
Organizations acting as sponsors for projects	48
Trade, technical or governmental bodies cooperating through representatives on special or sectional committees	275
Individuals on sectional committees	917

A plan for increasing the income of the committee has been adopted, which provides for a new class of membership known as sustaining-members. A sustaining-member is an organization, firm or individual interested in the work of the Committee and contributing directly to its support. Sustaining-members are given a special information service on the progress of industrial standardization work in this country and abroad. A schedule of recommended subscriptions has been arranged.

The Federal Specifications Board which has been organized to unify the specifications of the various departments and independent establishments of the Government has appealed to the Committee to aid it in its work, and cooperative arrangements between the Board and the A. E. S. C. have been made by which the Government specifications are being submitted, at first informally, for criticism by industry and to determine their acceptability to industry. It is expected that in future editions many, if not most, of the specifications adopted by the Federal Specifications Board will go through the regular procedure of the A. E. S. C. in order that the industrial and Government specifications may be unified, resulting in truly national specifications recognized by industry and Government alike.

Among specifications which have already been submitted to the A. E. S. C. under the cooperative arrangement are those for dry cells, snap switches, rigid conduits, and rubber insulated wires and cables. Other electrical projects which have an official status before the Committee are: electrical fire and safety code; safety code for electrical power control; safety code for lightning protection; terminal markings for electrical apparatus; rating of electrical machinery; insulated wires and cables (other than telephone and telegraph); symbols for electrical equipment of buildings and ships; electrical properties of aluminum; electrical installations on shipboard; specification for 600-volt d-c. overhead trolley construction; electrical safety rules for bituminous coal mines.

The cordial arrangement between the Committee and the Division of Simplified Practise of the Department of Commerce has been continued during the year. The work of the Committee generally has increased very rapidly but it is expected that the new financial arrangements will make it possible to care adequately for the new projects.

U. S. National Committee of the I. E. C.—This Committee has held two general meetings and the Executive Council of the Committee has held several meetings during the year. It has not been possible for the Committee to work on proposals to the I. E. C. on the Rating of Electrical Machinery pending further progress by the A. E. S. C. Sectional Committee on that subject. To the meeting of the I. E. C. Advisory Committee on Rating held in Geneva, Switzerland, in November, recognizing the great desirability of general international discussion on the subject for the adjustment of differences of attitude and policy in regard to the basis and specification of rating which had arisen shortly after the Brussels meeting of 1920, a delegation was sent representing all the different views held in this country. The Advisory Committee was therefore able to obtain a very clear conception of the different views outstanding here, although our Committee made no proposals and assumed a neutral open minded participation in the discussions of the subject.

No restrictions were necessary in regard to the participation of the delegation in the work of the three other Advisory Committees, namely the Committees on Edison Screw Lamp Bases and Sockets, on Standard Pressures and on Graphical Symbols, and it took an active part in the work of all these committees. The outcome of deliberations on a uniform Edison socket was very satisfactory, the prospect being opened of an internationally standardized socket. Complete report of the Committee has been filed with A. I. E. E.

U. S. National Committee, International Commission on Illumination—This Committee has held during the past year two formal meetings and a number of less formal conferences.

The principal question before the Committee during the past year has been the matter of holding a plenary session of the I. C. I. in this country in 1924. It was voted at the Paris meeting of 1922 that such a meeting should be held, but the troubled conditions in Europe have somewhat clouded the prospects of success of such a meeting. However, it is still hoped that the original plan can be successfully carried out. Preliminary arrangements to that end have been quite fully discussed and some of the necessary committees have been appointed, so that when the general question is once decided, there need be no loss of time in getting to work.

In the meantime the international committees of the Commission have been put into operation, and work is in progress which will come up for discussion at the next plenary session.

Three of the delegates of the A. I. E. E. have been abroad during the year and have had useful discussions with officers of other National Committees regarding the work, with the result that in several respects decided gains have been made.

Committee on Safety Codes.—The work of the Committee on Safety Codes, which formerly related chiefly to the Electrical Codes, is gradually broadening to include various kinds of codes as they are being developed under the organization of the American Engineering Standards Committee.

The Chairman of this Committee represents the Institute on the following committees:

(a) The National Electrical Committee of the National Electrical Protective Association, which has to do chiefly with interior wiring. (b) The Sectional Committee of the A. E. S. C., for the revision of part 2 of the National Electrical Safety Code. (c) The Sectional Committee of the A. E. S. C. on Safety Code for Electrical Power Control.

The National Electrical Code is revised biennially, the latest revision becoming effective during the year 1923.

The Sectional Committee, for the revision of part 2 of the National Electrical Safety Code, has been organized into four subcommittees during the past year and the work is proceeding in an orderly way.

The Sectional Committee on Safety Code for Electric Power Control has also been organized and is gathering data for the formulation of rules relating to this subject.

The Institute has been asked to designate representatives to act for it in the formulation of safety codes for electrical work in mines and for electrically operated elevators. In these cases the Safety Codes Committee has recommended the appointment of members of the Institute who are specialists in these subjects who are not members of the Safety Codes Committee.

Research Committee.—The principal efforts of the Committee have been directed to the preparation of a comprehensive statement of the experimental problem in the field of research on electrical insulation. The results of this work are embodied in a report which will be published in the JOURNAL for June 1923.

In addition, the Research Committee has, through correspondence, endeavored to stimulate experimental work, has urged the completion of several investigations already under way.

Technical Committees.—Reports of Technical Committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the Annual Convention and printed in the JOURNAL.

Membership.—The results of the Membership Committee's efforts this year are shown in the following table:

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1922	6	558	2,097	11,602	14,263
Additions:					
Transferred		25	110		
New Members Qualified		9	125	1,850	
Reinstated			11	84	
Deductions:					
Died		5	10	48	
Resigned		2	15	206	
Transferred			20	115	
Dropped		7	34	717	
Membership, April 30, 1923	6	578	2,264	12,450	15,298

Net increase in Membership during the year 1035

Deaths.—The following deaths have occurred during the year.

Fellows: Clare F. Beams, John F. Kelly, Arthur B. Stitzer, Schuyler Skaats Wheeler, M. Yokura.

Members: Frank J. Baker, Alexander Graham Bell, Harry H. Blades, Robert E. Chetwood, William Hamilton Garfield, William A. La Due, Alexander Lodyguine, William B. Vansize, George G. Ward, Valentine C. Wynne.

Associates: George J. Blum, William J. Clark, William W. Comstock, Charles W. Davis, F. G. Dieterich, B. H. Elkins, Henry W. Elsasser, J. T. Fahy, David D. Faris, Walter G. Franz, Frank W. Frueauff, George H. Guy, Thomas S. Haddaway, T. T. Haldeman, Roscoe S. Helvie, Robert W. Hemphill, Jr., James H. Jeans, J. J. Hunter Johnston, Thomas F. Judge, Sadaharu Katsuno, P. O. Keilholtz, Thomas W. Kennedy, J. E. Kershner, Frank P. Lewis, F. N. Maddux, F. L. Marsh, John Martin, Herbert S. Miller, Merritt B. Miller, O. B. Moorhead, L. B. Newell, H. J. Norregaard, Clyde Pattee, John C. Pennie, James M. Petrie, Byrl C. Rathburn, Luke Robinson, Dwight C. Rockwood, William Russin, Walter Schmidt, H. E. Shaw, Frank Warren Smith, Ralph W. Stearns, Edward R. Tracy, Frank S. Washburn, William P. White, F. G. Wiswell, Walker Gill Wylie.

Total deaths, 63.

Board of Examiners.—The Board of Examiners during the year held ten meetings, averaging about two hours and 45 minutes each. It considered and referred to the Board of Directors a total of 4137 applications for admission or transfer to the higher grades. This is an increase of about 20% over the figures of last year and is very close to the figures of the year ending April 30, 1921, the record year in Institute growth.

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate	1934	
Not recommended	3	1937
Recommended for grade of Member	104	
Not recommended for admission to this grade	51	155
Recommended for grade of Fellow	8	
Not recommended for admission to this grade	7	15
Recommended for enrolment as Students	1874	1874

APPLICATIONS FOR TRANSFER

Recommended for grade of Member	93	
Not recommended for transfer to this grade	34	127
Recommended for grade of Fellow	20	
Not recommended for transfer to this grade	9	29
Total number of applications considered		4137
Applications reconsidered		7
Total		4144

Committee on Code of Principles of Professional Conduct.—During the year a new edition of the code as adopted by the Board of Directors in 1912 has been published, distributed, and commented upon in a number of the leading technical publications of the country.

Since the adoption of the code by the Institute in 1912, several other engineering societies have also adopted codes of conduct, including the national societies of civil and mechanical engineers; and these other societies have also established committees, or other agencies, for the administration and enforcement of such codes.

American Committee on Electrolysis.—The American Committee on Electrolysis as a whole has been inactive during the past year, but the work of the Committee is going forward gradually through the compilation by its Research Committee of data that it is believed will be valuable in the work of the main Committee.

Scholarships.—The governing bodies of Columbia University placed at the disposal of the Institute a scholarship in Electrical Engineering beginning with the academic year 1922-23 and continuing until further notice. The scholarship pays $350 toward the annual tuition, and reappointment for completion of course is conditioned upon maintenance of good standing.

Two applications have been received for this scholarship for the year 1923-24 but no award has yet been made as neither applicant has submitted the information which will enable the committee to arrive at a decision.

The announcement of the availability of the scholarship was made in the monthly Institute JOURNAL and also by means of a circular letter addressed to the professors of electrical engineering in the principal educational institutions of the country in which electrical engineering courses are given.

Institute Prizes.—At the meeting of the Board of Directors of the Institute of April 16, 1921, recommendations were approved establishing two Institute prizes to be awarded yearly to authors of worthy papers. The 1921 Transmission Prize was awarded to F. C. Baum for his paper presented at the Salt Lake City Annual Convention, June 1921. Although all Sections were notified early in the year and announcements were printed in the JOURNAL, no "First Paper Prize" was awarded for the year 1921 as no paper was offered in competition. The awards for papers presented in 1922 will probably be announced in June 1923.

North Eastern District Prize.—In December 1922 the Executive Committee of the North Eastern District established a cash prize of $25.00, and certificate, to be awarded each year to the author of the most worthy paper presented before any Section in District No. 1 during the Institute year and who has never had a paper published in the JOURNAL or TRANSACTIONS.

For complete details and conditions of award see the JOURNAL for April 1923.

International Engineering Congress in Brazil.—On the invitation of the Brazilian Embassy to the engineering societies of this country the Institute appointed eight delegates to the International Engineering Congress, held in Rio de Janeiro, Brazil, September 7 to 30, 1922. A program calling for the presentation of a number of papers was arranged and carried out. For details see August, September and November JOURNALS, 1922.

Edison Medal.—The Edison Medal for 1922 has been awarded to Dr. Robert A. Millikan of Pasadena, Cal., for his experimental work in electrical science. The presentation will be made during the Pacific Coast Convention of the Institute at Del Monte, California, October 2-5, 1923.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of Civil, Mining, Mechanical and Electrical Engineers, awarded the medal to Senator Guglielmo Marconi "for the invention of wireless telegraphy." The medal was presented at a large gathering of engineers at the Engineering Societies Building, New York on the evening of July 6, 1922.

Employment Service.—The employment service maintained for many years at Institute headquarters and during the latter part of 1918 coordinated with the similar service of the other Founder Societies, and which on January 1, 1921 was transferred to the auspices of American Engineering Council, was returned to the control of the Founder Societies under date of July 1, 1922 and is now being conducted under the supervision of a Board consisting of the secretaries of these Societies for the benefit of members only. A committee was appointed by the Societies to study the whole question of conducting and developing the service and the report has recently been prepared for consideration by the governing boards of the Societies.

In addition to a direct service, the Bureau prepares an engineering service bulletin which is published each month in the Institute JOURNAL and it has served to place many members in positions of responsibility, both in this country and abroad. The bulletin is subdivided into two parts: one containing announcements of vacancies; and the other containing lists of men available, with condensed records of their experience. All announcements are published without charge either to the employers or to the members of the Institute seeking positions.

Federated American Engineering Societies.—The 1923 roster is made up of six national societies having a total membership of 42,627 and 22 local societies having a membership of 6920, an aggregate of 49,587. In 1921 the total number of members of the participating societies was 41,932. American Engineering Cluncil has functioned with 64 delegates and held its annual meeting in Washington on January 11-12. The Executive Board composed of 30 members has held 4 meetings, namely: Pittsburgh, May 26-27, 1922; Boston, Sept. 8-9, 1922; Washington, at the time of the Annual Council meeting, Jan. 11-12, 1923; and Cincinnati, March 23-24, 1923. In the interim between these meetings the Procedure Committee has held a number of meetings at irregular periods and whenever necessity required it.

The meeting of the Council and meetings of the Executive Board and of the Procedure Committee have been very well attended and while developing the liveliest discussions regarding the various subjects under consideration, have been marked with an earnestness and sincerity of purpose that is very gratifying and by an ultimate harmony of conclusions. The views of the Institute representatives have been treated with the utmost consideration, and it is safe to say that no policies to which they have objected have ever been adopted.

American Engineering Council has been active through its officers and Committees over a wide range during the year but as these activities have been chronicled from month to month in the JOURNAL, they will not be recapitulated here. It is believed that this organization is performing a very useful function. It is both serving the nation and increasing the prestige of engineers. The Institute's representatives believe that the Institute took a wise step in supporting American Engineering Council and that it should retain its membership therein and continue to participate vigorously in its activities.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1923 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining and Mechanical Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc.

An abstract of the annual report of the Engineering Societies Library covering the calendar year 1922 was published in the March 1923 JOURNAL.

Engineering Foundation.—Engineering Foundation is a trust fund established in 1914 by Ambrose Swasey, of Cleveland, Ohio, by gifts to United Engineering Society as a nucleus of a large endowment "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." It is administered by the Engineering Foundation Board upon which the Institute and other national engineering societies are represented. The Board is a Department of United Engineering Society.

During latter part of 1922 in order to meet expanding activities the office of Director of the Engineering Foundation was established and Alfred D. Flinn, who retired as Chairman of Engineering Division, National Research Council, was selected to fill the post.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and has appointed representatives upon a number of new Sectional Committees of American Engineering Standards Committee and upon the Charles A. Coffin Fellowship and Research Fund Committee. A complete list of representatives is published frequently in the JOURNAL.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report follows:

ATLANTA
BALTIMORE
BIRMINGHAM
BOSTON
BUFFALO
CHICAGO
CINCINNATI
CLEVELAND
DALLAS
DENVER
DETROIT
KANSAS CITY
LOS ANGELES
MINNEAPOLIS
NEWARK
NEW ORLEANS

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
37 WEST 39TH STREET
NEW YORK

NEW YORK
PHILADELPHIA
PITTSBURGH
PORTLAND
PROVIDENCE
SAINT LOUIS
SALT LAKE CITY
SAN DIEGO
SAN FRANCISCO
SEATTLE
TULSA
WATERTOWN

HAVANA
LONDON
PARIS
SHANGHAI

May 14, 1923.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

Pursuant to engagement, we have audited your books and accounts for the year ended April 30, 1923, and submit herewith our certificate and the following described exhibits and schedule:

Exhibit "A"—General Balance Sheet, April 30, 1923.
Schedule No. 1—Reserve Capital Fund—Securities.
Exhibit "B" Summary of Income and Profit & Loss for the Year ended April 30, 1923.

Yours truly,
HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE OF AUDIT

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1923, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly exhibits the financial condition of the Institute at April 30, 1923, that the Summary of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS

NEW YORK,
May 14, 1923.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Exhibit A. General Balance Sheet, April 30, 1923

ASSETS

Real Estate:			
One-Fourth Interest in United Engineering Society's Land, Building and Building Equipment, 25 to 33 West 39th Street (Depreciation carried on Books of United Engineering Society)			$489,785.17
Equipment:			
Library—Volumes and Fixtures		$40,383.06	
Works of Art, Paintings, etc		3,001.35	
Office Furniture and Fixtures	$15,309.25		
Less Reserve for Depreciation (including $1,500.00 funded)	10,540.43	4,768.82	
Total Equipment			48,153.23
Working Assets:			
Transactions, etc		$9,665.97	
Text and Cover Paper		755.73	
Badges		1,472.39	
Total Working Assets			11,894.09
Current Assets:			
Cash		$17,472.73	
Notes Receivable—Advertisers		109.50	
Accounts Receivable:			
Members—For Dues		14,903.12	
Advertisers		2,130.93	
Miscellaneous		1,453.48	
Accrued Interest on Investments		171.05	
Accrued Interest on Bank Balances		280.57	
Total Current Assets			36,521.38
Funds:			
Reserve Capital Fund—Securities—Schedule No. 1		$30,610.99	
Life Membership Fund:			
Cash	$1,438.67		
Chicago, Burlington & Quincy Railroad Company 4% Bonds, 1958 par value $5,000.00	4,868.75		
Accrued Interest	33.33		
		6,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$412.00		
New York City 4¼% Bonds, 1957, par value $2,000.00	2,216.81		
New York Telephone Company 4½% Bond, 1939, par value $1,000.00	878.75		
Accrued Interest	67.50	3,575.06	
Mailloux Fund:			
Cash	$51.38		
New York Telephone Company 4½% Bond, 1939, par value $1,000.00	1,000.00		
Accrued Interest	22.50	1,073.88	
Midwinter Convention Fund—Cash		143.41	
Depreciation of Furniture and Fixtures Fund—Cash		1,500.00	
Total Funds			43,244.09
Total			$629,597.96

LIABILITIES

Current Liabilities:		
Accounts Payable	$11,260.43	
Dues Received in Advance	2,984.11	
Entrance Fees and Dues Advanced by Applicants for Membership	431.50	
Subscriptions for "Transactions" received in Advance	1,203.00	
Total Current Liabilities		$15,899.04
Fund Reserves (Not Including Depreciation Reserve):		
Reserve Capital Fund	$30,610.99	
Life Membership Fund	6,340.75	
International Electrical Congress of St. Louis—Library Fund	3,575.06	
Mailloux Fund	1,073.88	
Midwinter Convention Fund	143.41	
Total Fund Reserves (not Including Depreciation Reserve)		41,744.09
Surplus: Per Exhibit "B"		$571,954.83
Total		$629,597.96

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Summary of Income and Profit & Loss

for the Year Ended April 30, 1923.

Exhibit B.

Income:			
Entrance Fees		$10,767.95	
Dues		*162,610.26	
Students' Dues		12,336.00	
Transfer Fees		1,285.00	
Advertising		57,524.37	
Journal Subscriptions		5,449.33	
Transactions Subscriptions		11,704.87	
Miscellaneous Sales		5,338.76	
Badges Sold	$5,410.50		
Less Cost	4,751.72		
		658.78	
Interest on Securities in Reserve Capital Fund		1,183.18	
Interest on Bank Balances		898.76	
Interest on Notes Receivable		7.60	
Total			$269,764.86
Expenses:			
Publications:			
Journal	$87,920.38		
Transactions	9,697.32		
Year Book	6,868.08	$104,485.78	
Meetings		12,017.13	
Administrative Expenses		44,519.56	
Sections Committee		22,715.55	
Membership Committee		9,042.66	
Standards Committee		362.68	
Finance Committee		235.60	
Headquarters Committee		85.00	
Code Committee		60.00	
International Electrotechnical Commission		493.95	
Interest on United Engineering Society Building Account		70.31	
President's Special Appropriation		710.00	
Honorary Secretary		4,000.00	
American Engineering Standards Committee		1,505.44	
John Fritz Medal Award		400.81	
Engineering Societies Library:			
Maintenance	$5,500.00		
Recataloging	2,500.00	8,000.00	
United Engineering Society Assessment		4,860.00	
Federated American Engineering Society		14,484.25	
Forward		$228,048.72	$269,764.86

*Includes $71,315.00 allocated to subscriptions for the Journal.

Total Income—(Forward)		$269,764.86
Expenses—(Forward)	$228,048.72	
Engineering Societies Employment Service	3,800.00	
International Annual Tables	100.00	
Total		231,948.72
Net Income		$37,816.14
Profit & Loss Charges:		
Uncollectible Dues Written off	$6,620.18	
Adjustment of Inventory of Furniture and Fixtures, April 30, 1923	74.75	
Adjustment of Inventory of Transactions, April 30, 1923	1,011.12	
Provision for Depreciation of Furniture and Fixtures	862.15	
Total		8,568.20
Surplus for the Year		$29,247.94
Surplus, May 1, 1922	$547,744.39	
Less Transferred to Capital Fund Reserve in Accordance with Resolution of Board of Directors	5,037.50	542,706.89
Surplus, April 30, 1923		$571,954.83

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Reserve Capital Fund—Securities, April 30, 1923

Exhibit A.

Schedule No. 1.

	Par Value	Book Value
The Detroit Edison Company 1st and Refunding 6%, Series "B", Gold Bonds, due 1940	$1,000.00	$1,040.00
The New York Central Railroad Company registered, 5%, Refunding and Improvement Mortgage Bonds, due 2013, Series "C"	1,000.00	980.00
Chicago, Burlington & Quincy Railroad Company 5%, 1st and Refunding Mortgage, Registered, Gold Bonds, Series "A", due 1971	1,000.00	1,010.00
Great Northern Railroad Company 5½%, General Mortgage, Registered, Gold Bonds, Series "B", due 1952	1,000.00	1,027.50
Southern Railway Company 5%, 1st Consolidated Mortgage, Registered, Gold Bonds, due 1994	1,000.00	980.00
City of Wilmington, Delaware, 4½% Bonds, due 1934	15,000.00	15,573.49
United States Third Liberty Loan 4¼% Bonds, due 1928	10,000.00	10,000.00
Total	$30,000.00	$30,610.99

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Statement of Cash Receipts and Disbursements for Designated Purposes, for the Year ended April 30, 1923.

Exhibit C.

Receipts:	
Life Membership Fund	$265.40
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	136.05
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Interest	11.02
Total	$457.47
Disbursements:	
Life Membership Fund	$265.40
Mailloux Fund	83.62
Midwinter Convention Fund	110.48
Total	$459.50

RECEIPTS AND DISBURSEMENTS PER MEMBER

During each fiscal year for the past seven years

Year ending April 30	1917	1918	1919	1920	1921	1922	1923
Membership, April 30, each year	8710	9282	10252	11345	13215	14263	15298
Receipts per Member	$13.30	$13.17	$13.18	$15.01	$17.87	$17.26	$17.45
Disbursements per Member	12.75	11.99	12.92	15.62	18.90	16.77	16.37
Credit Balance per Member	$.55	$1.18	$.26	*$.61	*$1.03	$.49	$1.08

*Deficit.

Respectfully submitted for the Board of Directors.

F. L. HUTCHINSON, *Secretary.*

New York, May 18, 1923.

Officers A. I. E. E. 1922-1923

PRESIDENT

(Term expires July 31, 1923)
FRANK B. JEWETT

JUNIOR PAST-PRESIDENTS

(Term expires July 31, 1923)	(Term expires July 31, 1924)
A. W. BERRESFORD	WILLIAM McCLELLAN

VICE-PRESIDENTS

(Terms expire July 31, 1923)	(Terms expire July 31, 1924)
N. W. STORER	G. FACCIOLI
C. G. ADSIT	W. I. SLICHTER
F. W. SPRINGER	R. F. SCHUCHARDT
ROBERT SIBLEY	H. W. EALES
F. R. EWART	H. T. PLUMB

MANAGERS

(Terms expire July 31, 1923)	(Terms expire July 31, 1924)
L. E. IMLAY	HAROLD B. SMITH
F. F. FOWLE	JAMES F. LINCOLN
L. F. MOREHOUSE	E. B. CRAFT
(Terms expire July 31, 1925)	**(Terms expire July 31, 1926)**
R. B. WILLIAMSON	H. M. HOBART
A. G. PIERCE	ERNEST LUNN
HARLAN A. PRATT	G. L. KNIGHT

TREASURER	SECRETARY
(Terms expire July 31, 1923)	
GEORGE A. HAMILTON	F. L. HUTCHINSON
HONORARY SECRETARY	**GENERAL COUNSEL**
RALPH W. POPE	PARKER & AARON, 30 Broad Street, New York.

PAST-PRESIDENTS—1884-1922

*NORVIN GREEN, 1884-5-6.
*FRANKLIN L. POPE, 1886-7.
T. COMMERFORD MARTIN, 1887-8.
EDWARD WESTON, 1888-9.
ELIHU THOMSON, 1889-90.
*WILLIAM A. ANTHONY, 1890-91.
*ALEXANDER GRAHAM BELL, 1891-2.
FRANK JULIAN SPRAGUE, 1892-3.
*EDWIN J. HOUSTON, 1893-4-5.
*LOUIS DUNCAN, 1895-6-7.
*FRANCIS BACON CROCKER, 1897-8.
A. E. KENNELLY, 1898-1900.
CARL HERING, 1900-1.
CHARLES P. STEINMETZ, 1901-2.
CHARLES F. SCOTT, 1902-3.
BION J. ARNOLD, 1903-4.
JOHN W. LIEB, 1904-5.
*SCHUYLER SKAATS WHEELER, 1905-6.
*SAMUEL SHELDON, 1906-7.
*HENRY G. STOTT, 1907-8.
LOUIS A. FERGUSON, 1908-9.
LEWIS B. STILLWELL, 1909-10.
DUGALD C. JACKSON, 1910-11.
GANO DUNN, 1911-12.
RALPH D. MERSHON, 1912-13.
C. O. MAILLOUX, 1913-14.
PAUL M. LINCOLN, 1914-15.
JOHN J. CARTY, 1915-16.
H. W. BUCK, 1916-17.
E. W. RICE, JR., 1917-18.
COMFORT A. ADAMS, 1918-19.
CALVERT TOWNLEY, 1919-20.
A. W. BERRESFORD, 1920-21.
WILLIAM McCLELLAN, 1921-22.

*Deceased.

LOCAL HONORARY SECRETARIES

Carrol M. Mauseau, Caixa Postal No. 571, Rio de Janeiro, Brazil, S. A.
Charles le Maistre, 28 Victoria St., London, S. W. England.
A. S. Garfield, 45 Bd. Beausejour, Paris 16 E, France.
H. P. Gibbs, Tata Sons, Ltd., Navsari Building, Fort Bombay, India.
Guido Semenza, N. 10 Via S. Radegonda, Milan, Italy.
Lawrence Birks, Public Works Department, Wellington, New Zealand.
W. Elsdon-Dew, P. O. Box 4563, Johannesburg, Transvaal, Africa.

A. I. E. E. Committees

GENERAL STANDING COMMITTEES

EXECUTIVE COMMITTEE

Va ewett, Chairman, 195 Broadway, New York.
W Berresford, H. M. Hobart, R. F. Schuchardt,
Wo amilton, L. F. Morehouse, W. I. Slichter.
To

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orehouse, Chairman, 195 Broadway, New York.
C. G. Adsit. G. L. Knight.

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k, John Mills, A. G. Pierce.
L. W. W. Morrow,
of Committee on Coordination of Institute Activities.
of Technical Committees.
of Sections.

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eighton, F. L. Hutchinson, Wm. McClellan.
L. F. Morehouse,

COMMITTEE ON COORDINATION OF INSTITUTE ACTIVITIES

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E. E. F. Creighton, F. L. Hutchinson, Donald McNicol.
L. F. Morehouse,

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Chairman of Working Committees of Standards Committee.

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(Term expires July 31, 1925)
H. M. Byllesby, D. E. Drake, W. L. R. Emmet.
(Term expires July 31, 1926)
B. A. Behrend, John H. Finney, C. S. Ruffner.
(Term expires July 31, 1927)
Gano Dunn, F. A. Scheffler, W. R. Whitney.

Elected by the Board of Directors from its own membership for term of two years

(Term expires July 31, 1923)
A. W. Berresford, L. F. Morehouse, R. B. Williamson.
(Term expires July 31, 1924)
E. B. Craft, G. Faccioli, William McClellan.

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P. V. Magalhaes,
Irving B. Smith,
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H. S. Vassar.

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Gordon Fox,
Eugene Friedlaender,
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D. M. Petty,
G. E. Stoltz,
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Wilfred Sykes,
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F. F. Fowle,
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E. G. Gallagher,
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H. Franklin Harvey, Jr.,
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Com. C. S. McDowell,
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H. A. Pratt,
H. M. Southgate,
W. E. Thau,
A. E. Waller.

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C. N. Beebe,
M. C. Benedict,
H. W. Eales,
L. C. Ilsley,
G. M. Kennedy,
R. L. Kingsland,
A. B. Kiser,
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F. L. Stone,
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J. T. Barron,
A. E. Bauhan,
E. T. Brandon,
D. H. Braymer,
H. W. Eales,
F. C. Hanker,
C. F. Hirshfeld,
H. A. Kidder,
W. H. Lawrence,
H. P. Liversidge,
D. C. McClure,
I. E. Moultrop,
A. L. Penniman,
F. A. Scheffler,
R. F. Schuchardt,
R. B. Williamson.

PROTECTIVE DEVICES

H. R. Woodrow, Chairman, 360 Pearl Street, Brooklyn, N. Y.
G. H. Bragg,
P. H. Chase,
L. B. Chubbuck,
F. C. Hanker,
S. E. M. Henderson,
E. A. Hester,
F. L. Hunt,
B. G. Jamieson,
M. G. Lloyd,
A. A. Meyer,
W. H. Millan,
A. G. Pierce,
N. L. Pollard,
F. E. Ricketts,
D. W. Roper,
C. H. Sanderson,
E. R. Stauffacher,
E. C. Stone,
A. H. Sweetnam,
John B. Taylor.

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G. A. Cellar,
H. P. Charlesworth,
L. W. Chubb,
E. H. Colpitts,
R. N. Conwell,
C. E. Davies,
H. W. Drake,
D. H. Gage,
H. Hulatt,
F. H. Kroger,
F. J. Mayer,
L. F. Morehouse,
W. O. Pennell,
F. A. Raymond,
Chester W. Rice,
J. K. Roosevelt,
Gen. Edgar Russell,
Arthur B. Smith,
I. J. Thorp,
F. A. Wolff,
C. A. Wright,

TRACTION AND TRANSPORTATION

N. W. Storer, Chairman, W. E. & M. Co., East Pittsburgh, Pa.
George Gibbs,
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W. S. Murray,
Hugh Pattison,
W. B. Potter.

TRANSMISSION AND DISTRIBUTION

E. B. Meyer, Chairman, 80 Park Place, Newark, N. J.
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Wallace S. Clark,
W. H. Cole,
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L. L. Elden,
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C. N. Rakestraw,
C. C. Robinson,
D. W. Roper,
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C. H. Shaw,
Robert Sibley,
W. K. Vanderpoel,
C. T. Wilkinson.

ON AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, COUNCIL

A. E. Kennelly,
John B. Taylor.

ON AMERICAN BUREAU OF WELDING

W. L. Merrill.

A. I. E. E. Representatives

ON AMERICAN COMMITTEE ON ELECTROLYSIS

Bion J. Arnold,
N. A. Carle,
F. N. Waterman.

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*C. G. Adsit,
John H. Finney,
F. L. Hutchinson,
*Frank B. Jewett,
W. A. Layman,
*William McClellan,
*L. F. Morehouse,
L. T. Robinson,
Charles S. Ruffner,
*Charles F. Scott,
C. E. Skinner,
L. B. Stillwell,
Calvert Townley.

*Members of Executive Board.

ON AMERICAN ENGINEERING STANDARDS COMMITT

H. M. Hobart,
Harold Pender,
L. T. Robinson (Alternate)
C. E. Skinner.

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H. H. Barnes, Jr.,
Bancroft Gherardi,
H. A. Lardner.

ON CHARLES A. COFFIN FELLOWSHIP AND RESEARCH F COMMITTEE

Frank B. Jewett.

ON ENGINEERING FOUNDATION BOARD

Frank B. Jewett,
E. W. Rice, Jr.

ON FRANCO-AMERICAN ENGINEERING COMMIT

A. M. Hunt,
L. B. Stillwell.

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ON JOSEPH A. HOLMES SAFETY ASSOCIATION

John H. Finney.

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The chairman of the Institute's Committee on Safety Codes.

ON NATIONAL RESEARCH COUNCIL, ENGINEERING DIVISION

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ON U. S. NATIONAL COMMITTEE OF THE INTERNATIONAL ILLUMINATION COMMISSION

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ON WASHINGTON AWARD, COMMISSION OF

John Price Jackson, Charles F. Scott.

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Baltimore	W.B.Kouwenhoven	F. T. Leilich
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Chicago	F. E. Goodnow	J. E. Kearns
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Cleveland	L. D. Bale	Chester L. Dows
Columbus	F. C. Caldwell	F. C. Nesbitt
Connecticut	E. H. Everit	A. E. Knowlton
Denver	H. B. Barnes	R. B. Bonney
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Erie	W. J. Seibert	M. W. Metzner
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Kansas City	Geo. C. Shaad	W. E. A. Nottorf
Lehigh Valley	D. M. Petty	H. G. Harvey
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Lynn	J. W. West	W. M. Howe
Madison	H. M. Crothers	R. G. Walter
Milwaukee	G. G. Post	F. A. Kartak
Minnesota	Fred A. Otto	H. W. Meyer
New York	Calvert Townley	A. E. Waller
Oklahoma	F. W. Insull	A. D. Stoddard
Panama	F. B. Coyle	M. P. Benninger
Philadelphia	E. B. Tuttle	Ross B. Mateer
Pittsburgh	E. C. Stone	O. Needham
Pittsfield	W. P. White	A. C. Stevens
Portland, Ore	D. W. Proebstel	E. F. Pearson
Providence	R. W. Adams	F. N. Tompkins
Rochester	G. A. Scoville	E. A. Roeser
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